MILP Based Hyper-Box Enclosure Approach to Multi-Class Data Classification

by

Fadime Üney Yüksektepe

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This is to certify that I have examined this copy of a master's thesis by

Fadime Üney Yüksektepe

and have found that it is complete and satisfactory in all respects, and that any and all revisions required by the final examining committee have been made

Committee Members:

Metin Türkay, Ph. D. (Advisor)

Kuban Altınel, Ph. D.

Ceyda Oğuz, Ph. D.

Serpil Sayın, Ph. D.

Deniz Yüret, Ph. D.

Date:

28.05.2009

I dedicate this thesis to my husband, Gökhan, and my parents for their constant support and unconditional love.

ABSTRACT

Data classification is an important data mining problem that aims to determine the membership of different instances to a number of different sets. Traditional approaches that are based on partitioning the data sets into two groups need some modifications for multi-class data classification problems. These modifications affect the efficiency and make the models more complex. In this thesis, a novel mixed integer programming based hyper-box enclosure approach is presented for multiclass data classification problems. In order to deal with large data sets, a three-stage mathematical programming based approach is developed for training part analysis of hyper-box enclosure method. Training set is preprocessed to identify the observations that are more difficult to classify, and seed finding and sub grouping algorithms are applied in the first stage. Then, optimization model is formulated considering these observations and seeds. Finally, assignments of non-problematic instances, intersection elimination and box combination algorithms are carried out. After training analysis with this three stage approach, the efficiency of the method is tested by the simple distance based testing algorithm. The efficiency of the proposed three-stage method is tested on two separate benchmark problems; the protein folding type prediction problem and the UCI Repository data sets. The computational results on the illustrative example and the benchmark problems show the accuracy of the proposed method.

ÖZET

Veri Sınıflandırma, farklı özelliklere sahip örneklerin bilinen sınıflara olan üyeliğini belirlemeye çalışan önemli bir veri madenciliği problemidir. Veri setini iki gruba ayıran geleneksel yöntemleri çok sınıflı veri sınıflandırma problemlerine uygulayabilmek için bazı düzenlemelere gerek vardır. Yapılan bu değişiklikler kullanılan yöntemin verimliliğini etkilemekte ve modeli daha karmaşık bir hale getirmektedir. Bu tezde çok gruplu veri sınıflandırma problemi için geliştirilmiş tamsayı karışık programlamaya dayalı yeni çok boyutlu kutu yaklaşımı anlatılmaktadır. Büyük veri kümeleri ile çalışabilmek için çok boyutlu kutu yaklaşımının eğitici bölümünde kullanılmak üzere üç aşamalı matematiksel programlamaya dayalı bir yöntem geliştirilmiştir. Birinci aşamada, eğitici kümedeki sınıflandırması zor olan örnekler belirlenerek, tohum bulma ve alt küme oluşturma algoritmaları uygulanmaktadır. Daha sonra edinilen bu gözlem ve tohumlar kullanılarak eniyileme modeli çözülmektedir. Son olarak da problemsiz örneklerin kutulara atanması, kesişme engelleme ve kutu birleştirme algoritmaları uygulanmaktadır. Bu üç aşamalı eğitici çalışmalar sonrasında, metodun verimliliği uzaklığa dayalı basit bir test algoritması ile ölçülmüştür. Bu üç aşamalı modelin verimliliği veri sınıflandırılmasında çok bilinen ve çok kullanılan veri setleri üzerinde test edilmiştir. Bunlar protein katlanma tahmin problemi ve UCI veri havuzu problemleridir. Örnek problem ve bilenen veri setleri kullanılarak elde edilen sonuçlar önerilen yöntemin çok sınıflı veri sınıflandırma problemine önemli bir katkıda bulunduğunu kanıtlamaktadır.

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NOMENCLATURE

- $INI_{ll'm}$ Binary variable to indicate the intersection of hyper-box l with hyper-box l' for attribute m
- $IN2_{ll'm}$ Binary variable to indicate the intersection of hyper-box l' with hyper-box l for attribute m
- $IO_{ll'}$ Binary variable that represents the intersection of hyper-box l and hyper-box l'
- CO_l Binary variable that represents an intersection related to hyper-box l'
- SO_l Binary variable that indicates that hyper-box l' could be obtained without causing any intersection
- $SI_{ll'}$ Binary parameter that gives the relationship with hyper-box l and hyper-box l'
- $SN_{l'l'}$ Binary parameter that represents the relationship between hyper-box l' and hyper-box l''
- DH_{il} Minimum distance between instance i and the normal of hyper-box l

$$
EP_{lj}
$$
  Extreme point *j* of hyper-box *l*

- EP_{lt} Extreme point t of hyper-box l
- EPP Set of extreme point combinations
- ep_{lim} The value of attribute *m* for extreme point *j* of hyper-box *l*
- ep_{lm} The value of attribute *m* for extreme point *t* of hyper-box *l*
- w_{ilitm} Difference between a_{im} and ep_{ljm}
- v_{ijtm} Difference vector between ep_{ijm} and ep_{lm}
- Cl_{iljtm} Dot product of w_{iljtm} and v_{iljtm}
- $C2_{\text{ilitm}}$ Dot product of v_{ilitm} by itself
- b_{ilitm} Ratio of $C1_{\text{ilitm}}$ to $C2_{\text{ilitm}}$
- pb_{iljtm} Point where a_{im} is perpendicular to the edge between two extreme points
- DED_{il} Minimum distance between instance i and the edges of hyper-box l
- DEP_{il} Minimum distance between instance i and the extreme points of hyper-box l
- $NDist_{il}$ Minimum distance from instance i to hyper-box l

Chapter 1

INTRODUCTION

Customer information becomes very important for companies as it is necessary to achieve power and success in the market. Due to recent advances in sophisticated hardware and software technologies, large quantities of data can be acquired, processed and stored. However, the amount of collected data frequently increases and constitutes large complicated databases. As a result of these structures, database management and data mining studies receive considerable attention. Data mining is the process of investigating and extracting implicit, previously unknown and potentially useful information form large data by using one or more computer-based learning techniques. The objective of data mining is to discover general patterns and similar characteristics of available data. Many different data mining methods exist; for example clustering, classification, association analysis, feature selection and characterization. Of these methods, data classification is the most important and widely studied topic [1].

1.1 Data Classification

Data classification, sometimes referred as pattern recognition or discriminant analysis, is a supervised learning strategy that analyzes the organization and categorization of data in distinct classes [2]. Generally, a training set, in which all objects are already associated with known class labels, is used by classification methods. The data classification algorithm works on this set by using the input attributes and builds a model to classify new objects. In other words, the algorithm predicts output attribute values. Output attribute of the developed model is categorical. For instance, a bank could attempt to understand the behavior of its customers via credit analysis, and customers can be assigned one of three possible labels; "safe", "risky", and "very risky". The generated model could be used either to accept or reject future credit requests [1].

Classification has several significant differences from clustering, a related data mining technique. The class labels and the number of classes are not known in clustering. On the other hand, the class labels and the number of classes are known a priori for classification. In addition, there is no output attribute in clustering, thus, clustering algorithms attempt to group instances into two or more classes by using some measure of cluster quality [3]. Unlike clustering, prediction has an output attribute. However, the purpose of prediction is to determine future outcome rather than current behavior. In classification, an output attribute is categorical, whereas the output attribute of a predictive model can be either categorical or numerical. In summary, classification places emphasize on building models that are able to assign new instances to one of a set of well-defined classes [2].

There are many applications of data classification in finance [2, 3], health care [2], sports [2], engineering [2, 4], and science [4]. In finance, especially in risk management, data classification is applied to determine insurance rates, manage investment portfolios, and differentiate between individuals who have good or poor credit risks [3]. Furthermore, financial institutions use data classification to detect which customers are using which products so they can offer the right mix of products and services to better meet customer needs. Another application used by financial institutions is fraud detection in credit card and large cash transactions [2].

Additionally, several health care studies such as medical diagnoses and treatment effectiveness can be analyzed by the help of classification [2]. For instance, information about patients who have had or not yet had a heart attack is collected. A person's risk for

heart attack can be predicted using data classification methods. By considering these risk values, precautions are taken and certain medical treatments are applied to high risk patients [2].

In case of sports, data classification studies are carried out for horse racing and lottery. Data related to past matches between the teams are collected. Then, while playing chance games, gamblers use these past data and estimate the result of the future match and the winner [2].

Customer Relationship Management (CRM) is a well-known application of data classification in business that involves the management of interactions with customers [3]. For this purpose, information related to each customer is collected and this data are used to increase the efficiency of interaction with the customers in all stages. In CRM, classification is generally used to assign a score to a particular customer or prospect indicating the likelihood that the individual will behave in such a way that revenues and customer satisfaction levels are improved. For example, the inclination to respond to a particular offer or to switch to a product from a competitor could be measured by a score. Moreover, characterization of customer segmentation into groups with similar behavior, such as buying a particular product, can be identified by classification. Consequently, data classification models can add tremendous value to organizations both in finance and business [2, 3].

Data classification has a wide range of security related applications as well: fingerprint and facial recognition are the most studied topics. Another widely used application of data classification is in the area of bioinformatics; classification methods are being used in order to get valuable information on the characteristics of genes and proteins. Many classification methods are used in micro array analysis to predict sample phenotypes based on gene expression patterns [4]. Another problem in bioinformatics that attracted a lot of attention in the literature is the prediction of secondary structure of a protein from its

amino acid sequence [4]. Moreover, protein folding type prediction is also studied with different classification methods [4]. In conclusion, data classification is an important problem that has applications in a diverse set of areas ranging from finance to bioinformatics.

1.2 Data Classification Methods

Typical classification algorithms have three basic steps; model construction, model evaluation, and model use [1]. Each instance in training set is assumed to belong to a predefined class. By examining input attributes of the training samples, a classification model defining the general characteristics of existing classes is obtained during the model construction step. Depending on the solution approach, the model can be represented in different forms such as mathematical formulae, rule, or a computer program. The next step, model evaluation, is the accuracy estimation of the model based on a test set. In this evaluation part, known labels of each of the test samples are compared with the results of the model. The percentage of test set samples that are correctly classified by the model constitutes the accuracy value of the method. Selecting the instances of the test set is very critical: the test set must be independent of training set in order to obtain reliable results. Finally, if the accuracy of the developed model is preferable, then it is used to classify the unseen samples by assigning labels for them.

A broad range of methods exists for data classification problems including Neural Networks (NN), Support Vector Machines (SVM), Mathematical Programming, Decision Trees, K-nearest Neighbor, Logistic Regression, Bayesian Networks, Genetic algorithms, Rough Set Theory, and Fuzzy Sets. An overall view of classification methods is published by Weiss and Kulikowski [5]. In this study, available classification and prediction methods from statistics, neural networks, machine learning and expert systems are reviewed.

 Widely studied data classification methods are explained briefly in the following subtitles.

1.2.1 Neural Networks

A neural network is a data structure that attempts to simulate the behavior of neurons in a biological brain. While the human brain consists of billions of neurons, a typical neural network is composed of layers of interconnected nodes up to 100. From one unit to another, messages are passed along these connections. Through this transfer, a message can change based on the weight of the connection and the value in the node. Neural networks operate in two phases; learning and output. During the network learning, attribute values of the training instances enter the network at the input layer. The network connection weights and attribute values are practiced to compute the output for each training instance. These output values are compared with the desired network output and any error between these two values is calculated to modify the weights of the interconnections. Learning phase terminates after a predetermined number of iterations or minimum error rate is achieved. Finally, network weights are fixed and the network is used to compute output values for new instances in the output phase [2].

A major shortcoming of the neural network approach is a lack of explanation of established model. Moreover, converting categorical values to numerical ones could be a challenging issue. In addition, although the prediction accuracy is generally high, neural networks need long training times [4, 6]. Moreover, the training procedures can lead to both over fitting problem [7, 8] and gets stuck at a local optimum of the cost function.

1.2.2 Support Vector Machines

Support Vector Machines (SVM) is a new classification technique developed by Vapnik and his group [9]. They operate by finding a hyper surface that will split the classes so that the distance between the hyper surface and the nearest of the points in the groups has the largest value. The main goal is to generate a separating hyper surface which maximizes the margin and produces good generalization ability [4]. In recent years, SVM

has been considered one of the most efficient methods for two-class classification problems [10].

On the other hand, the SVM has some important drawbacks. First, a combination of SVMs has to be used in order to solve the multi-group classification problems. Second, some approximation algorithms are used in order to reduce the computational time for SVMs while learning the large scale of data. However, this computational improvement could cause less efficient performance values. Additionally, choice of the Kernel Function and the values of parameters are important decisions that directly affect the performance.

To overcome the above problems, many variants of SVM have been suggested including the use of SVM ensemble with bagging or boosting rather than the use of a single SVM [11]. Hsu *et al.* [12] compared the performance values of "all-together" and binary classification based methods such as "one-against-all", "one-against-one" and direct acyclic graph (DAG) SVM.

The one-against-all method is the earliest used implementation for SVM multi-class classification. It constructs k SVM models where k is the number of classes. The i^{th} SVM is trained with all of the examples in the ith class with positive labels, and all other examples with negative labels. One piece at a time each class is separated from the others.

Conversely, one-against-one method constructs $k(k-1)/2$ classifiers where each one is trained on data from two classes. In the testing part, if sign of the model says x is in the ith class, then the vote for the ith class is added by one. Otherwise, the jth is increased by one. Finally, x is predicted to be in the class with the largest vote.

Direct acyclic graph SVM method's training phase is the same as the one-againstone method by solving $k(k-1)/2$ binary SVMs. However, in the testing phase, it uses a rooted binary directed acyclic graph which has $k(k-1)/2$ internal nodes and k leaves. Each node is a binary SVM of ith and jth classes. Given a test sample x, starting at root node, the binary decision function is evaluated. It then moves either left or right depending on the

output value. Therefore, it goes through a path before reaching a leaf node which indicates the predicted class.

Hsu et al. [12] conclude that "one-against-one" and DAG binary classification methods are more suitable for practical use than the other methods. Nevertheless, for solving multi-class SVM in one step, a much larger optimization problem is required so experiments are limited to small data sets.

1.2.3 Mathematical Programming Approaches

The mathematical programming approach to linear discriminant analysis was first introduced in early 1980's. Since then, numerous mathematical programming models have appeared in literature. As an extension of complement to these, Erenguc and Koehler made a comprehensive review [13]. In their research, they formulate a typical mathematical programming (MP) approach as follows:

$$
\text{minimize} \qquad f(w,c) \tag{1.1}
$$

subject to:
$$
X_l w \leq c l
$$
 (1.2)

$$
X_2 w \ge (c + \varepsilon)I \tag{1.3}
$$

$$
w \neq 0 \tag{1.4}
$$

By this general formulation MP approach tries to determine a scalar c and a nonzero vector $w \in \mathbb{R}^p$ such that the hyper plane $w'x = c$ partitions the *m*-dimensional (*m*: the number of attributes) Euclidean space R^m into a closed half-space $w \, x \leq c$ and an open halfspace $w'x > c$. In the formulation, ε represents an arbitrarily small positive number. An interior and exterior deviation term for each group are defined for MP approaches. An interior deviation is the deviation from the hyper plane of a properly classified point. An exterior deviation is the deviation from the hyper plane of an improperly classified point.

Many distinct MP methods with different objective functions are developed in literature. These include; minimizing the maximum exterior deviation, minimizing the

weighted sum of exterior deviations, minimizing a measure of exterior deviations while maximizing a measure of interior deviations, minimizing the number of misclassifications, and minimizing a generalized distance measure. Most of these methods modeled data classification as linear programming (LP) problems which optimize a distance function. Contrary to LP problems, mixed-integer linear programming (MILP) problems with minimizing the misclassifications on the design data set are also widely studied [12].

MP methods have certain advantages over the parametric ones. For instance, they are free from parametric assumptions and weights to be adjusted. Moreover, varied objectives and more complex problem formulations can easily be accommodated by using MP methods. On the other hand, obtaining a solution without any discriminating power, unbounded solutions and excessive computational effort requirement are some of the problems in MP based methods.

1.2.4 Decision Trees

Decision Trees are one of the most popular top-down induction techniques in data classification. One of the main reasons behind this popularity appears to be their transparency and relative advantage in terms of interpretability. Moreover, there exist two powerful implementations of decision trees; CART [14] and C4.5 [15]. Most decision tree induction algorithms construct a tree in a top-down manner by selecting attributes one at a time and splitting the data according to the values of those attributes. The most important attribute is selected as the top split node, and so forth. For example, in C4.5 attributes are chosen to maximize the information gain ratio in the split [15]. The basic steps of a decision tree algorithm are as follows [2]:

- 1. Let T be the set of training instances.
- 2. Choose an attribute that best differentiates the instances contained in T.
- 3. Create a tree node whose value is the chosen attribute. Create child links from this node where each link represents a unique value for the chosen

attribute. Use the child link values to further subdivide the instances into subclasses.

- 4. For each subclass created in step 3:
	- a. If the instances in the subclass satisfy predefined criteria or if the set of remaining attribute choices for this path of the tree is null, specify the classification for new instances following the decision path.
	- b. If the subclass does not satisfy the predefined criteria and there is at least one attribute to further subdivide the path of the tree, let T be the current set of subset classes and return to step 2.

Existing decision tree algorithms are computationally efficient and practically successful. However, the fact that they are limited to constructing axis-parallel separating planes limits their effectiveness in applications where some combinations of attributes are highly predictive of the class [16]. A further drawback lies in the fact that continuous variables are implicitly discretized by the splitting process, losing information along the way. Moreover, most decision tree algorithms are known to be unstable when dealing with a large data set where it can be impractical to access all data at once and construct a single decision tree [17].

1.2.5 K-Nearest Neighbor Algorithm

The nearest neighbor method is a non-parametric classification technique proposed by Fix and Hodges [18] and then modified by Cover and Hart [19]. The K-nearest neighbor (K-NN) classifies unlabeled samples based on their similarity with the observations in the training set. Thus, for a given unlabeled sample, we find the "Kclosest" labeled observations in the training set and assign the unlabeled samples to class that appears most frequently within k subset. Experimental studies show that K -nearest neighbor is computationally expensive for a large data set, but it is simple and running faster than other classification methods. Moreover, the misclassification rate of K-NN rule approaches the optimal error rate asymptotically as k increases.

The K-NN algorithm uses the metric properties of the data space. The most commonly used metrics in measuring the distance of a sample from a given training set $X = \begin{bmatrix} x_1, & x_2, & \cdots & x_m \end{bmatrix}$ are as follows:

• Euclidean Distance:

$$
d_2(X, X^*) = \sqrt{\sum_{i=1}^{m} (x_i - x_i^*)^2}
$$
 (1.5)

• Minkowski Distance:

$$
d_q(X, X^*) = \sqrt[q]{\sum_{i=1}^m |x_i - x_i^*|^q}
$$
\n(1.6)

• Elliptical Distance:

$$
d(X, X^*) = \sum_{i=1}^{m} |x_i - x_i^*| \tag{1.7}
$$

The major weakness of K -nearest neighbors lays in both choices the value of k and calculation of case neighborhood: for this one, one needs to define a metric that measures the distance between data items. In most application areas, it is not clear how to, other than by trial and error, define a metric in such a way that the relative importance of data components is reflected in the metric. Furthermore, as the size of the training set becomes large, distance calculation process becomes very expensive. Moreover, it needs a large storage, because it runs using the entire training set and highly sensitive to the curse of dimensionality.

1.2.6 Logistic Regression

Logistic Regression is a nonlinear regression technique that associates a conditional probability score with each data instance [2]. It is useful when the dependent variable is either binomial or multinomial values. Binomial logistic regression is a form of regression which is used when the dependent variable is a binary and the independent variables are continuous, categorical or both. On the other hand, multinomial logistic regression exists to handle the case of more than two dependent variables [20].

Generally, logistic regression produces a formula that predicts the probability of the dependent variable as a function of the independent variables. It produces Odds Ratios (Equation 1.8) by the help of the term $p(k=1|x)$, the probability of seeing the class associated with $k = 1$ given the values contained in the feature vector x. As it is producing odds ratios as functions of predictors, the regression coefficient in the logistic regression model has no interpretation of the linear correlation.

$$
\frac{p(k=1|x)}{1-p(k=1|x)}\tag{1.8}
$$

For any feature vector x, the odds indicate how often the class associated with $k = 1$ is seen relative to the frequency in which the class associated with $k = 0$ is observed for the binomial case. After taking the natural log of this odds ratio and some transformations, logistic regression model given in Equation 1.9 will be obtained. The method iteratively tries to determine the coefficient values for the exponent term $ax+c$ in Equation 1.9. Convergence occurs when the logarithmic summation is close to zero or when the value does not change from one iteration to the next [2].

$$
p(k=1 | x) = \frac{e^{ax+c}}{1 + e^{ax+c}}
$$
 (1.9)

1.2.7 Bayesian Networks

Bayes classifier is a simple but powerful data classification technique. The model assumes all input attributes to be of equal importance and independent of one another. The classifier is based on Bayes Theorem given in Equation 1.10 where H is a hypothesis to be tested and E is evidence associated with hypothesis. Hypothesis is the dependent variable and represents the class. The evidence is determined by input attributes. $P(E|H)$ is the conditional probability that H is true given evidence E. $P(H)$ is an a priori probability, which denotes the probability of the hypothesis before any evidence is given [2].

$$
P(H \mid E) = \frac{P(E \mid H)P(H)}{P(E)}
$$
(1.10)

A Bayesian network is a directed acyclic graph G that model probabilistic relationships among a set of random variables where each variable has specific classes. Each node in the graph represents a random variable and each edge captures the direct dependencies between variables. The network encodes the conditional independence relationships that each node is independent of its non-descendants given its parents [21]. The popular Bayesian network implementation is Naïve Bayes method.

1.2.8 Other Methods

Genetic Algorithms are used in data classification problems that are difficult to solve using conventional methods. It is based on Darwinian principle of natural selection; crossover and mutation are the most widely used genetic operators. In a basic genetic learning algorithm, a population P of n elements is initialized which often referred to as chromosomes. A fitness function is used to evaluate each element of current solution. If an element passes fitness criteria, it remains in P . By using genetic operators new elements are created and added to the population. This procedure is carried on until a specified termination condition is satisfied [4].

Rough Set Theory (RST) can be approached as an extension of the Classical Set Theory [2]. Rough sets are considered as the sets with fuzzy boundaries, in other words the sets that cannot be precisely characterized using the available set of attributes. In data classification, it is inconvenient to describe the similarity among data with the indiscerniblity relation because two data x and z cannot be guaranteed in the same class even though a couple of data x and y are contained in the same class and another couple of data y and z are also contained in the same class. In other words, the transitivity property is not always useful in the problem of data classification. This non-transitivity property is more salient for the data within the boundary region. For this reason, a tolerant relation appropriate for the data classification problem is studied by some researchers.

In contrast, Fuzzy Sets are based on Fuzzy Logic [4]. Fuzzy logic is an extension of Boolean logic (YES or NO) dealing with the concept of partial truth. Whereas, classical logic holds that everything can be expressed in binary terms (0 or 1, yes or no), fuzzy logic replaces Boolean truth values with degrees of truth.

1.3 Performance Evaluation

In evaluating the performances of classification methods, the percentage of correctly classified instances, accuracies, are estimated and compared. Accuracies estimated on the training set are called as self-consistency results. It is widely known that self-consistency test results tend to be biased. Hence, two different error estimation methods are recommended to have unbiased performance evaluation.

1.3.1 Training and Test Sets

Training set is a sample of data that is used to build classification rules and functions. In order to test the performance of the classification method, another independent data set, test set, is used. True classes of the instances in that test set are known but are not shown to the classifier. Finally, predicted and true classes of test set
instances are compared and classification performance is estimated by the number of correctly classified instances. As test set instances are unseen by the classifier, this performance estimate is unbiased. When a data set is given, conventionally a 2/3 of the data set is selected randomly and used as test set. The classifier is trained on the remaining data and then tested on the test data. There exists a small loss of efficiency due to not use the full sample as training but this is not a major problem for large data sets. Generally, this type of performance evaluation is adopted when the number of instances are much larger than 1000 [22].

1.3.2 Cross-validation

For moderate size samples, the cross validation is preferred. In cross-validation, data set is divided into *m* equal-sized sub samples randomly. Each sub sample is treated as a test set and predicted via the classification rule constructed from the remaining $(m-1)$ sub samples. The classification performance is estimated by taking the average of these m sub samples. In this way, the classification rate is calculated efficiently and in an unbiased way. Leave-one-out (LOO) rate is simply applying the cross-validation with m equal to the number of instances. LOO and 10 fold cross-validation (10FCV) are very popular performance evaluation methods [22].

1.3.3 Sensitivity and Specificity

In classification methods, giving only the accuracy values are not sufficient to analyze the results. There exist other values to be estimated and analyzed such as sensitivity, specificity, Mathews Correlation Coefficient and performance with respect to random prediction. In order to define these values easily, a representative confusion matrix given in Table 1.1 will be used. The values a, b, c and d are the number of correct predictions for the respective classes 1, 2, 3, and 4. Moreover, ab is the number of incorrect predictions where Class 1 instance is predicted as Class 2 and ba is the number of incorrect predictions where Class 2 instance is predicted as Class 1. The other values of the confusion matrix are similar definitions with ab and ba.

ACTUAL	PREDICTED CLASSES					
CLASSES Class 1 Class 2 Class 3 Class 4						
Class 1	a	ah	ac	ad		
Class 2	ha	h	hс	hd		
Class 3	ca	ch	\mathcal{C}	cd		
Class 4	da	dh	dc	d		

Table 1.1 A representative confusion matrix for a four-grouped data classification problem.

Furthermore, in order to simplify the equations of performance measures, we need to define five more parameters. Total number of instances in the data set is symbolized by N. In Table 1.1, N will be total sum of the values in each of the rows and columns of the confusion matrix. C_k represents the correctly classified instances in class k . For example, in Table 1.1, $C₁$ will equal to a. N C_k is used to give the number of correctly classified instance not in class k. In Table 1.1, NC_2 will equal to $(a+c+d)$. Additionally, the number of under-predicted instances and over-predicted instances for class k are defined by U_k and O_k , respectively. U₃ will be the sum (ac+bc+dc) and O_3 will be the sum (ca+cb+cd) from Table 1.1. Using these four new parameters, other performance measure definitions will be much simpler.

The sensitivity is the ratio of correct and all predictions for a given structural class [23]. The sensitivity value of class k is given in Equation (1.11).

$$
Sensitivity_k = \frac{C_k}{C_k + O_k} \tag{1.11}
$$

The specificity is the ratio between the correct and all predictions for proteins that should be excluded for a given class [23]. The specificity value of class k is given in Equations (1.12).

$$
Specificity_k = \frac{N - C_k - O_k - U_k}{N - C_k - O_k}
$$
\n(1.12)

Generally, average specificity and sensitivity values are given for classification methods. These values can be calculated taking the weighted averages of individual specificity and sensitivity values with respect to the class sizes. In Equations (1.13) and (1.14), formal definitions of average sensitivity and specificity values are presented, respectively.

$$
Sensitivity = \sum_{k} \frac{C_k + O_k}{N} sensitivity_k
$$
\n(1.13)

$$
Specificity = \sum_{k} \frac{C_k + O_k}{N} \cdot specificity_k \tag{1.14}
$$

1.3.4 Mathews Correlation Coefficient

Mathews Correlation Coefficient (MCC) is a limited number between -1 and 1. If there is no relationship between the predicted values and actual values, the MCC should be 0 or very low (the predicted numbers are not better than random numbers). In contrast, the MCC value would increase as the strength of the relationship between the predicted values and actual values increases. It is obvious that a perfect fit gives a coefficient of 1. The higher MCC indicates the better performance of the prediction [24]. The MMC value for class k can be calculated using Equation (1.15).

$$
MCC_k = \frac{[C_k NC_k - U_k O_k]}{\sqrt{(C_k + U_k)(C_k + O_k)(NC_k + U_k)(NC_k + O_k)}}
$$
(1.15)

1.3.5 Performance with Respect to Random Prediction

Performance with respect to random prediction can be calculated by Equation (1.17). For a perfect prediction, S_k should be equal to 1 while for the predictions that are no better than random it would be equal to zero [24].

$$
RTotal_{k} = \frac{(C_{k} + U_{k})(C_{k} + O_{k}) + (NC_{k} + U_{k})(NC_{k} + O_{k})}{N}
$$
(1.16)

$$
S_k = \frac{C_k + NC_k - RTotal_k}{N - RTotal_k} \tag{1.17}
$$

Besides giving the accuracy values of the studied data sets, we will investigate these performance measures and analyze the results deeper.

