Essays in Service Operations with Moral Hazard

by

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This is to certify that I have examined this copy of a master's thesis by

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To my parents

ABSTRACT

The main focus of this thesis is incentive design in service operations characterized by moral hazard with hidden action. We study two different problems: the incentive design problem for salespeople in settings where companies offer a bundle of products and services and a pricing problem for service co-production in knowledge intensive services. We develop analytical frameworks for both settings and derive comparative statics related to the problem parameters of both models. For the first problem we corroborate many of the results in the sales force compensation literature and also by modeling the customer state dynamics we analyze the effect of product and service characteristics of the firm on sales force compensation. For the second setting we provide a framework to analyze the pricing problem in service co-production by considering output as a joint effort by both the firm and the client, uncertainty in the environment, and the risk preferences of both the firm and the client.

ÖZETÇE

Bu tezin temel odak noktası saklı faaliyetli ahlaki tehlike problemi görülen servis hizmetlerinde teşvik sistemlerinin tasarımıdır. Bu tezde iki problem incelenmiştir: şirketlerin müşterilere ürün ve servis paketi sunduğu ortamlarda satış elemanları için teşvik sistemlerinin tasarımı ve bilgi yoğun sektörlerde servis hizmetlerinin fiyatlandırılması. İki problem için ayrı analitik modeller geliştirilmiş ve problem parametrelerine ait karşılaştırmalı istatistikler elde edilmiştir. İlk problem için satış elemanlarının ücretlerinin belirlenmesi ile ilgili literatürdeki pek çok sonucun doğrulanmasının yanında firmanın ürün ve servis özelliklerinin ücretler üzerindeki etkisi de incelenmiştir. İkinci problem için servis çıktısının firma ve müşterinin ortak çabasının bir sonucu olduğu, çevredeki belirsizlikler ve şirket ve müşterinin risk tercihleri göz önüne alınarak fiyatlandırma problemini analiz etmeye yardımcı olacak analitik bir model geliştirilmiştir.

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NOMENCLATURE

CLV	Customer Lifetime Value
MC	Markov Chain
MDP	Markov Decision Process

Chapter 1

INTRODUCTION

Traditionally, it is assumed that people are aware of the characteristics of the goods and services they buy or that they can observe the actions of the agent whose assistance they need to obtain a certain objective. However, this is not always the case and very often transactions are characterized by asymmetric information. For instance, it is not possible to determine the quality of a good or service acquired before using it or it is not possible to monitor all actions of an employee. There are many different situations where the problem of asymmetric information can arise. Some examples are, such as when an individual purchases a used car, when an employer hires workers with different skills, when insurance is provided to individuals characterized by different levels of risk, when credit is granted to different types of firms, or when shareholders rely on managers to produce profits for their corporation [25].

The principal-agent model is heavily used to analyze situations with asymmetric information. The two players are the principal and the agent, who are usually representative individuals. The principal hires an agent to perform a task, and the agent acquires an informational advantage about his type, his actions, or the outside world at some point in the game. It is assumed that a binding contract can be made among the players, i.e. the principal can commit to paying the agent an agreed amount if he observes a certain outcome. The information is not evenly distributed among the players. The principal (or the uninformed player) is the player who has the coarser information partition and the agent (or informed player) is the player who has the finer information partition [34].

The main problems that arise when there is asymmetric information among agents are investigated under three categories; moral hazard, adverse selection and signalling.

A moral hazard problem exists when the agent's actions are not verifiable or when the agent receives private information after the relationship has been initiated. In moral hazard problems, the participants have the same information when the relationship is established and information asymmetry arises since the principal cannot observe the action of the agent or cannot perfectly control the action. This type of situation is modeled by assuming that the agent's effort is not verifiable and therefore it cannot be included in terms of the contract. This means that the agent's payoff cannot depend on the effort that he offers or the effort that he has been contracted to offer [23].

We have a moral hazard problem with hidden action when the two parties agree to the terms of a contract but then the agent takes an action that is not observed by the principal. The problem in this case is that the agent can exploit his position to carry out actions that increase his welfare at the expense of the welfare of the principal. We have a moral hazard problem with hidden knowledge when the two parties agree to the terms of a contract but then a state of nature occurs that is observed by the agent but not by the principal. The problem in this case is that the agent may not show the true realization of the state of nature and may exploit the situation against the interest of the principal [25].

In the game theory literature, moral hazard with hidden action and moral hazard with hidden information are classified as games of complete information with uncertainty. The principal offers a contract, and after the agent accepts, Nature adds noise to the task being performed. In moral hazard with hidden action, the agent moves before the Nature, and in moral hazard with hidden information, the agent moves after the Nature and conveys a message to the principal about Nature's move [34].

An adverse selection problem appears when the agent holds private information before the relationship has begun. In adverse selection, the principal can verify the agent's behavior whereas, the agent is the only informed party on the production process, thus the optimal decision or the cost of the decision depends on the agent's type. Adverse selection refers to a situation in which goods of varying quality are exchanged while one party can observe the quality of the good perfectly and the other party does not have superior information but statistical knowledge like a probability distribution function. In this case, the actions of the less informed party may have undesirable results since his actions can adversely affect the quality of the goods. An example for the adverse selection case is the insurance sector in which it is difficult for a company to know the agent's type while organizing coverage. The insurance company is not indifferent between a careful driver and reckless driver. Moreover, the insurance company does not have true information about the driver's habits whether he is a heavy drinker, or whether he enjoys racing against others, or he drives many hours without rest [23, 25].

The signaling case is somewhat similar to the adverse selection case. The difference is that the agent can send a signal to the principal after learning his type and before signing the contract. In other words, the agent takes some sort of decision to influence the principal's beliefs about the agent's identity before the principal offers the contract. In some cases, the principal is the well-informed party and he has private information that affects the well-being of the agent. Therefore, the agent decides whether to accept the contract or not by looking at the signal that the principal sends via his behaviour. The principal's task is not only designing the contract. In this case, it is also signaling and in some sources it can be defined as screening. In the signaling case, the agent sends a signal to the principal about her type. However, in the screening case the principal designs the contract according to his type and uses it as a signal to the agent [23].

To summarize, if the principal knows the agent's ability but not his effort level, the problem is moral hazard with hidden action. If neither party knows the agent's ability at first, but the agent discovers it when the game starts, the problem is moral hazard with hidden information. The problem is adverse selection if the agent knows his ability from the start but the principal does not. If the agent knows his ability form the start and also he acquires credentials before he makes a contract, the problem is signaling. If the agent acquires credentials in response to a contract offer, the problem is screening. The categories are clarified in Table 1.1 [34].

The main focus of our study is the contracting problem in the case of moral hazard with hidden action. An example for this case is the situation in which the principal wishes to hire an agent for a one-time project. The project's profits are affected in part by the agent's actions. The contract between the principal and the agent would be straightforward if the

	Principal	**Agent**	**Effort or type and signal**
Moral hazard with hidden action	Insurance company	Policy Holder	Care to avoid theft
	Insurance company	Policy Holder	Drinking and smoking
	Plantation owner	Sharecropper	Farming effort
	Bondholders	Stockholders	Riskiness of corporate projects
	Tenant	Landlord	Upkeep of the building
	Landlord	Tenant	Upkeep of the building
	Society	Criminal	Number of robberies
Moral hazard with hidden information	Shareholders	Company president	Investment decision
	FDIC	Bank	Safety of loans
Adverse selection	Insurance company	Policy Holder	Infection with HIV virus
	Employer	Worker	Skill
Signalling and screening	Employer	Worker	Skill and education
	Buyer	Seller	Durability and warranty
	Investor	Stock issuer	Stock value and percentage retained

Source: Rasmusen [34]

Table 1.1: Applications of the principal-agent model

actions of the agent were observable to the principal. In that case, the contract would simply specify the action that should be taken by the agent and the compensation that the principal should provide to the agent. However, since the actions of the agent are not observable, the actions cannot be specified in the contract effectively. In this situation, the principal must design the compensation scheme in a way that gives the manager the incentive to take the correct actions [24].

Our study is compromised of two parts. The first part deals with an incentive design problem in a selling environment and the second part is related to a pricing problem in a service co-production setting.

The incentive design problem focuses on moral hazard with hidden action case in a sales environment. A new branch of economic analysis called agency theory is developed simultaneously with the development of the microeconomics-based analytic approach to sales force compensation. Agency theory is designed to analyze problems where a principal (e.g. the firm) hires an agent (e.g. the salesperson) to perform some actions for it (e.g. exert selling effort). In this framework, it is assumed that the responsiveness of the output (e.g. sales) to the agent's input (e.g. effort) is stochastic. Moreover, it is assumed that the principal can observe the agent's effort either imperfectly or not at all. It is also assumed that the principal and the agent have different attitudes toward risk in agency theory [8].

We assume a setting where firms are implementing servitization strategies which means that they offer the customers a bundle of products and service. Service offerings are important for developing relationships between the firm and the customers. The sales force of the firms are also effective in developing relationships between the firms and the customers. So, the firms which implement a servitization strategy need new ways to incentivize sales force such that the firm can develop both short term sales and long term relationships.

In the first part, we mainly model two relationship of the firm. The relationship between the firm and the agent is modeled using the agency theory. The firm's profit is affected by the agent's effort; however, the firm cannot perfectly observe the behaviour of the agent. The relationship between the firm and the customer is modeled via the help of the buyer decision process. By considering the steps of the buyer decision process, we model the customer state dynamics as a Markov Chain. The question explored is the optimal compensation scheme by considering the product and service characteristics of the firm as well as other parameters like risk aversion of the agent and uncertainty in the market environment.

We find that the product characteristics and the service characteristics of the firm should be considered while designing incentive schemes for the salespeople. Besides products characteristics and service characteristics, the number of households and the mean revenue generated from a customer are also effective on incentive schemes. We also observe that product and service characteristics positively affect the firm profit however there are several conditions of which one is more profitable for the firm.

In the second part, we focus on a knowledge-intensive business service firm. Service firms frequently need some inputs from their clients to be able to produce the service offered. Co-production can be defined as the joint effort by which the service provider and the client produce the service. Financial planning and management consulting are some examples where close cooperation is needed between the firm and the client. The client's contribution to the service delivery can be; to be present in the service production facilities, to interact with service employees, to provide information and personal belongings and to perform some of the operation themselves. The pricing problem in this thesis analyzes the situation where the output of the service is a result of joint effort by both the firm and the client. The client's effort is seen as being integral to the service success. Moreover it influences the satisfaction of the client with the resulting service output. The client's effort is necessary since the service provided is highly complex and customized.

The pricing problem also focuses on moral hazard with hidden action; however in a service co-production setting. In this part, we model the relationship between the firm and another client firm where the output of the project is a result of joint effort by the firm and the client firm and therefore the firm's profit is affected by the client firm's effort but it is impossible to perfectly measure the client's effort. The question explored in this part is to find the optimal pricing scheme to maximize the firm's profits.

We find that the skills and the capabilities of the customers are effective on the pricing scheme of knowledge-intensive service firms. We also observe that there are efficiency losses due to co-production. The efficiency loss also increases with the risk aversion of the firm and the client. Besides risk aversion of the firm, sensitivity of the firm to the cost of effort also affects the pricing scheme. Finally, the firm can develop different pricing schemes like charging common prices to all clients or charging customized prices to the clients and increase profits.

The outline of the thesis is as follows. In Chapter 2, technical background related to moral hazard with hidden action case is reviewed. In Chapter 3, we describe our model for sales force compensation and derive its solution. In Chapter 4, the model for service co-production is explained and solved. Finally, we present the concluding remarks in Chapter 5.

Chapter 2

THEORETICAL BACKGROUND

2.1 Basic Formulation

As described in the introduction, the principal-agent problem is found in many employer and employee relationships. The source of the problem is the inability of the employer to observe the effort of the employee perfectly. The profit of the firm is positively affected by the effort of the employee but also depends on some other random forces. Since the correlation between the employee effort and the profit is not perfect, the employer cannot design an incentive scheme to induce the highest possible effort. But the employer aims to design the incentive scheme to induce the profit maximizing effort. In this section, we focus on the principal-agent problem in moral hazard with hidden action case. We provide the mathematical formulations of the problem and the standard solution approaches that are developed to solve the problem.

Let us denote the monetary value of the output with $x(e)$. The output is an increasing function of agent effort e. The agent aims to maximize her utility $U(w, e)$. The agent's utility function is increasing in wage w but decreasing in effort e. The principal aims to maximize his profit. The principal's profit function is denoted with $B(x(e) - w)$ where w is the wage paid to the agent. The principal's profit is increasing with the difference between the output and the wage.

There is also an assumption that the principal or the agent is one of the many competitors in the market. Therefore, either the principals compete to hire one of the agents or the agents compete to work for the agent. If the principals are competing, the principal's net profit becomes 0 and if the agents are competing, the agent's utility becomes equal to the reservation utility which is denoted by $\bar{u}$. The agent should be paid at least the reservation utility since she can reject the contract and choose to remain unemployed. Reservation utility may also be called outside option of the agent or the minimum acceptable utility level.

The problem of the principal is mathematically modeled as;

$$\max_{w,e} \quad E[B(x(e) - w)] \tag{2.1}$$

$$\text{subject to} \quad E[U(w,e)] \geq \bar{u} \tag{2.2}$$

$$e \in argmax_{e' \in E} \quad E[U(w,e')] \tag{2.3}$$

The principal aims to maximize profit knowing that the agent is free to reject the contract and choose to remain unemployed and the contract should give the agent the right incentives to induce the desired effort level. These two constraints are present in all moral hazard problems. In the mathematical formulation, (2.2) is called the individual rationality constraint and guarantees the agent with the minimum expected utility level. Rationality constraint may also be called the participation constraint. (2.3) is called the incentive compatibility constraint and it reflects the fact that the principal can observe output x but not the agent effort e. Therefore, the effort level e should be the agent's choice (preferred action). The incentive compatibility constraint also reflects the fact that the agent moves second in the game and the right incentive should be provided to induce the desired effort level. So, it guarantees that the agent chooses to put the desired effort level.

2.2 Uncertainty and Risk Aversion

Uncertainty is a term that is used in many fields and it can be defined as having limited knowledge about a situation to exactly describe an existing state or future outcome. It is simply the lack of certainty.

In the moral hazard case, output is influenced by the effort devoted by the agent and a random variable. Lal and Srinivasan [22] state that the random variable in their model characterizes the stochastic effects of the market forces. For instance, the effects of competitive forces on the firm's sales are unpredictable. Similarly, the effects of the marketing mix variables of the firm, like advertising and promotion etc., on the sales amount are also unpredictable. To reflect the uncertainty in the market, the output is defined with probability distribution functions that are parameterized with agent effort.

Risk aversion is a concept in economics, finance, and psychology which explains the behaviour of consumers and investors under uncertainty. Risk aversion is the reluctance of a person to accept a bargain with an uncertain payoff rather than another bargain with a more certain, but possibly lower, expected payoff.

The current theory on sales force compensation is said to depend on the assumption that a risk-neutral firm hires a risk-averse agent [28]. The risk aversion of the agent is reflected in her utility function. Several classes of utility functions can be used for the agent. A commonly used utility function is the exponential utility function that reflects the property of constant risk aversion.

Let the utility of the agent be $U(w)$ where w is the net wealth that is equivalent to money in real terms. Thus, risk aversion of the agent increases as the curvature of the $U(w)$ increases. The Arrow-Pratt measure of absolute risk-aversion or coefficient of absolute risk aversion is defined as

$$r = -\frac{U''(w)}{U'(w)}. \tag{2.4}$$

The exponential utility function in the form

$$U(w) = 1 - e^{-rw}. \tag{2.5}$$

exhibits the constant risk aversion parameter r with respect to w.

When the net income of the agent is normally distributed and his utility function has the negative-exponential form, his expected utility has a simple expression. Let w be the normally distributed net income. The expected utility of the agent is thus $E[1 - e^{-rw}]$. The certainty equivalent of w, denoted by $CE[w]$, is the fixed net income that provides the agent with a utility level equal to $E[1 - e^{-rw}]$. Maximizing the expected utility is equivalent to maximizing the certainty equivalent [7].

The derivation is simple. By taking expectations of the agent's utility, we obtain the expected utility function.

$$E[U(w)] = \int_{-\infty}^{\infty} [1 - e^{-rw}] f(w) dw \tag{2.6}$$

Recognizing the relation between the above equation and the moment generating function of the normal distribution, it can be rewritten as

$$E[U(w)] \;=\; 1 - e^{-r\left\{\mu(w) - \frac{r}{2}Var(w)\right\}} \tag{2.7}$$

where $\mu(w)$ is the mean of the net wealth and $Var(w)$ is the variance. Maximizing the above equation is equivalent to maximizing the term inside the brackets. This is known as the certainty equivalent [28].

$$CE[w] \;=\; \mu(w) - \frac{r}{2}Var(w) \tag{2.8}$$

2.3 First-best Solution and Second-best Solution

The output of the project is affected in part by the agent's actions. Moral hazard becomes a problem when the output is not a one-to-one function because a single level of agent's effort may result in different levels of output depending on the nature. In other words, the principal cannot deduce the value of the agent's action perfectly by looking at the output. This means that the output function is not invertible and this feature is called lack of invertibility. The combination of unobservable effort and lack of invertibility means that no contract can induce the agent to put forth the efficient effort level without incurring extra cost, which usually takes the form of an extra risk imposed on the agent. The terms first-best and second-best are used to distinguish between two kinds of optimality since the contracting problem aims to maximize welfare given the information constraints. A first-best contract achieves the same allocation as the contract that is optimal when the principal and the agent have the same information set and all variables are contractible. A second-best contract is Pareto optimal given information asymmetry and constraints on writing contracts. The difference between the first-best world and the second-best world represents the cost in the agency problem [34].

In terms of the mathematical formulation, the solution of the principal-problem subject to only the incentive compatibility constraint (2.3), gives the first best solution; however, the solution of the problem subject to the individual rationality (2.2) and incentive compatibility constraints (2.3) gives the second best solution [15].

2.4 Solution Approaches

2.4.1 First-order Approach

The first-order approach is one of the solution procedures that are developed to solve the principal-agent problem. Holmstrom [15] develops the solution procedure by seeing the output x as a random variable with a distribution function $F(x,e)$ that is parameterized by the agent's effort. The idea is to substitute the incentive compatibility constraint of the problem with a first-order constraint that comes from the utility maximization problem of the agent. By assuming that $F(x,e)$ has a density function $f(x,e)$ with first and second derivatives well defined on (x,e) and the agent's utility function is separable in the form $U(w) - V(e)$, the problem is modeled as;

$$\max_{w,e} \int B(x(e) - w) f(x,e) dx \tag{2.9}$$

$$\text{subject to} \int [U(w) - V(e)] f(x,e) dx \geq \bar{u} \tag{2.10}$$

$$\int U(w) f'(x,e) dx = V'(e) \tag{2.11}$$

Letting λ be the multiplier for the individual rationality constraint (2.10) and μ be the multiplier for the incentive compatibility constraint (2.11), the first order condition of the Lagrangean with respect to the wage w characterizes the optimal solution for the contract.

$$\frac{B'(x(e) - w)}{U(w)} = \lambda + \mu \frac{f'(x,e)}{f(x,e)}.$$

The problem with this approach is that the optimal contract that is characterized may not always be correct, since the first order constraint that is substituted is only the necessary condition for the agent's maximization problem. The problem that can occur is that even though the utility function guarantees that it is concave in effort e, the expected utility function, in which effort e affects the distribution of the results, is concave in effort e under additional restrictions. Under these restrictions, the first-order condition is sufficient but in any other case, the first-order condition is necessary. When the first-order approach is used, too many points are considered as candidates; however, some of them are not optimal thus they do not maximize the agent's utility [27].

2.4.2 Two-step Approach

Another approach to solve the principal-agent problem is developed by Grossman and Hart [11]. The main idea of the approach is to break up the principal's problem into the computation of costs and benefits of the agents actions and therefore to avoid the difficulties of the first order approach. The first step is to find cost minimizing incentive schemes for possible effort levels. The second step is to find the effort levels that maximize profits given that the incentive scheme is from the ones that are found in the first step.

With this approach, it is possible to analyze the characteristics of the optimal contract independently of being able to identify the optimal effort or not [23].

