Pricing and Lot Sizing Decisions in a Two-Echelon Supply Chain with Transportation Costs

by

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This is to certify that I have examined this copy of a master's thesis by

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To my parents and my sister

ABSTRACT

In this thesis we consider a single-buyer single-supplier supply chain. The market demand is sensitive to the selling price set by the buyer. Constant-elasticity and linear demand functions are adopted. The buyer and the supplier operate with unit product costs, inventory holding costs, order placement costs, and transportation (freight) costs. We develop models for determining the optimal lot sizing and pricing decisions in centralized and decentralized systems under various transportation cost sharing schemes. Existing models for the problem do not consider the transportation costs with price sensitive market demand, and they determine the optimal decisions through an exhaustive search. A novel approximate solution procedure, along with simple search heuristic procedures, is proposed. We report computational results on the effectiveness of the proposed procedures and the available methods from the literature. In addition to supply chain modeling, we evaluate the effectiveness of coordination mechanisms in the same setting where the buyer is responsible for the transportation cost. We also study the effectiveness of transportation cost sharing contracts, quantity discounts, and volume discounts. We formulate models to determine optimal policies and evaluate their performance through a numerical study. The results of the study demonstrate that transportation cost sharing contract is not an effective mechanism. Volume discounts are more effective than quantity discounts, whereas the most effective coordination mechanism is to offer quantity and volume discounts simultaneously.

ÖZETÇE

Bu tezde bir tedarikçi ve bir alıcıdan oluşan temel bir tedarik zinciri ele alınmaktadır. Tedarikçi ve alıcı ürün maliyetleri, envanter taşıma maliyetleri, sipariş verme maliyeti ve taşıma maliyetleri ile çalışmaktadırlar. Bu ortamda sipariş miktarlarının ve ürün fiyatlarının (ve dolayısıyla ürün talebinin) belirlenmesi ile ilgili kararlar bütünleşik bir yapı içerisinde ele alınmakta ve bu kararların zincirin karlılığı üzerindeki etkileri incelenmektedir. Pazardaki talep satış fiyatına bağlıdır ve aralarındaki ilişki doğrusal ve esnekliği sabit talep fonksiyonları ile modellenmektedir. Modele göre tedarikçi veya alıcı taşıma maliyetini üstlenmektedir. Dağınık ve merkezi sistemlerde, en iyi sipariş miktarı ve fiyatlandırma kararlarının en iyi değerlerini bulmak üzere modeller geliştirilmektedir. Literatürdeki modeller taşıma maliyetlerini çoğunlukla ihmal etmekte ve en iyi kararları bulmak için etkin arama teknikleri sunmamaktadırlar. Bu çalışmada problemi çözmek için yeni bir yaklaşık çözüm metodu ve basit bir sezgisel yöntem önerilmektedir. Algoritmaların performanslarının hesaplamalı sonuçları, literatürdeki tekniklerle karşılaştırılarak sunulmuştur. Tedarik zincirinin modellenmesinin yanı sıra, koordinasyon mekanizmalarının performansları da değerlendirilmektedir. Miktara bağlı indirim, satış hacmine bağlı indirim ve taşıma maliyetleri paylaşma kontratı modellenmekte ve taşıma maliyetleri paylaşma kontratının fazla etkili olmadığı gösterilmektedir. Bunun yanında satış hacmine bağlı indirimin miktara bağlı indirimden daha etkili olduğu ve en etkili yöntemin miktara ve satış hacmine bağlı indirimlerin birlikte kullanılması olduğu sayısal bir çalışma ile gösterilmektedir.

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NOMENCLATURE

Q	regular shipment size (units),
p	selling price that the buyer sets for product (\$/unit).
v	wholesale price charged by the supplier (\$/unit),
n	lot size multiplier for the supplier,
v_{QD}	discounted wholesale price in quantity discount (\$/unit),
v_{VD}	discounted wholesale price in volume discount (\$/unit),
v_{QVD}	discounted wholesale price in quantity and volume discount (\$/unit),
p_{VD}	maximum market price in quantity discount offer (\$/unit),
p_{VQD}	maximum market price in quantity and volume discount offer (\$/unit),
Q_{QD}	minimum order quantity in quantity discount offer (unit),
Q_{QVD}	minimum order quantity in quantity and volume discount offer (unit),
α	the ratio of the transportation cost that the buyer pays in transportation cost sharing contract,
$\prod_S(v^*, n^*)$	profit of the supplier at the initial market equilibrium (\$),
$\prod_B(p^*, Q^*)$	minimum order quantity in quantity discount offer (\$),
$D(p)$	annual demand for the product (units/year), where $D(p) = ap^{-b}$,
a	demand function constant,
b	elasticity parameter of the demand function, where $b > 1$,
A	buyer's order placement cost (\$/order),
K	supplier's order placement cost (\$/order),
I	annual holding cost fraction (\$/\$/year),
h_s	holding cost for the supplier (\$/year),
h_b	holding cost for the buyer (\$/year),
m	unit manufacturing or procurement cost for the supplier (\$/unit),
$F(Q)$	freight charge for a shipment of size Q (\$),

C	maximum load that can be placed in a truck (units),
R	truckload charge ($),
T	maximum number of trucks available for a single shipment (units),
v_{max}	an upper bound on the wholesale price.

Chapter 1

INTRODUCTION

Supply Chain Management, according to Chopra and Meindl [1], involves the management of flows between and among stages in a supply chain to maximize total profitability. Since this broad term includes numerous subtopics, supply chain activities can be grouped into strategic, tactical, and operational levels of activities. One of the major purposes of supply chain management is to increase the level of coordination and collaboration among supply chain partners, thus increasing their overall profitability.

Supply chain management places a significant emphasis on lot sizing and pricing decisions, and lowering logistics costs has been of interest for enterprises. Lot sizing is one of the major considerations of inventory management, whereas pricing is crucial for revenue management. Hence, a global approach considering lot sizing and pricing decisions gained importance in reducing inventory costs and increasing revenue.

Transportation is another important consideration of logistics. Surveys of 1992 indicate that the average ratio of transportation costs to total sales dollar is 10.5% in the United States [2]. For certain industries transportation costs are relatively high. For instance, transportation costs account for 16.64% of the sales dollar of the products on the average in the food industry, whereas this ratio is 13.80% for chemicals, petroleum and rubber products, 11.10% for wood products [2]. For such items, transportation costs play a significant role in the market price.

Although lot sizing and pricing problems have been intensively studied in the literature, the effect of transportation costs has been generally neglected. Since transportation costs can be almost 50% of the total logistics cost [3], incorporation of transportation costs into lot sizing and pricing problems can have a significant value.

A basic two-stage supply chain consists of a manufacturer and a retailer, where the manufacturer sells the goods to the retailer and the retailer sells the goods in a market. The manufacturer represents the upstream and the retailer represents the downstream participant in the supply chain. Considering the pricing and lot sizing decisions, the manufacturer determines the wholesale price of the product. Consequently, the retailer determines the market price and the order frequency. Accordingly, the manufacturer determines his order frequency. Both enterprises maximize their overall profit independently and rationally. This process can be defined as a sequential game and can be modeled as a Stackelberg game. The Stackelberg leadership model is a strategic game in economics literature in which the leader firm moves first and then the follower firm moves sequentially. This system is called a decentralized supply chain where the enterprises independently determine their moves. An alternative model is the centralized supply chain where the manufacturer and the retailer are owned by the same enterprise and the decisions are made jointly.

Overall profit in a centralized supply chain is larger than the profit generated in a decentralized supply chain due to the double marginalization effect. This concept is introduced by Lerner [4] and can be defined as the exercise of market power at successive stages of a supply chain. Hence, in order to improve the independent profits, coordination mechanisms are exercised. Coordination in supply chain requires the partners of the supply chain to work together to improve overall profitability rather than concentrating on their own profitability. Channel coordination can be defined as the situation where the decentralized system offers equivalent outputs to the market, i.e. market price, with the centralized system. Coordination mechanisms are extensively applied in business and supply chain coordination is one of the most-studied topics of recent research in supply chain management.

Our main objective in this thesis is to analyze the impact of transportation costs on pricing and ordering decisions and modeling coordination mechanisms. We model a supply chain consisting of a single supplier supplying a product to a single buyer. Demand for the product is deterministic, and price-elastic. We consider linear and constant elasticity demand functions. The buyer buys the product from the supplier, sets the market price, and determines her order replenishment period. Meanwhile, the supplier sets the wholesale price, and determines his production quantity and frequency. They both incur procurement costs, inventory holding costs and order placement costs, whereas the partner who takes on the transportation cost additionally incurs freight cost, which depends on the lot shipment size. The transportation cost consists of freight cost and the term transportation cost, designates the freight cost in the thesis. We further assume that production rates of both the supplier and the buyer are infinitely large, which implies that production and storage capacities are unlimited.

Both parties act independently as rational agents maximizing their own profits. Given a wholesale price, the buyer determines the optimal retail price, and the order frequency and quantity. Full information is available for the supplier, and he knows the consequences of his wholesale price in the form of retail price and buyer's order frequency and quantity. Hence, the problem can be modeled as a Stackelberg game with the supplier acting as the leader and the buyer acting as the follower. The equilibrium point is determined by the solution of the Stackelberg game. Our first objective in this thesis is to develop algorithms for approximating optimal pricing and lot sizing decisions in this equilibrium point. We aim to develop an approximate procedure because the profit function of the supplier is non-concave, and the freight cost function adds a discontinuity to it. In the core of this approximate solution procedure is a novel approach for modeling the response of the buyer to different values of the wholesale price. This in turn helps us model the supplier's problem.

Our second objective in this thesis is to model the coordination mechanisms for the two stage supply chain. We model quantity and volume discounts, which are well-known coordination mechanisms in the literature. Additionally we propose and model a transportation cost sharing contract for coordination. We further provide a detailed analysis of the results and efficiencies of the mechanisms and compare them.

The remainder of the thesis is organized as follows: In Chapter 2, a review of the literature on pricing, lot sizing decisions and supply chain coordination is presented. In Chapter 3, problem description and solution procedures are presented. In Chapter 4, we describe our supply chain models and propose methods for solving optimal strategies. In Chapter 5, the coordination mechanisms are studied. Finally, we present some concluding remarks in Chapter 6.

Chapter 2

LITERATURE REVIEW

We categorize supply chain systems in the literature into three major groups:

- Lot Sizing Decisions in Supply Chain Management
- Lot Sizing and Pricing Decisions in Supply Chain Management
- Supply Chain Models Incorporating Transportation Costs

Subsequent to reviewing supply chain systems, supply chain coordination will be discussed. Finally, quantity discounts will be reviewed.

2.1 Lot Sizing Decisions in Supply Chain Management

Lot sizing is choosing a lot size for procurement or production where the main objective is to minimize the sum of ordering and holding costs. Lot sizing problem has been an important aspect of procurement operations of enterprises and, there is substantial research on the optimal lot sizing problem with deterministic demand, which are summarized in Table 2.1.

One of the essential papers in inventory theory is by Whitin [5], where the economic order quantity (EOQ) is introduced. In this paper he considers a retailer buying goods from a supplier and sells them in the market. He studies optimization of the total cost through balancing setup and inventory holding cost. Porteus [6] investigates the effects of investments in reducing setup costs in the EOQ model. Investment is modeled through a convex and strictly decreasing function. Additionally, sensitivity analysis is provided with respect to the parameters.

Roundy [11] introduces power-of-two and integer-ratio policies into the lot sizing problem for multi-echelon inventory systems. Power-of-two policy requires the buyer to select an order frequency in the form of $T_b = (2^{n_b})T_s$, where T_s is the order frequency of the supplier and n_b is an integer selected by the buyer. On the other hand, integer-ratio policy requires

Table 2.1: Papers on lot sizing decisions in supply chain.

Authors	Paper	Supply Chain	Pricing	Transportation	Product
Whitin	[5]	Retailer	-	-	Single
Porteus	[6]	Retailer	-	-	Single
Lu	[7]	Retailer	-	-	Single
Yao and Chiou	[8]	Retailer	-	-	Single
Khouja	[9]	M-M-M	-	-	Single
Chan and Kingsman	[10]	1-M	-	-	Single

the buyer to select an order frequency in the form of $T_b = (\frac{1}{n_b})T_s$ or n_bT_s, where n_b is a positive integer selected by the buyer. These policies have been extensively studied in inventory theory.

Lu [7] considers the lot sizing problem in a supplier-buyer relationship adopting integer ratio policy. Optimal solution for the single-supplier single-buyer case is derived, whereas a heuristic approach is proposed for the single-supplier multi-buyer case. Yao and Chiou [8] consider the study of Lu [7] and propose a heuristic providing shorter run time.

Khouja [9] formulates a three-stage, multi-customer, non-serial supply chain model, where a firm can supply more than one customer. Three mechanisms are considered for replenishment: Equal cycle time, integer multiplier, and integer powers-of-two multiplier mechanisms. The analysis shows that the integer powers-of-two-multiplier mechanism has the lowest cost, whereas the equal cycle time mechanism has the highest cost.

Chan and Kingsman [10] study a single-vendor multi-buyer supply chain model considering three replenishment policies: synchronized replenishment cycles, common order cycle model, and independent policy. Through examples the paper further claims that the synchronized cycles policy works better than independent optimization as well as restricting buyers to adopt a common order cycle.

Table 2.2: Papers on lot sizing and pricing decisions in supply chains.

Authors	Paper	Supply Chain	Pricing	Transportation	Product
Kunreuther and Richard	[12]	Retailer	+	-	Single
Arcelus and Srinivasan	[13]	Retailer	+	-	Single
Rosenberg	[14]	Retailer	+	-	Single
Cheng	[15]	Retailer	+	-	Multiple
Chen and Min	[16]	Retailer	+	-	Multiple
Kim and Lee	[17]	Retailer	+	-	Single

2.2 Lot Sizing and Pricing Decisions in Supply Chain Management

Lot sizing and pricing problems have been extensively studied for the last three decades. In this section, profit-maximizing models are reviewed and the papers are displayed in Table 2.2.

Kunreuther and Richard [12] extend Whitin's [5] study by incorporating pricing decision and introducing decentralized and centralized decision making within a retailer. In the paper, the marketing department decides on the price and purchasing department decides on the lot size. The paper investigates the influence of simultaneously and sequentially making the decisions. Besides, they show that the optimal lot size under finite production rate is larger than the optimal lot size with an infinitely large production rate.

Arcelus and Srinivasan [13] derive the optimal pricing and lot sizing decisions for a retailer facing a constant price-elasticity demand function under three different objectives: Total profit, return on inventory investment and residual income, which is defined as the difference between profit and the opportunity cost of the inventory investment. Rosenberg [14] derives optimal solutions for the pricing and lot sizing problem of a retailer with profit and return on inventory investment objectives.

Cheng [15] considers the optimal pricing and lot sizing decisions for multiple products of a retailer facing a linear demand function. He incorporates storage capacity and inventory investment constraints into the problem. The solution requires all of the products to be ordered with the same frequency. Chen and Min [16] reformulate Cheng's [15] model to satisfy equal replenishment cycle for each product in the optimal solution. They further derive the optimal solution for linear demand function.

Kim and Lee [17] incorporate capacity investment decisions into the pricing and lot sizing problem of a retailer. Geometric programming, which is a mathematical optimization technique characterized by an objective function and constraint functions that have a special form, is employed to derive optimal decisions for fixed and variable capacity models and analysis of these models is provided.

2.3 Supply Chain Models Incorporating Transportation Costs

Above mentioned studies do not explicitly consider transportation costs and their impact on supply chain. Carter and Ferrin [35] emphasize the explicit consideration of transportation costs in lot sizing. For the joint pricing and lot sizing problem, limited literature exists with explicit consideration of transportation costs, and the relevant papers are displayed in Table 2.3. In fact, transportation cost has been modeled in various structures in the literature. Mathematical formulations of the relevant structures studied in the pricing and lot sizing problems are discussed in Section 3.1. Moreover, Section 3.1 reviews the literature studying pricing and lot sizing problem including transportation cost.

Wehrman [18] considers the industries where transportation constitutes a significant part of the overall material costs and studies lot sizing problem for a retailer considering transportation costs. Transportation cost is determined by weight category, which is based on unit weight and order quantity.

Early studies try to incorporate transportation cost into the EOQ problem. Aucamp [19] incorporates freight cost, which is modeled by integer number of full truck loads, into the EOQ problem. Lee [20] introduces discounted freight cost into the EOQ problem. Freight cost is fixed at a certain interval and subject to economies of scale. Larson [21] introduces economic transportation quantity model. The setting includes a retailer facing deterministic demand incurs setup, holding and freight cost. Freight cost depends on the units transported and is subject to economies of scale. Transportation modes considered are air, full truck and less than truck load. Tersine et al. [22] consider a freight cost that is constant for a level in a particular interval and sustains economies of scale. Besides,

Table 2.3: Papers on supply chain models incorporating transportation costs.

Authors	Paper	Supply Chain	Pricing	Transportation	Product
Wehrman	[18]	Retailer	+	+	Single
Aucamp	[19]	Retailer	+	+	Single
Lee	[20]	Retailer	+	+	Single
Larson	[21]	Retailer	+	+	Single
Tersine et al.	[22]	Retailer	+	+	Single
Hwang et al.	[23]	Retailer	+	+	Single
Tersine and Barman	[24, 25]	Retailer	+	+	Single
Russell and Krajewski	[26]	Retailer	+	+	Single
Carter et al.	[27]	Retailer	+	+	Single
Shinn et al.	[28]	Retailer	+	+	Single
Burwell et al.	[29]	Retailer	+	+	Single
Swenseth and Godfrey	[3]	Retailer	+	+	Single
Abad and Aggarwal	[30]	Retailer	+	+	Single
Abad and Aggarwal	[31]	Retailer	+	+	Single
Hoque and Goyal	[32]	1-1	+	+	Single
Ertogral et al.	[33]	1-1	+	+	Single
Lei et al.	[34]	1-1-1	+	+	Single

all-weight and incremental freight discounts are considered within the framework of EOQ model.

Hwang et al. [23] introduces unit quantity discounts into the EOQ problem considering freight costs. Freight cost is again fixed at a certain interval and subject to economies of scale. Tersine and Barman [24, 25] study the lot sizing problem for a retailer facing all-units/incremental quantity discounts and all-weight/incremental freight discounts. Russell and Krajewski [26] study the same problem considering the effect of the transportation rate structure for less-than-truckload (LTL) shipments. They enable the buyer to have the over-declaring option, which refers to declaring a shipment amount that is larger than the real shipment size in order to take advantage of lower freight costs. Carter et al. [27] correct the algorithm of Russell and Krajewski [26], and avoid abnormalities due to the real-life freight schedules.

Transportation cost has been considered within pricing and lot sizing problems for the last decade. Shinn et al. [28] study optimal pricing and lot sizing decisions for a retailer under conditions of permissible delay in payments. The paper assumes a discounted freight cost scheme exerting economies of scale. Burwell et al. [29] consider a retailer that faces price dependent demand, and determine optimal shipment lot size and price. The retailer also receives quantity discount from supplier and freight discount from transporter. Freight cost structure does not include over-declaring option. Algorithms are proposed for different combinations of quantity and freight discounts. Swenseth and Godfrey [3] present a discussion of freight costs in literature and practice. Two freight rate functions, the inverse and the adjusted inverse, are incorporated into the inventory replenishment problem. The setting includes deterministic demand with setup, holding cost and freight costs. A heuristic algorithm is proposed and its performance is measured against the optimal decision. Abad and Aggarwal [30] consider a retailer that faces price sensitive demand with setup, holding cost and transportation cost. The transportation cost structure of Swenseth and Godfrey [3] is adopted, where the transportation cost structure is subject to economies of scale, and includes over-declaring option. The shipment size is limited to a truck load, and multiple truck load shipments are not allowed. The paper claims that the regular order quantity will be less than or equal to one full truckload, since shipments spanning different trucks can be timed so that they are received at different times to reduce inventory carrying costs. Moreover, optimal decisions are derived and an algorithm is proposed. The proposed algorithm requires the solution of a set of equations through optimality conditions. The equations are nonlinear and a methodology for solving the equations is not offered. The example mentioned in the paper uses GOAL SEEK function of Excel to solve the set of equations. Finally sensitivity analysis is generated in the paper. Abad and Aggarwal [31] correct their formulation by adding per unit transportation cost into the carrying cost.

Hoque and Goyal [32] consider a centralized system in a single-vendor single-buyer setting with deterministic demand and transportation cost. The vendor has a production capacity, determines his price and adopts lot-for-lot methodology for shipping. The transportation vehicle has a capacity. A model is derived to find the optimal centralized profit. Ertogral et al. [33] consider a centralized system in a single-vendor single-buyer setting. The buyer faces a price sensitive demand, determines his price and shipment size. The supplier has a production capacity, determines his price and lot size multiplier. Algorithms are devised for incremental freight discounts with and without over-declaring option.

Lei et al. [34] consider a supplier-transporter-buyer model with single participant at each supply chain stage. The transporter charges the supplier the transportation cost, which depends on shipment quantity. The supplier charges a unit price to the buyer and the buyer charges a unit price in the market. Demand is price-elastic and deterministic in the market. The supplier and the buyer operate under inventory holding costs, setup costs and unit costs whereas the transporter operates under fixed costs, and unit costs. The buyer determines his lot size, the supplier and transporter work with this lot size. The optimal prices and lot sizes are derived for centralized and decentralized case. Finally a numerical study is conducted and sensitivity analysis is provided.

2.4 Supply Chain Coordination

Supply chain coordination has been one of the major research topics in operations management. Major coordination mechanisms in the deterministic setting include profit sharing mechanisms, two part tariffs and discounts. Profit sharing mechanisms are designed to share the benefit of coordination between the supply chain members, whereas a two part tariff is a price discrimination technique in which the price is composed of a per-unit charge and an additional fixed fee. Quantity discount is the offer of price discount in return for increased order quantity. On the other hand, volume discount is the offer of price discount in return for increased annual demand. Moreover, simultaneous offering of quantity and volume discounts is also applicable.

Coordination mechanisms have been of interest in two-stage supply chains with price-elastic demand functions. Yano and Gilbert [36] provide a survey of research on supply chain coordination.

Zahir and Sarker [37] consider single-supplier multi-buyer model and derive optimal lot sizing and pricing decisions for the buyers and the supplier. Furthermore, the paper proposes and models a coordination mechanism through compensating profits. Then the supplier has the power to enforce the buyers to accept a higher order quantity to consequently increase his profit.

Abad [38] studies a single-supplier multi-buyer supply chain and derive the Pareto efficient and Nash bargaining solutions. He proposes a profit-sharing mechanism, two-part tariff and all-units quantity discount pricing schedules. The paper concludes that coordination can be achieved by the profit-sharing mechanism or the all-units quantity discount schedule. Ingene and Parry [39] consider a single-supplier and two competing buyers with a linear, downward-sloping demand curve. The costs of the enterprises entirely consist of fixed costs and variable costs. Three pricing mechanisms; a channel-coordinating quantity-discount schedule, a sophisticated Stackelberg two-part tariff, and a channel coordinating menu of two part tariffs are evaluated in various scenarios with varying degree of competition and relative size of the retailers. Coordination is achieved through quantity-discount schedule whereas channel coordinating wholesale-price strategy provides zero incremental profit for the buyers. The paper further analyzes the use of imperfect information and effects of the parameters.

Desai [40] considers coordination mechanisms in a single-supplier single-buyer supply chain with seasonal demand. He derives optimal solutions for three cases: the general case where they continuously change price and lot size, the case in which the supplier has a constant price, and the case where the buyer has a constant procurement rate. Reyniers [41] compares decentralized and centralized systems in the single-supplier single-buyer supply chain. She claims that market price can be larger or smaller in the centralized system depending on the market size. Ertek and Griffin [42] consider a single-supplier single-buyer supply chain and characterize the equilibrium solution to supplier-driven and buyer-driven versions of the game.

An important area of research in supply chain coordination is the supply chain contracts, which are studied in a stochastic setting. For recent surveys, see Cachon [43], Tsay et al. [44] and Lariviere [45], who concentrate on the issues of wholesale or transfer prices, buy-back provisions, franchise fees, and other financial agreements among the supply chain members. Cachon and Netessine [46] provide a comprehensive survey of game theory in supply chain analysis and outline game-theoretic concepts. Since we consider deterministic demand, this line of research is beyond the scope of the thesis.

2.4.1 Optimal Decision Making under Discount Offers

This section reviews the literature of optimal policies under discount offers. The models consider a retailer facing discounts and setting the market price and order quantity. The mathematical formulation of the discount offers differs across the papers.

Crowther [47] was the first to study the lot sizing problem for a retailer facing quantity discounts. Ladany and Sternlieb [48] study optimal product pricing and lot sizing problem for a retailer facing continuous-linear and continuous-hyperbolic quantity discounts. The paper concludes that the elasticity of the demand may change the optimal pricing policy at a break even point. Brahmbhatt and Jaiswal [49] extend this study by incorporating finite production rate.

Subramanyam and Kumaraswamy [50] study the optimal product pricing and lot sizing problem for a retailer facing continuous quantity discounts incorporating advertisement expenditures. Lee and Rosenblatt [51] further extend this model with modeling defective items in the procurement. Arcelus and Srinivasan [52] study the optimal product pricing and lot sizing problem for a retailer facing continuous quantity discounts who has a return on inventory investment objective.

Abad [53] and Abad [54] study the optimal product pricing and lot sizing problem for a retailer where there exist all-unit quantity discounts in the former and incremental quantity discounts in the latter. Burwell et al. [55] argue that the procedure of Abad [53] does not provide optimal solution for linear and constant elasticity demand functions.

Lee [56] derives optimal product pricing and lot sizing decisions for a retailer facing continuous quantity discounts through geometric programming approach.

Lee [57] studies optimal multi-product pricing and lot sizing problem for a retailer facing order quantity discounts. The model utilizes a power-form quantity discount function and incorporates storage capacity and inventory investment restrictions.

Abad [58] studies product pricing and lot sizing decisions for a retailer facing temporary discounts, which encourage forward-buying.

Munson and Rosenblatt [59] review the literature on quantity and volume discounts and report the industry practices of quantity discounts from 39 companies. It is a comprehensive study on discounts including motivations, assumptions, and industry trends. It is noted that there is a literature gap for the discounts in vendor-buyer setting with transportation cost. Sarmah et al. [60] provide a literature survey on quantity discounts in deterministic environment.

2.4.2 Optimal Discount Design in Supply Chain Management

This section reviews the literature for designing optimal policies of discount offers. The models consider sequential pricing and lot sizing decisions in two-echelon or three-echelon supply chain systems. Dolan [61] provides a literature review on quantity discounts with a managerial perspective, and illustrates the motivations of quantity discounts.

There are studies on quantity discounts in supply chains with constant demand. Lal and Staelin [62] study optimal quantity discount design in a two-stage supply chain with constant demand. They derive a closed form optimal solution for the single-supplier single-buyer case, and propose an algorithm for the single-supplier multi-buyer case. Drezner and Wesolowsky [63] derive the optimal all-units quantity discount scheme to be offered in a two-stage supply chain with constant demand. Kohli and Park [64] study the lot sizing problem in a single-supplier single-buyer supply chain with constant demand. They model quantity discounts through incorporating bargaining and utility theory. Chakravarty and Martin [65] study optimal quantity discount design in a two-stage supply chain. They further evaluate the centralized system and consider finite production rate, investments in setup costs and sensitivity analysis. Chiang et al. [66] study optimal quantity discount design in a single-supplier single-buyer supply chain with constant demand. The paper utilizes geometric programming and reveals the benefits of cooperation. Wang and Wu [67] study a single-supplier multi-buyer supply chain with constant demand. They propose a pricing policy for the supplier that offers price discounts based on the percentage increase from the buyers' order quantity before discount. Supplier's optimal decision, which is explicitly available, is a discrete all-unit quantity discount schedule with many break points and complies with general fair trade laws.

There are also studies on quantity discounts in supply chains with price-sensitive demand. Weng and Wong [68] investigate the effectiveness of all-unit quantity discount policies in supplier-buyer chains with deterministic demand. Single/multiple-buyer and constant/price sensitive-demand models are considered. Parlar and Wang [69] study optimal quantity discount design in a single-supplier single-buyer supply chain with linear demand function. The discount scheme considered in the paper is an all-unit discount scheme. Weng [70] develops optimal quantity discount policies and investigates their consequences in a single-supplier single-buyer supply chain with general price sensitive demand functions. The paper considers all-unit and incremental quantity discounts, and shows that both discount schemes are equivalent in terms of the benefits to the agents. Weng [71] considers the same setting, and models a profit sharing mechanism through franchise fees, together with quantity discounts. The study reveals that quantity discounts alone cannot achieve channel coordination.

Corbett and de Groote [72] consider a single-supplier single-buyer supply chain with constant demand and asymmetric information in a principal-agent framework. The paper derives the optimal quantity discount policy under asymmetric information and compare with the full information case.

Chen et al. [73] study a single-supplier multi-buyer system with price sensitive demand where the system operates using a power-of-two lot ratio policy. They show that quantity discounts only may not be sufficient to achieve channel coordination where each buyer's demand is price-sensitive. They further show that channel coordination can be achieved via periodically charged fixed fees and a nontraditional discount scheme that is based on annual sales volume, order quantity and order frequency. In the coordination mechanism, offers are special for each buyer, hence a buyer cannot choose to adopt another buyer's offer. Viswanathan and Wang [74] investigate effects of quantity and volume discounts in a single-supplier single-buyer system with price-sensitive demand. Discounts are applied separately and simultaneously. Algorithms are proposed for finding the optimal pricing and lot sizing decisions. The paper concludes that joint use of quantity and volume discounts leads to channel coordination.

Wang [75] studies a single-supplier multi-buyer system with constant demand where the system operates using a power-of-two lot ratio policy. He argues that the coordination mechanism of Chen et al. [73] may be conflicting with the Robinson-Patman Act, which is a United States federal law that prohibits anticompetitive practices through price discrimination, and proposes a common all-unit-quantity discount scheme for all of the buyers. He further confirms the results of Chen et al. [73]. Wang [76] studies a single-supplier multi-buyer system with constant demand and compares the effectiveness of quantity discounts with integer-ratio and power-of-two time coordination. He concludes that integer-ratio time coordination provides a better coordination mechanism for the supply chain.

Chen and Chen [77, 78] study a single-supplier multi-buyer system with multi-product and constant demand. They adopt an integer-ratio time policy and propose algorithms for finding optimal quantity discount schemes. Wang and Wang [79] analyze a supplier's optimal quantity discount policy (all-units and incremental) for a set of heterogeneous buyers with price sensitive demand. The supplier designs a single break point (which corresponds to a particular discount scheme consisting of discount and quantity) for each buyer such that they choose the designed break point desired by the supplier and higher break points are designed for larger (in annual demand) buyers. The supplier's all-units quantity discount policy is formulated as an NLP (nonlinear programming) model. A numerical algorithm is subsequently developed to obtain an optimal solution and finally, an optimal quantity discount policy is proposed for the supplier using a maximum lower bound formulation for its inventory related cost. The supplier adopts a simple heuristic inventory replenishment policy in the paper, and it is further claimed that this model provides a reasonably close approximation and a desirable solution. Wang [80] derives optimal quantity and volume discounts for a single-supplier multi-buyer system with constant demand. A buyer-specific discount scheme is designed instead of a common discount scheme. The paper further shows that discount policies are able to achieve nearly optimal system profit, hence provide efficient coordination.