1.3.6 P-value Analysis

When comparing supervised classification models, the P -value (paired t -test) analysis based on hypothesis testing need to be carried out in order to examine the differences in a statistical manner. P-value represents the difference between two models with 95% confidence. If P-value is greater than 2, the difference between the results of the models is not due to chance. Otherwise, the accuracies of the models are very close to each other and no significant improvement achieved. P-value can be calculated using Equation (1.18). In this equation, E_1 and E_2 are the error rates of two models; q is the average of two error rates; n_1 and n_2 are the number of instances in the test sets of two models.

$$
P = \frac{|E_1 - E_2|}{\sqrt{q(1-q)(1/n_1 + 1/n_2)}}
$$
(1.18)

1.4 Ideal Characteristics of Classification Methods

While evaluating the data classification methods, some important properties of the model have to be considered in detail. Firstly, methods are usually evaluated on the test data. Prediction accuracy, ability of the model to correctly predict the class label, is a very considerable point for evaluation. Most of the comparisons between the models are done by looking directly to these prediction accuracy values. On the other hand, time to construct the model and time to use it also has a big role in real life applications. For a preferable data classification model, computational time must be reasonable. Thirdly, for an ideal data classifier, it should have a few parameters to tune in the system as possible. In Neural Networks, the weights between the nodes have to be adjusted. Since all of the existing weights need to be optimized, it is not easy to incorporate the domain knowledge and they possess a long training time. Moreover, it is difficult to understand the learned function. Similarly, SVM method has the biggest limitation of choosing the kernel function. Once the kernel is fixed, SVM classifiers have only one user-chosen parameter, error penalty. However, kernel is a very important decision criterion. Another important characteristic of an ideal data classifier is the ability to form a decision boundary that minimizes the amount of misclassification for all of the overlapping classes in the training set.

Some of the methods mentioned above can only be used for the two class cases, such as yes (class1) or no (class 2). However, the number of classes to be classified is generally more than two in real life problems. Existing methods can be somehow modified or developed for multi-class case. In that situation, the accuracy values of the models decrease [4]. For instance, SVMs are originally a model for two class problems and are more effective. For multi-class case, combinations of SVMs should be used. Since SVMs use some approximation algorithms in order to reduce the computational time, increasing number of these approximation algorithms causes the degradation of classification performance. Thus, the performance does not improve as much as in binary case. Therefore, there is a need for new approaches that are able to address multi-group problems effectively. In this study, a novel mixed-integer programming approach for multi-class data classification problem has been developed. The proposed approach is based on the use of hyper-boxes for defining boundaries of the classes that include all or some of the

points in that set. The computational results on the studied datasets show that the suggested method is accurate and efficient on multi-class data classification problems.

1.5 Contributions

This thesis presents a novel three-stage mathematical programming based hyperbox enclosure approach for multi-class data classification problems. A mixed-integer programming model is developed for representing existence of hyper-boxes which define the boundaries of the classes for the training set. In order to overcome the computational difficulties for large data sets, a three-stage approach is developed for training part analysis of hyper-box enclosure approach. The performance of the model is tested by applying the testing part of the proposed method. Main contributions of this thesis can be summarized as follows:

One of the most important contributions is that the proposed data classification method based on mixed-integer programming allows the use of hyper-boxes for defining boundaries of the classes that enclose all or some of the points in that set. This approach in the training problem can indirectly effect and improve the prediction accuracy of the model. This may be one of the reasons behind the high classification accuracy values obtained by the proposed model.

The suggested model can be used for both binary and multi-class cases without any modifications or additions. High classification accuracies are observed for binary and multi-class problems.

The proposed model has only one parameter to initialize (big-M parameter) and this parameter does not require adjusting during the training of the model. Furthermore, the model can operate without a priori knowledge about the underlying distribution of the data.

From the computational time perspective, the proposed three-stage MILP approach is applicable to obtain solutions to large multi-class data classification problems. Furthermore, the testing algorithm is computationally tractable for high dimensional data sets. As observed from the examined data sets, total computational time for proposed approach is reasonable and less than the other methods used for these data sets.

The proposed approach in this thesis gives high accuracy values on the studied benchmark data sets. Hence, the developed multi-class data classification model is at least as accurate as the other models including NN, SVM, Decision Trees, K-Nearest Neighbor, Logistic Regression, Bayesian Classifier, etc.

In summary, by the development of this new approach, solutions to multi-class data classification problems can be obtained and the prediction accuracies can be improved. In addition to this, the simplicity and the understandability of the proposed model are preferable.

1.6 Outline

This thesis contains six chapters. Chapter 2 provides a literature review on data classification summarizing distinct methods reported. Moreover, existing mathematical programming based approaches to data classification are investigated in detail. The literature on protein folding type problem is also mentioned in Chapter 2. The developed three-stage MILP based hyper-box enclosure approach to multi-class data classification is presented in Chapter 3. The mixed-integer programming formulation, sub grouping algorithm, seed finding algorithm, intersection elimination algorithm and box combination algorithm for the training part of the problem are discussed in detail. In addition, original and new testing algorithms are explained and compared. The method is also illustrated on a small illustrative example in Chapter 3. The application of the proposed approach on existing protein folding type benchmark data sets are illustrated and results are examined in Chapter 4. Furthermore, the efficiency of the proposed method on existing eleven UCI Repository benchmark data sets is tested and results are given in Chapter 5. The thesis is

concluded with short summary, conclusions, and directions on future research work with Chapter 6.

Chapter 2

LITERATURE REVIEW

Data classification is a multidisciplinary problem which is a very active area of study and research. Classification problems have been intensively studied by a diverse group of researchers including statisticians, engineers, biologists, computer scientists. There are variety of methods for solving classification problems such as Neural Networks (NN), Support Vector Machines (SVM), Decision Trees, Bayesian Networks, Logistic Regression, K-nearest neighbor, tolerant rough sets, fuzzy logic and Mathematical Programming [25]. In this chapter, a literature review on data classification methods, mathematical programming based methods and an important problem, prediction of folding type of proteins, is provided.

2.1 Literature Review on Data Classification Methods

An overall view of classification methods is published by Weiss & Kulikowski [5]. In this book, available classification and prediction methods from statistics, neural networks, machine learning and expert systems are reviewed. Hand [26] investigates the statistical approach of data classification and pattern detection in the fields of medicine, psychology and finance. More recently, Webb provides an introduction to statistical pattern recognition theory and techniques in his book [27]. In that book, descriptions of today's pattern recognition techniques including many of the recent advances in nonparametric approaches to data classification in the statistics literature are provided. Moreover, the techniques are illustrated with examples of real-world applications. The

estimation of error rates in discriminant analysis is explored by Lachenbruch & Mickey [28]. In this study, leave-one-out cross-validations tests are proposed for error estimation. N (number of data points) separate times, the classification function is trained on all the data except for one point and a prediction is made for that point in leave-one-out crossvalidation tests. Average error is computed and used to evaluate the model. The evaluation given by this cross-validation test error is good, but computing the result of leave-one-out tests takes very long time. Kendall *et al.* [29] give a comprehensive exposition about the statistical approach of data classification and advance theory of statistics. Furthermore, McLachlan studied on a thorough treatment of statistical procedures in discriminant analysis and pattern recognition [30].

The study by Hertz *et al.* [31] is one of the most detailed and reliable information guides for neural network approach in data classification. They propose an introduction to neural computation and explain the theory of the neural network approach. Additionally, Simpson [32] developed a fuzzy min-max classification neural network in which pattern classes are utilized as fuzzy sets. In this study, learning in the neural network was performed by properly placing and adjusting hyper boxes in the pattern space. Simpson defines a fuzzy set hyper-box as an n-dimensional box defined by a min and a max point with a corresponding membership function. The min-max membership function defines a fuzzy set, hyper-box fuzzy sets are aggregated to form a single fuzzy set class, and the resulting structure fits naturally into a neural network framework. Therefore, this classification system is referred as fuzzy min-max classification neural network. Since it uses only a min and a max point in the *n*-dimensional space and combines fuzzy sets with the neural network idea, this model has a different approach as compared to the proposed model in this thesis. Moreover, Zhang [33] gave a review of the use of feed-forward neural networks for classification. In data classification problems, neural networks have the ability to learn nonlinear input or output relationships while propagating and adopting itself with a given training set by training procedures. The learning process involves updating network architecture and connection weights in order to achieve efficiency by the help of some learning algorithms. The most common types of neural networks that are used for data classification are feed forward neural networks (FFNN) which includes multilayer Perceptron and Radial Basis Functions (RBFs) [34-37]. In FFNNs, the neurons are organized in different layers and each of the neurons in one layer can receive an input from units in the previous layers without loss of generality. On the other hand, RBF network is capable to perform a nonlinear mapping between the input and output vector space. It is widely used in data classification problems such as speech recognition, medical diagnosis, handwriting recognition, image processing, and fault diagnosis. The other popular network is Kohonen network (self organizing map (SOM)) [38] in which two dimensional discretized representation of the input space of the training samples are produced during the training phase. SOMs are different than other neural networks in the sense that they use a neighborhood function to preserve the topological properties of the input space.

On the other hand, Devijver & Kittler [39] concentrate on the K-nearest neighbor approach for data classification problems from the perspective of statistical approach. A comprehensive review of K-NN and many of the important contributions to the literature are included in Dasarathy [40]. The performance of the K-NN depends on the choice of k . If the value of k is larger, the procedure is more robust but needs more computation. Hans [41] mentioned that k must be smaller than the minimum of n_j , the number of observations in class j. Otherwise, the neighborhood is no longer the local neighborhood of the sample. Other choices of k are $n^{2/8}$, $n^{3/8}$, and $n^{1/2}$, subject to rounding up to the nearest integer, where *n* is the total number of observations in the training set [37]. While the optimal value of k depends on the size and nature of the data, typical values are 3, 5, or 7.

One of the first papers published on data classification introduces fuzzy adaptive resonance theory (ART) which is a fast and reliable analog pattern clustering system. In this study, Carpenter and Grossberg combine the fuzzy logic with the idea of ART and try to develop an efficient classifier [42]. A general neural-network model for fuzzy logic control and decision systems including the data classification problem is discussed in [14].

Rough set theory introduced by Pawlak [43] is a mathematical tool to deal with vagueness and uncertainty in machine learning and pattern recognition. Two applications of logic for classification using rough set approach are presented in [44]. The multi-model logics is employed for automatic feature selection while a rough-set-based inductive reasoning is used for discovering optimal feature set with respect to the quality of classification as well as for improving the performance of decision algorithms. Another approach in data classification is to use rough sets by tolerating the relationships among the objects for pattern classification [45]. A data classification method based on the tolerant rough set that combines the use of logic and the tolerance relation among the objects is presented in [46]. The performance of this approach is tested on the UCI Repository data sets [47]. Furthermore, Castro et al. [48] presented a method to learn maximal structure rules in fuzzy logic to deal with the one of the UCI Repository data sets, Iris. Chen et al. [49], Hong et al. [50], Lin et al. [51] and Wu et al. [52] presented different methods to generate fuzzy rules from training instances based on genetic algorithms to study UCI Repository data sets. Most recently, Chen et al. [53] developed a new model based on distributions of training instances. Their proposed method achieves a higher average classification accuracy rate than existing methods. On the other hand, Uney and Turkay [54] proposed a mixed-integer linear programming approach and tested the performance of the method on Iris data set.

The training procedure of support vector machines (SVMs) usually requires huge memory space and significant computation time due to the enormous amounts of training data and quadratic programming problem [55]. Some of the researcher proposed incremental training or active learning to shorten the training time [56]. The main idea is to select a subset of training samples while preserving the performance as using all the training samples. Syed et al. [57] and Campbell et al. [58] proposed two different incremental learning procedures. On the other hand, multi-group data classification problems are solved either by constructing several two class classifier such as one-againstone, one-against-all, and DAG SVMs [12] or by constructing multi-class classifier directly such as k-SVM [59]. Recently, Zhu et al. [60] proposed a multi-class classification algorithm which adopted the minimum enclosing spheres to classify a new example and showed that the resulting classifier performed comparable to the standard SVMs. Based on Zhu *et al.* [60], Wang *et al.* [61] and Lee *et al.* [62] also proposed a new classification rule on the basis of Bayesian optimal decision theory.

Mathematical optimization techniques have been applied directly in the optimal construction of decision boundaries in the decision tree induction. Bennett [63] introduced an extension of linear programming techniques to decision tree construction for two class problems. Kennedy et al. [64] first developed a genetic algorithm for optimizing decision trees. In their approach, a binary tree is represented by a number of unit sub trees each having a root node and two branches. When using genetic algorithm to optimize the tree, the growth of the tree could not be controlled as genetic algorithm does not evaluate the size of the tree. Therefore, the resulting tree may become overly deep and complex or may be too simple. To address this problem, Niimi and Tazaki [65] combine genetic programming with association rule algorithm for decision tree construction. In this approach, rules generated by apriori association rule discovery algorithm are taken as the initial individual decision trees for a subsequent genetic programming algorithm.

In summary, a large number of data classification methods have been developed up to now; however each of them has some drawbacks which make them unattractive. Thus, researchers have been studying to develop more accurate and more efficient methods or to improve the existing methods.

2.2 Literature Review on Mathematical Programming Based Methods

Mathematical Programming (MP) based data classification models are used to generate linear discriminant functions, or separating hyper-planes, which optimally separate observations in a training set. Generally, two group data classification problems are considered by MP techniques and they can be extended to multi-group problems [66, 67]. Erenguc and Koehler [13] summarized the existing mathematical programming models and their experimental results.

Mangasarian [68] is the first researcher who proposed a linear programming model to determine separating hyper-planes, namely linear discriminant function, for two linearly separable classes. In the case of linearly inseparable classes, Freed and Glover [67] proposed a mathematical model which tries to minimize the sum of the deviations (MSD) of misclassified instances from the separating hyper-plane. In addition to that, Hand [69] developed a mathematical model with an objective function of maximization of the minimum deviation (MMD) of the misclassified instances from the separating hyper-plane. For multi-group problems, a model based on goal programming was also suggested by Freed and Glover [70]. An alternative LP approach for multi-group data classification problems has been proposed in [71]. In addition to being non-parametric, LP and other MP based approaches are also more flexible than statistical methods.

In LP based methods, deviations from the separating hyper-planes are used as measures of misclassification as mentioned above. On the other hand, the number of misclassifications can be considered directly in mixed integer linear programming (MILP) models in which binary variables are used to indicate whether instances are correctly or incorrectly classified. For two-group data classification problem, Bajgier and Hill [72] included the number of misclassifications and the deviations in the objective function of a

MILP model. On the other hand, Gehrlein [66] proposed a MILP approach for minimizing the number of misclassified instances in multi-group data classification problems, while Wilson [73] suggested an alternative MILP formulation and solution methods for these problems. Stam and Joachimsthaler [74] argued that these MILP based methods may be superior to both LP based techniques and statistical approaches. However, MILP approaches can be used to solve problems involving small number of instances due to computational reasons.

The problems that may appear in mathematical programming formulations for data classification are summarized by Koehler [75]. Specific problems include the choice of objective function, unacceptable or improper solutions, inconsistencies, gaps, and balancing of misclassifications. MP based data classification models must be normalized to prevent the generation of discriminant functions in which the variable coefficients and the constant term are zero. This normalization requirement can cause difficulties, and unlike statistical approaches, variables can not be selected in a computationally efficient way with MP models. Glen [76] developed two integer programming (IP) methods for normalizing MP discriminant analysis models. In the first method, binary variables are used to represent the constant term, but with this normalization functions with a zero constant term can not be generated. Moreover, the variable coefficients are not invariant under origin shifts. These limitations are overcome by the second method by using IP to constrain the sum of the absolute values of the variable coefficients to a constant [76]. Pavur and Loucopoulos [77] examined conditions under which degenerate solutions can occur in MILP models for the classification problem for more than two groups. They presented a multiple-group MSD model and a two-goal approach to the multiple-group data classification problem. Lam and Moy [78] proposed an aggregate model which simultaneously determines the cut-off values for the different classification functions in order to provide better estimates of the group boundaries.

Silva and Stam [79] introduced a computationally attractive algorithm, the Divide and Conquer $(D&C)$, for determining classification rules which minimize the number of misclassifications in the training set for two-group data classification problems. The D&C algorithm partitioned the problem in smaller and more easily handled sub problems and solved the problem to the exact optimal solution by allowing analysis of much larger training sets than previous methods. On the other hand, Glen [80] developed an iterative MILP model to allow classification accuracy maximizing discriminant functions to be generated for problems with many more instances that can be considered by the standard MILP formulations. First, a discriminant function is generated by using a MSD based mathematical programming formulation for the complete set of instances. Then, a neighborhood of instances is defined and a MILP model is used to generate a discriminant function that maximizes classification accuracy within this neighborhood. This procedure is repeated until there is no improvement in the total number of instances classified correctly. This iterative MILP method is applied to a two-group classification problem involving 690 observations.

There are some very good MP based heuristics [81, 82] that can solve real world two-group data classification problems fast. Although there exist ways to solve a multigroup data classification problem by means of solving several two-group problems, such approaches bring about new problems [83]. Hence, Adem and Gochet [25] presented a MP based heuristic that avoid these problems and can tackle with multi-group data classification problems directly. The basic idea is to improve an LP-generated classifier with respect to the number of misclassifications on the design data set. The performance of the proposed approach is tested on both simulated and real world data sets.

In addition to the standard MP based data classification methods in which discriminant functions are generated by solving a single MP model, two-stage based MP methods have also been developed. Stam and Ragsdale [84] proposed a two-stage method which is particularly suitable for data classification problems with outlier contaminated data. In the first stage, a discriminant function is generated by solving the MSD based model. In that model, some of the instances could be misclassified. In the second stage, the objective is to generate a new discriminant function that minimizes a measure of total misclassification while ensuring that the correctly classified instances in the first stage remain correctly classified. Detailed information related to two-stage MP based methods and comparisons with standard MP based methods are given by Glen [85]. The results from comparisons of methods on one real data set and six simulated data configurations indicate that a single technique will not produce good linear classifier under all data conditions. Several methods should consider in developing classification models, with the most appropriate method chosen for a particular problem.

2.3 Literature Review on Protein Folding Type Prediction

Proteins are the molecules of life that play a key role in realizing the functions of any biological organism. Discovery of the functions of proteins will enable us to understand the principles of life and working mechanisms of any organism. In the case of humans, this discovery will lead to the design of new drugs that will regulate the functions of proteins in order to improve the quality of life. Functions of proteins are highly correlated to their three dimensional structure. There exist some experimental methods to determine the protein structure including X-ray diffraction and nuclear magnetic resonance (NMR). These experimental methods require long experimental times and large amounts of resources. In order to overcome these shortcomings of experimental methods, researchers have developed a host of methods to predict the protein structures. Due to the importance of protein structure in understanding the biological and chemical activities in any biological system, protein structure determination and prediction has been a focal research subject in computational biology and bioinformatics. The knowledge of folding type of proteins is an important part of protein structure prediction and determination

studies. The results of the secondary structure prediction [86, 87] and the efficiency of searching the possible conformations of the tertiary structure [88, 89] could be significantly improved by incorporating the knowledge on folding types of protein. Another factor that motivates protein folding type prediction studies is the substantial gap between number of proteins for which structure is known and thus structural class can be assigned manually (approximately 30 000 proteins are stored in Protein Data Bank [90] and SCOP [91]) and the total number of currently known proteins (NCBI database contains over 2 million proteins). Therefore, development of a reliable method for prediction of folding types of proteins for new and undetermined protein sequences is very important.

A protein molecule is the chain(s) of amino acids (also called residues). There are 20 types of amino acids in nature and their names, three-letter representations and singleletter representations are provided in Table 2.1. Residue content and order in chain(s) is unique for each protein just like specificity of gene sequence.

Starting with the sequence of residues in the chain(s) making up protein, there are 4 basic structural phases: primary structure, secondary structure, tertiary structure and quaternary structure. The secondary structure (folding type) of a segment of polypeptide chain is the local spatial arrangement of its main-chain atoms without regard to the conformation of its side chains or to its relationship with other segments. This is the shape formed by amino acid sequences due to interactions between different parts of molecules. There are mainly three types of secondary structural shapes: α -helices, β -sheets and other structures connecting these such as loops, turns or coils. Alpha-helices are spiral strings formed by hydrogen bonds between CO and NH groups in residues. Beta-sheets are plain strands formed by stretched polypeptide backbone. When β -sheets come together, hydrogen bonds form between C=O and NH groups of residues of adjacent chains, keeping them together. Connecting structures do not have regular shapes; they connect α-helices and β -sheets to each other.

Amino Acid	Three	Single		Three	Single
	Letter	Letter	Amino Acid	Letter	Letter
alanine	ALA	A	leucine	LEU	L
arginine	ARG	\mathbb{R}	lysine	LYS	K
asparagine	ASN	N	methionine	MET	M
aspartic acid	ASP	D	phenylalanine	PHE	F
cysteine	CYS	\mathcal{C}	proline	PRO	P
glutamic acid	GLU	Q	serine	SER	S
glutamine	GLN	E	threonine	THR	T
glycine	GLY	G	tryptophan	TRP	W
histidine	HIS	H	tyrosine	TYR	Y
isoleucine	ШE	I	valine	VAL	V

Table 2.1 List of amino acids, their three-letter and single-letter representations.

The proportion of α -helices and β -sheets in the secondary structures of proteins are used to determine the folding type of proteins. Protein folding type definitions were initially developed in 1980s and redefined multiple times since then (Table 2.2).

Reference	Folding	Helix (a)	Strand (β)	Additional constraints
	Type	amount	amount	
	α proteins	>15%	$< 10\%$	
	β proteins	<15%	$>10\%$	
	$\alpha + \beta$ proteins	>15%	$>10\%$	Contains dominantly
$[92]$				antiparallel β -sheets
	α/β proteins	>15%	$>10\%$	Contains dominantly
				parallel β -sheets
	Irregular			Otherwise
	α proteins	$\geq 40\%$	$\leq 5\%$	
$[93]$	β proteins	$\leq 5\%$	$\geq 40\%$	
	$\alpha + \beta$ proteins	\geq 15%	\geq 15%	More than 60%
				antiparallel β -sheets
	α/β proteins	${\geq}15\%$	\geq 15%	More than 60% parallel
				β -sheets
	Irregular	\leq 10%	\leq 10%	
$[95]$	α proteins	>15%	<10%	
	β proteins	<15%	$>10\%$	
	Mixed proteins	>15%	$>10\%$	
	Irregular			Otherwise
SCOP[91]	α proteins	NA	NA	Manual classification
	β proteins	NA	NA	Manual classification
	$\alpha + \beta$ proteins	NA	NA	Manual classification
	α/β proteins	NA	NA	Manual classification
	+7 other classes	NA	NA	Manual classification

Table 2.2 Definitions of Protein Structural Classes.

The main differences were in the thresholds used to define amount of strands for all- α proteins, and amount of helices for all- β proteins. Nakashima and colleagues [92] defined five structural classes in 1986. Then, Chou [93] proposed classification into again five classes by using different thresholds in 1995. The change was due to Nakashima's classification, which set the thresholds for all- α proteins and all- β proteins that were not large enough to reflect the real features of the two structural classes. Chou also defined content of the secondary structures using the Dictionary of Secondary Structure of Proteins (DSSP) [94]. Eisenhaber and colleagues [95] proposed another definition which merges the $\alpha+\beta$ and the α/β classes into so-called mixed class and thus considers only four in 1996. In all above classifications, irregular proteins, ξ , are omitted from classification as they are small in numbers.

The threshold based classifications were replaced by the manually performed SCOP classification. The descriptions of the structural and evolutionary relationships of proteins from the Protein Data Bank (PDB) [90] are considered in the SCOP database [91]. The SCOP classifies proteins on multiple levels including structural classes, but also as belonging to different families, super families and containing different domains. Domain is defined as a structurally conserved part of a protein sequence, and together with the entire sequences is currently a target of structure prediction. The SCOP's classification does not incorporate hard coded rules for structural classes. Intuitively, it makes decisions based on structural elements that are located in individual domains that constitute the protein. Researchers claim that the SCOP classification is more "natural" and provides more reliable information to study protein structural classes when compared to classification based on the percentage amounts of the secondary structures [91, 96, 97]. The SCOP classification currently includes 11 classes [98]: (1) all- α proteins; (2) all- β proteins; (3) α/β proteins; (4) $\alpha+\beta$ proteins; (5) multi-domain proteins; (6) membrane and cell surface proteins; (7) small proteins; (8) coiled coils proteins; (9) low resolutions proteins; (10) peptides; and (11) designed proteins. Usually, only the first four categories are considered for computational prediction purposes as they include significant majority of the protein sequences.

It is postulated that overall folding type of a protein depends on its amino acid composition [92]. There have been several methods proposed to exploit this postulate for predicting folding type of a protein. Chou [99] developed a new prediction algorithm which incorporates coupling effect between different amino acid components. By the help of this component-coupled algorithm, prediction quality was significantly improved. Another important progress in this area was achieved by Bahar et al. [89]. In their study, a compact lattice model was proposed in predicting structural class from amino acid composition and 81% accuracy achieved using singular value decomposition method [89]. In this method, each protein is represented by a 19-dimensional array of fluctuations in fractions of residues of different types. The jth element of this vector is the difference between the composition of the amino acid type j and the average fraction of amino acid j in the group of n structures. The distance of a protein from the four type of structural classes are calculated using 19-dimensional array of the protein by applying singular value decomposition method. The smallest of the four distances obtained for each protein determines the structural class of that protein. Although they use the same data set and mathematically identical method with Chou, their accuracy is somehow less. They explore this puzzling difference and came up with the result that the data files used in these studies are different. Chou used files that contained fewer residues (chains of amino acids) compared with intact Protein Data Bank (PDB) files. Eisenhaber et al. [95] found that component coupling effect between amino acid components did not improve the class prediction, using a different dataset constructed according to their definition. In order to clarify this paradox, Zhou $[100]$, Chou *et al.* $[101]$ and Cai $[102]$ showed that componentcoupled algorithm significantly improved the prediction accuracy. The reasons why

Eisenhaber et al. come up with that result are misusing the component-coupled algorithm and using a conceptually incorrect rule to classify protein structural classes. On the other hand, Bu et al. [103] come up with a new idea, using amino acid index rather than composition in order to predict the structural classes. The overall predictive accuracy of the new proposed method for the jackknife test was 5-7% higher than the accuracy based only on the composition. However, many researchers continued studying on the first case, based on only the amino acid composition. Cai et al. [104] applied T. Kohonen's selforganization neural network on two data sets composed of 277 and 498 domains, respectively. They showed that this approach can be a powerful tool for protein structural class prediction. Furthermore, support vector machine (SVM) method was performed based on the same data sets by [100]. The SVM method applies for two class problems. Thus, "one-against other" method is used to transfer it into two class problems. Most recently, Kurgan and Homaeiang [23] provided a comprehensive literature survey and analyzed the impact of prediction algorithms and test procedures on accuracy. Consequently, the prediction of folding types from amino acid composition alone is an important topic, which has been the object of many recent researches. Existing data classification methods applied to protein folding type prediction is mainly appropriate for two-class problems. These methods can be modified for multi-class problems. Unfortunately, these modifications can cause the degradation of classification performance. Therefore, developed three-stage mathematical programming based hyper-box enclosure approach, which is capable of solving multi-class problems without any modification, can be used to classify a given primary protein structure into folding types according to its amino acid composition effectively.

 In conclusion, there exists restricted number of methods for multi class data classification problems in literature. This thesis addresses the need for efficient and reliable methods for multi-class problems by introducing a new mixed-integer programming approach. Moreover, the important and widely used data sets, the protein folding type data set and UCI Repository data sets are studied to analyze the performance of the developed model. The results on these data sets show that the prediction accuracy of the developed model is as good as the existing data classification models in literature. Furthermore, developed model gets rid of some drawbacks of the available multi-class data classification models with only one adjustable parameter, rather short learning and computational time, no need to know the underlying distribution of the data and wellconstruction of the class boundaries.

Chapter 3

MILP BASED HYPER-BOX ENCLOSURE APPROACH

The objective in data classification is to assign instances that are described by several attributes into a predefined number of classes. The use of hyper-boxes for defining boundaries of the sets that include all or some of the instances in that set as shown in Figure 3.1 can be very accurate on multi-class problems. If it is necessary, more than one hyper-box could be used in order to represent a class as shown in Figure 3.1. When the classes that are indicated by square and triangle instances are both represented with a single hyper-box respectively, the boundaries of these hyper-boxes overlap. Thus, two boxes are constructed in order to eliminate this overlapping. A very important consideration in using hyper-boxes is the number of boxes used to define a class. If the total number of hyperboxes is equal to the number of classes, then the data classification is very efficient. On the other hand; if there are as many hyper-boxes of a class as the number of instances in a class, then the data classification is inefficient.

