Is Subsidizing Students Uniformly A Pareto Improvement and/or Welfare Increasing?

by

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This is to certify that I have examined this copy of a master's thesis by

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and have found that it is complete and satisfactory in all respects,

and that any and all revisions required by the final

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To my altruistic parents

ABSTRACT

This paper analyzes a 3 period OLG model under the DSGE framework in which agents are heterogenous with respect to their ability and education level. Each young agent is assigned an ability level by nature randomly. Given the tuition determined by the government and bequest inherited from the parent, the young agent makes the choice of going to school or not. The consumption and education expenditures of the young agents are financed by middle-aged agents who leave bequests to them. The middle-aged agents also pay some fraction of their total income in the form of social security which is distributed to the current old agents. The redistribution rate is state contingent and determined within the model. The endogenously determined amount of bequest, uncertainty regarding the state of the economy and college completion, endogenous wage determination based upon the amount of schooling are other key characteristics of the paper. The paper investigates whether or not it is pareto optimal to subsidize students uniformly and considers both societal welfare and income inequality. It turns out that one of the two stable equilibria is a poverty trap which can be caused by the perpetuity of path dependence of instituitons and subsidizing students uniformly increases social welfare and decreases income inequaliy.

Keywords: Schooling, Human Capital, Social Security, OLG models, DSGE framework.

ÖZET

Bu makale dinamik belirsiz genel denge çerçevesi içerisinde 3 periyodluk bir çakışan nesiller modelini incelemektedir. Bu modelde insanlar yeteneklerine ve eğitim seviyelerine göre heterojendir. Her genç ajana rastlantısal olarak bir yetenek düzeyi verilmektedir. Devlet tarafından belirlenen harç ücretleri ve ailesinden aldığı miras verileri ışığında, genç ajan okula gidip gitmeme kararı almaktadır. Gençlerin eğitim ve tüketim harcamaları aileleri tarafından karşılanmaktadır. Orta yaşlı ajanlar ikinci periyodda kazandıkları gelirlerinin bir kısmını sosyal güvenlik vergisi olarak devlete ödemekte ve yaşlandıklarında da ödedikleri miktarla orantılı olarak geri ödeme almaktadırlar. Geri dağıtım oranı ekonominin şuanki ve bir periyod önceki durumuna bağlı olarak değişiklik göstermektedir ve modelin içinde endojen olarak bulunmaktadır. Endojen olarak belirnenen miras, ekonominin geleceğinin belirsizliği, edinilen eğitime göre maaşların belirlenmesi makalenin diğer ayırıcı özelliklerindendir. Bu makale, öğrencileri aynı oranda sübvanse etmenin toplumun refahına olan etkisini araştırmaktadır. Sonuç olarak ise, iki tane stabil denge bulunmakta ve bu stabil dengelerden birisinin de fakirlik tuzağı olduğu görülmektedir. Bu fakirlik tuzağının sebeblerinden birisi ise devletin kurumlarının geçmişin geleceğe taşıyıcısı durumunda olmalarındandır. Öğrencileri sübvanse etmek toplum refahını arttırmakta ve gelir eşitsizliğini azaltmaktadır.

Anahtar Kelimeler: Okullaşma, Beşeri Sermaye, Sosyal Güvenlik, Çakışan Nesiller Modeli, Dinamik Belirsiz Genel Denge Modeli.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

”Existing imperfections in the capital market tend to restrict the more expensive vocational and professional training to individuals whose parents or benefactors can finance the training required....**The result is to perpetuate inequalities in wealth and status.**”

Milton Friedman [8] in *Capitalism and Freedom*

Chapter 1

INTRODUCTION

Subsidization of education as a policy tool in order to increase welfare, efficiency and equality within a society has been a controversial issue for decades whereas increasing tuition fees usually causes a series of protests. The main question is to whom subsidy should be given and how should this process be carried out. If there is credit market imperfection and inability to borrow against human capital (Becker [2]) then poor parents may not finance the education expenditures of their children. This can cause persistence in income inequality among future generations of a country and inefficiency in the economy. Papers by Caucutt&Kumar [4],Fender&Wang [7] and Loury [11] also conclude that credit market constraints cause inefficiency which also can decrease aggregate welfare. Moreover Hanushek&Leung&Yilmaz [10] compares different tuition subsidy mechanisms with respect to their implications regarding efficiency, income equality and intergenerational mobility. These papers commonly agree that inability to borrow against human capital is one of the main reasons of inefficiency in the economy.

Being motivated by these considerations, we develop a 3 period OLG model in a stochastic general equilibrium framework. The main actors of the model are young, middle-aged and old agents together with a government and a firm. Middle-aged agents are assumed to be altruistic and so they give bequests to their children. The young agent in turn makes a decision of going to college or not depending on the tuition determined by the government and future expected utility of attending college or not. Agents are heterogenous with respect to their ability and education level. Each middle-aged agent pays some exogenously given fraction of her income in the form of social security to government and she gets paid by the government at a state contingent rate when she is old. This state contingent redistribution rate which is determined within the model is one of the main characteristics of the model differentiating it from other models in the literature. In the light of this setup, the paper considers some policy issues ,from an interim pareto optimality perspective, such as subsidizing young agents uniformly. Subsidizing these agents may increase the fraction of educated agents in the second period, but this may cause a decrease in the nominal wages of educated agents which in turn may decrease the aggregate welfare of agents. So there are opposing forces affecting aggregate welfare which necessiates a quantitative analysis.Indeed, the increase in the number of educated agents in the second period can increase the total social security revenue to be distributed among old agents. So the model gives some incentive for the middle-aged agents to pay social security tax which is used by the government as a subsidy tool for education. This model is calibrated to match the stylized facts of the U.S. college market, and it allows us to analyze the dynamics of income distribution along with government intervention in the college market.

The paper finds out four nontrivial equilibria with two of them being stable. One of the stable equilibria results in poverty trap (Azariadis&Stachurski[1]).The poverty trap can be caused by the perpetuity of path dependence of institutions. The effect of institutions can be exacerbated when combined with the capital market imperfection present in the model.

The paper is organized as follows. Section 2 describes the model. Section 3 calibrates parameters and some statistics of the model and compares those statistics with that of U.S. data. Section 3 also characterizes all equilibria. Section 4 contains some policy simulations and the last section includes concluding remarks.

Chapter 2

THE BENCHMARK MODEL

Each individual born at time t lives $3-$periods where each period corresponds to 20 years. Time is discrete and runs from $t = 0, 1, ..., \infty$. There is aggregate uncertainty in each period and shocks are modeled by the variable $s_t \in S = \{B, G\}$ where B, G denotes bad state and good state respectively. s_t is assumed to follow a Markov chain with transition probabilities given by $\Pr(s_t = k|s_{t-1} = l) = \pi_{kl}$ where $k, l \in S$ and it denotes the probability of reaching state s_t at time t given the previous state s_{t-1} at time $t-1$. There is only one storable good used for both consumption and investment and the price of the good is normalized to 1 at each possible state and the economy is assumed to be small and open.

From now on the following notation will be used.If m is any variable in the model, then m^t will mean that an agent born at time t is considered.A subscript $z \in Z = \{e, u\}$,where e, u denotes educated and uneducated agent respectively, such as m_z will tell the education level of the agent and a subscript x^t will denote the ability of the agent such as m_{x^t}.The current state of the economy s_t is given in parentheses such as $m(s_t)$. So the notation $m^t_{z,x^t}(s_t)$ will denote the current value of m for an agent born at time t with ability $x^t \in X$ and education level z.

2.1 Demographics

At any period there are 3 overlapping generations which are named as young,middle-aged and old. There are also families in the model each of which includes one young,one middle-aged and one old agent. The population growth rate is assumed to be zero for simplicity. The case at which population grows at a stochastic rate can be analyzed as an extension.

Young Agents: Young agents born at time t are heterogenous with respect to their initial exogenous endowment of ability $x^t \in X = \{H, L\}$ where $H \in [0, 1]$ denotes high ability and $L \in [0, 1]$ denotes low ability with $H > L$. H and L also equals the success probability of an agent attending school which is similar to the model by Ben-Porath [3].Let's denote the ability of the middle-aged agent in the same family as x^{t-1} where again $x^{t-1} \in X$. These abilities are distributed according to a Markov chain with a 4×4 transition matrix where $\Pr(x^t = j|x^{t-1} = i) = \widehat{\pi}_{ji}$ where $i, j \in X$. The middle-aged agent born at time $t-1$ leaves bequest to her offspring at time t which is denoted by $b^{t-1}_{z,x^t}(s_t)$ where $z \in Z$, $x^t \in X$. So the young agent uses this bequest to finance her consumption $c^t(s_t)$ and the tuition expense ϕ_t where ϕ_t is determined by the government within the model and is same for all young agents. Given the bequest by her parent, the young agent chooses to go to school or not. If $b^{t-1}_{z,x^t}(s_t) \leq \phi_t$ then the young agent does not go to school and becomes an unskilled worker with ability x^t in the next period. If $b^{t-1}_{z,x^t}(s_t) > \phi_t$ then she can consume all of her bequest without going to school or she can choose to go to school by paying ϕ_t to government and by consuming the remaining part of the bequest. If she goes to school she will successfully gradute with probability x^t and will fail with probability $1 - x^t$. If she successfully gradutes she will be an educated worker with ability x^t and if she fails then she will be an uneducated worker with ability x^t in the next period. The educated agent with ability x^t will receive $x^t w^e(s_{t+1})$ and uneducated agent with the same ability will receive $x^t w^u(s_{t+1})$ where $w^e(s_{t+1}), w^u(s_{t+1})$ denote the wages at time $t+1$ for educated and uneducated agents respectively.The wages $w^e(s_{t+1})$ and $w^u(s_{t+1})$ are determined endogenously by profit maximization condition of the firm which will be discussed later.

