OPTIMIZING MANUFACTURING PARTS SOURCING TO COORDINATE SUPPLIER SHIPMENTS WITH THE PRODUCTION SCHEDULE

by

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This is to certify that I have examined this copy of a master's thesis by

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and have found that it is complete and satisfactory in all respects,
and that any and all revisions required by the final
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to my family ...

ABSTRACT

It has been recognized that coordinating the shipments of manufacturing parts from the suppliers with the production schedule takes important roles on the smooth continuation of production in mass production systems. In this thesis, we investigate two problems that we encountered while analyzing the sourcing operations of a leading coach bus manufacturer in Turkey. The first problem addresses unorganized delivery operations while the second considers unreliable supplier capability. We optimize the parts ordering decisions under these two settings.

In the first part of the thesis, we study the multiple-item lot-sizing problem for a manufacturer that sources parts from a single supplier over a multi-period planning horizon. In order to operate more efficiently, the manufacturer controls its suppliers' delivery process in addition to its own parts ordering process. Since transportation costs are charged to the manufacturer, the manufacturer optimizes the ordering and shipment decisions. We consider the option of delaying transportation of a less-than-full truckload to the next period by allowing the use of items in the safety stock of the manufacturer. We develop a mixed integer programming model that minimizes the sum of transportation and inventory holding costs incurred to the manufacturer under the proposed policy. We investigate the effects of delaying shipments on both cost and service levels under stochastic environments by numerical experiments. The results indicate that the proposed policy is especially effective in reducing cost when frequent shipments with small sizes arise without creating much stock-out risk.

In the second part of the thesis, we study the dynamic lot-sizing problem under random supply where the supplier's shipment behavior is represented by a model that assumes a random portion of the current order is shipped in every period. To improve the sourcing process, we propose a method that enables the manufacturer to obtain more information about the supplier reliability throughout its ordering process. For this purpose, we develop a dynamic programming model with Bayesian Updates of supplier capability. There is no

information available about the supplier capability at the beginning of the planning horizon. We try to estimate the supplier's capability by using information on previous orders' ordered and received amounts. In this method the ordering decisions are optimized by considering the available information until that point. We then compare the proposed algorithm with the cases under Perfect Information as well as the case with No Information on supplier capability. In the Perfect Information case, the optimal sourcing decision is found by assuming that the supplier ships the given order with a binomial distribution and the supplier capability (reliability) parameter is known by the manufacturer. In the No Information case, the supplier shipment behavior is again binomially distributed but the reliability parameter is not known. By computational experiments, we show that the Bayesian Update approach provides significantly better expected total cost values than the No Information case. Furthermore, the optimal expected costs found by the Bayesian Update approach are close to those found in the Perfect Information case. With the proposed approach, the state space grows faster compared to the Perfect Information and No Information cases with problem size and input data magnitude. For this reason, problems with only moderate size can be solved in reasonable time with this approach. To overcome this computational difficulty, we develop an approach that reduces state space and solves larger problems approximately in reasonable solution time.

ÖZETÇE

Üretim parçalarının tedariğinin üretim planlamasıyla birlikte koordinasyonu sağlanarak yapılması, seri üretim sistemlerinin sürerli devamlılığında önemli bir rol oynamaktadır. Bu tezde çalışılan problemlerle Türkiye'de faaliyet gösteren bir otobüs üreticisinin tedarik zinciri incelenirken karşılaşılmıştır. İlk problem problem sevkiyattaki düzensizlik ile, ikinci problem tedarikçinin değişken sevkiyat miktarı ile ilgilidir. Bu iki tespit için üretim parçalarının sipariş verme kararları eniyilenmektedir.

Tezin ilk kısmında çok dönemli planlama çevreninde, üretim planlamasını sağlamak amacıyla tek tedarikçiden çok sayıda ürünün tedariğini gerçekleştiren üreticiler için sipariş büyüklüğünün belirlenmesi problemi üzerine çalışılmaktadır. Sevkiyat maliyetlerinin üretici tarafından ödendiği sistemlerde, üretici en iyi sipariş çizelgesini belirlerken aynı zamanda sevkiyat çizelgesini de kontrol altında tutmak istiyor. Verilen siparişleri taşımak için kullanılacak araçlar sipariş verildikten hemen sonra gönderilmek yerine, tam dolu olmayan araçların sevkiyatını ertelenebilme seçeneğini göz önünde bulunduruyoruz. Bir sonraki sevkiyat dönemine ertelenmiş olan ürünler için üretici kendi deposunda emniyet stoğu olarak tuttuğu ürünleri kullanabilmektedir. Önerilen politika altında üretici tarafından ödenen sevkiyat ve stokta tutma maliyetlerini enküçükleyen bir karmaşık tamsayı programlama modeli geliştirdik ve bu modeli kullanarak en iyi sipariş ve sevkiyat planını buluyoruz. Kısmi olarak ertelenmiş sevkiyattan doğabilecek maliyet ve hizmet kalitesi analizleri, bir otobüs üreticisinden alınmış verilerle ve benzetim koşumlarıyla yapılmıştır. Talep yüzünden sık ve küçük miktarlarla yapılması gereken sevkiyat sistemlerinde, önerilen politika daha verimli sonuçlar vermektedir.

Tezin ikinci kısmında rassal tedarik ortamında dinamik sipariş büyüklüğünün belirlenmesi problemi üzerinde çalışılmaktadır. Tedarikçi her sipariş döneminde verilen siparişin rassal olarak belirlenmiş bir bölümünü üreticiye göndermektedir. Sipariş sistemini geliştirmek için üreticinin planlama dönemi boyunca verdiği siparişlerin ve elde ettiği miktarların bilgisini kullanarak tedarikçisinin rassallığını tahmin etmeye çalıştığı bir modelleme öneriyoruz.

Bu sebeple dinamik sipariş büyüklüğünün belirlenmesi problemine tedarikçinin yeterliliğini güncelleyen bir Bayes güncellemesi modeli geliştirdik. Üretici planlama döneminin başında tedarikçinin yeterliliği ile ilgili olarak her hangi bir bilgi bilmemektedir. Daha önceki periyotlarda elde edilen başarılı ve başarısız sipariş miktarlarını kullanarak tedarikçinin yeterliliği tahmin edilmeye çalışılıyor. Önerdiğimiz modelin performansının nasıl olduğunu göstermek için Tam Bilgi ve Hiçbir Bilgi durumlarıyla toplam maliyetleri karşılaştırdık. Tam Bilgi durumunda, tedarikçinin gönderdiği miktarlar verilen sipariş miktarı ve tedarikçi yeterlilik parametresine bağlı olarak Binom dağılım ile belirlenmektedir. Tam Bilgi durumunda üretici tedarikçinin yeterlilik parametresini bilmekte ve sipariş miktarlarını ona göre belirlemektedir. Hiçbir Bilgi durumunda gelen miktarlar yine Binom dağılımla belirlenmekte ancak tedarikçinin yeterlilik parametresi üretici tarafından bilinmemektedir. Hesaplamalı deneylerle, önerdiğimiz Bayes güncellemesinin Hiçbir Bilgi durumundan çok daha iyi toplam beklenen maliyet verdiğini göstermekteyiz ve aynı zamanda Bayes güncellemesinin önerildiği model sonuçlarının Tam Bilgi durumuna çok yakın değerler vermektedir. Belirtilen üç durum için de dinamik programlama modelleri çözülmektedir. Önerilen yaklaşım ile dinamik programlamadaki durum kümesinin boyutu problem büyüdükçe Tam ve Hiçbir Bilgi durumlarına göre daha hızlı büyümektedir. Çözüm zamanındaki büyümeyi azaltabilmek için durum kümesini küçülten ve daha büyük problemlerin çözülmesini sağlayan bir yaklaşık çözüm modeli geliştirdik.

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NOMENCLATURE

BU	Bayesian Update
DP	Dynamic Programming
DTP	Delayed Transportation Policy
JIT	Just In Time
LTL	Less-than-full Truck Loads
MILS	Multi-Item Dynamic Lot-Sizing
MILS-DT	Multi-Item Dynamic Lot-Sizing with Delayed Transportation
MIP	Mixed Integer Programming
NI	No Information
PA	Percentage Approximation
PI	Perfect Information
TC	Total Cost

Chapter 1

INTRODUCTION

The need to coordinate the shipment of parts from suppliers with the production schedule has long been recognized by manufacturing companies. The smooth continuation of production is especially critical in mass production lines, such as those in the automotive and electrical home appliances industries, where the absence of a single part at the required time may lead to halts in the manufacturing process. It is important for the production that the purchased manufacturing parts are received on time and desired amount. Hence, manufacturers focus on controlling both their parts inventory and the shipment of their orders from the suppliers, in order to receive the required parts on time and in requested amounts, with minimum costs. Frequent shipments from suppliers lead to high transportation costs, whereas ordering in large quantities incurs high holding costs and may even be prohibited due to space constraints. As the number of end product types increases, these costs accumulate to significant figures and the underlying planning problem gets more complicated.

The relationship between manufacturers and suppliers are studied in the supply chain management problems a lot. In the first part of the thesis, we study the multi-item dynamic lot sizing problem for a manufacturer that sources parts from a single supplier over a multi-period planning horizon. We have a manufacturer that tries to source its manufacturing parts from a single supplier. According to the contract between the manufacturer and the supplier, the transportation costs of parts shipped from the supplier to the manufacturer are charged to the manufacturer. Since the transportation costs are not paid by the supplier, the supplier does not execute an efficient shipment plan and this causes a cost burden on the manufacturer. For this reason, the manufacturer wants to control its suppliers' delivery process in addition to its own parts ordering process. While deciding on the order quantities, the manufacturer also determines the shipment plan of parts from the supplier to the manufacturer. In the literature, there are lots of studies that combine inventory management and transportation. A detailed literature review on this topic is given in Section 2.1.

For the second chapter of the thesis we study dynamic lot sizing problem under random supply. We consider a stochastic environment where the amount of given order will not match with the received amount. According to its production plan the manufacturer tries to source its manufacturing parts with dynamic lot sizing. However, the supplier is not reliable and cannot send the entire amount ordered. There is a random difference between ordered and received amounts at each period an order is given. Because of this uncertainty in the supply, the manufacturer will face stockouts which is not a desired situation in the production systems. For this reason, the manufacturer should adjust its ordering plan so that he/she can deal with the uncertainties in the system. One way of dealing with the supply uncertainty is ordering more than the demand according to the distribution of randomness. A detailed literature review on the supplier uncertainty is given in Section 3.1.

Chapter 2

MULTI-ITEM DYNAMIC LOT-SIZING WITH DELAYED TRANSPORTATION POLICY

2.1 Introduction

In this chapter of the thesis, we propose a sourcing policy for the multi-item dynamic lot sizing problem.

In general, two kinds of purchasing situations exist: procurement from a single source (supplier) or multiple sourcing. In our setting, a manufacturer works with multiple suppliers but sources each part from a single supplier with whom a long-term relationship has been established. As opposed to Just-In-Time (JIT) sourcing, we consider a push approach with an MRP system. The manufacturer orders multiple parts to be used in production, from a single supplier over a multi-period planning horizon with known demands. When the manufacturer places an order with a supplier that consists of different quantities of multiple parts with given due dates, the supplier arranges a shipment plan. The supplier either takes care of the shipment using its own fleet, or as a common practice, it utilizes a third party logistics service provider. The parts that are to be shipped in the same period are packed into trucks and are directly shipped to the manufacturer. Here, we study the case where the incurred transportation cost is charged to the manufacturer. The transportation cost consists of a fixed cost per truck plus a variable cost that depends on the distance traveled, and thus, can be represented by a per truck per trip cost. The problem we focus on is to find a cost-efficient joint transportation and ordering policy for the manufacturer.

We encountered the problem we study while analyzing the sourcing operations of a leading coach bus manufacturer in Turkey, and since then have found it to be a common issue with manufacturers that have to pay for the costs of shipments from the suppliers. The bus manufacturer had been experiencing several problems with its local suppliers. Most local suppliers lack an organized delivery system. Some are even unaware of their shipping quantities and mostly ship their parts several times in a day in less-than-full truck loads to the manufacturer. While this frequent shipment has clear advantages in terms of inventory costs, it causes a significant financial burden on the manufacturer in the long run. In order to operate more efficiently, the manufacturer wanted to control its suppliers' delivery process in addition to its own parts ordering process.

The savings in total costs realized from optimization and coordination of inventory control and transportation planning in a multi-item inventory system motivated many researchers to analyze the joint problem in both retailing and production settings. We provide a classification of the studies in this area in Figure 2.1. Retail chain models differ from the ones in the manufacturing context mainly with respect to the demand characteristics while coordinating the ordering decisions with inbound or outbound logistics. In retail chain models, demand is either considered to be uncertain (for example, Cardos and Garcia-Sabater [2006]) or occurs at a constant rate (for example, Speranza and Ukovich [1994]). On the other hand, models studying the sourcing of manufacturing parts differ in pull and push systems. For example, Ben-Khendger and Yano [1994] proposed a heuristic solution procedure for the problem of scheduling the delivery of multiple items from a single supplier to a manufacturer in a JIT context. There is a fixed cost per truck and inventory holding cost on end-of-period inventory. Our focus in this study is on push systems, and as such, the ordering decisions are made for a multi-period planning horizon with given demand values.

Multi-period lot-sizing has been one of the most frequently studied problems in production and inventory management literature since Wagner and Whitin [1958]. Its extension to the case of multiple-items that are shipped together has also attracted significant interest, especially in the context of optimizing supply chain operations. As seen in Figure 1, the shipping can be in the form of 1) direct shipment from a single supplier, or 2) consolidation. In the latter, items at multiple suppliers are collected by vehicle routes. Thus, such sourcing problems fall under the extensively-studied inventory-routing problem (see Campbell et al. [1998] for a review on the subject).

Here, we focus on the case of single sourcing with direct shipment of the items from the supplier and review studies that fall under this category. The shipment costs have been modeled in various ways according to the contracts between the supplier and the manufacturer. Lee et al. [2002] analyzed the dynamic lot-sizing model with pre-shipping and late-shipping options, where the transportation costs are stepwise cargo cost functions. They character-

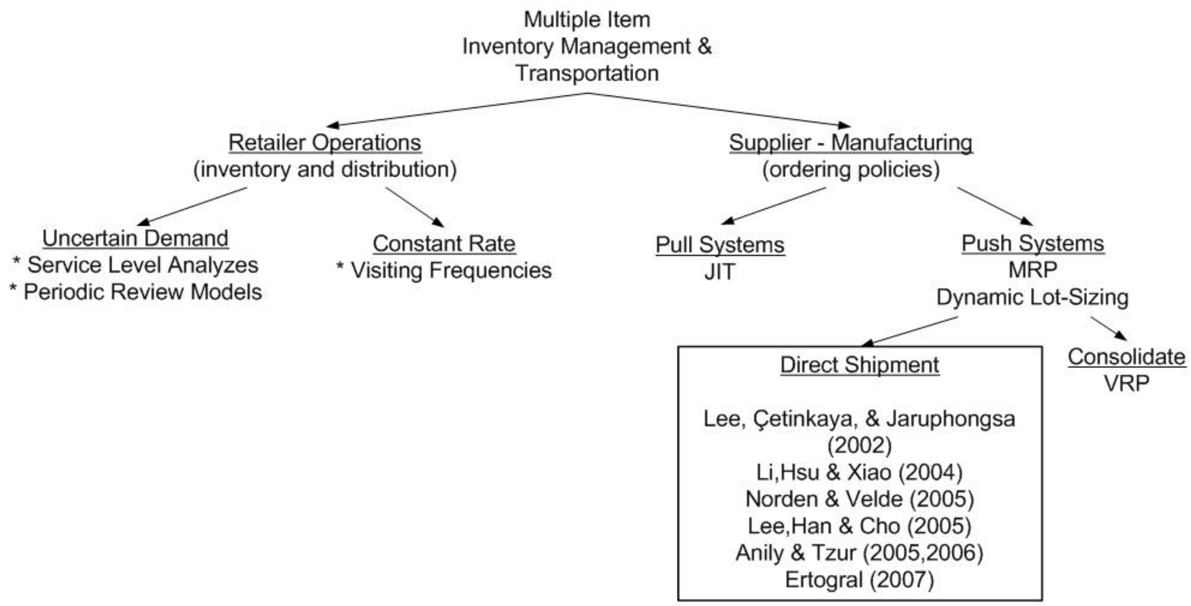

Figure 2.1: A classification of studies on multi-item transportation and inventory management

ized the properties of the problem and developed a polynomial time algorithm to compute the optimal solution. Li et al. [2004] developed a solution method for the lot-sizing problem where the cost structure includes a fixed charge for each order, a variable unit purchase cost, and a delivery cost with a truck load discount structure. Norden and Velde [2005] provided an integer linear programming formulation and developed a Lagrangian relaxation algorithm for the multi-product lot-sizing problem under a transportation capacity reservation contract with direct shipping. Lee et al. [2005] analyzed a dynamic lot-sizing problem in which the order size of multiple products is optimized to minimize the sum of production, inventory holding, and freight costs that are proportional to the number of containers used. A heuristic algorithm with an adjustment mechanism based on the properties of the optimal solution is proposed.

Several other studies with direct shipment of items have generalized the classical lot-sizing problem by making the fixed ordering cost dependent on the lot-size. Anily and Tzur [2005] studied the sourcing of multiple-items from a warehouse or a plant by a retailer. They assumed that items of identical size are packed into identical capacitated vehicles that incur a fixed cost per trip. Given dynamic deterministic demand of items over a planning horizon, they developed a dynamic programming algorithm that generates an optimal shipment plan. The proposed algorithm has polynomial time complexity for a fixed number of items. Consequently, Anily and Tzur [2006] proposed a different approach for the same problem: a search algorithm that generates different shipment schedules, and a shortest-path approach that selects a schedule among those. Ertogral [2008] suggested a Lagrangian decomposition based solution procedure for the multi-item uncapacitated dynamic lot-sizing problem with transportation costs. The transportation cost is assumed to be a piecewise linear function of the amount transported. Cost savings are observed with the integrated approach through numerical experiments.

