Network Optimization Problems for Disaster Mitigation: Network Reliability, Investment for Infrastructure Strengthening and Emergency Facility Location

by

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This is to certify that I have examined this copy of a master's thesis by

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and have found that it is complete and satisfactory in all respects,
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To my parents and brother

ABSTRACT

Three network optimization problems arising in disaster mitigation are addressed in this thesis. Each problem involves strategic decision-making in pre-disaster stage for effective post-disaster relief operations. The first problem is on assessing the reliability and performance of infrastructure networks under disaster risk. Here, a framework to represent link dependency in failures is proposed. Using this framework, a special type of dependency structure which is relevant for the disaster situation with lack of sufficient past data, is defined. Under this dependency structure, a novel polynomial-time algorithm is proposed to analyze the reliability and performance of a network. The second problem involves allocating a budget to links of an infrastructure network to increase the reliability and the performance of the network. With investment, each link can be strengthened structurally such that its probability of survival after a disaster increases. Then, the problem is to find which links to invest in so that most benefit is obtained in terms of post-disaster performance of the network with respect to accessibility and travel time between critical points in the network. The computational applicability of a previously developed method is demonstrated in the second problem by developing a Monte Carlo simulation algorithm and applying it to a real-life case study. The last problem addressed in this thesis is a facility location problem that seeks to identify locations of emergency response and distribution centers to provide effective post-disaster logistics operations such as the supply of relief commodities to the affected areas. For this problem, an uncapacitated facility location model is formulated with the objective of reaching a maximum number of people in minimum time possible after a disaster to distribute multiple commodities through the facilities under several disaster scenarios for demand and travel time. Average weighted travel time is minimized subject to constraints on the existence of a facility within a fixed distance from each district for each commodity. This model is used to solve a real-life problem. All of the three problems and the methodology developed for these problems are computationally tested on the case of Istanbul, a metropolitan under serious earthquake risk. Real-life data including the earth-

quake scenarios, the risk of the highway structures under these scenarios, expected damage at various districts, expected demand for relief commodities after an earthquake, travel time estimates between critical origin-destination points were collected. Our mathematical model inputs are generated with respect to the collected data, and our solution algorithms are used to solve the real-life problems relevant for the earthquake preparedness of the city of Istanbul.

ÖZETÇE

Bu tezde afet yönetimi ile ilgili üç ağ optimizasyonu problemi ele alınmıştır. Her problem etkin afet sonrası faaliyetler için afet öncesi süreçte stratejik planlamayı kapsamaktadır. İlk problem, alt yapı sistemlerinin güvenilirliğinin ve performansının değerlendirilmesi ile ilgilidir. Bu problemde, ağ bağlarının kopma olasılıklarının birbirlerine bağımlı olduğu durumları gösterebilmek için bir yaklaşım öne sürülmüştür. Bu yaklaşım kullanılarak, fazla verinin bulunmadığı afet durumlarında kullanılabilecek bir bağımlılık tanımı ortaya konmuştur. Bir ağın bu bağımlılık yapısı altında güvenilirliğini ve performansını ölçebilmek için yeni ve polinom-zamanlı bir algoritma geliştirilmiştir. İkinci problem, ağ bağlarını güçlendirmek için yapılan yatırımlarda belli bir bütçenin en iyi şekilde değerlendirilmesi icin kritik bağların belirlenmesi problemidir. Kritik bağlar belirlenirkenki amaç, güçlendirildiği takdirde ağın afet sonrası en yuksek performansını sergilediği bağı bulmaktır. İkinci problemde, daha önce geliştirilmiş olan bir metodun hesapsal uygulanabilirliği Monte Carlo yöntemi kullanılarak örnek bir olay üzerinde incelenmiştir. Tezde ele alınan son problem ise afet sonrası hizmet vermesi planlanan aktarma ve dağıtma merkezlerinin yer seçimine karar verilmesidir. Amaç fonksiyonu, farklı afet senaryoları altında en fazla sayıda insana en kısa zamanda çoklu malzeme taşınması ve dağıtılması olarak belirlenmiştir. Aktarma ve dağıtma merkezlerinin her bir malzeme için belli bir uzaklıkta olması koşulu altında taşıma zamanının ağırlıklı ortalaması minimize edilmeye çalışıldığı kapasitesiz bir matematiksel model kurulmuştur. Bu model gerçek bir problemi çözmek için kullanılmıştır. Bu üç problem de deprem riski altında olan İstanbul için uygulamaya konulmuştur. Deprem senaryolari, bu senaryolar altında ulaşım ağlarının durumları, belirli bölgelerdeki deprem riskleri ve bölgeler arasındaki uzaklıklar gibi gerçek veriler toplanılmıştır. Matematiksel modelin girdileri toplanan veriler kullanılarak türetilmiştir. Öne sürülen çözüm algoritmaları İstanbul şehrinin deprem hazırlıkları aşamasında karşılaştığı problemlerin çözülmesi yolunda kullanılmıştır.

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Chapter 1

INTRODUCTION

Disasters have posed a threat to human life from the beginning of life on earth. In most cases, it is not possible to forecast the time and the size/magnitude of a disaster. Consequently, the damage cannot be forecasted exactly. The situation should be handled in the after-math of a disaster.

Making decisions for the post-disaster stage in the pre-disaster planning stage is challenging due to the inherent uncertainty. On the other hand, decision-making in the post-disaster stage is also difficult since it requires assessing the current situation immediately and responding to it effectively in the chaos arising after the disaster by coordinating and dispatching the resources. The emergency and the post-disaster trauma complicate the construction of a systematic rational approach at this stage, which intensifies the significance of the strategic pre-disaster planning. The difficulty of the problem originates from this fact; the uncertainty of the consequences of the disaster. Pre-disaster decisions must be taken under uncertainty by considering the worst and the most likely conditions that might happen after a disaster, and the possible actions that may be taken after the disaster.

Disaster management can be considered to consist of three-stages [34]. The first stage is the pre-disaster stage which includes activities such as developing information systems, educating people and strengthening infrastructure systems. In this stage, providing maximum potential benefit to the society under cost effectiveness is the main objective. The second stage is the post-disaster emergency response stage which requires urgent intervention immediately to rescue people in the affected areas. In this stage effective coordination among different agencies and efficient use of resources is the main objective. The last stage is the post-disaster reconstruction and recovery stage. In this stage, rather than an emergency intervention, the disaster area is reconstructed in a longer period of time and relief operations take place.

Several Operations Research problems exist in each of these stages. Pre-disaster stage includes planning such as optimization of resource usage. Transportation of injured people to hospitals, transportation of search and rescue teams to the affected areas and distribution of food and necessity items are examples to such operational research problems in the post-disaster stage. The reconstruction and remedy of the city visage with optimal resource usage is a problem encountered in the long term post-disaster stage.

In this thesis, three problems are analyzed. The first problem is on assessing the reliability and performance of infrastructure networks under disaster risk. A framework to represent link dependency in failures is proposed. Using this framework, a special type of dependency structure, which is relevant for the disaster situation with lack of sufficient past data, is defined. Under this dependency structure, a novel polynomial-time algorithm is proposed to analyze the reliability and performance of a network and this original dependency structure is computationally tested on a case study for Istanbul. The second problem involves allocating a budget to links of an infrastructure network to increase the reliability and the performance of the network. With investment, each link can be strengthened structurally such that its probability of survival after a disaster increases. Then, the problem is to find which links to invest in so that most benefit is obtained in terms of post-disaster performance of the network with respect to accessibility and travel time between critical points in the network. The computational applicability of a previously developed method is demonstrated in the second problem by developing a Monte Carlo simulation algorithm and applying it to a real-life case study. The last problem addressed in this thesis is a facility location problem that seeks to identify locations of emergency response and distribution centers to provide effective post-disaster logistics operations such as the supply of relief commodities to the affected areas. For this problem, an uncapacitated facility location model is formulated with the objective of reaching a maximum number of people in minimum time possible after a disaster to distribute multiple commodities through the facilities under several disaster scenarios for demand and travel time. Average weighted travel time is minimized subject to constraints on the existence of a facility within a fixed distance from each district for each commodity. This model is used to solve a real-life problem for Istanbul against an anticipated earthquake.

A review of literature on disaster and emergency management with Operations Research approaches will be given in this chapter. A more detailed literature review for each of the problems can be found at the beginning of each related chapter.

For years, the studies on disaster management took place mostly in humanitarian science areas. Operations Research as a tool for decision makers in complex circumstances, has provided opportunities for the quantitative analysis of some problems arising in emergency management and there have been many applications on emergency management. There has been an increasing attention among researchers in Operations Research area on the analysis of disaster management and emergency response issues. The number of articles published since the beginning of the millennium support this fact as this number surpassed the total number of published articles in the 1990s [2]. Altay and Green [2] provide a survey that identifies the publication trends and the potential research directions in disaster operations. Wright et al. [36] categorize the literature on emergency preparedness and response into four parts as early work, location and resource allocation, evacuation models and disaster planning and response and investigates the topic in terms of homeland security. The reader is referred to Kolesar and Swersey [22] for a comprehensive literature survey and Green and Kolesar [15] for an overview of the developments in the last 15 years and possible future problems especially those relevant to emergency responsiveness published in the journal of Management Science. A systems view of emergency management, emphasizing the need for both pre-event and post-event strategies, policies and the role of advanced communications and computing technologies, coupled with analytic procedures and models are discussed by Tufekci and Wallace [33].

To sum up, disaster management and emergency preparedness topics have been extensively studied in the recent years. However, the three problems considered in this thesis bring different perspectives such as considering budgeting constraints for investments and using dependent survival link probabilities for the reliability of a network. An original dependency structure, vulnerability-based dependency, is introduced in this thesis. A polynomial-time algorithm for calculating reliability under this structure is given together with a case study which shows the practicality of the algorithm. A mathematical model and a solution to a real case study is completed with real-life data on Istanbul.

Disaster and emergency management will continue to be one of the broadly researched areas of operational research as it is linked to survival of many lives and the numbers and statistics reported after the disasters explain this link better. The Hurricane Katrina was reported to be one of the deadliest hurricanes in the history of United States. Katrina formed in late August during the 2005 Atlantic hurricane season and caused devastation along much of the north-central Gulf Coast of the United States. At least 1,836 people lost their lives. The storm is estimated to have been responsible for 81.2 billion (2005 U.S. dollars) in damage, making it the costliest natural disaster in U.S. history. There are many similar examples from all over the world. Turkey experienced the terrific Marmara Earthquake in 1999 that led to the death of over 17,000 people and a great damage to the economy. The words of Andrew Vorkink, Country Director of the World Bank for Turkey well introduces examples from all over the world and stresses the importance of disaster preparation. "In today's world, where the Marmara 1999 earthquake which killed 17,000 people and affected Turkey's economy by 2.5 percent of Gross National Product, the 2004 tsunami which killed more than 250,000 in Asia, Hurricane Katrina which is estimated to cost the U.S. well more than 100 billion dollars and more than 1000 lives, and the earthquake in South Asia which killed more than 30,000 people, emergency preparedness and hazard risk mitigation are essential roles of governments".

According to the reports of the World Bank, Istanbul is vulnerable to earthquakes due to its seismic-prone location on the North Anatolian Fault and its high population and commercial/industrial densities. It has been reported by Parsons et al.[26] in 2000 that the probability of a major earthquake in Istanbul in the next 30 years is $62.6 \mp 15\%$. If an earthquake of the same magnitude as that in 1999 were to occur in Istanbul, the human suffering as well as the social, economic, and the environmental impacts would be dramatically higher than the one in the Marmara region, as Istanbul is not only the financial, cultural and industrial center of the country, but is also a nexus of inter-continental importance and home to around 12 million people. Related to this declaration, the Turkish Government is in a collaboration with The World Bank on "Istanbul Seismic Risk Mitigation and Emergency Preparedness Project (ISMEP)" and is planning to invest in various districts of Istanbul.

In this major undertaking, the study in this thesis attempts to provide a theoretical framework for reliability analysis for optimal budget allocation in the pre-disaster stage, which is practical and can be applied to different networks in different disaster contexts.

Case studies are attained on highway networks in Istanbul supporting the fact that the methods proposed are practical. The rest of the thesis is organized as follows. The literature on general disaster management is given in the next section. The first problem, assessing the reliability and expected performance of a highway network, is given in Chapter 2. The second problem, strengthening the links of a stochastic highway network, is given in Chapter 3. The last problem is given in Chapter 4. Each chapter includes the related literature in detail and the computational results together with concluding remarks. The contributions of this study to the literature and future research ideas are in Chapter 5.

Chapter 2

ASSESSING THE RELIABILITY AND THE EXPECTED PERFORMANCE OF A NETWORK UNDER DISASTER RISK

The functionality of infrastructure networks after a disaster is critical for effective disaster response. In a disaster situation, the local and central government agencies as well as civil organizations should dispatch their resources immediately to rescue victims and to supply medical care, machinery, and relief commodities to the affected areas. As the search and rescue teams work in the field, the injured and dead people will be carried to nearby hospitals and shelters. In addition to the time-critical logistics operations carried out by the agencies, some residents will be on the roads trying to leave the affected area while others will try to reach the area to provide humanitarian aid. However, the disaster may render some of the links of the transportation network non-functional, leading to the blockage of some routes. As a result, the functionality and consequently the connectivity of the transportation network are vital factors for the success of the rescue and relief operations. In pre-disaster planning stage, it is important to assess the post-disaster performance of the network under possible disaster scenarios for the purpose of both strengthening the components of the network and for planning the post-disaster logistics activities.

In this problem, the aim is to measure the reliability and the expected post-disaster performance of a network under disaster risk. In post-disaster response, several nodes in the network act as supply points while affected areas are represented by demand points. As a result, these points constitute pairs of origin-destination (O-D) nodes in the network such that connectedness of these pairs of nodes in the post-disaster surviving network are essential under the foreseen disaster scenario. A set of reliability and expected performance measures that differ in terms of the number of disaster scenarios and the O-D pairs under consideration are proposed in this chapter.

An important aspect of this problem is the nature of uncertainty due to both how the disaster manifests itself and the vulnerability of the components in the network; hence how the network would be affected from a disaster event. It is assumed that each link of the network will be in one of two states after the disaster: i) operational (it survives), or ii) non-operational (it fails). In most cases, several likely disaster scenarios can be predicted by experts. Each disaster scenario is characterized in terms of its intensity and a geographic area of influence. In addition, the vulnerability of the components of the network can be measured by field engineers, mostly by structural analysis and statistical predictive methods. Hence, one needs to calculate the probability of failure/survival of each link under a given disaster scenario by incorporating all of these factors. One more concern is whether the links would fail independently or not. In the case of dependent link failures, one needs to identify the nature of dependence.

The exact calculation of any general network reliability measure, including the basic measures of two-terminal and all-terminal connectivity, is ♯P-Hard [23]. This is also the case for the measures proposed in this thesis due to the exponential number of possible states. To overcome this computational difficulty, methods are sought to reduce the state space by identifying relations among the link failures that would be pertinent to the particular disaster context. Furthermore, in case of a link failure dependency joint probability distributions of links are needed. As major disasters are rare events, in most cases sufficient amount of data does not exist to fit a joint probability distribution. In this chapter, a novel set-based link failure dependency scheme is conceptualized which may also be applicable in case of insufficient data. Next, a tractable link dependency structure that allows the existence of a polynomial-time algorithm when the number of O-D paths in the network is fixed is defined. In addition, Monte Carlo sampling based methods are present to estimate these measures under a general dependency structure. The use of this framework is then illustrated by a case study related to a highly anticipated earthquake in Istanbul.

The rest of the chapter is organized as follows. Section 2.1 provides a literature review of several areas related to network reliability. The necessary background and the problem characteristics are described in Section 2.2. Section 2.3 introduces the methodology used in this study and presents both the polynomial-time exact algorithm and the Monte Carlo simulation algorithm. In Section 2.4, computational results from the real-life Istanbul highway network case are reported and discussed together with a comparison between independent and dependent link failure cases. Concluding remarks are given in Section 2.5.

2.1 Literature Review

A degradation or a blockage of any infrastructure network, such as a telecommunication, power, water, energy and transportation network, etc., may cripple the performance of the network and complicate the daily life drastically. It is essential to identify the vulnerability of these networks in order to reduce the existing risk factors and to enhance the functionality of these networks. Researchers have contributed immensely on quantitative analysis to elicit information on the working principles and functionality of networks subject to failure risk over the last decades, mainly in the network reliability literature. In this section we discuss various network reliability and vulnerability definitions, the quantitative measures used to evaluate them and their role in the emergency/disaster management context.

2.1.1 Network Reliability

Network reliability is mainly the concern for a system to continue providing its function when component failures exist. Practical needs forced many researchers to study reliability of network systems, therefore there exists a wide and deep literature on network reliability. Networks have been studied for risk/reliability analysis where failures may originate from either natural or man-made disasters. Especially, increasing incidents of terrorism (e.g. September 11, 2001) and natural disasters (e.g. Kobe earthquake 1995, Katrina hurricane 2005) have led to heightened interest in the vulnerability of infrastructure network systems as a matter of national security. It has been recognized that it is essential to reinforce these network systems for better functionality after any type of destructive disaster.

However, how the reliability of a network system may be measured quantitatively is still not clear. This unclarity originates from the fact that the concepts of reliability and vulnerability do not yet have commonly accepted definitions and a theoretical basis on which to build [17]. The most general definition of reliability is that it is the probability of a specific pair of nodes to be connected in the network realization. There is an extensive survey on network reliability by Ball et al. [3] which includes related definitions, computational complexities, exact computation methods with performance analysis, in addition to Monte Carlo methods and computational techniques in practical use. Recently, Konak and Smith [23] reviewed network reliability optimization. They discussed the state-of-the-art techniques to solve the reliable and resilient network design problems and to evaluate reliability considering a survivability constraint. They also presented two original heuristic solution methods.

The evaluation of reliability requires several measures with respect to the structure of the system and concerns of the decision maker. Although it is possible to define them differently, types of measures may be collected in two categories by taking into account the connectivity and the performability aspects [3].

2.1.2 Connectivity Measures

Connectivity is defined as the probability that nodes of a network remain connected. It is mostly beneficial whenever the network performance is considered to be satisfactory as long as the network is connected [3]. Connectivity measures have special cases which narrows down the content/scope of the definition [23]. The following are the basic special cases of connectivity measures. i) *Two-terminal reliability* is the case in which the probability of a path existing between two specific nodes is considered. ii) *All-terminal reliability* is the probability that every node is connected with every other node in the network. iii) *K-terminal reliability* is the probability that every node in K is connected with every other node in K, where K is a subset of the total node set.

There are also *minimum over all two-terminal* and *average over all two-terminal* reliability measures which provide lower bounds and average values for the connectivity between specific pairs of nodes [3]. All of these measures may be important in different network systems. When there are specific target nodes that is desired to be connected, two-terminal would be a suitable measure. When traffic can be rerouted in case of a link failure for the packet switched networks, it is important that the all-terminal reliability of the network is high to keep the connectivity of the system. K-terminal measure is important for connectivity of local area networks of computers. The preference of minimum and average values for the two-terminal measures is up to the decision maker's risk attitude.

2.1.3 Performance Measures

Measurements for the network performance are based on the functionality of the network. Two measurements are defined below with respect to some requirements. i) *Travel-time reliability* is the probability that a trip between two specified nodes can be completed within a specified time interval [31]. ii) *Capacity reliability* performance index is the probability that a network can successfully accommodate a given level of travel demand.

Travel-time reliability and capacity-reliability terms have been proposed and analyzed in several cases where the networks are transportation/road networks. Sumalee and Watling [30] presented an algorithm for estimating bounds on the probability of a path travel time for the case of multi-mode link failures. Sumalee and Kurauchi [29] studied the network capacity reliability in terms of traffic regulations after a major disaster. Stochastic User's Equilibrium is assumed for the re-routing behavior effect due to changes in the traffic condition and the capacity reliability of the network is approximated with the use of Monte Carlo simulation. A capacity-reliability analysis has been conducted by Chen et al. [9] which combines reliability and uncertainty analysis.

There are also other measurements such as *behavioral-reliability* which is related to the attitude of the drivers in transportation systems and *potential reliability* which aims to find potential weak points and their effects on the whole network system [10]. Examples to applications of such measures may be transportation networks for the travel-time measurement and communication networks for the capacity reliability measurement.

Additionally, there are reliability measures concerning node failures. *Node-reliability* is obviously the case where nodes, rather than edges fail with certain probabilities. *Specified root-related reliability* is the probability that a specific number of roots are connected on the average.

When the literature is examined, it is observed that reliability is a flexible concept that can be modified with respect to the network characteristics in order to meet the decision maker's requirements. In addition to reliability, there are other concepts that help to characterize a network such as vulnerability, risk and accessibility. Well-defined definitions of these concepts are not available either. However, many researchers use the term vulnerability more closely aligned with network weaknesses and consequences of failure while reliability focuses on the probability of failure. Jenelius et al. [17] propose that vulnerability appears when the network is under pressure with full capacity, and a small amount of further stress may cause a major damage by magnifying itself and may cascade through the system. This implies that a network can be reliable, yet highly vulnerable at the same time.

2.1.4 Network Risk Assessment in Disaster Management

Emergency and disaster management is closely related with network reliability since disasters may cause damage to any one of the infrastructure systems. There are various studies on different types of disasters such as flood [28] and earthquake [27], [24], [8] with their effects on several network systems such as transportation, water or gas pipeline networks. An efficient pre-disaster strategy must contemplate reliability and vulnerability of such network systems. The knowledge on the status of a network in terms of the reliability measures prior to a disaster provides support for possible investment decisions on upgrading components of the network. Viswanath et al. [35] addressed the link upgrading problem under a limited budget and a disaster scenario with the purpose of effective post-disaster response. The objective is to maximize the expected performance of the network after a disaster and the study involves reducing the risk of disconnectedness of the O-D pairs via upgrading investment.

