# CAPACITY CONTRACTS IN SUPPLY CHAINS UNDER SYMMETRIC AND ASYMMETRIC INFORMATION

by

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This is to certify that I have examined this copy of a master's thesis by

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and have found that it is complete and satisfactory in all respects,

and that any and all revisions required by the final

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*To my family*

## ABSTRACT

In this study, we focus on a two-party supply chain composed of a single manufacturer and a single supplier where capacity is limited. We look into three different demand scenarios; price dependent deterministic demand, stochastic demand and stochastic price dependent demand. In all three cases we try to improve the efficiency of the decentralized channel by considering five well-known contracts, namely, simple wholesale price contract, bonus contract, linear contract, cost sharing contract and revenue sharing contract. In this thesis, we do not specifically look for a coordinating contract but focus on the non-coordinating contracts as well as the coordinating ones, find the optimal contract parameters and compare the performances of different contracts with each other. We aim to find which contracts would be better to use for the companies depending on the system parameters, even if these contracts can not fully coordinate the supply chain, and if it is worth to look for a more complex but coordinating contract in these situations. We also compare the performances of the contracts for the asymmetric information models and analyze the value of information in our models. Finally, through an extensive numerical study we investigate how the market variables will be affected under different contracts by the system parameters.

# ÖZETÇE

Bu çalışmada, tek üretici ve tek tedarikçiden oluşan iki kısımlı bir tedarik zincirini üç farklı talep senaryosu içinde inceledik. İlk senaryoda talep son ürün fiyatına bağlı ve belirli olarak varsayıldı. İkinci senaryoda ise son ürün fiyatı pazar tarafından belirlenmiş ürüne olan talep rastlantısal farzedildi. Son olarak da talep son ürün fiyatına bağlı ve raslantısal olarak alındı. Bütün senaryolarada tüm tedarik zincirinin verimliliğini arttırmak için bilinen beş çeşit kontrat kullandık. Bu kontratlar, primli kontrat, doğrusal kontrat, maliyet paylaşmalı kontrat ve gelir paylaşmalı kontratlardır. Bu tezde, sadece taraflar arasında koordinasyon sağlayan kontratlara değil koordinasyon sağlamayan ancak zincirin verimliliğini arttıran kontratları da inceledik. Koordine etmeyen kontratların optimal parametrelerini bulduk ve farklı kontratların performans karşılaştırmalarını yaptık. Amacımız sistem parametrelerine bağlı olarak koordinasyonu sağlamasa da şirketler için en uygun kontratları bulmak ve taraflar arasında koordinasyon sağlayan ancak daha karmaşık kontratları incelemenin gerekli olup olmadığını bulmaktı. Ayrıca taraflar arasında maliyetler ile ilgili asimetrik enformasyon olan durumu inceledik ve maliyet bilgisinin değerini ölçtük. Son olarak, pazar değişkenlerinin farklı kontrat kullanılan durumlarda farklı pazar sabitlerine göre değişimini sayısal bir çalışma ile inceledik.

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## NOMENCLATURE

| | |
|---|---|
| $a$ | Demand |
| $b$ | Price Dependency of Demand |
| $B$ | Capacity Investment Cost |
| $c(K)$ | Capacity Investment Cost Function of the Supplier |
| $c$ | Production Cost |
| $c_b$ | Bonus Payment to Supplier |
| $K$ | Invested Capacity |
| $t$ | Transfer Payment in Linear Contract |
| $T$ | Target Capacity in Bonus Contract |
| $p$ | Retail Price of the Product |
| $w$ | Wholesale Price of the Product |
| $\sigma$ | Portion of the Supplier's Capacity Investment Cost paid by the Manufacturer |
| $\rho$ | Protion of the Revenue Shared with the Supplier |
| $\gamma$ | Lowest value of the Capacity Investment Cost of the Supplier |
| $\delta$ | Highest value of the Capacity Investment Cost of the Supplier |
| $\phi$ | Probability of Being a High-Capacity-Cost Supplier |
| $\pi_m$ | Manufacturer's Profit in the Decentralized Case |
| $\pi_s$ | Supplier's Profit in the Decentralized Case |
| $\pi_{sres}$ | Supplier's Reservation Profit |
| $\Pi$ | Centralized Channel Profit |

Chapter 1

# INTRODUCTION

Manufacturing capacity plays an important role in a supplier-manufacturer relation. In a vertically decentralized supply chain composed of a single manufacturer (he) and a single supplier (she), manufacturer buys parts or products from the supplier and after adding value, he sells the new product to the market. To supply the manufacturer's demand the supplier should invest on her capacity prior to manufacturer's firm orders since the capacity installation has a significant lead time. Due to high volatility in demand the amount of invested capacity is a difficult decision for a supplier since she has to take the risk of over capacity. Thus, she has a tendency for low capacity installations which causes shortages in the channel. Several surveys show that especially in the high-tech industries manufacturers suffer due to the lost sales when suppliers do not build sufficient capacity. For instance, in the late 1980s in the US, computer makers lost huge amounts of revenues due to lost sales of 256-kilobite dynamic random access memory chips because of insufficient supply [1]. Also, in the biotech industry manufacturing facilities cost between $200- $400 million to build which poses a great risk for the supplier. Thus, despite of high market requirement of bio-drugs, invested capacity is always insufficient [2].

The inefficiency in the channels are due to the fact that each party in the supply chain make decisions in order to optimize its own objective with very little regard to the impact of its decisions on the other party or on supply chain performance. This inefficiency can be eliminated with the help of supply chain contracts which provide a risk share between the buyer and supplier. Contracts also provide a means for bringing the total expected profit of the decentralized model closer to the total expected profit of the centralized model, which is referred as the channel coordination objective. Additionally, contracts provide long-term partnerships between the parties in the supply chain which yield encouragement for the parties to engage in activities that are unfavorable in the short time but have great benefit

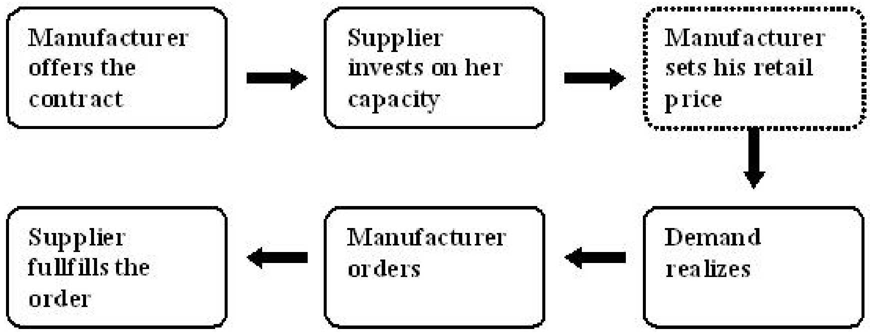


Figure 1.1: Sequence of Events

in the long term. Finally, contracts make the terms of the relationship explicit [3].

Information sharing between the parties also plays an important role in the supply chains. In practice full information between the parties in the channel case is rare. This is either because of information sharing is difficult and costly or because of the party having the superior information has an incentive to keep it private. For example a manufacturer can have a better information about demand and he may give the supplier an excessively optimistic demand forecast to induce him to invest more capacity [4]. Considering this fact, supplier may not believe the manufacturer's optimistic demand forecasts and invests an insufficient capacity which can cause huge losses as in the case of Boeing in 1997 [5].

Parties in a supply chain may have asymmetric information in terms of many parameters such as demand information, production costs and capacity investment costs. A branch of economic theory called the information economics deals with the issues due to various information asymmetries between the parties. Information economics mainly analyze consequences of informational asymmetries between the parties and optimal design of contracts to deal with them. There are mainly 4 heading studied under information economics: Moral hazard, screening , adverse selection and signalling.

Moral hazard heading refers to cases where a principle cannot observe the effort of his agent. There will be a conflict since principle prefers his agent to work hard while the agent dislikes it. In signalling, the party that has superior information tries to provide incentives by revealing his information. In screening and adverse selection, the less informed party offers a contract to the party that has superior information. Our information asymmetric models fall under the screening category.

In this thesis, we aim to improve the efficiency of a decentralized supply chain composed of a single supplier and a manufacturer using supply chain contracts under symmetric and asymmetric information. The sequence of events in all our decentralized models is as shown in Figure 1.1. At first, the manufacturer offers the contract and in case of contract acceptance the supplier sets her capacity according to contract parameters. Later, the manufacturer sets the retail price if it is endogenous and demand is realized. At this stage two cases are possible; the built capacity can be greater than the demand so the capacity cannot be fully utilized, or the demand can be greater than the invested capacity thus there will be some unmet demand. We assume that unused capacity has no value and unmet demand will be lost.

In our analysis of contracts in this thesis, we first look into the centralized model where the supplier and the manufacturer are owned by the same agent who makes all the decisions. Then, we analyze the decentralized model using contracts and try to determine whether these contracts can coordinate the channel or not. In some cases, finding a coordinating contract might not be possible or even if such a contract is found, it might be too complex, costly or hard to implement due to various reasons. Note that, the supply chain members need to implement new information technologies or control systems to facilitate an effective use of some of the contracts. In addition, some companies might prefer some contracts over the others due to the risks in the contracts and special circumstances of the relations in the supply chain. Thus, in those cases, there might exist simpler, less costly and easier to implement contracts which can not fully coordinate the supply chain but have a good enough performance for the system objectives. These non-coordinating contracts might be preferred by the supply chain members to the coordinating ones depending on the performances of the contracts, implementation costs and system characteristics. Although a contract can be very efficient in some cases, it might not perform as efficiently in some other situations. Thus, the choice of the contract to implement in the supply chain plays an important role in coordination. Companies need to determine the right type of contract to implement depending on the specific characteristics of their business structure and the parameters of their system.

In this thesis, we do not specifically look for a coordinating contract but focus on the non-coordinating contracts, as well as the coordinating ones, find the optimal contract parameters and compare the performances of different contracts with each other. We aim to find which contracts would be better to use for the companies depending on the system parameters, even if these contracts can not fully coordinate the supply chain, and if it is worth to look for a more complex but coordinating contract in these situations. For this purpose, we analyze several well-known contracts in a two-party supply chain composed of a single manufacturer and a single supplier, compare their performances and also investigate how the market variables will be affected with different contracts by the system parameters. We also compare the performances of the contracts for the asymmetric information models and analyze the value of information in our models.

We mainly consider five types of contracts in this thesis which are offered by the manufacturer: simple wholesale price contract, bonus contract, linear two part tariff contract, revenue sharing contract and cost sharing contract. In a simple wholesale price contract, only the wholesale price $w$ is agreed upon on the contract. In the bonus contract, the manufacturer offers a per unit bonus value to the supplier for each unit supplied over a certain value in order to induce the supplier to invest on a higher capacity. In a linear contract, the manufacturer offers a wholesale price $w$, and a fixed amount of money $t$ to the supplier. In a revenue sharing contract the manufacturer offers a portion of his revenue to the supplier and in a cost sharing contract he shares a portion of her capacity investment cost.

In the next chapter we conduct a literature survey on supply chain coordination with capacity allocation in the case of symmetric and asymmetric information between the parties. In Chapters 3, 4 and 5, we consider different settings for capacity investment decisions in a two party supply chain with full information and asymmetric information cases. In Chapter 3, we consider a simple setting in which the manufacturer faces a deterministic price dependent demand. The manufacturer sets the retail price and demand decreases linearly as retail price increases. For this case, at first we assume exogenous wholesale price. Later on, we modify the model where the wholesale price is adjustable and finally for this case we analyze the model in the presence of information asymmetry between the supplier and the manufacturer where the manufacturer does not know the supplier's capacity investment cost. In Chapter 4, we assume that the manufacturer faces a stochastic demand and the retail price for the end product is fixed. We also analyze the asymmetric information model in which we assume that the supplier has a better information about her capacity investment cost. In Chapter 5 we modify the setting in Chapter 4 by adding a retail price decision for the manufacturer. In this chapter we assume that the demand is both stochastic and price dependent. Finally, we performe a numerical study to compare the efficiencies of the contracts and to determine the effects of the parameters on our analysis in Chapter 6.

Chapter 2

# LITERATURE REVIEW

In this chapter we perform a literature survey about supply chain contracts related to capacity decisions. Contracts play a significant role in supply chains since they can improve the performances of supply chains and due to their importance, a broad literature on supply chain contracts exist. Whang [24] and Tsay et al. [3] provide a general review on supply chain contracts and Cachon [7] provides a more extensive survey on this area.

In practice, the most common contract between the parties in the channel is the simple wholesale price contract. Bresnahan and Reiss [25] consider the simple wholesale price contract with deterministic demand and Lariviere and Porteus [26] consider this contract in the context of the newsvendor problem. They show that maximum channel efficiency cannot be achieved with this contract because of the double marginalization phenomenon which is introduced by Splenger in [16] .Different contract types are proposed in the literature to coordinate the channel. For instance, Pasternak [17] proposes a buy-back contract which provides a risk sharing between the parties in the case of low demand. Cachon and Lariviere [18] consider a revenue sharing contract to coordinate the channel in fixed- price newsvendor and price setting newsvendor models. Tsay [19] proposes a quantity flexibility contract which couples the manufacturer's commitment to purchase no less than a certain percentage below the forecast with the supplier's guarantee to deliver up to a certain percentage above.

Some researchers relax the infinite capacity assumption and focus on coordinating capacitated supply chains. Wu et al.[2] concentrate on managing capacity in the high tech industry and they review emerging models in operations research, game theory and economics for high-tech capacity management. Cachon and Lariviere [4] consider a single manufacturer, single supplier setting with forced compliance and voluntary compliance regime and although coordination can be achieved under forced compliance regime with option contracts, under voluntary compliance regime it fails to coordinate the channel. Tomlin [10] enriches Cachon and Lariviere's approach by introducing partial compliance. He shows that nonlinear, price only contracts induce higher supplier capacity, contrary to result of Cachon and Lariviere. Also he shows that if the supplier's reservation profit is below a certain threshold, the manufacturer's optimal contract is quantity-premium price schedule.

Serel et al. [11] examine sourcing decision of a manufacturer in the presence of a long term capacity reservation contract. In the contract the supplier guarantees to deliver any order amount desired by the manufacturer up to a reserved fixed capacity and the buyer offers a guaranteed payment, and if the supplier fails to provide capacity the manufacturer has an alternative in the spot market. In their setting the wholesale price is charged by the supplier and its the main contract parameter.

Erkoc and Wu [12] propose a capacity reservation contract designed for a short-life cycle, make-to-order high-tech products under stochastic demand in the case of exogenous wholesale and retail price. They also consider the supplier's option of not complying with the contract. Jin and Wu [13] study a similar capacity reservation contract with Erkoc and Wu, however they consider the interaction among costumers and excess capacity expansion. They propose a deductible reservation contract where manufacturers reserve future capacity with a fee that is deductible from the purchasing price and show that coordination in the channel can be achieved with that contract. Mathur and Shah [1] analyze the impact of various penalty parameters on the supplier's capacity decision in a similar setting. They model the supply chain in a price compliance regime and they introduce two penalties, namely penalty for short supply and for short orders and one bonus, namely excess supply bonus. They showed that the manufacturer can influence the supplier's capacity decision with capacity commitment in the form of target capacity. Van Mieghem and Dada [20] consider a channel composed of a single supplier and single manufacturer in which retail price is also a decision variable for the manufacturer. In their setting demand is random and price dependent.

Afore mentioned studies consider common knowledge between the parties. However common knowledge assumption is a quite strong assumption and in practice parties in the channel has private information about demand and costs. Chen [9] provides an extensive survey on information asymmetries in supply chains. As Özer and Wei suggest in [5] the supply chain literature that explicitly models asymmetric information can be classified into two groups, namely studies that focus on the information asymmetry on market demand

and forecast and studies that concentrate on information asymmetry in costs.

Cachon and Lariviere [8] consider a single period game between one supplier and N independent retailers where retailers' demands are their private information. Retailers are assumed to be local monopolies in the consumer market, however they compete for the supplier's scarce capacity. They study different capacity allocation rules and their effects on players' strategic behavior. They show that some allocation mechanisms induce the retailers to reveal their private information whereas others lead them to inflate their orders to gain better allotment of stock.

Cachon and Lariviere [4] consider a single manufacturer, single supplier setting where the manufacturer has better information about the demand. In their setting, supplier's capacity investment cost function is separately considered. Manufacturer provides an initial forecast and a contract consisting of firm commitments and capacity options. This is basically a signalling problem in which manufacturer signals his private information to the supplier. They show that for a manufacturer with a high demand forecast, it is beneficial to share the forecast with the supplier however it is costly. Özer and Wei [5] analyze the same setting but they use two different models. In their first model, supplier screens the manufacturer's forecast information with capacity reservation contract and in their second model manufacturer signals his information with advance purchase contract. They compare the cost of screening and cost of signalling in their study.

Lee et al. [22] also consider a similar setting in which the retailer has better demand information and they show that information sharing alone could provide significant inventory reduction and cost savings to the manufacturer. They also suggest that the manufacturer would experience great savings when the demand correlation over time is high, the demand variance within each time period is high, or the lead times are long.

Chu [21] considers a distribution channel composed of a single manufacturer and a single retailer where retailer faces a deterministic demand that depends on the retail price and the manufacturer's advertising expenditure. In their first case, the retailer has better information about the demand and she offers the contract. In their second case, the manufacturer offers a contract to screen the demand information.

Desai and Srinivasan [23] study a two-sided information asymmetry model in which the principle has better demand information and agent can influence the demand by sales effort which is unobservable by the agent. This is basically a signalling problem in the presence of moral hazard. They consider linear and non-linear pricing contracts offered by the principle.

Ren et al. [32] study a model composed of a single retailer and a single supplier where the retailer has better information about demand and the supplier and the retailer have a long term relationship. Instead of screening or signalling models, they focus on trigger strategy which is a class of strategies employed in repeated non-cooperative games. A player using a trigger strategy initially cooperates but punishes the opponent if a certain level of defection is observed [33]. Ren et al. show that in a repeated forecast sharing game, coordination can be achieved with linear contract when the industry is stable, both parties value their long term relationship and overforecasting is easy to detect.

In the above paragraphs, we analyze information asymmetries on demand and forecast, however, information asymmetry can also occur on costs. Ha [27] studies a model composed of a single supplier and a single retailer who faces a stochastic price dependent demand. In his setting, the supplier does not know the retailer's marginal cost and she proposes a nonlinear contract with fixed pricing.

Corbett et al. [31] also consider a setting in which supplier does not know the retailer's marginal cost but they assume deterministic demand. They allow both sides to refuse the trade by introducing reservation profits for both parties and they focus on simple wholesale price, two-part linear and two-part non-linear contracts proposed by the supplier to examine the value of information.

Corbett and Groote [28] consider a model composed of a single supplier and a single retailer. Retailer faces a deterministic demand and the exact value of his inventory holding cost is unknown to the supplier. Supplier only knows the distribution of his inventory holding cost. In their setting the supplier tries to find the optimal menu of quantity discount contracts and tries to screen the retailer's inventory holding cost.

Corbett [29] considers stochastic demand on the contrary to Corbett and Groote. His setting is a classical (r,Q) model, in which, when the retailer's inventory falls under the reorder point r, the supplier provides a batch of Q units. In that model, supplier decides on the value of Q and she incurs a fixed cost $K$ for each batch she produces. In Corbett's first scenario, supplier privately observes $K$ and retailer screens $K$ by offering a contract. In his second scenario, the retailer's backorder penalty cost is his private information. In that case, supplier tries to find the optimal menu of contracts to screen the retailer's backorder penalty cost.

Kaya and Özer [30] address the quality risk in outsourcing and in a part of their study, they focus on a case in which manufacturer's quality cost is his private information and supplier screens his quality cost information.

Taylor and Plambeck [14] study a setting where the supplier and the manufacturer has common information about the demand and supplier's capacity investment cost is unobserved by the manufacturer. They assume that both firms employ *trigger strategy* and they focus on comparing the performances of relational contracts in which the manufacturer commits to buy a fixed quantity versus relational contracts that only specify per unit price. They also consider the extensions of their base model, including stochastic retail price and production cost, random production yield and private capacity cost information. Taylor and Plambeck look into the same setting in multiple periods in [15]

In this study, we focus on the capacitated supply chain composed of a single manufacturer and a single supplier and we analyze the model in the case of three different demand functions namely, price dependent deterministic demand, stochastic demand and price dependent stochastic demand. We consider five types of well-known contracts in our analysis: simple wholesale price contract, bonus contract, linear two part tariff contract, revenue sharing contract and cost sharing contract. Different from the studies in the literature on the capacitated settings we do not specifically look for a coordinating contract but focus on the non-coordinating contracts, as well as the coordinating ones, find the optimal contract parameters and compare the performances of different contracts with each other. Additionally, we compare the performance of a certain contract in different demand settings and investigate how the market variables will be effected with different contracts by the system parameters.

In our study we also consider information asymmetry between the parties such that we assume that the manufacturer's capacity investment cost is her private information as Taylor and Plambeck do in [14] and [15]. However in our setting we consider the model in a single period setting and we consider a screening problem. In the price dependent deterministic demand case we consider a single bonus contract offered by the manufacturer and in the stochastic demand case we analyze the information asymmetry when manufacturer screens the supplier's capacity investment cost with a menu of linear, bonus and cost sharing contracts. We determine the value of information in all those cases and we compare the efficiencies of the screening contracts.