Chapter 3

PROBLEM DESCRIPTION AND SOLUTION PROCEDURES

3.1 Problem Description

We analyze the pricing and lot sizing problem in two-echelon supply chain systems under deterministic demand. Figure 3.1 depicts the decisions and interactions within the supply chain. In the following chapters, technical derivations and numerical illustrations of the models are presented.

Two widely used demand functions in the literature are implemented in the thesis. We adopt a constant-elasticity demand function throughout the thesis, and models incorporating linear demand function are presented in Appendix B.

Transportation cost has been modeled in various structures in the literature. Section 2.3 reviewed relevant structures studied within pricing and lot sizing problems. We observe three major types of transportation cost as follows:

1. - Unit based: Transportation cost depends directly on the total shipped load, where distinct transportation loads cannot have the same costs.
   - Transportation range based: There exist different transportation cost categories and cost is determined by the category to which the total shipped load belongs. As long as the shipped loads belong to the same category, transportation costs are the same.

2. - Continuous: The transportation cost function is continuous.
   - Discontinuous: The transportation cost function has discontinuities.

3. - Regular: The marginal transportation cost is the same for distinct transportation loads.
   - Economies of scale in transportation costs: The incremental transportation cost decreases as the size of total shipped load increases, where per unit transportation

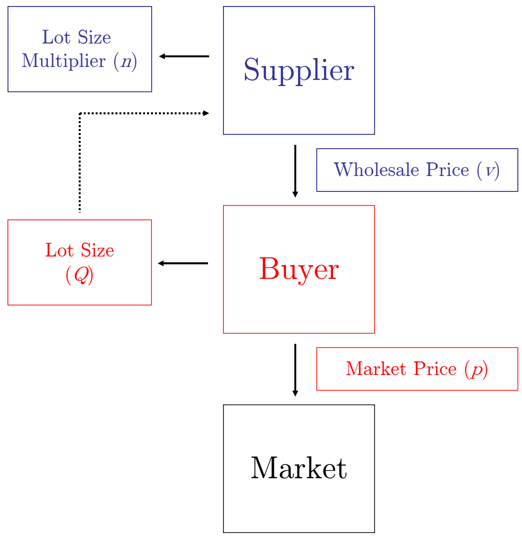

Figure 3.1: Supply chain structure and decision variables.

cost decreases as well.

Aucamp [19] adopts a transportation range based, discrete and regular cost structure, whereas Lee [20] and Hwang et al. [23] adopt a transportation range based, discrete cost structure with economies of scale. One of the essential studies concerning the models in this thesis is by Abad and Aggarwal [30], and they use a continuous transportation cost structure, which is subject to economies of scale. Besides, the structure allows over declaring option, which utilizes both unit and transportation range based cost structures within the model.

Moreover, the transportation sector in Turkey is heavily competitive. According to Turkish Statistical Institute's transportation statistics [81] total number of cargo vehicles by 2006 is 2,151,986 with 1,475,057 pick-up trucks and 676,929 trucks. This statistics is far above the average in European Union countries. Therefore, we can say that almost perfect competition exists in the sector and prices are close to marginal costs. Hence the economies of scale effect is not likely to be observed in the sector.

We model transportation cost through a stepwise linear function, which is adopted by Aucamp [19], as shown in Figure 3.2. We believe that this structure constitutes a relatively

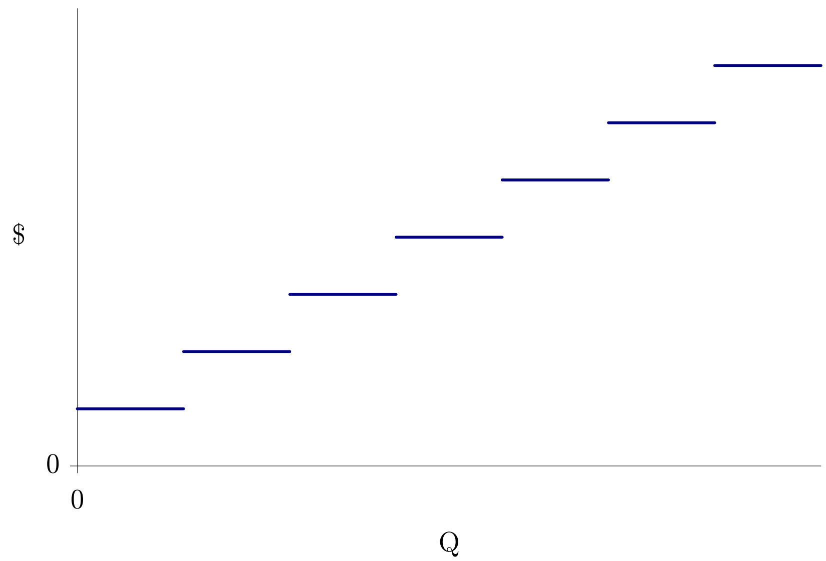

Figure 3.2: Freight cost as a function of the order quantity.

realistic representation of the transportation sector.

To compute $F(Q)$, per truck cost R is multiplied by the number of trucks required by the order quantity Q. We assume that the transportation cost is determined in terms of truck loads, therefore the freight cost is computed as:

$$F(Q) = \left\lceil \frac{Q}{C} \right\rceil R. \tag{3.1}$$

An interview with an expert from the transportation sector, (Arzu Aras, Quality and Management Systems Development Director of Borusan Logistics) also confirms that this transportation cost structure is a realistic approach for the market. She further notes that the economies of scale effect can exist in contracts with extremely high transportation loads, which is beyond the scope of this thesis.

We review the techniques from the literature and propose an approximation algorithm for the pricing and lot sizing problem in two-echelon supply chain systems. The following section briefly describes the solution methods considered in this thesis.

3.2 Solution Procedures

1. Approximation Algorithm

We propose an approximation algorithm for the optimal pricing and lot sizing problem. In the core of this approximate solution procedure is a novel approach for modeling the response of the buyer to different values of the wholesale price. This in turn helps us model the supplier's problem. We characterize the buyer's optimal order quantity response and propose a search algorithm over wholesale price range. The order quantity is constant in some of the wholesale price intervals whereas it is decreasing in the remaining intervals. We derive the pricing and lot sizing decisions in the constant order quantity ranges and approximate the solution in the remaining intervals.

2. Grid Search

   A common approach for solving the problem is simply adopting a grid search. It is the easiest way to solve the problem but a computationally intensive method since computations are generated for every single grid interval.

   Let us consider the optimization of a general function $f : R \rightarrow R$. If we have k test points $x_1, x_2, ..., x_k$ such that $L \leq x_1 \leq x_2 \leq ... \leq x_k \leq U$ where the grid lengths are constant as $\Im = x_i - x_{i-1}$, then the test point, x_j leading to the best value $f(x_j)$, is at maximum distance of $\Im$ to the optimal point x^*. Hence optimal solution can be achieved as an exact solution as $\Im$ approaches 0.

   The number of test points in each iteration step is crucial in the algorithm because the speed of the algorithm heavily depends on that number.

3. Myopic Approach

   Found in the literature, one way to solve the supplier's problem is to ignore the transportation cost. The supplier offers a wholesale price to the buyer assuming that the buyer does not incur transportation cost. When the buyer determines the market price, she takes transportation cost into account and does not move as the way supplier expects. Henceforth, we will refer to this approach as the myopic procedure.

4. Golden Section Search: A Heuristic

   Golden Section Search, which is introduced by Kiefer [82], is an unconstrained optimization technique, where a unimodal function is optimized by successively narrowing the range of values inside which the extremum point is known to exist.

Definition 1. *A function is quasi-concave $f : X \rightarrow R$ if*

$$f(\lambda x + (1 - \lambda)x') \geq min\{f(x), f(x')\} \tag{3.2}$$

for all $x, x' \in X$ and all $\lambda \in [0, 1]$.

The algorithm starts with two end points and two points, which are in between the end points and determined by the golden ratio $\frac{\sqrt{5}-1}{2}$. Through successive iterations, several intervals are eliminated and the algorithm terminates when the accuracy bound ϵ is achieved between the end points.

We propose using the golden section search algorithm as a heuristic procedure for the non-quasiconcave and sometimes discontinuous profit functions in the thesis.

Chapter 4

SUPPLY CHAIN MODELING

4.1 Decentralized System without Transportation Cost

The buyer's profit function can be written as

$$\prod_{B}(p,Q) = (p-v)D(p) - vI\frac{Q}{2} - A\frac{D(p)}{Q} \tag{4.1}$$

Likewise, the supplier's profit function is

$$\prod_{S}(v,n) = (v-m)D(p^*(v)) - mI(n-1)\frac{Q^*(v)}{2} - \left(\frac{K}{n}\right)\frac{D(p^*(v))}{Q^*(v)} \tag{4.2}$$

where $p^*(v)$ denotes the optimal price set by the buyer, and $Q^*(v)$ denotes the buyer's optimal replenishment quantity when the supplier sets the wholesale price as v.

The Buyer's Problem

In this section, we first discuss the solution of the pricing and lot sizing problems for the buyer assuming that the supplier sets the wholesale price as v. We will later incorporate these results into the supplier's problem within a Stackelberg setting.

Incorporating the demand function, and for a fixed value of v, the profit function of the buyer can be written as

$$\prod_{B}(p,Q) = (p-v)ap^{-b} - vI\frac{Q}{2} - A\frac{ap^{-b}}{Q} \tag{4.3}$$

The first-order condition for a local maximum with respect to p is obtained as below.

$$\frac{\partial \prod_B(p,Q)}{\partial p} = ap^{-b} - \frac{(p-v)\,ap^{-b}b}{p} + \frac{A\,ap^{-b}b}{pQ} = 0, \tag{4.4}$$

and we can state

$$p^*(Q) = \frac{b\,(A+Qv)}{Q\,(b-1)}. \tag{4.5}$$

We further take the second derivative and equate it to zero as:

$$\frac{\partial^2 \prod_B(p,Q)}{\partial p^2} = -\frac{abp^{-2-b}(A(1+b)+Q(p(1-b)+vQ(1+b)))}{Q} = 0 \tag{4.6}$$

Solving (4.6) gives the price

$$p^{'}(Q) = \frac{A + Qv + b\,(A + Qv)}{Q\,(b-1)}. \tag{4.7}$$

Since $p^{'}(Q) > p^{*}(Q)$, and $lim_{p \to \infty} \prod_B (p, Q) = 0$, it can be shown that (4.5) gives the global optimal solution. We can then substitute p by $p^{*}(Q)$ in Equation (4.3). We rewrite the buyer's profit function as

$$\prod_B (Q) = (p^{*}(Q) - v) a (p^{*}(Q))^{-b} - vI\frac{Q}{2} - A\frac{a(p^{*}(Q))^{-b}}{Q} \tag{4.8}$$

We write the first-order condition for a local maximum with respect to Q as

$$\frac{\partial \prod_B (Q)}{\partial Q} = -\frac{vI}{2} + \frac{A\,a(p^{*}(Q))^{-b}}{Q^2}, \tag{4.9}$$

and by setting it equal to zero we obtain

$$\left(-\frac{vI}{2} + \frac{Aa\left(\frac{b(A+Qv)}{Q(b-1)}\right)^{-b}}{Q^2} \right) = 0 \tag{4.10}$$

Equality (4.10) is the local optimality condition for the buyer's order quantity. Q^{*} can now be obtained through a line search over the buyer's profit function.

The Supplier's Problem

Viswanathan and Wang [74] claim that local maxima for the supplier's profit function (as given in Equation (4.2)) can occur. We have also encountered examples where the supplier's profit function is not a quasiconcave function of v. We will study the structural properties of the problem in the subsequent sections and review two solution procedures for the problem in this section.

4.1.1 Grid Search

In this section, we present the grid search approach for solving the supplier's problem. The optimal wholesale price v and corresponding profit for the supplier is determined numerically by a grid search within the range $[m, v_{max}]$.

4.1.1.1 Derivation of the optimal lot size multiplier under a fixed value of the wholesale price

We derive the optimal lot size multiplier in this section. The first and second derivatives of the supplier's profit function (4.2) with respect to n are

$$\frac{\partial \prod_S(v,n)}{\partial n} = -mI\frac{Q^*(v)}{2} + K\frac{D(p^*(v))}{n^2 Q^*(v)}, \tag{4.11}$$

and

$$\frac{\partial^2 \prod_S(v,n)}{\partial n^2} = -2K\frac{D(p^*(v))}{n^3 Q^*(v)}. \tag{4.12}$$

Since K, v, $D(p^*(v))$, $Q^*(v)$ are positive, $\frac{\partial^2 \prod_S(v,n)}{\partial n^2}$ is always negative. Therefore, the profit function (4.2) is strictly concave with respect to n. Since the optimal lot size multiplier n is an integer and the function is concave with respect to n, we can state the following two optimality conditions for n^*

$$\prod_S(v,n^*) \geq \prod_S(v,n^*+1) \tag{4.13}$$

and

$$\prod_S(v,n^*) \geq \prod_S(v,n^*-1) \tag{4.14}$$

n^* will be denoted as n in the remaining part of this section. Expanding (4.13) and (4.14) through (4.2) gives

$$mI(n-1)\frac{Q^*(v)}{2} + K\frac{D(p^*(v))}{nQ^*(v)} - mI(n)\frac{Q^*(v)}{2} - K\frac{D(p^*(v))}{(n+1)Q^*(v)} \leq 0 \tag{4.15}$$

and

$$mI(n-1)\frac{Q^*(v)}{2} + K\frac{D(p^*(v))}{nQ^*(v)} - mI(n-2)\frac{Q^*(v)}{2} - K\frac{D(p^*(v))}{(n-1)Q^*(v)} \leq 0 \tag{4.16}$$

Simplifying (4.15) and (4.16), we obtain

$$-mI\frac{Q^*(v)}{2} + K\frac{D(p^*(v))}{Q^*(v)n(n+1)} \leq 0 \tag{4.17}$$

and

$$mI\frac{Q^*(v)}{2} - K\frac{D(p^*(v))}{Q^*(v)n(n-1)} \leq 0 \tag{4.18}$$

Equating the inequality (4.17) to zero and solving for n gives the roots: $-\frac{1}{2}\left(1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$ and $\frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$. The inequality is not satisfied in between the roots. Likewise, equating the inequality (4.18) to zero and solving for n gives the roots: $\frac{1}{2}\left(1-\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$

and $\frac{1}{2}\left(1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$. The inequality is satisfied in between the roots. We can conclude that n is between $\frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$ and $\frac{1}{2}\left(1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$. Since the range is bounded to 1 and n is integer, optimal n can be written as

$$n^* = \left\lceil \frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil \text{ or } \left\lfloor \frac{1}{2}\left(1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rfloor. \tag{4.19}$$

For a fixed wholesale price v, optimal price can be stated as $p^*(Q) = \frac{b(A+Qv)}{Q(b-1)}$, which is given in (4.5). A line search for Q through solving $\left(-\frac{vI}{2} + \frac{(A)a\left(\frac{b(A+Qv)}{Q(b-1)}\right)^{-b}}{Q^2}\right) = 0$ gives the optimal order quantity, which is provided in (4.10). Finally, the optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$, which is stated in (4.19). We adopt a grid size $g = 10^{-3}$ in the computations.

The algorithm can be formally stated as follows:

Step 1: Let the optimal profit $\prod_S^*$ be 0 with $v = 0$ and $n = 0$.

Step 2: Start with $v = m$ and complete the following steps for each wholesale price from the set $\{m, m+g, m+2g, ..., v_{max}-2g, v_{max}-g, v_{max}\}$.

2.1: Given v, find Q through a line search solving (4.10), compute p as defined in (4.5) and $\prod_B(Q)$ as defined in (4.2).

2.2: Compute n and $\prod_S(v)$ as defined in (4.19) and (4.2) respectively. If $\prod_S(v)$ is greater than the optimal profit, update the optimal profit $\prod_S^*$, the wholesale price v^* and the lot size multiplier n^*.

4.1.2 Golden Section Search: A Heuristic

In this section, we present golden section search algorithm for solving the supplier's problem. The wholesale price v and corresponding profit for the supplier is determined numerically through interval eliminations within the range $[m, v_{max}]$. Since the profit function is not quasiconcave, convergence to optimality can not be guaranteed. However, Viswanathan and Wang [74] claim that local maxima, if any, seem to occur only close to the global maximum. Therefore, we present this approach as a heuristic procedure. We adopt the accuracy bound $\epsilon = 0.001$ in the computations.

For a fixed wholesale price v, optimal price can be stated as $p^*(Q) = \frac{b(A+Qv)}{Q(b-1)}$, which is given in (4.5). A line search for Q through solving $\left(-\frac{vI}{2} + \frac{(A)a\left(\frac{b(A+Qv)}{Q(b-1)}\right)^{-b}}{Q^2}\right) = 0$ gives the optimal order quantity, which is provided in (4.10). Finally, optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$, which is stated in (4.19).

The algorithm can be formally stated as follows:

Step 1: Let the optimal profit $\prod_S^*$ be 0 with $v = 0$ and $n = 0$. Start with $v_1 = m$, $v_2 = v_{max}$, $\lambda = v_1 + (1 - \frac{\sqrt{5}-1}{2})(v_2 - v_1)$, and $\mu = v_1 + \frac{\sqrt{5}-1}{2}(v_2 - v_1)$.

Step 2: Complete the following steps for wholesale prices v_1, v_2, λ, μ.

2.1: Find Q through a line search solving (4.10), compute p as defined in (4.5) and $\prod_B(Q)$ as defined in (4.8).

2.2: Compute n and $\prod_S$ as defined in (4.19) and (4.2) respectively.

Step 3: If $\prod_S(\lambda) < \prod_S(\mu)$, update the wholesale prices as follows

$v_1 = \lambda$

$\lambda = \mu$

$\mu = v_1 + \frac{\sqrt{5}-1}{2}(v_2 - v_1)$.

Otherwise, update the wholesale prices as follows

$v_2 = \mu$

$\mu = \lambda$

$\lambda = v_1 + (1 - \frac{\sqrt{5}-1}{2})(v_2 - v_1)$.

Step 4: If $v_2 - v_1 > \epsilon$ go to Step 2, otherwise terminate the algorithm. λ and associated $n, \prod_S(\lambda)$ are found to be the best solution.

4.2 Centralized System without Transportation Cost

Viswanathan and Wang [74] study this problem and we restate the derivation of optimal pricing and lot sizing decisions in this section for the sake of completeness.

The profit function can be stated as follows:

$$\prod_C(p, Q, n) = (p - m)D(p) - (h_s(n-1) + h_b)\frac{Q}{2} - \left(\frac{K}{n} + A\right)\frac{D(p)}{Q}. \tag{4.20}$$

The first order optimality condition with respect to Q is given by

$$\frac{\partial \prod_C(p, Q, n)}{\partial Q} = -\frac{h_s(n-1) + h_b}{2} + \left(\frac{K}{n} + A\right)\frac{D(p)}{Q^2} = 0, \tag{4.21}$$

and Q^* can be written as:

$$Q^* = \sqrt{\frac{2\left(\frac{K}{n} + A\right)D(p)}{h_s(n-1) + h_b}}. \tag{4.22}$$

Replacing Q^* with Q in (4.20),

$$\prod_C(p,n) = (p-m)D(p) - (h_s(n-1)+h_b)\frac{Q^*}{2} - \left(\frac{K}{n}+A\right)\frac{D(p)}{Q^*} \tag{4.23}$$

Rewriting the profit function,

$$\prod_C(p,n) = (p-m)D(p) - \sqrt{2D(p)\left(\frac{K}{n}+A\right)(h_s(n-1)+h_b)}. \tag{4.24}$$

Replacing $\left(\frac{K}{n}+A\right)(h_s(n-1)+h_b)$ with $L(n)$, we can write the profit function as follows:

$$\prod_C(p,n) = (p-m)D(p) - \sqrt{2D(p)L(n)} \tag{4.25}$$

Maximizing $\prod_C(p,n)$ is equivalent to minimizing $L(n)$ with respect to n . Since n is integer and $L(n)$ is convex, n^* satisfies the following equations. (n denotes n^* in this section)

$$L(n) \leq L(n-1) \text{ and } L(n) \leq L(n+1) \tag{4.26}$$

Replacing $L(n)$ and $L(n-1)$ in the first equality,

$$\left(\frac{K}{n}+A\right)(h_s(n-1)+h_b) - \left(\frac{K}{n-1}+A\right)(h_s(n-2)+h_b) \leq 0. \tag{4.27}$$

The roots of the equation,

$$\left(\frac{K}{n}+A\right)(h_s(n-1)+h_b) - \left(\frac{K}{n-1}+A\right)(h_s(n-2)+h_b) = 0 \tag{4.28}$$

are $\frac{1}{2}\left(1-\sqrt{1+4\frac{Ah_s}{K(h_b-h_s)}}\right)$ and $\frac{1}{2}\left(1+\sqrt{1+4\frac{Ah_s}{K(h_b-h_s)}}\right)$. The inequality is satisfied in between the roots.

Applying the same procedure to the second equality in (4.26), we can write the equality as

$$\left(\frac{K}{n}+A\right)(h_s(n-1)+h_b) - \left(\frac{K}{n+1}+A\right)(h_s(n)+h_b) \leq 0. \tag{4.29}$$

The roots of the equation,

$$\left(\frac{K}{n}+A\right)(h_s(n-1)+h_b) - \left(\frac{K}{n+1}+A\right)(h_s(n)+h_b) = 0 \tag{4.30}$$

are $\frac{1}{2}\left(-1-\sqrt{1+4\frac{Ah_s}{K(h_b-h_s)}}\right)$ and $\frac{1}{2}\left(-1+\sqrt{1+4\frac{Ah_s}{K(h_b-h_s)}}\right)$. The inequality is not satisfied in between the roots. Since both inequalities must be satisfied, n^* is in between

$\frac{1}{2}\left(-1+\sqrt{1+4\frac{Ah_s}{K(h_b-h_s)}}\right)$ and $\frac{1}{2}\left(1+\sqrt{1+4\frac{Ah_s}{K(h_b-h_s)}}\right)$. The interval is bounded to 1 and since n is integer, we can write n^* as

$$n^* = \left\lfloor \frac{1}{2}\left(1+\sqrt{1+4\frac{Ah_s}{K(h_b-h_s)}}\right) \right\rfloor \text{ or } \left\lceil \frac{1}{2}\left(-1+\sqrt{1+4\frac{Ah_s}{K(h_b-h_s)}}\right) \right\rceil \tag{4.31}$$

They return different values only if both of them have the same value $L(n)$. The former is proposed by Munson and Rosenblatt [83]. Replacing n^* and the demand function, we can now rewrite the profit function as

$$\prod_C(p) = (p-m)ap^{-b} - \sqrt{2ap^{-b}L(n^*)} \tag{4.32}$$

The first order optimality condition is given by

$$\frac{\partial \prod_C(p)}{\partial p} = \frac{p^{-1-b}}{2}\left(\sqrt{2}bp^b\sqrt{aL(n)p^{-b}} + 2a(b(m-p)+p)\right) = 0. \tag{4.33}$$

$\prod_C(p)$ is maximized at the minimum value of p for which $\frac{\partial \prod_C(p)}{\partial p} \geq 0$. Hence, the optimal price is found through a line search using (4.33)

4.3 Decentralized System where The Supplier Owns The Transportation Cost (Freight On Board)

The buyer's profit function can be written as

$$\prod_B(p,Q) = (p-v)D(p) - vI\frac{Q}{2} - A\frac{D(p)}{Q} \tag{4.34}$$

Likewise, the supplier's profit function is

$$\prod_S(v,n) = (v-m)D(p^*(v)) - mI(n-1)\frac{Q^*(v)}{2} - \left(\frac{K}{n} + F(Q)\right)\frac{D(p^*(v))}{Q^*(v)} \tag{4.35}$$

where $p^*(v)$ denotes the optimal price set by the buyer, and $Q^*(v)$ denotes the buyer's optimal replenishment quantity when the supplier sets the wholesale price as v.

The Buyer's Problem

Incorporating the demand function, and for a fixed value of v, the profit function of the buyer can be written as

$$\prod_B(p,Q) = (p-v)ap^{-b} - vI\frac{Q}{2} - A\frac{ap^{-b}}{Q} \tag{4.36}$$

The first-order condition for a local maximum with respect to p is obtained as below.

$$\frac{\partial \prod_B(p,Q)}{\partial p} = ap^{-b} - \frac{(p-v)\,ap^{-b}b}{p} + \frac{A\,ap^{-b}b}{pQ} = 0, \tag{4.37}$$

and we can state

$$p^*(Q) = \frac{b\,(A+Qv)}{Q\,(b-1)}. \tag{4.38}$$

We further take the second derivative and equate it to zero as:

$$\frac{\partial^2 \prod_B(p,Q)}{\partial p^2} = -\frac{abp^{-2-b}(A(1+b)+Q(p(1-b)+v(1+b)))}{Q} = 0 \tag{4.39}$$

Solving (4.39) gives the price

$$p^{'}(Q) = \frac{A+Qv+b\,(A+Qv)}{Q\,(b-1)}. \tag{4.40}$$

Since $p^{'}(Q) > p^*(Q)$, and $lim_{p\rightarrow\infty}\prod_B(p,Q) = 0$, it can be shown that $p^*(Q)$ is a global optimal solution. We can then substitute p by $p^*(Q)$ in Equation (4.36). We rewrite buyer's profit function as

$$\prod_B(Q) = (p^*(Q)-v)a(p^*(Q))^{-b} - vI\frac{Q}{2} - A\frac{a(p^*(Q))^{-b}}{Q} \tag{4.41}$$

We write the first-order condition for a local maximum with respect to Q as

$$\frac{\partial \prod_B(Q)}{\partial Q} = -\frac{vI}{2} + \frac{A\,a(p^*(Q))^{-b}}{Q^2}, \tag{4.42}$$

and by setting it equal to zero we obtain

$$\left(-\frac{vI}{2} + \frac{Aa\left(\frac{b(A+Qv)}{Q(b-1)}\right)^{-b}}{Q^2}\right) = 0 \tag{4.43}$$

Equality (4.43) is the local optimality condition for the buyer's order quantity. Q^* can now be obtained through a line search over the buyer's profit function.

The Supplier's Problem

Supplier's profit function is not quasi-concave, and it is difficult to obtain a global optimal solution. We will study the structural properties of the problem in the subsequent sections and review three solution procedures for the problem.

4.3.1 Approximation Algorithm

In this section we study the supplier's problem of finding the optimal wholesale price. As we have outlined in the previous section, a procedure that performs a finite number of line searches can be used to compute $Q^*(v)$. The supplier, however, wishes to determine v^*, and, to be able to solve his problem, he needs to incorporate the buyer's reaction as $Q^*(v)$ and $p^*(v)$ into his problem. Because $Q^*(v)$ is not available as an explicit function, and can be computed only through a line search, we propose an approach through which $Q^*(v)$ is approximated.

4.3.1.1 Approximating optimal order quantity response of the buyer

In this section, we analyze how the buyer's optimal order quantity, i.e., $Q^*(v)$, changes as we change v, and present an approximation of $Q^*(v)$.

Let us first take a wholesale price, v_a and corresponding order quantity $Q(v_a)$ obtained by (4.43). Any v_b larger than v_a will lead to $Q(v_b)$, which is strictly smaller than $Q(v_a)$. (see Proposition 2 in Appendix A)

Hence, an approximate procedure for determining the order quantity the buyer chooses can be formally stated as follows:

Initialization: Let $i = 1$. Determine the smallest value of v, v_1, for which Equality (4.43) holds for $t = 1$ and $Q = C$. Assuming that optimal wholesale price can be at most v_{max}, the order quantity response is linearly approximated in the interval $[v_1, v_{max}]$ through a linear function that crosses points $(v_1, Q^*(v_1) = C)$ and $(v_{max}, Q^*(v_{max}))$.

Step 1:

- Let $i = i + 1$.
- Determine the smallest value of v, v_i, for which (4.43) holds for $t = i$ and $Q = iC$.
- Linearly approximate the order quantity between $Q = iC$ and $Q = (i - 1)C$ in the interval $[v_i, v_{i-1}]$

Step 2: If $i > T$ (maximum number of trucks) combine approximated Q values characterizing the buyer's optimal response, and stop; otherwise go to Step 1.

4.3.1.2 Derivation of the supplier's optimal wholesale price under a fixed lot size multiplier

In the previous section, we have characterized the buyer's response in terms of her order quantity. In this section, we illustrate how the supplier can determine his optimal wholesale price.

Let us assume that n is constant but Q is not constant, where the order quantity is approximated as a linear decreasing function of v. Let this function be $Q(v) = c - dQ$ for a wholesale price range $[v_{j+1}, v_j]$. Replacing Q with $c-dQ$ in Equation (4.35), we approximate the best solution in this interval through a line search over v values in the $[v_{j+1}, v_j]$ range maximizing the supplier's profit.

4.3.1.3 Derivation of the optimal lot size multiplier under a fixed value of the wholesale price

After deriving the optimal wholesale price, we derive the optimal lot size multiplier. The first and second order derivatives of the profit function are given by (4.11) and (4.12) respectively. Hence the profit function is again strictly concave with respect to n. Besides, the optimality conditions (4.13) and (4.14) reduce to (4.15) and (4.16). Hence the remaining part of the derivation is the same with Section 4.1.1.1 and we conclude that optimal lot size multiplier is given by (4.19).

4.3.1.4 The Approximate Algorithm

In this section, we present the approximate algorithm for solving the supplier's problem by combining the results that have been developed in the previous sections.

In order to solve the supplier's problem, first the buyer's response has to be characterized. Since the buyer's optimal decision cannot be expressed in closed form, the supplier's decision cannot be immediately derived. Optimal market price as a function of the order quantity can be determined by using Equation (4.38). Substituting this into the profit function, we obtain Equation (4.41). Using this function, we can then characterize the order quantity as the wholesale price changes in Section 4.3.1.1. Hence we propose a search procedure with respect to the wholesale price. The algorithm can be formally stated as follows:

Step 1: Characterize the optimal response of the buyer utilizing the procedure outlined in Section 4.3.1.1.

Step 2: Let the optimal profit $\prod_S^*$ be 0 with $v = 0$.

Step 3: Consider each wholesale price interval where the truck option changes in the response profile generated in Step 1, and complete the following steps for each interval:

3.1: Replace the linear approximation function with Q in the supplier's profit function (4.35). Find the optimal wholesale price and lot size multiplier maximizing the profit function through a line search over (4.35) as an approximation. (We note that, as shown in Section 4.3.1.3, when the wholesale price and the order quantity are fixed, the optimal lot size multiplier can be readily computed in each objective function evaluation of the line search.)