The data classification problem is considered in two parts as training and testing. Determination of the characteristics of the instances that belong to a certain class and differentiating them from the instances that belong to other classes are the main objectives of the training part. The hyper-boxes that determine the characteristics of the classes are constructed in the training part by the help of mixed-integer linear programming (MILP) formulation. After the distinguishing characteristics of the classes are determined, then the effectiveness of the classification is observed by the help of distance-based testing

algorithm. Predictive accuracy of the developed model is performed on a test data set during the test part.

Figure 3.1 Schematic representation of multi-class data classification using hyper-boxes.

3.1 Training Algorithm: MILP Formulation

Training part studies are performed on a training data set composed of a number of instances *i*. The instances are represented by the parameter a_{im} that denotes the value of attribute m for the instance i . The class k that the instance i belongs to are given by the set D_{ik} . Each existing hyper-box l encloses a number of instances belonging to the class k. Moreover, bounds *n* (lower, upper) of each hyper-box is determined by solving the training problem. M and N represents the total number of attributes and bounds, respectively.

Given these parameters and the sets, the following binary and continuous variables are sufficient to model the data classification problem with hyper-boxes. The existence of hyper-box *l* is represented by binary variable yb_l . The binary variable ypb_{il} indicates the position (inside or outside) of the instance i with respect to box l . The binary variables ybc_{lk} and ypc_{ik} indicate the assigned class k of instance i and hyper-box l, respectively. If the instance i is within the bound n with respect to attribute m of hyper-box l , then the binary variable $yphn_{ilmn}$ is 1, otherwise 0. Similarly, $ypbm_{ilm}$ indicates whether the instance i is within the bounds of attribute m of hyper-box l or not. Finally, yp_{ik} indicate the misclassification of instance i to class k . In order to define the boundaries of hyper-boxes, two continuous variables are required: X_{lmn} is the one that models bounds n for box l on attribute m. Correspondingly, bounds n for box l of class k on attribute m are defined with the continuous variable XD_{lkmn} .

The following MILP problem models the training part of data classification method using hyper-boxes:

$$
\min \quad z = \sum_{i} \sum_{k} y p_{ik} + \sum_{l} y b_{l} \tag{3.1}
$$

subject to

$$
XD_{lkmn} \le a_{im} ypb_{il} + Q(1 - ypb_{il}) \quad \forall i, k, l, m, n \mid n = lower \tag{3.2}
$$

$$
XD_{lkmn} \ge a_{im} ypb_{il} \quad \forall i, k, l, m, n \mid n = upper \tag{3.3}
$$

$$
XD_{lkmn} \le Qybc_{lk} \quad \forall k, l, m, n \tag{3.4}
$$

$$
\sum_{k} X D_{lkmn} = X_{lmn} \quad \forall l, m, n \tag{3.5}
$$

$$
y p b n_{ilmn} \ge \frac{1}{Q} (X_{lmn} - a_{im}) \quad \forall i, l, m, n | n = upper \tag{3.6}
$$

$$
ypbn_{\text{ilmn}} \ge \frac{1}{Q}(a_{\text{im}} - X_{\text{lmn}}) \quad \forall i, l, m, n | n = lower \tag{3.7}
$$

$$
\sum_{l} y p b_{il} = 1 \quad \forall i \tag{3.8}
$$

$$
\sum_{k} ypc_{ik} = 1 \quad \forall i \tag{3.9}
$$

$$
\sum_{l} y p b_{il} = \sum_{l} y p c_{ik} \qquad \forall i \tag{3.10}
$$

$$
\sum_{k} ybc_{ik} = yb_{i} \quad \forall l \tag{3.11}
$$

$$
ybc_{ik} \le \sum_i ypb_{il} \quad \forall l, k \tag{3.12}
$$

$$
ybc_{lk} \le \sum_i ypc_{ik} \quad \forall l, k \tag{3.13}
$$

$$
\sum_{n} y p b n_{i l m n} - y p b m_{i l m} \le N - 1 \quad \forall i, l, m \tag{3.14}
$$

$$
\sum_{m} y p b m_{ilm} - y p c_{ik} \le M - 1 \quad \forall i, l, k \tag{3.15}
$$

$$
ype_{ik} \leq yp_{ik} \qquad \forall i, k \notin D_{ik} \tag{3.16}
$$

$$
X_{lmn}, \, XD_{lkmn} \ge 0 \tag{3.17}
$$

$$
yb_l, ypb_{il}, ypc_{ik}, ybc_{lk}, ypbn_{ilm}, ypbm_{ilm}, yp_{ik} \in \{0,1\}
$$
\n
$$
(3.18)
$$

Minimization of the misclassified instances in the data set with the minimum number of hyper-boxes is the objective of the MILP model given in (3.1). The lower and upper bounds of the hyper-boxes are determined by the instances that are enclosed within the hyper-boxes. Hence, lower and upper bounds of hyper-boxes are calculated by equations (3.2) and (3.3), respectively. Eq. (3.4) enforces the bounds of hyper-boxes exist if and only if this hyper-box is assigned to a class. The relationship between two continuous variables is given in Eq. (3.5). The position of an instance with respect to the bounds on attribute m for a hyper-box is given in Eqs. (3.6) and (3.7). The binary variable *ypbn*_{ilmn} helps to identify whether the instance i is within the hyper-box l. Two constraints,

one for the lower bound and one for the upper bound, are needed for this purpose (Eqs. (3.6) and (3.7)). Since these constraints establish a relation between continuous and binary variables, a large parameter, Q , is included. Q generally takes the maximum attribute value in the data set. The assignment of an instance to a single hyper-box l and a single class k is established by the equations (3.8) and (3.9), respectively. The equivalence between Eqs. (3.8) and (3.9) is given in Eq. (3.10) ; indicating that if there is an instance in the class k, then there must be a hyper-box l to represent the class k and vice versa. The existence of a hyper-box implies the assignment of that hyper-box to a class as shown in Eq. (3.11) . If a class is represented by a hyper-box, there must be at least one instance within that hyperbox as in Eq. (3.12). In the same manner, if a hyper-box represents a class, there must be at least an instance within that class as given in Eq. (3.13). The Eq. (3.14) represents the condition of an instance being within the bounds of a box in attribute m. If an instance is within the bounds of all attributes of a box, then it must be in the box as shown in Eq. (3.15). When an instance is assigned to a class that it is not a member of, a penalty applies as indicated in Eq. (3.16) . Finally, last two constraints Eq. (3.17) and (3.18) give nonnegativity and integrality of decision variables. The model has $LMN + LKMN$ continuous variables, $L + LK + 3IK + IL + ILMN + ILM$ binary variables and $O(IKLM)$ constraints.

3.2 Three-Stage Approach

Solving the proposed MILP problem to optimality is computationally expensive for large multi-group data classification problems. The major source of computational difficulty is the potentially large number of binary variables. Hence, we propose a threestage decomposition algorithm (shown in Figure 3.2) for obtaining optimal solutions to MILP model.

Figure 3.2 Flowchart of the decomposition algorithm for solving multi-class classification algorithm using hyper-boxes.

Instances that are difficult to classify are identified in the first stage that is referred to as preprocessing. Moreover, sub grouping and seed finding algorithms are applied to improve the computational efficiency. With greater emphasis given to these observations, solution to the problem is obtained in the second stage using the MILP formulation. Last, final assignments, elimination of box intersections and box combination procedures are carried out in the third step.

3.2.1 Preprocessing

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First, maximum and minimum attribute values for each class are determined. Then, the boundaries of the classes are compared to check whether they overlap or not. If the boundaries of the classes overlap, then the instances that are enclosed by other classes are identified. These instances are called as 'problematic' instances, since they are not separable from the instances of the other classes with a single hyper-box. In the case of having large number of 'problematic' instances, the same procedure is repeated to reduce the total number of such instances. In some cases, applying one or two times the same procedure do not reduce the number of problematic instances as we want. For those cases, we proposed a sub grouping algorithm in order to obtain small sub groups from the data sets efficiently.

The proposed MILP model has $O(LKMN)$ continuous variables, $O(LMN)$ binary variables and O(IKLM) constraints. For each instance removed in the preprocessing step, the binary variables and constraints in the MILP model are reduced by $O(LMN)$ and O(KLM), respectively.

3.2.2 Threshold Value for the Number of Problematic Instances

 In order to give more formal threshold value for the number of problematic instances, we perform some runs with different number of instances. For this purpose, sub problems of a protein folding type data set are used. By increasing the number of

instances, we try to observe the CPU times of the runs with respect to the change in the size of the problem (Figure 3.3.). For each problem size, we perform 10 different runs and give the average results. In this graph, I represents the number of instances, K represents the number of classes and M represents the number of attributes. The problem size is given by the products of cardinalities of I , M and K . As this product increases, the number of binary and continuous variables in the MILP model increases. Thus, the required solution time increases by the increase in the problem size. After some point, this increase is much more significant. As it could be observed from the graph, the threshold value is 2^{12} (4096). After card(IMK) achieves that value, the required CPU time is high and unfavorable.

Figure 3.3 Problem size versus CPU time of algorithm.

3.2.3 Sub Grouping Algorithm

j

For some of the data classification problems, the number of problematic instances is so high that this step does not make enough improvement in the computational time of the given problem. Hence, for this type of problems a sub grouping algorithm is proposed in order to improve the computational efficiency. Sub grouping is a method that constitutes a given number of subsets of the given data set by selecting instances considering some similarity-dissimilarity measure.

The determination of subsets is crucial: the instances for each subset must be chosen to ensure that they are separated well from other instances. We develop a pure integer programming (IP) formulation to accomplish this task.

As in the MILP, instances are represented by the parameter a_{im} that denotes the value of attribute *m* for the instances *i*. The class k of instance *i* belongs to is given by the set D_{ik} . NI_k represents the number of instances in class k. Moreover, DB_{ii} represents the distance between two data points i and i' . This distance is calculated using Euclidean distance in *m*-dimensional space as given in Equation (3.19).

$$
DB_{ii'} = \sqrt{\sum_{m} (a_{im} - a_{i'm})^2}
$$
 (3.19)

Given these parameters and the sets, the similarity, S_i , and dissimilarity, DS_i , of an instance *i* can be calculated as in Equations (3.20) and (3.21), respectively. Similarity, S_i , is the average distance from instance i to instances i' that exist in the same class with instance *i*. On the other hand, dissimilarity, DS_i , is the average distance from instance *i* to instances i' that are not in the same class with instance i .

$$
S_{i} = \frac{\sum_{i' \in D_{ik}} DB_{ii'}}{NI_{ki \in D_{ik}}} \tag{3.20}
$$

$$
DS_i = \frac{\sum_{i' \notin D_{ik}} DB_{ii'}}{\left(\sum_k NI_k\right) - NI_{k:i \in D_{ik}}} \tag{3.21}
$$

The binary variable SP_i , that indicates whether the instances *i* is selected for this sub group or not, is sufficient to model sub grouping problem. Furthermore, SS is the number of instances that exist in each of the constructed sub groups from the given data set D. TS is the number of sub groups that should be obtained. TS and SS can be determined by using the Equations (3.22) and (3.23) .

$$
TS = \left\lceil \frac{card(DMK)}{2^{12}} \right\rceil \tag{3.22}
$$

$$
SS = \frac{card(D)}{TS}
$$
 (3.23)

The following IP-Sub Group models the sub grouping problem and select SS number of instances to form a sub group: IP-Sub Group:

$$
\min z = \sum_{i} SP_i(S_i - DS_i) \tag{3.24}
$$

subject to

$$
\sum_{i} SP_i = SS \tag{3.25}
$$

$$
SP_i \in \{0,1\} \qquad \forall i \tag{3.26}
$$

The objective of the IP-Sub Group problem given in Eq. (3.24) is to minimize the similarities measures and maximize the dissimilarities measures of selected instances. Equation (3.25) states that the number of selected instances must be exactly SS. Finally, integrality of the decision variable SP_i is given by (3.26).

This IP-Sub Group model constitutes a single subset, S_l , from a given data set D. In order to obtain each subset, one should solve TS-1 consecutive IP-Sub Group model while in each case updating the new dataset D_{new} as $D_{old} \setminus S_i$. Hence, by solving IP-Sub Group models, we will obtain TS sub groups of data set D . As MILP is based on hyper-boxes approach, this sub group decomposition will not affect the inherent properties of this approach. Moreover, sub grouping will improve the computational efficiency of the overall data classification method.

Further investigation on the proposed IP-Sub Group model leads us to the following property.

Property 3.1: Total Unimodularity Property [105]

Let A be an mxn integer matrix with a rank of m . A is unimodular if the determinant of every basis matrix B of A has value $+1$ or -1 as given by Ahuja *et al.* [105]. Thus, relying on this, we can state that if an integer valued matrix A is unimodular, then every basic feasible solution of the polyhedron defined by the constraints $Ax = b$ where $x \ge 0$, is integer for every integer valued right hand side vector b . If every square submatrix of A has a determinant of 0 or ± 1 , then the matrix A is totally unimodular. Moreover, every totally unimodular matrix is unimodular since each basis matrix B of the matrix A has a determinant ± 1 [105].

Proposition 3.1: The constraint set of the IP-Sub Group model has the total unimodularity property.

Proof: For the equation 3.25, I is the total number of instances. The corresponding A matrix of the IP-Sub Group model can be stated algebraically as follows.

$$
SP1 SP2 ... SPi ... SP1-1 SP1
$$

$$
A = \begin{bmatrix} 1 & 1 & \cdots & 1 & \cdots & 1 & 1 \end{bmatrix}
$$

The rank of above 1xI matrix is equal to 1 since it consists of only one row. Moreover, every square submatrix of A has a determinant $+1$ and therefore it is a totally unimodular matrix. Thus, IP-Sub Group model has the total Unimodularity property. \Box

Using this property, we can conclude that every basic feasible solution of the LP relaxation of IP-Sub Group model defined by Equation 3.25 is integer. Therefore, optimal solution of LP-relaxation is the optimal solution of IP-Sub Group model which means that solution of IP-Sub Group model could be easily obtained in a small amount of time.

In order to clarify the sub grouping approach, we tested IP-Sub Group model on an illustrative example given in Figure 3.4. In this illustrative example, there exist 100 instances (25 from each of the four classes) represented by two attributes values.

Figure 3.4 Illustrative example for sub grouping algorithm.
When the IP-Sub Group problem is solved for this illustrative example, we constitute two sub groups with 50 instances. The resulting sub groups are shown in Figures 3.5 and 3.6. As it can be seen from obtained sub groups, IP-Sub Group model efficiently selects the instances and constitute easier sub problems for MILP model. Solving the overall problem takes much more computational time with respect to solving two sub group problems separately. Hence, by solving Sub Group 1 and Sub Group 2 instances one by one using MILP, we obtain the constructed hyper-boxes in a reasonable amount of time. In some cases, obtaining the optimal solution of the overall problem takes more than a week/month. Therefore, in those cases solving the IP-Sub Group model and decompose the overall problem into smaller sub groups is favorable and preferable.

Figure 3.5 Sub Group 1 of given illustrative example.

Figure 3.6 Sub Group 2 of given illustrative example.

3.2.4 Seed Finding Algorithm

Another method to improve the computational efficiency is determining representative seeds for each class. Seed finding is a method that selects an instance (seed) for each class and fixes assignments of these instances to their respective classes before solving the problem. The seeds improve the computational performance of the model without changing the optimal solution.

The determination of seeds is a critical task: the seeds for each class must be chosen to ensure that seeds are separated well from each other as well as being a good example of the group of instances in the same class. We develop a pure integer programming (IP) formulation to accomplish this task. As in the MILP formulation, instances are represented by the parameter a_{im} that denotes the value of attribute m for the instances i. The class k of instance *i* belongs to is given by the set D_{ik} . Moreover, $PP_{ii'}$ represents the distance

between two instances i and i'. This distance is calculated using Euclidean distance in m dimensional space as given in Equation (3.27).

$$
PP_{ii'} = \sqrt{\sum_{m} (a_{im} - a_{im})^2}
$$
 (3.27)

Given these parameters and the sets, the binary variable YP_i , that indicates whether the instance i is selected as seed or not, is sufficient to model the seed finding problem. The following IP-Seed models the seed finding problem:

IP-Seed:
$$
\min z = \sum_{k} \sum_{i \in k} \sum_{i' \in k} PP_{ii'}YP_i - \left(\frac{1}{card(i \in k)}\right) * \sum_{k} \sum_{i \in k} \sum_{i' \notin k} PP_{ii'}YP_i
$$
(3.28)

subject to

$$
\sum_{i \in k} Y P_i = 1 \quad \forall k \tag{3.29}
$$

$$
YP_i \in \{0,1\} \qquad \forall i \tag{3.30}
$$

The objective of the IP-Seed problem given in Eq. (3.28) is to minimize the distances from each seed to instance of its group (in-class distances) and maximize the average distances from each seed to the instances that belong to other classes (out-class distances). Equation (3.29) states that every class must have exactly one seed. Finally, integrality of the decision variable YP_i is given by (3.30).

We performed a set of experiments on MILP model without seeds to compare its results with the one initiated with seeds. One can observe the positive effect of seed finding algorithm on the solution of MILP model, in terms of improvement in the number of iterations, the number of nodes and the CPU times required to construct the hyper-boxes by comparing the results given in Table 3.1. In Table 3.1, i is the number of instances, Cons. is the number of constraints, *BVar* is the number of binary variables and *CVar* is the number of continuous variables in the model. When we analyze the Table 3.1, we see that

CPU times, number of iterations and nodes decrease significantly as introducing seeds to the model. Hence, seed finding algorithm improves the computational time requirement of the MILP model.

Problem Characteristics				MILP with seeds MILP without seeds					
	# of	# of	# of	# of	# of	CPU	# of	$#$ of	CPU
	Cons.	BVar	CVar	Iterations	nodes	(sec.)	Iterations	nodes	(sec.)
10	12,265	6,190	2,081	57,543	331	81.14	15	θ	0.468
20	22,435	12,330	2,161	114,470	239	458.843	1,152	θ	2.296
30	32,605	18,470	2,241	187,769	603	1062.90	3,467	10	3.796
40	42,775	24,610	2,321	297,133	350	2154.35	26,390	270	27.593
50	52,945	30,750	2,401	432,922	862	4786.1	22,945	283	29.343

Table 3.1 A comparison of MILP model with and without seeds.

The seeds found by IP-Seed model are given in Figure 3.7. As it can be observed, seeds found by IP-Seed well exemplify the class properties.

Figure 3.7 Seeds found by IP-Seed are circled on an illustrative example.

A

Further investigation on the proposed IP-Seed model leads us to the following property.

Proposition 3.2: The constraint set of the IP-Seed model has the total unimodularity property.

Proof: For the Equation 3.29, I is the total number of instances, K is the total number of classes, c is the total number of class 1 instances and t is the total number of class 2 instances. The corresponding A matrix of the IP-Seed model can be stated algebraically as follows.

The above KxI matrix is 0-1 matrix and its rank is equal to *n* since it is consists of *n* linearly independent rows. Moreover, every square submatrix of A has a determinant 0 or +1 and therefore it is a totally unimodular matrix. Thus, IP-Seed model has the total Unimodularity property. \square

By the help of this property, we can conclude every basic feasible solution of the LP relaxation of IP-Seed model defined by Constraint 3.29 is integer. Therefore, optimal solution of LP-relaxation is the optimal solution of IP-Seed model which means that solution of IP-Seed model could be easily obtained in a small amount of time.

3.2.5 MILP Model

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Once the k seeds to be assigned to the k classes are determined by IP-Seed model, we can solve MILP model for 'problematic instances' with these seeds. Assignment of the instances selected as seed in the MILP model means that we are setting the variables corresponding to these instances to a specific value. Hence, optimal values for associated variables are given and do not need to be optimized. This means that, some of the solutions in the solution space are eliminated by fixing these values. Thus, this approach is capable of obtaining alternative optimal solutions for MILP model with smaller computational effort.

3.2.6 Final assignment and Intersection Elimination

Since the MILP model is solved for 'problematic instances' only, the 'nonproblematic instances' are assigned to hyper-boxes in a straight forward way. We define k hyper-boxes for each class and assign a 'non-problematic instance' to corresponding newly defined hyper-box. Each 'non-problematic instance' is considered one by one until all of these instances are assigned to a hyper-box. Finally, the bounds of these new hyper-boxes are determined by considering the maximum and minimum attribute values of all instances in these hyper-boxes. It is possible that these hyper-boxes have intersections. Instances are separated from the original hyper-box until all intersections are eliminated. The eliminated instances are grouped in a new box and intersection checking and elimination procedure is repeated until no more intersections occur between all of the constructed and defined hyper-boxes. After intersection elimination, box combination algorithm is included in order to get tight hyper-boxes for each class.

3.2.7 Box Combination

Box combination is the last step in the three-stage hyper-box enclosure approach. Since we do not solve problematic and non-problematic instances together, we could have

some hyper-boxes that could be combined without causing any intersection. As we want to differentiate the class boundaries with minimum number of hyper-boxes, combination of these hyper-boxes and decreasing the number of overall hyper-boxes is preferable. Hence, we developed an integer programming (IP) formulation to accomplish this task. As in the MILP formulation, X_{lmn} represents the bounds of existing hyper-boxes or the hyper-boxes obtained at the end of intersection elimination algorithm. The index l represents the existing hyper-boxes and the index l' represents the hyper-boxes that are obtained by combinations of the existing ones. The class k of hyper-box l belongs to is given by the set BC_{lk} . N $X_{l'mn}$ represents the bounds of hyper-boxes l' that is obtained by combining the existing hyper-boxes that are in the same class. The class k of hyper-box l' belongs to is given by the set $NBC_{l'k}$. In order to define the box intersections, we need to use center and length of the hyper-boxes. The centers C_{lm} and C_{lm} can be calculated using the Equations (3.31) and (3.32) , respectively. The lengths L_{lm} and L_{lm} can be calculated using the Equations (3.33) and (3.34), respectively. If the difference between the centers of the hyper-boxes is greater than the average lengths of the hyper-boxes for an attribute, then there is no intersection between these hyper-boxes for that attribute. Otherwise, these hyper-boxes will intersect on that attribute (Figure 3.8).

$$
C_{lm} = \frac{X_{lmn|n = upper} + X_{lmn|n = lower}}{2}
$$
 (3.31)

$$
C_{l'm} = \frac{NX_{l'mn|n=upper} + NX_{l'mn|n=lower}}{2}
$$
 (3.32)

$$
L_{lm} = X_{lmn|n = upper} - X_{lmn|n = lower}
$$
\n(3.33)

$$
L_{l'm} = N X_{l'mn|n = upper} - N X_{l'mn|n = lower}
$$
\n(3.34)

Given these parameters and the sets, the binary variables $INI_{ll'm}$ and $IN2_{ll'm}$ are necessary to indicate the intersection of hyper-boxes l and l' for each attribute m. $IO_{ll'}$ is a

binary variable that represents the intersection of hyper-box l and l' . The binary variable CO_l is 1 if there is an intersection related to newly defined hyper-box l'. Finally, SO_l is a binary variable which takes the value 1 when the hyper-box l' could be obtained without causing any intersection. The parameter $SI_{ll'}$ is 1 if the hyper-box l' is not obtained by any combination of the hyper-box *l* with other hyper-boxes and 0 otherwise. This parameter is necessary to check intersection for only the rest of the hyper-boxes that are not combined. Furthermore, the parameter is $SN_{l'l'}$ is 1 if hyper-box l' and hyper-box l'' is obtained by combination of a common hyper-box and 0 otherwise. This parameter is necessary to eliminate the multiple selections of hyper-box l for combination.

Figure 3.8 Hyper-box intersection check via the centers and lengths of hyper-boxes.

Using these binary variables and the parameters, the following IP-Box Combine models the box combination problem:

IP-Box Combine:

j

$$
\max z = \sum_{l'} SO_{l'} \tag{3.35}
$$

subject to

$$
C_{l'm} - C_{lm} + Q \cdot IN1_{l'm} \ge \frac{L_{lm} + L_{l'm}}{2} + \varepsilon \qquad \forall l, l', m \mid SI_{l'} = 1 \tag{3.36}
$$

$$
C_{lm} - C_{l'm} + Q \cdot IN2_{ll'm} \ge \frac{L_{lm} + L_{l'm}}{2} + \varepsilon \qquad \forall l, l', m \mid SI_{ll'} = 1 \tag{3.37}
$$

$$
\sum_{m} (IN1_{ll'm} + IN2_{ll'm}) - 2 * card(m) + 1 \leq IO_{ll'} \qquad \forall l, l' \mid SI_{ll'} = 1
$$
 (3.38)

$$
IO_{ll'} \leq CO_{l'} \qquad \forall l, l' | SI_{ll'} = 1 \tag{3.39}
$$

$$
COl + SOl \le 1 \qquad \forall l'
$$
 (3.40)

$$
SO_{l'} + SO_{l''} \le 1 \qquad \forall l', l'' \mid SN_{l'''} = 1 \tag{3.41}
$$

$$
SO_{l'}, CO_{l'}, IO_{ll'}, IN1_{ll'm}, IN2_{ll'm} \in \{0,1\} \qquad \forall l, l', m \tag{3.42}
$$

The objective of the IP-Box Combine problem given in Eq. (3.35) is to maximize the number of newly obtained hyper-boxes that represents the combination of old ones. Equation (3.36) and (3.37) are necessary to count the intersections of existing and newly obtained hyper-boxes for an attribute. In order to give the relationship between the centers and lengths and intersections, a large parameter Q and ε are included in these constraints. If hyper-boxes intersect for all of the attributes, then the binary variable $IO_{ll'}$ is 1 with Equation (3.38) . If newly-obtained hyper-box l' has any intersection with existing ones,

then the corresponding binary variable CO_l will be 1 to represent the infeasibility of obtaining hyper-box l' (3.39). If obtaining the hyper-box l' is feasible, then the binary variable SO_l is 1, and 0 otherwise by Equation (3.40). The Equation (3.41) states that only one combination related to hyper-box l could be selected. Finally, integrality of the decision variables is given by (3.42).

The IP-Box Combine model tries to find the maximum number of hyper-box combinations and obtain combined hyper-boxes. It is not possible to get all of the hyperbox combinations after a single run. We should iteratively solve IP-Box Combine model until the objective function value is 0. In Figure 3.9, there is an artificial example to observe the behaviors of the IP-Box Combine model. After the first run of IP-Box Combine model, some of the hyper-boxes are combined but there are some more feasible combinations (Figure 3.10). After the second run of IP-Box Combine, all of the feasible combinations are obtained (Figure 3.11).

Figure 3.9 Artificial example for IP-Box Combine analysis.

Figure 3.10 Combined hyper-boxes after the first run of IP-Box Combine model.

Figure 3.11 Combined hyper-boxes after the second run of IP-Box Combine model.

3.3 Testing Algorithm

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3.3.1 Original Testing Algorithm

The original testing algorithm proposed in master thesis [106] is briefly explained in this section. If a new instance with an unknown class is given, it is necessary to assign this instance to one of the classes. There are two possibilities for a new instance when determining its class:

- i. the new instance is within the boundaries of a hyper-box,
- ii. the new instance is not enclosed in any of the hyper-boxes determined in the training problem.

When the first possibility is realized for the new instance, the classification is made by directly assigning this instance to the class that was represented by the hyper-box enclosing the data point. In the case when the second possibility applies, the assignment of the new instance to a class requires some analysis. If the instance is within the lower and upper bounds of all but not one of the attributes (i.e., m') defining the box, then the shortest distance between the new instance and the hyper-box is calculated using the minimum distance between hyper-planes defining the hyper-box and the new instance. The minimum distance between the new data point i and the hyper-box is calculated using Eq. (3.43) considering the fact that the minimum distance is given by the normal of the hyperplane.

$$
DH_{il} = \min_{n} \left\{ a_{im} - X_{lm'n} \right\} \tag{3.43}
$$

When the data point is between the bounds of smaller than or equal to M-2 attributes, then the smallest distance between the point and the hyper-box is obtained by calculating the minimum distance between edges of the hyper-box and the new point. An edge is a finite segment consists of the points of a line that are between two possible pairs of extreme points EP_{lj} and EP_{lt} where j and t represent the rank of extreme points. As the

number of extreme points for a given box is 2^M but the number of edges is $M2^{M-1}$, not all of the indexes will be used for edge calculation. This issue will be controlled by given the possible extreme point combinations as a set, EPP (extreme point pairs). Cardinality of EPP set is $M2^{M-1}$. The value of attribute *m* for data point *i* is represented by the parameter a_{im} and ep_{ljm} and ep_{ljm} are the values of attribute m for two possible pairs of extreme points j and t . The minimum distance between the new data point i and one of the segment of the hyper-box determined by two extreme points is calculated using Eq. (3.50).