In this chapter of the thesis, we study a multi-item lot-sizing problem where the shipments are from a single supplier to a manufacturer, and per truck-per trip transportation costs are incurred to the manufacturer. This problem is similar to the ones studied in Anily and Tzur [2005], Anily and Tzur [2006] and Ertogral [2008]. Here, we develop a Mixed Integer Programming (MIP) model to determine the order quantities that minimize total transportation and inventory holding costs over a multi-period planning horizon with given time-varying deterministic demand values. We propose and test a shipment strategy called the delayed transportation policy with the goal of utilizing truck capacities more efficiently, and optimize the order quantities under this policy. In finding the number of trucks needed, we do not assume uniform part size as in Anily and Tzur [2005] and Anily and Tzur [2006] but calculate the number of pallets required for each item based on its size. However, we still ignore the associated truck packing problem which would give a better estimate of the number of trucks needed.

Section 2.2 describes the problem and the developed mathematical model. In Section 2.3, benefits and risks of the delayed transportation policy are demonstrated by computational tests.

2.2 Lot-sizing under the Delayed Transportation Policy

The goal of the proposed policy is to optimize the order quantities so that the truck capacities are utilized efficiently. When an order is given to a supplier for the shipment of several items, these items are packed into trucks in pallets. Depending on the amounts to be shipped and the truck capacity, less-than-full truck loads (LTL) may arise frequently, while the full truck cost must be paid. To increase the truck load percentages, we consider delaying transportation of a portion of the order to the next period. That is, instead of shipping the required number of trucks whenever an order is given in a period, if a small truck load is left, we allow delaying the shipment of those items to the next period and keep them at the supplier. At the same time, we maintain a minimum safety stock of each part at the manufacturer so that the production plan is not disrupted due to the delays. We refer to this policy as the *Delayed Transportation Policy (DTP).*

2.2.1 Assumptions

Two main assumptions made for the sake of simplicity are the zero transportation lead time and volume based truck packing assumptions.

The proposed policy is suitable for a system with small transportation times. The manufacturer and its supplier are in close proximity to each other in the bus manufacturer case mentioned above. For this reason, the transportation lead time is assumed to be zero in the model. When the manufacturer puts in an order, the supplier sends the whole amount to be shipped with zero lead time because of the short distance and the make-to-stock strategy of the supplier.

The second assumption is about the packing of items into the trucks. Instead of modeling the three-dimensional packing of items and pallets, we model the utilization of the trucks by dividing the truck capacity by the volume of the pallets. This is a common assumption in similar work in the literature (for instance, Ben-Khendger and Yano [1994] and Ertogral [2008]). Each product is packed into a specific pallet type and how many products fit into its corresponding pallet type is known. The maximum number of a specific pallet type that can be packed into one truck is also known. By using these numbers, we calculate the number of trucks needed to carry the total order in a period.

2.2.2 Mathematical Models with and without DTP

We have per trip per truck shipment cost and holding cost, the initial stock level, pallet and truck capacities, and demand of each part in each period as inputs to our model. By using these parameters we calculate the optimal number of parts to be shipped, the inventory levels, the numbers of trucks to be used, and delayed amount for each product in each period. At each period, the model calculates the number of trucks needed with and without delayed transportation option. The mathematical model for the Multi-Item Dynamic Lot-Sizing Model with Delayed Transportation policy (MILS-DT) is given below.

Sets:

Set of parts I

Set of periods N

Decision variables:

x_{in}: number of units of part i ordered from the supplier at period n

y_{in}: number of units of part i stored in the manufacturer's warehouse at period n

z_n: number of trucks sent from supplier to the manufacturer at period n by using delayed transportation

f_n: number of trucks that would be sent from supplier to manufacturer at period n without considering delayed transportation

t_{in}: number of pallets for part i used at period n with delayed amount of period n

g_{in}: number of pallets for part i used at period n without delayed amount of period n

b_{in}: number of units of part i delayed at period n

Parameters:

p_i: number of part i per corresponding pallet type

r_i: number of pallets of part i that fit into one truck

c: cost of one truck

h_i: holding cost of part i in manufacturer's warehouse

S_i: safety stock for part i in manufacturer's warehouse

d_{in}: demand for part i at period n

I_i: initial stock for part i at the beginning of the planning horizon

Objective Function:

$$min \sum_{n=1}^{N} \left((cz_n) + \sum_{i=1}^{I} (y_{in} + b_{in}) h_i \right) . \tag{2.1}$$

s.t.

$$y_{in} = I_i \qquad \text{for all } i \in I\ , \tag{2.2}$$

$$b_{i0} = 0 \qquad \text{for all } i \in I\ , \tag{2.3}$$

$$b_{iN} = 0 \qquad \text{for all } i \in I\ , \tag{2.4}$$

$$y_{in} = y_{i(n-1)} + x_{in} - d_{in} - b_{in} + b_{i(n-1)} \qquad \text{for all } i \in I, n \in N\ , \tag{2.5}$$

$$b_{in} + y_{in} \geq S_i \qquad \text{for all } i \in I, n \in N\ , \tag{2.6}$$

$$g_{in} \geq \frac{x_{in} + b_{i(n-1)}}{p_i} \qquad \text{for all } i \in I, n \in N\ , \tag{2.7}$$

$$f_n \geq \sum_{i=1}^{I} \frac{g_{in}}{r_i} \qquad \text{for all } n \in N\ , \tag{2.8}$$

$$f_n \leq \sum_{i=1}^{I} \frac{g_{in}}{r_i} + 1 \qquad \text{for all } n \in N\ , \tag{2.9}$$

$$b_{in} \leq x_{in} + b_{i(n-1)} \qquad \text{for all } i \in I, n \in N\ , \tag{2.10}$$

$$\sum_{i=1}^{I} \frac{b_{in}}{p_i r_i} \leq \Big(1 - \Big(f_n - \sum_{i=1}^{I} \frac{g_{in}}{r_i}\Big)\Big) \qquad \text{for all } n \in N\ , \tag{2.11}$$

$$t_{in} \geq \frac{x_{in} + b_{i(n-1)} - b_{in}}{p_i} \qquad \text{for all } i \in I, n \in N\ , \tag{2.12}$$

$$z_n \geq \sum_{i=1}^{I} \frac{t_{in}}{r_i} \qquad \text{for all } n \in N\ , \tag{2.13}$$

$$z_n \leq \sum_{i=1}^{I} \frac{t_{in}}{r_i} + 1 \qquad \text{for all } n \in N\ , \tag{2.14}$$

$$x_{in}, y_{in}, z_n, f_n, t_{in}, g_{in}, b_{in} \geq 0 \qquad \text{for all } i \in I, n \in N\ , \tag{2.15}$$

$$b_{in}, x_{in}, y_{in}, z_n, f_n, t_{in}, g_{in} \text{ integers} \qquad \text{for all } i \in I, n \in N\ , \tag{2.16}$$

In the model, Constraints (2.2) set the initial stock for part i. Constraints (2.3) and (2.4) set the numbers of part i delayed at the initial and final periods to zero. Constraints (2.5) are the inventory balance equations, and calculate the number of part i stored in the manufacturer's stock in each period by using the previous period's inventory level, the number of part i shipped in that period, demand for part i and the delayed amount (if there is any). This is illustrated in Figure 2.2, where the material flow of part i in the n^{th} period is represented.

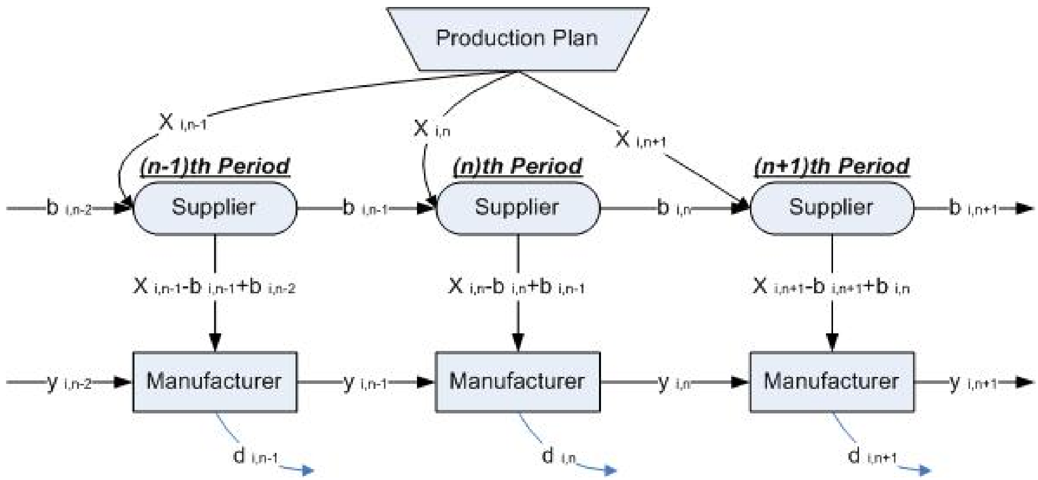

Figure 2.2: Representation of material flow for part i

Constraints (2.6) set the sum of stocks at manufacturer and delayed amount at the supplier to the safety stock limits so that the delayed amount can be used from the safety stock at the manufacturer. In constraints (2.7), the number of pallets for each part in a given period is calculated. The delayed amount of the previous period is also considered while calculating this number. Constraints (2.8) and (2.9) are used to find the number of trucks needed for the demand in period n without considering delaying. Constraints (2.10) set an upper bound for the maximum amount of delayed parts. The delayed amount cannot be more than the amount ordered in this period plus the delayed amount from the previous period. In Constraints (2.11), the total delayed amount is set to be less than or equal to the partial truck load. Constraints (2.12) calculate the number of pallets for each part in a given period while considering the delayed amount of previous period and current period. Constraints (2.13) and (2.14) are used to find the number of trucks needed for the demand in period n with delayed transportation policy.

The model described above is used for the Multi-item Dynamic Lot-sizing Model with Delayed Transportation Policy. A simplified model can be used to solve the same problem without allowing delayed transportation. This model, which we call Multi-item Dynamic Lot-sizing Model (MILS), is obtained by making the following modifications to the MILS-DT model. The decision variables b_{in}, z_n and t_{in} are removed. The objective function is changed as:

$$min \sum_{n=1}^{N} \left((cz_n) + \sum_{i=1}^{I} y_{in} h_i \right) . \tag{2.17}$$

Constraint set (2.5) is changed as:

$$y_{in} = y_{i(n-1)} + x_{in} - d_{in} . \tag{2.18}$$

Constraint set (2.6) is changed as:

$$y_{in} \geq S_i . \tag{2.19}$$

Constraint set (2.7) is changed as:

$$g_{in} \geq \frac{x_{in}}{p_i} , \tag{2.20}$$

and Constraints (2.3), (2.4), (2.10), (2.11), (2.12), (2.13), and (2.14) are removed from the model. In this form, this model solves a multiple-item lot sizing problem, where the setup costs are calculated with respect to the order quantities by figuring out how many trucks would be needed to ship the order amount in a period. Note that Florian et al. [1980] proved that this problem is NP-hard, which implies that the more general version with delayed transportation is also NP-hard. However, in our computational tests we found both models could be solved within reasonable time by a commercial solver for instances of realistic size. Hence, we were able to analyze the DTP computationally under different settings as presented in the next section.

2.3 Analysis of the Delayed Transportation Policy

In this section, we analyze the benefits and risks of using the delayed transportation policy numerically with a test bed of problem instances generated from the data of the bus manufacturer. We first investigate the benefits of adopting DTP under deterministic time-varying demand and explain the reasons for the cost savings. We then conduct computational experiments to evaluate the potential risks of adopting the proposed policy by evaluating the model solution in terms of service levels in various random settings such as uncertain demand pattern, contingency changes in the production plan, and an unreliable supplier with uncertain supply quantities. Next, we discuss how DTP can be implemented in practice, and test several versions of the policy for practical purposes. Finally, we investigate the solution of a capacitated version of the multi-item dynamic lot-sizing model with DTP.

Our test instances are based on the problem faced by the coach bus manufacturer in Turkey. The manufacturer refers to its suppliers that are within 150 kilometers distance to its production facility as the local suppliers and it works with approximately 60 local suppliers. For each supplier, the number of products sourced varies between 50 and 150. We obtained weekly demand data for 50 class A and class B items sourced from a problematic local supplier over a year. Having observed that part demands do not show seasonality or trend over the one-year period, we also generated instances with a planning horizon of 12 weeks.

The manufacturer has the policy of keeping one week's average demand as safety stock to avoid stock-outs in case of unexpected events. The holding costs are calculated on the basis of 20 % annual holding rate, and transportation cost is charged at 1 per kilometer travelled by each truck. In total we have 52 periods of demand data for 40 products, and we generated different data sets from this data by grouping it by products and periods. We divided the 40 products to 4 groups of 10 products randomly to obtain the 10 product instances. Similarly, we divided the 40 products randomly into 2 groups of 20 products. Thus, we obtained the instances with 52 weeks and 10, 20, 40 products listed in Table 2.1. Furthermore, we generated instances with a 12 week planning horizon by dividing the yearly demand data into four quarters. In Table 2.1, an instance type is defined for all instances with the same problem size and the total number of instances with the given problem size is listed.

Table 2.1: Test instances

Number of Products	**Number of Periods**	**Number of Instances**	**Instance Type**
10 Products	12 weeks	16	10P12N
20 Products	12 weeks	8	20P12N
40 Products	12 weeks	4	40P12N
10 Products	52 weeks	4	10P52N
20 Products	52 weeks	2	20P52N
40 Products	52 weeks	1	40P52N

Instances with 10, 20, and 40 products, and 12 periods could be solved to optimality with GAMS IDE (22.4) on a PC-platform using CPLEX 10.2 with default settings. The model solution with 20 items and 12 periods took on the average 16 seconds, and at most 26 seconds. In order to investigate how the model scales up, we solved instances with 40, 20, 10 items and 52 periods to optimality and found that the computation time was around 300 minutes on the average, with a maximum run time of 10 hours.

2.3.1 Benefits of the DTP

In the DTP, the delayed amounts are produced by the supplier in a given period but not shipped to the manufacturer to reduce the transportation costs. They are ready to be shipped in the next period and holding costs for these items are paid by the manufacturer. Since we know the amounts delayed are ready in the supplier stocks and they can be sent whenever requested in a short time, the manufacturer may utilize items kept in its safety stock instead of the delayed items. The amount used in this way from the safety stock is bounded by the delayed quantity.

To explain how cost savings are realized, let us consider a problem where 2 products are sourced from a supplier. The MILS and MILS-DT models are solved for an 8 period horizon with data and results reported in Table 2.2. The *Demand* row shows the requirements for products 1 and 2 in parentheses. The *Shipping Qts* row gives the optimal shipment quantities of the two products for the given model. In the # *Trucks Used* row, the number of trucks needed to carry the shipment amounts is given and the *% Load* row shows the truck utilizations for each period. For example, 2.25 in this row means that 2 trucks are fully loaded and one truck is 25 % loaded. The *Inv @Manuf* and *Inv @Supplier* rows indicate where the inventory is held in each period. For MILS model, all of the inventory is held at the manufacturer, whereas in MILS-DT we allow to hold some of the inventory at the supplier's warehouse if there is delaying in that period. For instance, for the 2^{nd} period, 10 units of product 1 and 15 units of product 2 are stored at the manufacturer's warehouse according the MILS model solution. In the MILS-DT model solution, 15 units of product 2 are stored at the manufacturer's warehouse, but 10 units of product 1 are stored at the supplier's warehouse. The total cost for the MILS and MILS-DT model solutions are €1649.13 and €1509.09, respectively. The relative cost saving from the proposed policy is about 8.5 %.

Table 2.2: Solutions of MILS and MILS-DT for 2 products, 8 periods example

	Periods							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Demand	(20,25)	(40,35)	(30,30)	(20,20)	(20,10)	(30,30)	(30,20)	(20,20)
	Optimal Plan for MILS							
Shipping Qts	(30,45)	(40,30)	(30,35)	(20,15)	(20,15)	(32,36)	(28,9)	(20,10)
# Trucks Used	3	2	2	1	1	2	1	1
% Load	2.25	2	1.92	1	1	2	1	0.83
Inv @ Manuf	(10,20)	(10,15)	(10,20)	(10,15)	(10,20)	(12,26)	(10,15)	(10,15)
Inv @ Supplier	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
	Optimal Plan for MILS-DT							
Shipping Qts	(28,39)	(32,36)	(40,30)	(16,18)	(24,12)	(20,15)	(40,30)	(20,15)
# Trucks Used	2	2	2	1	1	1	2	1
% Load	2	2	2	1	1	1	2	1
Inv @ Manuf	(8,14)	(0,15)	(10,15)	(6,13)	(10,15)	(10,10)	(10,15)	(10,15)
Inv @ Supplier	(2,1)	(10,0)	(0,0)	(4,2)	(0,0)	(10,15)	(0,5)	(0,0)

We investigate the cost savings that are realized with the DTP by comparing the solutions to the MILS and MILS-DT models for our test instances. The two models are solved to optimality and the average, minimum and maximum values of the transportation, holding and total cost of the optimal solutions over instances of the same problem size are reported in Table 2.3.