Chapter 3

# PRICE DEPENDENT DETERMINISTIC DEMAND

In this chapter we consider a single manufacturer and a single supplier in a single period setting where the manufacturer faces a deterministic and price dependent demand. We assume a linear demand function in the format $(a - bp)$ where $a$ is the market size and $b$ is the price elasticity of demand, which are commonly known by the manufacturer and the supplier. The supplier has a unit production cost $c$, and a separate capacity investment cost function $c(K)$ which is assumed to be a convex function. We consider linear and quadratic cost functions in this study such that $c(K) = BK$ and $c(K) = BK^2$ to analyze the effect of different cost structures on the model results. In the following analysis, first we consider the integrated channel problem where a single agent decides on both the amount of capacity investment and the retail price with the objective of maximizing the total supply chain profit. Later, we consider the decentralized models with symmetric information under two different assumptions. In the first case, we assume that the wholesale price, $w$, is exogenous and has been decided before the contract negotiations. For the exogenous wholesale price model, we consider a bonus contract offered by the manufacturer and determine whether it can coordinate the channel. Later, we introduce information asymmetry between the parties where the manufacturer does not know the supplier's capacity investment cost. Then, we consider the second model in which $w$ is also negotiable. We analyze the simple wholesale price, linear, revenue sharing and cost sharing contracts and determine if they can achieve channel coordination.

It is seen in the literature that when the supply chain is decentralized and each party tries to maximize its own profit, a considerable decrease in the total supply chain profit exists and the supply chain suffers from inefficiencies. Supply chain members use contracts to eliminate these inefficiencies in the supply chain and to achieve the first best solution. In this chapter, we analyze several different contracts and find the optimal contract parameters and optimal solutions with each contract. Note that, a contract that maximizes the total supply chain profit might not be in the best interest of one of the parties. One party might end up with a profit that is even less than the profit he might get with his outside option, and in that case, he chooses not to participate in such a contract. We assume that the manufacturer and the supplier have reservation profits $\pi_{sres}$ and $\pi_{mres}$, respectively, and they reject to participate in any contract that causes them to get a profit less than their reservation profits. Note that the reservation profits of the supply chain members depend on their powers in the industry and other business opportunities that they have in the market.

In this study, for all the contracts, we assume that the manufacturer designs the contract and solves the problem below to find the optimal contract parameters for himself while making sure that the supplier gets a profit that is at least as high as her reservation profit.

$$\begin{aligned} & \max \ \pi_m \\ s.t\ \pi_s &\geq \pi_{sres} \end{aligned} \tag{3.1}$$

Note that when the supplier gets more powerful in the supply chain, the manufacturer needs to solve the problem above using a higher $\pi_{sres}$ value since a powerful supplier can extract more profit from the manufacturer in the contract negotiations. In this study, we also analyze, how the efficiency of the contracts will be affected by the powers of the parties and which contracts would be better to use for each party depending on their powers and the parameters in the system. In our analysis, we present the optimal solutions under different contracts using various reservation profits and compare the performances of the contracts in each case. Note that, if a contract can fully coordinate the supply chain with arbitrary profit sharing, the solutions to the above problem for different $\pi_{sres}$ values all result in the same market price, capacity and total supply chain profit values and they are all equal to their values in the first best solution independent of the powers of the parties and the values of the reservation profits. But the profit of the manufacturer will be effected by the reservation profit. However, if the contract can not fully coordinate the supply chain, then the result of the above problem for different $\pi_{sres}$ values might be very different from each other and the powers of the parties play a much more important role in the determination of the final solution.

### 3.1 Centralized Channel

The centralized supply chain can be interpreted as a single firm that both makes the capacity investment and the pricing decisions. If the manufacturer and the supplier belong to the same firm

and the whole supply chain is controlled by a single decision agent, then this agent wants to maximize the total supply chain profit which is

$$\max_{p,K} \Pi = (p-c)min(K,(a-bp)) - c(K) \tag{3.2}$$

**Proposition 1** *For this model, the capacity is always equal to demand in the centralized channel i.e. $K = a - bp$ will always hold for the centralized channel.*

**Proof.** Assume on the contrary $K \neq a - bp$.

If $K > a - bp$ the amount that will be sold is $a - bp < K$. However, the profit can be increased by decreasing $K$. It is obvious that with parameters $(K - \varepsilon, p)$ the channel's profit will be higher than the profit with parameters $(K, p)$ when $K > a - bp$ holds. Thus, in the optimal solution $K > a - bp$ does not hold.

If $K < a - bp$ ,demand will be higher than capacity and since it depends on the retail price, $p$ will be increases to decrease the demand. Integrated channel will have higher profit with parameters $(K, p + \varepsilon)$ than with parameters $(K, p)$ when $K < a - bp$.

Thus, $K = a - bp$ should always hold. ■

Proposition 1 leads us the following equation.

$$\begin{aligned} \max_{p,K} \Pi &= [(p-c)(a-bp)] - c(K) \\ where\ K &= a - bp \end{aligned} \tag{3.3}$$

**Proposition 2** *The optimal retail price, $p^*$, for the integrated channel is the solution of $a + cb - 2pb - \partial c(K)/\partial p = 0$. Consequently, the optimal invested capacity and the total profit of the centralized channel will be $K^* = a - bp^*$ and $\Pi^* = (p^* - c)K^* - c(K^*)$.*

**Proof.** To find the optimal value of $p$ we use first order conditions since 3.3 is concave in terms of $p$ for increasing capacity investment cost functions. ■

Proposition 2 leads us that for $c(K) = BK$ the optimal retail price, invested capacity and total channel profit will be

$$
\begin{aligned}
p^*_{cent} &= \frac{1}{2b}\left(a + b(B+c)\right) \ , \ K^*_{cent} = \frac{a - b(c+B)}{2} \\
\Pi^*_{cent} &= \frac{(a - b(c+B))^2}{4b}
\end{aligned}
$$

and for $c(K) = BK^2$

$$
\begin{aligned}
p^*_{cent} &= \frac{a + cb + 2Bab}{2b(1+Bb)} \ , \ K^*_{cent} = \frac{a - bc}{2(1+Bb)} \\
\Pi^*_{cent} &= \frac{(a - cb)^2}{4b\,(Bb+1)}
\end{aligned}
$$

### 3.2 Coordination with Contracts for Exogenous Wholesale Price

In this part, we assume an exogenous wholesale price $w$ that is agreed upon beforehand and the parties are not allowed to change this unit wholesale price $w$ due to the market conditions. Exogenous wholesale price assumption is suitable for real life situations at the presence of an intensive competition between the suppliers [1]. In this model, in the decentralized case with no contract, the manufacturer will decide on his retail price whereas the supplier will decide on her capacity. The objective functions of the manufacturer and the supplier will be as follows respectively.

$$
\begin{aligned}
\max_p \ \pi_m &= (p - w)\min(K, a - bp) && (3.4) \\
\max_K \ \pi_s &= (w - c)\min(K, a - bp) - c(K) && (3.5)
\end{aligned}
$$

**Proposition 3** *The optimal capacity for the decentralized channel with no contract will be* $K^* = (a - bw)/2$ *for* $c(K) = BK$, *and for* $c(K) = BK^2$

$$K^* = \min\left\{\frac{w - c}{2B}, \frac{a - bw}{2}\right\}$$

**Proof.** If there were no capacity constraint in problem 3.4 $\min(K, a - bp) = a - bp$ and manufacturer solves the problem to find the optimal retail price, $p^*$ and he will order $a - bp^* = \frac{a-bw}{2}$. However, supplier decides on the capacity by solving 3.5. For $c(K) = BK$, 3.5 becomes

$$\max_K \pi_s = (w - c - B)K$$

and that is linear in terms of $K$. Thus, supplier sets her capacity equal to the maximum amount that the manufacturer will order which is $(a-bw)/2$ . And, for $c(K)=BK^2$ 3.5 becomes

$$\begin{aligned}\max_{K}\ \pi_s &= (w-c)K-BK^2\\ &\Longrightarrow \frac{\partial \pi_s}{\partial K}=w-c-2BK=0 \Longrightarrow K=\frac{w-c}{2B}\end{aligned}$$

since the supplier's objective function is strictly concave the optimal capacity will be

$$K^*=\min\left\{\frac{w-c}{2B},\frac{a-bw}{2}\right\}$$

■

### *3.2.1 Coordination with Bonus Contract*

In this part we consider a bonus contract proposed by the manufacturer and determine if coordination in the channel can be achieved with it. The sequence of events will be as follows; first the manufacturer being the first mover, offers a contract with parameters ($c_b$,$T$) where $T$ is specified as the target capacity and $c_b$ is the bonus retailer gives to the supplier per unit item supplied after $T$. According to contract parameters the supplier decides on her production capacity and finally, retailer specifies order amount and retail price. To find the optimal contract parameters the manufacturer solves the following problem

$$\begin{aligned}\max_{c_b,T}\ \pi_m &= (p-w)\min(K,a-bp)-c_b(\min(K,a-bp)-T)^+ \qquad (3.6)\\ s.t\ \pi_s &= (w-c)\min(K,a-bp)+c_b(\min(K,a-bp)-T)^+-c(K)\geq \pi_{sres}\end{aligned}$$

where $\pi_{sres}$ is the reservation profit of the supplier. We assumed that $\pi_{sres}$ is equal to the supplier's profit without contracting which is

$$\begin{aligned}\text{for } c(K) &= BK\\ \pi_{sres} &= (w-c-B)(\frac{a-bw}{2})\end{aligned}$$

and for $c(K) = BK^2$

$$\begin{aligned}
\text{for } c(K) &= BK^2 \\
\pi_{sres} &= (w-c)(\frac{w-c}{2B}) - B(\frac{w-c}{2B})^2 = \frac{(w-c)^2}{4B} \text{ if } \left\{\frac{w-c}{2B} < \frac{a-bw}{2}\right\} \\
\pi_{sres} &= (w-c)(\frac{a-bw}{2}) - B(\frac{a-bw}{2})^2 \\
&= \frac{(a-wb)\left(2(w-c) - B(a-wb)\right)}{4} \text{ if } \left\{\frac{w-c}{2B} > \frac{a-bw}{2}\right\}
\end{aligned}$$

The supplier will accept the bonus contract offered by the manufacturer if $\pi_{Sres} \leq \pi_{SC}$ where $\pi_{SC}$ is supplier's profit with contract.

**Proposition 4** *For $c(K) = BK$ a discount contract with parameters $c_d = w - B - c$ and $T = (a - bw)/2$ can coordinate the channel. For $c(K) = BK^2$ if $\frac{w-c}{2B} \leq \frac{a-bw}{2}$ holds coordination can be achieved in the channel with a bonus contract having parameters $c_b = (c - w + Ba - Bwb)/(Bb + 1)$ and $T = (c - w + 3Ba - 2Bcb - Bwb)/(4B(bB + 1))$ and if $\frac{w-c}{2B} > \frac{a-bw}{2}$ holds a quantity discount contract with parameters $c_d = (c - w + Ba - Bwb)/(Bb + 1)$ and $T = (c - w + 3Ba - 2Bcb - Bwb)/(4B(bB + 1))$ will achieve coordination.*

**Proof.** Consider the final stage of the sequence of moves where the manufacturer decides on his retail price for a given $c_b$ and $T$. His objective will be as in 3.6.

For $c(K) = BK$, the supplier will set her capacity equal to $a - bp$ for any price $p$ as long as $w \geq c + B$. Otherwise, the supplier will set the capacity equal to 0. Assume that for a given $T$, $a - bp = K > T$ . Otherwise, the bonus contract will have no effect on the solution. So, 3.6 becomes

$$\begin{aligned}
\max_{c_b,T} \pi_m &= (p-w)(a-bp) - c_b(K-T) \\
s.t\ \pi_m &= (w-c)K + c_b(K-T) - c(K) \geq \pi_{sres}
\end{aligned}$$

Then the optimal retail price can be found from the first order conditions

$$\begin{aligned}
\frac{\partial \pi_m}{\partial p} &= a - 2pb + wb + bc_b = 0 \\
&\Longrightarrow\ p^* = \frac{a + wb + bc_b}{2b} \Longrightarrow (a - bp^*) = \frac{a - wb - bc_b}{2}
\end{aligned}$$

Then, for $c_d = w - B - c$

$$\begin{aligned} p^* &= \frac{a + wb + bc_b}{2b} = \frac{(a + b(B + c))}{2b} = p^*_{cent} \\ and\ (a - bp^*) &= \frac{a - wb - bc_b}{2} = \frac{a - b(c + B)}{2} = (a - bp^*)_{cent}. \end{aligned}$$

For a given contract parameters supplier sets her capacity considering the following objective function

$$\max_K \pi_s = (w - c)K - c_d(K - T) - BK$$

which is linear in terms of $K$ and supplier sets her capacity equal to the manufacturer's maximum amount of order which is

$$(a - bp^*) = \frac{a - wb - bc_d}{2} = \frac{a - b(c + B)}{2} = (a - bp^*)_{cent}$$

when $c_d = w - B - c$.

Finally the manufacturer finds the optimal $T$ value from the supplier's individual rationality constraint in 3.6.

$$\begin{aligned} \pi_s &= c_d T = (w - c - B)(\frac{a - bw}{2}) \\ &\Longrightarrow T = (\frac{a - bw}{2}) \end{aligned}$$

For this value of $T$ we need to check if the assumption $K > T$ holds. $K = (a - b(c + B))/2 > (a - bw)/2 = T$ because $w > c + B$ should hold for supplier to make positive profit.

For $c(K) = BK^2$ two cases are possible with respect to the relation between the parameters

**Case 1** If $\frac{w-c}{2B} < \frac{a-bw}{2}$ holds manufacturer wants to order more so he wants the supplier to invest on more capacity. Manufacturer's optimal retail price previously found as $p^* = (a + wb + bc_b)/2b$. For $c_b = (c - w + Ba - Bwb)/(Bb + 1) \Longrightarrow p^* = (a + cb + 2Bab)/2b(1 + Bb) = p^*_{cent}$. Additionally, supplier's objective function with this contract will be

$$\begin{aligned} \pi_s &= (w - c)K - BK^2 + c_b(K - T) \\ \frac{\partial \pi_s}{\partial K} &= w - c - 2BK + c_b = 0 \Longrightarrow K^* = \frac{w - c + c_b}{2B} \\ &\Longrightarrow K^* = \frac{a - bc}{2(1 + Bb)} = K^*_{cent} \end{aligned}$$

Finally the manufacturer finds the optimal $T$ value from the supplier's individual rationality constraint in 3.6.

$$\begin{aligned}\pi_s &= (w-c)K^* - B(K^*)^2 + c_b^*(K-T) = \frac{(w-c)^2}{4B} \\ &\implies T = \frac{(w-c+Ba-2Bcb+Bwb)}{4B(bB+1)}\end{aligned}$$

For the assumption $a - bp = K > T$ check if the following inequality holds

$$\frac{a-bc}{2(1+Bb)} > \frac{(w-c+Ba-2Bcb+Bwb)}{4B(bB+1)}$$

after making simplifications and necessary rearrangements the inequality becomes

$$\frac{w-c}{B} < a - bw$$

which holds for this case.

**Case 2.** For the case $\frac{w-c}{2B} > \frac{a-bw}{2}$ supplier builds more capacity than retailer will order. Thus, retailer will pay $(w - c_d)$ for each unit that exceeds $T$,instead of giving extra $c_b$. Again, consider the final stage of the sequence of moves where manufacturer decides on his retail price for a given $c_d$ and $T$. Manufacturer's optimal retail price previously found as $p^* = (a + wb - bc_d)/(2b)$. For $c_d = (c - w + Ba - Bwb)/(Bb + 1) \implies p^* = (a + cb + 2Bab)/2b(1 + Bb) = p^*_{cent}$. Additionally, supplier's objective function with this contract will be

$$\begin{aligned}\pi_s &= (w-c)K - BK^2 - c_d(K-T) \\ \frac{\partial \pi_s}{\partial K} &= w - c - 2BK - c_d = 0 \implies K^* = \frac{w-c-c_d}{2B} \\ &\implies K^* = \frac{a-bc}{2(1+Bb)} = K^*_{cent}\end{aligned}$$

Similar to the previous case manufacturer determines the optimal value for $T$ from the supplier's individual rationality constraint.

$$\begin{aligned}\pi_s &= (w-c)K^* - B(K^*)^2 - c_d^*(K^*-T) = \frac{(a-wb)\left(2(w-c) - B(a-wb)\right)}{4} \\ &\implies T = \frac{\left(2a - 4cb + 2wb + B^2wb^3 - B^2ab^2 - 2Bab - Bcb^2 + 3Bwb^2\right)}{4Bb+4}\end{aligned}$$

For the assumption $a - bp = K > T$ check if the following inequality holds

$$\frac{a-bc}{2(1+Bb)} > \frac{\left(2a - 4cb + 2wb + B^2wb^3 - B^2ab^2 - 2Bab - Bcb^2 + 3Bwb^2\right)}{4Bb+4}$$

simplifying this inequality leads the following inequality

$$B(a - bw)(3 + Bb) > 2(w - c) + B(a - bc) \tag{3.7}$$

If 3.7 holds then

$$T = \frac{\left(2a - 4cb + 2wb + B^2wb^3 - B^2ab^2 - 2Bab - Bcb^2 + 3Bwb^2\right)}{4Bb + 4}$$

However if 3.7 does not hold $T = 0$ and the channel will act as the decentralized case. ■

For a coordinating bonus contract manufacturer's profit will be $\pi_m = \Pi^*_{cent} - \pi_{sres}$ and supplier's profit will be $\pi_{sres}$.

### *3.2.2 Case with Asymmetric Capacity Cost Coefficient*

In this section we will analyze the case in which manufacturer does not know the exact value of capacity cost coefficient, $B$. Similar to the previous part we use price dependent, deterministic demand, $D = a - bp$. The wholesale price,$w$ and marginal production cost for the supplier,$c$, are fixed and we consider that supplier's capacity cost function is $c(K) = BK$. In this case, manufacturer only knows the distribution of $B$ where $B$ is assumed to be uniformly distributed between $\gamma$ and $\delta$.The reservation profits are assumed to be equal to the the supplier's and the manufacturer's profits with no contract, which are calculated as follows.

$$\begin{aligned}
\pi_s &= (w - c - B)(\frac{a - bw}{2}) = \pi_{res} \\
\pi_m &= \frac{(a - bw)^2}{4b}
\end{aligned}$$

*Coordination with a Bonus Contract*

In this part we will analyze the model where retailer offers a bonus contract with parameters $(c_b, T)$. In this case manufacturer moves first and offers the contract parameters to the supplier, later on supplier decides on her capacity and finally manufacturer gives his order and sets his retail price. In this case we assume that the manufacturer offers only one contract to the supplier.

**Proposition 5** *For any $\gamma \leq w - c$ it is optimal for the manufacturer to offer a bonus contract with parameters $c_b = 2(w - c - \gamma)/3$ and $T = (a - bw)/2$. If $\gamma > w - c$, the supplier will set the capacity to 0.*

**Proof.** Consider the last step of the sequence of the moves, the manufacturer sets his retail price. For a given $(K, c_b, T)$ the manufacturer's objective is as follows

$$\pi_m = (p - w)(a - bp) + c_b(a - bp - T)^+$$

where $a - bp = K$ and assume $K > T$.

$$\begin{aligned} \frac{\partial \pi_m}{\partial p} &= a - 2pb + wb - bc_b = 0 \\ &\Longrightarrow p = \frac{(a + wb - bc_b)}{2b} \Longrightarrow a - bp = \frac{(a - wb + bc_b)}{2} \end{aligned}$$

Now,consider the second step, in which the supplier decides her production capacity. For a given $(c_b, T)$, her problem will be as follows

$$\pi_S = (w - c - B)K - c_b(K - T)$$

$$\frac{\partial \pi_S}{\partial K} = w - c - B - c_b$$

As a result supplier will increase $K$ up to $(a - wb + bc_b)/2$ as long as $w \geq c + B + c_b$ holds. Otherwise the supplier will act as in the decentralized case. Also, the supplier will not accept the contract if her reservation profit is more than her profit with the contract. The following equation shows the case in which the supplier accepts the contract for $B < w - c - c_b$.

$$\begin{aligned} (w - c - B)(\frac{a - bw}{2}) &\leq (w - c - B)(\frac{(a - wb + bc_b)}{2}) - c_b(\frac{(a - wb + bc_b)}{2} - T) \\ &\Longrightarrow B \leq \frac{2T + 2bw - cb - bc_b - a}{b} \end{aligned}$$

The manufacturer sets contract parameters considering his profit function

**Case 1.** For $w - c - c_b \geq \frac{2T + 2bw - cb - bc_b - a}{b}$, and assume $\delta$ is sufficiently large and $\gamma$ is sufficiently small

$$\begin{aligned} \pi_m &= \int_{\gamma}^{\frac{2T+2bw-cb-bc_b-a}{b}} (((\frac{a + bw - bc_b}{2b} - w)(\frac{a - bw + bc_b}{2}) + c_b(\frac{a - bw + bc_b}{2} - T))(\frac{1}{\delta - \gamma}))dB \\ &+ \int_{\frac{2T+2bw-cb-bc_b-a}{b}}^{\delta} (\frac{(a - bw)^2}{4b})(\frac{1}{\delta - \gamma})dB \end{aligned} \tag{3.8}$$

$$\begin{aligned} \pi_m &= \frac{(a - 2T + cb - 2wb + b\gamma + bc_b)\left(w^2b^2 - 2wab - 2wb^2c_b + a^2 + 2abc_b + b^2c_b^2 - 4Tbc_b\right)}{4b^2(\gamma - \delta)} \\ &- \frac{(a - wb)^2(a - 2T + cb - 2wb + b\delta + bc_b)}{4b^2(\gamma - \delta)} \end{aligned}$$

In order to find optimal value of $T$ that will maximize the objective function we take the derivative of the profit function with respect to $T$ and check the concavity of the objective function.

$$\begin{aligned}
\frac{\partial \pi_m}{\partial T} &= \frac{1}{2b}\frac{c_b}{\delta-\gamma}\left(4a-8T+2cb-6wb+2b\gamma+3bc_b\right)\\
\frac{\partial^2 \pi_m}{\partial T^2} &= \frac{4}{b}\frac{c_b}{\gamma-\delta} \text{ since } c_b \text{ and } b>0 \text{ and } \gamma-\delta<0\ \frac{\partial^2 \pi_R}{\partial T^2}<0\\
& \quad \text{thus } \pi_R \text{ is concave w.r.t } T.\\
&\implies T=\frac{1}{2}a+\frac{1}{4}cb-\frac{3}{4}wb+\frac{1}{4}b\gamma+\frac{3}{8}bc_b
\end{aligned}$$