The first step of the approach is to solve the cost minimizing problem.

$$C(e') = \min_{w(.)} \quad E[w(X(e'))] \tag{2.12}$$

$$\text{subject to} \quad E[U(w,e)] \geq \bar{u} \tag{2.13}$$

$$E[U(w,e')] \geq E[U(w,e)] \quad \forall e \in E \tag{2.14}$$

The cost minimizing problem is solved for every effort level e' in the effort set E. The next step is to choose which effort level to induce. Therefore, the principal solves

$$\max_{e'} E[B(X(e') - C(e'))] \tag{2.15}$$

In this thesis, we use the second approach for the solution of the models that are developed.

Chapter 3

SALES FORCE COMPENSATION IN SETTINGS WITH SERVITIZATION

3.1 Introduction to Sales Force Compensation

In the past a company was said to be in the goods sector or services sector. Today most companies need both goods and services and demonstrate inseparability of goods and services. Nowadays, companies offer customer-focused bundles of goods, services, support, self-service and knowledge. This trend in business is called *servitization* and is happening in almost all industries. Consequently, both service companies and manufacturers are moving towards services [37].

The trend in today's economy, especially for the developed countries, is to respond to the massive pressure from the emerging economies by seeking to innovate and coming up with more sophisticated products and services in order not to compete on costs alone. This strategy is called servitization. For instance, Rolls-Royce does not sell only engines any more but also retains the risk and maintenance to make the engine *available for use* and therefore offers a total solution by offering the product and service together. Similarly, IBM Corporation moves from being a pure manufacturing company to a company that offers business solutions. The point is that manufacturing companies move toward offering services and solutions that are delivered through their products [30]. Volvo for example essentially manufactured cars but today Volvo is in activities from finance, insurance to gas stations and roadside assistance networks. Even the classical service companies like banks use more products to facilitate and deliver services.

Servitization is driven by customers. Today customers demand more products and also want services to make the right decisions, fully utilize what they purchased and solve problems when things go wrong. They also want their product more quickly and conveniently. So, the emphasis is on building and maintaining a relationship between the firm and the customer and this pushes firms towards servitization [37]. Building relationship between the firms and their customers is important for the success of the servitization strategy and most of the tasks related to building maintaining relationship is done by selling activities and the sales force of the firm.

Selling is one of the oldest professions. Finding new customers, communicating information about the company's products and services, selling products by approaching customers, presenting the products and answering objections are done by the sales force.

Today, attracting new customers is more costly to the company than keeping the current customers. An important part of selling where the firm wants repeat sales is the relationship between the sales person and the person doing the buying. The main contribution in developing good relationships with the customers comes from the sales force. Today, sales forces are becoming more market focused and customer oriented. A market-oriented sales force is more effective in the long run than a sales-oriented sales force since it helps the company to have long-term and profitable relationships besides winning new customers and making sales. This is the type of sales force required for a successful servitization strategy.

In some industries, up to 60 percent of the workforce are composed of the sales teams. This constitutes one of the main operational costs of the company. It is natural for a company to look for ways to decrease this cost and maximize profit. However, selling is a task that has different characteristics since it is impossible to monitor all the activities of the sales force. So, the firm should provide the right incentives to get the optimal effort from the sales force. Moreover, today these incentives should be provided by taking into consideration the satisfaction of the customers.

Incentives given to the sales force may depend on the sales performance up to that point in the accounting period. Salespeople can also be given incentives by measuring the satisfaction level of customers. The old view was that salespeople should care about sales and the company should care about profit. Current view says that salespeople should also deal with how to produce customer satisfaction and company profit while being concerned about producing sales.

The main contribution of this chapter comes from explicit modeling of the dynamics of customer states while setting incentives for the sales force. We model customer state dynamics using Markov Chains and apply Principal-Agent theory for incentive design as done by many others in the sales force compensation literature.

We develop a model that considers individuals in two states, potential and customer. The transition probabilities between these states are determined by the salesperson effort as well as firm-specific parameters related to product characteristics and service characteristics. The effort exerted by the salesperson cannot be observed by the manager of the firm, since it is impossible to monitor all the activities of a salesperson and interactions with the customer, and the sales outcome is random. This creates a moral hazard, and the firm needs to provide the salesperson with appropriate incentives in order to get the optimal levels of effort.

The probability that an individual in potential state makes a purchase and becomes a customer increases with product characteristics and the effort of the salesperson. The probability that the customer will remain a customer increases with product characteristics, service characteristics, and salesperson's effort. The manager (principal) wants to maximize expected profits from sales, where sales from a customer have a Normal distribution. Our incentive model requires only sales as an outcome to be measured, and implicitly models the impact of service level on sales via customer state dynamics. The fact that operational service characteristics, captured by the service characteristic parameter, influence a customer's re-purchase decision constitutes another differentiation from the standard sales force compensation literature in marketing, and reflects characteristics of companies pursuing servitization strategy.

The remaining parts of the chapter is organized as follows. Relevant literature is reviewed in Section 3.2. Marketing information such as consumer markets, consumer buying behaviour, the buyer decision process, satisfaction and the role of the sales force are given in Section 3.3. The model is introduced in Section 3.4. We analyze the resulting principal-agent problem and derive comparative statics in Section 3.5.

3.2 Literature Review for Sales Force Compensation

The stream of literature related to this paper can be investigated under three parts. These are incentive contracts and agency problems, incentive effects in operational settings, and customer relationship dynamics and customer satisfaction.

There is a huge literature that deals with incentive contracts and agency problems. A comprehensive function for the output rather than using the models for the underlying operational system through which the effort leads to outcomes, is assumed among the classical papers on agency theory [15, 11, 17]. Holmstrom [15] uses the first-order approach to solve the contract design problem for moral hazard with hidden action case. However, Grossman and Hart [11] attacks the same problem with a different approach by breaking the principal's problem into computation of costs and benefits of agents actions and therefore to avoid the difficulties of the first order approach. The literature on sales force compensation has started from models with deterministic output functions in which the quantity sold is assumed to be a function of time devoted by the salesperson [9] and evolved into agency theoretic models. Various sales force compensation plans are considered in principal-agent framework. Basu et al. [2] presented several compensation plans like straight salary, straight commission or combination of both in environments where sales of a product not only depend on the salesperson's effort but also the uncertainty in the environment. The Holmstrom-Milgrom [16] result that shows that the linear compensation plan is optimal is applied to sales force compensation by Lal and Srinivasan [22]. In their model, the authors conduct analysis about the effort-rate decision of the sales-oriented salesperson. However, today salespeople are becoming more market focused and consumer oriented since they should deal with not only making sales but also with customer satisfaction. This necessitates designing incentives by considering the satisfaction of the customers.

A stream of studies considers incentive effects in different operational settings. The textbook by Laffont and Martimort [21] describes applications in wholesale contracts, financial contracts and insurance contracts. Bayiz and Corbett [3] use incentive contracting in project management and derive linear incentive contracts to be offered to the subcontractors of the project by the project manager. Gunes and Aksin [12] study the optimal design of incentives for cross-selling (selling related products or add-ons) related products which has received little attention in the operations management literature and therefore analyze the ties between market segmentation decisions, incentives and performance of the system. Plambeck and Zenios [33] analyze a maintenance problem modeled by a Markov chain as a dynamic principal-agent problem. We model customer dynamics in the market as a Markov chain.

Marketing science increasingly focuses on service and profitable customer relationships and customer satisfaction. Rust and Chung [35] provide a good review of this literature. In this paper, we try to model the customer state dynamics in a principal-agent framework and in a setting where service characteristics of the firm increases the probability of re-purchase, by using Markov Chains. Pfeifer and Carraway [32] proposed Markov Chain modeling in calculating the lifetime value of customers (LTV). They suggest the use of Markov Chains because of flexibility, ease for modeling uncertainty and being supported by a well-developed theory. Netzer [31] models customer relationship dynamics by constructing a hidden Markov model to relate relationship states to the observed buying behavior. Hauser et al. [14] provide a means of giving incentives to employees to increase customer satisfaction and thus increase the profitability of the firm. They suggest giving incentives on both sales and customer satisfaction and therefore encouraging employees to make short-versus-long term tradeoff, since the effect of satisfaction of customers is assumed to be realized in the future. Marketing science is also focusing on assessing the value of customer satisfaction. Rust and Zahorik [36] provide a mathematical framework that helps managers to determine the satisfaction elements that have the greatest impact and how much should be spent to improve satisfaction elements. Similar to our model, they use an individual-level model of loyalty and retention and then build-up the market share. Zeithaml et al. [40] also look for an answer to how much should be invested in service quality to receive the best return. In their work, they mention that marketing researchers distinguish between the offensive effects (capturing new customers) and defensive effects (retaining customers); and evaluating the defensive impact of service quality through customer retention helps companies to measure the financial impact of the service whereas considering service similar to advertising, pricing etc. does not guarantee results. This is consistent with our model since we use the effect of service attributes in the transitions from customer state to customer state and therefore for investigating the defensive effect (retaining customers). Mittal et al. [29] develop a theoretical model for conceptualizing satisfaction with consumption systems, which are defined as offerings characterized by a significant product and service subsystem, and empirically test it using real data. Our model also applies in the context of consumption systems.

3.3 Modeling Customer Relationships

3.3.1 Consumer Markets and Consumer Buying Behaviour

Consumer buying behaviour is defined as the buying behaviour of the final consumers. Final consumers are individuals and households who purchase goods and services for their personal consumption. All of these final customers together form the consumer market. Consumer market is defined as all the individuals and households that purchase and acquire goods and services for personal consumption.

3.3.2 The Buyer Decision Process

Every day a great deal of buying decisions are made by consumers. The stimulus-response model of the buyer behaviour, that consists of three stages, explains how consumers respond to marketing stimuli. These stages are marketing and other stimuli, buyer's black box, and buyer's responses. Marketing stimuli consists of product, price, place and promotion which are known as the four Ps in marketing literature [19].

Marketing stimuli enters the buyer's black box and are transformed into observable buyer responses: product choice, brand choice, dealer choice, purchase timing, purchase amount. The buyer's black box consists of two parts: buyer characteristics and the buyer decision process. Firstly, the buyer's characteristics affect how consumers perceive and react to marketing stimuli. We do not include the buyer's characteristics in our model. For simplicity we assume that all households in the consumer market are homogenous in these characteristics. Secondly, the buyer decision process itself affects the buyer behaviour.

The buyer decision process has five stages: need recognition, information search, evaluation of alternatives, purchase decision and post-purchase behaviour. Need recognition is the first stage of the buyer decision process in which the consumer recognizes a problem or need. Secondly information search, that is the stage of the buyer decision process in which the consumer is aroused to search for more information, comes. In this stage the consumer may simply have heightened attention or may go into active information search. The next stage is alternative evaluation which is the stage of the buyer decision process in which the consumer uses information to evaluate alternative brands in the choice set. After need recognition, information search and evaluation of alternatives the consumer can actually buy the product. Purchase decision is the stage of the buyer decision process in which the consumer actually buys the product. Final stage is the post-purchase behaviour in which consumers take further action after purchase based on their satisfaction or dissatisfaction.

3.3.3 Satisfaction

If the company considers long-term relationships with consumers, the job of the company does not end when the product is bought. After purchasing the product, the consumer can be satisfied or dissatisfied with it. Making customers satisfied is important for the company. Such satisfaction is important because a company's sales come from two basic groups: new customers and retained customers. It usually costs more to attract new customers than to retain current ones, and the best way to retain current customers is to keep them satisfied. In our model, the cost of attracting new customers is not included but the service level and the repurchase intention of consumers is taken into consideration.

3.3.4 The Role of the Sales force

A salesperson is defined as an individual acting for a company by performing one or more of the four activities: prospecting, communicating, servicing and information gathering.

Personal selling is a part of the company's promotion mix. Advertising provides a one-way nonpersonal communication with consumers, whereas personal selling provides a two-way personal communication between the salesperson and the consumer.

The role of personal selling changes from company to company. Some companies may

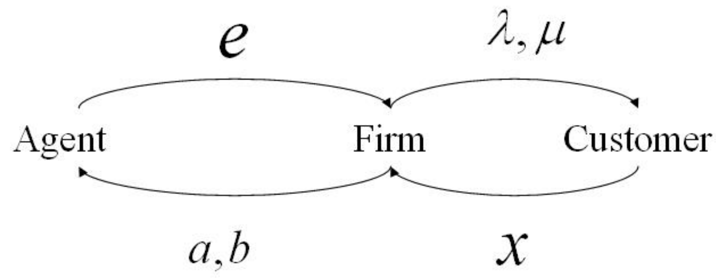

Figure 3.1: The firm's relationships with the agent and the customer

have no salespeople whereas sales people may be the only contact between the customer and the firm in others. The salesperson establishes an important link between the company and the customers. Finding new customers, communicating information about the company's products and services, selling products by approaching customers, presenting the products and answering objections are done by the sales force. Moreover, services are provided to customers by sales force.

3.4 The Model

3.4.1 Base Model

Our model basically depends on two relationships: relationship between the firm and the customer and the relationship between the firm and the agent (Figure 3.1). We model the firm-customer relationship in a principal-agent setting with a model consisting of a Markov chain. We assume there are n households in the market and these households can be either the firm's customers or not. The households can be in two states: potential and customer. Customers purchase the firm's products and re-purchase with a greater probability next time compared to a household in potential state. Potentials are households who never purchased a product, so never became a customer of the firm or decided not to re-purchase. Potentials can make a purchase and become customers but this has a smaller probability than a customer's re-purchase. The transition probability matrix is displayed in Figure 3.2.

Let us first introduce the problem parameters as displayed in Table 1.

a: Salary

b: Commission rate

e: Purchase intention from the agent's selling effort

t: Purchase intention from the agent's relationship effort

k: Effectiveness of selling effort

m: Effectiveness of relationship effort

d: Disutility for selling effort

f: Disutility for relationship effort

λ: Purchase intention from product characteristics in the marketing stimuli

μ: Re-purchase intention from service characteristics of the firm

n: Number of households in the market

x: Mean of sales generated from a consumer

σ^2: Variance of sales generated from a consumer

$\bar{u}$: Minimum expected utility (reservation utility)

c: Marginal cost of product to the firm

z: Expected income of the agent

R: Expected total sales to the principal

$p(C)$: Steady state probability of being in customer state

$p(P)$: Steady state probability of being in potential state

Table 3.1: Problem parameters

$$\begin{array}{c c} & \mathbf{P} \qquad\qquad\qquad \mathbf{C} \\ \begin{array}{c} \mathbf{P} \\ \mathbf{C} \end{array} & \begin{bmatrix} 1-\lambda-ke & \lambda+ke \\ 1-\lambda-ke-\mu & \lambda+ke+\mu \end{bmatrix} \end{array}$$

Figure 3.2: The transition probability matrix

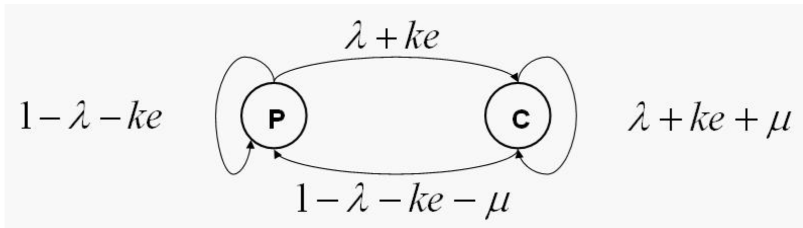

Figure 3.3: The transitions diagram

The transitions are characterized by parameters λ, μ, e which are; purchase intention from product characteristics in the marketing stimuli, re-purchase intention from service characteristics of the firm, purchase intention from the agent's effort for sales respectively. While both λ and μ actually characterize the customer's intentions, for brevity we use product characteristics for λ, and service characteristics or service level for μ. We also use service level and satisfaction interchangeably.

Marketing stimuli include: product, price, place and promotion. For simplicity, we do not consider the other stimuli, which are economic, technological, political and cultural, in the model that affects the buyer decision process. The basic idea that drives the transitions is the buyer decision process that has five stages. The first three stages are need recognition, information search and alternative evaluation. In our model, the parameter λ is related with these three stages. The fourth stage is the purchase decision in which the customer actually buys the product. In our model, the agent's effort e affects the buyer decision process in this fourth stage. After the purchase, a customer can be satisfied or dissatisfied with the product. In the model, satisfaction is modeled with parameter μ in the post-purchase behaviour stage. Based on their satisfaction or dissatisfaction customers take further action in the form of re-purchasing.

In order to better understand the meaning of parameters λ and μ, three levels of a product concept can be used. In this concept, a product is considered as three different products; the core, the actual and the augmented product. The core product is not the physical product but it is the benefit that makes the product valuable. The actual product is the tangible product that the consumer gets some use of it. It includes properties like

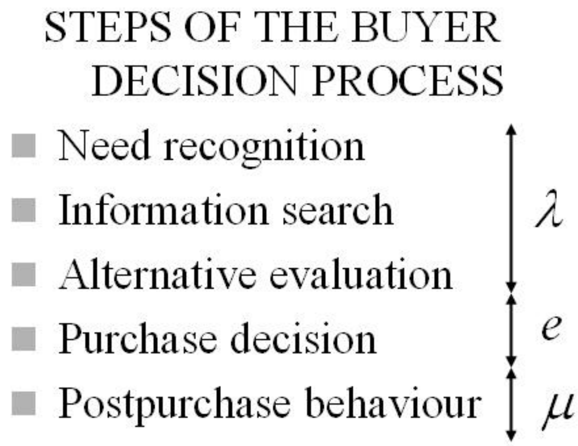

Figure 3.4: The problem parameters and the buyer decision process

colour, style, fashion, branding etc. In our model, λ models the actual product. Finally, the augmented product is the non-physical part of the product and it includes properties like warranty, customer service support, after-sales service etc. In our model, μ models the augmented product.

The parameters λ and μ can also be explained in the context of consumption systems. Consumption systems are offerings characterized by a significant product and service sub-system. It can also be defined as a bundle of goods and services that are consumed over time in multiple consumption episodes. We can view λ as capturing the purchase intention from product attributes, while μ represents the re-purchase intention from satisfaction due to service characteristics.

The second main relationship we use in our model is the relationship between the firm and the agent. We model the relationship between the firm and the agent in a principal-agent setting. The risk neutral firm declares a compensation plan for the risk averse salesperson. The salesperson decides on the effort level for selling business. The firm cannot verify the effort level exerted by the salesperson. The sales realized are observed by both parties. The effort level of the agent e together with product characteristics of the firm λ and service level of the firm μ influence the steady state probability of being in customer state. The uncertainty in the market conditions is captured through our usage of a Markov chain and the assumption that sales in a period are normally distributed.

The agent's activities for selling business is the sales effort. In the literature, satisfaction is assumed to be a post-purchase behaviour so, we assume that μ affects the permanency of the relationship. Therefore, transition probability from P to C (potential to customer) is not affected by service level of the firm μ. A household firstly purchases a product, becomes a customer, evaluates the product and service and then decides to make a purchase next time which means that the consumer decides to continue the relationship with the firm. We assume that effort e increases the transition probability from P to C together with product characteristics λ. The parameter λ is exogenous to the agent. The agent can only affect sales with e. The firm can change purchase intention λ by changing the product characteristics. Similarly, the parameter μ is exogenous to the agent. The agent does not know whether a household is customer or potential and exerts the same effort to all households. The parameter μ is the re-purchase intention from service characteristics and it provides the permanency of the relationship.

We can obtain steady state distributions by solving the following simple equations;

$$\begin{aligned} p(P) &= (1-\lambda-ke)p(P)+(1-\lambda-ke-\mu)p(C) \\ p(C) &= (\lambda+ke)p(P)+(\lambda+ke+\mu)p(C) \\ p(P)+p(C) &= 1 \end{aligned}$$

Therefore;

$$p(C) = \frac{\lambda+ke}{1-\mu}$$

We use the following assumptions. Households make purchases from the firm and revenue generated from a consumer is normally distributed with mean x and variance σ^2. The firm's reward system depends on total sales. We use linear compensation which is very common in the literature and is appropriate for a huge part of the current practice. The optimal employee behaviour is constant in effort. This means that optimal efforts are equal for all customers that have contact with the firm and with the agent. This is just because of the constant risk aversion of the agent assumption in our mathematical formulation. Similar to our model, constant employee effort is also assumed in [14]. Therefore, the agent exerts the same effort level to all households.