3.2: For the current wholesale price interval, compare the computed profit with the optimal profit. If it is greater than the optimal profit, update the optimal profit, the wholesale price and the lot size multiplier.

4.3.2 Grid Search

In this section, we present the grid search approach for solving the supplier's problem. The optimal wholesale price v and corresponding profit for the supplier is determined numerically by a grid search within the range $[m, v_{max}]$. For a fixed wholesale price v, optimal price can be stated as $p^*(Q) = \frac{b(A+Qv)}{Q(b-1)}$, which is given in (4.38). A line search for Q through solving $\left(-\frac{vI}{2} + \frac{(A)a\left(\frac{b(A+Qv)}{Q(b-1)}\right)^{-b}}{Q^2}\right) = 0$ gives the optimal order quantity, which is provided in (4.43). Finally, optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$, which is stated in (4.19). We adopt a grid size $g = 10^{-3}$ in the computations. The algorithm of Section 4.1.1 is used with the decision variables and profit functions mentioned above.

4.3.3 Golden Section Search: A Heuristic

In this section, we adopt the golden section search algorithm for solving the supplier's problem. The details of the procedure have been previously stated in Section 4.1.2. The algorithm, which is mentioned in the section is used with the decision variables and profit functions stated above.

4.4 Decentralized System where The Buyer Owns The Transportation Cost

The buyer's profit function can be written as:

$$\prod_B(p, Q) = (p - v)D(p) - (v + \frac{F(Q)}{Q})I\frac{Q}{2} - (A + F(Q))\frac{D(p)}{Q}. \tag{4.44}$$

We note that the $\frac{F(Q)}{Q}$ term in the holding cost reflects the impact of the transportation cost on the value of the product (see Abad and Aggarwal [31] for details). Likewise, the supplier's profit function is:

$$\prod_S(v,n) = (v-m)D(p^*(v)) - mI(n-1)\frac{Q^*(v)}{2} - K\frac{D(p^*(v))}{nQ^*(v)}, \tag{4.45}$$

where $p^*(v)$ denotes the optimal price set by the buyer, and $Q^*(v)$ denotes the buyer's optimal replenishment quantity when the supplier sets the wholesale price as v.

The Buyer's Problem

We first discuss the solution of the pricing and lot sizing problems for the buyer assuming that the supplier sets the wholesale price as v. We will later incorporate these results into the supplier's problem within a Stackelberg setting.

Abad and Aggarwal (2005) also derive the optimal pricing and lot sizing decisions for a slightly different yet structurally equivalent freight cost function. For the sake of completeness, we derive the optimal decisions here again.

Incorporating the demand function, and for a fixed value of v, the profit function of the buyer can be written as

$$\prod_B(p,Q) = (p-v)ap^{-b} - (v+\frac{F(Q)}{Q})I\frac{Q}{2} - (A+F(Q))\frac{ap^{-b}}{Q}, \tag{4.46}$$

or

$$\prod_B(p,Q) = (p-v)ap^{-b} - vI\frac{Q}{2} - I\frac{F(Q)}{2} - (A+F(Q))\frac{ap^{-b}}{Q}. \tag{4.47}$$

The first-order condition for a local maximum with respect to p is obtained as below.

$$\frac{\partial\prod_B(p,Q)}{\partial p} = ap^{-b} - \frac{(p-v)\,ap^{-b}b}{p} + \frac{(A+F(Q))\,ap^{-b}b}{pQ} = 0, \tag{4.48}$$

and we can state

$$p^*(Q) = \frac{b\,(A+Qv+F(Q))}{Q\,(b-1)}. \tag{4.49}$$

We further take the second derivative and equate it to zero to find the p value at which the second derivative changes sign:

$$\frac{\partial^2\prod_B(p,Q)}{\partial p^2} = -\frac{abp^{-2-b}((A+F(Q))(1+b)+Q(p(1-b)+vQ(1+b)))}{Q} = 0 \tag{4.50}$$

Solving (4.50) gives the price

$$p^{'}(Q) = \frac{A + F(Q) + Qv + b\,(A + Qv + F(Q))}{Q\,(b-1)}. \tag{4.51}$$

Since $p^{'}(Q) > p^{*}(Q)$, i.e., the second derivative changes sign after the local maximum obtained at $p^{*}(Q)$, and $lim_{p\to\infty} \prod_B (p, Q) = 0$, it can be shown that $p^{*}(Q)$ is actually a global optimal solution. We can then substitute p by $p^{*}(Q)$ in Equation (4.46). We rewrite buyer's profit function as

$$\prod_B(Q) = (p^{*}(Q) - v)a(p^{*}(Q))^{-b} - vI\frac{Q}{2} - (A + F(Q))\frac{a(p^{*}(Q))^{-b}}{Q}. \tag{4.52}$$

For a fixed number of trucks t, where $(t-1)C < Q \leq tC$, we can rewrite (4.52) as

$$\prod_B(Q) = (p^{*}(Q) - v)a(p^{*}(Q))^{-b} - vI\frac{Q}{2} - (A + tR)\frac{a(p^{*}(Q))^{-b}}{Q}. \tag{4.53}$$

As $F(Q)$ is a stepwise function of number of trucks used, t, where $t = \left\lceil \frac{Q}{C} \right\rceil$, we write the first-order condition for a local maximum with respect to Q for a particular t value as

$$\frac{\partial \prod_B(Q)}{\partial Q} = -\frac{vI}{2} + \frac{(A + tR)\,a(p^{*}(Q))^{-b}}{Q^2}, \tag{4.54}$$

and by setting it equal to zero we obtain

$$\left(-\frac{vI}{2} + \frac{(A + tR)\,a\left(\frac{b(A+Qv+tR)}{Q(b-1)}\right)^{-b}}{Q^2} \right) = 0 \tag{4.55}$$

Equality (4.55) is the local optimality condition for the buyer's order quantity when the number of trucks is fixed as t, and $(t-1)C < Q \leq tC$. The Q^{*} value that satisfies Equality (4.55) cannot be expressed in closed form, however it can be determined through a line search. Since T is the maximum number of trucks, the buyer can perform T line searches using (4.55) and obtain T many Q values. The buyer designates the order quantity that provides the highest profit as her order quantity.

The Supplier's Problem

The supplier's profit function (4.45) is not quasi-concave. We prove this claim through a counter example. Let us take an example with parameters $a = 100000$, $b = 5$, $A =$ \$125/order, $K =$ \$250/order, $I = 25\%$/year, $m =$ \$1/unit, $C = 1000$, $R =$ \$50 and examine

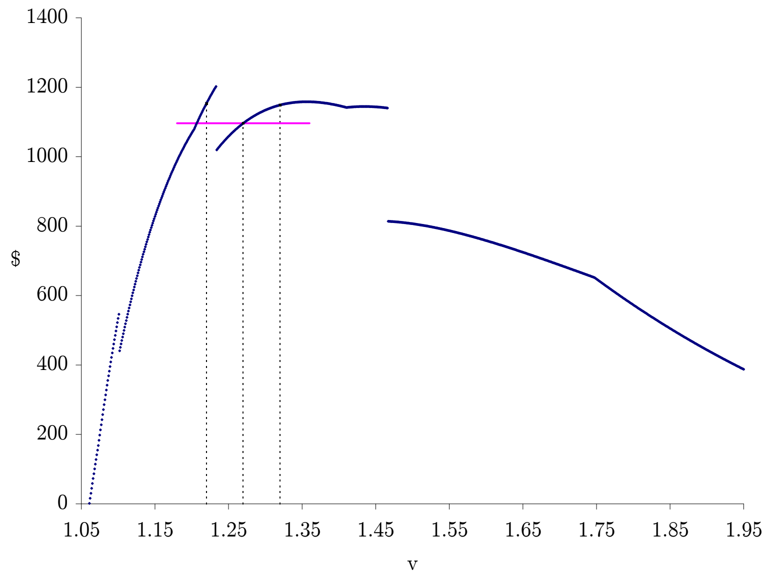

Figure 4.1: The supplier's profit function

quasi-concavity. We take two points $[v = 1.22, \prod_S(1.22) = 1153.44]$, $[v = 1.32, \prod_S(1.32) = 1148.80]$ and $\lambda = 0.5$. Since

$$\prod_S(1.27) = 1096.30 < min\{\prod_S(1.22), \prod_S(1.32)\} = 1148.80, \tag{4.56}$$

we conclude that the supplier's profit function is not quasi-concave and hence not concave, which is shown in Figure 4.1.

Viswanathan and Wang [74] claim that local maxima for the supplier's profit function (as given in Equation (4.45)) can occur even for the case where the buyer's profit function does not involve freight costs. We have also encountered examples where the supplier's profit function is not a quasiconcave function of v. Therefore, it is difficult to obtain a global optimal solution, and we will study the structural properties of the problem in the subsequent sections. We review four solution procedures for this problem.

4.4.1 Approximation Algorithm

In this section we study the supplier's problem of finding the optimal wholesale price. We will denote the order quantity selected by the buyer when the wholesale price is equal to v as $Q^*(v)$, and the price set by the buyer is $p^*(Q^*(v))$ or $p^*(v)$. As we have outlined in the previous section, a procedure that performs a finite number of line searches can be used to compute $Q^*(v)$. The supplier, however, wishes to determine v^*, and, to be able to solve his problem, he needs to incorporate the buyer's reaction as $Q^*(v)$ and $p^*(v)$ into his problem. Because $Q^*(v)$ is not available as an explicit function, and can be computed only through a line search, we propose an approach through which $Q^*(v)$ is approximated.

4.4.1.1 Approximating optimal order quantity response of the buyer

In this section, we analyze how the buyer's optimal order quantity, i.e., $Q^*(v)$, changes as we change v, and present an approximation of $Q^*(v)$. Let us first assume that the buyer uses only one truck, i.e. $t = 1$. Let v_1 denote the v value for which the optimality condition is satisfied with $Q = C$ and $t = 1$. For $v > v_1$, the optimal order quantity obtained from (4.55) will be smaller than C (see Proposition 1 in Appendix A), and using more than one truck would be more costly. Therefore, for $v > v_1$, the optimal response of the buyer would be to choose $Q^*(v)$ from (4.55) with $t = 1$.

Now let v_2 be the smallest value of v for which Equality (4.55) is satisfied with $Q = 2C$ and $t = 2$. From the results of Proposition 1 in Appendix A, $v_2 < v_1$, and for $v_2 < v < v_1$, the optimal order quantity obtained from (4.55) with $t = 2$ would be between $2C$ and C. The buyer has now two options for a given v value, where $v_2 < v < v_1$:

1. Order one full truck-load with $Q = C$,

2. Use two trucks, i.e., $t = 2$, and determine the order quantity from Equality (4.55) with $t = 2$.

It can be readily shown that the profit function of the buyer as expressed in Equation (4.53) is convex with respect to v when Q is fixed. In the second option with $t = 2$, as v changes, $Q^*(v)$ is determined through a line search, and it is not possible to express the buyer's profit function (4.53) in closed form. Therefore, as an approximation, we assume that, in the $v_2 < v < v_1$ range, the buyer's profit function (4.53) will be a linear function that crosses the points $(v_2, \prod_B(Q^*(v_2)))$ and $(v_1 - \epsilon, \prod_B(Q^*(v_1 - \epsilon)))$. The profit functions of the two options (the profit function of the second option being approximated as a linear function) may intersect in the $[v_2, v_1]$ interval. Note that when $v = v_1$, one full truck-load option dominates the options with two trucks; if that is also the case for $v = v_2$, then we will assume that the one full truck-load option dominates the second option in the $[v_2, v_1]$

range, and the buyer's optimal order quantity is equal to C. If the second option (i.e., use two trucks, determine Q from (4.55) with $t = 2$) dominates the first option when $v = v_2$, we then find the value of v where the two options generate the same profit for the buyer. Let $b^1_{t=2}$ be the intersection point. Then in the $v_2 < v < b^1_{t=2}$ range, the optimal order quantity is determined from (4.55) with $t = 2$, and for the $b^1_{t=2} < v < v_1$ range the optimal order quantity is equal to C.

By generalizing the above approach, an approximate procedure for determining the order quantity the buyer chooses can be formally stated as follows:

Initialization: Let $i = 1$. Determine the smallest value of v, v_1, for which Equality (4.55) holds for $t = 1$ and $Q = C$. For v values greater than v_1, the buyer will choose his order quantity according to equality (4.55) with $t = 1$. Assuming that optimal wholesale price can be at most v_{max}, the order quantity response is linearly approximated in the interval $[v_1, v_{max}]$ through a linear function that crosses points $(v_1, Q^*(v_1) = C)$ and $(v_{max}, Q^*(v_{max}))$.

Step 1:

- Let $i = i + 1$.

- Determine the smallest value of v, v_i, for which (4.55) holds for $t = i$ and $Q = iC$.

- Define the *upper envelope* as the combination of the profit functions of the truck options leading to the highest profit. Determine the upper envelope of the following profit functions of the buyer in the $v_i < v < v_{i-1}$ range:

  - $\prod_B(p^*(Q), Q)$ where $Q = jC$, and $t = j$, $j = 1, ..., i - 1$.
  - $\prod_B(p^*(Q(v)), Q(v))$ where $t = i$ and $Q(v)$ satisfies Equality (4.55).

- If $\prod_B(p^*(Q(v)), Q(v))$ at the point v_i is higher than any evaluated function $\prod_B(p^*(Q), Q)$ for $t = j$ where $j = 1, ..., i - 1$, first through a line search find the first break-point where the optimal truck option changes, and then linearly approximate the order quantity between the starting point of the interval and the break-point.

- Find the remaining break-points, if any, where the optimal truck option changes by a line search. Within these ranges, the order quantity is a multiple of C.

- Let b_i^k, $k = 1, .., l$, be the break-points generated by the upper envelope. Note that, from the fact that we are comparing i functions that are either convex or linear, $l < 2i$.

Step 2: If $i > T$ (maximum number of trucks) combine upper envelopes generated for intervals $[v_{j+1}, v_j]$ for $j = 1, 2, .., T-1$, and their corresponding Q values characterizing the buyer's optimal response, and stop; otherwise go to Step 1.

The computational complexity of the above outlined procedure lies with the generation of the upper envelope of at most T functions in Step 1. The upper envelope can be easily generated by a number of line searches under the assumption that Q linearly decreases when full truck option is not used. We provide an example to illustrate the steps of the algorithm in Section 4.4.1.4.

4.4.1.2 Derivation of the supplier's optimal wholesale price under a fixed lot size multiplier

In the previous section, we have characterized the buyer's response in terms of her order quantity. Accordingly, in wholesale price intervals that have been computed in Step 1 of the procedure presented in Section 5.1, the order quantity is either constant or linearly approximated. In this section, we illustrate how the supplier can determine his optimal wholesale price in each of these cases.

Case 1: If Q and n is constant for a wholesale price range $[v_{j+1}, v_j]$, then we can develop a search algorithm with respect to v in the range $[v_{j+1}, v_j]$. Replacing $p^*(v)$ with $p^*(Q)$ as given in Equation (4.49), we can rewrite the supplier's profit function as follows.

$$\prod_S(v) = (v-m)\,a\left(\frac{b\,(A+Qv+F(Q))}{Q\,(b-1)}\right)^{-b} - \frac{mI(n-1)Q}{2} - \frac{Ka\left(\frac{b(A+Qv+F(Q))}{Q(b-1)}\right)^{-b}}{nQ}. \tag{4.57}$$

Taking the derivative of (4.57) with respect to v, we can write

$$\frac{\partial \prod_S(v)}{\partial v} = \frac{a\left(\frac{b(Qv+F(Q)+A)}{Q(b-1)}\right)^{-b}(b(K+nQ(m-v))+n(A+F(Q)+vQ)}{n(A+F(Q)+vQ)}. \tag{4.58}$$

Since $\frac{a\left(\frac{b(Qv+F(Q)+A)}{Q(b-1)}\right)^{-b}}{n(A+F(Q)+vQ)}$ is always positive, the sign of $\frac{\partial \prod_S(v)}{\partial v}$ changes where

$$(b(K+nQ(m-v))+n(A+F(Q)+vQ)=0, \tag{4.59}$$

which gives the wholesale price

$$v^{'} = \frac{bK + An + F(Q)n + bmnQ}{nQ(b-1)}. \tag{4.60}$$

We further take the second derivative as follows:

$$\frac{\partial^2 \prod_S(v)}{\partial v^2} = -\frac{abQ((b+1)K + n(2A + 2F(Q) + Q(m + bm + v - bv)))}{\left(\frac{b(Qv+F(Q)+A)}{Q(b-1)}\right)^b n(A + F(Q) + vQ)^2}. \tag{4.61}$$

Since $\frac{abQ\left(\frac{b(Qv+F(Q)+A)}{Q(b-1)}\right)^{-b}}{n(A+F(Q)+vQ)^2}$ is always positive, the sign of $\frac{\partial^2 \prod_S(v)}{\partial v^2}$ changes at

$$v^{''} = \frac{(1+b)K + 2(A + F(Q))n + (1+b)mnQ}{(b-1)nQ}. \tag{4.62}$$

For the wholesale price range $[0, v^{''}]$ the profit function $\Pi_S(v)$ is convex with respect to v. Since the sign of $\frac{\partial \prod_S(v)}{\partial v}$ is negative for the wholesale price range $(v^{''}, \infty)$, optimal wholesale price v^* is given by $v^* = v^{'}$ if $v_{j+1} \leq v^{'} \leq v_j$. We can conclude that $v^* = v_{j+1}$ if $v^{'} < v_{j+1}$ and $v^* = v_j$ if $v_j < v^{'}$.

Case 2: If n is constant and Q is not constant, order quantity is approximated as a linear decreasing function of v. Let this function be $Q(v) = c - dQ$ for a wholesale price range $[v_{j+1}, v_j]$. Replacing Q with $c - dQ$ in Equation (4.57), we approximate the best solution in this interval through a line search over v values in the $[v_{j+1}, v_j]$ range maximizing the supplier's profit.

4.4.1.3 Derivation of the optimal lot size multiplier under a fixed value of the wholesale price

The first and second derivatives of the supplier's profit function (4.45) with respect to n are given by (4.11) and (4.12) respectively. Furthermore, the optimality conditions for n^* as given in (4.13) and (4.14) are the same with the equations (4.15) and (4.16). Hence, the optimal lot size multiplier is given by (4.19) and the remaining part of the derivation is not repeated here.

Note that above argument is valid for a fixed value of v. Since v^* depends on n from (4.60) and n^* depends on v from (4.19), an iterative procedure is required for finding the optimal decisions. We propose the following iterative process:

Step 1: Start with $n = 1$,

Step 2: Using n find v^* from (4.60) and check the optimality condition (4.19). If the

condition is not met, increment n by 1 and go to Step 2. Otherwise, optimal decisions are found.
We conjecture that the procedure provides the optimal lot size multiplier, although we cannot prove its optimality.

4.4.1.4 The Approximate Algorithm

In this section, we present the approximate algorithm for solving the supplier's problem by combining the results that have been developed in the previous sections.

In order to solve the supplier's problem, first the buyer's response has to be characterized. Since the buyer's optimal decision cannot be expressed in closed form, supplier's decision cannot be immediately derived. The optimal market price as a function of the order quantity can be determined by using Equation (4.49). Substituting this into the profit function, we obtain Equation (4.52). Using this function, we can then characterize the order quantity as the wholesale price changes in Section 4.4.1.1. Hence we propose a search procedure with respect to the wholesale price. The algorithm can be formally stated as follows:

Step 1: Characterize the optimal response of the buyer utilizing the procedure outlined in Section 4.4.1.1.

Step 2: Let the optimal profit $\prod_S^*$ be 0 with $v = 0$.

Step 3: Consider each wholesale price interval where the truck option changes in the response profile generated in Step 1, and complete the following steps for each interval:

3.1: If the order quantity is constant, let $n = 1$ and go to the next step. If not, replace the linear approximation function with Q in the supplier's profit function (4.57). Find the optimal wholesale price maximizing the profit function through a line search over (4.57) as an approximation, and go to Step 3.5. (We note that, as shown in Section 4.4.1.3, when the wholesale price and the order quantity is fixed, the optimal lot size multiplier can be readily computed in each objective function evaluation of the line search.)

3.2: Compute the optimal wholesale price by (4.60).

3.3: Let the endpoints of the interval be v_{start} and v_{end}. If the optimal wholesale price of Step 3.2 is less than v_{start}, equate the wholesale price to v_{start}, whereas if the wholesale is price greater than v_{end}, equate the wholesale price to v_{end}

3.4: If the optimality condition for the lot size multiplier holds by (4.19), go to Step 3.5. Else, increase n by 1 and go to Step 3.2.

3.5: For the current wholesale price interval, compute the profit with the optimal wholesale price and the lot size multiplier by (4.57) and compare it with the optimal profit. If it is greater than the optimal profit, update the optimal profit, the wholesale price and the lot size multiplier.

4.4.1.5 A Numerical Illustration of The Algorithm

Let us take an example with parameters $a = 54496$, $b = 1.5166$, $A = \$435.67$/order, $K = \$222.8$/order, $I = 35.572\%$, $m = \$1.9688$/unit, $C = 27.523$, $R = \$60$, $T = 20$. First, we find the optimal response of the buyer using the procedure outlined in Section 4.4.1.1.

Step 1:

- For the single-truck option, we find $v_1 = 56.58$ for solving (4.55) for $Q = C = 27.523$. Let $v_{max} = 1.2v_1$, which corresponds to $v_{max} = 67.89$. Solving the optimal order quantity for $v_{max} = 67.89$ leads to $Q = 21.64$ with profit 1733.98. Hence we linearly approximate the optimal order quantity between $[v_1 = 56.58, Q(v_1) = 27.523]$ and $[v_{max} = 67.89, Q^*(v_{max}) = 21.64]$ as $Q = 57.27 - 0.52v$.

- Next, the profit function for the two-truck option are evaluated at the endpoints of the interval. We find $v_2 = 34.54$ for solving $Q = 2 \times C = 55.046$ with profit 2535.35, whereas the single-truck option at the wholesale price 34.54 provides the profit 2480.82. Hence, in this interval best response is using the two-truck option for small values, and, after a break even point, using the single-truck option. Through a line search, we find that the break even point is $v = 40.39$ where the single-truck option with $Q = 27.523$ and two-truck option with $Q = 44.83$ both generate a profit of 2311.35. This is illustrated in Figure 4.2. Consequently we form the upper envelope by approximating the order quantity in the first part and using the full-truck option in the second part as it is graphically shown in Figure 4.3. Note that we approximate the order quantity as a linear function between the points $[v_2 = 34.54, Q(v_2) = 55.046]$ and $[b_1 = 40.39, Q^*(b_1) = 44.83]$. Additionally, the order quantity after $v = 40.41$ is constant and equals to 27.523.

- For the three-truck option, we find $v_3 = 26.16$ for solving $Q = 3 \times C = 82.569$ with profit 2946.08. In the interval $[26.16, 34.54)$ we now have three functions to compare. At $v = 34.54$, the three-truck option lead to $Q = 57.3$ with profit 2489.0. Comparing

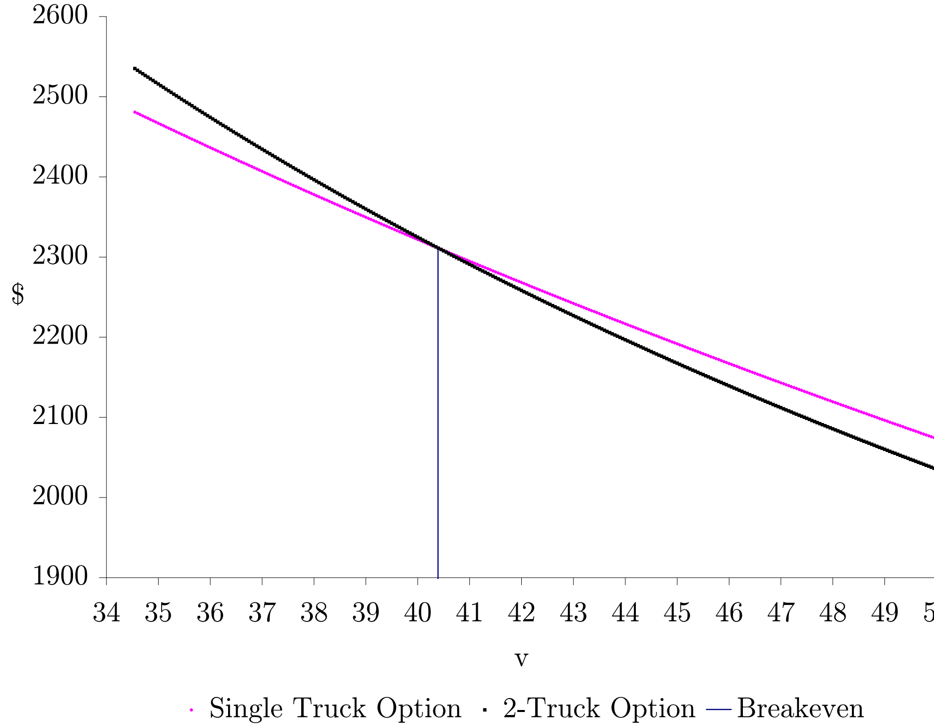

Figure 4.2: Break-even point in the analysis of buyer's response.

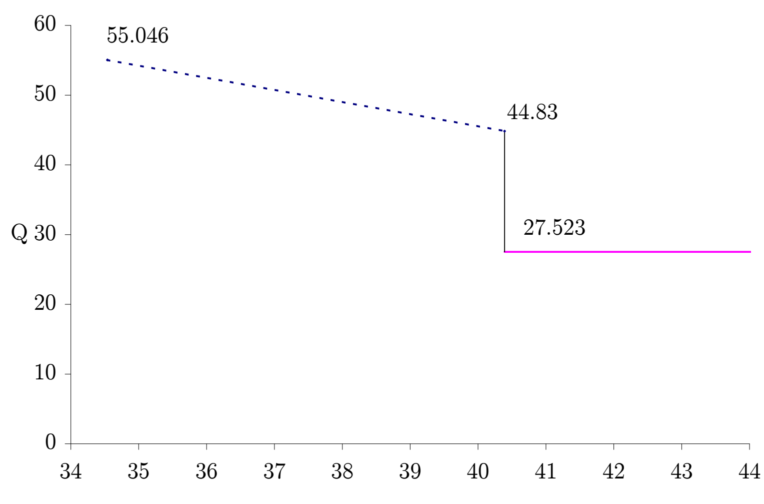

Figure 4.3: Characterization of the order quantity.

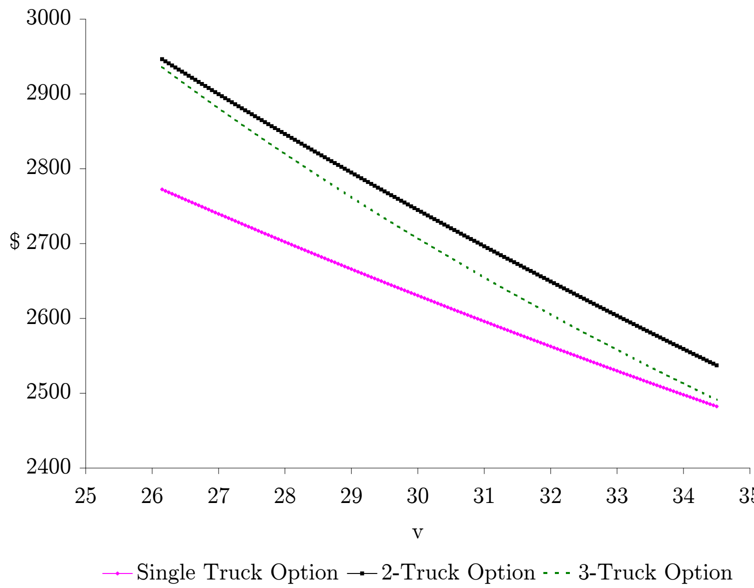

Figure 4.4: Profit of the buyer.

these endpoint values with the single-truck and two-truck options, which is illustrated in Figure 4.4, the two-truck option dominates the other options. Since the two-truck option leads to higher profit at both of the endpoints, optimal response is $Q = 55.046$ in this interval. Note that the profit function is convex and it decreases as the wholesale price increases. Therefore the three-truck option is dominated and we do not need to evaluate this function within this interval.

- After evaluating v_1, v_2, v_3 and finding the response of the buyer, we similarly evaluate $v_4, v_5, ..., v_{12}$ and find the optimal response. Finally, the buyer's order quantity response as a function of the wholesale price is illustrated in Figure 4.5. Note that the order quantity is approximated only in the intervals $[34.54, 40.39]$ and $[56.58, 67.89]$. In the remaining intervals, order quantity response is available as an exact solution.

Step 2:

- After characterizing the order quantity, we start searching for the optimal wholesale price. The first interval is $v \in [0, 3.33]$ where order quantity is constant as 550.46. For $n = 1$ we solve (4.60) and the resultant wholesale price is 12.72 satisfying (4.19).

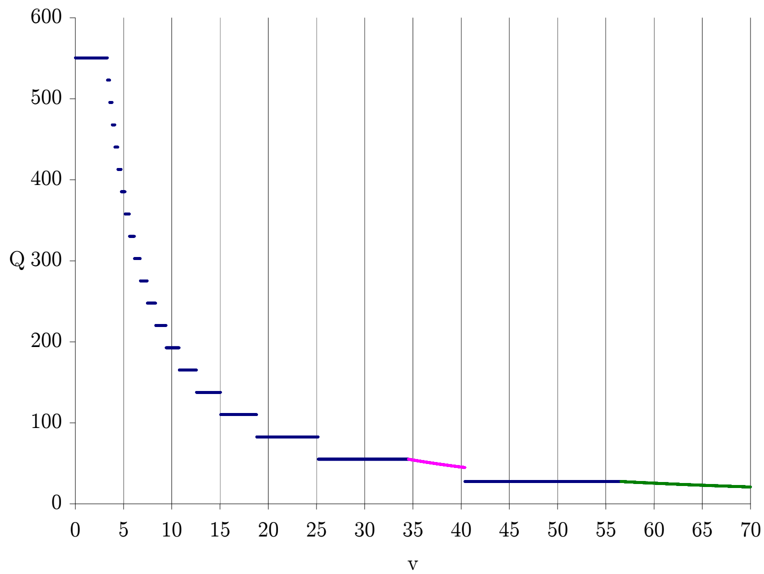

Figure 4.5: Optimal response of the buyer.

Since this value is greater than 3.33, optimal profit in this interval is obtained with the point $v = 3.33$ and $n = 1$. We update the current optimal profit as 624.10 with the optimal decisions.

- The next interval is $v \in (3.24, 3.56]$ where order quantity is constant as 522.937. For $n = 1$ we solve (4.60) and the resultant wholesale price is 12.86 satisfying (4.19). Since this value is greater than 3.56, optimal profit in this interval is the point $v = 4.56$ and $n = 1$. We update the optimal profit as 713.14 with the optimal decisions.