Similarly, to find the optimal value of $c_b$

$$\frac{\partial \pi_m}{\partial c_b}=\frac{\begin{pmatrix} 4w^2b^2-8Ta+2a^2+3b^2c_b^2+8T^2+2b^2\gamma c_b-4Tcb+12Twb-4Tb\gamma+\\ 2cab-6wab-12Tbc_b+2ab\gamma-2cwb^2+6abc_b-2wb^2\gamma+2cb^2c_b-8wb^2c_b\end{pmatrix}}{4b(\gamma-\delta)}$$

$$\implies c_b=\frac{\left(6T-3a-cb+4wb+\sqrt{c^2b^2+4w^2b^2-12Ta+3a^2+b^2\gamma^2+12T^2+12Twb-6wab-2cwb^2+2cb^2\gamma-2wb^2\gamma}-b\gamma\right)}{3b}$$

or

$$c_b=\frac{\left(6T-3a-cb+4wb-\sqrt{c^2b^2+4w^2b^2-12Ta+3a^2+b^2\gamma^2+12T^2+12Twb-6wab-2cwb^2+2cb^2\gamma-2wb^2\gamma}-b\gamma\right)}{3b}$$

For this $c_b$

$\implies T=\frac{1}{2}a-\frac{1}{2}wb$ or $T=\frac{1}{2}a-\frac{1}{2}cb-\frac{1}{2}b\gamma$ In this case we assume that $w-c-c_b\geq(2T+2bw-cb-bc_b-a)/b\implies T\leq(a-bw)/2$ should hold, which actually holds for $T=\frac{1}{2}a-\frac{1}{2}bw$, however for $T=\frac{1}{2}a-\frac{1}{2}cb-\frac{1}{2}b\gamma$ this condition does not hold due to the fact that $w>c+B$ and $\gamma$ is the smallest value that $B$ can take

For $c_b=\frac{\left(6T-3a-cb+4wb+\sqrt{c^2b^2+4w^2b^2-12Ta+3a^2+b^2\gamma^2+12T^2+12Twb-6wab-2cwb^2+2cb^2\gamma-2wb^2\gamma}-b\gamma\right)}{3b}$

$\implies T=\frac{1}{2}a-\frac{1}{2}wb$ or $T=\frac{1}{2}a-\frac{1}{2}cb-\frac{1}{2}b\gamma$. Using the same argument we can conclude that manufacturer will propose $T=(a-bw)/2$.For this case the optimal value of $c_b$ is found as

$$c_b=\frac{2(w-c-\gamma)}{3}>0$$

For these values of $c_b$ and $T$

$$p=\frac{(3a+2cb+wb+2b\gamma)}{6b}\implies q=\frac{a-bw}{2}+\frac{(w-c-\gamma)}{3}>T$$

**Case 2.** For $\frac{2T+2bw-cb-bc_b-a}{b}\geq w-c-c_b$, and assume $\delta$ is sufficiently large and $\gamma$ is sufficiently small, i.e. $T \geq \frac{a-bw}{2}$

$$\begin{aligned}\pi_m &= \int_{\gamma}^{w-c-c_b} (((\frac{a+bw-bc_b}{2b}-w)(\frac{a-bw+bc_b}{2})+c_b(\frac{a-bw+bc_b}{2}-T))(\frac{1}{\delta-\gamma}))dB \\ &+ \int_{w-c-c_b}^{\delta} (\frac{(a-bw)^2}{4b})(\frac{1}{\delta-\gamma})dB \qquad (3.9)\end{aligned}$$

$$\begin{aligned}\pi_m &= \frac{(c-w+\gamma+c_b)\left(w^2b^2-2wab-2wb^2c_b+a^2+2abc_b+b^2c_b^2-4Tbc_b\right)}{4b(\gamma-\delta)} \\ &-\frac{(a-wb)^2(c-w+\delta+c_b)}{4b(\gamma-\delta)}\end{aligned}$$

For the optimal value of $T$

$$\frac{\partial\pi_m}{\partial T} = \frac{c_b}{\delta-\gamma}(c-w+\gamma+c_b)$$

The manufacturer will decrease $T$ as long as $(c-w+\gamma+c_b) < 0 \implies w > (c+\gamma+c_b)$ which holds for the supplier to accept the contract, thus, he will propose $T = \frac{a-bw}{2}$.For the optimal value of $c_b$ first order condition is used

$$\begin{aligned}\frac{\partial\pi_m}{\partial c_b} &= \frac{\left((2a-4T)(c-w+\gamma+2c_b)+2w^2b+3bc_b^2-2cwb-2wb\gamma+2cbc_b-6wbc_b+2b\gamma c_b\right)}{4(\gamma-\delta)} \\ &= 0\end{aligned}$$

where $T=(a-bw)/2$.

$$\implies c_b = \frac{2(w-c-\gamma)}{3}$$

For these $c_b$ and $T$ values integrals in 3.8 and 3.9 are defined in the region $\gamma \leq w-c$ and $\delta \geq \frac{w-c}{3}+\frac{2\gamma}{3}$. Now, let us consider the other cases; for $\gamma > w-c$ the supplier will set her capacity to 0 and if $\delta < \frac{w-c}{3}+\frac{2\gamma}{3}$ the equation 3.8 becomes

$$\begin{aligned}\pi_m &= \int_{\gamma}^{\delta} (((\frac{a+bw-bc_b}{2b}-w)(\frac{a-bw+bc_b}{2})+c_b(\frac{a-bw+bc_b}{2}-T))(\frac{1}{\delta-\gamma}))dB \\ &= \frac{\left(w^2b^2-2wab-2wb^2c_b+a^2+2abc_b+b^2c_b^2-4Tbc_b\right)}{4b}\end{aligned}$$

It is seen that $\pi_m$ is linear in terms of $T$ and convex in terms of $c_b$since $\frac{\partial^2\pi_m}{\partial c_b^2} = \frac{1}{2}b > 0$. Thus, the optimal $c_b$ value should be on the boundary. In order to assure the supplier for accepting the contract $(w - c - c_b) > B$ should hold. And, since the maximum value of $B$ is $\frac{w-c}{3} + \frac{2\gamma}{3}$ the optimal $c_b = 2(w - c - \gamma)/3$ and the optimal $T = (a - bw)/2$. ■

The manufacturer will offer this contract with those parameters if his profit with the contract is greater than his profit without contracting.

For $\gamma \leq w - c$ and $\delta \geq \frac{w-c}{3} + \frac{2\gamma}{3}$

$\pi_m = \frac{(a-2T+cb-2wb+b\gamma+bc_b)\left(w^2b^2-2wab-2wb^2c_b+a^2+2abc_b+b^2c_b^2-4Tbc_b\right)}{4b^2(\gamma-\delta)} - \frac{(a-wb)^2(a-2T+cb-2wb+b\delta+bc_b)}{4b^2(\gamma-\delta)} > \frac{(a-bw)^2}{4b}$

Manufacturer's gain with the contract is

$$\begin{aligned}
\pi_{MG} &= \frac{(a-2T+cb-2wb+b\gamma+bc_b)\left(w^2b^2-2wab-2wb^2c_b+a^2+2abc_b+b^2c_b^2-4Tbc_b\right)}{4b^2(\gamma-\delta)} \\
&\quad - \frac{(a-wb)^2(a-2T+cb-2wb+b\delta+bc_b)}{4b^2(\gamma-\delta)} - \frac{(a-bw)^2}{4b} \\
&= \frac{1}{27}\frac{b}{\gamma-\delta}(c-w+\gamma)^3 > 0
\end{aligned}$$

For $\gamma \leq w - c$ and $\delta < \frac{w-c}{3} + \frac{2\gamma}{3}$

$\pi_m = \frac{\left(w^2b^2-2wab-2wb^2c_b+a^2+2abc_b+b^2c_b^2-4Tbc_b\right)}{4b}$

Manufacturer's gain with the contract is

$$\begin{aligned}
\pi_{MG} &= \frac{\left(w^2b^2 - 2wab - 2wb^2(\frac{2(w-c-\gamma)}{3}) + \left(a + b(\frac{2(w-c-\gamma)}{3})\right)^2 - 4(\frac{a-wb}{2})b(\frac{2(w-c-\gamma)}{3})\right)}{4b} \\
&\quad - \frac{(a-bw)^2}{4b} \\
&= \frac{1}{9}b(c-w+\gamma)^2
\end{aligned}$$

Supplier's gain from accepting this contract is

$$\begin{aligned}
\pi_{SG} &= (w-c-B)\left(\frac{\left(a + b(\frac{2(w-c-\gamma)}{3} - w)\right)}{2}\right) - \left(\frac{2(w-c-\gamma)}{3}\right) \\
&\quad \left(\frac{\left(a + b(\frac{2(w-c-\gamma)}{3} - w)\right)}{2} - \left(\frac{a-bw}{2}\right)\right) - (w-c-B)\left(\frac{a-bw}{2}\right) \\
&= \frac{b(c-w+\gamma)(3B+c-w-2\gamma)}{9}
\end{aligned}$$

### 3.3 Coordination with Contracts for Endogenous Wholesale Price

In this part, we analyze the case in which the supplier decides on both her capacity and wholesale price. The other parameters remain the same i.e. the supplier has a marginal production cost $c$, demand is price dependent and there is common knowledge between the parties in the channel.

In a vertically decentralized channel the manufacturer should determine the optimal retail price and corresponding order quantity where as the supplier should specify her production capacity and the wholesale price of the product. Since the actions of the parties in the channel are sequential , in the case of contract proposal manufacturer will set contract parameters considering the supplier's objective function where she decides on her capacity and supplier will announce her capacity and wholesale price for the product considering the manufacturer's objective function where he sets the retail price of the product.

If the manufacturer does not propose any contract the objective functions for the manufacturer and the supplier are as in the following equations respectively.

$$\begin{aligned} \max_{p} \pi_m &= (p-w)(\min(K, a-bp)) \\ \max_{K,w} \pi_s &= (w-c)(\min(K, a-bp) - c(K) \end{aligned} \tag{3.10}$$

**Proposition 6** *For this model, in the case of decentralized channel the supplier sets her capacity equal to optimal order quantity of the manufacturer where there is no capacity constraint, i.e.* $K = (a - bp^*) = (a - bw)/2$ *should always hold.*

**Proof.** Assume on the contrary $K \neq (a - bp^*)$.

If there were no capacity constraint equation 3.10 becomes

$$\pi_m = (p-w)(a-bp)$$

The optimal value of retail price that maximizes the manufacturer's profit is found from the first order conditions

$$\frac{\partial \pi_m}{\partial p} = a - 2pb + wb = 0 \Longrightarrow p^* = \frac{(a+wb)}{2b}$$

For this value of $p$, the optimal order quantity for the manufacturer will be

$$(a - bp^*) = \frac{a - bw}{2}$$

If $K > (a - bw)/2$ then, decreasing $K$ increases supplier's profit.

If $K < (a - bw)/2$ , increasing $w$ increases supplier's profit. ■

**Proposition 7** *The optimal wholesale price value for the supplier $w^*$, is the solution of*

$$\frac{a-2bw+bc}{2}-\frac{\partial c(K)}{\partial w}=0$$

*the optimal capacity is $K^*=(a-bw)/2$ and the manufacturer's optimal retail price ,$p^*=(a+bw^*)/2b$.*

**Proof.** In the proof of proposition 6 we found that manufacturer's optimal order quantity is equal to $(a-bw)/2$ which is equal to the supplier's optimal capacity. Thus, supplier's objective will be

$$\max_w \ \pi_s=(w-c)(\frac{a-bw}{2})-c(K) \tag{3.11}$$

3.11 is concave in terms of $w$ thus, First order conditions can be used to find the optimal wholesale price where

$$\frac{\partial^2\pi_s}{\partial w^2}=-b-\frac{\partial^2 c(K)}{\partial w^2}<0$$

since $b>0$ and $\partial c(K)/\partial w$is assumed to be constant or increasing in $w$.

$$\Longrightarrow \frac{\partial \pi_s}{\partial w}=\frac{a-2bw+bc}{2}-\frac{\partial c(K)}{\partial w}=0$$

Thus, the optimal wholesale price value is the solution of

$$\frac{\partial \pi_s}{\partial w}=\frac{a-2bw+bc}{2}-\frac{\partial c(K)}{\partial w}=0.$$

■

According to proposition 7 for the case $c(K)=BK$ the optimal parameter values and profits of the manufacturer's and supplier's profits will be

$$\begin{aligned} w^* &= \frac{(a+Bb+cb)}{2b}, p^*=\frac{3a+b(c+B)}{4b} \\ K^* &= \frac{a-b(c+B)}{4} \\ \pi_s^* &= \frac{(a-b(c+B))^2}{8b} \text{and } \pi_m^*=\frac{(a-b(c+B))^2}{16b} \end{aligned}$$

Similarly for the case $c(K) = BK^2$ the optimal parameter values and profits of the manufacturer's and supplier's profits will be

$$\begin{aligned}
w^* &= \frac{(a+cb+Bab)}{2b+Bb^2}, p^* = \frac{(3a+cb+2Bab)}{2Bb^2+4b} \\
K^* &= \frac{(a-cb)}{2Bb+4} \\
\pi_s^* &= \frac{(a-cb)^2}{4b\,(Bb+2)} \text{and } \pi_m^* = \frac{(a-cb)^2}{4b(Bb+2)^2}
\end{aligned}$$

### *3.3.1 Coordination with Contracts*

In this section we will try to identify whether the channel can be coordinated with supply chain contracts. We analyze the cases where the manufacturer offers a contract to the supplier. We first investigate the case where manufacturer proposes a wholesale price contract to the supplier, then we consider a linear contract, a revenue sharing contract and a cost sharing contract is proposed by the manufacturer. In every case the sequence of moves are the same. First, manufacturer proposes the contract and related parameters, then supplier decides on her capacity and finally sets his retail price. We assume that the supplier will accept the contract if her profit with the proposed wholesale price contract is greater than or equal to her profit in the decentralized channel, i.e.

$$\begin{aligned}
\text{For } c(K) &= BK \implies \pi_{sres} = \frac{(a-b(c+B))^2}{8b} \\
\text{For } c(K) &= BK^2 \implies \pi_{sres} = \frac{(a-cb)^2}{4b\,(Bb+2)}
\end{aligned}$$

*Linear Contract*

In this type of contract the manufacturer proposes $w$ and additional amount , $t$, to the supplier. According to this, the supplier decides on her capacity and finally manufacturer chooses his retail price and gives his order. With a given linear contract the manufacturer's and the supplier's profits are as follows respectively

$$\begin{aligned}
\pi_m &= (p-w)(a-bp)-t \\
\pi_s &= (w-c)(a-bp)-c(K)+t
\end{aligned}$$

**Proposition 8** *For this model, coordination in the channel can be achieved with linear contract offered by the manufacturer where*

$$w = c + B \quad , t = \frac{(a - b(c + B))^2}{8b}$$

*for the case* $c(K) = BK$ *and for the case* $c(K) = BK^2$ *a linear contract having parameters*

$$w = \frac{aB + c}{1 + Bb}, t = \frac{(a - cb)^2 \left(1 - 2B^2b^2 - 4Bb\right)}{4b\,(Bb + 1)^2\,(Bb + 2)}$$

*can coordinate the channel.*

**Proof.** According to proposition 6 we know that $K = q = a - bp$ should hold in the

optimal solution.

Now let us analyze the case where $c(K) = BK$.For this case manufacturer proposes $w = c + B$ to maximize his profit and also proposes $t = \pi_{sres} = (a - b(c + B))^2/8b$. Then order amount will become

$$q = K = \frac{a - bw}{2} = \frac{a - b(c + B)}{2}$$

which is equal to the order amount in centralized case. Consequently, we may conclude that a linear contract with given parameters will coordinate the channel.

For the case $c(K) = BK^2$ again $q = (a - bw)/2$ is true for the manufacturer. For the supplier

$$\begin{aligned} K &= a - bp \Rightarrow (w - c)K - BK^2 + t \\ &\Longrightarrow \frac{\partial \pi_s}{\partial K} = w - c - 2BK = 0 \\ &\Longrightarrow K = \frac{w - c}{2B}. \end{aligned}$$

We see that when $w = (aB + c)/(1 + Bb)$.

$$\begin{aligned} K^* &= \frac{a - bc}{2(1 + Bb)} = K^*_{cent} \\ t &= \pi_{sres} - (w - c)K - BK^2 \\ &= \frac{(a - cb)^2 \left(1 - 2B^2b^2 - 4Bb\right)}{4b\,(Bb + 1)^2\,(Bb + 2)} \end{aligned}$$

■

*Revenue Sharing Contract*

In this section we assumed that manufacturer offers a revenue sharing contract to the supplier with parameters $(w, \rho)$ where $\rho$ indicates the portion of his revenue that he gives to the supplier and $\rho$ is between [0,1]. The manufacturer's and the supplier's profit with a given revenue sharing contract is as follows

$$\begin{aligned}\pi_m &= ((1-\rho)p - w)\min\{K, (a-bp)\} \\ \pi_s &= (w - c + \rho p)\min\{K, (a-bp)\} - c(K)\end{aligned}$$

**Proposition 9** *A revenue sharing contract can coordinate the channel both for the case $c(K) = BK$ and for the case $c(K) = BK^2$.*

**Proof.** Let us consider the last step in the sequence of moves in other words, where

manufacturer sets his retail price.

If $K < a - bp$ then manufacturer will increase retail price until $p = (a-K)/b$ and $q = K$ will hold. So $K \geq q$ should always hold.

If $K \geq a - bp$ then, manufacturer's objective function becomes

$$\begin{aligned}\pi_m &= (1-\rho)p(a-bp) - w(a-bp) \Longrightarrow \frac{\partial \pi_R}{\partial p} = a - 2pb + wb - a\rho + 2pb\rho = 0 \\ \Longrightarrow & \quad p^* = \frac{(a(1-\rho) + wb)}{2b(1-\rho)} \Longrightarrow q = \frac{(1-\rho)a - bw}{2(1-\rho)} \leq K\end{aligned}$$

Now , for the second step in the sequence of moves, where supplier sets her capacity for a given $w$ and $\rho$. For the case $c(K) = BK$. Supplier will her set capacity from the solution of

$$\max_K \ \pi_s = (w-c)K - BK + \rho p K \tag{3.12}$$

It is seen that 3.12 is linear in terms of $K$ and as $K$ increases the supplier's profit also increase. However, supplier knows that the manufacturer's maximum order amount is $q = ((1-\rho)a - bw)/(2(1-\rho))$. Thus, she set her capacity to $K^* = ((1-\rho)a - bw)/(2(1-\rho))$.

For $w^* = (1-\rho^*)(c+B)$

the optimal capacity for the supplier is equal to the optimal capacity of the integrated channel, $K^*_{cent} = (a - b(c+B))/2$

The optimal value of $\rho$ can be found from supplier's reservation profit $\pi_{sres}$ where

$$\pi_{sres} = \frac{(a-b(c+B))^2}{8b} = (w^* - c - B + \rho p^*)K^*_{cent}$$

For this case with a revenue sharing contract having a wholesale price value $w^* = (1-\rho^*)(c+B)$ the manufacturer's profit becomes

$$\begin{aligned} \pi_m &= ((1-\rho^*)p - (1-\rho^*)(c+B))K - \pi_{sres} \\ &= (p-c-B)K - \pi_{sres} \end{aligned}$$

which is equivalent to the integrated channel's profit , so we may conclude that for the case $c(K) = BK$ a revenue sharing contract can coordinate the channel.

For the case $c(K) = BK^2$ the supplier sets her capacity considering the following objective

$$\begin{aligned} \max_K \ \pi_s &= (w-c)K - BK^2 + \rho(\frac{a-K}{b})K \\ s.t\ K &\leq (a-bp^*) \\ where\ p^* &= \frac{(a(1-\rho)+wb)}{2b(1-\rho)} \end{aligned} \tag{3.13}$$

The function $\pi_s$ in 3.13 is concave in terms of K and the optimal value of K is found from the first order conditions as $K^* = (a\rho + b(w-c))/(2\rho{+}2Bb)$. However, for a given contract parameters the supplier knows that the maximum amount of order that the manufacturer gives is $q = ((1{-}\rho)a{-}bw)/(2(1{-}\rho))$.Thus, she will not invest on more than $q$. Consequently, if $(a\rho + b(w-c))/(2\rho + 2Bb) \leq ((1-\rho)a - bw)/(2(1-\rho))$ holds the optimal K is $K^* = (a\rho + b(w-c))/(2\rho + 2Bb)$

and if $(a\rho + b(w-c))/(2\rho + 2Bb) > ((1-\rho)a - bw)/(2(1-\rho))$ holds the optimal K is $K^* = ((1-\rho)a - bw)/(2(1-\rho))$

A coordinating revenue sharing contract having parameters $w$ and $\rho$ should satisfy the following conditions simultaneously.

$$\begin{aligned} \frac{(a\rho + b(w-c))}{2\rho + 2Bb} &= \frac{(a-cb)}{2(Bb+1)} = K^*_{cent} \\ \text{and } \frac{(1-\rho)a - bw}{2(1-\rho)} &= \frac{(a-cb)}{2(Bb+1)} = K^*_{cent} \end{aligned}$$

$$
\begin{aligned}
\text{For } K^* &= \frac{(a\rho + b(w-c))}{2\rho + 2Bb} = \frac{(a-cb)}{2(Bb+1)} = K^*_{cent} \\
&\Longrightarrow\ w_1^* = \frac{1}{Bb+1}(c + Ba - c\rho - Ba\rho) \\
\text{For } K^* &= \frac{(1-\rho)a - bw}{2(1-\rho)} = \frac{(a-cb)}{2(Bb+1)} = K^*_{cent} \\
&\Longrightarrow\ w_2^* = \frac{1}{Bb+1}(c + Ba - c\rho - Ba\rho)
\end{aligned}
$$

Since $w_1^* = w_2^*$ we can claim that revenue sharing contract can also coordinate the channel when $c(K) = BK^2$. ■

*Cost Sharing Contract*

In this part we introduce a cost sharing contract where the manufacturer shares $(1-\sigma)$ portion of the supplier's capacity investment cost where $0 < \sigma < 1$. The manufacturer's and the supplier's profits will be as follows with a cost sharing contract.

$$
\begin{aligned}
\pi_m &= (p-w)\min\{K, (a-\beta p)\} - (1-\sigma)c(K) \\
\pi_s &= (w-c)\min\{K, (a-bp)\} - \sigma c(K)
\end{aligned}
$$

**Proposition 10** *For this setting cost sharing contract cannot coordinate the channel.*

**Proof.** Let us consider the last step in the sequence of moves in other words, where manufacturer sets his retail price.

