PRODUCT ASSORTMENT UNDER CUSTOMER-DRIVEN DEMAND SUBSTITUTION IN RETAIL OPERATIONS

by

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ABSTRACT

The problem of product assortment and inventory planning under customer-driven demand substitution is analyzed and a model for this problem is developed in this thesis. In addition, other realistic issues in a retail context such as supplier selection, shelf space constraints, and poor quality procurement are also taken into account. The model is analyzed for three different cases: (i) the deterministic demand, single-period case, (ii) the deterministic demand, multi-period case, and (iii) the stochastic demand, single period case. The characteristics of optimal assortment for different substitution costs is examined for each case. Then, the performance of the modified models, which neglects customers substitution behavior, which excludes supplier selection decision, and which ignores shelf space limitations, are analyzed separately. The results of the analysis demonstrate that neglecting customer-driven substitution or excluding supplier selection or ignoring shelf space limitations leads to inefficient assortments.

Empirical analysis for some of the input parameters of the model is also performed. During this analysis, we gather data on the substitution behavior of the customers in regard of product categories and retailer choices.

The main contribution of this thesis is the development of a practical and flexible model for retailer systems which results in optimal assortments in terms of total profit of the system.

ÖZETÇE

Bu tezde müşteri kaynaklı ürün ikamesi varlığında ürün portföyünün belirlenmesi ve envanter planlaması problemi incelenmiş ve problem için bir model geliştirilmiştir. Bunun yanı sıra, tedarikçi seçimi, raf kısıtlamaları, kalite ve kapasite açısından tedarikçi kısıtları gibi perakendeciler için geçerli olan diğer gerçekçi konular da dikkate alınmıştır. Geliştirilen model üç farklı durum için analiz edilmiştir: (i) determinist talep, tekil dönem durumu, (ii) determinist talep, çoklu dönem durumu, ve (iii) stokastik talep, tekil dönem durumu. Her durumda, farklı ikame bedelleri için önerilen en iyi ürün portföylerinin özellikleri incelenmiştir. Ayrıca, müşterilerin ikame davranışlarını ihmal eden, tedarikçi seçimi kararlarını dikkate almayan, ve raf kısıtlarını göz ardı eden değiştirilmiş modeller ayrı ayrı analiz edilmiştir. Bu analizlerin sonuçlarına göre, müşterilerin ikame davranışlarını ihmal etmenin, ya da tedarikçi seçimi kararlarını dikkate almamanın, veya raf kısıtlarını göz ardı etmenin daha düşük karlı ürün portföylerine neden olduğu görülmüştür.

Bunların yanı sıra, modelin bazı girdi parametreleri için deneysel çalışmalar yapılmıştır. Bu çalışmalar sırasında, müşterilerden ürün kategorisi ve perakendeci seçimi ile ilgili ikame davranşları üzerine bilgi toplanmıştır.

Bu tezin en önemli katkısı, perakende sistemleri için en kazançlı ürün portföyünü sunan pratik ve esnek bir model geliştirilmesidir.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

AHP	Analytical Hierarchy Process
ANP	Analytical Network Process
MP	Mathematical Programming
FST	Fuzzy Set Theory
K	the number of products
T	the number of periods
P	the set of products
S	a set of suppliers denoted
M	max. number of substitution levels
z_{it}	inventory position of product i at the end of period t [Z: vector representation]
x_{it}	quantity of product i to be ordered per order in period t [X: vector representation]
y_{it}	1, if product i is ordered in period t; 0, otherwise. [Y: vector representation]
ofs_j	number of orders placed with supplier j for all periods $t = 1, .., T$.
o_{jt}	1, if an order is placed with supplier j in period t; 0, otherwise.
ss_j	1, if any order is placed with supplier j in any period; 0, otherwise.
$x0_{it}$	the amount of satisfied demand for product i in period t.
xs_{mikt}	the amount of product i used to satisfy m^{th} substitution from product k in period t. ($m = 1, 2, ..., M$)
w_{ik}	the proportion of customers whose preference is product k that substitute product k with product i. ($w_{kk} = 0$) [W: Substitution probability matrix]

c_i	unit cost of purchasing plus transportation for product i.
oc_j	cost of ordering per order placed with supplier j.
ssc_j	cost of selecting supplier j as a supplier.
d_{it}	random demand for product i in period t.
	[D: vector representation]
$F(.)$	cumulative probability distribution function of d_{it}.
OQ_i	order quantity quota for product i.
SS_i	shelf space limitation quantity for product i.
a_{ij}	1, if product i can be supplied by supplier j; 0, otherwise.
	[A: vector representation]
h_i	inventory holding cost per unit of product i for one period.
pq_i	unit cost due to receiving poor quality products of type i.
q_i	percentage of defective products of type i.
p_{it}	unit price of product i in period t.
s_{mi}	cost penalty of m^{th} substitution from product i.
	(Lost sales are assumed to as substitution to a dummy product.)
$z0_i$	the initial inventory position for product i.
$z1_i$	the ending inventory position of product i.
x_i	the quantity of product i to be ordered per order.
y_i	1, if product i is ordered; 0, otherwise.
o_j	1, if an order is placed with supplier j; 0, otherwise.
$x0_i$	the amount of satisfied demand for product i.
xs_{mik}	the amount of product i used to satisfy m^{th} substitution from product k .
d_i	random demand for product i.
TP	Total Profit
TR	Total Revenue
TCO	Total Cost of Ordering

$TCSS$	Total Cost of Supplier Selection
TCP	Total Cost of Purchasing
TCI	Total Cost of Inventory Holding
$TCPQ$	Total Cost of Poor Quality Products
TCS	Total Cost of Substitution
$L(Z, X, D)$	the sum of all substitution costs among all periods
SC_i	the first level substitution cost from product i
A	supplier-product availability matrix
mg_i	profit margin of product i
θ	the multiplier of mg_i in the equation of SC_i ($SC_i = \theta * mg_i$)
%DS	the average percentage of demand satisfied with the original product
%LS	the average percentage of lost sales
%f	the average percentage of substituted demand to all other products in the first level of substitution
%s	the average percentage of substituted demand to all other products in the second level of substitution
%t	the average percentage of substituted demand to all other products in the third level of substitution
$[TP]$	the optimal total profit of the original model
$[TPw/osc]$	the optimal total profit of the model without substitution cost
$[TPw/oss]$	the optimal total profit of the model without supplier selection
$I[ssc]$	supplier selection cost(ssc) interval
$[N(S)]$	the number of selected suppliers
$[N(S)w/oss]$	the number of selected suppliers in the model without supplier selection

The Notation used by Smith and Agrawal(2000):

E	the set of products
n	the number of products
f_i	the probability that an arriving customer initially prefers product i from E
α_{ij}	the probability that a customer, who initially prefers an unavailable product i, switches to product j
L_i	the probability of lost sales for an initial choice product i
D	the total number of customers arriving per period for products in E
$P\{D=d\}$	the probability that D equals d
$\psi(d)$	$P\{D=d\}$
$D_i(f_i)$	the total number of customers per period initially preferring product i
$\psi(d_i\|f_i)$	the probability distribution for the initial demand for product i
$\Psi(d_i\|f_i)$	the cumulative probability distribution for the initial demand for product i
x_i	1, if product i is stocked; 0, otherwise.
r_i	service level for product i
q_i	the base stock level
$g_i(x,m)$	the probability that a product i is demanded by m_{th} arrival, either as an initial preference or as a substitute
V_i	the fixed cost associated with stocking product i
m_i	the unit profit margin for product i
c_{oi}	the unit overage cost for product i inventory left at the end of period
c_{ui}	the unit underage cost for product i
N	Negative binomial distribution parameter
p	Negative binomial distribution parameter

Chapter 1

INTRODUCTION

In order to survive in a competitive environment and maximize profits, retailers should use effective positioning strategies, which differentiate themselves from their competitors and satisfy the needs and wants of their customers. The differences and similarities in the positioning strategies lead to strategic groupings in the retail market. For instance, Gosh and McLafferty (1987) identify a number of different strategic groups in the department/discount store category, such as prestige stores, traditional department stores, mass merchandisers, and so on, each with a distinct price, quality, and product assortment strategy. In retailing terminology, *product assortment* is used for the collection and quantity of products in a category and *product line* is used for the number of product categories. It is common to classify retailers by the characteristics of their product line and product assortment: *Specialty stores* have narrow product line with deep product assortment, *Department stores* have a wide variety of product lines, *Supermarkets* have wide variety of food, laundry, and household products, *Convenience stores* have limited line of high-turnover goods, *Superstores* have large assortment of routinely purchased food and nonfood products, *Category killers* are giant specialty stores that carry a very deep assortment of a particular line, and *Hyperstores* are very large superstores.

Different price, quality, product line, and product assortment offerings of retailers attract customers with different demographic characteristics. For example, retailers that focus on quality are expected to attract different type of customers than retailers that compete primarily on a price basis. As an example of this from the Turkish retail market, the customers of Migros, that values the breadth and quality of the product assortment over price, have different expectations than that of the customers of BIM, which offers a limited assortment focusing on lower price. Therefore, while positioning themselves in the market and defining the customer segment that they target, retailers identify a general policy on the width and depth of their product assortment as well as their price and quality offerings at an aggregate level.

On the other hand, the product assortment decisions on a category basis are closely related with the operational activities. Therefore, while constructing a strong strategic position in the market, at the same time, retailers should be able to manage their operational activities in order to decrease operational costs while providing an adequate customer service-level. The operational costs are related to inventory management in the store, purchasing and ordering of products, establishing a relationship with the suppliers, and costs due to poor quality procurement. In addition, retailers are subject to store related limitations such as shelf space constraints. Therefore, maximizing the profit in the existence of strategic and operational issues is not straightforward for retailers.

The problem of optimal product assortment and inventory stocking policies for a given product category and a set of suppliers under customer-driven demand substitution in retail operations is examined in this thesis. Customer-driven demand substitution means that if a product type is unavailable, the customer might purchase a substitute product type or might not purchase anything which leads to a lost sale. Thus, when certain product types are not carried in the assortment or are stocked out, substitution causes the demand for the remaining product types to increase, affecting their optimal inventory levels. An important trade-off exists in retail operations in finding the right product assortment since increasing variety increases customer satisfaction but has a negative effect on operational costs. Therefore, the product assortment decision is impacted by several closely-related issues such as category management, selection of suppliers, and demand substitution. Since each product type corresponds to a brand, product assortment and selection of suppliers cannot be separated. In addition, as a result of product substitution behavior of customers, increasing the assortment reduces the demand per variant. Thus, customer-driven substitution should not be neglected in operational decision making. Due to shelf space limitations, inventory management should also be incorporated in the decision process.

Product assortment, demand substitution, supplier selection, and inventory management have been extensively studied separately. However, to the best of our knowledge, there is no previous work that considers all these aspects together. We introduce a mathematical model for the joint problem in order to specify which product types should be ordered from the suppliers, as well as the optimal ordering quantities, and the order frequencies for the ordered product types. The optimal policy is determined by maximizing expected total profit over a planning horizon.

In this problem, a number of issues complicate the optimization. First, customer demand for products is not known with certainty. Second, substitution affects the demand for all products in a category and their optimal inventory levels.

The main contribution of this study is providing an efficient tool to determine the product assortment for retailers, which considers supplier selection and inventory management decisions in the presence of shelf space limitations and substitution behavior of customers. With computational experiments, we identify the effect of substitution on the product assortment by varying substitution costs. Different substitution costs represent different expectations of the customers in terms of the product offerings of the retailer and the product category under consideration. In addition, we show that incorporating the supplier selection decision into the determination of product assortment may result in significantly increased profit and considering shelf space limitations in the decision process leads to more profitable assortments.

The organization of the remaining part of the thesis is as follows: Chapter 2 provides the necessary background and literature on product assortment, demand substitution, and supplier selection problems.

The mathematical model for the problem is presented in Chapter 3. A simple illustrative example is presented in order to make the model more understandable.

In addition, the model proposed by Smith and Agrawal (2000) is examined in detail and the performance of the model developed in this thesis is compared with that of Smith and Agrawal (2000).

In Chapter 4, three cases of the problem - the deterministic demand, multi-period case; the deterministic demand, single-period case; and the single-period, stochastic demand case - are analyzed and the test results for these cases with the comparison of the test results are provided.

The determination of input parameters in the model is explained in Chapter 5. In this context, the results of the survey that is conducted with 200 participants in order to understand the customer substitution behavior and the effects of retailer choice and product category in substitution cost are presented.

Chapter 6 concludes the thesis, summarizes the main contributions, and gives an outline of possible extensions of the thesis.

Chapter 2

LITERATURE REVIEW

2.1 Overview

In this chapter, a review of the literature on product assortment, inventory management, and supplier selection problems is given. Although each of these problems is extensively studied in the literature, to the best of our knowledge there does not exist any research on the joint problem of product assortment, demand substitution, supplier selection, and inventory management

2.2 Literature on Product Assortment and Inventory Management

Multi-product inventory management problem under a single resource constraint has been studied extensively in the literature (Hadley and Whitin (1963), Nahmias and Smith (1984), Downs et al. (2002), and others.) However, these models do not consider demand substitution or supplier selection.

Product assortment planning under demand substitution is defined as the selection of products in a category and determining inventory levels for the selected products when customers can meet their demand with another product in the same category if their first choice is unavailable. Product assortment and inventory management problem with stockout-based substitution was first introduced by McGillivary and Silver (1978). Later, Parlar and Goyal (1984), Pasternack and Drezner (1991), Moinzadeh and Ingene (1993), and Drezner et al. (1995) studied inventory management of two products in the existence of demand substitution. The solution approaches in these articles are complex and unsuitable for application to a larger number of products. In addition, researchers such as Netessine and Rudi (2003), Parlar (1985), Avsar and

Baykal-Gursoy (2002), and Rajaram and Tang (2001) studied this problem for centralized and decentralized regimes. However, the models provided by these studies have no resource constraint and focus on the stocking decisions for a given assortment, but not the selection of the assortment.

Pentico (1974), (1976), (1988), and Chand et al. (1994) studied inventory management problem of a multi-product system under deterministic, one-way substitution. They developed dynamic programming algorithms for finding the optimal product assortment. However, they considered only one-way substitution and neglected other cost items related to supplier selection or invisible costs related to substitution.

Matthews (1978) developed a linear programming formulation for the problem of inventory management under deterministic demand rates and deterministic substitution. Klein et al. (1993) studied resource allocation with deterministic demands where substitutions are represented by graphs. These articles consider only deterministic demand and do not consider supplier selection and substitution costs.

In addition, there exist several studies on modeling customers' choices under demand substitution (McFadden (1973), Guadagni and Little (1983), Bultez et al. (1989), Shugan (1989), Jain et al. (1994), and others). However, this group of studies does not consider inventory management and the proposed choice models can be used as an input for inventory management and product assortment problems.

One can consider two forms of demand substitution: In *assortment-based substitution*, a consumer might substitute when her favorite product is not in the assortment carried by the store, whereas in *stockout-based substitution*, a consumer might substitute when her favorite product is stocked-out at the moment of purchasing. Van Ryzin and Mahajan (1999) developed a model to determine the optimal assortment under stochastic demand, single-period setting with a utility-based approach, namely *the multinomial logit (MNL) choice decision model.* In utility-based model of substitution, a consumer chooses the alternative which maximizes her utility from a choice set. The choice set includes the products in the store and the no-purchase option. Their model allows assortment-based substitution, but does not consider stockout-based substitution. Later, Mahajan and van Ryzin (2001) proposed a stochastic sample path optimization method for the same model under both assortment-based and multiple rounds of stockout-based substitution. However, resource constraints are not considered in these articles. Kok and Fisher (2004) argued that the MNL model in its simplest form, which is used in both papers of Mahajan and van Ryzin, is unable to capture an important characteristic of the substitution behavior. They prove that it is not possible with this model to have two categories with the same penetration rate (purchase incidence) but different substitution rates. As an alternative for the MNL model, the probabilistic model of substitution is often used in inventory models (see Netessine and Rudi (2003)). Smith and Agrawal (2000) used the probabilistic model and studied the assortment planning problem with multi-period base-stock inventory models. They considered both assortment-based and stockout-based substitution, but allowed for one substitution attempt only. In their model, substitution is based on deterministic substitution probability matrices and they presented an approximation to the objective function of the resulting integer program. Our study differs from the previous two studies in the fact that we work on multi-way demand substitution including both assortment-based and stockout-based substitution. In addition as stated by Kok and Fisher (2004), the solution methodologies proposed by the last two papers have limited applicability for large problems.

Hsu and Bassok (1999) presented a single-period, multiproduct, downward substitution model. They determined the optimal production units to satisfy demand. In order to model random yield and random demand they use the technique of generating random scenarios. Rao et al. (2004) studied the same problem as Hsu and Bassok (1999) but they integrated setup costs in the system. These papers are relevant for production systems rather than retailing systems since substitution is not customer-driven. Kok and Fisher (2004) studied joint assortment selection and inventory planning problem under substitution. Their model of substitution is a probabilistic model with one substitution attempt only. They developed an estimation methodology for substitution rate and an iterative optimization heuristic for the assortment optimization problem which is a nonlinear, nonseperable, knapsack problem. They also considered realistic constraints such as discrete maximum inventory levels, batch sizes, delivery lead times, and perishability of products. In this paper, although we do not try to estimate substitution parameters, we also consider realistic constraints such as self space limitations and ordering quantity quotas for suppliers. As Kok and Fisher(2004) did, we also use probabilistic model for substitutions. In addition, our model does not neglect the supplier selection decision which is an important concern when maximizing total profit.

2.3 Literature on Supplier Selection

Supplier selection is a multi-criteria problem which includes both qualitative and quantitative factors. It is composed of four phases (Boer et al. (2001)): *(i).* Defining the goals of supplier selection (For example, profit maximization, increasing market share.), *(ii).* Defining the criteria (Historical data or forecasting might be used. Weights of each criterion should also be determined.), *(iii).* Supplier Qualification (A set of suppliers is sorted and/or ranked. Several objectives with several constraints exist. Qualification methods should be used.), *(iv).* Making the decision

The initial work for the supplier selection problem by Dickson (1966) underlines that cost, quality and delivery performance are the three most important criteria that need to be considered for vendor evaluation. The problem becomes different for the Single Sourcing case where the decision maker selects only the best supplier, and for the Multiple Sourcing case where the decision maker needs to make two decisions: which suppliers are the best and how much should be ordered from each selected supplier. Although the latter case is also important, not many articles focus on it.

The solution methods for the problem can be divided into two groups according to their objective: single objective and multiple objectives. Single objective techniques such as linear weighting, mixed integer programming usually consider cost as the unique criterion in the objective function. Other criteria are included as constraints. (See Gaballa (1974) and Pan (1989).) The problem with these techniques is that the criteria embedded in constraints are weighted equally. Also, qualitative factors are ignored. Multiple objectives techniques such as goal programming and multi-objective linear programming assign different weights to each criteria. (See Buffa and Jackson (1983), Sharma et al. (1989), and Weber and Current (1993)) Although these methods can assign different weights to different criteria, they are also weak in considering qualitative factors.

A distinction can also be made between single product and multiple product models. Single product models select suppliers for one product, that is the reason for that they do not consider the interdependencies between different products that the same supplier can offer.

The most important methods for supplier selection problem can be summarized as follows. *Rating/Linear Weighting Methods* use simple scoring methods which are very subjective and sensitive to different rating scales. (See Ghobadian et al. (1993)) Most of them are compensatory. In a compensatory model a low rating on a criterion can be compensated by a high rating on another criterion. In a non-compensatory model different minimum levels for each criterion are required.

Categorical Methods are qualitative and based on historical data. Suppliers are evaluated as *Positive*, *Negative*, and *Neutral* for each criteria and a total rate is calculated for each. Although these methods are very simple to implement, they are also subjective and depend on human judgment. Also, all the attributes are weighted equally.

Total Cost Methods outperform the rating models by objectifying the supplier selection process. They qualify all costs associated with the purchasing process throughout the entire value chain of the firm. Thus, they need extensive financial information. (See Timmerman (1986)) They are used for single-product models since it is hard to gather such information.

Analytical Hierarchy Process (AHP) method can be classified under linear weighting models. But it is somewhat different. It uses pairwise comparisons for obtaining scores of the suppliers and the weights of the attributes. It is used by Narasimhan

(1983). There is no imprecision according to decision maker's subjectivity. Analytical Network Process (ANP) is a more sophisticated version of AHP.

In *Data Envelopment Analysis*, suppliers are evaluated on benefit criteria (output) and cost criteria (input). The ratio of weighted sum of outputs to weighted sum of inputs gives the efficiency criteria for a supplier. (see Weber et al. (1998))

Cluster Analysis uses a classification algorithm to group a number of items that are described by a set of numerical attribute scores into a number of clusters such that for any two clusters there is low coupling and high cohesion. Therefore, Cluster Analysis can also be applied to a group of suppliers that are described by scores on some criteria. (see Holt (1998))

Case-based Reasoning Systems fall in the category of the so-called artificial intelligence approach. A case-based reasoning system is a software-driven database which provides decision-maker with useful information and experiences from similar, previous decision situations. This approach is very new and only a few systems have been developed for purchasing decision-making. (see Ng and Skitmore (1995))

Fuzzy Sets Theory is also a version of linear weighting model. Since supplier selection decision is hard due to high degree of fuzziness and uncertainties involved in the data set, some of the researchers focus on fuzzy set theory. Fuzzy set theory provides a framework for handling the uncertainties of this type. It offers a mathematical precise way of modeling vague preferences (e.g. weights of performance scores on criteria). Fuzzy Sets Theory can be combined with other methods. Morlacchi (1997) combines FST with AHP.