An analysis of the risk of the components can be an effective guide for any strategic/tactical level decisions. Moghtaderi-Zadeh [24] stated that systematic methods for efficient upgrading of lifelines for post-disaster earthquake serviceability had not been studied before 1983 and their paper is a first attempt at such a study. The aim of the paper is to determine the critical components in networks that would increase the connectivity-reliability of the network most. Selcuk and Yucemen [27] consider the reliability of lifeline networks with multiple sources under seismic hazard and propose a decision support system that presents a probabilistic model for the evaluation of the seismic reliability of a water distribution system. All of these previous studies support the fact that there is a need for a quantitative measure that will lead to mathematical analyses of network systems for an effective post-disaster response.

2.1.5 Computation of the Reliability Measures

Exact calculation of any general network reliability measure is known to be ♯P-Hard [23]. Exponential time exact algorithms are present for general networks and polynomial time exact algorithms for some restricted classes of networks [23]. With the assumption of two possible states of the links of a network, as functional and non-functional, the number of possible network realizations are 2^n, for a network of n links. Therefore, sampling and

Monte Carlo simulation are popular tools that are used for reliability calculations as the problem is mostly intractable with exponential complexity [9], [6]. Buchsbaum [6] presents a heuristic based on Monte Carlo and Markov Chain simulation techniques and proposes approximations and bounds on various reliability-related parameters. Sumalee and Kurauchi [29] utilize Monte Carlo simulation as well, however the authors generate the states of the links and the amount of degradation in functionality of the links after the disaster by random number generations. Karger [20] studies all-terminal connectedness using minimum cuts by means of a Fully Polynomial Randomized Approximation Scheme (FPRAS). Carey et al. [7] provide upper and lower bounds for the expected maximum flows in capacitated networks.

2.1.6 Link Failure Dependency

To the best of our knowledge, most of the studies in the literature consider the failure of the links of a network as independent events. However, it is often necessary to treat the link failures of a network as dependent events as they are subject to similar forces. This was also pointed out by Garg and Smith [14], who stated that in the context of emergency service deployment, disasters such as earthquakes may damage several roadways simultaneously, perhaps because of the presence of bridges or elevated roadways. Hence, contingency plans for providing relief to affected areas need to consider the potential failure of certain vulnerable links, some of which may simultaneously fail. Yet, how to define the dependency relationship among the link failures is a challenging problem.

There are a limited number of studies that take into account the dependency relationship among the links, such as [30], [31], [27]. Sumalee and Watling [30] also point out the lack of studies on the dependent case. In their study, the authors propose a scenario-based model where the causes of degradation of links change in each scenario. Each link is given a failure probability called the *conditional independence* due to each specific cause. Integrating these probabilities with the joint probabilities of the existence of these causes, dependency among the links is supplied implicitly. Most probable network states are generated among the most probable cause scenarios. Then, upper and lower bounds are calculated by the assumption that the possible network states, which are not generated, are all failing/surviving for the lower/upper bound. Taylor et al. [31] present a vulnerability analysis for the independent link failure case and suggest that the same procedure may be modified to a dependent case by considering the node failures which would lead to simultaneous failures of the links that are attached to it.

Selcuk and Yucemen [27] study the reliability of networks under seismic hazard. Although they use independent failures in their study, they propose a *spatial correlation* where the degree of spatial correlation between any two components depends on the distance separating them and generally decays with increasing distance. In this study our motivation is also related to earthquakes, and seismic risk of highway networks. In fact, distance to the epicenter of the earthquake are also identified as an important factor for risk. However, rather than deriving a function of risk with respect to distance, areas of high/low risk are defined. This approach seems to be more practical and allows a macro level analysis.

Joint probability distributions are necessary to incur a dependency relationship among link failures. However, this requires identifying the probability of each realization, where in the case of link failures, there exists an exponential number of realizations. Bayesian network analysis [18] facilitates the calculation of the joint probability distribution by eliminating a subset of the realizations. It first estimates the dependency relations by analyzing data and models them by a network. It then sets the required conditional probabilities by the network representation.

Bayesian networks are directed acyclic graphs where nodes represent the variables and arcs represent the immediate dependencies between two variables. Conditional independencies are defined among the variables that are dependent on each other but can become independent with the addition of new knowledge, ie. $\mathrm{P}(a|b) \neq \mathrm{P}(a)$ but $\mathrm{P}(a|b,c)=\mathrm{P}(a|c)$ for variables a, b and c. Hence, it is possible to use Bayesian network analysis to derive dependency relations among link failures when sufficient amount of data exists. Unfortunately, in the disaster context it is very difficult to obtain data on how the network components fail in dependence to each other.

2.1.7 Overview of the Proposed Method

For this problem, we propose an original framework for calculating the reliability and performance of a network under disaster risk. The disaster risk creates a dependency relationship among the link failures. One needs to identify first the set of links that show dependency among each other. Then, in each set the dependency relations should be modelled. This approach is neither network nor disaster specific and can be applied to different networks for different problems. We define the term "reliability" as the probability that two nodes, an origin and destination (O-D) pair, are connected and the "performance" of a network as the expected shortest path distance between an O-D pair, including a penalty cost for disconnectedness. It is already difficult to calculate the reliability under the independence assumption due to the intractable number of possible states. Therefore, we develop a solution approach for calculating/estimating the reliability and performance of a network under link dependency. We reduce the number of possible states by defining a particular dependency structure that is relevant in the disaster context. We then develop an exact polynomial-time algorithm under this structure. We also utilize a Monte Carlo Simulation algorithm to estimate the measures in cases where it is still intractable to obtain the exact calculations.

This new approach is applied to Istanbul highway network under earthquake risk, where the links are likely to fail due to the collapse of structures such as bridges and viaducts. In this context, we also provide a comparison between the independent and dependent link failure cases by analyzing how the performance measures vary under both cases.

2.2 Problem Definition

We consider the problem of assessing the reliability and travel distance of several O-D pairs on a given undirected graph $G = (V, E)$ under disaster risk, where $V = \{v_1, v_2, ..., v_n\}$ is the vertex set and $E = \{e_1, e_2, ..., e_m\}$ is the edge/link set. Several disaster scenarios are identified and each disaster scenario is characterized in terms of its intensity and a geographic area of influence. A disaster scenario is represented by ω_j that takes values from the possible set of disaster scenarios, $\Omega = \{\omega_1, \omega_2, \ldots, \omega_{|\Omega|}\}$. The probability that scenario ω_j occurs is denoted by $\mathrm{P}(\omega_j)$. Nodes are assumed to survive all the time, whereas each link, e_i, may exist in either the operational or the non-operational state after the disaster. Therefore, link e_i has a survival probability, $p_{e_i}(\omega_j)$ under disaster scenario ω_j. Each O-D pair is represented by a commodity $d \in C$ and has a positive weight r_d, representing estimated population traveling from its origin $O(d)$ to its destination $D(d)$. There is a travel cost t_{e_i} associated with each link $e_i \in E$ that represents travel distance along that link.

We represent the post-disaster state of link e_i by a random variable ξ_{e_i} that takes the value 1, if link e_i is operational after the disaster; and 0, otherwise. The vector of realizations of the random variables ξ_{e_i} over all links in E, denoted by $\xi = (\xi_{e_i})$, $\xi \subset \{0,1\}^{|E|}$ induces a subnetwork of G; $G(\xi) = (V, E(\xi))$ which we refer to as the "surviving network" where $E(\xi) = \{e_i \in E : \xi_{e_i} = 1\}$ denotes the surviving edges. The set of all network realizations is denoted by $\Xi = \{1, 2, ..., |\Xi|\}$. Travel distance of a shortest path from $O(d)$ to $D(d)$ in $G(\xi)$ is denoted by $T_d(\xi)$. If a particular O-D pair d is disconnected in any network realization, the shortest path length is equal to a penalty cost M_d for that pair. The probability of survival of link e_i under disaster scenario ω_j, i.e. $\mathrm{P}(\xi_{e_i} = 1|\omega_j)$ is denoted by $p_{e_i}(\omega_j)$ and the probability of occurrence of the vector realization ξ is denoted by $p(\xi)$.

2.2.1 Reliability and Performance Measures

In pre-disaster planning, one may be interested in looking into only the worst-case disaster scenario (single disaster scenario case) or a weighted combination of the desired measure over the most probable disaster scenarios (multiple disaster scenario case). Also, reliability and expected travel distance are possible measures of interest concerning either a single O-D pair or a weighted sum over multiple O-D pairs. We consider eight types of measures in this study with respect to these concerns. Each measure is characterized by three features and (./././) is used to represent these features. The first entry is used to show the number of O-D pairs, either S for single or M for multiple O-D pairs. The second entry denotes the number of scenarios, either S for single or M for multiple scenarios, and the third entry shows the type of the measure either R for reliability or P for performance. The measures we are interested in are:

1. O-D reliability under a single disaster scenario: For O-D pair d, disaster scenario w_j, $\mathrm{P}(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j$ occurs): (S/S/R).

2. O-D reliability under multiple disaster scenarios: For O-D pair d, $\sum_{j=1}^{|\Omega|} \mathrm{P}(\omega_j)\ \mathrm{P}(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j$ occurs): (S/M/R).

3. Multi O-D reliability under a single disaster scenario: For disaster scenario w_j,

$\sum_{d \in C} r_d$P($O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j$ occurs): (M/S/R).

4. Multi O-D reliability under multiple disaster scenarios:
   $\sum_{j=1}^{|\Omega|}$ P(ω_j)($\sum_{d \in C} r_d$P($O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j$ occurs): (M/M/R).

5. O-D performance under a single disaster scenario: For O-D pair d, and disaster scenario ω_j,
   $\sum_{\xi \in \Xi}$ P($\xi|\omega_j$) {P($O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j$) $T_d(\xi)$
   + (1-P($O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j$))M_d}: (S/S/P).

6. O-D performance under multiple disaster scenarios: For O-D pair d,
   $\sum_{j=1}^{|\Omega|}$ P(ω_j) $\sum_{\xi \in \Xi}$ P($\xi|\omega_j$) {P($O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j$) $T_d(\xi)$
   + (1-P($O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j$))M_d}: (S/M/P).

7. Multi O-D performance under a single disaster scenario: For scenario ω_j,
   $\sum_{\xi \in \Xi}$ P($\xi|\omega_j$) $\sum_{d \in D} r_d${P($O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j$)$T_d(\xi)$
   + (1-P($O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j$))M_d}: (M/S/P).

8. Multi O-D performance under multiple disaster scenarios:
   $\sum_{j=1}^{|\Omega|}$ P(ω_j) $\sum_{\xi \in \Xi}$ P($\xi|\omega_j$) $\sum_{d \in K} r_d${P($O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j$) $T_d(\xi)$
   + (1-P($O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j$))M_d}: (M/M/P).

The first four measures are reliability measures for connectivity. The next four are combined performance measures in which the reliability is also incorporated by means of a penalty cost for disconnectedness. Measures that consider multiple disaster scenarios incorporate the probability of occurrence of each scenario. In cases where estimating such a probability is difficult due to lack of data, one can interpret these probabilities as weights. Likewise, when multiple O-D pairs are considered, a weight can be assigned to each O-D pair, representing either the traffic demand between these locations in a disaster situation or the criticality of having connectivity between the particular O-D pair. Hence, the decision maker needs to determine which measures are relevant in their context based on the type of the disaster and the availability of data. The single scenario single O-D and multiple O-D measures are used on an investment problem in a previous study by Viswanath et al. [34] that will be mentioned in Chapter 3.

2.2.2 Link Failure Dependency

Exposed to a disaster, the links of a network may fail independently or depending on each other. This behavior is originated from the type of the disaster. The types of disasters may be concerned in two main parts according to their causes as natural and man-made. Usually, the natural disasters are more convenient to predictions in terms of happening time, area affected and intensity via scientific studies. Therefore, several likely disaster scenarios may be forecasted. On the other hand, the situation is a little different for man-made disasters. Man-made disasters may happen intentionally or accidentally. The intentional disasters such as terror attacks mostly target for specific links or nodes of a network such as critical structures and this decreases the stochastic nature of the problem whereas in the accidentally happening case, it is more difficult to predict both the source of the disaster and the aggrieved area. For highway networks in disaster context, link failures may be assumed to be independent for man-made intentional disasters or in cases for which the nature of dependencies of link failures may not be characterized. However, for cases in which a natural disaster affects an area enclosing the network, and the disaster hits certain links with more intensity, it is more reasonable to assume that the link failures are dependent.

The causes of this dependency are both internal and external to the network. Although the vulnerability of the link itself has an important effect on the failure of the link (such as the strength of a bridge), the consequences of the disaster (such as the collapse of buildings, an explosion or fire) have an effect that should not be neglected. The strength of a bridge on the link and the soil type on which the link stands are internal factors whereas the magnitude and the epicenter of an earthquake may be reckoned as external factors for a highway network under earthquake disaster risk. With these thoughts in mind, the dependencies of link failures tend to behave similarly for links sharing the same area. That is, a network embedded in a geographical region is under study and in this region there exist “areas” or “zones” with different characteristics that affect the link failures. Then, it is more probable that the links in the same area are operational or not together. This concept of an “area” on a network is to be specified with respect to the kind of the disaster it is subject to. It would be different for an earthquake than it is for a chemical explosion because diffusion of intensity/danger of chemical explosion is different than the intensity of earthquakes. So the characteristics of the disaster must be known for a better analysis. As a general observation, it may be said that there exist subsets of links that tend to act together within their sets after a disaster. Based on this idea, we propose a framework in which the set of links that have dependency are identified as mutually exclusive sets $A_l \subset E$, for $l = 1, \ldots, L$. Each *set of links* fail independently, while *links in a set* have dependence among each other. The independent failure case can be represented with singleton sets containing each link separately. The next section describes this concept of "area" utilized in this study in more detail and gives theoretical interpretations that lead to further investigation.

2.3 Methodology

Computing the proposed measures is of high computational complexity due to the exponential number of vector realizations when link failures are assumed to be independent. In the case of a natural disaster, to analyze this dependency related to spatial factors, one needs quantitative measures, such as the joint probability distributions of events relating to the dependent components. We assume a dependency relationship among the link failures which facilitates the computational effort. This relationship seems to be relevant and useful in the disaster context where the vulnerability of link components can be assessed.

2.3.1 Models of Dependency

The computational complexity of calculating the reliability originates from the number of conditional probabilities of components necessary to calculate the joint probability of the whole network. If joint probabilities among the components are available, then a Bayesian network can be used to determine the existing dependency structure. This would also help reduce the the number of conditional probabilities information needed to form the joint probability distribution. However, determination of the conditional probabilities requires analysis of previous data and the available data on disasters is insufficient. We propose an alternative approach for such cases where the dependency relationship among the components of a network can not be determined through historical data.

Set-Based Dependency:

Definition 2.3.1. *Given a network with link set E subject to failure due to a disaster event, the network is said to have* ***Set-Based Dependency (SB-dependency)****, if E can be partitioned into mutually exclusive sets $A_l \subset E$, for $l = 1, \ldots, L$ such that each set of links A_l fail independently with each other, while links within a set A_l have dependence among each other. If $L = 1$, that is $E = A_1$, then we have the all-dependent case. If each A_l consists of a single link, that is $|A_l| = 1$ for each l, then we have the all-independent case.*

This definition provides a framework to analyze link failures and can be applicable in cases where the network is embedded in a geographical region and the set of links are partitioned with respect to the effect of the disaster on the network under a particular disaster scenario. Thus, links in the same impact area of the disaster may constitute a set such that the set of links is partitioned into dependent subsets. In fact, this definition is sufficiently general to encompass the all-independent and all-dependent link failure cases as well. The all-independent case can be represented by considering each link as a single set and the all-dependent case can be obtained by using single set composed of all links. SB-dependency also allows the use of Bayesian network to determine the dependency relationships within each set.

In order to achieve further computational tractability, and a more specific structure on dependency in the disaster context, we define the following type of dependency within each set $A_l \subset E$. Here we assume that we are focusing on a particular disaster scenario and the link survival probabilities are given conditional on this scenario. However, for ease of presentation we change the notation from $p_{e_i}(\omega_j)$ to p_{e_i}.

Vulnerability-Based Dependency:

Definition 2.3.2. *Given two links i and j in set A_l with survival probabilities p_i and p_j, we say links i and j have* ***Vulnerability-based dependency (VB-dependency)****, if $p_i \leq p_j$ implies P(i fails | j fails)=1.*

Intuitively, one can think of this dependency relation as "the failure of the strongest link implies failure of all links". The links that have VB-dependency among each other are said to be in the same VB-dependency set. This concept of dependency allows us to determine the joint probability of failure or survival of a set of links. We denote the joint probability of survival of links i and j by $< p_i, p_j >$. Here we need to calculate the probability of connectedness of the O-D paths. Therefore, with respect to the VB-dependency definition above, the probability of link i surviving and link j failing in a vector realization of the links should be equal to zero. As a result, we obtain the following joint probability distribution for two links i, j in the same VB-dependency set, where $p_i \leq p_j$:

P(i survives, j fails)=$\langle p_i(1-p_j)\rangle = 0 = \langle p_i\rangle - \langle p_i p_j\rangle$.

Then, $\langle p_i p_j\rangle = \langle p_i\rangle$.

P(i fails, j survives)= $\langle (1-p_i)p_j\rangle = \langle p_j\rangle - \langle p_i p_j\rangle = p_j - p_i$.

P(i survives, j survives)= $\langle p_i p_j\rangle = p_i$.

P(i fails, j fails)= $\langle (1-p_i)(1-p_j)\rangle = 1-\langle p_j\rangle-\langle p_i\rangle+\langle p_i p_j\rangle = 1-\langle p_j\rangle-\langle p_i\rangle+\langle p_i\rangle = 1-p_j$.

To explain this concept more clearly, consider the case of a single disaster scenario occurring on a network with only 3 links of the network belonging to a single VB-dependency set. Assume that each link has a different survival probability, $p_i \neq p_j \ \forall \ i,j \in E$, and the links are ranked in the increasing order of their survival probabilities, that is $p_1 < p_2 < p_3$.

Table 2.1: Example of joint distribution of three links under VB-dependency

Case No	ξ_1	ξ_2	ξ_3	*Probability*
1	1	1	1	p_1
2	0	1	1	$p_2 - p_1$
3	0	0	1	$p_3 - p_2$
4	0	0	0	$1 - p_3$

Table 2.1 shows all possible network vector realizations together with their probabilities of occurrence. In the second, third and fourth columns, $\xi_i, i \in \{1,2,3\}$, equals 1 if the corresponding link survives after the disaster, and 0 otherwise. In case 1, all the links are functional. With respect to VB-dependency, probability of this vector realization equals to the survival probability of the weakest link, p_1, because it is guaranteed that if the weakest link has survived, the others will definitely survive. In the second case, where all but the first link survived, the probability is equal to $p_2 - p_1$. Only the third link survives in the third case, which makes the probability of such a case equal to $p_3 - p_2$. In the fourth case, all links are non-functional with probability equal to $1 - p_3$, because this is the probability that the strongest link will fail.

We next characterize the probability of realization of a particular surviving network configuration.

Proposition 2.3.1. *For networks with a single VB-dependency set, P(*$E(\xi)$*=E)* $= \min\{p_i\}_{1\le i\le m}$ *with respect to VB-dependency. For networks with multiple VB-dependency sets, P(*$E(\xi)$*=E)*= $\prod_{l=1}^{L}\min\{p_i\}_{i\in A_l}$ *with respect to VB-dependency.*

Proof. The first statement implies that the probability of all links surviving is equal to the minimum of the survival probabilities of the links in a single VB-dependency set. This can be shown using induction. First, note that P($E(\xi) = E$) = P(all links survive) = P(p_{i_1} survives, p_{i_2} survives,..., p_{i_m} survives).
Without loss of generalization, assume $p_{i_1} \le p_{i_2} \le ... \le p_{i_m}$ under VB-dependency. $\langle p_{i_1}\rangle = \langle p_{i_1}, p_{i_2}\rangle$ by definition of VB-dependency. Then,
P($E(\xi)$=E)= $\langle p_{i_1}, p_{i_2}, ..., p_{i_m}\rangle = \langle p_{i_1}\rangle\langle p_{i_2}\rangle, ..., \langle p_{i_m}\rangle = \langle p_{i_1}\rangle\langle p_{i_3}\rangle...\langle p_{i_m}\rangle$
$= \langle p_{i_1}\rangle\langle p_{i_4}\rangle...\langle p_{i_n}\rangle = ... = \langle p_{i_1}\rangle\langle p_{i_{m-1}}\rangle\langle p_{i_m}\rangle = \langle p_{i_1}\rangle\langle p_{i_m}\rangle = \langle p_{i_1}\rangle$
For the proof of the second statement of the proposition, simply consider that the probability of all links survive is equal to the product of probabilities of all links surviving in each set, since all VB-dependency sets are independent. □

Since the proposed measures include the connectivity of the O-D pairs, we need to concentrate on the joint probability distribution of links connecting each O-D pair. For connectivity, there must be at least one surviving path between an O-D pair and clearly, all of the links on a path should survive for a path to survive. Proposition 2.3.1 can be adapted for a single path as follows.

Corollary 2.3.2. *For a path with a single VB-dependency set, P(all links in the path survive)* $= \min\{p_i\}_{i\in path}$ *with respect to VB-dependency, where* $i \in path$ *represent the links on that path. For a path with links belonging to multiple VB-dependency sets, P(all links in the path survive)*= $\prod_{l=1}^{L'}\min\ \{p_i\}_{i\in A_l, i\in path}$ *with respect to VB-dependency, where* L' *is the number of different VB-dependency sets that the links on the path belong to.*

As a result, the following corollary provides computational efficiency in calculating the probabilistic measures that we are interested in.

