MODELING AND ANALYSIS OF RADIO FREQUENCY IDENTIFICATION (RFID) TECHNOLOGY WITHIN THE SUPPLY CHAIN

by

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ABSTRACT

It is known that inaccurate inventory records can lead to profit losses in a supply chain. Inventory records may not be correct due to various reasons such as transaction errors, misplacement, shrinkage, etc. Companies invest in information technologies to better manage their inventories. In order to eliminate the inventory inaccuracy and its reasons by increasing visibility, new technologies such as Radio Frequency Identification (RFID) can be used.

In this thesis, we consider a supply chain consisting of a retailer (distributor) and a supplier. We assume a single-period newsvendor-type setting where the retailer purchases the items from the supplier and distributes them to her regional warehouses. The thesis focuses on the problem of finding the optimal investment levels that maximize profit by decreasing inventory inaccuracy. The optimal level of investment is examined both for the centralized and the decentralized systems under two scenarios: inventory sharing between the warehouses is allowed and not allowed. The coordination problem is also considered for both scenarios. Finally, several extensions of the model are considered: asymmetric warehouse parameters, demand and inventory inaccuracy correlation, imperfect RFID implementation and multiple products.

ÖZETÇE

Tedarik zincirinde envanter kayıtlarının doğru olmaması kar kayıplarına neden olabilir. Envanter kayıtları bir çok nedene bağlı olarak yanlış olabilir, örneğin ürünlerin depoda yanlış yere konması, hırsızlık, ürünlerin depoya giriş veya çıkışı sırasındaki kontrollerde yapılan insana veya cihazlara bağlı hatalar, vb. Şirketler, envanter kayıtlarını ve tedarik zinciri içinde ürünlerini takip edebilmek için bilişim sistemleri yatırımları yapmaktadırlar. Son yıllarda ortaya çıkan ve ürünlerin tedarik zinciri içinde takibini kolaylaştıran Radyo Frekanslı Tanımlama (RFID) teknolojisi bu yöntemler arasındadır.

Bu tezde bir üretici ve bir toptancıdan oluşan bir tedarik zinciri, tek dönemlik bir zaman çizelgesinde ele alınmaktadır. Modelimizde toptancı ürünleri üreticiden alır ve kendi depolarına dağıtır. Amaç envanter kayıtlarındaki hataları azaltmak veya ortadan kaldırmak üzere kaç depoda yatırım yapılmasının en iyi sonuç olacağına karar vermektir. Merkezi ve merkezi olmayan sistemlerin yatırım davranışları, envanter paylaşımlı ve envanter paylaşımsız senaryolar altında incelenmektedir. Merkezi olmayan sistemin koordinasyon problemi de ele alınmıştır. Son olarak, modeldeki bazı varsayımlar esnekleştirilerek farklı durumlar altındaki yatırım davranışları da incelenmiştir.

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NOMENCLATURE

r	unit price
m	unit production cost
w	wholesale price
s	salvage value
D	demand
μ_D	mean of demand
σ_D	standard deviation of demand
Q	order quantity
$\bar{Q}$	actual available quantity after error occurs
N	number of warehouses
n	number of warehouses where the investment is made
X_i	inventory inaccuracy
μ_X	mean of inventory inaccuracy
σ_X	standard deviation of inventory inaccuracy
D'	$D - X$
k	variable investment cost defined per warehouse
K	fixed investment cost
Π_{W_i}	profit of warehouse i
Π_R	Profit of the retailer
Π_S	Profit of the supplier
Π_C	Profit of the centralized system
δ_i	1 if an investment is not made at warehouse i, 0 otherwise
$\phi(\cdot)$	standard normal density function
z_R	standard normal variable for the decentralized system
z_C	standard normal variable for the centralized system

α_R	critical fraction for the retailer
α_C	critical fraction for the centralized system
$\Phi^{-1}(\cdot)$	inverse cumulative distribution function
IS	inventory sharing
NIS	no inventory sharing
k_R^T	variable investment threshold for the retailer (NIS)
k_S^T	variable investment threshold for the supplier (NIS)
k_C^T	variable investment threshold for the centralized system (NIS)
β	revenue sharing proportion
θ	investment cost sharing fraction
$\varphi(\cdot)$	standard normal loss function
ϑ_R	gain of the retailer from making an investment
ϑ_S	gain of the supplier from making an investment
b	buyback price
$\bar{k}_R^T$	variable investment threshold for the retailer (IS)
$\bar{k}_S^T$	variable investment threshold for the supplier (IS)
$\bar{k}_C^T$	variable investment threshold for the centralized system (IS)
$\mathbf{y}$	a vector of 0,1 in R^N representing the RFID enabled warehouses
e_i	a unit vector in R^N representing RFID is implemented in warehouse i
e_j	a unit vector in R^N representing RFID is implemented in warehouse j
ρ_D	demand correlation
ρ_I	inventory inaccuracy correlation
τ_D^2	total demand variance when there is correlation
τ_I^2	total inventory inaccuracy variance when there is correlation
ω_i	actual inventory inaccuracy of warehouse i
t	the fraction of inventory inaccuracy eliminated by the RFID investment

Chapter 1

INTRODUCTION

Supply chain inventory management decisions depend on inventory data gathered from automated or manual control systems. Cheaper and faster computation have become available in 1980s and as a result of this, companies started to automate their inventory management processes and use inventory management softwares [26]. Although the use of IT made collecting and storing data about the flow of items through supply chain easier and less expensive, the tracking of inventory remains prone to error. The data collected by IT may not be accurate due to various reasons: incorrect product identification, transaction errors, inaccessibility of items due to improper usage of the depot (misplacement), shrinkage, etc. These may result in two problems: unplanned inventory depletion and addition. If the inventory records do not agree with the actual physical stock, either an order may not be placed in time or excessive inventory is held.

Inventory inaccuracy may be a significant issue as reported in a number of recent studies. Kang and Gershwin [23] report inventory accuracies of a global retailer's stores. It is seen that the inventory accuracy is only 51% on average for 500 stores. In other words the stores have the accurate records for only about a half of the SKUs (stock keeping units). The best performing store in the study knows its actual inventory with 75-80% accuracy. Raman et al. [34] report similar findings for a leading retailer. Almost 370,000 SKUs are investigated for the retailer, it is concluded that more than 65% of the inventory records do not match with the physical inventory.

To cope with inventory inaccuracy, different compensation methods can be used, e.g. periodical review of inventory, tracking of items, eliminating its reasons or making decisions by considering the inventory inaccuracy. In particular, RFID (Radio Frequency Identification) technology which has received considerable attention in recent years helps to track items through the supply chain. This technology is different from bar code technology in two ways: it does not require line of sight and RFID tags have unique codes. A more detailed comparison of the RFID technology with the bar code system is given in Chapter 2. Many companies consider investing in the RFID technology as pioneered by some major retailers such as Wal-Mart, Tesco and by organizations such as United States Department of Defense. In addition to reducing the inventory inaccuracy, the technology may also help to eliminate the reasons of inventory inaccuracy: e.g. shrinkage.

The three main components of the RFID technology are: tag (transponder), antenna and reader. A tag contains a computer chip that holds data related to a product. Different types of tags are available according to their shape, size, memory properties and frequencies. The readers broadcast signals via antenna. The tags receive the signals and and send the data to the readers by means of radio frequencies. The readers send the received data to the computer system for logging and processing. This identification provides tracking of items through the supply chain. However, using this technology requires a large investment. This investment consists of the cost of establishing the infrastructure as well as the costs of the tags and the readers. Tag price is the main issue of RFID; although pallet-level or case-level tagging is an option, tag prices are expected to be so low that they can be attached to every item. The price of a tag is expected to be 5 cents, which is now as low as 6 cents [21]. Besides tag prices, RFID implementations cost $400,000 per distribution center and $100,000 per store and $35-$40 million is required for the system integration of the entire organization [24].

Motivated by the RFID investment issue, this research mainly focuses on the decision of the optimal investment levels in order to decrease the inventory inaccuracy in a two-level supply chain consisting of a supplier and a retailer. We consider both the centralized and decentralized systems. In a centralized system a central planner decides on the investment while in a decentralized system the investment decision is made either by the retailer or the supplier. It is known that centralized system performs better than decentralized system. Particularly, we defined the following research questions:

- What are the optimal investment levels in centralized and decentralized supply chains (if at all)?

- What are the resulting benefits in terms of inventory costs?

- How does centralization affect investment decisions?

- What is the effect of inventory sharing on the investment decision?

In order to address the above questions, we analyze a supply chain consisting of a retailer and a supplier. The retailer has multiple warehouses and the demand for each warehouse is random. The optimal level of investment is examined both for the centralized and decentralized systems. The model is investigated under two scenarios: (1) the warehouses are able to share their inventories as needed, (2) inventory sharing is not allowed. In addition, we address the issue of how to share the investment within a given class of contracts and investigate the related coordination aspects. Finally, several extensions of our basic model are considered: asymmetric warehouse parameters, demand and inventory inaccuracy correlation, imperfect RFID implementation and multiple products.

The remainder of the thesis is organized as follows. In the following chapter, a brief explanation of the RFID technology is given. In Chapter 3, the related literature is reviewed. In Chapter 4 the model is described under two different scenarios and computational results are represented. Chapter 5 includes the extensions of our base model while the conclusion is presented in Chapter 6.

Chapter 2

A BASIC UNDERSTANDING OF RADIO FREQUENCY IDENTIFICATION (RFID) TECHNOLOGY

Today companies face challenges such as product/process complexity, product proliferation and uncertainty. These create inefficiency and increase the costs of companies [41]. In order to better manage companies, different systems are introduced recently. Auto ID technology (or RFID) is one of those systems that enable companies to track items within supply chain.

This chapter is dedicated to provide information about the radio frequency identification technology: its history, principles of working, areas of implementation, impact on supply chain systems, strengths and weaknesses. Furthermore, current applications and observed benefits of the pioneer applicants are presented in the last part of the chapter.

2.1 History

RFID is not a new technology; it has been used for about 50 years. Lon Theremin invented a spying tool which retransmitted incident radio waves with audio information for the Soviet government in 1945. Although this device was a passive covert listening device, it has been attributed as the predecessor to RFID technology [48]. Mario Cardullo received the first patent for a passive, read-write RFID tag in 1973 [6]. In 1970s, the railroads switched to the use of the technology by tagging every rail car in America. In addition, General Motors implemented RFID in its production process in 1984 and now most of the automobile manufacturers apply RFID tags for tracking automobiles and parts through the production process [33].

RFID has been used in many areas such as animal identification, waste management, baggage handling, time and attendance, postal tracking, document tracking, fare collection, laundry/textile identification and library information systems [41]. Today, RFID is also used to monitor items within supply chain and to improve the performance of supply chains in terms of decreasing costs and improving customer service levels.

As Wal Mart announced that its top 100 suppliers would use RFID tags on pallets and cases by January 1, 2005 and Gillette reported the purchase of 500 million tags, RFID has become a hot topic of 2000s [33]. RFID has attracted the attention of companies by the studies of Auto-ID Center. Auto ID Center which is a partnership between almost 100 global companies and five of the world's leading research universities was founded at Massachusetts Institute of Technology in October 1999. The center developed the EPC global Network system which is expected to perform better than bar code systems and replace it in the long run. The technology uses Electronic Product Codes (EPC) carried by RFID tags. Each item is identified by a 96-bit EPC, a number designed to uniquely identify an item in the supply chain. A typical object becomes unique by using a smart tag which carries an EPC code. (See Figure 2.1[1]) The EPC provides identifiers for 268 million companies, each with more than 1 million products (RFID Journal FAQ).

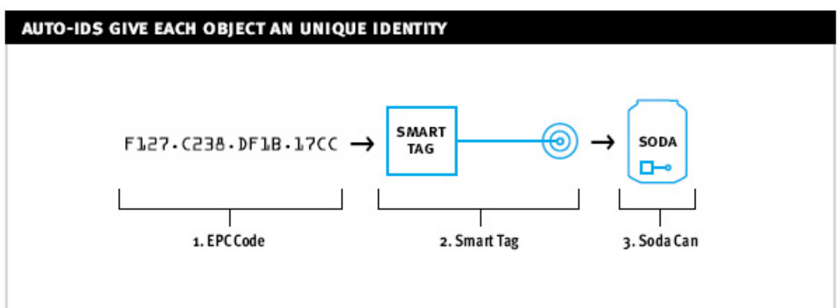

Figure 2.1: Auto-ID gives a unique code to each item.

Today, the price of tags is a significant focus within the RFID industry. The idea behind using the RFID technology to track items through the supply chain is that the price of the tags are expected to be so low that they can be attached to every product. In 2004, the price of EPC tags was 20 cents on all orders of over 1 million by Alien Technology [17]. In 2005, SmartCode announced that a Gen 2 tag for 7.5 cents in quantities over one million and for 7.2 cents for volumes over 10 million [40]. As reported in [21], a new report says that prices for RFID tags could plunge by 50 percent or more, but most likely not until 2008. The report also states that chip prices are currently as low as six cents, but analysts

[1] The figure is reproduced from [22].

expect that tag prices will decline further as volumes ramp into the billions in two years. It seems that the change in prices of RFID tags will define the future of the technology.

2.2 How does RFID work?

RFID technology has three main components: tag (transponder), reader (interrogator) and antenna. The other components are: encoder (printer) and middleware.

An RFID tag contains an integrated circuit and an antenna. A microchip is attached to the antenna that picks up signals from and signals to a reader. A reader sends signals to and receives signals from tags via antenna. The encoder is used to create a tag. Middleware refers to software that resides on a server between readers and enterprise applications. The middleware filters data and passes on only useful information to enterprise applications. They can also be used to manage readers on a network.

Without the technical details, the working process of the RFID technology is as follows. The first consideration when using RFID technology is how to apply the tags to a container, pallet, case or item. First of all, the tags are created by using an encoder (printer). The system associates an EPC code to each unit and encodes an RFID chip and creates a smart label. The failure points and resolutions of RFID tag application are given in detail in RFID Implementation Guideline of Auto-ID [12].

After creation, the tags can be applied manually or by using automated systems to a container, pallet, case or item. A read verification is performed before and after the application. After applying the tags, the container, pallet, case or item can communicate with the readers. The readers can be in different forms: free standing portal, conveyor, vehicle mounted or hand-held reader systems. In any case, the reader broadcasts signals via an antenna.[2] The electromagnetic field produced by an antenna can be constantly present or if constant interrogation is not required, a sensor device can activate the field [45]. The tag receives the signals and sends data. The reader sends received data to the computer system or internet for logging and processing. (See Figure 2.2)

Different types of tags are available according to their shape, size, memory properties and frequencies. There are basically two types of tags: active and passive. Active tags have batteries and can initiate the communication, while passive tags are activated by reader's

[2] Signals can even go through walls [27].

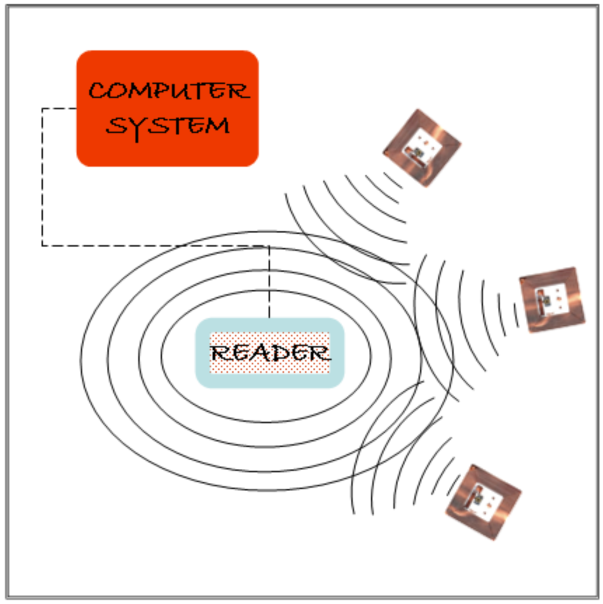

Figure 2.2: Representation of an RFID system

signals. According to their frequencies and application areas, there are four types of tags [43]:

1. Low frequency tags (1 foot range or less): These are appropriate for animal identification, auto key&lock, library books. Low frequency tags are more suitable for metal and liquid handling [36].

2. High frequency tags (3 feet) are used for item level tracking, airline baggage, building access.

3. Ultra high frequency tags (10-20 feet) are employed for case, pallet and container tracking, truck and trailer tracking.

4. Microwave tags are suitable for access control (vehicles).

Based on the working principles, the RFID technology is different from the bar code technology in many aspects:

- RFID tags do not require line of sight identification.

- RFID tags are unique for every item where as a bar code only represents a product number.
- The tags can store more information about a product than barcodes.[3]
- RFID technology provides the capability to scan multiple items at the same time so, it saves time and labor.
- Bar codes require manual handling whereas RFID enables automatic handling.

Since the technology is better than the bar code technology in many aspects, it is expected to replace the bar code technology in the long run.

2.3 Impact on Supply Chain

Expected benefits of the RFID technology within supply chain are given in many sources ([33], [41], [24]).

Those benefits are summarized in three levels in Figure 2.3. In the operational level, the operations that are affected by RFID are given. In the tactical level, benefits are represented resulting from the operational improvements. As known well, every company's final goal is improving profit, which is achieved by either decreasing costs or increasing revenue. Finally, at the strategic level, effects of benefits on strategic goals are depicted. It is seen that benefits mainly focus on decreasing costs.

The benefits of the retailers and suppliers can be classified separately as in the report of A.T. Kearney [24].

Retailers' benefits are:

(1) Reduced inventory

(2) Store and warehouse labor reduction

(3) Reduction of out of stocks

Benefits of manufacturers are:

[3] Data capacity of tags range from 2 KB to 1 MB [45]. Some active tags have data logging capabilities [36].

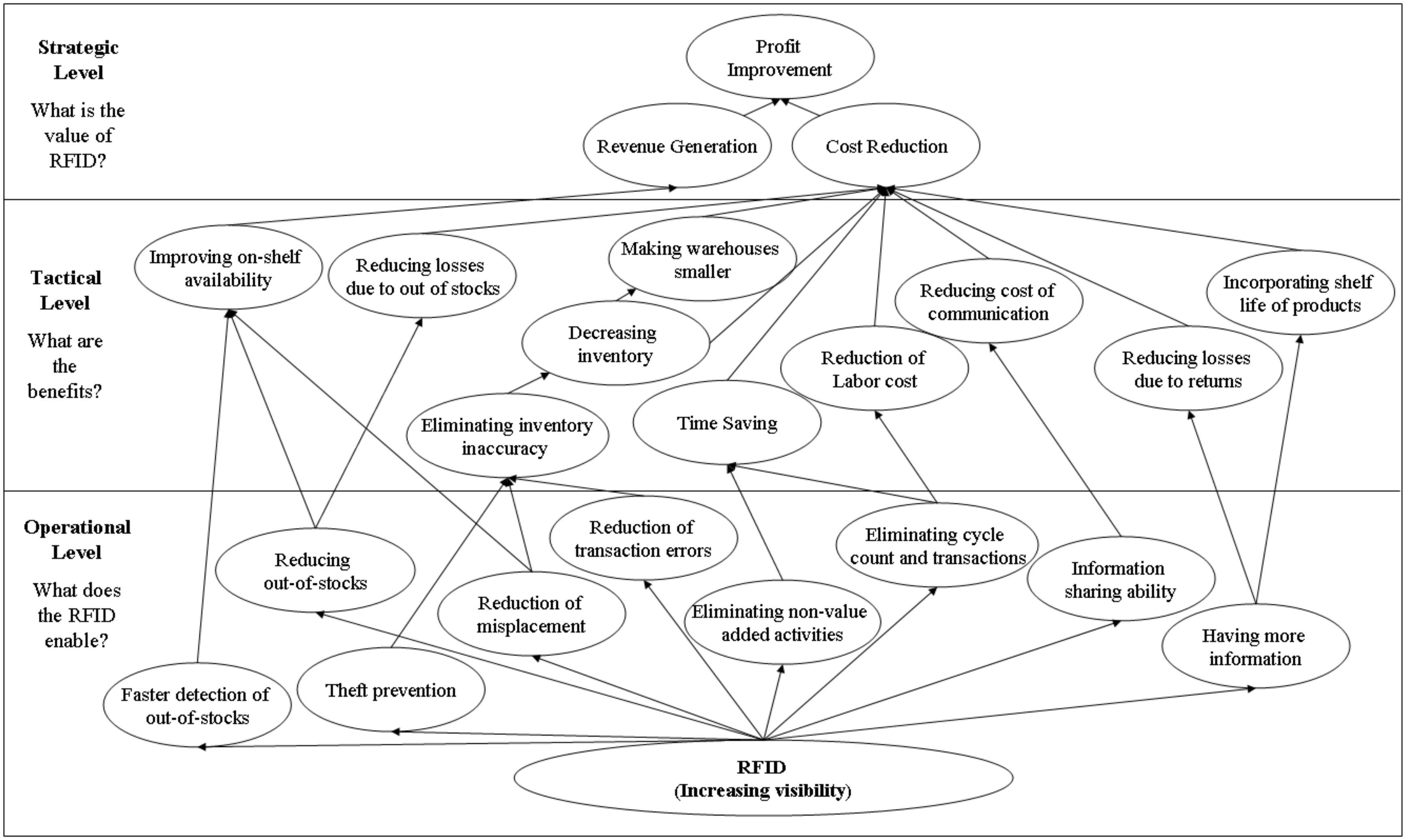

Figure 2.3: Benefits of the RFID technology

(1) Benefits under the control of the manufacturer: Inventory visibility, labor efficiency, improved fulfillment.