- We continue to apply the procedure. Note that we have not approximated the order quantity but exactly found the solution for the first two intervals. Omitting the next several interval evaluations, where the optimal profit is found to be 1342.41 with $v = 10.77$, $n = 2$ and $Q = 192.66$, let us consider the interval $v \in (34.54, 40.39]$ where the order quantity is linearly approximated between 55.046 and 44.83. Replacing Q with the linear approximation function in the supplier's profit function (4.57), optimal wholesale price is found by a line search. In this interval, the best profit is generated by $v = 34.54$ leading to 1007.60. Since $1007.60 < 1342.41$, the optimal profit is not

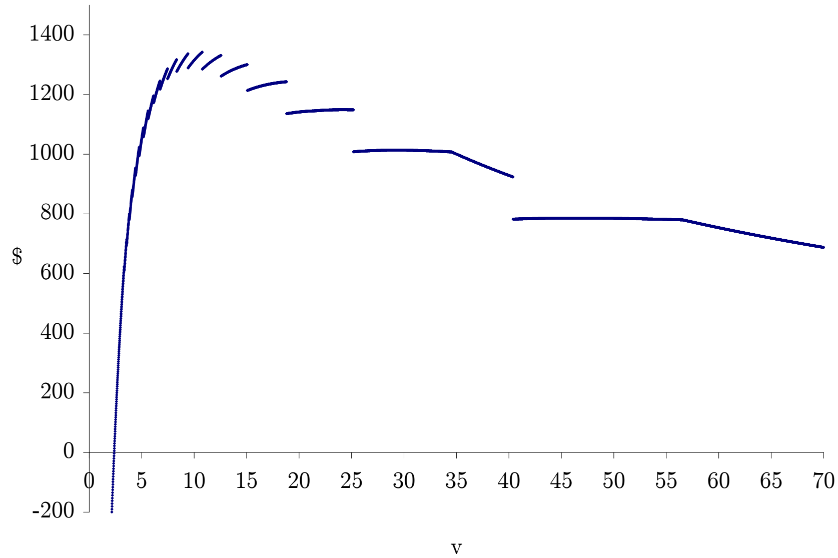

Figure 4.6: Profit of the supplier.

updated.

- After completing the evaluation of the remaining intervals, the optimal profit for the supplier is found to be 1342.41 with $v = 10.77$, $n = 2$. The supplier's profit function is shown in Figure 4.6 and Figure 4.7 illustrates the details from the wholesale price range $[5, 15]$ with additional information on the order quantity. Note that the algorithm determines the break-points where the order quantity changes and the supplier's problem is solved in each interval. The profit functions within these intervals are smooth and the order quantity changes cause the supplier's function to shift.

4.4.1.6 A Note on The Performance of The Algorithm

Let us take another example with parameters $a = 100000$, $b = 5$, $A = \$125$/order, $K = \$250$/order, $I = 25\%$/year, $m = \$1$/unit, $C = 1000$, $R = \$50$. Grid Search with 10^{-3} precision finds the best solution as $\prod_S(1.233) = \$1202.38$. Moreover, the approximation algorithm leads to the profit $\prod_S(1.233654) = \$1204.66$. Hence it leads to a slightly better solution then the grid search. As the precision of the grid search is decreased, the solution converges to the solution of the approximation algorithm. However, there are some cases that approximation algorithm perform better than the grid search regardless of the

Figure 4.7: Wholesale price intervals.

grid search precision. Since optimal wholesale price may be computed by (4.60) in the approximation algorithm, optimal solution can be explicitly known.

4.4.2 Grid Search

In this section, we present the grid search approach for solving the supplier's problem. The optimal wholesale price v and corresponding profit for the supplier is determined numerically by a grid search within the range $[m, v_{max}]$. For a fixed wholesale price v, optimal price can be stated as $p^*(Q) = \frac{b(A+Qv+F(Q))}{Q(b-1)}$, which is given in (4.49). A line search for Q through solving $\left(-\frac{vI}{2} + \frac{(A+tR)a\left(\frac{b(A+Qv+tR)}{Q(b-1)}\right)^{-b}}{Q^2}\right) = 0$ gives the optimal order quantity, which is provided in (4.55). Finally, optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$, which is stated in (4.19). We adopt a grid size $g = 10^{-3}$ in the computations. An algorithm similar to the one of Section 4.1.1 is used with the decision variables and profit functions mentioned above. Since the buyer needs to determine the profit function for T many truck options, T line searches are performed to find the order quantity in the algorithm.

4.4.3 Myopic Approach

In this section, we present the algorithm of myopic approach for solving the supplier's problem. The supplier optimizes his profit assuming that the buyer does not incur transportation cost. Hence, the supplier determines the wholesale price according to the results of Section 4.1. However, the buyer determines her decision variables taking the transportation cost into account for the wholesale price of the supplier. Finally, the supplier re-determines his lot size multiplier according to the buyer's actions.

The optimal wholesale price v and corresponding profit for the supplier is determined numerically by a grid search within the range $[m, v_{max}]$. For a fixed wholesale price v, optimal price can be stated as $p^*(Q) = \frac{b(A+Qv)}{Q(b-1)}$, which is given in (4.5). A line search for Q through solving $\left(-\frac{vI}{2} + \frac{(A)a\left(\frac{b(A+Qv)}{Q(b-1)}\right)^{-b}}{Q^2}\right) = 0$ gives the optimal order quantity, which is provided in (4.10). The optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$, which is stated in (4.19). After wholesale price is determined, optimal price is determined by $p^*(Q) = \frac{b(A+Qv+F(Q))}{Q(b-1)}$ which is stated in (4.49). Afterwards, optimal order quantity is determined by solving $\left(-\frac{vI}{2} + \frac{(A+tR)a\left(\frac{b(A+Qv+tR)}{Q(b-1)}\right)^{-b}}{Q^2}\right) = 0$, which is stated in (4.55). Finally, optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$. We adopt a grid size $g = 10^{-3}$ in the computations.

The algorithm can be formally stated as follows:

Step 1: Let the optimal profit $\prod_S^*$ be 0 with $v = 0$ and $n = 0$.

Step 2: Start with $v = m$ and complete the following steps for each wholesale price from the set $\{m, m+g, m+2g, ..., v_{max}-2g, v_{max}-g, v_{max}\}$.

2.1: Find Q through a line search solving (4.10), compute p as defined in (4.5) and $\prod_B(Q)$ as defined in (4.2).

2.2: Compute n and $\prod_S(v)$ as defined in (4.19) and (4.2) respectively. If $\prod_S(v)$ is greater than the optimal profit, update the optimal profit $\prod_S^*$, the wholesale price v^* and the lot size multiplier n^*.

Step 3: Solve (4.55) for v^* and determine $Q^*(v)$. Recalculate n^* by (4.19).

In order to emphasize the significance of the freight cost effect, we illustrate an extreme case, and compare the results. Let us take an example with parameters $a = 100000$, $b = 5$, $A = \$125$/order, $K = \$250$/order, $I = 25\%$/year, $m = \$1$/unit, $C = 200$, $R = \$100$. If the supplier does not consider the buyer's freight cost, he optimizes his profit with wholesale price $v^* = \$1.281$, and expects to achieve a profit of $\prod_S = \$1494.8$ assuming that the buyer will choose her order quantity as $Q = 2472$. With the wholesale price $v = \$1.281$, the buyer does not select $Q = 2472$ but $Q^* = 800$ since she considers the freight cost. After observing Q^*, the supplier sets the lot size multiplier $n = 2$. Finally he receives a profit of $\prod_S = \$49.82$. Had the supplier considered the freight cost, he would have optimized his profit at the wholesale price $v^* = \$1.63$. Consequently the buyer would have set $Q = 600$, and then he would have set $n = 2$. Ultimately he would have received $\prod_S = \$122.65$. Comparing \$122.65 and \$49.82, the supplier misses 59.57% of his optimal profit by ignoring the freight cost.

4.4.4 Golden Section Search: A Heuristic

In this section, we adopt the golden section search algorithm for solving the supplier's problem. The details of the procedure have been previously stated in Section 4.1.2. An algorithm similar to the one of the section is used with the decision variables and profit functions mentioned above. Since the buyer needs to determine the profit function for T many truck options, T line searches are performed to find the order quantity in the algorithm.

4.5 Centralized System with Transportation Cost

We can state the profit function as follows:

$$\prod_C(p,Q,n) = (p-m)D(p) - (h_s(n-1)+h_b)\frac{Q}{2} - \left(\frac{K}{n} + A + F(Q)\right)\frac{D(p)}{Q} - \frac{I\,F(Q)}{2} \tag{4.63}$$

Fixing $F(Q)$ as tR, first order optimality condition can be stated as below.

$$\frac{\partial \prod_C(p,Q,n)}{\partial Q} = -\frac{h_s(n-1)+h_b}{2} + \left(\frac{K}{n} + A + tR\right)\frac{D(p)}{Q^2} = 0 \tag{4.64}$$

Solving (4.64) gives the optimal order quantity as follows:

$$Q^* = \sqrt{\frac{2\left(\frac{K}{n} + A + tR\right)D(p)}{h_s(n-1)+h_b}} \tag{4.65}$$

Replacing Q^* with Q in (4.63) we can rewrite the profit function as

$$\prod_C(p,n) = (p-m)D(p) - (h_s(n-1)+h_b)\frac{Q^*}{2} - \left(\frac{K}{n} + A + tR\right)\frac{D(p)}{Q^*} - \frac{I\,tR}{2}. \quad (4.66)$$

The profit function can be restated as follows:

$$\prod_C(p,n) = (p-m)D(p) - \sqrt{2D(p)\left(\frac{K}{n} + A + tR\right)(h_s(n-1)+h_b)} - \frac{I\,tR}{2} \quad (4.67)$$

Replacing $\left(\frac{K}{n} + A + tR\right)(h_s(n-1)+h_b)$ with $L(n)$, we rewrite the profit function as

$$\prod_C(p,n) = (p-m)D(p) - \sqrt{2D(p)L(n)} - \frac{I\,tR}{2}. \quad (4.68)$$

Maximizing $\prod_C(p,n)$ is equivalent to minimizing $L(n)$ with respect to n . Since n is integer and $L(n)$ is convex, n^* satisfies the following equations. (n denotes n^* in this section)

$$L(n) \leq L(n-1) \;\&\; L(n) \leq L(n+1) \quad (4.69)$$

Replacing $L(n)$ and $L(n-1)$ in the first equality, we get

$$\left(\frac{K}{n} + A + tR\right)(h_s(n-1)+h_b) - \left(\frac{K}{n-1} + A + tR\right)(h_s(n-2)+h_b) \leq 0. \quad (4.70)$$

The roots of

$$\left(\frac{K}{n} + A + tR\right)(h_s(n-1)+h_b) - \left(\frac{K}{n-1} + A + tR\right)(h_s(n-2)+h_b) = 0 \quad (4.71)$$

are given by $\frac{1}{2}\left(1 - \sqrt{1 + 4\frac{(A+tR)h_s}{K(h_b-h_s)}}\right)$ and $\frac{1}{2}\left(1 + \sqrt{1 + 4\frac{(A+tR)h_s}{K(h_b-h_s)}}\right)$. The inequality is satisfied in between these roots. Applying the same procedure to the second equality,

$$\left(\frac{K}{n} + A + tR\right)(h_s(n-1)+h_b) - \left(\frac{K}{n+1} + A + tR\right)(h_s(n)+h_b) \leq 0 \quad (4.72)$$

The roots of

$$\left(\frac{K}{n} + A + tR\right)(h_s(n-1)+h_b) - \left(\frac{K}{n+1} + A + tR\right)(h_s(n)+h_b) = 0 \quad (4.73)$$

are given by $\frac{1}{2}\left(-1 - \sqrt{1 + 4\frac{(A+tR)h_s}{K(h_b-h_s)}}\right)$ and $\frac{1}{2}\left(-1 + \sqrt{1 + 4\frac{(A+tR)h_s}{K(h_b-h_s)}}\right)$. The inequality is not satisfied in between the roots. Since both inequalities must be satisfied, n is between $\frac{1}{2}\left(-1 + \sqrt{1 + 4\frac{(A+R_lB_l)h_s}{K(h_b-h_s)}}\right)$ and $\frac{1}{2}\left(1 + \sqrt{1 + 4\frac{(A+R_lB_l)h_s}{K(h_b-h_s)}}\right)$. The interval is bounded to 1 and since n is integer, we can write n^* as

$$n^* = \left\lfloor \frac{1}{2}\left(1 + \sqrt{1 + 4\frac{(A+tR)h_s}{K(h_b-h_s)}}\right)\right\rfloor \text{ or } \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + 4\frac{(A+tR)h_s}{K(h_b-h_s)}}\right)\right\rceil \quad (4.74)$$

Replacing n^*, we can now rewrite the profit function as

$$\prod_C(p) = (p-m)ap^{-b} - \sqrt{2ap^{-b}L(n^*)} - \frac{I\,tR}{2} \tag{4.75}$$

First order optimality condition is given as follows:

$$\frac{\partial \prod_C(p)}{\partial p} = \frac{p^{-1-b}}{2}\left(\sqrt{2}bp^b\sqrt{aL(n)p^{-b}} + 2a(b(m-p)+p)\right) = 0. \tag{4.76}$$

$\prod_C(p)$ is maximized at the minimum value of p for which $\frac{\partial \prod_C(p)}{\partial p} \geq 0$. Hence, optimal price is found through a line search using (4.76) Since T is the maximum number of trucks, the resultant profit functions of T many truck options are evaluated and the highest profit is selected with the associated decision variables.

4.6 Computational Results

4.6.1 Problem Generation

We test the algorithms' effectiveness over a set of problems that are created by considering various parameter values. Initially, we fix the unit procurement cost of the supplier to \$1, and express all other cost and price parameters in multiples of the unit procurement cost.

We let the supplier's order placement cost be 10 times, 25 times, 100 times, and 250 times the unit procurement cost. The buyer's order placement cost is designed to be 50%, 100%, 200% of the supplier's order placement cost. The capacity of a single truck is assumed to be 200, 400, and 1000 units. The truck cost is assumed to be \$10, \$20, \$50, and \$100. For the demand function, we fix the parameter a to 100000, and select the price elasticity parameter b as 1.25, 2, 3.5, and 5. Finally, we take the annual holding cost rate as 25% and v_{max} as $1.2v_1$. The combinations of these parameters lead to a set of 576 problems. The algorithms not incorporating transportation cost are tested with a set of 144 problems. They do not consider transportation cost however truck capacities are valid for the problems. Table 4.1 summarizes the parameter values used in creating the test problems. We solve five different problems with the algorithms mentioned. In the approximation algorithms, for finding the optimal shipment size for a given v, the line search procedure is terminated when the length of the uncertainty interval is less than 0.001. The golden section search algorithm for solving the optimal order quantity is used as a line search method in all of

Table 4.1: Parameter values in computational analysis.

m	1
a	100000
I	25%
K	$\{10, 25, 100, 250\}$
A	$\{0.5K, K, 2K\}$
b	$\{1.25, 2, 3.5, 5\}$
C	$\{200, 400, 1000\}$
R	$\{10, 20, 50, 100\}$

the problems. The length of the uncertainty interval is 0.001 in the line search. We also run the grid search with increments of size 0.001.

Under this precision scheme, the test problems have been solved in Matlab 6.5 in Microsoft Windows XP on a computer with an Intel Pentium M 1.60 GHz processor, and 512 MB of RAM.

4.6.2 Performance of the Algorithms Considering Profit Function

The summary of the computational results are provided in Table 4.2. It displays the ratios of the supplier's profit obtained by the algorithm to the supplier's profit obtained by the grid search for the decentralized case. This is an indicator of the quality of the solutions generated by our approximation approach. Cells include the minimum, maximum, average and standard deviation values for the performance measures. We also report the performance of the myopic approach, where the supplier chooses the wholesale price by neglecting the buyer's transportation cost. We have given a detailed example illustrating this approach in Section 4.4.3. The figures reported under the myopic approach column correspond to the percentage of the optimal profit (as computed by the grid search) that the supplier can capture when the transportation costs are neglected.

The maximum performance of the approximation algorithm is higher than golden section search algorithm for the supply chain modeling where the buyer owns the transportation

Table 4.2: Performance of the algorithms considering supplier's profit.

S. Chain	Method	Section	Minimum	Maximum	Average	S. Deviation
SC_1	Golden Section S.	4.1.2	0.999980	1.000140	1.000001	0.000013
SC_3	Approximation A.	4.3.1	0.000000	1.000000	0.940110	0.127242
	Golden Section S.	4.3.3	0.000000	1.001053	0.931347	0.229649
SC_4	Approximation A.	4.4.1	0.535505	1.001893	0.984540	0.050154
	Myopic A.	4.4.3	0.404301	1.000000	0.974522	0.048081
	Golden Section S.	4.4.4	0.814464	1.001607	0.995880	0.014674

SC_1: Decentralized system without transportation cost
SC_3: Decentralized system where the supplier owns the transportation cost
SC_4 :Decentralized system where the buyer owns the transportation cost

cost. The average values are similar for three algorithms, whereas the golden section search offers the best performance and the myopic approach naturally offers the worst performance. Besides, some values in Table 4.2 are over 1. The reason for this is the base performance is obtained by the grid search with the grid size 10^{-3}. Since approximation algorithm and golden section search algorithm may find better solutions, the results may outperform the grid search algorithm with the grid size 10^{-3}. Factor analysis for the computational results is provided in Table 4.3 and Table 4.4. The factor analysis is based on parameters K, A, b, C, and R. Treatment levels are stated in the second column. Moreover, minimum, mean and maximum values of the algorithms are displayed. Algorithms of Section 4.1 and Section 4.3 are evaluated in Table 4.3 whereas algorithms of Section 4.4 are evaluated in Table 4.4.

On average, the approximate algorithm achieves 98.45% and 94.01% of the profits that can be achieved by the grid search for the supply chains in Sections 4.3 and 4.4 respectively. The proposed algorithm works on an approximation with fixed truck options, and when the optimal shipment size is less than or equal to a single truck load, the performance of the algorithm decreases. This is a natural outcome, because, if the optimal shipment size is less

Table 4.3: Factor analysis for algorithms of Section 4.1 and Section 4.3.

	Section:	4.1.2 GSS			4.3.1 AA			4.3.3 GSS		
Factor	T. Level	MIN	MEAN	MAX	MIN	MEAN	MAX	MIN	MEAN	MAX
K	10	1.00	1.00	1.00	0.00	0.92	1.00	0.00	0.86	1.00
	25	1.00	1.00	1.00	0.00	0.93	1.00	0.00	0.94	1.00
	100	1.00	1.00	1.00	0.32	0.95	1.00	0.00	0.97	1.00
	250	1.00	1.00	1.00	0.00	0.96	1.00	0.00	0.96	1.00
A	$0.5K$	1.00	1.00	1.00	0.00	0.96	1.00	0.00	0.97	1.00
	K	1.00	1.00	1.00	0.00	0.94	1.00	0.00	0.95	1.00
	$2K$	1.00	1.00	1.00	0.00	0.92	1.00	0.00	0.87	1.00
b	1.25	1.00	1.00	1.00	0.52	0.96	1.00	0.90	1.00	1.00
	2	1.00	1.00	1.00	0.78	0.98	1.00	0.76	0.99	1.00
	3.5	1.00	1.00	1.00	0.59	0.95	1.00	0.00	0.93	1.00
	5	1.00	1.00	1.00	0.00	0.87	1.00	0.00	0.81	1.00
C	200	1.00	1.00	1.00	0.00	0.91	1.00	0.00	0.92	1.00
	400	1.00	1.00	1.00	0.16	0.95	1.00	0.00	0.93	1.00
	1000	1.00	1.00	1.00	0.52	0.96	1.00	0.00	0.94	1.00
R	10	1.00	1.00	1.00	0.52	0.98	1.00	0.96	1.00	1.00
	20	1.00	1.00	1.00	0.71	0.98	1.00	0.00	0.97	1.00
	50	1.00	1.00	1.00	0.63	0.94	1.00	0.00	0.96	1.00
	100	1.00	1.00	1.00	0.00	0.86	1.00	0.00	0.80	1.00

Table 4.4: Factor analysis for algorithms of Section 4.4.

	Section:	4.4.1 AA			4.4.4 GSS			4.4.3 MA		
Factor	T. Level	MIN	MEAN	MAX	MIN	MEAN	MAX	MIN	MEAN	MAX
K	10	0.54	0.98	1.00	0.96	1.00	1.00	0.89	0.99	1.00
	25	0.68	0.99	1.00	0.90	1.00	1.00	0.82	0.98	1.00
	100	0.79	0.99	1.00	0.90	0.99	1.00	0.76	0.97	1.00
	250	0.55	0.98	1.00	0.81	0.99	1.00	0.40	0.96	1.00
A	$0.5K$	0.78	0.99	1.00	0.90	1.00	1.00	0.76	0.98	1.00
	K	0.66	0.98	1.00	0.81	0.99	1.00	0.40	0.97	1.00
	$2K$	0.54	0.98	1.00	0.90	1.00	1.00	0.41	0.98	1.00
b	1.25	0.54	0.97	1.00	0.98	1.00	1.00	0.95	0.99	1.00
	2	0.97	1.00	1.00	0.97	1.00	1.00	0.93	0.99	1.00
	3.5	0.85	0.99	1.00	0.86	0.99	1.00	0.85	0.97	1.00
	5	0.55	0.97	1.00	0.81	0.99	1.00	0.40	0.95	1.00
C	200	0.55	0.99	1.00	0.94	1.00	1.00	0.40	0.96	1.00
	400	0.74	0.99	1.00	0.95	1.00	1.00	0.78	0.98	1.00
	1000	0.54	0.98	1.00	0.81	0.99	1.00	0.83	0.98	1.00
R	10	0.54	0.98	1.00	0.93	1.00	1.00	0.90	0.99	1.00
	20	0.73	0.98	1.00	0.93	1.00	1.00	0.83	0.99	1.00
	50	0.85	0.99	1.00	0.96	1.00	1.00	0.86	0.98	1.00
	100	0.55	0.98	1.00	0.81	0.99	1.00	0.40	0.95	1.00

than a truck load, there is no need to incorporate the transportation cost, and it can be taken as fixed. Therefore, omitting the problems having less than or equal to a single truck load shipment size the approximate algorithms achieves 94.58% and 98.94% of the profit that can be achieved with the grid search respectively.

Moreover, an important consideration is that the algorithm is specially designed for the supply chain there the buyer owns the transportation cost. Computational results show that the algorithm outperforms existing methods and when more than one truck load order is optimal, the algorithm provide excellent results.

When we consider the myopic approach, the average profit obtained deviates 2.25% from the profit obtained by the grid search. However, there are instances where the deviation is about 60%, such as the example in reported Section 4.4.3.

The third column of Table 4.3 shows that golden section search algorithm performs very well for the supply chain model without transportation cost. For the supply chain model with FOB origin, minimum performances of the algorithms are not promising. The main reason is that golden section search algorithm may be trapped in a local optimal point, whereas the approximation algorithm may provide loose approximations and miss the optimal solution. As capacity of a single truck increases, the minimum performance of approximation increases, whereas the does not change for golden section search algorithm. As the elasticity parameter of the demand function increases, the approximation algorithm outperforms the golden section search algorithm on the average.

Table 4.4 studies the supply chain modeling with transportation cost and the performance of golden section search algorithm is quite fine on the average. Furthermore, when the effect of the experimental factors are studied in detail, it can be observed that ignoring transportation costs may be more costly for large values of K, b, and R. In other words, when the supplier's order placement cost is large, or the demand is highly sensitive to price, or when the transportation costs are high, potential benefits of employing our approach might be larger.

4.6.3 Performance of the Algorithms Considering CPU Time

The CPU times of the algorithms are provided in Table 4.5. It displays the ratios of the CPU time of the algorithms to the CPU time of grid search algorithm for the decentralized

Table 4.5: Performance of the algorithms considering CPU times.

S. Chain	Method	Section	Minimum	Maximum	Average	S. Deviation
SC_1	Golden Section S.	4.1.2	0.000000	0.041899	0.005683	0.006196
SC_3	Approximation A.	4.3.1	0.000000	0.073111	0.005028	0.008180
	Golden Section S.	4.3.3	0.000000	0.051195	0.007038	0.007537
SC_4	Approximation A.	4.4.1	0.000000	0.009840	0.000442	0.000470
	Golden Section S.	4.4.4	0.002220	0.004376	0.002728	0.000208

case. The ratios reveal the efficiency of the algorithms utilized in the thesis. On the average grid search algorithm takes 1.44 time seconds for the supply chain model in Section 4.1, 1.25 time seconds for Section 4.3, and 34.60 time seconds for Section 4.4.

Although maximum CPU times of the golden section search algorithms are higher than maximum CPU time of the approximation algorithm for Sections 4.3 and 4.4, the approximation algorithm is faster on the average. On the average, it is 1.4 times faster for Section 4.3, and it is 6.17 times faster for Section 4.4, comparing with the golden section search algorithm.

Chapter 5

SUPPLY CHAIN COORDINATION

In this chapter, we evaluate the performance of the following coordination mechanisms

- Transportation Cost Sharing Contract (TCSC)
- Quantity Discounts (QD)
- Volume Discounts (VD)
- Simultaneous Offer of Quantity and Volume Discounts (QVD)

We propose the transportation cost sharing contract, which requires the supplier to share a predetermined portion of the transportation cost of the buyer. Quantity and volume discounts are the common coordination mechanisms that are extensively studied in the literature. We model these mechanisms together with the simultaneous offer of quantity and volume discounts considering transportation cost and provide numerical analysis.

The study in this chapter is extensively computational and numerical. Viswanathan and Wang [74] model discount mechanisms through a grid search. The paper neglects the transportation cost. Despite the difficulty of the problem, we show that transportation cost can be incorporated into the problem, more realistic and accurate results can be obtained.

5.1 Coordination

Section 4.4 considers the decentralized supply chain where the buyer owns the transportation cost, whereas Section 4.5 considers the centralized supply chain with transportation cost. Due to the double marginalization effect, there is an important gap between the total supply chain profits in these systems. The gap between $\prod_C$ and $(\prod_B + \prod_S)$ derive an important motivation for the supply chain members of the decentralized system to recover the additional profit generated by the centralized system.

The solution of the centralized system provides the coordinated price and order quantity. The coordination mechanisms tend to induce the supply chain members to choose coordinated price and order quantity and achieve coordination in the supply chain.

5.2 Initial Market Equilibrium

In this chapter, the decentralized supply chain considered in Section 4.4 is evaluated as the base supply chain. Moreover, the equilibrium point given by the solution of the Stackelberg game is considered as the initial market equilibrium. Let v^*, n^*, p^*, Q^* be the equilibrium decisions of the game and $\prod_S(v^*, n^*), \prod_B(p^*, Q^*)$ be the equilibrium profits.

5.3 Transportation Cost Sharing Contract

The transportation cost sharing contract is modeled in this section. The mechanism requires the supplier to share a predetermined portion of the transportation cost of the buyer. The supplier shares $(1-\alpha)$ portion of the total transportation cost, i.e., when α is 0, the system is based on freight on board (FOB) origin, whereas if α is 1, the buyer pays for the transportation. α is in between 0 and 1. Distinct α values are tested and the value providing the highest profit is selected as the optimal transportation cost sharing contract.

The Buyer's Problem

To obtain profit functions, we replace $F(Q)$ with $(\alpha)F(Q)$ in (4.44) and insert $(1-\alpha)F(Q)\frac{D(p)}{Q}$ into the supplier's profit function (4.45).

The buyer's profit function can be rewritten as follows:

$$\prod_B(p, Q, \alpha) = (p-v)D(p) - (v + \frac{\alpha F(Q)}{Q})I\frac{Q}{2} - (A + \alpha F(Q))\frac{D(p)}{Q}. \tag{5.1}$$

Likewise, the supplier's profit function is:

$$\prod_S(v, n, \alpha) = (v-m)D(p^*(v)) - mI(n-1)\frac{Q^*(v)}{2} - \left(\frac{K}{n} + (1-\alpha)F(Q)\right)\frac{D(p^*(v))}{Q^*(v)}, \tag{5.2}$$

We follow the derivation steps in Section 4.4 for the buyer's problem. We obtain first and second order optimality conditions with respect to p. The second derivative changes sign after the local maximum obtained at $p^*(Q)$, and $lim_{p\to\infty} \prod_B(p, Q, \alpha) = 0$, it can be shown that $p^*(Q)$ is actually a global optimal solution. Equating the first-order condition to zero we obtain the following optimal market price:

$$p^*(Q) = \frac{b\left(A + Qv + \alpha F(Q)\right)}{Q\left(b-1\right)}. \tag{5.3}$$

We can replace $p^*(Q)$ and rewrite the buyer's profit function as follows:

$$\prod_B(Q) = (p^*(Q) - v)a(p^*(Q))^{-b} - vI\frac{Q}{2} - (A + \alpha F(Q))\frac{a(p^*(Q))^{-b}}{Q} - \frac{\alpha F(Q)I}{2}. \tag{5.4}$$

For a fixed number of trucks t, where $(t-1)C < Q \leq tC$, we can rewrite (5.4) as

$$\prod_B(Q) = (p^*(Q) - v)a(p^*(Q))^{-b} - vI\frac{Q}{2} - (A + \alpha tR)\frac{a(p^*(Q))^{-b}}{Q} - \frac{\alpha tRI}{2}. \tag{5.5}$$

We write the first-order condition for a local maximum with respect to Q for a particular t value as

$$\frac{\partial \prod_B(Q)}{\partial Q} = -\frac{vI}{2} + \frac{(A + \alpha tR)\, a(p^*(Q))^{-b}}{Q^2}, \tag{5.6}$$

and by setting it equal to zero we obtain

$$\left(-\frac{vI}{2} + \frac{(A + \alpha tR)a\left(\frac{b(A+Qv+\alpha tR)}{Q(b-1)}\right)^{-b}}{Q^2}\right) = 0 \tag{5.7}$$

Equality (5.7) is the local optimality condition for the buyer's order quantity when the number of trucks is fixed as t, and $(t-1)C < Q \leq tC$. Q^* value that satisfies Equality (5.7) cannot be expressed in closed form, however it can be determined through a line search. Since T is the maximum number of trucks, the buyer can perform T line searches using (5.7) and obtain T many Q values. The buyer designates the order quantity that provides the highest profit as her order quantity.

The Supplier's Problem

We solve the problem utilizing the grid search. If Q and n are constant, replacing $p^*(v)$ with $p^*(Q)$ as given in Equation (5.3), we can rewrite the supplier's profit function as follows.

$$\begin{aligned}\prod_S(v) = (v - m)\, a\left(\frac{b\left(A + Qv + \alpha F(Q)\right)}{Q\left(b-1\right)}\right)^{-b} - \frac{mI(n-1)Q}{2} \\ - \frac{\left(\frac{K}{n} + (1-\alpha)F(Q)\right) a\left(\frac{b(A+Qv+\alpha F(Q))}{Q(b-1)}\right)^{-b}}{Q}.\end{aligned} \tag{5.8}$$

5.3.1 Derivation of the optimal lot size multiplier under a fixed value of the wholesale price

The first and second order derivatives of the profit function are given by (4.11) and (4.12) respectively. Hence the profit function is again strictly concave with respect to n. Besides, the optimality conditions (4.13) and (4.14) reduce to (4.15) and (4.16). Hence the remaining part of the derivation is the same with Section 4.1.1.1 and we conclude that optimal lot size multiplier is given by (4.19).