In Table 2.3, the *Holding Cost*, *Transportation Cost*, and *Total Cost* columns give the average values for each instance type, for both MILS-DT and MILS models. The *Average Relative Difference* column shows the percentage cost saving from the proposed policy, whereas the *Min Relative Difference* and the *Max Relative Difference* columns show the minimum and maximum relative difference over all instances of the same type. In all cases there is a significant cost saving from using DTP, but when a small number of products is ordered from the supplier, the DTP is more advantageous. As the number of products increases, the options for grouping the items ordered in the same period into trucks increases and thus products can be packed into trucks more efficiently in the MILS. Hence, the

Table 2.3: Results of MILS and MILS-DT model solutions

Instance Type	**Model**	**Holding Cost**	**Transportation Cost**	**Total Cost**	**Relative Differences Average**	**Min**	**Max**
	MILS	€1,307	€1,069	€2,376			
10P12N	MILS-DT	€1,240	€966	€2,206	7.15%	3.38%	13.38%
	MILS	€2,362	€1,819	€4,181			
20P12N	MILS-DT	€2,199	€1,781	€3,980	4.81%	2.38%	7.45%
	MILS	€4,283	€3,600	€7,883			
40P12N	MILS-DT	€4,207	€3,563	€7,770	1.44%	0.43%	2.44%
	MILS	€5,526	€5,526	€11,052			
10P52N	MILS-DT	€5,411	€4,826	€10,237	7.38%	6.93%	12.16%
	MILS	€9,631	€7,725	€17,356			
20P52N	MILS-DT	€8,949	€7,500	€16,449	5.22%	3.99%	7.67%
	MILS	€17,708	€14,850	€32,558			
40P52N	MILS-DT	€17,168	€14,700	€31,868	2.12%	2.12%	2.12%

small truck loads disappear and the opportunity for transportation cost savings with DTP decreases. When we compare the solutions of MILS-DT and MILS models, we see that DTP provides a 0.5 % to 13.5 % reduction in total cost over all instances. In all the runs, delayed transportation was advantageous, with gains ranging between around €100 to €440 for 12 week periods. Although this may seem to be small at first sight, if the proposed model is applied to all suppliers and their all products, the savings will accumulate to significant amounts. The manufacturer considered in this study has around 60 local suppliers providing on the average around 100 products each. If we realize similar savings for each supplier, the total gain is estimated to be around €150,000 per year. Since in the automotive and electrical home appliances sectors, the number of suppliers and parts supplied from each supplier are typically quite large, potential savings of using this policy may be significant.

The results in Table 2.3 also show that increasing the planning horizon provides slightly more cost savings from DTP, but to save from computation time, we used the 12 week instances only in the further analyses.

2.3.2 Risks of the DTP

In the proposed policy, since safety stock is used in place of delayed items, this may create risks in a random setting or when changes occur in the production plan. Although the demand for the parts in the manufacturer's system is essentially deterministic demand, the manufacturer keeps safety stocks against possible variability in demand and contingencies such as unexpected changes in the demand pattern as well as uncertainty in supplied quantities. To see how the proposed policy affects the service levels, we observe the stock-out percentages when MILS-DT and MILS solutions are subject to these random changes by a simulation study. Three types of randomness analysis are conducted: 1) demand uncertainty, 2) contingency changes in the production plan, and 3) supplier unreliability.

Demand Uncertainty

The demand uncertainty will not be observed frequently in a stable mass production system, such as the one pertinent to the coach bus manufacturer in Turkey, as the demand is predictable and the production quantities of the end items are fixed for a predefined planning horizon. However, because of the unforeseen changes the manufacturer may need to alter its production plan. Such changes will reflect on the demand for parts and components used in production. In this analysis, we investigate the robustness of the order schedule under random variations in demand by a simulation study.

We keep the ordering and transportation policy given in the model solutions and measure the Type I and Type II service levels under demand scenarios generated randomly. Note that in Type I service level the total number of periods with a stock-out situation is taken into account, and in Type II service level the total number of items that are stocked out is calculated (Nahmias [2005]). The following procedure is used to obtain the results shown in Table 2.4.

The Procedure for Evaluating the Robustness of the Solution under Demand Uncertainty

Step 1: Given an instance, for each product i define a truncated normal distribution for demand in a period by setting the mean, μ_i, to the average demand of product i over 52 weeks and the variance, ${\sigma_i}^2$, to the sample variance of 52 weeks' demand quantities.

Step 2: Generate K demand matrices, $D^k = (d_{in})^k$ for $k = 1, \dots, K$, with a size of 10 products and 12 periods, randomly using the demand distributions defined in Step 1.

Step 3: For $k = 1, \ldots, K$:

Step 3.1: For each demand matrix D^k, solve the MILS and MILS-DT models and report the k^{th} optimal shipment schedules.

Step 3.2: For each demand matrix D^k, generate N demand matrices from the distributions defined in Step 1. We denote each such matrix as ${D_n}^k$ for $n = 1, \ldots, N$.

Step 3.3: For $n = 1, \ldots, N$:

Step 3.3.1: Evaluate the performance of the k^{th} optimal solutions of MILS and MILS-DT under demand given as ${D_n}^k$:

Step 3.3.1.1: Calculate the positive or negative inventory in each period.

Step 3.3.1.2: Calculate the number of periods with a stock-out situation.

Step 3.3.1.3: Calculate the Type I and Type II service levels.

Step 3.4: Report average Type I and Type II service levels over all N scenarios for the k^{th} MILS and MILS-DT solutions.

Step 4: Report the average service levels of MILS and MILS-DT over all K demand matrices.

Table 2.4: Service levels for MILS and MILS-DT under demand uncertainty

	Service Level Type I			**Service Level Type II**		
	MILS	**MILS-DT**	**Difference**	**MILS**	**MILS-DT**	**Difference**
K=1	70.95%	65.71%	5.24%	93.46%	91.62%	1.84%
K=2	53.85%	50.88%	2.97%	86.26%	84.63%	1.63%
K=3	64.29%	60.83%	3.46%	90.45%	88.66%	1.79%
K=4	53.81%	51.49%	2.32%	85.75%	84.69%	1.06%
K=5	60.62%	55.44%	5.18%	89.43%	87.02%	2.41%
K=6	69.24%	67.55%	1.69%	92.17%	91.49%	0.68%
K=7	62.05%	56.62%	5.43%	89.43%	86.59%	2.84%
K=8	67.88%	64.87%	3.01%	90.21%	89.16%	1.05%
K=9	75.68%	71.83%	3.85%	94.99%	93.86%	1.13%
K=10	70.98%	66.36%	4.62%	93.93%	92.32%	1.61%
Average	64.94%	61.16%	3.78%	90.61%	89.00%	1.60%

In our analysis, K and N are set to 10 and 100, respectively. From the results given in Table 2.4, we see that MILS-DT solutions perform 3.78% and 1.6% worse on the average in terms of Type I and Type II service levels, respectively, compared to the corresponding MILS solutions. The results show that applying DTP decreases both types of service levels, but the differences are quite small, despite large demand variability as observed from the low service levels. In practice, demand variability will most likely be less than it is in the simulations. Furthermore, in cases where the distance between the manufacturer and the supplier is short, the delayed amount can be shipped immediately to the manufacturer when a stock-out occurs. In conclusion, the risks of DTP will most likely be at tolerable levels and could be weighed against the cost savings by a numerical analysis.

We next investigate the cost and service level differences in the MILS and MILS-DT models under varying safety stock levels. In practice, typically, the safety stock for each product is set with respect to demand variability during lead time or variance of the forecast error. In Figure 2.3, total costs and Type II service levels versus safety stock amounts are plotted. The X-axis in this graph shows the given safety stock amount for the manufacturer and the Y-axis is for the cost and service level values. For each level of the safety stock, which is a predefined parameter for the manufacturer, total costs and Type II service levels are calculated for both MILS and MILS-DT solutions. We observe that the difference in the total cost increases until the safety stock reaches a certain value. After that point increasing the safety stock level is not cost-advantageous for the manufacturer as the increase in holding costs dominates the cost saving from the proposed policy.

Contingency Changes in the Production Plan

Revisions in the planned order release values for parent items may affect the requirements for an item. We try to capture this kind of demand variation with lead time changes (shifts in demand to adjacent periods) or quantity adjustments (increase or decrease of demand quantity in a period) without changing the total demand of an item over the planning horizon. We created such changes in the demand pattern randomly and used the following procedure to evaluate service levels for both MILS and MILS-DT model solutions.

The Procedure for Evaluating the Robustness of the Solution under Contingency Changes

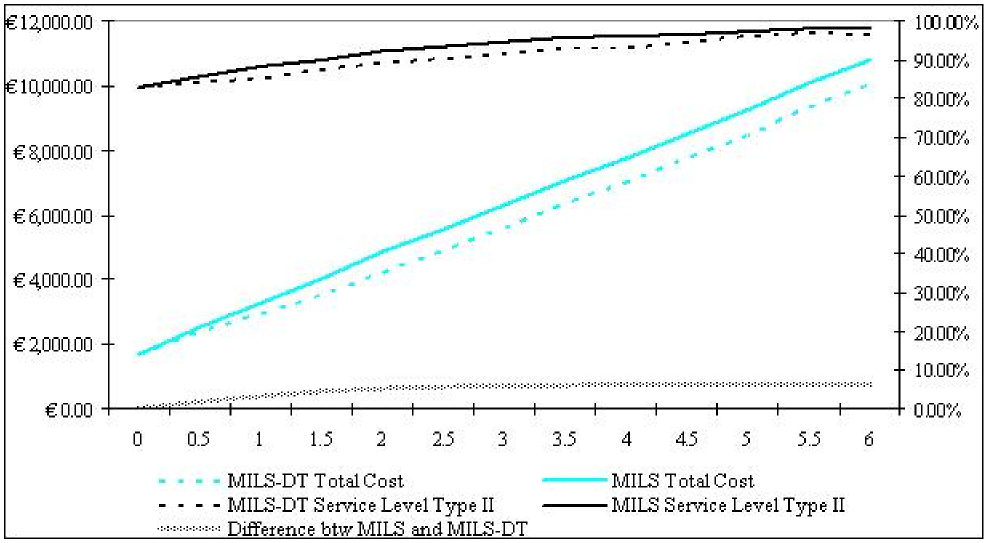

Figure 2.3: Total cost and type II service level vs. safety stock amount graph

Step 1: Define p_{sp}, p_{sn}, p_i, and p_d values between 0 and 1, according to the environment to be tested.

Step 2: For each 10P12N instance:

Step 2.1: Obtain the optimal solutions of MILS and MILS-DT models.

Step 2.2: Perturb the demand data to generate N scenarios with demand matrix D_n, $n = 1, \ldots, N$, according to the following rules:

Shift the demand of a given period to the previous period with probability p_{sp}.

Shift the demand of a given period to the next period with probability p_{sn}.

Increase the demand of each period by 20 % with probability p_i.

Decrease the demand of each period by 20 % with probability p_d.

Step 2.3: For $n = 1, \ldots, N$:

Step 2.3.1: Evaluate the performance of both optimal solutions of MILS and MILS-DT models in each scenario n, using order amounts from the optimal solutions and D_n.

Step 2.3.1.1: Calculate the positive and negative inventory amounts.

Step 2.3.1.2: Calculate the number of periods with a stock-out situation.

Step 2.3.1.3: Calculate Type I and Type II service levels.

Step 2.4: Calculate the average Type I and Type II service levels over all N scenarios.

Step 3: Report the average, minimum and maximum service level differences over all 10P12N instances.

In our tests, we used 3 levels of demand perturbation by setting the probability parameters as follows. Level 1 has the lowest probability values; p_{sp} and p_{sn} are set to 0.05, while both p_i and p_d are set to 0.1. For level 2, p_{sp} and p_{sn} are set to 0.1; p_i and p_d are set to 0.2. Finally, for level 3 we set p_{sp} and p_{sn} to 0.2; p_i and p_d to 0.4. The above procedure is applied for each level and for 16 instances of 10P12N with N equals to 100. The results are given in Table 2.5.

Table 2.5: Service levels for MILS and MILS-DT solutions under demand perturbation

		Service Level Type I			**Service Level Type II**		
		MILS	**MILS-DT**	**Difference**	**MILS**	**MILS-DT**	**Difference**
	Average	97.64%	89.80%	7.83%	99.77%	99.26%	0.52%
Level 1	**Min**	96.96%	92.83%	4.13%	99.70%	99.49%	0.21%
	Max	98.11%	84.83%	13.28%	99.82%	98.93%	0.89%
	Average	94.76%	84.52%	10.24%	99.51%	98.64%	0.87%
Level 2	**Min**	92.75%	87.47%	5.28%	99.28%	98.93%	0.35%
	Max	95.18%	77.84%	17.34%	99.55%	98.08%	1.47%
	Average	89.80%	78.59%	11.21%	98.96%	97.62%	1.34%
Level 3	**Min**	86.94%	81.23%	5.71%	98.58%	98.08%	0.50%
	Max	89.03%	70.14%	18.89%	99.08%	96.95%	2.13%

When we observe the percentage drop in the service level with MILS-DT in Table 2.5, we see that a difference of minimum 4.1% and maximum 18.9% exists for Type I service level, whereas the differences are much smaller for Type II. As the level of the perturbations increases, Type I service level decreases much faster than Type II service level, but the difference between the MILS and MILS-DT models does not increase as fast. The difference between the two models for Type I service level is at most 11.21% on the average. Note that this analysis measures how the initial optimal solution will perform under the stochastic environment. In real life, the shipment schedule would most likely be modified after several demand changes are experienced and a more favorable service level would be realized.

Supplier Unreliability

The received amount from the supplier may not be exactly the amount ordered because of factors such as capacity and production scheduling restrictions at the supplier, or unexpected disruptions at the supplier. We investigate how DTP will be affected when amounts received from the supplier may differ from the amounts ordered. We simulate a stochastic environment for the unreliable supply pattern as follows. There are 10 levels of unreliability and one fully reliable case (level 10). In each level $k = 0, 1, \ldots, 9$, with probability 0.5 the supplier sends all of the order quantity for each product at the given period, and with probability 0.5 a random percentage, which we refer to as the shipping percentage is shipped to the manufacturer. The shipping percentage is uniformly distributed between ($10k$, 100). We generate different shipping percentages for each product in the same period coming from the same distribution. Level 10 represents a 100% reliable supplier and we receive exactly the amount we order. In this stochastic environment, we test DTP by evaluating Type I and Type II service levels by means of a simulation study.

The Procedure for Evaluating the Robustness of the Solution under Supplier Unreliability

Step 1: For each of the 10P12N instances, simulate N supply pattern scenarios for each reliability level k, $k = 1, \ldots, 10$.

Step 2: Evaluate the performance of the optimal solutions of MILS and MILS-DT models in each scenario n, $n = 1, \ldots, N$, for each reliability level k, $k = 1, \ldots, 10$ by calculating Type I and Type II service levels.

Step 3: Calculate the average Type I and Type II service levels over all N scenarios for each set of 10P12N instances for each reliability level k, $k = 1, \ldots, 10$.

Step 4: Report the average service levels over all data sets of 10P12N instances for each reliability level k, $k = 1, \ldots, 10$.

The performance of the proposed policy under different supplier unreliability cases is investigated in this analysis. The scenario number N is set to 100, that is for each level of reliability and for each set of 10P12N instances, we generated 100 different shipping percentage matrices for 10 products and 12 periods. The average service levels for each reliability level k, $k = 1, \ldots, 10$ are reported in Table 2.6.

Table 2.6: Service levels for MILS and MILS-DT under supplier unreliability

	Service Level Type I			**Service Level Type II**		
	MILS	**MILS-DT**	**Difference**	**MILS**	**MILS-DT**	**Difference**
Level 0	92.84%	86.65%	6.19%	99.04%	98.29%	0.75%
Level 1	94.17%	87.70%	6.47%	99.32%	98.68%	0.64%
Level 2	95.38%	88.77%	6.60%	99.56%	99.01%	0.55%
Level 3	96.56%	89.75%	6.81%	99.72%	99.25%	0.46%
Level 4	97.61%	90.84%	6.78%	99.84%	99.47%	0.37%
Level 5	98.47%	91.90%	6.57%	99.92%	99.64%	0.28%
Level 6	99.18%	92.91%	6.27%	99.97%	99.76%	0.20%
Level 7	99.68%	93.63%	6.05%	99.99%	99.85%	0.14%
Level 8	99.92%	94.27%	5.65%	100.00%	99.92%	0.08%
Level 9	99.99%	95.06%	4.93%	100.00%	99.97%	0.03%
Level 10	100.00%	100.00%	0.00%	100.00%	100.00%	0.00%

For both MILS and MILS-DT models, Type I and Type II service levels are increasing as the reliability level increases because of the decrease in the uncertainty. The difference between Type I service level of MILS and MILS-DT is much higher than the difference between Type II service levels. For the Type I service level, using the DTP performs at most 6.81% worse on the average of 16 10P12N instances. There is at most 0.75% difference in the worst case reliability scenario. When there is a shipment with missing amounts because of the unreliability, safety stock absorbs part of the missing amount. Here, with the given safety stock levels the service levels turned to be high and applying DTP did not degrade them much.