Statistical Models consider stochastic uncertainty related to the supplier selection. Very few models handle this uncertainty. Soukoup (1987) introduces uncertainty with respect to the requirements patterns in a single item rating model without inventory management.

Mathematical Programming Models are more objective than rating models since they force the decision-maker to explicitly state the objective function. However, they are based on more quantitative criteria and have problems in including qualitative criteria that are very important in decision making (such as for partnership policies). This problem can be overcome by including AHP or ANP for treating qualitative factors. The literature survey reveals that in MP models, linear programming, (mixed) integer programming, and goal programming or multi-objective programming are the commonly used techniques (See Moore and Fearon (1972), Oliveria and Lourenco (2002), and Sharma et al. (1989)). Many of these models consider predetermined levels on quality, service and delivery constraints. Weber and Current (1993) overcome this by using more complex weighting and constraint methods and presenting trade off curves among multiple objectives as decision support. Weber and Desai (1996) combine multi-objective programming with Data Envelopment Analysis to offer buyers a negotiation with suppliers and to evaluate the number of suppliers to select. However, it only allows for negotiation with inefficient vendors but it is possible that some inefficient suppliers might perform better than some efficient suppliers. This is because an efficient unit may be excelling on only few dimensions and performing poorly on many other dimensions. That is the reason that Talluri (2002) proposed a buyer-seller game model for selection and negotiation of purchasing bids. Goal programming is used to minimize costs and maximize quality and delivery reliability. The drawback of goal programming and multi-objective programming is that it requires arbitrary aspiration levels and it fails in accommodation of subjective criteria. Ghoudsypour and O'Brien (1998) combine AHP and MP in order to consider tangible as well as intangible criteria and to optimize order allocation among suppliers.

In order to include performance variability measures in evaluating alternative suppliers, Talluri and Narasimhan (2002) proposed a Max-min Approach, which relies on maximizing the minimum performance of a supplier against the best target measures set by the buyer. It is formed of two models: the first model is structured in a way to identify the areas in which a supplier excells, the second model identifies the areas in which a supplier performs poorly. The main advantage of this approach is that it provides the buyer with effective alternative choices within a vendor group. Thus, final decision is based on also intangible factors. Price is considered as output and cost and delivery performance are considered as input for the model, where the objective function is defined as the ratio of weighted outputs to weighted inputs. This way, the objective function represents the productivity of suppliers and is maximized subject to a set of constraints.

Kasilingam and Lee (1996) proposed a mixed-integer programming model which includes the stochastic nature of demand, quality of supplied parts, the cost of purchasing and transportation, the fixed cost of establishing vendors, the cost of receiving poor quality parts, and the lead time requirements for the parts.

Kumar et al. (2004) proposed a fuzzy mixed integer goal programming approach which has three main goals: minimizing net cost, minimizing net rejections and minimizing net late deliveries subject to realistic constraints regarding buyer's demand, supplier's capacity, supplier's quota flexibility, purchase value of items, budget allocation to each supplier, etc. However, it has some non-realistic assumptions, such as, only one item is purchased from one supplier, quantity discounts are ignored, and no shortage in supplier side and demand is deterministic.

Most of the existing literature on supplier selection do not consider inventory management of the purchased items. Only some models incorporate decision to schedule orders over time with the supplier selection decision. In reality, the ordering policy and supplier choice affects one another. For instance, if frequent ordering is necessary due to inventory management reasons (e.g. perishable inventory), a supplier with low unit price but high order cost might generate a higher total cost than a supplier with a high unit price and low order cost. As another example, when suppliers offer quantity discounts, the trade-off between savings in purchasing and inventory holding costs should be considered. Bender, et al. (1985), Buffa and Jackson (1983)-but for single-item case-, and Degraeve, et al. (2000) consider inventory management in supplier selection problems. They propose mathematical programming methods for the problem. Degraeve and Roodhooft (1998) and Degraeve et al. (2000) developed a mathematical programming model, which minimizes the total cost of ownership of the supplier choice and inventory management policy using activity-based costing information.

Most of the existing literature on supplier selection are relevant for production systems rather than retail operations (see Chen and Munson (2004), Goyal et al. (2003), and Bender, et al. (1985)).

To the best of our knowledge, multiple products, multi-way product substitution, inventory planning, and supplier selection are not considered in an integrated model in the literature. In this thesis, we formulate the multi-period, multi-product inventory, product assortment, and supplier selection problem with multi-way demand substitution in order to maximize the total profit of a retail store. In that sense, our work provides the first step toward developing a guideline for retailers during their decision making process.

Chapter 3

THE MODEL

Our model aims to determine which product types should be ordered from the suppliers and the optimal ordering quantities in each period that maximize profit in the existence of operational costs. Operational costs include fixed ordering costs placed per order, fixed costs of supplier selection due to costs of establishing relations with suppliers, purchasing costs, inventory holding costs, costs incurred as a result of poor quality products received, and substitution costs. Constraints of the model include shelf space limitations and ordering quantity quotas of the suppliers.

The multi-period planning model considers one product category, such as shampoos or laundry detergents, consisting of a set of products, offered in the market, denoted by P and a set of suppliers, offering these products, denoted by S. There assumed to be no lead time.

3.1 Characteristics of the Model

The sequence of events in the system is as follows; first the suppliers declare their products and the order quantity quotas that they can supply. Next, the decision maker selects which products and how much to order with the knowledge of ordering costs, purchasing costs, lost sales costs, substitution costs, inventory holding costs, poor quality purchase costs, shelf space limitations and revenues associated with each product.

We assume stochastic demand and deterministic substitution behavior for customers. That is, customers are expected to substitute from one product to another with deterministic proportions. These proportions might be obtained by market research or by the methodology proposed by Kok and Fisher (2004). This is a deterministic approximation for probabilistic models of substitution.

As discussed in Chapter 2, it is possible to consider different levels and types of substitution. While Mahajan and van Ryzin (2001) allow multiple attempts of stockout-based substitution, Kok and Fisher (2004), Smith and Agrawal (2000) and Netessine and Rudi (2003) allow one substitution attempt only. In our model, we allow at most M levels of substitution and consider both stockout-based and assortment-based substitution. We consider multi-way demand substitution since we assume that customers know exactly what they want, and therefore, have deterministic substitution rates. In the formulation of the model, we use the same substitution matrix for all levels in order to make the model simpler, however, one can define different substitution rates for different levels and incorporate them to the model easily, as it is shown in Section 3.6.4.

We assume each product is supplied by exactly one supplier, whereas a supplier may supply more than one product.

Since a retailer cannot have an unlimited store space, retailers define limitations on product categories offered. In general form, these limitations are known as shelf space limitations. Shelf space limitations might limit the space that the assortment of the category covers, the number of products/brands that the assortment contains, the number of suppliers that the assortment selects, and/or the number of SKUs (Store Keeping Units) that the assortment contains. During our analysis, we concentrate on the number of SKUs in the assortment as the shelf space limitation, where one unit of each product/brand is assumed to cover one unit of SKU on the shelf. The reason that we select the maximum number of SKUs as the shelf space limitation is that this type of limitation is more generally handled in the literature. It should be noted that, implementation of other types of shelf space limitations requires slight modifications in the model.

In addition, in this thesis, the term *shelf space limitation* is used as a generalization of all of the space limitations such as store depot/warehouse limitations regarding the retailers. Such a generalization is meaningful, since we do not consider handling costs of inventory from store depot/warehouse to shelf inside a retailer store. Therefore, retailers are expected to distribute the available space in a store depot/warehouse to products proportional to the allocated shelf space for the products.

Substitution costs represent the cost of the loss of goodwill of customers that may reflect on the retailer at a future time point as lost sales. However, it is difficult to identify when and in what magnitude the loss of goodwill will incur an actual cost. Therefore, in our model we include a substitution cost per each substitution realized as if this cost is incurred to the retailer at the moment of the substitution or the lost sales. If the retailers know that their customers expect to find a wide product assortment, in general or in particular for the product category under consideration, or a high quality of service at their stores, then we assume that this implies that the retail will incur high substitution costs. During our analysis, which are provided in Chapter 4, we observed the effect of the level of the expectations of customers by changing the substitution costs.

3.2 Decision Variables

The decision variables of the model are as follows:

- z_{it} : inventory position of product i at the end of period t. [Z: vector representation]
- x_{it} : quantity of product i to be ordered per order in period t. [X: vector representation]
- y_{it} : 1, if product i is ordered in period t; 0, otherwise. [Y: vector representation]
- ofs_j : number of orders placed with supplier j for all periods $t = 1, .., T$.
- o_{jt} : 1, if an order is placed with supplier j in period t; 0, otherwise.
- ss_j : 1, if any order is placed with supplier j in any period; 0, otherwise.

- $x0_{it}$: the amount of satisfied demand for product i in period t.
- xs_{mikt} : the amount of product i used to satisfy m^{th} substitution from product k in period t. $(m = 1, 2, ..., M)$

We use Figure 3.1 to explain the role of the substitution variables, indicating first choice demands and substituted demands to the products in any period t. For each product, there are three sets of arcs in the figure:

- First choice demand arc, denoted by $x0_{it}$, incoming to the product i.
- Substitution demand arcs set, one for each level of substitution denoted by $\sum_j xs_{mijt}$, incoming to the product i.
- Substituted demand arcs set, one for each level of substitution denoted by $\sum_k xs_{mkit}$, outgoing from the product i.

3.3 Parameters

The model has the following parameters:

- w_{ik} : the proportion of customers whose preference is product k that substitute product k with product i. $(w_{kk} = 0)$ [W: Substitution probability matrix]
- c_i : unit cost of purchasing plus transportation for product i.
- oc_j : cost of ordering per order placed with supplier j.
- ssc_j : cost of selecting supplier j as a supplier.
- d_{it} : random demand for product i in period t. [D: vector representation]
- $F_{it}(.)$: cumulative probability distribution function of d_{it}.

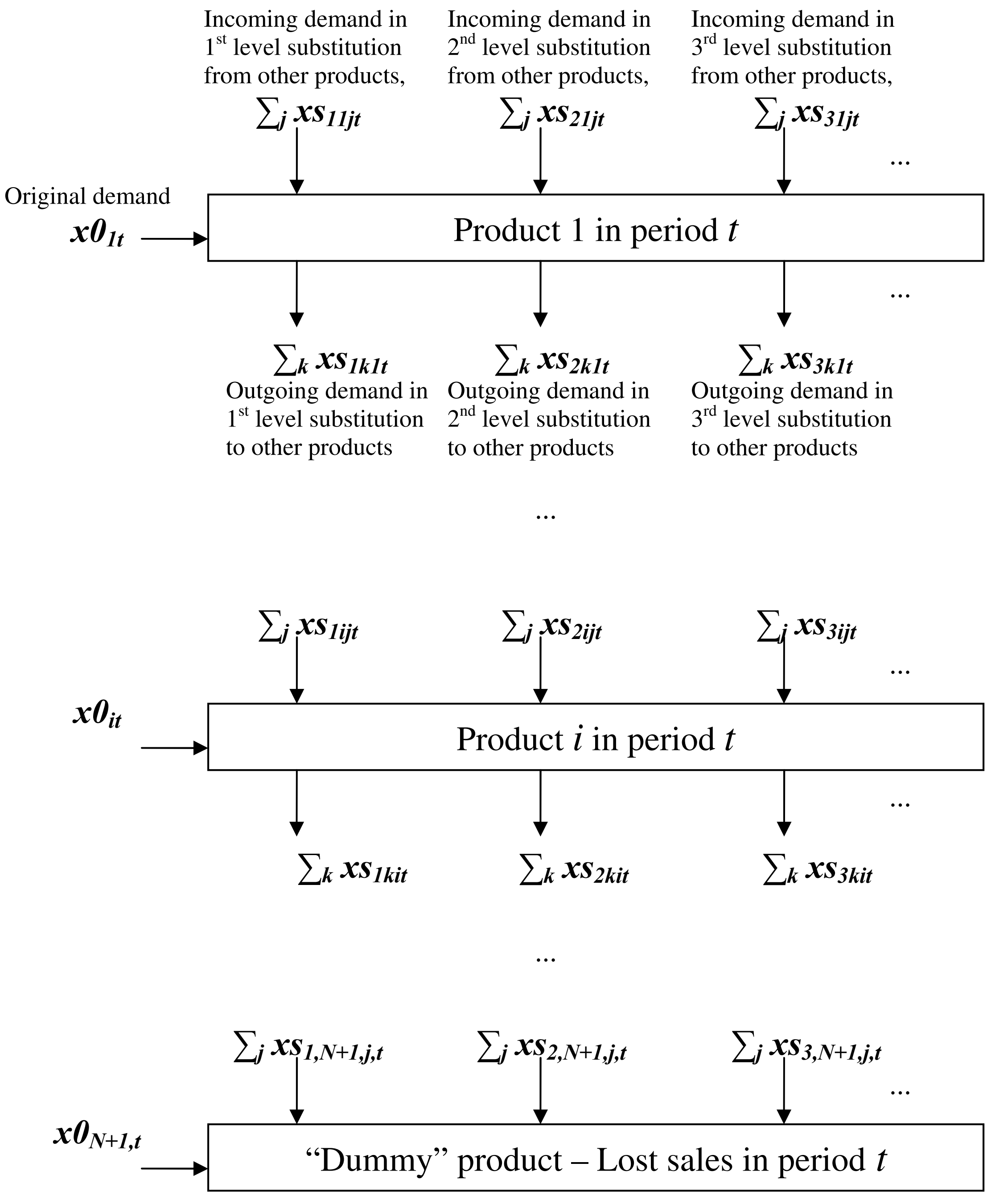

Figure 3.1: Demand Substitution

- OQ_i : order quantity quota for product i.
- SS_i : shelf space limitation quantity for product i.
- a_{ij} : 1, if product i can be supplied by supplier j; 0, otherwise. [A: matrix representation; product-supplier availability matrix]
- h_i : inventory holding cost per unit of product i for one period.
- pq_i : unit cost due to receiving poor quality products of type i.
- q_i : percentage of defective products of type i.
- p_{it} : unit price of product i in period t.
- s_{mi} : penalty cost of m^{th} substitution from product i. (Lost sales are assumed as substitution to a dummy product.)
- $z0_i$: initial inventory position for product i.

3.4 Objective Function

The objective of the model is to maximize the total profit.

$$Maximize\,TP = TR - TCO - TCSS - TCP - TCI - TCPQ - TCS \tag{3.1}$$

In Equation 3.1, TP is used for total profit, TR stands for total revenue, TCO is total cost of ordering, $TCSS$ is used for total cost of supplier selection, TCP stands for total cost of purchasing, TCI is total cost of inventory holding, $TCPQ$ is used for total cost of poor quality products, and TCS stands for total cost of substitution.

3.5 Constraints

The model tries to achieve the objective with the following constraints:

- The total revenue is equal to the sum of revenue generated in each period for each product. Revenue generated by a product in a period is the price multiplied by the amount sold in that period. The total revenue equation is expressed in Equation 3.2.

$$TR = \sum_{i \in P} p_{i1} \left(z0_i + x_{it} - z_{i1} \right) + \sum_{t=2}^{T} \sum_{i \in P} p_{it} \left(z_{i,t-1} + x_{it} - z_{it} \right) \tag{3.2}$$

- The total cost of ordering is equal to the sum of ordering cost for each supplier. Ordering cost for a supplier is the order frequency multiplied by the ordering cost of the supplier selected. The ordering frequency for a supplier is the sum of orders placed with that supplier during all periods. An order is placed for a supplier in a period if at least one of the products supplied by her is ordered in that period. The total cost of ordering is expressed in Equations 3.3 to 3.5, where ordering frequencies for suppliers are calculated by using the availability matrix A.

$$TCO = \sum_{j \in S} oc_j ofs_j \tag{3.3}$$

$$ofs_j = \sum_{t=1}^{T} o_{jt}, \forall j \in S \tag{3.4}$$

$$o_{jt} \geq a_{ij} y_{it}, \; \forall t = 1, .., T, \; \forall i \in P, \; \forall j \in S \tag{3.5}$$

- The total cost of supplier selection is equal to the sum of supplier selection costs of each selected supplier. Total cost of supplier selection is expressed in Equations 3.6 and 3.7, where selecting a supplier means at least one order is placed with that supplier.

$$TCSS = \sum_{j \in S} ssc_j ss_j \tag{3.6}$$

$$ss_j \geq o_{jt}, \; \forall t = 1, .., T, \; \forall j \in S \tag{3.7}$$

- The total purchasing cost is the sum of purchasing costs of each product in each period. Purchasing cost for a product in a period is unit purchasing cost for that product multiplied by the quantity of that product to be ordered. The total cost of purchasing is expressed in Equation 3.8.

$$TCP = \sum_{t=1}^{T} \sum_{i \in P} c_i x_{it} \tag{3.8}$$

- The total inventory holding cost is the sum of inventory holding costs of each product in each period. Inventory holding cost for a product in a period is unit inventory cost multiplied by the expected inventory of that product in that period. Expected inventory of a product in a period is calculated as the average of beginning inventory and ending inventory in that period. The total cost of inventory is expressed in Equation 3.9.

$$TCI = \sum_{i \in P} \frac{z0_i + x_{i1} + z_{i1}}{2} h_i + \sum_{t=2}^{T} \sum_{i \in P} h_i \left(\frac{z_{i,t-1} + x_{it} + z_{it}}{2} \right) \tag{3.9}$$

- The total cost of poor quality products is the sum of poor quality cost of each product in each period. Poor quality cost of a product in a period is unit cost of poor quality multiplied by the quantity of defective items in that period. The total cost of poor quality products is expressed in Equation 3.10 .

$$TCPQ = \sum_{t=1}^{T} \sum_{i \in P} pq_i q_i x_{it} \tag{3.10}$$

- The total substitution cost represents the expected substitution cost under the inventory position, Z, and the order quantity, X, for all demand realizations, D's. The substitution cost for a demand realization d under inventory position, Z, and order quantity, X is represented as $L(Z, X, d)$.

$$TCS = \int_D L(Z, X, d) dF(d) \tag{3.11}$$

$L(Z, X, d)$ is the sum of all substitution costs among all periods.

$$L(Z, X, d) = \sum_t \sum_m \sum_i \sum_{k, k \neq i} s_{mi} xs_{mikt} \tag{3.12}$$

Demand for a product is the sum of satisfied demand and all substitutions from other products to that product and is expressed in Equation 3.13.

$$x0_{it} + \sum_{m} \sum_{k \neq i} xs_{mkit} = d_{it}, \ \forall i \in P, \ \forall t = 1, .., T \tag{3.13}$$

In any period, for each product, the sum of the inventory level for that product at the end of the previous period and the quantity ordered for that product should be equal to the sum of substitutions to that product from any first choice product including itself and the inventory level of that product at the end of that period.
Then, for the first period:

$$x0_{i1} + \sum_{m} \sum_{k \neq i} xs_{mik1} + z_{i1} = z0_i + x_{i1}, \ \forall i \in P \tag{3.14}$$

For the remaining periods:

$$x0_{it} + \sum_{m} \sum_{k \neq i} xs_{mikt} + z_{it} = z_{i,t-1} + x_{it}, \ \forall i \in P, \ \forall t = 2, .., T \tag{3.15}$$

The substitution behavior of a customer is expressed in Equations 3.16- 3.18, which use the substitution probability matrix W. The amount of any level of substitution from product k to product i is less than or equal to a certain proportion of the satisfied demand for product k either by the stock of product k or by a substitution. This proportion is equal to the proportion obtained by multiplying the substitution probabilities in matrix W, which exist in that substitution chain. Substitution chain includes all the products that the customer tries to substitute from product k up to product i. Substitution inequalities are written for each level of substitution. Because of the complexity associated with higher substitution levels, we provide only the first three of them.

$$xs_{1ikt} \leq (d_{kt} - x0_{kt}) w_{ik}, \ \forall i, k \in P, \ \forall t = 1, .., T \tag{3.16}$$

$$xs_{2ikt} \leq (d_{kt} - x0_{kt} - \sum_{r \in P} xs_{1rkt}) \sum_{r \in P} w_{rk} w_{ir}, \ \forall i \in P, \ \forall k \in P \backslash \{i\}, \forall t = 1, .., T \tag{3.17}$$

$$xs_{3ikt} \leq (d_{kt} - x0_{kt} - \sum_{r;r\neq k,i} xs_{1rkt} - \sum_{r;r\neq k,i} xs_{2rkt}) \sum_{r;r\neq k,i} \sum_{p;p\neq k,i,r} w_{rk}w_{pr}w_{ip},$$
$$\forall i,k \in P\backslash\{i\},\ \forall t = 1,..,T \tag{3.18}$$

$$... \tag{3.19}$$

- There exist shelf space limitations for each product. For the first period:

$$z0_i + x_{i1} \leq SS_i,\ \forall i \in P \tag{3.20}$$

For the remaining periods:

$$z_{i,t-1} + x_{it} \leq SS_i,\ \forall t = 2,..,T,\ \forall i \in P \tag{3.21}$$

- If an order is not placed for a product in a period, the ordering quantity for that product in that period should be zero. If an order is placed for a product, then the quantity should be less than or equal to the maximum ordering quantity for that product.

$$0 \leq x_{it} \leq OQ_i y_{it},\ \forall t = 1,..,T,\ \forall i \in P \tag{3.22}$$

- In a period an order might or might not be placed.

$$y_{it} \in \{0,1\},\ \forall i \in P,\ \forall t = 1,..,T \tag{3.23}$$

3.6 Illustrative Example

In order to illustrate how this model can be used, let us work on a simple example. For this illustrative example, we consider a deterministic demand, single-period setting for simplicity. θ is considered to be 0.3. Assume there are 3 products/brands: $P1$, $P2$, and $P3$; and 2 suppliers : $S1$ and $S2$, where $S1$ supplies product $P2$ and $S2$ supplies products $P1$ and $P3$. Assume zero initial inventory and product $P4$ denotes lost sales and at most 3 levels of substitution exists($M = 3$). The substitution matrix is as provided in Table 3.1.