Let R denote the total sales during the accounting period. Therefore, expected total sales of the firm conditional on the agent effort becomes;

$$E[R] \;=\; \frac{nx(\lambda + ke)}{1-\mu}.$$

The following useful result is well known. When the net income of the agent is normally distributed and his utility function has the negative-exponential form, his expected utility has a simple expression. Let Y be the normally distributed net income. The agents expected utility is thus $E[-e^{-rY}]$. The certainty equivalent of Y, denoted by $CE[Y]$, is the fixed net income that provides the agent with a utility level equal to $E[-e^{-rY}]$. It can be easily verified that;

$$CE[Y] \;=\; \mu_Y - \frac{1}{2}r\sigma_Y^2$$

where μ_Y is the mean of Y and σ_Y^2 is the variance. Maximizing the expected utility is equivalent to maximizing the certainty equivalent [7]. In our model, we use the well-known certainty equivalent result with exponential utility function for the risk averse agent, since the total sales of the firm in a period is normally distributed.

When sales amount, x, has normal distribution for a customer, expected sales generated from an individual is a mixture of a normal random variable and a constant. It will be equal to x with probability $p(C)$ and 0 with probability $1 - p(C)$. In a market with n such customers, for large n, we will approximate the total sales by a Normal distribution with mean $nxp(C)$ and variance σ.

Another way to approximate the total sales by a Normal distribution is to assume that sales revenue generated from a purchase is fixed and it is equal to x. There are n households in the market and a household is the customer of the firm with probability $p(C)$ and is not a customer with probability $1 - p(C)$. So, number of customers is binomially distributed with mean $np(C)$ and variance $np(C)(1 - p(C))$ and consequently total sales is binomially distributed with mean $np(C)x$ and variance $np(C)(1 - p(C))x$. For large n we use the binomial approximation to Normal distribution. Also for simplicity we assume variance is fixed and therefore total sales is normally distributed with mean $np(C)x$ and with variance σ.

Therefore, we assume that total sales is normally distributed with mean $\frac{nx(\lambda+ke)}{1-\mu}$ and standard deviation $\sqrt{n}\sigma$. For simplicity, we assume that variance of total sales is not affected by the agent's effort e. We make the simplifying assumption that variance of outcome is independent of the agent's effort as in [22].

The sales volume is assumed to reflect the sales generated during the period of analysis. We assume that this period is not the first period of the firm in the market. Also the product is not launched recently in the market and it is not in the early stages of its life-cycle.

The firm chooses the compensation plan at the beginning of the period so as to maximize expected profits over the accounting period. The compensation plan is linear and specified in terms of the entire history of sales. The compensation plan is in terms of $y = a + bX$ where a is the salary component, b is the commission rate and X is the sales amount.

We use the two-step approach explained in Chapter 2. Hence we first find the optimal effort to maximize utility of the agent. The agent maximizes utility, which is a function of income $a + bX$ and cost of effort de^2.

We can write the certainty equivalent for the risk averse agent as

$$CE = a + b(\frac{nx(\lambda + ke)}{1-\mu}) - de^2 - \frac{r}{2}b^2 n\sigma^2. \tag{3.1}$$

By taking derivatives with respect to e, we find;

$$e^* = \frac{bknx}{2d(1-\mu)}. \tag{3.2}$$

Let the minimum expected utility to the agent be equal to $\bar{u}$. So the optimum value of a would be such that individual rationality constraint is satisfied:

$$a^* = u - b(\frac{nx(\lambda + ke)}{1-\mu}) + de^2 + \frac{r}{2}b^2 n\sigma^2. \tag{3.3}$$

The firm maximizes the expected profit, given by

$$\pi = (1-c)(\frac{nx(\lambda + ke)}{1-\mu}) - a - b(\frac{nx(\lambda + ke)}{1-\mu}). \tag{3.4}$$

Substituting e^* and a^* into the profit function and setting $\frac{d\pi}{db} = 0$, we obtain the optimal commission rate, as

$$b^* = \frac{1-c}{1+\frac{2dr\sigma^2(1-\mu)^2}{k^2nx^2}}. \tag{3.5}$$

3.4.2 First-best Solution

In this section, we characterize the effort level when the first-best solution is achieved and difference between the first-best effort level and the second-best effort level. The first-best solution can be achieved when there is no uncertainty in the environment. So, we assume that there is a social planner that aims to maximize the total profit π_t and can observe the effort of the agent e. Therefore we want to maximize the below profit function which is the sum of the firm profit and the utility of the salesperson.

$$\pi_t = (1-c)(\frac{nx(\lambda+ke)}{1-\mu}) - de^2. \tag{3.6}$$

The compensation related terms $a + b(\frac{nx(\lambda+ke)}{1-\mu})$ are not included in the above profit function since they cancel each other. So, the effort level of the agent to maximize the total profit becomes

$$e^*_{fb} = \frac{(1-c)knx}{2d(1-\mu)}. \tag{3.7}$$

The solution provided in the previous section characterizes the second-best effort level e^*_{sb}. The first-best effort level $e^*_{fb}$is always larger than the second-best effort level and the difference between them becomes

$$e^*_{fb} - e^*_{sb} = \frac{2dr\sigma^2(1-\mu)^2}{k^2nx^2 + 2dr\sigma^2(1-\mu)^2} e^*_{fb}. \tag{3.8}$$

The difference between the first-best and the second best effort level is the efficiency loss due to the inability to monitor the agent perfectly. It can be defined as a percentage of the first-best effort level. So, we can conclude that the efficiency loss is increased with d, r and σ and it is decreased with k, n, x and μ.

The efficiency loss can also be observed by looking at the difference between the profits. The profit function we consider is the sum of the firm profit and the utility of the agent.

The compensation related terms and the risk and uncertainty parameters are not involved in the profit function.

$$\begin{aligned}
\pi_{fb} &= (1-c)(\frac{nx(\lambda+ke^*_{fb})}{1-\mu}) - d(e^*_{fb})^2, \\
\pi_{sb} &= (1-c)(\frac{nx(\lambda+ke^*_{sb})}{1-\mu}) - d(e^*_{sb})^2.
\end{aligned}$$

The difference between the first-best and the second-best profit therefore becomes,

$$\pi_{fb} - \pi_{sb} = \frac{(-1+c)^2\, d\, k^2\, n^2\, r^2\, x^2\, (-1+\mu)^2\, \sigma^4}{\left(k^2\, n\, x^2 + 2\, d\, r\, (-1+\mu)^2\, \sigma^2\right)^2}.$$

3.4.3 Model with Two Efforts

In this section we analyze the problem of designing the compensation plan of a sales force which can exert two types of effort for affecting the decision of consumers. In our model, transitions from state to state are affected by product characteristics and service characteristics together with the agent's effort. The agent exerts effort in the interactions with both potentials and customers, therefore the agent's effort increases the probability of both transitions from potential state to customer state and from customer state to customer state. Consequently, the steady state probability of being customer $p(C)$ is affected by the agent's effort.

So far, we have considered a situation where the agent exerts effort only for sales activities. Today, customers have become more sophisticated and demanding of higher levels of customer service. This can be realized if they find someone who they can trust and understands their needs and wants during their interactions. People tend to do business with those they like and trust. Customer can walk away from a transaction because of not trusting the salesperson to deliver what was being promised, or because just plain not liking the salesperson. Conversely, customers can go back again and again to do business with helpful and honest salespeople.

Relationship selling is about building relationship with prospects and listening to their needs. Relationship selling can be applied to any kind of business, even if it is retail or business-to-business, product or service. While the steps to the sales process may vary slightly for each type, the overall purpose is building relationships.

There are several ways to build trust in a business environment. A salesperson can get involved in industry or neighborhood organizations where he can meet his prospects and customers in a different environment. Therefore, customers can experience another side of the salesperson, and get to know him as a person, not just as a vendor. While the salesperson is still selling the benefits of the product or service, he is also selling himself. Another way to build trust is to keep the word. From follow-up calls to delivering on time, keeping the word can be one of the most powerful sales tools.

To build trust, the salesperson should take the time to build relationships with customers, rather than just focusing on making the immediate sale. Although relationship selling may take longer to produce results, it is definitely worth it in the long run. The firm and therefore the salesperson will be well rewarded with high levels of repeat business and referrals from satisfied customers.

Therefore, we assume the agent decides on the level of two efforts; one for selling the product and one for developing relationship. These efforts together increase the steady state probability of being customer. In the meantime, the principal should design the optimal commission rate and the salary by taking into consideration the effectiveness and the disutility of the agent's efforts.

In this section we add two more parameters to the model: m and f. These parameters are effectiveness of relationship effort and disutility of relationship effort respectively. The agent's efforts e and t, sales effort and relationship effort, affect the buyer decision process together. The parameters k and m are effectiveness of sales effort and effectiveness of relationship effort respectively and the parameters d and f are disutility of selling effort and disutility of relationship effort respectively.

The salesperson decides on two effort levels: one for selling business and one for developing relationship with customers. The market share of the firm $p(C)$ is affected by these effort levels together with product characteristics and service level of the firm. With the same assumptions in the previous section we get

$$\begin{aligned} p(P) &= (1-\lambda-ke-mt)p(P)+(1-\lambda-ke-\mu-mt)p(C), \\ p(C) &= (\lambda+ke+mt)p(P)+(\lambda+ke+\mu+mt)p(C), \\ p(P)+p(C) &= 1. \end{aligned}$$

and

$$p(C) = \frac{\lambda+ke+mt}{1-\mu}.$$

Once again, we assume that total sales is normally distributed with mean $\frac{nx(\lambda+ke+mt)}{1-\mu}$ and standard deviation $\sqrt{n}\sigma$. For simplicity, we assume that variance of total sales is not affected by the agent's effort e and t.

We can write the certainty equivalent for the risk averse agent as

$$CE = a+b(\frac{nx(\lambda+ke+mt)}{1-\mu})-de^2-ft^2-\frac{r}{2}b^2n\sigma^2.$$

By taking derivatives with respect to e and t, we find the optimal effort levels

$$e^* = \frac{bknx}{2d(1-\mu)}, \tag{3.9}$$

$$t^* = \frac{bmnx}{2f(1-\mu)}. \tag{3.10}$$

The reader can refer to the appendix for the concavity of the functions.

Let the minimum expected utility to the agent be equal to $\bar{u}$. So the optimum value of a would be such that;

$$a^* = u-b(\frac{nx(\lambda+ke+mt)}{1-\mu})+de^2+ft^2+\frac{r}{2}b^2n\sigma^2. \tag{3.11}$$

The firm maximizes the expected profit by

$$\pi = (1-c)(\frac{nx(\lambda+ke+mt)}{1-\mu})-a-b(\frac{nx(\lambda+ke+mt)}{1-\mu}). \tag{3.12}$$

Substituting e^*,f^* and a^* into the profit function and setting $\frac{d\pi}{db}=0$, we obtain the optimal commission rate, as

$$b^* = \frac{1-c}{1+\frac{2dfr\sigma^2(1-\mu)^2}{nx^2(fk^2+dm^2)}}.$$

3.4.4 Optimal Effort Levels

In the previous sections we analyzed the problem of designing the compensation plan of a sales force which can exert two types of effort for affecting the decision of consumers. The steady state probability of being a customer, $p(C)$, is affected by the agent's efforts; one for selling the product e and one for developing relationship with the clients t. Therefore the efforts together increase the steady state probability of being customer. In the meantime, the principal should design the optimal commission rate and the salary by taking into account the effectiveness and the disutility of the agent's efforts since the optimal effort levels are dependent on the commission rate and the optimal commission rate is dependent on the effectiveness and the disutility of the effort types together with other parameters in our model.

We know that optimal commission rate is

$$b^* = \frac{1-c}{1+\frac{2dfr\sigma^2(1-\mu)^2}{nx^2(fk^2+dm^2)}}, \tag{3.13}$$

and optimal effort levels are

$$e^* = \frac{bknx}{2d(1-\mu)}, \tag{3.14}$$

$$t^* = \frac{bmnx}{2f(1-\mu)}. \tag{3.15}$$

By substituting b^* into e^* and f^* we get

$$e^* = \frac{(1-c)\, k\, n\, x}{2\, d\, (1-\mu)\left(1+\frac{2\, d\, f\, r\, (1-\mu)^2\, \sigma^2}{(f\, k^2+d\, m^2)\, n\, x^2}\right)}$$

and

$$t^* = \frac{(1-c)\, m\, n\, x}{2\, f\, (1-\mu)\left(1+\frac{2\, d\, f\, r\, (1-\mu)^2\, \sigma^2}{(f\, k^2+d\, m^2)\, n\, x^2}\right)}.$$

Simply we can say that

$$e^* = \frac{k}{d}K,$$

$$t^* = \frac{m}{f}K,$$

where

$$K = \frac{(1-c)\, n\, x}{2\,(1-\mu)\left(1+\frac{2\,d\,f\,r\,(1-\mu)^2\,\sigma^2}{(f\,k^2+d\,m^2)\,n\,x^2}\right)}.$$

It follows that

$$e^* > t^* \Longleftrightarrow (\frac{k}{d}) > (\frac{m}{f}).$$

The results can be stated in terms of Propositions 1-3.

Proposition 1 *Optimal effort rate is higher for the effort type with higher effectiveness, if they have the same disutility to the agent.*

Proposition 2 *Optimal effort rate is higher for the effort type with lower disutility, if they have the same effectiveness toward sales.*

Proposition 3 *The optimal effort levels for the effort types of the agent have the same rank order as the corresponding values of* e_i/d_i*, where* e_i *is the effectiveness of the effort* i *and* d_i *is the disutility of the effort* i*.*

The results show how the optimal commission rate depends on the effectivenesses and disutility of the effort types. It is clear that the firm should consider the agent's type in designing incentives because the disutility and the effectiveness of the efforts will change according to the agent. When $e^* > t^*$, i.e. when the effectiveness per disutility for the sales effort is higher than that of relationship development, the agent will be more transaction oriented. In the reverse case, the agent's relationship orientation will be dominant. This suggests that a firm desiring to transform their sales force from one with a heavy transactional sales focus to one with a relationship focus needs to increase their effectiveness m possibly through additional tools or training while trying to reduce their disutility f from these kinds of activities. In some cases, the latter may only be achieved through different hiring policies.

3.5 Results

3.5.1 Comparative Statics

In this section we examine how the endogenous variables that are optimal commission rate b, optimal effort levels e and t, and therefore optimal profit π, optimal salary a are affected by the exogenous parameters.

The exogenous parameters in our model are; marginal cost c, risk aversion parameter r, uncertainty σ^2, product characteristics λ, service characteristics μ, number of households in the market n, mean revenue generated from a customer x, minimum utility level $\bar{u}$, effectiveness of sales effort and relationship effort k, m, and disutility for effort d, f.

Proposition 4 *If the firm invests in the product characteristics, i.e. when λ is increased, the optimal commission rate is not changed but the optimal salary is decreased.*

Proposition 4 reflects the fact that if there is an increase in the product characteristics overall, the firm (the principal) should not deviate from the optimal commission rate and therefore from the optimal agent efforts. An increase in λ can be explained with an improvement in the product. This can be an investment in the product by the firm that causes a modification in the product.

Let's consider the following example for a car. In order to actively explore the nature of a product further, it is considered as three different products; the core product, the actual product, and finally the augmented product. These are known as the 'Three Levels of a Product.

The core product is not the tangible, physical product. It cannot be touched but it is the benefit of the product that makes it valuable to you. So with the car example, the benefit is convenience i.e. the ease at which you can go where you like, when you want to. Another core benefit is speed since you can travel around relatively quickly.

The actual product is the tangible, physical product. You can get some use out of it. Again with the car example, it is the vehicle that you test drive, buy and then collect.

The augmented product is the non-physical part of the product. It usually consists of lots of added value, for which you may or may not pay a premium. So when you buy a car, the warranty, the customer service support offered by the car's manufacture, and any after-sales service would be a part of the augmented product.

The actual product includes attributes like colour, style, fashion, quality, branding. An increase in parameter λ means, maintaining product quality and adding additional features for the above attributes or enhancing the current features to differentiate the product from competitors.

Proposition 4 says simply not to increase the commission rate if there is a change in the product characteristics that the firm believes that will positively affect sales. It is also optimal to decrease the fixed part of the compensation a since sales will increase and therefore a larger part of the wage will be delivered to the agent via commission.

The effect of λ on optimal contract parameters is the same as the effect of parameter base sales in [22]. Changes in base sales do not motivate the firm to induce a different level of effort, so an increase in base sales is accompanied by an decrease in salary without changing the commission rate.

Proposition 5 *If the firm invests in the service characteristics, i.e. when μ is increased, the optimal commission rate is increased and the optimal salary is decreased.*

Proposition 5 states that if the principal invests in the service level of the firm, it is optimal to increase the commission rate paid to the salesperson. The principal thereby induces more effort to the agent. This is necessary since the benefits from an increase in μ can only be reaped post-purchase, i.e. by individuals who are in the customer state. By increasing the incentives to the agent, the principal is able to increase the probability of being in the customer state. The increase in μ, in turn, will increase the probability of the customer to continue their relationship with the firm. Since the commission rate is increased, the fixed part of the salary is decreased to deliver the minimum expected utility.

Let's again consider the car example. The augmented product for a car includes warranties, customer care, installation, service, delivery. An increase in the parameter μ can be one of the following examples. In many car sales situations, the customer will have many more interactions after the sale with technical, service, or customer support people than they did with the sales people. So these interactions are much more important if the firm is serious about retaining customers. Because they can be considered as sales calls for repeat business in a way. They really should be handled with the same attention and focus that sales calls get. If we interpret the results in light of a servitization strategy we can say that a move towards more servitization necessitates redesign of incentive schemes providing higher performance driven pay.

Proposition 6 *If there is an increase in the number of households n, the optimal commission rate is increased and the optimal salary is decreased.*

Proposition 7 *If there is an increase in the mean revenue generated from a consumer x, the optimal commission rate is increased and the optimal salary is decreased.*

Proposition 6 and Proposition 7 suggest that the firm should increase the commission rate when there is a positive change in the parameter n or x. These parameters affect the optimal values of commission rate, effort levels and salary in the same way like the parameter effectiveness of effort. As a result of an increase in number of households or in mean revenue generated from a consumer, the firm increases the commission rate to induce greater effort. Because of increased commission rate, the fixed part of the salary is decreased to deliver the minimum expected utility.

Examples for an increase in number of households can be giving the agent responsibility for an extra region that has a customer pool having similar characteristics with the previous region. An increase in the mean revenue generated from a customer can be explained with good signals in the economy. For instance, a decrease in interest rates positively affects the sales volume of customers.

Proposition 8 *The comparative statics are as displayed in Table 3.2.*

Table 3.2 summarizes the effect of exogenous parameters of the model over endogenous parameters like b,e,t,π,a. The first four rows belongs to the parameters that are particular to our model. These are; λ,μ,n and x and comparative statics related to them are stated

	b	**e**	**t**	π	**a**
λ	**0**	**0**	**0**	+	**-**
μ	+	+	+	+	**-**
n	+	+	+	+	**-**
x	+	+	+	+	**-**
c	-	-	-	-	+
r	-	-	-	-	+
σ	-	-	-	-	+
u	0	0	0	-	+
k	+	+	+	+	-
m	+	+	+	+	-
d	-	-	-	-	+
f	-	-	-	-	+

Table 3.2: Comparative statics for sales force compensation model

in Propositions 4-7. The rest belongs to the parameters corresponding to the sales force compensation literature (e.g., Lal and Srinivasan [22]).

As a result of an increase in effectiveness of sales effort and in relationship effort, the firm finds it more profitable to sell more and induces greater effort from the agent. As a result of an increase in marginal cost, the firm finds it more profitable to sell less and induces less effort from the agent. Similarly, an increase in risk averseness of the agent and in product uncertainty and in disutility of effort for sales and for relationship makes the firm induce less effort. An increase in the minimum expected utility does not effect the optimal effort level.

Effects of parameters on fixed part of the compensation a and on the commission rate are correlated with the effect on optimal agent efforts. As a result of an increase in marginal cost, in risk averseness of the agent, in product uncertainty and in disutility of effort for sales and for relationship, the firm decreases the commission rate to induce lower effort. Because of decreased commission rate, the fixed part of the salary is increased to deliver the minimum expected utility.

As a result of an increase in effectiveness of sales effort and in relationship effort, the firm increases the commission rate to induce greater effort. Because of increased commission rate, the fixed part of the salary is decreased to deliver the minimum expected utility.

The minimum expected utility has no effect on commission rate. However, the fixed part of the salary is increased to deliver the minimum expected utility.

Proposition 9 *The effect of parameters on the steady state probability of customer $p(C)$ are as displayed in Table 3.3.*

Table 3.3 summarizes the effect of exogenous parameters of the model over the steady state probability of being in customer state. This probability can also be considered as the market share of the firm. Therefore, an increase in λ,μ,n,x,k and m positively affects the market share; whereas an increase in c,r,σ^2,d,f affects it negatively. The minimum expected utility of the agent has no effect on the market share.