Corollary 2.3.3. *If the links of a network with a single VB-dependency set are ranked with respect to their survival probabilities,* $p_{[1]} \le p_{[2]} \le ... \le p_{[m]}$ *in increasing order, then the* *probability of occurrence of a case in which a link with a higher index in this ordering fails while the one with a lower index survives is zero.*

Calculating any of the proposed measures defined in section 2.2 is computationally difficult due to the exponential number of possible realizations. However, the defined dependency relationship among the links, causes a decrease in the number of possible vector realizations (Corollary 2.3.3) and allows calculation of the reliability of a network in polynomial time for specified number of VB-dependency sets and number of shortest paths between O-D pairs.

We propose the following notation. The vector of realizations in which all links survive is represented by the vector $(1, 1, ..., 1)$ with size equal to the number of links in the network. Corollary 2.3.3 implies that, since the links are ordered with respect to their survival probabilities, a vector realization such as $(1, 0, ..., 1)$ can never exist because the failure of a link that is stronger in the survivability sense (with higher index) suggests that the weaker links (with lower index) must have already failed. This leads to the following proposition.

Proposition 2.3.4. *The maximum number of possible vector realizations for a network having a single VB-dependency set is equal to* $(m + 1)$*, where* m *is the number of links in the set.*

Proof. The number of possible vector realizations depends on the number of links which have the same probability of survival. It actually equals to $(m + 1)$ in the case where all links have different survival probabilities. When there are links with equal probabilities, then they behave as a single link surviving or failing together and this decreases the possible number of vector realizations. □

2.3.2 Calculation of the Reliability and Performance Measures

The reliability of the network is calculated as the summation of the probabilities of vector realizations which satisfy connectivity. If there is no functional link, ie. all links have failed, then this case does not contribute to the reliability of the network. However, the probability of occurrence of such vector realization is important because it will count towards the performance of the network due to the travel distance penalty, M_d, for the O-D pair d.

The connectivity of the surviving networks associated with the vector realizations are determined by checking the existence of the paths between the origin and destination nodes.

The number of paths between the O-D pairs may change according to the characteristics of the network. In this study, k-shortest paths between the origin and destination nodes are taken into account. π_s is the set of k-shortest paths determined in the preprocessing stage with $T(\pi_i)$ as the travel distance for path π_i. These k-shortest paths are considered as thresholds for the connectivity, ie. a vector realization is connected if any one of the k-shortest paths exists. In sparse networks, limiting the calculations to only k paths for each O-D pair is reasonable as the number of paths between any of the O-D pairs will be small. On the other hand, the number of paths connecting any two nodes may be exponential if the network is dense but using only the k-shortest paths is a practical approach in that case.

The pseudo-code to compute the reliability Rel, and the performance Per, of a single O-D pair in a network composed of a single VB-dependency set, A_1, under a single disaster scenario is given below. For the multi-scenario and the multi-O-D pair cases, each disaster scenario/O-D pair is given a weight. The weights for the disaster scenarios/O-D pairs reflect the importance given to that scenario/O-D pair. The O-D pairs are assumed to be connected for at least when all links are functional. Let p_i represent the survival probability of link i. (S/S/R) and (S/S/P) measures are calculated at the same time.

Algorithm for (S/S/R) and (S/S/P) Measures (Single VB-dependency Set)

Inputs: $p_i\ \forall i \in E$, $\{\pi_1, \pi_2, ..., \pi_k\}$

Outputs: Rel, Per

Step 1 Rank p_i such that $p_{[i]} \leq p_{[i+1]}$

Step 2 Compute initial reliability and performance (INITIAL)

Step 3 Generate a realization ξ_i (GENERATE)

Step 4 Check connectivity and calculate reliability and performance (CALCULATE)

Go to Step 3.

Step 5 Stop when all realizations are generated

INITIAL:

$\xi_i = 1$ for all i

$Rel = p_{[1]}$

$Per = p_{[1]} \cdot T(\pi_1)$

GENERATE:

For i=1 to n

$\xi_i = 0$

CALCULATE:

For s=1 to k repeat

Check if π_s survives in ξ_i

4.1 If yes, then

$Rel = Rel + p(\xi)$

$Per = Per + p(\xi) \cdot T(\pi_s)$

4.2 else

$Per = Per + p(\xi) \cdot M_d$

Computations of reliability and performance of the network are completed through an iterative approach. The first step is to sort the links with respect to their survival probabilities within the set. Reliability is computed as the summation of the probabilities of vector realizations which provide connectivity between the origin and destination nodes. In a similar manner, the performance is measured as the summation of the product of the traversal costs and the probabilities of the corresponding vector realizations. A penalty traversal cost is incurred whenever a vector realization is not connected. In the second step, the iteration starts with the full-functional case which is the case when all links are functional. Then, the initial reliability is the survival probability of the weakest link in the set, $p_{[1]}$, as shown in Proposition 2.3.1. The initial performance of the network is the shortest path between the origin and destination nodes since this path definitely exists when all the links are functional. In the third step, the next vector realization is generated in which the weakest link is non-functional. Connectivity check is completed in Step 4. This is done by checking the existence of the k-shortest paths. The first shortest path found to exist will be the traversal cost of that vector realization. Reliability of the network will be increased with the addition of the probability of that vector realization. When all the k-shortest paths are checked, if none of them exists within the given vector realization, then a penalty traversal cost is incurred which is a constant for each O-D pair d as M_d. The performance of the network is increased by the product of the traversal cost and the probability of the vector realization. If none of the shortest paths are found to exist, then the reliability stays the same, however, the performance of the network is increased by the amount of the product of the probability of the vector realization and M_d. The third step is repeated as many times as the number of links in the set. In m steps, all of the links will be failed one by one iteratively.

We next analyze the computational complexity of the proposed algorithm.

Proposition 2.3.5. *The computational complexity of the algorithm for a single O-D pair in a network composed of a single VB-dependency set with m links, under a single scenario is $O(km^2)$, where k is the number of shortest paths between the origin and destination nodes that are input to the algorithm.*

Proof. The time required for the algorithm to sort m elements is $O(mlogm)$. Then checking for connectedness is $O(m)$ because each link is checked for whether it is functional or not. This checking is repeated for the k-shortest paths in the worst case for a single step and this increases the complexity to $O(km^2)$ for m steps. □

Therefore, the computational complexity is polynomial time when k is a fixed number. The complexity differs for the case of multiple VB-dependency sets as follows.

Proposition 2.3.6. *The maximum number of possible vector realizations for a network of multi VB-dependency sets is equal to $(m_1 + 1) \cdot (m_2 + 1) \dots (m_L + 1)$, where m_l is the number of links in VB-dependency set A_l and L is the number of VB-dependency sets in the network.*

Let $p^l_{[i]}$ represent the survival probability of i-th link A_l when ranked with respect to survival probabilities. The pseudo-code for the single O-D pair, multiple VB-dependency sets is as follows:

Algorithm for (S/S/R) and (S/S/P) Measures with Multiple VB-dependency Sets

Step 1 Within each set A_l rank p_i such that $p_{[i]} \leq p_{[i+1]}$

Step 2 Compute initial reliability and performance (INITIAL)

Step 3 Generate a realization ξ_i (GENERATE)

Step 4 Check connectivity and calculate reliability and performance (CALCULATE)

Go to Step 3.

Step 5 Stop when all realizations are generated

INITIAL:

$\xi_i = 1$ for all i.

$Rel = p^1_{[1]} p^2_{[1]} ... p^L_{[1]}$.

$Per = (p^1_{[1]} p^2_{[1]} ... p^L_{[1]}) \cdot \pi_1$.

GENERATE:

For $i^1_1 = 1$ to m_1 repeat

Generate $\xi^l_i = 1$ for all i in all A_l

$\xi^l_i = 0$

For $i^2_1 = 1$ to m_2 repeat

Generate $\xi^l_i = 1$ for all i in all A_l

$\xi^l_i = 0$

...

For $i^L_1 = 1$ to m_L repeat

Generate $\xi^l_i = 1$ for all i in all A_l

$\xi^l_i = 0$

CALCULATE:

For s=1 to k repeat

Check if π_s exists

3.1 If yes, then

$Rel = Rel + p(\xi)$

$Per = Per + p(\xi) \cdot T(\pi_s)$

3.2 else

$Per = Per + p(\xi) \cdot M_d$

The algorithm starts with a full-functional state, similar to the algorithm for networks with a single VB-dependency set. The reliability and the performance are calculated iteratively. In this case, the initial reliability equals to the product of survival probabilities of the weakest links in each VB-dependency set, as shown in Proposition 2.3.1. The initial performance is then the product of this probability and the length of the shortest path, with the same reasoning as above. The major difference between the single VB-dependency set and the multiple VB-dependency set cases is the generation of the vector realizations. The number of possible realizations is $\prod_{l=1}^{L}(|A_l|+1)$ where the resulting vector realizations are combinations of vector realizations generated for each single VB-dependency set in the first algorithm. After generation of each vector realization, a connectivity check is done by searching for the k-shortest paths. If any of them exists, then the reliability is increased in the amount of the probability of that vector realization. The performance is calculated as the product of this probability and the shortest path length. If none of the k-shortest paths exist, then the traversal cost is taken as M_d, for the O-D pair d. The reliability stays the same, however the performance increases in the amount of the product of the traversal cost and the probability of the vector realization.

We give the following example to illustrate how the algorithms work. Consider the simple network in Figure 2.1. The survival probabilities of links p_i, $i \in \{1, ..., 5\}$ are given as $0.4, 0.4, 0.7, 0.7, 0.6$, respectively. The traversal costs are $10, 10, 5, 5, 15$ in the same order of links. The network is composed of two VB-dependency sets, namely $A_1 = \{p_1, p_2\}$ and $A_2 = \{p_3, p_4, p_5\}$. 4-shortest paths are determined as $\pi_1 = 1, 4, \pi_2 = 2, 3, 4, \pi_3 = 2, 5, \pi_4 = 1, 3, 5$ with travel distances between the O-D pair given as $T(\pi_1) = 15$, $T(\pi_2) = 20$, $T(\pi_3) = 25$, $T(\pi_4) = 30$. Here the penalty cost is taken as $M = 31$, just greater than the longest path.

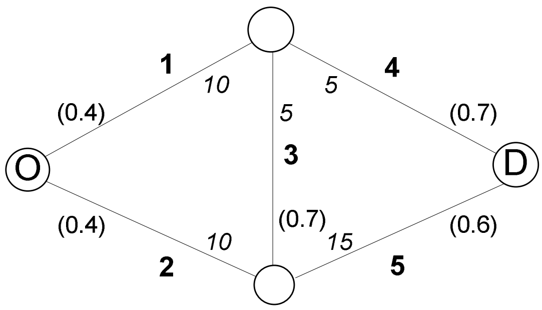

Figure 2.1: An example simple network

Preprocessing

Step 1 Sorting $p^1_{[1]} = 0.4$, $p^1_{[2]} = 0.4$, $p^2_{[1]} = 0.6$, $p^2_{[2]} = 0.7$, $p^2_{[3]} = 0.7$

Step 2 Start with $\xi = (1, 1, 1, 1, 1)$

$$Rel = p^1_{[1]} \cdot p^2_{[1]} = 0.4 \cdot 0.6 = 0.24$$

$$Per = (p^1_{[1]} \cdot p^2_{[1]}) \cdot T(\pi_1) = 0.24 \cdot 15 = 3.6$$

Iteration 1

Step 3 $\xi = (1, 1, 0, 1, 1)$

Step 4 $\pi_1 = (1, 0, 0, 0, 1)$. Check $\xi \geq \pi_1$ or not

TRUE

Step 4.1 $Rel = Rel + p(\xi) = 0.24 + (0.4 \cdot (0.7 - 0.6)) = 0.28$

$Per = Per + (p(\xi) \cdot T(\pi_1)) = 3.6 + (0.04 \cdot 15) = 4.2$

Go to Step 3.

Iteration 2

Step 3 $\xi = (1, 1, 0, 0, 0)$.

Step 4 $\pi_1 = (1, 0, 0, 0, 1)$. Check $\xi \geq \pi_1$ or not

FALSE

$\pi_2 = (0, 1, 0, 1, 1)$. Check $\xi \geq \pi_2$ or not

FALSE

$\pi_3 = (0, 1, 1, 0, 0)$. Check $\xi \geq \pi_3$ or not

FALSE

$\pi_4 = (1, 0, 1, 1, 0)$. Check $\xi \geq \pi_4$ or not

FALSE

Step 4.2 $Rel = 0.28$

$Per = Per + (p(\xi) + M \cdot (\pi_1)) = 4.2 + (0.12 \cdot 31) = 7.92$

Go to Step 3.

...

Step 5 Stop after ***Iteration 5*** (when all realizations are generated)

Next we analyze the computational complexity of the algorithm.

Proposition 2.3.7. *The computational complexity of this algorithm for a single O-D pair in a network composed of L VB-dependency sets with m_l links in each set, A_l, under a single disaster scenario is $O(km(m_{max}+1)^L)$, where m_{max} is the maximum of m_l and k is the number of input shortest paths between the origin and the destination nodes.*

As implied by Proposition 2.3.7, this algorithm calculates the reliability and performance measures in reasonable computational time for most practical cases. The computational time increases when the number of shortest paths or the number of VB-dependency sets increases. In the extreme case, if every VB-dependency set consists of a single link, ie. the all-independent link failure case, the complexity becomes exponential, $O(km2^m)$. Then, Monte Carlo simulation can be utilized to calculate these measures. The pseudo-code below summarizes such a Monte Carlo Simulation algorithm for networks with multiple VB-dependency sets, where N is the number of scenarios generated, $\mathcal{C}$ is the number of connected realizations and T is the shortest path length.

Monte Carlo Simulation Algorithm for (S/S/R) and (S/S/P) Measures for Multiple VB-dependency Sets

Step 1 For t=1:N, repeat

For each set $A_l,\ l = 1, 2, \ldots, L$

Generate a_t^l uniformly such that $0 \leq a_t^l \leq 1$

For each link $\xi_i \in A_l$, repeat

if $a_t^l \leq p_i$ then $\xi_i = 1$

else $\xi_i = 0$

Step 2 For s=1 to k

Check if π_s exists

Step 2.1 If yes, then

$C = C + 1$

$T = T + T(\pi_s)$

Step 2.2 else

$T = T + M_d$

Step 3 $Rel = C/N$

$Per = T/N$

In this algorithm, the reliability and the performance are calculated as the average over a sample of vector realizations which are generated using random numbers. In each iteration, the status of the links in each set A_l are determined by the same random number supporting the fact that the links belonging to the same VB-dependency set behave similarly. However, the random number used in each iteration is different. This is because every set is assumed to be independent from the other sets in the definition of SB-dependency. After generating the vector realizations, they are checked for connectedness and their performance in the same way as they were checked in the previous algorithms. This procedure is repeated for a fixed number of scenarios to be determined by the decision maker.

We have implemented all of the proposed algorithms and provide the computational results in the next section. The next section also demonstrates how to apply our proposed framework in a real-life case study.

2.4 Computational Results

The idea behind the proposed algorithms, with respect to the VB-dependency relationship, is a general approach that may be used for various applications; transportation, telecommunication or highway networks. These algorithms are applied to a case study on the urban highway network of Istanbul under earthquake risk. Istanbul, the cultural and industrial center of Turkey is under a serious risk of earthquake. Many studies have been completed on estimating the magnitude and the consequences of possible earthquakes and several earthquake scenarios have been developed [1] by the Government together with several universities in the city. The network considered in this thesis is constructed with respect to the two main highways in Istanbul and the bridges/viaducts located on them. The network includes 25 nodes and 30 links as depicted in Figure A.4. The O-D pairs are chosen based on detailed analysis of four most likely earthquake scenarios for the region of interest [1](Japan International Cooperation Agency Report, 2002). In these scenarios, the expected number of collapsed buildings, and the number of fatalities and injuries in each district of the region are estimated. The most-damaging earthquake scenario provides the basis for the selection of the O-D pairs. The origins correspond to the districts with the highest expected number of injured people. The destination nodes are the districts which have a large medical support capacity. The O-D pairs are (14-7), (12-18), (4-8), (9-7), (14-20).

The travel times are difficult to assess, especially in the case of disasters, as it is not possible to forecast the behavior of people. In this study, expected travelling distances are taken as the performance measures. We determined the possible k-shortest paths between the O-D pairs that will be needed to check connectivity. For each path, the total distance is calculated as the summation of the length of the links on that path. These distances are given in Table 2.2. If an O-D pair, d, turns out to be disconnected after the earthquake, then the travel distance for that pair is taken as a constant, M_d. This may be thought of using a helicopter for transportation. Using helicopters is one way in relief operations, however this is a costly way. The M_d value is taken as just higher than the longest shortest path between the O-D nodes for each pair so that it is always better to use one of the shortest paths rather than taking M_d as the travelling distance.

Table 2.2: k-shortest path distances for the O-D pairs

$O-D$	π_1	π_2	π_3	π_4	π_5	π_6	M
4-8	14.0012	17.9143	18.7937	21.511	26.7312	34.1542	35
14-7	11.1397	20.0876	25.4816	26.575	29.0784	30.1717	31
14-20	6.6489	20.4118	29.2	30.2664	–	–	31
12-18	9.5609	20.0476	20.2432	27.0592	–	–	28
9-7	9.4565	14.8505	16.8795	18,4473	–	–	19

2.4.1 Probability of Link Failures

The survival probabilities of the arcs are difficult to investigate. The likelihood of link failures vary with respect to the intensity and the location of the disaster experienced and the condition of the network components. Thus, a prior analysis of these factors is required. Still, it may be difficult to obtain a probability distribution for the network realizations or to use simulation. This is because the studies on disasters are mostly obliged to use historical data. However, sufficient data or statistics on link failures are not available in most cases because disasters are not frequently occurring natural events. Therefore, several assumptions have to be made and some parameters should be estimated using available information.

The factors that have been taken into account in this study for determining the link survival probabilities on the highway are the number of risky bridges on the roads including the strength of the bridges located on the links. The types of disasters is a crucial component in determining the failure probabilities of links as well because each type of disaster affects a different amount of area and with different severity. When a closer look is taken into each link, it is seen that each link may contain several vulnerable components and each component may withstand different levels of force depending on its structure, ie.

different survival probabilities appear due to different risk levels. These levels need to be translated into parameters, that will be input into a cumulative probability distribution function characterizing the failure probability of the component as a function of force on the component. Fragility curves can be used for this purpose. The intensity of a given disaster scenario determines the level of force at different areas, hence the probability of failure of a component under a given disaster scenario is calculated using this function. This incorporates the spatial factor into the probabilities as links in the same area are subject to the same disaster intensity in a given scenario. Scientific studies in Turkey agree on four different earthquake scenarios that Istanbul is mostly likely to experience. These scenarios are labeled as A, B, C and D where A is characterized as the most probable and C as the worst case scenario. The survival probabilities of the links change for each of these disaster scenarios; the survival probabilities are low for the risky scenarios whereas they are high in more risk averse scenarios.

2.4.2 Determination of VB-dependency Sets

For the earthquake application in this study, the idea of constructing "areas" for VB-dependency sets is administered using the measure of Peak Geographic Acceleration (PGA). PGA is a common measure used by earthquake engineers to evaluate the earthquake risk of a region. The PGA may be defined as the maximum acceleration experienced by an object in case of an earthquake. Various modeling of PGA distributions of Istanbul are done constituting of four different levels of PGA values [1] with respect to the disaster scenarios mentioned above. Based on previous studies [1], we were able to classify the links of the network as sets, A_l, $l = 1, ..., L$, for each PGA level under each disaster scenario. The classification for disaster scenario A is given Figure A.2. The links belonging to each set in each disaster scenario are summarized in Table 2.3.

Table 2.4 shows the survival probabilities for each link and how they are modified with respect to different PGA levels in each disaster scenario. Figure A.3 illustrates an example of risk levels for disaster scenario A by colors where the risk decreases as the color lightens.

Table 2.3: Links included in each VB-dependency set

Set	*Links*
	Scenario A
A_1	5
A_2	2, 3, 8, 9, 27
A_3	1, 4, 6, 7, 10, 11, 12, 13, 16, 20, 21, 22, 25, 26, 28, 29, 30
A_4	14, 15, 17, 18, 19, 23, 24
	Scenario B
A_1	2, 3, 5, 8, 9, 27
A_2	1, 4, 6, 7, 10, 12, 13, 16, 21, 22, 25, 28
A_3	11, 14, 15, 17, 18, 19, 20, 23, 24, 26, 29, 30
	Scenario C
A_1	2, 5, 8
A_2	3, 9, 12, 27
A_3	1, 4, 6, 7, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30
	Scenario D
A_1	5
A_2	2, 3, 8, 9, 27
A_3	1, 4, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30

2.4.3 Calculation of the Measures

The calculations for the eight measures defined in section 2.2 are completed with this data and the results are expressed in Table 2.5[1]. The first column shows the measure that is being evaluated as defined in Section 2.2. Measures (S/S/R) and (S/S/P) are given in the same row for the specified O-D pair in column 2. The third column contains the name of the earthquake scenario that is being considered. The last two columns give the reliability of the O-D pair and the performance, the expected shortest path distance between the O-D pair, respectively. The cpu time for these computations does not exceed a couple of seconds. To calculate the measures for the multiple O-D cases, weights are given to each

[1] All of the computations have been carried on the PC with 2x2.8 GHz Xeon Linux processor and 5 GB RAM memory with the algorithms implemented in Matlab 7.0.