5.3.1 The Algorithm for Transportation Cost Sharing Contract

In this section, we present the methodology, grid search approach, for solving the optimal transportation cost sharing contract for the supplier. The optimal contract and corresponding profit for the supplier is determined numerically by a grid search within the wholesale price range $[m, v_{max}]$ and sharing ratio $(1-\alpha)$ range [0,1]. For a fixed wholesale price v and sharing ratio $1-\alpha$, optimal price can be stated as $p^*(Q) = \frac{b(A+Qv+\alpha F(Q))}{Q(b-1)}$, which is given in (5.3). A line search for Q through solving $\left(-\frac{vI}{2} + \frac{(A+\alpha tR)a\left(\frac{b(A+Qv+\alpha tR)}{Q(b-1)}\right)^{-b}}{Q^2}\right) = 0$ gives the optimal order quantity, which is provided in (5.7). Finally, optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$, which is stated in (4.19). We adopt a grid size $g = 10^{-3}$ for the wholesale price and $g = 10^{-2}$ for the sharing ratio in the computations.

The algorithm can be formally stated as follows:

Step 1: Let the optimal profit $\prod_S^*$ be $\prod_S^*(v^*, n^*)$ with $\alpha = 1$.

Step 2: Start with $\alpha = 0$ and complete the following steps for each sharing ratio from the set $\{0, g, 2g, ..., 1-2g, 1-g, 1\}$.

2.1: Let the best profit $\prod_S^*$ be 0 with $v = 0$ and $n = 0$.

2.2: Start with $v = m$ and complete the following steps for each wholesale price from the set $\{m, m+g, m+2g, ..., v_{max}-2g, v_{max}-g, v_{max}\}$.

2.2.1: For each truck option $t = 1, 2, .., T$ find Q through a line search solving (5.7), compute p as defined in (5.3) and $\prod_B(Q)$ as defined in (5.5). Select (p, Q) corresponding to the highest profit $\prod_B(Q)$ among T many truck options.

2.2.2: Compute n and $\prod_S(v)$ as defined in (4.19) and (5.8) respectively. If $\prod_S(v)$ is greater than the best profit, update the best profit $\prod_S^*$, the wholesale price v^* and the lot size multiplier n^*.

Step 3: If $\prod_S(v)$ is greater than the optimal profit, update the optimal profit $\prod_S^*$, the wholesale price v^*, the lot size multiplier n^* and the sharing ratio α.

5.4 Quantity Discounts

The motivation for the supplier to offer quantity discount is to increase the buyer's order quantity and thereby decrease his own set up and holding costs. Although the main intention is to decrease cost, the quantity discount offer may also lead to a decreased price and increase the revenue of the supply chain members. In this section, we consider the optimal quantity discount offer of the supplier. The supplier offers a wholesale price v_{QD} if the order quantity of the buyer is at least Q_{QD}. In other words, if the order quantity is larger than or equal to Q_{QD}, a quantity discount is offered on all units.

The solution of the Stackelberg game is available for the supplier and the buyer a priori. Therefore, the buyer does not accept a wholesale price v_{QD} and an order quantity Q_{QD} that would lead to a profit less than $\prod_B(p^*, Q^*)$. Hence, the order quantity Q^*_{QD} is determined by finding the value of Q^*_{QD} where $Q^*_{QD} \geq Q^*_{v^*}$ for which $\prod_B(Q_{QD}) \geq \prod_B(p^*, Q^*)$. Likewise, the supplier will not offer a quantity discount if the optimal wholesale price offer v^*_{QD} leads to a profit less than $\prod_S(v^*, n^*)$. In such a situation, we may conclude that a feasible quantity discount does not exist. However, there exists a feasible quantity discount for the supply chain for all of the examples in our test in Section 5.8.

Let us restate the profit function of the buyer as a function of the market price. Since the supplier offers a discount based on the order quantity, the buyer does not select an order quantity but a market price.

$$\prod_B(p) = (p - v)D(p) - (v + \frac{F(Q)}{Q})I\frac{Q}{2} - (A + F(Q))\frac{D(p)}{Q}. \tag{5.9}$$

The supplier offers quantity discounts (v_{QD}, Q_{QD}) where $v_{QD} < v^*$ and $Q_{QD} > Q^*$. The optimization formulation for the optimal quantity discount offer of the supplier can be stated as follows:

$$\max \prod_S(v_{QD}, n, Q_{QD}) = (v_{QD} - m)D(p^*(v_{QD})) - mI(n-1)\frac{Q_{QD}}{2} - \left(\frac{K}{n}\right)\frac{D(p^*(v_{QD}))}{Q_{QD}} \tag{5.10}$$

$$\text{s.t.} \quad \prod_B(p^*(Q_{QD})) \geq \prod_B(p^*, Q^*), \text{ where} \tag{5.11}$$

$$\prod_B (p^*(Q_{QD})) = (p^*(Q_{QD}) - v_{QD})a(p^*(Q_{QD}))^{-b} - v_{QD}I\frac{Q_{QD}}{2} - (A + F(Q_{QD}))\frac{a(p^*(Q_{QD}))^{-b}}{Q_{QD}} \tag{5.12}$$

If the buyer accepts the discount scheme where $\prod_B(p^*(Q_{QD})) \geq \prod_B(p^*, Q^*)$, through (4.49) the optimal market price can be stated as follows:

$$p^*(Q_{QD}) = \frac{b\,(A + vQ_{QD} + F(Q_{QD}))}{Q_{QD}\,(b-1)} \tag{5.13}$$

Optimal quantity offer Q_{QD} and discounted wholesale price offer v_{QD} can be determined by a grid search through the ranges $(Q^*(v^*), TR]$ and (m, v^*) respectively. Furthermore, (4.19) provides the optimal lot size multiplier for a fixed whole price, which can be restated as

$$n^* = \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right) \right\rceil \text{ or } \left\lfloor \frac{1}{2}\left(1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right) \right\rfloor. \tag{5.14}$$

5.4.1 The Algorithm for Optimal Quantity Discount

We adopt a grid size $g = 5.10^{-3}$ for the wholesale price and $g = 10^0$ for the order quantity in the computations. The algorithm can be formally stated as follows:

Step 1: Let the optimal profit $\prod_S^*$ be $\prod_S^*(v^*, n^*)$.

Step 2: Start with $v = m$ and $Q = Q^*(v^*)$ and complete the following step for each wholesale price and order quantity from the sets $\{m, m+g, m+2g, ..., v^*-2g, v^*-g, v^*\}$ and $\{Q^*(v^*), Q^*(v^*)+g, Q^*(v^*)+2g, ..., TR-2g, TR-g, TR\}$ respectively.

2.1: Compute n, $\prod_S(v_{QD})$ and $\prod_B(p^*(Q_{QD})$ as defined in (4.19), (5.10) and (5.9) respectively. If $\prod_B(p^*(Q_{QD}))$ is higher than $\prod_B(Q^*)$ and $\prod_S(v_{QD})$ is greater than the optimal profit, update the optimal profit $\prod_S^*$, the wholesale price v_{QD}^*, order quantity Q_{QD}^* and the lot size multiplier n^*.

5.5 Volume Discounts

Since the market demand is sensitive to the price, a discount on the wholesale price given by the supplier based on the annual volume of demand motivates the buyer to decrease the market price. Therefore the motivation for the supplier to offer a volume discount is to increase the market demand and thereby increase his own revenue. Assuming that the market demand is deterministic, annual demand is determined by the market price. Thus, any discount based on annual volume of demand may be associated with the market price.

In this section, we consider the optimal volume discount offer of the supplier. The supplier offers a wholesale price v_{VD} if the market price is at most p_{VD}. In other words, if the market price is lower than or equal to p_{VD}, a volume discount is offered on all units.

The solution of the Stackelberg game is available for the supplier and the buyer a priori. Therefore, the buyer does not accept a wholesale price v_{VD} and a market price p_{VD} that would lead to a profit less than $\prod_B(p^*, Q^*)$. Hence, the market price p^*_{VD} is determined by finding the value of p^*_{VD} where $p^*_{VD} \leq p^*_{v^*}$ for which $\prod_B(p_{VD}) \geq \prod_B(p^*, Q^*)$. Likewise, the supplier will not offer a volume discount if the optimal wholesale price offer v^*_{VD} leads to a profit less than $\prod_S(v^*, n^*)$. In such a situation, we may conclude that a feasible volume discount does not exist. However, there exists a feasible volume discount for the supply chain for all of the examples in our test in Section 5.8.

Proposition 3 in Appendix A proves that as the wholesale price v increases, the optimal market price $p^*(v)$ increases. The opposite of this claim is also valid and the proof follows the same steps. As the wholesale price decreases, optimal market price decreases and consequently market demand increases. Therefore, the objective of the supplier is to reduce the wholesale price through offering a discount, and force the buyer to order at a demand rate higher than his initial optimal demand.

Let us restate the profit function of the buyer as a function of the order quantity, given the market price. Since the supplier offers a discount based on the market price, the supplier does not select a market price but an order quantity.

$$\prod_B(Q) = (p - v)D(p) - (v + \frac{F(Q)}{Q})I\frac{Q}{2} - (A + F(Q))\frac{D(p)}{Q}. \tag{5.15}$$

The supplier offers volume discounts (v_{VD}, p_{VD}) where $v_{VD} < v^*$ and $p_{VD} < p^*$. The optimization formulation for the optimal volume discount offer of the supplier can be stated as follows:

$$\begin{aligned} \max \prod_S(v_{VD}, p_{VD}, n) \quad &= \quad (v_{VD} - m)D(p_{VD}) - mI(n-1)\frac{Q^*(v_{VD})}{2} \\ &\quad - \left(\frac{K}{n}\right)\frac{D(p_{VD})}{Q^*(v_{VD})} \end{aligned} \tag{5.16}$$

$$\text{s.t.} \quad \prod_B(Q^*(v_{VD})) \geq \prod_B(p^*, Q^*), \text{ where} \tag{5.17}$$

$$\prod_B (Q^*(v_{VD})) = (p_{VD} - v_{QD})a(p_{VD})^{-b} - v_{VD} I \frac{Q^*(v_{VD})}{2} - (A + F(Q^*(v_{VD}))) \frac{a(p_{VD})^{-b}}{Q^*(v_{VD})} \quad (5.18)$$

Incorporating the demand function into (5.15), the first order optimality condition for a particular truck interval, t, can be stated as follows:

$$\frac{\partial \prod_B(Q)}{\partial Q} = \frac{a\,(A + tR)\,p^{-b}}{Q^2} - \frac{iv}{2} = 0 \quad (5.19)$$

Since the second derivative is always negative as follows, the function is convex.

$$\frac{\partial^2 \prod_B(Q)}{\partial p^2} = -\frac{2\,a\ (A + tR)}{p^b\,Q^3} < 0 \quad (5.20)$$

Hence, solving (5.19) for Q gives the optimal order quantity

$$Q^*(v) = \sqrt{\frac{2a(A + tR)}{Iv\,p^b}} \quad (5.21)$$

If the buyer accepts the discount scheme where $\prod_B(Q^*) \geq \prod_B(p^*, Q^*)$, the optimal order quantity can be stated as:

$$Q^*(v_{VD}) = \sqrt{\frac{2a(A + tR)}{I(v_{VD})\,(p_{VD}^b)}} \quad (5.22)$$

The optimal market price offer p_{VD} and the discounted wholesale price offer v_{VD} can be determined by a grid search through the ranges $(m, p^*]$ and (m, v^*) respectively. Furthermore, (4.19) provides the optimal lot size multiplier for a fixed whole price, which can be restated as

$$n^* = \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil \text{ or } \left\lfloor \frac{1}{2}\left(1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rfloor. \quad (5.23)$$

5.5.1 The Algorithm for Optimal Volume Discount

We adopt a grid size $g = 2.5.10^{-2}$ for the wholesale price and $g = 5.10^{-2}$ for the market price in the computations. The algorithm can be formally stated as follows:

Step 1: Let the optimal profit $\prod_S^*$ be $\prod_S^*(v^*, n^*)$.

Step 2: Start with $v = m$ and $p = m$ and complete the following step for each wholesale and market price from the sets $\{m, m+g, m+2g, ..., v^* - 2g, v^* - g, v^*\}$ and $\{m, m+g, m+2g, ..., p^* - 2g, p^* - g, p^*\}$ respectively.

2.1: Compute n, $\prod_S(v_{VD}, p_{VD}, n)$ and $\prod_B(Q^*(v_{VD})$ as defined in (4.19), (5.16) and (5.15) respectively. If $\prod_B(Q_{VD})$ is higher than $\prod_B(Q^*)$ and $\prod_S(v_{QD})$ is greater than the optimal profit, update the optimal profit $\prod_S^*$, the wholesale price v_{VD}^*, market price p_{VD}^* and the lot size multiplier n^*.

5.6 Simultaneous Offer of Quantity and Volume Discounts

We now consider the model, where the supplier offers both quantity and volume discounts simultaneously. By offering the discounts simultaneously, the supplier's motivation is to obtain higher through larger demand volume and lower inventory costs through larger order quantities. When offering both the discounts simultaneously, the supplier tends to control the supply chain. In this mechanism, the supplier sets the market price and order quantity as well as the wholesale price and the lot size multiplier. Hence the only decision for the buyer is to whether accept the discount offer or not. Since we assume that the buyer accepts any offer providing a profit equal to $\prod_B(p^*, Q^*)$ or higher, the supplier tend to offer her exactly $\prod_B(p^*, Q^*)$. Therefore, the buyer receives zero additional profit in this mechanism.

In this section, we consider the optimal quantity and volume discount offer of the supplier. The supplier offers a wholesale price v_{QVD} if the market price is at least p_{QVD} and the order quantity is at least Q_{QVD}.

The solution of the Stackelberg game is available for the supplier and the buyer a priori. The market price p_{QVD}^* and the order quantity Q_{QVD}^* are determined by finding the value of p_{QVD}^* and Q_{QVD}^* where $p_{QVD}^* \leq p_{v^*}^*$ and $Q_{QVD}^* \geq Q_{v^*}^*$ for which $\prod_B(p_{QVD}, Q_{QVD}) \geq \prod_B(p^*, Q^*)$. Likewise, the supplier will not offer a volume discount if the optimal wholesale price offer v_{QVD}^* leads to a profit less than $\prod_S(v^*, n^*)$. In such a situation, we may conclude that a feasible discount does not exist. However, there exists a feasible discount for the supply chain for all of the examples in our test in Section 5.8.

The supplier offers quantity and volume discounts $(v_{QVD}, p_{QVD}, Q_{QVD})$ where $v_{QVD} < v^*$, $p_{QVD} < p^*$, $Q_{QVD} > Q^*$. The optimization formulation for the optimal volume discount offer of the supplier can be stated as follows:

$$\max \prod_S (v_{QVD}, Q_{QVD}, p_{QVD}, n) = (v_{QVD} - m)D(p_{QVD}) - mI(n-1)\frac{Q_{QVD}}{2} - \left(\frac{K}{n}\right)\frac{D(p_{QVD})}{Q_{QVD}} \tag{5.24}$$

$$\text{s.t.} \ \prod_B (p_{QVD}, Q_{QVD}) \geq \prod_B (p^*, Q^*) \tag{5.25}$$

Since, the supply chain variables are controlled by the supplier, the buyer's profit function can be equated to the initial market equilibrium profit and the discounted wholesale price can be determined based on the price and order quantity. Solving $\prod_B(p_{QVD}, Q_{QVD}) = \prod_B(p^*, Q^*)$ gives the optimal discounted wholesale price as follows:

$$v^*_{QVD} = \frac{2\left(p^b \prod_B(p^*, Q^*)\, Q + a\,(p\,Q - A - F(Q))\right)}{Q\,(2\,a + I\,p^b\,Q)} \tag{5.26}$$

5.6.1 The Algorithm for Optimal Quantity and Volume Discount

We adopt a grid size $g = 10^0$ for the order quantity and $g = 10^{-2}$ for the market price in the computations. The algorithm can be formally stated as follows:

Step 1: Let the optimal profit $\prod_S^*$ be $\prod_S^*(v^*, n^*)$.

Step 2: Start with $Q = Q^*(v^*)$ and $p = m$ and complete the following step for each wholesale and market price from the sets $\{Q^*(v^*), Q^*(v^*)+g, Q^*(v^*)+2g, ..., tR-2g, tR-g, tR\}$ and $\{m, m+g, m+2g, ..., p^*-2g, p^*-g, p^*\}$ respectively.

2.1: Compute v^*, n, $\prod_S(v_{QVD}, Q_{QVD}, p_{QVD}, n)$ and $\prod_B(p_{QVD}, Q_{QVD})$ as defined in (5.26), (4.19), (5.24) and (4.44) respectively. If $\prod_S(v_{QVD}, Q_{QVD}, p_{QVD}, n)$ is greater than the optimal profit, update the optimal profit $\prod_S^*$, the wholesale price v^*_{QVD}, market price p^*_{QVD}, order quantity Q^*_{QVD} and the lot size multiplier n^*.

5.7 Myopic Approach

One way of modeling the discount mechanisms is simply to neglect the transportation cost. Viswanathan and Wang [74] model quantity discount offer, volume discount offer and their simultaneous offer without considering the transportation cost. We call this approach myopic approach because it takes the transportation cost as part of the fixed cost and neglects its effect. The following three discount mechanisms are modeled according to the procedures of Viswanathan and Wang [74].

- Quantity Discounts
- Volume Discounts

- Simultaneous Offer of Quantity and Volume Discounts

We assume that the initial market equilibrium is the solution of the Stackelberg game without transportation cost, which is considered in Section 4.1. Hence, the results of this section constitute the initial equilibrium.

After finding the optimal policies for the supplier (as in Viswanathan and Wang [74]), the buyer takes transportation cost into consideration and computes her profit with the discounted price. If the offer is not beneficial, the offer is not accepted. As a result, both the supplier and the buyer receive zero additional profit.

We do not review the derivations of the paper but the computational results are provided in the following sections.

5.8 Effectiveness of The Alternative Coordination Mechanisms

The relative performance of the alternative coordination schemes are evaluated through a numerical study. We use the same dataset of Section 4.6.

Table 5.1 provides a summary of the markup and profit results of the decentralized supply chain comparing with centralized supply chain. Considering the average values, the supplier's and the buyer's markups are very similar (2.63 and 2.60). At the same time, centralized system's markup is only slightly higher (2.83) than these values. Besides, standard deviation in decentralized system is very high (11.83) compared to the centralized system (1.84). Moreover, the maximum markup in decentralized system is close to 50%, which is extremely high.

On the average, the supplier receives about one third of the total profit, whereas the buyer receives about two thirds of the total profit. The supplier can get 94.35% of the total profit at most, whereas the buyer can get 85.05% of the total profit at most. On the average the decentralized system can achieve 73.23% of the total profit in the supply chain whereas these value ranges in between 28.14 and 83.50. Consequently, 26.77% of the centralized profit constitutes a potential for the supply chain members, since this is a strong motivation.

Table 5.2 provides a summary of the computational results of the coordination mechanisms. Some percentage values are above 100% because the inventories are valued differently

Table 5.1: Markup and profit sharing levels in supply chains.

	Minimum	Maximum	Mean	S. Deviation
Supplier's markup $\left(\frac{v}{m}\right)$	1.18	8.97	2.63	1.80
Buyer's markup $\left(\frac{p}{v}\right)$	1.27	5.78	2.60	1.57
Markup of decentralized s. $\left(\frac{p}{m}\right)$	1.52	48.92	9.59	11.83
Markup of centralized s. $\left(\frac{p}{m}\right)$	1.27	8.44	2.83	1.84
Profit % of the supplier $\left(100\frac{\Pi_S}{\Pi_B+\Pi_S}\right)$	14.95	94.35	33.68	11.56
Profit % of the buyer $\left(100\frac{\Pi_B}{\Pi_B+\Pi_S}\right)$	5.65	85.05	66.32	11.56
Centralization effect $\left(100\frac{\Pi_B+\Pi_S}{\Pi_C}\right)$	28.14	83.50	73.23	5.74

in decentralized and centralized supply chains. The results of the study evidently demonstrate that the effectiveness of the coordination mechanisms increase in the following order: transportation cost sharing contract, quantity discounts, volume discounts, simultaneous offer of quantity and volume discounts. Hence the least effective mechanism is the transportation cost sharing contract. Table 5.3 further displays a factor analysis on the results of the mechanism. Likewise, Tables 5.4, 5.5 and 5.6 provides a factor analysis for the results of the coordination mechanisms.

The simultaneous offer of quantity and volume discount is obviously the most effective coordination mechanism. On the other hand, volume discounts are more effective than quantity discounts. On the average, the simultaneous offer of quantity and volume discount can achieve 98.42% of the centralized profit. This means an additional profit of 25.18% of the centralized system is captured.

Moreover, the results show that as the effectiveness of the mechanism increases, the supplier's profit share significantly increases. Hence, we may conclude that a supplier dominant supply chain is more profitable. Since the supplier controls the supply chain decisions in the simultaneous offer of quantity and volume discount, a supplier controlled supply chain is more effective considering the coordination of the supply chain.

Table 5.2: Summary of the computational results of the coordination mechanisms.

		CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7
TCSC	Minimum	28.14	0.00	0.00	0.00	0.00	0.00	0.00
	Maximum	83.50	4.97	20.77	0.06	2.89	12.77	7.37
	Mean	73.25	0.01	2.08	0.00	0.01	0.03	0.02
	S. Deviation	5.73	0.21	2.29	0.00	0.12	0.53	0.31
QD	Minimum	65.11	0.00	0.00	0.00	0.00	0.00	0.00
	Maximum	86.98	36.97	20.77	152.64	1.12	77.29	131.40
	Mean	76.55	3.31	2.08	11.35	0.00	1.03	5.03
	S. Deviation	3.33	3.79	2.29	13.44	2.29	3.55	8.14
VD	Minimum	76.62	15.21	0.00	25.95	0.00	0.00	18.37
	Maximum	100.07	48.48	25.35	226.98	1.85	52.25	172.31
	Mean	97.47	24.24	14.56	103.21	0.00	3.84	33.88
	S. Deviation	2.41	4.35	3.03	33.10	3.03	5.86	10.64
QVD	Minimum	86.76	15.15	3.82	49.20	0.00	0.00	18.18
	Maximum	103.43	58.62	27.70	260.71	0.00	0.00	208.35
	Mean	98.41	25.18	16.51	111.73	0.00	0.00	35.29
	S. Deviation	1.39	4.96	1.92	28.88	1.92	0.00	12.41

CR_1: Centralization effect	$\left(100\frac{\Pi_B+\Pi_S}{\Pi_C}\right)$
CR_2: Incremental centralization effect	$\left(100\frac{\Pi_B^{CM}+\Pi_S^{CM}-\Pi_B-\Pi_S}{\Pi_C}\right)$
CR_3: Suppliers incremental profit ratio in the supply chain	$\left(\frac{\Pi_S^{CM}}{\Pi_S^{CM}+\Pi_B^{CM}}-\frac{\Pi_S}{\Pi_S+\Pi_B}\right)$
CR_4: Incremental profit of the supplier as a percentage	$\left(100\frac{(\Pi_S^{CM}-\Pi_S)}{\Pi_S}\right)$
CR_5: Buyers incremental profit ratio in the supply chain	$\left(\frac{\Pi_B^{CM}}{\Pi_S^{CM}+\Pi_B^{CM}}-\frac{\Pi_B}{\Pi_S+\Pi_B}\right)$
CR_6: Incremental profit of the buyer as a percentage	$\left(100\frac{(\Pi_B^{CM}-\Pi_B)}{\Pi_B}\right)$
CR_7: Total Increment of the supply chain profit	$\left(100\frac{\Pi_S^{CM}+\Pi_B^{CM}-\Pi_S-\Pi_B}{\Pi_S+\Pi_B}\right)$
$\prod_S^{CM}$: The supplier's profit in the coordination mechanism	
$\prod_B^{CM}$: The buyer's profit in the coordination mechanism	

Table 5.3: Factor analysis of transportation cost sharing contract.

		CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7
K	10	74.63	0.01	0.01	0.00	0.00	0.02	0.01
	25	74.21	0.00	0.01	0.00	0.00	0.01	0.01
	100	73.03	0.00	0.03	0.00	0.00	0.00	0.00
	250	71.11	0.04	0.04	0.00	0.00	0.09	0.05
A	$0.5K$	73.49	0.00	0.02	0.00	0.00	0.01	0.00
	K	73.37	0.03	0.02	0.00	0.00	0.07	0.04
	$2K$	72.87	0.00	0.02	0.00	0.00	0.01	0.01
b	1.25	79.57	0.00	0.01	0.00	0.00	0.00	0.00
	2	73.38	0.00	0.01	0.00	0.00	0.00	0.00
	3.5	71.18	0.00	0.03	0.00	0.00	0.01	0.01
	5	68.86	0.04	0.04	0.00	0.00	0.11	0.06
C	200	73.31	0.03	0.02	0.00	0.00	0.07	0.04
	400	73.00	0.00	0.02	0.00	0.00	0.01	0.01
	1000	73.43	0.00	0.02	0.00	0.00	0.01	0.00
R	10	74.07	0.00	0.02	0.00	0.00	0.01	0.01
	20	73.93	0.04	0.02	0.00	0.00	0.10	0.06
	50	72.96	0.00	0.02	0.00	0.00	0.01	0.00
	100	72.02	0.00	0.02	0.00	0.00	0.01	0.00

Table 5.4: Factor analysis of quantity discounts.

		CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7
K	10	75.99	1.37	0.01	3.88	-0.01	0.67	1.91
	25	76.30	2.09	0.01	6.37	-0.01	0.78	2.93
	100	76.89	3.85	0.03	13.06	-0.03	0.96	5.55
	250	77.00	5.93	0.04	22.08	-0.04	1.73	9.72
A	$0.5K$	76.56	3.08	0.02	10.06	-0.02	0.97	4.43
	K	76.69	3.34	0.02	11.50	-0.02	1.23	5.26
	$2K$	76.38	3.51	0.02	12.47	-0.02	0.91	5.38
b	1.25	80.20	0.63	0.01	4.80	-0.01	0.04	0.79
	2	75.12	1.75	0.01	6.79	-0.01	0.31	2.41
	3.5	75.17	4.00	0.03	12.51	-0.03	1.11	5.75
	5	75.69	6.87	0.04	21.29	-0.04	2.67	11.15
C	200	76.85	3.57	0.02	11.99	-0.02	1.58	5.73
	400	76.45	3.45	0.02	11.86	-0.02	0.91	5.11
	1000	76.34	2.91	0.02	10.19	-0.02	0.62	4.24
R	10	76.86	2.79	0.02	9.48	-0.02	0.80	3.95
	20	76.74	2.85	0.02	9.72	-0.02	0.76	4.04
	50	76.39	3.43	0.02	11.91	-0.02	0.80	5.06
	100	76.20	4.18	0.02	14.28	-0.02	1.77	7.05

Table 5.5: Factor analysis of volume discounts.

		CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7
K	10	98.88	24.26	0.15	100.07	-0.15	2.94	32.83
	25	98.42	24.21	0.14	100.72	-0.14	3.54	33.07
	100	97.10	24.07	0.14	102.85	-0.14	4.16	33.55
	250	95.49	24.42	0.15	109.22	-0.15	4.73	36.07
A	$0.5K$	97.57	24.08	0.14	101.21	-0.14	4.11	33.24
	K	97.30	23.95	0.15	102.89	-0.15	3.48	33.68
	$2K$	97.56	24.69	0.15	105.54	-0.15	3.94	34.72
b	1.25	98.97	19.40	0.16	151.50	-0.16	0.05	24.41
	2	98.34	24.96	0.16	101.19	-0.16	1.53	34.15
	3.5	97.15	25.97	0.14	83.20	-0.14	4.24	36.79
	5	95.44	26.62	0.12	76.96	-0.12	9.55	40.17
C	200	97.46	24.19	0.15	103.88	-0.15	3.93	34.10
	400	97.38	24.38	0.15	103.85	-0.15	3.74	34.02
	1000	97.58	24.15	0.14	101.91	-0.14	3.86	33.53
R	10	97.88	23.81	0.14	101.07	-0.14	3.79	32.52
	20	97.79	23.89	0.14	101.16	-0.14	3.98	32.74
	50	97.30	24.34	0.15	103.86	-0.15	3.76	34.00
	100	96.94	24.92	0.15	106.77	-0.15	3.85	36.25

Table 5.6: Factor analysis of quantity and volume discounts.

		CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7
K	10	99.43	24.80	0.16	105.87	-0.16	0.00	33.59
	25	99.09	24.88	0.16	107.89	-0.16	0.00	34.01
	100	98.12	25.09	0.17	112.34	-0.17	0.00	35.01
	250	97.01	25.94	0.17	120.81	-0.17	0.00	38.57
A	$0.5K$	98.19	24.70	0.16	108.52	-0.16	0.00	34.13
	K	98.42	25.08	0.16	111.48	-0.16	0.00	35.41
	$2K$	98.63	25.76	0.17	115.18	-0.17	0.00	36.34
b	1.25	98.97	19.40	0.16	151.73	-0.16	0.00	24.41
	2	98.70	25.32	0.17	105.79	-0.17	0.00	34.64
	3.5	98.28	27.10	0.16	93.28	-0.16	0.00	38.39
	5	97.71	28.89	0.16	96.10	-0.16	0.00	43.73
C	200	98.37	25.10	0.16	112.09	-0.16	0.00	35.55
	400	98.36	25.36	0.17	112.64	-0.17	0.00	35.47
	1000	98.50	25.08	0.16	110.45	-0.16	0.00	34.86
R	10	98.71	24.64	0.16	109.40	-0.16	0.00	33.71
	20	98.61	24.71	0.16	109.76	-0.16	0.00	33.91
	50	98.33	25.37	0.17	112.85	-0.17	0.00	35.52
	100	98.00	25.98	0.17	114.90	-0.17	0.00	38.04

5.8.1 Myopic Approach Results

Myopic approach for the coordination mechanisms does not provide promising results. The main problem is that since the supplier neglects the transportation cost, the buyer does not accept the discount offer because it does not offer a higher profit than the profit of the solution of the Stackelberg game.

The buyer accepts the offer in 49 problems of the 576 test problems for quantity discounts, 2 problems for volume discounts and does not accept any of the offers for the simultaneous offer of quantity and volume discount. Since the feasible discount offers are very few, we do not provide detailed analysis in this section.