	Steady state probability of customer
λ	+
μ	+
n	+
x	+
k	+
m	+
u	0
c	-
r	-
σ	-
d	-
f	-

Table 3.3: Effect of problem parameters on the steady state probability of being in customer state

3.5.2 What is More Effective, Product or Service?

In this section, we analyze the effect of the parameters λ and μ on the profit of the firm. The profit of the firm can be defined as what is left to the company after subtracting marginal cost of the firm activities and the compensation cost. The profit function is

$$\pi \;=\; (1-c)(\frac{nx(\lambda+ke^*)}{1-\mu}) - a^* - b^*(\frac{nx(\lambda+ke^*)}{1-\mu}).$$

Proposition 10 *a-) Profit is affected positively by an increase in product characteristics or service characteristics* $\frac{\partial\pi}{\partial\lambda} \geq 0$, $\frac{\partial\pi}{\partial\mu} \geq 0$; *b-) Effect of product characteristics and service characteristics on the profit is complementary* $\frac{\partial\pi^2}{\partial\lambda\partial\mu} \geq 0$, $\frac{\partial\pi^2}{\partial\mu\partial\lambda} \geq 0$

The first and the second part of the proposition is obvious as the derivatives of the profit with respect to the parameters show. The profit increases as a result of an increase in λ and μ. Moreover, product and service are complementary on the profit. This means higher λ increases the effect of service level on the profit and higher μ increases the effect of product characteristics on the profit. These complementarities suggest that product and service characteristics should be designed jointly, which implies that successful servitization strategies require well integrated design and operations for the firm.

Proposition 11 *There are two regions; in one of them the firm finds it more profitable to increase product characteristics, i.e.* $\frac{\partial\pi}{\partial\lambda} > \frac{\partial\pi}{\partial\mu}$ *and in the other the firm finds it more profitable to increase service characteristics, i.e.* $\frac{\partial\pi}{\partial\lambda} < \frac{\partial\pi}{\partial\mu}$.

The above proposition pertains to the question whether the product or service characteristics are more effective on the profit.

We look for regions where $\frac{\partial\pi}{\partial\lambda} > \frac{\partial\pi}{\partial\mu}$. This is also similar to look for regions where $\frac{\partial\pi}{\partial\lambda} - \frac{\partial\pi}{\partial\mu} > 0$. Below, we define these two regions in terms of problem parameters.

By simplifying $\frac{\partial\pi}{\partial\lambda} - \frac{\partial\pi}{\partial\mu}$ $(=\frac{A-B}{C})$, where C is a squared term, we get,

$$\begin{aligned} A - B \;=\; & k^4\, n^2\, x^4\, \left((-1+c)\; k^2\, n\, x + 2\, d\, (-1+\mu)\, (-1+\lambda+\mu)\,\right) \\ & +4\, d\, k^2\, n\, r\, x^2\, (-1+\mu)^2\, \left((-1+c)\; k^2\, n\, x + 2\, d\, (-1+\mu)\, (-1+\lambda+\mu)\,\right)\sigma^2 \\ & +8\, d^3\, r^2\, (-1+\mu)^5\, (-1+\lambda+\mu)\, \sigma^4 \end{aligned}$$

By re-arranging terms we get,

$$A - B = (2d(-1+\mu)(-1+\lambda+\mu) + (-1+c)k^2nx)(k^2nx^2 + 2dr(-1+\mu)^2\sigma^2)^2$$
$$-(-1+c)k^2nx4d^2r^2(-1+\mu)^4\sigma^4$$

Remember that,

$$b^* = \frac{(1-c)k^2nx^2}{k^2nx^2 + 2dr\sigma^2(-1+\mu)^2}.$$

We simply say,

$$(1-c)k^2nx^2 = b^*D,$$

where

$$D = k^2nx^2 + 2dr\sigma^2(-1+\mu)^2.$$

Therefore,

$$A - B = (2d(-1+\mu)(-1+\lambda+\mu) - \frac{b^*D}{x})D^2 + \frac{b^*D}{x}E,$$

where

$$E = 4d^2r^2(-1+\mu)^4\sigma^4.$$

The three regions can be determined by looking at the following three conditions;

I- $A - B > 0$ if $2d(-1+\mu)(-1+\lambda+\mu) - \frac{b^*D}{x} > 0$

II- $A - B > 0$ if $2d(-1+\mu)(-1+\lambda+\mu) - \frac{b^*D}{x} < 0$ and $-[(2d(-1+\mu)(-1+\lambda+\mu) - \frac{b^*D}{x})D^2] < \frac{b^*D}{x}E$

III- $A - B < 0$ if $2d(-1+\mu)(-1+\lambda+\mu) - \frac{b^*D}{x} < 0$ and $-[(2d(-1+\mu)(-1+\lambda+\mu) - \frac{b^*D}{x})D^2] > \frac{b^*D}{x}E$

The regions can be shown on a graph. For simplicity, we use $1 - \lambda - \mu$ as X-axis and D as Y-axis (Figure 3.5). Below we show how these axes define the above conditions.

Region I:

Region I is defined by

$2d(1-\mu)(1-\lambda-\mu)-\frac{b^*D}{x}>0$

By rearranging we get $2d(1-\mu)(1-\lambda-\mu)>\frac{b^*D}{x}$

$\frac{2d(1-\mu)x}{b^*}(1-\lambda-\mu)>D$

Remembering X axis is defined by $1-\lambda-\mu$, we can write

$$AX \quad > \quad Y \tag{3.16}$$

Where;

$$A=\frac{2d(1-\mu)x}{b^*}$$

Note that x is the mean revenue generated from a customer and $X=1-\lambda-\mu$ is the x-axis.

Region II and Region III

Region II and III satisfy,

$-[(2d(1-\mu)X-\frac{b^*D}{x})D^2]<\frac{b^*D}{x}E$

$(2d(-1+\mu)X+\frac{b^*D}{x})D^2<\frac{b^*D}{x}E$

$(2d(-1+\mu)xX+b^*D)D^2<b^*DE$

$2d(-1+\mu)xD^2X+b^*D^3<b^*DE$

$2d(-1+\mu)xD^2X<b^*DE-b^*D^3$

$$-AX \quad < \quad \frac{E}{Y}-Y \tag{3.17}$$

I and II are regions where the effect of product characteristics on the profit are higher than the effect of service characteristics. III is the region where the effect of the service is higher. I is the region below $Y<AX$, II is the area below $-AX<\frac{E}{Y}-Y$ and above $Y<AX$ and finally III is the area above $-AX<\frac{E}{Y}-Y$.

Although, I and II are characterized by different inequalities, they have the same property of showing the region where $\frac{\partial\pi}{\partial\lambda}-\frac{\partial\pi}{\partial\mu}>0$. Moreover, mathematically they do not intersect when solved simultaneously. Also $Y=AX$ always lies below $-AX=\frac{E}{Y}-Y$ since $Y=AX$

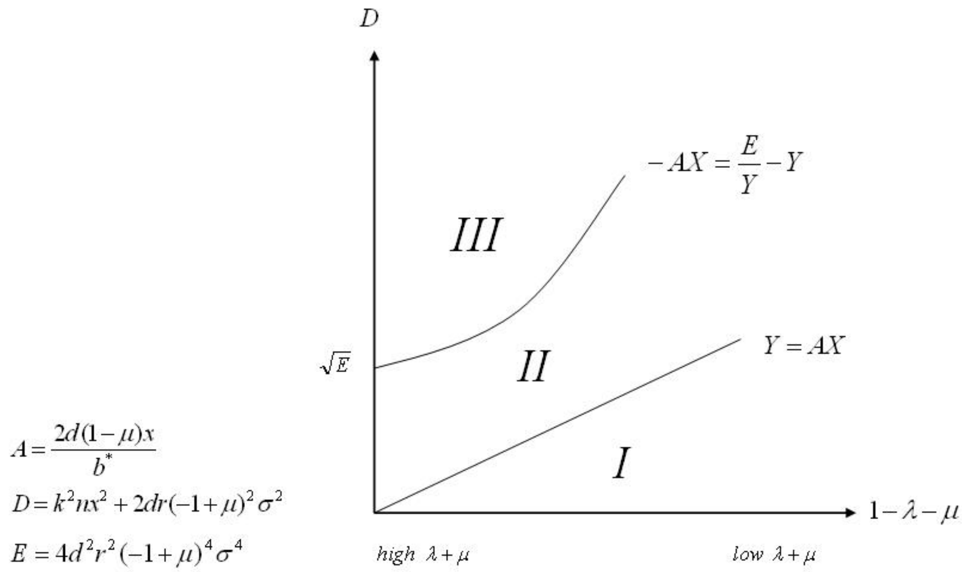

Figure 3.5: The graph with three regions

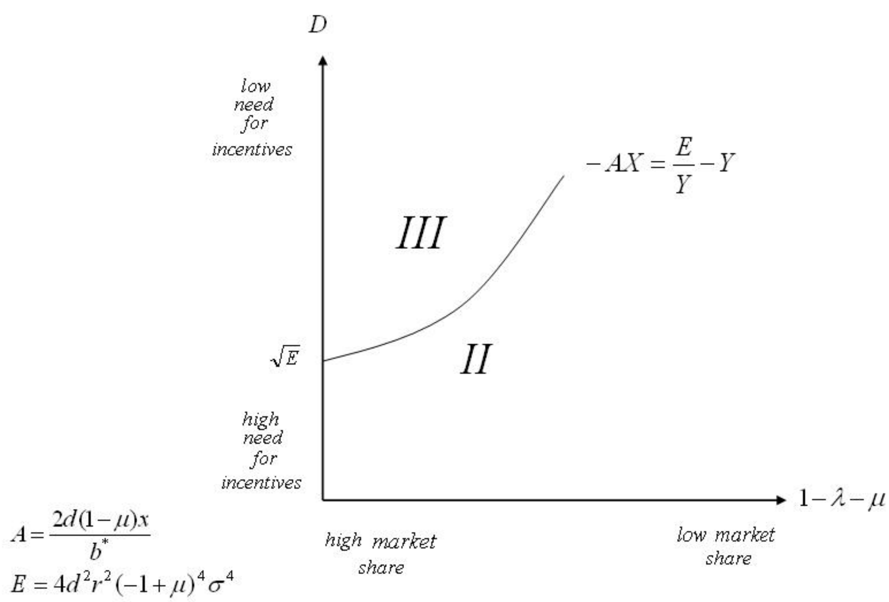

Figure 3.6: The graph with two regions

starts from the origin and $Y = \sqrt{E}$ when $X = 0$ for $-AX = \frac{E}{Y} - Y$. Therefore, in fact there is one inequality that characterizes the region in which $\frac{\partial \pi}{\partial \lambda} - \frac{\partial \pi}{\partial \mu} > 0$ (Figure 3.6).

II is the region where $\frac{\partial \pi}{\partial \lambda} - \frac{\partial \pi}{\partial \mu} > 0$ and III is the region where $\frac{\partial \pi}{\partial \lambda} - \frac{\partial \pi}{\partial \mu} < 0$. Regions are characterized by $-AX = \frac{E}{Y} - Y$.

Remember that steady state probability of customer equals $p(C) = \frac{\lambda + ke}{1-\mu}$. Since, the X-axis is characterized by $1 - \lambda - \mu$ it can be seen as a proxy for market share. As we move towards the left on the X-axis, the value of $\lambda + \mu$ increases. As a result of an increase in $\lambda + \mu$, $p(C)$ increases. Y-axis is characterized by $D = k^2nx^2 + 2dr(-1+\mu)^2\sigma^2 = \frac{(1-c)k^2nx^2}{b^*}$. Similarly, there is a relationship between D and b^*. As b^* decreases, the value of D increases and we move upwards on Y-axis. So, Y-axis is characterized by the required level of incentives.

The main conclusion is that the effect of service on profit is higher when market share of the firm is high and required level of incentives is low. In other words when λ and μ are already set at high values and uncertainty and/or disutility of the agent is low and effectiveness of the agent is high, the firm finds it profitable to increase the service characteristics in region III. However, the firm finds it profitable to increase the product characteristics in region II.

The above graphs are aimed to show the parameters of our model and the relationships between them in a 2-dimensional space so as to gain better insights about the problem. The drawings are snapshots of certain values and as a result of change in a parameter, the slope of the inequality gets steeper or flatter and the areas gets smaller or larger.

From the above results, the changes in parameters, therefore the changes in A and E can be examined as to see how these two regions are changed. An increase in A increases the slope of $-AX = \frac{E}{Y} - Y$ and therefore service becomes more effective than the product in higher market share levels. Similarly, as a result of an increase in E service becomes more effective in lower levels of incentives.

By expressing in terms of the problem parameters we find that if there is an increase in the marginal cost, the area of region III gets smaller and service becomes more effective in higher market shares. As a result of an increase in risk aversion of the agent, product uncertainty and disutility of effort the area of region III gets smaller and the firm finds it profitable to increase service characteristics in higher market share levels and with lower commission rates. Finally, if there is an increase in the effectiveness of the effort of the agent, the region III gets bigger and it becomes possible to increase profit by increasing service characteristics in lower market share levels.

Chapter 4

MODELING CO-PRODUCTION IN KNOWLEDGE INTENSIVE SERVICE OPERATIONS

4.1 Introduction to Co-production

Management and technical consulting services constitute a significant sector in developed economies like U.S. and UK. Statistics show that these services contribute up to 30 per cent of the total value added in the economy [4]. Many large companies seek for cost-effective and high quality solutions with the help and the expertise of consulting firms. Moreover 30 per cent of the workforce is estimated to be engaged in knowledge work in U.S. [38] and on the EU level, these kind of services account up to 60 per cent of the employment. Their overall impact on the economy also increases the attention to these services in the literature.

Management and technical consulting services are classified as Knowledge-Intensive Business Services (KIBS). KIBS are defined as the business service firms that rely on knowledge and expertise for a specific domain to supply intermediate services that are knowledge-based and support the knowledge processes in the client companies [20]. IBM, McKinsey, Accenture are among the famous consulting firms that can be classified as KIBS. Another definition of KIBS is given as enterprises whose primary value-added activities are accumulation or dissemination of knowledge in order to develop a customized service to satisfy the client's needs [4].

Service delivery activities of KIBS are complex, unstructured and customized to meet a specific client's unique needs and clients must perform a variety of roles since they possess much of the knowledge and competence that a KIBS needs [4]. Co-production is a key element in service delivery of KIBS firms since it involves mutual exchanges in which the client accesses the service provider's expertise and the service provider at the same time needs to access the client's situational knowledge in order to deliver successful service [20]. A challenge for a KIBS firm is to have the client perform the necessary effort to produce the service output. If the clients do not have the right incentives, the performance of the co-production process and the quality of the output may be affected negatively.

A service production process is defined as a combination of tasks and decisions to some extent performed by customers or by the firm. The effect of customer involvement in service can be seen in the heterogeneity of service performance and service outcomes. Frei [10] states that customers introduce tremendous variability, since customers themselves are key inputs to the service production process, and defines five types of variability. We consider two of these variability types in our model. These are capability variability, which occurs since service businesses should work with customers whose own capabilities differ, and effort variability, which reflects the fact that it is up to customers how much effort they apply to the process. Therefore, customer participation is described as troubling and its inputs are seen as a source of uncertainty. But the value gained from adjusted customization and the increase in process productivity are seen as positive consequences of customer participation [1, 5].

Transaction cost economics is a means of explaining to what extent the firm will perform some tasks and to what extent the customer will. If the firm transfers the tasks to the customer, there are also costs like providing the customer operational knowledge, use of facilities and costs of monitoring the customer. If the firm performs the tasks internally, information inputs are needed to achieve customization. The more specific and complex the inputs transferred are, the higher the transaction cost will be [1].

Bowen and Jones [5] state that the sources of transaction costs are performance ambiguity and goal incongruence. The reasons for performance ambiguity are intangibility of services and the way services are produced. The more intangible the service, the more this problem occurs. For instance it is not easy for the customer to envision what is obtained while buying insurance. The goal incongruence is also a source of transaction costs and it occur because the goals and motives of the firm and the client may be different. This may be a major problem when the firm and the customer have unequal information about the object exchanged.

The main focus of our study is solving the pricing problem of a knowledge-intensive business service firm whose service delivery is jointly produced by the effort of the firm and the client. In this research co-production is defined as the joint effort by which the service provider (the firm) and the client produce the service.

Differently than the existing literature, we consider the utility from the service as an output of the joint effort by the firm and the client. In the literature, there are several studies where the output of the service co-production is assumed to be fixed. However, there are also empirical papers that emphasize that the success of co-production is highly dependent on the involvement of the client.

Besides considering the service utility as an output of the joint effort by the firm and the client, we assume that knowledge is also an output of the service co-production. In the literature, knowledge is also mentioned among the outputs of the service co-production. However, the analytical models that are developed to model service co-production do not include knowledge as an output.

The risk aversion differences of the firm and client is also considered in this study. Modeling risk aversion also enables modeling the uncertainty in the co-production outputs.

The remaining parts of the chapter is organized as follows. Relevant literature is reviewed in Section 4.2. The model is introduced in Section 4.3. We analyze the resulting pricing problem and derive comparative statics in Section 4.4. We compare the common price and customized price case in Section 4.5.

4.2 Literature Review for Co-production

The stream of literature related to this paper is co-production in service operations management. The service co-production literature can be investigated in two parts: analytical models and exploratory and empirical studies.

To my knowledge, the study by Karmarkar and Pittbladdo [18] is among the first analytical frameworks for modeling co-production in service operations. In their paper, they provide a comprehensive literature on service markets on competition by focusing on topics like characteristics of service operations, service markets and contracts, service transactions and processes and service competition. They also argue that using the analytical models in manufacturing context for analyzing the service operation can be misleading. The important point is that they assume that the service is jointly produced by the supplier and the buyer of the service. So, they define the problem faced by the supplier as

$$\begin{aligned} \max \quad & P - c_s t_s - C \\ \text{s.t.} \quad & U(x) - c_b t_b - P \geq u_m. \end{aligned}$$

Where P is the price charged to the buyer, c_s, c_b are costs per unit time for buyer and seller, t_s, t_b are time spent by buyer and seller, C is other costs to seller, u_m is the minimum acceptable utility level for the buyer and $U(x)$ is the utility for the service. The firm solves the above problem and decides on P by considering that utility for service is a function of both buyer and the seller.

Xue and Field [38] discuss a theoretical framework and develop analytical models for service co-production in consulting firms. They derive results depending upon information stickiness of knowledge management literature, service co-production of service operations management literature and incomplete contracts of transactions cost economics literature. In their work, they define the client effort as the self-service level which is assumed between 0 and 1. The client's and the firm's information transfer and information processing costs depend on the self-service level. The price for the service is composed of two parts: fixed fee and price discount depending on the self-service level. The firm's profit function is similar to the model in Karmarkar and Pittbladdo [18]. They analyze three models; in one of them the socially optimal process efficiency is maximized, in the second the client decides on the self-service level and the firm decides on the discount rate, and in the third the firm decides on the self-service level and the client approves the discount rate. They also assume that utility from service is constant and does not depend on the self-service level.

Xue and Harker [39] develop an analytical model to investigate the relationship between a firm's capacity decision, pricing decision and the self-service level offered by the firm. In their model, the firm decides on the self-service level and the price for the service. The problem is solved and analyzed for various market conditions like monopoly, duopoly and oligopoly. The client's time spent while doing the self-service and time spent in service with employee depends on the self-service level offered. The client has also different costs for the time spent in self-service or in service with employee. The full price is a function of cost of these times and the price charged by the firm. The market demand is assumed to be a function of the full price. The firm's cost structure involves fixed cost and variable cost that both depend on the self-service level that is determined by the firm.

Cachon and Harker [6] provide a model that analyzes the competition between two firms that have scale economies. They present a framework that incorporates the competition between the two firms in a queuing game and in an economic order quantity game. Modeling the competition between the two firms with price and time sensitive demand is called the queuing game and modeling competition between two retailers with price sensitive customers is called the economic order quantity game. The paper is not directly related with co-production in service operations but with outsourcing to another firm. The authors make comments about the possibility of interpreting outsourcing to customers in service context as similar to outsourcing to another firm in their model. Moreover, Xue and Field [38] state that self-service level in their paper is the same with number of tasks outsourced to customer of Cachon and Harker [6]. Therefore, they define the self-service level as the proportion of tasks outsourced to customer in a service co-production context.