The parameter values for products are set as provided in Table 3.2 and the parameter values for suppliers are set as provided in Table 3.3 .

	$i+1^{st}$ preference			
i^{th} preference	$P1$	$P2$	$P3$	$P4$
$P1$	-	0.1	0.2	0.7
$P2$	0.2	-	0.5	0.3
$P3$	0.1	0.5	-	0.4

Table 3.1: The substitution matrix

Parameter	$P1$	$P2$	$P3$
c	10	8	6
d	3,000	4,000	5,000
OQ	12,000	10,000	20,000
SS	10,000	12,000	9,000
h	0.7	0.5	0.4
pq	4	3	2
q	0.05	0.10	0.09
p	19	14	12

Table 3.2: Parameter values for products

The model for this example can be formulated as provided in Table 3.4.

The optimal assortment provided by this model is provided in Table 3.5 with total profit of $10,825$.

As seen, the supplier $S1$ is not selected. With this assortment, all of demands for products $P1$ and $P3$ are satisfied, where 40% of demand for product $P2$ is lost and 60% of demand for product $P2$ is substituted. The total profit is $10,825$.

3.6.1 The Importance of Substitution Behavior

When we ignore the consumers substitution behavior in this example, the model proposes an assortment, which generates a total profit of $8,333$, thus resulting in 23%

Parameter	$S1$	$S2$
oc	40	45
ssc	35,000	50,000

Table 3.3: Parameter values for suppliers

profit loss.

3.6.2 The Importance of Supplier Selection

When we ignore the supplier selection decision in this example, the model proposes an assortment, which generates total profit of $7,565$, thus resulting in 30% of profit loss.

3.6.3 The Importance of Shelf Space Limitations

If the assortment is decided without considering shelf space limitations, most probably this might cause inconvenience since the optimal ordering quantities might not fit into the reserved shelf space for that category. In this case, one strategy the retailer may take is to distribute the limited shelf space proportionally among the products according to the product quantities of proposed assortment, which we call *method 1.* Another simple strategy that comes to mind is dividing the excess amount of quantity to the number of products in the assortment and each product's quantity is subtracted by that amount, which we call *method 2.*

In order to analyze the importance of shelf space limitations, we first introduced an effective shelf space limitation of $8,800$ units for the category and found the optimal assortment with this constraint. Then, we excluded shelf space constraint from the model and found the optimal assortment for this new setting. Then, we applied the shelf space limitations to this assortment in both methods described in the previous paragraph and found the total profits that these assortments generate for the case with shelf space limitation. Table 3.6 shows the assortments and the profits that these

---

$$
\begin{aligned}
&\textbf{\textit{Maximize}}\ \ TP = TR - TCO - TCSS - TCP - TCI - TCPQ - TCS \\
&\textbf{\textit{subject to}} \\
&\quad TR = 19x_1 + 14x_2 + 12x_3 \\
&\quad TCO = 40o_1 + 45o_2 \\
&\quad TCSS = 35000o_1 + 50000o_2 \\
&\quad TCP = 10x_1 + 8x_2 + 6x_3 \\
&\quad TCI = 0.7x_17/2 + 0.5x_2/2 + 0.4x_3/2 \\
&\quad TCP = 4 * 0.05 * x_1 + 3 * 0.1 * x_2 + 2 * 0.09 * x_3 \\
&\quad TCS = 0.3\,(9xs_{112} + 9xs_{113} + 6xs_{121} + 6xs_{123} + 6xs_{131} + 6xs_{132}) \\
&\quad +0.3 * 2\,(9xs_{212} + 9xs_{213} + 6xs_{221} + 6xs_{223} + 6xs_{231} + 6xs_{232}) \\
&\quad\ \ +0.3 * 3\,(9xs_{312} + 9xs_{313} + 6xs_{321} + 6xs_{323} + 6xs_{331} + 6xs_{332}), \\
&\quad xs_{112} \leq 0.1(3000 - x0_1),\ xs_{113} \leq 0.2(3000 - x0_1),\ xs_{114} \leq 0.7(3000 - x0_1), \\
&\quad xs_{121} \leq 0.2(4000 - x0_2),\ xs_{123} \leq 0.5(4000 - x0_2),\ xs_{124} \leq 0.3(4000 - x0_2), \\
&\quad xs_{131} \leq 0.1(5000 - x0_3),\ xs_{132} \leq 0.5(5000 - x0_3),\ xs_{134} \leq 0.4(5000 - x0_3), \\
&\quad xs_{212} \leq 0.1(3000 - x0_1 - xs_{112} - xs_{113} - xs_{114}), \\
&\quad xs_{213} \leq 0.05(3000 - x0_1 - xs_{112} - xs_{113} - xs_{114}), \\
&\quad xs_{214} \leq 0.11(3000 - x0_1 - xs_{112} - xs_{113} - xs_{114}), \\
&\quad xs_{221} \leq 0.05(4000 - x0_2 - xs_{121} - xs_{123} - xs_{124}), \\
&\quad xs_{223} \leq 0.04(4000 - x0_2 - xs_{121} - xs_{123} - xs_{124}), \\
&\quad xs_{224} \leq 0.34(4000 - x0_2 - xs_{121} - xs_{123} - xs_{124}), \\
&\quad xs_{231} \leq 0.1(5000 - x0_3 - xs_{131} - xs_{132} - xs_{134}), \\
&\quad xs_{232} \leq 0.01(5000 - x0_3 - xs_{131} - xs_{132} - xs_{134}), \\
&\quad xs_{234} \leq 0.22(5000 - x0_3 - xs_{131} - xs_{132} - xs_{134}), \\
&\quad xs_{314} \leq 0.05(3000 - x0_1 - xs_{112} - xs_{113} - xs_{114} - xs_{212} - xs_{213} - xs_{214}), \\
&\quad xs_{324} \leq 0.051(4000 - x0_2 - xs_{121} - xs_{123} - xs_{124} - xs_{221} - xs_{223} - xs_{224}), \\
&\quad xs_{334} \leq 0.073(5000 - x0_3 - xs_{131} - xs_{132} - xs_{134} - xs_{232} - xs_{231} - xs_{234}), \\
&\quad x_1 \leq 10000,\ x_2 \leq 12000,\ x_3 \leq 9000,\ x_1 \leq 12000y_1,\ x_2 \leq 10000y_2,\ x_3 \leq 20000y_3, \\
&\quad o_1 \geq y_2,\ o_2 \geq y_1,\ o_2 \geq y_3, \\
&\quad x_1, x_2, x_3, xs_{112}, xs_{113}, xs_{121}, xs_{123}, xs_{131}, xs_{132}, xs_{212}, xs_{213}, xs_{221}, xs_{223}, xs_{231} \geq 0, \\
&\quad xs_{232}, xs_{313}, xs_{321}, xs_{323}, xs_{331}, xs_{332} \geq 0,\ y_1, y_2, y_3, o_1, o_2 \in \{0, 1\}.
\end{aligned}
$$

Table 3.4: The model for the illustrative example

	$P1$	$P2$	$P3$
x	3,400	-	7,000

Table 3.5: Optimal ordering quantities for products

assortments generate.

Method	The Assortment			[TP]
	$P1$	$P2$	$P3$	
The original model	3,400	-	5,400	1,833
Without shelf space limitations with *method 1*	2,877	-	5,923	353
Without shelf space limitations with *method 2*	2,600	-	6,200	-431

Table 3.6: The effect of shelf space limitations

In Table 3.6, we see that the assortments of the model without self space constraint either with *method 1* or *method 2* generate lower profit than the optimal assortment of the model with shelf space constraints generates.

3.6.4 Different Substitution Matrices for Different Substitution Levels

In the formulation of the model, we use the same substitution matrix for all levels in order to make the model simpler, however, one can define different substitution rates for different levels and incorporate them to the model easily. In order to show the applicability of such a case, let us consider the substitution matrix provided in Table 3.1 for the first level of substitution, and two other substitution matrices, which are provided in Table 3.7 and Table 3.8, for the second and third levels of substitutions, respectively. As seen in Table 3.8, in the third level of substitution lost sales must occur since with 3 products it is impossible to substitute to another product.

	3^{rd} preference			
2^{nd} preference	$P1$	$P2$	$P3$	$P4$
$P1$	-	0.05	0.15	0.8
$P2$	0.1	-	0.4	0.5
$P3$	0.05	0.35	-	0.6

Table 3.7: The substitution matrix for the second level

	4^{th} preference			
3^{rd} preference	$P1$	$P2$	$P3$	$P4$
$P1$	-	0	0	1
$P2$	0	-	0	1
$P3$	0	0	-	1

Table 3.8: The substitution matrix for the third level

The optimal assortment provided by the modified model, which considers different substitution matrices for different levels, for this example is provided in Table 3.9 with total profit of $9,612$. In the new optimal assortment, compared to that of the single substitution matrix case, ordering amount of product $P1$ is higher and the total profit is lower. The reason for this is that since the substitution matrices for second and third levels have high lost sales probability, the system orders more in order not to pay more for lost sales.

	$P1$	$P2$	$P3$
x	3,800	-	7,000

Table 3.9: Optimal ordering quantities for products

3.6.5 Different Demand Scenarios with Different Probabilities

Continuing with the same example, we can consider different demand scenarios for individual products with different probabilities rather than assuming deterministic demand. There is not any change in all of the parameters, except we introduce two scenarios for demand as follows: for *scenario 1*, same as the deterministic demand case, the individual demands for products $P1$, $P2$, and $P3$ are $3,000$, $4,000$, and $5,000$, respectively, and for *scenario 2*, $2,500$, $4,300$, and $5,200$. The total demand in both of the scenarios are the same. The probability of *scenario 1* is 0.3, whereas the probability of *scenario 2* is 0.7.

The variables that need to be modified to handle these two scenarios are xs, $x0$, and $z1$, and we should add them one more index denoting the scenario. In addition, for this case, the objective function is maximization of the expected profit over scenarios. The generalized version of handling different demand scenarios with different probabilities when modeling the stochastic demand case in Section 4.2.

The optimal assortment provided by this model is provided in Table 3.10 with total profit of $7,130$.

	$P1$	$P2$	$P3$
x	2,930	-	7,350

Table 3.10: Optimal ordering quantities for products with different demand scenarios

Compared to the optimal solution provided for the original example, we observe in Table 3.10 that the system orders more and results in reduced profit. This observation about the effects of increasing demand variability will be generalized in Section 4.5.

3.6.6 Adding a New Product into the Assortment

An important decision should be taken when a new product is introduced to the market. While deciding on the entrance of such a new product into the assortment, retailers are interested in the profit margin of this product, its forecasted market share, its effect on the category sales, and the revenue that it is supposed to generate. In addition to these issues, incoming substitution rates to this product from other products is also another important concern since it affects the assortment sales amounts.

In this subsection, continuing with the same example, we want to analyze new product introduction and concentrate on the issues about profit margins, category sales, and incoming substitution rates. Related to the position of this new product in the whole category, we consider three cases:

1. *New Product with Higher Incoming Substitution Rate:* The profit margin of the new product is not worse than any product in the assortment, but the rate of substitutions to this product is more than that of the lowest profit margin product in the assortment.

2. *New Product with Higher Profit Margin:* The profit margin of the new product is greater than the lowest profit margin product in the assortment, but the rate of substitutions to this product is the same as that of the lowest profit margin product in the assortment.

3. *New Product that Increases Category Sales:* Including the new product in the assortment increases the demand for the category and the profit margin of the new product. The rate of substitutions to this product are same as that of the lowest profit margin product in the assortment.

3.6.6.1 Case 1: New Product with Higher Incoming Substitution Rate

In the optimal assortment we have two products $P1$ and $P3$. The profit margins of these products are $9 = (p_{P1} - c_{P1})$ for $P1$ and $6 = (p_{P3} - c_{P3})$ for $P3$. Then, let us assume that the new product, say P^*, is being introduced as a competitor for product $P3$ and therefore, has a profit margin of 6. As a competitor of $P3$, P^* is introduced by supplier $S1$. Considering that the rate of substitutions to P^* is more than $P3$, assume that according to survey data the new substitution matrix is as provided in Table 3.11.

	$i+1^{st}$ preference				
i^{th} preference	$P1$	$P2$	$P3$	$P4$	P^*
$P1$	-	0.05	0.10	0.7	0.15
$P2$	0.09	-	0.225	0.3	0.385
$P3$	0.05	0.25	-	0.4	0.3
P^*	0.05	0.25	0.3	0.4	-

Table 3.11: The new substitution matrix - parameter w

Note that the previous lost sales rates are not changed and the substitution rates are adjusted so that P^* has more incoming substitution rates than $P3$ and the proportions of substitution rates of the initial products remain as they were. For instance; before the introduction of P^*, the lost sales rate of $P1$ was 0.7 and it does not change after the introduction of P^*. Before the introduction of P^*, the substitution rate from $P1$ to $P3$ was two times the substitution rate from $P1$ to $P2$ and it does not change after the introduction of P^*. In addition, the substitution rate from $P1$ to $P3$ is less than the substitution rate from $P1$ to P^*.

The optimal assortment for this new setting is shown in Table 3.12.

	$P1$	$P2$	$P3$	P^*
x	-	4,833	-	3,400

Table 3.12: Optimal ordering quantities for products

Note that, in the previous assortment only supplier $S2$ was selected, in this assortment only supplier $S1$ is selected and the assortment is changed completely. Therefore, the system prefers the products that have more incoming substitution rates when profit margins are same.

3.6.6.2 Case 2: New Product with Higher Profit Margin

In this case, the new product has higher profit margin than the product with the lowest profit margin in the assortment. In the optimal assortment we have two products $P1$ and $P3$. Then, let us assume that the new product, say P^*, has a profit margin of 7. Again, P^* is introduced by supplier $S1$. Considering that the rate of substitutions to P^* is the same as that of $P3$, assume that according to survey data the new substitution matrix is as provided in Table 3.13.

	$i+1^{st}$ preference				
i^{th} preference	$P1$	$P2$	$P3$	$P4$	P^*
$P1$	-	0.06	0.12	0.7	0.12
$P2$	0.117	-	0.2915	0.3	0.2915
$P3$	0.0545	0.2725	-	0.4	0.2725
P^*	0.0545	0.2725	0.2725	0.4	-

Table 3.13: The new substitution matrix

Note that the previous lost sales rates are not changed and the substitution rates are adjusted so that P^* has the same substitution rates with $P3$ and the proportions of substitution rates of the initial products remain as they were. For instance, before the introduction of P^*, the lost sales rate of $P1$ was 0.7 and it does not change after the introduction of P^*. Likewise, before the introduction of P^*, the substitution rate from $P1$ to $P3$ was two times the substitution rate from $P1$ to $P2$ and it does not change after the introduction of P^*. In addition, the substitution rate from $P1$ to $P3$ is the same as the substitution rate from $P1$ to P^*.

The optimal assortment for this new setting is shown in Table 3.14.

Again, the assortment is changed completely and instead of supplier $S2$, supplier $S1$ is selected. Therefore, the system prefers the products that have more profit margins when incoming substitution are the same.

	$P1$	$P2$	$P3$	P^*
x	-	5,080	-	3,345

Table 3.14: Optimal ordering quantities for products

3.6.6.3 Case 3: New Product that Increases Category Sales

In this case, the new product increases the category sales and has the same profit margin and incoming substitution rates as the product with the lowest profit margin in the assortment. Then, let us assume that the new product, say P^*, has a profit margin of 6. Again, P^* is introduced by supplier $S1$. Since the rate of substitutions to P^* is the same as that of $P3$, assume that the new substitution matrix is as provided in Table 3.13. According to the survey results, the category demand is expected to increase by 5 %.

The optimal assortment for this new setting is shown in Table 3.15.

	$P1$	$P2$	$P3$	P^*
x	-	5,334	-	3,512

Table 3.15: Optimal ordering quantities for products

Again, the assortment is changed completely and instead of supplier $S2$, supplier $S1$ is selected. Therefore, the system prefers the products that increases category demand when profit margins and incoming substitution are same.

This example illustrates that using our model, retailers can decide on new product introduction, given the estimate of the profit margin of the product, substitution rates from/to this product, and its expected effect on the category demand.

3.7 The Performance of the Model - Advantages and Limitations

As seen in the formulation of the model, there exist demand realizations, denoted by d, which specify the individual demands for that realization, and the probabilities of the demand realizations, denoted by $dF(d)$. Therefore, for any realization of the total demand of the product category (except the case when the total demand is equal to zero), there exist more than one demand realization for that product category, in which the total demand is distributed in different ways. If we consider each demand realization as a scenario, then the set of possible scenarios and their probabilities should be provided as input for the model. However, when the total demand is defined as a random variable with a probability distribution function and the product preference proportions/frequencies are given, it is hard to generate demand scenarios and their probabilities. In such cases, in the model defining the total demand as a random variable with given probability distribution function and distributing the total demand to individual products according to given product preference proportions/frequencies is an approximation of the real case and facilitates the implementation of random demand in the model. By this approximation, for each realization of the total demand, rather than having more than one demand scenario with different probabilities, in which the total demand is distributed in different ways, we only consider one scenario, in which the total demand is distributed according to given frequencies. In this section, we investigate whether this approximation may cause significant profit loss with a benchmark comparison with the computational results of Smith and Agrawal (2000), in which the demand is modeled as originating from a random number of arriving customers, who select randomly with known frequencies from the product category. We compare the solutions proposed by our model for the same example set analyzed by Smith and Agrawal (2000) with the results generated by the model of Smith and Agrawal (2000).

In the following subsections, first, their methodology is summarized, then, the solutions proposed by our model for the same example set analyzed by Smith and Agrawal (2000) are provided and compared with the results generated by the model of

Smith and Agrawal (2000). Finally, according to these comparisons, the advantages and limitations of our model are explained. During our analysis, we used GAMS with Cplex 9.1 as the computational environment.

3.7.1 The Solution Methodology of Smith and Agrawal (2000)

As briefly summarized in Section 2.2, Smith and Agrawal (2000) studied the assortment planning problem with multi-period base-stock inventory models. They constructed a customer demand model in which the demand per period originates from a random number of arriving customers, who wish to purchase exactly one product of a particular type. The probability that an arriving customer initially prefers product i from a choice set of products, E , where $E = 1, 2, ..., n$, is denoted by f_i, and the probability that a customer, who initially prefers product i and sees that it is unavailable, switches to product j is denoted by α_{ij}. Then, a lost sales for an initial choice product i occurs with probability $L_i = 1 - \sum_j \alpha_{ij}$. This model of substitution is known as the logit model (Guadagni and Little (1983)) and is widely used in marketing studies. According to this model, when the products in a set $R \subset E$ are removed, the substitution probabilities are: $\alpha_{ij} = f_i(1 - L_i)/\sum_{k \notin R} f_k$, where $i \in R$, $j \notin R$.

In their model, D is expressed as the total number of customers arriving per period for products in E, where $\psi(d) = P\{D = d\}$, meaning that the probability that D equals d is given by $\psi(d)$. They assume that for a total demand of $D = d$, the total number of customers per period initially preferring product i, which is denoted by $D_i(f_i)$, has a binomial distribution. Then, the probability distribution for the initial demand for product i is expressed as $\psi(d_i|f_i) = \sum_{d=d_i}^{\infty} \binom{d}{d_i} f_i^{d_i}(1 - f_i)^{d-d_i}\psi(d)$.

By using this model, they decide on either stocking a product i or not, which is expressed by variable x_i, and the inventory level for product i at the beginning of each period. They assume that in a given set of products to stock, specified by x, the base stock level q_i is set to achieve a fixed service level $r_i = P\left\{\bar{D}_i(q, x) \leq q_i\right\}$, where $\bar{D}_i(q, x)$ stands for the demand for product i, given the inventory policy q, x. As a result of this assumption, for given service levels $r_1, ..., r_n$, they determine q_i by using the inequalities: $P\left\{\bar{D}_i(x) \leq q_i\right\} \geq r_i$ and $P\left\{\bar{D}_i(x) \leq q_i - 1\right\} \leq r_i$.

Before they construct their model, they observe that the likelihood of substitution due to stockouts increases as the cycle progresses. Therefore, they focus on the number of previous arrivals and denote the probability that product i is demanded by m^{th} arrival, either as an initial preference or as a substitute, by $g_i(x, m)$. As a result of their assumption on service level, $r_i \leq P\{last\ arriving\ customer\ finds\ i\ still\ in\ stock\}$, then, they indicate that $f_i + \sum_{j \neq i} f_j(1 - x_j)\alpha_{ji} \leq g_i(x, m) \leq f_i + \sum_{j \neq i} f_j(1 - x_j r_j)\alpha_{ji}$. They use the lower bound of $g_i(x, m)$, which is denoted by $h_i(x)$ and $h_i(x) = g_i(x, 1)$, as an approximation for its value in computing the demand distribution for $\bar{D}_i(x)$. Therefore, the probability distribution for the demand for product i is now expressed as $\psi(d_i | h_i(x)) = \binom{N+d_i-1}{N-1} y_i^N (1 - y_i) d_i$ where $y_i = \frac{p}{p + h_i(x)(1-p)}$, $d = 0, 1, 2, ...,$ and negative binomial distribution is used.