The major reason for the inferiority of the myopic approach is the neglected transportation cost. Since the supplier designs the contracts according to the break even points where the buyer receives the profit provided by the solution of the Stackelberg game, with the incorporation of the transportation cost the offer becomes unprofitable for the buyer. Hence, most of the myopic discount offers are not feasible offers for the buyer.

In conclusion, we can claim that the incorporation of transportation cost into the coordination problem significantly change the nature of the problem.

Chapter 6

CONCLUSION

Operations management literature places a significant emphasis on lot sizing and pricing decisions. A global approach considering lot sizing and pricing decisions is essential in reducing inventory costs and increasing revenue in supply chain management.

Moreover, transportation is another important consideration of inventory management. Although lot sizing and pricing problems have been intensively studied in the literature, the effect of transportation costs has been generally neglected. Incorporation of transportation costs into lot sizing and pricing problems can have a significant value.

In this thesis, we analyzed the impact of transportation costs on pricing and ordering decisions in a two stage supply chain. We incorporate transportation costs into the lot sizing and pricing problems, and present an approximate algorithm together with conventional techniques and heuristics to solve the resulting optimization problem. Although the emphasis on lowering the logistics costs has been of interest, the problem stated in this thesis has not been addressed in the literature, possibly due to its complexity. The approximate solution procedure we propose generates acceptable solutions in less than one CPU second. Since neglecting the transportation cost in pricing and lot sizing decisions may cause a noticeable decrease in the profit, it is worthwhile to include transportation costs, and the approximate algorithm constitutes a practical approach with a good performance. Additionally, we proposed alternative solution procedures for different supply chain models depending on the ownership of the transportation cost.

Besides, we considered supply chain coordination in this setting. We modeled quantity and volume discounts, which are well-known coordination mechanisms in the literature. Additionally, we propose and model a transportation cost sharing contract for coordination. The transportation cost sharing contracts do not provide promising performance for supply chain coordination, whereas quantity discounts significantly improve the supply chain profit. Volume discounts, on the other hand, provide better results comparing with the performance of the quantity discounts. Furthermore, the simultaneous offer of quantity and volume discounts is found to be the most effective coordination mechanism. We further provide a detailed analysis of the results and efficiencies of the mechanisms.

On the other hand, the demand structure and the sensitivity of the price are the most determinant factors. They drastically affect the performance of the algorithms and coordination mechanisms.

Moreover, the golden section search algorithm is used as a heuristic in the thesis. Although heuristics for the problem are not popular in the literature, we show that it may outperform existing techniques in the literature. More importantly, the algorithm is significantly faster to implement.

We show that, incorporation of transportation cost into the lot sizing and pricing problems and supply chain coordination problem can be modeled. In addition to the complication of introducing the transportation cost into the problem, structural properties of the complex problem lets us use efficient algorithms. This can be considered as the major contribution of the thesis.

The performance of the approximation algorithm can be improved in several ways. In the thesis, we only consider linear approximation of the buyer's response and complicated techniques may lead to better results. Besides, the region of approximation can be divided into smaller intervals and the quality of the approximation can be increased. Implementation of such procedures would be beneficial for increasing the performance of the algorithm.

We considered a two-stage supply chain with single participants at each stage. As a future research area, the model can be improved with a complex transportation structure. In the thesis, transportation costs are taken as fixed, however the transportation company can be integrated into the problem as another decision making entity. Another improvement can be extending the supply chain structure. Multiple buyers can be introduced into the problem and the effects of the coordination mechanisms can be observed.

Appendix A

STRUCTURAL RELATIONS OF THE OPTIMAL DECISIONS

Proposition 1. *For a fixed number of trucks t, as the wholesale price increases, regular shipment size decreases.*

Proof: Suppose we fix the truck option t. Although there is a feasibility consideration for the shipment size, let us first show that there are distinct optimal shipment sizes for a pair of distinct wholesale prices. Let us state the optimality condition (4.55) for a pair $[v_1, Q_1]$ and $[v_2, Q_2]$. Q_i corresponds to the optimal shipment quantity when wholesale price is v_i. Let $v_1 < v_2$ and using (4.49) we can write the demand function as follows:

$$D(p^*(v)) = a\left(\frac{b\,(A + Qv + F(Q))}{Q\,(b-1)}\right)^{-b}. \tag{A.1}$$

We state the optimality condition (4.55) for the pairs as follows:

$$-\frac{v_1 I}{2} + \frac{(A + F(Q_1))D(p^*(v_1))}{Q_1^2} = 0, \tag{A.2}$$

and

$$-\frac{v_2 I}{2} + \frac{(A + F(Q_2))D(p^*(v_2))}{Q_2^2} = 0. \tag{A.3}$$

Subtracting (A.3) from (A.2) gives the following

$$\frac{v_2 I}{2} - \frac{v_1 I}{2} + \frac{(A + F(Q_1))D(p^*(v_1))}{Q_1^2} - \frac{(A + F(Q_2))D(p^*(v_2))}{Q_2^2} = 0. \tag{A.4}$$

Given that $-\frac{v_1 I}{2} + \frac{v_2 I}{2} > 0$, we can claim $\frac{(A+F(Q_1))D(p^*(v_1))}{Q_1^2} - \frac{(A+F(Q_2))D(p^*(v_2))}{Q_2^2} < 0$. After simplification, we obtain

$$\frac{D(p^*(v_1))}{Q_1^2} - \frac{D(p^*(v_2))}{Q_2^2} < 0, \tag{A.5}$$

or

$$D(p^*(v_1))Q_2^2 < D(p^*(v_2))Q_1^2. \tag{A.6}$$

Since $p^*(v)$ increases as v increases, (proof in Proposition 3) $D(p^*(v))$ decreases as v increases. It is known that $D(p^*(v_1)) > D(p^*(v_2))$, hence (A.6) implies $Q_1 > Q_2$.

The previous argument is valid for Q satisfying $(t-1)C < Q \leq tC$. From Lemma 1, it is known that there are distinct wholesale prices v_t and v_{t-1} that solve (4.55) for $(t-1)C$ and tC. For the range $[v_t, v_{t-1}]$, the argument is valid and the optimal shipment size decreases as the wholesale price increases.

Now, let us take a wholesale price $v^{'}$ lower than v_t. Previous argument suggests that $Q^{'} > tC$ should hold; however the feasibility consideration and the concavity of the buyer's profit force $Q^{'} = tC$. Hence the optimal shipment size is constant in the wholesale price interval $(0, v_t)$. A similar argument is valid for the wholesale price interval (v_{t-1}, ∞) where optimal shipment size $Q^{'}$ equals to $(t-1)C$.

Proposition 2. *As the wholesale price increases, regular shipment size decreases. (Decentralized system where the buyer owns the transportation cost)*

Proof: Let us state the optimality condition (4.43) for a pair $[v_1, Q_1]$ and $[v_2, Q_2]$. Q_i corresponds to the optimal shipment quantity when wholesale price is v_i. Let $v_1 < v_2$ and using (4.38) we can write the demand function as follows:

$$D(p^*(v)) = a\left(\frac{b\,(A + Qv)}{Q\,(b-1)}\right)^{-b}. \tag{A.7}$$

We state the optimality condition (4.43) for the pairs as follows:

$$-\frac{v_1 I}{2} + \frac{(A)D(p^*(v_1))}{Q_1^2} = 0, \tag{A.8}$$

and

$$-\frac{v_2 I}{2} + \frac{(A)D(p^*(v_2))}{Q_2^2} = 0. \tag{A.9}$$

Subtracting (A.9) from (A.8) gives the following

$$\frac{v_2 I}{2} - \frac{v_1 I}{2} + \frac{(A)D(p^*(v_1))}{Q_1^2} - \frac{(A)D(p^*(v_2))}{Q_2^2} = 0. \tag{A.10}$$

Given that $-\frac{v_1 I}{2} + \frac{v_2 I}{2} > 0$, we can claim $\frac{(A)D(p^*(v_1))}{Q_1^2} - \frac{(A)D(p^*(v_2))}{Q_2^2} < 0$. After simplification, we obtain

$$D(p^*(v_1))Q_2^2 < D(p^*(v_2))Q_1^2. \tag{A.11}$$

Since $p^*(v)$ increases as v increases, (proof in Proposition 4) $D(p^*(v))$ decreases as v increases. It is known that $D(p^*(v_1)) > D(p^*(v_2))$, hence (A.11) implies $Q_1 > Q_2$.

Proposition 3. *As wholesale price v increases, optimal market price $p^*(v)$ increases.*

Proof: Restating (4.49),

$$p^*(v) = \frac{b\,(A + Qv + F(Q))}{Q\,(b-1)} \tag{A.12}$$

Taking the first derivative,

$$\frac{\partial p^*(v)}{\partial v} = \frac{b}{b-1} \tag{A.13}$$

Since $b > 1$, first derivative is positive. Hence $p^*(v)$ increases as v increases.

Proposition 4. *As wholesale price v increases, optimal market price $p^*(v)$ increases. (Decentralized system where the buyer owns the transportation cost)*

Proof: Restating (4.38),

$$p^*(v) = \frac{b\,(A + Qv)}{Q\,(b-1)} \tag{A.14}$$

Taking the first derivative,

$$\frac{\partial p^*(v)}{\partial v} = \frac{b}{b-1} \tag{A.15}$$

Since $b > 1$, first derivative is positive. Hence $p^*(v)$ increases as v increases.

Lemma 1. *For a full truck load shipment, there is a unique wholesale price satisfying (4.55) when $F(Q)$=tR is fixed.*

Proof: Let us restate (4.55) here as:

$$f(v) = -\frac{vI}{2} + \frac{(A+tR)\,a\left(\frac{b(A+Qv+tR)}{Q(b-1)}\right)^{-b}}{Q^2} \tag{A.16}$$

Both $-\frac{vI}{2}$ and $\frac{(A+tR)a\left(\frac{b(A+Qv+tR)}{Q(b-1)}\right)^{-b}}{Q^2}$ decreases as v increases since $b > 1$. Since $lim_{v\to 0} f(v) = \frac{(A+tR)a\left(\frac{b(A+tR)}{Q(b-1)}\right)^{-b}}{Q^2} > 0$ and $lim_{v\to\infty} f(v) = -\infty$, we conclude that for $Q = tR$, there is a unique v satisfying $f(v) = 0$.

Lemma 2. *For a full truck load shipment, there is a unique wholesale price satisfying (4.43).*

Proof: Let us restate (4.43) here as:

$$f(v) = -\frac{vI}{2} + \frac{(A)a\left(\frac{b(A+Qv)}{Q(b-1)}\right)^{-b}}{Q^2} \tag{A.17}$$

Both $-\frac{vI}{2}$ and $\frac{(A)a\left(\frac{b(A+Qv)}{Q(b-1)}\right)^{-b}}{Q^2}$ decreases as v increases since $b > 1$. Since $lim_{v\to 0}f(v) = \frac{(A)a\left(\frac{b(A)}{Q(b-1)}\right)^{-b}}{Q^2} > 0$ and $lim_{v\to\infty}f(v) = -\infty$, we conclude that for any Q, there is a unique v satisfying $f(v) = 0$.

Appendix B

SUPPLY CHAIN MODELING WITH LINEAR DEMAND FUNCTION

In this chapter, we review the models in Chapter 4 and the coordination mechanisms of Chapter 5 with linear demand function. Following the same notation, demand function $D(p) = a - bp$ is used, where a is the scale parameter and $-b$ is the slope of demand.

B.1 Decentralized System without Transportation Cost

The buyer's and the supplier's profit functions can be written as (4.1) and (4.2).

The Buyer's Problem

In this section, we first discuss the solution of the pricing and lot sizing problems for the buyer assuming that the supplier sets the wholesale price as v. We will later incorporate these results into the supplier's problem.

Incorporating the linear demand function, and for a fixed value of v, the profit function of the buyer can be written as

$$\prod_B(p, Q) = (p - v)(a - bp) - vI\frac{Q}{2} - A\frac{(a - bp)}{Q} \tag{B.1}$$

The first-order condition for a local maximum with respect to p is obtained as below.

$$\frac{\partial \prod_B(p, Q)}{\partial p} = a + b\left(-2\,p + \frac{A}{Q} + v\right) = 0, \tag{B.2}$$

and we can state

$$p^*(Q) = \frac{A\,b + a\,Q + b\,Q\,v}{2\,b\,Q}. \tag{B.3}$$

since the second derivative $\frac{\partial^2 \prod_B(p,Q)}{\partial p^2} = -2b$, (B.3) provides the global optimal price. We can then substitute p by $p^*(Q)$ in Equation (B.1). We rewrite buyer's profit function as

$$\prod_B(Q) = (p^*(Q) - v)(a - bp^*(Q)) - vI\frac{Q}{2} - A\frac{(a - bp^*(Q))}{Q} \tag{B.4}$$

We write the first-order condition for a local maximum with respect to Q as

$$\frac{\partial \prod_B(Q)}{\partial Q} = \frac{(A)\ (-(b\ (A)) + a\,Q) - Q\ \left(b\ (A) + I\,Q^2\right)\ v}{2\,Q^3}. \tag{B.5}$$

and by setting it equal to zero we obtain three roots. The following root is the only real and positive root.

$$Q^*(v) = \frac{\left(1 + i\sqrt{3}\right)\,(a\,A - A\,b\,v)}{2^{\frac{2}{3}}\left(27\,A^2\,b\,I^2\,v^2 + \sqrt{729\,A^4\,b^2\,I^4\,v^4 - 108\,I^3\,v^3\,(a\,A - A\,b\,v)^3}\right)^{\frac{1}{3}}} + \frac{\left(1 - i\sqrt{3}\right)\left(27\,A^2\,b\,I^2\,v^2 + \sqrt{729\,A^4\,b^2\,I^4\,v^4 - 108\,I^3\,v^3\,(a\,A - A\,b\,v)^3}\right)^{\frac{1}{3}}}{6\,2^{\frac{1}{3}}\,I\,v} \qquad \text{(B.6)}$$

Note that i is the complex number $\sqrt{-1}$. Although a complex number appears in the equation, complex numbers are cancelled and a real number is obtained when computed. Hence, the optimal order quantity can be expressed as closed form expression. An important fact is that, it cannot be explicitly stated with a constant elasticity demand function, where we have stated an optimality condition for the order quantity. This constitutes the major difference of the models between constant elasticity and linear demand functions.

The Supplier's Problem

The supplier's problem is not quasi-concave, however we follow the method in Chapter 4 to approximate the optimal solution.

B.1.1 Grid Search

In this section, we present the grid search approach for solving the supplier's problem. The optimal wholesale price v and corresponding profit for the supplier is determined numerically by a grid search within the range $[m, v_{max}]$.

B.1.1.1 Derivation of the optimal lot size multiplier under a fixed value of the wholesale price

Since we derive the optimal lot size multiplier without replacing the demand function, the derivation is the same with Section 4.4.1.3. Hence we may state that (4.19) gives the optimal lot size multiplier.

For a fixed wholesale price v, optimal price can be stated as $p^*(Q) = \frac{A\,b + a\,Q + b\,Q\,v}{2\,b\,Q}$, which is given in (B.3). Q^* provides the optimal order quantity, which is provided in (B.6). Finally, optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$, which is stated in (4.19). We adopt a grid size $g = 10^{-3}$ in the computations. The algorithm of Section 4.1.1 is used with the decision variables and profit functions mentioned above.

B.2 Centralized System without Transportation Cost

We can state the profit function as follows:

$$\prod_C(p,Q,n) = (p-m)D(p) - (h_s(n-1)+h_b)\frac{Q}{2} - \left(\frac{K}{n}+A\right)\frac{D(p)}{Q}. \tag{B.7}$$

First order optimality condition can be stated as below.

$$\frac{\partial \prod_C(p,Q,n)}{\partial p} = a - b\,p - b\,(-m+p) + \frac{b\,\left(A+\frac{K}{n}\right)}{Q} = 0 \tag{B.8}$$

Solving (B.8) gives price below:

$$p^* = \frac{a\,n\,Q + b\,(K + n\,(A + m\,Q))}{2\,b\,n\,Q} \tag{B.9}$$

Since the second derivative is $-2b$ and b is always positive, the function is concave and the p^* is the optimal market price. Replacing p^* with p in (B.7), we can rewrite the profit function as

$$\begin{aligned}\prod_C(p,n) &= \frac{a^2\,n^2\,Q^2 + b^2\,(K+n\,(A+m\,Q))^2}{4\,b\,n^2\,Q^2} \\ &\quad - \frac{2\,b\,n\,Q\,\left(I\,m\,n^2\,Q^2 + a\,(K+n\,(A+m\,Q))\right)}{4\,b\,n^2\,Q^2}.\end{aligned} \tag{B.10}$$

We write the first-order condition for a local maximum with respect to Q as

$$\frac{\partial \prod_B(Q)}{\partial Q} = \frac{-\left(b\,(K+(A)\,n)^2 - (a-b\,m)\,n\,(K+(A)\,n)\,Q + I\,m\,n^3\,Q^3\right)}{2\,n^2\,Q^3}. \tag{B.11}$$

Setting (B.11) equal to zero we obtain three roots where the following root is the only real and positive root.

$$\begin{aligned}Q^*(v) &= \frac{\left(1+i\sqrt{3}\right)\,\left(a\,K\,n - b\,K\,m\,n + a\,A\,n^2 - A\,b\,m\,n^2\right)}{2^{\frac{2}{3}}\,Z} \\ &\quad + \frac{\left(1-i\sqrt{3}\right)\,Z}{6\,2^{\frac{1}{3}}\,I\,m\,n^3} \text{ where} \\ Z &= \left(M + \sqrt{M^2 - 108\,I^3\,m^3\,n^9\,(a\,K\,n - b\,K\,m\,n + a\,A\,n^2 - A\,b\,m\,n^2)^3}\right)^{\frac{1}{3}}, \text{ and} \\ M &= 27\,b\,I^2\,K^2\,m^2\,n^6 + 54\,A\,b\,I^2\,K\,m^2\,n^7 + 27\,A^2\,b\,I^2\,m^2\,n^8\end{aligned} \tag{B.12}$$

Note that i is the complex number $\sqrt{-1}$. Although a complex number appears in the equation, complex numbers are canceled and a real number is obtained when computed.

Hence, the optimal order quantity can be expressed as a closed form expression. Finally, we can claim that the optimal lot size multiplier does not depend on the demand function. Hence the derivation is the same with Section 4.2. We can state the result of this section here as:

$$n^* = \left\lfloor \frac{1}{2}\left(1+\sqrt{1+4\frac{Ah_s}{K(h_b-h_s)}}\right)\right\rfloor \text{ or } \left\lceil \frac{1}{2}\left(-1+\sqrt{1+4\frac{Ah_s}{K(h_b-h_s)}}\right)\right\rceil \tag{B.13}$$

B.3 Decentralized System where The Supplier Owns The Transportation Cost

The buyer's and supplier's profit functions can be written as (4.34) and (4.35).

The Buyer's Problem

Incorporating the demand function, and for a fixed value of v, the profit function of the buyer can be written as

$$\prod_B(p,Q) = (p-v)(a-bp) - vI\frac{Q}{2} - A\frac{(a-bp)}{Q} \tag{B.14}$$

The first-order condition for a local maximum with respect to p is obtained as below.

$$\frac{\partial \prod_B(p,Q)}{\partial p} = a + b\left(-2\,p + \frac{A}{Q} + v\right) = 0, \tag{B.15}$$

and we can state

$$p^*(Q) = \frac{A\,b + a\,Q + b\,Q\,v}{2\,b\,Q}. \tag{B.16}$$

since the second derivative $\frac{\partial^2 \prod_B(p,Q)}{\partial p^2} = -2b$, (B.16) provides the global optimal price. We can then substitute p by $p^*(Q)$ in Equation (B.14). We rewrite buyer's profit function as

$$\prod_B(Q) = (p^*(Q)-v)(a-bp^*(Q)) - vI\frac{Q}{2} - A\frac{(a-bp^*(Q))}{Q} \tag{B.17}$$

We write the first-order condition for a local maximum with respect to Q as

$$\frac{\partial \prod_B(Q)}{\partial Q} = \frac{(A)\,\left(-\left(b\,(A)\right) + a\,Q\right) - Q\,\left(b\,(A) + I\,Q^2\right)\,v}{2\,Q^3}. \tag{B.18}$$

and by setting it equal to zero we obtain three roots. The following root is the only real and positive root.

$$Q^*(v) = \frac{\left(1+i\,\sqrt{3}\right)\,(a\,A - A\,b\,v)}{2^{\frac{2}{3}}\left(27\,A^2\,b\,I^2\,v^2 + \sqrt{729\,A^4\,b^2\,I^4\,v^4 - 108\,I^3\,v^3\,(a\,A - A\,b\,v)^3}\right)^{\frac{1}{3}}}$$
$$+\frac{\left(1-i\,\sqrt{3}\right)\left(27\,A^2\,b\,I^2\,v^2 + \sqrt{729\,A^4\,b^2\,I^4\,v^4 - 108\,I^3\,v^3\,(a\,A - A\,b\,v)^3}\right)^{\frac{1}{3}}}{6\,2^{\frac{1}{3}}\,I\,v} \tag{B.19}$$

Note that i is the complex number $\sqrt{-1}$. Although a complex number appears in the equation, complex numbers are cancelled and a real number is obtained when computed. Hence, the optimal order quantity can be expressed as a closed form expression. An important fact is that, it cannot be explicitly stated with a constant elasticity demand function, where we have stated an optimality condition for the order quantity. This constitutes the major difference of the models between constant elasticity and linear demand functions.

The Supplier's Problem

The supplier's problem is not quasi-concave. Therefore, it is difficult to obtain a global optimal solution, and we will study the structural properties of the problem in the subsequent sections. Besides, we review three solution procedures for the problem.

B.3.1 Approximation Algorithm

In this section we study the supplier's problem of finding the optimal wholesale price. The supplier wishes to determine v^*, and, to be able to solve his problem, he needs to incorporate the buyer's reaction as $Q^*(v)$ and $p^*(v)$ into his problem. We propose an approach through which $Q^*(v)$ is approximated.

B.3.1.1 Approximating optimal order quantity response of the buyer

In this section, we analyze how the buyer's optimal order quantity, i.e., $Q^*(v)$, changes as we change v, and present an approximation of $Q^*(v)$.

Let us first take a wholesale price, v_a and corresponding order quantity $Q(v_a)$ obtained by (B.19). Any v_b larger than v_a will lead to $Q(v_b)$, which is strictly smaller than $Q(v_a)$.

Hence, an approximate procedure for determining the order quantity the buyer chooses can be formally stated as follows:

Initialization: Let $i = 1$. Determine the smallest value of v, v_1, for which Equality (B.19) holds for $t = 1$ and $Q = C$. Assuming that optimal wholesale price can be at most v_{max}, the order quantity response is linearly approximated in the interval $[v_1, v_{max}]$ through a linear function that crosses points $(v_1, Q^*(v_1) = C)$ and $(v_{max}, Q^*(v_{max}))$.

Step 1:

- Let $i = i + 1$.
- Determine the smallest value of v, v_i, for which (B.19) holds for $t = i$ and $Q = iC$.

- Linearly approximate the order quantity between $Q = iC$ and $Q = (i-1)C$ in the interval $[v_i, v_{i-1}]$

Step 2: If $i > T$ (maximum number of trucks) combine approximated Q values characterizing the buyer's optimal response, and stop; otherwise go to Step 1.

B.3.1.2 Derivation of the supplier's optimal wholesale price under a fixed lot size multiplier

In the previous section, we have characterized the buyer's response in terms of her order quantity. In this section, we illustrate how the supplier can determine his optimal wholesale price.

Let us assume that n is constant and Q is not constant, where the order quantity is approximated as a linear decreasing function of v. Let this function be $Q(v) = c - dQ$ for a wholesale price range $[v_{j+1}, v_j]$. Replacing Q with $c-dQ$ in Equation (4.35), we approximate the best solution in this interval through a line search over v values in the $[v_{j+1}, v_j]$ range maximizing the supplier's profit.

B.3.1.3 Derivation of the optimal lot size multiplier under a fixed value of the wholesale price

Since we derive the optimal lot size multiplier without replacing the demand function, the derivation is the same with Section 4.3.1.3. Hence (4.19) gives the optimal lot size multiplier.

B.3.1.4 The Approximate Algorithm

In this section, we present the approximate algorithm for solving the supplier's problem by combining the results that have been developed in the previous sections.

In order to solve the supplier's problem, first the buyer's response has to be characterized. Optimal market price as a function of the order quantity can be determined by using Equation (B.16). Substituting this into the profit function, we obtain Equation (B.17). Using this function, we can then characterize the order quantity as the wholesale price changes in Section B.3.1.1. Hence we propose a search procedure with respect to the wholesale price. The algorithm of Section 4.3.1.4 is used with the decision variables and profit functions mentioned above.

B.3.2 Grid Search

In this section, we present the grid search approach for solving the supplier's problem. The optimal wholesale price v and corresponding profit for the supplier is determined numerically by a grid search within the range $[m, v_{max}]$. For a fixed wholesale price v, optimal price can be stated as $p^*(Q) = \frac{A\,b+a\,Q+b\,Q\,v}{2\,b\,Q}$, which is given in (B.16). Optimal order quantity Q^* is computed through (B.19). Finally, optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$, which is stated in (4.19). We adopt a grid size $g = 10^{-3}$ in the computations. The algorithm of Section 4.3.2 is used with the decision variables and profit functions mentioned above.

B.3.3 Golden Section Search: A Heuristic

In this section, we adopt the golden section search algorithm for solving the supplier's problem. The details of the procedure have been previously stated in Section 4.1.2. The algorithm, which is mentioned in the section is used with the decision variables and profit functions stated above.

B.4 Decentralized System where The Buyer Owns The Transportation Cost

The buyer's and supplier's profit functions can be written as (4.44) and (4.45).

The Buyer's Problem

Incorporating the demand function, and for a fixed value of v, the profit function of the buyer can be written as

$$\prod_B(p, Q) = (p - v)(a - bp) - (v + \frac{F(Q)}{Q})I\frac{Q}{2} - (A + F(Q))\frac{(a - bp)}{Q}, \tag{B.20}$$

or

$$\prod_B(p, Q) = (p - v)(a - bp) - vI\frac{Q}{2} - I\frac{F(Q)}{2} - (A + F(Q))\frac{(a - bp)}{Q}. \tag{B.21}$$

The first-order condition for a local maximum with respect to p is obtained as below.

$$\frac{\partial \prod_B(p, Q)}{\partial p} = a + \frac{b\ (A + F(Q) - 2\,p\,Q + Q\,v)}{Q} = 0, \tag{B.22}$$

and we can state

$$p^*(Q) = \frac{a\,Q + b\ (A + F(Q) + Q\,v)}{2\,b\,Q}. \tag{B.23}$$

We further check the second derivative as follows:

$$\frac{\partial^2 \prod_B(p,Q)}{\partial p^2} = -2b \tag{B.24}$$

Since b is positive, second derivative is always negative and (B.23) provides the optimal price. We rewrite buyer's profit function as

$$\prod_B(Q) = (p^*(Q) - v)(a - b(p^*(Q))) - vI\frac{Q}{2} - (A + F(Q))\frac{(a - b(p^*(Q)))}{Q}. \tag{B.25}$$

For a fixed number of trucks t, where $(t-1)C < Q \leq tC$, we can rewrite (B.25) as

$$\prod_B(Q) = (p^*(Q) - v)(a - b(p^*(Q))) - vI\frac{Q}{2} - (A + tR)\frac{(a - b(p^*(Q)))}{Q}. \tag{B.26}$$

As $F(Q)$ is a stepwise function of number of trucks used, t, where $t = \left\lceil \frac{Q}{C} \right\rceil$, we write the first-order condition for a local maximum with respect to Q for a particular t value as

$$\frac{\partial \prod_B(Q)}{\partial Q} = \frac{(A+tR)\ (-(b\ (A+tR)) + a\,Q) - Q\ \left(b\ (A+tR) + I\,Q^2\right)\ v}{2\,Q^3}. \tag{B.27}$$

and by setting it equal to zero we obtain three roots. The following root is the only real and positive root.

$$Q^*(v) = \frac{\left(1 - i\,\sqrt{3}\right)\ G^{\frac{1}{3}}}{6\,2^{\frac{1}{3}}\ I\,v} + \frac{\left(1 + i\,\sqrt{3}\right)\ (a\,A + a\,tR - A\,b\,v - b\,tR\,v)}{2^{\frac{2}{3}}\ G^{\frac{1}{3}}}, \text{ where}$$

$$G = 27\,b\,(A+tR)^2\ I^2\ v^2$$

$$+\sqrt{27}\,\sqrt{(A+tR)^3\ I^3\ v^3\ \left(-4\,a^3 + 3\,b\ (4\,a^2 + 9\,b\ (A+tR)\ I)\ v - 12\,a\,b^2\,v^2 + 4\,b^3\,v^3\right)} \tag{B.28}$$

Note that i is the complex number $\sqrt{-1}$. Although a complex number appears in the equation, complex numbers are cancelled and a real number is obtained when computed. Hence, the optimal order quantity can be expressed as closed form expression. An important fact is that, it cannot be explicitly stated with a constant elasticity demand function, where we have stated an optimality condition for the order quantity. This constitutes the major difference of the models between constant elasticity and linear demand functions.

Since T is the maximum number of trucks, the buyer obtains T many Q values. The buyer designates the order quantity that provides the highest profit as her order quantity.

The Supplier's Problem

The supplier's profit function (4.45) is not quasi-concave. However, we follow the method in Chapter 4 to approximate the optimal solution. We review four solution procedures for this problem.

B.4.1 Approximation Algorithm

In this section we study the supplier's problem of finding the optimal wholesale price. We follow the same notation and propose an approach through which $Q^*(v)$ is approximated. An important consideration is that the optimal order quantity and the wholesale price where the truck option changes can be computed by an equation, whereas a line search is performed in the supply chain with constant-elasticity demand.

B.4.1.1 Approximating optimal order quantity response of the buyer

In this section, we analyze how the buyer's optimal order quantity, i.e., $Q^*(v)$, changes as we change v, and present an approximation of $Q^*(v)$. Let us first assume that the buyer uses only one truck, i.e. $t = 1$. Let v_1 denote the v value for which the optimality condition is satisfied with $Q = C$ and $t = 1$. For $v > v_1$, the optimal order quantity obtained from (B.28) will be smaller than C, and using more than one truck would be more costly. Therefore, for $v > v_1$, the optimal response of the buyer would be to compute $Q^*(v)$ from (B.28) with $t = 1$.

Now let v_2 be the smallest value of v for which Equality (B.28) is satisfied with $Q = 2C$ and $t = 2$. Since $v_2 < v_1$, and for $v_2 < v < v_1$, the optimal order quantity obtained from (B.28) with $t = 2$ would be between $2C$ and C. The buyer has now two options for a given v value, where $v_2 < v < v_1$:

1. Order one full truck-load with $Q = C$,
2. Use two trucks, i.e., $t = 2$, and determine the order quantity from Equality (B.28) with $t = 2$.