Middle-aged Agents: They are heterogenous with respect to their education and ability level. For instance, two educated middle-aged agents can have different ability levels given by nature to them. Each middle-aged agent is endowed with 1 unit of labor. The wage income of an educated individual with ability x^t is $x^t w^e(s_{t+1})$. The wage income of an uneducated agent with ability x^t is $x^t w^u(s_{t+1})$. Define the indicator function I^{t+1}_e for the middle-aged agent born at time t as follows:

$$I^{t+1}_e = \begin{cases} 1 & \text{if agent is educated at time } t+1 \\ 0 & \text{if agent is uneducated at time } t+1 \end{cases}$$

Let $f(x^t, I^{t+1}_e)$ represent the number middle-aged agents at time $t+1$ with ability x^t and education outcome I^{t+1}_e.For instance, $f(x^t = L, I^{t+1}_e = 0), f(x^t = L, I^{t+1}_e = 1), f(x^t = H, I^{t+1}_e = 0), f(x^t = H, I^{t+1}_e = 1)$ represent the number of low ability-uneducated,low ability-educated,high ability-uneducated and high ability-educated middle-aged agents at time

$t+1$ respectively. It can be said that to have $f(x^t, I_e^{t+1} = 1)$ educated middle-aged agents at time $t+1$ with ability level x^t, there should be an enrollment at school in the amount of $\frac{f(x^t, I_e^{t+1}=1)}{x^t}$ at time t. Therefore, a total enrollment at time t in the amount of $N_t^r = \sum_{x^t=H,L} \frac{f(x^t, I_e^{t+1}=1)}{x^t}$ yields a total supply of educated middle-aged agents at time $t+1$ in the amount of $N_{t+1}^s = \sum_{x^t=H,L} f(x^t, I_e^{t+1} = 1)$.To educate young agents, government hires educated middle-aged agents in the amount of γN_t^r where $0 < \gamma < 1$[1].The proportion of low ability educated and high ability educated agents hired by government are given by exogenous parameters γ_H, γ_L where $\gamma = \gamma_H + \gamma_L$ .So the number of high ability-educated and low ability-educated teachers will be $\gamma_H N_t^r, \gamma_L N_t^r$ respectively. All middle-aged agents pay a social security tax collected by government as part of the social security system at the rate of τ which is given exogenously.The total revenue is distributed among the current old generation at a rate $\tau_d(s_t, s_{t-1})$ of their respective second period wage earning. For instance if an agent's second period wage income at time t is $x^{t-1}w^e(s_t)$ then she will receive $\tau_d(s_{t+1}, s_t)x^{t-1}w^e(s_t)$ as social security payment when she becomes old at time $t+1$. The redistribution rate depends on both current and previous state and it is determined within the model. The consumption of middle-aged agents born at time t with ability $x^t \in X$ and education $z \in Z$ is denoted by $c_{z,x^t}^t(s_{t+1})$ and middle-aged agents are assumed to save some part of their income. Denote the saving of a middle-aged agent by $k_{z,x^t}^t(s_{t+1})$ and bequest to her offspring by $b_{z,x^t}^t(s_{t+1})$.

Old Agents: When the agent becomes old,she will receive an interest payment at the rate r from her previous period saving which is $k_{z,x^t}^t(s_{t+1})$.The gross interest rate r is determined by the world financial market and therefore it is exogenous because the economy in the model is assumed to be small and open. The consumption of an old agent born at time t is $c_{z,x^t}^t(s_{t+2})$. The old agent also receives a share of the total payroll taxes collected from current middle-aged agents which was explained above. At initial date $t = 0$, there are families consisting of one middle-aged and one old agent. Each old agent possesses $k_{z,x^{-2}}^{-2}(s_0)$ units of capital good and each middle-aged agent is assumed to have already acquired an education $z \in Z$ and possesses an exogenously given ability $x^{-1} \in X$.

[1]Our education model follows closely Hanushek,Leung and Yilmaz[10]. For more justification regarding γ see Hanushek,Leung and Yilmaz[9]

2.2 Government

The functions of the government are collecting tuition payments and social security payments. The government has different budgets for education and social security system. Both budgets are assumed to be balanced at all periods. Government's education budget at time t is as follows:

$$\phi(s_t)N_t^r = \gamma_H N_t^r H w^e(s_t) + \gamma_L N_t^r L w^e(s_t)$$

The right hand side in the above expression is the total payment made to middle-aged teachers who are educated agents with different abilities and the left hand side equals tuition revenue. The corresponding social security budget at time t is:

$$TR(s_t) = \tau_d(s_t, s_{t-1})TW(s_{t-1})$$

where $TW(s_{t-1})$ is the sum of wage earnings of all old agents when they were middle-aged. So one can write:

$$TW(s_{t-1}) = \sum_{x^{t-2}=L,H} \sum_{z=e,u} x^{t-2} w^z(s_t)$$

2.3 Preferences

There are 4 types of young agents at time t according to their financial constraints, school attendance decision and future education level. The first type consists of agents who are high or low ability and cannot finance the tuition expense so they will be uneducated workers. The second type consists of agents who are high or low ability and they can finance the tuition and they choose to go to school and they successfully graduate from school.The third type consists of agents who are high or low ability and they can finance the tuition and they choose to go to school but they fail. The last type consists of agents who are high or low ability and they can finance the tuition, but they choose not to go to school. If one also considers the heterogeneity with respect to ability then there are 8 types of agents. In this section, the optimization problems of these 8 types of agents will be formulated together with their respective lifetime budget constraints. The Figure 2.1 illustrates these 8 groups.

In the following optimization problems the utility functions for each period are CRRA form and

Cannot Finance $\nearrow$High ability / $\searrow$Low Ability

Finance&Attend $\nearrow$High ability $\nearrow$Success / $\searrow$Fail; $\searrow$Low ability $\nearrow$Success / $\searrow$Fail

Can Finance&Not Attend $\nearrow$High ability / $\searrow$Low Ability

Figure 2.3.1: Different Groups

they are specified as follows for $0 < \delta < 1$ where δ is the risk aversion parameter:

$$
\begin{aligned}
U_1(c^t(s_t)) &= \frac{(c^t(s_t))^{1-\delta}}{1-\delta} \\
U_2(b^t_{z,x^t}(s_{t+1}), c^t_{z,x^t}(s_{t+1})) &= \frac{\left(\frac{(c^t_{z,x^t}(s_{t+1}))^{\alpha}(b^t_{z,x^t}(s_{t+1}))^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{1-\delta}}{1-\delta} \\
U_3(c^t_{z,x^t}(s_{t+2})) &= \frac{(c^t_{z,x^t}(s_{t+2}))^{1-\delta}}{1-\delta}
\end{aligned}
$$

Note that the utility functions are strictly concave, strictly increasing and twice continously differentiable in their arguments. In all of the following optimization problems $b^{t-1}_{z,x^t}(s_t)$ is a constant since bequest inherited by the young agent is a choice variable of her mother. Moreover the distribution of different types of agents is captured by distribution vector $F_t(x^t, e, s_t)$ such that :

$$F_{t+1}(x^t, e, s_t) = \Pi_t F_t(x^t, I^t_e, s_t)$$

where Π_t is the transition matrix consisting of transitions of the type $(x^{t+1}, I^{t+1}_e, s_{t+1}|x^t, I^t_e, s_t)$.

1. **First&Second Types:**(High or low ability and can't finance tuition)

$$\max_{c^t(s_t),b^t_{u,x^t}(s_{t+1}),c^t_{u,x^t}(s_{t+1}),c^t_{u,x^t}(s_{t+2})} U_1(c^t(s_t))+\beta_2 E[U_2(b^t_{u,x^t}(s_{t+1}),c^t_{u,x^t}(s_{t+1}))]+\beta_3 E[U_3(c^t_{u,x^t}(s_{t+2}))]$$

s.t.

$$c^t(s_t) = b^{t-1}_{z,x^t}(s_t)$$

$$c^t_{u,x^t}(s_{t+1}) + b^t_{u,x^t}(s_{t+1}) + k^t_{u,x^t}(s_{t+1}) = (1-\tau)x^t w^u(s_{t+1})$$

$$c^t_{u,x^t}(s_{t+2}) = rk^t_{u,x^t}(s_{t+1}) + \tau_d(s_{t+1},s_{t+2})x^t w^u(s_{t+1})$$

2. **Third&Fourth Types:**(High or low ability who finance tuition expense and choose to go to school and they successfully graduate from school)

$$\max_{c^t(s_t),b^t_{e,x^t}(s_{t+1}),c^t_{e,x^t}(s_{t+1}),c^t_{e,x^t}(s_{t+2})} U_1(c^t(s_t))+\beta_2 E[U_2(b^t_{e,x^t}(s_{t+1}),c^t_{e,x^t}(s_{t+1}))]+\beta_3 E[U_3(c^t_{e,x^t}(s_{t+2}))]$$

s.t.

$$c^t(s_t) + \phi(s_t) = b^{t-1}_{z,x^t}(s_t)$$

$$c^t_{e,x^t}(s_{t+1}) + b^t_{e,x^t}(s_{t+1}) + k^t_{e,x^t}(s_{t+1}) = (1-\tau)x^t w^e(s_{t+1})$$

$$c^t_{e,x^t}(s_{t+2}) = rk^t_{e,x^t}(s_{t+1}) + \tau_d(s_{t+1},s_{t+2})x^t w^e(s_{t+1})$$

3. **Fifth&Sixth Types:**(High or low ability who finance tuition expense and choose to go to school but they fail at school)

$$\max_{c^t(s_t),b^t_{u,x^t}(s_{t+1}),c^t_{u,x^t}(s_{t+1}),c^t_{u,x^t}(s_{t+2})} U_1(c^t(s_t))+\beta_2 E[U_2(b^t_{u,x^t}(s_{t+1}),c^t_{u,x^t}(s_{t+1}))]+\beta_3 E[U_3(c^t_{u,x^t}(s_{t+2}))]$$

s.t.

$$c^t(s_t) + \phi(s_t) = b^{t-1}_{z,x^t}(s_t)$$

$$c^t_{u,x^t}(s_{t+1}) + b^t_{u,x^t}(s_{t+1}) + k^t_{u,x^t}(s_{t+1}) = (1-\tau)x^t w^u(s_{t+1})$$

$$c^t_{u,x^t}(s_{t+2}) = rk^t_{u,x^t}(s_{t+1}) + \tau_d(s_{t+1}, s_{t+2})x^t w^u(s_{t+1})$$

4. **Seventh&Eighth Types:**(High or low ability who can finance tuition expense but do not choose to attend school) The optimization problem of these agents is same as the optimization problem of first&second types given above.

2.4 Firm

There is a single firm which uses educated and uneducated labor to produce output.Supply of total educated,uneducated effective labor to be used in production at time t is given by

$$S^t_e = \sum_{x^{t-1}=H,L} (f(x^{t-1}, I^t_e = 1) - \gamma_{x^{t-1}} N^r_t)x^{t-1}$$

and

$$S^t_u = \sum_{x^{t-1}=H,L} f(x^{t-1}, I^t_e = 0)x^{t-1}$$

respectively.The demand of the firm for uneducated and educated labor is denoted by D^t_u, D^t_e.The production function is as follows:

$$F(D^t_e, D^t_u) = (D^t_u)^{\beta-\xi(s_t)}(D^t_e)^{1-\beta+\xi(s_t)}$$

where $\xi(s_t)$ is the stochastic shock to productivity distributed identically with s_t so that $\xi(s_t)$ also follows a Markov Chain.The firm is assumed to make its decision regarding D^t_e, D^t_u by maximizing the time t profit which is given by:

$$\max_{D^t_e, D^t_u} \Pi(s_t) = (D^t_u)^{\beta-\xi(s_t)}(D^t_e)^{1-\beta+\xi(s_t)} - w^e(s_t)D^t_e - w^u(s_t)D^t_u$$

First order conditions imply:

$$w^e(s_t) = (1-\beta+\xi(s_t))\frac{F(D^t_e, D^t_u)}{D^t_e}$$

$$w^u(s_t) = (\beta - \xi(s_t))\frac{F(D_e^t, D_u^t)}{D_u^t}$$

The Figure 2.2 actually summarizes the interaction between different agents in the model as seen below.