As the parameters of the model, V_i denotes the fixed cost associated with stocking product i, m_i denotes the unit profit margin for product i, c_{oi} denotes the unit overage cost for product i's inventory left at the end of period, and c_{ui} denotes the unit underage cost for product i, including m_i and possibly other factors. Then, for the negative binomial demand distribution, using $h_i(x)$ as an approximation in computing product demands, the expected profit per period is written as

$$\pi(q, x) = \sum_i \pi_i(q_i, x) x_i - V_i x_i, \tag{3.24}$$

where $\pi_i(q_i, x) = m_i \mu_i(x) - c_{oi} \sum_{d=0}^{q_i} (q_i - d)\psi_i(d | h_i(x)) - c_{ui} \sum_{d=q_i}^{\infty} (q_i - d)\psi_i(d | h_i(x))$.

They rearrange terms and find that

$$\pi(q, x) = (m_i - c_{ui})\mu_i(x) + (c_{ui} + c_{oi}) \sum_{d=0}^{q_i - 1} r_i - \Psi_i(d | h_i(x)) \tag{3.25}$$

By taking the first differences with respect to q_i, the first order necessary conditions are $\Psi_i(q_i | h_i(x)) \geq r_i$[1] and $\Psi_i(q_i - 1 | h_i(x)) \leq r_i$. Then, each q_i can be written as $q_i(h_i(x))$, $i = 1, ..., n$, and then be substituted into Equation 3.25 to obtain an

[1] Ψ denotes the cumulative probability distribution for the demand for product i

optimization problem in x only. Since total profit for all products can be maximized by maximizing profit of each product individually due to Equation 3.24, the problem turns out to be maximization of $\pi(q(x), x)$ with respect to x. Since $x_i = 0, 1$, for all i, Equation 3.24 is a nonlinear integer programming problem. For small values of n - number of products-, solution by enumeration of all combinations of the x_i is possible. For larger values of n, Equation 3.24 is approximated by a $0, 1$ linear programming problem.

Smith and Agrawal (2000) analyzes this solution methodology by using a set of numerical examples. These examples and the results obtained by our model for these examples are provided in the following subsection. The results of Smith and Agrawal (2000) are provided in Appendix A.

3.7.2 The Results of Our Model for The Examples of Smith and Agrawal (2000)

All examples have 5 substitutable products. The total number of customers per cycle has a negative binomial distribution with $N = 3$ and $p = 1/6$, i.e., a mean of 15 and a standard deviation of 9.5. The shortage costs c_{ui} are set to the profit margins m_i and all products have the same target in-stock probability r and fixed cost V. In the first example set, all products have identical costs, profit margins, and initial preferences. In the second example set, products have different profit margins m_i. In the third example set, products have different initial preferences f_i.

Three different substitution matrices (α_{ij}) shown below are evaluated and compared to the case $L = 1$, that corresponds to no substitution, where L denotes the lost sales probability. The *Random Substitution Matrix* describes the case in which all products have equal market shares resulting in equal substitution rates. The *Adjacent Substitution Matrix* describes the case in which products are rank ordered by customers according to some criterion, and in case of an unavailability in first choice product, customers choose a product that is adjacent in rank to their first preference. The *One-Item Substitution Matrix* describes the case where one product is a common second choice for all customers.

The Random Substitution Matrix

$$\begin{pmatrix} 0 & \frac{1-L}{n-1} & \frac{1-L}{n-1} & \frac{1-L}{n-1} & \frac{1-L}{n-1} \\ \frac{1-L}{n-1} & 0 & \frac{1-L}{n-1} & \frac{1-L}{n-1} & \frac{1-L}{n-1} \\ \frac{1-L}{n-1} & \frac{1-L}{n-1} & 0 & \frac{1-L}{n-1} & \frac{1-L}{n-1} \\ \frac{1-L}{n-1} & \frac{1-L}{n-1} & \frac{1-L}{n-1} & 0 & \frac{1-L}{n-1} \\ \frac{1-L}{n-1} & \frac{1-L}{n-1} & \frac{1-L}{n-1} & \frac{1-L}{n-1} & 0 \end{pmatrix} \tag{3.26}$$

The Adjacent Substitution Matrix

$$\begin{pmatrix} 0 & 1-L & 0 & 0 & 0 \\ \frac{1-L}{2} & 0 & \frac{1-L}{2} & 0 & 0 \\ 0 & \frac{1-L}{2} & 0 & \frac{1-L}{2} & 0 \\ 0 & 0 & \frac{1-L}{2} & 0 & \frac{1-L}{2} \\ 0 & 0 & 0 & 1-L & 0 \end{pmatrix} \tag{3.27}$$

The One-Item Substitution Matrix

$$\begin{pmatrix} 0 & 0 & 1-L & 0 & 0 \\ 0 & 0 & 1-L & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-L & 0 & 0 \\ 0 & 0 & 1-L & 0 & 0 \end{pmatrix} \tag{3.28}$$

In the following subsections the results generated by our model for all three example sets with these different substitution cases and the no substitution case are provided. In order to compare these results with that of Smith and Agrawal (2000), the results of Smith and Agrawal (2000) are provided in Appendix A.

3.7.2.1 Example Set 1: Identical Products

The following parameter values are used for this example set.

Mean total demand per period (per store)	μ	15
Initial preference probabilities	f_i	0.2, $i = 1, ..., 5$
Critical ratio (target in-stock probability)	r	0.95
Unit profit margin	m_i	5, $i = 1, ..., 5$
Fixed cost per cycle of stocking product i	V	10
$P\{lostsale \mid productisunavailable\}$	L	0.2

Table 3.16: Example Set 1 Parameter Values

The optimal results generated by our model for the three different substitution matrices and the no substitution case are provided in Table 3.17.

Products	No Subs. Assort. Size	Random Subs. Assort. Size	Adjacent Subs. Assort. Size	One-Item Subs. Assort. Size
1	8	12		
2	8	12	22	
3	8	12		42
4	8	12	22	
5	8			
Opt. Profit	16.4	22.6	37.6	45.1

Table 3.17: Optimal assortment generated by our model for Example Set 1

3.7.2.2 Example Set 2: Products with Different Profit Margins

This example set has the same parameter values with the Example Set 1, except for the fixed cost and unit profit margins, which are different for each product. The fixed costs are set to zero in order to focus only on the effect of different profit margins.

Unit profit margin	$m_1 = 5$, $m_2 = 4$, $m_3 = 3$, $m_4 = 2$, $m_5 = 1$
Fixed cost per cycle of stocking product i	$V = 0$

Table 3.18: Example Set 2 Parameter Values

The optimal results generated by our model for the three different substitution matrices and the no substitution case are provided in Table 3.19.

Products	No Subs. Assort. Size	Random Subs. Assort. Size	Adjacent Subs. Assort. Size	One-Item Subs. Assort. Size
1	8	9	8	8
2	8	9	9	8
3	7	9	8	30
4	7	8	12	
5	6	8	4	
Opt. Profit	41.1	40.9	40.4	44.6

Table 3.19: Optimal assortment generated by our model for Example Set 2

3.7.2.3 Example Set 3: Products with Different Initial Preferences

This example set also has the same parameter values for Example Set 1, except for the initial probabilities, which are different for each product. The fixed costs are set to zero in order to focus only on the effect of different profit margins.

Initial preference probabilities	$f_1 = 0.5,\ f_2 = 0.2,\ f_3 = 0.15,\ f_4 = 0.1,\ f_5 = 0.05$

Table 3.20: Example Set 3 Parameter Values

The optimal results generated by our model for three different substitution matrices and the no substitution case are provided in Table 3.21.

Products	No Subs. Assort. Size	Random Subs. Assort. Size	Adjacent Subs. Assort. Size	One-Item Subs. Assort. Size
1	19	26	19	19
2	8	11	19	
3	6	8		27
4	4	6	10	
5				
Opt. Profit	23.4	30.3	33.4	40.6

Table 3.21: Optimal assortment generated by our model for Example Set 3

3.7.2.4 The Analysis of Results

In order to benchmark our model using the model of Smith and Agrawal (2000), we tailored our model to a similar setting. Specifically, we excluded supplier selection, shelf space limitations, poor quality procurement, and order quotas of suppliers from the model and considered the same objective function with that of Smith and Agrawal (2000). Therefore, the model of Smith and Agrawal (2000) constitutes a pessimistic benchmark for our model. In addition, the model of Smith and Agrawal (2000) is not guaranteed to provide the best solutions for the systems.

When the results generated by our model provided in Tables 3.17, 3.19, and 3.21 and the results of Smith and Agrawal (2000) provided in Appendix A are compared, we see that our model generates solutions with lower profit. The difference between the profits is %14.4 in the worst case and %7.5 in the average case. The median of the difference is %7.2. The reason for this difference is that, as stated in the first paragraph of Section 3.7, in our model, for each realization of total demand, rather than having more than one demand scenario with different probabilities, in which the total demand is distributed to individual product in different ways, we only consider one scenario, in which the total demand is distributed according to given frequencies. This easy approximation results in profit loss. In order to prevent the profit loss, the demand scenarios/realizations should be generated. According to these results, we can state that using the approximation for random demand does not result in high profit losses.

3.7.3 The Advantages and Limitations of Our Model compared to Smith and Agrawal (2000)

When we analyze the solution methodology of Smith and Agrawal (2000), we see that it has the following advantages:

- The demand is modeled as originating from a random number of arriving customers, who select randomly with known frequencies from the product category. Therefore, when the total demand is defined as a random variable with a probability distribution function and the frequencies are given, all demand realizations are considered.

- It gives an optimal solution for small values of n, the number of products.

However, this solution methodology has also some limitations such as:

- For large values of n, the solution methodology provides an approximate solution rather than an optimal solution.

- Even for small values of n, finding the optimal solution by enumerating all combinations of the $\{v_i\}$ is hard to implement.

- For small values of n, that is, when optimal solution is obtainable, adding new constraints, such as shelf space constraints, to the system makes the implementation harder. When the number of constraints increases, the complexity also increases.

Considering the advantages and limitations of Smith and Agrawal (2000), our solution methodology has also a limitation: in the cases when the total demand is defined as a random variable with a probability distribution function and the frequencies are given, in our model it is easier to define the total demand as a random variable with given probability distribution function and distribute the total demand to individual products according to given frequencies. However, it results in profit loss. In order to prevent this profit loss, each possible demand scenario should be generated, which is hard to implement.

Although our methodology has these limitations, it has the following advantages:

- Our method is much more practical for all values of n, much more understandable and easier to implement.

- It is simple to add new constraints, such as limitations on the number of store keeping units (SKU), to the system. The number of new constraints does not affect the complexity.

- In addition to adding new constraints, decision makers can modify the objective function. For example, new revenues or costs, such as supplier selection costs, can be added easily.

As a conclusion of this section, we can say that in general, both methodologies propose an approximate solution. However, our solution methodology is more flexible and practical when the changing requirements of retailers are considered.

Chapter 4

EXPERIMENTAL STUDY I : THE ANALYSIS FROM THE RESULTS OF THE MODEL

In this chapter, we analyze the results of the model for certain settings such as the multi-period problem with deterministic demand and the single-period problem with stochastic demand in an experimental study. We investigate the effect of ignoring supplier selection decision, the effect of changing substitution cost, the effect of ignoring substitution cost, and the effect of ignoring shelf space limitations. First, the analyses are performed for the multi-period problem with deterministic demand. Next, the single-period problem with stochastic demand is considered. In order to perform comparisons between the stochastic demand case and the deterministic demand case, analysis on single-period problem with deterministic demand is also performed.

4.1 *Analysis of the Multi-period Problem with Deterministic Demand*

In this section, first, the deterministic demand, multi-period case is analyzed to observe the effects of varying substitution costs in optimal order quantities. Second, the performance of a model, which neglects customers' substitution behavior is examined. Third, the analysis on the performance of another model, which excludes the supplier selection decision, is performed. Finally, the importance of shelf space constraints is analyzed.

With deterministic demand assumption, the parameter d_{it} represents the forecasted demand for product i in period t. In the model, Equations 3.11 and 3.12 are combined in Equation 4.1 :

$$TCS = \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{i \in P} \sum_{k \in P} s_{mi} xs_{mikt} \tag{4.1}$$

In the deterministic demand setting, it is possible to focus on demand variation over time, such as seasonality.

4.1.1 Experimental Data

In our experiments we generated instances with four time periods ($T = 4$), 10 products (with the 11^{th} product representing lost sales), and 5 suppliers. We assumed that customers perform at most 3 levels of substitution ($M = 3$). This is a reasonable value for the maximum substitution level since we can both consider multiple-way of substitution and also get rid of the complexity caused by extremely small numerals of substitution probabilities of higher substitution levels. In addition to this, for many customers, substitution becomes meaningless after the third attempt.

The substitution cost is assumed to be a linear function of the substitution level, m. That is, we let $s_{mi} = SC_i * m$, where SC_i denotes the first level substitution cost from product i. The first level substitution cost for a product i is a linear function of its margin, mg_i, and is calculated as $SC_i = \theta * mg_i$. We vary θ in our experiments.

The supplier-product availability matrix, A, is given in Table 4.1. Note that, products correspond to brands so that a product cannot be supplied by more than one supplier, but a supplier can supply more than one product/brand.

	Product									
Supplier	1	2	3	4	5	6	7	8	9	10
1	1	0	1	0	1	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	1
3	0	0	0	1	0	0	0	0	0	0
4	0	0	0	0	0	1	1	1	0	0
5	0	0	0	0	0	0	0	0	1	0

Table 4.1: The supplier-product availability matrix, A

The parameter values are generated according to Table 4.2. We generate 100 random data sets according to the provided distributions. The average of investigated values over these 100 data sets are provided as test results. GAMS with Cplex version 9.1 is used as the computational environment.

Parameter	Distribution
w_{ik}	Uniform distribution where $\sum_{k \in P} w_{ik} = 1$ and $0 \leq w_{ik} \leq 1$, $\forall i, k \in P$
c_i	Uniform distribution, where $5 \leq c_i \leq 10$, $\forall i \in P$
oc_j	Uniform distribution, where $30 \leq oc_j \leq 50$, $\forall j \in S$
ssc_j	Uniform distribution, where $15,000 \leq ssc_j \leq 50,000$, $\forall j \in S$
d_{it}	$d^{it} = 10,000 * \alpha_i$, where $\sum_i d_{it} = 10,000$ and $0 \leq \alpha_i \leq 1$, $\forall i \in P$, $\forall t = 1,..,T$ (Total demand for products is assumed to be $10,000$ for each period.)
OQ_i	Uniform distribution, where $1,000 \leq OQ_i \leq 8,500$, $\forall i \in P$
SS_i	Uniform distribution, where $2,000 \leq SS_i \leq 10,000$, $\forall i \in P$
h_i	Uniform distribution, where $0.3 \leq h_i \leq 1$, $\forall i \in P$
pq_i	Uniform distribution, where $2 \leq pq_i \leq 4$, $\forall i \in P$
q_i	Uniform distribution, where $0.0 \leq q_i \leq 0.15$, $\forall i \in P$
p_i	$p_i = c_i + mg_i$, where mg_i has a normal distribution with mean 6 and deviation 2, $\forall i \in P$. (Price for products is assumed to be constant over all periods and mg_i can be considered as the margin of product i.)

Table 4.2: Experimental Data for multi-period problem with deterministic demand

4.1.2 The Impact of Changing Substitution Costs

The model is solved for different θ values that result in different substitution costs. In these experiments it is observed that all types of costs, total revenue, and the percentage of demand satisfied are sensitive to the substitution cost changes. Table 4.3 shows the results obtained for $0.0 \leq \theta \leq 1$. In the table, TP is used for the optimal total profit, %ds denotes the average percentage of demand satisfied with the original product, %ls is the average percentage of lost sales, %f, %s, %t are the average percentage of substituted demand to all other products in the first, second and third level of substitutions, respectively, %subs denotes the average percentage of sum of all levels of substitutions, where $\%subs = \%f + \%s + \%t$, TR is the total revenue, TCO is used for the total cost of ordering, TCSS is the total cost of supplier selection, TCP denotes the total cost of purchasing, TCI is used for the total cost of inventory, TCPQ denotes the total cost of poor quality parts, TCS is the total cost of substitution, and SOC is used for the sum of operating costs.

θ	TP	%ds	%ls	%f	%s	%t	%subs	TR	TCO	TCSS	TCP	TCI	TCPQ	TCS	SOC
0.0	239,669	36.2	4.8	26	20.5	12.6	59	828,434	535	101,523	440,845	33,009	12,853	0	588,765
0.1	224,871	58.5	3.6	21.3	11.1	5.5	37.9	833,071	544	103,658	445,178	34,502	13,029	11,291	608,200
0.2	215,651	66.2	2.7	19.6	7.9	3.5	31	845,000	570	109,905	454,286	35,548	13,315	15,724	629,349
0.3	208,683	70.7	2.3	18.3	6.2	2.5	27	850,293	580	113,116	459,910	36,105	13,443	18,456	641,610
0.4	203,249	75.2	2.1	16.4	4.6	1.7	22.7	855,833	600	117,130	465,437	36,784	13,678	18,956	652,584
0.5	198,979	78.7	1.7	14.8	3.6	1.2	19.6	860,864	616	121,755	469,922	37,415	13,871	18,306	661,885
0.6	195,716	81.7	1.7	13.1	2.6	0.8	16.6	862,191	632	124,911	472,070	37,744	13,969	17,148	666,475
0.7	193,197	83.9	1.9	11.7	1.9	0.6	14.2	862,047	641	126,721	473,116	38,023	14,006	16,344	668,850
0.8	190,984	85.7	1.9	10.5	1.4	0.4	12.4	863,121	649	128,937	474,923	38,176	14,048	15,404	672,137
0.9	189,223	86.6	1.9	9.9	1.3	0.4	11.5	862,380	654	129,657	474,555	38,185	14,012	16,094	673,157
1	187,330	88.1	1.9	8.7	1.0	0.3	10	865,640	667	132,546	477,131	38,495	14,146	15,325	678,310

Table 4.3: The effect of changing substitution cost in deterministic demand, multi-period case

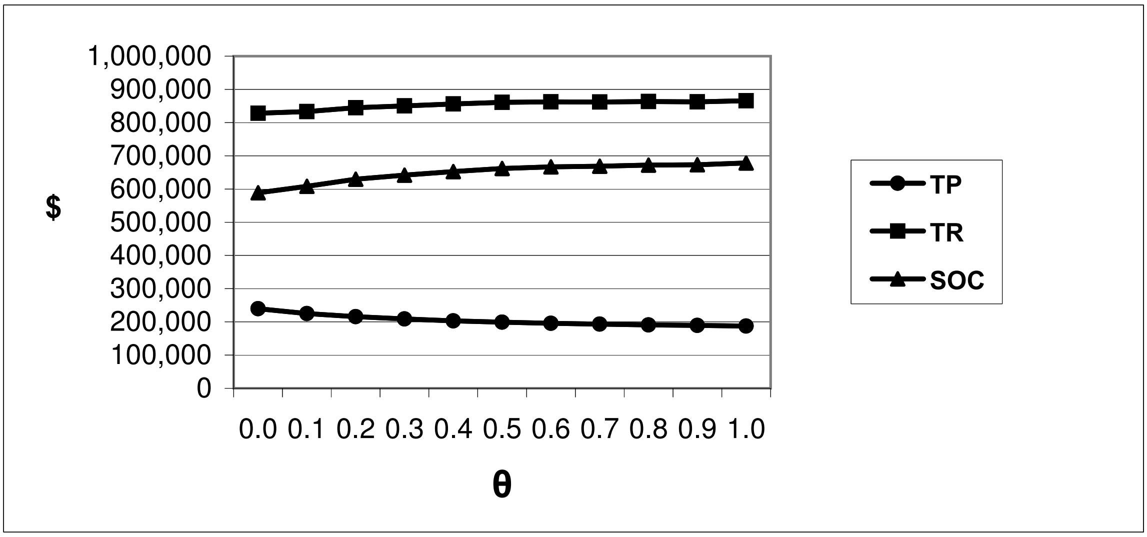

Figure 4.1: Change in the profit, revenue, and operating costs as θ increases

The effect of substitution cost on system parameters such as total revenue (TR), total profit (TP), and total operating costs (SOC) is shown in Figure 4.1, Figure 4.2, and Figure 4.3 are provided.

The optimal system has limited assortment with $\theta = 0$, selecting a small subset of suppliers and favors substitution since it generates no cost, as shown in Table 4.3. During the analysis, we set high supplier selection costs in order to observe this effect. When $\theta \geq 0.1$, the optimal system extends its assortment, increasing the number of selected suppliers. Therefore, supplier selection costs increase as seen in Figure 4.2. This shows that substitution costs significantly affect ordering and supplier selection decisions.

As it is expected, the increase in the substitution and lost sales costs results in satisfying more demand and decreasing substitutions, especially for higher levels of substitution, as seen in Figure 4.3. In order to achieve this, the system purchases more, resulting in increased inventory. Therefore, purchasing and inventory costs increase as observed in Figure 4.2. This shows that substitution costs affect purchasing and inventory decisions, as well. Hence, it is important to estimate the substitution cost accurately.