∑

$$
w_{ijtm} = a_{im} - ep_{ijm} \tag{3.44}
$$

$$
v_{iljtm} = ep_{ljm} - ep_{lm} \tag{3.45}
$$

$$
C1_{ijtm} = \frac{\sum_{m} W_{ijtm} v_{ijtm}}{\sqrt{\sum_{m} W_{ijtm}^2} \sqrt{\sum_{m} v_{ijtm}^2}}
$$
(3.46)

$$
C2_{ijtm} = \frac{\sum_{m} v_{ijtm} v_{ijtm}}{\sqrt{\sum_{m} v_{ijtm}^2} \sqrt{\sum_{m} v_{ijtm}^2}}
$$
(3.47)

$$
b_{ijim} = C1_{ijim} / C2_{ijim}
$$
 (3.48)

$$
pb_{iljtm} = ep_{ljm} + b_{iljtm}v_{iljtm}
$$
\n(3.49)

$$
DED_{il} = \min_{\substack{j,t:\\(j,t)\in EPP}} \left\{ \sqrt{\sum_{m} (a_{im} - pb_{iljtm})^2} \right\}
$$
(3.50)

When data point is not within the lower and upper bounds of any attributes defining the box, then the shortest distance between the new point and the hyper-box is calculated using the minimum distance between extreme points of the hyper-box and the new data. The minimum distance between the new data point i and one of the extreme points ep_{lim} of the hyper-box is calculated using Eq. (3.51).

$$
DEP_{il} = \min_{j} \left\{ \sqrt{\sum_{m} (a_{im} - ep_{ljm})^2} \right\}
$$
 (3.51)

The following algorithm assigns a new data point *i* with attribute values a_{im} to class *k*:

Step 0: Initialize $inAtt_{lm}=0$.

j

- **Step 1:** For each l and m, if $X_{lmn} \le a_{im} \le X_{lmn}$ $\forall n = lower, n' = upper$, set inAtt_{lm} = inAtt_{lm} $+1$
- Step 2: If in $Att_{lm} = M$, then go to Step 3. Otherwise, continue. If in $Att_{lm} \leq M-1$, then go to Step 4.
- Step 3: Assign the new data point to class k where ybc_{lk} is equal to 1 for the hyper-box in Step 2. Stop.
- **Step 4:** If $inAtt_{lm} = M-1$, then $dist_{il} = DH_{il}$. If $0 \leq inAtt_{lm} \leq M-1$, then $dist_{il} = DED_{il}$. If $inAtt_{lm} = 0$, then $dist_{il} = DEP_{il}$.
- **Step 5:** Select the minimum between $\min_{l} \{dist_{il}\}\}$ to determine the hyper-box *l* that is closest to the new data point *i*. Assign the new data point to class *k* where ybc_{lk} is equal to 1 for the hyper-box l. Stop.

After finding the assigned classes of test instances, we must compare the assigned and original classes in order to calculate the accuracy of the proposed model. The proportion of correctly classified instances will give the efficiency and accuracy of the algorithm.

3.3.2 Improved Testing Algorithm

The original testing algorithm is computationally intractable for high-dimensional problems due to high number of extreme point calculations. Hence, an improved testing algorithm that approximates the original algorithm is developed. The testing results for large data classification problems can be computed in a very smaller amount of time with the improved testing algorithm compared to the original algorithm. The following new algorithm assigns a new data point *i* with attribute values a_{im} to class k :

Step 1: For each l and m ,

j

If
$$
a_{im} > X_{lmn}
$$
 where $n = upper$, then $d_{ilm} = (a_{im} - X_{lmn})^2$.
\nIf $a_{im} < X_{lmn}$ where $n = lower$, then $d_{ilm} = (X_{lmn'} - a_{im})^2$.
\nIf $X_{lmn} \le a_{im} \le X_{lmn}$ where $n = upper$ and $n = lower$, then $d_{ilm} = 0$.

Step 2: Calculate distance from data point *i* to box *l* by using Equation 3.1.

$$
Ndist_{il} = \sqrt{\sum_{m} d_{ilm}}
$$
\n(3.52)

Step 3: Select the minimum between $\min_{l} \{ Ndist_{il} \}$ to determine the hyper-box *l* that is closest to the new data point *i*. Assign the new data point to class *k* where ybc_{lk} is equal to 1 for the hyper-box l. Stop.

3.3.3 Comparison of Original and Improved Testing Algorithms

There exists four possible cases for the position of an instance i with respect to a hyper-box l in the original testing algorithm (Figure 3.12). These cases can be listed as follows:

Case I: Instance i is enclosed by the hyper-box l .

Case II: Instance i is within the lower and upper bounds of all but not one of the attributes (m') of hyper-box l.

Case III: Instance i is between the bounds of smaller than or equal to M-2 attributes of hyper-box l.

Case IV: Instance i is not within the lower and upper bounds of any attributes of hyperbox l.

Figure 3.12 The possible positions of an instance with respect to a hyper-box.

The following analysis can be done for each case.

Case I: If instance i is inside the hyper-box l , then it is directly assigned to the corresponding class of hyper-box l in the original testing algorithm. Similarly, in the improved testing algorithm $X_{lmn} \le a_{lm} \le X_{lmn}$ holds and d_{ilm} will be 0 for each attribute m. This will result in $d_{il} = 0$. Hence, the closest hyper-box to that instance i will be hyper-box l and instance i will be assigned to the corresponding class of hyper-box l . Therefore, improved testing algorithm gives the same results as the original algorithm for Case I.

Case II: If an instance i is within the lower and upper bounds of all but not one of the attributes (m') of hyper-box l, minimum distance form that instance i to the hyper-box l is calculated by using Equation 3.43 in the original testing algorithm. For the improved testing algorithm, as $X_{lmn} \le a_{lm} \le X_{lmn}$ holds for all attributes except m', d_{ilm} will be zero

for those attributes and d_{lim} will be greater than zero (Equation 3.53). Hence, distance from instance i to the hyper-box l is calculated using Equation 3.54 in the improved testing algorithm. As Eq. (3.54) and Eq. (3.43) are identical, both of the testing algorithms are identical for Case II.

$$
d_{ilm'} = \min_{n} \left\{ (a_{im'} - X_{lm'n})^2 \right\} \tag{3.53}
$$

$$
d_{il} = \sqrt{d_{ilm}} = \min_{n} |a_{im} - X_{lmn}|
$$
 (3.54)

Case III: If an instance i is between the bounds of smaller than or equal to $M-2$ attributes of hyper-box l , the original algorithm will calculate the distances from instance i to each edge of the hyper-box l. Then, it selects the smallest one from $m2^{m-1}$ edges as given in Eq. (3.50). On the other hand, the improved algorithm will find out the closest extreme point of the hyper-box l that is the one of the extreme points of the closest edge found with the original algorithm. Then, the improved algorithm calculates the Euclidean distance from instance i to that extreme point. Hence, the improved algorithm's distance value will always be greater than the distance value of the old algorithm.

In order to prove this more formally, assume that the closest extreme point of hyper-box *l* to instance *i* is $(X_{l_1wper}, X_{l_2wper}, ..., X_{l_1wper}, ..., X_{l_mwper})$. For the improved algorithm, distance from instance i to hyper-box l is calculated as in Eq. (3.55).

$$
d_{il} = \sqrt{(a_{il} - X_{l \text{lupper}})^2 + ... + (a_{ik} - X_{l \text{kupper}})^2 + ... (a_{im} - X_{l \text{mupper}})^2}
$$
(3.55)

As neighboring extreme points have (m-1) attribute values in common, the closest edge to instance i will be the one with an end point of(X_{l_1} _{Lupper}, X_{l_2} _{lupper}, ..., X_{l_2} _{lpupper}, ..., X_{l_2} _{lpupper}). Assume the other end point of this edge is $(X_{l_1wper}, X_{l_2wper}, \ldots, X_{l_kdower}, \ldots, X_{l_mwper})$ as only one attribute value changes for neighbor extreme points. Then, the closest point on that edge to instance i is

 $(X_{l1upper}, X_{l2upper}, \ldots, (X_{lklower} + b(X_{lkupper} - X_{lklower})), \ldots, X_{lmupper})$ where b is that ratio that shows how far instance i from start point of that edge. This b is given in Eq. (3.48). Hence, from Eq. (3.50) , the original algorithm gives the minimum distance from instance i to hyper-box l as follows:

$$
origd_{il} = \sqrt{(a_{il} - X_{l1upper})^{2} + ... + (a_{ik} - X_{lklower} - k(X_{lkupper} - X_{lklower}))^{2} + ... (a_{im} - X_{lmupper})^{2}}
$$
\n(3.56)

All terms of d_{il} and $origd_{il}$ are equal to each other except $(a_{ik} - X_{lkupper})^2$ and $(a_{ik} - X_{lklower} - k(X_{lkupper} - X_{lklower}))^2$. We only need to compare these terms to give the superiority relationship between d_{il} and origd_{il}.

Claim: $d_{il} \geq \text{origd}_{il}$.

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Proof: As mentioned before, all terms are equal in these distance values except $(a_{ik} - X_{lkupper})^2$ and $(a_{ik} - X_{lklower} - k(X_{lkupper} - X_{lklower}))^2$. Hence, we need to compare these two terms in order to conclude. As closest extreme point consists of $X_{lkupper}$, then $0.5 \le k \le 1$ and $a_{im} > X_{lmupper}$ holds.

$$
(a_{ik} - X_{lkupper})^2 \geq (a_{ik} - X_{lklower} - k(X_{lkupper} - X_{lklower}))^2
$$
 (3.57)

$$
\left| a_{ik} - X_{\text{lkupper}} \right| \geq \left| a_{ik} - X_{\text{lklower}} - k(X_{\text{lkupper}} - X_{\text{lklower}}) \right| \tag{3.58}
$$

$$
a_{ik} - X_{lkupper} \ge a_{ik} - X_{lklower} - k(X_{lkupper} - X_{lklower})
$$
\n(3.59)

$$
(k-1)X_{\text{lkupper}}^2 \geq (k-1)X_{\text{lklower}} \tag{3.50}
$$

$$
X_{\text{lkupper}} \overset{?}{\geq} X_{\text{lklower}} \tag{3.61}
$$

Since X_{known} is always greater than or equal to X_{known} , the claim $d_i \geq \text{orig } d_i$ is true. In the same manner, the case where instance i is closer to X_{kkower} can be proved. Therefore, the improved testing algorithm gives distance values greater than or equal to the original distance value for Case III.

Case IV: If an instance i is not within the lower and upper bounds of any attributes of hyper-box l , the original algorithm calculates the distances from instance i to each extreme points of hyper-box l . Then, it will select the smallest one from 2^m extreme points as given in Eq. (3.51). On the other hand, the proposed improved algorithm tries to find the closest bound (either lower or upper) for each attribute. Then, the closest extreme point will be found out by these closest bounds. Hence, the same distance value will be obtained as in the original testing algorithm. Both algorithms give identical distance values for Case IV.

Therefore, the improved testing algorithm is an approximation of the original testing algorithm. In Cases I, II and IV, calculated distance values will be same. On the other hand, for Case III improved testing algorithm will give a higher distance value. Hence, the improved testing algorithm is an approximation of the original one.

3.3.4 Computational Complexities of the Original and Improved Testing Algorithms

The original testing algorithm has a poor computational performance on data sets with large number of attributes. The improved algorithm is an approximation of the original algorithm. Therefore, a worse performance can be expected from the new algorithm. However, the computational complexity of the improved algorithm is far superior to the original one. Therefore, we compare the computational complexities of two testing algorithms. The number of algebraic operations for the original testing algorithm is $O(M2^{M-1})$ whereas that for the new testing algorithm is $O(LM)$ (see Table 3.2). Thus, the original testing algorithm is an exponential algorithm that depends one the number of attributes M. However, the improved testing algorithm is a polynomial algorithm that depends on the number of hyper-boxes L and number of attributes M . Hence, the improved

testing algorithm is preferable in the case of data classification problems with large number of attributes.

	Original Testing Algorithm	New Testing Algorithm			
Place	Computation Time	Place	Computation Time		
Step 1	O(LM)	Step 0	O(LM)		
Step 4	$O(N)$, $O(2^M)$, $O(M2^{M-1})$	Step 3	O(L)		
Step 5	$\mathrm{O}(L)$				
Overall	$O(M2^{M-1})$	Overall	O(LM)		
Complexity		Complexity			

Table 3.2. Computational complexities of two testing algorithms.

3.4 Illustrative Example

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We applied the proposed three-stage MILP based approach on set of 105 training data points in four different classes given in Figure 3.13.

3.4.1 Training Part

When we apply proposed three-stage algorithm, we first calculate the boundaries of classes and compare whether they overlap or not. As shown in Figure 3.14, overlapping between the classes exists. The instances that are enclosed by other classes are identified as 'problematic instances'. For this data set, there exist 18 data points which fall into the bounds of other classes. These problematic instances are enclosed by dashed points in Figure 3.15. Using these problematic instances, IP-Seed model is solved to find a seed for each class. Seeds are indicated with circles in Figure 3.15. Once four seeds to be assigned to the four classes are determined, we solve MILP model for these 'problematic instances' with fixed assignment of these seeds. The constructed hyper-boxes for these problematic instances are shown in Figure 3.16.

Figure 3.13 Data points in the illustrative example and their graphical representation.

Figure 3.14 Maximum and minimum attribute values for each class.

Figure 3.15 Problematic instances are enclosed by dashed points and seeds with circles.

Figure 3.16 Constructed hyper-boxes for problematic instances.

Figure 3.17 Defined and constructed hyper-boxes for illustrative example.

Figure 3.18 Hyper-boxes after intersection elimination for illustrative example.

Figure 3.19 Final solution for illustrative example.

The next step is the assignment of non-problematic instances. New hyper-boxes for each class are defined and remaining 87 non-problematic instances are assigned to the hyper-boxes that correspond to their own classes. Then, the bounds of the newly defined hyper-boxes are calculated by obtaining the maximum and minimum attribute values of instances belonging to them (Figure 3.17). As it can be seen from Figure 3.17, there are some intersections between constructed and defined hyper-boxes. In order to get rid of these intersections, instances in the defined hyper-boxes are separated one by one until intersections are eliminated. Then, the eliminated instances are grouped in a new hyperbox. Resulting hyper-boxes do not intersect each other as shown in Figure 3.18. After that, IP-Box Combine model is studied and the feasible combination of hyper-boxes is obtained (Figure 3.19). The final solution for this illustrative example is found. At last, without any misclassifications of training set instances, 8 hyper-boxes are obtained.

Hence, the proposed three-stage MILP approach categorized the 105 training instances into their corresponding classes with a training accuracy value of 100%. The characteristics of each of the steps of proposed approach on illustrative example are given in Table 3.3.

Steps of	Problem Characteristics							
3-Stage Approach	# of	# of	# of	# of	# of	CPU		
	Nodes	Iterations	Constraints	BVar	CVar	(sec.)		
Problematic Instances						0.093		
Seed Finding	θ	θ	59	72	θ	0.078		
MILP with Seeds	Ω	22	1509	858	265	0.265		
Defined Hyper-boxes						0.063		
Intersection						0.203		
Elimination								
Box Combination	θ	θ	4045	222	θ	0.109		
Testing						0.016		

Table 3.3 Problem characteristics for illustrative example.

3.4.2 Testing

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After classifying the training data perfectly, the 52 test instances (shown in Figure 3.20) are assigned to the constructed hyper-boxes by applying the improved testing algorithm. After improved test set analysis, it is observed that all of test instances are assigned to their original classes. Hence, accuracy of the proposed three-stage approach is 100% for this illustrative example using the testing algorithm.

Figure 3.20 Test instances for illustrative example.

On the other hand, the same illustrative example is studied with different types of classifiers available in the well-known Weka. Weka is a collection of machine learning algorithms for data mining tasks including data classification [107]. In Table 3.4, different classification methods and their accuracy values are listed. The best accuracy value is 96.1% received by the classifier NNge (Nearest neighbor like algorithm using non-nested generalized exemplars).

Classifier	Accuracy	Classifier	Accuracy
BayesNet	84.6%	Decorate	90.3%
NaiveBayes	92.3%	END	88.4%
NaiveBayesSimple	94.2%	FilteredClassifier	84.6%
NaiveBayesUpdateable	92.3%	LogitBoost	82.6%
Logistic	86.5%	MultiClassClassifier (RBF)	88.4%
MultiLayerPerceptron	88.4%	MultiClassClassifier	86.5%
		(MultiLayerPerceptron)	
RBFNetwork	94.2%	RandomCommittee	92.3%
SimpleLogistic	88.4%	BFTree	92.3%
SMO	92.3%	J48	84.6%
IB1	92.3%	NBTree	94.2%
IB2	92.3%	RandomForest	92.3%
IB ₃	94.2%	RandomTree	88.4%
IB4	92.3%	REPTree	94.2%
IB ₅	94.2%	SimpleCart	92.3%
IB ₆	94.2%	NNge	96.1%
KStar	94.2%	Bagging	94.2%
LWL	94.2%	Ridor	92.3%
AttributeSelectedClassifier	88.4%	ClassificationviaRegression	90.3%

Table 3.4 Accuracies of different data classification methods for illustrative example.

As a result, suggested three-stage approach performs better than other data classification methods that are listed in Table 3.4 for this illustrative example. Thus, this new method can be attractive for real life data classification problems. For further

investigation to the performance of the developed MILP based algorithm, distinct benchmark problems are examined in the next chapter of the thesis.

3.4.3 The Original and New Testing Algorithms' Performances on New Thyroid Data Set

In this part of the study, the efficiencies of original and new testing algorithms are compared on new thyroid dataset [108]. This data set is composed of 215 samples with 5 different attribute values and 3 different classes: euthyroidism (class 1), hypothyroidism (class 2), or hyperthyroidism (Class 3). In this dataset, 150 of instances belong to class 1, 35 of them belong to class 2 and remaining 30 belong to class 3.

For thyroid data set, 10-fold cross-validation approach is used to estimate the performance of three-stage MILP based approach with both original and new testing algorithms.

In Table 3.5, results for the new and original testing algorithms are listed. As it is seen in Table 3.5, the new testing algorithm has better in overall accuracy for thyroid dataset. For the runs 2, 3, 5, 7 and 8, both algorithms give the same accuracy values. On the other hand, in runs 1, 4, 6 and 10 the new testing algorithm has a higher accuracy value. Interestingly, the original testing algorithm has 100% accuracy for run 9, which is more accurate than the new testing algorithm. To sum up, we could not conclude that the new testing algorithm is always better than the original algorithm with respect to accuracy. However, it gives better results on most of the cases and has higher average classification accuracy for thyroid data set.

# of run	Accuracy with the original testing algorithm	Accuracy with the new testing algorithm
	90.90%	95.45%
2	95.45%	95.45%
3	95.45%	95.45%
4	86.36%	90.90%
5	100%	100%
6	95.23%	100%
	100%	100%
8	95.23%	95.23%
9	100%	90.47%
10	76.19%	80.95%
Overall	93.48%	94.39%

Table 3.5 Prediction results for Thyroid data set for original and new testing algorithm.

Chapter 4

COMPUTATIONAL RESULTS ON PROTEIN FOLDING TYPE PREDICTION

The performance of proposed three-stage approach is evaluated on distinct protein folding type prediction benchmark data sets. The prediction results and comparisons with other data classification methods are examined in this chapter.

4.1 Protein Folding Type Prediction Problem

 The prediction of protein folding type is a typical multi-group data classification problem. The are four different classes; all-alpha (α), all-beta (β), alpha+beta (α + β), alpha/beta (α/β) . 20 amino acid compositions constitute the attributes of protein folding type prediction problem.

4.2 Protein Folding Type Data Sets

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In order to observe the performance of the proposed approach, the following four data sets from [97] are tested: 138 domains in Table A.1, 253 domains in Table A.2, 359 domains in Table A.3, 1601 domains in Table A.4, 225 Domains in Table A.5, 510 Domains in Table A.6, 2438 Domains in Table A.7. Moreover, two data sets from [100] are studied: 277 Domains in Table A.8 and 498 Domains in Table A.9. Finally, two more data sets from [23] are tested: 1189 Domains in Table A.10 and 25PDB in Table A.11. Each of these data sets is constructed from SCOP [91] and Protein Data Bank [90]. The unit of classification in the SCOP database is usually the protein domain. Small proteins and most medium-size proteins have single domain. Domains in large proteins are usually classified individually. Therefore, the sequence of a domain considered here is either the

whole chain or a partial chain of a protein. Each domain is represented by a symbol of X|Y, where first four character of X is the corresponding PDB code and the fifth character indicates the specific chain of the protein. If it is , then the corresponding protein has only one chain. If Y=W.C., it means the domain is constituted by the whole chain. Otherwise, Y contains two number to indicate starting and end points along the sequence.

In the SCOP database, protein domains are classified into the following 11 categories [91]: (1) all-α proteins; (2) all-β proteins; (3) α/β proteins; (4) $\alpha+\beta$ proteins; (5) multi-domain proteins; (6) membrane and cell surface proteins; (7) small proteins; (8) coiled coils proteins; (9) low resolution proteins; (10) peptides; and (11) designed proteins. Usually, only the first four categories are considered for computational prediction purposes as they include significant majority of the protein sequences.

For 138, 253, 359, 225, 510, 277, 498, 1189 and 25PDB protein data sets, they are assumed to have four different classes. On the other hand, for 1601 and 2438 protein domains seven different structural classes, i.e. all α , all β, α +β, α /β, multi domain (μ), small protein (σ) and peptides (ρ), were used. Details related to these seven classes were given in [23, 97].

The leave-one-out (LOO) results of 138, 253, 359 and 1601 data sets are given in [97] and [109]. Moreover, the prediction quality is also examined by independent training and test data sets as in [97] and [110]. The training data set is composed of 225 protein domains and the corresponding test data set contains 510 protein domains. Furthermore, 1601 protein domains are used as training set in order to test the performance on 2438 protein domains. On the other hand, LOO results of 277 and 498 domain data sets are given in [100], [104] and [109]. Finally, 10-fold cross-validation (10FCV) results of 1189 and 25PDB data sets are mentioned in [23].

4.3 Classification Algorithms

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In order to compare the results of proposed MILP approach, WEKA classification algorithms J48, RBF Network, Logistic, Naïve Bayes (NB), SMO, Random Forest (RF) and IB1 are also studied (Table 4.1). Optimized parameter values of these WEKA classifiers given by [23] are used to perform the studies on the given data sets. Moreover, well-known support vector machine implementation LibSVM given by [111] is also studied to observe the accuracy values. For each of the data sets, parameters related to SVM algorithm are optimized by performing 10FCV validation with different combinations of cost and gamma values. The optimal values that achieve the highest 10FCV accuracy are used to obtain the LOO results for each data set (Table 4.2).

Data Sets	Kernel Type	c	g
		(Cost)	(Gamma)
138 Protein Domains	Radial Basis Function	2048	8
253 Protein Domains	Radial Basis Function	8192	8
359 Protein Domains	Radial Basis Function	512	8
277 Protein Domains	Radial Basis Function	2048	8
498 Protein Domains	Radial Basis Function	2048	8
225&510 Protein Domains	Radial Basis Function	32	\mathcal{D}_{\cdot}
1601&2438 Protein Domains	Radial Basis Function	128	8
1189 Protein Domains	Radial Basis Function	512	0.5
25PDB Protein Domains	Radial Basis Function	8	8

Table 4.2 Optimal parameter values of LibSVM for each of the data sets.

Classifier	Reference	Short Description
Naïve Bayes	$[112]$	• Class for a Naive Bayes classifier using estimator classes. • Numeric estimator precision values are chosen based on
		analysis of the training data.
RBF Network	[113]	• Class that implements a normalized Gaussian radial basis function network.
		• It uses the k-means clustering algorithm to provide the basis functions and learns either a logistic regression (discrete class problems) or linear regression (numeric class problems) on top of that.
		• It standardizes all numeric attributes to zero mean and unit variance.
IB1	[114]	\bullet IB1-type classifier.
		• Uses a simple distance measure to find the training instance closest to the given test instance, and predict the same class as this training instance.
		• If multiple instances are the same (smallest) distance to
J48	$[115]$	the test instance, the first one found is used. • Class for generating an unpruned or a pruned C4.5 decision tree.
Random Forest	$[116]$	• Decision tree type algorithm
JRip	[117]	• Class for constructing random forests. • This class implements a propositional rule learner, Incremental Pruning Produce Repeated to Error Reduction (RIPPER), which is proposed by William W.
		Cohen as an optimized version of IREP.
SMO	$[118]$	John Platt's \bullet Implements C_{\cdot} sequential minimal optimization algorithm for training a support vector classifier using polynomial kernels.
		• Transforms output of SVM into probabilities by applying a standard sigmoid function that is not fitted to the data.
Logistic	[119]	• Class for building a logistic regression model using LogitBoost.
		selection • Incorporates attribute by fitting simple regression functions in LogitBoost.

Table 4.1 Summary of the applied classification algorithms of WEKA.

Furthermore, the existing results of distance-based classification methods based on Hamming Distance (HD), Euclidean Distance (ED) and Component-coupled (CC) algorithms given in [97] and [100], the reported results of SVM algorithm used in [110] and [109], and the existing result of Neural Networks method given in [104] are also investigated for comparison.

4.4 Results for Independent Data Sets

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Using the 225 training set samples given in [97] (Table A.5), the proposed threestage MILP model is solved in GAMS [120] using ILOG CPLEX Solver version 10.0 [121] on a notebook computer with Inter Pentium M 1.73 Ghz Processor and 512 MB of RAM. The characteristics of the constructed model for 225 training samples are listed in Table 4.3

After classifying the training data perfectly (self-consistency test result is 100%), the test set given in Table A.6 is assigned to constructed hyper-boxes by applying the testing algorithm. The assignment of data in the test set to structural classes is done without a prior knowledge on their membership to a class. For each member of the test data set, testing algorithm is applied and an assignment to a structural class is done. After all, the accuracy of the developed model is checked by comparing the original and assigned structural classes of proteins. At the end of the testing, it is realized that 489 proteins in the test set are correctly classified. On the other hand, 21 proteins are misclassified.

ITEM	VALUE
$\overline{\#}$ of continuous variables	2401
# of binary variables	30750
$#$ of constraints	52495
$#$ of nodes	283
# of iterations	22945
Solver Memory (MB)	12
CPU time (sec)	29.343

Table 4.3 Characteristics of the MILP model for 225 training samples.

		Class-based Accuracy				
Methods	α	ß	$\alpha + \beta$	α/β	Accuracy	
MILP	93.58%	96.15%	96.32%	97.04%	95.88%	
IB1	90.83%	94.62%	98.53%	97.78%	95.68%	
SVM	NA.	NA	NA	NA.	94.90%	
Random Forest	93.58%	96.15%	86.76%	95.56%	92.94%	
Component-coupled	74.31%	90.00%	87.50%	91.85%	86.47%	
J48	80.73%	59.23%	82.35%	79.26%	75.29%	
LibSVM	63.30%	78.46%	50.00%	42.22%	58.04%	
RBF Network	46.79%	58.46%	51.47%	66.67%	56.27%	
Logistic	68.81%	77.69%	30.15%	50.37%	55.88%	
Naïve Bayes	45.87%	69.23%	24.26%	77.78%	54.50%	
SMO	50.46%	49.23%	49.26%	50.37%	49.80%	
JRip	18.35%	66.15%	74.26%	26.67%	47.64%	
Euclidean Distance	50.46%	75.38%	23.53%	41.48%	47.25%	
Hamming Distance	60.55%	73.08%	22.06%	36.30%	47.06%	

Table 4.4 Performance results for the 510 protein domains in the test set.

 The overall accuracy of the proposed model on 510 protein domains is 95.88%. The results of distance-based classification methods Hamming Distance, Euclidean Distance and Component-coupled algorithms [97] and the result of SVM algorithm [110] are listed in Table 4.4. Moreover, LibSVM and classifiers found in WEKA are also studied to observe the accuracy values. Proposed three-stage MILP approach gives the highest accuracy for this test set as shown in Table 4.4. IB1, instance-based classifier, has the closest accuracy value to MILP approach. SVM result given in [110] has a higher accuracy value compared to well-known support vector machine implementations SMO and LibSVM. As Cai et al. [110] did not provide individual accuracy values of classes and

detailed confusion matrix; we could not compare classed-based accuracies. Hamming Distance and Euclidean Distance algorithm has the worst accuracy values for this data set.

 In the same manner, 1601 domains data set (Table A.4) is studied by proposed three-stage MILP approach. After classifying the training data perfectly self-consistency test result is 100%), the test set composed of 2438 domains given in Table A.7 is assigned to constructed hyper-boxes by applying the testing problem algorithm. The accuracy of the developed model is checked by comparing the original and assigned structural classes of proteins. At the end of the testing, it is realized that 2318 proteins in the test set are correctly classified. On the other hand, 120 proteins are misclassified.