Table 2.4: Survival probabilities of each link in different disaster scenarios

Link	A	B	C	D
1	0.8	0.8	0.8	0.84
2	0.76	0.76	0.72	0.8
3	0.76	0.76	0.76	0.8
4	0.7	0.7	0.7	0.735
5	0.72	0.76	0.72	0.76
6	0.6	0.6	0.6	0.63
7	0.8	0.8	0.8	0.84
8	0.57	0.57	0.54	0.6
9	0.76	0.76	0.76	0.8
10	0.7	0.7	0.7	0.735
11	0.55	0.5775	0.55	0.5775
12	0.8	0.8	0.76	0.84
13	0.6	0.6	0.6	0.63
14	0.525	0.525	0.5	0.525
15	0.84	0.84	0.8	0.84
16	0.55	0.55	0.55	0.5775
17	0.735	0.735	0.7	0.735
18	0.63	0.63	0.6	0.63
19	0.84	0.84	0.8	0.84
20	0.55	0.5775	0.55	0.5775
21	0.8	0.8	0.8	0.84
22	0.7	0.7	0.7	0.735
23	0.84	0.84	0.8	0.84
24	0.63	0.63	0.6	0.63
25	0.7	0.7	0.7	0.735
26	0.6	0.63	0.6	0.63
27	0.5225	0.5225	0.5225	0.5775
28	0.8	0.8	0.8	0.84
29	0.7	0.735	0.7	0.735
30	0.6	0.63	0.6	0.63

Table 2.5: Results for the reliability and performance measures for single O-D

Measure	$O-D$	*Scenario*	*Rel*	*Per*
$(S/S/R), (S/S/P)$	4-8	A	0.573	23.4556
		B	0.5890	23.1688
		C	0.7264	21.4638
		D	0.6359	22.1951
$(S/S/R), (S/S/P)$	14-7	A	0.550	20.0768
		B	0.423	23.9651
		C	0.550	20.0768
		D	0.577	19.5307
$(S/S/R), (S/S/P)$	14-20	A	0.7	13.9542
		B	0.757	13.9119
		C	0.7	13.9542
		D	0.735	13.1019
$(S/S/R), (S/S/P)$	12-18	A	0.4358	20.421
		B	0.5775	19.3215
		C	0.6	17.6330
		D	0.63	17.1147
$(S/S/R), (S/S/P)$	9-7	A	0.4358	15.0753
		B	0.6878	12.9097
		C	0.6	13.2739
		D	0.63	12.9876

O-D pair. In this application, the weights of all of the O-D pairs are taken to be equal. For the multiple disaster scenario measures, each disaster scenario is given a weight to emphasize its relative significance rather than its probability of occurrence. We know from previous studies that propose these disaster scenarios have labeled scenario A as the most probable disaster scenario and C as the worst case scenario, therefore we have chosen to give 0.4 weight for scenario C and $0.3, 0.2, 0.1$ for the scenarios A, B and D, respectively. The results for the multiple O-D pairs and multiple disaster scenarios are given in Table 2.6.

Table 2.6: Results for the reliability and performance measures.

$Measure$	$O-D$	$Scenario$	Rel	Per
$(S/M/R),(S/M/P)$	4-8	multi	0.5520	23.8449
	14-7	multi	0.5270	20.7998
	14-20	multi	0.7150	13.8605
	12-18	multi	0.5492	18.7553
	9-7	multi	0.5713	13.7129
$(M/S/R),(M/S/P)$	multi	A	0.5389	18.5966
	multi	B	0.6071	18.6554
	multi	C	0.5894	17.9651
	multi	D	0.6417	16.9860
$(M/M/R),(M/M/P)$	multi	multi	0.5830	18.1947

2.4.4 Comparison of Various Dependency Structures

To reach a general conclusion of whether reliability of a network is higher or lower in the all-independent or the VB-dependent link failure cases, we made a comparison of the results of this method with the results of the calculations of the two extreme cases which are the all-independent and the all-dependent cases.

Since it is computationally difficult to calculate the exact reliability value for the all-independent case, we used the Monte Carlo Simulation Algorithm given in the previous section. Several sample sizes are chosen. The estimations are subject to error and a confidence interval is computed for each estimated performance value by the standard formula $Per \mp z \frac{\sigma}{\sqrt{n}} = Per \mp \Delta$, where Per is taken as the estimated performance value, σ as the sample standard deviation, n as 4,000,000 for the sample size and for a desired confidence of %90, z is taken to be 1.645. With these parameters, the representative Δ values for one O-D pair (4,8) are provided in Table 2.7, along with the confidence interval limits. It illustrates that we obtain robust accuracy with the selected sample size and within reasonable computation time relative to pre-disaster planning context.

The results of the comparison are given in Table 2.8. The first column shows the link dependency structure. The structure given in the first row is when the VB-dependency sets are based on the PGA values of the region. The case with only one VB-dependency set is given in the second row as all-dependent link failure. In the all-independent case, the number of VB-dependency sets is equal to the number of links in the network.

In this comparison, it is observed that the reliability has the lowest value in the 10-sets case and the all-independent case is less reliable then the all-dependent case. However, no pattern can be observed among reliability. The reason is as follows. Note that for a path to be connected, it is necessary that all the links on that path should survive. If all the links on the path belong to the same VB-dependency set, then reliability of the network is $\min\{p_i\}_{i\in E}$. If the link failures are assumed independent, then the reliability becomes $\prod_{i\in E} p_i$. $\prod_{i\in E} p_i \leq \min\{p_i\}_{i\in E}$, since $0 \leq p_i \leq 1$. Therefore, the single VB-dependency set is always more reliable. On the other hand if the links of the path belong to more than one VB-dependency set, then the reliability becomes $\prod_{l=1}^{L'} \min\{p_i\}_{i\in A_l, i\in path}$. The extreme case now is that each link belongs to a different VB-dependency set which is equal to the case of all-independent link failures. However, the reliability of an O-D is the summation of the probabilities of the existing k-shortest paths. So, every time a vector realization is checked,

Table 2.7: Results for Monte Carlo Simulation for the O-D pair $4-8$, Disaster Scenario C and M=35

Sample Size	*Rel*	Δ	*Lower Limit*	*Upper Limit*	*Cpu Time*
500000	0.6664	0.0211	22.6340	22.6762	11.71
750000	0.6669	0.0173	22.6193	22.6539	17.62
1000000	0.6667	0.0149	22.6285	22.6583	23.69
4000000	0.6668	0.0075	22,6321	22,6471	23.53

Table 2.8: Results for the O-D pair $4-8$, Disaster Scenario C and M=35

Link Dependency Structure	*Rel*	*Per*
1-set (all-dependent)	0.7000	20.7801
3-sets (PGA)	0.7264	21.4638
10-sets (PGA)	0.6031	24.0035
30-sets (all-independent)	0.6668	22.6396

it is checked for more than one path. In this case, the effect of survival probability of a link changes with how many different shortest paths it belongs to. Then the reliability becomes a characteristic for the specific network and this prevents us from reaching a general conclusion on the comparison of reliability of the network in the all-independent or VB-dependent link failures. Additionally, the number of links that have the same survival probability affects these measures.

The example network in Figure 2.1 is an example which shows that there is no general behavior for the change in the reliability of the network when the link failures are assumed either to be VB-dependent or independent. The reliability for several cases for the network

Table 2.9: Comparison of Various Dependency Structures

Case No	p_1	p_2	p_3	p_4	p_5	Rel_{1-VB}	Rel_{2-VB}	Rel_{ind}
1	0.4	0.5	0.7	0.3	0.6	0.5	0.30	0.46800
2	0.4	0.4	0.7	0.7	0.6	0.4	0.28	0.53008

is given in Table 2.9 where p_i show the survival probability of link i. The seventh column, Rel_{1-VB} is the reliability in the case where the network consists of only one VB-dependent set. The next column, Rel_{2-VB}, gives the reliability in the case where two VB-dependent sets are present, where the first VB-dependency set includes links $1, 2$ and the second one includes the remaining links $3, 4, 5$. The last column, Rel_{ind}, gives the reliability in the all-independent link failure case. In Case 1, the reliability in the all-dependent case is higher than the reliability in the all-independent case. Although this is consistent with the results in Table 2.8, the all-independent case has the highest reliability in case number 2. This case is a counter example to the statement that the all-independent link failure structure is less reliable than the all-dependent link failure structure.

2.5 Concluding Remarks

In this chapter, a network system is examined for its reliability and performance after a disaster. Concentrating on eight different reliability measures a transportation network is examined for status in the pre-disaster stage. The paths between several O-D pairs are examined. The link failures are assumed to be dependent on each other and a general conclusion about reliability is attempted to be found out. However, it is shown that reliability is a characteristic feature of a network and it is not possible to reach a general conclusion on the reliability in the dependent or independent link failure cases.

Chapter 3

STRENGTHENING THE LINKS OF A STOCHASTIC NETWORK FOR DISASTER RESPONSE

In this chapter, the optimal budget allocation for pre-disaster strengthening of infrastructure networks in disaster management problem is discussed. This chapter is an application of the paper by Viswanath et al. [35] and provides computational results using a real-world case study. It illustrates the quality of the local optimum solution compared to the global optimum for instances in which the global optimum could be obtained by enumeration. It also demonstrates the practical applicability of the approach proposed.

3.1 Summary of the Previous Study

A strategic planning problem that seeks to strengthen, under a limited budget, an infrastructure network whose links are subject to independent and random failures due to a disaster is addressed in this study. The objective is to optimize post-disaster response in terms of network reliability and accessibility of nodes through investment which increases the survival probabilities of the links invested in. A network with several origin-destination (O-D) pairs is considered with the objective of minimizing the expected value of the weighted traversal costs between the origin and destination nodes, across post-disaster network realizations. The traversal cost under a realization is the least path cost among the surviving paths of the O-D pairs. If no surviving path exists for an O-D, the traversal cost is a fixed penalty cost. The problem is modelled as a two-stage stochastic program in which the probability distribution of the network realization in the second stage is dependent on the investment decisions of the first stage. The objective is to strengthen the links in the pre-disaster stage so that the expected value of the shortest paths after the disaster is minimized for an efficient post-disaster response including the transportation of people, food, medicine, etc. from/to the affected areas. The approximate equivalent deterministic program of the two-stage stochastic program, constructed using a path-based approach, is used to show that there exists a monotone decreasing multilinear function of the investment variables that coincides with the objective function. It is shown that using the first order terms of the multilinear function leads to a knapsack problem whose solution is a local optimum to the original problem.

The two-stage stochastic model of the problem is given below and the related notation can be found below.

Notations:

c_e	:	Cost of investing in link e
B	:	Budget
y_e	:	1, If there is an investment in link e; 0, otherwise
$y = (y_e)$	:	The investment decision vector for all links in E
ξ_e	:	1, If link e is operational after the disaster; 0, otherwise
$\xi = (\xi_e)$	:	1, The vector of the random variables ξ_e for all links in E
$\widetilde{\xi_e}$	:	a specific realization of ξ_e
$\widetilde{\xi} = (\widetilde{\xi_e})$	:	a specific realization of ξ
t_e	:	Non-negative traversal cost for link e
$x_e(\widetilde{\xi})$	:	1, if there is a unit flow through link e in the network realization ; 0, otherwise
$x(\widetilde{\xi}) = (x_e(\widetilde{\xi}))$	:	The flow vector
$X(\xi)$	:	The set of paths from O to D in the network realization
$f(\xi)$	:	Least path cost if it exists, or the penalty cost M if O-D is not connected
$F(y)$	:	Expectation of $f(\xi)$ wrt the random variable ξ for a given investment vector y
u_e	:	Unit vector of dimension $\|E\|$ having 1 at component e and 0 at the remaining.

Program P:

1st Stage:

$$Z = \min_y F(y) = \min_y E_{\xi|y}(f_i(\xi)) \tag{3.1}$$

subject to

$$\sum_{e \in E} c_e y_e \leq B \tag{3.2}$$

$$y_e = 0 \text{ or } 1 \qquad \forall e \in E \tag{3.3}$$

2nd Stage:

$$\min f(\xi) = \begin{cases} \min\limits_{x(\xi) \in X(\xi)} \sum\limits_{e \in E} t_e x_e(\xi), & \text{if } X(\xi) \neq \emptyset \\ M, & \text{otherwise} \end{cases} \tag{3.4}$$

subject to

$$\sum_{e=(i,j) \in E} x_e(\xi) = \sum_{e=(j,i) \in E} x_e(\xi) = \begin{cases} 1 & \text{if } i = O, \\ -1 & \text{if } i = D, \\ 0 & \text{otherwise.} \qquad \forall i \in N \end{cases} \tag{3.5}$$

$$x_e(\xi) \leq \xi_e \qquad \forall e \in E \tag{3.6}$$

$$0 \leq x_e(\xi) \leq 1 \qquad \forall e \in E \tag{3.7}$$

Here $X(\xi)= \{x(\xi)|x(\xi)$ satisfies constraints $(4), (5)$ and $(6)\}$ is the set of paths in the network realization $\widetilde{\xi}$ for the O-D pair d. The second-stage objective function $f(\xi)$ is a function of the random variable ξ whose probability distribution is determined by the investment vector y. Its value is equal to the least path cost in the network realization, if such a path exists, or the penalty cost M if O-D is disconnected. The objective function, $F(y) = E_{\xi|y}(f(\xi))$, is the expectation of $f(\xi)$ with respect to the random variable ξ for a given investment vector y and can be expanded as $F(y) = \sum\limits_{\widetilde{\xi} \in E} P(\xi = \widetilde{\xi}|y) \cdot f(\widetilde{\xi})$ where $P(\xi = \widetilde{\xi}|y)$ is the probability that is realized given that the investment vector is y.

The $P-approx$ program given below is proven to give the local optima of the two-stage stochastic program.

$P - approx$:

$$\min_y \sum_{e \in E} g_e(\mathbf{0}) y_e \tag{3.8}$$

subject to

$$\sum_{e \in E} c_e y_e \leq B \tag{3.9}$$

$$y_e = 0 \text{ or } 1 \qquad \forall e \in E \tag{3.10}$$

where $g_e(0)$ are the first order term Taylor series expansion coefficients for $F(y)$ in the neighborhood of point 0, which is the no-investment case. $P - approx$ is a 0-1 knapsack problem that can be solved efficiently either in pseudo-polynomial time by dynamic programming (see Martello and Toth, 1990), or by branch and bound using a standard solver.

Lemma 3.1.1. [35] $g_e(0) = F(u_e) - F(0)$.

Proposition 3.1.2. [35] $g_e(0)$ *denotes the marginal system-level benefit of investing in link* e *alone.*

Proof. See Viswanath et al.[35] for the proofs. □

3.2 Monte Carlo Sampling-Based Implementation Procedure

This section describes a Monte Carlo sampling procedure to calculate $g_e(\mathbf{0})$ using the equality in Lemma 3.1.1. It is important to note that while computation time is not a key factor in the deployment of this method due to its pre-disaster planning context, we nevertheless need a procedure that is efficient for tractability.

It is shown in Section 4.3 of [35] that the objective coefficients $g_e(\mathbf{0})$ of the approximate integer program $P - approx$ are computed as $F(u_e) - F(\mathbf{0})$ for a given O-D pair. However, calculating $F(y)$ for any y requires exploring an exponential number of possible network realizations. For example, for the 30-link network, $30 \cdot 2^{30}$ ($\approx$ 32 billion) cases should be explored. As a practical approach to overcome this difficulty, Monte Carlo Sampling is used to estimate the $F(u_e)$ and $F(\mathbf{0})$ values. First, the k-shortest paths are determined in advance for the O-D pair under consideration. To estimate $F(u_e)$ for each link e, one million random network realizations are generated such that the links are either operational or non-operational according to the probabilities determined by the investment vector u_e. For each realization, the O-D connectivity of the predetermined k-shortest paths is checked in terms of the increasing order of traversal cost to find the minimum cost operational path. If the O-D pair is not connected in that realization, the traversal cost is taken as

M_d, for the O-D pair d. Then, the average of these 1 million traversal costs is assumed to be the estimated $F(u_e)$ value. The procedure is repeated for each link with a different set of realizations generated for the corresponding post-investment probabilities. Finally, one million random network realizations are generated to estimate $F(\mathbf{0})$, and $g_e(\mathbf{0})$ estimates are obtained. This computation takes about 380 seconds of CPU time for one O-D pair in the 30-link network.

3.3 Computational Studies

Numerical experiments on real-life data related to the strengthening of Istanbul's urban highway system against earthquake risk both illustrates the applicability of the method and suggests insights and trade-offs between the expected least path cost and reliability.

The computational study is based on highway networks from Istanbul, Turkey. Istanbul has been affected by two major earthquakes in 1999 with epicenters about 250km from it that caused $10-$25 billion in damage [26]. The Turkish government is planning to invest $400 million to strengthen critical public infrastructure for earthquake resistance. A key element of this plan is to retrofit the highway system to ensure maximum accessibility and functionality after an earthquake. This implies the seismic retrofit of bridges/viaducts which tend to be the weakest structural elements in the highway system. For this study, it provides a very relevant setting to analyze the practical use of the model proposed by Viswanath et al.[35]. The relevant data is obtained from the 2003 Master Earthquake Plan (MEP) of the Istanbul municipality, and focuses on the two main highways TEM and E-5 in the city and the bridges/viaducts located on them. The associated map is depicted in Figure A.1. It shows the southern part of the city, which is its most densely populated and seismically risk-prone region. The city is separated into the European and Asian sides by the Strait of Bosphorus. In our experiments, we consider two networks to analyze our model. The first one consists of 25 nodes and 30 links and includes both sides as shown in Figure A.4. It is used to analyze the performance and the computational scalability of the model. The second network has 8 nodes and 9 links, shown in Figure A.4, and represents only the Asian side. It is considered because the global optimum can be obtained by enumeration, thereby providing a benchmark for the quality of the local optimum obtained through our approach. The link traversal costs are chosen proportional to the distances between the nodes.

The initial link survival probabilities are typically determined by structural engineers using domain-specific information. In this study, we use data from the MEP that classifies bridges/viaducts as less "risky" and "very risky", to determine the probabilities as shown in Figure A.1. This is done by identifying the numbers of less risky and very risky bridges/viaducts on each link, and determining a weighted score which is then translated into the survival probabilities. In this context, very risky structures are weighted with 3 points while less risky ones are weighted with 1 point. Five initial link survival probability levels are assigned ranging from 0.5 to 0.8, based on link scores as illustrated in Table 3.1. Post-investment, the link survival probabilities are assumed to be 1 based on feedback from the structural engineers involved in the retrofitting plan. The investment cost for each link is calculated as a weighted score proportional to the link length and the number of bridges/viaducts located on it. The survival probabilities and the investment costs are given in Table 3.2.

Table 3.1: Scale used to determine the survival probabilities

Link Score	< 1	< 5	< 10	< 15	< 20
Probability	0.8	0.7	0.6	0.55	0.5

The total budget needed to invest in all links is 11640 units. We consider three budget levels for the experiments: $B_1 = 1164$, $B_2 = 2328$, and $B_3 = 3492$. They correspond to strengthening approximately 10%, 20% and 30% of the links, respectively. The O-D pairs are chosen as the pairs in Chapter 2 for the 30-link network. For the 9-link network, (15,22) and (17,19) are considered as the O-D pairs.

3.3.1 Sample Size and Convergence

We investigate the convergence of the estimated $F(u_e)$ values by increasing sample sizes of the generated realizations for the 30-link network. Table 3.3 shows the representative estimated $F(u_e)$ values, denoted by $F(u_e)'$, for a subset of links, for sample sizes varying from 2 to 1,000,000. Table 3.4 shows the difference between the $F(u_e)'$ values for consecutive sample sizes for the same subset of links, where the lower case letters, $a, b, c, d, e, f, g, h, i, j$ denote the differences between $F(u_e)$ values obtained by consecutive sample sizes. Also the mean square error (MSE), the average of the squared differences over all 30 links, is reported for each sample size. They indicate that the estimated values converge rapidly as the sample size changes between 10,000 and 100,000. Hence, the computation time could be reduced more than tenfold with an insignificant loss in accuracy. Even if 1,000,000 samples were used, they would represent about 0.009 of the possible realizations, while providing high levels of accuracy.

3.3.2 Confidence Intervals

The estimations are subject to error and a confidence interval is computed for each estimated value by the standard formula given in Chapter 2 Section 2.4.4. With the same parameters, the representative Δ values for one O-D pair (14,7) are provided in Table 3.5, along with the confidence interval limits. It illustrates that we obtain robust accuracy with the selected sample size and within reasonable computation time relative to the pre-disaster planning context.

3.3.3 Insights on the Solution Method and Parameters

This section provides insights on the interpretation of the $g_e(\mathbf{0})$ values and the effect of parameter M on the solution using the 30-link network with the five O-D pairs.

Table 3.2: Link investment costs and survival probabilities

Link	c_e	p_e	*Link*	c_e	p_e	*Link*	c_e	p_e
1	80	0.8	11	940	0.55	21	40	0.8
2	80	0.8	12	160	0.8	22	160	0.7
3	320	0.8	13	620	0.6	23	40	0.8
4	260	0.7	14	1180	0.5	24	620	0.6
5	160	0.8	15	40	0.8	25	260	0.7
6	420	0.6	16	940	0.55	26	780	0.6
7	160	0.8	17	300	0.7	27	800	0.55
8	620	0.6	18	520	0.6	28	120	0.8
9	120	0.8	19	40	0.8	29	220	0.7
10	340	0.7	20	800	0.55	30	500	0.6

Interpretation of the $g_e(0)$ values

The $g_e(\mathbf{0})$ values for the five O-D pairs are summed up to form the objective coefficients of the knapsack problem $P - approx$, and are shown in Table 3.6. Table 3.8 and Table 3.9 show the solutions to the knapsack problems for the 3 budget levels. Solving the knapsack problem takes less than one second on our computing platform.

The $F(\mathbf{0})$ value for each O-D pair are again determined by Monte Carlo sampling with sample size 200,000 for the first case with $M = 120$ and 500,000 for the second case in which each O-D pair has different M values and the following values are obtained.