2.3.3 Implementation of the Policy in Practice

Our numerical experiments on the benefits and risks of the proposed policy show that using the DTP will be advantageous for the manufacturer. The implementation of the policy requires the manufacturer to provide the supplier with a shipment plan indicating the actual quantities to be shipped and delayed. An easy way to implement this policy may be by defining a control parameter R, the minimum allowable truckload percentage, according to the cost structures and the capacity constraints of the system. This control parameter implies that the trucks dispatched from the supplier should not be loaded with less than R % of the capacity. Then, the manufacturer needs to provide only the order quantities to the supplier, and the parameter R. For the bus manufacturer case, we tried the control parameter values of 50 % and 100 %. We modified the MILS-DT model to solve the problem with control parameter R by adding the constraint set 2.21 and named the resulting model as MILS-DT-R.

$$z_n - \sum_{i=1}^{I} \frac{t_{in}}{r_i} \leq 1 - R \qquad \text{for all } n \in N \ . \tag{2.21}$$

Table 2.7 gives the total cost values for MILS, DTP with "full truck" or with "at least half-load", and MILS-DT. With the data of the coach bus manufacturer, 50 % truck load results in total costs close to the MILS-DT costs, whereas 100 % truck load results in slightly better solutions than MILS. In this case, the "at least half-load" restriction can be used for ease of implementation.

Table 2.7: Total costs for MILS, MILS-DT, and truckload percentages 50 % and 100 %

10P12N Instances	Model	R Value	Holding Cost	Transportation Cost	Total Cost	Relative Difference	% of Order Amounts Delayed
Set 1	MILS	-	€1,713	€1,650	€3,363	-	-
	MILS-DT-R	100%	€1,869	€1,350	€3,219	4.27%	12.32%
	MILS-DT-R	50%	€1,705	€1,350	€3,055	9.15%	20.12%
	MILS-DT	-	€1,705	€1,350	€3,055	9.15%	22.37%
Set 2	MILS	-	€2,060	€1,200	€3,260	-	-
	MILS-DT-R	100%	€1,822	€1,050	€2,872	11.91%	22.50%
	MILS-DT-R	50%	€1,774	€1,050	€2,824	13.38%	27.06%
	MILS-DT	-	€1,774	€1,050	€2,824	13.38%	26.63%

2.3.4 Capacitated Version of the Problem

It is possible to incorporate various capacity limitations to the MIP models. In fact, one may add a capacity constraint for each type of arc in Figure 2.2; namely, capacity for the units produced, capacity for the stock level in the manufacturer and the supplier, and capacity for the units shipped. To see how the solution complexity increases and how the results change, we solved the capacitated version of the problem. With respect to the average four weeks demand, a production upper limit, U_i, is defined for each product i, and the following constraint is added into both MILS and MILS-DT models:

$$x_{in} \leq U_i \qquad \text{for all } i \in I, n \in N \ . \tag{2.22}$$

For the 20-product data set, we found a slight increase in the solution time of the *Capacitated* MILS (C-MILS) and *Capacitated* MILS-DT (C-MILS-DT) models. Solution of C-MILS took about 30 seconds on the average, whereas without the capacity limit this was 16 seconds. The C-MILS-DT took at most 72 seconds, as opposed to 26 seconds of the uncapacitated case. As a result, the solution times were still in reasonable range. For 10-product and 20-product instances, we compare the optimal costs of the C-MILS-DT and C-MILS models in Table 2.8. For the capacitated case of the problem, the relative cost differences and the delayed amounts in solutions of the models with and without DTP are generally smaller, compared to the uncapacitated case, but there is still a significant cost saving from using the proposed policy.

2.4 Conclusion

The computational tests show that the proposed policy is advantageous for the multi item dynamic lot sizing problems with short lead times and frequent shipments. Over all data sets, *Delayed Transportation Policy* provides significant improvement on the total costs with less significant decrease on the service levels in case of unexpected changes in the manufacturer's procurement plans. The risk analysis will give different results for different cost structures and demand patterns. However, implementing such an analysis for different systems is not difficult, and the risks and benefits can be easily evaluated as demonstrated in this study.

Table 2.8: Results for the capacitated MILS-DT and MILS models

Demand	Data Used Product	Period #	Model	Total Cost	Relative Difference	Model	Total Cost	Relative Difference
Quarter 1	10 Products	12 weeks	C-MILS	€3,518		MILS	€3,363	
	Set 1		C-MILS-DT	€3,224	8.35%	MILS-DT	€3,055	9.15%
Quarter 2	10 Products	12 weeks	C-MILS	€3,376		MILS	€3,260	
	Set 1		C-MILS-DT	€2,958	12.38%	MILS-DT	€2,824	13.38%
Quarter 3	10 Products	12 weeks	C-MILS	€3,061		MILS	€3,038	
	Set 1		C-MILS-DT	€2,860	6.57%	MILS-DT	€2,819	7.22%
Quarter 4	10 Products	12 weeks	C-MILS	€2,703		MILS	€2,629	
	Set 1		C-MILS-DT	€2,531	6.37%	MILS-DT	€2,445	7.01%
Quarter 1	20 Products	12 weeks	C-MILS	€5,259		MILS	€5,093	
	Set 1		C-MILS-DT	€5,049	3.99%	MILS-DT	€4,880	4.19%
Quarter 2	20 Products	12 weeks	C-MILS	€5,630		MILS	€5,596	
	Set 1		C-MILS-DT	€5,441	3.35%	MILS-DT	€5,380	3.87%
Quarter 3	20 Products	12 weeks	C-MILS	€5,599		MILS	€5,481	
	Set 1		C-MILS-DT	€5,346	4.52%	MILS-DT	€5,249	4.24%
Quarter 4	20 Products	12 weeks	C-MILS	€6,247		MILS	€5,813	
	Set 1		C-MILS-DT	€5,969	4.44%	MILS-DT	€5,526	4.94%

Chapter 3

DYNAMIC LOT-SIZING UNDER RANDOM SUPPLY WITH BAYESIAN UPDATES OF SUPPLIER CAPABILITY

3.1 Introduction

In this chapter of the thesis, we address the dynamic lot sizing problem under random supply and propose a Bayesian update approach to obtain and use information about the supplier's capability of fulfilling the requested orders while optimizing the ordering decisions of the manufacturer

Different from Chapter 2, in this chapter we do not consider the transportation of the sourcing operation. However, while determining the order quantities we consider that the manufacturer faces uncertainty in the sourcing because of the randomness in the supply. When an order is given to the supplier, the manufacturer does not know the exact amount and arrival time of the order to be received. There are different kinds of supply uncertainty encountered in practice and studied in the literature, as mentioned in Snyder and Shen [2008]. Lead time uncertainty is the first type of supply uncertainty. In studies addressing the lead time uncertainty, it is assumed that the time between giving and receiving an order is a random variable and the distribution of the lead time is known. Another source of uncertainty is due to the supply disruptions arising from several reasons such as machine breakdowns or stopping of production at economical crisis times. In this kind of uncertainty, the requested parts are unavailable at random time intervals and any given order for this portion may not be satisfied by the supplier at the desired time. The final uncertainty type analyzed in the literature is yield uncertainty where the quantity produced or received is different from the ordered amount due to mostly quality and supplier capability problems. These random supply environments are investigated under various inventory models. In this part of the thesis, we focus on lot sizing under random yield, which is observed while sourcing manufacturing parts under push production control systems.

The problem studied in this chapter is encountered while analyzing the ordering and receiving process of a leading coach bus manufacturer in Turkey. When the order amounts and the received amounts are compared over long time periods, a large gap between these amounts is recognized for some periods. Often the bus manufacturer gives an order but cannot receive the desired amounts from most of its suppliers. An example of the real life case analysis is given in Figure 3.1. For some periods, the received amount is much below the ordered amount. To prevent shortages, it is essential to include this yield uncertainty into the lot sizing decisions. For this purpose, a distribution is defined for the yield uncertainty and it is assumed that the supplier sends each order according to this distribution while considering order quantities.

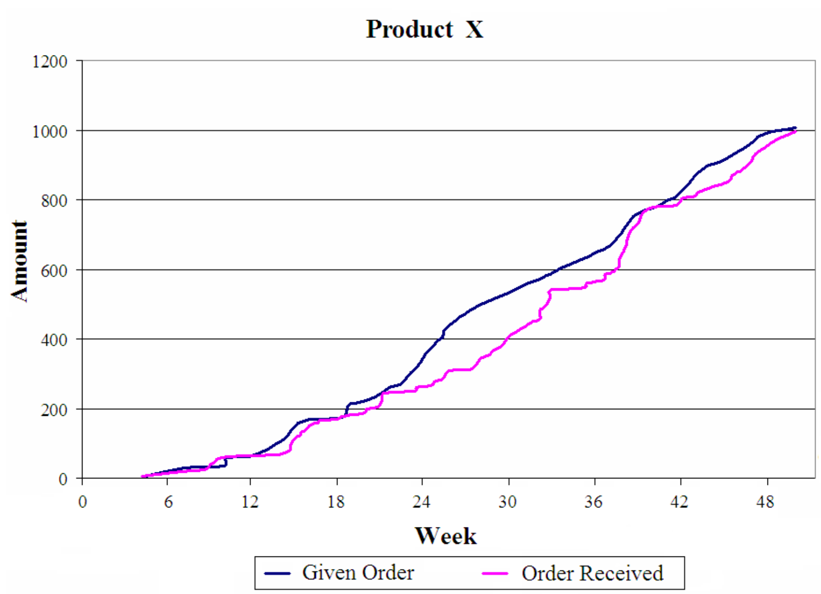

Figure 3.1: Given orders and received amounts for Product X

We next provide a literature review for the supply uncertainty classification defined above. Since in this part of the thesis we focus on lot sizing under yield randomness, we only mention a few studies on lead time uncertainty and supply disruption uncertainty, and place more emphasis on studies on yield randomness.

In case of lead time uncertainty. Nevison and Burstein [1984] suggest a dynamic programming (DP) approach for the dynamic lot sizing problem with deterministic demands but stochastic lead times. Alp et al. [2003] also study lot sizing under dynamic deterministic demand and stochastic lead time. They minimize sum of holding, shortage, and stepwise fixed transportation cost by suggesting a DP algorithm solution method. Brennan and Gupta [1993] investigate the effects of demand and lead time uncertainties on MRP systems by a simulation model. In addition to these studies, the performance of many others exist on inventory management under lead time uncertainty.

The significance of supply disruptions has been recognized in the last decade in the supply chain management domain (Sheffi [2005]). Arreola-Risa and DeCroix [1998] proposed a modified (s,S) policy for the management of inventory with stochastic demand and random supply disruptions. The source of disruption is defined as process related or market related. The inter arrival time of supply disruptions and the length of supply disruptions are exponentially distributed. Güllü et al. [1999] propose a stochastic DP model for a periodic review, single item inventory model under supply uncertainty. The supply uncertainty is modeled in terms of completely available, partially available, and unavailable orders. Optimality of order-up-to policies are investigated. Schmitt et al. [2008] examine the optimal base stock inventory policies for a single supplier and single retailer model where the retailer is subject to stochastic disruptions. Infinite-state discrete time Markov chain with state representing the number of consecutive disrupted periods is used to model the disruptions in their model.

An extended literature review on random yield was given by Yano and Lee [1995] and Mula et al. [2006]. While investigating the literature in yield uncertainty, we classify the studies according to the inventory models used. Parlar and Wang [1993] analyze sourcing from two suppliers where shipments are a random function of the amount requisitioned for both EOQ and newsboy models. Fadiloglu et al. [2008] characterize the optimal replenishment policy for the EOQ type inventory setting with multiple suppliers each with binomial yield. Maddah et al. [2009] extend the classical single period and EOQ models by accounting for randomness in the supply process. There are two types of items with different qualities: perfect and imperfect quality items. The percentage of perfect quality items is assumed to be a random variable with known probability distribution.

Henig and Gerchak [1990] provide an analysis of a general periodic review production/inventory model with yield uncertainty, where the amount of material received is a random multiple of the amount ordered. Wang and Gerchak [1996] formulate a stochastic DP model for a periodic review production planning problem to minimize the total discounted expected costs under the uncertainties of variable production capacity, random yield, and uncertain demand. Bollapragada and Morton [1999] proposed a myopic policy for a single item periodic review inventory problem with random yield and stochastic demand and showed by numerical experiments that their policy is a very good approximation to the optimal policy under fairly general conditions. However, later Inderfurth and Transchel [2007] showed that the evaluation of the optimality condition is not correct in the Bollapragada and Morton [1999] article under some specific conditions. Erdem and Özekici [2002] consider a single item whose yield is random due to the random capacity of the vendor and develop general periodic-review inventory model that incorporates both random yield and random environment parameter. Finally, for the periodic review inventory models, Li et al. [2008] construct upper and lower bounds for the optimal threshold value considering the structure of the optimal policy under uncertain yield and demand.

Yield uncertainty has been also studied in MRP systems. Mazzola et al. [1987] formulated a DP model for dynamic lot-sizing under binomial yield with time varying deterministic demand and developed several heuristic algorithms. Our DP model with Bayesian Updates is built on the model given in Mazzola et al. [1987]. We define the yield randomness with a binomial distribution as in Mazzola et al. [1987] and propose a Bayesian update approach to update the parameter of binomial yield. Tomlin [2009] characterizes the optimal sourcing policy with Bayesian updating for a firm that works with unreliable suppliers. Each order is recieved either in full or not at all for all suppliers. There is one supplier with known reliability parameter and there are multiple available suppliers with unknown parameters that manufacturer selects one of them according to their cost and reliability parameters as second source. In each period the model decides on with which supplier to place an order for deterministic demand. For the finite horizon problem, inventory left at the end of each period is salvaged and unfilled sales are lost. This property enables decomposition of the finite horizon problem into T single period problems. Then in each period, the order quantity must be equal to the demand value. The supplier from which the order is requested can be determined by a comparison of costs. This approach generalizes to the case with two periods, where inventory is carried from the first period to the second period without much additional computational burden. However, the approach does not scale to the general multi-period problem with inventory due to the computational difficulty. In the article, general yield distribution case is handled by calculating the expected cost for all possible order quantities. Hence, an efficient algorithm is not proposed to solve the general yield problem. Chen et al. [2009] proposed a Bayesian model for the same problem described in Tomlin [2009] with inventory carryover. But the uncertainty in the supply process is defined as supply disruptions and the supplier will be up or down with some probability. When the supplier is up, the the order is delivered in full amount and on time. On the other hand, when supplier is down, some portion of the ordered amount will be delivered. For the multiple period analysis a DP algorithm is given with states described by inventory level at the beginning of each period, the availability of the suppliers, and the total number of observations for each possible disruption value. According to the recieved amounts from each supplier the inventory level is updated and expected cost calculated for each period. With these state descriptions the DP algorithm is not efficient to solve for the large data instances.

Until recently, Bayesian update was used in inventory models to update the parameters of demand distributions to represent the learning effect in demand forecasting. Crowston et al. [1973] proposed a forecast revision to determine a posterior distribution for total demand by applying the Bayes' theorem for multistage production planning of seasonal goods. Azoury [1985] gave a DP algorithm for the periodic review inventory problem with a Bayesian formulation to estimate one or more parameters of the demand distribution. Bradford and Sugrue [1990] used a Bayesian procedure for two-period style goods inventory problem for a firm which stocks items having Poisson demands. Hill [1997] applied the Bayesian approach to estimate the parameters of the demand distribution in the single period inventory model. Lee [2008] used the Bayesian approach to forecast demand in a newsboy problem. While Tomlin [2009] and Chen et al. [2009] use Bayesian Update approach for yield randomness and supply disruption, to the best of our knowledge there is no solution time efficient Bayesian update approach to update the parameters of the yield distributions for the lot sizing models. We provide a DP solution algorithm for dynamic lot sizing problem under binomial yield with Bayesian update of the yield parameter to capture the learning process of the manufacturer by tracking the received orders from the supplier with an approximation algorithm that performs significantly efficient solution time.

Section 3.2 describes the problem and the developed DP models for different cases of information availability. In Section 3.3 results of computational experiments are summarized and an approximation algorithm is proposed to overcome the inherent computational difficulty. Extensions of the results are given in Section 3.4.

3.2 Problem Description

Consider periodic ordering where at the beginning of each period, the inventory level is observed and a replenishment quantity is determined. We assume that the demand in each period is known in advance, and the objective is to minimize the sum of holding and shortage costs in a finite horizon of length N. Different from the usual setting, we consider the case of an unreliable supplier, who only delivers a random portion of the order. To model the supplier's unreliability, we assume that the delivered number of items in a period is binomially distributed with parameters x and p, where x denotes the original replenishment quantity and p denotes the probability that an ordered unit will be delivered. Note that this assumption implies that each unit in the order is delivered with probability p, and that the delivery of each unit is independent of other units. This may be considered as a restrictive assumption; however, the same assumption for yield randomness was also made in Mazzola et al. [1987] and other studies including Fadiloglu et al. [2008]. The delivered quantity, Y is a random variable where

$$P(Y = y|x) = \binom{x}{y} p^y (1-p)^{x-y} \ . \tag{3.1}$$

We assume that the supplier enforces a minimum order quantity of m, and a maximum order quantity of M to the manufacturer. This indicates that receiving orders of less than m units is not economical for the supplier and furthermore, because of capacity limitations, the supplier can supply at most M units. Together m, M, and p values represent the capability of the supplier. Furthermore, we assume that the manufacturer is subject to a warehouse space constraint; hence, the on-hand inventory of the part cannot exceed w units.

In our setting, the manufacturer treats shortages as backorders, which incur a charge of s dollars per unit per period. The holding costs are calculated using a unit holding cost of h dollars per unit per period. The variable costs incurred per each received item, such as purchasing and transportation, are modeled using a linear cost term with a unit cost of c

dollars.

Let D_k denote the known demand in period k, where $k = 0, 1, \ldots, N-1$. The on hand inventory at the beginning of period k is denoted by i_k. With the given information, we would like to minimize the total expected cost over N periods. The total cost function contains the holding cost of positive inventory, shortage cost of negative inventory, and unit purchasing and transportation cost of items received at each period. At the beginning of each period k, we observe the inventory level i_k and determine the optimal order quantity, x_k. The lead time is assumed to be zero, as the items can be shipped in less than a period because of the short distance between the manufacturer and the supplier. Hence, as the manufacturer puts in an order of x_k units to the supplier, the supplier sends y_k items with zero lead time. Then, a demand of D_k units is realized by the manufacturer during that period. The decision epochs and actions are shown in Figure 3.2.