If $K < a - bp$ then manufacturer will increase retail price until $p = (a-K)/b$ and $q = K$ will hold. So $K \geq q$ should always hold. And when $K > a - bp$ the manufacturer will decrease his retail price to equalize $q$ and $K$ since he also pays for the excess capacity.

If $K = a - bp$ then, for $c(K) = BK$ the manufacturer's objective function becomes

$$
\begin{aligned}
\pi_m &= (p - w - (1-\sigma)B)(a-bp) \Longrightarrow \frac{\partial \pi_m}{\partial p} = a + Bb - 2pb + wb - B\sigma b = 0 \\
&\Longrightarrow\ p^* = \frac{(a + Bb + wb - B\sigma b)}{2b} \Longrightarrow q^* = \frac{a - b(B + w - B\sigma)}{2} = K
\end{aligned}
$$

and for $c(K) = BK^2$ the manufacturer's objective is

$$
\begin{aligned}
\pi_m &= (p-w)(a-bp) - (1-\sigma)B(a-bp)^2 \\
&\Longrightarrow\ p^* = \frac{(a + wb + 2Bab - 2Ba\sigma b)}{2b + 2Bb^2 - 2B\sigma b^2} \Longrightarrow q^* = \frac{a - wb}{2Bb - 2B\sigma b + 2} = K
\end{aligned}
$$

Now , for the second step in the sequence of moves, where supplier sets her capacity for a given $w$ and $\sigma$. For the case $c(K) = BK$. Supplier will her set capacity from the solution of

$$\max_{K} \ \pi_s = (w - c - \sigma B)K \tag{3.14}$$

It is seen that 3.14 is linear in terms of $K$ and as $K$ increases the supplier's profit also increase. However, supplier knows that the manufacturer's maximum order amount is $q = (a - b(B + w - B\sigma))/2$. Thus, she set her capacity to $K^* = (a - wb - B(1 - \sigma)b)/2$.

For $w^* = c + B\sigma$ the optimal capacity for the supplier is equal to the optimal capacity of the integrated channel, $K^*_{cent} = (a - b(c + B))/2$. However for $w^* = c + B\sigma$ the supplier's profit becomes zero and she will not accept this contract although it can coordinate the channel. The optimal value of $w$ and $\sigma$ is the solution of the following problem

$$\begin{aligned} \max_{w,\sigma} \ \pi_m &= \left( \left( \frac{a + Bb + wb - B\sigma b}{2b} \right) - w - (1 - \sigma)B \right) \left( \frac{a - b(B + w - B\sigma)}{2} \right) &(3.15)\\ s.t\ \pi_s &= (w - c - \sigma B) \left( \frac{a - b(B + w - B\sigma)}{2} \right) \geq \pi_{sres} &(3.16) \end{aligned}$$

The function 3.15 is convex in both $w$ and $\sigma$. Consequently, for the optimal $w, \sigma$ pair 3.16 should bind.

$$\begin{aligned} \pi_s &= (w - c - \sigma B) \left( \frac{a - b(B + w - B\sigma)}{2} \right) = \frac{(a - b(c + B))^2}{8b} \\ &\implies w^* = \frac{(a - Bb + cb + 2B\sigma^* b)}{2b} \end{aligned}$$

with the optimal cost sharing contract the manufacturer's profit is equal to his profit with simple wholesale price contract $\pi_m = (Bb - a + cb)^2 / (16b)$. As a result cost sharing contract cannot coordinate the channel for the case $c(K) = BK$.

For $c(K) = BK^2$ Supplier will her set capacity from the solution of the following problem

$$\max_{K} \ \pi_s = (w - c)K - \sigma BK^2 \tag{3.17}$$

The solution of 3.17 gives us

$$\begin{aligned} K^* &= \frac{w - c}{2B\sigma} \text{ if } \frac{w - c}{2B\sigma} < \frac{a - wb}{2Bb - 2B\sigma b + 2} \\ K^* &= \frac{a - wb}{2Bb - 2B\sigma b + 2} \quad \text{if } \frac{w - c}{2B\sigma} \geq \frac{a - wb}{2Bb - 2B\sigma b + 2} \end{aligned}$$

We see that when $w = (c + Bcb + Ba\sigma - Bc\sigma b)/(Bb+1)$ both $K^*$ values are equal to $K^*_{cent}$. For this $w$ the manufacturer determines the optimal $\sigma$ from the supplier's individual rationality constraint.

$$\begin{aligned}(w-c)K^*_{cent} - \sigma B (K^*_{cent})^2 &= \frac{(a-cb)^2}{4b\,(Bb+2)} \\ \sigma &= \frac{\left(B^2b^2+2Bb+1\right)}{B^2b^2+2Bb} > 1\end{aligned}$$

This $\sigma$ value is greater than 1 thus, a cost sharing contract cannot coordinate the channel in this case. The optimal value of $w$ and $\sigma$ is the solution of the following problem

$$\begin{aligned}\max_{w,\sigma}\ \pi_m &= \left(\frac{(a+wb+2Bab-2Ba\sigma b)}{2b+2Bb^2-2B\sigma b^2} - w\right)K^* - (1-\sigma)(K^*)^2 \\ s.t\ \pi_s &= (w-c)K^* - \sigma B (K^*)^2 \geq \pi_{sres}\end{aligned}$$

where $K^* = (w-c)/(2B\sigma)$ if $(w-c)/(2B\sigma) < (a-wb)/(2Bb-2B\sigma b+2)$ holds and $K^* = (a-wb)/(2Bb-2B\sigma b+2)$ if $(w-c)/(2B\sigma) \geq (a-wb)/(2Bb-2B\sigma b+2)$ holds. ■

Chapter 4

# STOCHASTIC DEMAND MODELS

In this chapter we analyze a single manufacturer and a single supplier model in a single period setting as in the previous chapter. However, now we modify the model by assuming that the manufacturer faces a stochastic demand having continuous distribution $F(x)$ with density function $f(x)$. In this model, we assume that the supplier produces the product at a unit cost $c$ and then sells the product to the manufacturer with a wholesale price, $w$. Then the supplied product is worked upon by the manufacturer and a value-added finished product is sold to the market with a constant price $p$. In this chapter, we assume that the market price $p$ is fixed, however, we relax that assumption in the next chapter. In order to make the production, the supplier needs to build her capacity some time before the actual demand will be realized, since building a capacity has a lead time and capacity cannot be built on a incremental basis. The capacity investment cost function is denoted by $c(K)$.

When the demand is realized, two cases are possible; the built capacity can be greater than the demand so the capacity cannot be fully utilized ,or the demand can be greater than the invested capacity thus there will be some unmet demand. We assume there will be no backorder and the unmet demand is lost in the case of insufficient capacity.

In this chapter at first we analyze the base model where there is common knowledge between the parties and then, we assume that the supplier's capacity investment cost is her private information. We also perform a numerical study to analyze the effect of each parameter to the solution and to compare the efficiencies of the contracts in the asymmetric information case.

## *4.1 Symmetric Information*

In this section we analyze the base model where all parameters in the model is known by both the manufacturer and the supplier. At first we look into the centralized channel. Later, we consider decentralized channel in two cases. At first case we assumed exogenous wholesale price and determine if coordination can be achieved with a bonus contract offered by the manufacturer. In the second case we assumed endogenous wholesale price and determine if coordination can be achieved with linear, revenue sharing and cost sharing contracts.

### 4.1.1 Centralized Channel

We first consider the centralized supply chain in which all decisions are made to maximize the integrated channel profit. For any given capacity $K$ the integrated channel profit can be written as

$$\begin{aligned} \max_K \; E[\Pi(K)] &= (p-c)\min\{K, D\} - c(K) \\ &\Rightarrow \; E[\Pi(K)] = (p-c)E[D] - E[(p-c)(D-K)^+] - E[c(K)] \\ &= (p-c)E[D] - (p-c)\int_K^a (x-K)f(x)dx - c(K) \end{aligned} \tag{4.1}$$

First order conditions is used to find the optimal value of $K$ because 4.1 is strictly concave in $K$ due to the fact that

$$\frac{\partial^2 E[\Pi(K)]}{\partial K^2} = -(p-c)f(K) - c''(K) < 0$$

where $F(K)$ is increasing in $K$ and $c''(K)$ is assumed to be less than or equal to 0.

$$\frac{\partial E[\Pi(K)]}{\partial K} = (p-c)(1-F(K)) - c'(K) = 0$$

For $c(K) = BK$ then $F(K^*) = (p-c-B)/(p-c) \Rightarrow K^c = F^{-1}(\frac{p-c-B}{p-c})$

For $c(K) = BK^2$ then $K^c$ is the solution of $(p-c)(1-F(K)) - 2BK = 0$.

It is known that the channel efficiency is maximum for $K^c$. However in the decentralized case the supplier solves her own newsvendor problem.

### 4.1.2 Decentralized Case

It is known that the channel efficiency is maximum for $K^c$. However in the decentralized case, the supplier solves her own newsvendor problem. In this model, we assume that the manufacturer offers the supplier a unit wholesale price when the wholesale price is negotiable and then the supplier decides on her capacity level according to the wholesale price. The manufacturer's problem in this case is:

$$\max_K E[\pi_s(K)] = (w - c)E[\min\{K, D\}] - c(K)$$

and the supplier's problem is:

$$\max_K E[\pi_S(K)] = (w - c)E[\min\{K, D\}] - c(K)$$

The optimal capacity for the supplier is found to be $K^d = F^{-1}(\frac{w-c-B}{w-c})$ for the case $c(K) = BK$ and when the capacity cost function is in the form $c(K) = BK^2$, the solution of the equation $(w - c)(1 - F(K)) - 2BK = 0$ gives the optimal capacity level for the supplier. It is seen that for both cases $K^d < K^c$ since $p > w$ should hold and $F$ is an increasing function. Thus, it is optimal for the supplier to build less capacity than what is optimal for the centralized channel since the supplier's marginal profit is less than the marginal profit of the integrated channel and the manufacturer takes no risk for the capacity investment. As a result, maximum channel efficiency cannot be achieved with the simple wholesale price contract. In the next section we will try to find the contracts that can coordinate the channel or increase the channel efficiency. Channel efficiency is defined as the ratio of the total channel profit to the profit of the integrated channel.

$$Eff = \frac{E[\pi_S] + E[\pi_M]}{E[\Pi]}$$

### *4.1.3 Coordination with Contracts*

In this section we are trying to determine whether the parties in the channel will act as they are integrated when a contract is offered by the manufacturer. At first we assumed fixed wholesale price and we try to coordinate the channel with bonus contract. Then for the adjustable wholesale price settings we try to coordinate or increase the channel efficiency with linear contract, revenue sharing contract and cost sharing contract. In all cases the sequence of moves is identical. First, the manufacturer offers the contract parameters. We assume that the supplier's reservation profit, $\pi_{sres}$, is her profit in the decentralized channel for exogenous wholesale price and for adjustable wholesale price cases we assumed that her reservation profit is equal to her profit with the simple wholesale price contract offered by the manufacturer. Thus, she will accept the contract if $\pi_{sres} \leq \pi_{sc}$ where $\pi_{sc}$ is the supplier's profit with the proposed contract.

After the contract proposal the supplier invest on her capacity, then demand is realized and the manufacturer places the amount of order equals to the demand finally the supplier fulfills the order. As stated before if demand does not equal to the capacity, lost sales or excess capacity might be observed.

*Bonus Contract with Exogenous wholesale price*

In this part we introduce a bonus contract where the manufacturer rewards the supplier whenever she supplies for a demand above the target level $T$, by paying excess supply bonus $c_b$ on this quantity. In this section we assumed fixed wholesale price.

Let $S(K)$ be the expected sales for a given capacity.

$$\begin{aligned} S(K) &= E[\min\{D,K\}] = K(1-F(K)) + \int_0^K xf(x)dx \\ &= K - \int_0^K F(x)dx \end{aligned}$$

Similarly, let $S_b(K)$ the amount of product that the supplier provides above the target level, $T$.

$$S_b(K) = (S(K) - T)^+$$

**Proposition 11** *For the base model a bonus contract can coordinate the channel with parameters $c_b^* = p - w$ and $T = T^*$ where $T$ is the solution of*

$$T - \int_0^T F(x)dx = \frac{(p-c)S(K_c) - c(K_c) - \pi_{sres}}{p-w} \tag{4.2}$$

**Proof.** For a given $c_b$ and $T$ the supplier sets her capacity. Depending on the value of $T$ two cases are possible

Case 1 $K \leq T$ For this case the supplier sets her capacity less than the target level then $S_b(K) = 0$ and her problem is

$$\max_K E[\pi_s] = (w-c)S(K) - c(K)$$

and consequently this case is the same as the decentralized case.

Case 2 $K \geq T$ For this case the manufacturer's problem becomes

$$\begin{aligned} \max_{c_b,T} E[\pi_m] &= (p-w)S(K) - c_b S_b(K) \\ s.t\ E[\pi_s] &= (w-c)S(K) - c(K) + c_b S_b(K) \geq \pi_{sres} \end{aligned} \tag{4.3}$$

where $S_b(K) = S(K) - S(T)$

From 4.3 it is seen that for a positive $c_b$ as $T$ increases the manufacturer's objective increases thus, the supplier's individual rationality constraint should hold as equality. Then, $c_b S_b(K) = \pi_{sres} - (w-c)S(K) - c(K)$ and the manufacturer's problem becomes

$$\max_{c_b} E[\pi_m] = (p-c)S(K) - c(K) - \pi_{sres} \tag{4.4}$$

It is seen that 4.4 is equivalent to the integrated channel problem and we know that for the given bonus contract the supplier solves her own problem to find the optimal value of $K$ solving the following problem

$$\max_{K} E[\pi_s] = (w-c)S(K) - c(K) + c_b(S(K) - S(T)) \tag{4.5}$$

$\partial^2 E[\pi_s]/\partial K^2 < 0$ since $F$ is an increasing function, $w > c$ and $c_b > 0$ which implies 4.5 is concave in terms of $K$. Thus,the optimal amount of capacity invested can be found from the first order conditions

For $c(K) = BK$, the optimal $K$ is found as $K = F^{-1}(\frac{w-c-B+c_b}{w-c})$ . For a coordinating bonus contract

$$\begin{aligned} F^{-1}\left(\frac{w-c-B+c_b}{w-c+c_b}\right) &= F^{-1}\left(\frac{p-c-B}{p-c}\right) \\ &\implies c_b^* = p-w \end{aligned}$$

The optimal $T$ value can be found from the supplier's individual rationality constraint.

$$\begin{aligned} E[\pi_s] &= (w-c)S(K_c) - BK_c + c_b(S(K_c) - S(T)) = \pi_{sres} \\ &\implies S(T) = \frac{(p-c)S(K_c) - BK_c - \pi_{sres}}{p-w} \end{aligned}$$

And optimal value of $T$ is the solution of Problem 4.2

For $c(K) = BK^2$, the optimal value of $K$ is the solution of $(w-c+c_b)(1-F(K))-2BK$. Similarly for a coordinating bonus contract

$$\begin{aligned} (p-c)(1-F(K)) - 2BK &= (w-c+c_b)(1-F(K)) - 2BK \\ &\implies c_b^* = p-w \end{aligned}$$

The optimal $T$ value can be found from the supplier's individual rationality constraint.

$$\begin{aligned} E[\pi_s] &= (w-c)S(K_c) - BK_c^2 + c_b(S(K_c) - S(T)) = \pi_{sres} \\ \Longrightarrow & \quad S(T) = \frac{(p-c)S(K_c) - BK_c^2 - \pi_{sres}}{p-w} \end{aligned}$$

And optimal value of $T$ is the solution of Problem 4.2 . ■

*Linear Contract*

In this part we assumed that the manufacturer proposes a linear contract to the supplier having parameters $(w, t)$. The manufacturer offers wholesale price,$w$ and he gives the supplier a fixed amount of payment $t$.

**Proposition 12** *For the base model, a linear contract having parameters*

$$w^* = p \text{ and } t^* = (p-c)\left(E[D] - \int_{K^c}^{a} (x - K^c) f(x) dx\right) - BK^c - \pi_{Sres}$$

*coordinates the channel when the supplier has a linear capacity cost function, and when she has a quadratic capacity cost function coordinating linear contract has parameters*

$$w^* = p \text{ and } t^* = (p-c)\left(E[D] - \int_{K^c}^{a} (x - K^c) f(x) dx\right) - B(K^c)^2 - \pi_{Sres}$$

**Proof.** The manufacturer's problem is as follows

$$\begin{aligned} \max_{w,t} E[\pi_m] &= (p-w)E[\min\{D, K\}] - t \\ s.t\ E[\pi_m] &= (w-c)E[\min\{D, K\}] - c(K) + t \geq \pi_{Sres} \end{aligned} \quad (4.6)$$

It is obvious that 4.6 should hold as equality since decreasing $t$ improves the manufacturer's objective. Consequently, for $c(K) = BK$ the manufacturer's problem becomes

$$\begin{aligned} \max_{w} E[\pi_m] &= (p-c)E[\min\{D, K\}] - BK - \pi_{Sres} \\ where\ K &= F^{-1}\left(\frac{w-c-B}{w-c}\right) \end{aligned}$$

the manufacturer's new problem is the same as the integrated channel problem. Thus, the optimal value of $w$ will make $K = F^{-1}(\frac{w-c-B}{w-c}) = K^c$ for $c(K) = BK$.From here the $w^*$ is found as follows

$$\begin{aligned} F^{-1}(\frac{w^* - c - B}{w^* - c}) &= F^{-1}(\frac{p - c - B}{p - c}) \\ &\Rightarrow w^* = p \end{aligned}$$

Here, the manufacturer sets the wholesale price equal to the retail price consequently, he will take $t$ from the supplier instead of giving her. The optimal value of $t$ is found as

$$\begin{aligned} t^* &= (p - c)\left(E[D] - \int_{K^c}^{a} (x - K^c) f(x) dx\right) - BK^c - \pi_{Sres} \\ where\ K^c &= F^{-1}(\frac{p - c - B}{p - c}) \end{aligned}$$

Similarly, for $c(K) = BK^2$

$$\begin{aligned} \max_w E[\pi_m] &= (p - c)E[\min\{D, K\}] - BK^2 - \pi_{Sres} \\ & \quad where\ K \text{ is the solution of} \\ & \quad (w - c)(1 - F(K)) - 2BK = 0 \end{aligned}$$

the manufacturer's new problem is the same as the integrated channel problem as in the previous case. Thus $w^* = p$, and the manufacturer takes $t$ from the supplier.

$$\begin{aligned} t^* &= (p - c)\left(E[D] - \int_{K^c}^{a} (x - K^c) f(x) dx\right) - B(K^c)^2 - \pi_{Sres} \\ & \quad where\ K^c \text{ is the solution of } (p - c)(1 - F(K)) - 2BK = 0 \end{aligned}$$

■

With the given coordinating linear contracts expected profit of the manufacturer and the supplier will be as follows respectively

$$\begin{aligned} E[\pi_S] &= \pi_{Sres} \\ E[\pi_m] &= t^* \end{aligned}$$

*Revenue Sharing Contract*

In this contract the manufacturer sets the wholesale price and offers $\rho$ portion of his revenue to the supplier where $0 < \rho < 1$.

the manufacturer's problem with the contract is

$$\begin{aligned}\max_{w,\rho} E[\pi_m] &= (1-\rho)pE[\min\{D,K\}] - wE[\min\{D,K\}] \\ s.t\ E[\pi_s] &= (w-c)E[\min\{D,K\}] - c(K) + \rho pE[\min\{D,K\}] \geq \pi_{Sres}\end{aligned} \tag{4.7}$$

**Proposition 13** *Revenue sharing contract with parameters that satisfies the conditions $\rho^* = (p - w^*)/p$ coordinates the channel for the base model. However, manufacturer will have zero profit with this contract.*

**Proof.** In the manufacturer's problem the constraint should hold as equality since the manufacturer's objective improves as $\rho$ decreases. Thus $\rho pE[\min\{D,K\}] = \pi_{Sres} - (w - c)E[\min\{D,K\}] + c(K)$ . If we substitute this into the manufacturer's objective and make necessary simplifications his problem will become

$$\max_{w,\rho} E[\pi_m] = (p-c)E[\min\{D,K\}] - BK - \pi_{Sres} \tag{4.8}$$

It is seen that 4.8 is equivalent to the integrated channel problem and we know that for the given revenue sharing contract the optimal value of $K$ is the solution of the following problem

$$\max_{K} E[\pi_s] = (w-c+\rho p)E[\min\{D,K\}] - c(K) \tag{4.9}$$

$$\frac{\partial E[\pi_s]^2}{\partial K^2} = -(w-c+\rho p)F'(K) - c''(K) < 0$$

since $F$ is an increasing function, $w > c$ and $\rho p > 0$ which implies 4.9 is concave in terms of $K$. Thus,the optimal amount of capacity invested can be found from the First order conditions

For $c(K) = BK$, the optimal $K$ is found as $K = F^{-1}(\frac{w-c-B+\rho p}{w-c+\rho p})$ . For a coordinating revenue sharing contract

$$\begin{aligned}F^{-1}\left(\frac{w-c-B+\rho p}{w-c+\rho p}\right) &= F^{-1}\left(\frac{p-c-B}{p-c}\right) \\ &\implies \rho^* = \frac{p-w^*}{p}\end{aligned}$$

For $c(K) = BK^2$, the optimal value of $K$ is the solution of $(w-c+\rho p)(1-F(K))-2BK$. Similarly for a coordinating bonus contract

$$\begin{aligned}(p-c)(1-F(K)) - 2BK &= (w-c+\rho p)(1-F(K)) - 2BK \\ &\implies \rho^* = \frac{p-w^*}{p}\end{aligned}$$

The coordinating revenue sharing contract parameters for both the case of linear and quadratic capacity investment cost will satisfy $\rho^* = (p - w^*)/p$. However, the coordinating revenue sharing contract with parameters $\rho$ and $w$ which satisfy $(p - w^*)/p$, the manufacturer's profit will become zero and supplier gets the whole profit. Thus, the manufacturer will not offer the coordinating revenue sharing to the supplier. ■

In the proof of the proposition 13 we show that for a given contract parameters $w$ and $\delta$ the optimal capacity for the supplier is $K^* = F^{-1}\left(\frac{w-c-B+\rho p}{w-c+\rho p}\right)$ for the case $c(K) = BK$.