(2) Related with retailers and distribution centers: Store level of stocks, finished goods inventory and unsaleables.

It is reported in [24] that RFID seems profitable for high impact manufacturers. High impact (drug and general merchandise) manufacturers sell low volumes of expensive goods, so they experience significant out-of-stocks and shrinkage. Low impact manufacturers are not expected to realize important benefits from RFID. Low impact (food and grocery) manufacturers sell high volumes of less expensive goods.

RFID technology seems very useful for tracking items through supply chain. However, cost of the technology is high. Besides the costs of RFID readers and tags, the technology requires a well-established infrastructure. Since RFID just captures data, the systems such as Warehouse Management System (WMS), Enterprise Resource Planning (ERP), and Transportation Management System (TMS) are needed to use the technology efficiently. Since it is estimated that only 20,000 warehouses out of 600,000 use WMS in the U.S. [33], the infrastructures of the companies for the RFID investment should be revised.

2.4 Shortcomings

Despite its benefits, RFID has some shortcomings ([33], [43]). Those can be summarized as follows:

Cost: The cost of the implementation is the main issue. Although the price of the tags drop, they are still too high and besides tag cost RFID requires investment for infrastructure.

Tag Readability: Pilot programs have shown that errors such as misread and no-read occur too often. 80%-90% success rate in reading is identified. Radio frequencies are absorbed by liquids and reflected by metals. (There are studies about this shortcoming.)

Read Range Most chips have range of specific feet. (Wal-Mart have a maximum range of about ten feet.)

Consumer backlash: The fear of being tracked all over the world.

Data Sharing and Security: Sharing of data and security are potential obstacles (Competitors may monitor data.).

2.5 Pioneer Applications and Announced Benefits

Up to 2004, Wal Mart, Woolworth, Marks&Spencer, Procter&Gamble, Metro, Carrefour and Gillette were the most known companies that use or pilot RFID technology to evaluate its benefits [41]. However today, many companies have started using the RFID technology to track the products within supply chain.

The literature is full of references on industrial trials (for example, articles of RFID Journal, Frontline Solutions, Logistics Management, Information Week, RFID Gazette, Computerworld, Supply Chain Systems). In addition, consultant or solution provider companies are good sources to gain information about the existing applications. Those companies may include: Savi Technology, Zebra Technology, Texas Instruments, Alien Technology, SAP, Symbol Technologies, Sun Microsystems. Furthermore, many white papers are available as published by consultancy groups namely AMR Research, Accenture, IBM Business Consulting Services, A.T. Kearney, AMB Property.

In this section, several current applications, the challenges that the companies or organizations have faced and the gained benefits are presented. Note that the applications and pilot studies are so many that only several interesting and informative examples are given below.

- In June 2003, Wal-Mart as the leading company in retail industry mandated its top 100 manufacturers (e.g. Procter and Gamble, Gillette, Kraft, Unilever, etc.) to supply RFID tagged products by January 2005. The total cost for the implementation is estimated to be more than $2 billion dollars. Tags and readers are estimated to cost $5 million to $10 million, while system integration, which includes changing the current supply chain applications, upgrading storage systems and software, costs $8 million to $13 million per manufacturer [7]. Today, Wal-Mart has 500 stores and clubs and 5 distribution centers equipped with RFID technology. In addition to existing 130 suppliers, next 200 suppliers were expected to have the technology in January 2006. By the end of 2006, more than 1000 stores, clubs and distribution centers will use RFID. In January 2007, the next 300 suppliers will join the RFID journey of Wal-Mart. ([44],[13]).

  To evaluate the benefits of Wal-Mart, a study was conducted by the RFID center of the

University of Arkansas beginning in February 2005. The first results were obtained in October, 2005. For 29 weeks, researchers observed 12 pilot stores and 12 stores without the technology as control units. It is found that there is a 16% reduction in out-of-stocks at Wal-Mart stores that use RFID tags. The study also shows that the stores equipped with the technology are 63% more effective in replenishing out-of-stock products. Furthermore, manual orders are reduced by approximately 10% [13]. Although the study did not quantify the savings, it is reported that using RFID technology also saves time and labor [20].

- As the worlds third largest retailer, Metro Group also uses RFID technology to enable visibility within supply chain since 2004. They reported a 11-18% reduction in losses and theft, a 17% reduction in labor cost and a 9-14% increase in available merchandise on store shelves at their test sites in 2005 [26].

- Tesco, the largest retailer of the United Kingdom, put RFID tags on cases of non-food items at its distribution centers and tracks them through to stores since April 2004. It is reported that item-level RFID trial in two stores, increased on-shelf availability by 50% [39].

- Migros is the pioneer in Turkey in implementing RFID. They started pilot studies and research and development activities in the Gebze plant in October 2004. The RFID project is studied in two parts in Migros: stores and logistics. First of all they are planning to focus on the logistics part, since implementing RFID in store level requires item-level tagging which is really hard due to the immaturity of the technology [32]. Recently, Migros also works on a project which aims to understand the behavior of shoppers in stores by locating RFID tags to the shopping carts.

Even before the companies, some organizations started using RFID technology. Well-known organizations using the RFID technology are : North Atlantic Treaty Organization (NATO) and U.S. Department of Defense (DoD)

- NATO initiated Phase 1 RFID pilot in 2004 and signed a contract with Savi for the second phase in December 2005. It currently works on the implementation and installation of CMS (Consignment Management Solution). NATO tracks multinational defense consignments. The goal of the project is increasing the visibility by building an RFID infrastructure whereby member countries can have interactive and inter-operable supply chains between themselves. With this project, member nations can have their own consignment tracking systems which are inter-operable with NATO's RF (radio frequency) system that is needed in case of multi-national joint-force operations. NATO has an agreement to share the cost of the overall project among its 26 member nations and nations that are in NATO observer status. In addition to sharing the costs, those nations are supposed to integrate the NATO's RF system by the end of 2006 [14].

- DoD has an In-Transit Visibility (ITV) system which is the world's largest active RFID-enabled cargo tracking system, operating at 800 nodes and 1,400 installations [46]. The system is built and maintained by Savi Technology.

  In addition to having the world's largest active RFID-enabled cargo tracking system, RFID tags became mandatory in DoD contracts beginning October 1, 2004 for delivery of material on or after January 1, 2005 [38]. DoD already has had measurable benefits from using the technology. For instance implementing the technology in the Marine supply chain decreased the inventory values from $127 million to $70 million. Average delivery times have reduced from 28 days to 16 days and backlogs have been dropped from 92,000 shipments to 11,000 shipments. It is expected that the DoD will spend $500 million for the implementation of RFID in six years, but it is also estimated that DoD will save $70 million or more optimistically $1.7 billion in seven years [8].

As clarified above, the world's largest companies and organizations started using the RFID technology and began to get benefits from the RFID applications. Those benefits generally focus on decreased out-of-stocks, increased shelf-availability, decreased inventories and labor savings.

In parallel to those benefits, academic world also focuses on the effects of RFID investment on inventories and specifically on inventory inaccuracy. Since decreased inventory and stock-outs and increased shelf-availability are results of decreasing inventory inaccuracy, reduction in inventory inaccuracy seems to be the main benefit gained by RFID technology. If the inventory inaccuracy is eliminated from the system, uncertainty on inventory is reduced which results in decreasing safety stock (or inventory). Facing with out-of-stocks is caused by inventory inaccuracies, as well. If the stock level is not known accurately, the orders are not given on time which results in out-of-stocks.

Particularly, in this thesis, we focus on the RFID investment decision under the assumption that using the technology eliminates the inventory inaccuracy.

Chapter 3

LITERATURE REVIEW

This thesis builds on three streams of literature: inventory inaccuracy, information technology investment and supply chain coordination. The literature about the RFID investment is mainly built on working papers, since the RFID technology is an emerging technology and it has recently taken the attention of the researchers. Hence, the studies on this subject are not mature yet. The literature is given in three parts: inventory inaccuracy, the RFID investment and related literature.

3.1 Inventory Inaccuracy

Empirical studies have made clear the existence of the inventory inaccuracy problem. The first empirical study that addresses the inventory inaccuracy problem is performed by Rinehart [37]. The paper reports on a case study of discrepancies of a Federal government supply facility. In the study, 2000 discrepant items are discovered among 6000 items, it is also reported that 80% of the discrepancies were caused by the discrepancy correcting procedure itself.

Recently, Raman et al. [34] performed an empirical analysis to reveal the inventory inaccuracy problem. They reported that 65% of nearly 370,000 inventory records from 37 stores of a large retailer are inaccurate. That is, the inventory record of an item fails to match the physical quantity found in the store. The profit lost due to inventory inaccuracy is reported to be 10%. In addition, misplacement can be observed even when the inventory records are accurate. For another leading retailer, it is reported that 16% of the items cannot be found in the store due to misplacement. It is also reported that misplaced items reduced profits by %25. DeHoratius and Raman [11] investigate the problem and find that the variation in inventory inaccuracy record is associated with the cost of an item, its annual selling quantity and the distribution method used to ship that product to the stores.

Furthermore, Kang and Gershwin [23] report similar findings for a global retailer's stores.

The inventory accuracy is 51% on average for 500 stores and the best performing store in the study knows its actual inventory with 75-80% accuracy.

The above empirical studies identify the magnitude of the inventory inaccuracy problem. Although there is a considerable amount of research that focuses on inventory management in the literature, most of this research assumes perfect knowledge of the inventory data. There are relatively few studies considering inventory error.

Iglehart and Morey [19] is an early paper that studies the inventory inaccuracy problem. The objective of the paper is to select the proper frequency of inventory counts and additional safety stock by minimizing the sum of holding and counting costs when there is random demand and inventory inaccuracy. Morey [30] studies poor service level resulting from inventory inaccuracy and suggests actions to improve service level: increasing the safety stock, increasing the frequency of inventory counts and reducing the errors by focusing on its reasons. There are other articles studying counting frequencies and counting techniques to eliminate inventory inaccuracy: e.g. Buck and Sadowski [2], Martin and Goodrich [28] and Morey and Dittman [31].

In a recent study, DeHoratius et al. [10] propose probabilistic inventory records. They show that the distribution of inventory records can be used as the basis for replenishment and inventory audit policies and the needed parameters can be estimated from past sales and replenishment observations. They use data from a large retailer to illustrate the estimation of the parameters. They suggest a Bayesian procedure to periodically update the inventory record.

There are studies in which a specific reason of inventory inaccuracy is the focus: misplacement in Camdereli and Swaminathan [5], transaction errors in Kok and Shang [25] and shrinkage in Kang and Gershwin [23].

Camdereli and Swaminathan [5] study the supply chain coordination issue under misplaced inventory. They analyze the effect of misplaced inventory on the ordering decision and compare the performance of the decentralized system with the centralized system and suggest coordinating the decentralized system by means of revenue sharing and buy-back contracts.

Kok and Shang [25] consider the inventory inaccuracy problem. They work on finding a counting policy for an inventory replenishment problem to correct transaction errors.

Inventory inaccuracy is modelled as random and additive errors as in our model. The errors accumulate through periods until the inventory record is updated by an inspection. The tradeoff in the problem is either dealing with uncertainty or incurring the cost of inspection. They develop a joint inspection and replenishment policy that minimizes total costs in a finite horizon and show that an inspection adjusted base-stock policy is near-optimal. They also quantify the true value of RFID systems. Numerically, they show that potential benefit of RFID systems is on average 5% of the inventory costs. However, the proposed heuristic method compensates almost two thirds of this potential benefit. So, the true value of RFID systems is 1.8% of the total cost.

There are papers that use simulation to observe the effects of inventory inaccuracy on the supply chain. Kang and Gershwin [23] examine the problems related to information inaccuracy in inventory systems. They use simulation to see the effects of stock loss on stock outs and conclude that even a small rate of stock loss can create severe out-of-stocks. According to their results, the effect of stock loss is greater when short lead times and small order quantities are considered and the inventory inaccuracy problem can be controlled if the behavior of the stock loss is known.

Our research differs from the above mentioned papers, since they do not model the investment decision in the decentralized system with multiple decision makers. Also, some of them focuses on specific reasons of inventory inaccuracy. However, our model is built on a more general framework and focuses on investment decision under inventory inaccuracy.

3.2 The RFID Investment

Initial papers considering the RFID investment through the supply chain are emerging. Lee and Ozer [26] review some of the ongoing research on RFID and suggest future research opportunities on the subject. They argue that there is a huge credibility gap of the value of RFID and call the academic community to produce models to obtain realistic estimates of the RFID value.

There are several papers specifically focusing on tag prices. Gaukler et al. [16] study the introduction of item-level RFID in a decentralized supply chain and argue that the cost of item-level RFID should be allocated among the retailer and the supplier. They investigated the impact of RFID technology on the centralized and decentralized supply chains and found that an uncoordinated introduction of item-level RFID generates suboptimal benefits for the supply chain. Kok et al. [9] considers shrinkage (specifically theft) as the source of the inventory inaccuracy. By comparing shrinkage case with and without shrinkage case the break-even prices for an RFID tag is found. It is reported that the break-even prices are strongly correlated with the value of the items that are lost and the shrinkage fraction. Models of Gaukler et al. [16] and Kok et al. [9] are similar to our model, however there are some differences. Firstly, they assume that inventory inaccuracy is always negative. Secondly, they do not consider the costs of scanners, infrastructure and IT investments.

Sahin [41] studies a single-stage inventory system under inventory inaccuracy and builds several mathematical models. She finds value of RFID system by considering built models. Sahin and Dallery [42] explore the benefit of using the Auto-ID technology in improving the inventory accuracy in three stages including a supplier, a wholesaler and a retailer without considering the centralized case. A similar analysis to ours is performed by Rekik et al. [35]. Our research is similar in spirit but in contrast with [35], we explicitly model the investment costs in the RFID infrastructure that depends on how the technology is deployed.

Fleisch and Tellkamp [15] use simulation to examine the relationship between inventory inaccuracy and performance in a three stage supply chain. In a base model physical inventory and information system inventory differ due to low process quality, theft and items becoming unsaleables. The results of the paper show that an elimination of inventory inaccuracy can reduce supply chain costs and the out-of-stock level.

More recently, Heese [18] studied the inventory inaccuracy problem by considering RFID investment. His model and results are quite similar to ours. However, in his model, the demand is normally distributed and error is multiplicative.

3.3 Related Literature

One objective of our model is to find the optimal number of warehouses where the technology is applied and the optimal order quantity, such as the problem of sharing information with customers. In those problems the variance of demand is decreased by communicating with customers.

Milgrom and Roberts [29] investigate an information acquisition model for reducing demand uncertainty. They study the effects of communication with the customers on inventories and investigate the situation where the demand variance can be decreased by means of customer surveys. In the paper, the optimum amount of investment on obtaining demand information through customer surveys is found.

Zhu and Thonemann [49] considerably extend the framework of Milgrom and Roberts [29] by investigating the benefits of sharing future demand information when customer demands are correlated and the information given by the customers is imperfect. Although it is optimal to contact all or none of the customers if demand is not correlated and the information is perfect (Milgrom and Roberts [29]), it is often optimal to share information with some customers if the demands are correlated and the information is imperfect (Zhu and Thonemann [49]). Our model resembles the one in Zhu and Thonemann but we focus on the effect of multiple decision makers. In addition, we assume reduction of inventory inaccuracy but not demand variance. It will be seen later that in our models, the implementation of the technology lowers the variance of inventory inaccuracy. This makes our problem similar to the other variance reduction problems.

The literature in the field of supply chain coordination by contracts is vast. Cachon [3] presents an extensive literature review about supply chain coordination with contracts. Cachon and Lariviere [4] study strengths and limitations of revenue sharing contracts.

This study analyzes the investment decision that considers the total investment costs including the infrastructure through the supply chain for the centralized and the decentralized systems and tries to find the intuition behind the investment decisions of the centralized and decentralized systems under different scenarios.

Chapter 4

THE MODEL

Consider a supply chain consisting of a retailer (distributor) and a supplier. We assume a single-period newsvendor-type setting where the retailer purchases the items from the supplier and distributes them to her regional warehouses. It is assumed that the retailer sells the items to the customer at a unit price of r and the supplier's unit production cost is m. The wholesale price that the supplier charges the retailer is w, and the retailer has the chance to sell the unsold items at the end of the period, the salvage value of an unsold item is s.

More precisely, we consider a single selling period with random demand at each of the retailer's warehouses. In particular, it is assumed that the retailer has N regional warehouses and the regional demand for each warehouse has an independent normal distribution with mean μ_D and standard deviation σ_D. This assumption is made to keep the analysis tractable. In Chapter 5, we investigate several extensions that include correlated demand and asymmetric demand structures. There are two scenarios for our model: no inventory sharing (NIS) and inventory sharing (IS). Under the IS scenario, the warehouses are able to share their inventories as needed by lateral transshipments in order to avoid stockouts. In contrast inventory sharing is not allowed under the NIS scenario. The retailer decides on the total amount of inventory needed for her warehouses Q, $Q \geq 0$. At the beginning of the period, there is no inventory in the warehouses. After the retailer receives Q from the supplier, inventory inaccuracy problem occurs and then the demand is realized. The sequence of events is represented in Figure 4.1 and the structure of the model is depicted in Figure 4.2.

Clearly, under the above assumption, the total demand of the retailer is normally distributed with mean $N\mu_D$ and standard deviation $\sqrt{N}\sigma_D$ but the retailer should also take into account inventory inaccuracy to decide on the order quantity. The discrepancy between actual inventory and inventory records is known as inventory inaccuracy. Inventory

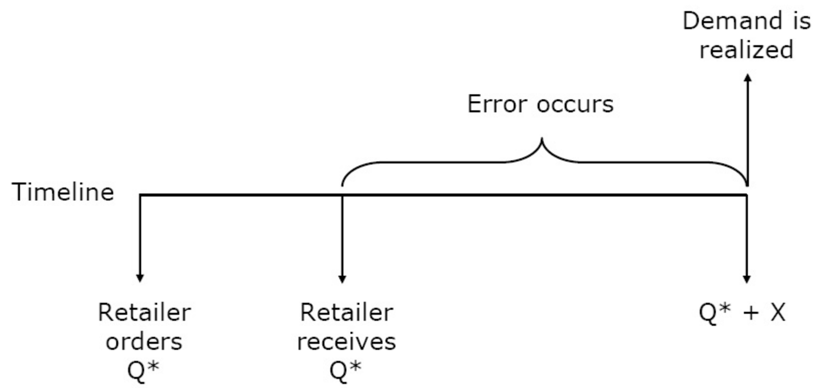

Figure 4.1: The sequence of events

inaccuracy may be caused by many reasons. Those reasons can be summarized under three categories: misplacement, shrinkage (stock loss) and transaction errors [26].

Misplacement occurs when the products are somewhere in the facility but cannot be found. Generally, the inaccessible products eventually are found and become available for sale. The inventory misplacement can be corrected implying that the inventory can be greater than the inventory records. In our model, the misplaced products can be found or some products may be misplaced during the period. So, the inventory inaccuracy may cause an increase or a decrease in the number of products available in the warehouses.

Stock loss, which is also known as shrinkage, is caused by all forms of loss of the products. The well-known way of stock loss is theft. The products may be stolen by internal (employees) or external thefts. Also, the products may be lost by becoming obsolete, damaged or spoiled (unsaleables). The inventory records are higher than the actual inventory in case of stock loss.

Transaction errors may occur at the inbound or outbound of a facility during the registration of products. The products may be wrongly identified or the products may not be counted correctly. So, the actual inventory and shipment records may not match. Transaction errors only affect the inventory records but not physical inventory.

It is stated in Lee and Ozer [26] that shrinkage and misplacement are more challenging than transaction errors. Shrinkage and misplacement would remain unnoticed without tracking the items by a technology such as RFID. As our main focus is RFID, our model considers shrinkage and misplacement as the main causes of inventory inaccuracy. So, the inventory records may be less than or greater than the actual inventory level. However, the case when the inventory discrepancy is negative is more challenging in general, since the products are lost for the current period and cannot be sold which is an important problem in our single period setting.