Class-based Accuracy							Overall
α	β	$\alpha + \beta$	α/β	μ	σ	$\boldsymbol{\rho}$	Accuracy
96.44%	95.74%	95.72%	97.25%	71.74%	87.34%	85.00%	95.08%
95.17%	94.18%	97.20%	95.48%	89.13%	94.30%	65.00%	95.03%
NA	NA	NA.	NA	NA	NA	NA	94.50%
93.38%	92.76%	94.74%	92.14%	86.96%	96.84%	75.00%	93.23%
83.72%	87.93%	87.34%	88.41%	71.74%	85.44%	35.00%	86.54%
79.39%	92.90%	79.11%	87.43%	90.00%	96.20%	0.0%	84.58%
68.70%	78.27%	69.74%	86.44%	76.09%	90.51%	75.00%	77.03%
64.12%	91.05%	44.57%	51.47%	13.04%	89.24%	60.00%	65.01%
65.14%	70.17%	52.63%	68.76%	41.30%	85.44%	35.00%	64.84%
58.78%	72.44%	60.36%	64.83%	0.00%	76.58%	0.00%	63.94%
63.87%	76.28%	51.64%	60.90%	0.00%	79.11%	0.00%	63.04%
58.78%	67.05%	26.64%	72.69%	28.26%	80.38%	40.00%	56.72%
56.23%	57.10%	23.52%	49.51%	50.00%	77.22%	5.00%	47.74%
47.08%	58.81%	10.36%	45.58%	47.83%	74.05%	0.00%	42.38%

Table 4.5 Performance results for the 2438 protein domain in the test set.
The overall accuracy of the proposed model on 2438 protein domains is 95.08%. In Table 4.5, accuracy results given in [97] and [110] are listed. Moreover, LibSVM and the same classifiers found in WEKA are also studied. Proposed three-stage MILP approach gives the highest accuracy for this test data set than Hamming Distance, Euclidean Distance, Component-coupled and SVM methods. However, the accuracy values of MILP approach is much closer to IB1 accuracy value. SVM result compared to well-known support vector machine classifiers LibSVM and SMO.

4.5 Results for Self-consistency Tests

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For self-consistency tests, data sets with 138, 253, 359, 1601, 277 and 498 protein domains given in Appendix are used. Using these data sets, the proposed three-stage MILP model is solved in GAMS [120] using ILOG CPLEX Solver version 10.0 [121] on a notebook computer with Inter Pentium M 1.73 Ghz Processor and 512 MB of RAM. For each data set, as we will perform LOO tests, training runs are carried out. Average selfconsistency test results for 138, 253, 359 and 1601 data sets are given in Table 4.6. Moreover, average self-consistency test results for 277 and 498 domains are listed in Table 4.7.

Methods	138 Domains	253 Domains	359 Domains
Hamming Distance [97]	55.8%	52.57%	55.15%
Euclidean Distance [97]	57.25%	53.36%	52.37%
Component-coupled [97]	97.83%	95.26%	94.43%
SVM [110]	100%	100%	93%
3-Stage MILP Approach	100%	100%	100%

Table 4.6 Self-consistency test results for 138, 253, 359 and 1601 Domains.

Methods	277 Domains	498 Domains
Hamming Distance [100]	62.8%	65.5%
Euclidean Distance [100]	58.8%	64 3%
Component-coupled [100]	94 2%	95.8%
NN [104]	93.5%	94.6%
SVM [109]	100%	100%
3-Stage MILP Approach	100%	100%

Table 4.7 Self-consistency test results for 277 and 498 Domains.

 Self-consistency test results indicate the percentage of information grasped during the training studies that captures the relationship between amino acid composition and protein folding type. As it could be observed from Table 4.6 and Table 4.7., proposed three-stage MILP approach gives highest self-consistency results for each one of the data sets. Hence, the relationship between amino acid composition and protein folding type is fully grasped by the developed approach.

4.6 Results for Leave-one-out Tests

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 In this part, structural classes of leaved-out proteins are predicted by the results derived using all other proteins in the training set. LOO test results for 138 protein domains are given in Table 4.8. LibSVM method has the highest LOO test result for 138 protein domains data set with accuracy of 70.29%. Proposed MILP approach has the second best LOO accuracy value, 67.39%, for 138 protein domains data set. IB1 classifier of WEKA also has a very close result to MILP approach. Detailed comparison of these methods based on hypothesis testing is given in Section 4.8.

Methods	Class-based Accuracy	Overall			
	α	β	$\alpha + \beta$	α/β	Accuracy
LibSVM	80.56%	75.86%	65.85%	59.40%	70.29%
MILP	83.33%	79.31%	63.41%	43.80%	67.39%
IB1	61.11%	79.31%	58.54%	71.90%	66.67%
Component-coupled	77.78%	55.17%	85.37%	28.12%	63.77%
SMO	63.89%	75.86%	53.66%	53.10%	60.87%
J48	63.89%	72.41%	58.54%	50.00%	60.87%
Random Forest	66.67%	65.52%	56.10%	53.10%	60.14%
RBF Network	63.89%	62.07%	56.10%	50.00%	57.97%
SVM	52.77%	75.86%	58.50%	43.75%	57.24%
Naïve Bayes	63.89%	65.52%	34.15%	56.30%	53.62%
Logistic	61.11%	65.52%	46.34%	40.60%	52.90%
Hamming Distance	61.11%	55.17%	36.59%	43.75%	48.55%
Euclidean Distance	61.11%	51.72%	34.15%	40.62%	46.38%
JRip	50.00%	58.62%	48.78%	18.80%	44.20%

Table 4.8 LOO test results for 138 protein domains.

 Existing and calculated LOO test results for 253 protein domains are given in Table 4.9. Proposed three-stage MILP approach has the highest LOO test result for 253 protein domains with accuracy of 87.65%. Instance-based classifier IB1 has the second best result with accuracy value of 86.45%. Random Forest classifier and LibSVM have also high classification accuracy values with respect to other methods.

Methods			Class-based Accuracy		Overall
	α	β	$\alpha + \beta$	α/β	Accuracy
MILP	91.94%	85.96%	92.96%	78.69%	87.65%
IB1	90.32%	85.96%	80.28%	90.16%	86.45%
Random Forest	87.10%	80.70%	80.28%	83.61%	82.86%
LibSVM	88.71%	77.19%	76.06%	85.25%	81.67%
J48	80.65%	68.42%	67.61%	73.77%	72.51%
Component-coupled	84.13%	79.31%	70.49%	81.69%	63.77%
JRip	66.13%	61.40%	63.38%	55.74%	61.75%
SMO	67.74%	70.18%	52.11%	52.46%	60.15%
RBF Network	66.13%	66.67%	53.52%	52.46%	59.36%
Naïve Bayes	69.35%	59.65%	40.85%	68.85%	58.96%
SVM	84.12%	79.31%	81.96%	87.32%	57.24%
Logistic	61.29%	63.16%	49.30%	37.70%	52.58%
Hamming Distance	60.32%	60.34%	47.54%	29.58%	48.55%
Euclidean Distance	58.73%	62.07%	47.54%	35.21%	46.38%

Table 4.9 LOO test results for 253 protein domains.

 Table 4.10 shows the LOO test results for 359 protein domains. Proposed threestage MILP based approach has the highest LOO test result for 359 protein domains with accuracy of 96.38%. The accuracy value of the SVM method given in [110] is the second best result. However, the well-known support vector machine classifiers LibSVM and SMO have surprisingly lower results than this SVM result. Instance-based classifier IB1 and LibSVM has also higher classification accuracy values than other existing methods.

	Class-based Accuracy	Overall			
Methods	α	β	$\alpha + \beta$	α/β	Accuracy
MILP	98.78%	97.65%	92.47%	96.97%	96.38%
SVM	92.68%	96.47%	96.77%	94.94%	95.26%
IB1	93.90%	94.12%	88.17%	97.98%	93.59%
LibSVM	92.68%	90.59%	86.02%	96.97%	91.64%
Random Forest	89.02%	88.24%	82.80%	94.95%	88.85%
Component-coupled	89.02%	83.53%	78.49%	85.85%	84.12%
J48	76.83%	88.24%	69.89%	85.86%	80.22%
JRip	76.83%	74.12%	63.44%	77.78%	72.98%
RBF Network	67.07%	65.88%	53.76%	69.70%	64.06%
SMO	65.85%	69.41%	45.16%	70.71%	62.67%
Naïve Bayes	68.29%	67.06%	36.56%	73.74%	61.28%
Logistic	57.32%	65.88%	47.31%	53.54%	55.71%
Hamming Distance	57.32%	60.00%	33.33%	59.60%	52.37%
Euclidean Distance	62.20%	60.00%	34.41%	43.43%	41.22%

Table 4.10 LOO test results for 359 protein domains.

 LOO test results for 277 protein domains are given in Table 4.11. LibSVM method has the highest LOO test result for 277 protein domains with accuracy value of 84.48%. Ib1 has a very close accuracy value of 84.11% for 277 protein data set. Proposed threestage MILP based approach has the third highest LOO test result for 277 protein domains data set with accuracy value of 81.50%.

Methods	Class-based Accuracy	Overall			
	α	β	$\alpha + \beta$	α/β	Accuracy
LibSVM	82.86%	88.52%	75.83%	90.12%	84.48%
IB1	80.00%	88.52%	73.85%	92.59%	84.11%
MILP	87.14%	75.41%	72.31%	88.89%	81.50%
SVM	74.30%	82.00%	72.30%	87.70%	79.40%
Component-coupled	84.30%	82.00%	67.70%	81.50%	79.10%
Random Forest	75.71%	83.61%	70.77%	85.19%	79.06%
J48	77.14%	77.05%	64.62%	85.19%	76.53%
Neural Network	68.60%	85.20%	56.90%	86.40%	74.70%
RBF Network	77.14%	68.85%	53.85%	77.78%	70.03%
SMO	72.86%	75.41%	44.62%	77.78%	68.23%
JRip	64.29%	75.41%	55.38%	76.54%	68.23%
Naïve Bayes	74.29%	57.38%	47.69%	77.78%	65.34%
Logistic	71.43%	67.21%	44.62%	58.02%	60.28%
City-block Distance	72.90%	62.30%	43.10%	60.50%	59.90%
Euclidean Distance	71.40%	54.10%	41.50%	53.10%	55.20%

Table 4.11 LOO test results for 277 protein domains.

 Table 4.12 shows the LOO test results for 498 protein domains. The overall accuracy of the proposed MILP model on 498 protein domains is 92.97%. On the other hand, the best accuracy value is 93.20% received by SVM given in [110]. However, the accuracy value of MILP approach is closer to SVM accuracy value. Moreover, the accuracy values of LibSVM and SMO classifiers are 92.17% and 76.30%, respectively, which are lower with respect to SVM result given in [110]. As they did not give any

detailed information related to predicted results for 498 data sets, we could not investigate the results in deeper.

Methods	Class-based Accuracy	Overall			
	α	β	$\alpha + \beta$	α/β	Accuracy
SVM	88.80%	95.20%	91.50%	96.30%	93.20%
MILP	91.59%	94.44%	93.80%	91.91%	92.97%
IB1	89.72%	96.83%	88.37%	95.59%	92.77%
LibSVM	91.59%	94.44%	89.92%	92.65%	92.17%
Random Forest	89.72%	92.86%	89.92%	94.12%	91.76%
Component-coupled	93.50%	88.90%	84.50%	90.40%	89.20%
Neural Network	86.00%	96.00%	86.00%	88.20%	89.20%
JRip	87.85%	88.89%	83.72%	88.24%	87.14%
J48	84.11%	88.89%	86.82%	87.50%	86.94%
SMO	71.03%	71.43%	74.42%	86.76%	76.30%
Logistic	68.22%	79.70%	65.89%	82.35%	74.29%
RBF Network	68.22%	75.40%	68.22%	74.26%	71.68%
Naïve Bayes	76.64%	72.22%	55.81%	75.00%	69.67%
Euclidean Distance	73.80%	65.10%	56.60%	60.30%	63.50%
City-block Distance	64.50%	68.30%	50.40%	67.70%	62.70%

Table 4.12 LOO test results for 498 protein domains.

4.7 Results for 10-Fold Cross-validation Tests

For the 1189 and 25PDB data sets, there exists 10-fold cross validation results in literature. Therefore, we investigate the performance of these data sets by applying 10-fold cross-validation (10FCV). The 10FCV test results for 1189 protein domains are given in

Table 4.13. The overall accuracy of the proposed model on 1189 protein domains is 53.30% with the highest accuracy value. LibSVM and Logistic classifiers has second and third best results for 1189 data set. On the other hand, the IB1 classifier which gives generally better results for the above data sets has the worst accuracy value for 1189 data set. This is a surprising result.

Methods	Class-based Accuracy	Overall			
	α	β	$\alpha + \beta$	α/β	Accuracy
MILP	76.23%	59.86%	36.52%	44.31%	53.30%
LibSVM	47.09%	65.99%	12.03%	74.55%	52.84%
Logistic	51.57%	67.35%	15.35%	66.17%	52.29%
SMO	46.19%	63.61%	8.29%	75.15%	51.37%
RBF Network	45.74%	53.40%	24.07%	71.86%	51.01%
Naïve Bayes	45.74%	50.68%	14.11%	79.04%	50.27%
Random Forest	47.53%	58.50%	21.99%	48.50%	45.15%
JRip	25.56%	45.24%	1.66%	82.63%	43.04%
J48	41.26%	48.30%	24.48%	51.20%	42.49%
IB1	39.46%	46.60%	19.08%	54.79%	41.57%

Table 4.13 10FCV test results for 1189 protein domains.

10FCV test results for 25PDB protein domains are given in Table 4.14. The overall accuracy of the proposed model on 1189 protein domains is 51.82%. The highest accuracy value is achieved by LibSVM method with 52.54%. SMO classifier has a very close accuracy value to LibSVM. MILP approach has the third best accuracy value as Logistic classifier. On the other hand, the IB1 classifier which gives generally better results for the above data sets has the second worst accuracy value for 25PDB data set.

Methods		Class-based Accuracy								
	α	β	$\alpha + \beta$	α/β	Accuracy					
LibSVM	65.69%	59.37%	29.48%	56.36%	52.54%					
SMO	67.49%	63.66%	34.01%	40.17%	52.00%					
MILP	60.95%	56.43%	53.47%	36.73%	51.82%					
Logistic	66.82%	62.75%	34.24%	41.04%	51.82%					
RBF Network	57.11%	52.37%	29.93%	60.69%	49.43%					
Naïve Bayes	51.02%	45.82%	29.25%	69.36%	47.69%					
Random Forest	58.24%	52.60%	27.44%	36.42%	44.11%					
J48	49.21%	42.44%	31.29%	38.15%	40.40%					
IB1	40.18%	35.89%	27.44%	49.13%	37.53%					
JRip	42.89%	39.50%	2.49%	19.94%	26.59%					

Table 4.14 10FCV test results for 25PDB protein domains.

4.8 Statistical Analysis of the Results

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 In order to analyze the results in detail, sensitivity (SEN), specificity (SPE), MCC and S values of each of the protein data sets are calculated and examined (Table 4.15 - Table 4.24). The specificity values are always significantly greater compared to sensitivity. High average specificity means that the number of under predicted proteins is low. Thus, low accuracy is a result of relatively low sensitivity values. Moreover, as sensitivity values increases, the difference between sensitivity and specificity decreases. Therefore, observing high specificity values do not mean that the values of classification accuracy are good as expected.

MCC value gives the strength of relationship between the actual and predicted values. A perfect fit will give a MCC value of 1. Due to the low sensitivity for 138 Domains data set, MCC and S values are low for each of the classes. This means that the classifier could not effectively capture the characteristics of that class. For a perfect

prediction, S value should be equal to 1 and 0 for vice versa. On the other hand, when we observe the results of each data set in overall, each of the classes have higher and lower MCC and S values with respect to the remaining classes. Hence, we could not say that MILP based hyper-box enclosure approach performs rather purely for any of the classes. Depending on the data sets, proposed data classification approach works well for each of the classes.

Classifier	SEN	SPE			MCC		S			
			α	β	$\alpha + \beta$	α/β	α	β	$\alpha + \beta$	α/β
LibSVM	70.29%	89.79%	0.62	0.66	0.53	0.51	0.51	0.5	0.47	0.45
MILP	67.39%	88.59%	0.65	0.58	0.49	0.38	0.5	0.46	0.44	0.36
IB1	66.67%	89.35%	0.54	0.66	0.55	0.42	0.44	0.48	0.45	0.39
Component-coupled	63.77%	NA	NA.	NA	NA.	NA	NA	NA.	NA	NA.
SMO	60.87%	86.55%	0.48	0.64	0.34	0.31	0.4	0.44	0.33	0.32
J48	60.87%	86.38%	0.45	0.56	0.35	0.38	0.39	0.42	0.34	0.35
Random Forest	60.14%	86.01%	0.41	0.54	0.34	0.42	0.37	0.41	0.33	0.37
RBF Network	57.97%	85.54%	0.47	0.44	0.31	0.33	0.38	0.37	0.31	0.32
SVM	57.24%	NA.	NA.	NA.	NA	NA	NA	NA	NA.	NA
Naïve Bayes	53.62%	85.06%	0.44	0.41	0.23	0.24	0.35	0.34	0.26	0.26
Logistic	52.90%	84.21%	0.38	0.42	0.24	0.17	0.33	0.34	0.27	0.24
Hamming Distance	48.55%	NA	NA.	NA	NA	NA	NA	NA	NA.	NA
Euclidean Distance	46.38%	NA	NA	NA.	NA	NA	NA	NA.	NA	NA
JRip	44.20%	79.72%	0.22	0.41	0.03	-0.03	0.25	0.3	0.12	0.16

Table 4.15 Values of performance measures for the 138 protein domains.

Classifier	SEN	SPE	MCC				S			
			α	β	$\alpha + \beta$	α/β	α	β	$\alpha + \beta$	α/β
MILP	87.65%	95.87%	0.84	0.84	0.88	0.76	0.74	0.72	0.76	0.7
IB1	86.45%	95.47%	0.87	0.84	0.78	0.79	0.73	0.71	0.71	0.71
Random Forest	82.86%	94.11%	0.79	0.78	0.71	0.78	0.67	0.65	0.65	0.66
LibSVM	81.67%	93.86%	0.79	0.71	0.7	0.8	0.65	0.61	0.63	0.65
J48	72.51%	90.71%	0.7	0.6	0.55	0.61	0.54	0.5	0.49	0.51
Component-coupled	63.77%	NA	NA	NA	NA	NA.	NA.	NA	NA	NA
JRip	61.75%	86.54%	0.59	0.54	0.33	0.37	0.44	0.42	0.33	0.36
SMO	60.15%	86.54%	0.53	0.58	0.34	0.27	0.41	0.42	0.33	0.29
RBF Network	59.36%	86.08%	0.49	0.54	0.3	0.33	0.4	0.41	0.31	0.33
Naïve Bayes	58.96%	86.40%	0.56	0.46	0.3	0.35	0.41	0.38	0.3	0.33
SVM	57.24%	NA	NA	NA	NA	NA	NA	NA	NA	NA
Logistic	52.58%	84.07%	0.38	0.43	0.26	0.12	0.33	0.35	0.28	0.21
Hamming Distance	48.55%	NA	NA	NA	NA	NA	NA	NA	NA	NA
Euclidean Distance	46.38%	NA	NA	NA	NA	NA	NA	NA	NA	NA

Table 4.16 Values of performance measures for the 253 protein domains.

For 2438 Domain data set, there are 7 different classes. Similar to above observations, specificity values are higher than the sensitivity values (Table 4.21 & Table 4.22). Furthermore, MCC and S values of classes α, β, α+β and α/β are greater than the MCC and S values of classes μ , σ and ρ . As the number of proteins belongs to the classes α, β, α+β and α/β are higher, proposed approach grasped the characteristics of these classes well. On the other hand, instances in μ , σ and ρ classes are very low with respect to the other classes. Hence, MCC and S values of these classes are low.

Classifier	SEN	SPE	MCC				S			
			α	β	$\alpha + \beta$	α/β	α	β	$\alpha + \beta$	α/β
MILP	96.38%	98.77%	0.97	0.97	0.92	0.95	0.91	0.91	0.91	0.91
SVM	95.26%	NA	NA	NA	NA	NA	NA	NA	NA.	NA
IB1		93.59% 97.81%	0.93	0.94	0.84	0.95	0.84	0.85	0.83	0.86
LibSVM	91.64%	97.20%	0.88	0.9	0.84	0.92	0.8	0.8	0.8	0.83
Random Forest	88.85%	96.29%	0.82	0.87	0.81	0.89	0.74	0.75	0.74	0.78
Component-coupled	84.12%	NA.	NA	NA.	NA	NA	NA.	NA.	NA.	NA.
J48	80.22%	93.31%	0.73	0.78	0.65	0.75	0.61	0.63	0.58	0.63
JRip	72.98%	90.77%	0.69	0.64	0.55	0.61	0.54	0.52	0.48	0.52
RBF Network		64.06% 87.70% 0.58		0.52	0.38	0.47	0.44	0.42	0.36	0.41
SMO		62.67% 87.35%	0.6°	0.54	0.16	0.62	0.44	0.43	0.18	0.46
Naïve Bayes	61.28%	86.55%	0.58	0.52	0.27	0.41	0.43	0.41	0.29	0.37
Logistic		55.71% 84.87%	0.4	0.47	0.3	0.25	0.35	0.37	0.31	0.27
Hamming Distance	52.37%	NA.	NA.	NA.	NA	NA	NA	NA.	NA	NA.
Euclidean Distance	41.22%	NA	NA	NA	NA	NA	NA	NA	NA.	NA

Table 4.17 Values of performance measures for the 359 protein domains.

Classifier	SEN	SPE		MCC			S				
			α	β	$\alpha + \beta$	α/β	α	β	$\alpha + \beta$	α/β	
LibSVM		84.48% 94.72%	0.76	0.82	0.75	0.82	0.67	0.68	0.65	0.71	
IB1		84.11% 94.19%	0.8	0.82	0.79	0.75	0.67	0.68	0.66	0.68	
MILP		81.50% 93.74%	0.7	0.73	0.73	0.82	0.63	0.61	0.61	0.67	
SVM	79.40%	NA	NA	NA	NA	NA	NA	NA	NA	NA	
Component-coupled	79.10%	NA	NA	NA	NA	NA	NA	NA	NA	NA	
Random Forest		79.06% 92.91%	0.69	0.73	0.68	0.74	0.59	0.6	0.57	0.62	
J48		76.53% 92.18%	0.66	0.69	0.56	0.76	0.56	0.56	0.5	0.61	
NN	74.70%	NA	NA	NA	NA	NA	NA	NA	NA	NA	
RBF Network	70.03%	89.54%	0.64	0.6	0.48	0.57	0.51	0.48	0.43	0.49	
SMO	68.23%	88.74%	0.68	0.61	0.41	0.51	0.5	0.48	0.38	0.45	
JRip	68.23%	88.95%	0.52	0.63	0.49	0.55	0.45	0.49	0.42	0.47	
Naïve Bayes		65.34% 87.62%	0.63	0.51	0.39	0.48	0.47	0.42	0.37	0.42	
Logistic		60.28% 86.31%	0.55	0.51	0.3	0.34	0.42	0.4	0.31	0.33	
Hamming Distance	59.90%	NA	NA	NA	NA	NA	NA	NA	NA	NA.	
Euclidean Distance	55.20%	NA	NA	NA	NA	NA	NA	NA	NA	NA	

Table 4.18 Values of performance measures for the 277 protein domains.

Classifier	SEN	SPE		MCC			S				
			α	β	$\alpha + \beta$	α/β	α	β	$\alpha + \beta$	α/β	
SVM	93.20%	NA	NA	NA	NA	NA	NA	NA	NA	NA	
MILP	92.97%	97.65%	0.89	0.93	0.9	0.9	0.82	0.84	0.84	0.84	
IB1	92.77%	97.53%	0.89	0.93	0.89	0.9	0.81	0.84	0.83	0.84	
LibSVM	92.17%	97.40%	0.88	0.92	0.88	0.89	0.8	0.83	0.82	0.83	
Random Forest	91.76%	97.25%	0.87	0.9	0.87	0.91	0.79	0.81	0.81	0.82	
Component-coupled	89.20%	NA	NA.	NA	NA	NA	NA	NA	NA	NA	
NN	89.20%	NA	NA	NA	NA	NA	NA	NA.	NA	NA	
JRip	87.14%	95.63%	0.84	0.83	0.82	0.81	0.72	0.73	0.72	0.73	
J48	86.94%	95.62%	0.79	0.87	0.81	0.81	0.7	0.73	0.72	0.73	
SMO	76.30%	91.84%	0.65	0.68	0.72	0.65	0.54	0.56	0.57	0.56	
Logistic	74.29%	91.25%	0.6	0.73	0.59	0.64	0.51	0.56	0.51	0.54	
RBF Network	71.68%	90.30%	0.62	0.62	0.6	0.56	0.5	0.51	0.5	0.49	
Naïve Bayes	69.67%	89.74%	0.61	0.65	0.54	0.52	0.49	0.5	0.45	0.46	
Euclidean Distance	63.50%	NA	NA	NA	NA	NA	NA	NA	NA	NA	
Hamming Distance	62.70%	NA	NA	NA	NA	NA	NA	NA	NA	NA	

Table 4.19 Values of performance measures for the 498 protein domains.

Classifier	SEN	SPE			MCC		S				
			α	β	$\alpha + \beta$	α/β	α	β	$\alpha + \beta$	α/β	
MILP	95.88%	98.60%	0.93	0.94	0.95	0.95	0.89	0.9	0.9	0.9	
IB1	95.68%	98.52%	0.92	0.93	0.96	0.95	0.88	0.89	0.9	0.9	
SVM	94.90%	NA	NA.	NA.	NA	NA.	NA.	NA	NA.	NA	
Random Forest	92.94%	97.69%	0.89	0.93	0.87	0.92	0.82	0.84	0.83	0.84	
Component-coupled	86.47%	NA	NA	NA	NA	NA	NA	NA	NA	NA	
J48	75.29%	91.97%	0.63	0.6	0.81	0.61	0.53	0.51	0.6	0.54	
LibSVM	58.04%	86.02%	0.4	0.61	0.27	0.28	0.36	0.43	0.29	0.3	
RBF Network	56.27%	84.93%	0.4	0.33	0.25	0.46	0.35	0.32	0.28	0.37	
Logistic	55.88%	83.60%	0.39	0.54	0.07	0.34	0.34	0.39	0.15	0.32	
Naïve Bayes	54.50%	84.26%	0.42	0.39	0.1	0.42	0.34	0.34	0.2	0.35	
SMO	49.80%	82.24%	0.48	0.41	0.07	0.2	0.34	0.32	0.13	0.25	
JRip	47.64%	81.27%	0.32	0.4	0.18	0.14	0.27	0.32	0.18	0.22	
Euclidean Distance	47.25%	NA.	NA	NA	NA	NA	NA.	NA	NA	NA	
Hamming Distance	47.06%	NA.	NA	NA	NA	NA	NA	NA	NA	NA	

Table 4.20 Values of performance measures for the 510 protein domains.

Classifier	ACC	SPE				MCC			
	(SEN)		α	β	$\alpha + \beta$	α/β	ρ	σ	μ
MILP	95.08%	98.51%	0.91	0.94	0.94	0.96	0.9	0.93	0.82
IB1	95.03%	98.78%	0.94	0.95	0.96	0.9	0.78	0.95	0.85
SVM	94.50%	NA	NA	NA	NA	NA	NA	NA	NA
Random Forest	93.23%	98.11%	0.92	0.91	0.93	0.88	0.79	0.95	0.93
J48	86.54%	96.27%	0.79	0.84	0.83	0.82	0.57	0.9	0.8
LibSVM	84.58%	95.46%	0.78	0.86	0.77	0.75	0.86	0.97	$\boldsymbol{0}$
Component-coupled	77.03%	NA	NA	NA	NA	NA	NA	NA	NA
JRip	65.01%	87.34%	0.66	0.46	0.56	0.53	0.58	0.86	0.29
RBF Network	64.84%	90.49%	0.58	0.57	0.44	0.46	0.36	0.79	0.32
SMO	63.94%	88.99%	0.51	0.52	0.47	0.45	$\boldsymbol{0}$	0.79	$\boldsymbol{0}$
Logistic	63.04%	89.22%	0.47	0.55	0.42	0.45	$\boldsymbol{0}$	0.75	$\boldsymbol{0}$
Naïve Bayes	56.72%	88.79%	0.52	0.49	0.26	0.35	0.33	0.77	0.16
Euclidean Distance	47.74%	NA	NA	NA	NA	NA	NA	NA	NA
Hamming Distance	42.38%	NA	NA	NA	NA	NA	NA	NA	NA

Table 4.21 Values of performance measures-1 for the 2438 protein domains.