Let the amount of expected sales be

$$S(K^*) = K^* - \int_0^{K^*} F(x)dx$$

. Then, the manufacturer's problem becomes

$$\begin{aligned} \max_{w,\rho} E[\pi_m] &= ((1-\rho)p - w)K^* - \int_0^{K^*} F(x)dx \qquad (4.10) \\ s.t\ E[\pi_s] &= (w - c + \rho p)\left(K^* - \int_0^{K^*} F(x)dx\right) - c(K^*) \geq \pi_{Sres} \end{aligned}$$

The optimal revenue sharing contract for the manufacturer is the solution of 4.10 where $K = F^{-1}\left(\frac{w-c-B+\rho p}{w-c+\rho p}\right)$ for the case $c(K) = BK$ and for the case $c(K) = BK^2$ $K$ is the solution of $(w - c + \rho p)(1 - F(K)) - 2BK = 0$.

*Cost Sharing Contract*

In this part we introduce a cost sharing contract where the manufacturer shares $(1 - \sigma)$ portion of the supplier's capacity investment cost where $0 < \sigma < 1$. In this case the manufacturer's problem is

$$\begin{aligned} \max_{w,\sigma} E[\pi_m] &= (p - w)E[\min\{D, K\}] - (1 - \sigma)c(K) \\ s.t\ E[\pi_m] &= (w - c)E[\min\{D, K\}] - \sigma c(K) \geq \pi_{Sres} \end{aligned}$$

**Proposition 14** *Cost sharing contract having parameters*

$$w^* = c + \frac{\pi_{sres}(p - c)}{(p - c)E[\min\{D, K\}] - BK} \quad and \sigma^* = \frac{\pi_{sres}}{(p - c)E[\min\{D, K\}] - BK}$$

*coordinates the channel for the case $c(K) = BK$ and for the case $c(K) = BK^2$ a cost sharing contract having parameters that satisfy $(p - c)(1 - F(K^c)) - 2BK^c = (w^* - c)(1 - F(K^c)) - 2\sigma^* BK^c$ coordinates the channel.*

**Proof.** In the manufacturer's problem the supplier's individual rationality constraint should hold as equality since the manufacturer's objective improves as $\sigma$ decreases. Thus $\sigma c(K) = (w-c)E[\min\{D, K\}] - \pi_{Sres}$. If we substitute this into the manufacturer's objective and make necessary simplifications his problem will become

$$\max_{\sigma} E[\pi_m] = (p-c)E[\min\{D, K\}] - c(K) - \pi_{Sres}$$

The maximization problem is equivalent to the integrated channel problem. And it is known that for the given cost sharing contract the supplier solves her own problem to find the optimal value of $K$

$$\max_{K} E[\pi_s] = (w-c)E[\min\{D, K\}] - \sigma c(K) \tag{4.11}$$

$$\frac{\partial E[\pi_s]^2}{\partial K^2} = -(w-c)F'(K) - \sigma c''(K) < 0$$

since $F$ is an increasing function and $w > c$ which imply 4.11 is concave in terms of $K$. Thus,the optimal amount of capacity invested can be found from the First order conditions

For $c(K) = BK$, the optimal $K$ is found as $K = F^{-1}(\frac{w-c-\sigma B}{w-c})$ . For a coordinating cost sharing contract

$$\begin{aligned} \frac{w-c-\sigma B}{w-c} &= \frac{p-c-B}{p-c} \\ \sigma^* &= \frac{w^*-c}{p-c} \end{aligned}$$

We also know that

$$\begin{aligned} \sigma^* &= \frac{(w^*-c)E[\min\{D, K^c\}] - \pi_{Sres}}{BK^c} = \frac{w^*-c}{p-c} \Longrightarrow \\ w^* &= c + \frac{\pi_{sres}(p-c)}{(p-c)E[\min\{D, K\}] - BK} \\ and\ \ \sigma^* &= \frac{\pi_{sres}}{(p-c)E[\min\{D, K\}] - BK} \end{aligned}$$

.

For $c(K) = BK^2$, the optimal value of $K$ is the solution of $(w-c)(1-F(K)) - 2\sigma BK$. Similarly for a coordinating bonus contract

$$(p-c)(1-F(K^c)) - 2BK^c = (w^*-c)(1-F(K^c)) - 2\sigma^* BK^c \tag{4.12}$$

Optimal contract parameters in the quadratic capacity cost function case comes from simultaneous solution of 5.12 and $\sigma^* c(K_c) = (w-c)E[\min\{D, K_c\}] - \pi_{sres}$. ∎

For coordinating cost sharing contract the manufacturer's and the supplier's profits will be as follows, respectively.

$$\begin{aligned} \pi_m &= \Pi_{cent} - \pi_{sres} \\ \pi_s &= \pi_{sres} \end{aligned}$$

## *4.2 Asymmetric Information*

In the previous models, we study the symmetric capacity cost information scenario in which the manufacturer knows exact value of the supplier's capacity investment cost. In this chapter, we address the case in which capacity cost is the supplier's private information. We assume that the supplier can be of two types with respect to capacity cost: a low-capacity-cost supplier with capacity cost coefficient, $B_L$, or a high-capacity-cost supplier with capacity cost coefficient, $B_H$ where $B_H > B_L$. the manufacturer believes that the supplier is a high-capacity-cost supplier with probability $\phi$ or low-capacity-cost supplier with probability $(1 - \phi)$. In this setting the manufacturer proposes a menu of contracts instead of a single contract to screen the supplier's capacity investment cost. A menu of contract consists of different contract schemes such that supplier type $i$ chooses the contract designed for her.

As stated before screening is a contracting problem under hidden information where uninformed party offers the contract to the informed party. Screening is first formally analyzed within the context of optimal income taxation by Mirrlees in 1971. To illustrate a typical screening application let us consider a setting with an agent (she) and a principle (he). Assume agent knows whether she is high skilled $\theta_H$ or low skilled $\theta_L$, where $\theta_H > \theta_L$. However, principle only knows that she can be either high or low skilled employee with probability $p_H$ and $p_L$ relatively. Also, assume that the output of the agent is unobservable. Suppose the principle offers offers a total payment of $t(l)$ in exchange of $(1-l)$ units of work and his utility function will be $U[\alpha\theta_i(1 - l) - t_i(l)]$ where $\alpha > 1$, and the agent's utility function will be $u(\theta_i l - t_i(l))$. If the agent does not work she will get $u(\theta_i)$. Although the principle offers a simple contract compared to the real-life cases it is quite difficult to find the optimal contracts in the set of all nonlinear functions $t(l)$. Fortunately, the *revelation principle* offers a key simplification. Revelation principle suggests that the principle needs to determine a menu of two "point contracts": $(t_L, l_L)$ and $(t_H, l_H)$, where by convention, $(t_j, l_j)$ is the contract chosen by type $j$. The reason why the principle does not need to specify a full (nonlinear) contract $t(l)$ is that each type of agent would pick only one point in the full schedule $t(l)$ anyway. So, she may as well pick that point directly. However each point has to be *incentive compatible,* namely type $\theta_H$ should prefer contract $(t_H, l_H)$ over $(t_L, l_L)$ and type $\theta_L$ should prefer contract $(t_L, l_L)$ over $(t_H, l_H)$.

Thus, the principle finds the optimal menu of contracts from the following problem

$$\max_{l_j, t_j} \; p_L U[\alpha\theta_L(1-l_L) - t_L] + p_H U[\alpha\theta_H(1-l_H) - t_H]$$

$$s.t$$

$$\begin{aligned} u(\theta_L l_L - t_L) &\geq u(\theta_L) \\ u(\theta_H l_H - t_H) &\geq u(\theta_H) \\ u(\theta_H l_H - t_H) &\geq u(\theta_L l_L - t_L) \\ u(\theta_L l_L - t_L) &\geq u(\theta_H l_H - t_H) \end{aligned}$$

The first two constraints are the individual rationality constraints of the agents and the last two constraints are the incentive compatibility constraints.

Generally in the optimal solutions of the screening problems, information rent is given to the type that mimics the other type(s), and there is distortion for the pretended types. Thus, the main trade-off in the contracting problems with hidden information is giving information rent and allocating efficiency. [6]

For computational simplicity, in this chapter, we assume that the demand is uniformly distributed between 0 and $a$ and the supplier has a linear capacity cost function.

When we consider the centralized supply chain model for this setting, there will be no information asymmetry between the parties and the maximum expected channel profit will be as follows depending on the supplier type $j \in \{L, H\}$.

$$\begin{aligned} E[\Pi] &= \left( (p-c)(K_j^* - \frac{(K_j^*)^2}{2a}) - B_j K_j^* \right) \\ where\ K_j^* &= \frac{(p-c-B_j)a}{p-c} \end{aligned}$$

In this section we first consider the case of exogenous wholesale price and the manufacturer offers a menu of bonus contract to screen the supplier's capacity investment cost.

Then ,we consider the case where wholesale price is adjustable and the manufacturer tries to screen the supplier's capacity investment cost with linear and cost sharing contracts.

### *4.2.1 Bonus Contract with Exogenous Wholesale Price*

In this section we are trying to determine whether the coordination can be achieved with a bonus contract having parameters $(c_b, T)$ offered by the manufacturer where wholesale price is exogenous. Similar to the previous section let $S(K)$ be the expected sales for a given capacity.

$$\begin{aligned} S(K) &= E[\min\{D, K\}] = K(1 - F(K)) + \int_0^K x f(x) dx \\ &= K - \int_0^K F(x) dx \end{aligned}$$

and let $S_b(K)$ the amount of product that the supplier provides above the target level, $T$.

$$S_b(K) = (S(K) - T)^+$$

For uniform demand distribution where $K > T$ holds

$$S(K) = (K - \frac{K^2}{2a}) \text{ and } S_b(K) = (K - \frac{K^2}{2a}) - (T - \frac{T^2}{2a})$$

the supplier's expected profit in terms of the contract parameters in the case of contract acceptance is as follows;

$$\begin{aligned} \pi_s(c_b, T, B) &= (w - c + c_b)(K - \frac{K^2}{2a}) - c_b(T - \frac{T^2}{2a}) - BK \\ where\ K &= \frac{a\,(w + c_b - B - c)}{w + c_b - c} \end{aligned}$$

As we stated before the manufacturer does not know the supplier's capacity investment cost and he tries to maximize his profit by designing a menu of contracts. According to revelation principle he can restrict his attention to the class of truth telling mechanisms to find the optimal menu of contracts. Also, we normalized the reservation profits of both type of suppliers to zero for simplicity. As a result the manufacturer objective becomes as follows.

$$(P) \quad \max\ \phi[(p-w-c_{bH})S(K)_H + c_{bH}S_b(T_H)] + (1-\phi)[(p-w-c_{bL})S(K)_L + c_{bL}S_b(T_L))]$$

$$\begin{aligned}
s.t\ (w-c+c_{bH})S(K)_H - c_{bH}S_b(T_H) - B_HK_H &\geq (w-c+c_{bL})S(K)_{HL} - c_{bL}S_b(T_L) - B_HK_{HL} \\
(w-c+c_{bL})S(K)_L - c_{bL}S_b(T_L) - B_LK_L &\geq (w-c+c_{bH})S(K)_{LH} - c_{bH}S_b(T_H) - B_LK_{LH} \\
(w-c+c_{bH})S(K)_H - c_{bH}S_b(T_H) - B_HK_H &\geq \pi_{sres} \\
(w-c+c_{bL})S(K)_L - c_{bL}S_b(T_L) - B_LK_L &\geq \pi_{sres}
\end{aligned}$$

where

$$\begin{aligned}
K_H &= \frac{a\,(w+c_{bH}-B_H-c)}{w+c_{bH}-c}\ ,\ K_L = \frac{a\,(w+c_{bL}-B_L-c)}{w+c_{bL}-c} \\
K_{HL} &= \frac{a\,(w+c_{bL}-B_H-c)}{w+c_{bL}-c}\ ,\ K_{LH} = \frac{a\,(w+c_{bH}-B_L-c)}{w+c_{bH}-c} \\
S(K)_H &= \frac{1}{2}\frac{a\left((w-c+c_{bH})^2 - B_H^2\right)}{(w-c+c_{bH})^2},\ S(K)_L = \frac{1}{2}\frac{a\left((w-c+c_{bL})^2 - B_L^2\right)}{(w-c+c_{bL})^2} \\
S(K)_{HL} &= \frac{1}{2}\frac{a\left((w-c+c_{bL})^2 - B_H^2\right)}{(w-c+c_{bL})^2}\ ,\ S(K)_{LH} = \frac{1}{2}\frac{a\left((w-c+c_{bH})^2 - B_L^2\right)}{(w-c+c_{bH})^2}
\end{aligned}$$

In problem $P$ the first two constraints are incentive compatibility constraints *(IC)* and they are added to the manufacturer's problem to assure that each supplier type choose the contract designed for her and the last two constraints are individual rationality constraints *(IR)* which assure the supplier's contract acceptance.

At first let us consider the case in which the manufacturer asks the supplier to report her type i.e. the first best case. According to the supplier's report the manufacturer will solve the following problem

$$\begin{aligned}
\max_{c_b,T} E[\pi_m] &= (p-w)(K-\frac{K^2}{2a}) - c_b\left((K-\frac{K^2}{2a}) - (T-\frac{T^2}{2a})\right) \qquad (4.13) \\
s.t\ E[\pi_s] &= (w-c)(K-\frac{K^2}{2a}) - B_iK + c_b\left((K-\frac{K^2}{2a}) - (T-\frac{T^2}{2a})\right) \geq \pi_{sres} \\
where\ K &= \frac{a\,(w+c_b-B_i-c)}{w+c_b-c} \qquad i = L, H
\end{aligned}$$

We solved the manufacturer's problem 4.13 in the symmetric information case and we found that the optimal bonus contract for high cost supplier has parameters $c_{bH}^* = p - w$

and

$$T_H^* = \left( a - \sqrt{\frac{a\left(p(2B_H - w) - (B_H + c)^2 + c(p+w)\right) - 2\pi_{sres}(p-c)}{(p-c)\,(p-w)}} \right)$$

and for low cost supplier optimal bonus contract has parameters $c_{bL}^* = p - w$ and

$$T_L^* = \left( a - \sqrt{\frac{a\left(p(2B_L - w) - (B_L + c)^2 + c(p+w)\right) - 2\pi_{sres}(p-c)}{(p-c)\,(p-w)}} \right)$$

**Proposition 15** *The function $\pi_s(c_b, T, B)$ has decreasing differences in $B$, that is $\pi_s(c_{b1}, T, B) - \pi_s(c_{b2}, T, B)$ is strictly decreasing in $B$ for all $c_{b1} > c_{b2} > (c - w)$. As a result, a low cost supplier values an increase in $c_b$ more than a high cost supplier.*

**Proof.** $\pi_s(c_{b1}, T, B) - \pi_s(c_{b2}, T, B) = \frac{a(c_{b1}-c_{b2})\left((w-c)(c_{b1}+c_{b2})+c_{b1}c_{b2}+(w-c)^2-B^2\right)}{2(w-c+c_{b1})(w-c+c_{b2})}$
$\frac{\partial(\pi_s(c_{b1},T,B)-\pi_s(c_{b2},T,B))}{\partial B} = -Ba\frac{c_{b1}-c_{b2}}{(w-c+c_{b1})(w-c+c_{b2})} < 0$ ■

**Corollary 1** If the manufacturer asks the supplier for her capacity cost report, a low-capacity-cost supplier has an incentive to report high capacity cost.

**Proof.** We see that $T_L > T_H$ since $B_L < B_H$ and the function $\pi_s(c_b, T, B)$ is decreasing in $T$ since $\frac{\partial \pi_s(c_b,T,B)}{\partial T} = \frac{1}{a}c_b\,(T-a) < 0$ where $a > T$. Thus, it is beneficial for the low capacity cost supplier to report high capacity cost. ■

Corollary 1 proves that the first best solution is not optimal for the manufacturer since each supplier type claim to have high capacity cost. Thus, the manufacturer should consider the second best case.

**Proposition 16** *The following problem is the same as problem $(P)$*

$$(RP) \quad max\ \phi[(p-w-c_{bH})S(K)_H + c_{bH}S_b(T_H)] + (1-\phi)[(p-w-c_{bL})S(K)_L + c_{bL}S_b(T_L))]$$

$$s.t$$

$$\begin{aligned}
(w-c+c_{bL})S(K)_L - c_{bL}S_b(T_L) - B_LK_L &= (w-c+c_{bH})S(K)_{LH} - c_{bH}S_b(T_H) - B_LK_{LH} \\
(w-c+c_{bH})S(K)_H - c_{bH}S_b(T_H) - B_HK_H &= \pi_{sres} \\
c_{bL} &\geq c_{bH}
\end{aligned}$$

**Proof.** In problem $P$ IR$_L$ is redundant due to the fact that $(w-c+c_{bL})S(K)_L - c_{bL}S_b(T_L) - B_LK_L \geq (w-c+c_{bH})S(K)_{LH} - c_{bH}S_b(T_H) - B_LK_{LH}$

$> (w-c+c_{bH})S(K)_H - c_{bH}S_b(T_H) - B_H K_H \geq \pi_{sres}$. The left hand side of the inequality comes from the IC$_L$ and right hand side comes from claim 15. As a result, only the high-capacity-cost supplier's IR constraint should be considered. Additionally IR$_H$ should hold as equality since increasing both $T_H$ and $T_L$ without violating the IR$_H$ will improve the the manufacturer's objective.

Also, we can claim that IC$_L$ must be binding since the manufacturer's objective can be improved by increasing $T_L$ a small amount which will preserve IC$_L$, relax IC$_H$ and does not effect IR$_H$. Finally we may argue that IC$_H$ will be redundant. We proved that IC$_L$ and IR$_H$ constrains should be binding and from these constraints we found the values of $S_b(T_H)$ and $S_b(T_L)$ such as

$$S_b(T_H) = \frac{(w-c+c_{bH})S(K)_H - B_H K_H - \pi_{sres}}{c_{bH}}$$

and

$$S_b(T_L) = \frac{(w-c+c_{bL})S(K)_L - B_L K_L - (w-c+c_{bH})S(K)_{LH} + B_L K_{LH} + (w-c+c_{bH})S(K)_H - B_H K_H - \pi_{sres}}{c_{bL}}.$$

Substituting these values in IC$_H$ and making the necessary simplifications leads us to the following inequality $u(w_L, B_L) - u(w_H, B_L) > u(w_H, B_H) - u(w_L, B_H)$ which is proven to be true in claim 15 ■

According to the previous proposition the manufacturer's problem becomes

$$\begin{aligned}
&\max \phi[(p-c)S(K)_H - B_H K_H] + (1-\phi)[(p-c)S(K)_L - B_L K_L - (w-c+c_{bH})S(K)_{LH} \\
&+B_L K_{LH} + (w-c+c_{bH})S(K)_H - B_H K_H)] \\
&s.t \\
c_{bL} \quad &\geq \quad c_{bH}
\end{aligned}$$

The function is separable in $c_{bH}$ &$c_{bL}$, so we separate the function into two sections; Section H and Section L where

Section H: $H = \phi[(p-c)S(K)_H - B_H K_H] + (1-\phi)[B_L K_{LH} + (w-c+c_{bH})S(K)_H - B_H K_H) - (w-c+c_{bH})S(K)_{LH}]$

$$= \phi\left[\frac{a}{2}(p-c)\left(1 - \frac{B_H^2}{(w-c+c_{bH})^2}\right) - B_H a(1 - \frac{B_H}{w+c_{bH}-c})\right] +$$

$$(1-\phi)\left[\begin{array}{c} B_L a(1 - \frac{B_L}{w+c_{bH}-c}) + \frac{a}{2}\left((w-c+c_{bH}) - \frac{B_H^2}{(w-c+c_{bH})}\right) \\ -B_H a\left(1 - \frac{B_H}{w+c_{bH}-c}\right) - \frac{a}{2}\left((w-c+c_{bH}) - \frac{B_L^2}{(w-c+c_{bH})}\right) \end{array}\right]$$

and

Section L :$L = (1-\phi)[(p-c)S(K)_L - B_L K_L]$
$= (1-\phi)[(p-c)(\frac{1}{2}\frac{a\left((w-c+c_{bL})^2 - B_L^2\right)}{(w-c+c_{bL})^2}) - B_L(\frac{a(w+c_{bL}-B_L-c)}{w+c_{bL}-c})]$

Let us consider Section L first. $L$ is concave with respect to $c_{bL}$ if $(3p-2w-2c_b-c)>0$ holds since
$\frac{\partial^2 L}{\partial c_{bL}^2} = aB_L^2 \frac{(1-\phi)(c-3p+2w+2c_b)}{(w-c+c_b)^4}$. The optimal value of $c_{bL}$ is found as follows

$$\begin{aligned}\frac{\partial L}{\partial c_{bL}} &= aB_L^2 \frac{\phi - 1}{(w-c+c_b)^3}(w-p+c_b) = 0 \\ &\Longrightarrow c_{bL}^* = p - w\end{aligned}$$

This $c_{bL}$ value is optimal since $(3p-2w-2c_b-c) = (p-c) > 0$ and concavity of $L$ with respect to $c_{bL}$ is satisfied. It is seen that the optimal value of $c_{bL}$ in the asymmetric information case is the same as the optimal $c_{bL}$ in the symmetric information case.