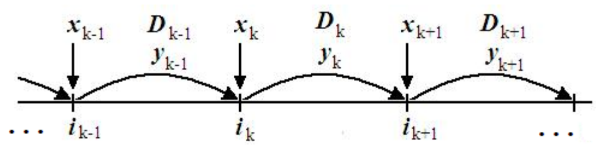

Figure 3.2: Decision epochs and actions

3.2.1 Perfect Information on Supplier Reliability (The PI Case)

In this case, we assume that the manufacturer knows the supplier reliability parameter, p. The parameter may change with time and order quantity, but for the simplicity of presentation we use constant p and discuss the case where p is a function of the order quantity in Section 3.4.2. The manufacturer determines his/her orders according to this known p value. The DP model defined in this section is similar to the one defined in Mazzola et al. [1987], but it differs in that we do not have a setup cost for each order given and we pay purchase cost just for the received amount, not for the ordered amount. This cost structure was the actual case in our motivating application at the bus manufacturer.

The system evolves according to the equation

$$i_{k+1} = i_k + y_k - D_k \ . \tag{3.2}$$

Then, the DP formulation is

$$J_N(i_N) = 0 \qquad \forall i_N \in \left\{ -\sum_{t=0}^{N-1} D_t, \ldots, w \right\}, \tag{3.3}$$

$$\begin{aligned} J_k(i_k) = \min_{x_k \in \{0,1,2,\ldots,M\}} \Big[\sum_{y=0}^{x_k} \binom{x_k}{y} p^y (1-p)^{x_k-y} \Big(c\,y + h(i_k - D_k + y)^+ \\ + s(-i_k + D_k - y)^+ + J_{k+1}(i_k - D_k + y) \Big) \Big] \\ \text{for all } k = 0, 1, \ldots, N-1, \quad \forall i_k \in \left\{ -\sum_{t=0}^{k-1} D_t, \ldots, w \right\}. \end{aligned} \tag{3.4}$$

3.2.2 No Information on Supplier Reliability (The NI Case)

In this case the manufacturer calculates the replenishment quantities without any information on the supplier's reliability. That is, the manufacturer assumes that the supplier reliability parameter, p is uniformly distributed between 0 and 1. Then, the DP formulation becomes

$$J_N(i_N) = 0 \qquad \forall i_N \in \left\{ -\sum_{t=0}^{N-1} D_t, \ldots, w \right\}, \tag{3.5}$$

$$\begin{aligned} J_k(i_k) = \min_{x_k \in \{0,1,2,\ldots,M\}} \int_0^1 \Big[\sum_{y=0}^{x_k} \binom{x_k}{y} q^y (1-q)^{x_k-y} \Big(c\,y + h(i_k - D_k + y)^+ \\ + s(-i_k + D_k - y)^+ + J_{k+1}\Big(i_k - D_k + y\Big) \Big) \Big] dq \\ \text{for all } k = 0, 1, \ldots, N-1, \quad \forall i_k \in \left\{ -\sum_{t=0}^{k-1} D_t, \ldots, w \right\}. \end{aligned} \tag{3.6}$$

3.2.3 Learning about Supplier Reliability through Bayesian Updates (The BU Case)

Finally, we consider the possibility of using past supplier performance while determining the order quantities.

We assume that at the beginning of the period no information on the supplier is known, although we can also consider situations in which certain expectations on the value of p have already been formed. To reflect the fact that nothing on the supplier's reliability is known, we take the uniform distribution as a prior distribution. That is, we assume that the binomial parameter, p is distributed uniformly in $[0, 1]$ range (i.e., $f_0(p) = 1$, $0 \le p \le 1$). Note that $p = 1$ represents a perfectly reliable supplier who always delivers the quantity ordered. The uniform distribution is actually a special case of the Beta distribution with parameters $m_0 = n_0 = 1$. We know that the posterior distribution of the Beta distribution is also Beta with updated parameters of $m + m_0$ and $n + n_0$, where $m + n$ is the number observations and m denotes the number of successes in $m + n$ observations.

We will carry information on the observed delivered quantities. That is, the state is described by i_k and n_k. For the Beta distribution two shape parameters should be defined and updated at each period. For our model, these parameters are the successful observations, m_k, and failed observations, n_k. The successful amounts m_k will be calculated using inventory level and cumulative demand:

$$m_k = i_k + \sum_{t=0}^{k-1} D_t + m_0 \ . \tag{3.7}$$

The state variables are updated according to the following

$$i_{k+1} = i_k + y_k - D_k \ , \tag{3.8}$$

$$n_{k+1} = n_k + x_k - y_k \ . \tag{3.9}$$

Then, the DP formulation is:

$$\begin{aligned} J_N(i_N, n_N) \quad &= \quad 0 \qquad \forall i_N \in \left\{ -\sum_{t=0}^{N-1} D_t, \ldots, w \right\}, \ \forall n_N \in \{n_0, \ldots, n_0 + NM\} \ , \qquad (3.10) \\ J_k(i_k, n_k) \quad &= \quad \min_{x_k \in \{0,1,2,\ldots,M\}} \int_0^1 \Big[\sum_{y=0}^{x_k} \binom{x_k}{y} q^y (1-q)^{x_k - y} \Big(c\, y + h(i_k - D_k + y)^+ \\ &\quad + s(-i_k + D_k - y)^+ + J_{k+1}(i_k - D_k + y, n_k + (x_k - y)) \Big) \Big] \\ &\quad \frac{\Gamma(m_k + n_k)}{\Gamma(m_k)\Gamma(n_k)} q^{m_k - 1} (1-q)^{n_k - 1} dq \qquad \text{for all } k = 0, 1, \ldots, N-1, \\ &\qquad \forall i_k \in \left\{ -\sum_{t=0}^{k-1} D_t, \ldots, w \right\}, \ \forall n_k \in \{n_0, \ldots, n_0 + kM\} \ . \qquad (3.11) \end{aligned}$$

To solve the three DP formulations given above in Sections 3.2.1, 3.2.2, and 3.3, *Matlab R2008a* codes were prepared. The Algoritms 4, 5, and 6, given in the Appendix, are used for the PI, NI, and BU cases, respectively.

3.2.4 Numerical Example on PI, NI, and BU

To understand the DP algorithm solutions and the differences between the solution of each model, let us consider a problem where the manufacturer wants to source a demand of $D = [2\ 0\ 1\ 2]$ over 4 periods from its supplier. For this small example, holding, shortage and unit variable purchasing/transportation costs are set to 1, 6, and 3, respectively. Maximum order quantity limit is 5 units and inventory limit is defined as 5 units. The reliability parameter of the supplier, p, is set to be 0.7. While solving the DP algorithm for the PI case, this reliability parameter is known. However, for the NI and BU cases, there is no information about the exact value of the reliability parameter. For the BU model initial parameters of Beta distribution are set to 1 ($m_0 = 1$ and $n_0 = 1$). Tables 3.1, 3.3, 3.5, 3.6, and 3.7 give the optimal order quantities for the corresponding cases.

Table 3.1: Optimal order quantities for the PI case

	Stage 0	**Stage 1**	**Stage 2**	**Stage 3**
i	$\pi_0(i)$	$\pi_1(i)$	$\pi_2(i)$	$\pi_3(i)$
-3				5
-2		4	5	5
-1		2	4	4
0	4	0	3	2
1		0	0	1
2		0	0	0
3		0	0	0
4			0	0
5			0	0

In Table 3.1, for each stage k and for each possible inventory level i, the optimal ordering quantities are given in the $\pi_k(i)$ columns. For this Perfect Information case, the reliability parameter is known to be $p = 0.7$. The manufacturer will determine its order level by using this table according to its inventory level at each stage k. To see the actions of the manufacturer, we run a simulation where we generate the received amounts for the corresponding order amounts, and the given reliability parameter. Assume at the beginning of the planning horizon the manufacturer has 0 initial inventory and wants to order using

Table 3.2: Sample Path of order schedule for the PI case

	Stage 0	**Stage 1**	**Stage 2**	**Stage 3**
Demand, D_k	2	0	1	2
Starting Inventory, i_k	0	1	1	0
Given Order, $\pi_k(i_k)$	4	0	0	2
Received Amount, y_k	3	0	0	2
Next Inventory Level, i_{k+1}	1	1	0	0

the DP solutions. According to Table 3.1, the manufacturer orders $\pi_0(0) = 4$. After the manufacturer gives this order, the supplier sends y_0 which is binomially distributed with parameter 4 and 0.7. If the received amount that is generated in this simulation is $y_0 = 3$, then the inventory level becomes $i_1 = 1$ since $D_0 = 2$. So, for the next stage the starting inventory level becomes 1, and the manufacturer gives an order of $\pi_1(1) = 0$. Since no order is given, nothing is received and the inventory level is updated according to the demand of that period, $D_1 = 0$. The inventory level of the second stage becomes $i_2 = 1$, and the corresponding order quantity is again $\pi_2(1) = 0$. After the demand $D_2 = 1$ is realized, the inventory level i_3 becomes 0. Finally, in the last stage the order quantity is determined from $\pi_3(0)$ as 2. And the received amount y_3 is binomially distributed with parameters 2 and 0.7. The simulation results for this example are summarized in Table 3.2. There is no single ordering plan in this stochastic environment as the order schedule changes according to the realized received amounts, y_k for all $k = 0, 1, ..., N-1$.

In Table 3.3, the optimal ordering quantities, $\pi_k(i)$, are given for the No Information case. The order quantities are determined with respect to the expected total cost over p, where p is uniformly distributed between 0 and 1. The actual reliability parameter, will be a number between 0 and 1 and the realized received amounts are distributed with the binomial distribution using the actual reliability parameter and order quantity. As an example of how the manufacturer orders according to the NI results, assume an initial inventory of 0. From the results of Table 3.3, the manufacturer orders $\pi_0(0) = 5$. According to the actual supplier reliability p (assume that $p = 0.7$) and order quantity 5, the manufacturer will receive $y_0 = 4$ (a simulation result). The inventory level becomes $i_1 = 2$ and the manufacturer orders $\pi_1(2) = 0$. Since there is no demand and order for this stage the inventory level for the

Table 3.3: Optimal order quantities for the NI case

	Stage 0	**Stage 1**	**Stage 2**	**Stage 3**
i	$\pi_0(i)$	$\pi_1(i)$	$\pi_2(i)$	$\pi_3(i)$
-3				5
-2		5	5	5
-1		4	5	4
0	5	1	3	2
1		0	2	1
2		0	1	0
3		0	0	0
4			0	0
5			0	0

Table 3.4: Sample Path of order schedule for the NI case

	Stage 0	**Stage 1**	**Stage 2**	**Stage 3**
Demand, D_k	2	0	1	2
Starting Inventory, i_k	0	2	2	2
Given Order, $\pi_k(i_k)$	5	0	1	0
Received Amount, y_k	4	0	1	0
Next Inventory Level, i_{k+1}	2	2	2	0

next stage remains at $i_2 = 2$. For this inventory level, the manufacturer orders $\pi_2(2) = 1$ and receives $y_2 = 1$. The inventory level becomes $i_3 = 2$ and the order quantity is set to be $\pi_3(2) = 0$. The results of this simulation are summarized in Table 3.4.

Table 3.5: Optimal order quantities for the BU case, Stages 0 and 1

Stage 0		**Stage 1**						
	$\pi_0(i,n)$		$\pi_1(i,n)$					
(i,n)	**1**	**(i,n)**	**1**	**2**	**3**	**4**	**5**	**6**
-3		**-3**						
-2		**-2**	5	5	5	5	5	5
-1		**-1**	2	4	5	5	5	
0	5	**0**	0	0	1	1		
1		**1**	0	0	0			
2		**2**	0	0				
3		**3**	0					
4		**4**						
5		**5**						

For the BU case, to include the learning effect of the supplier capability into the DP model, we need to keep the data of unsuccessful orders. For this reason, at each stage the states are described with inventory level i and the number of unsuccessful orders up to that stage, n. Tables 3.5, 3.6, and 3.7 gives the optimal order quantities $\pi_k(i,n)$ for each stage $k = 0, 1, ..., N-1$, for each possible inventory level i, and for each possible unsuccessful order size n. The manufacturer will determine its order level using these tables according to the current inventory and unsuccessful order levels. To show how the manufacturer will act by using these order quantities, assume at the beginning of the planning horizon the manufacturer has zero initial inventory and wants to order using the solutions of Tables 3.5, 3.6, and 3.7. At the beginning, the parameters of the Beta distribution, m_0 and n_0, are both set to 1. Then, in the first stage the manufacturer orders for $i_0 = 0$ and $n_0 = 1$ which is $\pi_0(0,1) = 5$. After the manufacturer gives this order, the supplier sends y_0 units which is binomially distributed with parameter $\pi_0(0,1)$ and actual reliability parameter p. Assume that the received amount is $y_0 = 4$, then the inventory level becomes $i_1 = 2$ and the unsuccessful order amount becomes $n_1 = 2$. By using this information, the manufacturer

Table 3.6: Optimal order quantities for the BU case, Stage 2

Stage 2

	$\pi_2(i,n)$										
(i,n)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
-3											
-2	5	5	5	5	5	5	5	5	5	5	5
-1	4	5	5	5	5	5	5	5	5	5	
0	3	3	3	4	4	5	5	5	5		
1	0	0	2	2	2	2	2	2			
2	0	0	0	1	1	1	1				
3	0	0	0	0	0	0					
4	0	0	0	0	0						
5	0	0	0	0							

Table 3.7: Optimal order quantities for the BU case, Stage 3

Stage 3

	$\pi_3(i,n)$															
(i,n)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
-3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
-2	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	
-1	3	4	4	5	5	5	5	5	5	5	5	5	5	5		
0	2	2	3	3	3	3	4	4	4	5	5	5	5			
1	1	1	1	1	1	1	1	1	2	2	2	2				
2	0	0	0	0	0	0	0	0	0	0	0					
3	0	0	0	0	0	0	0	0	0	0						
4	0	0	0	0	0	0	0	0	0							
5	0	0	0	0	0	0	0	0								

Table 3.8: Sample Path of order schedule for the BU case

	Stage 0	**Stage 1**	**Stage 2**	**Stage 3**
Demand, D_k	2	0	1	2
Unsuccessful Order Amount, n_k	1	2	2	2
Starting Inventory, i_k	0	2	2	1
Given Order, $\pi_k(i_k)$	5	0	0	1
Received Amount, y_k	4	0	0	1
Next Inventory Level, i_{k+1}	2	2	1	0

orders nothing, $\pi_1(2,2) = 0$. Because of no order and demand, the inventory level and the unsuccessful order amount do not change. With $i_2 = 2$ and $n_2 = 2$, the manufacturer again orders nothing ($\pi_2(2,2) = 0$). After realizing $D_2 = 1$, the inventory level decreases by 1. For the final stage, with $i_2 = 1$ and $n_2 = 2$ the optimal order quantity is $\pi_3(1,2) = 1$. And the received amount y_3 will be binomially distributed with parameters $x = 1$ and $p = 0.7$. The simulation results with $p = 0.7$ are summarized in Table 3.8.

3.3 Insights on the Value of Information and Learning

In this section we investigate the value of information for the dynamic lot-sizing problem under random supply by computational experiments. In a situation where the supplier reliability parameter, p, is not known, we show that implementing the learning aspect into the DP algorithm improves the total expected cost. In our problem setting we assume that the manufacturer knows that the supplier will send some portion of the given order. The perfect and no information cases differ in terms of the information about the success probability of each item to be sent (i.e. the supplier reliability). When p is unknown at the beginning of the planning horizon, the NI and BU approaches defined in Section 3.2.2 and are used. To see how effective the BU model with respect to the NI model is, we compare the total expected cost values of each model with the PI model.

3.3.1 Computational Experiments

To test the models, we use three different demand sets. The first demand set is generated using a triangular distribution with parameters $a = 3$, $b = 7$, and $c = 5$ (Figure 3.3). The second demand set is again generated using a triangular distribution, but with a high variance, with parameters $a = 0$, $b = 10$, and $c = 5$. The third demand set is obtained by modifying the data taken from the coach bus manufacturer. We normalized the demand values so that the mean over the planning horizon is close to 5. This demand set has the highest variance. The planning horizon for the test cases are defined as 12 periods and 10 instances for each data set are generated using these parameters to test the performance of the BU model. The summary of the demand sets are given in Table 3.9.

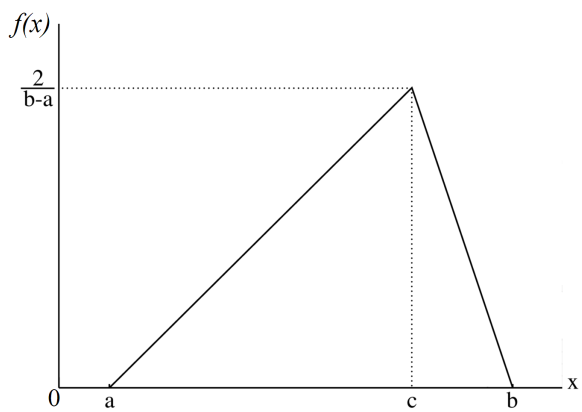

Figure 3.3: Probability density function of the triangular distribution

Table 3.9: Summary of demand sets used for calculations

Instance Set	**Number of Instances**	**Number of Periods**	**Triangular Distribution Parameters** a	b	c	μ	σ
Set 1	10	12	3	7	5	5	0.817
Set 2	10	12	0	10	5	5	2.041
Set 3	10	12	– *Real Life Data* –			5.03	6.314

The holding cost, shortage cost, and unit variable transportation and purchase cost is defined as \$1, \$6, and \$3, respectively. The warehouse limit, w, is set to 10 and the minimum order quantity, m, is defined as 0. For the maximum order quantity, we set $M = [10, 15, 20, 30]$, to see the effect of the order size on the total expected cost.