From Table 3.6, the objective coefficients with the most negative values appear in links 10, 11, 13, 16, 20, and 22, for $M = 120$, and links 10, 16, 20, 22, and 25, for $M = 31$. In

Table 3.3: $F(u_e)'$ values for different sample sizes

	Sample Size										
$F(u_e)'$	2	10	100	1000	10000	100000	200000	400000	600000	800000	1000000
$F(u_1)'$	65.5699	68.5476	99.7284	90.4219	89.2341	89.0514	88.8871	88.915	88.932	89.0732	89.0106
$F(u_2)'$	120	87.3419	87.2498	91.5962	89.1253	88.8526	89.0942	88.9732	88.9392	89.0564	89.0394
$F(u_3)'$	120	89.7802	87.2862	88.5659	88.8772	88.7807	88.9159	89.1911	89.0184	88.9748	88.9921
$F(u_4)'$	120	68.0082	90.0928	88.2565	89.3811	89.0233	89.163	88.9458	89.0149	88.9408	89.0019
$F(u_5)'$	120	109.114	96.9687	87.635	88.8266	88.6577	89.1829	89.0444	89.0423	88.8787	89.0009
$F(u_6)'$	72.7408	87.3419	95.512	88.5946	88.9952	88.6965	88.6423	88.7613	88.7024	88.6339	88.6864
$F(u_7)'$	120	99.6621	90.5332	92.2611	88.2813	88.6443	88.8855	88.8078	88.7379	88.9932	88.8845
$F(u_8)'$	73.2875	109.114	85.3278	89.9902	88.8237	88.6805	88.8231	88.9457	88.9264	89.0716	89.0601
$F(u_9)'$	120	78.8942	81.6857	87.9703	88.2132	87.7325	88.0062	87.8183	87.8783	87.9139	87.9015
$F(u_{10})'$	120	43.7978	83.1665	78.2807	80.2595	80.452	80.6969	80.6232	80.4943	80.5887	80.5564
$F(u_{11})'$	120	99.1227	83.5378	88.5106	85.8299	85.4444	85.5188	85.4745	85.4122	85.3958	85.519
$F(u_{12})'$	72.7408	88.2367	86.1721	87.061	88.7283	88.5252	88.5086	88.3917	88.4047	88.3975	88.5037
$F(u_{13})'$	120	67.1134	80.4669	85.2427	86.0284	86.7632	86.776	86.5705	86.6048	86.6385	86.6015
$F(u_{14})'$	120	109.114	81.3062	82.9552	83.9622	82.9159	83.0438	83.1116	82.984	82.8841	83.0678
$F(u_{15})'$	72.7408	67.8988	82.0287	88.6072	89.3701	88.8141	88.8234	88.9852	88.9606	88.96	88.9048
$F(u_{16})'$	65.5699	77.8901	73.7856	74.6744	73.9044	73.6204	73.7129	73.3949	73.4572	73.638	73.6412
$F(u_{17})'$	65.5699	87.3419	89.7963	88.7252	86.8622	86.5872	86.4109	86.4782	86.3446	86.438	86.3217
$F(u_{18})'$	65.5699	77.8901	78.0111	86.2035	88.9627	88.8299	88.9062	88.8545	88.9165	89.0947	88.9528
$F(u_{19})'$	120	98.2279	98.0498	89.8511	88.6053	88.8844	89.1825	89.0061	88.9759	88.9371	89.0143
$F(u_{20})'$	65.5699	88.7761	56.8557	65.4956	62.9872	63.7297	63.711	63.6033	63.6056	63.5462	63.5135
$F(u_{21})'$	120	109.114	86.0642	86.2938	88.7158	89.1216	89.232	88.923	88.9883	88.9444	89.0111
$F(u_{22})'$	65.5699	69.5517	88.5146	89.3242	89.0866	89.1916	88.9105	88.9923	89.041	88.8572	88.9114
$F(u_{23})'$	65.5699	109.114	92.791	91.4769	88.5238	89.1625	89.0753	89.0216	88.8955	88.9874	88.9389
$F(u_{24})'$	65.5699	87.3419	85.8584	87.0283	88.8355	89.0397	88.908	88.9805	89.0266	88.8764	89.0375
$F(u_{25})'$	15.6137	78.7849	85.9173	88.3581	88.2472	89.1627	88.8867	89.0481	89.0217	88.867	88.9848
$F(u_{26})'$	23.3313	110.009	91.3178	85.3852	89.1894	88.9621	88.9525	88.9939	89.0257	88.9291	88.9411
$F(u_{27})'$	65.5699	110.548	80.1848	90.2803	88.7972	88.7075	88.9966	88.9967	89.0298	88.9585	88.9354
$F(u_{28})'$	11.1397	98.2279	91.3822	88.6903	88.643	88.8142	88.8531	89.0243	88.9878	89.0187	89.0185
$F(u_{29})'$	65.5699	65.5699	83.5401	88.2562	88.7485	89.0428	89.1445	88.983	89.0426	89.0936	89.0118
$F(u_{30})'$	120	76.4559	95.9406	88.9307	88.6466	89.0565	88.9696	89.0181	88.9746	88.9642	88.9543

Proposition 1 in [35], the objective coefficients for links were interpreted as the marginal system-level benefit of investing in that link alone, indicating that the links with the most negative objective coefficients provide the maximum improvement under investment. When the solutions are examined in Table 3.8 and 3.9, we note that links 10, 20, 21, 22, 23 and 25 have been invested in under most budget levels. We also note that some links, such as 16, which have high negative coefficients, are not invested in. Others, with very low coefficients, such as 23, are invested in. This highlights the systems perspective inherent in our method, where the marginal benefits are traded-off with budget limitations and unit investment costs. It also illustrates the fallacies of some existing approaches that focus on

Table 3.4: Difference in $F(u_e)'$ values between consecutive sample sizes. and MSE across all links

	Sample Size									
Link	a	b	c	d	e	f	g	h	i	j
1	2.9777	31.1808	-9.3065	-1.1878	-0.1827	-0.1643	0.0279	0.017	0.1412	-0.0626
2	-32.658	-0.0921	4.3464	-2.4709	-0.2727	0.2416	-0.121	-0.034	0.1172	-0.017
3	-30.22	-2.494	1.2797	0.3113	-0.0965	0.1352	0.2752	-0.1727	-0.0436	0.0173
4	-51.992	22.0846	-1.8363	1.1246	-0.3578	0.1397	-0.2172	0.0691	-0.0741	0.0611
5	-10.886	-12.145	-9.3337	1.1916	-0.1689	0.5252	-0.1385	-0.0021	-0.1636	0.1222
6	14.6011	8.1701	-6.9174	0.4006	-0.2987	-0.0542	0.119	-0.0589	-0.0685	0.0525
7	-20.338	-9.1289	1.7279	-3.9798	0.363	0.2412	-0.0777	-0.0699	0.2553	-0.1087
8	35.8265	-23.786	4.6624	-1.1665	-0.1432	0.1426	0.1226	-0.0193	0.1452	-0.0115
9	-41.106	2.7915	6.2846	0.2429	-0.4807	0.2737	-0.1879	0.06	0.0356	-0.0124
10	-76.202	39.3687	-4.8858	1.9788	0.1925	0.2449	-0.0737	-0.1289	0.0944	-0.0323
11	-20.877	-15.585	4.9728	-2.6807	-0.3855	0.0744	-0.0443	-0.0623	-0.0164	0.1232
12	15.4959	-2.0646	0.8889	1.6673	-0.2031	-0.0166	-0.1169	0.013	-0.0072	0.1062
13	-52.887	13.3535	4.7758	0.7857	0.7348	0.0128	-0.2055	0.0343	0.0337	-0.037
14	-10.886	-27.808	1.649	1.007	-1.0463	0.1279	0.0678	-0.1276	-0.0999	0.1837
15	-4.842	14.1299	6.5785	0.7629	-0.556	0.0093	0.1618	-0.0246	-0.0006	-0.0552
16	12.3202	-4.1045	0.8888	-0.77	-0.284	0.0925	-0.318	0.0623	0.1808	0.0032
17	21.772	2.4544	-1.0711	-1.863	-0.275	-0.1763	0.0673	-0.1336	0.0934	-0.1163
18	12.3202	0.121	8.1924	2.7592	-0.1328	0.0763	-0.0517	0.062	0.1782	-0.1419
19	-21.772	-0.1781	-8.1987	-1.2458	0.2791	0.2981	-0.1764	-0.0302	-0.0388	0.0772
20	23.2062	-31.92	8.6399	-2.5084	0.7425	-0.0187	-0.1077	0.0023	-0.0594	-0.0327
21	-10.886	-23.05	0.2296	2.422	0.4058	0.1104	-0.309	0.0653	-0.0439	0.0667
22	3.9818	18.9629	0.8096	-0.2376	0.105	-0.2811	0.0818	0.0487	-0.1838	0.0542
23	43.5441	-16.323	-1.3141	-2.9531	0.6387	-0.0872	-0.0537	-0.1261	0.0919	-0.0485
24	21.772	-1.4835	1.1699	1.8072	0.2042	-0.1317	0.0725	0.0461	-0.1502	0.1611
25	63.1712	7.1324	2.4408	-0.1109	0.9155	-0.276	0.1614	-0.0264	-0.1547	0.1178
26	86.6775	-18.691	-5.9326	3.8042	-0.2273	-0.0096	0.0414	0.0318	-0.0966	0.012
27	44.9783	-30.363	10.0955	-1.4831	-0.0897	0.2891	0.0001	0.0331	-0.0713	-0.0231
28	87.0882	-6.8457	-2.6919	-0.0473	0.1712	0.0389	0.1712	-0.0365	0.0309	-0.0002
29	0	17.9702	4.7161	0.4923	0.2943	0.1017	-0.1615	0.0596	0.051	-0.0818
30	-43.544	19.4847	-7.0099	-0.2841	0.4099	-0.0869	0.0485	-0.0435	-0.0104	-0.0099
MSE	1504.458	318.6208	28.81053	3.275041	0.184238	0.035594	0.022299	0.004928	0.012244	0.006714

individual link-level "criticality" measures.

Effect of Parameter M

Table 3.5: Confidence intervals for the O-D pair (14,7) with $M = 120$ for the 30-link network

Link	σ^2	σ	Δ	*Lower Limit*	*Upper Limit*
1	1389.188	37.27181	0.061312	89.07191	88.94929
2	1388.248	37.2592	0.061291	89.10069	88.97811
3	1389.632	37.27777	0.061322	89.05342	88.93078
4	1389.9	37.28136	0.061328	89.06323	88.94057
5	1390.436	37.28855	0.06134	89.06224	88.93956
6	1395.676	37.35875	0.061455	88.74786	88.62494
7	1390.828	37.29381	0.061348	88.94585	88.82315
8	1388.664	37.26478	0.061301	89.1214	88.9988
9	1409.804	37.54736	0.061765	87.96327	87.83973
10	1603.168	40.03958	0.065865	80.62227	80.49053
11	1449.236	38.06883	0.062623	85.58162	85.45638
12	1401.328	37.43432	0.061579	88.56528	88.44212
13	1439.58	37.9418	0.062414	86.66391	86.53909
14	1497.128	38.69274	0.06365	83.13145	83.00415
15	1391.292	37.30003	0.061359	88.96616	88.84344
16	1697.32	41.19854	0.067772	73.70897	73.57343
17	1441.884	37.97215	0.062464	86.38416	86.25924
18	1389.452	37.27535	0.061318	89.01412	88.89148
19	1389.652	37.27804	0.061322	89.07562	88.95298
20	1668.3	40.84483	0.06719	63.58069	63.44631
21	1389.76	37.27948	0.061325	89.07242	88.94978
22	1392.068	37.31043	0.061376	88.97278	88.85002
23	1390.492	37.2893	0.061341	89.00024	88.87756
24	1390.828	37.29381	0.061348	89.09885	88.97615
25	1391.308	37.30024	0.061359	89.04616	88.92344
26	1390.884	37.29456	0.06135	89.00245	88.87975
27	1390.764	37.29295	0.061347	88.99675	88.87405
28	1389.2	37.27197	0.061312	89.07981	88.95719
29	1388.216	37.25877	0.061291	89.07309	88.95051
30	1390.32	37.287	0.061337	89.01564	88.89296

Table 3.8 and 3.9 illustrate the effect of M on the solution. As discussed earlier, larger M implies greater emphasis on connectivity. In Table 3.8 and 3.9, we note that link 9 is invested in under all budget levels when $M = 120$, and under none when $M = 31$. Similar results are also noted for link 4. In both cases, the links provide key options for connectivity as seen in Figure A.4.

Table 3.6: Objective coefficients of $P - approx$ model for the 30-link network for $M = 120$ and $M = 31$

Link	M	1	2	3	4	5	6	7	8
Obj. Coeff.	120	0.4796	0.0909	-7.5522	-12.9623	-4.6874	-10.1104	-3.5077	-12.6736
Obj. Coeff.	31	0.2411	0.2723	0.2486	0.2546	0.2583	0.2665	0.2172	0.3085
Link	M	9	10	11	12	13	14	15	16
Obj. Coeff.	120	-9.62	-23.4098	-22.5208	-7.959	-20.1996	-10.7251	-0.2549	-19.1451
Obj. Coeff.	31	0.0025	-3.0429	-0.5788	0.061	-2.3111	-0.5424	0.2497	-3.0092
Link	M	17	18	19	20	21	22	23	24
Obj. Coeff.	120	-12.0672	-11.7344	0.299	-45.7208	-16.122	-27.9923	-4.6131	-12.0463
Obj. Coeff.	31	-2.8857	-1.1547	0.2678	-9.7512	-4.0536	-7.1832	-0.2923	-1.1597
Link	M	25	26	27	28	29	30		
Obj. Coeff.	120	-16.4982	-9.5883	0.0067	-1.7972	-2.5347	-4.4025		
Obj. Coeff.	31	-3.4785	-1.3181	0.2753	0.1773	0.1187	0.0333		

Table 3.7: $F(\mathbf{0})$ values for different O-D pairs

O	D	M	F(0)	O	D	M	F(0)
14	7	120	88.774	14	7	31	26.1843
14	20	120	69.2556	14	20	31	21.0518
9	7	120	46.264	9	7	19	14.0559
4	8	120	50.999	4	8	35	22.6365
12	18	120	79.669	12	18	28	22.9214

3.4 Quality of the Local Optimum Solution

As stated earlier, the 9-link network was considered primarily to analyze the solution quality as it allows for the enumeration of the solution. Tables 3.10, 3.11 and 3.12 report investment vectors, y_a, obtained by solving $P - approx$ when its objective coefficients are calculated with respect to sample sizes ranging from 1 to 100,000, for three budget levels. $F(y_a)$ is the objective value of the problem P at the proposed approximate solution y_a. $F(y_a)$ is compared to $F(y_{opt})$, where the optimal solution y_{opt} is found by enumeration. The values of $F(y_{opt})$ are 16.082, 12.085 and 8.7546 for budget levels 100, 200, and 800, respectively. We note that the proposed method reaches the optimal solution with a sample size of 1,000 for B=100, 100 for B=200, and 10 for B=800. Therefore, smaller sample sizes are sufficient as the budget constraint is relaxed. Also, the proposed approach obtains the optimal solution with little computational effort in this case.

Table 3.8: The knapsack problems solutions for the 30-link network for $M = 120$

Link	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B1	0	0	0	0	0	0	0	0	1	1	0	1	0	0	1
B2	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0
B3	0	0	0	1	1	0	1	0	1	1	0	1	1	0	1
Link	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B1	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0
B2	0	1	0	0	1	1	1	1	0	1	0	0	0	0	0
B3	0	1	0	0	1	1	0	1	0	1	0	0	0	0	0

Table 3.9: The knapsack problem solutions for the 30-link network for $M = 30$

Link	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
B3	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0
Link	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B1	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
B2	0	1	0	0	1	1	1	1	0	1	0	0	0	0	0
B3	1	1	0	0	1	1	1	0	0	1	0	0	0	0	0

3.5 Making investment decisions under VB-dependency

In this section, VB-dependency structure defined in the previous chapter is used to determine the criticality of the links in the dependent link failure structure. The O-D pair 9-7

Table 3.10: Solution of $P - approx$ model for various sample sizes and comparison with the global optimum. ($B = 100$ and $c_e = 100$ for each link e)

Sample size	1	2	3	4	5	6	7	8	9	$F(y_a)$	$F(y_a) - F(y_{opt})$
1	0	0	1	0	0	0	0	0	0	16.812	-0.73
10	0	0	0	0	0	1	0	0	0	17.99	-1.908
100	0	0	1	0	0	0	0	0	0	16.812	-0.73
1000	1	0	0	0	0	0	0	0	0	16.082	0
10000	1	0	0	0	0	0	0	0	0	16.082	0
100000	1	0	0	0	0	0	0	0	0	16.082	0

Table 3.11: Solution of $P - approx$ model for various sample sizes and comparison with global optimum. ($B = 200$ and $c_e = 100$ for each link e)

Sample size	1	2	3	4	5	6	7	8	9	$F(y_a)$	$F(y_a) - F(y_{opt})$
1	0	0	0	0	0	0	0	1	1	18.026	-5.941
10	1	0	0	0	0	1	0	0	0	13.75	-1.665
100	1	0	1	0	0	0	0	0	0	12.085	0
1000	1	0	1	0	0	0	0	0	0	12.085	0
10000	1	0	1	0	0	0	0	0	0	12.085	0
100000	1	0	1	0	0	0	0	0	0	12.085	0

Table 3.12: Solution of $P - approx$ for various sample sizes and comparison with global optimum. ($B = 800$ and $c_e = 100$ for each link e)

Sample Size	1	2	3	4	5	6	7	8	9	$F(y_a)$	$F(y_a) - F(y_{opt})$
1	0	1	1	1	0	0	0	1	1	14.515	-5.7604
10	1	0	1	0	1	1	1	0	0	8.7546	0
100	1	0	1	0	0	1	0	1	0	8.7546	0
1000	1	0	1	1	1	1	1	1	1	8.7546	0
10000	1	1	1	0	0	1	1	1	1	8.7546	0
100000	1	1	1	1	0	1	1	1	1	8.7546	0

is used as an example to make a comparison with the independent failure case. The results are given in Table 3.13.

Table 3.13: Link investment under VB-dependency structure

e	$F(u_e)$	$F(u_e) - F(0)$	$F(u_e)$	$F(u_e) - F(0)$
1	14.0627	0.0068	13.2739	0
2	14.0515	-0.0044	13.2739	0
3	14.0696	0.0137	13.2739	0
4	14.0624	0.0065	13.2739	0
5	14.0571	0.0012	13.2739	0
6	14.049	-0.0069	13.2739	0
7	14.0548	-0.0011	13.2739	0
8	14.0616	0.0057	13.2739	0
9	13.8598	-0.1961	13.2739	0
10	12.5746	-1.4813	13.2739	0
11	13.3921	-0.6638	12.0125	-1.2614
12	13.8743	-0.1816	13.2739	0
13	11.8684	-2.1875	12.3196	-0.9543
14	13.9823	-0.0736	13.2739	0
15	14.0594	0.0035	13.2739	0
16	13.9984	-0.0575	13.2739	0
17	14.0264	-0.0295	13.2739	0
18	14.056	0.0001	13.2739	0
19	14.061	0.0051	13.2739	0
20	14.061	0.0051	13.2739	0
21	14.0589	0.0030	13.2739	0
22	14.0688	0.0129	13.2739	0
23	14.0507	-0.0052	13.2739	0
24	14.0676	0.0117	13.2739	0
25	14.0605	0.0046	13.2739	0
26	14.0612	0.0053	13.2739	0
27	14.0594	0.0035	13.2739	0
28	14.06	0.0041	13.2739	0
29	14.0634	0.0075	13.2739	0
30	14.0642	0.0083	13.2739	0

In Table 3.13, the first column show the link under consideration. The second column gives the $F(u_e)$ value which is the expectation for investing on link e. The next column gives the difference between $F(u_e)$ and $F(\mathbf{0})$ values which represent the decrease in the expected value. The last two columns are the similar values obtained by considering dependent link failures and using the VB-based approach. The results in the third column show that the most benefit is gained by investing in link 13 since it has the largest decrease. Links 10 and 11 are the next two beneficial links. It is observed that in the dependent link failure case, it is not beneficial to invest in any of the links except link 13 and link 11. The reason why it does not help to invest in the other links may be due to the characteristics of the network or the links that form the paths between the O-D pairs. However, the fact that link 13 and 11 are the most beneficial links in the VB-dependent case supports the results in the independent case. In both approaches, it is most beneficial to invest in link 13 and 11.

3.6 Concluding Remarks

We presented numerical experiments on a real-world case related to strengthening Istanbul's urban highway system against earthquake risk. The problem was solved on a 30-link in less than 7 minutes by utilizing Monte Carlo sampling of the network realizations. An investigation of the convergence suggests that much lesser computational effort, in terms of sample size, yields the same solutions. The experiments also provided insights on the effects of problem parameters on the solutions. The quality of the solutions was investigated on a 9-link network by comparing the solutions using the proposed approach with those obtained through enumeration. The proposed method found the global optimum in all cases explored in negligible computation time.

The proposed model and the solution approach can be used by local and central government agencies to aid investment decisions in the context of response to natural calamities and protection against terrorist attacks.

Chapter 4

A FACILITY LOCATION MODEL FOR EMERGENCY RESPONSE AND DISTRIBUTION CENTERS

Disaster response starts with the transportation of food, medicine as well as search and rescue (SAR) teams to help the people blocked in the affected area as soon as possible. Afterwards, the injured people should be moved to safer locations and provided medical aid. One of the problems in this post-disaster stage is distribution of goods and necessity items. However, demand for these services and the status of the resources are not known in advance because the needs and the functionality to reach the resources depend on the impact of the disaster. Therefore, pre-disaster planning is to take due precautions.