Now let us consider Section H, $H$ is said to be concave in terms of $c_{bH}$ if
$\frac{\partial^2 H}{\partial c_{bH}^2} = \frac{a((w-c+c_{bH})(B_H^2-(1-\phi)B_L^2)-\phi B_H^2(3p-2c-w-c_{bH}))}{(w-c+c_b)^4} < 0$ holds.
Thus,

$$c_{bH} < \frac{3\phi B_H^2}{(1+\phi)B_H^2 - (1-\phi)B_L^2} - (w-c)$$

should be satisfied for $H$ to be concave. $c_{bH}$ is found from the first order conditions assuming $H$ is concave.

$$\begin{aligned}\frac{\partial H}{\partial c_{bH}} &= -\frac{1}{2}\frac{a\left((w-c+c_{bH})(B_H^2 - B_L^2 + \phi B_L^2) + \phi B_H^2(c_{bH}+c-2p+w)\right)}{(w-c+c_b)^3} = 0 \\ \Longrightarrow c_{bH} &= \frac{\left((c-w)(B_H^2-B_L^2) - c\phi(B_H^2-B_L^2) + 2p\phi B_H^2 - w\phi(B_H^2+B_L^2)\right)}{(1+\phi)B_H^2 - (1-\phi)B_L^2}\end{aligned}$$

This $c_{bH}$ value satisfies the concavity of $H$ with respect to $c_{bH}$. However, $c_{bH} > (B_H - w + c)$ for nonnegative capacity.

Thus, the optimal value of $c_{bH}$ is as follows
$c_{bH} = \frac{\left((c-w)(B_H^2-B_L^2)-c\phi(B_H^2-B_L^2)+2p\phi B_H^2-w\phi(B_H^2+B_L^2)\right)}{(1+\phi)B_H^2-(1-\phi)B_L^2}$ if $\frac{\left((c-w)(B_H^2-B_L^2)-c\phi(B_H^2-B_L^2)+2p\phi B_H^2-w\phi(B_H^2+B_L^2)\right)}{(1+\phi)B_H^2-(1-\phi)B_L^2} > (B_H - w + c)$
$c_{bH} = (B_H - w - c)$ if $\frac{\left((c-w)(B_H^2-B_L^2)-c\phi(B_H^2-B_L^2)+2p\phi B_H^2-w\phi(B_H^2+B_L^2)\right)}{(1+\phi)B_H^2-(1-\phi)B_L^2} < (B_H - w + c)$

Than, the optimal value of $S_b(T_H)$ and $S_b(T_L)$ becomes

$$S_b(T_H)^* = \frac{(w-c+c_{bH}^*)S(K)_H^* - B_H K_H^* - \pi_{sres}}{c_{bH}^*}$$

$$S_b(T_L)^* = \frac{(w-c+c_{bL}^*)S(K)_L^* - B_L(K_L^* - K_{LH}^*) - (w-c+c_{bH}^*)(S(K)_{LH}^* + S(K)_H^*) - B_H K_H^* - \pi_{sres}}{c_{bL}^*}$$

for $S_b(T_H)^* < S(K)_H^*$ and $S_b(T_L)^* < S(K)_L^*$.

the manufacturer's expected profit when he offers both type of contract is

$$E[\pi_{m1}] = \phi[(p-w-c_{bH}^*)S(K)_H^* + c_{bH}^* S_b(T_H)^*] + (1-\phi)[(p-w-c_{bL}^*)S(K)_L^* + c_{bL}^* S_b(T_L)^*)]$$

And the expected profits of high and low capacity cost suppliers' are relatively as follows

$$\begin{aligned} E[\pi_{sH}] &= (w-c)S(K)_H^* - c_{bH}^*(S(K)_H^* - S_b(T_H)^*) - B_H K_H \\ E[\pi_{sL}] &= (w-c)S(K)_L^* - c_{bL}^*(S(K)_L^* - S_b(T_L)^*) - B_L K_L \end{aligned}$$

And if he only offers the contract designed for the low capacity cost supplier, high capacity cost supplier will not accept the contract since we proved that $\text{IC}_H$ is redundant. And knowing this fact the manufacturer designs the low capacity cost supplier's contract as in the first best case. His expected profit becomes with this optimal bonus contract is

$$E[\pi_{m2}] = (1-\phi)\left(\frac{(a(p-w)+w-c-2B_L)(p-c)+B_L^2)}{2\,(p-c)} + \frac{\pi_{sres}}{a}\right)$$

**Corollary 2** The manufacturer offers both contract types if $\max\{\pi_{m1}, \pi_{m2}\} = \pi_{m1}$. If $\max\{\pi_{m1}, \pi_{m2}\} = \pi_{m2}$ the manufacturer only offers the contract designed for the low capacity cost supplier

**Proof.** Under the assumption that the manufacturer is rational he decides the menu of contact to offer considering his profit function and he chooses the contract type which maximizes his profit. ■

The efficiency of the channel with asymmetric information case is

$$\begin{aligned} Eff &= \frac{E[\pi_{m(i)}] + E[\pi_{s(i)}]}{E[\Pi]} \\ where\ \pi_{m(i)} &= \max\{\pi_{m1}, \pi_{m2}\} \end{aligned}$$

### *4.2.2 Linear Contract*

In this section we are trying to determine whether the coordination can be achieved with a linear contract having parameters $(w, t)$ offered by the manufacturer. The sequence of order is the same as the symmetric capacity cost information case. First the manufacturer offers the linear contract with parameters $(w, t)$ then the supplier either rejects the contract or accept it and sets her capacity and finally demand realizes and the manufacturer orders the demand. If the supplier rejects the contract her profit will be $\pi_{sres}$. In the case of contract acceptance let us represent the the supplier's expected profit in terms of the contract parameters;

$$\pi_{s=}u(w, B) + t$$

where

$$\begin{aligned} u(w, B) &= (w-c)E[\min\{D, K\}] - BK \\ u(w, B) &= (w-c)(K - \frac{K^2}{2a}) - BK \\ where\ K &= \frac{(w-c-B)a}{w-c} \quad and\ u(0, B) = \pi_{Sres} \end{aligned}$$

Also for the asymmetric case we use the notation $S(K)$ for $E[\min\{D, K\}]$, which is the expected amount of sales for a given capacity $K$.

As we stated before the manufacturer does not know the supplier's capacity investment cost and he tries to maximize his profit by designing a menu of contracts. According to revelation principle he can restrict his attention to the class of truth telling mechanisms to find the optimal menu of contracts. Also, we normalized the reservation profits of both type of suppliers to zero for simplicity. As a result the manufacturer objective becomes as follows.

$$(P) \qquad \max\ \phi[(p - w_H)S(K)_H + t_H] + (1-\phi)[(p - w_L)S(K)_L + t_L]$$

$$s.t$$

$$\begin{aligned} u(w_H, B_H) - t_H &\geq u(w_L, B_H) - t_L && \text{IC}_H \\ u(w_L, B_L) - t_L &\geq u(w_H, B_L) - t_H && \text{IC}_L \\ u(w_H, B_H) - t_H &\geq \pi_{res} && \text{IR}_H \\ u(w_L, B_L) - t_L &\geq \pi_{res} && \text{IR}_L \end{aligned}$$

The first two constraints are the incentive compatibility constraints for the high and low-capacity-cost supplier relatively and the last two constraints are the individual rationality constraints for them. Incentive compatibility constraints assure that the supplier will choose the contract type that is designed for her.

**Proposition 17** *The function $u(w,B)$ has decreasing differences in $B$, that is, $u(w_1,B)-u(w_2,B)$ is strictly decreasing in $B$ for all $w_1 > w_2 > c$. As a result, low-capacity-cost supplier values an increase in the per-unit payment $w$ more than a high-capacity cost supplier*

**Proof.**

$$u(w_1,B)-u(w_2,B) = -\frac{1}{2}a\frac{w_1-w_2}{(c-w_1)(c-w_2)}\left(cw_1+cw_2-w_1w_2+B^2-c^2\right)$$

Thus, $\frac{\partial(u(w_1,B)-u(w_2,B))}{\partial B} = -Ba\frac{w_1-w_2}{(c-w_1)(c-w_2)} < 0$ since, $w_1 > w_2 > c$. ■

As a first best scenario we may consider that the manufacturer asks the supplier to report her type and according to her report he proposes a contract. In this case the manufacturer's problem becomes

$$\begin{aligned} \max \;\; & (p-w)\left(\frac{(w-c-B)a}{w-c} - \frac{\left(\frac{(w-c-B)a}{w-c}\right)^2}{2a}\right) - t \\ s.t \;\; & u(w,B)+t \geq \pi_{res} \qquad\qquad \text{(IR)} \end{aligned}$$

IR constraint must hold as equality since the manufacturer improves his objective by decreasing $t$ without violation. Thus, we may conclude that $t = \pi_{res} - u(w,B)$ and with this result the manufacturer's problem becomes

$$\max \;\; (p-c)\left(\frac{(w-c-B)a}{w-c}\right)\left(1-\frac{\left(\frac{(w-c-B)a}{w-c}\right)}{2a}\right) - B\frac{(w-c-B)a}{w-c} - \pi_{res}$$

according to that the manufacturer sets the wholesale price to maximize his profit.

$$\frac{\partial \pi_m}{\partial w} = -B^2\frac{a}{(c-w)^3}(p-w) \Longrightarrow w^* = p$$

This result is the unique global solution of the problem since the objective function is concave.

$$\frac{\partial^2 \pi_m}{\partial w^2} = B^2\frac{a}{(c-w)^4}(c-3p+2w) < 0 \text{ since } p \geq w > c \text{ holds.}$$

This means that manufacturer will propose the same wholesale price to both low-capacity-cost and high-capacity-cost supplier which is equal to the retail price $p$. In this case, in order to make profit the manufacturer takes $t$ from supplier instead of giving her. Thus, the IR constraint of the supplier becomes

$$
\begin{aligned}
t^* &= u(w^*, B) - \pi_{res} \\
&= (p-c)(\frac{(p-c-B)a}{p-c})(1 - \frac{(\frac{(p-c-B)a}{p-c})}{2a}) - B(\frac{(p-c-B)a}{p-c}) - \pi_{res}
\end{aligned}
$$

As a result if the supplier report herself as a high-capacity-cost supplier the manufacturer will propose a linear contract with parameters

$$w = p \ , \ t = \frac{1}{2}\frac{a\,(B_H + c - p)^2}{p-c} - \pi_{res}$$

and he will propose a linear contract with parameters

$$w = p \ , \ t = \frac{1}{2}\frac{a\,(B_L + c - p)^2}{p-c} - \pi_{res}$$

if the supplier report herself as a low-capacity-cost supplier.

For this case we may claim the following corollary.

**Corollary 4.**If the manufacturer asks the supplier for her capacity cost report, a low-capacity-cost supplier has an incentive to report high capacity cost.

**Proof.** Let us analyze the function $t(B)$

$\frac{\partial t}{\partial B} = \frac{(B+c-p)a}{p-c} < 0$ since $p = w > c$ and $w \geq c + B$ should hold for the supplier to produce. This implies that as $B$ increases $t$ will decrease. Thus, $t_L > t_H$. Consequently we may say that $u(p, B_L) - t_H > u(p, B_L) - t_L$ which implies that it will be beneficial for the low-capacity-cost supplier to report high capacity cost. ■

**Proposition 18** *The previous problem will be the same as the following problem*

$$
\begin{aligned}
(RP) \qquad & \max \ \phi[(p - w_H)S(K)_H + t_H] + (1-\phi)[(p - w_L)S(K)_L + t_L] \\
& s.t \\
u(w_L, B_L) - t_L &= u(w_H, B_L) - t_H \qquad IC_L \\
u(w_H, B_H) - t_H &= \pi_{res} \qquad IR_H
\end{aligned}
$$

**Proof.** In problem $P$ $\mathrm{IR}_L$ is redundant due to the fact that $u(w_L, B_L) - t_L - u(0, B_L) \geq u(w_H, B_L) - t_H - u(0, B_L) > u(w_H, B_H) - t_H - u(0, B_H) \geq \pi_{res}$. Left hand side of the inequality comes from the $\mathrm{IC}_L$ constraint and the right hand side of the inequality comes from proposition 17. Thus, we may only consider the high-cost-capacity supplier's IR constraint. Additionally $\mathrm{IR}_H$ should hold as equality since increasing both $t_H$ and $t_L$ without violating the $\mathrm{IR}_H$ will improve the manufacturer's objective. Also, we can claim that $\mathrm{IC}_L$ must be binding since the manufacturer's objective can be improved by increasing $t_L$ a small amount which will preserve $\mathrm{IC}_L$, relax $\mathrm{IC}_H$ and does not effect $\mathrm{IR}_H$. Finally we may argue that $\mathrm{IC}_H$ will be redundant. We proved that $\mathrm{IC}_L$ and $\mathrm{IR}_H$ constrains should be binding and from these constraints we found the values of $t_H$ and $t_L$ such as ;$t_H = u(w_H, B_H) - \pi_{sres}$ and $t_L = u(w_L, B_L) - u(w_H, B_L) + u(w_H, B_H) - \pi_{sres}$. Substituting these values in $\mathrm{IC}_H$ and making the necessary simplifications leads us to the following inequality $u(w_L, B_L) - u(w_H, B_L) > u(w_H, B_H) - u(w_L, B_H)$ which is proven to hold in proposition 17. ■

**Proposition 19** *The optimal linear contracts designed for the low and high-capacity-cost supplier are $(p, t_L^*)$, and $(w_H^*, t_H^*)$ respectively.*

**Proof.** the manufacturer's problem becomes

$\max \phi[(p - w_H)S(K)_H + u(w_H, B_H) - u(0, B_H)] + (1 - \phi)[(p - w_L)S(K)_L + u(w_L, B_L)$
$-u(w_H, B_L) + u(w_H, B_H) - u(0, B_H)]$

here we may ignore $u(0, B_H)$ and since the function is additively separable in $w_H$ &$w_L$ we can separate the function into two sections; Section H and Section L where

Section H: $H = \phi[(p - w_H)S(K)_H + u(w_H, B_H)] + (1 - \phi)[u(w_H, B_H) - u(w_H, B_L)]$ and

Section L :$L = (1 - \phi)[(p - w_L)S(K)_L + u(w_L, B_L)]$

Let us consider Section L first. $L$ is concave with respect to $w_L$ since

$\frac{\partial^2 L}{\partial w_L^2} = aB_L^2 \frac{1-\phi}{(c-w_L)^4} (c - 3p + 2w_L) < 0$ since $p \geq w_L > c$ holds. The optimal value of $w_L$ is found as follows

$$\frac{\partial L}{\partial w_L} = aB_L^2 \frac{\phi - 1}{(c - w_L)^3} (p - w_L) \Longrightarrow w_L = p$$

It is seen that the optimal value of $w_L$ in the asymmetric information case is the same as the optimal $w_L$ in the symmetric information case.

Now let us consider Section H, the open form of section H is as follows

$H = \phi[(p-c)(\frac{(w_H-c-B_H)a}{w_H-c})(1-\frac{\frac{(w_H-c-B_H)a}{w_H-c}}{2a}) - B_H(\frac{(w_H-c-B_H)a}{w_H-c})] + (1-\phi)$
$[(w_H-c)(\frac{(w_H-c-B_H)a}{w_H-c})(1-\frac{\frac{(w_H-c-B_H)a}{w_H-c}}{2a}) - B_H(\frac{(w_H-c-B_H)a}{w_H-c}) - (w_H-c)(\frac{(w_H-c-B_L)a}{w_H-c})(1-$
$\frac{\frac{(w_H-c-B_L)a}{w_H-c}}{2a}) - B_L(\frac{(w_H-c-B_L)a}{w_H-c})]$

$H$ is said to be concave if

$$\frac{\partial H^2}{\partial w_H} = \frac{a\left((w_H-c)(B_H^2+3B_L^2) + \phi B_H^2 w_H - 3\phi B_L^2 w_H + 2c\phi B_H^2 + 3c\phi B_L^2 - 3p\phi B_H^2\right)}{(c-w_H)^4} < 0$$

holds. This means

$\left(B_H^2 w_H + 3B_L^2 w_H - cB_H^2 - 3cB_L^2 + \phi B_H^2 w_H - 3\phi B_L^2 w_H + 2c\phi B_H^2 + 3c\phi B_L^2 - 3p\phi B_H^2\right) =$ $A < 0$ should hold since $(c-w_H)^4$ and $a > 0$. Thus, H is concave where

$$w_H < \frac{3(1-\phi)cB_L^2 + B_H^2(3p\phi - 2c\phi + c)}{B_H^2 + 3B_L^2 + \phi B_H^2 - 3\phi B_L^2}$$

holds.

To find the optimal value of $w_H$

$$\begin{aligned}\frac{\partial H}{\partial w_H} &= \frac{1}{2}\frac{a\left(B_H^2 w_H + 3B_L^2 w_H - cB_H^2 - 3cB_L^2 + \phi B_H^2 w_H - 3\phi B_L^2 w_H + c\phi B_H^2 + 3c\phi B_L^2 - 2p\phi B_H^2\right)}{(c-w_H)^3} \\ \Longrightarrow \quad w_H &= \frac{\left(cB_H^2 + 3cB_L^2 - c\phi B_H^2 - 3c\phi B_L^2 + 2p\phi B_H^2\right)}{B_H^2 + 3B_L^2 + \phi B_H^2 - 3\phi B_L^2}\end{aligned}$$

by substituting this value of $w_H$ to $A$ we obtain $A = \phi B_H^2(c-p) < 0$ thus, the concavity of $H$ function is satisfied.

But $w_H$ should also be greater than $(B_H + c)$. Thus, the optimal value of $w_H$ is as follows

$$\begin{aligned} w_H^* &= \frac{\left(cB_H^2 + 3cB_L^2 - c\phi B_H^2 - 3c\phi B_L^2 + 2p\phi B_H^2\right)}{B_H^2 + 3B_L^2 + \phi B_H^2 - 3\phi B_L^2} \\ \text{if } B_H + c &< \frac{\left(cB_H^2 + 3cB_L^2 - c\phi B_H^2 - 3c\phi B_L^2 + 2p\phi B_H^2\right)}{B_H^2 + 3B_L^2 + \phi B_H^2 - 3\phi B_L^2}\end{aligned}$$

$$\begin{aligned} w_H^* &= B_H + c \\ \text{if } B_H + c &> \frac{\left(cB_H^2 + 3cB_L^2 - c\phi B_H^2 - 3c\phi B_L^2 + 2p\phi B_H^2\right)}{B_H^2 + 3B_L^2 + \phi B_H^2 - 3\phi B_L^2} \\ \text{and } (p-c) &< \frac{(1+\phi)}{3\phi} + \frac{(1-\phi)B_L^2}{\phi B_H}\end{aligned}$$

$$
\begin{aligned}
w_H^* &= \frac{3(1-\phi)cB_L^2 + B_H^2(3p\phi - 2c\phi + c)}{B_H^2 + 3B_L^2 + \phi B_H^2 - 3\phi B_L^2} \\
\text{if } B_H + c &> \frac{\left(cB_H^2 + 3cB_L^2 - c\phi B_H^2 - 3c\phi B_L^2 + 2p\phi B_H^2\right)}{B_H^2 + 3B_L^2 + \phi B_H^2 - 3\phi B_L^2} \\
\text{and } (p-c) &> \frac{(1+\phi)}{3\phi} + \frac{(1-\phi)B_L^2}{\phi B_H}
\end{aligned}
$$

And from this value of $w_H^*$ the optimal $t_L^*$ and $t_H^*$ is as follows

$t_L^* = u(w_L^*, B_L) - u(w_H^*, B_L) + u(w_H^*, B_H) - \pi_{res}$ and $t_H^* = u(w_H^*, B_H) - \pi_{res}$. ■

the manufacturer's profit when he offers both type of contract is

$E[\pi_{m1}] = \phi[(p - w_H^*)S(K)_H^* + t_H^*] + (1-\phi)[(p - w_L^*)S(K)_L^* + t_L^*]$ and suppliers' profits will be $E[\pi_{s1(j)}] = u(w_j^*, B_j) + t_j^*$ where $j = L, H$

And if he only offers the contract designed for the low capacity cost supplier, high capacity cost supplier will not accept the contract since we proved that IC$_H$ is redundant. And knowing this fact the manufacturer designs the low capacity cost supplier's contract as in the first best case. His profit becomes with this optimal linear contract is

$$E[\pi_{m2}] = \frac{(1-\phi)}{2}\frac{a\,(B_L + c - p)^2}{p - c} - \pi_{res}$$

and supplier's profit will be $E[\pi_{s2}] = (1-\phi)\pi_{res}$

**Corollary 5** The manufacturer offers both contract types if $\max\{\pi_{m1}, \pi_{m2}\} = \pi_{m1}$. If $\max\{\pi_{m1}, \pi_{m2}\} = \pi_{m2}$ the manufacturer only offers the contract designed for the low capacity cost supplier