We denote X_i as a random variable representing the inaccuracy of the inventory record. X_i represents the discrepancy between what is thought to be available and what is really available at the end of a period. In our context, X_i s the number of items that are lost or found between the reception of an order and the sales. Please note that this makes the error additive as in Kok and Shang [25] and Sahin and Dallery [42]. To generalize, X_i is assumed to be normally distributed with mean μ_X and standard deviation σ_X for each warehouse. We assume that the retailer is aware of the inventory inaccuracy for each warehouse and gives her order by considering this in her ordering decision.

To model the above discussed structure, assume that the retailer orders the optimal order quantity Q^* from the supplier. However, due to the inaccuracy of the inventory records, the total actual quantity available in the warehouses $\bar{Q}^*$ is $Q^* + X$ where $X = \sum_{i=1}^{N} X_i$. Therefore, the total actual inventory available to the retailer through the season is $\sum_{i=1}^{N} Q_i^* + X_i$. As a result, the system can not satisfy the demand if $Q^* + X < D$ (equivalently $Q^* < D - X$) and has overstocked items if $Q^* + X > D$ (equivalently $Q^* > D - X$). The model with inventory inaccuracy X and demand D is then equivalent to a model with no inventory accuracy and demand $D - X$. To simplify the notation, we let $D' = D - X$. In other words, inventory inaccuracy affects the order quantity decision of the retailer in a similar way as demand uncertainty does. To summarize, with inventory inaccuracy, the total equivalent demand of the retailer is normally distributed with mean $N\mu_D - N\mu_X$ and standard deviation $\sqrt{N(\sigma_D^2 + \sigma_X^2)}$ (IS) or $N\sqrt{\sigma_D^2 + \sigma_X^2}$ (NIS). On the other hand, if RFID is applied at warehouse i then the actual inventory level at that warehouse is known with complete certainty and $\bar{Q}_i^* = Q_i^*$. We assume that RFID technology eliminates the inventory inaccuracy problem. The random variable X_i is removed if the technology is applied in warehouse i (we consider the case of imperfect error removal in Chapter 5).

It is well known that RFID technology investment requires fixed and variable costs.

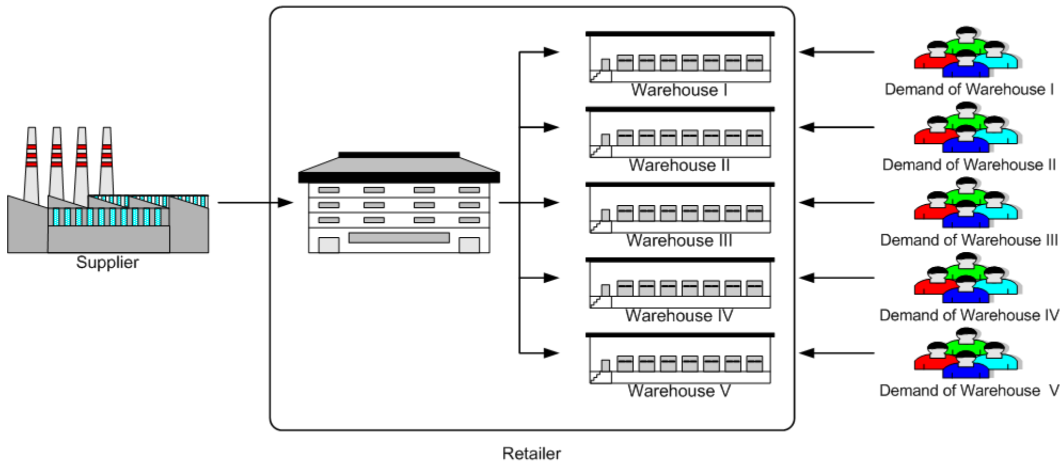

Figure 4.2: The proposed model

The fixed cost includes establishing the infrastructure for the technology, whereas variable costs include cost of tags and maintenance cost. According to A.T. Kearney [24], EPC (Electronic Product Code) and RFID implementations cost $400,000 per distribution center and $100,000 per store and additional costs for system integration range from $35 to $40 million for entire organization.

Motivated by the above structure, our model considers both the fixed and the variable investment costs. In our model, the fixed investment cost consists of the costs required to establish infrastructure of the entire system while the variable investment costs include the costs of investment required for each warehouse to eliminate its inaccuracy. Although these costs may be very large as a one-time investment, since our model considers a single period, we interpret both fixed costs and variable costs as equivalent amortized costs per single selling season.

In short, decreasing the inventory inaccuracy of every warehouse has a cost, k (per warehouse) and making an investment requires a fixed cost, K. The fixed investment cost incurs when the technology is applied in one or more warehouses. The function $g(n)$ represents the variable cost incurred by the investment, n is the number of warehouses where the technology is applied to eliminate inventory inaccuracy. The variable investment costs depend on the number of warehouses where the new technology is used. So,

$$K_{\{n>0\}} = \begin{cases} K & \text{if } n > 0 \\ 0 & \text{o.w.} \end{cases} \quad \text{and} \quad g(n) = kn$$

The objective of our model is maximizing the expected profit by finding the optimum number of warehouses to apply the technology. Making an investment decreases the number of warehouses that have inventory inaccuracy. The optimum number of warehouses n^* must be less than or equal to the total number of warehouses N and greater than zero. To simplify the analytical expression, we treat n as a continuous variable.

We solve the problem in two steps:

1. The optimum number of warehouses and the corresponding increase in profit are found ignoring the fixed cost.

2. If the increase in profit is greater than the fixed investment cost, then it is optimal to invest. Otherwise, the optimum solution is to make no investment.

In the remainder of this chapter, firstly the NIS scenario and then IS scenario is considered. At the end of the chapter computational results are represented.

4.1 No Inventory Sharing (NIS) Scenario

Through this section, it is presumed that there is no inventory sharing between the warehouses. The retailer decides on Q_i, the amount of inventory needed for warehouse i, $i = 1, ..., N$ and orders $\sum_{i=1}^{N} Q_i$ from the supplier.

4.1.1 The Decentralized System

In this section, we focus on the case of a decentralized supply chain under two extreme scenarios: either the supplier makes the investment without any cost sharing support from the retailer or the retailer makes the investment without any support from the supplier.

The Retailer Invests

For each warehouse, the retailer selects the optimal order quantity Q_i^*. When the investment is made at warehouse i, $Q_i^* = \mu_D + z_R\sigma_D$ otherwise $Q_i^* = \mu_D - \mu_X + z_R\sqrt{\sigma_D^2 + \sigma_X^2}$, where $z_R = \Phi^{-1}(\alpha_R)$. Φ^{-1} is the inverse cumulative distribution function of the standard normal distribution and α_R is the critical fraction for the retailer (see Zipkin [50]) and for the decentralized system, it is given by: $\alpha_R = (r-w)/(r-s)$. This fraction is found by dividing underage cost by the sum of underage and overage costs. Underage cost incurs when the system cannot satisfy the demand due to stock-out. Then, the system loses $(r-w)$. Overage cost incurs due to the excess inventory and is equal to $(w-s)$.

The profit of warehouse i is:

$$\Pi_{W_i} = \begin{cases} rD_i - wQ_i + s(Q_i + X_i - D_i) & if\ D_i < Q_i + X_i \\ r(Q_i + X_i) - wQ_i & if\ D_i \geq Q_i + X_i \end{cases}$$

The above expression can be written as:

$$\Pi_{W_i} = rmin(D_i, Q_i + X_i) - w(Q_i + X_i) + s(Q_i + X_i - D)^+ + wX_i$$

where $(Q_i+X_i-D)^+$ is equal to (Q_i+X_i-D) if $D < Q_i+X_i$ and 0 otherwise. Replacing $min(D_i, Q_i+X_i)$ by $D_i-(D_i-(Q_i+X_i))^+$ and Q_i+X_i by $D_i-(D_i-(Q_i+X_i))^+ + (Q_i+X_i-D_i)^+$ and taking the expectation results in:

$$E\left(\Pi_{W_i}\right) = (r-w)E\left[D_i\right] - \left[(r-w)E\left[(D_i-(Q_i+X_i))^+\right] + (w-s)E\left[(Q_i+X_i-D_i)^+\right]\right] + wE[X_i]$$

To incorporate inventory inaccuracy to our model, let us define δ_i such that:

$$\delta_i = \begin{cases} 1 & \textit{if an investment is not made at warehouse } i \\ 0 & o.w. \end{cases}$$

The expected profit of each warehouse is found under investment decision (using Zipkin [50]).

$$E\left(\Pi_{W_i}\right) = (r-w)\mu_D - (r-s)\phi(z_R)\sqrt{\sigma_D^2 + \delta_i\sigma_X^2} + \delta_i w\mu_X - K_{\{n>0\}} - k$$

where $\phi(z_R)$ denotes the standard normal density function for the decentralized system.

To find the expected profit of the retailer, we sum the expected profits of N warehouses. This gives:

$$E\left(\Pi_R\right) = \sum_{i=1}^{N}\left[(r-w)\mu_D - (r-s)\phi(z_R)\sqrt{\sigma_D^2 + \delta_i\sigma_X^2} + \delta_i w\mu_X\right] - K_{\{n>0\}} - kn$$

The number of warehouses where the technology is applied is denoted by n. For n warehouses the optimal order quantity is equal to $Q_i^* = \mu_D + z_R\sigma_D$, $i = 1, ..., n$ and for

$(N-n)$ warehouses it is equal to $Q_j^* = \mu_D - \mu_X + z_R\sqrt{\sigma_D^2 + \sigma_X^2}$, $j = n+1, ..., N$. Then, the expected profit function of the retailer is written as:

$$E(\Pi_R) = (r-w)N\mu_D - (r-s)\phi(z_R)\left[n\sigma_D + (N-n)\sqrt{\sigma_D^2 + \sigma_X^2}\right] + w(N-n)\mu_X - K_{\{n>0\}} - kn$$

The expected profit consists of five terms. The first term, $(r-w)N\mu_D$, is the sure profit. The second term, $(r-s)\phi(z_R)\left[n\sigma_D + (N-n)\sqrt{\sigma_D^2 + \sigma_X^2}\right]$, represents the cost associated with demand uncertainty (underage and overage costs). The cost of the items which are lost or found during the period, $w(N-n)\mu_X$, is added to the expected profit function, since the cost of those items are not paid in the current period. The last two terms, $K_{\{n>0\}}$ and kn are the fixed and variable investment costs of implementing the technology respectively.

The retailer's expected profit function is linear in n. So, if the function is increasing, the optimal investment decision is making the full investment, otherwise the optimal decision is making no investment. The first derivative of the expected profit function is:

$$\frac{\partial E(\Pi_R)}{\partial n} = (r-s)\phi(z_R)\left(\sqrt{\sigma_D^2 + \sigma_X^2} - \sigma_D\right) - w\mu_X - k$$

We observe that there is a threshold for the variable investment cost k such that making an investment becomes beneficial for the retailer. We define k_R^T as the variable investment threshold value where the retailer starts making a positive profit from making an investment. k_R^T for no inventory sharing scenario is equal to:

$$k_R^T = (r-s)\phi(z_R)\left[\sqrt{\sigma_D^2 + \sigma_X^2} - \sigma_D\right] - w\mu_X \tag{4.1}$$

Proposition 1 *If $k_R^T > k$ then $n^* = N$, otherwise $n^* = 0$.*

Proof. If k_R^T is greater than k, the expected profit of the retailer increases in n. The sign of the first derivative of the expected profit function with respect to n is positive. Since the maximum feasible n for our model is N, the optimal decision is making the full investment. Otherwise the expected profit function of the retailer decreases in n and the minimum feasible n is 0, so the optimal decision is making no investment.

□

The optimal solution is: $n^* = N$ or $n^* = 0$ ignoring fixed costs. This is similar to the corresponding result in Milgrom and Roberts [29] in a different context.

Corollary 1 examines the impacts of the parameters σ_X, σ_D and μ_X on the investment threshold.

Corollary 1 *The investment threshold k_R^T increases in σ_X and decreases in σ_D and μ_X.*

Proof. The effects of σ_X and μ_X follow directly from (4.1). As σ_X increases, the variable investment threshold increases. When μ_X gets smaller, k_R^T increases.

The claim on the demand variance follows from (4.1). The expression $\sqrt{\sigma_D^2 + \sigma_X^2} - \sigma_D$ is decreasing in σ_D, so the increase in σ_D causes a decrease in the variable investment cost k_R^T. The first derivative of $\sqrt{\sigma_D^2 + \sigma_X^2} - \sigma_D$ with respect to σ_D is equal to:

$$\frac{\sigma_D}{\sqrt{\sigma_D^2 + \sigma_X^2}} - 1 \leq 0$$

□

Remark: In our numerical results, k_R^T is also increasing in price, r.

According to Corollary 1, the retailer is more likely to make the investment as the initial inventory inaccuracy increases and the demand variance decreases. When the inventory inaccuracy is an important problem for the retailer, the retailer is more likely to make an investment to decrease it. In contrast, if there is high demand variance in the market, the retailer does not prefer spending much to decrease inventory inaccuracy, since the demand variance behaves like the inventory inaccuracy variance. This means decreasing inventory inaccuracy variance will not help decreasing uncertainty in the system due to the demand variance. As the mean of the inventory inaccuracy decreases, the problem becomes so important for the retailer that she can pay more to make the investment.

The Supplier Invests

When the supplier makes the investment, the expected profit function of the supplier is:

$$E(\Pi_S) = \left[n\left(\mu_D + z_R\sigma_D\right) + (N-n)\left(\left(\mu_D - \mu_X\right) + z_R\sqrt{\sigma_D^2 + \sigma_X^2}\right)\right](w-m) - K_{\{n>0\}} - kn$$

The expected profit of the supplier is found by multiplying the optimal order quantity of the retailer by the profit margin of the supplier. The expected profit function of the supplier is a linear function in n. Its first derivative is given by:

$$\frac{\partial E(\Pi_S)}{\partial n} = \left[z_R\left(\sigma_D - \sqrt{\sigma_D^2 + \sigma_X^2}\right) + \mu_X\right](w-m) - k$$

Without considering μ_X, the expected profit function of the supplier is decreasing if underage cost is greater than overage cost ($z_R > 0$) and for this case the optimal decision is making no investment. In any case, if $\left[z_R\left(\sigma_D - \sqrt{\sigma_D^2 + \sigma_X^2}\right) + \mu_X\right](w-m) > k$, the first derivative of the expected profit function is increasing and the optimal decision is making the full investment.

There is a threshold for the supplier, k_S^T, which is:

$$k_S^T = \left[z_R\left(\sigma_D - \sqrt{\sigma_D^2 + \sigma_X^2}\right) + \mu_X\right](w-m) \tag{4.2}$$

As in Corollary 1, as σ_D decreases and σ_X increases, the investment threshold for the supplier increases as it is the case for the retailer (when the underage cost is less than the overage cost. ($z_R < 0$)). As the production cost m decreases, the investment threshold increases as well. On the other hand, the number of warehouses does not have an impact on the investment threshold.

4.1.2 The Centralized System

In the centralized supply chain, it is assumed that a central planner determines the amount of investment made by the entire supply chain and the order quantity to maximize the total profit. The optimal order quantity for the centralized system is:

$$Q^* = \sum_{i=1}^{N}\left(\mu_D - \delta_i\mu_X + z_C\sqrt{\sigma_D^2 + \delta_i\sigma_X^2}\right)$$

Similar to the decentralized case, z_C is $\Phi^{-1}(\alpha_C)$ and α_C is the critical fraction of the centralized system given by: $\alpha_C = (r-m)/(r-s)$.

The expected profit function of the centralized system is shown to be similar to the expected profit function of the retailer and it is equal to:

$$E(\Pi_C) = (r-m)N\mu_D - (r-s)\phi(z_C)\left[n\sigma_D + (N-n)\sqrt{\sigma_D^2 + \sigma_X^2}\right] + m(N-n)\mu_X - K_{\{n>0\}} - kn$$

Similar to the expected profit function of the retailer, the expected profit function of the centralized system is linear in n and it is increasing when the first derivative of the function is greater than 0.

$$\frac{\partial E(\Pi_C)}{\partial n} = (r-s)\phi(z_C)\left(\sqrt{\sigma_D^2+\sigma_X^2}-\sigma_D\right) - m\mu_X - k$$

Then, the threshold for the variable investment cost k_C^T is:

$$k_C^T = (r-s)\phi(z_C)\left[\sqrt{\sigma_D^2+\sigma_X^2}-\sigma_D\right] - m\mu_X \tag{4.3}$$

Similar to the decentralized case, if the variable investment cost k is less than k_C^T, then the optimal solution for the centralized system is making the full investment when the fixed investment cost is ignored. As proposed in Corollary 1, the investment threshold increases as the variance of demand decreases or the variance of inventory inaccuracy increases.

The Effects of Centralization on Investment

Comparing the variable investment threshold values of the retailer and the centralized system gives insight about the investment decisions of the retailer and the centralized system. To make the comparison easier, the value of μ_X is assumed to be 0. In order to compare the investments made by the centralized system and retailer, we should compare (4.1) and (4.3). Since all the parameters are equal except the density functions, $\phi(z_C)$ and $\phi(z_R)$, the value of the density functions must be compared to analyze the investment decision. Since $\phi(z_C)$ can be greater than or less than or equal to $\phi(z_R)$, the investment threshold of the centralized system can be greater than or less than or equal to the investment threshold of the retailer. The conditions for the comparison of the investment thresholds are presented in Proposition 2.

Proposition 2 *The threshold values of the centralized system and retailer differ according to following conditions:*

1. *If $r-w=m-s$, then the investment threshold values are equal,*

2. *If $r-w>m-s$, then k_C^T is less than k_R^T,*

3. *If $r-w<m-s$, then k_C^T is greater than k_R^T.*

Proof. If the sum of α_C and α_R is equal to 1, then the density functions are equal for both systems. So, it is concluded that if $r - w = m - s$, then the investment thresholds are equal. Both the centralized system and retailer are willing to make an investment, if the variable investment cost of eliminating inventory inaccuracy for each warehouse is less than the investment threshold ignoring the fixed investment cost.

$r - w > m - s$ means that sum of α_C and α_R is greater than 1. Since $w > m$, α_C is always greater than α_R. When the density functions are compared, the centralized system has a smaller density function value if $r - w > m - s$ holds. That is $\phi(z_C) < \phi(z_R)$ and from (4.1) and (4.3), $k_C^T < k_R^T$. As a result, the retailer has a higher tendency to make an investment. The third condition can be shown similarly.

□

According to Proposition 2, the retailer has a higher tendency to make an investment to decrease inventory variance when her profit margin is high. In a way, she tends to make an unnecessary investment for the supply chain in that case since making an investment may not be the optimal strategy for the centralized system. In contrast, when her profit margin is lower, she may not want to make an investment even though it is beneficial for the centralized system.

4.1.3 Coordination of the Supply Chain

In this subsection, we investigate investment cost sharing structures between the retailer and the supplier through simple contracts. We consider the well-known revenue sharing and buyback contracts and investigate coordination issues under these contracts. Throughout the coordination section, μ_X is assumed to be 0, since having positive or negative μ_X can cause problems such as: how to share the revenue generated from the items which are not purchased in the current period (under the revenue sharing contract) or how to give the unsold items to the supplier if they are not purchased in the current period (under the buyback contract).

In this section, we seek the coordinating parameters which make the optimal order quantity and the investment decision (i.e. making the full investment or no investment) of the decentralized system equivalent to the optimal order quantity and the investment decision of the centralized system. For the investment decision coordination, the variable investment thresholds are considered. However, it should be noted that the fixed investment costs may affect the system. It is assumed that when the decision is making an investment by considering the variable investment threshold, the fixed investment cost is compensated.

Revenue Sharing Contract

Revenue sharing contracts coordinate the supply chain by dividing the revenue according to a given proportion, β and adjusting the wholesale price w accordingly. Under these contracts, the retailer keeps β portion of all revenue while the supplier takes $(1-\beta)$ portion. A conventional revenue sharing contract coordinates the system by forcing the retailer to give the (centralized) optimum order quantity. However, in our model the fixed and variable investment costs are also needed to be shared to coordinate the system. We assume that the retailer pays θ_1 portion of the fixed investment cost and θ_2 portion of the variable investment cost and the supplier pays $(1-\theta_1)$ portion of the fixed investment cost and $(1-\theta_2)$ portion of the variable investment cost.