2.5 Competitive Equilibrium

In our model, there are 2 markets which are goods market for consumption and labor market. Both of them are perfectly competitive.Before defining competitive equilibrium let us define some notation. Y^t denotes the total population of young agents at time t.Also assume that $M^t_{z,x^{t-1}}, O^t_{z,x^{t-2}}$ denote the total population of middle-aged and old agents with education $z \in Z$ and ability level $x^{t-1}, x^{t-2} \in X$ respectively.

Definition 2.5.1. *For given initial state s_{-1}, s_0 and $k^{-2}_{z,x^{-2}}(s_0)$ a **competitive equilibrium** is an allocation $c^{-2}_{z,x^{-2}}(s_0)$,$(c^{-1}_{z,x^{-1}}(s_0), b^{-1}_{z,x^{-1}}(s_0), k^{-1}_{z,x^{-1}}(s_0), c^{-1}_{z,x^{-1}}(s_1))$, $(c^t(s_t), c^t_{z,x^t}(s_{t+1}), b^t_{z,x^t}(s_{t+1}), k^t_{z,x^t}(s_{t+1}), c^t_{z,x^t}(s_{t+2}))_{t=0}^{\infty}$,a policy vector for government $\{(\phi(s_t), \tau_d(s_t, s_{t-1}))_{t=0}^{\infty}, \tau_d(s_0, s_{-1})\}$, a series of distributions $\left\{F_t(x^t, I_e^t, s_t)\right\}_{t=0}^{\infty}$ and prices $\{(w^e(s_t), w^u(s_t))_{t=0}^{\infty}, (w^e(s_{-1}), w^u(s_{-1}))\}$ such that $\forall t \in \{0, 1, ..., \infty\}$ and $\forall s_t \in S_t$:*

i) Given $\{w^e(s_t), w^u(s_t)\}_{t=0}^{\infty}$ for each $t \geq 0$ $(c^t(s_t), c^t_{z,x^t}(s_{t+1}), b^t_{z,x^t}(s_{t+1}), k^t_{z,x^t}(s_{t+1}),$ $c^t_{z,x^t}(s_{t+2}))$ solves:

$$\max_{c^t(s_t), b^t_{z,x^t}(s_{t+1}), c^t_{z,x^t}(s_{t+1})} U_1(c^t(s_t)) + \beta_2 E[U_2(b^t_{z,x^t}(s_{t+1}), c^t_{z,x^t}(s_{t+1}))] + \beta_3 E[U_3(c^t_{z,x^t}(s_{t+2}))]$$

s.t.

$$c^t(s_t) = b^{t-1}_{z,x^t}(s_t)$$

$$c^t_{z,x^t}(s_{t+1}) + b^t_{z,x^t}(s_{t+1}) + k^t_{z,x^t}(s_{t+1}) = (1-\tau)x^t w^z(s_{t+1})$$

$$c^t_{z,x^t}(s_{t+2}) = rk^t_{z,x^t}(s_{t+1}) + \tau_d(s_{t+1}, s_{t+2})x^t w^z(s_{t+1})$$

for all $z \in Z$ and $x^t \in X$

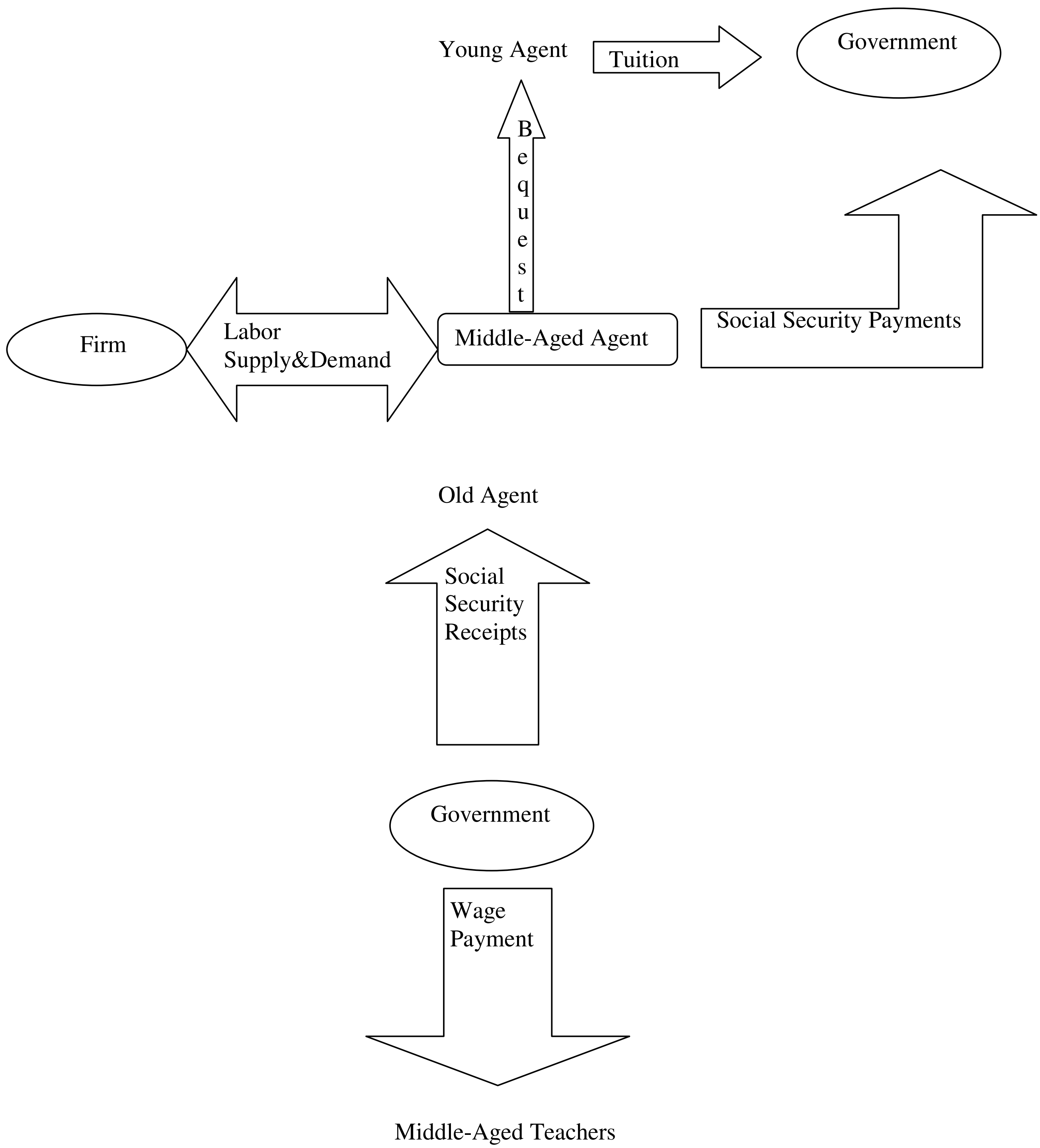

Figure 2.4.1: Interaction of Agents

ii) Given $(w^e(s_{-1}), w^u(s_{-1}))$, $c^{-2}_{z,x^{-2}}(s_0)$ *solves:*

$$\max_{c^{-2}_{z,x^{-2}}(s_0)} U_3(c^{-2}_{z,x^{-2}}(s_0))$$

s.t.

$$c^{-2}_{z,x^{-2}}(s_0) = rk^{-2}_{z,x^{-2}}(s_0) + \tau_d(s_0, s_{-1})x^{-2}w^z(s_{-1})$$

for all $z \in Z$ *and* $x^{-2} \in X$

iii) Given $(w^e(s_0), w^u(s_0))$, $(c^{-1}_{z,x^{-1}}(s_0), b^{-1}_{z,x^{-1}}(s_0), k^{-1}_{z,x^{-1}}(s_0), c^{-1}_{z,x^{-1}}(s_1))$ *solves:*

$$\max_{b^{-1}_{z,x^{-1}}(s_0), c^{-1}_{z,x^{-1}}(s_0), c^{-1}_{z,x^{-1}}(s_1)} U_2(b^{-1}_{z,x^{-1}}(s_0), c^{-1}_{z,x^{-1}}(s_0)) + \beta_3 E[U_3(c^{-1}_{z,x^{-1}}(s_1))]$$

s.t.

$$c^{-1}_{z,x^{-1}}(s_0) + b^{-1}_{z,x^{-1}}(s_0) + k^{-1}_{z,x^{-1}}(s_0) = (1-\tau)x^{-1}w^z(s_0)$$

$$c^{-1}_{z,x^{-1}}(s_1) = rk^{-1}_{z,x^{-1}}(s_0) + \tau_d(s_1, s_0)x^{-1}w^z(s_0)$$

for all $z \in Z$ *and* $x^{-1} \in X$

iv) Firm maximizes its profit :

$$\max_{D^t_e, D^t_u} \Pi(s_t) = (D^t_u)^{\beta-\xi(s_t)}(D^t_e)^{1-\beta+\xi(s_t)} - w^e(s_t)D^t_e - w^u(s_t)D^t_u$$

v) Government's budgets balance :

$$\phi(s_t)N^r_t = \gamma_H N^r_t H w^e(s_t) + \gamma_L N^r_t L w^e(s_t)$$

$$TR(s_t) = \tau_d(s_t, s_{t-1})TW(s_{t-1})$$

where

$$TW(s_{t-1}) = \sum_{x^{t-2}=L,H} \sum_{z=e,u} x^{t-2}w^z(s_t)$$

vi) Goods market clears:

$$Y^t c^t(s_t) + \sum_{x^{t-1}=L,H} \sum_{z=e,u} M^t_{z,x^{t-1}} c^{t-1}_{z,x^{t-1}}(s_t) + \sum_{x^{t-2}=L,H} \sum_{z=e,u} O^t_{z,x^{t-2}} c^{t-2}_{z,x^{t-2}}(s_t) = F(D^t_e, D^t_u)$$

vii) Labor market clears:

$$D^t_e = S^t_e$$

$$D^t_u = S^t_u$$

Chapter 3

CALIBRATION OF THE BENCHMARK ECONOMY

The exogenous variables of the model are ε_{good},low&high ability levels L&H,risk aversion coefficient δ,gross interest rate r,relative wage ratios $\frac{w^u}{w^e}good$&$\frac{w^u}{w^e}bad$,discount factors β_2&β_3,social security tax rate τ and fraction of educated agents needed to work in the schools γ. The values of these variables are given Table 1 below. These values are determined by considering U.S. labor and college markets. The production parameter β is calibrated in the following manner. The model estimates the demand for educated labor divided by uneducated labor which is $\frac{D_e}{D_u}$ as 0.487804878 . By using the following two relations derived from the production function, the value of the bad shock and beta can be found:

$$\frac{\beta - \varepsilon_{bad}}{1 - \beta + \varepsilon_{bad}} \frac{D_e^t}{D_u^t} = \frac{w^u}{w^e} bad = 0.6$$

$$\frac{\beta - \varepsilon_{good}}{1 - \beta + \varepsilon_{good}} \frac{D_e^t}{D_u^t} = \frac{w^u}{w^e} good = 0.55$$