Ha [13] studies the incentive problem of a firm in a service context where the service is jointly produced by the firm and the customer. In his paper, the facility is modeled as a queue with customer-chosen service rates. The customer decides on his effort level which decreases the amount of work required to complete the service. It is assumed that by choosing the effort and therefore the amount of work to complete the service, the customer chooses his own service time. The full price is composed of price, delay costs, delay costs in the queue and the service cost. The firm decides on the optimal price and therefore decides on the arrival rate. The author derives a pricing scheme that is optimal and incentive-compatible to induce the optimal effort for the system. Optimal price charged to the customer is increasing in service time. Mendelson and Whang [26] also seek for incentive-compatible pricing schemes for a system that is modeled as a queuing system, where the queuing system is modeled with multiple user classes.

The study by Bettencourt et al. [4] is among the empirical papers in the service co-production literature. They present a co-production model that is relevant to a KIBS firm especially providing complex, unstructured and customized services. They explain the client co-production model in detail. Clients' contribution to the service delivery is seen integral to service success by affecting both the quality of the service outcome and the client satisfaction. In their paper, client role responsibilities and how the firm should manage those behaviors are explained. Research is conducted by 25 in-depth interviews done with 12 firm associates and 13 clients. It is also mentioned that there is some research showing that extensive client monitoring can decrease the client cooperation.

Kuusisto and Viljamaa [20] conducted a study to investigate how the KIBS can benefit clients. In their paper, they provide detailed explanation about KIBS that rely heavily on knowledge and expertise. The paper explains the three steps of co-production process: the entry in the co-production process, actual service delivery process and the outcomes of the service co-production process. Co-production is defined as the joint effort by which the firm and the customer produce the service. The research is conducted with a total of 105 interviews done with client companies. They state that the co-production outcomes are the service, knowledge, capability and learning. An important point is that knowledge is explained as an outcome of co-production process in this paper.

4.3 Modeling Co-production

In this section, we develop a conceptual framework for service co-production process, especially for knowledge-intensive business service firms. First of all, we use the same underlying assumption that both the client and the firm own a certain amount of information that is needed for the completion of the service, however; this is not shared common knowledge and it needs to be transferred so that the other can process the information [38]. Together with this assumption, Xue and Field [38] define the self-service level as the proportion of total amount of information that is to be processed by the client. Differently than their work, we define the self-service level as the amount of effort exerted on the service co-production and this amount is a proportion of the total effort that is to be exerted on the service co-production. Therefore, we assume that the amount of effort exerted on the project by the client is e and accordingly, the amount of effort to be exerted by the firm is $(1 - e)$. Xue and Field [38] claim that this definition of client effort also captures the number of tasks outsourced to the customer that is the service operations process design variable defined by Cachon and Harker [6].

By focusing on the amount of effort exerted by the client on the service delivery process, we consider the client's contribution to the service delivery. Client's contribution to the service delivery process is integral to the service success since it affects both the quality of service of the service outcome and the client's satisfaction with service delivered. Also, service delivery processes of the knowledge-intensive service business firms are complex and customized to a specific client's needs and therefore, clients should effectively perform their roles as co-producers since they own a part of the knowledge that the firm needs to deliver the service solution successfully [4]. To sum up, both the firm and the client benefits from the effort exerted by the client during the service co-production process due to the nature of the service co-production.

Besides the client's satisfaction with the service delivery, some other client firm outcomes have been discussed. Kuusisto and Viljaama [20] assume that four types of outcomes are obtained as a result of co-production in KIBS firms. The outcomes are innovation, capability development, knowledge accumulation/learning, and service delivery. They assume that in a co-production process involving mutual efforts, it is more likely that client gains knowledge during the process. Although from the client perspective learning is a potential but not a necessary outcome, it may result in a change if the new knowledge touches upon a critical juncture. Therefore, we assume that as a result of a co-production process that involves mutual efforts of both the client and the firm, the utility gained from the service outcomes are different to each party although they took part in the same project. So, we model the quality of the service outcome for the firm and the satisfaction of the client as functions of the client's effort. We define the client satisfaction of the service as:

$$s = h_0 + k_0 e, \tag{4.1}$$

Where h_0 is the base satisfaction level that the client will get from the service and k_0 is the effectiveness of the client's effort for satisfaction. We also define the service quality for the firm as:

$$q = h_1 + k_1 e \tag{4.2}$$

Where h_1 is the base quality level of the project that will be obtained without client effort and k_1 is the effectiveness of the client's effort for the service quality.

In this model we also focus on the pricing problem in the service co-production context together with the effort decision of the client. In his model, Ha [13] seeks for incentive-compatible pricing schemes for a service facility with joint production. The joint production concept is similar to our service co-production problem. In his work, he assumes that although customers are homogenous with all characteristics known to the manager, a moral hazard problem arises since effort levels are not observable or verifiable ex-post. He also claims that one way to induce the customer optimal effort is to charge a price that is increasing in the observed service time. Similarly, we look for pricing schemes under which the client firm will choose the effort level that maximizes the firm's profit. We assume that the firm cannot observe the client's effort exerted on the service delivery but the project completion time is observed. Therefore, we use the pricing scheme that charges a fixed fee together with a variable fee that is proportional to the project completion time. We define the full price as:

$$p = a + bT, \tag{4.3}$$

where a is the fixed fee, b is the variable fee that is proportional to service completion time T. We also assume that project completion time is a function of the client's effort. The project completion time decreases with an increase in the client effort. So, the full price can be defined as:

$$p = a + b(t - me), \tag{4.4}$$

where t is the service completion time with no client effort and m is the effectiveness of the client effort for the service time. For this pricing scheme, we assume that there is no trade-off between output quality and time which implies that less time spent on the service co-production does not imply lower service quality.

The above scheme is the expected price, since project completion time is assumed to be distributed normally with mean $t - me$ and variance σ^2.

Our model also involves the expense functions of both the firm and the client. By our assumptions the effort exerted on the service delivery by the client is e. Accordingly, the effort exerted by the firm is $(1 - e)$. Xue and Field [38] claim that in a consulting context a squared term can be included for the cost of effort to relate to the complexity of knowledge transfer. Since, we model the co-production process especially in the context of KIBS; we define the cost of the effort to the client as:

$$c_c = de^2, \tag{4.5}$$

where d is the disutility of effort to the client. The cost function of the firm is defined as:

$$c_f = c(1 - e). \tag{4.6}$$

Where c is a parameter for the firm's cost. We assume that the firm is a larger organization than the client firm and the project with this firm may be the one of the many projects that belong to the consulting firm. So, we assume that the firm's cost is linear in terms of effort. We also consider a model with convex cost function for the firm in the later sections.

The model depends on the assumptions that the risk-neutral firm provides service delivery to the risk-averse client firm. Consequently, the firm's preferences towards risk have no impact on the design of the pricing scheme. We assume that the utility function of the client to be exponential to exhibit the property of constant risk aversion. The client's utility for income increases at a decreasing rate and satisfies the property of constant risk aversion r. We also assume that there are uncertainties in the project completion time and the actual project time is normally distributed with mean $(t - me)$ and standard deviation σ.

Problem parameters are summarized in Table 1.

e: Effort exerted on the service delivery by the client

$1-e$: Effort exerted on the service delivery by the firm

h_0: Base satisfaction level that the client gets from the service

h_1: Base service outcome quality that the firm gets form the service

k_0: Effectiveness of the client effort for the satisfaction of the client

k_1: Effectiveness of the client effort for the service quality level

a: Fixed fee charged to the client

b: Variable fee that is proportional to the service completion time

t: Base service completion time

m: Effectiveness of the client effort for service completion time

c: Cost of effort to the firm

d: Disutility of the client to the effort

r: Risk aversion of the client

σ: Standard deviation of the service completion time

$\bar{u}$: Minimum acceptable utility level for the client

Table 4.1: Problem parameters of co-production model

The firm's profit function can be stated as:

$$\pi = h_1 + k_1 e + a + b(t - me) - c(1 - e) \tag{4.7}$$

Meanwhile, assume the client's certainty equivalent of the utility function of the service as:

$$CE = h_0 + k_0 e - a - b(t - me) - de^2 - \frac{r}{2} b^2 \sigma^2 \tag{4.8}$$

using the first order condition, the client sets e to maximize the certainty equivalent;

$$e^* = \frac{bm + k_0}{2d}. \tag{4.9}$$

Let the minimum acceptable utility level to the client be equal to $\bar{u}$. So the optimum value of the fixed fee would be such that;

$$a^* = -\left(d\, e^2\right) + b\, e\, m - b\, t - \frac{b^2\, r\, \sigma^2}{2} - \bar{u} + h_0 + e\, k_0. \tag{4.10}$$

Substituting e^* and a^* into the profit function and setting $\frac{d\pi}{db} = 0$, we obtain the optimal variable fee, as;

$$b^* = \frac{m(c + k_1)}{m^2 + 2dr\sigma^2}. \tag{4.11}$$

Substituting b^* into e^*, we obtain the optimal client effort as;

$$e^* = \frac{k_0 + \frac{m^2\,(c+k_1)}{m^2 + 2\,d\,r\,\sigma^2}}{2\,d}. \tag{4.12}$$

4.3.1 Deterministic Model

In this section we present a model that solves the effort decision of the client by maximizing the social utility. The main assumption is that if there is a social planner who observes everything, optimal distribution of effort is set to maximize the total social welfare (sum of utilities). Thus we would like to understand if there is any efficiency loss in the system because of co-production of service by different parties and because of the unobserved effort e by the client. Xue and Field [38] also solve a socially optimal process efficiency model with the assumptions that there are no uncertainties in the service need or the contract is complete at the time it is signed and the two parties have equal power in the negotiations.

The difference in this case is that there is no uncertainty, hence there is no risk involved for the client. So, we do not include the uncertainty term in the utility function of the client.

The firm's profit function can be stated as:

$$\pi = h_1 + k_1 e + a + b(t - me) - c(1 - e) \tag{4.13}$$

Meanwhile, assume the client's utility function of the service as:

$$U = h_0 + k_0 e - a - b(t - me) - de^2 \tag{4.14}$$

By taking derivatives with respect to e, we find the optimal client effort as;

$$e^* = \frac{bm + k_0}{2d}. \tag{4.15}$$

Let the minimum acceptable utility level to the client be equal to $\bar{u}$. So the optimum value of the fixed fee would be such that;

$$a^* = -\left(d\,e^2\right) + b\,e\,m - b\,t - \bar{u} + h_0 + e\,k_0. \tag{4.16}$$

Substituting e^* and a^* into the profit function and setting $\frac{d\pi}{db} = 0$, we obtain the optimal variable fee, as;

$$b^* = \frac{c + k_1}{m}. \tag{4.17}$$

Substituting b^* into e^*, we obtain the optimal client effort as;

$$e^* = \frac{c + k_0 + k_1}{2\,d}. \tag{4.18}$$

The optimal fixed fee becomes;

$$a^* = \frac{c\,(c\,m - 4\,d\,t) - 4\,d\,m\,\bar{u} + 4\,d\,m\,h_0 + m\,k_0\,(2\,c + k_0) + 2\,(c\,m - 2\,d\,t + m\,k_0)\,k_1 + m\,{k_1}^2}{4\,d\,m}. \tag{4.19}$$

The optimal firm profit is;

$$\pi^* = \frac{c\,(c - 4\,d) + 4\,d\,(-\bar{u} + h_0 + h_1) + (k_0 + k_1)\,(2\,c + k_0 + k_1)}{4\,d}. \tag{4.20}$$

4.3.2 Convex Cost Function for the Firm

In this section, we assume that the cost function of the firm has a specific form. This assumption depends on our model's relation with the co-production in KIBS. Xue and Field [38] state that the complexity of an item of knowledge includes the number of elements contained or the amount of information required to be characterized. They also state that the complexity increases the difficulty of knowledge transfer and likelihood that the problems will occur and literature has suggested that the complexity of knowledge has curvilinear relationship with the efficiency of its use.

So, in this section, the firm's cost function is assumed as:

$$c_f \;=\; c(1-e)^2. \tag{4.21}$$

Therefore, we define the firm's utility function as;

$$\pi \;=\; h_1 + k_1 e + a + b(t - me) - c(1-e)^2. \tag{4.22}$$

The client's utility function, the derivation of the optimal client effort and the derivation of the optimal fixed fee is the same as in the base model.

Substituting e^* and a^* into the profit function and setting $\frac{d\pi}{db} = 0$, we obtain the optimal variable fee, as;

$$b^* \;=\; \frac{m\,(-(c\,k_0) + d\,(2\,c + k_1))}{(c+d)\,m^2 + 2\,d^2\,r\,\sigma^2}. \tag{4.23}$$

Substituting b^* into e^*, we obtain the optimal client effort as;

$$e^* \;=\; \frac{(m^2 + 2\,d\,r\,\sigma^2)\,k_0 + m^2\,(2\,c + k_1)}{2\,(c+d)\,m^2 + 4\,d^2\,r\,\sigma^2}. \tag{4.24}$$

Note that the parameter related to the cost of the firm, c, is included in the optimal client effort, unlike the effort in the base case. The effort in (4.12) is larger than the effort in (4.24) and the effort in (4.24) decreases with an increase in c while the effort in (4.12) increases in c. We can say that the form of the cost function influences the qualitative behaviour in the model as well as the absolute values of e^*, b^* and π^*.

4.3.3 Risk Averse Firm

In this section, we derive a model to analyze the pricing and effort decisions of the parties in a co-production context in which the firm and the client are both risk averse. Misra et al. [28] state that the current theory on sales force compensation is based on the assumptions that a risk neutral firm employs a risk averse salesperson. The framework in the previous sections of the paper is based on the same assumptions since the firm cannot observe the effort of the client firm and the utility of the client is decreasing with an increase in the uncertainty of the income. Besides that, the firm's profit function is based on the assumption that firm's preferences towards risk have no impact on the design of the pricing scheme.

In this section, we assume that the firm's utility for income also satisfies the property of constant risk aversion, denoted by R. We define the firm's utility function as;

$$\pi = h_1 + k_1 e + a + b(t - me) - c(1 - e) - \frac{R}{2} b^2 \sigma^2. \tag{4.25}$$

The client's utility function, the derivation of the optimal client effort and the derivation of the optimal fixed fee is the same as in the base model.

Substituting e^* and a^* into the profit function and setting $\frac{d\pi}{db} = 0$, we obtain the optimal variable fee, as;

$$b^* = \frac{m\,(c + k_1)}{m^2 + 2\,d\,(r + R)\,\sigma^2}. \tag{4.26}$$

Substituting b^* into e^*, we obtain the optimal client effort as;

$$e^* = \frac{k_0 + \frac{m^2\,(c+k_1)}{m^2+2\,d\,(r+R)\,\sigma^2}}{2\,d}. \tag{4.27}$$

4.4 Comparative Statics for Co-production

4.4.1 Base Model

In this section, we investigate how the optimal parameters b, e, and optimal profit π is affected by the endogenous parameters in our model.

Proposition 12 *The comparative statics are as displayed in Table 4.2 .*

	k_0	k_1	**m**	**c**	**d**	**r**	σ
b	0	+	+/-	+	-	-	-
e	+	+	+	+	-	-	-
π	+	+	+	-	-	-	-

Table 4.2: Comparative statics for the base case

The parameter b (variable fee charged to the client) is negatively affected by an increase in d (disutility of the client), r (risk aversion of the client) and σ (uncertainty in the project completion time). These results are intuitive. The client dislikes exerting effort and it is optimal to lower the fee charged if there is an increase in disutility. Similarly, when there is an increase in risk aversion and uncertainty it is optimal to lower the variable fee charged according to the project time that has a probability distribution. Naturally, the variable fee is not affected by the parameter k_0 (Effectiveness of client effort for the satisfaction of the client). However, as a result of an increase in k_1 (Effectiveness of the client effort for the service quality), it is optimal to increase the variable fee. It is also optimal for the firm to charge a higher price if there is an increase in his costs c. Finally, we expect the firm to charge a higher variable fee (increase b) if there is an increase in m. However, this result is true if and only if;

$$2dr\sigma^2 - m^2 > 0 \tag{4.28}$$

$$2dr\sigma^2 > m^2 \tag{4.29}$$

The firm should increase the variable fee b as a result of a increase in m by considering the other characteristics of the client like d, r and uncertainty σ. Therefore, we can say that b increases with an increasing m but after a threshold level for m, b starts to decrease with an increasing m. The threshold level depends on d, r and σ.

The effects of k_1, c, d, r, σ on the optimal effort are similar to their effect on the optimal variable fee. This is due to the fact that optimal client effort is a function of the optimal variable fee. The difference is that optimal client effort is also affected by an increase in k_0.

	k_0	k_1	**m**	**c**	**d**	**r**	σ
b	0	+	-	+	0	0	0
e	+	+	0	+	-	0	0
π	+	+	0	-	-	0	0

Table 4.3: Comparative statics for the deterministic case

This is intuitive since it directly increases the utility level of the client. Another difference is that we can certainly say that an increase in the effectiveness of the client effort for project completion time m, increases the optimal client effort.

We can simply say that an increase in k_0, k_1, m increases the profit of the firm whereas, an increase in d, r, σ decreases the optimal profit. Finally, firm's profit decreases as a result of an increase in c.

4.4.2 Deterministic Case

In this section, we investigate how the optimal parameters b, e, and optimal profit π is affected by the endogenous parameters in our model for the deterministic case.

Proposition 13 *The comparative statics are as displayed in Table 4.3 .*

The comparative statics results are similar to the base model case. One of the differences is that the optimal b, e and π are not affected by r and σ due to no uncertainty in the environment. The effect of k_0, k_1 and c on the optimal b, e and π are completely the same as in the base model. Another difference is that the optimal variable fee b is not affected by disutility d. The last difference is that m has a negative effect on b in this case. There is no threshold level for a condition that shows when an increase in m should decrease the variable fee. This happens since there are no uncertainties in the environment and an increase in m decreases firm profits.

	k_0	k_1	**m**	**c**	**r**	σ
b	-	+	+/-	+	-	-
e	+	+	+	+	-	-
π	+/-	+	+	-	-	-

Table 4.4: Comparative statics for the convex cost case

4.4.3 Convex Cost Function for the Firm

Proposition 14 *The comparative statics are as displayed in Table 4.4 .*

Firstly, we make some assumptions about the parameters. As in the previous models, we assume that e is between 0 and 1.

$$\frac{bm + k_0}{2d} < 1$$

We also assume that variable fee b cannot be negative. Therefore the following must hold.

$$-ck_0 + d(2c + k_1) > 0$$

Together with these assumptions, the comparative statics table is the same as in the base model.

One difference is that an increase in k_0 results with a decrease in b. It is optimal for the firm to charge lower variable fee in this case, whereas in the model with linear firm cost it is optimal not to change the variable fee as a result of an increase in k_0. The intuition for this is that the firm is becoming more sensitive to cost. Since total effort is allocated between the firm and the client, an increase in client effort results with a decrease in firm's cost. The firm also knows that an increase in k_0 will induce the client to exert more effort since k_0 directly increases the client's utility. Therefore, the firm also charges lower variable fee to the client to induce more effort.

Another difference is in the effect of m on b. An increase in m results with an increase in b. However, the condition is a bit different than the condition (4.29) in the model with

	k_0	k_1	**m**	**c**	**d**	**r**	σ	**R**
b	0	+	+/-	+	-	-	-	-
e	+	+	+	+	-	-	-	-
π	+	+	+	-	-	-	-	-

Table 4.5: Comparative statics for the risk-averse firm case

linear cost.

$$\begin{aligned} 2d^2 r\sigma^2 - m^2(c+d) &> 0 \\ \frac{2dr\sigma^2}{\frac{c}{d}+1} &> m^2 \end{aligned}$$

The above threshold is smaller than the threshold in (4.29). The firm becomes more sensitive to profit in this case and it is optimal to increase the variable fee as a result of an increase in client effectiveness for lower levels of m.

4.4.4 Risk Averse Firm

Proposition 15 *The comparative statics are as displayed in Table 4.5 .*

The comparative statics for this model are completely the same with those of the base model except for the condition on the effect of m on b. The inequality is

$$2d(r+R)\sigma^2 - m^2 > 0 \tag{4.30}$$

$$2d(r+R)\sigma^2 > m^2 \tag{4.31}$$

This the highest threshold in all of the cases. The firm is also risk averse in this case and it becomes optimal to increase the variable fee as a result of an increase in m for higher levels of m.