Under the revenue sharing contract, the expected profit functions of the retailer and supplier are as follows:

$$E\left(\Pi_R\right) = (\beta r - w)N\mu_D - \beta(r-s)\phi(z_R)\left(n\sigma_D + (N-n)\sqrt{\sigma_D^2+\sigma_X^2}\right) - \theta_1 K_{\{n>0\}} - \theta_2 kn$$

and

$$\begin{aligned} E(\Pi_S) = \quad & \left[n\left(\mu_D + z_R\sigma_D\right) + (N-n)\left(\mu_D + z_R\sqrt{\sigma_D^2+\sigma_X^2}\right)\right](w-m) \\ & +(1-\beta)rE\left[\min(Q,D')\right] + (1-\beta)sE\left[(Q-D')^+\right] \\ & -(1-\theta_1)K_{\{n>0\}} - (1-\theta_2)kn \end{aligned}$$

$E\left[\min(Q,D')\right]$ is replaced by $E\left[D' - (D'-Q)^+\right]$. According to Zipkin [50]:

$$E\left[(D'-Q)^+\right] = \left(\sqrt{\sigma_D^2+\sigma_X^2}\right)\varphi(z_R)$$

and

$$E\left[(Q-D')^+\right] = \left(\sqrt{\sigma_D^2+\sigma_X^2}\right)(z+\varphi(z_R))$$

where φ is the standard normal loss function, $\varphi(z_R) = -z_R[1-\Phi(z_R)] + \phi(z_R)$.

Then the expected profit function of the supplier is:

$$\begin{aligned} E(\Pi_S) = \quad & \left[n\left(\mu_D + z_R\sigma_D\right) + (N-n)\left(\mu_D + z_R\sqrt{\sigma_D^2+\sigma_X^2}\right)\right](w-m) \\ & +(1-\beta)r\left[(n\mu_D + (N-n)\mu_X) - \varphi(z_R)\left(n\sigma_D + (N-n)\sqrt{\sigma_D^2+\sigma_X^2}\right)\right] \\ & +(1-\beta)s\left[z_R + \varphi(z_R)\left(n\sigma_D + (N-n)\sqrt{\sigma_D^2+\sigma_X^2}\right)\right] \\ & -(1-\theta_1)K_{\{n>0\}} - (1-\theta_2)kn \end{aligned}$$

Proposition 3 establishes the optimal retailer portions of the fixed and variable costs θ_1^* and θ_2^* that coordinate the system. For system coordination, the investment decisions for the centralized and decentralized systems must be the same and the order quantities must be equal.

Proposition 3 *The coordinating contract parameters are as follows:* $\theta_1^* = \theta_2^* = \beta$ *and* $w = \beta m$.

Proof. As it is seen in (4.1) and (4.3), the investment decision is only affected by the standard normal density function and the standard normal density function depends on the critical ratio, α. (when μ_X is equal to 0.) The optimal order quantity is also obtained by using the critical ratio. It is known that the revenue sharing contract coordinates the system if the wholesale price w is equal to β percent of the unit production cost m (see Cachon and Lariviere [4]).

$$z_C = \frac{r-m}{r-s} = \frac{\beta r - \beta m}{\beta r - \beta s},\ \Phi^{-1}(z_C) = \Phi^{-1}(z_R) \text{ and } \phi(z_C) = \phi(z_R)$$

as a result, $Q_C^* = Q_D^*$ and $k_C^T = k_R^T$

If the wholesale price w is equal to β percent of the unit production cost m, the system makes the optimum investment and gives the optimum order quantity. We also know that the revenue sharing contract shares both the profit and the revenue according to the proportion β (Cachon and Lariviere [4]). Since, the profit function is multiplied by β, sharing the total investment cost according to the same ratio coordinates the system. So, we conclude that:

$$\theta_1^* = \theta_2^* = \beta$$

The revenue sharing contract coordinates our system where $w = \beta m$ and $\theta_1^* = \theta_2^* = \beta$.

□

Proposition 3 implies that there are coordinating revenue sharing contracts. Interestingly, the revenue sharing contract may coordinate the supply chain, even if the investment cost is not shared according to the ratio β. Below we investigate conditions under which coordination can be achieved. Note that, the following conditions coordinate the system only if making an investment is optimal for the centralized system. The gain of the retailer and supplier without considering the investment cost when the investment is made are equal to:

$$\vartheta_R = \beta(r-s)\phi(z_R)\left[N\sqrt{(\sigma_D^2 + \sigma_X^2)} - N\sigma_D\right]$$

and

$$\vartheta_S = \left[N\sqrt{\sigma_D^2 + \sigma_X^2} - N\sigma_D\right]\left[(m-w)z_R + (1-\beta)(r-s)\varphi(z_R)\right]$$

Consider a case where the supplier is stronger than the retailer. An extreme example of this case is the automotive industry. In the automotive industry, the manufacturer or the supplier is stronger than the distributors. In such an industry, the supplier may be more willing to pay for the RFID investment. In this case, the following conditions may hold.

- The supplier pays all variable cost, $\theta_2 = 0$ and θ_1 is negotiated. The system is coordinated if:

  $$\vartheta_R \geq \theta_1 K \text{ and } \vartheta_S \geq (1-\theta_1)K + kN$$

  Since the supplier is the strongest member of the supply chain, he can own all the variable investment cost and even he can share the fixed investment cost required for the infrastructure to support the retailer to make an investment.

- The supplier pays all fixed cost, $\theta_1 = 0$ and θ_2 is negotiated. The system is coordinated if:

$$\vartheta_R \geq \theta_2 kN \text{ and } \vartheta_S \geq K + (1-\theta_2)kN$$

The supplier establishes the infrastructure of the system and supports the retailer to use the technology by paying a fraction of the variable investment cost.

Now, consider a case where the retailer is stronger and benefits more from making an investment on RFID than the supplier. As reported an article by Jonathan Collins [7], CPG (Consumer Packaged Goods) manufacturers worry that RFID would bring little benefit to their current operations. It is also reported that most suppliers' inventory accuracy is already %99.9. In such a case, the retailer may want to own the greater part of the investment cost.

- $\theta_2 = \beta$ and θ_1 is negotiated. The system is coordinated if:

$$\vartheta_R \geq \theta_1 K + \beta kN \text{ and } \vartheta_S \geq (1-\theta_1)K + (1-\beta)kN$$

Since the retailer is the strongest member of the supply chain, he can own all the variable cost and shares the fixed investment cost required for the infrastructure to force the supplier to make an investment.

- $\theta_1 = \beta$ and θ_2 is negotiated. The system is coordinated if:

$$\vartheta_R \geq \beta K + \theta_2 kN \text{ and } \vartheta_S \geq (1-\beta)K + (1-\theta_2)kN$$

The retailer establishes the infrastructure of the system and forces the supplier to use the technology by paying a fraction of the variable investment cost.

Finally, the supplier and the retailer may have equivalent strengths and both of them may benefit from making an investment. In that case, both sides may negotiate on sharing the variable investment cost or the fixed investment cost.

- The retailer pays all variable costs, $\theta_2 = 1$ and θ_1 is negotiated. The system is coordinated if:

$$\vartheta_R \geq \theta_1 K + kN \text{ and } \vartheta_S \geq (1-\theta_1)K$$

- The retailer pays all fixed costs, $\theta_1 = 1$ and θ_2 is negotiated. The system is coordinated if:

$$\vartheta_R \geq K + \theta_2 kN \text{ and } \vartheta_S \geq (1-\theta_2)kN$$

  The supplier and retailer negotiate on the investment decision. Since we initially assumed that making an investment is profitable. They can find fractions both for the variable investment cost and fixed investment cost such that making an investment will be beneficial for both parties.

In the above conditions, the number of warehouses and the variance of inventory inaccuracy have positive effects on supply chain coordination. An increase in those parameters causes an increase in the gains of both the retailer and the supplier. On the other hand, the variance of demand has a negative effect on the gain.

Buyback Contract

A buyback contract can also coordinate the system when there is an investment decision. However, the conventional buyback contract needs some modification for our model. Under a buyback contract, the supplier takes back unsold items from the retailer at a unit buyback price of b. In our model, an additional parameter is needed to share investment between two partners.

Cachon and Lariviere [4] show that in the newsvendor setting with a fixed price, for any coordinating buyback contract $\{b, w\}$ there exists a unique revenue sharing contract $\{\beta, w\}$ that generates the same profit for the retailer and supplier. By using this property, the investment can be shared between the supplier and the retailer by adding an extra parameter θ to buyback contract.

Under a buyback contract, the profit of the retailer is:

$$\Pi_R = \begin{cases} rD' - wQ + b(Q - D') - \theta \left[K_{\{n>0\}} + kn \right] & if\ D' < Q \\ (r - w)Q - \theta \left[K_{\{n>0\}} + kn \right] & if\ D' \geq Q \end{cases}$$

Then, the expected profit function of the retailer is:

$$E(\Pi_R) = (r-w)N\mu_D - (r-b)\phi(z_R)\left[n\sigma_D + (N-n)\sqrt{\sigma_D^2+\sigma_X^2}\right] - \theta\left[K_{\{n>0\}} + kn\right]$$

Under a buyback contract, the profit of the supplier is:

$$\Pi_S = \begin{cases} wD' - mQ + (w-b)(Q-D') - (1-\theta)\left[K_{\{n>0\}} + kn\right] & if\ D' < Q \\ (w-m)Q - (1-\theta)\left[K_{\{n>0\}} + kn\right] & if\ D' \geq Q \end{cases}$$

The structure of the supplier's profit function is similar to the structure of the profit function of the retailer. r is replaced by w, w is replaced by m and b is replaced by $w-b$. So, the expected profit function of the supplier is written as:

$$E(\Pi_S) = (w-m)N\mu_D - b\phi(z_R)\left[n\sigma_D + (N-n)\sqrt{\sigma_D^2+\sigma_X^2}\right] - (1-\theta)\left[K_{\{n>0\}} + kn\right]$$

For coordination:

$$\frac{r-m}{r-s} = \frac{r-w}{r-b} \text{ and } b = \frac{r(w-m)+s(r-w)}{r-m}$$

and

$$\frac{\beta r - \beta m}{\beta r - \beta s} = \frac{r-w}{r-b},\ \beta = \theta,\ \theta = \frac{r-b}{r-s} \text{ or } \theta = \frac{r-w}{r-m}$$

The coordination conditions proposed in Section 4.1.3 for the revenue sharing contract are valid for the buyback contract as well, since it is established by Cachon and Lariviere [4] for each coordinating buyback contract there is an equivalent revenue sharing contract.

4.2 Inventory Sharing (IS) Scenario

In this section, it is assumed that the warehouses are able to share their inventories as needed by lateral transshipments in order to avoid stockouts. The retailer decides on the total optimal order quantity, Q^* for N warehouses and orders Q^* from the supplier.

4.2.1 The Decentralized System

As in Section 4.1.1, two cases will be analyzed for the decentralized system: the retailer makes the investment and the supplier makes the investment.

The Retailer Invests

When the retailer considers the investment decisions without any support from the supplier, she selects the optimal total quantity Q^* to maximize her individual profits. This quantity is given by:

$$Q^* = N\mu_D - (N-n)\mu_X + z_R\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2}$$

When the investment decision is made by the retailer and the total equivalent demand is normally distributed with mean $N\mu_D - (N-n)\mu_X$ and standard deviation $\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2}$, the expected profit function of the retailer is found to be [50]:

$$E(\Pi_R) = (r-w)N\mu_D - (r-s)\phi(z_R)\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2} + w(N-n)\mu_X - K_{\{n>0\}} - kn$$

The retailer's expected profit function is convex in n since:

$$\frac{\partial E(\Pi_R)^2}{\partial^2 n} = \frac{\sigma_X^4 (r-s)\phi(z_R)}{4\ {}^{3/2}\!\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2}} \geq 0$$

Since the expected profit function is convex, the optimal solution is either making no investment or making an investment to eliminate the inventory inaccuracy in all warehouses. Therefore, if we ignore the fixed costs: $n^* = N$ or $n^* = 0$.

As in the previous section, we observe that there is a threshold for the variable investment cost k such that making an investment becomes beneficial for the retailer. We define $\bar{k}_R^T$ as the investment threshold value where the retailer starts making a positive profit from making an investment.

Proposition 4 *If $\bar{k}_R^T > k$ then $n^* = N$, otherwise $n^* = 0$.*

Proof. Since the expected profit function is convex, it is argued that if the full investment case results in lower cost than no investment case, then the investment is made to eliminate the inventory inaccuracy in all warehouses. This corresponds to:

$$(r-s)\phi(z_R)\sigma_D\sqrt{N} + kN < (r-s)\phi(z_R)\sqrt{N(\sigma_D^2 + \sigma_X^2)} - wN\mu_X$$

According to the above comparison, the investment threshold is found to be:

$$\bar{k}_R^T = \frac{(r-s)\phi(z_R)\left[\sqrt{N(\sigma_D^2 + \sigma_X^2)} - \sigma_D\sqrt{N}\right]}{N} - w\mu_X \tag{4.4}$$

If the variable investment cost k is less than $\bar{k}_R^T$, then the optimal solution for the retailer is making the full investment when the fixed investment cost is ignored. In other words, the retailer would then make the investment if the total increase in profit compensates the fixed costs.

$\square$

The effects of parameters σ_X, σ_D and N on the investment threshold are given in Corollary 2.

Corollary 2 *The investment threshold $\bar{k}_R^T$ increases in σ_X and decreases in N, σ_D and μ_X.*

Proof. The effects of σ_X, μ_X and N follow directly from (4.4).

The claim on the demand variance follows from (4.4). The expression $\sqrt{\sigma_D^2 + \sigma_X^2} - \sigma_D$ is decreasing in σ_D, so the increase in σ_D causes a decrease in the variable investment cost $\bar{k}_R^T$. The first derivative of $\sqrt{\sigma_D^2 + \sigma_X^2} - \sigma_D$ with respect to σ_D is equal to:

$$\frac{\sigma_D}{\sqrt{\sigma_D^2 + \sigma_X^2}} - 1 \leq 0$$

$\square$

Remark: In our numerical results, k_R^T is also increasing in price r.

The parameters σ_X and σ_D affect the investment threshold in the same way as in the NIS scenario. On the other hand, if there is a large number of warehouses, to decide on the investment, lower variable investment costs are expected by the retailer, because the total amount of investment is higher when the number of warehouses is large.

The Supplier Invests

When the supplier considers the investment without any support from the retailer, his expected profit function is:

$$E(\Pi_S) = \left(N\mu_D - (N-n)\mu_X + z_R\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2}\right)(w-m) - K_{\{n>0\}} - kn$$

where the optimal ordering quantity is equal to:

$$Q^* = N\mu_D - (N-n)\mu_X + z_R\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2}$$

The expected profit function of the supplier is known to be convex when z_R is negative and concave when z_R is positive, since:

$$\frac{\partial E(\Pi_S)^2}{\partial^2 n} = -\frac{\sigma_X^4 (w-m) z_R}{4 \sqrt[3/2]{N\sigma_D^2 + (N-n)\sigma_X^2}}$$

As in the retailer's case, we find that there exists an investment threshold for the variable investment cost $\bar{k}_S^T$ such that there is a positive benefit for the supplier for all $k < \bar{k}_S^T$. The supplier prefers making an investment if:

$$\left(N\mu_D + z_R\sigma_D\sqrt{N}\right)(w-m) - kN > \left(N(\mu_D - \mu_X) + z_R\sqrt{N(\sigma_D^2 + \sigma_X^2)}\right)(w-m)$$

Then, $\bar{k}_S^T$ is found to be:

$$\bar{k}_S^T = \frac{(w-m) z_R \left[\sigma_D\sqrt{N} - \sqrt{N(\sigma_D^2 + \sigma_X^2)}\right]}{N} + (w-m)\mu_X \tag{4.5}$$

Note that if underage cost is greater than overage cost ($z_R > 0$) in Equation 4.5, $\bar{k}_S^T$ becomes negative if μ_X is 0. If underage cost of the retailer is greater than overage cost, the supplier make an investment if the inventory inaccuracy μ_X is different than 0. If z_R is negative, implying small profit margins for the retailer, from (4.5), we can observe the tradeoffs for the supplier when the investment decision is considered. Similar to the retailer, as in Corollary 2, the supplier is more likely to invest as the number of warehouses and the demand variance decrease and the variance of inventory inaccuracy increases (when z_R is negative.). In addition, as the production cost m decreases, the supplier is more likely to pay more for the investment.

4.2.2 The Centralized System

The optimal order quantity for the centralized system is:

$$Q^* = N\mu_D - (N-n)\mu_X + z_C\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2}$$

As in Section 4.1.2, the expected profit function of the centralized system is shown to be similar to the retailer's expected profit function and it is equal to:

$$E(\Pi_C) = (r-m)N\mu_D - (r-s)\phi(z_C)\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2} + m(N-n)\mu_X - K_{\{n>0\}} - kn$$

The convexity of the expected profit function of the centralized system can be verified since:

$$\frac{\partial E(\Pi_C)^2}{\partial^2 n} = \frac{\sigma_X^4 (r-s)\phi(z_C)}{4\sqrt[3/2]{N\sigma_D^2 + (N-n)\sigma_X^2}} \geq 0$$

Just like in the decentralized case, due to convexity, the optimal solution is either making no investment or making an investment to eliminate the inventory inaccuracy in all warehouses.

Like $\bar{k}_R^T$, the threshold for the centralized system $\bar{k}_C^T$ is:

$$\bar{k}_C^T = \frac{(r-s)\phi(z_C)\left[\sqrt{N(\sigma_D^2+\sigma_X^2)} - \sigma_D\sqrt{N}\right]}{N} - m\mu_X \tag{4.6}$$

Similar to the decentralized case, if the investment variable cost k is less than $\bar{k}_C^T$, then the optimal solution for the centralized system is making the investment when the fixed investment cost is not considered.

As proposed in Corollary 2, $\bar{k}_C^T$ increases in the inventory inaccuracy variance, σ_X and decreases in the number of warehouses, N and the demand variance, σ_D. In contrast to the NIS scenario, the variable investment threshold decreases in the number of warehouses for the retailer and the centralized system.

The analysis made to compare the investment decisions of the centralized system and the retailer in Section 4.1.2 is valid for the IS scenario. (see Proposition 2)

When NIS and IS scenarios are compared, it is concluded that k_R^T is always greater than $\bar{k}_R^T$ and k_C^T is always greater than $\bar{k}_C^T$ (see equations (4.1) and (4.4) and (4.3) and (4.6)). Under the IS scenario, the retailer and the centralized system have less tendencies to make an investment. A general conclusion is that the inventory sharing decreases the need for the RFID investment.

For all the scenarios proposed, after finding the optimum number of warehouses, the second step is making a comparison of the increase in the expected profit with the fixed investment cost and to decide whether to invest or not.

4.2.3 Coordination of the Supply Chain

As in the NIS scenario, to coordinate the supply chain under the investment decision, a revenue sharing contract or a modified version of the buyback contract is used. The mean of inventory inaccuracy, μ_X is assumed to be 0.

Revenue Sharing Contract

Under the revenue sharing contract, the expected profit functions of the retailer and supplier are as follows:

$$E\left(\Pi_R\right) = (\beta r - w)N\mu_D - \beta(r-s)\phi(z_R)\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2} - \theta_1 K_{\{n>0\}} - \theta_2 kn$$

and

$$\begin{aligned} E(\Pi_S) = \quad & \left(N\mu_D + z_R\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2}\right)(w-m) + (1-\beta)rE\left[\min(Q, D')\right] \\ & +(1-\beta)sE\left[(Q-D')^+\right] - (1-\theta_1)K_{\{n>0\}} - (1-\theta_2)kn \end{aligned}$$

$E\left[\min(Q, D')\right]$ is replaced by $E\left[D' - (D'-Q)^+\right]$. Then the expected profit function of the supplier is:

$$\begin{aligned} E(\Pi_S) = \quad & \left(N\mu_D + z_R\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2}\right)(w-m) \\ & +(1-\beta)r\left(N\mu_D - \varphi(z_R)\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2}\right) \\ & +(1-\beta)s\left(z_R + \varphi(z_R)\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2}\right) - (1-\theta_1)K_{\{n>0\}} - (1-\theta_2)kn \end{aligned}$$

For system coordination, the investment decisions must be the same and the order quantities must be equal for the centralized and decentralized systems. The optimum θ_1^* and θ_2^* that coordinate the system are equal to β as proposed in Proposition 3.