As shown in Section 3.2.4, the optimal order quantities of each model differ according to the states defined for that model and each model has different total expected cost value J_0. We cannot compare the J_0 values of the PI, NI, and BU cases since the actual p value is not known while solving the NI and BU cases and J_0 is found by taking expectation over all possible values of p. For any given actual p value the realized cost will be different than the total expected cost. But there is no single decision path for any given actual p, at each period the inventory level will change according to the realized received amounts, which is binomailly distributed with given order quantity and p, and for each different inventory level the order quantities will be different. So, it will not be reasonable to compare the model solutions by a single sample path as the path changes with respect to the probability distribution of the received amounts. For this reason, to compare the models with each other we define a simulation run that calculates the average total cost of a given solution over K replications. We set the number of replications as $K = 10,000$. In each replication, the received amounts change; hence, the actions taken and the associated cost also change. After completing the 10,000 replications, we report the average of the total costs of these replications and compare PI, NI, and BU models with respect to this average total cost. The algorithms for the PI, NI and BU models are given in the Algorithm 1, 2, and 3, respectively. The results of the simulation runs are summarized in the Tables 3.10, 3.11, 3.12, 3.13, 3.14, 3.15, 3.16, 3.17, 3.18, and 3.19.

In Tables 3.10, 3.11, 3.12, and 3.13, the average total cost of 10,000 scenarios are reported for one instance of the demand Set 2. The p column gives which reliability parameter is used in the simulation. For the NI and BU models, the reliability factor is not used while calculating the optimal order quantities, but in the simulations the received amounts are calculated using these reliability factors. In each table, p starts from a different level. We saw in our numerical tests that the results depend on the maximum order size M and the reliability level p. As p decreases, the manufacturer tries to order more to satisfy its demand. However, after some level of p, the order sizes given by the three models (PI,NI, and BU)

Algorithm 1 The Simulation Procedure to Calculate the Average Total Cost for the PI model

Define the maximum order size M, M will be $[10, 15, 20, 30]$
for each supplier reliability parameter p, $p = [0.1, 0.2, \dots, 1]$ **do**
  for each demand set D^j, $j \in \{\text{Set }1, \text{Set }2, \text{Set }3\}$ **do**
    Solve the PI model and report the optimal order quantities $\pi^{PI,j,p}$
    for $t = 1, \dots, K$ **do**
      for each period $k = 0, 1, \dots, N-1$ in the planning horizon **do**
        Determine the optimal order quantity ${x_k}^t = {\pi_k}^{PI,j,p}({i_k}^t)$
        Generate a received amount ${y_k}^t = \text{Binomial}({x_k}^t, p)$
        Update ${i_{k+1}}^t = {i_k}^t + {y_k}^t - {D_k}^j$
      end for
      Calculate the total cost for scenario t
    end for
    Report the average total cost over K scenarios
  end for
  Report the average total cost over all instances of demand Set 1, Set 2, and Set 3
end for
Report Results for each p, $p = [0.1, 0.2, \dots, 1]$

Table 3.10: Simulation results of first instance for the demand Set 2, for $M = 10$

p	**Avg TC (PI)**	**Avg TC (NI)**	**Avg TC (BU)**	**% Diff of NI from PI**	**% Diff of NI from BU**	**% Diff of BU from PI**
0.6	291.58	293.58	292.56	0.68%	0.35%	0.33%
0.7	232.28	249.80	235.01	7.54%	6.29%	1.18%
0.8	222.68	263.13	229.19	18.16%	14.81%	2.92%
0.9	214.77	273.98	221.33	27.57%	23.79%	3.05%
1	198.00	282.00	209.00	42.42%	34.93%	5.56%

Algorithm 2 The Simulation Procedure to Calculate the Average Total Cost for the NI model

Define the maximum order size M, M will be $[10, 15, 20, 30]$
for each demand set D^j, $j \in \{\text{Set 1}, \text{Set 2}, \text{Set 3}\}$ **do**
  Solve the NI model and report the optimal order quantities $\pi^{NI,j}$
  for each supplier reliability parameter p, $p = [0.1, 0.2, \ldots, 1]$ **do**
    for $t = 1, \ldots, K$ **do**
      for each period $k = 0, 1, \ldots, N-1$ in the planning horizon **do**
        Determine the optimal order quantity ${x_k}^t = {\pi_k}^{NI,j}({i_k}^t)$
        Generate a received amount ${y_k}^t = \text{Binomial}({x_k}^t, p)$
        Update ${i_{k+1}}^t = {i_k}^t + {y_k}^t - {D_k}^j$
      end for
      Calculate the total cost for scenario t
    end for
    Report the average total cost over K scenarios
  end for
  Report Results for each p, $p = [0.1, 0.2, \ldots, 1]$
end for
Report the average total cost over all instances of demand Set 1, Set 2, and Set 3

Table 3.11: Simulation results of first instance for the demand Set 2, for $M = 15$

p	Avg TC (PI)	Avg TC (NI)	Avg TC (BU)	% Diff of NI from PI	% Diff of NI from BU	% Diff of BU from PI
0.4	339.27	341.89	342.40	0.77%	-0.15%	0.93%
0.5	238.18	244.07	240.45	2.47%	1.51%	0.95%
0.6	228.43	251.81	233.53	10.23%	7.83%	2.23%
0.7	223.21	262.25	229.67	17.49%	14.19%	2.89%
0.8	218.61	270.50	225.93	23.74%	19.73%	3.35%
0.9	213.04	277.23	222.90	30.13%	24.37%	4.63%
1	198.00	283.00	216.00	42.93%	31.02%	9.09%

Algorithm 3 The Simulation Procedure to Calculate the Average Total Cost for the BU model

Define the maximum order size M, M will be $[10, 15, 20, 30]$
for each demand set D^j, $j \in \{\text{Set } 1, \text{Set } 2, \text{Set } 3\}$ **do**
  Solve the BU model and report the optimal order quantities $\pi^{BU,j}$
  for each supplier reliability parameter p, $p = [0.1, 0.2, \ldots, 1]$ **do**
    for $t = 1, \ldots, K$ **do**
      for each period $k = 0, 1, \ldots, N-1$ in the planning horizon **do**
        Determine the optimal order quantity ${x_k}^t = {\pi_k}^{BU,j}({i_k}^t)$
        Generate a received amount ${y_k}^t = \text{Binomial}({x_k}^t, p)$
        Update ${i_{k+1}}^t = {i_k}^t + {y_k}^t - {D_k}^j$
        Update ${n_{k+1}}^t = {n_k}^t + {x_k}^t - {y_k}^t$
      end for
      Calculate the total cost for scenario t
    end for
    Report the average total cost over K scenarios
  end for
  Report Results for each p, $p = [0.1, 0.2, \ldots, 1]$
end for
Report the average total cost over all instances of demand Set 1, Set 2, and Set 3

Table 3.12: Simulation results of first instance for the demand Set 2, for $M = 20$

p	**Avg TC (PI)**	**Avg TC (NI)**	**Avg TC (BU)**	**% Diff of NI from PI**	**% Diff of NI from BU**	**% Diff of BU from PI**
0.3	463.04	474.47	467.71	2.47%	1.45%	1.01%
0.4	256.18	259.73	258.88	1.39%	0.33%	1.06%
0.5	231.06	241.17	233.71	4.37%	3.19%	1.15%
0.6	225.66	251.98	229.94	11.66%	9.59%	1.90%
0.7	222.40	262.02	227.75	17.81%	15.05%	2.40%
0.8	218.62	269.52	226.31	23.28%	19.09%	3.52%
0.9	213.13	275.54	224.91	29.29%	22.52%	5.53%
1	198.00	280.00	219.00	41.41%	27.85%	10.61%

Table 3.13: Simulation results of first instance of demand Set 2, for $M = 30$

p	**Avg TC (PI)**	**Avg TC (NI)**	**Avg TC (BU)**	**% Diff of NI from PI**	**% Diff of NI from BU**	**% Diff of BU from PI**
0.2	812.75	818.35	812.71	0.69%	0.69%	0.00%
0.3	405.28	416.74	406.94	2.83%	2.41%	0.41%
0.4	254.64	258.06	256.40	1.34%	0.65%	0.69%
0.5	230.84	241.01	233.14	4.40%	3.37%	1.00%
0.6	225.43	251.28	229.42	11.47%	9.53%	1.77%
0.7	222.40	260.08	227.43	16.94%	14.36%	2.26%
0.8	218.60	266.37	226.35	21.85%	17.68%	3.54%
0.9	213.19	271.07	224.92	27.15%	20.52%	5.50%
1	198.00	275.00	219.00	38.89%	25.57%	10.61%

will usually be equal to M. This level of p is determined from the average demand over the planning horizon (μ_D) and the maximum order size (M). On the average, the manufacturer tries to receive at least μ_D units to satisfy its demand in a period and can order a maximum of M at each period. We know that the supply quantity is binomially distributed with given order x and the reliability p. The expected supply quantity for this binomially distributed supply will be xp and this should be close to μ_D on the average to satisfy the demand. We have a relation between p and μ_D as $xp = \mu_D$. Since x will be at most M, the minimum p value that gives order quantities different from M is:

$$p > \frac{\mu_D}{M} \ . \tag{3.12}$$

Since the order quantities are fixed to M, and there is no difference between the three policies for the p values that are smaller than the lower bound defined in Equation 3.12, we just report the results of p values that yield order sizes smaller than M. For this reason, possible outcomes of p increases as we increase M used in the calculations. Together p and M parameters define the capability of the supplier. If one of these parameters is low, than the other parameter should be high enough to satisfy the demand. For a supplier with both low values of p and M, the cost incurred to the manufacturer will be high because of the unsatisfied demand and working with such a supplier will not be preferred.

Avg TC (PI), Avg TC (NI), and *Avg TC (BU)* columns in Tables 3.10, 3.11, 3.12, and 3.13 give the average total cost of 10,000 replications for the PI, NI, and BU models, respectively. Since the total cost over 12 periods is different for each simulation run, we give the average comparisons. *% Diff of NI from PI*, *% Diff of NI from BU* and *% Diff of BU from PI* colums represent the percentage difference between the total cost of NI model from PI and BU models and BU model from the PI model. The base model for the comparisons is the PI model since we consider that there is a supplier reliability parameter but the manufacturer does not know it.

From the results of Tables 3.10, 3.11, 3.12, and 3.13, it can be concluded that the proposed BU model gives better average total cost than the NI model for each $p > \mu_D/M$. The lower bound for p decreases by increasing M. On the average the difference from the perfect information case is decreased to 3.08 % from 16.99 %. This improvement is for just one instance of the Set 2. The average results over 10 instances of Set 1, Set 2, and Set 3 are summarized in the Tables 3.14, 3.15, 3.16, 3.17, 3.18, and 3.19 for $M = 10$ and $M = 15$.

Table 3.14: Average results for 10 instances of Set 1, $M = 10$

	% Diff of NI from PI			**% Diff of BU from PI**		
p	**Avg TC**	**min TC**	**max TC**	**Avg TC**	**min TC**	**max TC**
0.6	7.55%	2.28%	11.91%	1.56%	0.89%	3.04%
0.7	18.52%	12.35%	22.34%	2.29%	0.99%	4.18%
0.8	26.55%	21.59%	29.79%	2.54%	1.39%	3.71%
0.9	34.07%	30.01%	37.08%	2.65%	1.72%	3.79%
1	48.30%	44.28%	51.72%	5.89%	4.48%	7.53%

The first column in Tables 3.14, 3.15, 3.16, 3.17, 3.18, and 3.19 gives the possible p values. The average demand on each data set is equal to 5. For $M = 10$, lower bound for p is greater than $5/10 = 0.5$ ($p \geq 0.6$) and for $M = 15$, results are interesting only for p greater than $5/15 = 0.33$ ($p \geq 0.4$). The next three columns give the average, minimum, and maximum total cost differences of the NI model from the PI model for the corresponding demand set. The last three columns give the average, minimum, and maximum total cost differences of the BU model from the PI model for the corresponding demand set.

The results of the simulation runs show that, the proposed BU approach performs better

Table 3.15: Average results for 10 instances of Set 2, $M = 10$

	% Diff of NI from PI			**% Diff of BU from PI**		
p	**Avg TC**	**min TC**	**max TC**	**Avg TC**	**min TC**	**max TC**
0.6	4.17%	0.54%	8.30%	1.35%	0.33%	2.21%
0.7	13.63%	7.54%	18.48%	2.04%	1.06%	3.59%
0.8	22.60%	18.02%	28.17%	2.49%	1.05%	4.23%
0.9	30.64%	24.38%	37.22%	2.53%	0.84%	3.68%
1	44.71%	37.59%	51.61%	5.08%	3.65%	6.67%

Table 3.16: Average results for 10 instances of Set 3, $M = 10$

	% Diff of NI from PI			**% Diff of BU from PI**		
p	**Avg TC**	**min TC**	**max TC**	**Avg TC**	**min TC**	**max TC**
0.6	2.96%	0.56%	12.20%	2.44%	0.34%	7.42%
0.7	8.48%	1.20%	24.04%	4.11%	-0.12%	10.91%
0.8	14.55%	2.54%	31.03%	5.04%	0.17%	11.37%
0.9	21.82%	4.41%	38.23%	5.65%	-0.02%	11.71%
1	35.43%	12.41%	54.59%	8.60%	0.00%	15.34%

Table 3.17: Average results for 10 instances of Set 1, $M = 15$

	% Diff of NI from PI			**% Diff of BU from PI**		
p	**Avg TC**	**min TC**	**max TC**	**Avg TC**	**min TC**	**max TC**
0.4	2.11%	1.63%	2.60%	1.01%	0.24%	1.55%
0.5	7.16%	5.65%	8.51%	1.53%	0.92%	2.39%
0.6	13.51%	11.95%	15.66%	2.09%	1.24%	2.78%
0.7	18.88%	16.47%	21.42%	2.74%	1.84%	3.56%
0.8	23.95%	21.07%	26.68%	3.80%	2.50%	4.49%
0.9	29.70%	26.44%	32.68%	5.28%	3.38%	6.07%
1	42.53%	39.18%	46.24%	10.26%	7.46%	11.83%

Table 3.18: Average results for 10 instances of Set 2, $M = 15$

	% Diff of NI from PI			% Diff of BU from PI		
p	**Avg TC**	**min TC**	**max TC**	**Avg TC**	**min TC**	**max TC**
0.4	1.83%	0.77%	3.53%	1.14%	0.70%	1.64%
0.5	5.09%	2.47%	7.42%	1.46%	0.81%	2.16%
0.6	11.78%	8.27%	14.43%	2.02%	1.28%	2.70%
0.7	17.80%	13.17%	20.72%	2.41%	1.21%	3.17%
0.8	23.25%	18.20%	27.42%	3.03%	1.67%	4.05%
0.9	29.10%	23.06%	34.06%	4.32%	2.40%	6.16%
1	41.67%	34.75%	47.06%	8.71%	5.82%	11.85%

Table 3.19: Average results for 10 instances of Set 3, $M = 15$

	% Diff of NI from PI			% Diff of BU from PI		
p	**Avg TC**	**min TC**	**max TC**	**Avg TC**	**min TC**	**max TC**
0.4	1.06%	0.37%	1.76%	1.33%	0.15%	3.16%
0.5	3.26%	0.62%	9.35%	2.80%	0.57%	7.34%
0.6	7.24%	0.76%	16.79%	3.81%	0.40%	8.30%
0.7	12.53%	1.65%	25.51%	4.58%	0.36%	8.85%
0.8	18.40%	4.05%	32.59%	5.26%	0.45%	9.77%
0.9	25.52%	6.05%	39.08%	6.22%	0.21%	11.28%
1	37.44%	5.09%	54.95%	9.21%	0.00%	15.76%

than the NI approach on the defined data sets over 10 instances and for each possible p value. In all instances, the percentage difference of the average total cost of the BU model is smaller than the NI model; that is the BU model gives closer results to the PI model than the NI model. For $M = 10$, on the average the NI model gives 22.27 % worse results than the PI model where as, the BU model gives 3.62 % worse results. For $M = 15$,the NI model gives 17.8 % worse results than the PI mode, and the BU model gives 3.95 % worse results than the PI on the average. For larger p values, the improvement in the average total cost is noteworthy since the BU model updates the reliability parameter. At the beginning of the planning horizon the NI and BU models assume that p is uniformly distributed between 0 and 1. The expected reliability parameter ($\tilde{p}$) is 0.5 for this distribution. If the actual p value is larger than $\tilde{p}$, the BU model observes this difference after some period and updates p according to its observations. Observing and updating the value of p in the BU model reduces the average total cost to become closer to the PI case.