This chapter presents an uncapacitated emergency facility location problem in pre-disaster planning for effective post-disaster logistics. The selection of locations of Emergency Response and Distribution Centers (ERDC) is considered for post-disaster transportation of commodities such as search and rescue teams, medical teams, food, water, necessity items and machinery. The mathematical model formulated will be utilized for the site selection of ERDC in Istanbul with earthquake risk considerations.

Post-disaster logistics problems have more distinct characteristics compared to a conventional logistics problem. There exist multiple objectives with different priority levels. In emergency logistics, the main goal is to reach the affected areas as soon as possible, rather than minimizing transportation costs. Minimizing access time is the most vital objective because people in the affected area are in need of urgent help. Furthermore, different time periods have different significance in terms of response objectives. The post-disaster time frame is typically divided into the first 4, 8, 12, 16, 24 hours and so on as the chances of saving lives, as well as the services needed differ at each interval. This study aims to provide the need in the first 24 hours. In addition to time restrictions, the actions needed to be taken after a disaster require cooperation of different departments such as the fire department, independent search and rescue teams, military rescue personnel, emergency relief centers, hospitals, etc. The problem may become intractable if the location under consideration is large-scaled. The dispatching of multiple resources which are under the control of multiple decision makers clearly needs well organization and coordination. For successful results, the planning should be done before disaster happens.

The Municipality of Istanbul is preparing to locate Emergency Response and Distribution Centers (ERDC) throughout the city. These centers will be used as coordination centers for different departments of the Municipality that will be in charge in case of an emergency. As the name implies, these centers will be used for organization and distribution of resources after the disaster. More specifically, the problem considered in this paper is the selection of locations of the Emergency Response and Distribution Centers (ERDC) among suitable regions, determined by the municipality in Istanbul, so that the demand for the distribution of goods and services is satisfied as much as possible in the shortest time right after a disaster occurs. An uncapacitated facility location model is formulated with the objective of reaching a maximum number of people in minimum time possible after a disaster to distribute multiple commodities through the facilities under several scenarios for demand and travel time. Average weighted travel time is minimized subject to constraints on the existence of a facility within a fixed distance from each district for each commodity.

The aim of this study is to provide a guide for the Local Municipality of Istanbul by preparing policies and providing recommendations on the selection of locations of the ERDC in the city. At the same time our objective is to provide a realistic mathematical model and a solution method that can be applicable for decision making under other disaster-planning circumstances.

4.1 Literature Review

The literature on strategic planning for emergency logistics is rather rare but there are considerably many studies on different components of the problem such as the location of emergency services and dispatching of commodities.

One of the earliest studies conducted on location of emergency service facilities is by Toregas et al. [32]. The problem is modelled as a set covering problem where the affected areas are represented as demand nodes and the emergency service facilities are the supply nodes. The objective is to minimize the maximum time/distance of a demand node to its closest supply node. An upper bound is determined as a constraint for a time/distance between any demand and supply node and linear programming is applied to solve the covering problem.

Consignment of goods are typically examined in the literature as a multi-commodity network flow problem, with a multi-period and/or multi-modal setting. Haghani and Oh [16] formulated a multi-commodity, multi-modal network flow model with time windows for disaster response. The objective is to minimize the costs under the assumptions that all the cost functions are linear and all the commodity quantities at supply and demand nodes are known in advance. Therein the time concern is handled by including penalty costs for late deliveries in the objective function. The authors put emphasis on the diversity of transportation modes to optimize the shipping efficiency. Two heuristic algorithms are proposed.

The flow of goods over an urban transportation network is modelled as a multi-commodity, multi-modal network flow problem by Barbarosoglu and Arda [4]. The demand and the resources are treated as random variables. The vulnerability of the transportation system is considered with random arc capacities based on sample earthquake and impact scenarios. A two-stage stochastic programming framework is formed as the solution approach.

Another study on the topic, conducted by Fiedrich et al. [13], model the problem similarly to a machine scheduling problem. The authors develop an optimization model for allocating search and rescue resources to appropriate operational areas. The paper aims to build a model to be effective especially for the first 3 days after the disaster because the SAR work peaks in efficacy for the aforesaid period. Minimizing the total number of fatalities, which changes over all relevant time intervals, in the SAR period is the main objective of the model. Two heuristics, Simulated Annealing and Tabu Search, are used as the solution methods.

As seen from the previous work discussed above, mathematical modelling of disaster recovery operations has been limited in the literature, whereas as stated in Bryson et al. [5] decision makers could benefit from the application of quantitative decision-making techniques. A systems view of emergency management, emphasizing the need for both pre-event and post-event strategies, policies and the role of advanced communications and computing technologies, coupled with analytic procedures and models is also discussed by Tufekci and

Wallace [33].

In addition to the above-mentioned studies, there are some applications of such mathematical models to real-life cases. Ozdamar et al. [25] analyze the dispatching of commodities to distribution centers as part of emergency logistics planning for the Marmara region. Their contribution to this literature is by a complex model in which the problem is considered as a hybrid that integrates the multi-commodity network flow problem and the vehicle routing problem. This study concentrates mainly on the detailed planning of pick-up, load and delivery time of commodities and usage of vehicles. The solution approach proposed includes Lagrangean relaxation on the capacity constraints. The model is run repetitively at given time intervals with updated information at each interval. The model and the solution methodology is implemented on a scenario based on the 1999 Marmara Earthquake. Similarly, Yi and Ozdamar [37] consider a dynamic and fuzzy logistics coordination model for conducting disaster response activities for which they rerun the corresponding model in each planning period to handle the new information communicated from affected areas. The authors define the problem in detail, taking into consideration the uncertain demand, supply, injured people and hospital service rates. The injured people are classified with respect to their health priorities and different transportation modes are considered for carriage. Fuzzy parameters are used to cope with the uncertainties while minimizing the risk of unserved medical requirements and unsatisfied commodity demand. The model is illustrated on an earthquake data set from Istanbul.

The location of emergency facility locations have been considered by Dekle et al. [12] for a real application study for Florida. A covering location model with a two-stage approach is used to determine the locations of disaster recovery centers that will be used by the Federal Emergency Management Agency(FEMA). The first stage in the solution approach is the determination of the number of such facilities with respect to the objective that each residence in the county to be within a specified miles of the closest facility. In the second stage, other criteria have been taken into consideration while relaxing the objective in the first stage.

The reader is referred to [19] for a review of facility location models for emergency services. The review is separated into three sections depending on the objective function of the location models which are covering models, p-median models and p-center models.

When the literature is reviewed, most of the models stated deal with predetermined supply nodes. In none of the above, the decision maker tries to optimize the location of transshipment nodes to maximize the commodity transportation. The models consider arc capacities which may actually be realized at different levels in case of emergency. The stochastic nature of the problem is handled via sampled scenarios in a two-stage stochastic model by Barbarosoglu and Arda [4] and via fuzzy variables by Yi and Ozdamar [25]. In this study we cope with the uncertainty by considering several disaster scenarios. We represent the functionality of the highway system and the facilities via using different travel times under different disaster scenarios with estimated potential damage. The demand nodes are also determined according to the most likely disaster scenario and estimated damage in terms of population. We do not consider detailed post-disaster dispatching since our main goal is to determine the location of the ERDC facilities. However, we consider accessibility after the disaster as one of the main constraints for the selection of sites.

This study attempts to provide a systematic approach for decision makers to minimize the detriments of the disaster by fast access to the affected areas with necessary commodities immediately after the disaster. The time restrictions in the problem are dealt by assigning higher weights to the commodities with urgency rather than assigning penalty costs for delays in the objective function as done in the literature. Our model includes a rather simple setting for distribution of commodities, excluding a detailed study of the dispatching and the loading/unloading of vehicles since the main aim of this study is to optimize the selection of facility locations and construct a pre-disaster strategy. The detailed scheduling and vehicle routing is to be carried out after the disaster dynamically, as soon as information on the damage is available.

4.2 Problem Definition

The problem considered is the location of Emergency Response and Distribution Centers (ERDC) for post-disaster emergency response. In particular, the aim is to solve for the selection of locations of ERDC among forty possible sites determined by the Municipality of Istanbul. ERDC are facilities which will be used both for transshipment and storage of commodities.

The first 24 hours after a disaster are critical for saving lives. The search and rescue teams have the greatest chance in this period to rescue the people trapped under debris with minimum injury. The fastest transportation of the medical teams and machinery are the top priority objectives. The transportation of water, food and necessities, shelter equipment and the personnel to service the affected areas form the second level priority, because if the transportation of both of these medical and rescue teams are not successful then the transportation of the rest would be unnecessary. The ERDC are considered as transshipment points. Since the timing and the magnitude of an earthquake is not known in advance with the existing technology, if the goods and commodities necessary to be distributed in the post-disaster stage were to be stored in a depot, they would perish. Therefore, the centers will function as transshipment nodes between suppliers and the districts of population after the disaster for the distribution of perishable goods and also equipment and personnel that will be called on duty right after the disaster.

4.3 The Mathematical Approach

The uncertainties due to the emergency situation complicates the large-scale facility location problem which is already computationally difficult to solve. Therefore, the following assumptions are made on different aspects of the problem to formulate an uncapacitated facility location model.

4.3.1 Assumptions

The assumptions are made on travel time between nodes, the capacities of the facilities, cost of opening a facility, demand of the affected people and the earthquake risk of both facilities and districts.

Travel time: The travel time considered in the model involves uncertainty with respect to the disaster scenarios. This uncertainty is incurred in the model by means of failure of the links and the nodes connecting the supplier, facility and the demand points. The expected distance are thus calculated with respect to the measure proposed in Chapter 3. However, the number of binary variables necessary to represent the functional links and nodes is exponential. In this large-scale case study, the travel times between two nodes are calculated as Euclidean distances. This simplification allows modification of the travel times/distances for each disaster scenario by means of coeffients.

Capacity: The capacity is considered both as a parameter for the suppliers and a decision variable for the capacity of the ERDC. However, under emergency conditions, it is difficult to predict the amount available at each supplier for every commodity. The decision for the capacities of the ERDC are not considered in the model because, as the suppliers are assumed uncapacitated and the demand is assumed to be fully satisfied.
Cost: The model includes fixed cost of opening facilities together with holding costs for storing durable commodities. The costs in the new model include only the fixed cost of opening new facilities. Since the capacities are not decision variables any more, the cost of opening an ERDC are assumed to be equal. Therefore, rather than an available budget, the number of facilities that can be opened are limited by a constant. This is similar to p-center and p-median problems where the aim is to open p facilities under additional restrictions.
Demand: The demand is obtained by data generation with respect to previous data. The details of demand for each commodity at each district are given in the next section.
Disaster risk: It is important that ERDC remain functional after the disaster. Therefore, the structure of the facility must be strong and the location should be chosen with respect to the disaster risk of the region. Rather than including this factor in the mathematical model, the disaster risk for the supplier nodes and the potential sites are searched in a preprocessing stage which prevents risky locations to be potential sites for ERDC. The municipality has already determined the possible sites with respect to risk levels and proximity to entrances and highways of Istanbul.

In case of disasters, the most critical issue is time management. The problem analyzed in this chapter is a facility location problem to minimize the total time to reach every district. Therefore, rather than a complex setting for capacities, costs and transportation amounts, a model is constructed that optimizes the locations of ERDC to reach the disaster area as soon as possible. The following are the notations used in the model.

P : Set of potential sites

D : Set of districts

S : Set of suppliers

W : Set of possible scenarios

$p(w)$: Probability of scenario w occurring, $w \in W$

NC : Set of non-durable commodities

I_c : importance factor of commodity c, $c \in NC$

T^w_{kjc} : Total distance for c to reach k using node j

s^w_{ijc} : 1 if i is the supplier of j for commodity c in scenario w

a^w_{jkc} : 1 if j can serve k for commodity c in scenario w

t^w_{jc} : distance from supply of c to j

R_c : Allowed distance for commodity c for every district

B : number of facilities to be opened

o_j : 1, if a facility will be open at site j; 0, otherwise, $j \in P$

R_c is a parameter that controls the allowed distance for a facility, which can serve commodity c, to be away from any district. This parameter is stated by the decision maker and may change for every commodity. This is reasonable because it may be more important to have a facility that can serve water than a facility which serves food and hygiene items for the first hours after the disaster. Therefore the allowed distance for facility with food and hygiene items are greater than the allowed distance for a facility that can serve water. a^w_{jkc} is dependent on this parameter R_c that is set by the decision maker.

4.3.2 The Uncapacitated Mathematical Model

$$\min \sum_{w \in W} p(w)(\sum_{c \in NC} I_c \sum_{k \in D} d^w_{kc} T^w_{kjc}) \tag{4.1}$$

subject to

$$\sum_{w \in W} p(w) s_{ijc}^{w} \leq o_j a_{jkc}^{w} \qquad \forall i \in S, \forall j \in P, \forall c \in NC \tag{4.2}$$

$$\sum_{j \in P} s_{jkc}^{w} = 1 \qquad \forall j \in P, \forall k \in D, \forall c \in NC, \forall w \in W \tag{4.3}$$

$$T_{kjc}^{w} = t_{jc}^{w} + t_{jc}^{w} s_{jkc}^{w} \qquad \forall j \in P, \forall k \in D, \forall c \in NC, \forall w \in W \tag{4.4}$$

$$\sum_{j \in P} o_j \leq B \tag{4.5}$$

$$o_j \in \{0,1\}, \ \forall j \in P$$

The objective function 4.1 is the minimization of total travel distances multiplied by the amount of demand at each district. Constraint 4.2 shows that a facility j can not serve district k if j is not opened or j is not in the allowed distance for district k to serve commodity c. Constraint 4.3 forces the model to assign a facility for every district. Constraint 4.4 shows the total distance travelled by a commodity, from the supplier plus the distance from the facility to district if that facility serves it. Constraint 4.5 is the budget constraint which determines the number of facilities to be opened.

4.4 Computational Studies

In this section, we report our computational studies and discuss the insights obtained. This study aims to provide a portfolio of solutions for the decision maker to reach the affected areas in minimum possible time. All of the computations have been carried on the PC with 1.60 GHz Intel Pentium processor and 504 MB RAM memory with the algorithms implemented in GAMS 22.5 with OSL solver. The computations are based on the most risky regions of Istanbul, with the data generated as described in Section 4.4.1.

4.4.1 Data Collection

The Municipality has cooperated with several universities in Turkey and prepared a detailed report containing four disaster scenarios ([1]). Most likely, some of the people will rush to hospitals, shelters and so on while others will prefer to stay by their houses. The travel behavior of the people under a disaster is still an open issue that needs to be researched as mentioned in Chapter 3 and Chapter 2. To construct the data three types of locations are needed; potential sites, demand points and supplier points. Additionally, the goods to be transported and stored should be determined, together with the demand for the districts for each commodity.

The locations are chosen from a selected number of sites. Several characteristics have been taken into account in determining these possible sites. One of these is of being reachable by at least two highways to be used alternatively in case one of them fails. Another consideration is the available area which is not easy to find in a metropolitan like Istanbul since it is not only the most populated city but also the industrial center in Turkey. The size of an ERDC is approximated based on the facilities it will contain such as a storage area, a large enough car park, several offices, a medical crisis unit and loading/unloading platforms.

Potential Locations:

The forty potential sites are determined by Municipality of Istanbul, the decision maker, with respect to the available areas and their proximities to highways in the city. Population densities throughout the city is available by previous studies [1]. This data is used as a reference and changed into a point-wise representation. District points are then aggregated to construct 84 demand points. The aggregation is completed taking weighted combinations of points in the same region to decrease the number of demand points for computational complexity. The eighty four districts are classified into 5 parts according to their population densities. The average number of people at each point and the number of points with such density are given in Table 4.1.

Table 4.1: District nodes with estimated population values

	Num. of Nodes	*Avg Num. of People*
District 1	19	35842
District 2	20	71685
District 3	22	107527
District 4	15	215054
District 5	8	286738

Suppliers:

The supplier nodes are determined separately for each commodity. The following are considered as the commodities to be transported; water, food and necessity items, medicine and search and rescue (SAR) teams. The obvious diversification is the shelf-life of each commodity which determines whether it can be stored or not. Sometimes the commodity may not easily perish but it may still be impractical to hold it in large amounts due to capacity limitations. Water is a good example of this. The fastest existence of the SAR teams in the affected area are highly important and it is considered as the commodity to be distributed after the disaster with highest priority. After this, the transportation of bread and water form the second priority level. It is assumed that direct transportation from the supplier nodes to the affected districts is not possible. This is a rational assumption because the outgoing goods from the suppliers will be in large amounts. The airport and highways to city entrances are considered to be suppliers node for resources brought in by international aid offices. The transportation of injured people is also an important aspect of post-disaster logistics. However, this issue will not be coordinated through the ERDC, whose site selection is the main concern in this study. For all kinds of aid and assistance to reach to the devastated area, connectivity to the affected districts is the most important criterion. As the links between any two nodes is subject to failure, the maximum survivability must be satisfied when choosing the facility locations so that alternative paths are available to reach every region. If the access to a district by highways is totally blocked after the disaster, helicopters may be utilized. However, helicopters are not suitable for transportation of heavy commodities and additionally not many helicopters are available in the Local Municipality. To prevent helicopter usage as much as possible in the mathematical model, a penalty cost will be added if a district can not be reached from any of the ERDC. Therefore, when selecting the sites, accessibility is an important criterion.

Commodities:

Five types of commodities are chosen that can be transported through ERDC; bread, drinking water, food and hygiene items, medical help and search and rescue teams. The commodities to be stored in ERDC are chosen to be stored as a unite package and therefore considered as one single commodity. The suppliers for each commodity are shown as separate points on the network. The suppliers for bread are chosen as the large bakeries in the city that are operated by the local government. Municipality has 27 drinking water tanks in the city. For food and hygiene packages, the depots of three large supermarkets in the city are considered together with the communal kitchen operated by Turkish Red Crescent. The scope of the medical help in our problem includes the blood supply in the first 24 hours, and not in detail of all hospitals in the city. The blood center operated by Turkish Red Crescent and the medical military hospital are taken as suppliers for medical help. Past experience connote that a lot of national and international help accrue out of city and out of country for search and rescue operations. Therefore, the entry points to the city are taken as supply points for search and rescue activities together with the 25 local offices of voluntary organizations. These entry points are chosen as the 2 airports and 2 national bus stations one in each side of Bosphorus, 1 national train station and 2 national ferry ports.

Demand:

The demand for each commodity at each district point is calculated with respect to the average number of people at that point. The requirements of one person for one day is determined as 2 breads, 3 liters of water. For the other commodities 1 package of food and hygiene items for 8 people, 1 medical supply for 50 people, 1 package of durable goods for 4 people and 1 search and rescue team for 20 people are determined as needs for the first 24 hours. These demands are adapted for each disaster scenario, increasing with a more intense scenario, as given in Table 4.2.

Travel time:

Measuring real travelling time requires to focus on a limited region. However, this study considers a large-scale problem and uses the southern part of Istanbul for computations. Therefore, it is difficult to measure the traveling times between the potential sites, suppliers and the district points. Therefore, direct distances between each point is taken as the travelling distance between any two nodes. Since it would not be reasonable to make a center further away from a point to serve that point, an upper bound is determined which prevents a center to be assigned to a very far away district. The available distance lengths are then adapted by coefficients for each disaster scenario. So rather than an assumption of travelling times, the distances are increased or decreased with respect to the expected impact of the disaster.

Budget:

The budget restrictions are the least important constraints in case of emergency. The only cost considered in this problem is the fixed cost of opening an ERDC. The cost of constructing such a center depends on many facts such as its location, equipments or the area used. All the fixed costs are assumed to be the same together with same capacities in this study. The capacities are taken as the maximum number of durable commodities in disaster scenario C. Four levels are determined for the total budget to control the number of open facilities.

In case of an emergency, since time is the most critical issue and it is difficult to supply all the demand, priorities are given to districts and commodities. This approach is incurred

Table 4.2: Demand amounts at each district

		Bread (*units*)	*Water* (*lt*)	*Food* (*units*)	*Blood* (*units*)	*SaR* (*units*)	*Durable Goods* (*units*)
Scenario A	District 1	86022	129032	5376	860	2151	10753
	District 2	172043	258065	10753	1720	4301	21505
	District 3	258065	387097	16129	2581	6452	32258
	District 4	516129	774194	32258	5161	12903	64516
	District 5	688172	1032258	43011	6882	17204	86022
Scenario B	District 1	78853	118280	4928	789	1971	9857
	District 2	157706	236559	9857	1577	3943	19713
	District 3	236559	354839	14785	2366	5914	29570
	District 4	473118	709677	29570	4731	11828	59140
	District 5	630824	946237	39427	6308	15771	78853
Scenario C	District 1	93190	139785	5824	932	2330	11649
	District 2	186380	279570	11649	1864	4659	23297
	District 3	279570	419355	17473	2796	6989	34946
	District 4	559140	838710	34946	5591	13978	69892
	District 5	745520	1118280	46595	7455	18638	93190
Scenario D	District 1	71685	107527	4480	717	1792	8961
	District 2	143369	215054	8961	1434	3584	17921
	District 3	215054	322581	13441	2151	5376	26882
	District 4	430108	645161	26882	4301	10753	53763
	District 5	573477	860215	35842	5735	14337	71685

into the mathematical model with importance factors for each district and commodity. These importance factors are given in Table 4.3. As it can be seen from the table, the highest priorities are given to the cities with most population and to the most urgent needed commodities.

Table 4.3: Importance factors for districts and commodities

Disaster Scenarios				Districts					Commodities					
A	B	C	D	D1	D2	D3	D4	D5	C1	C2	C3	C4	C5	C6
0.3	0.2	0.4	0.1	0.1	0.1	0.2	0.2	0.4	0.2	0.2	0.1	0.1	0.3	0.1

4.4.2 Results

The objective function of the model gives the summation of the weighted travel time over all districts and commodities. For a better analysis, the weighted travel time over all districts is calculated for each commodity. Then, this value is divided by the weighted total demand of that commodity and the number of districts which gives the average weighted travel time for a single commodity. The solutions of this model for $R_1 = 20, R_2 = 20, R_3 = 20, R_4 = 40, R_5 = 40$ are reported in Table 4.4.