**Proof.** Under the assumption that the manufacturer is rational he decides the menu of contact to offer considering his profit function and he chooses the contract type which maximizes his profit. ■

The efficiency of the channel with asymmetric information case is the total profit of the channel over the maximum profit of the centralized channel.

$$
\begin{aligned}
Eff &= \frac{E[\pi_{m(i)}] + E[\pi_{s(i)}]}{E[\Pi]} \\
where\ \pi_{m(i)} &= \max\{\pi_{m1}, \pi_{m2}\}
\end{aligned}
$$

*Cost Sharing Contract*

In this section we analyze the case where the manufacturer offers a cost sharing contract to the supplier having parameters $(w, \sigma)$. The sequence of order is the same as the symmetric capacity cost information case. First the manufacturer offers the contract then the supplier either rejects the contract or accept it and sets her capacity and finally demand realizes and the manufacturer orders the demand. If the supplier rejects the contract her profit will be $\pi_{sres}$. In the case of contract acceptance let us represent the supplier's expected profit in terms of the contract parameters;

$$\pi_s = u(w, B, \sigma)$$

where

$$\begin{aligned}
u(w, B, \sigma) &= (w - c)E[\min\{D, K\}] - \sigma BK \\
u(w, B, \sigma) &= (w - c)S(K) - \sigma BK \\
where\ K &= \frac{(w - c - \sigma B)a}{w - c}
\end{aligned}$$

As we stated before the manufacturer does not know the supplier's capacity investment cost and he tries to maximize his profit by designing a menu of contracts. According to revelation principle he can restrict his attention to the class of truth telling mechanisms to find the optimal menu of contracts. Also, we normalized the reservation profits of both type of suppliers to zero for simplicity. As a result the manufacturer objective becomes as follows..

$$(P) \quad \max\ \phi[(p - w_H))S(K)_H - (1 - \sigma_H)B_H K_H)] + (1 - \phi)[(p - w_L)S(K)_L + (1 - \sigma_L)B_L K_L))]$$

$$\begin{aligned}
s.t\ (w_H - c)(K_H - \frac{K_H^2}{2a}) - \sigma_H B_H K_H &\geq (w_L - c)(K_{HL} - \frac{K_{HL}^2}{2a}) - \sigma_L B_H K_{HL} && \mathrm{IC}_H \\
(w_L - c)(K_L - \frac{K_L^2}{2a}) - \sigma_L B_L K_L &\geq (w_H - c)(K_{LH} - \frac{K_{LH}^2}{2a}) - \sigma_L B_L K_{LH} && \mathrm{IC}_L \\
(w_H - c)(K_H - \frac{K_H^2}{2a}) - \sigma_H B_H K_H &\geq \pi_{sres} && \mathrm{IR}_H \\
(w_L - c)(K_L - \frac{K_L^2}{2a}) - \sigma_L B_L K_L &\geq \pi_{sres} && \mathrm{IR}_L
\end{aligned}$$

where

$$
\begin{aligned}
K_H &= \frac{(w_H - c - \sigma_H B_H)a}{w_H - c} \; K_L = \frac{(w_L - c - \sigma_L B_L)a}{w_L - c} \\
K_{HL} &= \frac{(w_L - c - \sigma_L B_H)a}{w_L - c} \; K_{LH} = \frac{(w_H - c - \sigma_H B_L)a}{w_H - c}
\end{aligned}
$$

**Proposition 20** *The function $\pi_s(w, B, \sigma)$ has decreasing differences in $B$, that is, $\pi_s(w_1, B, \sigma) - \pi_s(w_2, B, \sigma)$ is strictly decreasing in $B$ for all $w$. As a result, low-capacity-cost supplier values an increase in the wholesale price $w$, more than a high-capacity cost supplier.*

**Proof.**

$$u(w_1, B, \sigma) - u(w_2, B, \sigma) = -\frac{a(w_1 - w_2)\left(B^2\sigma^2 + cw_1 + cw_2 - w_1 w_2 - c^2\right)}{2\left(w_1 - c\right)\left(w_2 - c\right)}$$

$\frac{\partial(u(w_1,B,\sigma) - u(w_2,B,\sigma))}{\partial B} = -Ba\sigma^2 \frac{w_1 - w_2}{(w_1-c)(w_2-c)} < 0$ since $w_1 > w_2 > c$. ■

In the previous section, we solved the first best case in which the manufacturer asks supplier to report her type. Thus, if the supplier report herself as a high-cost supplier the manufacturer offers her a cost sharing contract with parameters $w_H^* = c + \frac{2\pi_{sres}(p-c)^2}{a(p-c-B_H)^2}$ and $\sigma_H^* = \frac{2\pi_{sres}(p-c)}{a(p-c-B_H)^2}$ and if the supplier report herself as a low-cost supplier the manufacturer offers a revenue sharing contract with parameters $w_L^* = c + \frac{2\pi_{sres}(p-c)^2}{a(p-c-B_L)^2}$ and $\sigma_L^* = \frac{2\pi_{sres}(p-c)}{a(p-c-B_L)^2}$

**Corollary 6.** A low capacity cost supplier has an incentive to report high capacity cost.

**Proof.** If a low capacity cost supplier report herself as a high capacity cost supplier her expected profit will be

$$
\begin{aligned}
E[\pi_{LH}] &= (w_H - c)(K_{LH} - \frac{K_{LH}^2}{2a}) - \sigma_H B K_{LH} \\
K_{LH} &= \frac{(w_H - c - \sigma_H B_L)a}{w_H - c}
\end{aligned}
$$

And if she reports herself as a low capacity cost supplier her profit will be

$$
\begin{aligned}
E[\pi_L] &= (w_L - c)(K_L - \frac{K_L^2}{2a}) - \sigma_L B K_L \\
K_L &= \frac{(w_L - c - \sigma_L B_L)a}{w_L - c}
\end{aligned}
$$

$E[\pi_{LH}] - E[\pi_L] = \frac{\pi(B_H - B_L)(2p - 2c - B_H - B_L)}{(p-c-B_H)^2} > 0$ since $p > c + B_i$ where $i = H, L$. Thus, it will be beneficial for low capacity cost supplier to mimic high capacity cost supplier. ■

**Proposition 21** *The following problem is the same as problem $(P)$*

$$(RP)\ \max\ \phi[(p-w_H)(K_H-\frac{K_H^2}{2a})-(1-\sigma_H)B_HK_H)]+(1-\phi)[(p-w_L)(K_L-\frac{K_L^2}{2a})-(1-\sigma_L)B_LK_L))]$$

$$s.t$$

$$\begin{aligned}(w_L-c)(K_L-\frac{K_L^2}{2a})-\sigma_LB_LK_L &= (w_H-c)(K_{LH}-\frac{K_{LH}^2}{2a})-\sigma_HB_LK_{LH} \quad IC_L\\ (w_H-c)(K_H-\frac{K_H^2}{2a})-\sigma_HB_HK_H &= \pi_{sres} \quad IR_H\end{aligned}$$

**Proof.** In problem $P$ IR$_L$ is redundant due to the fact that $(w_L-c)(K_L-\frac{K_L^2}{2a})-\sigma_LB_LK_L \geq (w_H-c)(K_{LH}-\frac{K_{LH}^2}{2a})-\sigma_HB_LK_{LH} > (w_H-c)(K_H-\frac{K_H^2}{2a})-\sigma_HB_HK_H \geq \pi_{sres}$. The left hand side of the inequality comes from the IC$_L$ and right hand side comes from claim 20.As a result, only the high-capacity-cost supplier's IR constraint should be considered. Additionally IR$_H$ should hold as equality since increasing both $\sigma_H$ and $\sigma_L$ without violating IR$_H$ will improve the manufacturer's objective. In addition, IC$_L$ must be binding since a small increase in $\sigma_L$ preserve IC$_L$, relax IC$_H$ and does not effect IR$_H$. Finally IC$_H$ is redundant. Binding IC$_L$ gives us the equality $\sigma_L(w_H-c)=\sigma_H(w_L-c)$ if we plug this in to IC$_H$ we will end up with the inequality $w_H-c > w_L-c$, which is the final constraint of the relaxed problem. ■

According to proposition 21 the manufacturer's problem becomes

$$\begin{aligned}&\max_{w_H,w_L}\ \phi[(p-w_H)(K_H-\frac{K_H^2}{2a})-(1-\sigma_H)B_HK_H)]\\ &+(1-\phi)[(p-w_L)(K_L-\frac{K_L^2}{2a})-(1-\sigma_L)B_LK_L))]\end{aligned}$$

$$where\ \sigma_H = \frac{\sqrt{\frac{2\pi_{sres}(w_H-c)}{a}}-(w_H-c)}{B_H} \text{ and } \sigma_L = \frac{(w_L-c)\left(\sqrt{\frac{2\pi_{sres}(w_H-c)}{a}}-(w_H-c)\right)}{(w_H-c)B_H}$$

We performed a numerical study in the last chapter to solve this problem.

the manufacturer's profit when he offers both type of contract is

$E[\pi_{m1}] = \phi[(p-w_H^*))S(K)_H^*-(1-\sigma_H^*)B_HK_H^*)]+(1-\phi)[(p-w_L^*)S(K)_L^*-(1-\sigma_L^*)B_LK_L^*))]$ supplier's profits will be $\pi_{s1(j)=}u(w_j^*,B_j,\sigma_j^*)+t_j^*$ where $j=L,H$

And if he only offers the contract designed for the low capacity cost supplier, high capacity cost supplier will not accept the contract since we proved that IC$_H$ is redundant. And knowing this fact the manufacturer designs the low capacity cost supplier's contract as in the first best case. His profit becomes with this optimal linear contract is

$$E[\pi_{m2}] = \frac{(1-\phi)}{2}\frac{a\left((p-c)^2 - 2B_L(p-c-B_L)\right)}{(p-c)} - \pi_{res}$$

and supplier's profit will be $E[\pi_{s2}] = (1-\phi)\pi_{res}$

**Corollary 7** The manufacturer offers both contract types if $\max\{\pi_{m1}, \pi_{m2}\} = \pi_{m1}$. If $\max\{\pi_{m1}, \pi_{m2}\} = \pi_{m2}$ the manufacturer only offers the contract designed for the low capacity cost supplier

**Proof.** Under the assumption that the manufacturer is rational he decides the menu of contact to offer considering his profit function and he chooses the contract type which maximizes his profit. ■

The efficiency of the channel with asymmetric information case is

$$\begin{aligned} Eff &= \frac{E[\pi_{m(i)}] + E[\pi_{s(i)}]}{E[\Pi]} \\ where\ \pi_{m(i)} &= \max\{\pi_{m1}, \pi_{m2}\} \end{aligned}$$

Chapter 5

# CAPACITY CONTRACTS IN A PRICE DEPENDENT STOCHASTIC DEMAND MODEL

In this chapter, we extend the previous chapter by allowing the manufacturer to decide on the market price $p$. Similar to the previous setting, the supplier sells the product to the manufacturer with a wholesale price, $w$. Supplied product is worked upon by the manufacturer and a value-added finished product is sold to the market with a retail price $p$ determined by the manufacturer. We again assume that the supplier has a unit production cost $c$ and the supplier needs to build her capacity, $K$, by incurring the capacity investment cost $c(K)$, before the realization of the demand, since building capacity has a lead time and capacity cannot be built on an incremental basis.

Throughout this chapter we assume that the market demand is stochastic with mean $(a - bp)$.Moreover, in our analysis, we assume that the demand is uniformly distributed between 0 and $2(a-bp)$ and thus the density function of the demand is $f(x) = 1/(2(a-bp))w$ for $x \in [0, 2(a - bp)]$. We also assume that the supplier has a linear capacity cost where $c(K) = BK$. Prior to demand realization, the supplier sets her capacity, then considering this capacity the manufacturer sets his retail price and the demand is realized depending on the market price.

Similar to the previous chapters, we first analyze the centralized supply chain in which all decisions are made to maximize the integrated channel profit. Later, we consider the decentralized channel in two cases. In the first case we assume that the wholesale price is exogenous and we determine whether the coordination can be achieved with bonus contract offered by the manufacturer. In the second case we assume endogenous wholesale price and we determine whether the coordination can be achieved with linear, cost sharing and revenue sharing contracts offered by the manufacturer.

## 5.1 Centralized Model

In this section we consider the integrated channel where all decisions are made by a single agent who has all the information and aims to maximize the total supply chain profit. The profit function of the integrated channel is as in 5.1.

$$E[\Pi] = (p-c)E[\min\{K, D\}] - BK \tag{5.1}$$

Here, we analyze the problem in two different cases. In the first case, assume that $K \leq 2(a-bp)$. Then, 5.1 becomes

$$E[\Pi] = (p-c)E[D] - (p-c)\int\limits_{K}^{2(a-bp)} (x-K)f(x)dx - BK$$

As the second case assume $K > 2(a-bp)$ holds then $E[\min\{K, D\}] = D$ and 5.1 becomes

$$E[\Pi] = (p-c)E[D] - BK \tag{5.2}$$

**Proposition 22** *For the integrated channel, the optimal solution has to satisfy the condition $K \leq 2(a-bp)$.*

**Proof.** The maximum value of demand is $2(a-bp)$ and in the integrated channel one party decides to both the amount of capacity investment and the retail price. So, it will not be rational for him to invest more capacity then the maximum value of demand. ■

**Proposition 23** *The optimal $p$ and $K$ values for the centralized channel is the simultaneous solution of 5.3 and 5.4*

$$\begin{aligned}
\frac{\partial E[\Pi]}{\partial K} &= (p-c)\int\limits_{K}^{2(a-bp)} f(x)dx - B = 0 \\
&\Longrightarrow 1 - F(K) = \frac{B}{p-c} \Longrightarrow F(K^*) = \frac{p-c-B}{p-c} \qquad (5.3) \\
\frac{\partial E[\Pi]}{\partial p} &= (p-c)\left(\int\limits_{0}^{K} xf'(x)dx + K\int\limits_{K}^{2(a-bp)} f'(x)dx - 2bf(2(a-bp))\right) \\
&\quad + \int\limits_{0}^{K} xf(x)dx + K\int\limits_{K}^{2(a-bp)} f(x)dx \qquad (5.4)
\end{aligned}$$

**Proof.** It is obvious that $E[\Pi]$ is concave in terms of $p$ and $K$ thus first order conditions is used to find the optimal values for the retail price and capacity. Thus the simultaneous solution of 5.3 and 5.4 gives us the optimal $K$ and $p$ values. ■

The above result holds for any distribution for the demand function. When we use uniform distribution for the distribution of the demand, we get the following result.

**Proposition 24** *For uniformly distributed price dependent demand the optimal retail price and the optimal invested capacity for the integrated channel is*

$$\begin{aligned} p^* &= \frac{1}{2b}\left(\frac{1}{2}a + \frac{1}{2}\sqrt{(a-bc)(a+8Bb-bc)} + \frac{3}{2}bc\right) \\ \textit{and } K^* &= \frac{2(p^*-c-B)(a-bp^*)}{(p^*-c)} \end{aligned}$$

*respectively.*

**Proof.** Proposition 22 states that at the optimal solution $K \leq 2(a-bp)$ should hold. The profit function of the integrated channel for uniformly distributed demand between 0 and $2(a-bp)$ is as follows

$$\begin{aligned} E[\Pi] &= (p-c)\min\{K, D\} - BK \\ &= (p-c)E[D] - (p-c)\int_K^{2(a-bp)} (x-K)f(x)dx - BK \\ &= (p-c-B)K - \frac{(p-c)K^2}{4(a-bp)} \end{aligned}$$

Since the objective function is jointly concave w.r.t. $K$ and $p$, the optimal $K$ and $p$ values is found from the first order conditions

$$\begin{aligned} \max_{p,K}\ E[\Pi] &= (p-c-B)K - \frac{(p-c)K^2}{4(a-bp)} \\ \frac{\partial E[\Pi]}{\partial K} &= (p-c-B) - \frac{(p-c)K}{2(a-bp)} = 0 \\ &\Longrightarrow K = \frac{2(p-c-B)(a-bp)}{(p-c)} \\ \frac{\partial E[\Pi]}{\partial p} &= K - \frac{1}{4}\frac{K^2(a-cb)}{(a-bp)^2} = 0 \\ where\ K &= \frac{2(p-c-B)(a-bp)}{(p-c)} \Longrightarrow \frac{2(p-c-B)(a-bp)}{(p-c)} - \end{aligned}$$

$$\frac{1}{4}\frac{(\frac{2(p-c-B)(a-bp)}{(p-c)})^2(a-cb)}{(a-bp)^2} = 0$$

There are two values of $p$ as the solution of this equation which are

$$\begin{aligned} p &= \frac{1}{2b}\left(\frac{1}{2}a + \frac{1}{2}\sqrt{(a-bc)(a+8Bb-bc)} + \frac{3}{2}bc\right) \\ \text{and } p &= \frac{1}{2b}\left(\frac{1}{2}a - \frac{1}{2}\sqrt{(a-bc)(a+8Bb-bc)} + \frac{3}{2}bc\right) \end{aligned}$$

with the optimal $p$ value both $(a-bp) > 0, (p-c-B) > 0$ should hold. The first $p$ value satisfies both conditions however the second $p$ value fails to satisfy $(p-c-B) > 0$ since this inequality reduces to $0 > (a-b(B+c))$ which cannot hold. Thus, the optimal value of $p^* = \frac{1}{2b}\left(\frac{1}{2}a + \frac{1}{2}\sqrt{(a-bc)(a+8Bb-bc)} + \frac{3}{2}bc\right)$ and the optimal value of $K^* = \frac{2(p^*-c-B)(a-bp^*)}{(p^*-c)}$
■

According to the optimal values of $p$ and $K$ the expected profit of the integrated channel is:

$$E[\Pi] = \frac{(p-c-B)^2(a-bp)}{(p-c)}$$

### 5.2 Decentralized Case with Exogenous Wholesale Price

In this section we consider the decentralized case with exogenous wholesale price. In this setting, $w$ is assumed to be fixed and agreed upon beforehand. So, first the supplier invests on her capacity based on this $w$ and then the manufacturer sets the retail price based on $w$ and $K$. Finally, the demand is realized and the supplier tries to fulfill the demand using her capacity.

**Proposition 25** *In the decentralized case the supplier's invested capacity will not exceed the maximum amount of demand i.e.* $K^d \leq 2(a-bp)$ *should always hold.*

**Proof.** In this game there is common knowledge between the supplier and the manufacturer i.e. the supplier knows the optimal retail price for the manufacturer in terms of her invested capacity. Thus, she will not invest on a capacity higher than the maximum amount of demand. ■

**Proposition 26** *For any given capacity level* $K$, *the optimal market price for the manufacturer is* $p^* = \left(2a - \sqrt{K(a-bw)}\right)/(2b)$ *if* $K \leq a-bw$ *and* $p^* = (a+bw)/(2b)$ *if* $K \geq a-bw$.

**Proof.** If $K$ is high enough such that there were no capacity constraint, where $E[\min\{D, K\}] = E[D]$, i.e. if $K \geq 2(a - bp)$, the manufacturer's problem will be:

$$\begin{aligned} \max_p \; E[\pi_m] &= (p - w)(a - bp) \\ s.t \; K &\geq 2(a - bp) \end{aligned}$$

and the optimal $p$ value is found as follows

$$\begin{aligned} \frac{\partial E[\pi_m]}{\partial p} &= a - 2bp + bw \\ &\Longrightarrow \; p^* = \frac{(a + bw)}{2b} \end{aligned}$$

However, this $p^*$ is the optimal solution only if the capacity level $K$ is high enough such that $K \geq 2(a - bp^*) = a - bw$. Due to the capacity restriction, in the region where $K \leq 2(a - bp)$, the manufacturer's problem is as follows:

$$\begin{aligned} \max_p \; E[\pi_m] &= (p - w)\left(K - \frac{K^2}{4(a - bp)}\right) \\ s.t \; K &\leq 2(a - bp) \end{aligned}$$

It is obvious that $E[\pi_m]$ is concave in terms of $p$ since $\partial^2 E[\pi_m]/\partial p^2 < 0$. To find the optimal value of $p$ for a given $K$

$$\begin{aligned} \frac{\partial E[\pi_m]}{\partial p} &= K - \frac{K^2(a - bw)}{4\,(a - bp)^2} = 0 \\ &\Longrightarrow \; p = \frac{\left(2a - \sqrt{K(a - bw)}\right)}{2b} \text{ or } p = \frac{\left(2a + \sqrt{K(a - bw)}\right)}{2b} \end{aligned}$$

$p = \left(2a + \sqrt{K(a - bw)}\right)/(2b)$ gives negative $(a - bp)$ value, so the optimal $p$ will be $p^* = \left(2a - \sqrt{K(a - bw)}\right)/(2b)$ if this $p^*$ satisfies the condition $K \leq 2(a - bp^*) = \sqrt{K(a - bw)}$, which is true for any $K \leq a - bw$. So, if $K \leq a - bw$, the optimal market price for the manufacturer is $p^* = \left(2a - \sqrt{K(a - bw)}\right)/(2b)$ and if $K \geq a - bw$, $p^* = (a + bw)/(2b)$.