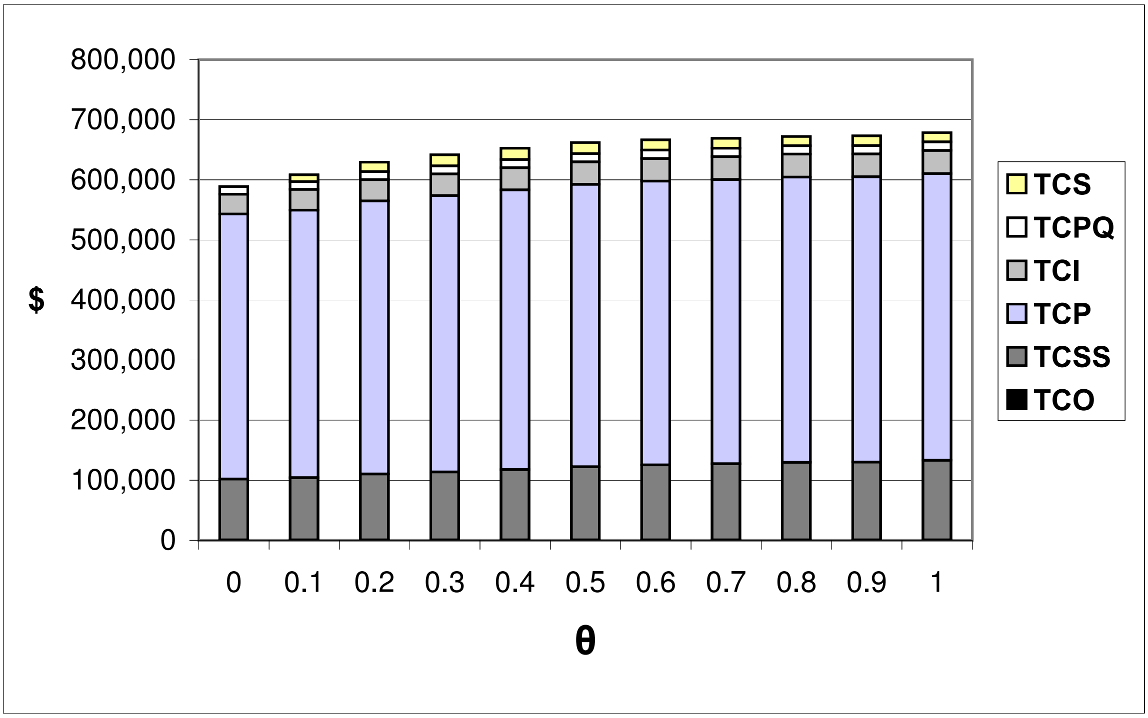

Figure 4.2: Change in each operating cost as θ increases

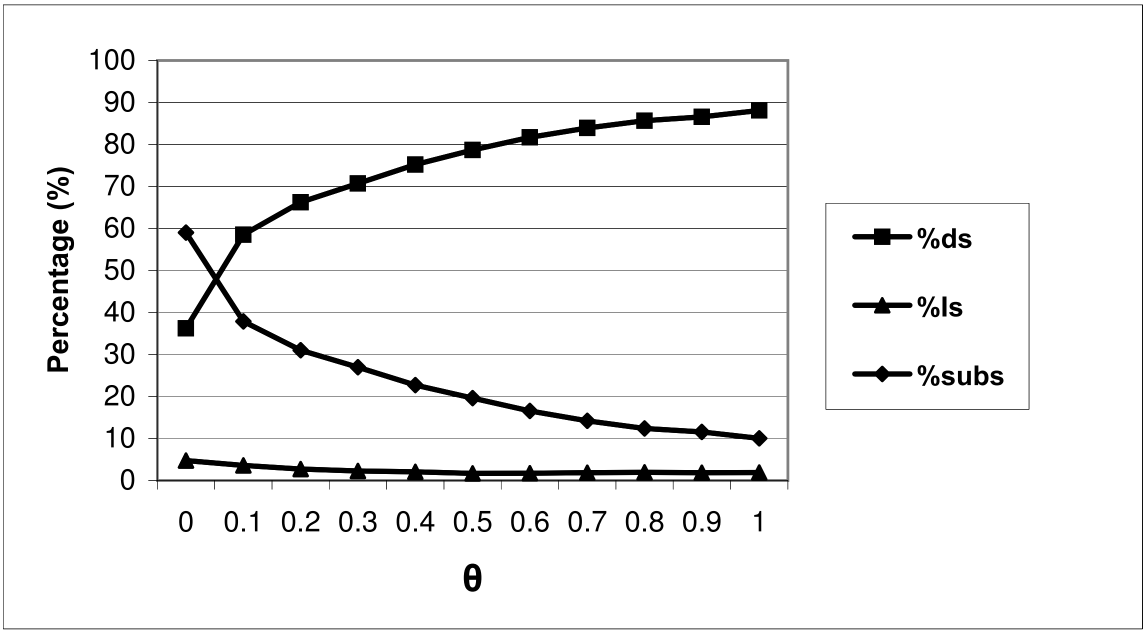

Figure 4.3: Change in the percentages of satisfied demand, lost sales and substitution as θ increases

The decision maker may vary θ, and observe the change in profit as well as customer service level in terms of the percentage of demand satisfied as shown in Figures 4.1 and 4.3. In Figure 4.1, it is observed that as substitution costs increase, total revenue increases, however, the sum of operational costs increases more than total revenue, resulting in decreased total profit.

By using Figure 4.2, we can also state that the desired service level might be used set the value of the parameter θ as will be discussed in Section 5.2. In addition, we should note that the value of θ depends on the product category and which customer segment the retailer attracts, in terms of their product assortment expectations.

4.1.3 The Importance of Substitution Behavior

In order to analyze the importance of considering customers' substitution behavior in this problem, the total cost of substitution (TCS) is excluded from the objective function and a modified model is obtained. Next, the total profit is calculated by subtracting the total cost of substitution from the optimal objective value of modified model.

In this way, we compared the optimal total profit of the model without substitution costs with that of the original model. Since the optimal solution of the original model is also feasible for the model without substitution costs, the optimal total profit of the former is at least as good as the latter. Then, it is clear that excluding substitution costs cannot provide better results. Table 4.4 shows the results obtained with the same experimental data used in Section 4.1.1. In the table, [TP] is used for the optimal total profit of the original model, [TP w/o sc] is given for the optimal total profit of the model without substitution cost, [Difference] denotes the difference between these values, where Difference = [TP] - [TP w/o sc], and [%Difference] is the percentage of the difference compared to the optimal total profit of the original model, where %Difference = $\frac{([TP]-[TPw/osc])*100}{[TP]}$. In order to illustrate the results of Table 4.4 graphically, Figure 4.4 is provided.

The results show that when substitution costs increase, neglecting customers'

θ	[TP]	[TP w/o sc]	[Difference]	[% Difference]
0	239,669	239,669	0	0
0.1	224,871	217,406	7,465	3.3
0.2	215,651	195,144	20,508	9.5
0.3	208,683	172,881	35,802	17.2
0.4	203,249	150,619	52,630	25.9
0.5	198,979	128,357	70,622	35.5
0.6	195,717	106,094	89,622	45.8
0.7	193,197	83,832	109,365	56.6
0.8	190,984	61,569	129,415	67.8
0.9	189,223	39,307	149,916	79.2
1	187,330	17,045	170,285	90.9

Table 4.4: The effect of substitution behavior in deterministic demand, multi-period case

substitution behavior results in increased profit loss which cannot be neglected as seen in Figure 4.4.

4.1.4 The Importance of Supplier Selection Decision

In order to analyze the importance of supplier selection in this problem, the total cost of supplier selection (TCSS), which is equal to the sum of supplier selection costs for each selected supplier, is excluded from the objective function and a modified model is obtained. Similar to the previous case, in order to obtain the total profit, the total cost of ordering of selected suppliers is subtracted from the optimal objective value of the modified model.

We compared the optimal total profit of the model without supplier selection decision with that of the original model. As before, it is clear that, excluding supplier selection decision cannot provide better results. When the same experimental data

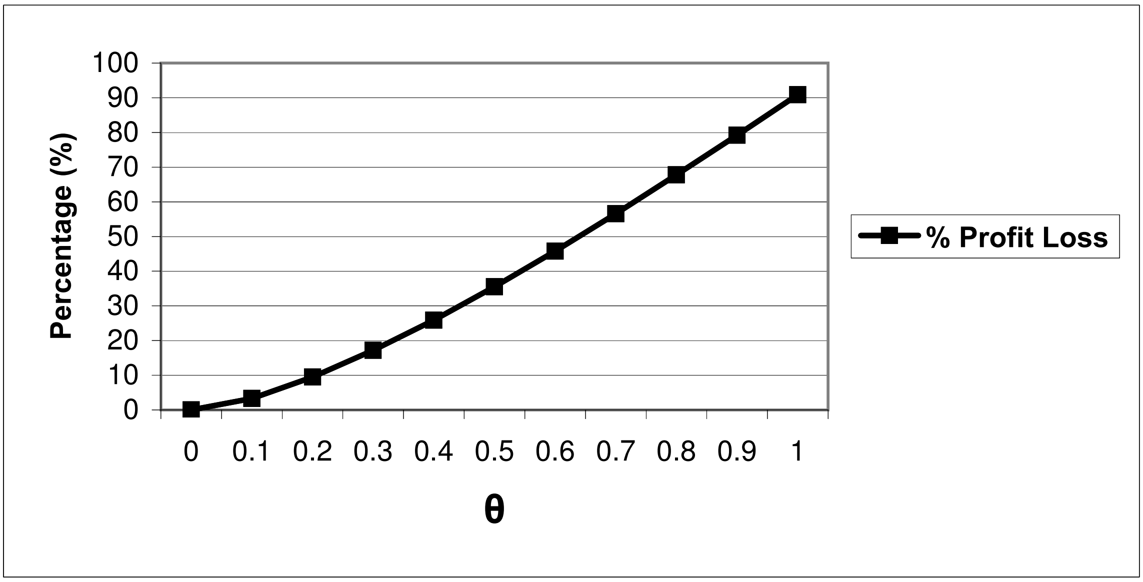

Figure 4.4: Change in the percentage of profit loss as θ increases in case of ignored substitution behavior

of Section 4.1.1 is used, we observe that excluding supplier selection decision results in full assortment, that is, all of the suppliers are selected, hence, profit decreases. Table 4.5 shows these results. In the table, [TP] is used for the optimal total profit of the original model, [TP w/o ss] is given for the optimal total profit of the modified model, which excludes supplier selection decision, [Difference] denotes the difference between these values, where Difference = [TP] - [TP w/o ss], and [%Difference] is the percentage of the difference compared to the optimal total profit of the original model, where %Difference = $\frac{([TP]-[TPw/oss])*100}{[TP]}$.

In order to illustrate the results of Table 4.5 graphically, Figure 4.5 is provided. According to the Figure 4.5, it is observed that as substitution costs increase, the profit loss due to ignoring supplier selection decision decreases, in other words, the importance of integrated supplier selection decreases. The reason of this is that as substitution costs increase, the percentage of the total substitution cost in total operating costs increases so much that the percentage of total supplier selection cost in total operating costs decreases. As a result of this, the importance of integrated

θ	[TP]	[TP w/o ss]	[Difference]	[% Difference]
0	239,668	212,035	27,634	11.5
0.1	224,871	198,511	26,360	11.7
0.2	215,651	191,477	24,175	11.2
0.3	208,683	186,844	21,840	10.5
0.4	203,249	183,655	19,594	9.6
0.5	198,979	181,458	17,520	8.8
0.6	195,716	179,862	15,854	8.1
0.7	193,197	178,646	14,551	7.5
0.8	190,984	177,699	13,285	7.0
0.9	189,222	177, 015	12,208	6.5
1	187,330	176,475	10,854	5.8

Table 4.5: The effect of supplier selection decision as substitution costs change

supplier selection decreases.

In addition, the effect of including supplier selection decision in the model is analyzed with increasing supplier selection costs. Table 4.6 shows the results obtained with the same experimental data used in Section 4.1.1 except that the interval for uniform distribution of supplier selection cost is changed during the analysis and θ is considered to be 0.3. This time, the number of selected suppliers for both problems are provided in the table for comparison purposes. In the table, I[ssc] is used for the supplier selection cost(ssc) interval, [N(S)] denotes the number of selected suppliers, and [N(S) w/o ss] is given for the number of selected suppliers in the model without supplier selection. Figure 4.6 is provided in order to illustrate the results of Table 4.6 graphically. As seen in Figure 4.6 when supplier selection costs increase, excluding supplier selection decision results in increased profit loss since neglecting supplier selection cost during decision making results in selecting more suppliers.

Another point that can be observed in Table 4.6 is that as the difference between

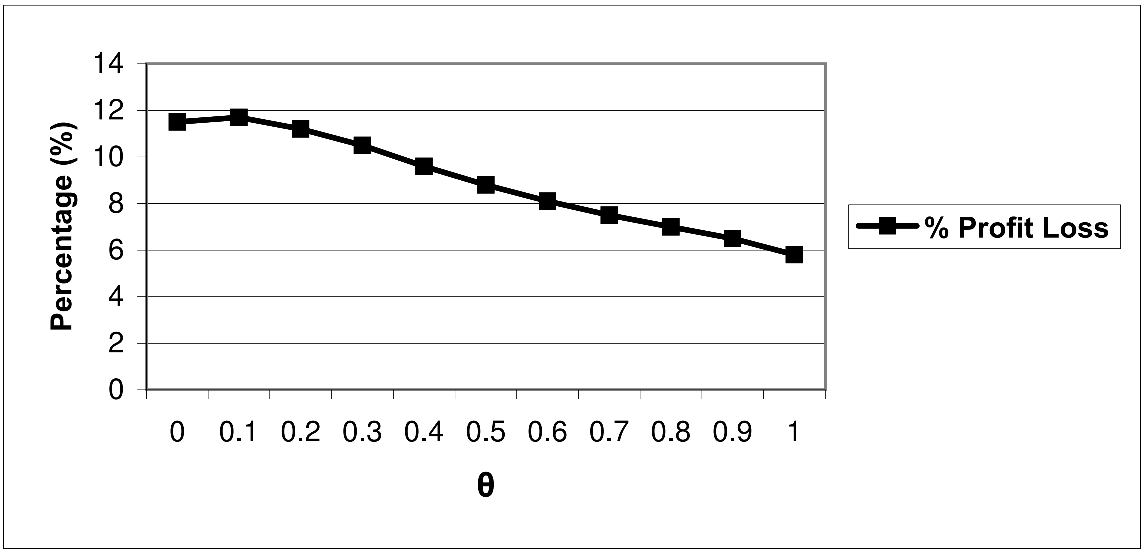

Figure 4.5: Change in the percentage of profit loss as θ increases in case of ignored supplier selection

I[ssc]	[TP]	[N(S)]	[TP w/o ss]	[N(S) w/o ss]	[% Difference]
$15,000 - 50,000$	208,688	3.6	186,844	5	10.5
$25,000 - 60,000$	208,688	3.6	136,844	5	34.4
$35,000 - 70,000$	173,590	3.3	86,844	5	50
$45,000 - 80,000$	142,777	2.9	36,844	5	74.2
$55,000 - 90,000$	115,301	2.6	-13,156	5	111.4
$65,000 - 1000,000$	90,993	2.3	-63,156	5	169.4

Table 4.6: The effect of supplier selection decision as supplier selection costs change

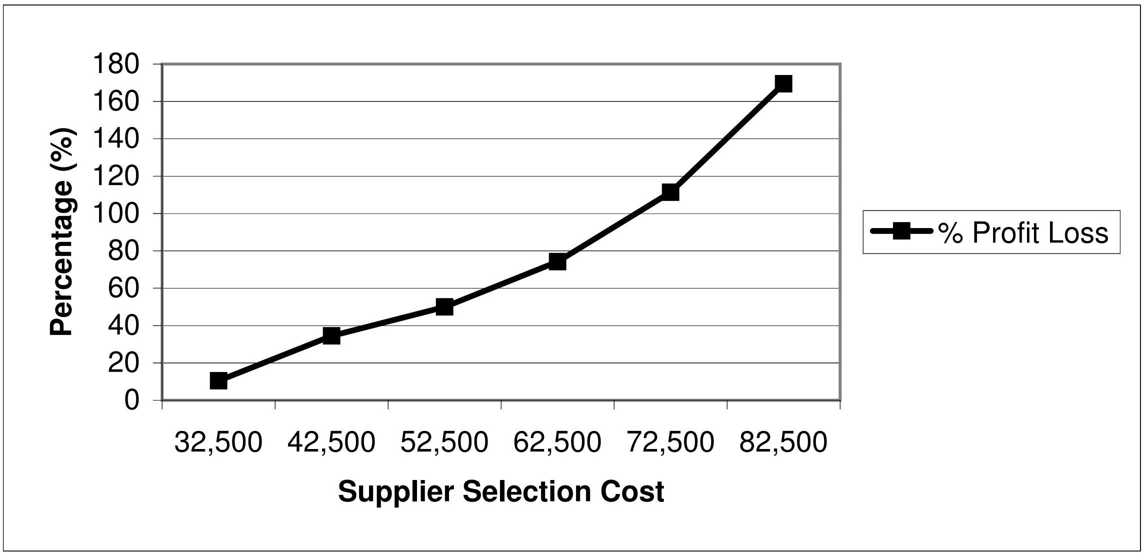

Figure 4.6: Change in the percentage of profit loss as supplier selection costs increase in case of ignored supplier selection

the number of suppliers that the model, which initially neglects supplier selection costs, selects and the number of suppliers that the original model selects increases, the profit loss increases if supplier selections costs are not considered initially. This implies that when supplier selection decision is not integrated in the assortment selection decision, as the number of suppliers increases, the importance of integrated supplier selection decision increases.

4.1.5 The Importance of Shelf Space Limitations

In order to analyze the importance of shelf space limitations in this problem, we first introduced an effective shelf space limitation of $8,000$ units for the category and found the optimal profit with this constraint. Then, we excluded the shelf space constraint from the model and found the optimal assortment for this new setting. Later, we applied the shelf space limitation to this assortment in both ways described in Section 3.6.3 and found the total profits that these optimal assortments generate with shelf space limitations. Lastly, we compared these profits with the total profit generated when shelf space limitations are included in the model. We performed these

The assortment	[TP]	[% Profit Loss]
The assortment of the original model	22,312	-
The assortment of the model w/o shelf space limitations applied with *method 1*	12,209	45.3
The assortment of the model w/o shelf space limitations applied with *method 2*	16,292	27

Table 4.7: The effect of shelf space limitations on total profit

with the same experimental data used in Section 4.1.1, where θ is considered to be 0.3, and we observed that excluding shelf space limitation in the model results in significant decreases in profit. Table 4.7 shows these results. In this table, the total profit of the assortment proposed by original model, the total profit of the assortment obtained by applying *method 1* to the optimal assortment of the model without shelf space constraints, and the total profit of the assortment obtained by applying *method 2* to the optimal assortment of the model without shelf space constraints are provided in the second column. The percentages of profit loss compared to the optimal total profit found by the original model are given in the third column.

According to these results, the assortments of the model without shelf space constraints either with *method 1* or *method 2* generate lower profit than the optimal assortment of the model with shelf space constraints generates. In addition, there exist high profit losses when shelf space limitations are not considered. When we analyzed the results, we see that the main deficiency of neglecting shelf space limitations is that in that case system selects more suppliers because of unlimited space and this incurs additional cost.

In order to see the effect of shelf space limitations on the number of selected suppliers, we also performed experiments on the original model and observed the number of suppliers for changing shelf space limitations. Table 4.8 shows the results obtained for the same experimental data except that the shelf space limitation is

Shelf Space Limitation [units]	[N(S)]
$8,000$	1.38
$20,000$	1.98
$32,000$	2.87

Table 4.8: The effect of shelf space limitations on the number of selected suppliers

changing. Again, θ is considered to be 0.3. In the table, [N(S)] denotes the average number of selected suppliers. The results show that as shelf space limitations becomes looser, the number of selected suppliers increases.

We can relate the results in Table 4.8 with the results obtained in Section 4.1.4. According to Table 4.6 when supplier selection decision is not integrated in the assortment selection decision, as the number of suppliers increases, the importance of integrated supplier selection decision increases. Therefore, it is implied that as shelf space limitations becomes looser, the importance of integrated supplier selection decision increases.

4.2 Analysis of the Single-period Problem with Stochastic Demand

As the next step of our analysis, we consider the stochastic demand, single-period case. This version of the problem results in modifications in decision variables and constraints. In order to model random demand, we can formulate the problem in a way that demand distributions are represented by a collection of random scenarios. The technique of generating random scenarios is equivalent to Monte Carlo sampling. The objective of the formulation is, then, to minimize the expected cost over the scenarios. Let B denote the number of possible demand realizations and β_b denote the probability of demand realization b such that $\sum_{b=1}^{B} \beta_b = 1$. We denote the demand for product i under the demand realization b by d_{ib}. In this new formulation, we have the following decision variables:

- $z1_{ib}$: ending inventory position of product i under demand realization b. [Z: vector representation]

- x_i : quantity of product i to be ordered per order. [X: vector representation]

- y_i : 1, if product i is ordered; 0, otherwise. [Y: vector representation]

- o_j : 1, if an order is placed with supplier j; 0, otherwise. (o_j also indicates whether supplier j is selected or not.)

- $x0_{ib}$: the amount of satisfied demand for product i under demand realization b.

- xs_{mikb} : the amount of product i used to satisfy m^{th} substitution from product k under demand realization b. ($m = 1, 2, ..., M$ where M is a constant)

The parameters that are modified are:

- $z0_i$: the initial inventory position of product i.

- p_i : unit price of product i.