There is also one more different parameter R in this model. An increase in the firm's risk aversion R results with a decrease in b, e and π.

4.4.5 Comparisons

In this section we compare the optimal effort, optimal firm profit, optimal variable fee and optimal fixed fee for the three models. The three models are, the base model which is represented with subscript b, the deterministic model that is represented with subscript d and the risk averse firm model that is represented with subscript R.

By comparing the optimal variable fees, we get

$$b_d > b_b > b_R.$$

The fee that the firm charges to the client is the highest in the deterministic case and the variable fee in base model is higher than the fee in risk averse firm case. The reason for this ranking is the risk aversion of the parties. We know that as risk aversion increases the client will be more prone to decrease his effort. So, the firm charges higher variable fees in models with less uncertainty.

By comparing the optimal efforts, we get

$$e_d > e_b > e_R.$$

The ranking of the optimal efforts is consistent with the ranking of the variable fees. An increase in variable fee causes an increase in client effort. Therefore, the client effort in the deterministic case is the highest. Second is the effort in the base model and the smallest is the client effort in the risk averse firm model.

The risk and disutility related terms do not appear in e_d (4.18) and optimal client effort is larger than e_b (4.12). This means that there is more co-production if there is no uncertainty. Also note that there is less co-production when the firm is risk averse $e_b > e_R$ ((4.12) > (4.27)). This can also be seen from the variable fees. The risk averse firm induces less effort from the client $b_b > b_R$ and as a result there is less co-production. Finally, by comparing the optimal firm profits, we get

$$\pi_d > \pi_b > \pi_R.$$

The reason for this ranking is also consistent with the rankings for variable fees and optimal efforts. Naturally, the variable fee charged to the client positively affects the firm's profits.

Moreover, an increase in client effort also affects the firm's profits positively by decreasing the cost of the firm due to the fact that total effort is allocated between the firm and the client. Therefore, total profit is highest in the deterministic case in which the variable fee and the client effort are highest. The next is the profit in the base model and the smallest profit is in the risk averse firm case in which the variable fee and the client effort are also the smallest. The profit loss due to increasing uncertainty and risk aversion are found as follows:

$$\pi_d - \pi_b = \frac{r\,\sigma^2\,(c+k_1)^2}{2\,\left(m^2+2\,d\,r\,\sigma^2\right)}.$$

$$\pi_b - \pi_R = \frac{m^2\,R\,\sigma^2\,(c+k_1)^2}{2\,\left(m^2+2\,d\,r\,\sigma^2\right)\,\left(m^2+2\,d\,(r+R)\,\sigma^2\right)}.$$

These differences represent the efficiency loss in the system due to co-production. This loss increases with the importance of the client effort for the firm's utility k_1, firm's cost of effort c and decreases with m. The main driver of the differences between the profits for the three cases is the risk aversion of the parties. The difference between the deterministic case and the base case $\pi_d - \pi_b$ increases with an increase in risk aversion of the client r. However, the difference between the base case and the risk averse firm case $\pi_b - \pi_R$ increases with the risk aversion of the firm R, but decreases with the risk aversion of the client r. We can say that the risk aversion of the client becomes less important when the firm is also risk averse in terms of differences between profits. However, the difference between base case and risk averse firm case can never be 0 although an increase in r decreases the difference.

4.5 Customized price vs. common price

Firms in the real world typically have multiple clients and client firms are generally offered the same pricing scheme regardless of their skills and capabilities. In this section we use the pricing framework developed so far to explore the pricing problem when there are multiple clients of the firm. We consider a framework for the pricing problem of a knowledge-intensive service firm in a setting that enables us to compare the profits of the firm for two cases: the firm charges the common price to all clients and the firm charges customized prices to the clients.

Let us consider a firm with two clients. We assume that the firm has two clients that are different in their effectivenesses of effort m, n and minimum utility levels u_1, u_2. Therefore, the effort level choice of the client also differs e, f. For simplicity, we assume that the risk aversion of the clients and the cost of effort for the clients are the same.

4.5.1 Common Price to Clients

The problem is mathematically modeled as;

$$\max_{a,b} \quad h_1 + k_1 e + a + b(t - me) - c(1 - e) + h_1 + k_1 f + a + b(t - nf) - c(1 - f) \qquad (4.32)$$

$$\text{subject to} \quad h_0 + k_0 e - a - b(t - me) - de^2 - \frac{r}{2} b^2 \sigma^2 \geq u_1 \qquad (4.33)$$

$$h_0 + k_0 f - a - b(t - nf) - df^2 - \frac{r}{2} b^2 \sigma^2 \geq u_2 \qquad (4.34)$$

$$e \in argmax_{e' \in E} \quad h_0 + k_0 e - a - b(t - me) - de^2 - \frac{r}{2} b^2 \sigma^2 \qquad (4.35)$$

$$f \in argmax_{f' \in F} \quad h_0 + k_0 f - a - b(t - nf) - df^2 - \frac{r}{2} b^2 \sigma^2 \qquad (4.36)$$

In the first case, the firm decides on a common price to be charged to all clients whatever their characteristics are. The firm aims to maximize the total profit to be earned from the service. The first client decides on the effort level e and the second decides on the effort level f. The constraints (4.33) and (4.35) show the individual rationality and incentive compatibility constraints for client 1 respectively and (4.34) and (4.36) show the individual rationality and incentive compatibility constraints for client 2.

Certainty equivalents of the utility functions of the clients are

$$\begin{aligned} CE_1 &= h_0 + k_0 e - a - b(t - me) - de^2 - \frac{r}{2} b^2 \sigma^2 \\ CE_2 &= h_0 + k_0 f - a - b(t - nf) - df^2 - \frac{r}{2} b^2 \sigma^2. \end{aligned}$$

By taking derivatives with respect to e and f, we find the optimal client efforts as;

$$\begin{aligned} e^* &= \frac{bm + k_0}{2d} \\ f^* &= \frac{bn + k_0}{2d}. \end{aligned}$$

Let the minimum acceptable utility levels to the clients be equal to u_1 and u_2. So the optimum value of the fixed fee would be such that one of the individual rationality constraints will be binding. The right hand side of the second individual rationality constraint will be greater than zero. In practice, this means that the firm aims to do business with both clients. To achieve this, the individual rationality constraints must be satisfied so that the clients accept the pricing scheme.

Therefore, the firm may have two choices of fixed fee a.

$$a_1^* = -\left(d\,e^2\right) + b\,e\,m - b\,t - \frac{b^2\,r\,\sigma^2}{2} + h_0 + e\,k_0 - u_1.$$

if u_1 is binding and

$$a_2^* = -\left(d\,f^2\right) + b\,f\,n - b\,t - \frac{b^2\,r\,\sigma^2}{2} + h_0 + f\,k_0 - u_2.$$

if u_2 is binding.

The firm chooses one of the above depending on m, n, u_1, u_2. Substituting e^*, f^* and a^* into firm's profit function and setting $\frac{d\pi}{db} = 0$, we obtain the common optimal variable fee, as;

$$b_1^* = \frac{(m-n)\;k_0 + (m+n)\;(c+k_1)}{2\;(n^2 + 2\,d\,r\,\sigma^2)}.$$

if u_1 is binding and

$$b_2^* = \frac{(-m+n)\;k_0 + (m+n)\;(c+k_1)}{2\;(m^2 + 2\,d\,r\,\sigma^2)}.$$

if u_2 is binding.

Finally, the total profit that the firm earns from the two clients becomes

$$\pi^* = h_1 + k_1 e^* + a_1^* + b_1^*(t - me^*) - c(1-e^*) + h_1 + k_1 f + a_1^* + b_1^*(t - nf^*) - c(1-f^*).$$

if u_1 is binding and

$$\pi^* = h_1 + k_1 e^* + a_2^* + b_2^*(t - me^*) - c(1-e^*) + h_1 + k_1 f + a_2^* + b_2^*(t - nf^*) - c(1-f^*).$$

if u_2 is binding. The firm chooses pricing scheme a_1, b_1 if

$$h_0 + k_0 e - a_1 - b_1(t - me) - de^2 - \frac{r}{2} b_1^2 \sigma^2 \geq u_1$$
$$h_0 + k_0 f - a_1 - b_1(t - nf) - df^2 - \frac{r}{2} b_1^2 \sigma^2 \geq u_2$$

are both satisfied and chooses pricing scheme a_2, b_2 if

$$h_0 + k_0 e - a_2 - b_2(t - me) - de^2 - \frac{r}{2} b_2^2 \sigma^2 \geq u_1$$
$$h_0 + k_0 f - a_2 - b_2(t - nf) - df^2 - \frac{r}{2} b_2^2 \sigma^2 \geq u_2$$

are satisfied.

4.5.2 Customized Price to Clients

In this case the firm solves separate pricing problems for all clients. The model and the solution is the same as the framework developed in the previous sections. For consistency of the optimal results with the common price case and simplicity to make comparisons, we re-write the optimal parameters with the assumption that the firm has two clients with effectivenesses m, n and the clients decide on efforts e, f.

The effort levels of the clients, when the firm solves the pricing problem separately, become;

$$e = \frac{k_0 + \frac{m^2\,(c+k_1)}{m^2+2\,d\,r\,\sigma^2}}{2\,d}.$$

$$f = \frac{k_0 + \frac{n^2\,(c+k_1)}{n^2+2\,d\,r\,\sigma^2}}{2\,d}.$$

The fixed fee charged to the first client is

$$a_1^* = -\left(d\,e^2\right) + b\,e\,m - b\,t - \frac{b^2\,r\,\sigma^2}{2} - u_1 + h_0 + e\,k_0.$$

and the fixed fee charged to the second client is

$$a_2^* = -\left(d\,f^2\right) + b\,f\,n - b\,t - \frac{b^2\,r\,\sigma^2}{2} - u_2 + h_0 + f\,k_0.$$

Respectively, the variable fees charged to the clients become

$$b_1^* = \frac{m(c+k_1)}{m^2 + 2dr\sigma^2}$$

and

$$b_2^* = \frac{n(c+k_1)}{n^2 + 2dr\sigma^2}.$$

The profit earned from the first client therefore becomes

$$\pi_1^* = h_1 + k_1 e^* + a + b(t - me^*) - c(1 - e^*)$$

and the profit earned from the second client is

$$\pi_2^* = h_1 + k_1 f^* + a + b(t - nf^*) - c(1 - f^*).$$

Total profit earned from the two clients therefore becomes

$$\pi_t^* = \pi_1^* + \pi_2^*.$$

4.5.3 Common Price vs. Customized Price

In this section, we analyze the difference between the total profits of the firm for the two cases: common price and customized price. Numerically, we look for regions where $\pi_t - \pi$ is greater than zero or smaller than zero, where π_t is the total profit when customized price is charged to clients and π is the total profit when common price charged. We run several instances of different client characteristics and derive numerical results. Five scenarios $\Delta u = [-0.04, -0.02, 0, 0.02, 0.04]$ are analyzed. Parameter m is fixed to 2 and n floats between 0,5 and 3,5.

Difference between profits

The profit of the firm is always greater when customized price is given to clients, i.e. $\pi_t - \pi > 0$. The difference between the profits depends on the difference in effectivenesses of the clients and the difference in their minimum utility levels (Figure 4.1). In the graph, Δu represents $u_2 - u_1$.

When $\Delta u = 0$ the firm is better off when customized prices are charged. The difference between profits become 0 when $m = n$ and starts to increase as $|m - n|$ increases. The firm may choose to charge a common price to the clients if their minimum utility levels are equal, $u_1 = u_2$, and difference between their effectivenesses are small. This is true if there is cost for the firm to calculate or implement customized price to clients since we do not include the cost of calculating pricing schemes.

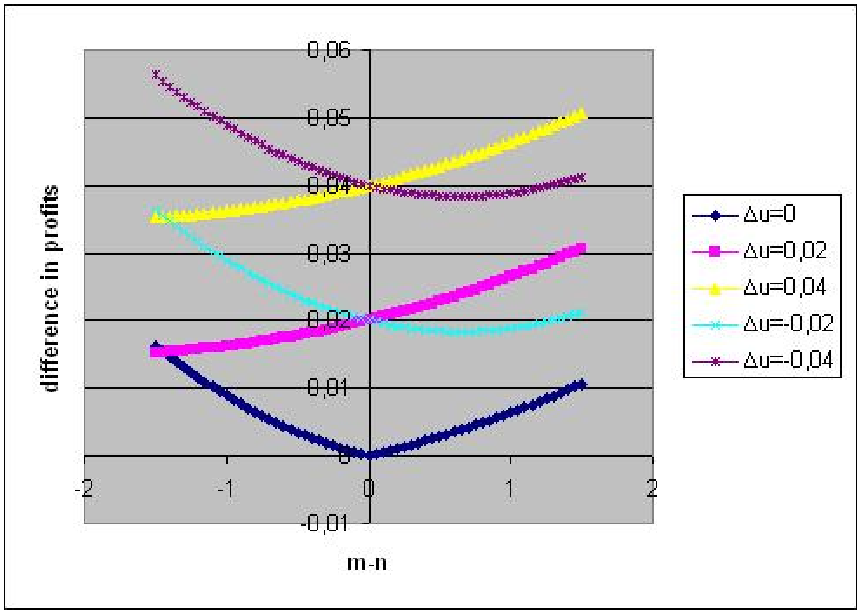

Figure 4.1: Loss in the firm profit when common price charged

When $\Delta u = 0,02$ or $\Delta u = -0,02$ the firm again chooses to charge customized prices. The difference between minimum utility levels $|u_1 - u_2|$ also affects the difference between profits. As $|u_1 - u_2|$ increases the difference between profits also increases. The reason for that is obvious. The firm lowers the fixed fee to make all clients accept the pricing scheme offered and the amount of discount on the fixed fee equals the difference between minimum utility levels $u_1 - u_2$. So, we can say that the firm is better off when giving customized prices if the difference between the clients minimum utility levels is high. However, there are point where the difference in profits are minimized. For instance, when both the effectiveness and minimum utility level of a client is higher than the other client's effectiveness and minimum utility level, i.e. $m > n$ and $u_1 > u_2$ or $m < n$ and $u_1 < u_2$; the difference between profits can be in minimum levels.

The results in the above paragraph are similar for cases when $\Delta u = 0,04$ or $\Delta u = -0,04$. Obviously, difference between profits in cases $\Delta u = 0,04$ or $\Delta u = -0,04$ are higher than differences in cases when $\Delta u = 0,02$ or $\Delta u = -0,02$.

We see that the differences in the two attributes, the effectiveness and minimum utility levels can compensate for each other if they go in the opposite directions. For example, we expect an increase in the profit difference when Δu increases from -0,02 to 0,04. However for $m - n = -1,5$, profit difference is the same for $\Delta u = -0,02$ and $\Delta u = 0,04$ which means that the two curves intersect. So, both the direction and magnitude of the difference in minimum utility level plays a role.

Another result is that the difference between the profits also depends on the magnitude of m and n. The difference between profits is higher for the case $m = 3, n = 2$ than the case $m = 2, n = 1$. A strategy for a KIBS firm may be to group clients and charge common prices to the group with lower effectiveness and charge customized price to the group with higher effectiveness.

Difference in variable fees

There are also differences between the variable fees charged for the two cases. Let b_1 be the customized variable fee for the client with effectiveness m and b_2 be the customized variable fee for the client with effectiveness n. The common variable fee is b. The client with higher effectiveness is charged lower price ($b_1 - b > 0$ when $m > n$ and $b_2 - b > 0$ when $m < n$) compared to the customized price case and the client with lower effectiveness is charged higher price ($b_2 - b < 0$ when $m > n$ and $b_1 - b < 0$ when $m < n$). The Results are shown in figure 4.2. The differences between variable fees do not change when difference between minimum utility levels $u_2 - u_1$ change. This is due to the fact that variable fee is not affected by the minimum utility level.

The variable fee charged to the client with higher effectiveness is decreased and the fee charged to the client with lower effectiveness is increased when a common price is given. The amount decreased is always larger than the amount increased. The client with higher effectiveness is better off and the client with lower effectiveness is worse off. This can also be seen from the utilities obtained. The client that is worse off is paid the minimum utility level however the other client gets utility that is higher than his/her minimum utility level. When minimum utility levels of the clients differ, amount decreased becomes smaller than the amount increased. However, the profit of the firm cannot exceed the customized price

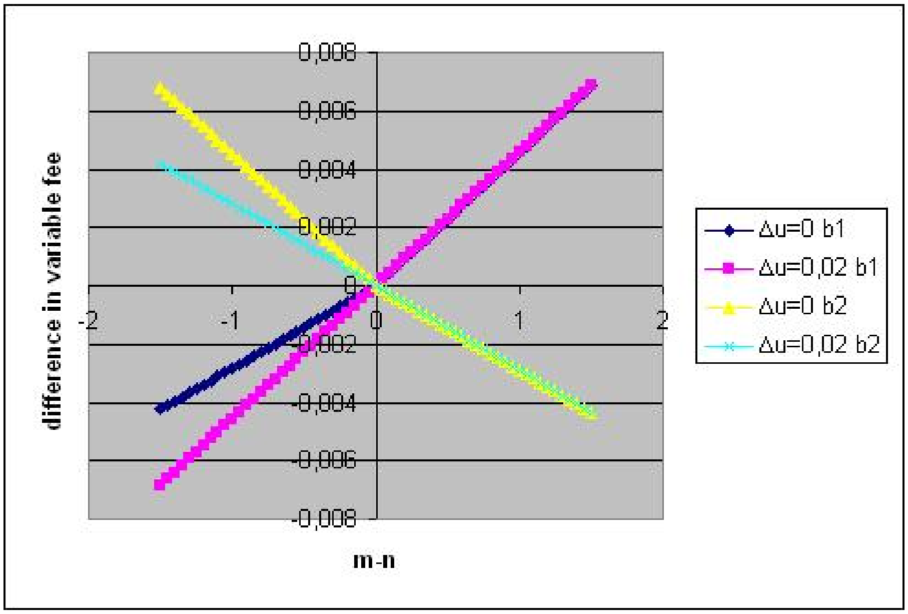

Figure 4.2: Differences in the fees when common price charged

profit even in this case.

Chapter 5

CONCLUSION

In this thesis, we studied problems in service industry, where existence of moral hazard constitutes a challenge to maximize profits. In this section, we provide conclusions obtained from each model and discuss avenues for future research.

5.1 *Model for Sales Force Compensation in Settings with Servitization*

We developed a framework to analyze the sales force compensation decisions of a firm which implements a servitization strategy. The trend in today's economy is to offer the customers a bundle of product and service irrespective of being a manufacturing or a service firm. This bundle can also be expanded to product, service, support, self-service and knowledge. This strategy is called servitization.

The servitization strategy of the firm is reflected in the model with parameters of product characteristics λ and service characteristics μ. To be more precise, these parameters can be defined in terms of consumption systems which are offerings characterized by a significant product and a service subsystem. According to these concepts, λ reflects the intention of customers towards purchase as a result of the product characteristics and μ reflects the intention of customers towards purchase as a result of service characteristics.

Reflecting the intention of customers towards purchase is important since our model also depends on the buyer decision process in marketing theory. The buyer decision process explains reactions of customers to marketing stimuli in the environment. Reactions of customers such as purchase or purchase frequency occur after the five steps in the buyer decision process. These steps include need recognition, information search, evaluation, actual purchase and post-purchase behaviour. Our model assumes that product characteristics are effective in the first three steps and service characteristics are effective in the final step. The reactions of customers in the final step are explained with satisfaction which is a post-purchase behaviour. We assume that improving service characteristics and therefore satisfying customers is important in developing long term relationships between the firm and the customers. To model the long term relationships we model the customer state dynamics with a Markov Chain and for simplicity we assume that customer can be in two states and transition between the states are affected by satisfaction. We also assume that effort of the salespeople is effective in the actual purchase step.

Sales teams of firms typically establish a large portion of the workforce and therefore become one of the key elements in the operational costs of the firm. So, a rational decision maker looks for ways to minimize the sales force compensation costs of the firm. Our model attacks the problem of sales force compensation by using the agency theory which has its roots in the informational asymmetry between the two parties: the principal and the agent. In our model the firm is modeled as the principal and the salesperson as the agent. The main source of the problem in this case is the inability of the firm to observe the actions of the salesperson perfectly.

The inability of one party to observe the actions of the other party is called the moral hazard problem in which the party whose actions are unobservable has the chance to shirk. In our model the salesperson has the chance to shirk since the sales of the firm are positively affected by the effort of the agent but also depend on some other random forces. In this setting the firm has to come up with an incentive scheme to maximize profits but also to induce the maximum possible effort from the client.