3.3.2 Approximation for Bayesian Updates (PA)

From the simulation results of Section 3.3.1, we show that when there is imperfect information about the supplier reliability, in terms of the average total cost BU model is more advantageous than the NI model. Although the cost saving from the BU model is considerable, there is a solution time disadvantage of the proposed model. To implement the learning effect into the DP algorithm, we need to keep track of the failed order amounts as states. This additional state description results with an increase in the state space. For the PI and NI algorithms, the states at each stage are described using just i_k and the total number of states at stage k is

$$1 + w + \sum_{t=0}^{k-1} D_t \ . \tag{3.13}$$

where w is an upper bound and $\sum_{t=0}^{k-1} D_t$ is a lower bound for inventory $(-\sum_{t=0}^{k-1} D_t \leq i_k \leq w)$. Then the size of the state space over all stages is

$$N + N\ w + \sum_{k=1}^{N} \sum_{t=0}^{k-1} D_t \ . \tag{3.14}$$

In the worst case scenario, the cumulative demand at each period will be equal to the total demand over N periods, $D = \sum_{t=0}^{N-1} D_t$, and the total size of the state space becomes

$$N + N\ w + \sum_{k=1}^{N} D\ . \tag{3.15}$$

The order of the state space is

$$O(N + N\ w + N\ D)\ . \tag{3.16}$$

Since the on-hand inventory level, w, will be bigger than one, the order of state space becomes

$$O(N\ max(w, D))\ . \tag{3.17}$$

On the other hand, the states at each stages of the BU algorithm are described using both i_k and n_k and the total number of states at stage k is

$$(k\ M)\Big(1 + w + \sum_{t=0}^{k-1} D_t\Big)\ . \tag{3.18}$$

since $n_0 \leq n_k \leq n_0 + kM$ and $-\sum_{t=0}^{k-1} D_t \leq i_k \leq w$. Then, the state space over all stages has size

$$M\ \sum_{k=1}^{N} k + M\ w\ \sum_{k=1}^{N} k + M\ \Big(\sum_{k=1}^{N} k \sum_{t=0}^{k-1} D_t\Big)\ . \tag{3.19}$$

In the worst case scenario, the cumulative demand at each period will be equal to D, and the total state space becomes

$$M\ \sum_{k=1}^{N} k + M\ w\ \sum_{k=1}^{N} k + M\ D\ \Big(\sum_{k=1}^{N} k\Big)\ . \tag{3.20}$$

The order of the state space is

$$O(M\ N^2 + M\ w\ N^2 + M\ D\ N^2)\ . \tag{3.21}$$

Since the on-hand inventory level, w, will be bigger than one, the order of state space becomes

$$O(N^2\ M\ max(w, D))\ . \tag{3.22}$$

With this complexity, it is only possible to solve instances with small values of M, N, and D. For large values of M, N, and D the solution time grows rapidly. To solve large data sets, we propose an approximation algorithm (Percentage Approximation, (PA), Algorithm) which gives close results to the BU algorithm with significantly less solution time. We try to estimate the number of failed orders by defining and updating a success percentage for the past orders.

To reduce the state space, we modify the DP algorithm defined in Section 3.3. In the original DP algorithm we carry information on the failed order quantities and the states are described by i_k, and n_k. Possible outcomes of n_k is between n_0 and $n_0 + M\ k$ for the k^{th} period and possible outcomes of n_{k+1} is between n_0 and $n_0 + M\ (k+1)$ for the $(k+1)^{th}$ period. We will give an approximation algorithm to carry this information with smaller state sizes. Instead of keeping the amount of failed orders, n_k, we suggest to keep the ratio of the successful orders to the total order. From the definition of the states, we know the inventory level at each state, i_k. Since we know the starting inventory in period k, i_k, and the total demand during k periods, the received amount, m_k, can be easily calculated by Equation 3.7. When the percentage of received amount until period k, say κ_k, is known, then the failed amount will be

$$n_k = n_0 + \frac{(i_k + \sum_{t=0}^{k-1} D_t)\ (1-\kappa_k)}{\kappa_k}\ . \tag{3.23}$$

For each given m_k and n_k, there is a ratio of $\kappa_k = (m_k - m_0)/((m_k - m_0) + (n_k - n_0))$ and this ratio will be any real number between 0 and 1. But keeping the real number values for ratios is not suitable to solve the DP algorithm. To limit the state space explosion in the DP algorithm, we need to define states of ratios discretely. For this reason in our approximation algorithm, we divide the possible outcomes of the success percentage into $R = 2^l$, $l \in Z^+$ intervals and assign each κ_k to a specific interval, r_k, with the following comparison,

$$\frac{r_k - 1}{R} < \kappa_k \le \frac{r_k}{R} \qquad r_k \in \{1, 2, \ldots, R\}\ . \tag{3.24}$$

For each κ_k in the same interval r_k, we use a common percentage value ρ_k which is the maximum percentage value in r_k. That is, if an ratio κ_k is in r_k, then in the next period the information about the the past successful observation will be ρ_k.

To understand this approximation approach by an example, assume that in period k, $\sum_{t=0}^{k-1} D_t = 8$, $i_k = 1$ and $n_k = 7$. This information is carried by the state $(i_k, n_k) = (1, 7)$

for the BU algorithm and $(i_k, r_k) = (1, 3)$ for the PA algorithm with $R = 4$. In the exact algorithm we know that $m_k = 10$ and $n_k = 7$ and $\kappa_k = 0.6$. Since

$$\frac{2}{4} < \kappa_k \leq \frac{3}{4} \ , \tag{3.25}$$

r_k is set to be 3 and the corresponding percentage approximation, ρ_k, will be $r_k/R = 0.75$. So, we represent $(i_k, n_k) = (1, 7)$ as $(i_k, r_k) = (1, 3)$ in the PA algorithm. With state $(i_k, r_k) = (1, 3)$, n_k will be estimated as

$$\begin{aligned} \hat{n_k} &= n_0 + \left[\frac{(i_k + \sum_{t=0}^{k-1} D_t) \ (1-\rho_k)}{\rho_k}\right] \\ &= 1 + \left[\frac{(1+8) \ (1-0.75)}{0.75}\right] \\ &= 4 \ . \end{aligned}$$

The estimated failed amount, $\hat{n_k}$, will always be less than or equal to the actual failed amount n_k since for each κ_k within r_k we use the same percentage value ρ_k in the approximation algorithm. By increasing the number of intervals defined in $[0, 1]$, the error $\epsilon = n_k - \hat{n_k}$ will be decreased.

Proposition 1. *If the number of intervals that describe the κ_k in the PA algorithm, R, increases, the error $\epsilon = n_k - \hat{n_k}$ will decrease.*

Proof. Assume that the actual success percentage is κ_k and $\frac{r_k-1}{R} < \kappa_k \leq \frac{r_k}{R}$ for some $r_k \in \{1, 2, \ldots, R\}$ and $R = 2^l$ and ρ_k is the estimation for this κ_k in the approximation algorithm.

In the PA algorithm the success probability is taken into account as $\rho_k = r_k/R$. To calculate $\hat{n_k}$, we multiply the received amount with $(1-\rho_k)/\rho_k$, where in the exact algorithm we use $(1-\kappa_k)/\kappa_k$. Since $\kappa_k \leq \rho_k$,

$$\frac{1-\rho_k}{\rho_k} \leq \frac{1-\kappa_k}{\kappa_k} \ . \tag{3.26}$$

The error term ϵ results from the difference of ρ_k and κ_k. If the difference between ρ_k and κ_k decreases, the error ϵ will also decrease.

When we increase $R = 2^l$ to $R = 2^{l+1}$, the ρ_k will be updated to two possible cases:
The first case is where κ_k is closer to the lower bound of r_k, $\frac{r_k-1}{R}$. As we increase R, the intervals will be changed as shown in Figure 3.4. In Figure 3.4 (a), $R = 2^l$ and in Figure 3.4

(b) $R = 2^{l+1}$. As seen from the figure, for the first case increasing R decreases the upper bound of the corresponding interval from ρ_k to ρ'_k. Thus, the difference between ρ'_k and κ_k decreases, which also decreases the $\epsilon = n_k - \hat{n_k}$.

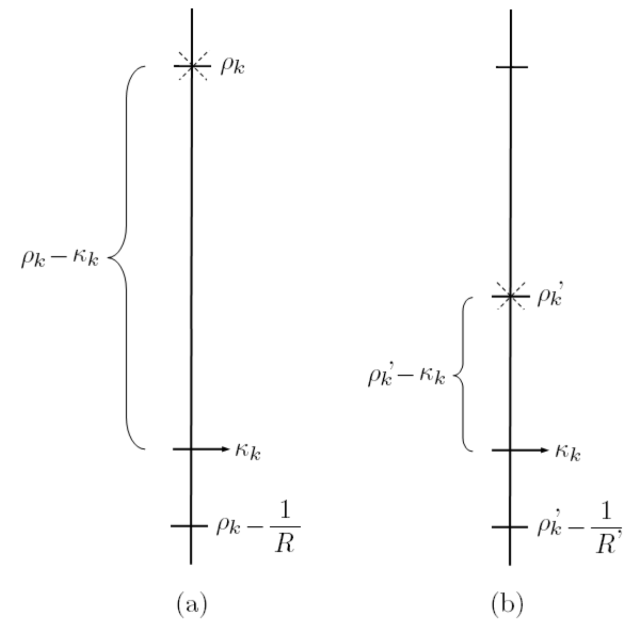

Figure 3.4: The effect of increasing R on $\epsilon = n_k - \hat{n_k}$, Case 1: (a) $R = 2^l$, (b) $R = 2^{l+1}$.

The second case is for κ_k closer to ρ_k. For this case increasing R may not improve the error. But at some point if we continue to increase R, and create fine intervals, the ρ_k value will improve and eventually the error term will decrease. Figure 3.5 shows an example of this case.

□

After defining an appropriate R value, the percentage approximations, ρ_k, successful observations, m_k, and the estimated failed amount, n_k, will be calculated as following in the Percentage Approximation Algorithm:

$$\rho_k = \frac{r_k}{R} \ , \tag{3.27}$$

$$m_k = i_k + \sum_{t=0}^{k-1} D_t + m_0 \ , \tag{3.28}$$

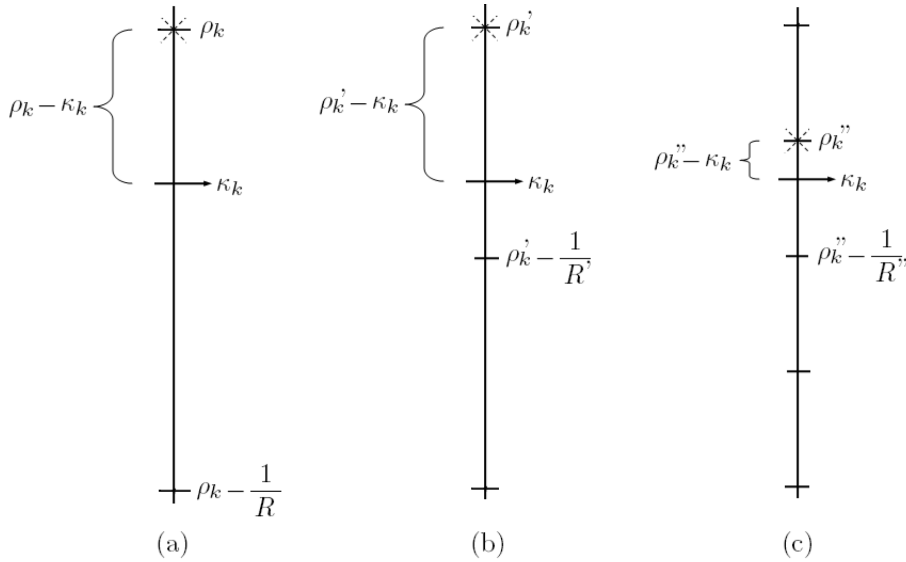

Figure 3.5: The effect of increasing R on $\epsilon = n_k - \hat{n_k}$, Case 2. (a) $R = 2^l$, (b) $R = 2^{l+1}$, (c) $R = 2^{l+2}$.

$$\hat{n_k} = n_0 + \left\lceil \frac{(i_k + \sum_{t=0}^{k-1} D_t)\ (1-\rho_k)}{\rho_k} \right\rfloor \ . \tag{3.29}$$

To describe the parameters of the Beta distribution, we need to have integer values for m and n, so we round the estimation of the failed amount to the nearest integer. The state variables i_k and ρ_k are updated according to

$$i_{k+1} = i_k + y_k - D_k \ , \tag{3.30}$$

$$r_{k+1} = \left\lceil \frac{(m_k - m_0) + y_k}{(m_k - m_0) + (\hat{n_k} - n_0) + x_k} R \right\rceil \ . \tag{3.31}$$

In Equation 3.31, we update the interval for the next period. The actual successful observations $(m_k - m_0)$ and the actual failed observations $(\hat{n_k} - n_0)$ are calculated according to the given state. For each order quantity x_k and received amount y_k, we update the interval from the ratio between total actual successful observation and the total given order. Then, the approximate DP algorithm is:

$$J_N(i_N, r_N) = 0 \ , \tag{3.32}$$

$$\begin{aligned} J_k(i_k, r_k) \quad = \quad & \min_{x_k \in \{0,1,2,\ldots,M\}} \int_0^1 \Big[\sum_{y=0}^{x_k} \binom{x_k}{y} q^y (1-q)^{x_k - y} \\ & \Big(c\, y + h(i_k - D_k + y)^+ + s(-i_k + D_k - y)^+ \\ & + J_{k+1}(i_k + y - D_k, \Big\lceil \frac{(m_k - m_0) + y}{(m_k - m_0) + (\hat{n_k} - n_0) + x_k} R \Big\rceil)\Big)\Big] \\ & \frac{\Gamma(m_k + \hat{n_k})}{\Gamma(m_k)\Gamma(\hat{n_k})} q^{m_k - 1}(1-q)^{\hat{n_k}-1} dq \end{aligned} \tag{3.33}$$

For this algorithm, the states at each stage are described using i_k and r_k and the total number of states at stage k is

$$R\Big(1 + w + \sum_{t=0}^{k-1} D_t\Big) \ , \tag{3.34}$$

since $1 \leq r_k \leq R$ and $-\sum_{t=0}^{k-1} D_t \leq i_k \leq w$. Then the state space over all stages has size

$$R\ N + N\ w\ R + R\ \sum_{k=1}^{N} \sum_{t=0}^{k-1} D_t \ . \tag{3.35}$$

In the worst case scenario, the cumulative demand at each period will be equal to D, and the total state space has size

$$R\ N + N\ w\ R + R\ \sum_{k=1}^{N} D \ . \tag{3.36}$$

The order of the state space is

$$O(R\ N + R\ w\ N + R\ D\ N) \ . \tag{3.37}$$

Since the on-hand inventory level, w, will be bigger than one, the order of the state space becomes

$$O(R\ N\ max(w, D)) \ . \tag{3.38}$$

When R is set to a constant value, we have

$$O(N\ max(w, D)) \ . \tag{3.39}$$

Then, the following proposition can be concluded:

Proposition 2. *For a fixed value of R, the size of the state space of the proposed approximation algorithm is decreased to $O(N\ max(w, D))$, which is the same for the PI and NI algorithms.*

It is important to set the R parameter. Clearly, for large R values, the solution time will increase and for small R values the solution of the PA algorithm will deviate from the exact solution. To test how the approximation algorithm works and how to set the R values, we solved both BU and PA algorithms for the instances defined in Table 3.9 with different R values and report the total expected cost and solution time of each algorithm. The results are summarized in Tables 3.20, 3.21, 3.22, 3.23, 3.24, and 3.25.

Table 3.20: Percentage difference between the expected total costs of BU and PA models for data Set 1 and $M = 10$

R	**Average Difference**	**Minimum Difference**	**Maximum Difference**	**Average Solution Time**	**Minimum Solution Time**	**Maximum Solution Time**
4	2.77%	1.79%	3.95%	0.18 h	0.17 h	0.20 h
8	2.09%	1.51%	2.49%	0.37 h	0.34 h	0.41 h
16	1.31%	0.96%	1.59%	0.81 h	0.75 h	0.90 h
BU	-	-	-	2.22 h	2.03 h	2.40 h

Table 3.21: Percentage difference between the expected total costs of BU and PA models for data Set 2 and $M = 10$

R	**Average Difference**	**Minimum Difference**	**Maximum Difference**	**Average Solution Time**	**Minimum Solution Time**	**Maximum Solution Time**
4	3.25%	1.18%	4.90%	0.18 h	0.14 h	0.19 h
8	2.19%	1.66%	2.73%	0.36 h	0.28 h	0.40 h
16	1.32%	1.04%	1.64%	0.79 h	0.59 h	0.90 h
BU	-	-	-	2.07 h	1.77 h	2.21 h

The R column in Tables 3.20, 3.21, 3.22, 3.23, 3.24, and 3.25 represents which value of R is used such that that there are 4,8, and 16 possible percentage intervals in the calculations. The success percentages are approximated with these intervals. BU represents the solutions of the exact algorithm. *Average Difference* column gives the percentage difference of the total expected cost of the approximation algorithm PA with parameter R and the exact

Table 3.22: Percentage difference between the expected total costs of BU and PA models for data Set 3 and $M = 10$

R	**Average Difference**	**Minimum Difference**	**Maximum Difference**	**Average Solution Time**	**Minimum Solution Time**	**Maximum Solution Time**
4	3.28%	0.38%	5.36%	0.14 h	0.07 h	0.18 h
8	2.98%	1.81%	4.29%	0.28 h	0.14 h	0.36 h
16	1.88%	1.11%	2.54%	0.62 h	0.28 h	0.78 h
BU	-	-	-	1.90 h	1.16 h	2.14 h

Table 3.23: Percentage difference between the expected total costs of BU and PA models for data Set 1 and $M = 15$

R	**Average Difference**	**Minimum Difference**	**Maximum Difference**	**Average Solution Time**	**Minimum Solution Time**	**Maximum Solution Time**
4	1.81%	1.42%	2.35%	0.31 h	0.29 h	0.35 h
8	0.67%	0.43%	1.02%	0.64 h	0.60 h	0.73 h
16	0.49%	0.28%	0.60%	1.38 h	1.28 h	1.55 h
BU	-	-	-	7.33 h	6.74 h	8.00 h

Table 3.24: Percentage difference between the expected total costs of BU and PA models for data Set 2 and $M = 15$

R	**Average Difference**	**Minimum Difference**	**Maximum Difference**	**Average Solution Time**	**Minimum Solution Time**	**Maximum Solution Time**
4	1.84%	1.43%	2.32%	0.31 h	0.24 h	0.37 h
8	0.74%	0.42%	1.06%	0.64 h	0.49 h	0.74 h
16	0.51%	0.30%	0.74%	1.39 h	1.01 h	1.63 h
BU	-	-	-	6.90 h	5.56 h	7.62 h

Table 3.25: Percentage difference between the expected total costs of BU and PA models for data Set 3 and $M = 15$

R	**Average Difference**	**Minimum Difference**	**Maximum Difference**	**Average Solution Time**	**Minimum Solution Time**	**Maximum Solution Time**
4	3.13%	0.80%	6.04%	0.24 h	0.11 h	0.31 h
8	1.69%	0.56%	3.59%	0.50 h	0.23 h	0.64 h
16	1.43%	0.27%	3.79%	1.07 h	0.47 h	1.39 h
BU	-	-	-	6.08 h	3.43 h	7.05 h

algorithm over 10 instances of the data set. *Minimum Difference* and *Maximum Difference* shows the minimum and maximum difference values over 10 instance sets. *Average Solution Time* , *Minimum Solution Time*, and *Maximum Solution Time* columns report the average, minimum and maximum solution times of the algorithms over 10 instance sets.