In addition to the above case, it is argued that under the coordination conditions explained in Section 4.1.3, the revenue sharing contract can coordinate the supply chain, even if investment cost is not shared according to the ratio β. However, the conditions coordinate the system only if making an investment is optimal for the centralized system. The gain of the retailer and supplier without considering the investment cost when investment is made are equal to:

$$\vartheta_R = \beta(r-s)\phi(z_R)\left[\sqrt{N(\sigma_D^2 + \sigma_X^2)} - \sigma_D\sqrt{N}\right]$$

and

$$\vartheta_S = \left[\sqrt{N(\sigma_D^2 + \sigma_X^2)} - \sigma_D\sqrt{N}\right]\left[(m-w)z_R + (1-\beta)(r-s)\varphi(z_R)\right]$$

Just like the previous model, for the given conditions, the number of warehouses and the variance of inventory inaccuracy have positive effects and the variance of demand has a negative effect on supply chain coordination.

Buyback Contract

Under a buyback contract, the expected profit function of the retailer is:

$$E\left(\Pi_R\right) = (r-w)N\mu_D - (r-b)\phi(z_R)\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2} - \theta\left[K_{\{n>0\}} + kn\right]$$

Under a buyback contract, the expected profit of the supplier is:

$$E\left(\Pi_S\right) = (w-m)N\mu_D - b\phi(z_R)\sqrt{N\sigma_D^2 + (N-n)\sigma_X^2} - (1-\theta)\left[K_{\{n>0\}} + kn\right]$$

For coordination:

$$\frac{r-m}{r-s} = \frac{r-w}{r-b} \text{ and } b = \frac{r(w-m)+s(r-w)}{r-m}$$

and

$$\frac{\beta r - \beta m}{\beta r - \beta s} = \frac{r-w}{r-b},\ \beta = \theta,\ \theta = \frac{r-b}{r-s} \text{ or } \theta = \frac{r-w}{r-m}$$

The coordination conditions proposed in Section 4.1.3 for the revenue sharing contract are valid for the buyback contract as well.

4.3 Computations and Discussion of the Results

In this section, the findings of our study are shown by numerical examples. Figures represent the results for both IS and NIS scenarios. As the base case in the numerical examples, we used the following parameters: $\mu_D = 100$, $\sigma_D = 30$, $\mu_X = -2$, $\sigma_X = 10$, $r = 10$, $w = 5$, $m = 3$, $s = 2$ and $N = 10$. (The variable investment cost, k changes for every example, since the profit improvements are compared. A k value which results in an investment decision is chosen.)

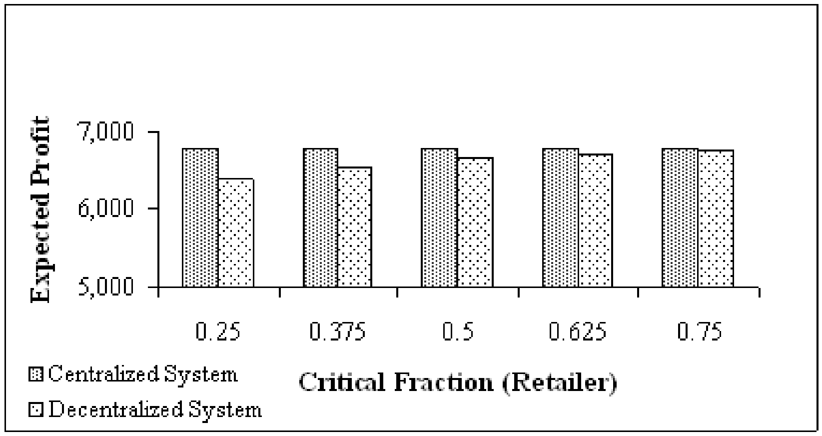

Figure 4.3: The effect of investment decision on decentralization under the IS scenario.

In Figures 4.3 and 4.4, the x axis corresponds to the critical fraction α and the y axis corresponds to the expected profit when the investment is made. As stated before α is the critical fraction and is equal to $(r-m)/(r-s)$ for the centralized system and $(r-w)/(r-s)$ for the decentralized system. If the critical fraction is low, it can be interpreted as a low profit margin and vice versa. The results are given for the centralized and decentralized systems under NIS and IS scenarios. It is known that the centralized system performs better than the decentralized system, optimizing entire system is better than optimizing its parts independently.

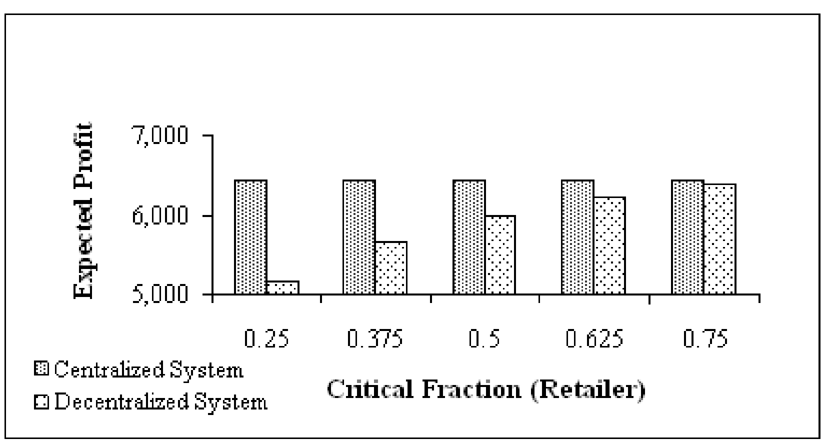

Figure 4.4: The effect of investment decision on decentralization under the NIS scenario.

In Figures 4.3 and 4.4 it is observed that decentralization penalty which is defined as the difference between the expected profits of the centralized system and decentralized system can be severe when the critical fraction is low for the retailer (see for examples $\alpha_R = 0.25$ and 0.375). This is the well-known effect of double marginalization. The decentralization penalties of the NIS scenario are worse than the ones of the IS scenario. This is due to the fact that the IS scenario performs better than the NIS scenario. Under both scenarios, the retailer may not make any investment when her profit margin is low, even though it is optimal for the centralized system (when $\mu_X = 0$, see Proposition 2). When $\mu_X < 0$, the retailer has a higher variable investment threshold than the centralized system (see Equations 4.4 and 4.6 for IS scenario, $w > m$). This is logical since the problem directly affects the retailer.

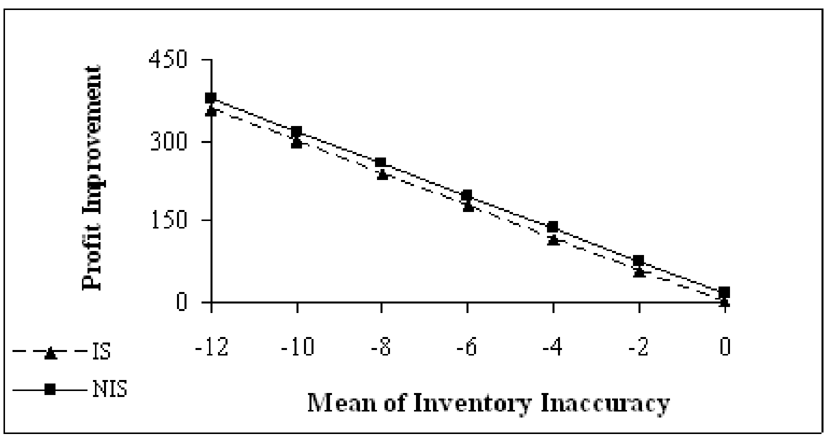

Figure 4.5: The effect of the mean inventory inaccuracy on investment decision.

In Figure 4.5, the effects of the mean and in Figure 4.6 the effects of the standard deviation of the inventory inaccuracy on investment decision are investigated. In the figures, the y axis corresponds to the profit improvement which is the difference of the expected profits of the systems that make a full investment and does not make any investment. It is observed that as the standard deviation of the inventory inaccuracy increases and mean of the inventory inaccuracy decreases, the system (the centralized system and the retailer) benefits more from making an investment.

If the variance of demand is high for a company, trying to decrease the variance of inventory inaccuracy may be meaningless; since the profit improvement may not compensate

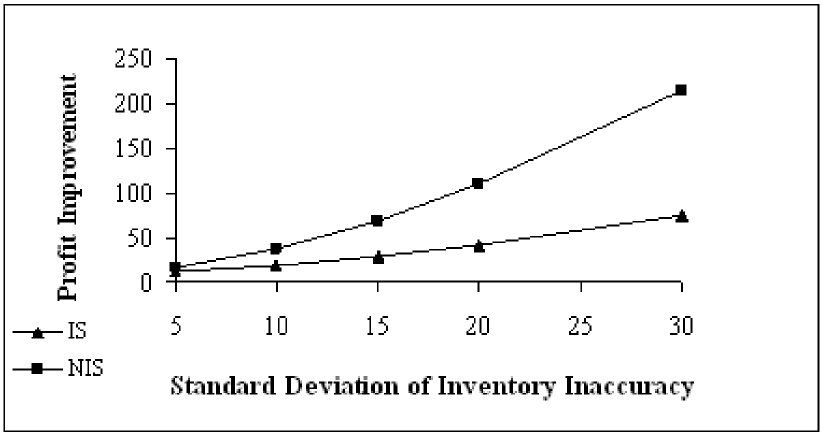

Figure 4.6: The effect of the standard deviation of the inventory inaccuracy on investment decision.

the fixed investment cost. It is easier to make an investment if the standard deviation of demand is low (see Corollaries 1 and 2). Figures 4.7 and 4.8 depict that property with the numerical examples for the NIS and the IS scenarios. As the standard deviation of demand increases, the profit improvement decreases (for the centralized system and the retailer). The effect of demand variation on the investment decision decreases if the mean of the inventory inaccuracy is negative, since under this situation the inventory inaccuracy problem is more critical.

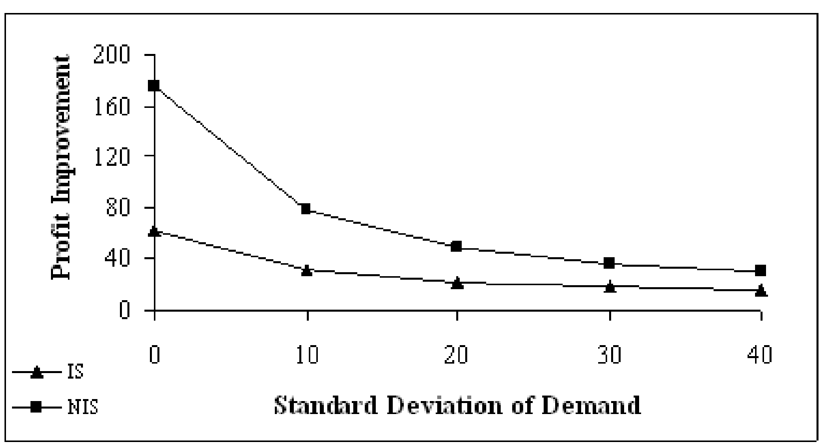

Figure 4.7: The effect of standard deviation of demand on investment decision ($\mu_X < 0$).

In summary, several intuitive results are obtained by our model. One of those is related

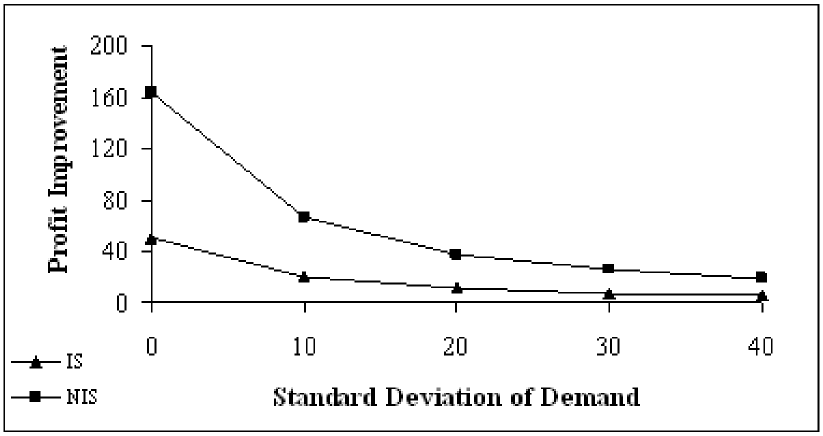

Figure 4.8: The effect of standard deviation of demand on investment decision ($\mu_X = 0$).

to the investment decisions of the retailer and the supplier. It is argued that the retailer makes an investment when the mean of inventory inaccuracy, μ_X is negative and the supplier makes an investment if μ_X is positive. (see equations (4.1), (4.2), (4.3), (4.4), (4.5) and (4.6)).

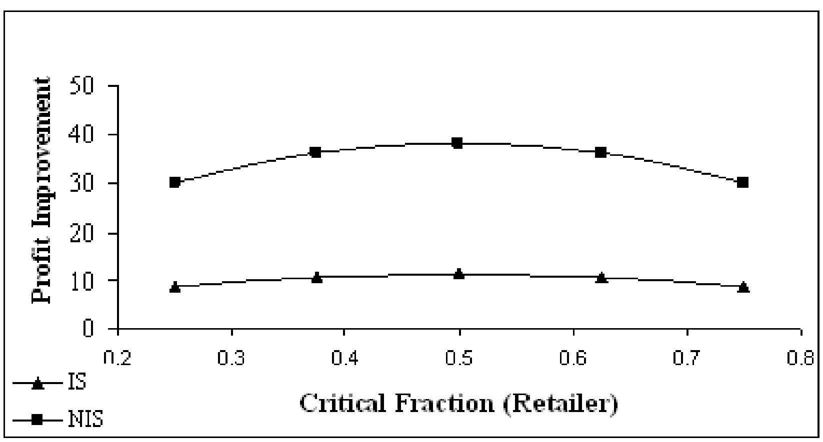

Figure 4.9: The profit improvements for the retailer ($\mu_x = 0$ and the retailer makes the investment decision and pays for the investment).

These findings are not surprising, since if there is continuous shrinkage, the retailer wants to eliminate inventory inaccuracy. The same result is relevant for the centralized system. The centralized system behaves rationally, since having a negative mean of inven-

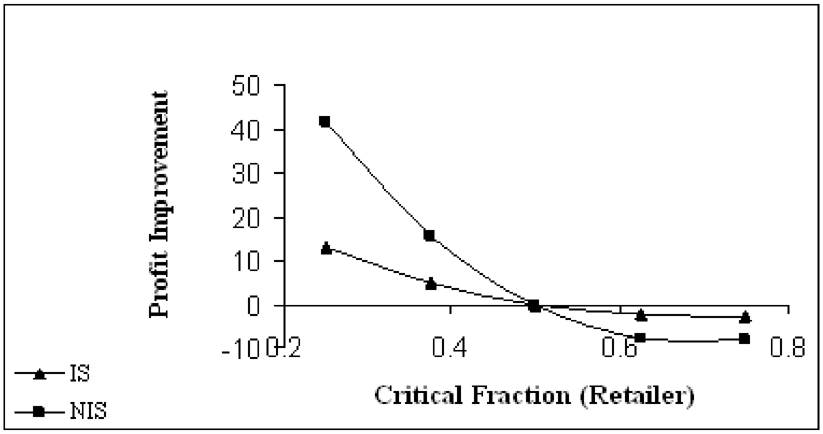

Figure 4.10: The profit improvements for the supplier ($\mu_x = 0$ and the retailer makes the investment decision and pays for the investment).

tory inaccuracy is more critical when the whole system is considered, it wants to make an investment in this situation, as well. In contrast, if the retailer always orders less since she finds extra items, the supplier wants to eliminate this situation, so the supplier wants to make an investment when the mean of the inventory inaccuracy is positive.

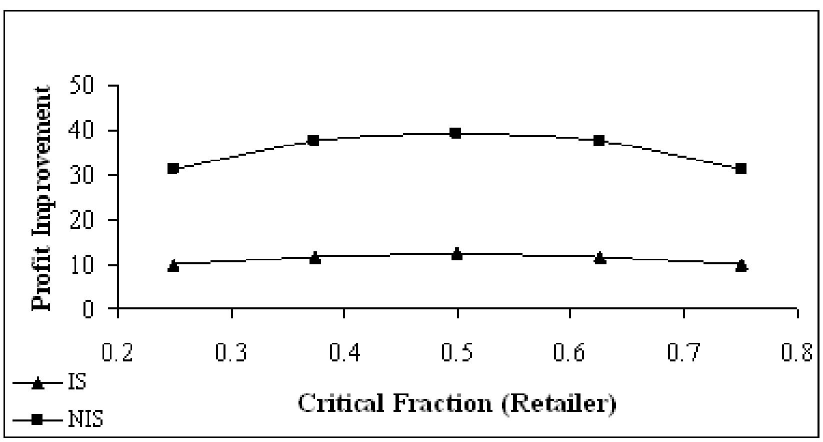

Figure 4.11: The profit improvements for the retailer($\mu_x = 0$ and the supplier makes the investment decision and pays for the investment).

When an investment decision is considered, one of the major questions is: Who benefits more from making an investment? Depending on the above result, the supplier does not benefit from using the technology if the retailer makes an investment, since the retailer

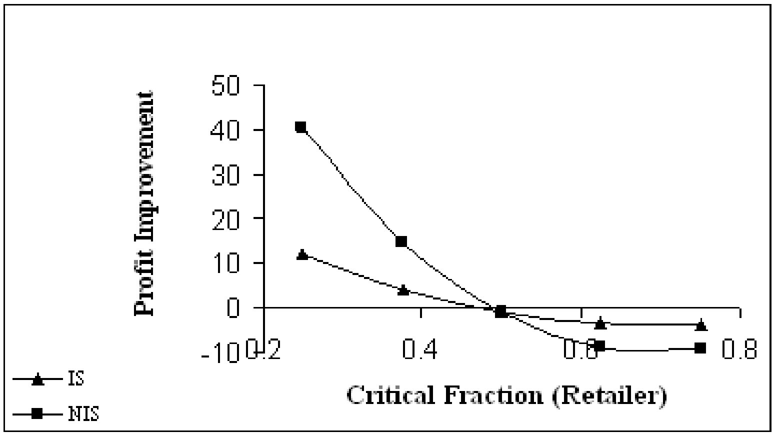

Figure 4.12: The profit improvements for the supplier ($\mu_x = 0$ and the supplier makes the investment decision and pays for the investment).

makes an investment when the mean of inventory inaccuracy is negative and this causes to decrease the profit of the supplier. Similarly, the retailer does not benefit from the investment when the supplier wants to make it (when $\mu_X > 0$).

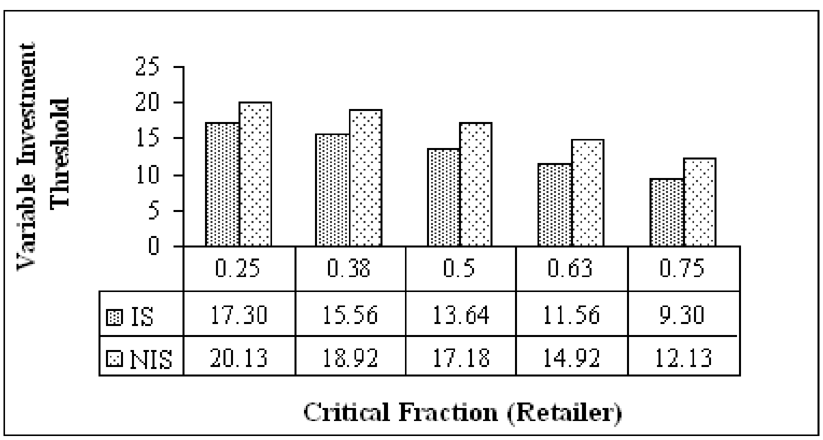

Figure 4.13: Comparison of the variable investment thresholds of the retailer under IS and NIS scenarios, $\mu_X < 0$.

The benefits of the retailer and the supplier can be compared when the mean of the inventory inaccuracy is 0. When the retailer pays high wholesale prices, the profit improvements for the supplier are higher. Figures 4.9, 4.10 (the retailer makes the investment) and 4.11 and 4.12 (the supplier makes the investment) clarify this structure by numerical

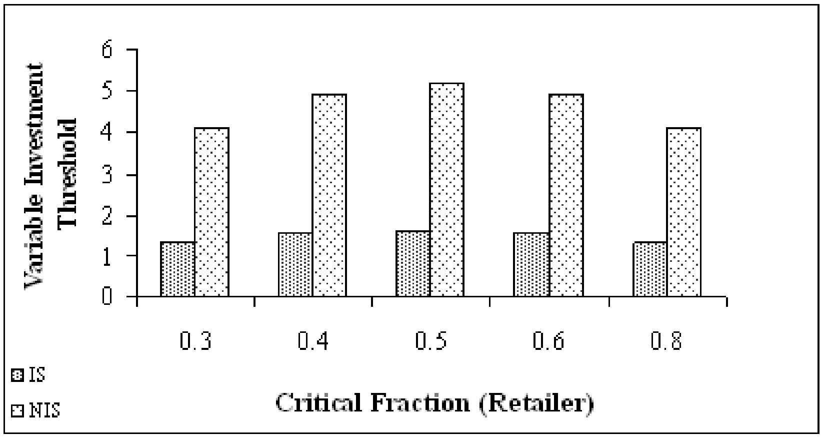

Figure 4.14: Comparison of the variable investment thresholds of the retailer under IS and NIS scenarios, $\mu_X = 0$.

examples. If the profit margin of the retailer is low and the products are really valuable, then making an investment is important. The profit improvement follows a pattern like the standard normal distribution function for the retailer. In this situation, either too low or too high profit margins result in low profit improvements for the retailer. If the supplier does not have a high profit margin, he does not want to make an investment. For examples see $z_R = 0.5$, 0.6 and 0.7 in Figures 4.11 and 4.12, in those cases the supplier does not want to make an investment.