Therefore $\beta = 0.63$ and $\varepsilon_{bad} = 0.0784$. An average ability level of 0.5 is substituted in the government's budget for tuition. As of solving the model, the value for tuition is found to be 0.025 and the values for redistribution rates turn out to be:

$$\tau_d(G, G) = 0.1$$

$$\tau_d(G, B) = 0.0977$$

$$\tau_d(B, G) = 0.1032$$

$$\tau_d(B, B) = 0.1$$

These values are consistent with intuition because the mother who lives in bad state when middle-aged and in good state when old receives more compared to a mother who lives in good state when middle-aged and in bad state when old. The nominal wages are calibrated by the model as:

$$w^e(G) = 0.6876$$

$$w^e(B) = 0.6662$$

$$w^u(G) = 0.3782$$

$$w^u(B) = 0.3998$$

This finding is also consistent with the production function specified because bad shock negatively affects the educated agents and positively effects the uneducated ones.The model finds the correlation between mother's income and child's income as 0.4346 which is consistent with the finding of Solon [13]. The probability that the child is uneducated and low ability given that mother is uneducated and low ability is 0.4754 and this falls to 0.2421 after five generations. The proportion of financially constrained students is found to be 0.0715 which is consistent with the finding of Carneiro&Heckman [6]. Because of financial constraints, the overall quality of students in the schools is lower compared to zero borrowing constraint case which in turn causes failure rate to be $\%65$. This clearly causes an efficiency loss in the economy. The degree of this efficiency loss will be calculated in the next section. Furthermore, the aggregate expected utility of all 8 types of agents turns out to be 52.3425 in the benchmark case. The results are summarized in Table 1 3.1.

3.1 Characterization of Equilibrium

In equilibrium, the choice variables such as consumption,bequest and saving are expressed in terms of relative wage ratios $\frac{w^u}{w^e}$ in both good and bad states. Therefore finding the equilibrium value of relative wages corresponds to finding the equilibrum of the model. The equilibrium of relative wage ratios is found through a grid search on the interval $[0, 1]$. Relative wage ratios greater than 1 are not considered since it means the nominal wage rate of the educated agents is less than those of the uneducated agents which in turn can cause individuals not to attend school. The grid search is done on the $\frac{w^u}{w^e}good$. The equilibrium value of the $\frac{w^u}{w^e}bad$ is then calculated by the relation:

$$\frac{w^u}{w^e}bad = (\frac{w^u}{w^e}good)\frac{(1-\beta+\varepsilon_{good})(\beta-\varepsilon_{bad})}{(1-\beta+\varepsilon_{bad})(\beta-\varepsilon_{good})}$$

The result of this grid search is seen in Figure 3.1 . The horizontal axis represents the interval grid search is done and the vertical axis represents the corresponding values of the $\frac{w^u}{w^e} good$. The 45-degree line is also drawn in order to configure the equilibrium. The intersections of these two curves are equilibria.

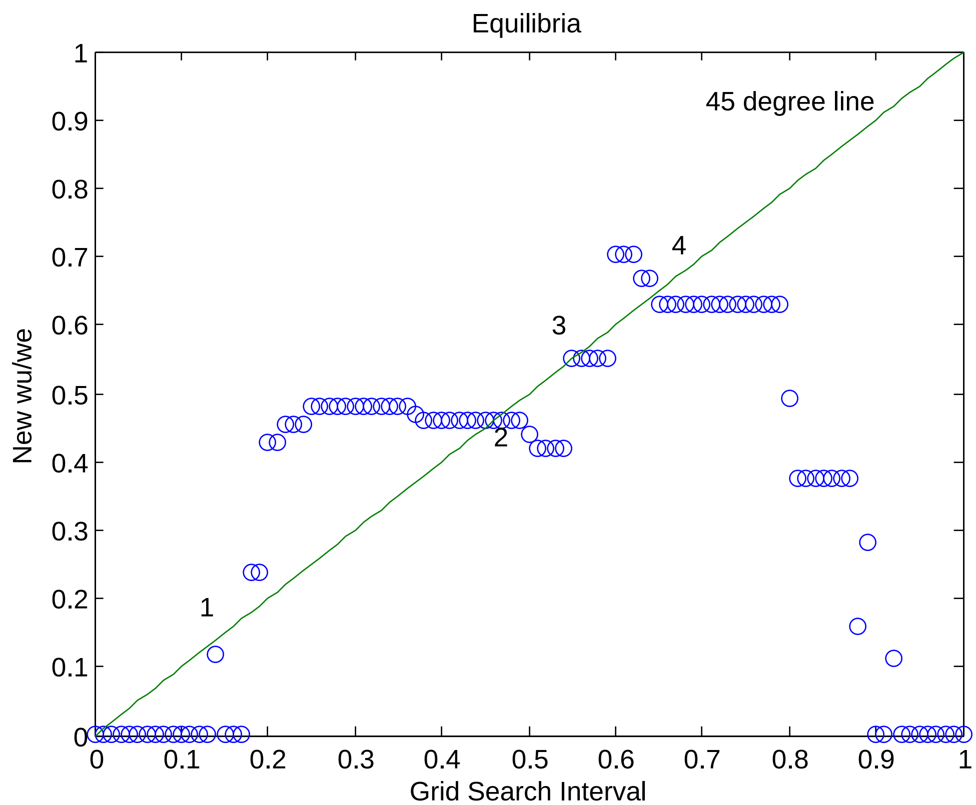

Figure 3.1.1: All Equilibria

As seen from the Figure 3.1, there are 4 equilibria for the relative wage ratio in the good state. Fortunately, not all equilibria are stable. Only $\frac{w^u}{w^e} good = 0.55$ and $\frac{w^u}{w^e} good = 0.19$ equilibrium points are stable which correspond to number 1 and 3 in the above figures. Stability is analyzed in Figure 3.2. Let us first analyze equilibrium 1. If the economy's relative wage ratio is below equilibrium 1 then the nominal wage rate for the uneducated agents is smaller than the educated agents' nominal wage rate. Therefore, agents will have an incentive to attend school which will in turn decrease the relative return for the educated agents. So economy will move up to equilibrium 1.

If the economy is above of equilibrium 1 then nominal wage rate of uneducated agents will be greater compared to educated agents which will in turn decrease the demand for education. As a result, return to schooling will increase and the economy will move to equilibrium 1. The movement of the economy around the possible equilibria is demonstrated in Figure 3.2. By the same reasoning the third equilibrium is also stable. On the other hand, second and fourth equilibria are unstable. For instance, if the economy is on the right hand side of second equilibria then nominal wage of uneducated agents is smaller than the nominal wage of educated agents. So the demand for education will increase causing the return for education to decrease. Therefore, the relative wage ratio will increase causing the new equilibrium to go further from second equilibrium. Again, if the economy is above of second equilibrium then nominal wage of uneducated agents is greater than the nominal wage of educated agents. So the demand for education will decrease causing the return for education to increase. Therefore, the relative wage ratio will decrease causing the new equilibrium to go again further from second equilibrium. Because of the same reasoning, the fourth equilibrium is also unstable.

The first equilibrium is related with poverty trap (Azariadis&Stachurski[1]) because the relative wage ratio of uneducated agents with respect educated agents is very low. This can be caused by perpetuity of path dependence of institutions. Since the model considers borrowing constraints, the effect of path dependence can be magnified. Therefore, institutions as ”carriers of history” (David[5]) can cause countries to converge to different equilibria.