It can be readily shown that the profit function of the buyer as expressed in Equation (B.26) is concave with respect to v when Q is fixed. In the second option with $t = 2$, as v changes, $Q^*(v)$ is determined through an approximation. Therefore, as an approximation, we assume that, in the $v_2 < v < v_1$ range, the buyer's profit function (B.26) will be a linear function that crosses the points $(v_2, \prod_B(Q^*(v_2)))$ and $(v_1 - \epsilon, \prod_B(Q^*(v_1 - \epsilon)))$. The profit functions of the two options (the profit function of the second option being approximated as a linear function) may intersect in the $[v_2, v_1]$ interval. Note that when $v = v_1$, one full truck-load option dominates the options with two trucks; if that is also the case for $v = v_2$, then we will assume that the one full truck-load option dominates the second option in the $[v_2, v_1]$ range, and the buyer's optimal order quantity is equal to C. If the second option (i.e., use two trucks, determine Q from (B.28) with $t = 2$) dominates the first option when $v = v_2$, we then find the value of v where the two options generate the same profit for the buyer. Let $b^1_{t=2}$ be the intersection point. Then in the $v_2 < v < b^1_{t=2}$ range, the optimal order quantity is determined from (B.28) with $t = 2$, and for the $b^1_{t=2} < v < v_1$ range the optimal order quantity is equal to C.

By generalizing the above approach, an approximate procedure for determining the order quantity the buyer chooses can be formally stated as follows:

Initialization: Let $i = 1$. Determine the smallest value of v, v_1, for which Equality (B.28) holds for $t = 1$ and $Q = C$. For v values greater than v_1, the buyer will choose his order quantity according to equality (B.28) with $t = 1$. Assuming that optimal wholesale price can be at most v_{max}, the order quantity response is linearly approximated in the interval $[v_1, v_{max}]$ through a linear function that crosses points $(v_1, Q^*(v_1) = C)$ and $(v_{max}, Q^*(v_{max}))$.

Step 1:

- Let $i = i + 1$.
- Determine the smallest value of v, v_i, for which (B.28) holds for $t = i$ and $Q = iC$.
- Define the *upper envelope* as the combination of the profit functions of the truck options leading to the highest profit. Determine the upper envelope of the following profit functions of the buyer in the $v_i < v < v_{i-1}$ range:
  - $\prod_B(p^*(Q), Q)$ where $Q = jC$, and $t = j$, $j = 1, ..., i-1$.
  - $\prod_B(p^*(Q(v)), Q(v))$ where $t = i$ and $Q(v)$ satisfies Equality (B.28).
- If $\prod_B(p^*(Q(v)), Q(v))$ at the point v_i is higher than any evaluated function $\prod_B(p^*(Q), Q)$ for $t = j$ where $j = 1, ..., i-1$, first through a line search find the first break-point where the optimal truck option changes, and then linearly approximate the order quantity between the starting point of the interval and the break-point.
- Find the remaining break-points, if any, where the optimal truck option changes by a line search. Within these ranges, the order quantity is a multiple of C.

- Let b_i^k, $k = 1, .., l$, be the break-points generated by the upper envelope. Note that, from the fact that we are comparing i functions that are either convex or linear, $l < 2i$.

Step 2: If $i > T$ (maximum number of trucks) combine upper envelopes generated for intervals $[v_{j+1}, v_j]$ for $j = 1, 2, .., T-1$, and their corresponding Q values characterizing the buyer's optimal response, and stop; otherwise go to Step 1.

The computational complexity of the above outlined procedure lies with the generation of the upper envelope of at most T functions in Step 1. The upper envelope can be easily generated by a number of line searches under the assumption that Q linearly decreases when full truck option is not used.

B.4.1.2 Derivation of the supplier's optimal wholesale price under a fixed lot size multiplier

In the previous section, we have characterized the buyer's response in terms of her order quantity. Accordingly, in wholesale price intervals that have been computed in Step 1 of the procedure presented in Section 5.1, the order quantity is either constant or linearly approximated. In this section, we illustrate how the supplier can determine his optimal wholesale price in each of these cases.

Case 1: If Q and n is constant for a wholesale price range $[v_{j+1}, v_j]$, then we can develop a search algorithm with respect to v in the range $[v_{j+1}, v_j]$. Replacing $p^*(v)$ with $p^*(Q)$ as given in Equation (B.23), we can rewrite the supplier's profit function as follows.

$$\begin{aligned} \prod_S(v) &= \frac{(v-m)\left(a - \frac{b\,(A+F(Q)+Q\,v)}{Q}\right)}{2} - \frac{(I\,m\,(-1+n)\,Q)}{2} \\ &- \frac{K\left(a - \frac{b\,(A+F(Q)+Q\,v)}{Q}\right)}{2\,n\,Q} \end{aligned} \tag{B.29}$$

Taking the derivative of (B.29) with respect to v, we can write

$$\frac{\partial \prod_S(v)}{\partial v} = \frac{a\,n\,Q + b\,(K - n\,(A + F(Q) - m\,Q + 2\,Q\,v))}{2\,n\,Q}. \tag{B.30}$$

Since the second derivative is $-b$ and always negative, we can equate (B.30) to zero and find the optimal wholesale price as follows:

$$v^* = \frac{b\,(K - (A + F(Q))\,n) + (a + b\,m)\,n\,Q}{2\,b\,n\,Q}. \tag{B.31}$$

Case 2: If n is constant and Q is not constant, order quantity is approximated as a linear decreasing function of v. Let this function be $Q(v) = c - dQ$ for a wholesale price range $[v_{j+1}, v_j]$. Replacing Q with $c - dQ$ in Equation (B.29), we approximate the best solution in this interval through a line search over v values in the $[v_{j+1}, v_j]$ range maximizing the supplier's profit.

B.4.1.3 Derivation of the optimal lot size multiplier under a fixed value of the wholesale price

Since we derive the optimal lot size multiplier without replacing the demand function, the derivation is the same with Section 4.4.1.3. Hence (4.19) gives the optimal lot size multiplier.

B.4.1.4 The Approximate Algorithm

In this section, we present the approximate algorithm for solving the supplier's problem by combining the results that have been developed in the previous sections.

In order to solve the supplier's problem, first the buyer's response has to be characterized. Optimal market price as a function of the order quantity can be determined by using Equation (B.23). Substituting this into the profit function, we obtain Equation (B.25). Using this function, we can then characterize the order quantity as the wholesale price changes in Section B.4.1.1. Hence we propose a search procedure with respect to the wholesale price. The algorithm can be formally stated as follows:

Step 1: Characterize the optimal response of the buyer utilizing the procedure outlined in Section B.4.1.1.

Step 2: Let the optimal profit $\prod_S^*$ be 0 with $v = 0$.

Step 3: Consider each wholesale price interval where the truck option changes in the response profile generated in Step 1, and complete the following steps for each interval:

3.1: If the order quantity is constant let $n = 1$ and go to the next step. If not, replace the linear approximation function with Q in the supplier's profit function (B.29). Find the optimal wholesale price maximizing the profit function through a line search over (B.29) as an approximation, and go to Step 3.5.

3.2: Compute the optimal wholesale price by (B.31).

3.3: Let the endpoints of the interval be v_{start} and v_{end}. If the optimal wholesale price of Step 3.2 is less than v_{start}, equate the wholesale price to v_{start}, whereas if the wholesale is price greater than v_{end}, equate the wholesale price to v_{end}.

3.4: If the optimality condition for the lot size multiplier holds by (4.19), go to Step 3.5. Else, increase n by 1 and go to Step 3.2.

3.5: For the current wholesale price interval, compute the profit with the optimal wholesale price and the lot size multiplier by (B.29) and compare it with the optimal profit. If it is greater than the optimal profit, update the optimal profit, the wholesale price and the lot size multiplier.

B.4.2 Grid Search

In this section, we present the grid search approach for solving the supplier's problem. The optimal wholesale price v and corresponding profit for the supplier is determined numerically by a grid search within the range $[m, v_{max}]$. For a fixed wholesale price v, optimal price can be stated as $p^*(Q) = \frac{a\,Q + b\,(A+F(Q)+Q\,v)}{2\,b\,Q}$, which is given in (B.23). $Q^*(v)$ is computed by (B.28). Finally, optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$, which is stated in (4.19). We adopt a grid size $g = 10^{-3}$ in the computations. An algorithm similar to the one of Section B.1.1 is used with the decision variables and profit functions mentioned above. Since the buyer needs to determine the profit function for T many truck options, T line searches are performed to find the order quantity in the algorithm.

B.4.3 Myopic Approach

In this section, we present the algorithm of myopic approach for solving the supplier's problem. The supplier optimizes his profit assuming that the buyer does not incur transportation cost. Hence, the supplier determines the wholesale price according to the results of Section B.1. However, the buyer determines her decision variables taking the transportation cost into account for the wholesale price of the supplier. Finally, the supplier re-determines his lot size multiplier according to the buyer's actions.

The optimal wholesale price v and corresponding profit for the supplier is determined numerically by a grid search within the range $[m, v_{max}]$. For a fixed wholesale price v, optimal price can be stated as $p^*(Q) = \frac{A\,b + a\,Q + b\,Q\,v}{2\,b\,Q}$, which is given in (B.3). Q is computed through (B.6). The optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$, which is stated in (4.19). After wholesale price is determined, optimal order quantity is re-determined through (B.28). Afterwards, optimal lot size multiplier is recomputed by $n^* = \left\lceil \frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$. We adopt a grid size $g = 10^{-3}$ in the computations.

The algorithm can be formally stated as follows:

Step 1: Let the optimal profit $\prod_S^*$ be 0 with $v = 0$ and $n = 0$.

Step 2: Start with $v = m$ and complete the following steps for each wholesale price from the set $\{m, m+g, m+2g, ..., v_{max} - 2g, v_{max} - g, v_{max}\}$.

2.1: Find Q through (B.6), compute p as defined in (B.3) and $\prod_B(Q)$ as defined in (4.2).

2.2: Compute n and $\prod_S(v)$ as defined in (4.19) and (4.2) respectively. If $\prod_S(v)$ is greater than the optimal profit, update the optimal profit $\prod_S^*$, the wholesale price v^* and the lot size multiplier n^*.

Step 3: Compute (B.28) for v^* and determine $Q^*(v)$. Recalculate n^* by (4.19).

B.4.4 Golden Section Search: A Heuristic

In this section, we adopt the golden section search algorithm for solving the supplier's problem. The details of the procedure have been previously stated in Section 4.1.2. An algorithm similar to the one of the section is used with the decision variables and profit functions mentioned above. Since the buyer needs to determine the profit function for T many truck options, T function evaluations are performed to find the order quantity in the algorithm.

B.5 Centralized System with Transportation Cost

We can state the profit function as follows:

$$\begin{aligned}\prod_C(p,Q,n) &= (p-m)D(p) - (h_s(n-1)+h_b)\frac{Q}{2} - \left(\frac{K}{n} + A + F(Q)\right)\frac{D(p)}{Q} \\ &\quad -\frac{I\,F(Q)}{2}. \end{aligned} \tag{B.32}$$

First order optimality condition can be stated as below.

$$\frac{\partial \prod_C(p,Q,n)}{\partial p} = a - b\,p - b\,(-m+p) + \frac{b\,\left(A + F(Q) + \frac{K}{n}\right)}{Q} = 0 \tag{B.33}$$

Solving (B.33) gives the price below:

$$p^* = \frac{a\,n\,Q + b\,(K + n\,(A + F(Q) + m\,Q))}{2\,b\,n\,Q} \tag{B.34}$$

Since the second derivative is $-2b$ and b is always positive, the function is convex and the p^* is the optimal market price. Replacing p^* with p in (B.32), we can rewrite the profit function as

$$\begin{aligned}\prod_C(p,n) &= \frac{a^2\,n^2\,Q^2 + b^2\,(K + n\,(A + F(Q) + m\,Q))^2}{4\,b\,n^2\,Q^2}\\ &-\frac{2\,b\,n\,Q\,\left(I\,m\,n^2\,Q^2 + a\,(K + n\,(A + F(Q) + m\,Q))\right)}{4\,b\,n^2\,Q^2} - \frac{I\,F(Q)}{2}.\end{aligned} \tag{B.35}$$

Considering a truck interval, we replace $F(Q)$ with tR and write the first-order condition for a local maximum with respect to Q as

$$\begin{aligned}\frac{\partial \prod_B(Q)}{\partial Q} &= -\frac{b\,(K + (A + tR)\,n)^2 - (a - b\,m)\,n\,(K + (A + tR)\,n)\,Q}{2\,n^2\,Q^3}\\ &-\frac{I\,m\,n}{2}.\end{aligned} \tag{B.36}$$

Setting (B.36) equal to zero we obtain three roots where the following root is the only real and positive root.

$$\begin{aligned}Q^*(v) &= \frac{\left(1 + i\,\sqrt{3}\right)\,\left(a\,K\,n - b\,K\,m\,n + a\,A\,n^2 + a\,F\,n^2 - A\,b\,m\,n^2 - b\,F\,m\,n^2\right)}{2^{\frac{2}{3}}\,Z}\\ &\qquad + \frac{\left(1 - i\,\sqrt{3}\right)\,Z}{6\,2^{\frac{1}{3}}\,I\,m\,n^3} \text{ where}\\ Z &= \left(M + \sqrt{M^2 + 108\,I^3\,m^3\,(-a + b\,m)^3\,n^{12}\,(K + (A + F(Q))\,n)^3}\right)^{\frac{1}{3}}, \text{ and}\\ M &= 27\,b\,I^2\,m^2\,n^6\,(K + (A + F(Q))\,n)^2\end{aligned} \tag{B.37}$$

Note that i is the complex number $\sqrt{-1}$. Although a complex number appears in the equation, complex numbers are canceled and a real number is obtained when computed. Hence, the optimal order quantity can be expressed as closed form expression. Finally, we can claim that optimal lot size multiplier does not depend on the demand function. Hence the derivation is the same with Section 4.4.1.3. We can state the result of this section here as:

$$n^* = \left\lfloor \frac{1}{2}\left(1 + \sqrt{1 + 4\frac{(A + tR)h_s}{K(h_b - h_s)}}\right)\right\rfloor \text{ or } \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + 4\frac{(A + tR)h_s}{K(h_b - h_s)}}\right)\right\rceil \tag{B.38}$$

Since T is the maximum number of trucks, the resultant profit functions of T many truck options are evaluated and the highest profit is selected with the associated decision variables.

Table B.1: Demand parameter values in the computational analysis.

a	20000
b	{625, 1000, 1750, 2500}

B.6 Computational Results

B.6.1 Problem Generation

We test the algorithms' effectiveness over a set of problems similar to the one in Section 4.6. The combinations of these parameters again lead to a set of 576 problems. The algorithms not incorporating transportation cost are tested with a set of 144 problems. They do not consider transportation cost however truck capacities are valid for the problems. We take the maximum wholesale price, v_{max}, as 20 and fixed. Table B.1 displays the parameter values for the demand function used in creating the test problems. The remaining part of the parameter values are adopted from Table 4.1. We solve five different problems with the algorithms mentioned. In the approximation algorithms, for finding the optimal shipment size for a given v, the line search procedure is terminated when the length of the uncertainty interval is less than 0.001. Golden section search algorithm for solving the optimal order quantity is used as a line search method in all of the problems. The length of the uncertainty interval is 0.001 in the line search. We also run the grid search with increments of size 0.001.

Under this precision scheme, the test problems have been solved in Matlab 6.5 in Microsoft Windows XP on a computer with an Intel Pentium M 1.60 GHz processor, and 512 MB of RAM.

B.6.2 Performance of the Algorithms Considering Profit Function

The summary of the computational results are provided in Table B.2. It displays the ratios of the supplier's profit obtained by the algorithm to the supplier's profit obtained by the grid search for the decentralized case. This is an indicator of the quality of the solutions generated by our approximation approach. Cells include the minimum, maximum, average and standard deviation values for the performance measures.

Table B.2: Performance of the algorithms considering supplier's profit (linear demand).

Method	Appendix	Minimum	Maximum	Average	S. Deviation
Approximation A.	B.4.1	0.048201	1.000001	0.927405	0.185793
Myopic A.	B.4.3	0.990023	1.000024	0.999857	0.000907
Golden Section S.	B.4.4	1.000396	1.173842	1.020948	0.030862

Furthermore, the performance of the myopic approach is significantly satisfactory. Hence, neglecting the transportation cost in linear demand environment may be effective. On the other hand, golden section search algorithm's performance is outstanding. The performance of the approximation algorithm is not better than golden section search algorithm. The quality of its performance can be considered as moderately acceptable. On average, the approximate algorithm achieves 92.74% of the profits that can be achieved by the grid search. The proposed algorithm works on an approximation with fixed truck options, and when the optimal shipment size is less than or equal to a single truck load, the performance of the algorithm decreases. This is a natural outcome, because if the optimal shipment size is less than a truck load, there is no need to incorporate the transportation cost, and it can be taken as fixed. Therefore, omitting the problems having less than or equal to a single truck load shipment size the approximate algorithm achieves 99.83% of the profit that can be achieved with the grid search. Hence, the performance of the approximation algorithm is promising if less than truck load shipments are discarded. Factor analysis for the computational results is provided in Table B.3. The factor analysis is based on parameters K, A, b, C, and R. Treatment levels are stated in the second column. Moreover, minimum, mean and maximum values of the algorithms are displayed. Algorithms of Section B.4 are evaluated. An important consideration is that, the performance of the approximation algorithm increases as the slope of the demand increases. Hence, we can say that potential benefits of employing the approximation algorithm become larger in price sensitive environments.

Table B.3: Factor analysis for algorithms of Appendix B.4 with linear demand.

	Appendix:	B.4.1			B.4.3			B.4.4		
Factor	T. Level	MIN	MEAN	MAX	MIN	MEAN	MAX	MIN	MEAN	MAX
K	10	0.05	0.85	1.00	0.99	0.99	1.00	1.00	1.02	1.17
	25	0.13	0.90	1.00	1.00	1.00	1.00	1.00	1.02	1.17
	100	0.45	0.97	1.00	0.99	0.99	1.00	1.00	1.02	1.17
	250	0.81	0.99	1.00	0.99	0.99	1.00	1.00	1.02	1.17
A	$0.5K$	0.20	0.95	1.00	0.99	0.99	1.00	1.00	1.02	1.17
	K	0.10	0.93	1.00	0.99	0.99	1.00	1.00	1.02	1.17
	$2K$	0.05	0.90	1.00	0.99	0.99	1.00	1.00	1.02	1.17
b	625	0.05	0.88	1.00	0.99	0.99	1.00	1.00	1.01	1.03
	1000	0.07	0.92	1.00	0.99	0.99	1.00	1.00	1.01	1.06
	1750	0.11	0.95	1.00	0.99	0.99	1.00	1.00	1.02	1.11
	2500	0.13	0.96	1.00	0.99	0.99	1.00	1.00	1.04	1.17
C	200	0.97	1.00	1.00	0.99	0.99	1.00	1.00	1.04	1.17
	400	0.65	0.99	1.00	0.99	0.99	1.00	1.00	1.02	1.08
	1000	0.05	0.79	1.00	0.99	0.99	1.00	1.00	1.01	1.04
R	10	0.05	0.87	1.00	1.00	1.00	1.00	1.00	1.00	1.02
	20	0.16	0.90	1.00	1.00	1.00	1.00	1.00	1.01	1.03
	50	0.42	0.95	1.00	0.99	0.99	1.00	1.00	1.02	1.08
	100	0.70	0.98	1.00	0.99	0.99	1.00	1.00	1.05	1.17

Appendix C

SUPPLY CHAIN COORDINATION WITH LINEAR DEMAND FUNCTION

In this chapter, we evaluate the performance of the following coordination mechanisms under linear demand function.

- Transportation Cost Sharing Contract (TCSC)
- Quantity Discounts (QD)
- Volume Discounts (VD)
- Simultaneous Offer of Quantity and Volume Discounts (QVD)

We model these mechanisms considering transportation cost and provide numerical analysis. Besides, we further model the simultaneous offer of quantity and volume discounts.

C.1 Initial Market Equilibrium

In this chapter, the decentralized supply chain considered in Section B.4 is evaluated as the base supply chain. Moreover, the equilibrium point given by the solution of the Stackelberg game is considered as the initial market equilibrium. Let v^*, n^*, p^*, Q^* be the equilibrium decisions of the game and $\prod_S(v^*, n^*), \prod_B(p^*, Q^*)$ be the equilibrium profits.

C.2 Transportation Cost Sharing Contract

The transportation cost sharing contract is modeled in this section. The mechanism requires the supplier share a predetermined portion of the transportation cost of the buyer. The supplier shares $(1-\alpha)$ portion of the total transportation cost and the α value providing the highest profit is selected as the optimal transportation cost sharing contract.

The Buyer's Problem

To obtain profit functions, we replace $F(Q)$ with $(\alpha)F(Q)$ in (4.44) and insert $(1-\alpha)F(Q)\frac{D(p)}{Q}$ into the supplier's profit function (4.45).

The buyer's profit function can be rewritten as follows:

$$\prod_B(p,Q,\alpha) = (p-v)D(p) - (v + \frac{\alpha F(Q)}{Q})I\frac{Q}{2} - (A + \alpha F(Q))\frac{D(p)}{Q}. \tag{C.1}$$

Likewise, the supplier's profit function is:

$$\prod_S(v,n,\alpha) = (v-m)D(p^*(v)) - mI(n-1)\frac{Q^*(v)}{2} - \left(\frac{K}{n} + (1-\alpha)F(Q)\right)\frac{D(p^*(v))}{Q^*(v)}, \tag{C.2}$$

We follow the derivation steps in Section B.4 for the buyer's problem. We obtain first and second order optimality conditions with respect to p. Equating the first-order condition to zero we obtain the following global optimal solution for optimal market price:

$$p^*(Q) = \frac{aQ + b(A + F(Q) + Qv)}{2bQ}. \tag{C.3}$$

We can replace $p^*(Q)$ and rewrite the buyer's profit function as follows:

$$\prod_B(Q) = (p^*(Q)-v)(a-bp^*(Q)) - vI\frac{Q}{2} - (A + \alpha F(Q))\frac{(a-bp^*(Q))}{Q} - \frac{\alpha F(Q)I}{2}. \tag{C.4}$$

For a fixed number of trucks t, where $(t-1)C < Q \leq tC$, we can rewrite (C.4) as

$$\prod_B(Q) = (p^*(Q)-v)(a-bp^*(Q)) - vI\frac{Q}{2} - (A + \alpha tR)\frac{(a-bp^*(Q))}{Q} - \frac{\alpha tRI}{2}. \tag{C.5}$$

We write the first-order condition for a local maximum with respect to Q for a particular t value as

$$\frac{\partial \prod_B(Q)}{\partial Q} = \frac{(A+\alpha tR)\ (-(b\ (A+\alpha tR)) + a\,Q) - Q\ \left(b\ (A+\alpha tR) + I\,Q^2\right)\ v}{2\,Q^3}. \tag{C.6}$$

and by setting it equal to zero we obtain the single positive and real root

$$Q^*(v) = \frac{\left(1 - i\,\sqrt{3}\right)\,G^{\frac{1}{3}}}{6\,2^{\frac{1}{3}}\,I\,v} + \frac{\left(1 + i\,\sqrt{3}\right)\,(a\,A + a\,\alpha tR - A\,b\,v - b\,\alpha tR\,v)}{2^{\frac{2}{3}}\,G^{\frac{1}{3}}}, \text{ where}$$
$$G = 27\,b\,(A+\alpha tR)^2\,I^2\,v^2$$
$$+\sqrt{27}\,\sqrt{(A+\alpha tR)^3\,I^3\,v^3\ (-4\,a^3 + 3\,b\ (4\,a^2 + 9\,b\ (A+\alpha tR)\ I)\ v - 12\,a\,b^2\,v^2 + 4\,b^3\,v^3)} \tag{C.7}$$

Equality (C.7) provides the optimal order quantity when the number of trucks is fixed as t, and $(t-1)C < Q \leq tC$. Since T is the maximum number of trucks, the buyer can perform

T evaluations using (C.7) and obtain T many Q values. The buyer designates the order quantity that provides the highest profit as her order quantity.

The Supplier's Problem

We solve the problem utilizing the grid search. If Q and n is constant, replacing $p^*(v)$ with $p^*(Q)$ as given in Equation (C.3), we can rewrite the supplier's profit function as follows.

$$\prod_S(v) = (v-m)D(p^*(Q)) - mI(n-1)\frac{Q}{2} - \left(\frac{K}{n} + (1-\alpha)F(Q)\right)\frac{D(p^*(Q))}{Q}. \qquad \text{(C.8)}$$

C.2.1 Derivation of the optimal lot size multiplier under a fixed value of the wholesale price

As in Section 5.3, we state first and second derivatives of the supplier's profit function (C.2) with respect to n, and state two optimality conditions. Since the equations do not depend on the demand function, the derivation is the same. Hence, we do not repeat the derivation and state the optimal lot size multiplier as follows:

$$n^* = \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil \text{ or } \left\lfloor \frac{1}{2}\left(1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rfloor. \qquad \text{(C.9)}$$

The optimal contract and corresponding profit for the supplier is determined numerically by a grid search within the wholesale price range $[m, v_{max}]$ and sharing ratio $(1-\alpha)$ range [0,1]. For a fixed wholesale price v and sharing ratio $1-\alpha$, optimal price can be stated as $p^*(Q) = \frac{b(A+Qv+\alpha F(Q))}{Q(b-1)}$, which is given in (C.3). Optimal order quantity is given by (C.7). Finally, optimal lot size multiplier is given by $n^* = \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil$, which is stated in (C.9). We adopt a grid size $g = 10^{-3}$ for the wholesale price and $g = 10^{-2}$ for the sharing ratio in the computations. The algorithm of Section 5.3.1 is used with the decision variables and profit functions mentioned above.

C.3 Quantity Discounts

In this section, we model the optimal quantity discount offer of the supplier with linear demand. The solution of the Stackelberg game is available for the supplier and the buyer as a priori. Therefore, the buyer does not accept a wholesale price v_{QD} and an order quantity Q_{QD} that would lead to a profit less than $\prod_B(p^*, Q^*)$. Hence, the order quantity Q^*_{QD} is determined by finding the value of Q^*_{QD} where $Q^*_{QD} \geq Q^*_{v^*}$ for which $\prod_B(Q_{QD}) \geq$

$\prod_B(p^*, Q^*)$. Likewise, the supplier will not offer a quantity discount if the optimal wholesale price offer v^*_{QD} leads to a profit less than $\prod_S(v^*, n^*)$. In such a situation, we may conclude that a feasible quantity discount does not exist. However, there exists a feasible quantity discount for the supply chain for all of the examples in our test in Section C.6.

Let us restate the profit function of the buyer as a function of the market price. Since the supplier offers a discount based on the order quantity, the supplier does not select an order quantity but a market price.

$$\prod_B(p) = (p - v)D(p) - (v + \frac{F(Q)}{Q})I\frac{Q}{2} - (A + F(Q))\frac{D(p)}{Q}. \tag{C.10}$$

The supplier offers quantity discounts (v_{QD}, Q_{QD}) where $v_{QD} < v^*$ and $Q_{QD} > Q^*$. The optimization formulation for the optimal quantity discount offer of the supplier can be stated as follows:

$$\begin{aligned} \max \prod_S(v_{QD}, n, Q_{QD}) &= (v_{QD} - m)D(p^*(v_{QD})) - mI(n-1)\frac{Q_{QD}}{2} \\ &\quad - \left(\frac{K}{n}\right)\frac{D(p^*(v_{QD}))}{Q_{QD}} \end{aligned} \tag{C.11}$$

$$\text{s.t.} \ \prod_B(p^*(Q_{QD})) \geq \prod_B(p^*, Q^*), \text{ where} \tag{C.12}$$

$$\begin{aligned} \prod_B(p^*(Q_{QD}) &= (p^*(Q_{QD}) - v_{QD})(a - bp_{QD}) - v_{QD}I\frac{Q_{QD}}{2} \\ &\quad -(A + F(Q_{QD}))\frac{(a - bp_{QD})}{Q_{QD}} \end{aligned} \tag{C.13}$$

If the buyer accepts the discount scheme where $\prod_B(p^*(Q_{QD})) \geq \prod_B(p^*, Q^*)$, through (B.23) the optimal market price can be stated as follows:

$$p^*(Q_{QD}) = \frac{a\,Q_{QD} + b\ (A + F(Q_{QD}) + Q_{QD}\,v)}{2\,b\,Q_{QD}} \tag{C.14}$$

Optimal quantity offer Q_{QD} and discounted wholesale price offer v_{QD} can be determined by a grid search through the range $(Q^*(v^*), TR]$ and (m, v^*) respectively. Furthermore, (4.19) provides the optimal lot size multiplier for a fixed whole price, which can be restated as

$$n^* = \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil \text{ or } \left\lfloor \frac{1}{2}\left(1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rfloor. \tag{C.15}$$

We adopt a grid size $g = 5.10^{-3}$ for the wholesale price and $g = 10^0$ for the order quantity in the computations. The algorithm of Section 5.4 is used with the decision variables and profit functions mentioned above.

C.4 Volume Discounts

In this section, we consider the optimal volume discount offer of the supplier. The supplier offers a wholesale price v_{VD} if the market price is at least p_{VD}. In other words, if the market price is lower than or equal to v_{VD}, a volume discount is offered on all units.

The solution of the Stackelberg game is available for the supplier and the buyer as a priori. Therefore, the buyer does not accept a wholesale price v_{VD} and a market price p_{VD} that would lead to a profit less than $\prod_B(p^*, Q^*)$. Hence, the market price p^*_{VD} is determined by finding the value of p^*_{VD} where $p^*_{VD} \leq p^*_{v^*}$ for which $\prod_B(p_{VD}) \geq \prod_B(p^*, Q^*)$. Likewise, the supplier will not offer a volume discount if the optimal wholesale price offer v^*_{VD} leads to a profit less than $\prod_S(v^*, n^*)$. In such a situation, we may conclude that a feasible volume discount does not exist. However, there exists a feasible volume discount for the supply chain for all of the examples in our test in Section C.6.