Classifier	ACC	SPE				S			
	(SEN)		α	β	$\alpha + \beta$	α/β	ρ	σ	μ
MILP	95.08%	98.51%	0.85	0.89	0.88	0.87	0.54	0.74	0.57
IB1	95.03%	98.78%	0.85	0.89	0.88	0.86	0.52	0.75	0.6
SVM	94.50%	NA	NA	NA	NA	NA	NA	NA	NA
Random Forest	93.23%	98.11%	0.8	0.85	0.84	0.82	0.51	0.71	0.57
J48	86.54%	96.27%	0.66	0.73	0.72	0.7	0.47	0.59	0.5
LibSVM	84.58%	95.46%	0.63	0.72	0.67	0.66	0.48	0.59	0.43
Component-coupled	77.03%	NA	NA	NA	NA	NA	NA	NA	NA
JRip	65.01%	87.34%	0.45	0.39	0.42	0.42	0.4	0.45	0.39
RBF Network	64.84%	90.49%	0.43	0.46	0.39	0.4	0.39	0.44	0.38
SMO	63.94%	88.99%	0.41	0.43	0.41	0.39	0.38	0.43	0.38
Logistic	63.04%	89.22%	0.39	0.44	0.38	0.39	0.38	0.43	0.37
Naïve Bayes	56.72%	88.79%	0.38	0.39	0.28	0.32	0.36	0.4	0.34
Euclidean Distance	47.74%	NA	NA	NA	NA	NA	NA	NA	NA
Hamming Distance	42.38%	NA	NA	NA	NA	NA	NA	NA	NA

Table 4.22 Values of performance measures-2 for the 2438 protein domains.

Classifier	ACC	SPE		MCC			S		
	(SEN)		α	$\alpha + \beta$	α/β	α	ß	$\alpha + \beta$	α/β
MILP		53.30% 84,91% 0.37 0.36 0.285 0.28 0.31 0.33 0.3							0.28
LibSVM		52.84% 81,90% 0.37 0.39 0.025 0.3 0.33 0.33 0.2							0.28
Logistic		52.28% 82,45% 0.36 0.39 0.045 0.27 0.32 0.33 0.21 0.27							
SMO		51.37% 81,30% 0.35 0.35 0.028 0.27 0.31 0.32 0.21 0.26							
RBF Network 51.00% 82,29% 0.31 0.31 0.07 0.31 0.3 0.3 0.21									0.29
Naïve Bayes		50.27% 81,26% 0.33 0.3 0.019 0.3 0.3 0.29 0.2 0.27							
Random Forest 45.14% 80,85% 0.24 0.23 -0.01 0.14 0.26 0.25 0.17 0.21									
JRip		43.04% 76.81% 0.18 0.23 -0.08 0.14 0.24 0.25 0.19 0.14							
J48		42.49% 80,22% 0.14 0.13 -0.02 0.14 0.22 0.21 0.17 0.21							
IB1		41.57% 79.49% 0.1 0.22 -0.07 0.07 0.21 0.24 0.15 0.16							

Table 4.23 Values of performance measures for the 1189 protein domains.

Table 4.24 Values of performance measures for the 25PDB protein domains.

Classifier	ACC	SPE			MCC			S		
	(SEN)		α	B	$\alpha + \beta$	α/β	α	ß	$\alpha + \beta$	α/β
LibSVM	52.54%	84.20%	0.42	0.35	0.082	0.32	0.34	0.32	0.19	0.31
SMO		52.00% 83.64%	0.42		0.35 0.096	0.25	0.34	0.31	0.19	0.28
MILP		51 82% 83 74%	0.39		0.29 0.322 0.13		0.33	0.29	0.31	0.21
Logistic		51.82% 83.54%	0.4	0.33	0.107	0.26	0.34	0.31	0.2	0.29
RBF Network		49.43% 83.30%			0.36 0.29 0.038	0.3	0.31	0.29	0.16	0.29
Naïve Bayes		47.69% 83.31% 0.32 0.26 0.054 0.27					0.29	0.27	0.18	0.26
Random Forest 44.11% 80.87% 0.22 0.18					-0	0.16	0.24	0.23	0.15	0 24
J48		40.40\% 79.75\%	0.15	0.07	-0.04	0.12	0.21	0.18	0.12	0.21
IB1	37.53%	79.79%	0.11	0.07	-0.08	0.04	0.2	0.18	0.11	0.16
JRip		26.59% 74.05% -0.26		-0.3	-0.1	0.03	-0	-0.01	0.14	0.16

In order to evaluate if there is any statistical significant difference between the existing and proposed data classification approaches tested on the same data sets, P-value (paired test) analysis are carried out. The results of P-value test results are given in Table 4.25 and in Table 4.26.

	138	253	359	277	498	510	2438	1189	25PDB
Compared Methods	P	P	P	P	P	P	P	P	P
	Value								
MILP vs HD	2.91	9.44	13.5	5.58	10.77	16.15	39.23	NA	NA
MILP vs ED	3.27	9.89	15.95	6.65	10.53	16.1	36.09	NA	NA
MILP vs CC	0.37	6.29	5.53	0.71	1.17	3.63	17.57	NA	NA
MILP vs SVM	1.48	1.39	0.74	0.62	1.06	1.06	0.12	NA	NA
MILP vs LibSVM	0.52	1.87	2.68	0.93	0.48	14.35	12.13	0.22	0.42
MILP vs SMO	1.13	7.04	11.19	4.09	7.29	16.54	26.94	0.9	0.1
MILP vs NN	NA	NA	NA	1.93	1.17	NA	NA	NA	NA
MILP vs IB1	0.13	0.4	1.71	0.81	0.12	0.16	0.08	5.49	8.31
MILP vs J48	1.13	4.26	6.74	1.63	3.16	9.36	10.32	5.06	6.63
MILP vs Random Forest	1.25	1.52	3.86	0.82	0.72	2.04	2.75	3.81	4.64
MILP vs RBF Network	1.62	7.21	10.87	3.58	8.81	14.83	26.38	1.08	1.38
MILP vs JRip	3.88	6.7	8.7	4.09	3.07	17.11	26.27	4.79	14.95
MILP vs NaiveBayes	2.34	7.29	11.51	4.9	9.43	15.3	31.31	1.42	2.39
MILP vs Logistic	2.46	8.62	12.77	6.25	7.29	14.93	27.49	0.48	$\boldsymbol{0}$

Table 4.25 The results of P-value analyses.

The accuracy values of MILP approach on each of the data sets is statistically significant than the accuracies of the distance based algorithms HD and ED given in [97] and [100]. Since there are not any existing literature results of these methods for 1189 and 25PDB data sets, P-value analysis for these data sets are not available. On the other hand, there is no statistical difference between the CC algorithm and MILP approach for 138, 277

and 498 data sets. However, MILP approach is statistically significant than CC algorithm for the data sets 253, 359, 510 and 2438. There is no statistically significant difference between the accuracy values of SVM given in [109] and [110] and proposed MILP approach. However, the results given in [110] are not consistent with the results achieved by LibSVM and SMO.

	138	253	359	277	498	510	2438	1189	25PDB
Compared Methods	P	\mathbf{P}	\mathbf{P}	P	\mathbf{P}	P	\mathbf{P}	P	P
	Test	Test	Test	Test	Test	Test	Test	Test	Test
				Result Result Result Result Result Result Result Result					Result
MILP vs HD	$++$	$++$	$++$	$++$	$++$	$++$	$++$	NA	NA
MILP vs ED	$++$	$++$	$++$	$++$	$++$	$++$	$++$	NA	NA
MILP vs CC	$=$ $=$	$++$	$++$	$=$ $=$	$=$ $=$	$++$	$++$	NA	NA
MILP vs SVM	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	NA	NA
MILP vs LibSVM	$=$ $=$	$=$ $=$	$++$	$=$ $=$	$=$ $=$	$++$	$++$	$=$ $=$	$=$ $=$
MILP vs SMO	$=$ $=$	$++$	$++$	$++$	$++$	$++$	$++$	$=$ $=$	$=$ $=$
MILP vs NN	NA	NA	NA	$=$ $=$	$=$ $=$	NA.	NA	NA	NA
MILP vs IB1	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	$++$	$++$
MILP vs J48	$=$ $=$	$++$	$++$	$=$ $=$	$++$	$++$	$++$	$++$	$++$
MILP vs Random Forest	$=$ $=$	$=$ $=$	$++$	$=$ $=$	$=$ $=$	$++$	$++$	$++$	$++$
MILP vs RBF Network	$=$ $=$	$++$	$++$	$++$	$++$	$++$	$++$	$=$ $=$	$=$ $=$
MILP vs JRip	$++$	$++$	$++$	$++$	$++$	$++$	$++$	$++$	$++$
MILP vs NaiveBayes	$++$	$++$	$++$	$++$	$++$	$++$	$++$	$=$ $=$	$++$
MILP vs Logistic	$++$	$++$	$++$	$++$	$++$	$++$	$++$	$++$	$=$ $=$

Table 4.26 The results of P-test.

 $++$ denotes that the first method is statistically significantly better than the second method. - - represents that the second method is statistically significantly better than the first method. $=$ = indicates that there is no significant difference between the results of the methods. HD: Hamming Distance. ED: Euclidean Distance. CC: Component-coupled. SVM: Support Vector Machines. NN: Neural Networks.

MILP approach is statistically significantly better than the support vector machine algorithms implemented in LibSVM and WEKA for some of the data sets. Similarly, there is no statistically significant difference between the LOO results of Neural Network given in [104] and MILP approach on 277 and 498 data sets. On the other hand, MILP approach is statistically significant than the Neural Network classifier found in WEKA (RBF Network) for most of the data sets. There is no statistically significant difference between the results of IB1 classifier and MILP approach for each of the data sets except 1189 and 25PDB. Surprisingly, IB1 has worse accuracy value with respect to MILP approach for data sets 1189 and 25PDB. Finally, MILP approach has statistically significant accuracy values for the methods J48, Random Forest (RF), JRip, Naïve Bayes (NB) and Logistic for most of the data sets.

In order to compare the existing data classification methods with MILP, some of the ordered P-value graphs are shown in Figure 4.1 to Figure 4.6. In Figure 4.1, the ordered P-values of MILP versus LibSVM for each of the nine data sets are shown. For three data sets, the P-values are greater than 2 and very close to 15 which is a considerably high Pvalue. In general, MILP is preferable since it performs quite well for each of the existing benchmark data sets. However, LibSVM method performs poorly with respect to MILP approach for 3 of the data sets. Hence, we could say that MILP approach is significantly better than LibSVM method in general. We could come up with the same conclusion for IB1 and MILP methods (Figure 4.2). In a similar way, IB1 method performs worse for two of the data sets despite its high efficiency for the rest of the data sets. Thus, MILP approach is statistically better than IB1 method in general. MILP approach is statistically significant than SMO, Logistic and RBF Network algorithms found in WEKA in most of the data sets (Figure 4.3, 4.4 and 4.5). Moreover, the highest P-value for these methods is close to 30 which mean that the difference between the performances of the methods and MILP is highly significant. Finally, proposed MILP approach is statistically significantly

better than Random Forest algorithm for half of the data sets (Figure 4.6). For the rest, the difference between the accuracies of two methods is not significant. Moreover, P-values are not very high for Random Forest algorithm compared to the rest of the listed methods in Figure 2. For each of the existing protein folding type benchmark data sets, MILP approach generally achieves high accuracy values and mostly ranks in first three positions with respect to accuracy. Furthermore, MILP is statistically significantly better than the existing data classification methods for protein folding type prediction problems on given nine distinct benchmark data sets.

Figure 4.1 P-value graph of MILP versus LibSVM.

Figure 4.2 P-value graph of MILP versus IB1.

Figure 4.3 P-value graph of MILP versus SMO.

Figure 4.4 P-value graph of MILP versus Logistic.

Figure 4.5 P-value graph of MILP versus RBF Network.

Figure 4.6 P-value graph of MILP versus Random Forest.

4.9 Problematic Instance Analysis

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 In order to analyze whether there exists any relation between the performance of the proposed approach and the number of problematic instances in the data sets, we investigate the results of these data sets in detail. In Table 4.27, the number of problematic instances is given by average, maximum and minimum values for each of the protein folding type benchmark data sets. As 225 and 1601 data sets are used for training sets for the test sets 510 and 2438 data sets, they do not have any maximum and minimum number of problematic instances. On the other hand, since for the data sets 138, 253, 359, 277 and 498 LOO tests are carried, their problematic instance analyses are comprehensive (Figure 4.7 and 4.11). Furthermore, the number of problematic instances for 1189 and 25PDB data sets change from one run to another as 10FCV results are obtained for them (Figure 4.12 – Figure 4.13).

 As it can be observed from the Table 4.27, the number of problematic instances does not affect the performance of the proposed approach. For the same percentage of problematic instances as in 359 and 498 data sets, the proposed approach could achieve the best and second best results for 359 and 498 data sets, respectively. Moreover, for the data sets that have high percentage of problematic instances as 1601 data set, proposed approach could very high accuracy value, 95.88%. Hence, considering only the number of problematic instances could not be sufficient to analyze the difficulty of the data sets.

Data Accuracy	Accuracy	$%$ of Av.	Number of Problematic				
Set	(%)	Rank	Problematic		Instances		
Name			Instances	Average	Max.	Min.	
138	67.39%	$\overline{2}$	53%	73	74	69	
253	87.65%	1	65%	164	165	157	
359	96.38%	1	65%	233	234	228	
277	81.50%	3	60%	167	168	161	
498	92.97%	$\overline{2}$	65%	322	323	312	
225	95.88%	1	78%	179	N/A	N/A	
1601	95.08%	1	97%	1554	N/A	N/A	
1189	53.30%	1	88%	959	962	956	
25PDB	51.82%	3	89%	1486	1489	1482	

Table 4.27 Number of problematic instances for each of the protein folding type data sets.

Figure 4.7 The number of problematic instances for 138 data set.

Figure 4.8 The number of problematic instances for 253 data set.

Figure 4.9 The number of problematic instances for 359 data set.

Figure 4.10 The number of problematic instances for 277 data set.

Figure 4.11 The number of problematic instances for 498 data set.

Figure 4.12 The number of problematic instances for 1189 data set.

Figure 4.13 The number of problematic instances for 25PDB data set.

Chapter 5

COMPUTATIONAL RESULTS ON UCI REPOSITORY DATA SETS

The performance of proposed three-stage approach is evaluated on eleven UCI repository benchmark data sets [108]. The prediction results and comparisons with other data classification methods are examined in this chapter.

5.1 UCI Repository Data Sets

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The UCI Machine Learning Repository is a collection of databases, domain theories, and data generators that are used by the machine learning community for the empirical analysis of machine learning algorithms. The archive was created in 1987 by David Aha and fellow graduate students at UC Irvine. Since that time, it has been widely used by students, educators, and researchers all over the world as a primary source of machine learning data sets [108].

In order to observe the performance of the proposed MILP based hyper-box enclosure approach, the following eleven data sets from [108] are tested. First five of them are binary-class data classification data sets and the rest are multi-class data sets.

5.1.1 Johns Hopkins University Ionosphere Database

This database contains the radar data collected by a system in Goose Bay, Labrador. This system consists of a phased array of 16 high-frequency antennas with a total transmitted power on the order of 6.4 kilowatts. Free electrons in the ionosphere are the targets of this study. "Good" radar returns are those showing evidence of some type of

structure in the ionosphere. On the other hand, "Bad" radar returns are those that do not show any evidence and their signals pass through the ionosphere.

 Received signals were processed using an autocorrelation function that depends on the time of the pulse and the pulse number. There were 17 pulse numbers for the Goose Bay system. Instances in this database are described by 2 attributes per pulse number, corresponding to the complex values returned by the function resulting from the complex electromagnetic signal. The overall characteristics of the database are given in Table 5.1. This data set is referred as "Ionosphere".

Data Set Name	# of Attributes	# of Classes	# of Instances	# of Instances in Class 1	# of Instances in Class 2
Ionosphere	34	$\overline{2}$	351	225	126
Pima	8	2	768	500	268
Blood	4	2	748	570	178
WDBC	9	2	683	444	239
Liver	6	2	345	200	145

Table 5.1 Binary-class UCI Repository data sets and their characteristics.

5.1.2 Pima Indians Diabetes Database

 This database consists of female patients at least 21 years old who have Pima Indian Heritage. The given 8 properties related to the patients are used to test the diabetes for each one of them. The overall characteristics of the database are given in Table 5.1. This data set is referred as "Pima".

5.1.3 Blood Transfusion Service Center Data Set

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 This data set is taken from the donor database of Blood Transfusion Service Center in Hsin-Chu City in Taiwan. 748 donors are randomly selected from the databases with information related to the months since last donation, months since first donation, total blood donated in c.c., total number of donation. The class variable represents whether she/he donated blood in March 2007. The overall characteristics of the data set are given in Table 5.1. This data set is referred as "Blood".

5.1.4 Wisconsin Diagnostic Breast Cancer (WDBC)

 This breast cancer database was obtained from the University of Wisconsin Hospitals, Madison from Dr. William H. Wolberg. Using the 9 different information related to the patients, one is trying to find out whether the patient has a breast cancer or not. The overall characteristics of the database are given in Table 5.1. This data set is referred as "WDBC".

5.1.5 Liver Disorders Data Set

 This data set is consists of the records of male individuals with 5 blood test values which are thought to be sensitive to liver disorders that might arise from excessive alcohol consumption. Moreover, each individual have an attribute value related to the number of half-pint equivalents of alcoholic beverages drunk per day. The class variable represents whether he has a liver disorder or not. The overall characteristics of the data set are given in Table 5.1. This data set is referred as "Liver".

5.1.6 Wine Recognition Data

 These data are the results of a chemical analysis of wines grown in the same region in Italy but derived from three different cultivars. The analysis determined the quantities of

13 constituents found in each of the three types of wines. The overall characteristics of the data set are given in Table 5.2. This data set is referred as "Wine".

Data Set	#	#		$#$ and $#$					# of Instances in each class			
	Att.	Classes	Ins. C1		C2	C3 C4 C5 C6 C7				C8	C9	C10
Wine	13	3	178	59	71	48	$---$					
Iris	4	3	150	50	50	50	$---$					
Thyroid	5	3	215 150		35	30	$---$					
Glass	9	6	214	70	76	-17	-13	9 29				
Ecoli		8	336	143	77		2 2 35 20			$5 \t 52 \t ---$		
Yeast	8	10	1484 244 429 463					44 35 51 163		30	- 20	

Table 5.2 Multi-class UCI Repository data sets and their characteristics.

5.1.7 Iris Data Set

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 Iris data is the best known data set to be found in the pattern recognition literature. The sepal length, sepal width, petal length, and petal width are measured in centimeters on 50 iris specimens from each of three species, Iris setosa, I. versicolor, and I. virginica. The overall characteristics of the data set are given in Table 5.2. This data set is referred as "Iris".

5.1.8 Thyroid Gland Data

 This data set composed of five laboratory tests of patients to predict whether a patient's thyroid to the class euthyroidism, hypothyroidism or hyperthyroidism. The diagnosis (the class label) was based on a complete medical record, including anamnesis, scan, etc. The overall characteristics of the data set are given in Table 5.2. This data set is referred as "Thyroid".

5.1.9 Glass Identification Database

 This database is composed of 6 different types of glasses with having some chemical properties to differentiate. The aim is to classify the glasses using the 9 characteristics of instances exist in the data set. The overall characteristics of the data set are given in Table 5.2. This data set is referred as "Glass".

5.1.10 Ecoli Data Set

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 This data set is composed of proteins with 7 different score values and a localization site. There are 8 different sites that proteins are localized. The overall characteristics of the data set are given in Table 5.2. This data set is referred as "Ecoli".

5.1.11 Yeast Data Set

 This data set is also composed of proteins with 8 different score values and a cellular localization site. There are 10 different cellular sites that proteins are localized. The overall characteristics of the data set are given in Table 5.2. This data set is referred as "Yeast".

5.2 Classification Algorithms

In order to compare the results of proposed MILP approach, WEKA classification algorithms J48, RBF Network, Logistic, Naïve Bayes (NB), SMO, Random Forest (RF) and IB1 are studied (Table 5.3). Optimized parameter values of these WEKA classifiers are determined and used to perform the studies on the given data sets. Moreover, wellknown support vector machine implementation LibSVM given by [111] is also studied to observe the accuracy values. For each of the data sets, parameters related to SVM algorithm are optimized by performing 10FCV validation with different combinations of cost and gamma values. The optimal values that achieve the highest 10FCV accuracy are used to obtain the results for each data set (Table 5.4).

Data Sets	Kernel Type	c (Cost)	g (Gamma)
Ionosphere	Radial Basis Function	8192	0.5
Pima	Radial Basis Function	8	0.00003
Blood	Radial Basis Function	2048	0.00012
WDBC	Radial Basis Function	32768	0.00003
Liver	Radial Basis Function	2	0.00012
Wine	Radial Basis Function	8192	0.00003
Iris	Radial Basis Function	2	0.125
Thyroid	Radial Basis Function	512	0.00003
Glass	Radial Basis Function	32768	0.03125
Ecoli	Radial Basis Function	0.5	8
Yeast	Radial Basis Function	0.5	8

Table 5.4 Optimal parameter values of LibSVM for each of the data sets.

5.3 10-Fold Cross-validation Results for Binary-Class Data Sets

For 10-fold cross-validation tests (10FCV), five binary-class data sets including Ionosphere, Pima, Blood, WDBC, and Liver are used. Using these data sets, the proposed three-stage MILP model is solved in GAMS [120] using ILOG CPLEX Solver version 10.0 [121] on a notebook computer with Inter Pentium M 1.73 Ghz Processor and 512 MB of RAM. For each data set, 10 different runs are carried out and average 10FCV results are given.

10FCV test results for Ionosphere data set are given in Table 5.5. The overall accuracy of the proposed model on Ionosphere data set is 94.59%. MILP approach has the second best accuracy value. The highest accuracy value is achieved by LibSVM method with 94.87%. Random Forest classifier has a very close accuracy value to LibSVM and

MILP with 93.45%. On the other hand, the Naïve Bayes classifier has the worst accuracy value for Ionosphere data set. The rest of the classifiers have moderate accuracy values.

Methods		Class-based Accuracy					
	C1	C2	Accuracy				
LibSVM	95.56%	93.65%	94.87%				
MILP	97.33%	89.68%	94.59%				
Random Forest	96.44%	88.10%	93.45%				
RBF Network	93.33%	90.48%	92.31%				
J48	96.44%	82.54%	9145%				
JRip	91.56%	86.51%	89.74%				
Logistic	94.22%	79.37%	88.89%				
SMO	96.89%	73.81%	88.60%				
IB1	96.89%	67.46%	86.32%				
Naïve Bayes	80.44%	86.51%	82.62%				

Table 5.5 10FCV results for Ionosphere data set.

10FCV test results for Pima data set are given in Table 5.6. The highest accuracy value is achieved by proposed MILP approach with 81.25%. SMO and Logistic classifiers are the ones that have the closest accuracy value to the MILP approach's accuracy. As expected, LibSVM has also high accuracy with respect to other classifiers with 76.43%. On the other hand, decision tree based classifiers J48 and Random Forest has low accuracy values compared to the MILP approach. Furthermore, the IB1 classifier has the worst accuracy value for Pima data set.

Table 5.7 gives the 10FCV test results for Blood data set. The highest accuracy value is achieved by proposed MILP approach with 79.95%. The neural network based

classifier RBF Network has a very close accuracy value to the MILP. Rule based classifier JRip and decision tree based classifier J48 have also high accuracy value for Blood data set. Support vector machine based classifier LibSVM and SMO have relatively low classification accuracies. The nearest neighborhood based classifier IB1 has the lowest accuracy value, 68.58%, for Blood data set.

Methods	Class-based Accuracy	Overall	
	C1	C ₂	Accuracy
MILP	62.69%	91.20%	81.25%
SMO	54.10%	89 80%	77.34%
Logistic	57.09%	88.00%	77.21%
LibSVM	52.24%	89.40%	76.43%
Naïve Bayes	61.19%	77 80%	76.30%
RBF Network	54.10%	86.80%	75.39%
JRip	5746 ^o	84.20%	74.87%
J48	59 70%	81.40%	73.83%
Random Forest	61.19%	77.80%	72.01%
IB1	52.99%	79 40%	70.18%

Table 5.6 10FCV results for Pima data set.

10FCV test results for WDBC data set are given in Table 5.8. The highest accuracy value is achieved by proposed MILP approach with 97.36%. SMO, LibSVM and Logistic classifiers are the ones that have the closest accuracy value to the MILP approach's accuracy. Naïve Bayes, Random Forest, JRip, IB1 and J48 have moderate accuracy values approximately 96%. Furthermore, neural network based classifier RBF Network has the worst accuracy value for WDBC data set, 95.75%.

Methods	Class-based Accuracy	Overall	
	C1	C ₂	Accuracy
MILP	42.13%	91.75%	79.95%
RBF Network	25.84%	96.49%	79.68%
JRip	41.57%	90.53%	78.88%
J48	43 26%	88.60%	77.81%
LibSVM	34 83%	91.23%	77 81%
Logistic	12.36%	97.37%	77.14%
SMO	0.00%	100.00%	76.20%
Naïve Bayes	20.22%	92.63%	75.40%
Random Forest	32.58%	84.74%	72.33%
IB1	37.08%	78.42%	68.58%

Table 5.7 10FCV results for Blood data set.

Table 5.8 10FCV results for WDBC data set.

Methods	Class-based Accuracy	Overall	
	C1	C ₂	Accuracy
MILP	98.87%	94.56%	97.36%
SMO	97.30%	96.65%	97.07%
Logistic	97.75%	94.98%	96.78%
LibSVM	97.52%	95.40%	96.78%
Naïve Bayes	95.72%	97.49%	96.34%
Random Forest	97.52%	93.72%	96.19%
JRip	$96\,40\%$	95.82%	96.19%
IB1	97.52%	93.31%	96.05%
J48	96.40%	95.40%	96.05%
RBF Network	95.72%	95.82%	95.75%

Methods	Class-based Accuracy	Overall	
	C1	C ₂	Accuracy
LibSVM	64.83%	81%	74.20%
MILP	65.52%	79%	73.33%
J48	53.10%	80%	68.70%
Logistic	53.10%	79%	68.12%
Random Forest	62.76%	69%	66.38%
JRip	46.90%	77%	64.35%
RBF Network	51.72%	73.50%	64.35%
IB1	56.55%	67.50%	62.90%
SMO	0.69%	100%	58.26%
Naïve Bayes	76.55%	40%	55.36%

Table 5.9 10FCV results for Liver data set.

10FCV test results for Liver data set are given in Table 5.9. The overall accuracy of the proposed model on Liver data set is 73.33%. MILP approach has the second best accuracy value. The highest accuracy value is achieved by LibSVM method with 74.20%. J48 and Logistic classifiers have a closer accuracy values with 68.70% and 68.12%, respectively. On the other hand, the famous probabilistic classifier Naïve Bayes has the worst accuracy value for Liver data set. The rest of the classifiers have moderate accuracy values ranging from 58% to 66%.

5.4 10-Fold Cross-validation Results for Multi-Class Data Sets

For 10-fold cross-validation tests (10FCV) of multi-class problems, six data sets including Wine, Iris, Thyroid, Glass, Ecoli and Yeast are used. Using these data sets, the proposed three-stage MILP model is solved in GAMS [120] using ILOG CPLEX Solver version 10.0 [121] on a notebook computer with Inter Pentium M 1.73 Ghz Processor and

512 MB of RAM. Similar to two-class case, 10 different runs are carried out and average 10FCV results are given for each of the data sets.

Methods	Class-based Accuracy	Overall		
	C ₂ $\bf C1$		C ₃	Accuracy
SMO	100%	95.77%	100%	98.31%
Random Forest	100%	95.77%	100%	98.31%
RBF Network	96.61%	100%	97.92%	98.31%
Logistic	98.31%	95.77%	97.92%	97.19%
Naïve Bayes	94.92%	95.77%	100%	96.63%
MILP	94.92%	95.77%	93.75%	94.94%
IB ₁	100%	87.32%	100%	94.94%
JRip	91.53%	94.37%	95.83%	93.82%
J48	98.31%	94.37%	87.50%	93.82%
LibSVM	94.92%	90.14%	91.67%	92.13%

Table 5.10 10FCV results for Wine data set.

10FCV test results for Wine data set are given in Table 5.10. The overall accuracy of the proposed model on Ionosphere data set is 94.94%. MILP approach has the fourth best accuracy value as distance based classifier IB1. The highest accuracy value is achieved by SMO, Random Forest and RBF Network methods with 98.31%. Logistic and Naïve Bayes classifiers have also higher accuracy values than MILP approach with 97.19% and 96.63%, respectively. On the other hand, the famous decision tree classifier J48 and rule based classifier have the same accuracy value, 93.83%, for Wine data set. Surprisingly, LibSVM has the worst accuracy value, 92.13%, for this data set.