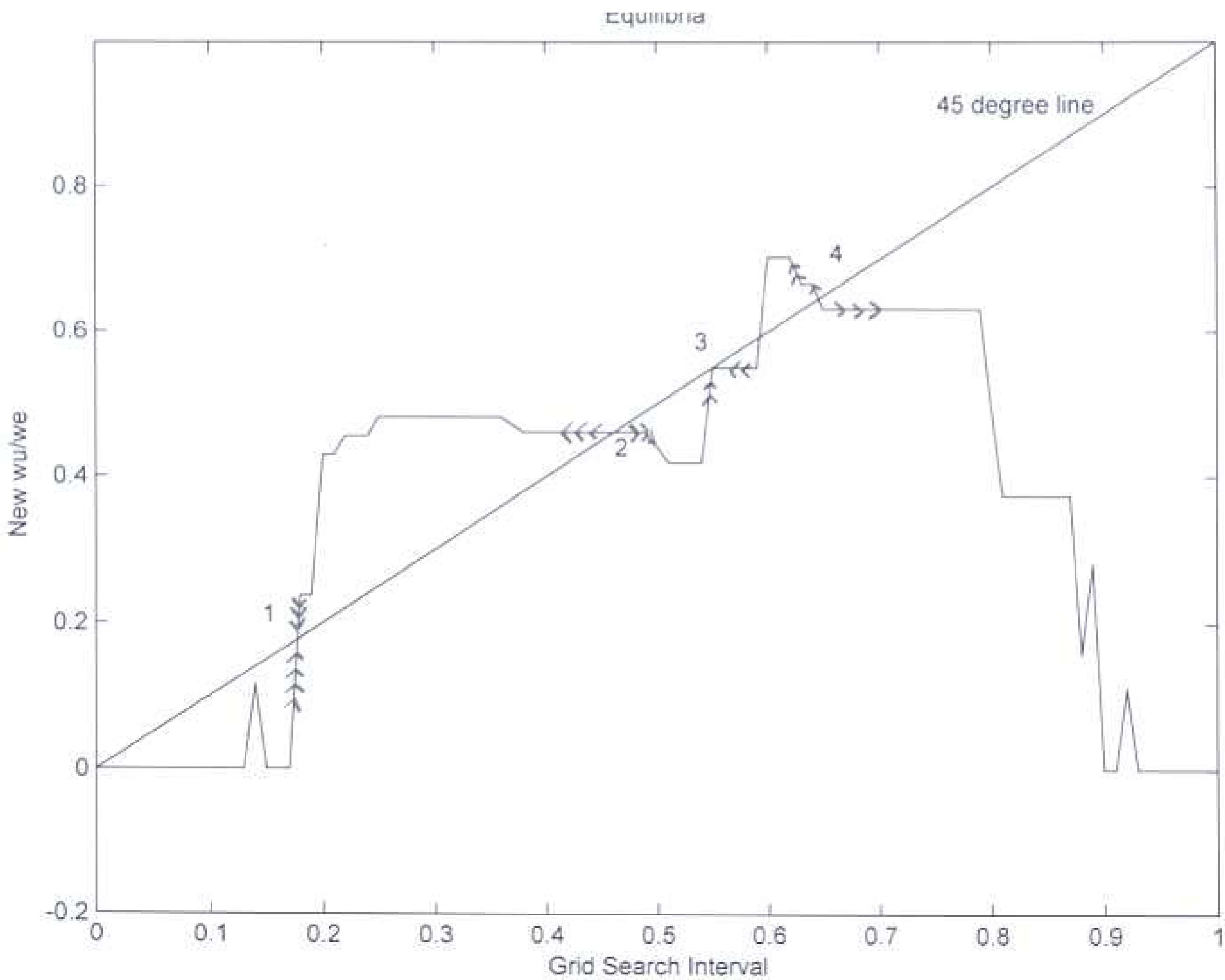

Figure 3.1.2: Stability Analysis

Exogenous Variables	Value	Calibration Results	Value
ε_{good}	0.1	ε_{bad}	0.0784
$Low Ability$	0.25	$beta$	0.63
$High Ability$	0.75	$\tau_d(G, G)$	0.1
$delta$	0.5	$\tau_d(G, B)$	0.0977
r	1.03^{20}	$\tau_d(B, G)$	0.1032
$\frac{w^u}{w^e} good$	0.55	$\tau_d(B, B)$	0.1
$\frac{w^u}{w^e} bad$	0.6	$tuition$	0.025
β_2	$\frac{1}{1.03^{20}}$	$N_t^r(enrollment)$	0.5250
β_3	$\frac{1}{1.03^{20}}$	$Successrate$	0.3455
τ	0.1	$Corr(MotherIncome, ChildIncome)$	0.4346
γ	0.05	$w^e(bad)$	0.6662
		$w^e(good)$	0.6876
		$w^u(bad)$	0.3998
		$w^u(good)$	0.3782
		$Financially\ Const.\ Std.Rate$	0.0715
		$Aggregate\ Expected\ Utility$	52.3425

Table 3.1: Calibration of the Benchmark Model

Chapter 4

THE EXPERIMENT

The next step is to make the experiment of subsidizing tuition uniformly. To carry on this experiment the budget constraints of the government will be changed as follows:

$$\phi(s_t)N_t^r + \tau_s TR(s_t) = \gamma_H N_t^r H w^e(s_t) + \gamma_L N_t^r L w^e(s_t)$$

$$(1-\tau_s)TR(s_t) = \tau_d(s_t, s_{t-1})TW(s_{t-1})$$

The first one is the new education budget and the second one is the new social security budget. Note that some fraction τ_s of the social security revenue is added to education budget to ensure that tuition is uniformly is subsidized. The remaining part of the social security revenue is distributed among current old agents which is $(1-\tau_s)TR(s_t)$. The results of this experiment are summarized in the table below. There are two opposing forces affecting the societal welfare. The first one is the negative effect of increasing total number of educated middle aged agents which will drive the wage rate of middle aged educated agents down because of general equilibrium effects. This will also negatively affect the total social security revenue. On the other hand, government subsidization of education is expected to increase the number of educated agents which will positively affect the total amount of social security revenue collected because of scale effect. Therefore to determine the effect of this experiment to societal welfare more precisely we have to make a computational study.

The school subsidization rate is selected as $\%6$ because tuition rates become less than half of the bencmark case when $\tau_s = \%6$.

4.1 Results

It should be noted that the social welfare function is assumed to be utilitarian. Therefore the expected utility of all agents is summed in order to calculate Aggregate Expected Utility which is seen in Table 2. Subsidizing education by $\%6$ has increased the societal welfare by $\%2$.

Moreover, relative tuition rates has decreased more than $\%50$ as expected. Although the fall in relative tuition rates, the enrollment rate in schools has not changed which in turn caused relative wage ratios in both good and bad state to be constant. Another reason that is responsible for the constancy of relative wages is the tinyness of the ratio of social security revenue with respect to total production which is 0.0595 as seen from Table 2.

The redistribution rates have decreased in the subsidy case compared to benchmark case since the total amount to be distributed is less in the subsidy case. The relation between redistribution rates is same as in benchmark case. Again the relation $\tau_d(B,G) > \tau_d(B,B) = \tau_d(G,G) > \tau_d(G,B)$ holds.

In addition to the above remarks, the gini coefficient which is the most widely used measure of inequality is smaller in the subsidy case compared to benchmark case. This is also consistent with the increase in the aggregate welfare in the economy. The experiment results are summarized in Table 2 4.1. Now let us turn to the question of whether subsidizing education uniformly is Pareto

	$\tau_s = 0$	$\tau_s = 0.06$
$Aggregate\ Welfare$	52.3425	53.4273
$(\phi/w^e)_{good}$	0.025	0.011
$(\phi/w^e)_{bad}$	0.025	0.0113
$\tau_d(B,G)$	0.1032	0.097
$\tau_d(G,B)$	0.0977	0.0918
$\tau_d(B,B)$	0.1	0.094
$\tau_d(G,G)$	0.1	0.094
N_t^r	0.525	0.525
$\frac{w^u}{w^e} good$	0.55	0.55
$\frac{w^u}{w^e} bad$	0.6	0.6
TR/F	0	0.0595
$Gini\ Coefficient$	0.3357	0.3349

Table 4.1: Experiment Results

Improving. To do this the lifetime expected utility matrices in both cases should be compared. In order to understand the entries of the lifetime expected utility matrix, some more explanation is needed which is given below.

The lifetime expected utility matrices regarding the 8 types of individuals are given below. First define the characteristics vector as

$$v = [(L,u,B),(H,u,B),(L,u,G),(H,u,G),(L,e,B),(H,e,B),(L,e,G),(H,e,G)]$$

where each element of c represents one type of agent when she is middle-aged.For instance (H,u,G) represents a middle-aged agent who is high ability,uneducated and living in good state. The $(i,j)^{th}$ element in the following matrices gives the lifetime expected utility for the middle-aged agent whose characteristics is given by j^{th} element of vector c and her mother's characteristics is i^{th} element of vector v.

$$\begin{pmatrix} 0.5083 & 0.9207 & 0.5103 & 0.9209 & 0.5083 & 0.9207 & 0.5103 & 0.9209 \\ 0.5985 & 1.1012 & 0.6019 & 1.1042 & 0.5985 & 1.1012 & 0.6019 & 1.1042 \\ 0.5016 & 0.7824 & 0.5034 & 0.7843 & 0.5016 & 0.7824 & 0.5034 & 0.7843 \\ 0.5879 & 1.0965 & 0.5911 & 1.0993 & 0.5879 & 1.0965 & 0.5911 & 1.0993 \\ 0.5442 & 1.0201 & 0.5467 & 1.0219 & 0.5442 & 1.0201 & 0.5467 & 1.0219 \\ 0.7310 & 1.1807 & 0.7353 & 1.1851 & 0.7310 & 1.1807 & 0.7353 & 1.1851 \\ 0.5427 & 1.0278 & 0.5452 & 1.0296 & 0.5427 & 1.0278 & 0.5452 & 1.0296 \\ 0.7315 & 1.1885 & 0.7358 & 1.1928 & 0.7315 & 1.1885 & 0.7358 & 1.1928 \end{pmatrix}$$

Lifetime Expected Utility Matrix (Benchmark)

$$\begin{pmatrix} 0.5075 & 1.0088 & 0.5095 & 1.0104 & 0.5075 & 1.0088 & 0.5095 & 1.0104 \\ 0.5975 & 1.1244 & 0.6009 & 1.1278 & 0.5975 & 1.1244 & 0.6009 & 1.1278 \\ 0.5008 & 0.7812 & 0.5026 & 0.7830 & 0.5008 & 0.7812 & 0.5026 & 0.7830 \\ 0.5870 & 1.1206 & 0.5902 & 1.1239 & 0.5870 & 1.1206 & 0.5902 & 1.1239 \\ 0.5433 & 1.0575 & 0.5458 & 1.0599 & 0.5433 & 1.0575 & 0.5458 & 1.0599 \\ 0.7480 & 1.1971 & 0.7527 & 1.2017 & 0.7480 & 1.1971 & 0.7527 & 1.2017 \\ 0.5418 & 1.0644 & 0.5443 & 1.0668 & 0.5418 & 1.0644 & 0.5443 & 1.0668 \\ 0.7481 & 1.2044 & 0.7527 & 1.2090 & 0.7481 & 1.2044 & 0.7527 & 1.2090 \end{pmatrix}$$

Lifetime Expected Utility Matrix (Subsidy Equilibrium)

By analyzing the Lifetime Expected Utility matrices, it can be concluded that subsidizing education is not a Pareto Improving step since the Lifetime Expected Utility of some agents fall whereas some others' increase compared to no subsidy case.

Chapter 5

CONCLUDING REMARKS

In this paper, we have developed a heterogenous stochastic dynamic general equilibrium framework to analyze the effects of subsidizing students uniformly on aggregate welfare and income inequality. The intergenerational bequest and social security transfers capture the interaction between consecutive generations in an OLG framework. The benchmark model is calibrated so as to match the certain features of the U.S. college market. It turned out that there are multiple equilibria with two of them being stable. One of these stable equilibria is a so called ”poverty trap” which is some ”vicious circle of poverty” (Nurkse[12]) at which the uneducated agents’ wage is approximately one-fifth of educated agents’ wage. The effect of this poverty is amplified when combined with the borrowing constraint present in the model. This capital market imperfection adds to the persistence of this poverty trap. One other cause of this persistence can be associated with the perpetuity of path dependence of institutions in which the past completely determines the present and future.

The paper then analyzes a subsidy experiment on the non-poverty trap, stable equilibrium. In this simulation, government subsidizes students uniformly through some fraction of total social security revenue. The subsidy is chosen in such way that the new tuition rate is as half as the benchmark case. The paper concludes that subsidizing students uniformly increases aggregate welfare and decreases gini coefficient in an exante sense.

For future research, the model can be extended to incorporate endogenously determined effort level by students (young agents), a stochastic population growth, a continuum of ability levels in the interval $[0, 1]$. The experiment can be changed as subsidizing only high ability and constrained students. And the effects of this experiment on the poverty trap equilibrium can be analyzed in order to see if the poverty trap is still persistent.