Let us restate the profit function of the buyer as a function of the order quantity, given the market price. Since the supplier offers a discount based on the market price, the supplier does not select an order quantity but a market price.

$$\prod_B(Q) = (p - v)D(p) - (v + \frac{F(Q)}{Q})I\frac{Q}{2} - (A + F(Q))\frac{D(p)}{Q}. \tag{C.16}$$

The supplier offers volume discounts (v_{VD}, p_{VD}) where $v_{VD} < v^*$ and $p_{VD} < p^*$. The optimization formulation for the optimal volume discount offer of the supplier can be stated as follows:

$$\begin{aligned} \max \prod_S(v_{VD}, p_{VD}, n) &= (v_{VD} - m)D(p_{VD}) - mI(n-1)\frac{Q^*(v_{VD})}{2} \\ &\quad - \left(\frac{K}{n}\right)\frac{D(p_{VD})}{Q^*(v_{VD})} \end{aligned} \tag{C.17}$$

$$\text{s.t.} \prod_B(Q^*(v_{VD})) \geq \prod_B(p^*, Q^*), \text{ where} \tag{C.18}$$

$$\begin{aligned} \prod_B(Q^*(v_{VD})) &= (p_{VD} - v_{QD})(a - bp_{VD}) - v_{VD}I\frac{Q^*(v_{VD})}{2} \\ &\quad -(A + F(Q^*(v_{VD})))\frac{(a - bp_{VD})}{Q^*(v_{VD})} \end{aligned} \tag{C.19}$$

Incorporating the demand function into (C.16), first order optimality condition for a particular truck interval, t, can be stated as follows:

$$\frac{\partial \prod_B(Q)}{\partial Q} = \frac{(A + tR)\ (a - b\,p)}{Q^2} - \frac{I\,v}{2} = 0 \tag{C.20}$$

Since the market demand is positive, second derivative is always negative as follows.

$$\frac{\partial^2 \prod_B(Q)}{\partial Q^2} = \frac{-2\ (A + tR)\ (a - b\,p)}{Q^3} < 0 \tag{C.21}$$

Hence, solving (C.20) for Q gives the optimal order quantity

$$Q^*(v) = \sqrt{\frac{2\ (A + tR)\ (a - b\,p)}{I\,v}} \tag{C.22}$$

If the buyer accepts the discount scheme where $\prod_B(Q^*) \geq \prod_B(p^*, Q^*)$, the optimal order quantity can be stated as:

$$Q^*(v_{VD}) = \sqrt{\frac{2\ (A + tR)\ (a - b\,p_{VD})}{I\,v_{VD}}} \tag{C.23}$$

Optimal market price offer p_{VD} and discounted wholesale price offer v_{VD} can be determined by a grid search through the range $(m, p^*]$ and (m, v^*) respectively. Furthermore, (4.19) provides the optimal lot size multiplier for a fixed whole price, which can be restated as

$$n^* = \left\lceil \frac{1}{2}\left(-1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil \text{ or } \left\lfloor \frac{1}{2}\left(1 + \sqrt{1 + \frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rfloor. \tag{C.24}$$

We adopt a grid size $g = 2.5.10^{-2}$ for the wholesale price and $g = 5.10^{-2}$ for the market price in the computations. The algorithm of Section 5.5 is used with the decision variables and profit functions mentioned above.

C.5 Simultaneous Offer of Quantity and Volume Discounts

In this section, we consider the optimal quantity and volume discount offer of the supplier. The supplier offers a wholesale price v_{QVD} if the market price is at least p_{QVD} and the order quantity is at least Q_{QVD}.

The solution of the Stackelberg game is available for the supplier and the buyer as a priori. The market price p^*_{QVD} and the order quantity Q^*_{QVD} are determined by finding the value of p^*_{QVD} and Q^*_{QVD} where $p^*_{QVD} \leq p^*_{v^*}$ and $Q^*_{QVD} \geq Q^*_{v^*}$ for which $\prod_B(p_{QVD}, Q_{QVD}) \geq$

$\prod_B(p^*, Q^*)$. Likewise, the supplier will not offer a volume discount if the optimal wholesale price offer v^*_{QVD} leads to a profit less than $\prod_S(v^*, n^*)$. In such a situation, we may conclude that a feasible discount does not exist. However, there exists a feasible discount for the supply chain for all of the examples in our test in Section C.6.

The supplier offers quantity and volume discounts $(v_{QVD}, p_{QVD}, Q_{QVD})$ where $v_{QVD} < v^*$, $p_{QVD} < p^*$, $Q_{QVD} > Q^*$. The optimization formulation for the optimal volume discount offer of the supplier can be stated as follows:

$$\begin{aligned} \max \prod_S(v_{QVD}, Q_{QVD}, p_{QVD}, n) &= (v_{QVD} - m)D(p_{QVD}) - mI(n-1)\frac{Q_{QVD}}{2} \\ &\quad - \left(\frac{K}{n}\right)\frac{D(p_{QVD})}{Q_{QVD}} \end{aligned} \tag{C.25}$$

$$\text{s.t.} \prod_B(p_{QVD}, Q_{QVD}) \geq \prod_B(p^*, Q^*) \tag{C.26}$$

Since, the supply chain variables are controlled by the supplier, the buyer's profit function can be equated to the initial market equilibrium profit and the discounted wholesale price can be determined based on the price and order quantity. Solving $\prod_B(p_{QVD}, Q_{QVD}) = \prod_B(p^*, Q^*)$ gives the optimal discounted wholesale price as follows:

$$v^*_{QVD} = \frac{2\,(a - b\,p)\,(-A - F(Q) + p\,Q) - 2\,Q\,\prod_B(p^*, Q^*)}{Q\,(2\,a - 2\,b\,p + I\,Q)} \tag{C.27}$$

We adopt a grid size $g = 10^0$ for the order quantity and $g = 10^{-2}$ for the market price in the computations. The algorithm of Section 5.6 is used with the decision variables and profit functions mentioned above.

C.6 Effectiveness of The Alternative Coordination Mechanisms

The relative performance of the alternative coordination schemes are evaluated through a numerical study. We use the same dataset of Section B.6.

Table C.1 provides a summary of the markup and profit results of the decentralized supply chain comparing with centralized supply chain. The results for the supply chain are significantly different than the results of the supply chain with constant elasticity demand function. Comparing the results of the supply chain with constant elasticity demand function, markup of the supplier is significantly higher than the buyer. Centralized system's

Table C.1: Markup and profit sharing levels in supply chains.

	Minimum	Maximum	Mean	S. Deviation
Supplier's markup $\left(\frac{v}{m}\right)$	3.82	16.46	9.25	4.60
Buyer's markup $\left(\frac{p}{v}\right)$	1.40	1.59	1.47	0.03
Markup of decentralized s. $\left(\frac{p}{m}\right)$	6.01	24.47	13.68	6.95
Markup of centralized s. $\left(\frac{p}{m}\right)$	4.51	16.84	9.52	4.63
Profit % of the supplier $\left(100\frac{\Pi_S}{\Pi_B+\Pi_S}\right)$	60.77	70.39	67.17	0.94
Profit % of the buyer $\left(100\frac{\Pi_B}{\Pi_B+\Pi_S}\right)$	29.61	39.23	32.83	0.94
Centralization effect $\left(100\frac{\Pi_B+\Pi_S}{\Pi_C}\right)$	68.05	79.43	73.40	1.47

markup is higher than the decentralized system. On the average, the buyer receives one third of the total profit and the supplier receives two thirds of the total profit. The situation was vise versa for the supply chain with constant elasticity demand function. Hence, the results seem to depend on the functional form of the demand. Moreover, decentralized system can cover 73.40% of the centralized system's profit, therefore there is an important motivation for the supply chain members to coordinate.

Moreover, Table C.2 provides a summary of the computational results. On the other hand, Tables C.3, C.4, C.5 and C.6 provides a factor analysis for the results of the coordination mechanisms. The results of the study evidently demonstrate that the effectiveness of the coordination mechanisms increase in the following order: transportation cost sharing contract, quantity discounts, volume discounts, simultaneous offer of quantity and volume discounts. Hence the least effective mechanism is transportation cost sharing contract. Quantity discounts are better than the transportation cost sharing contract. However, the performance of the simultaneous offer of quantity and volume discounts is very similar to the performance of the volume discounts. When constant elasticity demand function is used, there is a significant difference between them but for the linear demand case, the simultaneous offer of quantity and volume discounts do not have a significant superiority over volume discounts.

Table C.2: Summary of the computational results of the coordination mechanisms with linear demand.

		CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7
TCSC	Minimum	68.05	0.00	-0.16	0.00	0.00	0.00	0.00
	Maximum	79.43	2.79	1.02	0.66	2.54	12.23	3.96
	Mean	73.44	0.03	0.18	0.00	0.03	0.14	0.05
	S. Deviation	1.44	0.24	0.18	0.03	0.21	1.01	0.34
QD	Minimum	69.57	0.05	-0.16	0.10	-1.02	0.00	0.07
	Maximum	79.98	2.78	1.02	5.43	0.16	1.82	4.08
	Mean	74.07	0.66	0.18	1.18	-0.18	0.35	0.91
	S. Deviation	1.19	0.43	0.18	0.84	0.18	0.39	0.62
VD	Minimum	82.79	10.43	4.19	21.03	-8.45	0.00	13.62
	Maximum	99.43	26.54	8.45	54.95	-4.19	1.95	38.69
	Mean	92.46	19.05	6.54	38.47	-6.54	0.47	26.03
	S. Deviation	5.23	5.59	1.56	11.39	1.56	0.36	7.85
QVD	Minimum	82.79	10.20	4.10	20.68	-8.53	0.00	13.46
	Maximum	99.43	26.54	8.53	55.52	-4.10	0.00	38.97
	Mean	92.46	19.06	6.66	38.70	-6.66	0.00	26.03
	S. Deviation	5.28	5.64	1.61	11.58	1.61	0.00	7.92

Table C.3: Factor analysis of transportation cost sharing contract.

		CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7
K	10	74.34	0.00	0.00	0.00	0.00	0.00	0.00
	25	74.01	0.00	0.00	0.00	0.00	0.00	0.00
	100	73.31	0.07	0.00	0.01	0.00	0.31	0.10
	250	72.09	0.06	0.00	0.01	0.00	0.25	0.08
A	$0.5K$	73.09	0.04	0.00	0.00	0.00	0.17	0.06
	K	73.37	0.05	0.00	0.01	0.00	0.22	0.07
	$2K$	73.85	0.01	0.00	0.00	0.00	0.03	0.01
b	625	73.93	0.02	0.00	0.00	0.00	0.08	0.03
	1000	73.73	0.01	0.00	0.00	0.00	0.06	0.02
	1750	73.38	0.06	0.00	0.01	0.00	0.27	0.09
	2500	72.70	0.04	0.00	0.00	0.00	0.16	0.05
C	200	73.51	0.00	0.00	0.00	0.00	0.00	0.00
	400	73.42	0.05	0.00	0.00	0.00	0.21	0.07
	1000	73.38	0.05	0.00	0.01	0.00	0.21	0.07
R	10	73.44	0.05	0.00	0.00	0.00	0.20	0.07
	20	73.44	0.04	0.00	0.00	0.00	0.18	0.06
	50	73.47	0.03	0.00	0.00	0.00	0.10	0.04
	100	73.41	0.02	0.00	0.00	0.00	0.08	0.03

Table C.4: Factor analysis of quantity discounts.

		CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7
K	10	74.74	0.39	0.00	0.65	0.00	0.30	0.53
	25	74.53	0.52	0.00	0.89	0.00	0.30	0.70
	100	74.01	0.77	0.00	1.39	0.00	0.38	1.06
	250	72.99	0.97	0.00	1.80	0.00	0.41	1.36
A	$0.5K$	73.81	0.76	0.00	1.38	0.00	0.36	1.05
	K	73.98	0.66	0.00	1.20	0.00	0.32	0.91
	$2K$	74.41	0.57	0.00	0.98	0.00	0.37	0.78
b	625	74.30	0.39	0.00	0.73	0.00	0.13	0.53
	1000	74.22	0.50	0.00	0.92	0.00	0.21	0.69
	1750	74.05	0.74	0.00	1.33	0.00	0.35	1.01
	2500	73.69	1.02	0.00	1.76	0.00	0.71	1.42
C	200	74.01	0.50	0.00	0.81	0.00	0.43	0.69
	400	74.02	0.65	0.00	1.12	0.00	0.41	0.89
	1000	74.17	0.84	0.00	1.62	0.00	0.21	1.16
R	10	74.06	0.67	0.00	1.23	0.00	0.29	0.93
	20	74.06	0.67	0.00	1.21	0.00	0.33	0.92
	50	74.10	0.66	0.00	1.17	0.00	0.37	0.91
	100	74.04	0.65	0.00	1.13	0.00	0.40	0.89

Table C.5: Factor analysis of volume discounts.

		CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7
K	10	93.45	19.11	0.07	38.18	-0.07	0.46	25.72
	25	93.10	19.09	0.07	38.27	-0.07	0.46	25.82
	100	92.18	18.95	0.07	38.38	-0.07	0.49	25.96
	250	91.09	19.06	0.07	39.05	-0.07	0.46	26.61
A	$0.5K$	92.02	18.97	0.07	38.42	-0.07	0.47	26.06
	K	92.44	19.12	0.07	38.63	-0.07	0.44	26.14
	$2K$	92.91	19.07	0.07	38.37	-0.07	0.49	25.87
b	625	85.12	11.20	0.04	22.51	-0.04	0.23	15.16
	1000	90.28	16.56	0.06	33.35	-0.06	0.39	22.48
	1750	96.77	23.45	0.08	47.40	-0.08	0.56	32.02
	2500	97.66	25.00	0.08	50.62	-0.08	0.69	34.45
C	200	92.63	19.12	0.07	38.57	-0.07	0.50	26.08
	400	92.47	19.09	0.07	38.54	-0.07	0.48	26.09
	1000	92.28	18.95	0.07	38.30	-0.07	0.43	25.90
R	10	92.43	19.04	0.07	38.48	-0.07	0.47	26.01
	20	92.41	19.01	0.07	38.43	-0.07	0.43	25.97
	50	92.47	19.03	0.07	38.42	-0.07	0.46	25.98
	100	92.53	19.14	0.07	38.55	-0.07	0.51	26.15

Table C.6: Factor analysis of quantity and volume discounts.

		CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7
K	10	93.44	93.44	0.07	38.38	-0.07	0.00	25.71
	25	93.08	19.07	0.07	38.47	-0.07	0.00	25.80
	100	92.20	18.97	0.07	38.65	-0.07	0.00	25.98
	250	91.12	19.09	0.07	39.31	-0.07	0.00	26.64
A	$0.5K$	92.02	18.97	0.07	38.65	-0.07	0.00	26.06
	K	92.44	19.12	0.07	38.84	-0.07	0.00	26.15
	$2K$	92.92	19.08	0.07	38.62	-0.07	0.00	25.88
b	625	85.06	11.15	0.04	22.52	-0.04	0.00	15.08
	1000	90.24	16.52	0.06	33.45	-0.06	0.00	22.42
	1750	96.82	23.51	0.08	47.78	-0.08	0.00	32.09
	2500	97.72	25.05	0.08	51.06	-0.08	0.00	34.53
C	200	92.62	19.11	0.07	38.80	-0.07	0.00	26.07
	400	92.46	19.09	0.07	38.77	-0.07	0.00	26.09
	1000	92.30	18.97	0.07	38.54	-0.07	0.00	25.93
R	10	92.44	19.05	0.07	38.74	-0.07	0.00	26.03
	20	92.41	19.02	0.07	38.66	-0.07	0.00	25.98
	50	92.46	19.02	0.07	38.63	-0.07	0.00	25.97
	100	92.52	19.13	0.07	38.79	-0.07	0.00	26.14

BIBLIOGRAPHY

[1] S. Chopra and P. Meindl, *Supply chain management*, Prentice Hall Upper Saddle River, NJ, 2001.

[2] R.H. Ballou, *Business Logistics Management*, New Jersey: Prentice Hall, 1992.

[3] S.R. Swenseth and M.R. Godfrey, "Incorporating transportation costs into inventory replenishment decisions," *International Journal of Production Economics*, vol. 77, no. 2, pp. 113–130, 2002.

[4] A.P. Lerner, "The Concept of Monopoly and the Measurement of Monopoly Power," *The Review of Economic Studies*, vol. 1, no. 3, pp. 157–175, 1934.

[5] T.M. Whitin, "Inventory Control and Price Theory," *Management Science*, vol. 2, no. 1, pp. 61–68, 1955.

[6] E.L. Porteus, "Investing in Reduced Setups in the EOQ Model," *Management Science*, vol. 31, no. 8, pp. 998–1010, 1985.

[7] L. Lu, "A one-vendor multi-buyer integrated inventory model," *European Journal of Operational Research*, vol. 81, no. 2, pp. 312–323, 1995.

[8] M. Yao and C. Chiou, "On a replenishment coordination model in an integrated supply chain with one vendor and multiple buyers," *European Journal of Operational Research*, vol. 159, no. 2, pp. 406–419, 2004.

[9] M. Khouja, "Optimizing inventory decisions in a multi-stage multi-customer supply chain," *Transportation Research Part E: Logistics and Transportation Review*, vol. 39, no. 3, pp. 193–208, 2003.

[10] C.K. Chan and B.G. Kingsman, "Coordination in a single-vendor multi-buyer supply chain by synchronizing delivery and production cycles," *Transportation Research Part E: Logistics and Transportation Review*, vol. 43, no. 2, pp. 90–111, 2007.

[11] R. Roundy, "98%-Effective Integer-Ratio Lot-Sizing for One-Warehouse Multi-Retailer Systems," *Management Science*, vol. 31, no. 11, pp. 1416–1430, 1985.

[12] H. Kunreuther and J.F. Richard, "Optimal Pricing and Inventory Decisions for Non-Seasonal Items," *Econometrica*, vol. 39, no. 1, pp. 173–175, 1971.

[13] F.J. Arcelus and G. Srinivasan, "Inventory Policies under Various Optimizing Criteria and Variable Markup Rates," *Management Science*, vol. 33, no. 6, pp. 756–762, 1987.

[14] D. Rosenberg, "Optimal price-inventory decisions: profit vs. ROII," *IIE Transactions*, vol. 23, no. 1, pp. 17–22, 1991.

[15] T.C.E. Cheng, "An EOQ model with pricing consideration," *Computers and Industrial Engineering*, vol. 18, no. 4, pp. 529–534, 1990.

[16] C.K. Chen and K.J. Min, "A multi-product EOQ model with pricing considerationTCE Cheng's model revisited," *Computers and Industrial Engineering*, vol. 26, no. 4, pp. 787–794, 1994.

[17] D. Kim and W.J. Lee, "Optimal joint pricing and lot sizing with fixed and variable capacity," *European Journal of Operational Research*, vol. 109, no. 1, pp. 212–227, 1998.

[18] J.C. Wehrman, "Evaluating the total cost of a purchase decision," *Production and Inventory Management*, vol. 25, pp. 86–91, 1984.

[19] D.C. Aucamp, "Nonlinear Freight Costs in the EOQ Problem.," *European Journal of Operational Research*, vol. 9, no. 1, pp. 61–63, 1982.

[20] C. Lee, "The economic order quantity for freight discount costs," *IIE Transactions*, vol. 18, no. 3, pp. 318–320, 1986.

[21] P.D. Larson, "The economic transportation quantity," *Transportation Journal*, vol. 28, no. 2, pp. 43–48, 1988.

[22] R.J. Tersine, P.D. Larson, and S. Barman, "An Economic Inventory/Transport Model with Freight Rate Discounts," *Logistics and Transportation Review*, vol. 25, no. 4, pp. 291–305, 1989.

[23] H. Hwang, D.H. Moon, and S.W. Shinn, "An EOQ model with quantity discounts for both purchasing price and freight cost," *Computers and Operations Research*, vol. 17, no. 1, pp. 73–78, 1990.

[24] R.J. Tersine and S. Barman, "Economic inventory/transport lot sizing with quantity and freight rate discounts," *Decision Sciences*, vol. 22, no. 5, pp. 1171–1179, 1991.

[25] R.J. Tersine and S. Barman, "Optimal lot sizes for unit and shipping discount situations," *IIE Transactions*, vol. 26, no. 2, pp. 97–101, 1994.

[26] R.M. Russell and L. Krajewski, "Optimal purchase and transportation cost lot sizing for a single item," *Decision Sciences*, vol. 22, no. 4, pp. 940–952, 1991.

[27] J.R. Carter, B.G. Ferrin, and C.R. Carter, "On extending Russell and Krajewskis algorithm for economic purchase order quantities," *Decision Sciences*, vol. 26, pp. 819–830, 1995.

[28] S.W. Shinn, H. Hwang, and S.S. Park, "Joint price and lot size determination under conditions of permissible delay in payments and quantity discounts for freight cost," *European Journal of Operational Research*, vol. 91, pp. 528–542, 1996.

[29] T.H. Burwell, D.S. Dave, K.E. Fitzpatrick, and M.R. Roy, "Economic lot size model for price-dependent demand under quantity and freight discounts," *International Journal of Production Economics*, vol. 48, no. 2, pp. 141–155, 1997.

[30] P.L. Abad and V. Aggarwal, "Incorporating transport cost in the lot size and pricing decisions with downward sloping demand," *International Journal of Production Economics*, vol. 95, pp. 297–305, 2005.

[31] P.L. Abad and V. Aggarwal, “Erratum to: ”Incorporating transport cost in the lot size and pricing decisions with downward sloping demand”: [International Journal of Production Economics 95, 297-305],” *International Journal of Production Economics*, vol. 103, no. 2, pp. 893–894, 2006.

[32] M.A. Hoque and S.K. Goyal, “An optimal policy for a single-vendor single-buyer integrated production-inventory system with capacity constraint of the transport equipment,” *International Journal of Production Economics*, vol. 65, no. 3, pp. 305–315, 2000.

[33] K. Ertogral, M. Darwish, and M. Ben-Daya, “Production and shipment lot sizing in a vendor–buyer supply chain with transportation cost,” *European Journal of Operational Research*, vol. 176, no. 3, pp. 1592–1606, 2007.

[34] L. Lei, Q. Wang, and C. Fan, “Optimal business policies for a supplier-transporter-buyer channel with a price-sensitive demand,” *Journal of the Operational Research Society*, vol. 57, no. 3, pp. 281–289, 2006.

[35] J.R. Carter and B.G. Ferrin, “Transportation costs and inventory management: why transportation cost matter,” *Production and Inventory Management Journal*, vol. 37, no. 3, pp. 58–62, 1996.

[36] C.A. Yano and S.M. Gilbert, *Chakravarty, and Eliashberg, ”Coordinated pricing and production/procurement decisions: A review” Managing Business Interfaces: Marketing, Engineering, and Manufacturing Perspectives*, Kluwer Academic Publishers, Boston, MA, 2004.

[37] S. Zahir and R. Sarker, “Joint Economic Ordering Policies of Multiple Wholesalers and a Single Manufacturer with Price-Dependent Demand Functions,” *The Journal of the Operational Research Society*, vol. 42, no. 2, pp. 157–164, 1991.

[38] P.L. Abad, “Supplier pricing and lot sizing when demand is price sensitive,” *European Journal of Operational Research*, vol. 78, no. 3, pp. 334–354, 1994.

[39] C.A. Ingene and M.E. Parry, "Is channel coordination all it is cracked up to be," *Journal of Retailing*, vol. 76, no. 4, pp. 511–547, 2000.

[40] V.S. Desai, "Interactions between members of a marketing-production channel under seasonal demand," *European Journal of Operational Research*, vol. 90, no. 1, pp. 115–141, 1996.

[41] D.J. Reyniers, "The Effect of Vertical Integration on Consumer Prices in the Presence of Inventory Costs," *European Journal of Operational Research*, vol. 130, no. 1, pp. 83–89, 2001.

[42] G. Ertek and P.M. Griffin, "Supplier-and buyer-driven channels in a two-stage supply chain," *IIE Transactions*, vol. 34, no. 8, pp. 691–700, 2002.

[43] G.P. Cachon, "Supply chain coordination with contracts," *Handbooks in Operations Research and Management Science: Supply Chain Management*, 2003.

[44] A.A. Tsay, S. Nahmias, and N. Agrawal, "Modeling supply chain contracts: A review," *Quantitative Models for Supply Chain Management*, pp. 299–336, 1999.

[45] M.A. Lariviere, *Supply Chain Contracting and Coordination with Stochastic Demand*, Kluwer Academic Publishers, Boston, 1999.

[46] G. Cachon and S. Netessine, "Game theory in supply chain analysis. D. Simchi-Levi, SD Wu, ZM Shen, eds," *Supply Chain Analysis in the eBusiness Era*, 2003.

[47] J. Crowther, "Rationale for quantity discounts," *Harvard Business Review*, vol. 42, no. 2, pp. 121–127, 1964.

[48] S. Ladany and A. Sternlieb, "The interaction of economic ordering quantities and marketing policies," *AIIE Transactions*, vol. 6, no. 1, pp. 35–40, 1974.

[49] A.C. Brahmbhatt and M.C. Jaiswal, "An order-level-lot-size model with uniform replenishment rate and variable mark-up of prices," *International Journal of Production Research*, vol. 18, no. 5, pp. 655–664, 1980.

[50] E.S. Subramanyam and S. Kumaraswamy, "EOQ formula under varying marketing policies and conditions," *AIIE Transactions*, vol. 13, pp. 312–314, 1981.

[51] H.L. Lee and M.J. Rosenblatt, "A Generalized Quantity Discount Pricing Model to Increase Supplier's Profits," *Management Science*, vol. 32, no. 9, pp. 1177–1185, 1986.

[52] F.J. Arcelus and G. Srinivasan, "A roi-maximizing eoq model under variable demand and markup rates," *Engineering Costs and Production Economics*, vol. 9, no. 1-3, pp. 113–117, 1985.

[53] P.L. Abad, "Determining optimal selling price and lot size when the supplier offers all-unit quantity discounts," *Decision Sciences*, vol. 19, no. 3, pp. 622–634, 1988.

[54] P.L. Abad, "Joint Price and Lot-Size Determination When Supplier Offers Incremental Quantity Discounts," *The Journal of the Operational Research Society*, vol. 39, no. 6, pp. 603–607, 1988.

[55] T. Burwell, D. Dave, K. Fitzpatrick, and M. Roy, "A Note on Determining Optimal Selling Price and Lot Size under All-Unit Quantity Discounts," *Decision Sciences*, vol. 21, no. 2, pp. 471–474, 1990.

[56] W.J. Lee, "Determining Order Quantity and Selling Price by Geometric Programming: Optimal Solution, Bounds, and Sensitivity," *Decision Sciences*, vol. 24, no. 1, pp. 76–87, 1993.

[57] W.J. Lee, "Optimal order quantities and prices with storage space and inventory investment limitations," *Computers and Industrial Engineering*, vol. 26, no. 3, pp. 481–488, 1994.

[58] P.L. Abad, "Optimal policy for a reseller when the supplier offers a temporary reduction in price," *Decision Sciences*, vol. 28, no. 3, pp. 637–653, 1997.

[59] C.L. Munson and M.J. Rosenblatt, "Theories and realities of quantity discounts: An exploratory study," *Production and Operations Management*, vol. 7, no. 4, pp. 352–369, 1998.

[60] S.P. Sarmah, D. Acharya, and S.K. Goyal, "Buyer vendor coordination models in supply chain management," *European Journal of Operational Research*, vol. 175, no. 1, pp. 1–15, 2006.

[61] R.J. Dolan, "Quantity Discounts: Managerial Issues and Research Opportunities," *Marketing Science*, vol. 6, no. 1, pp. 1–22, 1987.

[62] R. Lal and R. Staelin, "An Approach for Developing an Optimal Discount Pricing Policy," *Management Science*, vol. 30, no. 12, pp. 1524–1539, 1984.

[63] Z. Drezner and G.O. Wesolowsky, "Multi-buyer discount pricing.," *European Journal of Operational Research*, vol. 40, no. 1, pp. 38–42, 1989.

[64] R. Kohli and H. Park, "A Cooperative Game Theory Model of Quantity Discounts," *Management Science*, vol. 35, no. 6, pp. 693–707, 1989.

[65] A.K. Chakravarty and G.E. Martin, "Operational economies of a process positioning determinant," *Computers and Operations Research*, vol. 18, no. 6, pp. 515–530, 1991.

[66] W.C. Chiang, J. Fitzsimmons, Z. Huang, and S.X. Li, "A Game-Theoretic Approach to Quantity Discount Problems," *Decision Sciences*, vol. 25, no. 1, pp. 153–168, 1994.

[67] Q. Wang and Z. Wu, "Improving a supplier's quantity discount gain from many different buyers," *IIE Transactions*, vol. 32, no. 11, pp. 1071–1079, 2000.

[68] Z.K. Weng and R.T. Wong, "General models for the supplier's all-unit quantity discount policy," *Naval Research Logistics*, vol. 40, no. 7, pp. 971–991, 1993.

[69] M. Parlar and Q. Wang, "Discounting decisions in a supplier-buyer relationship with a linear buyer's demand," *IIE Transactions*, vol. 26, no. 2, pp. 34–41, 1994.

[70] Z.K. Weng, "Modeling quantity discounts under general price-sensitive demand functions: Optimal policies and relationships," *European Journal of Operational Research*, vol. 86, no. 2, pp. 300–314, 1995.

[71] Z.K. Weng, "Channel Coordination and Quantity Discounts," *Management Science*, vol. 41, no. 9, pp. 1509–1522, 1995.

[72] C.J. Corbett and X. de Groote, "A Supplier's Optimal Quantity Discount Policy Under Asymmetric Information," *Management Science*, vol. 46, no. 3, pp. 444–450, 2000.

[73] F. Chen, A. Federgruen, and Y.S. Zheng, "Coordination Mechanisms for a Distribution System with One Supplier and Multiple Retailers," *Management Science*, vol. 47, no. 5, pp. 693–708, 2001.

[74] S. Viswanathan and Q. Wang, "Discount pricing decisions in distribution channels with price-sensitive demand," *European Journal of Operational Research*, vol. 149, no. 3, pp. 571–587, 2003.

[75] Q. Wang, "Coordinating Independent Buyers in a Distribution System to Increase a Vendor's Profits," *Manufacturing & Service Operations Management*, vol. 3, no. 4, pp. 337–348, 2001.

[76] Q. Wang, "Coordinating independent buyers with integer-ratio time coordination and quantity discounts," *Naval Research Logistics*, vol. 51, no. 3, pp. 316–331, 2004.

[77] J.M. Chen and T.H. Chen, "The multi-item replenishment problem in a two-echelon supply chain: the effect of centralization versus decentralization," *Computers and Operations Research*, vol. 32, no. 12, pp. 3191–3207, 2005.

[78] T.H. Chen and J.M. Chen, "Optimizing supply chain collaboration based on joint replenishment and channel coordination," *Transportation Research Part E: Logistics and Transportation Review*, vol. 41, no. 4, pp. 261–285, 2005.

[79] Q. Wang and R. Wang, "Quantity Discount Pricing Policies for Heterogeneous Retailers with Price Sensitive Demand," *Naval Research Logistics*, vol. 52, pp. 645–658, 2005.

[80] Q. Wang, "Discount Pricing Policies and the Coordination of Decentralized Distribution Systems," *Decision Sciences*, vol. 36, no. 4, 2005.

[81] Turkish Statistical Institute - Prime Ministry Republic of Turkey, "Turkey's statistical yearbook," 2006, http://www.tuik.gov.tr/yillik/yillik.pdf.

[82] J. Kiefer, "Sequential Minimax Search for a Maximum," *Proceedings of the American Mathematical Society*, vol. 4, no. 3, pp. 502–506, 1953.

[83] C.L. Munson and M.J. Rosenblatt, "Coordinating a three-level supply chain with quantity discounts," *IIE Transactions*, vol. 33, no. 5, pp. 371–384, 2001.

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