The constraints are also modified as follows:

$$TR = \sum_{i \in P} \sum_{b=1}^{B} p_i \left(z0_i + x_i - z1_{ib} \right) \beta_b \tag{4.2}$$

$$TCO = \sum_{j \in S} oc_j o_j \tag{4.3}$$

$$o_j \geq a_{ij} y_i, \, \forall i \in P, \, \forall j \in S \tag{4.4}$$

$$TCSS = \sum_{j \in S} ssc_j o_j \tag{4.5}$$

$$TCP = \sum_{i \in P} c_i x_i \tag{4.6}$$

$$TCI = \sum_{b=1}^{B} \sum_{i \in P} \frac{(z0_i + x_i + z1_{ib})\beta_b}{2} h_i \tag{4.7}$$

$$TCPQ = \sum_{i \in P} pq_i q_i x_i \tag{4.8}$$

$$TCS = \sum_{b=1}^{B} \sum_{m=1}^{M} \sum_{i \in P} \sum_{k \in P} s_{mi} \beta_b xs_{mikb} \tag{4.9}$$

$$x0_{ib} + \sum_{m} \sum_{k \in P} xs_{mkib} = d_{ib},\ \forall i \in P,\ \forall b \in B \tag{4.10}$$

$$x0_{ib} + \sum_{m} \sum_{k \in P} xs_{mikb} + z1_{ib} = z0_i + x_i,\ \forall i \in P,\ \forall b \in B \tag{4.11}$$

$$xs_{1ikb} \le (d_{kb} - x0_{kb}) w_{ik},\ \forall i, k \in P \backslash \{i\},\ \forall b \in B \tag{4.12}$$

$$xs_{2ikb} \le (d_{kb} - x0_{kb} - \sum_{r \in P} xs_{1rkb}) \sum_{r \in P} w_{rk} w_{ir},\ \forall i \in P,\ \forall k \in P \backslash \{i\},\ \forall b \in B \tag{4.13}$$

$$\ldots^{1} \tag{4.14}$$

$$z0_i + x_i \le SS_i,\ \forall i \in P \tag{4.15}$$

$$0 \le x_i \le OQ_i y_i,\ \forall i \in P \tag{4.16}$$

$$y_i \in \{0, 1\},\ \forall i \in P \tag{4.17}$$

The same analysis explained in Section 4.1 is also performed for this case.

4.2.1 Experimental Data

In order to facilitate comparisons among different cases of the problem, experimental data for all cases are taken to be consistent with experimental data provided in Section 4.1.1. Again, 10 products (with the 11^{th} product representing the lost sales) and 5 suppliers are considered. Customers perform at most 3 levels of substitution ($M = 3$). Since in this case we consider single-period, distributions for some of the parameters are modified. The updated parameter distributions are shown in Table 4.9 .

In order to make relevant comparisons, demand data values of 100 data sets, which are generated initially, are considered as possible demand realizations. Thus, B is set to 100. Since we take the average of results during the previous analysis, here, we consider equal probability for each demand realization setting $\beta_b = 0.01$ for each b.

[1] Because of the complexity associated with higher substitution levels, we provide only the first two of the substitution inequalities.

Parameter	Description	Distribution
OQ_i	Order quantity quota of product i	Uniform distribution, where $4,000 \leq OQ_i \leq 34,000$, $\forall i \in P$
SS_i	Shelf space limitation quantity of product i	Uniform distribution, where $8,000 \leq SS_i \leq 40,000$, $\forall i \in P$

Table 4.9: Experimental Data for single-period problem with stochastic demand

During our analysis, we assume that the total demand for the product category is a random variable, with a known probability distribution function, and in order to reduce the computational complexity, we distribute the total demand to individual products according to given deterministic customer preference proportions, which is an approximation of the real case.

4.2.2 The Impact of Changing Substitution Costs

Table 4.10 shows the results obtained for $0.0 \leq \theta \leq 1$.

θ	TP	%ds	%ls	%f	%s	%t	%subs	TR	TCO	TCSS	TCP	TCI	TCPQ	TCS	SOC
0.0	199,711	48.3	3.7	23	16.2	8.8	48	978,934	835	151,027	565,876	43,609	17,876	0	779,223
0.1	190,110	67.5	2.6	18.3	7.9	3.7	29.9	1,019,464	944	153,058	605,780	44,672	17,889	7,011	829,354
0.2	185,194	74.3	2.0	16.0	5.6	2.7	24.3	1,095,064	980	165,905	669,286	45,568	17,987	10,144	909,870
0.3	182,266	77.0	1.6	14.8	4.2	2.4	21.4	1,110,036	1,050	170,896	679,810	47,005	18,003	11,006	927,770
0.4	181,089	80.2	1.2	14.0	3.4	1.2	18.6	1,139,104	1,200	178,030	700,437	48,004	18,108	12,136	958,015
0.5	180,009	83.1	1.0	13.1	2.1	0.7	15.9	1,149,834	1,261	179,999	709,976	48,112	18,171	12,306	969,825
0.6	178,716	85.1	1.0	12.0	1.4	0.5	13.9	1,174,248	1,332	184,111	732,078	48,994	18,249	10,768	995,532
0.7	176,997	87.6	0.9	10.2	1.0	0.3	11.5	1,209,345	1,488	187,687	766,116	49,123	18,296	9,644	1,032,348
0.8	175,987	89.2	0.9	9.0	0.7	0.2	9.9	1,238,356	1,549	188,037	786,175	49,356	18,348	8,904	1,062,369
0.9	173,843	90.4	0.7	8.3	0.6	0.2	9.1	1,278,200	1,660	205,789	818,875	49,985	18,459	9,589	1,104,357
1	170,990	91.3	0.7	7.3	0.5	0.2	8	1,300,004	1,671	214,746	835,031	50,075	18,496	8,995	1,129,014

Table 4.10: The effect of changing substitution cost in stochastic demand, single-period case

62

In order to illustrate the results of Table 4.10 graphically, Figure 4.7, Figure 4.8, and Figure 4.9 are provided.

Similar to the deterministic demand, multi-period case, with $\theta = 0$, the optimal system has limited assortment, selecting a subset of suppliers and favors substitution since it generates no cost. As θ increases, the optimal system extends the assortment, increasing the number of selected suppliers. Therefore, supplier selection costs increase as seen in Figure 4.8. Again, this shows that substitution costs significantly affect ordering and supplier selection decisions.

Increase in substitution and lost sales costs leads to increases in the amount of purchases, resulting in decreased amounts of substitution as seen in Figure 4.9 and increased inventory. Therefore, purchasing and inventory costs increase as seen in Figure 4.8. Again, this shows that substitution costs affect purchasing and inventory decisions, as well.

Similarly, in Figure 4.7, it is observed that as substitution costs increase, although total revenue increases, total profit decreases since the sum of operational costs increase more than total revenue.

4.2.3 The Importance of Substitution Behavior

The results shown in Table 4.11 and illustrated in Figure 4.10 are similar to the deterministic demand, multi-period case and show that increase in substitution costs increases profit loss if customers' substitution behavior is neglected.

4.2.4 The Importance of Supplier Selection Decision

Similar to the deterministic demand, multi-period case, in this case excluding supplier selection decision results in decreased profit as shown in Table 4.12. However, in this case the profit loss is more than 50% .

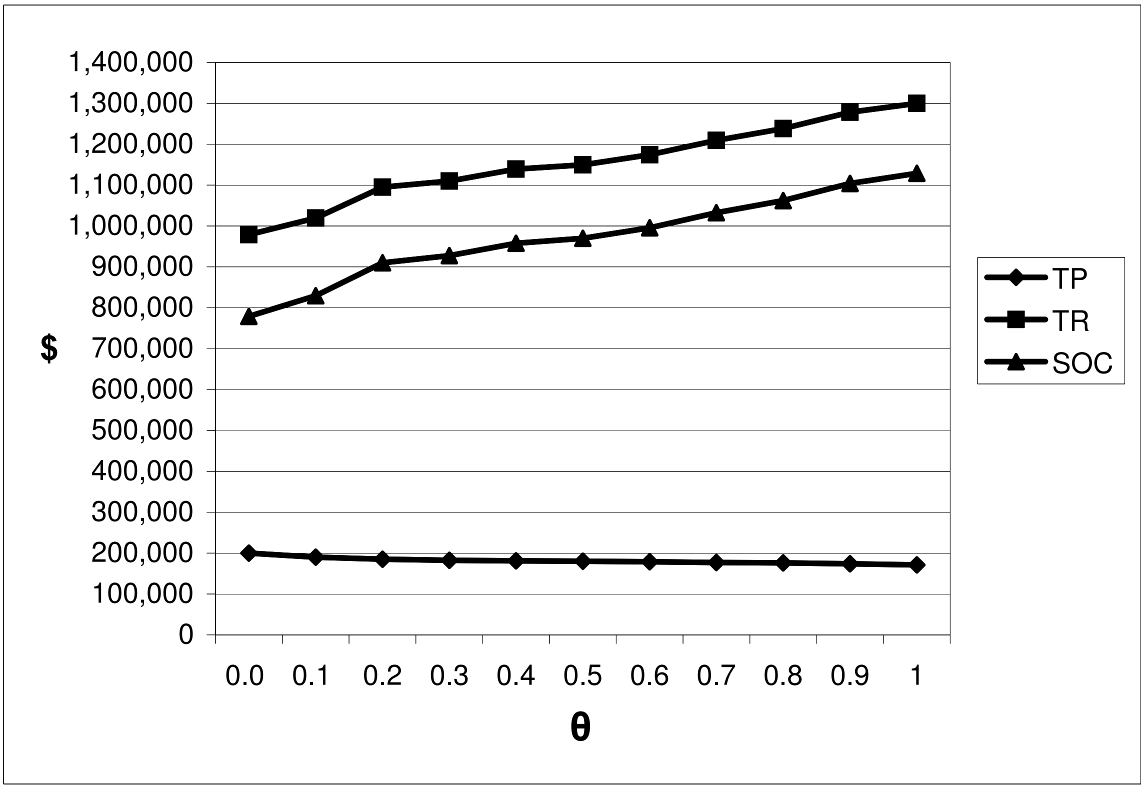

Figure 4.7: Change in the profit, revenue, and operating costs as θ increases

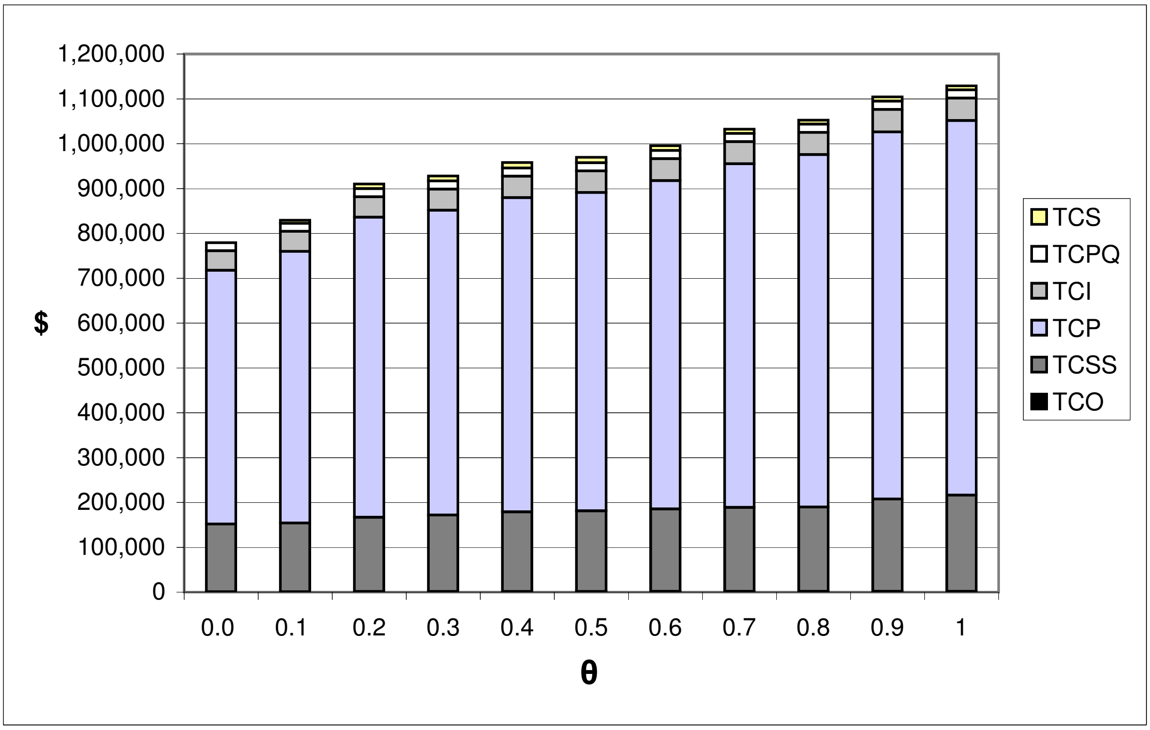

Figure 4.8: Change in each operating cost as θ increases

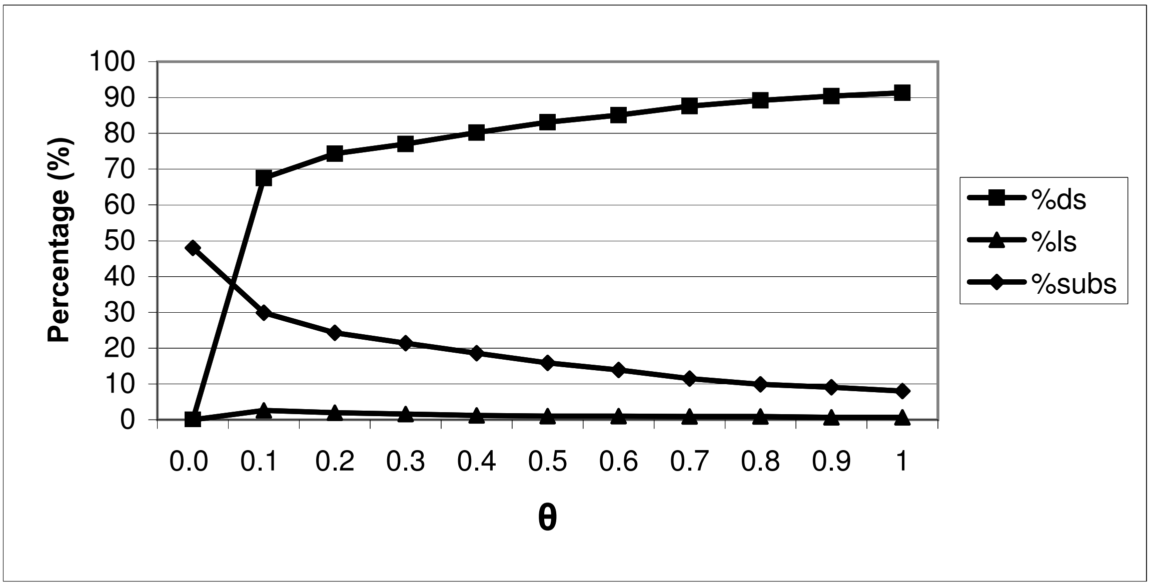

Figure 4.9: Change in the percentages of satisfied demand, lost sales and substitution as θ increases

θ	[TP]	[TP w/o sc]	[Difference]	[% Difference]
0	199,711	199,711	0	0
0.1	190,110	181,936	8,174	4.3
0.2	185,194	175,564	9,630	5.2
0.3	182,266	162,217	20,049	11.0
0.4	181,089	108,835	72,254	39.9
0.5	180,009	62,109	117,900	65.5
0.6	178,716	-7,668	186,384	104.3
0.7	176,997	-100,003	277,000	156.5
0.8	175,987	-196,809	373,796	212.4
0.9	173,843	-327,867	501,710	288.6
1	170,990	-376,550	547,540	320.2

Table 4.11: The effect of substitution behavior in stochastic demand, single-period case

4.2.5 The Importance of Shelf Space Limitations

As discussed in Section 3.6.3, in order to analyze the importance of shelf space limitation in this problem, we first introduced an effective shelf space limitation of $8,000$ units for the category and found the optimal profit with this constraint. Then, we excluded shelf space constraint from the model and found the assortment for this new setting. Then, we applied the shelf space limitation to this assortment in both ways described in Section 3.6.3 and found the total profits that these assortments generate for the case with shelf space limitations. Lastly, we compared these profits with the total profit generated when shelf space limitations are included in the model. We performed these for the same experimental data used in Section 4.1.1, where θ is considered to be 0.3, and we observed that excluding shelf space limitations results in decreased profit. Table 4.13 shows these results.

Similar to the deterministic demand, multi-period case, the assortments of the

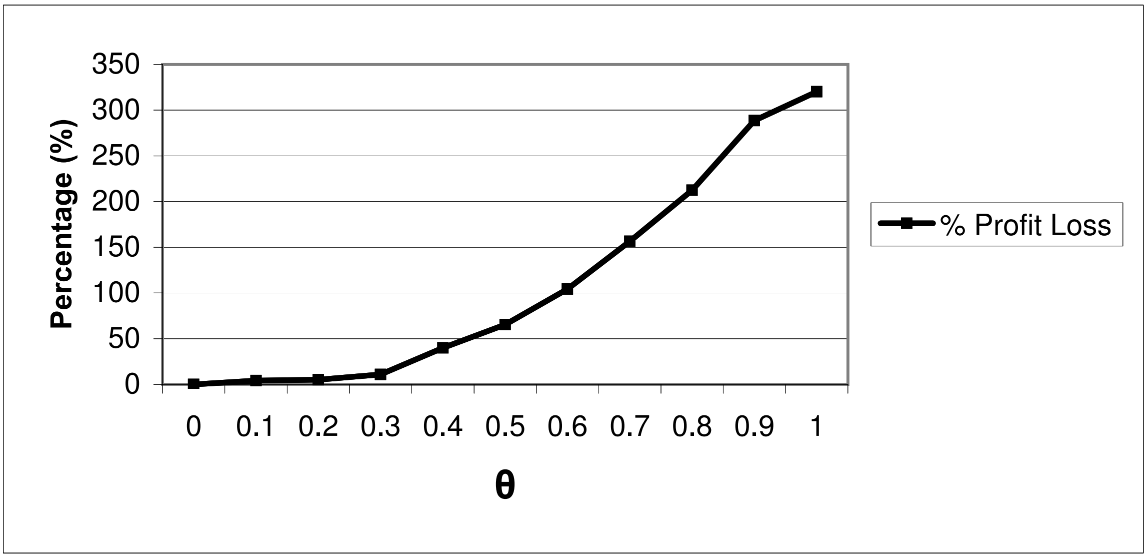

Figure 4.10: Change in the percentage of profit loss as θ increases in case of ignored substitution behavior

model without self space constraints either with *method 1* or *method 2* generate lower profit than the optimal assortment of the model with shelf space constraints generates, and there exists high profit loss when shelf space limitations are not considered as a result of higher supplier selection costs.

In order to see the effect of shelf space limitations on the number of selected suppliers, we also performed experiments on the original model and observed the number of suppliers for changing shelf space limitations. Table 4.14 shows the results obtained for the same experimental data except that the shelf space limitation is changing. Again, θ is considered to be 0.3. The results show that similar to the deterministic demand and multi-period case, as shelf space limitations becomes looser, the number of selected suppliers increases. Therefore, also for stochastic demand case, as shelf space limitations becomes looser, the importance of integrated supplier selection decision increases.

θ	[TP]	[TP w/o ss]	[Difference]	[% Difference]
0	199,711	95,662	104,049	52.1
0.1	190,110	86,125	103,985	54.7
0.2	185,194	83,337	101,857	55.0
0.3	182,266	82,936	99,330	54.5
0.4	181,089	80,947	100,142	55.3
0.5	180,009	80,364	99,645	55.3
0.6	178,716	79,388	96,328	55.5
0.7	176,997	77,012	99,985	56.4
0.8	175,987	76,946	99,041	56.2
0.9	173,843	76,266	89,877	56.1
1	170,990	75,975	95,015	55.5

Table 4.12: The effect of supplier selection decision as substitution costs change in stochastic demand, single-period case

4.3 The Benchmark Case: The Single-period Problem with Deterministic Demand

As the next step of our analysis, we consider the deterministic demand, single-period case in order to make comparisons with the stochastic demand, single-period case. The deterministic demand, single-period problem is a special case of both the deterministic demand, multi-period problem, where the number of periods is equal to 1 ($T = 1$) and the stochastic demand, single-period problem, where there exists one possible scenario. This special case requires modifications in decision variables and constraints of the model. In the new formulation, forecasted demand for product i is parameterized as d_i and we have the following decision variables:

- $z1_i$: ending inventory position of product i.
- x_i : quantity of product i to be ordered per order.

The assortment	[TP]	[% Profit Loss]
The assortment of the original model	19,872	-
The assortment of the model w/o shelf space limitations with *method 1*	15,453	22.2
The assortment of the model w/o shelf space limitations with *method 2*	13,983	29.6

Table 4.13: The effect of shelf space limitations

Shelf Space Limitation [units]	[N(S)]
$8,000$	1.20
$20,000$	1.89
$32,000$	2.68

Table 4.14: The effect of shelf space limitations on the number of selected suppliers

- y_i : 1, if product i is ordered; 0, otherwise.
- o_j : 1, if an order is placed with supplier j; 0, otherwise. (o_j also indicates whether supplier j is selected or not.)
- $x0_i$: the amount of satisfied demand for product i.
- xs_{mik} : the amount of product i used to satisfy m^{th} substitution from product k .

The parameters that are modified are:

- $z0_i$: the initial inventory position of product i.
- p_i : unit price of product i.