Appendix A

APPENDIX A

The matlab codes found in this appendix can be requested by email from muharremysl@gmail.com. This appendix includes the matlab code used in calibrating the model and making the experiment.

```
clear all;n=8;tolerance=10^(-6);
ability=[0.25;0.75];
k=5.5;delta=0.5;grossinterestrate=1.03^20; beta3=1/1.03^20; alpha=0.05; gamma=0.05;
goodshock=0.1;badshock=0.0784;
wu_we_good=0.55;wu_we_bad=0.6;
temp=ones(1,8);
social_security_tax_rate=0.1;
schoolsubsidy=0.06;
%state matrice mother and child
state_economy_mother1=cat(1,zeros(2,1),ones(2,1));
state_economy_mother2=cat(1,state_economy_mother1,state_economy_mother1);
state_economy_mother=state_economy_mother2*temp;
state_economy_mother_short=state_economy_mother(:,1);
state_economy_child=(state_economy_mother)';
tuitionrange=0:10^(-4):0.025;
counttui=1;
tuitionbudgoodrange=[];
tuitionbudbadrange=[];
while counttui<=length(tuitionrange)
    tuition1=tuitionrange(counttui);
    tuition2=tuitionrange(counttui);
    tuition=tuition1*state_economy_mother+tuition2*(1-state_economy_mother);
a1=0; b1=1;a2=0;b2=1;
tao1=a1+(b1-a1).*rand(1,1);
tao2=a2+(b2-a2).*rand(1,1);
taodiff=10;
row_1=[0.1 0.1 0.1031 0.1031 0.1 0.1 0.1031 0.1031];
row_2=[0.0976 0.0976 0.1 0.1 0.0976 0.0976 0.1 0.1];
redistributionrate=cat(1,row_1,row_1,row_2,row_2,row_1,row_1,row_2,row_2);
counttao=1;
while taodiff>tolerance
repaymentratebadbad=redistributionrate(1,1);
repaymentratebadgood=redistributionrate(2,3);
repaymentrategoodbad=redistributionrate(3,1);
repaymentrategoodgood=redistributionrate(3,3);
ability2=cat(1,ability',ability');
HH=0.7;LH=0.3;HL=0.3;LL=0.7;
transition_ability=[LL HL;LH HH];
BB=0.6792;GB=0.3208;GG=0.6792;BG=0.3208;
beta2=1/grossinterestrate;beta3=1/1.03^20;
beta=0.63;
%define state matrices
ability_mother=cat(1,ability,ability,ability,ability)*temp;
ability_child=(ability_mother)';
education_mother=cat(1,zeros(4,1),ones(4,1))*temp;
education_child=(education_mother)';
%second period income
wu_we=state_economy_mother.*education_mother+wu_we_good*state_economy_mother.*(1-↙
education_mother)+(1-state_economy_mother).*education_mother+wu_we_bad*(1-↙
education_mother).*(1-state_economy_mother);
y2mother=(1-social_security_tax_rate)*k*ability_mother.*wu_we;
%second period saving problem
saving_lower=-(1/grossinterestrate)*redistributionrate.*y2mother;
saving_upper=y2mother;
savingdiff=10*ones(n,n);
saving=saving_lower+(saving_upper-saving_lower).*rand(n,n);
```

```matlab
while max(max(savingdiff))>tolerance
    indic=((-1*(y2mother-saving).^(-delta)+beta3*grossinterestrate*(state_economy_mother.*(GB*(repaymentratebadgood*y2mother+grossinterestrate*saving).^(-delta)+GG*(repaymentrategoodgood*y2mother+grossinterestrate*saving).^(-delta))+(1-state_economy_mother).*(BG*(repaymentrategoodbad*y2mother+grossinterestrate*saving).^(-delta)+BB*(repaymentratebadbad*y2mother+grossinterestrate*saving).^(-delta))))>0);
    saving_upper=saving_upper.*indic+saving.*(1-indic);
    saving_lower=saving.*indic+saving_lower.*(1-indic);
    savingdiff=saving_upper-saving_lower;
    saving=(saving_lower+saving_upper)/2;

end
bequest2mother=alpha*(y2mother-saving);
%expected discounted utility in the second period
u2mom=((y2mother-saving).^(1-delta)/(1-delta))+(beta3/(1-delta))*(state_economy_mother.*(GB*(repaymentratebadgood*y2mother+grossinterestrate*saving).^(1-delta)+GG*(repaymentrategoodgood*y2mother+grossinterestrate*saving).^(1-delta))+(1-state_economy_mother).*(BG*(repaymentrategoodbad*y2mother+grossinterestrate*saving).^(1-delta)+BB*(repaymentratebadbad*y2mother+grossinterestrate*saving).^(1-delta)));
u2child=(u2mom)';
%first period problem
u2college_success_good_child1=cat(2,u2child(:,7),u2child(:,8));
u2college_success_good_child=repmat(u2college_success_good_child1,1,4);
u2college_success_bad_child1=cat(2,u2child(:,5),u2child(:,6));
u2college_success_bad_child=repmat(u2college_success_bad_child1,1,4);
u2college_fail_good_child1=cat(2,u2child(:,3),u2child(:,4));
u2college_fail_good_child=repmat(u2college_fail_good_child1,1,4);
u2college_fail_bad_child1=cat(2,u2child(:,1),u2child(:,2));
u2college_fail_bad_child=repmat(u2college_fail_bad_child1,1,4);
thirdperiodtransitionrow1=[BB BB GB GB BB BB GB GB];
thirdperiodtransitionrow2=1-thirdperiodtransitionrow1;
thirdperiodtransition1=cat(1,thirdperiodtransitionrow1,thirdperiodtransitionrow1,thirdperiodtransitionrow2,thirdperiodtransitionrow2);
thirdperiodtransition=cat(1,thirdperiodtransition1,thirdperiodtransition1);
thirdperiodtransitionbad=thirdperiodtransition(:,1)*temp;
thirdperiodtransitiongood=1-thirdperiodtransitionbad;
%child's decision problem
expectedutilitycollege=((1/(1-delta))*(bequest2mother-tuition).^(1-delta))+beta2*(ability_child.*thirdperiodtransitiongood.*u2college_success_good_child+ability_child.*thirdperiodtransitionbad.*u2college_success_bad_child+(1-ability_child).*thirdperiodtransitiongood.*u2college_fail_good_child+(1-ability_child).*thirdperiodtransitionbad.*u2college_fail_bad_child);
expectedutilitynotcollege=((1/(1-delta))*(bequest2mother).^(1-delta))+beta2*(thirdperiodtransitiongood.*u2college_fail_good_child+thirdperiodtransitionbad.*u2college_fail_bad_child);
indicator=((bequest2mother>=tuition)&(expectedutilitycollege>=expectedutilitynotcollege));
utilityindicator=(expectedutilitycollege>=expectedutilitynotcollege);
tuitionindicator=(bequest2mother>=tuition);
row1=cat(2,BB*transition_ability.*(1-ability2),BG*transition_ability.*(1-ability2),BB*transition_ability.*ability2,BG*transition_ability.*ability2);
row2=cat(2,GB*transition_ability.*(1-ability2),GG*transition_ability.*(1-ability2),GB*transition_ability.*ability2,GG*transition_ability.*ability2);
T_college=cat(1,row1,row2,row1,row2);
row1=cat(2,BB*transition_ability,BG*transition_ability,zeros(2,2),zeros(2,2));
row2=cat(2,GB*transition_ability,GG*transition_ability,zeros(2,2),zeros(2,2));
```

```
T_not_college=cat(1,row1,row2,row1,row2);
%setting up the budgets
TRANSITION=indicator.*T_college+(1-indicator).*T_not_college;
B=eye(n)-TRANSITION;
B(:,n)=1;
stationarydist=[zeros(1,n-1),1]/B;
dynamicmatrix=(stationarydist'*temp).*TRANSITION;
dynamicmatrixbadgood=(dynamicmatrix.*(1-state_economy_mother).*state_economy_child)/sum
(sum(dynamicmatrix.*(1-state_economy_mother).*state_economy_child));
dynamicmatrixgoodbad=(dynamicmatrix.*state_economy_mother.*(1-state_economy_child))/sum
(sum(dynamicmatrix.*state_economy_mother.*(1-state_economy_child)));
workeducatedrate=sum(sum(dynamicmatrix.*education_mother));
workuneducatedrate=sum(sum(dynamicmatrix.*(1-education_mother)));
enrollmentrate=sum(sum(dynamicmatrix.*indicator));
fin_const_std=sum(sum(dynamicmatrix.*utilityindicator.*(1-tuitionindicator)));
workeducated_teacher=gamma*enrollmentrate;
workeducated_production=workeducatedrate-workeducated_teacher;
%wage rates
wubad=(beta-badshock)*(workuneducatedrate^(beta-1-badshock))*((workeducated_production)
^(1-beta+badshock));
wugood=(beta-goodshock)*(workuneducatedrate^(beta-1-goodshock))*
((workeducated_production)^(1-beta+goodshock));
webad=(1-beta+badshock)*(workuneducatedrate^(beta-badshock))*((workeducated_production)
^(-beta+badshock));
wegood=(1-beta+goodshock)*(workuneducatedrate^(beta-goodshock))*
((workeducated_production)^(-beta+goodshock));
wubad/webad;
wugood/wegood;
%total revenues
TRgood=(wegood)*social_security_tax_rate*k*ability_child.*(education_child+wu_we_good*
(1-education_child));
TR_wegood=social_security_tax_rate*k*ability_child.*(education_child+wu_we_good*(1-
education_child));
TRbad=(webad)*social_security_tax_rate*k*ability_child.*(education_child+wu_we_bad*(1-
education_child));
TR_webad=social_security_tax_rate*k*ability_child.*(education_child+wu_we_bad*(1-
education_child));
TRgood=sum(sum(dynamicmatrixbadgood.*TRgood));
TR_wegood=sum(sum(dynamicmatrixbadgood.*TR_wegood));
TRbad=sum(sum(dynamicmatrixgoodbad.*TRbad));
TR_webad=sum(sum(dynamicmatrixgoodbad.*TR_webad));
%bisection on redistribution rates
indic21=(((1-schoolsubsidy)*TR_wegood-sum(sum((webad/wegood)
*tao1*k*dynamicmatrixbadgood.*ability_mother.*(education_mother+wu_we_bad*(1-
education_mother)))))*((1-schoolsubsidy)*TR_wegood-sum(sum((webad/wegood)
*a1*k*dynamicmatrixbadgood.*ability_mother.*(education_mother+wu_we_bad*(1-
education_mother)))))>0);
indic22=(((1-schoolsubsidy)*TR_webad-sum(sum((wegood/webad)
*tao2*k*dynamicmatrixgoodbad.*ability_mother.*(education_mother+wu_we_good*(1-
education_mother)))))*((1-schoolsubsidy)*TR_webad-sum(sum((wegood/webad)
*a2*k*dynamicmatrixgoodbad.*ability_mother.*(education_mother+wu_we_good*(1-
education_mother)))))>0);
a1=tao1.*indic21+a1.*(1-indic21);
b1=b1.*indic21+tao1.*(1-indic21);
a2=tao2.*indic22+a2.*(1-indic22);
b2=b2.*indic22+tao2.*(1-indic22);
```

```
taodiff1=b1-a1;
taodiff2=b2-a2;
taodiff=max([taodiff1 taodiff2]);
tao1=(a1+b1)/2;tao2=(a2+b2)/2;
repaymentratebadgood=tao1;
repaymentrategoodbad=tao2;
repaymentrategoodgood=social_security_tax_rate*(1-schoolsubsidy);
repaymentratebadbad=repaymentrategoodgood;
redistributionrate=repaymentratebadgood*(1-state_economy_mother).↙
*state_economy_child+repaymentrategoodbad*state_economy_mother.*(1-state_economy_child)↙
+repaymentrategoodgood*(state_economy_mother.*state_economy_child+(1-↙
state_economy_mother).*(1-state_economy_child));
counttao=counttao+1;
end
%gridsearch on tuition rates
tuitionbudgood=gamma*0.5*enrollmentrate-tuition1*enrollmentrate-↙
schoolsubsidy*TR_wegood;
tuitionbudbad=gamma*0.5*enrollmentrate-tuition2*enrollmentrate-schoolsubsidy*TR_webad;
tuitionbudgoodrange=[tuitionbudgoodrange tuitionbudgood];
tuitionbudbadrange=[tuitionbudbadrange tuitionbudbad];
counttui=counttui+1;
end
abstuitiongoodrange=abs(tuitionbudgoodrange);
abstuitionbadrange=abs(tuitionbudbadrange);
i=find(abstuitiongoodrange==min(abstuitiongoodrange));
j=find(abstuitionbadrange==min(abstuitionbadrange));
tuition1=tuitionrange(i(1));
tuition2=tuitionrange(j(1));
tuition=tuition1*state_economy_mother+tuition2*(1-state_economy_mother);
%income correlation of mother and child
y2child=y2mother';
expectedincomemother=sum(sum(dynamicmatrix.*y2mother));
expectedincomechild=sum(sum(dynamicmatrix.*y2child));
expectedincomemother2=sum(sum(dynamicmatrix.*y2mother.^2));
expectedincomechild2=sum(sum(dynamicmatrix.*y2child.^2));
varincomemother=expectedincomemother2-expectedincomemother^2;
varincomechild=expectedincomechild2-expectedincomechild^2;
expectedincomemotherchild=sum(sum(dynamicmatrix.*y2mother.*y2child));
correlationincome_mother_child=(expectedincomemotherchild-↙
expectedincomemother*expectedincomechild)/sqrt(varincomemother*varincomechild);
%two ,five and ten period ahead transition matrices
stationarybig=(stationarydist)'*temp;
transitionmatrix2=TRANSITION*TRANSITION;
transitionmatrix5=transitionmatrix2*transitionmatrix2*TRANSITION;
transitionmatrix10=transitionmatrix5*transitionmatrix5;
dynamicmatrix2=stationarybig.*transitionmatrix2;
dynamicmatrix5=stationarybig.*transitionmatrix5;
dynamicmatrix10=stationarybig.*transitionmatrix10;
%aggregate expected utility
nominal=temp'*[webad webad wegood wegood webad webad wegood wegood];
expectedutilitycollege_nominal=((1/(1-delta))*((bequest2mother-tuition).*nominal).^(1-↙
delta))+beta2*(wegood*ability_child.*thirdperiodtransitiongood.↙
*u2college_success_good_child+webad*ability_child.*thirdperiodtransitionbad.↙
*u2college_success_bad_child+wegood*(1-ability_child).*thirdperiodtransitiongood.↙
*u2college_fail_good_child+webad*(1-ability_child).*thirdperiodtransitionbad.↙
*u2college_fail_bad_child);
```

```
expectedutilitynotcollege_nominal=((1/(1-delta))*(bequest2mother.*nominal).^(1-delta))
+beta2*(wegood*thirdperiodtransitiongood.
*u2college_fail_good_child+webad*thirdperiodtransitionbad.*u2college_fail_bad_child);
aggregatelifetimeutility=enrollmentrate*indicator.*expectedutilitycollege_nominal+(1-
enrollmentrate)*(1-indicator).*expectedutilitynotcollege_nominal;
%gini coefficient
gini_aggregatelifetimeutility=reshape(aggregatelifetimeutility,64,1);
gini_dynamicmatrix=reshape(dynamicmatrix,64,1);
anteAEU=sum(gini_aggregatelifetimeutility);
tempa=gini_aggregatelifetimeutility;
antegini=0;
z=1;gini=[];
while z<=64
antegini=antegini+sum((abs(gini_dynamicmatrix(z)*tempa-gini_dynamicmatrix*tempa(z))));
z=z+1;
end
gini_coefficient=1-antegini/(2*anteAEU);
```