10FCV test results for Iris data set are given in Table 5.11. The overall accuracy of the proposed model on Iris data set is 96%. MILP approach has the second best accuracy value. The highest accuracy value is achieved by LibSVM method with 98%. Logistic, J48, Naïve Bayes and SMO classifiers have the same accuracy value with the MILP approach. On the other hand, Random Forest, IB1 and RBF Network have equal accuracy

value, 95.33%, for Iris data set. The lowest accuracy value, 94%, is achieved by the rule based classifier JRip.

Methods	Class-based Accuracy	Overall		
	C1	C ₂	C ₃	Accuracy
LibSVM	100%	96%	98%	98%
MILP	100%	94%	94%	96%
Logistic	100%	92%	96%	96%
J48	98%	94%	96%	96%
Naïve Bayes	100%	96%	92%	96%
SMO	100%	98%	90%	96%
Random Forest	100%	96%	90%	95.33%
IB1	100%	94%	92%	95.33%
RBF Network	100%	92%	94%	95.33%
JRip	100%	90%	92%	94%

Table 5.11 10FCV results for Iris data set.

Table 5.12 gives the 10FCV test results for Thyroid data set. The highest accuracy value is achieved by proposed MILP approach with 97.21%. The nearest neighborhood based classifier IB1 has the same accuracy value with MILP approach. The famous probabilistic classifier Naïve Bayes and Logistic classifier has the second best results with the accuracy value of 96.74%. RBF Network has 95.35% accuracy and stand at the third order. Random forest and LibSVM has the same accuracy values, 93.95%. The support vector machine based classifier SMO has the lowest accuracy value, 89.77%, for Thyroid data set.

Methods	Class-based Accuracy	Overall		
	C1	C2	C ₃	Accuracy
MILP		98.67% 91.43% 93.33%		97.21%
IB1		99.33% 94.29% 86.67%		97.21%
Naïve Bayes		97.33% 94.29% 86.67%		96 74%
Logistic		100\% 57.14\% 76.67\%		96.74%
RBF Network		98% 97.14% 93.33%		95.35%
Random Forest 94.67% 88.57% 83.33%				93.95%
LibSVM		97.33% 85.71% 86.67%		93.95%
JRip		94.67% 85.71% 93.33%		93.02%
J48		99.33% 82.86% 80.00%		92.09%
SMO		98% 94.29% 96.67%		89.77%

Table 5.12 10FCV results for Thyroid data set.

10FCV test results for Glass data set are given in Table 5.13. The overall accuracy of the proposed model on Iris data set is 76.17%. MILP approach has the second best accuracy value. The highest accuracy value is achieved by Random Forest classifier with 77.57%. LibSVM and IB1 have accuracy value greater than 70% and are following the MILP approach. On the other hand, JRip, Logistic, J48 and RBF Network have some how closer accuracies to each other for Glass data set. The lowest accuracy value, 49.53%, is achieved by the probabilistic classifier Naïve Bayes.

Methods		Overall					
	C ₁	C ₂	C ₃	C ₄	C ₅	C6	Accuracy
Random Forest 82.86% 78.95% 29.41% 76.92% 88.89% 86.21%							77.57%
MILP			75.71% 78.95% 47.06% 76.92% 77.78% 86.21%				76.17%
LibSVM			70\% 78.95\% 17.65\% 76.92\% 66.67\% 86.21\%				71.50%
IB1			77.14% 67.11% 35.29% 76.92% 66.67% 82.76%				70.56%
JRip		61.43% 76.32%		5.88% 69.23% 77.78% 82.76%			66.36%
Logistic		67.14% 67.11%		5.88% 76.92% 88.89% 86.21%			66.36%
J48			71.43% 56.58% 29.41% 84.62% 88.89% 82.76%				65.89%
RBF Network		72.86% 63.16%	11.76% 53.85% 77.78% 89.66%				65.89%
SMO		44.29% 85.53%		0.00% 15.38%		0.00% 86.21%	57.48%
Naïve Bayes			71.43% 19.74% 35.29% 23.08% 88.89% 82.76%				49.53%

Table 5.13 10FCV results for Glass data set.

10FCV test results for Ecoli data set are given in Table 5.14. The overall accuracy of the proposed model on Iris data set is 86.61%. MILP approach has the second best accuracy value. The highest accuracy value is achieved by LibSVM classifier with 87.50%. Logistic classifier has a very close accuracy value to the MILP approach with 86.31%. As the number of instances in the classes C3 and C4 are very low (Table 5.2), the class-based accuracy values for these classes are 0 for each of the methods. As two instances are not sufficient to capture the class characteristic for this large Ecoli data set, these results are not surprising. On the other hand, Naïve Bayes, RBF Network, J48, SMO, and Random Forest have some how closer and moderate accuracies to each other for Ecoli data set. The lowest accuracy value, 80.36%, is achieved by the instance based classifier IB1 and the rule based classifier JRip.

Methods		Overall						
	C1	C ₂	$C3$ $C4$	C ₅	C6	C ₇	C8	Accuracy
LibSVM				98.60% 84.42% 0% 0% 62.86% 80%			80\% 88.46\%	87.50%
MILP		97.90% 83.12% 0% 0%			60% 80\%		80\% 88.46\%	86.61%
Logistic		96.50% 84.42% 0% 0%			60% 80\%		100% 86.54%	86.31%
Naïve Bayes				95.80% 72.73% 0% 0% 82.86% 90%			60\% 84.62\%	85.42%
RBF Network				96.50% 80.52% 0% 0% 54.29% 75%			80\% 88.46\%	84.52%
J48				95.10\% 84.42\% 0\% 0\% 60.00\% 70\%			60% 84 62\%	84.23%
SMO				98.60% 83.12% 0% 0% 25.71% 75% 100% 90.38%				83.63%
Random Forest				95.80% 81.82% 0% 0% 45.71% 90%			60\% 84.62\%	83.63%
IB1				93.01% 72.73% 0% 0% 48.57% 75% 100% 84.62%				80.36%
JRip				95.80% 75.32% 0% 0% 51.43% 75%			20\% 78.85\%	80.36%

Table 5.14 10FCV results for Ecoli data set.

Table 5.15 10FCV results for Yeast data set.

Methods	Class-based Accuracy									Overall	
	C1	C ₂	C ₃	C ₄	C5	C6	- C7	C8	C9	C10	Accuracy
MILP							57% 50% 76% 89% 49% 45% 84% 10% 45%			40%	63%
LibSVM		55\% 50\% 74\% 82\% 49\% 37\% 78\%							3% 35%	20%	60%
RBF Network		56\% 53\% 64\% 80\% 63\% 29\% 80\%							0% 45%	100%	59%
Random Forest 61% 57% 60% 75% 49% 35% 80%									0% 25%	20%	59%
Logistic		57\% 46\% 70\% 64\% 49\% 37\% 81\%							0% 45%	80%	59%
JRip		50\% 53\% 64\% 73\% 49\% 35\% 80\%							0% 55%	80%	58%
Naïve Bayes		61% 40\% 70\% 61\% 69\% 39\% 80\%							0% 45%	40%	58%
SMO		56\% 35\% 78\% 80\% 29\% 20\% 78\%							0% 55%	60%	57%
J48		54\% 51\% 56\% 82\% 43\% 43\% 83\%							0% 25%	80%	56%
IB1		48\% 48\% 56\% 68\% 49\% 33\% 68\%							7% 45%	100%	52%

Table 5.15 gives the 10FCV test results for Yeast data set. The highest accuracy value is achieved by proposed MILP approach with 63%. The support vector machine based classifier LibSVM has the closest accuracy value to MILP approach with 60%. RBF Network, Random forest and Logistic classifiers achieved the third best results, 59%, for Yeast data set. The JRip, Naïve Bayes, SMO and J48 have moderate results with 58%, 58%, 57%, and 56%, respectively. The instance based classifier IB1 has the lowest accuracy value, 52%, for Yeast data set.

5.5 Statistical Analysis of the Results

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In order to analyze the results in detail, average sensitivity (SEN), average specificity (SPE), MCC and S values of each of the protein data sets are calculated and examined (Table 5.16 - Table 5.28). The average sensitivity values are same as the overall accuracy values. Hence, each of the tables are arranged so as to show the ordering of the methods based on sensitivity values. The average specificity values are generally significantly lower compared to average sensitivity values for the two-class data sets (Table 5.16 – Table 5.20). On the other hand, this observation is not valid for multi-class problems. For six multi-class benchmark data sets, the average specificity values are significantly higher than average sensitivity values (Table 5.17 – Table 5.28). High average specificity means that the number of under predicted proteins is low. Thus, low accuracy is a result of relatively low sensitivity values. Moreover, as average sensitivity values increases, the difference between average sensitivity and average specificity decreases for multi-class problems. Nevertheless, for two-class benchmark data sets, the difference between average sensitivity and average specificity values decreases, as average sensitivity value decreases.

MCC value gives the strength of relationship between the actual and predicted values. A perfect fit will give a MCC value of 1. For two-class benchmark data sets, MCC and S values are equal to the each other for each one of the classes (Table 5.16 – Table

5.20). Moreover, MCC values are always higher than the S values for both two-class and multi-class data sets. On the other hand, each one of the classes in multi-class benchmark data sets has different MCC and S values (Table 5.17 – Table 5.28). Due to the low accuracy values for Blood and Yeast data sets, MCC and S values are low for each of the classes (Table 5.18, Table 5.27, and Table 5.28). This means that the classifier could not effectively capture the characteristics of that class. As accuracy values of data sets including Ionosphere, WDBC, Wine, Iris, Thyroid and Ecoli are high, the MCC and S values are also high for these data sets (Table 5.16, Table 5.19, Table 5.21, Table 5.22, Table 5.23, Table 5.25, and Table 5.26). Furthermore, the data sets Pima, Liver and Glass have moderate MCC and S values as they have moderate accuracy values (Table 5.17, Table 5.20, and Table 5.24).

Classifier	Average	Average	MCC		S		
	Sensitivity	Specificity	C1	C ₂	C1	C ₂	
LibSVM	94.87%	94.33%	0.89	0.89	0.89	0.89	
MILP	94.59%	92.43%	0.88	0.88	0.88	0.88	
Random Forest	93.45%	91.09%	0.86	0.86	0.86	0.86	
RBF Network	92.31%	91.50%	0.83	0.83	0.83	0.83	
J48	91.45%	87.53%	0.81	0.81	0.81	0.81	
JRip	89.74%	88.32%	0.78	0.78	0.78	0.78	
Logistic	88.89%	84.70%	0.76	0.76	0.75	0.75	
SMO	88.60%	82.09%	0.75	0.75	0.74	0.74	
IB1	86.32%	78.02%	0.70	0.70	0.68	0.68	
Naïve Bayes	82.62%	84.33%	0.65	0.65	0.64	0.64	

Table 5.16 Values of performance measures for the Ionosphere data set.

For a perfect prediction, S value should be equal to 1 and 0 for vice versa. Depending on the characteristics of the data sets such as complexity and dimensionality, the prediction accuracies could be low as in the Blood and Yeast data sets (Table 5.18, Table 5.27, and Table 5.28). Hence, the S values for these data sets are very low. On the other hand, when we observe the results of each data set in overall, each of the classes have higher and lower MCC and S values with respect to the remaining classes. Hence, we could not say that MILP based hyper-box enclosure approach performs rather purely for any of the classes. Depending on the data set characteristics, proposed data classification approach works well for each of the classes.

Classifier	Average	Average	MCC		S	
	Sensitivity Specificity		C1	C ₂	C1	C2
MILP	81.25%	72.64%	0.57	0.57	0.56	0.56
SMO	77.34%	66.56%	0.48	0.48	0.47	0.47
Logistic	77.21%	67.88%	0.48	0.48	0.47	0.47
LibSVM	76.43%	65.21%	0.46	0.46	0.45	0.45
Naïve Bayes	76.30%	69.29%	0.47	0.47	0.47	0.47
RBF Network	75.39%	65.51%	0.44	0.44	0.43	0.43
JRip	74.87%	66.79%	0.43	0.43	0.43	0.43
J48	73.83%	67.27%	0.42	0.42	0.42	0.42
Random Forest	72.01%	66.99%	0.39	0.39	0.39	0.39
IB ₁	70.18%	62.20%	0.33	0.33	0.33	0.33

Table 5.17 Values of performance measures for the Pima data set.

Classifier	Average	Average	MCC		S		
	Sensitivity	Specificity	C1	C ₂	C1	C2	
MILP	79.95%	75.58%	0.39	0.39	0.38	0.38	
RBF Network	79.68%	42.65%	0.34	0.34	0.29	0.29	
JRip	78.88%	53.22%	0.36	0.36	0.36	0.36	
J48	77.81%	54.05%	0.35	0.35	0.34	0.34	
LibSVM	77.81%	48.25%	0.31	0.31	0.30	0.30	
Logistic	77.14%	32.59%	0.19	0.19	0.13	0.13	
SMO	76.20%	23.80%	NA	NA	0.00	0.00	
Naive Bayes	75.40%	37.46%	0.18	0.18	0.16	0.16	
Random Forest	72.33%	44.99%	0.19	0.19	0.19	0.19	
IB1	68.58%	46.92%	0.15	0.15	0.15	0.15	

Table 5.18 Values of performance measures for the Blood data set.

Table 5.19 Values of performance measures for the WDBC data set.

Classifier	Average	Average	MCC		S		
	Sensitivity	Specificity	C1	C ₂	C1	C ₂	
MILP	97.36%	96.07%	0.94	0.94	0.94	0.94	
SMO	97.07%	96.88%	0.94	0.94	0.94	0.94	
Logistic	96.78%	95.95%	0.93	0.93	0.93	0.93	
LibSVM	96.78%	96.14%	0.93	0.93	0.93	0.93	
Naïve Bayes	96.34%	96.87%	0.92	0.92	0.92	0.92	
Random Forest	96.19%	95.05%	0.92	0.92	0.92	0.92	
JRip	96.19%	96.02%	0.92	0.92	0.92	0.92	
IB1	96.05%	94.78%	0.91	0.91	0.91	0.91	
J48	96.05%	95.75%	0.91	0.91	0.91	0.91	
RBF Network	95.75%	95.78%	0.91	0.91	0.91	0.91	

Classifier	Average	Average	MCC		S		
	Sensitivity	Specificity	C1	C ₂	C1	C ₂	
LibSVM	74.20%	71.62%	0.47	0.47	0.46	0.46	
MILP	73.33%	71.18%	0.45	0.45	0.45	0.45	
J48	68.70%	64.41%	0.35	0.35	0.34	0.34	
Logistic	68.12%	63.99%	0.33	0.33	0.33	0.33	
Random Forest	66.38%	65.38%	0.32	0.32	0.32	0.32	
JRip	64.35%	59.55%	0.25	0.25	0.25	0.25	
RBF Network	64.35%	60.88%	0.26	0.26	0.26	0.26	
IB ₁	62.90%	61.15%	0.24	0.24	0.24	0.24	
SMO	58.26%	42.43%	0.06	0.06	0.01	0.01	
Naïve Bayes	55.36%	61.19%	0.17	0.17	0.15	0.15	

Table 5.20 Values of performance measures for the Liver data set.

Table 5.21 Values of performance measures for the Wine data set.

Classifier	Average	Average	MCC			S	
	Sensitivity	Specificity C1	C ₂	C ₃	C ₁	C ₂	C ₃
RBF Network	98.31%	98.88% 0.97	0.97	0.99	0.96	0.97	0.96
SMO	98.31%	99.31% 0.99 0.97 0.97 0.96 0.96 0.96					
Random Forest	98.31%	99.31% 0.99 0.97 0.97 0.96 0.96 0.96					
Logistic	97.19%	98.56% 0.97 0.94 0.96 0.94 0.94 0.93					
Naïve Bayes	96.63%	98.26% 0.96 0.93 0.96 0.93 0.93 0.92					
IB ₁	94.94%	97.78% 0.94 0.90 0.95 0.90 0.89 0.89					
MILP	94.94%	97.16% 0.94 0.91 0.93 0.89 0.90 0.88					
J48	93 82%	96.44\% 0.94 0.88 0.90 0.87 0.87 0.85					
JRip	93.82%	96.44% 0.88 0.88 0.96 0.86 0.87 0.86					
LibSVM	92 13%	95 82% 091 085 088 083 084 081					

Classifier	Average	Average		MCC		S			
		Sensitivity Specificity C1 C2			C ₃	C ₁	C ₂	C ₃	
LibSVM	98%	99%	1.00	0.95		0.96 0.96 0.95		0.96	
Naïve Bayes	96%	98%				1.00 0.91 0.91 0.92 0.91		0.91	
Logistic	96%	98%				1.00 0.91 0.91 0.92 0.91		0.91	
SMO	96%					98\% 1.00 0.91 0.91 0.92 0.91		0.91	
J48	96%					98% 0.98 0.91 0.93 0.92 0.91		0.91	
MILP	96%					98\% 1.00 0.91 0.91 0.92 0.91		0.91	
Random Forest	95.33	97.67% 1.00 0.90 0.89 0.91 0.90 0.89							
IB ₁	95.33	97.67% 1.00 0.90 0.89 0.91					0.90 0.89		
RBF Network	95.33	97.67% 1.00 0.89 0.90 0.91 0.89 0.90							
JRip	94%	97%				0.98 0.86 0.88 0.88 0.86 0.87			

Table 5.22 Values of performance measures for the Iris data set.

Classifier		MCC SEN SPE					S					
		C1	C ₂		$C3 \tC4 \tC5 \tC6$		- C1	C2			$C3$ $C4$ $C5$ $C6$	
Random Forest 78% 91% 0.67 0.63 0.32 0.75 0.94 0.87 0.59 0.58 0.39 0.50 0.50 0.58												
MILP	76% 91% 0.63 0.63 0.42 0.72 0.82 0.87 0.56 0.57 0.41 0.48 0.48 0.57											
LibSVM	71% 89% 0.54 0.59 0.13 0.72 0.74 0.83 0.48 0.52 0.32 0.46 0.45 0.53											
IB1	71% 89% 0.59 0.51 0.27 0.66 0.69 0.83 0.51 0.47 0.36 0.45 0.44 0.52											
JRip	66% 85% 0.45 0.45 0.08 0.63 0.68 0.85 0.41 0.41 0.33 0.42 0.42 0.49											
Logistic	66% 86% 0.43 0.43 0.03 0.69 0.88 0.85 0.40 0.41 0.32 0.43 0.44 0.49											
J48	66% 87% 0.48 0.39 0.21 0.74 0.79 0.81 0.43 0.37 0.34 0.44 0.43 0.48											
RBF Network	66% 86% 0.47 0.44 0.08 0.50 0.88 0.80 0.43 0.41 0.32 0.40 0.43 0.49											
SMO	57% 78% 0.30 0.33 NA 0.24 NA 0.84 0.30 0.28 0.32 0.34 0.34 0.44											
Naïve Bayes	50\% 82\% 0.21 0.11 0.14 0.16 0.75 0.79 0.22 0.18 0.27 0.30 0.35 0.38											

Table 5.24 Values of performance measures for the Glass data set.

Classifier	SEN	SPE				S				
				$\mathbf{C2}$	C ₃	C ₄	C ₅	C6	C ₇	C8
SMO	87.50%	96.14%	0.79	0.70	0.45	0.45	0.56	0.59	0.50	0.69
Naïve Bayes	86.61%	95.81%	0.78		0.68 0.45	0.45	0.54	0.57	0.50	0.68
MILP	86.31%	96.37%	0.77		0.69 0.45		$0.43 \quad 0.53$	0.56 0.51		0.67
Random Forest	85.42%	96.35%	0.76		$0.66 \quad 0.44$		0.44 0.57	0.58	0.48	0.65
LibSVM	84.52%	96.01%	0.75		0.65 0.44	0.43	0.49	0.55	0.48	0.65
Logistic	84.23%	96.04%	0.74		0.66 0.45	0.43		$0.52 \quad 0.52$	0.47	0.64
IB1	83.63%	94.81%	0.74		$0.63 \quad 0.45$		$0.45 \quad 0.38$		0.54 0.48	0.64
J48	83.63%	95.39%	0.73		0.64 0.45		$0.45 \quad 0.45$	0.55	0.47	0.62
JRip	80.36%	94 59%	0.69		0.57 0.44	0.43	0.42	0.52	0.48	0.59
RBF Network	80.36%	93.87%	0.68		0.60 0.44	0.43	0.43	0.51	0.42	0.59

Table 5.26 Values of performance measures II for the Ecoli data set.

Table 5.27 Values of performance measures I for the Yeast data set.

Classifier	SEN	SPE						MCC				
			C1	C2	C ₃	C ₄	$C5$ $C6$		C7	\mathbf{C}		$C9$ $C10$
LibSVM		63.14\% 86.56\% 0.53 0.40 0.40 0.78 0.59 0.49 0.81									0.31 0.55 0.63	
MILP		60.38% 85.51% 0.49 0.36 0.35 0.73 0.59 0.42 0.76 0.18 0.46 0.45										
Logistic		59.10% 86.04% 0.51 0.33 0.32 0.69 0.57 0.34 0.71 -0.01 0.55 0.91										
Naïve Bayes		58.89% 86.48% 0.48 0.34 0.33 0.70 0.51 0.37 0.73 -0.02 0.42 0.25										
RBF Network		58.63% 85.68% 0.48 0.33 0.32 0.61 0.51 0.37 0.75 -0.01 0.57 0.73										
J48		58.09% 85.08% 0.48 0.34 0.28 0.68 0.54 0.39 0.71 -0.01 0.63 0.89										
SMO		57.61% 86.29% 0.50 0.31 0.32 0.59 0.48 0.34 0.75 -0.01 0.55 0.63										
Random Forest 57.01% 84.14% 0.46 0.29 0.32 0.69 0.44 0.28 0.75											NA 063 077	
IB ₁		55.86% 85.53% 0.39 0.27 0.27 0.78 0.45 0.40 0.74 -0.02 0.39 0.80										
JRip		52.29% 85.07% 0.33 0.23 0.24 0.66 0.43 0.31 0.64 0.04 0.39 1.00										

Classifier	SEN	SPE	S									
			C1	C2	$C3$ $C4$ $C5$ $C6$				$\mathsf{C}7$	C8	C9	C ₁₀
LibSVM		63.14% 86.56% 0.41 0.37 0.37 0.41 0.39 0.39 0.46 0.38 0.39 0.39										
MILP		60.38% 85.51% 0.39 0.34 0.33 0.39 0.38 0.37 0.43 0.36 0.38 0.38										
Logistic		59.10% 86.04% 0.39 0.32 0.32 0.39 0.38 0.36 0.42 0.36 0.37 0.37										
Naïve Bayes		58.89% 86.48% 0.38 0.33 0.33 0.38 0.37 0.36 0.42 0.36 0.37 0.37										
RBF Network		58.63% 85.68% 0.38 0.32 0.31 0.38 0.37 0.36 0.42 0.36 0.37 0.37										
J48		58.09% 85.08% 0.37 0.33 0.29 0.38 0.37 0.36 0.41 0.35 0.37 0.37										
SMO		57.61% 86.29% 0.38 0.31 0.31 0.37 0.36 0.35 0.42 0.35 0.37 0.37										
Random Forest 57.01% 84.14% 0.37 0.29 0.29 0.38 0.36 0.35 0.41 0.35 0.37 0.36												
IB ₁		55.86% 85.53% 0.34 0.29 0.29 0.38 0.36 0.35 0.41 0.34 0.36 0.36										
JRip		52.29% 85.07% 0.31 0.26 0.26 0.35 0.34 0.33 0.37 0.33 0.34 0.35										

Table 5.28 Values of performance measures II for the Yeast data set.

In order to evaluate if there is any statistical significant difference between the existing and proposed data classification approaches tested on the same data sets, P-value (paired test) analysis are carried out. The results of P-value test for two-class benchmark data sets are given in Table 5.29 and Table 5.30. There is no statistical significant difference between the results LibSVM and MILP approach on two-class problems except Pima data set. MILP approach is statistically significant than the LibSVM method for Pima data set. On the other hand, there is no significant difference between the results of SMO, support vector implementation of WEKA, and MILP approach for the data sets Pima, Blood and WDBC. However, MILP approach is statistically significant than the accuracies of the SMO classifier on Ionosphere and Liver data sets. The accuracy values of MILP approach on each of the data sets is statistically significant than the accuracies of the distance based algorithm IB1 except the WDBC data set. There is no statistically significant difference between the accuracies of the decision tree based classifier J48 and

MILP approach except the Pima data set. MILP approach is statistically significant than the J48 classifier for Pima data set. There is no statistical significant difference between the results of Random Forest and RBF Network classifier with the MILP approach except the two data sets. MILP approach is statistically significant than the results of Random Forest classifier for Pima and Blood data sets. Furthermore, MILP approach is statistically significant than the results of RBF Network classifier for Pima and Liver data sets. Fro most of the data sets; MILP approach has statistically significant accuracy values from the classifiers JRip and Naïve Bayes. Finally, there is no significant difference between the results of MILP approach and Logistic classifier for each of the data sets except Ionosphere. When we observe the results from the data sets one by one, each of the methods applied to WDBC data set have statistically equivalent results. For the rest of the data sets, MILP approach statistically dominates the results of some of the classifiers.

Compared	Ionosphere Pima Blood WDBC Liver				
Methods	P	P	P	P	P
	Value		Value Value	Value	Value
MILP vs LibSVM	0.17	2.31	1.02	0.64	0.26
MILP vs SMO	2.86	1.89	1.75	0.33	4.17
MILP vs IB1	3.73	5.06	5.03	1.36	2.94
MILP vs J48	1.63	3.49	1.02	1.36	1.34
MILP vs Random Forest	0.64	4.28	3.46	1.22	1.99
MILP vs RBF Network	1.22	2.79	0.13	1.63	2.55
MILP vs JRip	2.39	3.02	0.51	1.22	2.55
MILP vs Naïve Bayes	4.99	2.37	2.11	1.08	4.93
MILP vs Logistic	2.74	1.95	1.32	0.64	1.50

Table 5.29 The results of P-value analyses for two-class data sets.

	Ionosphere	Pima	Blood	WDBC	Liver	
Compared Methods	P Test	P Test	P Test	P Test	P Test	
	Result	Result	Result	Result	Result	
MILP vs LibSVM	$=$ $=$	$++$	$=$ $=$	$=$ $=$	$=$ $=$	
MILP vs SMO	$++$	$=$ $=$	$=$ $=$	$=$ $=$	$++$	
MILP vs IB1	$++$	$++$	$++$	$=$ $=$	$++$	
MILP vs J48	$=$ $=$	$++$	$=$ $=$	$=$ $=$	$=$ $=$	
MILP vs Random Forest	$=$ $=$	$++$	$++$	$=$ $=$	$=$ $=$	
MILP vs RBF Network	$=$ $=$	$++$	$=$ $=$	$=$ $=$	$++$	
MILP vs JRip	$++$	$++$	$=$ $=$	$=$ $=$	$++$	
MILP vs Naïve Bayes	$++$	$++$	$++$	$=$ $=$	$++$	
MILP vs Logistic	$++$	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	

Table 5.30 The results of P-test for two-class data sets.

 $+$ + denotes that the first method is statistically significantly better than the second method. $$ represents that the second method is statistically significantly better than the first method. $=$ $=$ indicates that there is no significant difference between the results of the methods.

The results of P-value test for multi-class benchmark data sets are given in Table 5.31 and Table 5.32. There is no statistical significant difference between the results LibSVM and MILP approach on each one of the multi-class benchmark problems. On the other hand, MILP approach is statistically significant than the accuracies of the SMO classifier on half of the data sets (Thyroid, Glass and Yeast) and there is no significant difference between the results of SMO and MILP approach on half of the data sets (Wine, Iris and Ecoli). There is no statistically significant difference between the performances of the methods IB1 and MILP on each of the multi-class data sets except Ecoli and Yeast. The result of MILP approach is significantly better than the IB1 classifier fro the data sets

Ecoli and Yeast. Similar to SMO classifier, MILP approach is statistically significant than the results of the J48 classifier on half of the data sets (Thyroid, Glass and Yeast) and there is no significant difference between the results of J48 and MILP approach on half of the data sets (Wine, Iris and Ecoli).

Compared	Wine	Iris	Thyroid Glass		Ecoli Yeast	
Methods	P	P	P	P	P	P
		Value Value	Value		Value Value	Value
MILP vs LibSVM	1.08	1.02	1.64	1.10	0.34	1.55
MILP vs SMO	1.76	$\overline{0}$	3.13	4.11	1.08	3.41
MILP vs IB1	θ	0.28	θ	1.31	2.18	5.98
MILP vs J48	0.46	θ	2.36	2.34	0.88	4.04
MILP vs Random Forest	1.76	0.28	1.64	0.34	1.08	2.37
MILP vs RBF Network	1.76	0.28	1.02	2.34	0.77	2.26
MILP vs JRip	0.46	0.79	2.01	2.24	2.18	2.82
MILP vs Naïve Bayes	0.79	θ	0.28	5.70	0.45	3.08
MILP vs Logistic	1.09	$\overline{0}$	0.28	2.24	0.11	2.52

Table 5.31 The results of P-value analyses for multi-class data sets.