The constraints are also modified as follows:

$$TR = \sum_{i \in P} p_i \left(z0i + x_i - z1_i \right) \tag{4.18}$$

$$TCO = \sum_{j \in S} oc_j o_j \tag{4.19}$$

$$o_j \geq a_{ij} y_i, \forall i \in P, \forall j \in S \tag{4.20}$$

$$TCSS = \sum_{j \in S} ssc_j o_j \tag{4.21}$$

$$TCP = \sum_{i \in P} c_i x_i \tag{4.22}$$

$$TCI = \sum_{i \in P} \frac{(z0_i + x_i + z1_i)}{2} h_i \tag{4.23}$$

$$TCPQ = \sum_{i \in P} pq_i q_i x_i \tag{4.24}$$

$$TCS = \sum_{m=1}^{M} \sum_{i \in P} \sum_{k \in P} s_{mi} xs_{mik} \tag{4.25}$$

$$x0_i + \sum_{m} \sum_{k \in P} xs_{mki} = d_i, \forall i \in P \tag{4.26}$$

$$x0_i + \sum_{m} \sum_{k \in P} xs_{mik} + z1_i = z0_i + x_i, \forall i \in P \tag{4.27}$$

$$xs_{1ik} \leq (d_k - x0_k) w_{ik}, \forall i, k \in P \backslash \{i\} \tag{4.28}$$

$$xs_{2ik} \leq (d_k - x0_k - \sum_{r \in P} xs_{1rk}) \sum_{r \in P} w_{rk} w_{ir}, \forall i \in P, \forall k \in P \backslash \{i\} \tag{4.29}$$

$$...^2 \tag{4.30}$$

$$z0_i + x_i \leq SS_i, \forall i \in P \tag{4.31}$$

$$0 \leq x_i \leq OQ_i y_i, \forall i \in P \tag{4.32}$$

$$y_i \in \{0, 1\}, \forall i \in P \tag{4.33}$$

The analysis carried out for the deterministic demand, multi-period case given in Section 4.1 is also performed for this case.

[2] Because of the complexity associated with higher substitution levels, we provide only the first two of the substitution inequalities.

4.3.1 Experimental Data

The experimental data used for the deterministic demand, single-period case is taken to be consistent with experimental data provided in Section 4.1.1 and the updated parameter distributions, which are shown in Table 4.9. The deterministic demand for products, d_i, is determined as $d_i = 40,000 * \alpha_i$, where $\sum_i d_i = 40000$ and $0 \leq \alpha_i \leq 1$, $\forall i \in P$. Here, note that, the total demand for products is assumed to be $40,000$.

4.3.2 The Impact of Changing Substitution Costs

Table 4.15 shows the results obtained for different θ values in the range $0 \leq \theta \leq 1$.

θ	TP	%ds	%ls	%f	%s	%t	%subs	TR	TCO	TCSS	TCP	TCI	TCPQ	TCS	SOC
0.0	152,164	36.3	11.7	24.4	17.5	10.1	51.97	481,894	85.5	62,793	249,031	10,621	7,198	0	329,730
0.1	132,000	44.5	8.5	24.0	15.2	7.9	47.11	493,155	90.0	66,749	257,855	10,907	7,464	18,090	361,156
0.2	115,876	51.2	6.2	23.5	12.8	6.3	42.58	501,911	99.8	74,644	264,085	11,302	7,614	28,289	386,035
0.3	103,259	56.5	5.3	23.1	10.4	4.8	38.24	505,203	108.2	81,414	267,962	11,452	7,753	33,254	401,944
0.4	93,596	62.0	4.8	21.6	8.1	3.5	33.17	505,699	114.7	87,574	270,394	11,621	7,803	34,594	412,103
0.5	86,061	66.7	5.5	19.5	5.8	2.2	27.67	499,122	120.1	92,058	268,960	11,655	7,814	32,452	413,062
0.6	80,262	69.8	5.6	18.7	4.1	1.5	24.50	497,326	125.6	96,861	268,981	11,644	7,826	31,623	417,064
0.7	75,486	73.5	5.5	16.6	3.1	1.1	20.87	496,198	131.1	102,132	269,689	11,710	7,812	29,236	420,713
0.8	71,593	74.9	6.1	15.9	2.2	0.7	19.05	493,422	133.8	104,175	268,879	11,667	7,767	29,203	421,829
0.9	68,172	76.4	5.7	15.2	1.8	0.6	17.78	493,383	137.1	107,252	269,317	11,691	7,793	29,019	425,211
1	65,161	78.1	5.4	14.4	1.4	0.5	16.44	494,442	140.4	109,983	270,761	11,724	7,833	28,838	429,281

Table 4.15: The effect of changing substitution cost in deterministic demand, single-period case

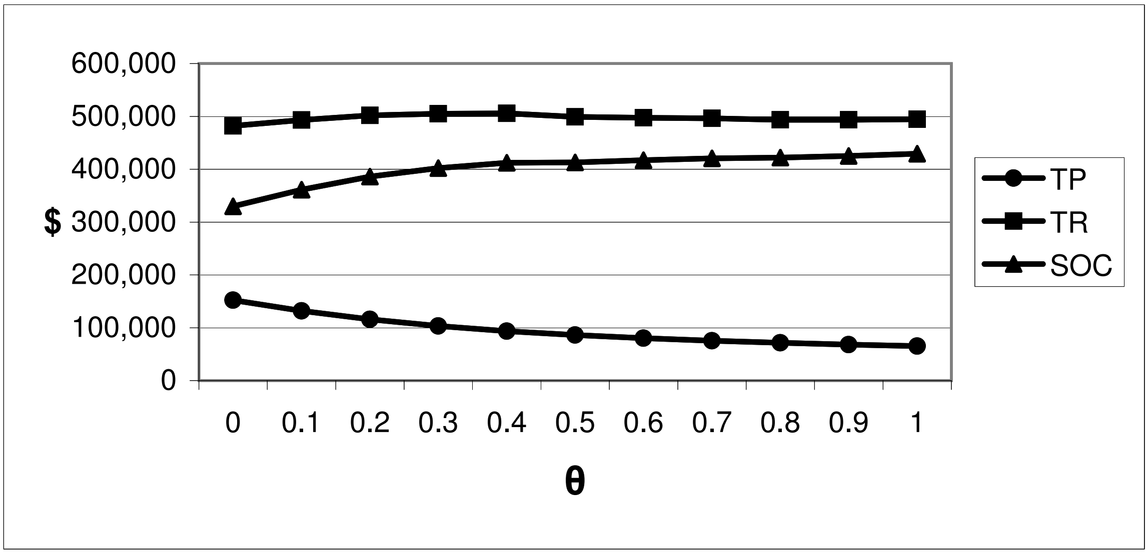

Figure 4.11: Change in the profit, revenue, and operating costs as θ increases

In order to illustrate the results of Table 4.15 graphically, Figure 4.11, Figure 4.12, and Figure 4.13 are provided.

According to these results, for smaller θ values, the optimal system has limited assortment, selecting a subset of suppliers and favors substitution since it generates no cost. However, for greater θ values, the optimal system extends its assortment, increasing the number of selected suppliers. Therefore, supplier selection costs increase as seen in Figure 4.12. Similar to the deterministic demand and multi-period case, this shows that substitution costs significantly affect ordering and supplier selection decisions.

As substitution and lost sales costs increase, the system tries to satisfy more demand and decreases substitutions as shown in Figure 4.13. Thus, similar to the deterministic demand, multi-period case, this shows that substitution costs affect purchasing and inventory decisions, as well.

When we compare these results with the results obtained for previous cases, it is observed that stochastic demand case generates higher costs and lower profit. In the stochastic demand case, less substitution and lost sales are preferred, and more

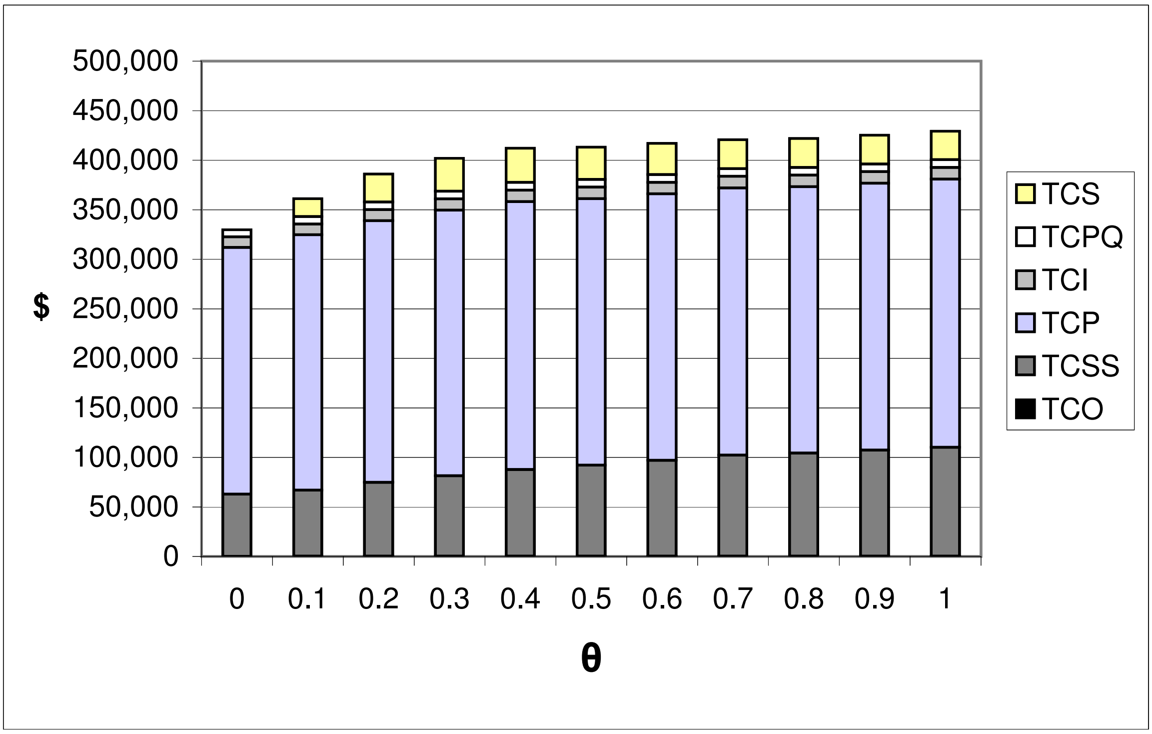

Figure 4.12: Change in each operating cost as θ increases

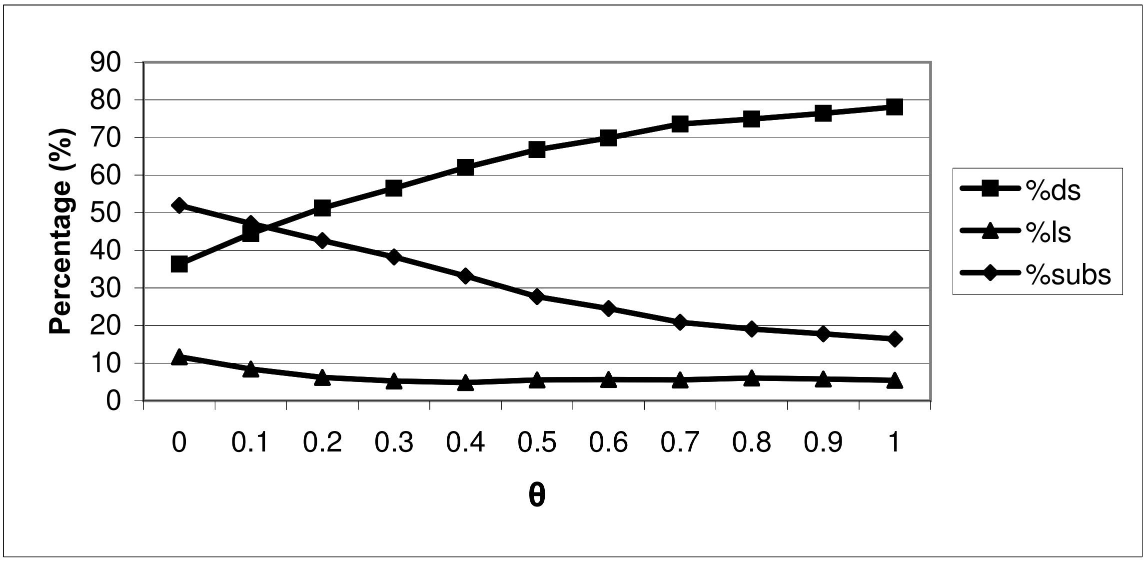

Figure 4.13: Change in the percentages of satisfied demand, lost sales and substitution as θ increases

products are ordered. As a result of these higher costs, for the stochastic demand case, proper identification of substitution costs has higher importance.

4.3.3 The Importance of Substitution Behavior

The results, shown in Table 4.16, are similar to the deterministic demand, multi-period case. In Figure 4.14, we observe that as substitution costs increase, profit loss increases if customers' substitution behavior is neglected. According to the results Table 4.16, in single-period case the profit loss becomes more critical compared to that of multi-period case especially in case of high substitution costs.

More importantly, when we compare Table 4.16 with Table 4.11 , we observe that for the stochastic demand case, the profit loss becomes much more critical compared to that of the deterministic demand case especially when substitution costs are high.

θ	[TP]	[TP w/o sc]	[Difference]	[% Difference]
0	152,164	152,164	0	0
0.1	132,000	129,608	2,392	1.8
0.2	115,876	107,052	8,824	7.6
0.3	103,259	84,496	18,763	18.2
0.4	93,596	61,940	31,656	33.8
0.5	86,061	39,384	46,677	54.2
0.6	80,262	16,828	63,485	79
0.7	75,486	-5,728	81,214	107.6
0.8	71,593	-28,284	99,878	139.5
0.9	68,172	-50,840	119,012	174.6
1	65,161	-73,396	138,557	212.6

Table 4.16: The effect of substitution behavior in deterministic demand, single-period case

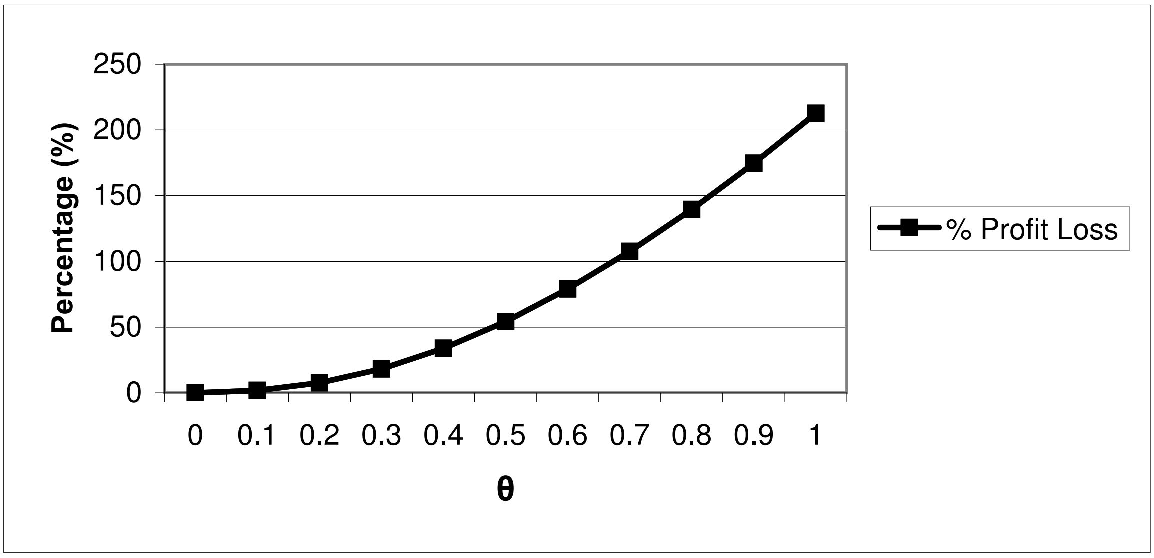

Figure 4.14: Change in the percentage of profit loss as θ increases in case of ignored substitution behavior

4.3.4 The Importance of Supplier Selection Decision

Similar to the multi-period case, in the single-period case excluding supplier selection decision results in decreased profit of more than 40% . Table 4.17 shows these results.

More importantly, when we compare Table 4.17 with Table 4.12 , we observe that for stochastic demand case, the profit loss becomes much more critical compared to that of previous cases especially when supplier substitution costs are high.

If we compare the single-period and the multi-period, deterministic demand cases, we see that the percentage of profit loss as a result of excluding supplier selection decision in single-period case is greater than that of multi-period case. This shows that in the single-period case, supplier selection decision is more important.

4.3.5 The Importance of Shelf Space Limitations

As discussed in Section 3.6.3 in order to analyze the importance of shelf space limitation in this problem, we first introduced an effective shelf space limitation of

θ	[TP]	[TP w/o ss]	[Difference]	[% Difference]
0	152,164	82,669	69,495	45.7
0.1	132,000	66,318	65,683	49.8
0.2	115,876	57,398	58,478	50.5
0.3	103,259	51,935	51,324	49.7
0.4	93,596	46,685	46,911	50.1
0.5	86,061	43,815	42,246	49.1
0.6	80,262	41,741	38,522	48
0.7	75,486	40,546	34,940	46.3
0.8	71,593	39,335	32,259	45.1
0.9	68,172	38,098	30,075	44.1
1	65,161	37,431	27,731	42.6

Table 4.17: The effect of supplier selection decision as substitution costs change for deterministic demand, single-period case

$8,000$ units for the category and found the optimal profit with this constraint. Then, we excluded shelf space constraint from the model and found the assortment for this new setting. Next, we applied the shelf space limitation to this assortment in both ways described in Section 3.6.3 and found the total profits that these assortments generate for the case with shelf space limitations. Lastly, we compared these profits with the total profit generated when shelf space limitations are included in the model. We performed these for the same experimental data used in Section 4.1.1, where θ is considered to be 0.3, and we observed that excluding shelf space limitations results in decreased profit. Table 4.18 shows these results.

According to these results, similar to the multi-period, deterministic demand case, the assortments of the model without self space constraints either with *method 1* or *method 2* generate lower profit than the optimal assortment of the model with shelf space constraints generates. In addition, there exist high profit losses when shelf space limitations are not considered. Again, it is as a results of that the system selects more

The assortment	[TP]	[% Profit Loss]
The assortment of the original model	16,187	
The assortment of the model w/o shelf space limitations with *method 1*	12,389	23.5
The assortment of the model w/o shelf space limitations with *method 2*	11,456	29.2

Table 4.18: The effect of shelf space limitations

suppliers because of unlimited space in case of unlimited shelf space.

When we compare the deterministic demand case with the stochastic demand case considering Tables 4.18 and 4.13, we observe that profit losses are greater in stochastic demand case. Therefore, when the demand is stochastic, including shelf space constraints in the model gains more importance.

In order to see the effect of shelf space limitations on the number of selected suppliers, we also performed experiments on the original model and observed the number of suppliers for changing shelf space limitations. Table 4.19 shows the results obtained for the same experimental data except that the shelf space limitation is changing. Again, θ is considered to be 0.3. The results show that similar to the multi-period case, as shelf space limitations becomes looser, the number of selected suppliers increases. Therefore, also for the deterministic demand and single-period case, as shelf space limitations becomes looser, the importance of integrated supplier selection decision increases. In addition, since the number of selected suppliers in the stochastic demand case is more than that of deterministic demand case, the importance of integrated supplier selection decision is also greater for stochastic demand case.

Shelf Space Limitation [units]	[N(S)]
$8,000$	1
$20,000$	1.57
$32,000$	2.41

Table 4.19: The effect of shelf space limitations on the number of selected suppliers

4.4 *Comparisons of Test Results for Deterministic Demand and Stochastic Demand Cases*

If we summarize the comparisons among the stochastic demand and the deterministic demand cases, we make the following observations provided in Table 4.20.

	Single-period, Determistic Demand	Single-period, Stochastic Demand
Original demand satisfied		Higher
Substitution percentages	Higher	
The Amount of Lost Sales	Higher	
Ordering Quantitites per period		Higher
Optimal Profit	Higher	
Importance of Substitution		Higher
Importance of Supplier Selection		Higher
Importance of Shelf Space Limitations		Higher

Table 4.20: Comparisons of Test Results for Deterministic Demand and Stochastic Demand Cases

These results are based on the following facts: in the stochastic demand case, since the demand is not known with certainty, the system orders more in order to satisfy the future demand. As a results of this, more demand can be satisfied compared to the deterministic demand case. Although percentage of the lost sales and substitution amounts are lower, ordering more decreases the total profit.

In addition, since the system is more sensitive due to demand uncertainty in stochastic demand case, the impact of substitution, supplier selection, and shelf space limitations on the total cost are greater. An important point needs clarification here: although substitution and lost sales percentages are lower in the stochastic demand case, the effect of substitution on the total cost is higher. This can be explained by the fact that the incurred substitution costs might be lower in stochastic demand case, however, the profit is also lower, and therefore the percentage of substitution cost in profit and/or revenue is higher than that of the deterministic demand case.

4.5 The Impact of Demand Variability

In order to see the impact of demand variability on substitution amounts, the optimal profit, ordering quantities, and supplier selection costs, we change the demand variability and observe the corresponding parameters.

In our experiments, we consider the single-period problem with $\theta = 0.3$ and the parameter values are generated according to Table 4.21.

The mean of total demand for products is assumed to be constant, $40,000$, for all experiments, where it is uniformly distributed according to changing variances provided in Table 4.22. Again Monte Carlo sampling is used and 100 scenarios are generated for total demand.