Appendix B

APPENDIX B

This appendix contains the matlab code used in finding the new relative wage equilibrium in the subsidy case. It is also used to generate Figure 3.1 and 3.2 .

```
clear all;n=8;tolerance=10^(-6);
ability=[0.25;0.75];
k=5.5;delta=0.5;grossinterestrate=1.03^20; beta3=1/1.03^20; alpha=0.05; gamma=0.05;
goodshock=0.1;badshock=0.0784;
wu_we_good=0.55;wu_we_bad=0.6;
temp=ones(1,8);
social_security_tax_rate=0.1;
schoolsubsidy=0.06;
%state matrice mother and child
state_economy_mother1=cat(1,zeros(2,1),ones(2,1));
state_economy_mother2=cat(1,state_economy_mother1,state_economy_mother1);
state_economy_mother=state_economy_mother2*temp;
state_economy_mother_short=state_economy_mother(:,1);
state_economy_child=(state_economy_mother)';
count=1;
wu_wegoodnew=[];
wu_wegoodrange=0:10^(-2):1;
while count<=length(wu_wegoodrange)
    wu_we_good=wu_wegoodrange(count);
    wu_we_bad=(wu_we_good*(1-0.63+goodshock)*(0.63-badshock))/((0.63-goodshock)*(1-0.63
+badshock));
%grid search on tuition rates
tuitionrange=0:10^(-4):0.025;
counttui=1;
tuitionbudgoodrange=[];
tuitionbudbadrange=[];
while counttui<=length(tuitionrange)
    tuition1=tuitionrange(counttui);
    tuition2=tuitionrange(counttui);
    tuition=tuition1*state_economy_mother+tuition2*(1-state_economy_mother);
%bisection on redistribution rates
a1=0; b1=1;a2=0;b2=1;
tao1=a1+(b1-a1).*rand(1,1);
tao2=a2+(b2-a2).*rand(1,1);
taodiff=10;
row_1=[0.1 0.1 0.1031 0.1031 0.1 0.1 0.1031 0.1031];
row_2=[0.0976 0.0976 0.1 0.1 0.0976 0.0976 0.1 0.1];
redistributionrate=cat(1,row_1,row_1,row_2,row_2,row_1,row_1,row_2,row_2);
counttao=1;
while taodiff>tolerance
repaymentratebadbad=redistributionrate(1,1);
repaymentratebadgood=redistributionrate(2,3);
repaymentrategoodbad=redistributionrate(3,1);
repaymentrategoodgood=redistributionrate(3,3);
ability2=cat(1,ability',ability');
HH=0.7;LH=0.3;HL=0.3;LL=0.7;
transition_ability=[LL HL;LH HH];
BB=0.6792;GB=0.3208;GG=0.6792;BG=0.3208;
beta2=1/grossinterestrate;beta3=1/1.03^20;
beta=0.63;
%define state matrices
ability_mother=cat(1,ability,ability,ability,ability)*temp;
ability_child=(ability_mother)';
education_mother=cat(1,zeros(4,1),ones(4,1))*temp;
education_child=(education_mother)';
%second period income
```

```
wu_we=state_economy_mother.*education_mother+wu_we_good*state_economy_mother.*(1-
education_mother)+(1-state_economy_mother).*education_mother+wu_we_bad*(1-
education_mother).*(1-state_economy_mother);
y2mother=(1-social_security_tax_rate)*k*ability_mother.*wu_we;
%second period saving problem
saving_lower=-(1/grossinterestrate)*redistributionrate.*y2mother;
saving_upper=y2mother;
savingdiff=10*ones(n,n);
saving=saving_lower+(saving_upper-saving_lower).*rand(n,n);
while max(max(savingdiff))>tolerance

    indic=((-1*(y2mother-saving).^(-delta)+beta3*grossinterestrate*
(state_economy_mother.*(GB*(repaymentratebadgood*y2mother+grossinterestrate*saving).^(-
delta)+GG*(repaymentrategoodgood*y2mother+grossinterestrate*saving).^(-delta))+(1-
state_economy_mother).*(BG*(repaymentrategoodbad*y2mother+grossinterestrate*saving).^(-
delta)+BB*(repaymentratebadbad*y2mother+grossinterestrate*saving).^(-delta))))>0);
    saving_upper=saving_upper.*indic+saving.*(1-indic);
    saving_lower=saving.*indic+saving_lower.*(1-indic);
    savingdiff=saving_upper-saving_lower;
    saving=(saving_lower+saving_upper)/2;

end
bequest2mother=alpha*(y2mother-saving);
%expected discounted utility in the second period
u2mom=((y2mother-saving).^(1-delta)/(1-delta))+(beta3/(1-delta))*(state_economy_mother.
*(GB*(repaymentratebadgood*y2mother+grossinterestrate*saving).^(1-delta)+GG*
(repaymentrategoodgood*y2mother+grossinterestrate*saving).^(1-delta))+(1-
state_economy_mother).*(BG*(repaymentrategoodbad*y2mother+grossinterestrate*saving).^
(1-delta)+BB*(repaymentratebadbad*y2mother+grossinterestrate*saving).^(1-delta)));
u2child=(u2mom)';
%first period problem
u2college_success_good_child1=cat(2,u2child(:,7),u2child(:,8));
u2college_success_good_child=repmat(u2college_success_good_child1,1,4);

u2college_success_bad_child1=cat(2,u2child(:,5),u2child(:,6));
u2college_success_bad_child=repmat(u2college_success_bad_child1,1,4);

u2college_fail_good_child1=cat(2,u2child(:,3),u2child(:,4));
u2college_fail_good_child=repmat(u2college_fail_good_child1,1,4);

u2college_fail_bad_child1=cat(2,u2child(:,1),u2child(:,2));
u2college_fail_bad_child=repmat(u2college_fail_bad_child1,1,4);

thirdperiodtransitionrow1=[BB BB GB GB BB BB GB GB];
thirdperiodtransitionrow2=1-thirdperiodtransitionrow1;
thirdperiodtransition1=cat(1,thirdperiodtransitionrow1,thirdperiodtransitionrow1,
thirdperiodtransitionrow2,thirdperiodtransitionrow2);
thirdperiodtransition=cat(1,thirdperiodtransition1,thirdperiodtransition1);
thirdperiodtransitionbad=thirdperiodtransition(:,1)*temp;
thirdperiodtransitiongood=1-thirdperiodtransitionbad;
%child's decision problem
expectedutilitycollege=((1/(1-delta))*(bequest2mother-tuition).^(1-delta))+beta2*
(ability_child.*thirdperiodtransitiongood.*u2college_success_good_child+ability_child.
*thirdperiodtransitionbad.*u2college_success_bad_child+(1-ability_child).
*thirdperiodtransitiongood.*u2college_fail_good_child+(1-ability_child).
*thirdperiodtransitionbad.*u2college_fail_bad_child);
```

```
expectedutilitynotcollege=((1/(1-delta))*(bequest2mother).^(1-delta))+beta2*
(thirdperiodtransitiongood.*u2college_fail_good_child+thirdperiodtransitionbad.
*u2college_fail_bad_child);
indicator=((bequest2mother>=tuition)&
(expectedutilitycollege>=expectedutilitynotcollege));
utilityindicator=(expectedutilitycollege>=expectedutilitynotcollege);
tuitionindicator=(bequest2mother>=tuition);
row1=cat(2,BB*transition_ability.*(1-ability2),BG*transition_ability.*(1-ability2),
BB*transition_ability.*ability2,BG*transition_ability.*ability2);
row2=cat(2,GB*transition_ability.*(1-ability2),GG*transition_ability.*(1-ability2),
GB*transition_ability.*ability2,GG*transition_ability.*ability2);
T_college=cat(1,row1,row2,row1,row2);
row1=cat(2,BB*transition_ability,BG*transition_ability,zeros(2,2),zeros(2,2));
row2=cat(2,GB*transition_ability,GG*transition_ability,zeros(2,2),zeros(2,2));
T_not_college=cat(1,row1,row2,row1,row2);
TRANSITION=indicator.*T_college+(1-indicator).*T_not_college;
B=eye(n)-TRANSITION;
B(:,n)=1;
stationarydist=[zeros(1,n-1),1]/B;
dynamicmatrix=(stationarydist'*temp).*TRANSITION;
dynamicmatrixbadgood=(dynamicmatrix.*(1-state_economy_mother).*state_economy_child)/sum