In accordance with Proposition 1, for both $M = 10$ and $M = 15$ cases increasing R improves the performance of the PA algorithm and decreases the gap between the PA and BU models. We start to solve the PA algorithm with $R = 4$. The average percentage difference between the expected costs of BU and PA models over three demand sets with 10 instances are 3.10% for $M = 10$ and 2.26% for $M = 15$, where the maximum differences are 5.36% and 6.04%, respectively. Since we know that increasing R will improve the model, we increase R to 8 and 16. For $R = 16$, the average cost difference over the three demand sets with 10 instances becomes 1.50% for $M = 10$ and 0.81% for $M = 15$, where the maximum differences are 2.54% and 3.79%, respectively. We think that on the average 1.50% and 0.81% are sufficient to say that the PA algorithm performs well because increasing the R increases the solution time.

For small values of M, the difference between the solution times of BU and PA models is not noteworthy. For $M = 10$, the average solution time of in the BU approach is 2.06 h, while it is decreased to 0.74 h by the PA approach. However, as M increases, using the PA approach becomes more beneficial. For $M = 15$, the solution time is decreased to 1.28 h from 6.77 h with the PA approach. If the M value would be large for a specific problem, then the PA algorithm provides better benefit in terms of solution time. To show this, we solve one instance of demand Set 2 with $M = 10, 15, 20,$ and 30. The results are shown in Table 3.26.

Table 3.26: The performance of PA algorithm with different M values.

R	**M**	J_0	$(J_0^{PA} - J_0^{BU})/J_0^{BU}$	**Solution Time**
4	10	976.4	3.94%	0.19 h
8	10	959.6	2.15%	0.39 h
16	10	949.7	1.10%	0.78 h
BU	10	939.4	-	2.21 h
4	15	723.6	1.68%	0.37 h
8	15	719.2	1.06%	0.74 h
16	15	715.1	0.49%	1.63 h
BU	15	711.6	-	7.62 h
4	20	643.5	3.69%	0.57 h
8	20	643.0	3.62%	1.14 h
16	20	632.4	1.92%	2.45 h
BU	20	620.5	-	17.89 h
4	30	574.9	2.50%	1.02 h
8	30	571.4	1.87%	2.01 h
16	30	570.5	1.73%	4.24 h
BU	30	560.9	-	55.00 h
4	40	562.3	-	1.44 h
8	40	560.6	-	2.94 h
16	40	560.1	-	6.11 h

In Table 3.26, the J_0 column gives the total expected costs. In the $(J_0^{PA} - J_0^{BU})/J_0^{BU}$ column the percentage difference of PA algorithms with respect to the exact BU algorithm is given. As seen from the results, as the R value increases, J_0 takes closer values to the BU algorithm and the percentage differences decreases. From the solution time improvements, it can be concluded that the proposed approximation algorithm performs better for large values of M. For $M = 10$ and $R = 16$, the solution time of PA is 35.18% of the BU algorithm, while this ratio decreases to 7.7% for $M = 30$ and $R = 16$.

3.4 Extensions

3.4.1 Bayesian Update Approach with Stochastic Demand

The model defined in Section 3.3 is for the deterministic demand case. When demand is deterministic, the received amount can be calculated using Equation 3.7. In case of stochastic demand, we cannot calculate the success amount m_k since $\sum_{t=0}^{k-1} d_t$ is unknown. For this reason, we need to keep the observed delivered quantities and modify the proposed DP algorithm where the states are described by i_k, m_k, and n_k, and updated according to the following:

$$i_{k+1} = i_k + y_k - d_k \ , \tag{3.40}$$

$$m_{k+1} = m_k + y_k \ , \tag{3.41}$$

$$n_{k+1} = n_k + (x_k - y_k) \ . \tag{3.42}$$

Then, the DP algorithm is:

$$J_N(i_N, \quad) = 0 \ , \tag{3.43}$$

$$\begin{aligned} J_k(i_k, m_k, n_k) \quad = \quad & \min_{x_k \in \{0,1,2,\ldots,M\}} \int_0^1 E_{d_k} \Big[\sum_{y=0}^{x_k} \binom{x_k}{y} q^y (1-q)^{x_k - y} \\ & (c\, y + h(i_k - d_k + y)^+ + s(-i_k + d_k - y)^+ \\ & + J_N(i_k - d_k + y, m_k + y, n_k + (x_k - y))) \Big] \\ & \frac{\Gamma(m_k + n_k)}{\Gamma(m_k)\Gamma(n_k)} q^{m_k - 1} (1-q)^{n_k - 1} dq \ . \end{aligned} \tag{3.44}$$

Since the states at each stage of the BU algorithm are described using i_k, m_k and n_k, total number of states at stage k is

$$(k\ M)(k\ M)\Big(1 + w + \sum_{t=0}^{k-1} D_t\Big) \ . \tag{3.45}$$

since $n_0 \leq n_k \leq n_0 + kM$, $m_0 \leq m_k \leq m_0 + kM$ and $-\sum_{t=0}^{k-1} D_t \leq i_k \leq w$. Then the state space over all stages has size

$$M^2 \sum_{k=1}^{N} k^2 + M^2\ w \sum_{k=1}^{N} k^2 + M^2 \Big(\sum_{k=1}^{N} k^2 \sum_{t=0}^{k-1} D_t\Big) \ . \tag{3.46}$$

In the worst case scenario, the cumulative demand at each period will be equal to D, and the total state space has size

$$M^2 \sum_{k=1}^{N} k^2 + M^2 \ w \sum_{k=1}^{N} k^2 + M^2 \ D \left(\sum_{k=1}^{N} k^2 \right) . \tag{3.47}$$

Then, the order of the state space is

$$O(M^2 \ N^3 + M^2 \ w \ N^3 + M^2 \ D \ N^3) \ . \tag{3.48}$$

Since the on-hand inventory level, w, will be bigger than one, the order of state space becomes

$$O(N^3 \ M^2 \ max(w, D)) \ . \tag{3.49}$$

The complexity of this algorithm is too high for practical purposes, but if the demand is not deterministic, the information about the received amount should be included into the states and the DP algorithm should be solved with respect to that.

3.4.2 The Case When p Varies with Order Quantity

As mentioned before, the reliability parameter may change with time and order quantity. In this section we define reliability parameter as a function of the order quantity x_k, $p(x_k)$. For each order size $m \leq x_k \leq M$, the reliability parameter changes in $[1, p]$ with the function defined in Figure 3.6.

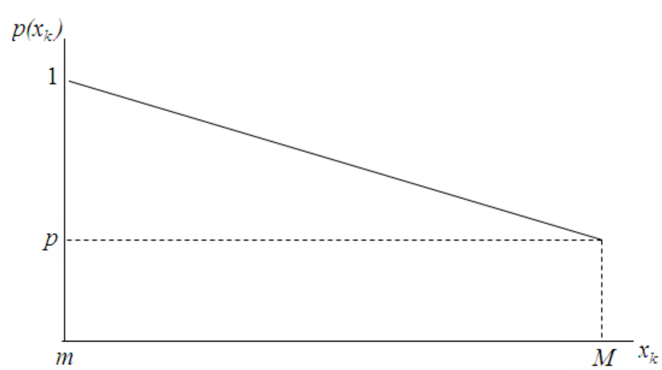

Figure 3.6: Function of the reliability parameter with order quantity

To solve the PI case we modify the Equation 3.4 as following:

$$\begin{aligned} J_k(i_k) \quad &= \quad \min_{x_k \in \{0,1,2,\ldots,M\}} \Big[\sum_{y=0}^{x_k} \binom{x_k}{y} p(x_k)^y (1-p(x_k))^{x_k - y} \Big(c\, y + h(i_k - D_k + y)^+ \\ &\qquad + s(-i_k + D_k - y)^+ + J_{k+1}(i_k - D_k + y) \Big) \Big] \\ &\qquad \text{for all } k = 0, 1, \ldots, N-1, \quad \forall i_k \in \left\{ -\sum_{t=0}^{k-1} D_t, \ldots, w \right\} . \qquad (3.50) \end{aligned}$$

For the other cases, the DP formulations are not modified since the ordering quantities are calculated without knowing the actual reliability parameter. The order quantity dependent reliability parameter $p(x_k)$ is used in the simulation runs for these cases. The results for the PI, NI and PA with $R = 16$ are summarized in Table 3.27.

Table 3.27: Average total cost values for the PI, PA, and NI cases with order quantity dependent reliability parameter.

Model	M	p	Avg TC	Cost Difference	Percentage Difference
PI	30	0.5	213.43	-	-
PA w/ R=16	30	0.5	221.95	8.51	3.99%
NI	30	0.5	269.03	55.59	26.05%
PI	30	0.1	224.52	-	-
PA w/ R=16	30	0.1	240.33	15.81	7.04%
NI	30	0.1	256.99	32.48	14.46%

In Table 3.27, M and p columns show the maximum order quantity and the lower bound for the reliability parameter as shown in Figure 3.6. *Avg TC* column gives the average total expected cost over all simulation runs in which the reliability parameter depends on the order quantity. Finally, *Cost Difference* and *Percentage Difference* columns report the differences between the PA and NI models with the PI model with absolute and percentage gaps. From the results of Table 3.27, it can be concluded that for the model in which the reliability parameter depends on the order quantity, the proposed Bayesian update approach performs considerably better than the NI case. For the large values of p, the difference between PA and PI cases are close, but as p decreases this difference increases.

3.5 Conclusion

Suppliers may not be capable of satisfying all of the order amount given by the manufacturers at the requested time. In this study we model the capability of the supplier to fulfill the orders by modeling the amount received in a period with a Binomial distribution. Each item ordered is delivered with a probability that we call supplier reliability. When the manufacturer has no information about the reliability parameter of its supplier, we propose a Bayesian update approach to estimate the supplier reliability at each period using the data of past orders' ordered and recieved amounts. The developed Dp algorithm has a computational complexity that depends on input data values such as the maximum order quantity and total demand. We show by computational experiments that the Bayesian update approach can be used successfully for instances with small input data size. For problem instances with large parameter values, we propose an approximation algorithm which gives expected costs close to those found by the exact algorithm and the order of its computational complexity is smaller than the exact algorithm. Hence, in our experiments the solution times of the approximation algorithm were much less compared to the exact algorithm.

Chapter 4

CONCLUSIONS

We studied the problem of finding a cost-efficient ordering and a shipment plan for a manufacturer that sources multiple items from a supplier in the first part of the thesis. We determine how many units of each part should be ordered and shipped from the supplier in each period, and how many trucks to ship over a planning horizon to minimize the inventory holding and transportation costs by solving a MIP model.

We proposed a simple policy that allows delaying the shipment of trucks with a small truckload percentage to increase the average vehicle utilization. We investigated its benefits and risks, as well as its practicality. Our numerical experiments indicate that the proposed policy can be beneficial for the multiple-item sourcing problems with short lead times and frequent shipments. In all data sets tested, the DTP improves the total costs with a small decrease in the service level in case of unexpected changes in the manufacturer's procurement plans. The risk analysis will give different results for different cost structures and demand patterns. However, implementing such an analysis for different systems is not difficult, and the risk and benefits can be easily evaluated as demonstrated in this study.

The proposed policy can be extended to cases where vehicle types having different capacities and costs are available by modifying the proposed MIP formulation, albeit with a slight increase in the number of integer variables, to find an optimal mix of the vehicle types to be used. Truck packing considerations can also be incorporated into the model for more accurate estimates of the number of trucks needed. Consolidation of the loads from multiple suppliers as an alternative means of truck load efficiency may also be compared against the DTP.

For the second part of the thesis, we studied the dynamic lot-sizing problem under random supply where the supplier's shipment behavior is represented by a model that assumes a random portion of the given order is shipped in every period. In this stochastic environment, we determine how many units should be ordered from the supplier in each period to minimize the sum of expected inventory holding, shortage and purchase costs by solving DP algorithms.

We investigated different cases in terms of availability of information about the supplier's capability. The supply process of the supplier is known to be binomially distributed, but the success probability of each item to be sent (supplier reliability) may not be known by the manufacturer. We are interested in the case where the supplier reliability is not known. To improve the sourcing process, we propose a method that enables the manufacturer to obtain more information about the supplier reliability throughout its ordering process and we develop a dynamic programming model with Bayesian updating of the supplier reliability parameter. From the information of previous orders' ordered and received amounts, the supplier's capability is estimated and the ordering decisions are optimized by considering the available information until that point. The proposed algorithm is compared with the case under perfect information (PI) as well as the one with no information (NI) on supplier reliability. By computational experiments we show that the proposed algorithm with Bayesian Updates (BU), gives better expected total cost values than the NI case and the optimal expected costs found by the BU approach are close to those found in the PI case.

To overcome the computational difficulty due to the increase in the state space size of the BU approach, we define an approximation algorithm. With this algorithm we decreased the order of the state space from $O(N^2\ M\ max(w, D))$ to $O(N\ max(w, D))$ by using a fixed number of success percentage intervals, R. The computational analysis of the PA approach shows that the proposed approximation algorithm performs significantly better when the maximum order size, M, is large. We both proved and showed computationally that as R increases, the expected total cost values in the PA approach will be close to the expected total cost values of exact algorithm, BU.

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APPENDIX

Algorithm 4 DP algorithm for the PI case

Define deterministic demand D_k, $k = 0, \ldots, N-1$, cost parameters h, s, and c, warehouse limit w, minimum and maximum order quantities m and M, and initial stock i_0

for each period $k = N, N-1, \ldots, 1$ **do**

  for each inventory level $-\sum_{t=0}^{k-1} D_t \leq i_k \leq w$ **do**

    if $k = N$ **then**

      Set terminal cost to 0, $J_k(i_k) = 0$

      Set ordering quantity to 0, $\pi_k(i_k) = 0$

    else

      Using Equation 3.4 find the minimum expected cost and equate to $J_k(i_k)$

      Set $\pi_k(i_k)$ to the x_k that minimizes the expected cost

    end if

  end for

end for

for initial period, $k = 0$ **do**

  Using Equation 3.4 and i_0 find the minimum expected cost and equate to $J_0(i_0)$

  Set $\pi_0(i_0)$ to the x_0 that minimizes the expected cost

end for

Algorithm 5 DP algorithm for the NI case

Define deterministic demand D_k, $k = 0, \ldots, N-1$, cost parameters h, s, and c, warehouse limit w, minimum and maximum order quantities m and M, and initial stock i_0

for each period $k = N, N-1, \ldots, 1$ **do**

  for each inventory level $-\sum_{t=0}^{k-1} D_t \leq i_k \leq w$ **do**

    if $k = N$ **then**

      Set terminal cost to 0, $J_k(i_k) = 0$

      Set ordering quantity to 0, $\pi_k(i_k) = 0$

    else

      Using Equation 3.6 find the minimum expected cost and equate to $J_k(i_k)$

      Set $\pi_k(i_k)$ to the x_k that minimizes the expected cost

    end if

  end for

end for

for initial period, $k = 0$ **do**

  Using Equation 3.6 and i_0 find the minimum expected cost and equate to $J_0(i_0)$

  Set $\pi_0(i_0)$ to the x_0 that minimizes the expected cost

end for

Algorithm 6 DP algorithm for the BU case

Define deterministic demand D_k, $k = 0, \ldots, N-1$, cost parameters h, s, and c, warehouse limit w, minimum and maximum order quantities m and M, initial Beta distribution parameters m_0 and n_0, and initial stock i_0

for each period $k = N, N-1, \ldots, 1$ **do**

  for each inventory level $-\sum_{t=0}^{k-1} D_t \leq i_k \leq w$ **do**

    Calculate the successful observations using Equation 3.7

    for each failed observations $n_0 \leq n_k \leq n_0 + kM$ **do**

      if $k = N$ **then**

        Set terminal cost to 0, $J_k(i_k, n_k) = 0$

        Set ordering quantity to 0, $\pi_k(i_k, n_k) = 0$

      else

        Using Equation 3.11 find the minimum expected cost and equate to $J_k(i_k, n_k)$

        Set $\pi_k(i_k, n_k)$ to the x_k that minimizes the expected cost

      end if

    end for

  end for

end for

for initial period, $k = 0$ **do**

  Calculate the successful observations using Equation 3.7 and i_0

  Using Equation 3.11, i_0, m_0, and n_0 find the minimum expected cost and equate to $J_0(i_0, n_0)$

  Set $\pi_0(i_0, n_0)$ to the x_0 that minimizes the expected cost

end for

VITA

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