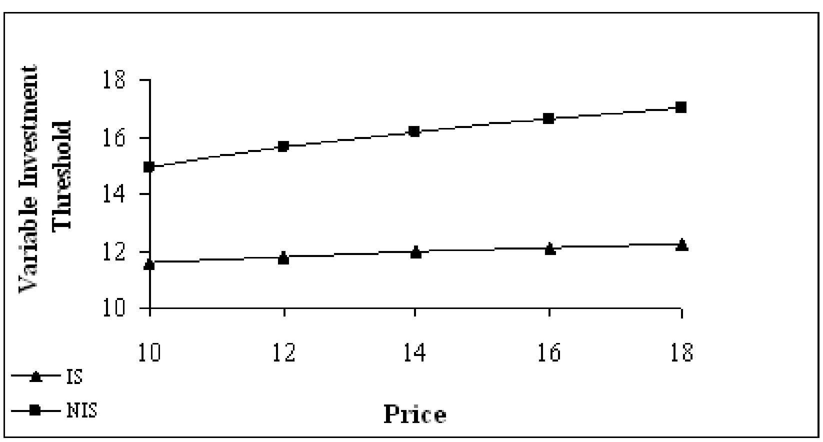

Figure 4.15: The effect of price, r on the variable investment threshold, $\mu_X < 0$.

In Figures 4.13 and 4.14, the variable investment thresholds of the retailer are depicted.

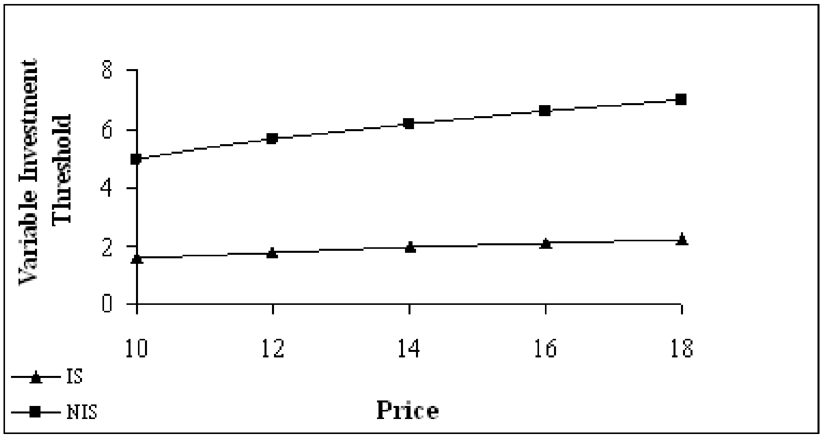

Figure 4.16: The effect of price, r on the variable investment threshold, $\mu_X = 0$.

The retailer has a higher variable investment threshold when there is no inventory sharing and the mean of the inventory inaccuracy is negative. The same result is relevant for the centralized system. If the profit margin is either too low or too high, the variable investment threshold is low both for the centralized system and the retailer (when $\mu_X = 0$).

As we remarked in Sections 4.1.1 and 4.2.1, the variable investment threshold increases in price, r. In Figures 4.15 and 4.16, this property is observed. This result is stated in a number of studies [24].

Finally, a general conclusion from our numerical studies and analytical model is that if there is no inventory sharing between the warehouses, making an investment to decrease inventory inaccuracy is more valuable. Also the following results are observed:

- The optimum solution in our model for the centralized system and the decentralized systems under different scenarios is either making no investment or full investment.

- The decentralized system may be worse of if the profit margin of the retailer is low.

- The supplier does not want to make an investment if his profit margin is low.

- The centralized system and the retailer wants to make an investment if the mean of the inventory inaccuracy is negative whereas the supplier wants to make an investment if the mean of the inventory inaccuracy is positive.

- Unnecessary investment may be made if retailers margin is high.
- Making an investment is justified when:
  - The variable and the fixed investment costs are low,
  - The variance of demand is low,
  - The variance of the inventory inaccuracy is high and the mean of the inventory inaccuracy is low,
  - The number of warehouses is small (under the IS scenario),
  - The price of the product is high.

In the following chapter several extensions of our model are considered and the results of relaxing some assumptions are represented.

Chapter 5

EXTENSIONS

In this section, we consider some extensions of the model introduced in Chapter 4. We try to observe the effects of relaxing some assumptions. In Section 5.1, we analyze the case where the system parameters are asymmetric for every warehouse so that partial investment decisions can be optimal. In Section 5.2, there are two parts. In the first part, we allow the demands to be correlated and in the second part we analyze the situation where the inventory inaccuracies are correlated. A different approach for the inventory inaccuracy correlation is considered in Section 5.3. In Section 5.4 imperfect RFID implementation is analyzed. In the last section, the multi product case is considered. Each section is analyzed both for NIS and IS scenarios. The analysis is performed by considering the centralized case, but the results are relevant for the decentralized case as well (the retailer makes the investment decision.). Throughout this chapter, the fixed investment cost is not considered since it is easy to take into account and μ_X is assumed to be zero to facilitate the comparison.

5.1 Asymmetric Parameters

NIS Scenario

The statistical and financial parameters for each warehouse were identical in the model in Chapter 4. Here, we investigate the situation with non-identical parameters. Since non-identical warehouse parameters result in different investment thresholds, partial investment decisions can be optimal in the case of asymmetric parameters.

Let us relax the assumption of identical warehouse demands, inventory inaccuracies, prices, wholesale prices and production costs. Let μ_{D_i} and σ_{D_i} denote the mean and standard deviation of warehouse i's demand, σ_{X_i} denote the standard deviation of warehouse i's inventory inaccuracy, and r_i, w_i and m_i denote price, wholesale price and production cost for warehouse i, respectively. The variable investment cost threshold for warehouse i is k_i^T. Proposition 5 identifies the optimum investment level when the parameters are non-identical for the warehouses.

Proposition 5 *If the system parameters (μ_D, σ_D, σ_X, r, w, m and s) are not identical for the warehouses, the optimal policy is ordering the warehouses according to the thresholds k_1^T, k_2^T, ..., k_N^T in decreasing order and making the investment in the warehouses whose threshold values are greater than the actual variable investment cost, k.*

Proof. In our initial model, we make the investment in warehouses for which the variable investment threshold is greater than the actual variable investment cost (Invest if $k < k_C^T$ (without considering fixed investment cost), otherwise do not invest).

For warehouse i:

if $k < k_C^T$, then the expected profit function increases by making an investment,

if $k \geq k_C^T$, then the expected profit function decreases by making an investment.

If the warehouses are ordered in the decreasing order according to the their variable investment thresholds, there will be a warehouse where making an investment becomes non-profitable. The optimal decision is investing up to this warehouse.

□

Proposition 5 defines the characteristic of optimal investment decision in case of non-identical warehouse parameters. The optimal policy is making no investment or full investment if the parameters are identical, because the investment thresholds of every warehouse are equal. In the asymmetric parameters case, the optimal solution is again implementing the technology in the warehouses that have greater variable investment thresholds than the actual variable investment cost. However, in that case the investment thresholds are not equal. So, a partial investment decision can be optimal. It should be noted that nonidentical means of the warehouse demands do not affect the optimal decision.

In case of non-identical parameters, the decision in which warehouse to implement the technology is affected in the following ways:

1. If demands for the warehouses are non-identical, the optimal policy is implementing the technology at warehouse for which the variance of the demand is the smallest,

2. If the inventory inaccuracies of warehouses are not identical, the optimal policy is implementing the technology in the warehouse for which the variance of inventory inaccuracy is the highest.

The above results are obtained using Corollaries 1 and 2.

IS Scenario

In the IS scenario, the ordering decision of a warehouse affects the other warehouses since the inventory is shared. So, when deciding on investment, all investment alternatives must be considered.

Throughout this section, only the asymmetry of the parameters σ_X and k is considered to simplify the analysis. If r, w and m were asymmetric for the warehouses, the expected profit function could not be written in the form that we used in the previous sections. In that case, some other problems occur such as how to share the items that have different prices (Should we sell the item which has the highest price first, then how should we share the items?). Throughout the analysis it is observed that the asymmetry of the mean μ_D and the standard deviation σ_D of demand do not have an effect on the investment decision.

In this section, we perform a marginal analysis to find out the optimal investment decision. We use supermodularity and complementarity concepts of economics. The complementarity suggests that having more of one variable increases the marginal returns to having more of the other variable. A function $f : E_i \times E_j \rightarrow R$ has strictly increasing differences in (e_i, e_j) if the following inequality holds (Amir [1]).

$$f(y + e_i + e_j) - f(y + e_i) \geq f(y + e_j) - f(y) \quad \forall y, e_i, e_j \in R \tag{5.1}$$

Let us define $\Pi_C(\cdot)$ as the expected profit of the centralized system, $\mathbf{y}$ as a vector representing the warehouses where the RFID is implemented, e_i and e_j as the unit vectors showing that the RFID technology is implemented in warehouse i and j. Please note that $\mathbf{y}$ is a vector of 0,1 in R^N and e_i and e_j are unit vectors in R^N. In Proposition 6, the inequality (5.1) is shown to be true for our model under some conditions.

Proposition 6

$$\Pi_C(\boldsymbol{y} + e_i + e_j) - \Pi_C(\boldsymbol{y} + e_i) \geq \Pi_C(\boldsymbol{y} + e_j) - \Pi_C(\boldsymbol{y}) \quad \forall \boldsymbol{y}, e_i, e_j \in R^N \tag{5.2}$$

Proof. Let us denote I as the set of warehouses where RFID is implemented and NI as the set of warehouses where RFID is not implemented and select two warehouses i and j such that $i, j \in NI$. Then, $\Pi_C(\mathbf{y} + e_i + e_j)$ is the expected profit when the technology is implemented in warehouses that belong to set I and in warehouses i and j, $\Pi_C(\mathbf{y} + e_i)$ and $\Pi_C(\mathbf{y} + e_j)$ are the expected profits when the technology is implemented in warehouses which belong to set I and additionally warehouse i and warehouse j respectively and $\Pi_C(\mathbf{y})$ is the expected profit when no additional investment is made (RFID is implemented in warehouses that belong to I).

To show that the inequality holds, we write the expected profits explicitly.

$$
\begin{aligned}
&(r-m)(\sum_{q=1}^{N} \mu_{D_q}) - (r-s)\phi(z_C)\sqrt{\sum_{q=1}^{N} \sigma^2_{D_q} + \sum_{p \in NI, p \neq i,j} \sigma^2_{X_p}} - k_i - k_j \\
&-(r-m)(\sum_{q=1}^{N} \mu_{D_q}) + (r-s)\phi(z_C)\sqrt{\sum_{q=1}^{N} \sigma^2_{D_q} + \sum_{p \in NI, p \neq i,j} \sigma^2_{X_p} + \sigma^2_{X_j}} + k_i \\
&\geq (r-m)(\sum_{q=1}^{N} \mu_{D_q}) - (r-s)\phi(z_C)\sqrt{\sum_{q=1}^{N} \sigma^2_{D_q} + \sum_{p \in NI, p \neq i,j} \sigma^2_{X_p} + \sigma^2_{X_i}} - k_j \\
&-(r-m)(\sum_{q=1}^{N} \mu_{D_q}) + (r-s)\phi(z_C)\sqrt{\sum_{q=1}^{N} \sigma^2_{D_q} + \sum_{p \in NI, p \neq i,j} \sigma^2_{X_p} + \sigma^2_{X_i} + \sigma^2_{X_j}}
\end{aligned}
$$

By simplifying the above inequality, we obtain:

$$
\begin{aligned}
&\sqrt{\sum_{q=1}^{N} \sigma^2_{D_q} + \sum_{p \in NI, p \neq i,j} \sigma^2_{X_p} + \sigma^2_{X_j}} - \sqrt{\sum_{q=1}^{N} \sigma^2_{D_q} + \sum_{p \in NI, p \neq i,j} \sigma^2_{X_p}} \\
&\geq \sqrt{\sum_{q=1}^{N} \sigma^2_{D_q} + \sum_{p \in NI, p \neq i,j} \sigma^2_{X_p} + \sigma^2_{X_i} + \sigma^2_{X_j}} - \sqrt{\sum_{q=1}^{N} \sigma^2_{D_q} + \sum_{p \in NI, p \neq i,j} \sigma^2_{X_p} + \sigma^2_{X_i}}
\end{aligned}
$$

In the above form it is observed that the asymmetry of μ_D and σ_D do not affect the inequality. Multiplying and dividing the both sides by their conjugates gives the following form:

$$
\frac{\sigma^2_{X_j}}{\sqrt{\sum_{q=1}^{N} \sigma^2_{D_q} + \sum_{p \in NI, p \neq i,j} \sigma^2_{X_p} + \sigma^2_{X_j}} + \sqrt{\sum_{q=1}^{N} \sigma^2_{D_q} + \sum_{p \in NI, p \neq i,j} \sigma^2_{X_p}}}
$$

$$\geq \frac{\sigma^2_{X_j}}{\sqrt{\sum_{q=1}^{N}\sigma^2_{D_q} + \sum_{p\in NI, p\neq i,j}\sigma^2_{X_p} + \sigma^2_{X_i} + \sigma^2_{X_j}} + \sqrt{\sum_{q=1}^{N}\sigma^2_{D_q} + \sum_{p\in NI, p\neq i,j}\sigma^2_{X_p} + \sigma^2_{X_i}}}$$

Looking at the above form of the inequality, we see that the inequality holds, since the denominator of the left part is less than the denominator of the right part.

□

Proposition 6 establishes that investing in warehouse j is more profitable when an investment is made at warehouse i. In that case, when all the pairs are compared, the optimum solution is making the full investment without considering the fixed investment cost.

At first sight, Proposition 6 seems to suggest that implementing the technology at all warehouses is optimal, since implementation has a complementarity property. However, in the situations where σ_X is higher for a warehouse relatively and/or the variable investment costs for the warehouses are high, the profit differences should be investigated carefully. In those situations, partial investment or no investment decisions may be optimal, since the increases in profits can be negative.

Corollary 3 *If (1) σ_{X_i} is strictly higher than σ_{X_j} and (2) k_i and k_j are too large (for any given i and j), the optimal policy may not be making the full investment.*

Proof. The values of the parameters may affect the optimal solution, since the inequality (5.2) may hold since investing at a warehouse may not be profitable in some cases. Not to ignore those cases, the expected profit increases should be investigated carefully. To show that the optimum investment decision can be affected by the relative values of σ_X and k, let us examine the following inequality which is equal to the inequality (5.2).

$$\Pi_C(\mathbf{y}+e_i+e_j)-\Pi_C(\mathbf{y}) \geq \Pi_C(\mathbf{y}+e_i)-\Pi_C(\mathbf{y})+\Pi_C(\mathbf{y}+e_j)-\Pi_C(\mathbf{y}) \quad \forall \mathbf{y}, e_i, e_j \in R^N \quad (5.3)$$

In this inequality, the profit improvement when full investment is made is compared with the profit improvements of making individual investments. It is argued that the inequality (5.3) holds since it is equivalent to the inequality (5.2). However, by looking at this inequality, it is realized that making an investment may decrease the expected profit of warehouse i due to high variable investment cost. Although the equation holds, making the investment in two warehouses may not be optimal. Relative values of inventory inaccuracy variances may affect the inequality in the same way. As a result, it is certain that if all the expected profit increases are positive, the optimal investment decision is still making the full investment. Otherwise, profit improvements should be checked for each pair.

$\square$

For examples to the Corollary 3, the following four cases which result in different solutions can be outlined:

- $N = 2$, $\sigma_{X_1} = 20$, $\sigma_{X_2} = 10$, $\sigma_{D_1} = 10$, $\sigma_{D_2} = 10$, $r = 5$, $m = 3$, $s = 0$ and $k_1 = k_2 = 10$. The difference between no investment and full investment cases is $\Pi_C(\mathbf{y} + e_1 + e_2) - \Pi_C(\mathbf{y}) = 3.8$ and investing in the first warehouse increases the profit by $\Pi_C(\mathbf{y} + e_1) - \Pi_C(\mathbf{y}) = 7.7$ and investing in the second warehouse increases the profit by $\Pi_C(\mathbf{y} + e_2) - \Pi_C(\mathbf{y}) = -6.2$. Although it seems like investing in two warehouses increases the expected profit much ($3.8 > 7.7 - 6.2 = 1.5$), it should be noticed that in such a case investing in only warehouse 1 is the optimal decision. In this case, the inventory inaccuracy variance of warehouse 1 is higher than the the inventory inaccuracy variance of warehouse 2.

- $N = 2$, $\sigma_{X_1} = 20$, $\sigma_{X_2} = 14$, $\sigma_{D_1} = 10$, $\sigma_{D_2} = 10$, $r = 5$, $m = 3$, $s = 0$ and $k_1 = k_2 = 20$. $\Pi_C(\mathbf{y} + e_1 + e_2) - \Pi_C(\mathbf{y}) = -12.8$, $\Pi_C(\mathbf{y} + e_1) - \Pi_C(\mathbf{y}) = -3.9$ and $\Pi_C(\mathbf{y} + e_2) - \Pi_C(\mathbf{y}) = -12.8$. As it is seen, investing in the warehouses decreases the expected profit (the variable investment costs are too large), the optimal decision is making no investment. In this case, the variable investment costs are large.

- $N = 2$, $\sigma_{X_1} = 20$, $\sigma_{X_2} = 20$, $\sigma_{D_1} = 10$, $\sigma_{D_2} = 10$, $r = 5$, $m = 3$, $s = 0$ and $k_1 = 5, k_2 = 15$. $\Pi_C(\mathbf{y} + e_1 + e_2) - \Pi_C(\mathbf{y}) = 13.8$, $\Pi_C(\mathbf{y} + e_1) - \Pi_C(\mathbf{y}) = 8.8$ and $\Pi_C(\mathbf{y} + e_2) - \Pi_C(\mathbf{y}) = -1.2$. Investing in warehouse 1 is profitable since the variable investment cost is low for warehouse 1. Although investing in warehouse 2 is not profitable, the optimal decision is still making the full investment.

- $N = 2$, $\sigma_{X_1} = 30$, $\sigma_{X_2} = 20$, $\sigma_{D_1} = 10$, $\sigma_{D_2} = 10$, $r = 5$, $m = 3$, $s = 0$ and $k_1 = k_2 = 10$. $\Pi_C(\mathbf{y} + e_1 + e_2) - \Pi_C(\mathbf{y}) = 27.5$, $\Pi_C(\mathbf{y} + e_1) - \Pi_C(\mathbf{y}) = 17.5$ and

$\Pi_C(\mathbf{y}+e_2) - \Pi_C(\mathbf{y}) = 0.75$, the optimal decision is making the full investment.

5.2 Demand/Inventory Inaccuracy Correlation

In our model, demands and inventory inaccuracies for the warehouses are not correlated. This section shows that our results obtained for independent demands and independent inventory inaccuracies may not be optimal for correlated demands and inventory inaccuracies.

There are two scenarios considering correlation. Firstly, the demands for the warehouses can be correlated. Secondly, the inaccuracies of the warehouses can be correlated. Those two scenarios will be analyzed in the following two subsections.

5.2.1 Demand Correlation

NIS Scenario

Let us relax the assumption of independent demands. For the NIS scenario, the demand correlation does not have an effect on the system. Under the NIS scenario, each warehouse is considered separately and the demand correlation does not change the ordering decisions and expected profits. Since inventories of the warehouses are separate, the ordering decision is based on the marginal demand distributions.

IS Scenario

The demand for warehouse i is normally distributed with mean μ_D and variance σ_D^2 and demands for any two warehouses are correlated with correlation coefficient ρ_D, $-1/(N-1) < \rho_D < 1$[1]. By recalling that:

$$Var\left(\sum_{i=1}^{N} D_i\right) = \sum_{i=1}^{N} \sigma_{D_i} + 2\sum_{i=1}^{N}\sum_{j<i} Cov(D_i, D_j)$$

and

$$Cov(D_i, D_j) = \rho_D \times \sigma_{D_i} \times \sigma_{D_j}$$

the total demand variance is $\tau_D^2 = [N + N(N-1)\rho_D]\sigma_D^2$ and total variance affecting the system is $\tau_D^2 + N\sigma_X^2$.