Our model enables us to incorporate the product characteristics and service characteristics of the firm to the incentive design problem and analyze the effects of product characteristics and service characteristics on the compensation plan of the firm as well as other parameters that are already developed in the sales force literature. We find that it is optimal for the firm to change the incentive scheme when there is a change in product characteristics or service characteristics. If intention of customers toward purchase is increased as a result of product characteristics, the firm should decrease the optimal salary while optimal commission rate remains unchanged. An increase in the intention of customers toward purchase is increased as a result of product characteristics means an improvement in the tangible, physical product which will effect the reactions of customers positively. However, if intention of customers toward purchase as a result of service characteristics is increased, it is optimal for the firm to set the commission rate at higher values while optimal salary is decreased. A change in service characteristics which will affect the reactions of customers positively means a change in the non-physical part of the product which usually consists of lots of added value. In terms of servitization strategy, the firm should design incentive scheme given to the salespeople according to the product and service bundle offered to the customers.

Differently than the sales force compensation literature, we can analyze the effects of parameters like number of households in the market and mean revenue generated from a customer on the incentive scheme of the firm. We find that the firm should increase the commission rate and decrease the salary when there is an increase in the number of households or in the mean revenue generated from a customer. A managerial result is that the firm can offer different incentive schemes in different regions. For instance, a salesperson can be responsible for two different regions in the market and therefore incentives offered to the salesperson should differ. The firm should also change the incentive scheme when there is a change in the economy. The trend in the economy can change over time and when there is a change that will increase the purchase power of the customer, the firm should change the incentive scheme accordingly.

We also observe the effects of product characteristics and service characteristics on the profit of the firm. Naturally, firm profit is positively affected by the product and service characteristics. First of all, we find that effects of product and service are complementary. This result suggests the firm that uses servitization strategy to jointly design products and services. This implies that a successful servitization strategy requires well integrated product design and operations functions. Secondly, neither the product nor the service not always more profitable than the other one. We can observe the effects of product and service characteristics on a two-dimensional space: need for incentives and market share. There are two regions and in one of them improving product characteristics is more profitable and in the other improving service characteristics is more profitable. When the market share is high and need for incentives is low the firm finds it more profitable to improve service characteristics. The market share is characterized by product and service characteristics and higher market share means to set λ and μ at high values. Need for incentives are characterized by skills and capabilities of the salesperson and the uncertainty in the market. Need for incentives is low if effectiveness of the salesperson is high, disutility of the salesperson is low and uncertainty in the market is low.

For future work related to sales force compensation in settings with servitization, a model that enables us to incorporate the costs of improving product characteristics and service characteristics into the firm profit function can be developed. Results in this thesis ignore the costs of improving product and service characteristics. Therefore, our analysis only considers the costs of improving product and service characteristics in terms of sales force compensation costs. Including costs of improving product and service characteristics would produce more realistic results.

Another suggestion for future work is to involve more complex and detailed customer states in the model. In this thesis, we assume that customers can be in two states, however number customer states can be increased to reflect other marketing strategies. For instance, customer states that reflect the effects of loyalty programs can be added to the model. Developing more complex customer states would enable us to gain more insights about customer reactions. Questions like how an unsatisfied customer reacts, and if the cost of gaining unsatisfied customers is different can thus be answered.

Finally, a model that assumes that total sales amount has a binomial distribution can be formulated. In this thesis customers can be in one of two states which means that he/she is the customer of the firm or not. In our model, we can find the steady state probability of being in customer state. Let this probability be p. According to the normal approximation for binomial distribution , total sales in the accounting period is normally distributed with mean np and variance $np(1-p)$. This setting is not tractable analytically with our current model. However, this modeling can also overcome the limitation that variance of the total sales is not affected from the agent effort.

5.2 Model for Co-production in Knowledge Intensive Service Operations

Service production processes and manufacturing processes have different characteristics. One of the reasons to analyze service processes different than manufacturing processes is the involvement of clients in the service production process. The important point is that characteristics of service processes and manufacturing processes are different and tools that are developed to analyze the operations in the manufacturing processes may not be useful for analyzing service processes.

We developed a framework to analyze the pricing problem of a firm that produces knowledge intensive services. Management and technical consulting services are classified as Knowledge Intensive Business Services (KIBS). The main point in the service production of KIBS firms is that service is jointly produced by the effort from both the firm and the client. Contribution of the clients in the service co-production is important because the service provided in KIBS firms is highly customized and therefore effort from the client is needed for high process performance and satisfaction.

Although the effort of clients is needed for output performance and client satisfaction, involvement of clients brings high variability to the outcomes of the service production. Two types of variability, capability variability and effort variability, will affect co-production results. Capability variability is a term used to explain that clients that are involved in co-production process will differ in skills and capabilities. Effort variability means that clients will also differ in how much effort they exert in the co-production process.

The effort decision of the client is explained using transaction cost economics which is a means to explain how much the firm will perform some of the tasks and how much the client will. If the firm performs the tasks internally, information input from the client is needed to achieve customization. If the firm transfers some of the tasks to the client, a transaction between the firm and the client is again needed since the firm should provide the client operational knowledge. There are also costs due to need for monitoring the client. Since service outputs are functions of client effort and the firm cannot perfectly observe if the client exerts required effort to the service co-production there is a moral hazard situation.

We model the relationship between the KIBS firm and the client by using the agency theory. KIBS firm and the client are parts of a contract and the firm aims to design a pricing scheme to maximize profits and to induce the client to exert optimal effort. We consider a linear pricing scheme, which charges a variable fee for project completion time, which is affected from client effort, in addition to a fixed fee. Our model enables us to analyze the effect of parameters like client effectiveness, client disutility, client risk aversion and uncertainty on the pricing decisions of KIBS firms. The effects of various parameters on price, effort and firm profit are presented in the thesis and can be useful in managing pricing decisions.

We find that as client disutility, client risk aversion and uncertainty increases the firm charges a lower variable fee and therefore induces lower effort from the client. However an increase in client effectiveness and cost of firm effort leads the firm to induce more effort from the client. To induce higher effort, the firm charges a higher variable fee to the client.

We also observe the effect of the parameters on different cases in which there is no uncertainty in the environment, the firm is also risk averse or the cost function of firm effort is different. We find that the effort induced to the client and the fee charged is the highest when there is no uncertainty. This is the case in which the client contributes the most to the co-production. As we move toward a more uncertain environment and to more risk averse parties, the client contribution to co-production decreases. The co-production level is the lowest when the firm is also risk averse. The efficiency loss in the system can also be seen in the difference between the profits in these cases. The profit of the firm decreases as risk aversion of the parties increase. Finally, a change in the sensitivity of the firm to the transaction costs in the service co-production is also important while managing pricing decision. We find that the form of the cost function influences the qualitative behaviour in the model as well as the values of the optimal variable fee and client effort.

Firms in the real world typically have multiple clients and these firms are offered the same pricing scheme. Based on the above model, we also analyze the pricing decision of the firm and firm profit when there are several clients of the firm and the firm charges the same price to all. The pricing strategy of the firm can be to charge a common price to the clients when difference between their effectiveness and difference between their utility levels are low.

This result needs the assumption that the cost of implementing a customized price strategy is higher than implementing a common price strategy. Another strategy for the firm may be to group the clients and charge a common price to the group with low effectiveness and charge customized prices to the group with high effectiveness, since the difference between the profits for the two cases also depend on the value of effectiveness. When a common price is charged, the variable fee charged to the client with higher effectiveness is decreased and the variable fee charged to the client with lower effectiveness is increased. The amount decreased is always larger than the amount increased except the case where the minimum utility levels of the clients differ. Hence a common pricing strategy makes the more effective clients better off while less effective customers would be worse off.

For future research, other pricing schemes can be integrated into the model. In knowledge intensive service business firms price can be charged by only looking at total the consulting time or another scheme is charging only fixed fee. New ways should be developed for giving incentives to the clients under these pricing schemes. Another suggestion for future work is involving asymmetric information case in the model. We assume that the firm can observe the type of the clients and therefore designs the common price that will be charged to the client. If the firm cannot observe their effectivenesses, different pricing schemes should be developed to induce the clients to choose the pricing scheme available for their type.

Appendix A

CONCAVITY OF THE FUNCTIONS

A.1 Concavity of the Agent's Utility Function

Maximizing the agent's expected utility is equal to maximizing the certainty equivalent.

$$f(e,t) = a + b(\frac{nx(\lambda + ke + mt)}{1-\mu}) - de^2 - ft^2 - \frac{r}{2}b^2 n\sigma^2$$

We firstly show that $f(e,t)$ has a local maximum at stationary points (e^*,t^*). Thus, we look at the Hessian matrix.

The determinant of the Hessian matrix is $D = f''_{ee}f''_{tt} - (f''_{et})^2$, Then ($e^*$,$t^*$) is a maximum point since;

$$D(e,t) > 0, f''_{ee} < 0$$
$$4df - 0 > 0, -2d < 0$$

A.2 Concavity of the Principal's Profit Function

Substituting e^*,t^* and a^* and simplifying, the principal's profit function becomes;

$$p(b) = (1-c)(\frac{nx(\lambda + k(\frac{bknx}{2d-2d\mu}) + m(\frac{bmnx}{2f-2f\mu}))}{1-\mu}) - d(\frac{bknx}{2d-2d\mu})^2 - f(\frac{bmnx}{2f-2f\mu})^2 - \frac{r}{2}b^2\sigma^2 - \bar{u}$$

$p(b)$ is twice differentiable and $p(b)$ is concave since $p''(b) \leq 0$

$$p''(b) = -\frac{n^2x^2(fk^2 + dm^2)}{2df(1-\mu)^2} - nr\sigma^2$$

Appendix B

PROOFS OF PROPOSITIONS

PROPOSITION 4

$$\frac{\partial b}{\partial \lambda} = 0$$

$$\begin{aligned}\frac{\partial a}{\partial \lambda} &= -\left(\frac{(-1+c)\ k^2\, n^2\, x^3}{k^2\, n\, x^2\ (-1+\mu)+2\, d\, r\, (-1+\mu)^3\, \sigma^2}\right)\\ &\leq 0\end{aligned}$$

$$\frac{\partial e}{\partial \lambda} = 0$$

PROPOSITION 5

$$\begin{aligned}\frac{\partial b}{\partial \mu} &= \frac{4\ (-1+c)\ d\, k^2\, n\, r\, x^2\ (-1+\mu)\ \sigma^2}{\left(k^2\, n\, x^2+2\, d\, r\, (-1+\mu)^2\, \sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial a}{\partial \mu} &= \frac{(-1+c)\ k^2\, n^2\, x^3}{4\, d\left(k^2\, n\, x^2\ (-1+\mu)+2\, d\, r\, (-1+\mu)^3\, \sigma^2\right)^3}\\ &\quad \times 2\ (-1+c)\ k^6\, n^3\, x^5+48\, d^3\, r^2\, \lambda\, (-1+\mu)^5\, \sigma^4\\ &\quad +4\, d\, k^4\, n^2\, x^3\ (-1+\mu)\ \left(x\, \lambda+3\ (-1+c)\ r\ (-1+\mu)\ \sigma^2\right)\\ &\quad +16\, d^2\, k^2\, n\, r\, x\, (-1+\mu)^3\, \sigma^2\ \left(2\, x\, \lambda+r\ (-1+c+\mu-c\, \mu)\ \sigma^2\right)\\ &\leq 0\end{aligned}$$

if

$$2x\lambda > r(-1+c+\mu-c\mu)\sigma^2$$

$$\begin{aligned}\frac{\partial e}{\partial \mu} &= \frac{(-1+c)\;k^3\,n^2\,x^3\;\left(-\left(k^2\,n\,x^2\right)-6\,d\,r\,(-1+\mu)^2\,\sigma^2\right)}{2\,d\left(k^2\,n\,x^2\;(-1+\mu)+2\,d\,r\,(-1+\mu)^3\,\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

PROPOSITION 6

$$\begin{aligned}\frac{\partial b}{\partial n} &= \frac{-2\;(-1+c)\;d\,k^2\,r\,x^2\,(-1+\mu)^2\,\sigma^2}{\left(k^2\,n\,x^2+2\,d\,r\,(-1+\mu)^2\,\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial a}{\partial n} &= \frac{(-1+c)\;k^2\,n\,x^3}{2\,d\,(-1+\mu)^2\left(k^2\,n\,x^2+2\,d\,r\,(-1+\mu)^2\,\sigma^2\right)^3}\\ &\quad\times-\left(k^4\,n^2\,x^4\;\left((-1+c)\;k^2\,n\,x+2\,d\,\lambda\;(-1+\mu)\right)\right)\\ &\quad-3\,d\,k^2\,n\,r\,x^2\;\left((-1+c)\;k^2\,n\,x+4\,d\,\lambda\;(-1+\mu)\right)\;(-1+\mu)^2\,\sigma^2\\ &\quad-2\,d^2\,r^2\;\left(-3\;(-1+c)\;k^2\,n\,x+8\,d\,\lambda\;(-1+\mu)\right)\;(-1+\mu)^4\,\sigma^4\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial n} &= \frac{(-1+c)\;k^3\,n\,x^3\;\left(k^2\,n\,x^2+4\,d\,r\,(-1+\mu)^2\,\sigma^2\right)}{2\,d\;(-1+\mu)\;\left(k^2\,n\,x^2+2\,d\,r\,(-1+\mu)^2\,\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

PROPOSITION 7

$$\begin{aligned}\frac{\partial b}{\partial x} &= \frac{-4\;(-1+c)\;d\,k^2\,n\,r\,x\,(-1+\mu)^2\,\sigma^2}{\left(k^2\,n\,x^2+2\,d\,r\,(-1+\mu)^2\,\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial a}{\partial x} &= \frac{(-1+c)\;k^2\,n^2\,x^2}{2\,d\,(-1+\mu)^2\left(k^2\,n\,x^2+2\,d\,r\,(-1+\mu)^2\,\sigma^2\right)^3}\\ &\quad\times(1-c)\;k^6\,n^3\,x^5-24\,d^3\,r^2\,\lambda\,(-1+\mu)^5\,\sigma^4\\ &\quad+8\,d^2\,k^2\,n\,r\,x\,(-1+\mu)^3\,\sigma^2\;\left(-2\,x\,\lambda+(-1+c)\;r\;(-1+\mu)\;\sigma^2\right)\\ &\quad-2\,d\,k^4\,n^2\,x^3\;(-1+\mu)\;\left(x\,\lambda+3\;(-1+c)\;r\;(-1+\mu)\;\sigma^2\right)\\ &\leq 0\end{aligned}$$

$$\frac{\partial e}{\partial x} = \frac{(-1+c)\; k^3\, n^2\, x^2\; \left(k^2\, n\, x^2 + 6\, d\, r\, (-1+\mu)^2\, \sigma^2\right)}{2\, d\; (-1+\mu)\; \left(k^2\, n\, x^2 + 2\, d\, r\, (-1+\mu)^2\, \sigma^2\right)^2}$$
$$\geq \; 0$$

PROPOSITION 8

The proofs are straightforward and are done by taking derivatives of optimal contract parameters and optimal profit with respect to exogenous parameters and checking the sign of the derivative.

$$\frac{\partial b}{\partial c} = -\left(\frac{k^2\, n^2\, x^2}{k^2\, n^2\, x^2 + 2\, d\, r\, (-1+\mu)^2\, \sigma^2}\right)$$
$$\leq \; 0$$

$$\frac{\partial b}{\partial r} = \frac{2\; (-1+c)\; d\, k^2\, n^2\, x^2\, (-1+\mu)^2\, \sigma^2}{\left(k^2\, n^2\, x^2 + 2\, d\, r\, (-1+\mu)^2\, \sigma^2\right)^2}$$
$$\leq \; 0$$

$$\frac{\partial b}{\partial \sigma} = \frac{4\; (-1+c)\; d\, k^2\, n^2\, r\, x^2\, (-1+\mu)^2\, \sigma}{\left(k^2\, n^2\, x^2 + 2\, d\, r\, (-1+\mu)^2\, \sigma^2\right)^2}$$
$$\leq \; 0$$

$$\frac{\partial b}{\partial d} = \frac{2\; (-1+c)\; k^2\, n^2\, r\, x^2\, (-1+\mu)^2\, \sigma^2}{\left(k^2\, n^2\, x^2 + 2\, d\, r\, (-1+\mu)^2\, \sigma^2\right)^2}$$
$$\leq \; 0$$

$$\frac{\partial b}{\partial k} = \frac{-4\; (-1+c)\; d\, k\, n^2\, r\, x^2\, (-1+\mu)^2\, \sigma^2}{\left(k^2\, n^2\, x^2 + 2\, d\, r\, (-1+\mu)^2\, \sigma^2\right)^2}$$
$$\geq \; 0$$

$$\frac{\partial b}{\partial \bar{u}} = 0$$

$$\begin{aligned}\frac{\partial e}{\partial c} &= \frac{k^3\,n^3\,x^3}{2\,d\,\left(-1+\mu\right)\,\left(k^2\,n^2\,x^2+2\,d\,r\,\left(-1+\mu\right)^2\sigma^2\right)}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial r} &= -\left(\frac{\left(-1+c\right)\,k^3\,n^3\,x^3\,\left(-1+\mu\right)\,\sigma^2}{\left(k^2\,n^2\,x^2+2\,d\,r\,\left(-1+\mu\right)^2\sigma^2\right)^2}\right)\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial \sigma} &= \frac{-2\,\left(-1+c\right)\,k^3\,n^3\,r\,x^3\,\left(-1+\mu\right)\,\sigma}{\left(k^2\,n^2\,x^2+2\,d\,r\,\left(-1+\mu\right)^2\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial d} &= \frac{-\left(\left(-1+c\right)\,k^3\,n^3\,x^3\,\left(k^2\,n^2\,x^2+4\,d\,r\,\left(-1+\mu\right)^2\sigma^2\right)\right)}{2\,d^2\,\left(-1+\mu\right)\,\left(k^2\,n^2\,x^2+2\,d\,r\,\left(-1+\mu\right)^2\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial k} &= \frac{\left(-1+c\right)\,k^2\,n^3\,x^3\,\left(k^2\,n^2\,x^2+6\,d\,r\,\left(-1+\mu\right)^2\sigma^2\right)}{2\,d\,\left(-1+\mu\right)\,\left(k^2\,n^2\,x^2+2\,d\,r\,\left(-1+\mu\right)^2\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

$$\frac{\partial e}{\partial \bar{u}} = 0$$

$$\begin{aligned}\frac{\partial \pi}{\partial c} &= \frac{k^2\,n^3\,x^3\,\left(\left(-1+c\right)\,k^2\,n\,x+2\,d\,\lambda\,\left(-1+\mu\right)\right)+4\,d^2\,n\,r\,x\,\lambda\,\left(-1+\mu\right)^3\sigma^2}{2\,d\,\left(-1+\mu\right)^2\,\left(k^2\,n^2\,x^2+2\,d\,r\,\left(-1+\mu\right)^2\sigma^2\right)}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial r} &= \frac{-\left((-1+c)^2\,k^4\,n^4\,x^4\,\sigma^2\;\right)}{2\left(k^2\,n^2\,x^2+2\,d\,r\,(-1+\mu)^2\,\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial \sigma} &= -\left(\frac{(-1+c)^2\,k^4\,n^4\,r\,x^4\,\sigma}{\left(k^2\,n^2\,x^2+2\,d\,r\,(-1+\mu)^2\,\sigma^2\right)^2}\right)\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial d} &= \frac{-\left((-1+c)^2\,k^4\,n^4\,x^4\,\left(k^2\,n^2\,x^2+4\,d\,r\,(-1+\mu)^2\,\sigma^2\right)\right)}{4\,d^2\,(-1+\mu)^2\left(k^2\,n^2\,x^2+2\,d\,r\,(-1+\mu)^2\,\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial k} &= \frac{(-1+c)^2\,k^3\,n^4\,x^4\,\left(k^2\,n^2\,x^2+4\,d\,r\,(-1+\mu)^2\,\sigma^2\right)}{2\,d\left(k^2\,n^2\,x^2\,(-1+\mu)+2\,d\,r\,(-1+\mu)^3\,\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

$$\frac{\partial \pi}{\partial \bar{u}} = -1$$

PROPOSITION 9

The proofs are straightforward and are done by taking derivatives of steady state probability of being in customer state with respect to exogenous parameters and checking the sign of the derivative.