Table 4.22 provides the results of the experiments. Under the *Total Substitution Percentage* column, the sum of first, second, third levels of substitution percentages and lost sales percentages for all products is given. The substitution percentage of a product at a certain level is calculated as the ratio of the sum of substitutions from that product in that level to the demand for that product. Similarly, the lost sales percentage of a product is calculated as the ratio of lost sales amount from that product to the demand for that product.

Parameter	Value
w_{ik}	Random, where $\sum_{k \in P} w_{ik} = 1$ and $0 \leq w_{ik} \leq 1$, $\forall i, k \in P$
c_i	Random, where $5 \leq c_i \leq 10$, $\forall i \in P$
oc_j	Random, where $30 \leq oc_j \leq 50$, $\forall j \in S$
ssc_j	Random, where $15,000 \leq ssc_j \leq 50,000$, $\forall j \in S$
d_i	$d_i = (\sum_i d_i) * \alpha_i$, where $\sum_i \alpha_i = 1$ and $0 \leq \alpha_i \leq 1$, $\forall i \in P$, $\forall t = 1, .., T$ (Total demand distribution varies from one experiment to another and is provided in Table 4.22.)
OQ_i	Random, where $4,000 \leq OQ_i \leq 34,000$, $\forall i \in P$
SS_i	Random, where $8,000 \leq SS_i \leq 40,000$, $\forall i \in P$
h_i	Random, where $0.3 \leq h_i \leq 1$, $\forall i \in P$
pq_i	Random, where $2 \leq pq_i \leq 4$, $\forall i \in P$
q_i	Random, where $0.0 \leq q_i \leq 0.15$, $\forall i \in P$
p_i	$p_i = c_i + mg_i$, where mg_i is random with $4 \leq mg_i \leq 8$, $\forall i \in P$.

Table 4.21: Experimental Data

Mean of Demand	Uniform Distribution Variance	Total Substitution Percentage	Optimal Profit	Cost of Ordering	Cost of Supplier Selection	Cost of Substitution
$40,000$	$variance = 0$ (deterministic)	43.4 %	103,259	108.2	81,414	33,254
$40,000$	$39,000 \leq \sum_i d_i \leq 41,000$	29.6 %	81,034	156.4	102,344	26,840
$40,000$	$35,000 \leq \sum_i d_i \leq 45,000$	20.1 %	66,309	203.5	102,344	22,397
$40,000$	$30,000 \leq \sum_i d_i \leq 50,000$	11.3 %	55,183	289.2	125,198	19,172

Table 4.22: The Impact of Demand Variability

According to Table 4.20, it is seen that substitution amounts of the stochastic demand case are lower than that of the deterministic demand case. Due to the results provided in Table 4.22, it is observed that as the demand variability increases, the substitution percentages decrease. This generalizes the comparison result related to substitution amounts in stochastic and deterministic cases stated in Table 4.20.

According to the results provided in Table 4.22, increasing demand variability decreases optimal profit, which generalizes the comparison result related to optimal profit stated in Table 4.20.

When ordering amounts are observed for increasing demand variability, it is seen in Table 4.22 that as demand variability increases, the system orders more in order to be able to meet the demand and pay more for ordering. By ordering more, the system makes less substitution and pay less substitution cost. However, since optimal profit decreases as demand becomes more variable, the proportion of substitution cost to total profit increases. Therefore, the importance of substitution increases as demand variability increases. These observations are the generalization of the the comparison result related to ordering amounts and the importance of substitution provided in Table 4.20.

Another result of increasing demand variability is increased supplier selection cost as observed in Table 4.22. In order to be able to satisfy the demand for products in case of variability in demand, on behalf of ordering more, the system extends its assortment and works with more suppliers. Therefore, as demand variability increases, the importance of integrated supplier selection decision increases.

Chapter 5

EXPERIMENTAL STUDY II: EMPIRICAL OBSERVATIONS

In our model, there are some input parameters that need to be set carefully, since we have observed that the model solution is sensitive to these parameters. These parameters are

- w_{ik} : the proportion of customers whose preference is product k and who substitute product k with product i in case of unavailability, that is *substitution rates*, and

- s_{mi} : the penalty cost of m^{th} substitution from product i.

5.1 Determination of Substitution Rates, w_{ik}'s

Previous research on customer response to stockouts have shown that stockouts might result in substantial losses from a brand sales perspective (Schary and Christopher (1979)) and from a category sales perspective (Bell and Fitzsimons (1999)). Although there is some researches on the assortment and/or inventory planning in the existence of substitution in the literature, there is not so much research on the estimation of substitution rates. Anupindi et al. (1998) develop a methodology in order to estimate customer demand with stockout-based substitution. In this method, the arrival process for products is presumed to be a Poisson process and they solve for the arrival rates and substitution rates by maximizing the likelihood function by using the expected-maximization method. Their method requires partial or complete inventory transaction data. Campo et al. (2003) apply the logit model estimation of when-to-buy, what-to-buy, and how-much-to-buy decisions of customers by dynamically changing the product assortment and observes the impact of stockouts on category sales and purchase quantity. Kok and Fisher (2004) develop an new methodology for assortment-based demand estimation under an assumed relatively simple substitution structure and use the estimations for the stockout-based substitution. Smith and Agrawal (2000) use a probabilistic model and determine substitution rates as described in Section 3.7.1.

Any one of the above methods can be used to determine the input parameters, substitution rates, of our model.

5.2 Determination of Substitution Cost Penalties, s_{mi}'s

As stated in Section 4.1.1, we assume $s_{mi} = SC_i * m$, where SC_i denotes the first level substitution cost from product i, SC_i is a linear function of its margin, mg_i, and is calculated as $SC_i = \theta * mg_i$. Therefore, in order to find SC_i, we need to determine θ. Since θ is specific for our model, we prepared a survey to understand the substitution behavior of customers, and used the results of the survey to determine the characteristics of θ.

5.2.1 The Customer Survey on the Substitution Behavior

In the survey, we tried to analyze the effect of retailer choice in substitution cost, and the effect of product category in substitution. Rather than to provide specific results, with this survey we aimed to gain insights on the substitution behavior. There are 250 people who participated in the survey. The survey questions and the results that we obtained are provided in Appendix B.

The results of the survey can be stated as follows:

- Customers incur less substitution cost for retailers with limited assortment. That is, the customers of the retailers with limited assortment are more likely to substitute to available products whereas the customers of the retailers with larger assortment are intolerant to product unavailabilities.

- The sensitivity of customers to product unavailabilities is not same for all product categories. They are more sensitive in categories of milk products, meat products, and cosmetics, and the sensitivity to these product categories is independent of the retailer choice of the customer.

- The expectation of female customers on the product variety and availability is more than that of male customers. Females are less price-sensitive than male customers.

- The lost sales penalty and the probability of losing customers forever are smaller for limited assortment, low price retailers than that for retailers with large assortment and higher price.

- The customers of limited assortment, low price retailers have less brand loyalty than the customers of retailers with large assortment and higher price.

Due to these results, we can say that the following two factors determine the value of θ for a product category in a retailer, and explain how the value of θ for a product category can be set.

1. *The characteristics of the assortment for that product category.* Here, we use the term *characteristics* since not only the size of the assortment but also its content are important factors. For instance, a retailer might have two products/brands in a category but these two products/brands might span 90% of the market, whereas another retailer might have two products/brands in a category which spans only 30% of the market share. However, the amount of share that the assortment spans in the market is not only the determinant factor as the next instance shows. For instance, there might be two retailers, A and B, both having assortments with size 2 that span 50 % of the market in a product category. In this product category, retailer A's assortment is composed of market-leader brand, which has a share of 40 %, and the brand with lowest market share, 10 %. Retailer B's assortment is composed of two brands with market shares

of 25 % each. Clearly, the value of θ will be different for retailer B in that product category. Therefore, although θ is used to determine the assortment, the assortment determines the value of θ, as well. Then, we can say that the retailer sets a mean value of θ for all product categories when he determines its position in the market, for instance as being a hard discounter or a department store.

2. *The product category itself.* After the retailer determines its position in the market and sets the mean value of θ for all product categories, say as θ_R, the value of θ for each product category can be set so that their mean gives θ_R according to the sensitivity scales of the customers that the customer surveys provide.

Chapter 6

CONCLUSION

The problem of product assortment under customer-driven demand substitution in retail operations is analyzed in this thesis. The complexity of the problem comes from several factors. First, increasing variety in product assortment increases customer satisfaction but has a negative effect on operational costs such as the cost of ordering and the cost of supplier selection. Second, each product type is supplied by a supplier; therefore, product assortment and selection of suppliers cannot be separated. Third, since increase in variety of the assortment reduces demand per variant as a result of substitution, customer-driven substitution should not be neglected while deciding the right assortment. Fourth, since a product category cannot have unlimited shelf space in the store, inventory management should also be incorporated in the decision process. Therefore, in order to decide on the right product assortment, a retailer should consider several closely-related issues such as category management, selection of suppliers, and demand substitution.

In this thesis, we addressed the above problem and developed an optimization model for the multi-period, multi-product inventory, product assortment and supplier selection problem with multi-way demand substitution. The behavior of the solution provided by the model is analyzed for three cases: *(i)* the deterministic demand, multi-period case, *(ii)* the deterministic demand, single-period case, and *(iii)* the stochastic demand, single-period case. The analysis for each case is performed to examine the effects of three parameters - substitution cost, supplier selection cost, and shelf space limitations -, separately. The analysis and the results of the analysis can be summarized as follows:

- The solution of the model is analyzed in case of different substitution costs. It is observed that substitution affects purchasing, ordering, inventory management, and supplier selection decisions. In addition, high substitution and lost sales costs for retailers resulted in extended assortments and increased service level, but reduced profit due to increase in operational costs.

- The effect of neglecting customers' substitution behavior in the model is analyzed and it is observed that ignoring substitution in product assortment decision results in reduced profit.

- The effect of including supplier selection decision to the model is examined and it is concluded that excluding supplier selection decision leads to significant profit loss.

- The effect of ignoring shelf space limitations is analyzed and it is concluded that considering shelf space limitations in the model results in more profitable assortment.

- The results are compared for the deterministic demand and the stochastic demand cases and the impact of variability on the profit, operational costs, and substitution amounts is observed. According to these analyses, it can be stated that as demand variability increases, retailers should order more in order to satisfy the demand and pay more operational costs. In addition, they might need to extend the width of their assortment by working with more suppliers. It is seen that as demand variability increases, substitution behavior, supplier selection decision, and shelf space limitations become more important in the product assortment decision.

The main contributions of this thesis can be listed as follows:

- To the best of our knowledge, the problem of multiple products, multi-way product substitution, inventory planning, and supplier selection in the existence of shelf space limitations is not considered in an integrated model in the literature.

Therefore, our study provides an efficient tool to determine the product assortment for retailers, which considers supplier selection and inventory management decisions in the existence of shelf space limitations and customers' substitution behavior. In addition, using our model, retailers can position themselves in the market by solving the model for different customer segments that have different substitution behaviors. The model can also be used to decide on new product introduction, given the estimate of the profit margin of the new product, substitution rates from/to this product, and its expected effect on the category demand.

- The analysis of the developed model gives insights about the effect of the substitution on the product assortment, the importance of incorporating supplier selection decision to the product assortment problem, and the significance of shelf space limitations on determining the right assortment.

An extension of this study is to integrate vendor-managed inventory into the system since it is an emerging trend in retailing. It reduces supply chain costs and inventory levels and increases profit. However, it requires the rapid and accurate transfer of information between the retailer and its suppliers. In this setting, suppliers create the purchase orders based on the demand information exchanged by the retailer. Therefore, while integrating vendor-managed inventory into our system, inventory related issues should be standardized according to the agreements between the retailer and its suppliers.

In this thesis, it was assumed that before visiting the retail store, customers have clear and stable preferences for the product categories that they are going to shop. However, as stated in Simonson (1999), in many situations, customers construct their preferences when faced with a specific product assortment, rather than having constructed and stable preferences. Therefore, a possible extension of our study might be considering different customer preferences for different assortments. However, deciding product assortment in the existence of flexible customer preferences will not be trivial as it requires understanding consumer behavior.

Moreover, an attractive future work direction is considering promotional activities in the model. Retailers also position themselves in the market according to their nature of promotional activities. The studies show that promotions such as price cuts have significant effects on product choice (e.g. Guadagni and Little (1983)) and product substitution (Dodson, Tybout, and Sternthal (1978)). However, similar to the previous possible extension, considering promotional activities in the model might not be trivial since it will require understanding the reaction of customers to price discounts and assessing the effect of brand loyalty.

Appendix A

THE RESULTS OBTAINED BY SMITH AND AGRAWAL (2000)

Products	No Subs. Assort. Size	Random Subs. Assort. Size	Adjacent Subs. Assort. Size	One-Item Subs. Assort. Size
1	8	10		
2	8	10	15	
3	8	10		28
4	8		15	
5	8			
Opt. Profit	16.4	26.3	39.6	47.3

Table A.1: Optimal assortment generated by the model Smith and Agrawal(2000) for Example Set 1

Products	No Subs. Assort. Size	Random Subs. Assort. Size	Adjacent Subs. Assort. Size	One-Item Subs. Assort. Size
1	8	10	8	8
2	8	10	8	8
3	8	10	8	18
4	8		13	
5	8			
Opt. Profit	41.1	45.1	41.6	45.1

Table A.2: Optimal assortment generated by the model Smith and Agrawal(2000) for Example Set 2

Products	No Subs. Assort. Size	Random Subs. Assort. Size	Adjacent Subs. Assort. Size	One-Item Subs. Assort. Size
1	17	19		
2	8	10	22	
3				28
4			8	
5				
Opt. Profit	27.2	35.4	38.2	46.7

Table A.3: Optimal assortment generated by the model Smith and Agrawal(2000) for Example Set 3

Appendix B

THE RESULTS OF SURVEYS

B.1 Survey Questions

1. What is your sexuality?

   ◯ Female

   ◯ Male

2. Which type of retailers do you prefer in order perform your shopping?

   ◯ The ones with large product-brand variety but high price, say A

   ◯ The ones with restricted product-brand variety but low price, say B

3. Which one is more important for you when you're shopping?

   ◯ Affordable prices

   ◯ Large product-brand variety

   ◯ Availability of the product-brand that you want to buy

4. Which product categories are you more selective? (Mark all that are appropriate.)

   ◯ Baby products

   ◯ Beverages

   ◯ Milk and milk products (yogurt, cheese, ice-cream, etc.)

   ◯ Frozen nutrients

   ◯ Meat and meat products

◯ Cigarettes and alcoholic drinks

◯ Sugary nutrients (biscuit, wafer, etc.)

◯ Light nutrients

◯ Legumes

◯ Flour and floury nutrients (pasta, bread, etc.)

◯ Oil

◯ Delicatessen

◯ Soaps

◯ Detergents

◯ House-cleaning products/tools

◯ Cosmetics (deodorant, shampoo, moisturizer, etc.)

5. When you need to buy a shampoo, what do you do if the brand you want is unavailable?

   ◯ I choose another brand which is available.

   ◯ I do never choose another brand, but I continue shopping.

   ◯ I leave the shopping basket as is, and go to the retailer that I can find the brand I want.

6. As a result of this :

   ◯ In the future, in my shoppings that incurs shampoo I can choose this retailer.

   ◯ In the future, in my shoppings that incurs shampoo I will never choose this retailer.

   ◯ In the future, in all my shoppings I will never choose this retailer.

7. When you need to buy a toilet paper, what do you do if the brand you want is unavailable?

   ◯ I choose another brand which is available.

◯ I do never choose another brand, but I continue shopping.

◯ I leave the shopping basket as is, and go to the retailer that I can find the brand I want.

8. As a result of this :

   ◯ In the future, in my shoppings that incurs toilet paper I can choose this retailer.

   ◯ In the future, in my shoppings that incurs toilet paper I will never choose this retailer.

   ◯ In the future, in all my shoppings I will never choose this retailer.

9. When you need to buy flour or floury nutrients (pasta, bread, etc.), what do you do if the brand you want is unavailable?

   ◯ I choose another brand which is available.

   ◯ I do never choose another brand, but I continue shopping.

   ◯ I leave the shopping basket as is, and go to the retailer that I can find the brand I want.

10. As a result of this :

    ◯ In the future, in my shoppings that incurs flour or floury nutrients I can choose this retailer.

    ◯ In the future, in my shoppings that incurs flour or floury nutrients I will never choose this retailer.

    ◯ In the future, in all my shoppings I will never choose this retailer.

11. Please rate the retailer that you generally go in regard of the following fields: (1: very bad, 5: very good)

	1	2	3	4	5
Price	○	○	○	○	○
Product-brand variety	○	○	○	○	○
Availability of the product-brand that you want to buy	○	○	○	○	○

B.2 Survey Results

In Table B.1 the statistical results obtained by analyzing the answers of 250 participants are provided. In the table, the first column refers to a choice of a question, the second column, **% Participants**, refers to the percentage of participants who selected that choice, the third column, **% Female**, refers to the percentage of females who selected that choice, the fourth column, **% Male**, refers to the percentage of males who selected that choice, the fifth column, **% Type-A**, refers to the percentage of Retailer Type-A customers who selected that choice, and the last column, **% Type-B**, refers to the percentage of Retailer Type-B customers who selected that choice.

Table B.1: Survey Results

	% Participants	% Female	% Male	% Type-A	% Type-B
→ **What is your sexuality?**					
Female	56%			60%	44%
Male	44%			40%	56%
→ **Which type of retailers do you prefer in order perform your shopping?**					
The ones with large product-brand variety but high price, say Type-A	77%	82%	71%		
The ones with restricted product-brand variety but low price, say Type-B	23%	18%	29%		
→ **Which one is more important for you when youre shopping?**					
Affordable prices	31%	25%	38%	23%	56%
Large product-brand variety	36%	39%	34%	39%	27%
Availability of the product-brand that you want to buy	33%	36%	28%	38%	17%
→ **Which product categories are you more selective?**					
Baby products	13%	14%	12%	15%	5%
Beverages	26%	27%	26%	26%	27%
Milk and milk products (yogurt, cheese,...)	76%	83%	67%	78%	71%

Table B.1: Survey Results (continued)

	%Participants	Female	Male	Type-A	Type-B
Frozen nutrients	25%	28%	21%	25%	22%
Meat and meat products	72%	75%	67%	72%	71%
Cigarettes and alcoholic drinks	11%	6%	17%	11%	10%
Sugary nutrients (biscuit, wafer, etc.)	26%	22%	32%	24%	34%
Light nutrients	12%	18%	4%	14%	5%
Legumes	6%	7%	4%	4%	12%
Flour and floury nutrients (bread, etc.)	15%	17%	12%	13%	20%
Oil	31%	35%	26%	34%	22%
Delicatessen	36%	37%	36%	37%	34%
Soaps	23%	24%	23%	26%	15%
Detergents	36%	42%	28%	38%	30%
House-cleaning products/tools	26%	26%	27%	27%	24%
Cosmetics (deodorant, shampoo, etc.)	66%	73%	56%	69%	59%
→ When you need to buy a shampoo, what do you do if the brand you want is unavailable?					
I choose another brand which is available.	31%	24%	41%	28%	41%
I do never choose another brand, but I continue shopping.	65%	72%	55%	70%	49%

Table B.1: Survey Results (continued)

	%Participants	Female	Male	Type-A	Type-B
I leave the shopping basket as is, and go to the retailer that I can find the brand I want.	4%	4%	4%	2%	10%
→ **As a result of this :**					
In the future, in my shoppings that incurs shampoo I can choose this retailer.	52%	45%	62%	48%	66%
In the future, in my shoppings that incurs shampoo I will never choose this retailer.	46%	54%	34%	49%	34%
In the future, in all my shoppings I will never choose this retailer.	2%	1%	4%	3%	0%
→ **When you need to buy a toilet paper, what do you do if the brand you want is unavailable?**					
I choose another brand which is available.	87%	84%	90%	86%	88%
I do never choose another brand, but I continue shopping.	12%	15%	9%	13%	10%
I leave the shopping basket as is, and go to the retailer that I can find the brand I want.	1%	1%	1%	1%	2%

Table B.1: Survey Results (continued)

	%Participants	Female	Male	Type-A	Type-B
→ **As a result of this :**					
In the future, in my shoppings that incurs toilet paper I can choose this retailer.	85%	83%	87%	84%	85%
In the future, in my shoppings that incurs toilet paper I will never choose this retailer.	14%	16%	13%	15%	15%
In the future, in all my shoppings I will never choose this retailer.	1%	1%	0%	1%	0%
→ **When you need to buy flour or floury nutrients, what do you do if the brand you want is unavailable?**					
I choose another brand which is available.	74%	69%	79%	72%	80%
I do never choose another brand, but I continue shopping.	23%	28%	18%	27%	13%
I leave the shopping basket as is, and go to the retailer that I can find the brand I want.	3%	3%	3%	1%	7%
→ **As a result of this :**					

Table B.1: Survey Results (continued)

	%Participants	**Female**	**Male**	**Type-A**	**Type-B**
In the future, in my shoppings that incurs flour or floury nutrients I can choose this retailer.	79%	75%	85%	78%	83%
In the future, in my shoppings that incurs flour or floury nutrients I will never choose this retailer.	20%	24%	15%	21%	17%
In the future, in all my shoppings I will never choose this retailer.	1%	1%	0%	1%	0%
→ Please rate the retailer that you generally go in regard of the following fields: (1: very bad, 5: very good)					
Price	3,3	3,3	3,3	3,2	3,7
Product-brand variety	3,8	3,9	3,6	3,8	3,5
Availability of the product-brand that you want to buy	3,7	3,9	3,6	3,8	3,6

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