Depending on the above structure, the optimal total order quantity of the centralized system is:

[1]To ensure that the total demand variance is positive (Zhu and Thonemann [49]).

$$Q^* = N\mu_D + z_C\sqrt{(N + N(N-1)\rho_D)\sigma_D^2 + (N-n)\sigma_X^2} \tag{5.4}$$

If the demands for warehouses are correlated, the expected profit function of the centralized system is:

$$E\left(\Pi_C\right) = (r-m)N\mu_D - (r-s)\phi(z_C)\sqrt{(N + N(N-1)\rho_D)\sigma_D^2 + (N-n)\sigma_X^2} - K_{\{n>0\}} - kn$$

It is seen in equation (5.4) that when z_C is positive (underage cost is higher than overage cost), as the demand correlation increases the centralized system's total order quantity increases. When z_C is negative (underage cost is lower than overage cost), as the demand correlation increases the centralized system's total order quantity decreases. On the other hand, the expected profit function decreases in ρ_D and the negative correlation is beneficial for the system, since the system has the ability to share the inventories between warehouses.

As it is seen, the demand correlation between the warehouses does not affect the structure of the expected profit function. The expected profit function is still convex in n, since:

$$\frac{\partial E(\Pi_C)^2}{\partial^2 n} = \frac{(r-s)\phi(z_C)\sigma_X^4}{4\ \sqrt[3/2]{(N + N(N-1)\rho_D)\sigma_D^2 + (N-n)\sigma_X^2}} \geq 0$$

The optimal solution is still making the full investment or no investment. However, the variable investment threshold is affected by the correlation of demands. The variable investment threshold value when demands are correlated is:

$$\bar{k}_C^T = \frac{(r-s)\phi(z_C)\left[\sqrt{(N + N(N-1)\rho_D)\sigma_D^2 + N\sigma_X^2} - \sqrt{(N + N(N-1)\rho_D)\sigma_D^2}\right]}{N}$$

The variable investment threshold is decreasing in ρ_D, it is verified by taking its first derivative.

$$\frac{\partial \bar{k}_C^T}{\partial \rho_D} = \frac{1}{2}\phi(z_C)(N-1)(r-s)\sigma_D^2\left(\frac{1}{\sqrt{(N + N(N-1)\rho_D)\sigma_D^2 + N\sigma_X^2}} - \frac{1}{\sqrt{(N + N(N-1)\rho_D)\sigma_D^2}}\right) \leq 0$$

As the demand correlation increases, the effective demand variance also increases. Since an increase in the demand variance decreases the variable investment threshold, an increase in the demand correlation has the same effect on the variable investment threshold.

5.2.2 Inventory Inaccuracy Correlation

NIS Scenario

If RFID enables having information about the inventory inaccuracy and its reasons, this data can be used to decrease the inventory inaccuracies of the other warehouses. However, in our setting, RFID eliminates the inventory inaccuracy. So, the correlation between the warehouses cannot be used to obtain information on inventory discrepancy even if the inventory inaccuracies of the warehouses are dependent.

For the NIS scenario, the inventory inaccuracy correlation does not have an effect on the system. Under the NIS scenario, each warehouse is considered separately and the inventory inaccuracy correlation does not change the ordering decisions and expected profits.

IS Scenario

It is presumed that the inventory inaccuracy of the warehouse i, X_i is normally distributed with mean 0 and variance σ_X^2. Inventory inaccuracies are correlated with correlation coefficient ρ_I and $-1/(N-1) < \rho_I < 1$. The retailer has the ability to eliminate the inventory inaccuracy of a warehouse by paying variable investment cost k. The variance of total inventory inaccuracy is $\tau_I^2 = [N + N(N-1)\rho_I]\sigma_X^2$ and the total variance affecting the system is $\tau_I^2 + N\sigma_D^2$.

Depending on the above structure, the optimal total order quantity of the centralized system is:

$$Q^* = N\mu_D + z_C\sqrt{N\sigma_D^2 + ((N-n) + (N-n)(N-n-1)\rho_I)\sigma_X^2} \tag{5.5}$$

If the inventory inaccuracies for the warehouses are correlated, the expected profit function of the centralized system is:

$$E(\Pi_C) = (r-m)N\mu_D - (r-s)\phi(z_C)\sqrt{N\sigma_D^2 + ((N-n) + (N-n)(N-n-1)\rho_I)\sigma_X^2} - K_{\{n>0\}} - kn$$

It is seen in equation (5.5) that when underage cost is higher than overage cost, as the correlation increases the centralized system's total order quantity increases (If investment is not made in all warehouses, ($n \neq N$)). When underage cost is lower than overage cost, as the correlation increases the centralized system's total order quantity decreases. In contrast, the expected profit function decreases in ρ_D, since the negative correlation is beneficial for the system.

The first derivative of the expected profit function with respect to n is:

$$\frac{\partial E(\Pi_C)}{\partial n} = -\frac{\phi(z_C)(\rho_I - 1 + 2\rho_I(n-N))(r-s)\sigma_X^2}{2\sqrt{N\sigma_D^2 + (n-N)(-1+(1+n-N)\rho_I)\sigma_X^2}} - k$$

Note that to have an increasing function, $(\rho_I - 1 + 2\rho_I(n-N))$ must be negative. If $\rho_I \geq 0$, the function can be increasing depending on the value of k.

The second derivative of the expected profit function with respect to n is:

$$\frac{\partial E(\Pi_C)^2}{\partial^2 n} = \frac{(r-s)\phi(z_C)\sigma_X^2(-4N\rho_I\sigma_D^2 + (\rho_I - 1)^2\sigma_X^2)}{4\sqrt[3/2]{N\sigma_D^2 + (n-N)(-1+(1+n-N)\rho_I)\sigma_X^2}}$$

Let us check the sign of the second derivative of the expected profit function with respect to n. Note that $(-4N\rho_I\sigma_D^2 + (\rho_I - 1)^2\sigma_X^2)$ defines the sign of it. The sign of the derivative can be positive if:

$$\sigma_X^2 \geq \frac{4N\rho_I}{(\rho_I - 1)^2}\sigma_D^2$$

Note that the above equation holds for the very large σ_X. So, the expected profit function is convex for very large σ_X. Otherwise, it is concave. Since the inventory inaccuracy variance is not likely to be much higher than the demand variance, the expected profit function is concave for a reasonable situation.

The investment decision structure is depicted in Figure 5.1 by using the parameters: $r = 10$, $m = 5$, $s = 0$, $N = 10$, $\mu_D = 100$, $\sigma_D = 20$ and $\sigma_X = 10$.

Observed results in Figure 5.1 are:

- Making the full investment is optimal if the variable investment cost is small. For negative correlation examples, the variable investment cost is expected to be lower compared to positive correlation examples to justify the investment. The intuition is that negative correlation is beneficial for the system under the IS scenario.

- Making a partial investment is optimal if the variable investment cost is medium and greater than a threshold value. The threshold observed in the figure is equal to the variable investment threshold of the system without inventory inaccuracy correlation under the IS scenario.

  The intuition behind this observation is: if there is no correlation between the warehouses and the variable investment cost is less than a threshold value, the system

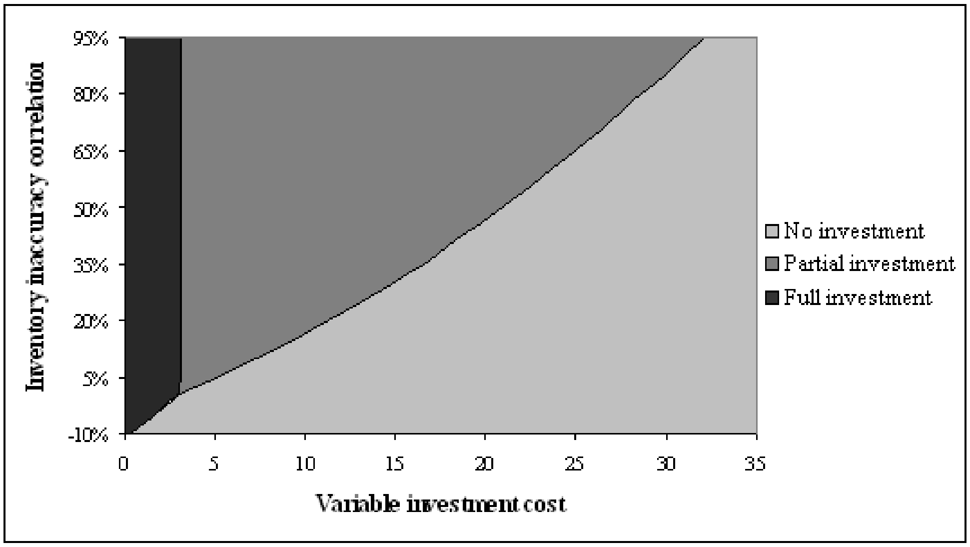

Figure 5.1: The effects of the inventory inaccuracy correlation, ρ_I and the variable investment cost, k on investment decision under IS scenario.

make an investment to eliminate the inventory inaccuracy in all warehouses. If there is correlation between the warehouses, the system still makes the full investment, since the variable investment cost is low enough even for no correlation situation. If the variable investment cost is higher than this threshold value, then the system makes a partial investment since there is a positive correlation between the warehouses. Positive correlation increases the effective variance in the system.

- Making no investment is optimal if the variable investment cost is high and the inventory inaccuracy correlation is low. Low inventory inaccuracy correlation is beneficial for the supply chain.

5.3 Inventory Inaccuracy Correlation - A different point of view

How can decreasing a warehouse's inventory inaccuracy help improving the others' inventory inaccuracies? If the RFID implementation can give insights about the processes, reasons of inventory inaccuracy, implementing the technology in a warehouse can be beneficial for the other warehouses as well. Consider the situation where RFID is implemented in a warehouse and the inventory inaccuracy is known with complete certainty without eliminating its reasons. For instance, let us assume that the inventory inaccuracy of a warehouse is -10 and it is known with certainty after the implementation. Since the inventory inaccuracies of the warehouses are correlated, this information can improve the inventory inaccuracy knowledge of the other warehouses. This setting is similar to the model of Zhu and Thonemann [49]. In their model, the demand uncertainty can be decreased by sharing information with customers. Different from their model, in our model uncertainty in the system cannot be eliminated, since there is demand uncertainty besides the uncertainty due to inventory inaccuracy.

In this section, we check the situation where inventory inaccuracies of the warehouses are correlated and when the above structure is considered and try to see if partial investment decisions are obtained as it is the case of Zhu and Thonemann [49].

It is presumed that the inventory inaccuracy of the warehouse i, X_i is normally distributed with mean 0 and variance σ_X^2. Inventory inaccuracies are correlated with correlation coefficient ρ_I and $-1/(N-1) < \rho_I < 1$. The exact inventory inaccuracy is known if RFID is implemented. The system decides on the order quantity and investment at the beginning of the period. After the RFID implementation, ω_i which is the inventory inaccuracy of the current period is known with complete certainty. The actual inventory inaccuracy after the implementation, X_i is normally distributed with mean 0 and variance 0. The retailer has the ability to eliminate the inventory inaccuracy of a warehouse by paying variable investment cost k. The variance of total inventory inaccuracy is $\tau_I^2 = [N + N(N-1)\rho_I]\sigma_X^2$ and the total variance affecting the system is $\tau_I^2 + N\sigma_D^2$.

Assume that the investment is made at n warehouses, so their inventory inaccuracies are known. To update the inventory inaccuracy distributions, the joint vector $(\sum_{i=1}^N \gamma_i X_i; \omega_1, ..., \omega_n)$, $\gamma_i \in 0, 1$ is found (Zhu and Thonemann [49] (Proposition 1 of the paper)).

Then, the results of Tong [47] are used to show the distribution of $(\sum_{i=1}^N \gamma_i X_i | \omega_1, ..., \omega_n)$ Based on the results of Zhu and Thonemann [49] (Proposition 2 of the paper) for a warehouse i where the technology is implemented, the inventory inaccuracy is:

$$X_i = 0 \text{ with probability 1. } (1 \leq i \leq n)$$

and for a warehouse i where the technology is not implemented, X_i is normally distributed:

$$X_i|(\omega_1, ..., \omega_n) \sim N\left(0, \left(1 - \frac{n\rho_I^2}{1+(n-1)\rho_I}\right)\sigma_X^2\right)\ (n+1 \leq i \leq N)$$

The variance of inventory inaccuracy where the technology is not implemented decreases as n and ρ_I increases. The effect of n is verified by taking the first derivative of the variance with respect to n:

$$\frac{(\rho_I - 1)\rho_I^2\sigma_X^2}{(1+(n-1)\rho_I)^2} \leq 0$$

In the variance formula, it is observed that the nominator of the fraction, $n\rho_I^2/(1+(n-1)\rho_I)$ grows faster as ρ_I increases. So, the variance decreases as ρ_I increases.

Therefore, the inventory accuracy of warehouses improves if the number of warehouses where the technology is implemented and the inventory inaccuracy correlation increase.

It should be noted that inventory accuracy improves in two ways: (1) eliminating the inventory inaccuracy of the warehouses where the technology is implemented and (2) decreasing the inventory inaccuracies of the warehouses where the technology is not implemented due to correlation of inventory inaccuracies.

NIS Scenario

Under the no inventory sharing and inventory inaccuracy correlation scenario, the optimal total order quantity is:

$$Q^*|(\omega_1, ..., \omega_n) = N\mu_D + z_C\left[n\sigma_D + (N-n)\sqrt{1 - \frac{n\rho_I^2}{1+(n-1)\rho_I}\sigma_X^2 + \sigma_D^2}\right]$$

and the expected profit function of the centralized system is:

$$E(\Pi_C) = (r-m)N\mu_D - (r-s)\phi(z_C)\left[n\sigma_D + (N-n)\sqrt{1 - \frac{n\rho_I^2}{1+(n-1)\rho_I}\sigma_X^2 + \sigma_D^2}\right] - K_{\{n>0\}} - kn$$

The optimal number of warehouses where the technology is implemented can be determined by maximizing the expected profit or by minimizing the expected cost. The expected cost function is:

$$E(Cost_C) = (r-s)\phi(z_C)\left[n\sigma_D + (N-n)\sqrt{1 - \frac{n\rho_I^2}{1+(n-1)\rho_I}\sigma_X^2 + \sigma_D^2}\right] + K_{\{n>0\}} + kn$$

The second derivative of the expected cost function with respect to n is:

$$\frac{\partial E\left(Cost_C\right)^2}{\partial^2 n} = \left(\frac{\phi(z_C)(\rho_I - 1)\rho_I^2(r-s)\sigma_X^2}{4 + (1+(n-1)\rho_I)^3((-1+\rho_I - n\rho_I)\sigma_D^2 + (\rho_I - 1)(1+n\rho_I)\sigma_X^2)\sqrt{\sigma_D^2 + \left(1 + \frac{n\rho_I^2}{\rho_I - 1 - n\rho_I}\right)\sigma_X^2}} \right)$$
$$\times(4(1+(n-1)\rho_I)(1+(N-1)\rho_I)\sigma_D^2 - (\rho_I - 1)(4 + 4(-1+n+N)\rho_I + (-3n - N + 4Nn)\rho_I^2)\sigma_X^2)$$

The convexity or concavity of the expected profit function cannot be verified.

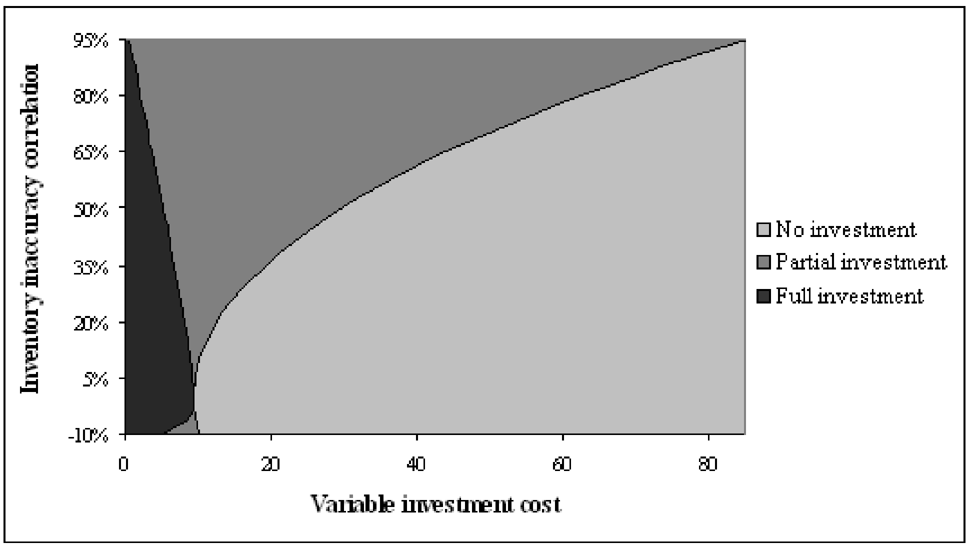

Figure 5.2: The effects of the inventory inaccuracy correlation, ρ_I and the variable investment cost, k on investment decision under NIS scenario.

We analyze how the optimal investment decision is affected by the variable investment cost k and inventory inaccuracy correlation ρ_I. Closed form expressions cannot be determined for the threshold values of k and ρ_I. So, numerical examples are performed and the results are summarized in Figure 5.2. The parameters used for calculations are: $r = 10$, $m = 5$, $s = 0$, $N = 10$, $\mu_D = 100$, $\sigma_D = 20$ and $\sigma_X = 10$.

The following results are observed in Figure 5.2.

- Making the full investment is optimal if the variable investment cost is small. For large values of ρ_I, the variable investment threshold must be smaller to satisfy the full investment. If the variable investment cost is too low, making the full investment is optimal, which is an intuitive result.

- Making a partial investment is optimal if the variable investment cost is medium. In this case, making the investment is relatively inexpensive. If inventory inaccuracy correlation is high, making the investment is possible even for higher values of the variable investment cost.

- Making no investment is optimal if the variable investment cost is high and demand correlation is low.

IS Scenario

Under the IS scenario, to determine the total order quantity and the expected profit, the total variance of the system should be known. The total inventory inaccuracy given that the RFID technology is implemented in n warehouses is normally distributed (Zhu and Thonemann [49]):

$$\textstyle\sum_{i=1}^{N} X_i|(\omega_1,...,\omega_n) \sim N\left(0, \frac{(1-\rho_I)(N-n)}{N+N(n-1)\rho_I}\tau_I^2\right)$$

Then, the optimal total order quantity is:

$$Q^*|(\omega_1,...,\omega_n) = N\mu_D + z_C\sqrt{\frac{(1-\rho_I)(N-n)}{N+N(n-1)\rho_I}\tau_I^2 + N\sigma_D^2}$$

The expected profit function is:

$$E\left(\Pi_C\right) = (r-m)N\mu_D - (r-s)\phi(z_C)\sqrt{\frac{(1-\rho_I)(N-n)}{N+N(n-1)\rho_I}\tau_I^2 + N\sigma_D^2} - K_{\{n>0\}} - kn$$

The optimal number of warehouses where the technology is implemented can be determined by maximizing the expected profit function or minimizing the expected cost. The expected cost is:

$$E\left(Cost_C\right) = (r-s)\phi(z_C)\sqrt{\frac{(1-\rho_I)(N-n)}{N+N(n-1)\rho_I}\tau_I^2 + N\sigma_D^2} + K_{\{n>0\}} + kn$$

The second derivative of the expected cost function with respect to n is:

$$\frac{\partial E\left(Cost_C\right)^2}{\partial^2 n} = \left(\frac{\phi(z_C)(1+(N-1)\rho_I)^2(r-s)\sigma_X^2}{4(1+(n-1)\rho_I)^4\left(N\sigma_D^2 + \frac{(n-N)(-1+\rho_I)(1+(N-1)\rho_I)\sigma_X^2}{1+(n-1)\rho_I}\right)^{3/2}}\right)$$
$$\times(\rho_I-1)[-4N\rho_I(1+(n-1)\rho_I)\sigma_D^2 + (\rho_I-1)(1+(N-1)\rho_I)(-1+\rho_I-4n\rho_I+3N\rho_I)\sigma_X^2]$$

Let us define $c(n) = (\rho_I - 1)[-4N\rho_I(1+(n-1)\rho_I)\sigma_D^2 + (\rho_I - 1)(1+(N-1)\rho_I)(-1+\rho_I - 4n\rho_I + 3N\rho_I)\sigma_X^2]$. Note that the sign of the second derivative of the expected cost function with respect to n is the same as the sign of $c(n)$. There is at most one sign change, since $c(n)$ is linear in n. So, we consider the signs of $c(0)$ and $c(N)$.