The first column in Table 4.4 shows the available budget for opening facilities. This number determines the number of facilities to be opened since all the facilities have the same fixed cost. (1000 distance unit corresponds to approximately 1 km in real life). Facilities to be opened are shown with a + sign. When there is enough budget to open all facilities, all of the facilities are opened, as expected. When the budget is decreased step by step, it is observed that some of the facilities remain open while some are closed. For example, facilities 2, 3, 4, 5, 8, 11, 12, 13, 18, 22, 23, 27, 28, 29, 32 and 33 are open in every solution and facilities 7, 14, 19, 20 and 36 are only open when there is budget to open every facility. Thus, the results are robust in the sense that the open facilities in different budget levels show consistency.

The computational time needed to solve the model increases as the number of facilities to be opened approaches to 10. In Table 4.5 it can be observed how cpu times change for each number of facilities. For 10 open facilities, the model has been run for more than 14 hours and a solution with %0.68 gap is obtained. The model is infeasible for the case when only 5 open facilities are allowed.

The $R_c, c \in NC$, values can be modified with respect to the risk aversion of the decision maker. Solutions to several uniform and non-uniform R values are given in Table 4.6, Table 4.7 and Table 4.8. The solutions for different R_c values support the fact that some facilities are open in almost all cases.

The change in the expected average weighted travel distance for each commodity with respect to the number of open facilities is given in Figure A.5. All of these results are weighted over the four possible earthquake scenarios. When only the worst case disaster scenario C is considered with probability 1, the model is infeasible for $R_1 = 20, R_2 = 20, R_3 = 20, R_4 = 20, R_5 = 20$. If the R_c values are relaxed, then the results in Table 4.10 are obtained. It is interesting that for R_c values that are just smaller than 40, the solutions

Table 4.4: Solutions for several budget limits for $R_1 = 20, R_2 = 20, R_3 = 20, R_4 = 40, R_5 = 40$

No of Open Facilities	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
40	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
35	+	+	+	+	+	+	+	-	+	+	+	+	+	+	-	+	+	+	+	-
30	-	+	+	+	+	+	-	-	+	+	+	+	+	+	-	+	+	+	+	-
25	-	+	+	+	+	+	-	-	+	+	+	+	+	+	-	+	+	+	+	-
20	-	-	+	+	+	+	-	-	+	+	+	+	+	+	-	+	+	+	+	-
15	-	-	+	+	+	+	-	-	+	-	-	+	+	+	-	-	-	-	+	-
	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
40	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
35	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	+	+	+
30	-	-	+	+	+	+	+	+	+	+	-	+	+	+	+	-	-	+	+	+
25	-	-	+	+	-	+	+	+	+	+	-	+	+	-	+	-	-	-	-	-
20	-	-	+	+	+	+	+	+	+	+	-	+	+	+	+	-	-	+	+	+
15	-	-	+	+	-	-	-	+	+	+	-	-	+	+	-	-	-	-	-	-

Table 4.5: Cpu times for different number of open facilities for $R_1 = 20, R_2 = 20, R_3 = 20, R_4 = 40, R_5 = 40$

Num. of open facilities	40	35	30	25	20	15
Cpu Time	2.125	19.187	31.609	79.265	243.328	1194.654

Table 4.6: Solutions for several budget limits for $R_1 = 40, R_2 = 40, R_3 = 40, R_4 = 40, R_5 = 40$

No of Open Facilities	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
40	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
30	-	+	+	+	+	+	-	-	+	+	+	+	+	+	-	+	+	+	+	-
20	-	+	+	+	+	+	-	-	+	+	+	+	+	+	-	+	-	-	+	-
	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
40	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
30	-	-	+	+	+	+	+	+	+	+	-	+	+	+	+	-	-	+	+	+
20	-	-	+	+	+	+	+	+	-	+	-	-	+	-	+	-	-	-	-	-

Table 4.7: Solutions for several budget limits for $R_1 = 20, R_2 = 20, R_3 = 20, R_4 = 20, R_5 = 20$

Num. of Open facilities	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
40	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
30	-	+	+	+	+	+	-	-	+	+	+	+	+	+	-	+	+	+	+	-
	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
40	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
30	-	-	+	+	+	+	+	+	+	+	-	+	+	+	+	-	-	+	+	+

Table 4.8: Solutions for several budget limits for $R_1 = 30, R_2 = 30, R_3 = 40, R_4 = 30, R_5 = 20$

Num. of Open Facilities	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
40	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
30	-	+	+	+	+	+	-	-	+	+	+	+	+	+	-	+	+	+	+	-
	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
40	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
30	-	-	+	+	+	+	+	+	+	+	-	+	+	+	+	-	-	+	+	+

are infeasible as shown in Table 4.11.

The facilities which are open in almost all the cases, when the budget is available, are shown on the Figure A.6. There are more open facilities on the European side of the city. This is reasonable as the demand in the European side is greater than the Asian side due

Table 4.9: Average weighted travel distance for each commodity for different R_c

R_1	R_2	R_3	R_4	R_5	No of Open Facilities	Average Weighted Travel Distance				
						Bread	Water	Food	Blood	SaR
20	20	20	40	40	40	140.28	180.92	256.27	153.56	154.26
					35	140.58	181.85	262.14	154.40	155.11
					30	141.32	183.70	265.08	155.27	157.16
					25	143.35	186.01	268.89	156.86	161.84
					20	147.57	190.71	275.40	160.41	167.68
					15	155.65	199.24	316.35	173.70	174.25
40	40	40	40	40	40	140.28	180.92	152.78	153.56	154.26
					30	141.17	183.66	153.65	155.27	157.16
					20	146.63	190.51	158.89	158.54	166.31
20	20	20	20	20	40	140.28	180.92	256.27	153.57	154.26
					30	141.32	183.70	265.08	169.17	157.31
30	30	40	30	20	40	140.28	180.92	152.78	153.56	154.26
					30	141.17	183.66	153.65	155.27	157.31

Table 4.10: Solutions for the single disaster scenario C for $R_1 = 40, R_2 = 40, R_3 = 40, R_4 = 40, R_5 = 40$

Num. of Open Facilities	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
40	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
30	+	+	+	+	+	+	-	-	+	+	+	+	+	+	-	+	+	-	+	-
	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
40	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
30	-	+	+	+	+	+	+	+	+	+	+	+	-	-	+	-	+	+	+	-

Table 4.11: Average weighted travel distance for each commodity for $R_1 = 40, R_2 = 40, R_3 = 40, R_4 = 40, R_5 = 40$ for disaster scenario C

R_1	R_2	R_3	R_4	R_5	No of Open Facilities	Avrg Weighted Travel Distance				
						Bread	Water	Food	Blood	SaR
40	40	40	40	40	40	267.63	518.77	17.75	2.92	7.34
					30	267.63	519.23	17.76	2.93	7.35

to the population densities.

4.5 Concluding Remarks

A pre-disaster facility location problem for disaster relief is formulated in this study. An application of the problem is considered for Istanbul to provide guidance to the Local Municipality for the location of ERDC within the scope of planning for post-disaster logistics, especially against an expected major earthquake in the area. Further sensitivity analysis on parameters (distance tolerated for each commodity, number of facilities to be opened) can be completed for better analysis of the marginal benefits of each facility.

The uncertainties in the problem data such as the magnitude and impact of the disaster, the number of injured people, the functionality of transportation links, and so on could also be handled with several approaches. One which is a two-stage stochastic programming model is given in Appendix B.2. We have constructed a capacitated facility location model that decides on both the locations of the facilities to be opened and the capacities of the facilities together with the amount of commodity stored at each facility is given in that can be used in a further study.

Chapter 5

CONCLUSIONS

In this thesis, three problems in a disaster management context are analyzed. All of the three problems take place in the pre-disaster stage and include pre-disaster planning strategies for effective post-disaster relief operations. The first two problems consider transportation networks and the last problem is a facility location problem.

In the first problem, network reliability under dependent link failures is considered. Up to our knowledge, studies considering dependent link failures for a highway network under disaster risk are limited. Reliability concept is attracting an increasing attention by researchers however not many quantitative measures are present. The algorithm proposed in this thesis attempts to fulfill the deficiency of such quantitative measures by providing an efficient algorithm, that is polynomial time for special cases, for calculating the reliability and expected performance of a network. An original framework is designed for dependent link failures that evaluate the network in independent sets of links where the links in a set are dependent. This framework can be adapted in different contexts where dependent sets can be determined by the decision maker with respect to the network and the way environment affects the dependency relationship. Within the proposed framework, in order to identify the dependencies within a set, a vulnerability-based link dependency structure is defined that ranks the links according to their probabilities of survival. Then, the failure of a stronger link, that is the link with higher probability of survival, implies the failure of weaker links (links with smaller probability of survival) certainly. This approach seems reasonable in the earthquake context where links within the same area are exposed to similar risk.

The second problem is an optimal budget allocation problem for strengthening the links of a transportation network under disaster risk. The scope of the problem is limited to computational studies of a previous study [35]. The study constitutes of two parts; data collection and computational studies. Data collection process revealed that data on disaster related issues are difficult to collect and more attention should be given to numerical reports after disasters. In this chapter of the thesis, the applicability of the proposed approach is illustrated by means of a case study of the Istanbul highway system. The aim here is to identify the most critical links in terms of reliability. A Monte Carlo Simulation algorithm is developed where link failures are assumed to be independent. The performance of the algorithm is measured by using a smaller size network for which the reliability and the expected performance can be enumerated. For the intractable cases, where the exact values of reliability results are not available, convergence of the results are taken into account for analysis. These results support that reliability and performance of a network of realistic size can be estimated with high accuracy in moderate computation time with the proposed Monte Carlo simulation method. Hence, very promising results are obtained in terms of proving the practicality of the proposed approach. The algorithm is also tested in a VB-dependent link failure environment. It is interesting that the critical links in the independent and dependent cases for the same O-D pair match each other. This can be due to several reasons such as the characteristics of the network.

The third problem is an emergency facility location problem. The uncertainties in the problem originate from the uncertainties of consequences and impact of the disaster. The problem is formulated as an uncapacitated mathematical model under several assumptions and solved with data related to Istanbul against an earthquake. The aim is to reach maximum number of people in minimum time possible. The selection of ERDC locations are completed under different budgets restrictions and different scenarios.

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Appendix A

NETWORKS

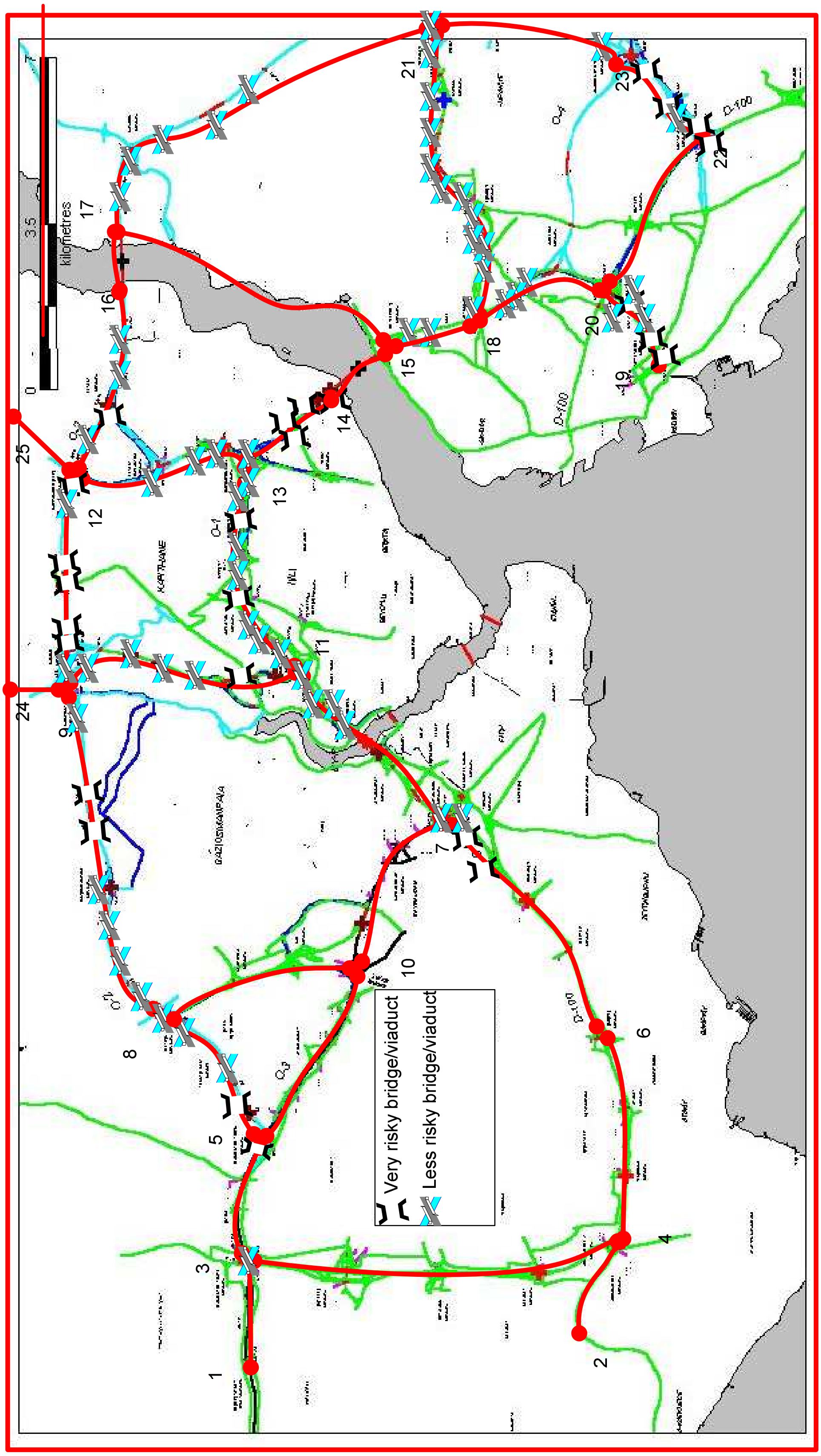


Figure A.1: Bridges and viaducts on main highway networks of Istanbul

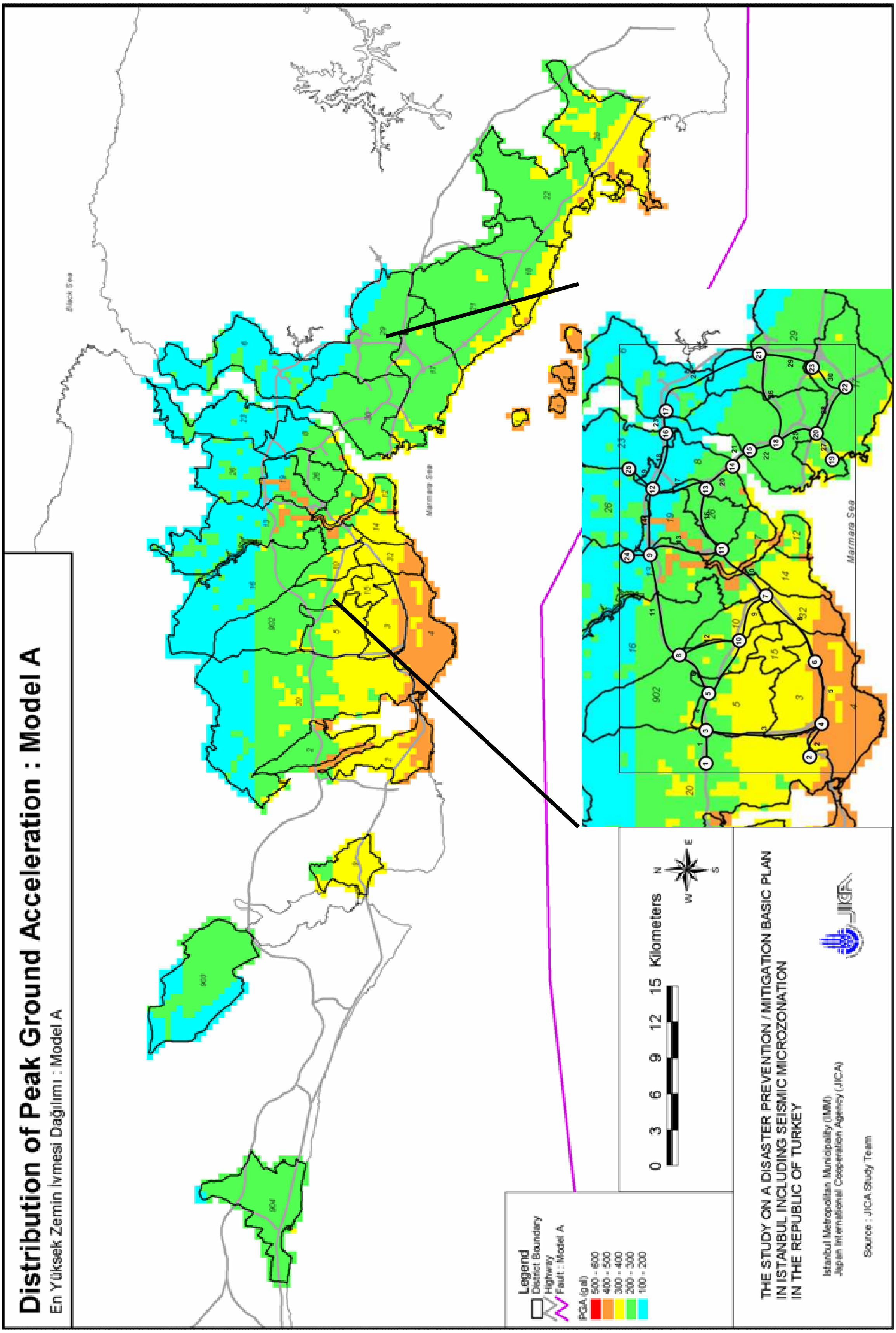


Figure A.2: Network generation out of PGA distribution for disaster scenario A for Istanbul

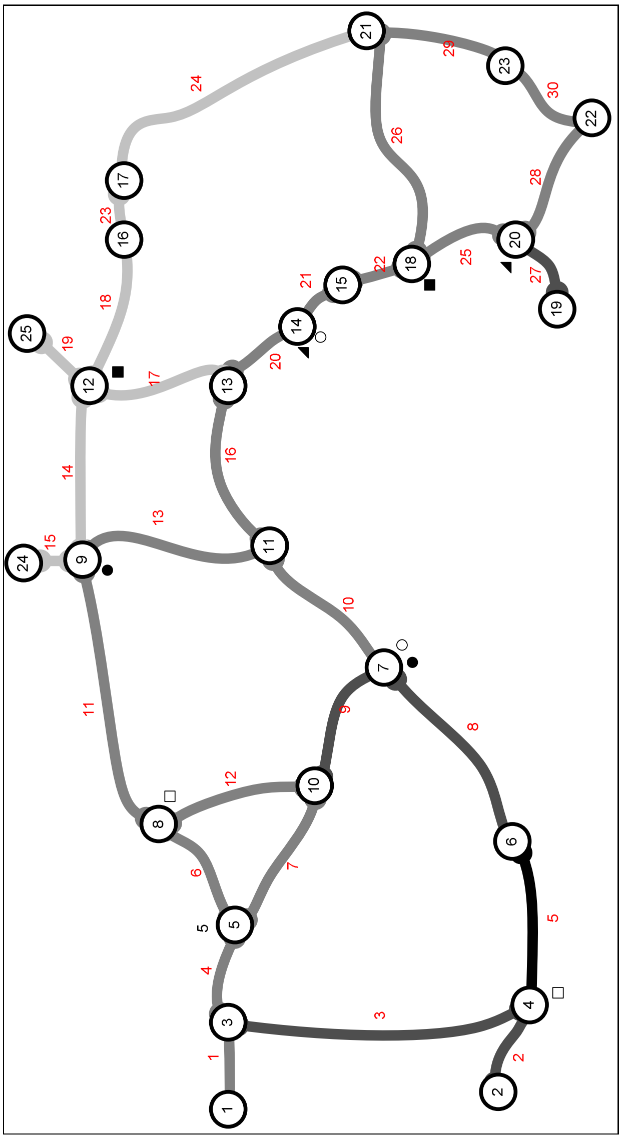
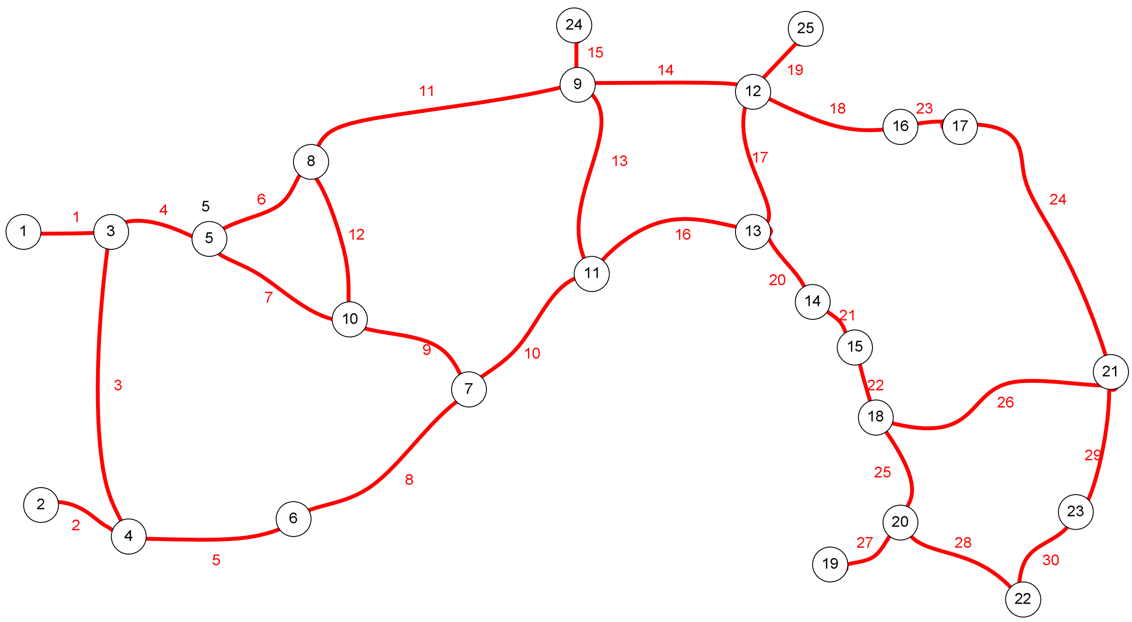