(sum(dynamicmatrix.*(1-state_economy_mother).*state_economy_child));
dynamicmatrixgoodbad=(dynamicmatrix.*state_economy_mother.*(1-state_economy_child))/sum
(sum(dynamicmatrix.*state_economy_mother.*(1-state_economy_child)));
workeducatedrate=sum(sum(dynamicmatrix.*education_mother));
workuneducatedrate=sum(sum(dynamicmatrix.*(1-education_mother)));
enrollmentrate=sum(sum(dynamicmatrix.*indicator));
fin_const_std=sum(sum(dynamicmatrix.*utilityindicator.*(1-tuitionindicator)));
workeducated_teacher=gamma*enrollmentrate;
workeducated_production=workeducatedrate-workeducated_teacher;
%wage rates
wubad=(beta-badshock)*(workuneducatedrate^(beta-1-badshock))*((workeducated_production)
^(1-beta+badshock));
wugood=(beta-goodshock)*(workuneducatedrate^(beta-1-goodshock))*
((workeducated_production)^(1-beta+goodshock));
webad=(1-beta+badshock)*(workuneducatedrate^(beta-badshock))*((workeducated_production)
^(-beta+badshock));
wegood=(1-beta+goodshock)*(workuneducatedrate^(beta-goodshock))*
((workeducated_production)^(-beta+goodshock));
%total revenues
TRgood=(wegood)*social_security_tax_rate*k*ability_child.*(education_child+wu_we_good*
(1-education_child));
TR_wegood=social_security_tax_rate*k*ability_child.*(education_child+wu_we_good*(1-
education_child));
TRbad=(webad)*social_security_tax_rate*k*ability_child.*(education_child+wu_we_bad*(1-
education_child));
TR_webad=social_security_tax_rate*k*ability_child.*(education_child+wu_we_bad*(1-
education_child));
TRgood=sum(sum(dynamicmatrixbadgood.*TRgood));
TR_wegood=sum(sum(dynamicmatrixbadgood.*TR_wegood));
TRbad=sum(sum(dynamicmatrixgoodbad.*TRbad));
TR_webad=sum(sum(dynamicmatrixgoodbad.*TR_webad));
%bisection on redistribution rates
indic21=(((1-schoolsubsidy)*TR_wegood-sum(sum((webad/wegood)
*tao1*k*dynamicmatrixbadgood.*ability_mother.*(education_mother+wu_we_bad*(1-
education_mother)))))*((1-schoolsubsidy)*TR_wegood-sum(sum((webad/wegood)
```

```
*a1*k*dynamicmatrixbadgood.*ability_mother.*(education_mother+wu_we_bad*(1-
education_mother)))))>0);
indic22=(((1-schoolsubsidy)*TR_webad-sum(sum((wegood/webad)
*tao2*k*dynamicmatrixgoodbad.*ability_mother.*(education_mother+wu_we_good*(1-
education_mother)))))*((1-schoolsubsidy)*TR_webad-sum(sum((wegood/webad)
*a2*k*dynamicmatrixgoodbad.*ability_mother.*(education_mother+wu_we_good*(1-
education_mother)))))>0);
a1=tao1.*indic21+a1.*(1-indic21);
b1=b1.*indic21+tao1.*(1-indic21);
a2=tao2.*indic22+a2.*(1-indic22);
b2=b2.*indic22+tao2.*(1-indic22);
taodiff1=b1-a1;
taodiff2=b2-a2;
taodiff=max([taodiff1 taodiff2]);
tao1=(a1+b1)/2;tao2=(a2+b2)/2;
repaymentratebadgood=tao1;
repaymentrategoodbad=tao2;
repaymentrategoodgood=social_security_tax_rate*(1-schoolsubsidy);
repaymentratebadbad=repaymentrategoodgood;
redistributionrate=repaymentratebadgood*(1-state_economy_mother).
*state_economy_child+repaymentrategoodbad*state_economy_mother.*(1-state_economy_child)
+repaymentrategoodgood*(state_economy_mother.*state_economy_child+(1-
state_economy_mother).*(1-state_economy_child));
counttao=counttao+1;
end
%gridsearch on tuition rates
tuitionbudgood=gamma*0.5*enrollmentrate-tuition1*enrollmentrate-
schoolsubsidy*TR_wegood;
tuitionbudbad=gamma*0.5*enrollmentrate-tuition2*enrollmentrate-schoolsubsidy*TR_webad;
tuitionbudgoodrange=[tuitionbudgoodrange tuitionbudgood];
tuitionbudbadrange=[tuitionbudbadrange tuitionbudbad];
counttui=counttui+1;
end
abstuitiongoodrange=abs(tuitionbudgoodrange);
abstuitionbadrange=abs(tuitionbudbadrange);
i=find(abstuitiongoodrange==min(abstuitiongoodrange));
j=find(abstuitionbadrange==min(abstuitionbadrange));
tuition1=tuitionrange(i(1));
tuition2=tuitionrange(j(1));
tuition=tuition1*state_economy_mother+tuition2*(1-state_economy_mother);
wu_wegoodnew=[wu_wegoodnew (wugood/wegood)];
count=count+1;
end
wagevectordiff=abs(wu_wegoodnew-wu_wegoodrange);
wagevectordiff=wagevectordiff(2:1:length(wagevectordiff));
index=find(wagevectordiff==min(wagevectordiff));
wu_we_good=wu_wegoodrange(index+1)
wu_we_bad=(wu_we_good*(1-beta+goodshock)*(beta-badshock))/((beta-goodshock)*(1-
beta+badshock))
plot(wu_wegoodrange,wu_wegoodnew,'o',wu_wegoodrange,wu_wegoodrange);
%income correlation of mother and child
y2child=y2mother';
expectedincomemother=sum(sum(dynamicmatrix.*y2mother));
expectedincomechild=sum(sum(dynamicmatrix.*y2child));
expectedincomemother2=sum(sum(dynamicmatrix.*y2mother.^2));
expectedincomechild2=sum(sum(dynamicmatrix.*y2child.^2));
```

```
varincomemother=expectedincomemother2-expectedincomemother^2;
varincomechild=expectedincomechild2-expectedincomechild^2;
expectedincomemotherchild=sum(sum(dynamicmatrix.*y2mother.*y2child));
correlationincome_mother_child=(expectedincomemotherchild-
expectedincomemother*expectedincomechild)/sqrt(varincomemother*varincomechild);
%two ,five and ten period ahead transition matrices
stationarybig=(stationarydist)'*temp;
transitionmatrix2=TRANSITION*TRANSITION;
transitionmatrix5=transitionmatrix2*transitionmatrix2*TRANSITION;
transitionmatrix10=transitionmatrix5*transitionmatrix5;
dynamicmatrix2=stationarybig.*transitionmatrix2;
dynamicmatrix5=stationarybig.*transitionmatrix5;
dynamicmatrix10=stationarybig.*transitionmatrix10;
%aggregate lifetime expected utility
nominal=temp'*[webad webad wegood wegood webad webad wegood wegood];
expectedutilitycollege_nominal=((1/(1-delta))*((bequest2mother-tuition).*nominal).^(1-
delta))+beta2*(wegood*ability_child.*thirdperiodtransitiongood.
*u2college_success_good_child+webad*ability_child.*thirdperiodtransitionbad.
*u2college_success_bad_child+wegood*(1-ability_child).*thirdperiodtransitiongood.
*u2college_fail_good_child+webad*(1-ability_child).*thirdperiodtransitionbad.
*u2college_fail_bad_child);
expectedutilitynotcollege_nominal=((1/(1-delta))*(bequest2mother.*nominal).^(1-delta))
+beta2*(wegood*thirdperiodtransitiongood.
*u2college_fail_good_child+webad*thirdperiodtransitionbad.*u2college_fail_bad_child);
aggregatelifetimeutility=enrollmentrate*indicator.*expectedutilitycollege_nominal+(1-
enrollmentrate)*(1-indicator).*expectedutilitynotcollege_nominal;
%gini coefficient
gini_aggregatelifetimeutility=reshape(aggregatelifetimeutility,64,1);
gini_dynamicmatrix=reshape(dynamicmatrix,64,1);
anteAEU=sum(gini_aggregatelifetimeutility);
tempa=gini_aggregatelifetimeutility;
antegini=0;
z=1;gini=[];
while z<=64
antegini=antegini+sum((abs(gini_dynamicmatrix(z)*tempa-gini_dynamicmatrix*tempa(z))));
z=z+1;
end
gini_coefficient=1-antegini/(2*anteAEU);
```

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