The following properties are shown to be true:

- If ρ_I is in the range of:

$$-\frac{1}{N-1} < \rho_I \leq \frac{1}{1+N+\frac{2N\sigma_D^2+2\sqrt{N(N\sigma_D^2\sigma_X^2+N\sigma_X^4)}}{\sigma_X^2}}$$

  then $c(0) \leq 0$ and $c(N) \leq 0$ hold. The optimal expected cost is concave in n, so the optimal solution is either making the full investment or no investment. In this case the correlation coefficient is negative or positive but small.

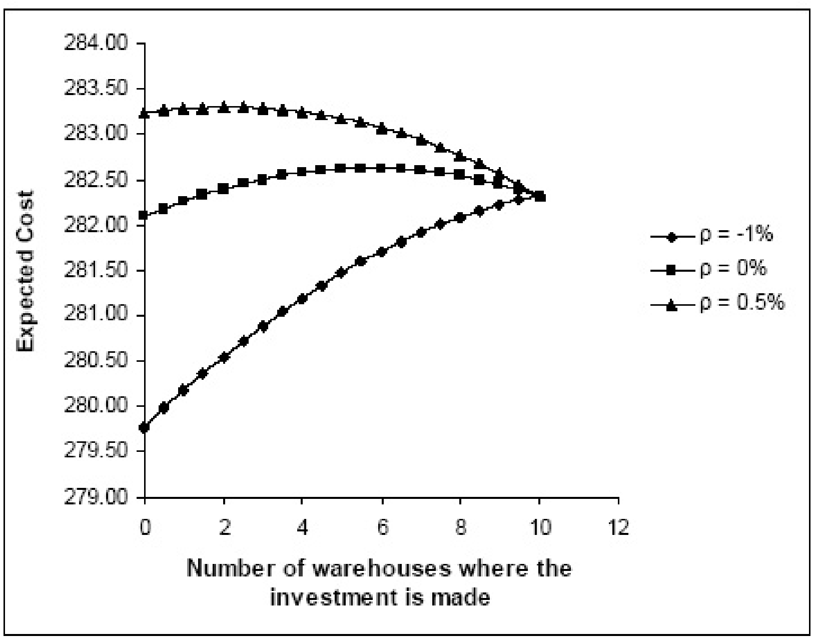

Figure 5.3: For negative and positive but small values of ρ_I, expected cost is concave.

- If ρ_I is in the range of:

$$\frac{1}{1+N+\frac{2N\sigma_D^2+2\sqrt{N(N\sigma_D^2\sigma_X^2+N\sigma_X^4)}}{\sigma_X^2}} < \rho_I \leq \frac{-4N\sigma_D^2+(N-2)\sigma_X^2+\sqrt{N(16\sigma_D^2\sigma_X^2+N(-4\sigma_D^2+\sigma_X^2)^2)}}{2(N-1)\sigma_X^2}$$

then $c(0) \geq 0$ and $c(N) \leq 0$ hold. $E\left(Cost_C\right)$ is convex-concave in n. The optimal solution for this case is to make full investment, partial investment or no investment depending on the parameters. In this case the correlation coefficient is positive and medium.

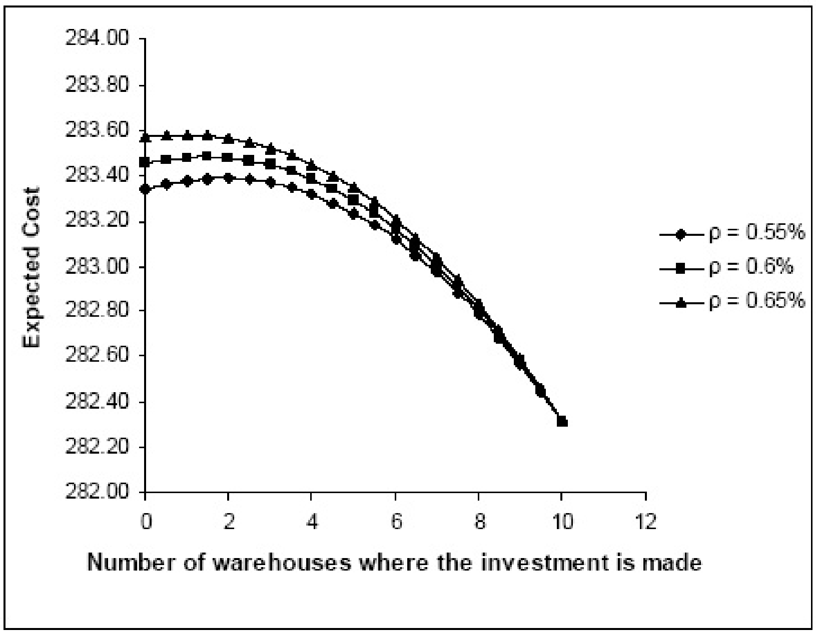

Figure 5.4: For positive and medium values of ρ_I, expected cost is convex-concave.

- If ρ_I is in the range of:

$$\frac{-4N\sigma_D^2 + (N-2)\sigma_X^2 + \sqrt{N(16\sigma_D^2\sigma_X^2 + N(-4\sigma_D^2 + \sigma_X^2)^2)}}{2(N-1)\sigma_X^2} < \rho_I < 1$$

  then $c(0) \geq 0$ and $c(N) \geq 0$ hold. $E\left(Cost_C\right)$ is convex in n. The optimal solution for this case is to make full investment, partial investment or no investment depending on the parameters. In this case the correlation coefficient is positive and large.

The numerical examples for the above cases are represented in Figures 5.3, 5.4 and 5.5 ($r = 10$, $m = 5$, $s = 0$, $N = 10$, $\mu_D = 100$, $\sigma_D = 20$, $\sigma_X = 10$ and $k = 3$). By using the given parameters for NIS scenario, the ranges for ρ_I are calculated as: (-0.1111, 0.0054)

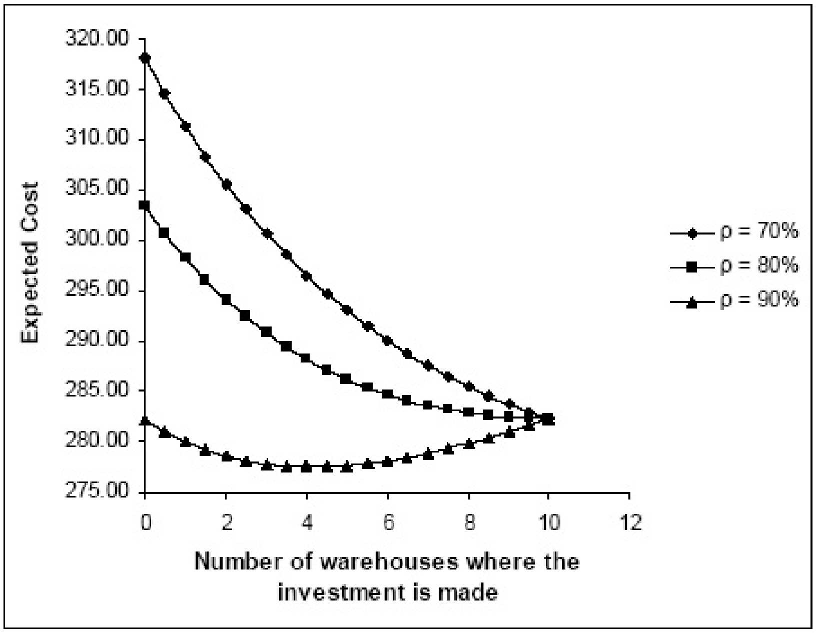

Figure 5.5: For large values of ρ_I, expected cost is convex.

(The expected cost is concave.), (0.0054, 0.0066) (The expected cost is convex-concave) and (0.0066, 1) (The expected cost is convex.).

Finally, we analyze how the optimal investment decision is affected by the variable investment cost k and inventory inaccuracy correlation ρ_I as in the NIS scenario. Numerical examples are done and the results are summarized in Figure 5.6.

- Making the full investment is optimal if the variable investment cost is small. For too low and too high values of ρ_I, the variable investment threshold must be smaller to satisfy full investment.

- Making a partial investment is optimal if the variable investment cost is medium and demand correlation is medium or high. In this case, making the investment is relatively inexpensive and high inventory inaccuracy correlation ensures that a significant portion of inventory inaccuracy will be eliminated.

- Making no investment is optimal if the variable investment cost is high and demand correlation is low.

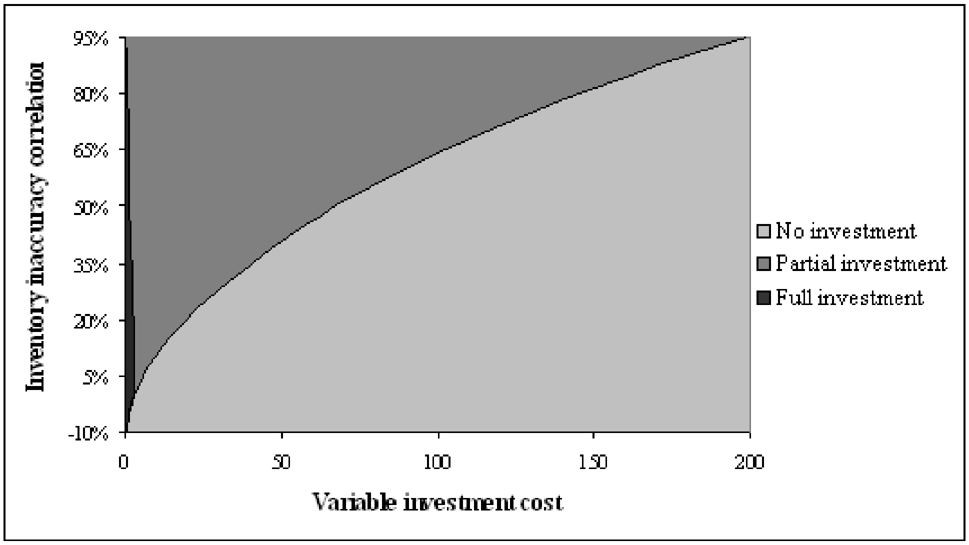

Figure 5.6: The effects of the inventory inaccuracy correlation, ρ_I and the variable investment cost, k on investment decision under IS scenario.

Compared to the NIS scenario, in the IS scenario the variable investment cost must be smaller to satisfy the full investment. This result is similar to our previous results. Recall that the variable investment cost threshold for the NIS scenario is greater than the variable investment cost threshold of the IS scenario. However, partial investment is justified for the IS scenario for higher values of the variable investment cost.

5.4 Imperfect RFID Implementation

NIS Scenario

Although RFID technology is a very hot topic and most of the companies are ready to implement the technology, it has some shortcomings. First of all, pilot programs have shown that errors such as misread and no-read occur too often. 80% success rate in reading is being identified in the report of AMB Property [33]. Secondly, radio frequencies are absorbed by liquids and reflected by metals. Such problems in implementation may result in imperfect implementation of the technology. In our initial model, we assume that when the technology is implemented in a warehouse, the inventory inaccuracy is completely eliminated. However, it may not be the case in real applications. We relax the assumption of perfect implementation in this section. Let t denote the fraction of inventory inaccuracy that is eliminated by investing on the RFID technology.

The optimum order quantity for the warehouse in which the RFID is implemented is:

$$Q^* = \mu_D + z_C\sqrt{(1-t)\sigma_X^2 + \sigma_D^2}$$

and for the warehouse in which the RFID is not implemented is:

$$Q^* = \mu_D + z_C\sqrt{\sigma_X^2 + \sigma_D^2}$$

The expected profit function is:

$$E\left(\Pi_C\right) = (r-m)N\mu_D - (r-s)\phi(z_C)\left[n\sqrt{\sigma_D^2 + (1-t)\sigma_X^2} + (N-n)\sqrt{\sigma_D^2 + \sigma_X^2}\right] - K_{\{n>0\}} - kn$$

It is observed that as the efficiency of implementation increases (as t increases), the expected profit increases, which is an intuitive result.

The imperfect implementation does not affect the structure of the expected profit function. The function is increasing if:

$$\frac{\partial E(\Pi_C)}{\partial n} = (r-s)\phi(z_C)\left(\sqrt{\sigma_D^2 + \sigma_X^2} - \sqrt{\sigma_D^2 + (1-t)\sigma_X^2}\right) - k \geq 0$$

If the first derivative is greater than 0, the optimum solution is making the full investment, otherwise making no investment.

The expected profit function is increasing when:

$$k \geq (r-s)\phi(z_C)\left[\sqrt{\sigma_D^2 + \sigma_X^2} - \sqrt{\sigma_D^2 + (1-t)\sigma_X^2}\right] \tag{5.6}$$

The above expression characterizes the variable investment threshold. The variable investment threshold increases in t, which means the system can pay more for investment if the implementation is perfect.

IS Scenario

For IS scenario, similar results to NIS scenario are obtained.

The total optimum order quantity and the expected profit function in case of imperfect implementation are:

$$Q^* = N\mu_D + z_C\sqrt{(N-nt)\sigma_X^2 + N\sigma_D^2}$$

and

$$E\left(\Pi_C\right) = (r-m)N\mu_D - (r-s)\phi(z_C)\sqrt{(N-nt)\sigma_X^2 + N\sigma_D^2} - K_{\{n>0\}} - kn$$

Convexity of the expected profit function is verified since:

$$\frac{\partial E(\Pi_C)^2}{\partial^2 n} = \frac{\sigma_X^4 (r-s)\phi(z_C)t^2}{4\ \sqrt[3/2]{N\sigma_D^2 + (N-nt)\sigma_X^2}} \geq 0$$

In case of imperfect implementation under IS scenario, the variable investment threshold is:

$$k_C^T = \frac{(r-s)\phi(z_C)\left[\sqrt{N(\sigma_D^2+\sigma_X^2)} - \sqrt{N\sigma_D^2 + (N-Nt)\sigma_X^2}\right]}{N}$$

As in the NIS scenario, the expected profit function and the investment threshold is increasing in t under the IS scenario.

5.5 Multi Product Problem

Our last extension considers the multi product case. Recall that in our initial model, we assumed that there is single product in our system.

The multi product case can be analyzed under different scenarios (see Figure 5.7). There could be single or multiple warehouses. If there is a single warehouse, the variable investment cost can be defined per product type. If there are multiple warehouses, the variable investment cost can be per product type or per warehouse. If there are multi products in a single warehouse, then inventory sharing scenario cannot be considered assuming that the products are not substitutes. If there are multiple warehouses, both inventory sharing and no inventory sharing scenarios can be considered.

The multi product problem under the NIS scenario can be solved as in Section 5.1. See the following cases.

- Consider a case where there are multiple products in a single warehouse and the variable investment cost is defined per product type.

  - Compare the variable investment thresholds of the products by the actual variable investment costs to find the optimum investment level.

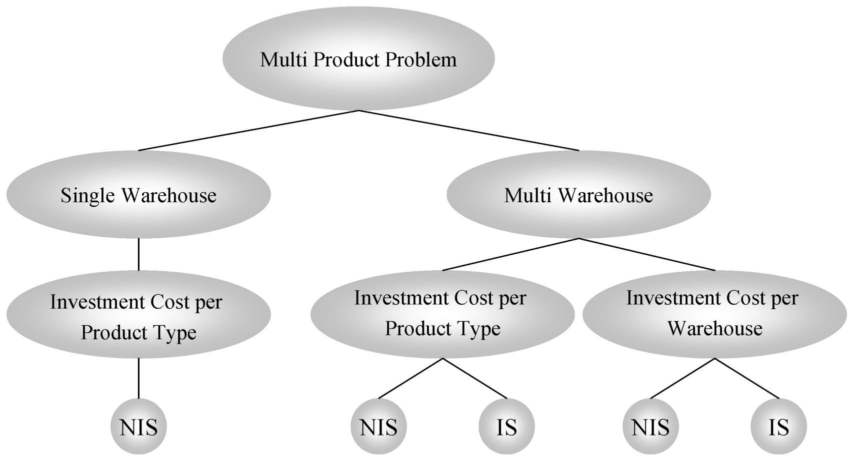

Figure 5.7: Alternative scenarios for multi product problem

- Consider a case such that there are multiple warehouses and each warehouse is dedicated to a product type and there is no inventory sharing assuming that the products are not substitutes.
  - Compare the variable investment thresholds of the products by the actual variable investment costs to find the optimum investment level.
- Consider the multi product and multi warehouse case, where the variable investment cost is defined per warehouse under the NIS scenario.
  - Compare the variable investment thresholds of the warehouses by the actual variable investment costs to find the optimum investment level.

On the other hand, under the IS scenario, the multi product problem under the investment decision is a hard problem. Under the IS scenario, all the investment alternatives must be considered before making the investment decision. There are 2^N (total number of subsets of N, N is the number of warehouses or product types.) alternative solutions for the problem, the number of alternative solutions increases exponentially. To solve this problem, a heuristic method can be developed.

Chapter 6

CONCLUSION

One of the important premises of the RFID technology is decreasing the inventory inaccuracy. We focused on the problem of how fixed and variable investment costs related to RFID affect a decentralized supply chain. We considered two main scenarios for the supply chain which may be appropriate in different practices: when there is inventory sharing between the warehouses and when there is no inventory sharing between the warehouses. In addition, two different means of coordinating supply chain under investment decision were investigated using revenue sharing and buyback type contracts modified to include an investment cost sharing component. The coordinating investment cost sharing component was found for two types of contracts and it is argued that the investment costs may be shared under different scenarios. Several extensions of the model are considered. Insights and intuitions about the model are obtained by relaxing some assumptions.

Our model yields several insights on RFID investment cost sharing in a supply chain under different situations. Obviously, the RFID investment improves the supply chain efficiency by decreasing inventory inaccuracy under the two proposed scenarios if the variable investment cost is under the threshold and the increase in expected profit compensates the fixed investment cost. The thresholds have different characterizations for the supplier and the retailer and different decisions may emerge when only one of the parties make the investment. If the profit margin of the retailer is too low, she may not make an investment to decrease inventory inaccuracy although it is optimal for the centralized system. Also, the penalty of decentralization can be severe in cases where the profit margin of the retailer is low. Finally, the effect of the investment on supply chain efficiency is much more significant when there is no inventory sharing between the warehouses.

We can also characterize the important factors for the investment decision. Clearly, making an investment is easier when the variable and the fixed investment costs are low. In addition, as the demand variance increases, the tendency of the system to make an investment decreases. If the market is characterized by highly uncertain demand, making an investment on the RFID technology to decrease inventory inaccuracy may not be reasonable. Furthermore, making an investment to decrease inventory inaccuracy becomes more important for high inventory inaccuracy. In contrast, as the number of warehouses increases, it becomes more difficult to make an investment under the inventory sharing scenario. This is in contrast with the no inventory sharing scenario where the number of warehouses does not have any effect on the investment decision. A general conclusion is that any initiative towards better supply chain efficiency such as increased demand pooling or inventory sharing between retailers diminishes the need for RFID investment.

By relaxing several assumptions of the base model, we checked the different variations of our model. First of all, we have seen that partial investment decisions may be optimal when the parameters of the warehouses are not identical and when the inventory inaccuracies of the warehouses are correlated. Secondly, it is seen that making an investment is justified when: the demand correlation is low and the implementation is perfect. If the implementation is perfect, the system is willing to pay more for the investment which is an intuitive result. However, the intuition behind making the investment when demand correlation is low is not clear enough. Since negative demand correlation increases the efficiency of the supply chain, it is expected that the system may not want to pay more for the investment. We should note that as the demand correlation increases, the effective uncertainty influencing the system increases, which decreases the need for RFID in our setting. Remember that making an investment is justified when the demand inaccuracy is low.

Finally, it is known and observed in initial applications that RFID has high potential to decrease inventory inaccuracy. However, in certain settings such as inventory sharing and imperfect tag readability, the RFID implementation may be unnecessary. Depending on the conditions, the retailer or the supplier may lead the investment. The supplier may lead the investment if the inventory inaccuracy is inclined to be positive, otherwise the retailer may lead the investment.

To gain basic insights on the investment decisions, this thesis employed several simplifying assumptions in order to keep the underlying models simple. It would be interesting to consider models that take into account other complications such as multiple periods, multiplicative error and to analyze their effects on the investment decision.

To conclude, the benefits of RFID are clear as demystified by several applications. RFID application improves supply chain performance in many aspects. In our model, we assumed that RFID eliminates the inventory inaccuracy problem. However, it may provide other benefits such as making warehouses smaller, improving shelf availability, decreasing out of stocks, etc. To find the true value of RFID and to decide on the investment, all the benefits should be considered. Our model gives insights about the RFID investment decision by considering inventory inaccuracy problem.

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