Figure A.3: PGA distribution for disaster scenario A for the links of the network

Figure A.4: Highway networks of Istanbul



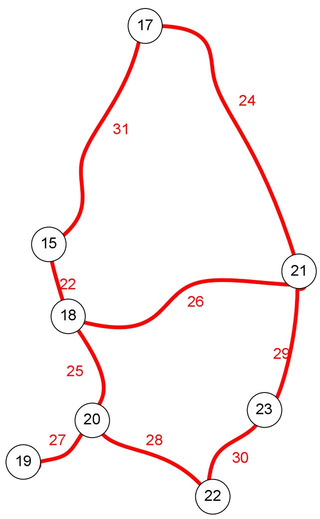


Figure A.5: Expected average travel distance for each commodity

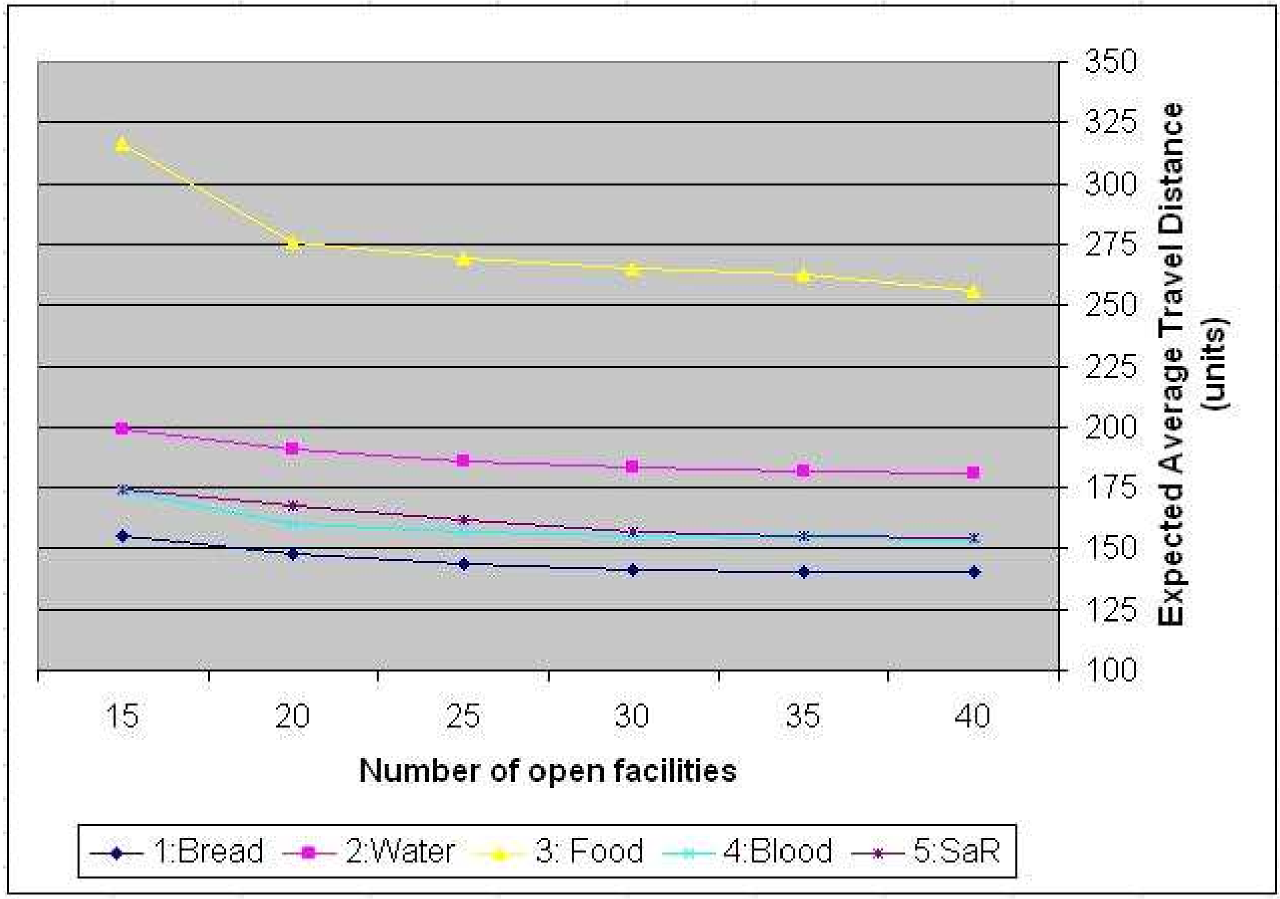
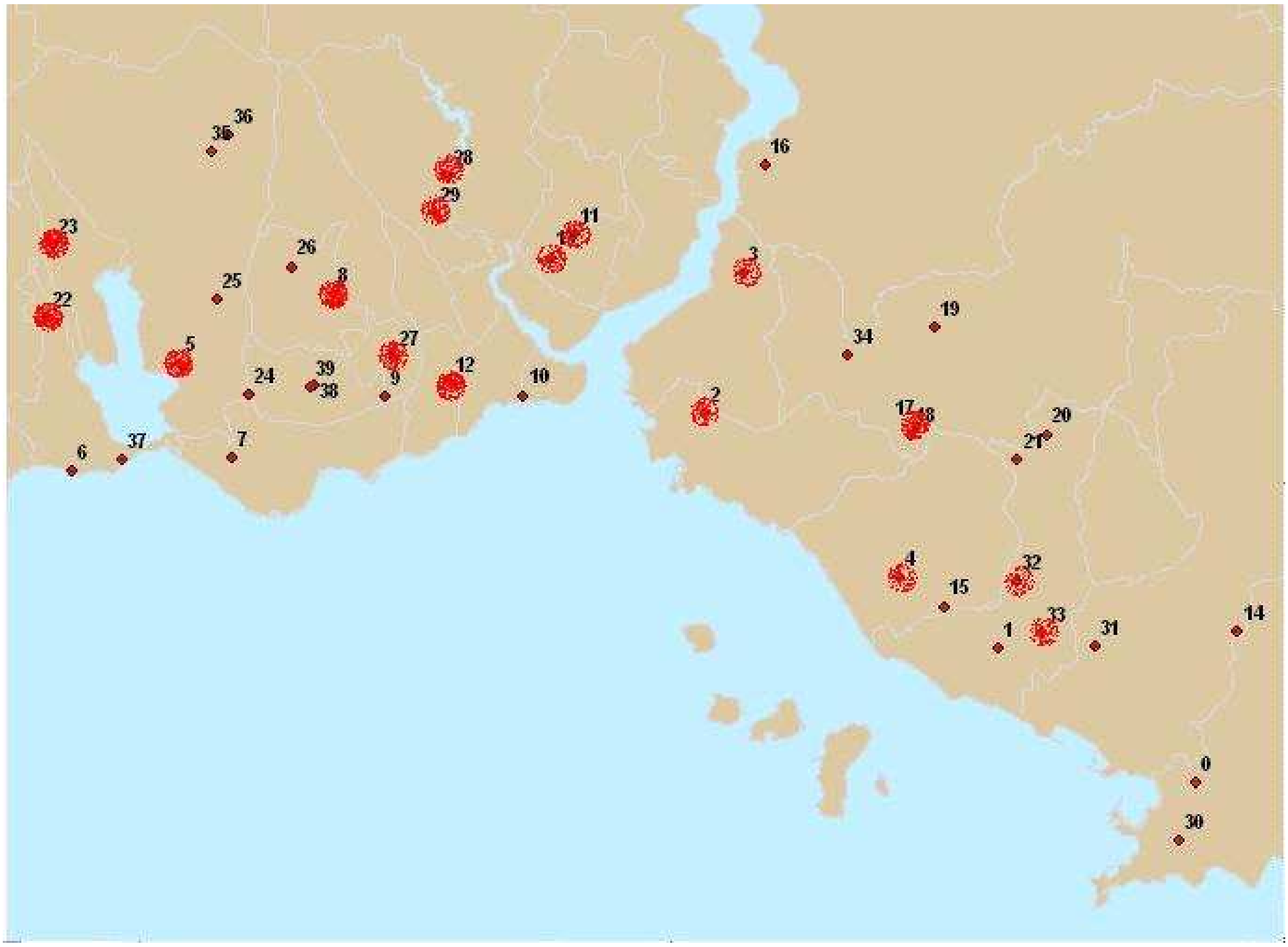


Figure A.6: Facilities that are open in most of the budget levels



Appendix B

A CAPACITATED GOAL PROGRAMMING MODEL FOR LOCATION OF EMERGENCY RESPONSE AND DISTRIBUTION CENTERS

B.1 Problem Difficulty

The uncapacitated fixed-charge facility location problem is known to be an NP-Hard problem [21], [11]. In addition, the problem in this paper requires deciding on the capacity of the facilities and the storage quantities for durable commodities. Our model differs from classical location models in that uncertainty due to a disaster has to be considered. This uncertainty is handled via a two-stage stochastic programming model. In the first stage the facilities to be opened are determined together with their capacities and the storage quantities of each commodity. In the second stage, the distribution of the commodities are optimized. The transportation variables and binary variables indicating whether each link is being used by each commodity need to be duplicated for each scenario. According to the goal programming approach, the model will be solved for each priority level with corresponding goals. For a specific objective in a priority level, we need to solve a two-stage stochastic program. Monte Carlo Sampling may be utilized to cope with the large number of possible scenarios. However, the availability of data is still limited.

B.2 The Mathematical Framework

The problem is formulated as a two-stage multi-criteria stochastic programming model. The first-stage decisions are whether to open a facility at a site and if so, to determine the capacity of the facility and how much to store of the durable commodities. In the second stage, a transshipment problem is solved with two assumptions: 1) The links connecting the regions are uncapacitated but might have failed in a given disaster scenario; 2) The transhipment nodes serve as depots for commodities. The decision of opening such centers are taken with respect to the expected value of several objectives over the possible disaster scenarios with target values determined by the decision maker. The goals are given below with their corresponding priority levels.

(Priority 1)

Goal 1: Total expected weighted time to transport all commodities to the affected areas should not exceed G1, the target level for Goal 1.

Goal 2: The expected maximum time for each commodity to reach a district must be less than G2, the target level for Goal 2.

(Priority 2)

Goal 3: The average risk associated with the locations of open facilities should not exceed G3, the target level for Goal 3.

(Priority 3)

Goal 4: Total expected weighted unsatisfied demand should not exceed G4, the target level for Goal 4.

(Priority 4)

Goal 5: The sum of the fixed opening costs and holding costs over the planning horizon should not exceed G5, the budget.

Four priority levels are determined to reach the affected areas as soon as possible with the necessary commodities. The expectations are taken over all possible disaster scenarios. This first level priority includes two goals. One of these is to minimize the expected weighted time to reach the affected areas. The weights are with respect to both the criticality of a commodity and the criticality of a district. The importance of a commodity is determined by the urgency of requirement. The importance of a district depends on aspects such as the population and the number of industrial centers present, as well as its risk. The second goal in this priority level is the minimization of the expected maximum time for each commodity to reach a district. This is to enforce that all the commodities are delivered to the districts in need as soon as possible. The second level priority goal is minimizing the average risk associated with locations of open facilities. For example, based on most likely earthquake scenarios and the geological condition of the region, a risk index can be assigned to each location. It is crucial that these facilities are functional after an earthquake, therefore the average risk is attempted to be kept below a target value. Since it is very difficult to satisfy all of the demand, a dummy node is created in the model to represent the supply of unsatisfied demand. The third level priority is to minimize total weighted unsatisfied demand. Clearly, some of the commodities such as SAR teams, water and medical supplies are critical for the survival of the victims. The fifth goal is in the fourth priority level. It is the total cost of opening the facilities and the inventory cost for holding the durable commodities in these facilities over a specified duration. This goal has the least priority compared to the others. Although budget is an important concern in many cases, in a disaster situation both the government and the people mobilize their resources without hesitation. Still, in pre-disaster planning stage agencies have to operate under a budget limit.

Formulation

The following additional notation is used in the model.

Decision Variables

o_j : 1, if a facility will be open at site j; 0, otherwise, $j \in P$

x^w_{cij} : amount of commodity type c sent from supply i to facility j in scenario w

y^w_{cjk} : amount of commodity type c sent from facility j to district k in scenario w

z_{cj} : amount of commodity type c stored in facility j

c_j : capacity of the facility to be opened at site j

m^w_{ck} : amount of unsatisfied commodity c for district k in scenario w

T_{max} : upper limit for the maximum time for any commodity to reach to any district

Parameters

T^w_{1ijc}	:	time to transport commodity type c from supply i to facility j in scenario w
T^w_{2jkc}	:	time to transport commodity type c from facility j to district k in scenario w
R_j	:	earthquake risk index of potential site j
R	:	average risk that can be tolerated
I_{1k}	:	importance factor of district k, $k \in D$
n^w_{1i}	:	1, if supply node i is operational in scenario w; 0, otherwise
n^w_{2j}	:	1, if potential site node j is operational in scenario w; 0, otherwise
l^w_{1ij}	:	1, if link between i and j is operational in scenario w; 0, otherwise
l^w_{2jk}	:	1, if link between j and k is operational in w; 0, otherwise
S_{ci}	:	supply amount of commodity c present at supply node i
C_j	:	capacity of site j, $j \in P$
f_j	:	fixed charge of opening a facility at site j
d^w_{ck}	:	demand for commodity type c at district k in scenario w
h_{cn}	:	holding cost for one unit of commodity c in year n
a_k	:	cost of carrying one unit of commodity k by helicopter
M	:	a big number
N	:	number of years for which the holding costs are considered
G_i :	:	target level of goal i
z_1	:	maximum risk that can be tolerated

Goal Formulation 1:

$$\sum_{w \in W} p(w)(\sum_{c \in NC} I_{2c} \sum_{i \in S} \sum_{j \in P} x^w_{cij} T_{1ijc} + \sum_{c \in DC,NC} I_{2c} \sum_{j \in P} \sum_{k \in D} I_{1k} y^w_{cjk} T_{2jkc}) \leq G_1$$

Goal Formulation 2:

$$\sum_{w \in W} p(w)(v^w_{cjk} T_{2jkc}) \leq T_{max}, \ \forall c \in DC, \text{ and } \forall j \in P, \ \forall k \in D$$

$$\sum_{w \in W} p(w)(\sum_{i \in S}\sum_{j \in P} u^w_{cij} T_{1ijc} + \sum_{j \in P} v^w_{cjk} T_{2jkc}) \leq T_{max}, \forall c \in NC \text{ and } \forall k \in D$$

$$T_{max} \leq G_2$$

Goal Formulation 3:

$$\sum_{j \in P} R_j o_j \leq G_3 R$$

Goal Formulation 4:

$$\sum_{w \in W} p(w) \sum_{k \in D} I_{1k} \sum_{c \in DC,NC} I_{2c} m^w_{ck} \leq G_4$$

Goal Formulation 5:

$$\sum_{j \in P} f_j c_j + \sum_{n=1}^{N}(\sum_{j \in P}\sum_{c \in DC,NC} h_{cn} z_{cj}) \leq G_5$$

The Model

$$\min \;\; P1(d_1^+) + P1(d_2^+) + P2(d_3^+) + P3(d_4^+) + P4(d_5^+) \tag{B.1}$$

subject to

$$\sum_{w \in W} p(w)(\sum_{c \in NC} I_{2c} \sum_{i \in S}\sum_{j \in P} x^w_{cij} T_{1ijc} + \sum_{c \in DC,NC} I_{2c} \sum_{j \in P}\sum_{k \in D} I_{1k} y^w_{cjk} T_{2jkc}) = G1 + d_1^+ - d_1^- \text{(B.2)}$$

$$T_{max} = G2 + d_2^+ - d_2^-, \;\; \forall w \text{(B.3)}$$

$$\sum_{j \in P} R_j o_j = G3R + d_3^+ - d_3^- \text{(B.4)}$$

$$\sum_{w \in W} p(w) \sum_{k \in D} I_{1k} \sum_{c \in DC,NC} I_{2c} m^w_{ck} = G4 + d_4^+ - d_4^- \text{(B.5)}$$

$$\sum_{j \in P} f_j c_j + \sum_{n=1}^{N}(\sum_{j \in P}\sum_{c \in DC,NC} h_{cn} z_{cj}) = G5 + d_5^+ - d_5^- \text{(B.6)}$$

$$\sum_{i \in S} x^w_{cij} + z_{cj} \geq \sum_{k \in D} y^w_{cjk}, \qquad \forall w \in W, \forall j \in P, \forall c \in DC \tag{B.7}$$

$$\sum_{i \in S} x^w_{cij} \geq \sum_{k \in D} y^w_{cjk}, \qquad \forall w \in W, \forall j \in P, \forall c \in NC \tag{B.8}$$

$$\sum_{c \in DC} z_{cj} \leq c_j \leq C_j o_j, \qquad \forall j \in P \tag{B.9}$$

$$y^w_{cjk} \leq M n^w_{2j}, \quad \forall w \in W, \forall j \in P, \forall k \in D, \forall c \in DC, NC \tag{B.10}$$

$$x^w_{cij} \leq M l^w_{1ij}, \quad \forall w \in W, \forall i \in S, \forall j \in P, \forall c \in DC, NC \tag{B.11}$$

$$y^w_{cjk} \leq M l^w_{2jk}, \quad \forall w \in W, \forall j \in P, \forall k \in D, \forall c \in DC, NC \tag{B.12}$$

$$\sum_{j \in P} y^w_{cjk} + m^w_{ck} = d^w_{ck}, \qquad \forall w \in W, \forall k \in D, \forall c \in DC, NC \tag{B.13}$$

$$\sum_{j \in P} x^w_{cij} \leq S_{ci} n^w_{1i}, \qquad \forall w \in W, \forall i \in S, \forall c \in DC, NC \tag{B.14}$$

$$\sum_{w \in W} p(w) (\sum_{j \in P} v^w_{cjk} T_{2jkc}) \leq T_{max}, \quad \forall k \in D, \forall c \in DC \tag{B.15}$$

$$\sum_{w \in W} p(w) (\sum_{i \in S} \sum_{j \in P} u^w_{cij} T_{1ijc} + \sum_{j \in P} v^w_{cjk} T_{2jkc}) \leq T_{max}, \quad \forall k \in D, \forall c \in NC \tag{B.16}$$

$$o_{ij} \in \{0, 1\}, \forall i \in S, \forall j \in P$$

$$u^w_{cij}, v^w_{cjk} \in \{0, 1\}, \ \forall w \in W, \forall i \in S, \forall j \in P, \forall k \in D, \forall c \in DC, NC$$

$$x^w_{cjk}, y^w_{cij}, m^w_{ck} \geq 0, \forall w \in W, \forall i \in S, \forall j \in P, \forall k \in D, \forall c \in NC$$

$$z_{cj} \geq 0, \forall j \in P, \forall c \in DC$$

$$c_j \geq 0, \forall j \in P$$

$$T_{max} \geq 0$$

$$d^+_i, d^-_i \geq 0, \ \ i = 1, ..., 5$$

The objective function B.1 is the minimization of deviations from the five goals with four priority levels. The constraints (B.2), (B.3), (B.4), (B.5) and (B.6) are the goal equations. In constraint (B.4) the target value is multiplied with the constant R, which can be set by the decision maker depending on his/her risk attitude. Constraint (B.7) and (B.8) are the balance equations at the ERDC for the durable and non-durable commodities, respectively. In constraint (B.9), the first and the second inequalities guarantee that the amount stored in an ERDC is less than its capacity and the capacity of an ERDC is equal to 0 if that ERDC is not opened, respectively. Constraint (B.10) represents the fact that the transshipment nodes may not be functional after the disaster. Constraints (B.11) and (B.12) force the amount of commodity carried from link i to j and j to k to be 0 if those links have failed in a realization, respectively. Constraint (B.13) is the demand satisfaction constraint. Constraint(B.14) is the capacity restriction on the supplier, where the capacity is zero if the supplier has failed in a disaster scenario. Lastly constraints (B.15), (B.16) force the expected maximum time for each commodity to reach a district to be less than the variable T_{max}.

The problem is formulated as a two-stage stochastic programming model. The first-stage decisions are whether to open a facility at a site and if so, to determine the capacity of the facility and how much to store of the durable commodities. In the second stage, a transshipment problem is solved with two assumptions: 1) The links connecting the regions are uncapacitated. 2) The transhipment nodes serve as depots for commodities. The decision of opening such centers are taken with respect to the expected value of the objective over the possible disaster scenarios.

It is crucial that these facilities are functional after an earthquake, therefore the average risk is attempted to be kept below a target value. Since it is very difficult to satisfy all of the demand, a dummy node is created in the mathematical model to represent the supply of unsatisfied demand together with an upper bound to limit the unsatisfied demand. There is also a budget constraint. Although budget is an important concern in many cases, in a disaster situation both the government and the people mobilize their resources without hesitation. Still, in pre-disaster planning stage agencies have to operate under a budget limit.

VITA

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