$$\begin{aligned}\frac{\partial p(C)}{\partial \lambda} &= \frac{1}{1-\mu}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial p(C)}{\partial \mu} &= \frac{\lambda + \frac{(-1+c)\, k^4\, n^3\, x^3\, \left(k^2\, n^2\, x^2 + 4\, d\, r\, (-1+\mu)^2\, \sigma^2\right)}{d\, (-1+\mu)\, \left(k^2\, n^2\, x^2 + 2\, d\, r\, (-1+\mu)^2\, \sigma^2\right)^2}}{(-1+\mu)^2} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial p(C)}{\partial n} &= \frac{(-1+c)\; k^4\, n^2\, x^3\; \left(-\left(k^2\, n^2\, x^2\right) - 6\, d\, r\, (-1+\mu)^2\, \sigma^2\right)}{2\, d\, \left(k^2\, n^2\, x^2\; (-1+\mu) + 2\, d\, r\, (-1+\mu)^3\, \sigma^2\right)^2} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial p(C)}{\partial x} &= \frac{(-1+c)\; k^4\, n^3\, x^2\; \left(-\left(k^2\, n^2\, x^2\right) - 6\, d\, r\, (-1+\mu)^2\, \sigma^2\right)}{2\, d\, \left(k^2\, n^2\, x^2\; (-1+\mu) + 2\, d\, r\, (-1+\mu)^3\, \sigma^2\right)^2} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial p(C)}{\partial k} &= \frac{(-1+c)\; k^3\, n^3\, x^3\; \left(-\left(k^2\, n^2\, x^2\right) - 4\, d\, r\, (-1+\mu)^2\, \sigma^2\right)}{d\, \left(k^2\, n^2\, x^2\; (-1+\mu) + 2\, d\, r\, (-1+\mu)^3\, \sigma^2\right)^2} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial p(C)}{\partial c} &= \frac{-\left(k^4\, n^3\, x^3\right)}{2\, d\, (-1+\mu)^2\; \left(k^2\, n^2\, x^2 + 2\, d\, r\, (-1+\mu)^2\, \sigma^2\right)} \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial p(C)}{\partial r} &= \frac{(-1+c)\; k^4\, n^3\, x^3\, \sigma^2}{\left(k^2\, n^2\, x^2 + 2\, d\, r\, (-1+\mu)^2\, \sigma^2\right)^2} \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial p(C)}{\partial \sigma} &= \frac{2\; (-1+c)\; k^4\, n^3\, r\, x^3\, \sigma}{\left(k^2\, n^2\, x^2 + 2\, d\, r\, (-1+\mu)^2\, \sigma^2\right)^2} \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial p(C)}{\partial d} &= \frac{(-1+c)\,k^4\,n^3\,x^3\,\left(k^2\,n^2\,x^2+4\,d\,r\,(-1+\mu)^2\,\sigma^2\right)}{2\,d^2\,(-1+\mu)^2\left(k^2\,n^2\,x^2+2\,d\,r\,(-1+\mu)^2\,\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\frac{\partial p(C)}{\partial \bar{u}} = 0$$

PROPOSITION 10

a-)

$$\begin{aligned}\frac{\partial \pi}{\partial \lambda} &= \frac{(1-c)nx}{1-\mu}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial \mu} &= \frac{(1-c)nx}{2d(1-\mu)^3(k^2nx^2+2dr(-1+\mu)^2\sigma^2)^2}\\ &\quad \times k^4n^2x^4((-1+c)k^2nx^2+2d\lambda(-1+\mu))\\ &\quad +4dk^2nrx^2((-1+c)k^2nx^2+2d\lambda(-1+\mu))(-1+\mu)^2\sigma^2\\ &\quad +8d^3r^2\lambda(-1+\mu)^5\sigma^4\\ &\geq 0\end{aligned}$$

b-)

$$\begin{aligned}\frac{\partial^2 \pi}{\partial \lambda \partial \mu} &= \frac{(1-c)nx}{(1-\mu)^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 \pi}{\partial \mu \partial \lambda} &= \frac{(1-c)nx}{2d(1-\mu)^3(k^2nx^2+2dr(-1+\mu)^2\sigma^2)^2}\\ &\quad \times 2dk^4n^2x^4(-1+\mu)\\ &\quad +8d^2k^2nrx^2(-1+\mu)^3\sigma^2\\ &\quad +8d^3r^2(-1+\mu)^5\sigma^4\\ &\geq 0\end{aligned}$$

PROPOSITION 12

$$\frac{\partial b}{\partial k_0} = 0$$

$$\begin{aligned}\frac{\partial b}{\partial k_1} &= \frac{m}{m^2 + 2\,d\,r\,\sigma^2} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial b}{\partial m} &= -\left(\frac{\left(m^2 - 2\,d\,r\,\sigma^2\right)\,\left(c + k_1\right)}{\left(m^2 + 2\,d\,r\,\sigma^2\,\right)^2}\right) \\ &\geq 0\end{aligned}$$

if

$$2dr\sigma^2 - m^2 > 0$$

$$\begin{aligned}\frac{\partial b}{\partial c} &= \frac{m}{m^2 + 2\,d\,r\,\sigma^2} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial b}{\partial d} &= \frac{-2\,m\,r\,\sigma^2\,\left(c + k_1\right)}{\left(m^2 + 2\,d\,r\,\sigma^2\right)^2} \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial b}{\partial r} &= \frac{-2\,d\,m\,\sigma^2\,\left(c + k_1\right)}{\left(m^2 + 2\,d\,r\,\sigma^2\right)^2} \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial b}{\partial \sigma} &= \frac{-4\,d\,m\,r\,\sigma\,\left(c + k_1\right)}{\left(m^2 + 2\,d\,r\,\sigma^2\right)^2} \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial k_0} &= \frac{1}{2d} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial k_1} &= \frac{m^2}{2\,d\,m^2 + 4\,d^2\,r\,\sigma^2} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial m} &= \frac{2\,m\,r\,\sigma^2\;(c + k_1)}{\left(m^2 + 2\,d\,r\,\sigma^2\right)^2} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial c} &= \frac{m^2}{2\,d\,m^2 + 4\,d^2\,r\,\sigma^2} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial d} &= \frac{-\left(\left(m^2 + 2\,d\,r\,\sigma^2\right)^2 k_0 + m^2\;\left(m^2 + 4\,d\,r\,\sigma^2\right)\;(c + k_1)\right)}{2\,d^2\left(m^2 + 2\,d\,r\,\sigma^2\right)^2} \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial r} &= -\left(\frac{m^2\,\sigma^2\;(c + k_1)}{\left(m^2 + 2\,d\,r\,\sigma^2\right)^2}\right) \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial \sigma} &= \frac{-2\,m^2\,r\,\sigma\;(c + k_1)}{\left(m^2 + 2\,d\,r\,\sigma^2\right)^2} \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial k_0} &= \frac{c+k_0+k_1}{2\,d}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial k_1} &= \frac{\left(m^2+2\,d\,r\,\sigma^2\right)\,k_0+m^2\,\left(c+k_1\right)}{2\,d\,\left(m^2+2\,d\,r\,\sigma^2\right)}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial m} &= \frac{m\,r\,\sigma^2\left(c+k_1\right)^2}{\left(m^2+2\,d\,r\,\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial c} &= \frac{\left(c-2\,d\right)\,m^2-4\,d^2\,r\,\sigma^2+\left(m^2+2\,d\,r\,\sigma^2\right)\,k_0+m^2\,k_1}{2\,d\,\left(m^2+2\,d\,r\,\sigma^2\right)}\\ &\leq 0\end{aligned}$$

An interesting result is about the firm's cost c. We expect that an increase in c should decrease profits. However, an increase in c may increase profits if;

$$\begin{aligned}(c-2d)m^2-4d^2r\sigma^2+(m^2+2dr\sigma^2)k_0+m^2k_1 &> 0\\ (c+k_1)m^2+k_0(m^2+2dr\sigma^2) &> 2d(m^2+2dr\sigma^2)\\ \frac{(c+k_1)m^2}{m^2+2dr\sigma^2}+k_0 &> 2d\\ mb+k_0 &> 2d\end{aligned}$$

Which is impossible since we assume that e is between 0 and 1. Therefore, we can certainly say that firm's profit decreases as a result of an increase in c.

$$\begin{aligned}\frac{\partial \pi}{\partial d} &= \frac{-\left(\left(m^2+2\,d\,r\,\sigma^2\right)^2{k_0}^2+2\left(m^2+2\,d\,r\,\sigma^2\right)^2k_0\,\left(c+k_1\right)+m^2\,\left(m^2+4\,d\,r\,\sigma^2\right)\,\left(c+k_1\right)^2\right)}{4\,d^2\left(m^2+2\,d\,r\,\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial r} &= \frac{-\left(m^2\,\sigma^2\,(c+k_1)^2\right)}{2\,\left(m^2+2\,d\,r\,\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial \sigma} &= -\left(\frac{m^2\,r\,\sigma\,(c+k_1)^2}{\left(m^2+2\,d\,r\,\sigma^2\right)^2}\right)\\ &\leq 0\end{aligned}$$

PROPOSITION 13

$\frac{\partial b}{\partial k_0}=0$, $\frac{\partial b}{\partial k_1}=\frac{1}{m}\geq 0$, $\frac{\partial b}{\partial m}=-\frac{c+k_1}{m^2}\leq 0$, $\frac{\partial b}{\partial c}=\frac{1}{m}\geq 0$, $\frac{\partial b}{\partial d}=0$, $\frac{\partial b}{\partial r}=0$, $\frac{\partial b}{\partial \sigma}=0$, $\frac{\partial e}{\partial k_0}=\frac{1}{2d}\geq 0$, $\frac{\partial e}{\partial k_1}=\frac{1}{2d}\geq 0$, $\frac{\partial e}{\partial m}=0$, $\frac{\partial e}{\partial c}=\frac{1}{2d}\geq 0$, $\frac{\partial e}{\partial d}=-\frac{c+k_0+k_1}{2d^2}\leq 0$, $\frac{\partial e}{\partial r}=0$, $\frac{\partial e}{\partial \sigma}=0$, $\frac{\partial \pi}{\partial k_0}=\frac{c+k_0+k_1}{2d}\leq 0$, $\frac{\partial \pi}{\partial k_1}=\frac{c+k_0+k_1}{2d}\leq 0$, $\frac{\partial \pi}{\partial m}=0$, $\frac{\partial \pi}{\partial c}=\frac{c-2\,d+k_0+k_1}{2\,d}\leq 0$, $\frac{\partial \pi}{\partial d}=\frac{-(c+k_0+k_1)^2}{4\,d^2}\leq 0$, $\frac{\partial \pi}{\partial r}=0$, $\frac{\partial \pi}{\partial \sigma}=0$

PROPOSITION 14

$$\begin{aligned}\frac{\partial b}{\partial k_0} &= -\left(\frac{c\,m}{(c+d)\;m^2+2\,d^2\,r\,\sigma^2}\right)\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial b}{\partial k_1} &= \frac{d\,m}{(c+d)\;m^2+2\,d^2\,r\,\sigma^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial b}{\partial m} &= -\left(\frac{\left((c+d)\;m^2-2\,d^2\,r\,\sigma^2\right)\,\left(-(c\,k_0)+d\;(2\,c+k_1)\right)}{\left((c+d)\;m^2+2\,d^2\,r\,\sigma^2\right)^2}\right)\\ &\geq 0\end{aligned}$$

if

$$2d^2r\sigma^2-m^2(c+d) \;>\; 0$$

$$\begin{aligned}\frac{\partial b}{\partial c} &= \frac{d\,m\,\left(m^2+2\,d\,r\,\sigma^2\right)\,\left(2\,d-k_0\right)-d\,m^3\,k_1}{\left(\left(c+d\right)\,m^2+2\,d^2\,r\,\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial b}{\partial r} &= \frac{-2\,d^2\,m\,\sigma^2\,\left(-\left(c\,k_0\right)+d\,\left(2\,c+k_1\right)\right)}{\left(\left(c+d\right)\,m^2+2\,d^2\,r\,\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial b}{\partial \sigma} &= \frac{-4\,d^2\,m\,r\,\sigma\,\left(-\left(c\,k_0\right)+d\,\left(2\,c+k_1\right)\right)}{\left(\left(c+d\right)\,m^2+2\,d^2\,r\,\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial k_0} &= \frac{m^2+2\,d\,r\,\sigma^2}{2\,\left(c+d\right)\,m^2+4\,d^2\,r\,\sigma^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial k_1} &= \frac{m^2}{2\,\left(c+d\right)\,m^2+4\,d^2\,r\,\sigma^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial m} &= \frac{2\,d\,m\,r\,\sigma^2\,\left(-\left(c\,k_0\right)+d\,\left(2\,c+k_1\right)\right)}{\left(\left(c+d\right)\,m^2+2\,d^2\,r\,\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial c} &= \frac{m^2\,\left(m^2+2\,d\,r\,\sigma^2\right)\,\left(2\,d-k_0\right)-m^4\,k_1}{2\,\left(\left(c+d\right)\,m^2+2\,d^2\,r\,\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial r} &= \frac{d\,m^2\,\sigma^2\,\left(c\,k_0 - d\,\left(2\,c + k_1\right)\right)}{\left(\left(c + d\right)\,m^2 + 2\,d^2\,r\,\sigma^2\right)^2} \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial \sigma} &= \frac{-2\,d\,m^2\,r\,\sigma\,\left(-\left(c\,k_0\right) + d\,\left(2\,c + k_1\right)\right)}{\left(\left(c + d\right)\,m^2 + 2\,d^2\,r\,\sigma^2\right)^2} \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial k_0} &= \frac{\left(m^2 + 2\,\left(-c + d\right)\,r\,\sigma^2\right)\,k_0 + \left(m^2 + 2\,d\,r\,\sigma^2\right)\,\left(2\,c + k_1\right)}{2\,\left(c + d\right)\,m^2 + 4\,d^2\,r\,\sigma^2} \\ &\geq 0\end{aligned}$$

if

$$(2c + k_1)d > ck_0$$

$$\begin{aligned}\frac{\partial \pi}{\partial k_1} &= \frac{\left(m^2 + 2\,d\,r\,\sigma^2\right)\,k_0 + m^2\,\left(2\,c + k_1\right)}{2\,\left(c + d\right)\,m^2 + 4\,d^2\,r\,\sigma^2} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial m} &= \frac{m\,r\,\sigma^2\left(c\,k_0 - d\,\left(2\,c + k_1\right)\right)^2}{\left(\left(c + d\right)\,m^2 + 2\,d^2\,r\,\sigma^2\right)^2} \\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial c} &= \frac{-\left(\left(m^2 + 2\,d\,r\,\sigma^2\right)\,\left(2\,d - k_0\right) - m^2\,k_1\right)^2}{4\left(\left(c + d\right)\,m^2 + 2\,d^2\,r\,\sigma^2\right)^2} \\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial r} &= \frac{-\left(m^2\,\sigma^2\left(c\,k_0 - d\,\left(2\,c + k_1\right)\right)^2\right)}{2\left(\left(c + d\right)\,m^2 + 2\,d^2\,r\,\sigma^2\right)^2} \\ &\leq 0\end{aligned}$$

$$\frac{\partial \pi}{\partial \sigma} = -\left(\frac{m^2\, r\, \sigma\, (c\, k_0 - d\, (2\, c + k_1))^2}{\left((c+d)\, m^2 + 2\, d^2\, r\, \sigma^2\right)^2}\right)$$
$$\leq 0$$

PROPOSITION 15

$$\frac{\partial b}{\partial k_0} = 0$$

$$\frac{\partial b}{\partial k_1} = \frac{m}{m^2 + 2\, d\, (r+R)\, \sigma^2}$$
$$\geq 0$$

$$\frac{\partial b}{\partial m} = -\left(\frac{(m^2 - 2\, d\, (r+R)\, \sigma^2\,)\, (c + k_1)}{\left(m^2 + 2\, d\, (r+R)\, \sigma^2\right)^2}\right)$$
$$\geq 0$$

$$\frac{\partial b}{\partial c} = \frac{m}{m^2 + 2\, d\, (r+R)\, \sigma^2}$$
$$\geq 0$$

$$\frac{\partial b}{\partial d} = \frac{-2\, m\, (r+R)\, \sigma^2\, (c + k_1)}{\left(m^2 + 2\, d\, (r+R)\, \sigma^2\right)^2}$$
$$\leq 0$$

$$\frac{\partial b}{\partial r} = \frac{-2\, d\, m\, \sigma^2\, (c + k_1)}{\left(m^2 + 2\, d\, (r+R)\, \sigma^2\right)^2}$$
$$\leq 0$$

$$\begin{aligned}\frac{\partial b}{\partial \sigma} &= \frac{-4\,d\,m\,(r+R)\,\sigma\,(c+k_1)}{\left(m^2+2\,d\,(r+R)\,\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial b}{\partial R} &= \frac{-2\,d\,m\,\sigma^2\,(c+k_1)}{\left(m^2+2\,d\,(r+R)\,\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial k_0} &= \frac{1}{2d}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial k_1} &= \frac{m^2}{2\,d\,\left(m^2+2\,d\,(r+R)\,\sigma^2\,\right)}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial m} &= \frac{2\,m\,(r+R)\,\sigma^2\,(c+k_1)}{\left(m^2+2\,d\,(r+R)\,\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial c} &= \frac{m^2}{2\,d\,\left(m^2+2\,d\,(r+R)\,\sigma^2\,\right)}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial d} &= \frac{-\left(k_0+\frac{m^2\left(m^2+4\,d\,(r+R)\,\sigma^2\right)(c+k_1)}{\left(m^2+2\,d\,(r+R)\,\sigma^2\right)^2}\right)}{2\,d^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial r} &= -\left(\frac{m^2\,\sigma^2\;(c+k_1)}{\left(m^2+2\,d\;(r+R)\;\sigma^2\right)^2}\right)\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial \sigma} &= \frac{-2\,m^2\;(r+R)\;\sigma\;(c+k_1)}{\left(m^2+2\,d\;(r+R)\;\sigma^2\right)^2}\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial R} &= -\left(\frac{m^2\,\sigma^2\;(c+k_1)}{\left(m^2+2\,d\;(r+R)\;\sigma^2\right)^2}\right)\\ &\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial k_0} &= \frac{c+k_0+k_1}{2d}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial k_1} &= \frac{\left(m^2+2\,d\;(r+R)\;\sigma^2\right)\;k_0+m^2\;(c+k_1)}{2\,d\;\left(m^2+2\,d\;(r+R)\;\sigma^2\right)}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial m} &= \frac{m\;(r+R)\;\sigma^2\,(c+k_1)^2}{\left(m^2+2\,d\;(r+R)\;\sigma^2\right)^2}\\ &\geq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi}{\partial c} &= \frac{(c-2\,d)\;m^2-4\,d^2\;(r+R)\;\sigma^2+\left(m^2+2\,d\;(r+R)\;\sigma^2\right)\;k_0+m^2\,k_1}{2\,d\;\left(m^2+2\,d\;(r+R)\;\sigma^2\right)}\\ &\leq 0\end{aligned}$$

$$\frac{\partial \pi}{\partial d} = \frac{-\Big(\left(m^2 + 2\,d\,(r+R)\,\sigma^2\right)^2 {k_0}^2 \; + 2\left(m^2 + 2\,d\,(r+R)\,\sigma^2\right)^2 k_0\,(c+k_1) + m^2\,(m^2 + 4\,d\,(r+R)}{4\,d^2\left(m^2 + 2\,d\,(r+R)\,\sigma^2\right)^2}$$
$$\leq 0$$

$$\frac{\partial \pi}{\partial r} = \frac{-\left(m^2\,\sigma^2\,(c+k_1)^2\right)}{2\left(m^2 + 2\,d\,(r+R)\,\sigma^2\right)^2}$$
$$\leq 0$$

$$\frac{\partial \pi}{\partial \sigma} = -\left(\frac{m^2\,(r+R)\,\sigma\,(c+k_1)^2}{\left(m^2 + 2\,d\,(r+R)\,\sigma^2\right)^2}\right)$$
$$\leq 0$$

$$\frac{\partial \pi}{\partial R} = \frac{-\left(m^2\,\sigma^2\,(c+k_1)^2\right)}{2\left(m^2 + 2\,d\,(r+R)\,\sigma^2\right)^2}$$
$$\leq 0$$

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