The performances of the methods Random Forest and MILP are not statistically significant for each one of the data sets except Yeast. For the Yeast data set, MILP approach is significantly better than the classifier Random Forest. The neural network based classifier RBF Network and MILP approach do not have statistically significant results for each one of the data sets except Glass and Yeast. However, the performance of MILP approach is statistically significant than the RBF Network classifier for the data sets Glass and Yeast. The accuracy values of MILP approach on each of the data sets is statistically significant than the accuracies of the rule based classifier JRip except the data

sets Wine and Iris. Similar to RBF Network classifier, Naïve Bayes and Logistic classifiers have no statistically significant results than the MILP approach for the data sets Glass and Yeast. On the other hand, the performances of the methods MILP and Naïve Bayes and Logistic classifiers are not significant in statistical manner for the rest of the multi-class benchmark data sets.

Compared	Wine	Iris	Thyroid	Glass	Ecoli	Yeast
	P Test	P Test	P Test	P Test	P Test	P Test
Methods	Result	Result	Result	Result	Result	Result
MILP vs LibSVM	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$
MILP vs SMO	$=$ $=$	$=$ $=$	$++$	$++$	$=$ $=$	$++$
MILP vs IB1	$=$ $=$	$=$	$=$ $=$	$=$ $=$	$++$	$++$
MILP vs J48	$=$ $=$	$=$ $=$	$++$	$++$	$=$ $=$	$++$
MILP vs Random Forest	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	$=$ $=$	$++$
MILP vs RBF Network	$=$ $=$	$=$ $=$	$=$ $=$	$++$	$=$ $=$	$++$
MILP vs JRip	$=$ $=$	$=$ $=$	$++$	$++$	$++$	$++$
MILP vs Naïve Bayes	$=$ $=$	$=$ $=$	$=$ $=$	$++$	$=$ $=$	$++$
MILP vs Logistic	$=$ $=$	$=$ $=$	$=$ $=$	$++$	$=$ $=$	$++$

Table 5.32 The results of P-test for multi-class data sets.

 $++$ denotes that the first method is statistically significantly better than the second method. $--$ represents that the second method is statistically significantly better than the first method. $=$ = indicates that there is no significant difference between the results of the methods.

In order to compare the existing data classification methods with MILP, some of the ordered P-value graphs are shown in Figure 5.1 to Figure 5.9. In Figure 5.1, the ordered P-values of MILP versus LibSVM for each of the eleven data sets are shown. For only one data set, the P-value is greater than 2. In general, MILP is preferable since it performs quite well for each of the existing benchmark data sets. However, LibSVM method

performs poorly with respect to MILP approach for one of the data sets (Pima). Hence, we could say that MILP approach is significantly better than LibSVM method in general. For more than half of the data sets, MILP approach is statistically significant than SMO, IB1, JRip, and Naïve Bayes classifiers (Figure 5.2, Figure 5.3, Figure 5.7, Figure 5.8). Thus, MILP approach is statistically better than these 4 methods in general. For three data sets, the P-values are greater than 2 for the classifiers Random Forest and Logistic (Figure 5.5 and Figure 5.9). For the rest of the data sets, the difference between the accuracies of two methods is not significant. Finally, proposed MILP approach is statistically significantly better than J48 and RBF Network classifiers for four of the data sets (Figure 5.4 and Figure 5.6).

Figure 5.1 P-value graph of MILP versus LibSVM for UCI Benchmark data sets.

For each of the eleven UCI Repository data sets, MILP approach generally achieves high accuracy values and mostly ranks in first three positions with respect to accuracy. Furthermore, MILP is statistically significantly better than the existing data classification methods for these benchmark problems on given eleven distinct benchmark data sets.

Figure 5.2 P-value graph of MILP versus SMO for UCI Benchmark data sets.

Figure 5.3 P-value graph of MILP versus IB1 for UCI Benchmark data sets.

Figure 5.4 P-value graph of MILP versus J48 for UCI Benchmark data sets.

Figure 5.5 P-value graph of MILP versus Random Forest for UCI Benchmark data sets.

Figure 5.6 P-value graph of MILP versus RBF Network for UCI Benchmark data sets.

Figure 5.7 P-value graph of MILP versus JRip for UCI Benchmark data sets.

Figure 5.8 P-value graph of MILP versus Naïve Bayes for UCI Benchmark data sets.

Figure 5.9 P-value graph of MILP versus Logistic for UCI Benchmark data sets.

5.6 Problematic Instance Analysis

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 In order to analyze whether there exists any relation between the performance of the proposed approach and the number of problematic instances in the data sets, we investigate the results of these data sets in detail. In Table 5.33, the number of problematic instances is given by average, maximum and minimum values for each of the protein folding type benchmark data sets. Furthermore, the number of problematic instances for 1189 and 25PDB data sets change from one run to another as 10FCV results are obtained for them (Figure 5.10 – Figure 5.20).

 As it can be observed from the Table 5.33, the number of problematic instances does not affect the performance of the proposed approach. For example, for the data sets WDBC and thyroid which have 67% and 10% problematic instances proposed approach

achieved approximately 97% accuracy. Hence, considering only the number of problematic instances could not be sufficient to analyze the difficulty of the data sets.

Data Set			$%$ of Av.	Number of Problematic			
	Accuracy	Accuracy Rank	Problematic	Instances			
Name	$(\%)$		Instances	Average	Max.	Min.	
Ionosphere	94.59%	$\overline{2}$	60%	212	217	206	
Pima	81.25%		87%	666	677	641	
Blood	79.95%		89%	664	666	662	
WDBC	97.36%		67%	458	471	430	
Liver	73.33%	2	80%	275	279	269	
Wine	94.94%	4	6%	10	12	9	
Iris	96.00%	$\overline{2}$	15%	22	25	16	
Thyroid	97.21%		10%	21	24	15	
Glass	76.17%	2	65%	140	143	134	
Ecoli	87.50%	2	48%	162	170	151	
Yeast	63.00%		88%	1299	1307	1290	

Table 5.33 Number of problematic instances for each of UCI repository data sets.

Figure 5.10 The number of problematic instances for Ionosphere data set.

Figure 5.11 The number of problematic instances for Pima data set.

Figure 5.12 The number of problematic instances for Blood data set.

Figure 5.13 The number of problematic instances for WDBC data set.

Figure 5.14 The number of problematic instances for Liver data set.

Figure 5.15 The number of problematic instances for Wine data set.

Figure 5.16 The number of problematic instances for Iris data set.

Figure 5.17 The number of problematic instances for Thyroid data set.

Figure 5.18 The number of problematic instances for Glass data set.

Figure 5.19 The number of problematic instances for Ecoli data set.

Figure 5.20 The number of problematic instances for Yeast data set.

Chapter 6

CONCLUSION

With the rapid increase in the availability of data for exploration and analysis, it is important to develop techniques that efficiently perform data mining studies. As data classification is one of the important issues in these studies, many researchers study this concept. Classification involves the supervised assignment of data points to predefined or known classes. Here, there exists a collection of classes with labels and the problem is to label a new instance as belonging to one or more of the classes. The field of data classification is wide and covers a broad range of areas including bioinformatics, decision sciences, finance, sports and health care. A large number of data classification methods have been developed to date; however, each of them has several drawbacks which make them unattractive. Thus, researchers have been studying to develop more accurate and more efficient methods or to improve the existing methods.

In this thesis, a new three-stage mathematical programming based hyper-box enclosure approach for multi-class data classification problem is proposed. A mixedinteger programming model is developed for representing existence of hyper-boxes which define the boundaries of the classes for the training set. In order to overcome the computational difficulties for large data sets, a three-stage approach is developed for training part analysis of hyper-box enclosure approach. The performance of the model is tested by the testing part of the proposed method and compared with existing multi-class data classification methods on two widely used challenging problems; the protein folding type prediction problem and UCI Repository benchmark problems.
The developed three-stage MILP based hyper-box enclosure approach to multiclass data classification is described in Chapter 3. In the training part of the proposed approach, the characteristics of data points belonging to a certain class are determined by the construction of hyper-boxes. The hyper-boxes define the boundaries of the classes that include all or some of the points in that set. In order to represent the existence of hyperboxes and their boundaries, a mixed-integer programming model is developed.

Solving the proposed MILP formulation to optimality is computationally expensive for large multi-group data classification problems. The major source of computational difficulty is the potentially large number of binary variables. Hence, we proposed a threestage decomposition algorithm for obtaining solutions to MILP model. Instances that are difficult to classify are identified in the first stage that is referred to as preprocessing. Moreover, sub grouping and seed finding algorithms are applied to improve the computational efficiency. With greater emphasis given to these observations, solution to the problem is obtained in the second stage using the MILP formulation. Last, final assignments, elimination of box intersections and box combination procedures are carried out in the third step.

After distinguishing characteristics of the classes are determined in the training part, the performance of the model is tested by the distance based algorithm introduced in testing problem formulation part. While the original and proposed testing algorithms are compared and investigated in detail, the advantages of proposed testing algorithm are shown. If a new data point with an unknown membership arrives, it is necessary to assign this data point to one of the classes. For each member of the test data set, testing algorithm is applied and assignments to a class are done. After all, by checking the original classes of the test set samples the performance of the developed model is evaluated.

The proposed model is illustrated on a small illustrative example. By this illustrative example, the main steps of the developed three-stage MILP based hyper-box

enclosure approach are understood. Moreover, the comparison of the results of distinct models available for data classification is performed. The suggested model's result is accurate and efficient in this small example with regard to the other methods listed in Table 3.3.

In Chapter 4, proposed three-stage MILP approach is applied to the protein folding type prediction problem. Different performance evaluation techniques and measures are examined in order to investigate the details of results and compare different algorithms. The performance of proposed three-stage MILP based approach is compared with the results in [97], [100] and [23] for nine distinct data sets. Two independent datasets (225 training - 510 testing and 1601 training - 2438 testing) results are calculated and pretty good results are obtained. Furthermore, LOO test results are given for 138, 253, 277, 35 and 498 protein data sets and 10FCV results are studied for 1189 and 25PDB data sets. Results indicate that proposed MILP approach gives generally high accuracy values and mostly rank in the first or second position. Moreover, P-value analyses show that MILP approach is statistically significantly better than the existing distance based algorithms HD, ED and CC algorithm. Moreover, MILP approach is statistically better than the LibSVM and well-known WEKA classifiers for protein folding type prediction problems on given nine distinct benchmark data sets. In summary, proposed MILP based hyper-box enclosure approach is a powerful and efficient computational method for predicting folding types of proteins with its favorable results and characteristics.

In Chapter 5, the performance of proposed three-stage approach is evaluated on eleven UCI repository benchmark data sets [108]. In order to observe the performance of the proposed MILP based hyper-box enclosure approach, the eleven data sets including Ionosphere, Pima, Blood, WDBC, Liver, Wine, Iris, Thyroid, Glass, Ecoli and Yeast are tested. First five of them are binary-class data classification data sets and the rest are multi-class data sets. In order to compare the results of proposed MILP approach, WEKA

classification algorithms J48, RBF Network, Logistic, Naïve Bayes (NB), SMO, Random Forest (RF) and IB1 are studied (Table 5.3). Moreover, well-known support vector machine implementation LibSVM given by [111] is also studied. Using these data sets, the proposed three-stage MILP model is solved in GAMS [120] using ILOG CPLEX Solver version 10.0 [121] on a notebook computer with Inter Pentium M 1.73 Ghz Processor and 512 MB of RAM. For each data set, 10 different runs are carried out and average 10FCV results are calculated. In order to analyze the results in detail, average sensitivity (SEN), average specificity (SPE), MCC and S values of each of the protein data sets are investigated (Table 5.16 - Table 5.28). Depending on the data set characteristics, proposed data classification approach works well for each of the classes. In order to evaluate if there is any statistical significant difference between the existing and proposed data classification approaches tested on the same data sets, P-value (paired test) analysis are carried out. For each of the eleven UCI Repository data sets, MILP approach generally achieves high accuracy values and mostly ranks in first three positions with respect to accuracy. Furthermore, MILP is statistically significantly better than the existing data classification methods for these benchmark problems on given eleven distinct benchmark data sets.

In conclusion, this thesis introduces a new three-stage mathematical programming based hyper-box enclosure method for multi-class data classification problem. One of the most important characteristics of the proposed approach is allowing the use of hyper-boxes for defining the boundaries of the classes that enclose all or some of the points in that set. In other words, if necessary, more than one hyper-box is constructed for a specific class in the training part. Moreover, well-construction of the boundaries of each class provides the lack of misclassifications in the training set and indirectly improves the accuracy of the model. In addition, the model does not need to know the underlying distribution of the training data set and learns from the training set in a reasonable time. With only one parameter to be initialized, the suggested model is simple and easily understandable.

Furthermore, the proposed model can be used for both binary and multi-class data classification problems without any modifications or additions. The accuracy, simplicity and understandability of the proposed model are favorable. Proposed three-stage MILP approach is applicable to obtain solutions to large multi-class data classification problems. These characteristics make the proposed approach efficient, simple and easily implementable.

The advantage of the mathematical programming approach in the context of supervised classification lies in its power to model more complex real world problems. Future studies should further evaluate the performance of the proposed approach on data sets with categorical attributes. Since the proposed approach depends on a geometrical idea, it is efficient for data sets including continuous and integer valued attributes. In literature, there exist data classification problems which include both categorical and numerical attributes. Hence, MILP approach could be modified in order to deal with categorical attributes. Moreover, overall method could be implemented in a computer package and could be parallelized. Finally, proposed data classification approach could be implemented in WEKA. In that case, there will be some solver related problems since MILP approach needs an IP solver such as GAMS. If these problems could be solved, MILP approach could be tested by many researchers by the help of WEKA.

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APPENDIX A: PROTEIN FOLDING TYPE PREDICTION DATA SETS

Table A.1 The 138 Protein Domains.

* Each domain is represented by a symbol of X|Y, where first four character of X is the corresponding PDB code and the fifth character indicates the specific chain of the protein. If it is _, then the corresponding protein has only one chain. If Y=W.C., it means the domain is constituted by the whole chain. Otherwise, Y contains two number to indicate starting and end points along the sequence.

Table A.2 The 253 Protein Domains.

Table A.3 The 359 Protein Domains.

Table A.4 The 1601 Protein Domains.

61 all- α domains									
3sdhA W.C.	1grj 2-79	1 ycc $ W.C.$	1enh W.C.	1dtr 65-191	2tct W.C.				
1 tns $ W.C.$	2spcA W.C.	1fc2C/W.C.	1bal W.C.	lerl W.C.	1rtm1 73-104				
lacp W.C.	2abd W.C.	2end W.C.	1 lis $ W.C.$	laab W.C.	1 fps $ W.C.$				
1mmoG W.C.	1 lpe $ W.C.$	1bcfA/W.C.	1rhgA W.C.	1ropA/W.C.	1 oelA $ 2-136$				
leciA W.C.	$1octC$ 5-75	$1\text{coo}_\text{N.C.}$	1 mdy A W.C.	4icb W.C.	lecmA W.C.				
1parA W.C.	1dsbA 65-128	2gstA 85-217	1bmtA 651-740	$1c5a$ W.C.	2sblB 150-838				
1hyp W.C.	$2bpfA 9-91$	1 olg $A/W.C.$	$1hnr$ W.C.	laep W.C.	$1poc$ W.C.				
$laxn$ W.C.	1tadA 57-177	1ezm 154-298	2ts1 228-319	2hmx W.C.	1bvp1 1-120				
1llp_W.C.	2abk_ W.C.	1gln 306-468	1lla 2-379	2pgd 177-473	laorA 211-605				
lutg W.C.	1 glm $ W.C.$	1csh W.C.	1phc $ W.C.$	1fiaA W.C.	2wrpR W.C.				
1prcC W.C.									
45 all- β domains									
1ddt 381-535	1cdg 582-686	Ihoe W.C.	laac W.C.	2bpal W.C.	$2pec$ [W.C.				
4gcr 1-85	2sblB 7-149	1pgs 4-140	1 gof $1-150$	$1byp1 1-217$	1 lxa W.C.				
1knb W.C.	1tnrA W.C.	1thw W.C.	$1scs$ W.C.	1bglA 731-1023	2phlA W.C.				
1bia 271-317	1 ltsD $ W.C.$	1prcH W.C.	4fgf W.C.	1fnb 19-154	1 htp $ W.C.$				
1eft 313-405	1 arb W.C.	1bco 481-560	1difA W.C.	1gtrA 339-547	2kauC 2-129				
1btn $ W.C.$	1pkn 116-217	1hbq W.C.	1sriA W.C.	2cpl W.C.	1kapP 247-470				
1 hxn_ $ W.C.$	$2sil$ W.C.	1gof 151-537	3aahA W.C.	lcdg 383-495	1msaA/W.C.				
1hcb W.C.	3bcl W.C.	1vmoA W.C.							
56 α / β domains									
1cdg 1-382	1tml W.C.	2tmdA 490-645	1dik 377-505	1bta W.C.	3ecaA W.C.				
1bnh W.C.	liceA W.C.	1mla 3-127	1fnb 155-314	2ts1 1-217	1 ctt 1-150				
2tmdA 341-489	2ohxA 175-324	1bncA 1-114	$1pydA 2-181$	1nbaA W.C.	1pfkA W.C.				
1deaA W.C.	$1gky$ $ W.C.$	1 chd W.C.	1 cse $E W.C.$	3cla W.C.	2dri W.C.				
$1phr$ W.C.	2 hnq W.C.	2trxA W.C.	1trkA 535-680	1pkm 396-530	2olbA W.C.				
1 lcpA $ 1-159$	1eriA/W.C.	2rslA/W.C.	1hpm 4-188	3 pgm W.C.	2 bgu $ W.C.$				
1gph1235-465	11faA/W.C.	1 gar A W.C.	1hmy W.C.	7aatA W.C.	1pxtA 28-293				
1 ulb W.C.	1 gpmA $ 3-207$	$lack$ W.C.	2ctb W.C.	8dfr $ W.C.$	1mioA W.C.				
1aliA W.C.	1xaa W.C.	4at1A 1-150	lubsB W.C.	lora 1-149	3 pgk W.C.				
7acn 2-528	3pmgA 1-190								
63 α + β domains									
1fus $ W.C.$	1cnsA W.C.	2act W.C.	7rsa W.C.	2kauA W.C.	$1\,\text{mri}$ W.C.				
lnapA/W.C.	1 sso $ W.C.$	1pkp_ 78-147	ligd W.C.	3cox 319-450	1 ltsA $ W.C.$				
$1 \text{mol} \text{A}$ W.C.	1fruA 1-178	laak W.C.	1 fkd $ W.C.$	1dhy 1-132	1esl 1-118				
$lcsel$ [W.C.	1mngA 93-203	1ctf W.C.	2reb 269-328	1stu $ W.C.$	1mldA 145-313				
1 vih $ W.C.$	1gpmA 405-525	2mnr 3-132	loelA 137-190	1 fxd W.C.	1pyaA W.C.				
1tig W.C.	1kptA W.C.	3rubS W.C.	1dchA/W.C.	2chsA W.C.	1gph1 1-234				
1 otfA $ W.C.$	1gadO 149-312	$10acA$ 5-90	3sicl W.C.	2ms2A/W.C.	$1aorA 1-210$				
3grs 364-478	lezm 1-153	1lkkA W.C.	1hid W.C.	1 puc $ W.C.$	2dnjA W.C.				
1sryA 111-421	1vil W.C.	$2prf$ W.C.	1 mut $ W.C.$	1tlcA W.C.	2glt 123-316				
1 lba $ W.C.$	1 cyo $ W.C.$	1 vcc $ W.C.$	1ordA 570-730	1 smn $A W.C.$	$2polA 1-122$				
1chmA 157-402	1ytbA 61-155	$1xrb$ $ 1-101$							

Table A.5 The 225 Protein Domains.

Table A.6 The 510 Protein Domains.

393 all- α domains									
laca W.C.	1cpe_ W.C.	1hbbB W.C.	1mbi W.C.	1pra W.C.	2dhbA W.C.				
$1afb1 73-104$	1 cpf $ W.C.$	1hbhA/W.C.	$1mbj$ W.C.	1prhA 74-586	2dhbB/W.C.				
lafd1 73-104	$lcpg$ W.C.	1hbhB/W.C.	$1mbk$ W.C.	$1pru$ W.C.	2fal W.C.				
$lagm$ W.C.	l crc $A W.C.$	1hbsA/W.C.	$1mbo$ W.C.	$1prv$ W.C.	2 fam $ W.C.$				
l anw $A W.C.$	l crg $ W.C.$	1hbsB/W.C.	1 mcy $ W.C.$	1pvaA W.C.	2frc $ W.C.$				
l anx A W.C.	1 crh $ W.C.$	$1hcl$ W.C.	1 mgn W.C.	$1r36$ W.C.	2glrA 79-209				
lapc $ W.C.$	1 cri W.C.	lhc3 5-398	$1mIf$ W.C.	$1 \text{rap } W.C.$	2hbdA W.C.				
$1arp$ W.C.	$1crj$ $ W.C.$	$1hc4$ W.C.	$1mlg$ W.C.	1raq W.C.	2hbdB W.C.				
$1 \text{arg} A[W.C.$	lcsgA/W.C.	$1hc5$ W.C.	$1mlh$ W.C.	$ $ Ircc $ $ W.C.	2hbeA/W.C.				
1arrA/W.C.	1 csi $ W.C.$	$1hc6$ 5-398	$1mlj$ W.C.	l rce $ W.C.$	2hbeB/W.C.				
1 arv W.C.	$lcsr$ W.C.	1 hcy $\lfloor W.C. \rfloor$	$1mlk$ W.C.	$lrcg$ W.C.	2hbfA W.C.				
l arw $ W.C.$	$less$ W.C.	1hdbA/W.C.	1 mll $ W.C.$	$lrci$ W.C.	2hbfB W.C.				
l arx $ W.C.$	$lcsu$ W.C.	1hdbB/W.C.	$1mlm$ W.C.	$lres$ W.C.	2hcoA W.C.				
$lary_{W.C.}$	$lcsx$ W.C.	1 hgaA $ W.C.$	$1mln$ W.C.	$lret$ W.C.	2hcoB W.C.				
lavhA/W.C.	lctaA/W.C.	l hgaB $ W.C.$	1mlo W.C.	1 mrA $ W.C.$	2hhdA/W.C.				
1 avr W.C.	lctdA/W.C.	1 hgbA $ W.C.$	$1mlq$ $ W.C.$	1 rpo $ W.C.$	2hhdB/W.C.				
laypA W.C.	1ctr W.C.	1 hgb $B/W.C.$	$1mlr$ W.C.	1 rpr $A W.C.$	2hmqA W.C.				
1 babA $ W.C.$	$lety$ W.C.	l hgcA $ W.C.$	$1mlu$ W.C.	IsaeA/W.C.	2hoa $ W.C.$				
1 bab $B/W.C.$	$letz$ W.C.	1 hgcB $ W.C.$	$1mnh$ W.C.	1safA/W.C.	$2ifn$ W.C.				
1bbbBA W.C.	$lcxa$ W.C.	1 hgu $ W.C.$	1mnA/W.C.	1 sag $A W.C.$	$2int$ W.C.				
1bbbBB W.C.	leyf W.C.	1hhoA/W.C.	1 mnk $A W.C.$	lsahA W.C.	2lh2 W.C.				
$1bb1$ W.C.	$1cyj$ W.C.	1hhoB/W.C.	1 moa $ W.C.$	1saiA W.C.	2lh6 W.C.				
$1bbn$ W.C.	$1cyl$ $ W.C.$	1hij W.C.	1mob W.C.	1sajA W.C.	$2lh7$ W.C.				
1bcn $ W.C.$	1dec W.C.	1 hik $ W.C.$	1moc W.C.	1salA/W.C.	2mgb_ W.C.				
1 bgd $ W.C.$	1dog W.C.	1 hkt $ W.C.$	1m rA/W.C.	1 san $ W.C.$	2mgc_ W.C.				
1bib $ 2-63 $	$1dprA 3-64$	1 hlm_ $ W.C.$	$1msdA 1-83$	1setA/W.C.	2mgd W.C.				
$1boc$ W.C.	$1dprA 65-136$	1 hmd $A W.C.$	$1mti$ W.C.	1 ses $A W.C.$	$2mge$ W.C.				
$1bold \mid W.C.$	1 dvh $ W.C.$	1 hme $ W.C.$	$1mtj$ W.C.	1setA/W.C.	$2mgf$ W.C.				
$1bpd$ W.C.	$1dx$ t A W.C.	$1hmf$ W.C.	1 mtk $ W.C.$	1 spe $ W.C.$	$2mgg$ W.C.				
1bpq_ W.C.	1dxtB W.C.	1 hmoA $ W.C.$	1 myf $ W.C.$	1 swm $ W.C.$	$2mgh$ W.C.				
1 _{bvd} $ W.C.$	1dxuA/W.C.	1 hnbA $ 85-217$	1 myh $A W.C.$	$ltag$ W.C.	$2mgi$ W.C.				
1cbIA/W.C.	1 dxuB W.C.	1 hnc $A W.C.$	1 myi $A W.C.$	1 thbA $ W.C.$	$2mgi$ W.C.				
1cbIB/W.C.	1 dxvA W.C.	$1h$ M.C.	1 myj $A W.C.$	1 thbB $ W.C.$	2 mgk $ W.C.$				
1cbmA/W.C.	1 dxvB W.C.	1 hrm $ W.C.$	1 myk $A W.C.$	1thl $ W.C.$	$2mgl$ W.C.				
1cbmB/W.C.	$1 \cdot \text{ecd}$ W.C.	1 hsm $ W.C.$	lmyIA/W.C.	1tlpE 156-316	$2mgm$ W.C.				
$lcca$ W.C.	1 ecn $ W.C.$	1 _{hsn} W.C.	1 mym $ W.C.$	1 tndA W.C.	2mm1 W.C.				
1ccb W.C.	$1eco$ W.C.	1 hsy_ $ W.C.$	$1ner$ W.C.	1 tnp $ W.C.$	2 mya $ W.C.$				
l cce $ W.C.$	leni $ W.C.$	1 huw_ $ W.C.$	1 nhm $ W.C.$	1 tnq $ W.C.$	2 myb_ $ W.C.$				
$lccg$ W.C.	lenj $ W.C.$	1 hve_ $ W.C.$	$1nhn$ W.C.	1 tnt_ $ W.C.$	2myd W.C.				
1 cch $ W.C.$	$lenk$ $ W.C.$	1hvf W.C.	1nihA/W.C.	1 tnw $ W.C.$	2 mye $ W.C.$				
$1cep$ W.C.	$lerc$ W.C.	1 hvg $ W.C.$	1 nih $B/W.C.$	1 tnx $ W.C.$	2pac_ W.C.				
$1cd1A_$ W.C.	l esp $ W.C.$	1hyt 156-316	$1nol$ W.C.	1trf W.C.	2 pas $ W.C.$				
$1cdmA$ W.C.	1 fcs $ W.C.$	1 ifd $ W.C.$	$1noo$ W.C.	ltrlA W.C.	2pcbA W.C.				
$1cdn$ W.C.	1 fhb $ W.C.$	1 ifi W.C.	1 olh $A W.C.$	1tyaE 228-319	2pcbB W.C.				
1 ceh $ W.C.$	1 fipA $ W.C.$	1 isbA $ 1-82$	1 omd $ W.C.$	1tybE 228-319	2pccA W.C.				
1 cfc $ W.C.$	1 fw4 W.C.	1 isc A 1-82	$10xy_{1}-379$	1 tyc $ W.C.$	2pecB/W.C.				

Table A.7 The 2438 Protein Domains.

Table A.8 The 277 Protein Domains.

Table A.9 The 498 Protein Domains.

1anu_|W.C. 1cuk_|1-64 1hoe_|W.C. 1occB|91-227 1tdtA|W.C. 2cpl_|W.C.

Table A.10 The 1189 Protein Domains.

Table A.11 The 25PDB Protein Domains.

VITA

Fadime Üney Yüksektepe completed the high school in Salihli Sekine Evren Anatolian High School, Manisa, in 1998. She received her high honor B. Sc. and M. Sc. degrees in Chemical Engineering from Istanbul Technical University, in 2003 and in Industrial Engineering from Koç University, in 2005, respectively. Since 2005, she is in the Ph.D. program in Industrial Engineering & Operations Management at Koç University as a teaching and research assistant. For the completion of the program, she has studied the thesis "MILP Based Hyper-Box Enclosure Approach to Multi-Class Data Classification".