**Proposition 27** *If $w - c \leq 4B$, the optimal invested capacity for the supplier is*

$$K^* = \frac{16(a - bw)(w - c - B)^2}{9(w - c)^2}$$

*and the optimal market price is*

$$p^* = \frac{1}{2b}\left(2a - \frac{4(a - bw)(w - c - B)}{3(w - c)}\right)$$

*. However, if $w - c \geq 4B$, the optimal capacity investment for the supplier is $K^* = (a - bw)$ and $p^* = (a + bw)/(2b)$.*

We know from proposition 25 that in the optimal solution, $K \leq 2(a - bp)$. Using the results from proposition 26, we conclude that in the optimal solution $K \leq a - bw$ should hold since the maximum realization of demand which is $2(a-bp)$ is at most equal to $a-bw$ due to the pricing decision of the manufacturer. Considering the optimal value of $p$ for the manufacturer as a function of $K$, the supplier solves her own problem to find the optimal amount of capacity and the optimal wholesale price for her. In the region $K \leq a - bw$, supplier's problem is as follows:

$$\begin{aligned}
\max_K \; E[\pi_s] &= (w-c)\left(K - \frac{K^2}{4(a-bp^*)}\right) - BK \\
s.t\; K &< 2(a-bp^*) \\
where\; p &= \frac{\left(2a - \sqrt{K(a-bw)}\right)}{2b} \\
&\Longrightarrow \max_K \; E[\pi_s] = (w-c)\left(K - \frac{K^2}{2\sqrt{Ka-Kbw}}\right) - BK \\
s.t\; K &< (a-bw)
\end{aligned}$$

The function $E[\pi_s]$ is concave in terms of $K$ since $\partial^2 E[\pi_s]/\partial K^2 < 0$. To find the optimal value of $K$, we look at the first order conditions.

$$\begin{aligned}
\frac{\partial E[\pi_s]}{\partial K} &= (w-c)\left(1 - \frac{3}{4}\frac{\sqrt{K}}{\sqrt{a-bw}}\right) - B = 0 \\
&\Longrightarrow K = \frac{16(a-bw)(w-c-B)^2}{9(w-c)^2} \\
&\Longrightarrow p = \frac{1}{2b}\left(2a - \frac{4(a-bw)(w-c-B)}{3(w-c)}\right)
\end{aligned}$$

If the above value of $K$ satisfies the condition that $K \leq (a-bw)$, then it is the optimal solution. So, if $K = 16(a-bw)(w-c-B)^2/(9(w-c)^2) \leq a - bw \Rightarrow 4(w-c-B) \leq 3(w-c) \Rightarrow w - c \leq 4B$, the optimal solution is

$$K^* = \frac{16(a-bw)(w-c-B)^2}{9(w-c)^2}$$

and

$$p^* = \frac{1}{2b}\left(2a - \frac{4(a-bw)(w-c-B)}{3(w-c)}\right)$$

.Otherwise, the constraint will be binding and the optimal $K^* = (a-bw)$ and the optimal $p^* = (a+bw)/(2b)$. ■

### 5.2.1 Coordination with a Bonus Contract

In this section we consider a bonus contract and determine whether we can achieve the supply chain coordination with it. In this section, we assume that the manufacturer offers a bonus contract with parameters $(c_b, T)$ and the supplier accepts the contract if her profit is greater than or equal to her reservation profit.

For a given contract parameters $(c_b, T)$ and $K$ the manufacturer's and the supplier's problems are:

$$\max_p \; E[\pi_m] = (p-w)E[\min(D,K)] - c_b E[(\min(D,K)-T)^+]$$

$$\max_K \; E[\pi_s] = (w-c)E[\min(D,K)] - BK + c_b E[(\min(D,K)-T)^+]$$

Let us denote $S(K)$ as the expected amount of sales for a given capacity level $K$ and $S_b(K)$ as the amount of product that the supplier provides above the target level, $T$.

$$\begin{aligned} S_b(K) &= (S(K)-T)^+ \\ &= S(K) - S(T) \end{aligned}$$

When $K < T$ holds then $S_b(K) = 0$ and the channel will act as the decentralized channel.

When $K > T$ holds and demand is uniformly distributed
$S(K) = \left(K - \frac{K^2}{4(a-bp)}\right)$ and $S_b(K) = \left(K - \frac{K^2}{4(a-bp)}\right) - \left(T - \frac{T^2}{4(a-bp)}\right)$

**Proposition 28** *For any given capacity level $K$, and contract parameters $(c_b, T)$ the optimal market price for the manufacturer is* $p^* = \frac{\left(2Ka - \sqrt{K^3a - K^3bw - K^3bc_b + KT^2bc_b}\right)}{2Kb}$ *if* $K \leq a - bw - bc_b + \frac{T^2}{K}bc_b$ *and* $p^*$ *is the solution of Equation 5.5*

$$\frac{\partial E[\pi_m]}{\partial p} = \frac{\left(4Ka^2 - K^2a + K^2bw + K^2bc_b - T^2bc_b + 4Kb^2p^2 - 8Kabp\right)}{4\,(a-bp)^2} = 0 \qquad (5.5)$$

*if it satisfies* $K \geq 2(a-bp)$.

**Proof.** If $K$ is high enough such that there were no capacity constraint, where $E[\min\{D,K\}] = E[D]$, i.e. if $K \geq 2(a-bp)$, the manufacturer's problem will be:

$$\begin{aligned} \max_p \; E[\pi_m] &= (p-w)\,(a-bp) - c_b((a-bp) - \left(T - \frac{T^2}{4(a-bp)}\right)) \\ s.t\; K &\geq 2(a-bp) \end{aligned}$$

The function $E[\pi_m]$ is concave in terms of $p$ since $\partial^2 E[\pi_m]/\partial p^2 < 0$. Thus, the optimal value of $p$ is found from the first order conditions

$$\frac{\partial E[\pi_m]}{\partial p} = \frac{\left(4a^3-8b^3p^3+4b^3p^2c_b-16a^2bp+4a^2bw-T^2bc_b+4a^2bc_b+20ab^2p^2+4b^3p^2w-8ab^2pw-8ab^2pc_b\right)}{4(a-bp)^2} = 0$$

Let $p^*$ is the solution of $\partial E[\pi_m]/\partial p = 0$. However, this $p^*$ is the optimal solution only if the capacity level $K$ is high enough such that $K \geq 2(a-bp^*)$

Due to the capacity restriction, in the region where $K \leq 2(a-bp)$, the manufacturer's problem is as follows:

$$\begin{aligned}\max_p\ E[\pi_m] &= (p-w)\left(K-\frac{K^2}{4(a-bp)}\right)-c_b\left(\left(K-\frac{K^2}{4(a-bp)}\right)-\left(T-\frac{T^2}{4(a-bp)}\right)\right)\\ s.t\ K &\leq 2(a-bp)\end{aligned}$$

$E[\pi_m]$ is concave in terms of $p$ if

$$\frac{\partial^2 E[\pi_m]}{\partial p^2} = -\frac{1}{2}\frac{b}{(a-bp)^3}\left(K^2a-K^2bw-K^2bc_b+T^2bc_b\right) < 0$$

holds. If $\left(K^2a-K^2bw-K^2bc_b+T^2bc_b\right) > 0$ holds then $E[\pi_m]$ is concave in terms of $p$ and the optimal value of $p$ for a given $K$ can be found from the first order conditions.

$$\begin{aligned}\frac{\partial E[\pi_m]}{\partial p} &= \frac{\left(4Ka^2-K^2a+K^2bw+K^2bc_b-T^2bc_b+4Kb^2p^2-8Kabp\right)}{4(a-bp)^2} = 0\\ &\Longrightarrow\ p = \frac{\left(2Ka-\sqrt{K^3a-K^3bw-K^3bc_b+KT^2bc_b}\right)}{2Kb}\\ \text{or } p &= \frac{\left(2Ka+\sqrt{K^3a-K^3bw-K^3bc_b+KT^2bc_b}\right)}{2Kb}\end{aligned}$$

$p = \left(2Ka+\sqrt{K^3a-K^3bw-K^3bc_b+KT^2bc_b}\right)/(2Kb)$ gives negative $(a-bp)$ value, so the optimal $p$ will be $p^* = \left(2Ka-\sqrt{K^3a-K^3bw-K^3bc_b+KT^2bc_b}\right)/(2Kb)$

if this $p^*$ satisfies the condition

$K \leq 2(a-bp^*) = (\sqrt{K^3a-K^3bw-K^3bc_b+KT^2bc_b})/K$, which is true for any $K$ that satisfies $K \leq a-bw-bc_b+\frac{T^2}{K}bc_b$. So, for $K$ that satisfies $K \leq a-bw-bc_b+\frac{T^2}{K}bc_b$ the optimal retail price is $p^* = \left(2Ka-\sqrt{K^3a-K^3bw-K^3bc_b+KT^2bc_b}\right)/(2Kb)$ and the optimal retail price is the solution of 5.5 if it satisfies $K \geq 2(a-bp^*)$. ■

**Proposition 29** *For given contract parameters $(c_b, T)$ the optimal invested capacity is found from the solution of problem*

$$\begin{aligned}
\max_K \; E[\pi_s] &= (w - c + c_b)\left(K - \frac{K^2}{4(a - bp^*)}\right) - BK - c_b\left(T - \frac{T^2}{4(a - bp)}\right) \\
s.t\; K &< 2(a - bp^*) \\
where\; p &= \frac{\left(2Ka - \sqrt{K^3a - K^3bw - K^3bc_b + KT^2bc_b}\right)}{2Kb} \\
&\Longrightarrow \max_K \; E[\pi_s] = (w - c + c_b)\left(\frac{2K\sqrt{K^3a - K^3bw - K^3bc_b + KT^2bc_b} - K^3}{2\sqrt{K^3a - K^3bw - K^3bc_b + KT^2bc_b}}\right) - BK \\
&\quad -c_bT\left(\frac{2\sqrt{K^3a - K^3bw - K^3bc_b + KT^2bc_b} - KT}{2\sqrt{K^3a - K^3bw - K^3bc_b + KT^2bc_b}}\right) \\
s.t\; K &\leq a - bw - bc_b + \frac{T^2}{K}bc_b
\end{aligned} \tag{5.6}$$

**Proof.** We know from proposition 25 that in the optimal solution, $K \leq 2(a - bp)$ holds. Using the results from proposition 28, we conclude that in the optimal solution $K \leq a - bw - bc_b + \frac{T^2}{K}bc_b$ should hold since the maximum realization of demand which is $2(a - bp)$ is at most equal to $a - bw - bc_b + \frac{T^2}{K}bc_b$ due to the pricing decision of the manufacturer. Considering the optimal value of $p$ for the manufacturer as a function of $K$, the supplier solves her own problem to find the optimal amount of capacity and the optimal wholesale price for her. In the region $K \leq a - bw - bc_b + \frac{T^2}{K}bc_b$, the supplier's problem is as in problem 5.6.

The function $E[\pi_s]$ is concave in terms of $K$ since $\frac{\partial^2 E[\pi_s]}{\partial K^2} < 0$. To find the optimal value of $K$, we look at the first order conditions. Let $K^*$ be the solution of Problem 5.6. ■

**Proposition 30** *The optimal contract parameters $(c_b, T)$ is found from the solution of 5.7.*

**Proof.** The optimal retail price and optimal capacity in terms of contract parameters are given in Proposition 28 and 29. Considering those value the manufacturer will solve the following problem to find the optimal $(c_b, T)$.

$$\begin{aligned}
\max_{c_b,T}\ E[\pi_m] &= (p^*-w)\left(K^*-\frac{(K^*)^2}{4(a-bp^*)}\right)-c_b\left(\left(K^*-\frac{(K^*)^2}{4(a-bp^*)}\right)-\left(T-\frac{T^2}{4(a-bp^*)}\right)\right)\\
s.t\ K^* &\leq 2(a-bp^*)\\
where\ p^* &= \frac{\left(2K^*a-\sqrt{(K^*)^3(a-bw-bc_b)+K^*T^2bc_b}\right)}{2K^*b} \qquad (5.7)\\
&\quad \text{and } K^* \text{ is the solution of Problem 5.6.}
\end{aligned}$$

■

## 5.3 Decentralized Case with Endogenous wholesale price

In this section we assumed that the wholesale price is adjustable. As in the previous setting, the manufacturer does not commit for a retail price at the beginning of the game. At first we analyze the case where the supplier sets both the wholesale price and her capacity. Later, we set the manufacturer as the Stackelberg leader in this game and we analyze the wholesale price, linear ,cost sharing and revenue sharing contracts offered by him. In all cases we assume that the supplier accepts the contract if her profit with the contract is at least as good as her profit with the wholesale price contract that she offers.

### 5.3.1 Simple Wholesale Price Contract offered by the supplier

In this section we consider a simple wholesale price contract offered by the supplier. The sequence of events is as follows; first the supplier invests on her capacity and sets the wholesale price, after that the manufacturer sets the retail price. Finally, demand realizes and the manufacturer gives his order which is equal to the amount of demand.

**Proposition 31** *The optimal capacity and wholesale price for the supplier is the solution of problem*

$$\begin{aligned}
\max_{K,w}\ E[\pi_s] &= (w-c)\left(K-\frac{K^2}{4(a-bp^*)}\right)-BK\\
s.t\ K &\leq 2(a-bp^*)\\
where\ p^* &= \frac{\left(2a-\sqrt{K(a-bw)}\right)}{2b}\\
&\Longrightarrow \max_{K,w}\ E[\pi_s]=(w-c)\left(K-\frac{K^2}{2\sqrt{Ka-Kbw}}\right)-BK \qquad (5.8)\\
s.t\ K &\leq (a-bw)
\end{aligned}$$

**Proof.** From propositions 26 we know that for any given capacity level $K$, the optimal market price for the manufacturer is $p^* = \frac{\left(2a - \sqrt{K(a-bw)}\right)}{2b}$ if $K \leq a - bw$ and $p^* = \frac{(a+bw)}{2b}$ if $K \geq a - bw$. Considering the optimal value of $p$ for the manufacturer as a function of $K$, the supplier solves her own problem to find the optimal amount of capacity and the optimal wholesale price for her.

In the region $K \leq a - bw$, the supplier's problem is as in Problem 5.8

Optimal $K$ and $w$ is found from the solution of Problem 5.8. ■

### *5.3.2 Coordination with Contracts*

In this part we will try to coordinate the channel with linear, bonus, revenue sharing and cost sharing contracts. In all contract types the sequence of events is the same. First the manufacturer proposes the contract, later the supplier invests on her capacity, then the manufacturer decides on his retail price after that demand realizes and finally the manufacturer gives his order and supplier fulfills his order. In all contract types obtaining the analytical results is difficult thus we perform a numerical study at the end to see whether the coordination can be achieved and to understand the effect of each parameters.

#### *Simple Wholesale Price*

In this part we assume that the manufacturer proposes a simple wholesale price contract to the supplier. The sequence of events is as follows. First, the manufacturer proposes the wholesale price then,the supplier invest on her capacity and after that the manufacturer sets the retail price. After than demand realizes, the manufacturer gives his order and supplier fulfills the demand.

In propositions 26 and 27 we found the optimal $K$ and $p$ values for given $w$ such that; if $w - c \leq 4B$, the optimal invested capacity for the supplier is $K^* = \frac{16(a-bw)(w-c-B)^2}{9(w-c)^2}$ and the optimal market price is $p^* = \frac{1}{2b}\left(2a - \frac{4(a-bw)(w-c-B)}{3(w-c)}\right)$.

According to those values the manufacturer sets the wholesale price

For the case $w - c \leq 4B$ the manufacturer's problem will be as follows

$$\begin{aligned}
\max_{w}\ E[\pi_m] &= (p^*-w)\left(K^d-\frac{K^{d2}}{4(a-bp^*)}\right) \qquad (5.9)\\
s.t\ E[\pi_s] &= (w-c)\left(K^d-\frac{K^{d2}}{4(a-bp^*)}\right)-BK^d\geq \pi_{sres}\\
w-c &\leq 4B\\
where\ K^d &= \frac{16(a-bw)(w-c-B)^2}{9(w-c)^2} \text{ and } p^*=\frac{1}{2b}\left(2a-\frac{4(a-bw)(w-c-B)}{3(w-c)}\right)
\end{aligned}$$

The optimal value of $w$ is found by solving this problem. And the manufacturer's and the supplier's profits will be as follows.

$$\begin{aligned}
E[\pi_m] &= (p^*-w^*)\left(K^d-\frac{K^{d2}}{4(a-bp^*)}\right)\\
E[\pi_s] &= (w^*-c)\left(K^d-\frac{{K^d}^2}{4(a-bp^*)}\right)-BK^d
\end{aligned}$$

where $K^d=\frac{16(a-bw)(w-c-B)^2}{9(w-c)^2}$ and $p^*=\frac{1}{2b}\left(2a-\frac{4(a-bw)(w-c-B)}{3(w-c)}\right)$ and $w^*$ is the solution of 5.9

*Linear Contract*

In this part we consider a linear contract offered by the manufacturer contract with parameters $(w,t)$. For a given contract parameters and $K$ the manufacturer sets his retail price as follows

$$\begin{aligned}
\max_{p}\ E[\pi_m] &= (p-w)E[\min(D,K)]-t\\
s.t\ K &\leq 2(a-bp)
\end{aligned}$$

For given contract parameters the manufacturer's optimal retail price and the supplier's optimal capacity is found in Propositions 26 and 27.

For the case $w-c\leq 4B$ the manufacturer's problem will be as follows

$$\begin{aligned}
\max_{w}\ E[\pi_m] &= (p-w)\left(K-\frac{K^2}{4(a-bp)}\right)-t\\
s.t\ E[\pi_s] &= (w-c)\left(K-\frac{K^2}{4(a-bp)}\right)-BK+t\geq \pi_{sres}\\
w-c &\leq 4B\\
where\ K &= \frac{16(a-bw)(w-c-B)^2}{9(w-c)^2} \text{ and } p=\frac{1}{2b}\left(2a-\frac{4(a-bw)(w-c-B)}{3(w-c)}\right)
\end{aligned}$$

The manufacturer's objective improves as $t$ decreases so the constraint should hold as equality and $t = \pi_{sres} - (w-c)\left(K - \frac{K^2}{4(a-bp)}\right) + BK$. Substituting the values of $t, K$ and $p$ into manufacturer's objective function

$$\begin{aligned}
\max_{w}\ E[\pi_m] &= (p-c)\left(K - \frac{K^2}{4(a-bp)}\right) - BK - \pi_{sres} \\
&= (\frac{16(a-bw)(w-c-B)^2(w-c+2B)(a-bc)}{27(w-c)^3 b} \\
&\quad - \frac{32(a-bw)^2(w-c-B)^3(w-c+2B)}{81(w-c)^4 b} \qquad (5.10)\\
&\quad - B\left(\frac{16(a-bw)(w-c-B)^2}{9(w-c)^2}\right) - \pi_{sres} \\
s.t\ w-c &\leq 4B \qquad (5.11)
\end{aligned}$$

Optimal $w$ value is the solution of Problem 5.11.

*Cost Sharing Contract*

In this section we consider the case that the manufacturer offer a cost sharing contract with parameters $(w, \sigma)$ and with this contract he shares $(1-\sigma)$ portion of the supplier's capacity investment cost.

**Proposition 32** *For given contract parameters $(w, \sigma)$ and $K$ the manufacturer sets his retail price $p^* = \frac{\left(2a - \sqrt{K(a-bw)}\right)}{2b}$ if $K \leq a - bw$ and $p^* = \frac{(a+bw)}{2b}$ if $K \geq a - bw$.*

**Proof.** According to Proposition 25 we know that $K \leq 2(a-bp)$ should hold for the optimal solution.

For given contract parameters $(w, \sigma)$ and $K$ the manufacturer sets his retail price as follows

$$\begin{aligned}
\max_{p}\ E[\pi_m] &= (p-w)E[\min(D,K)] - (1-\sigma)BK \\
s.t\ K &\leq 2(a-bp)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E[\pi_m]}{\partial p} &= K - \frac{K^2(a-bw)}{4\,(a-bp)^2} \\
&\implies p^* = \frac{\left(2a - \sqrt{K(a-bw)}\right)}{2b}
\end{aligned}$$

$p^* = \frac{\left(2a - \sqrt{K(a-bw)}\right)}{2b}$ if this $p^*$ satisfies the condition $K \leq 2(a - bp^*) = \sqrt{K(a-bw)}$, which is true for any $K \leq a - bw$. So, if $K \leq a - bw$, the optimal market price for the manufacturer is $p^* = \frac{\left(2a - \sqrt{K(a-bw)}\right)}{2b}$ and for the case $K \geq a - bw$ the optimal retail price is found as follows

$$\begin{aligned} \frac{\partial E[\pi_m]}{\partial p} &= a + 2Bb - 2bp + bw - 2Bb\sigma \\ &\implies p^* = \frac{(a + b(2B(1-\sigma) + w))}{2b} \end{aligned}$$

■

**Proposition 33** *If $w - c \leq 4\sigma B$, the optimal invested capacity for the supplier is $K^* = \frac{16(a-bw)(w-c-B)^2}{9(w-c)^2}$ and the optimal market price is $p^* = \frac{1}{2b}\left(2a - \frac{4(a-bw)(w-c-B)}{3(w-c)}\right)$. However, if $w - c \geq 4\sigma B$, the optimal capacity investment for the supplier is $K^* = (a - bw)$ and $p^* = \frac{(a+b(2B(1-\sigma)+w))}{2b}$.*

**Proof.** We know from proposition 25 that in the optimal solution, $K \leq 2(a - bp)$. Using the results from proposition 32, we conclude that in the optimal solution $K \leq a - bw$ should hold since the maximum realization of demand which is $2(a - bp)$ is at most equal to $a - bw$ due to the pricing decision of the manufacturer. Considering the optimal value of $p$ for the manufacturer as a function of $K$, the supplier solves her own problem to find the optimal amount of capacity and the optimal wholesale price for her. In the region $K \leq a - bw$, supplier's problem is as follows:

$$\begin{aligned} &\implies \max_K \; E[\pi_s] = (w-c)\left(K - \frac{K^2}{2\sqrt{Ka - Kbw}}\right) - BK \\ s.t\; K &\leq (a - bw) \end{aligned}$$

To find the optimal value of $K$ we can use the first order conditions since $E[\pi_s]$ is concave in terms of $K$

$$\begin{aligned} \frac{\partial E[\pi_s]}{\partial K} &= (w-c)\left(1 - \frac{3}{4}\frac{\sqrt{K}}{\sqrt{a-bw}}\right) - \sigma B = 0 \\ &\implies K^d = \frac{16(a-bw)(w-c-\sigma B)^2}{9(w-c)^2} \\ &\implies p = \frac{1}{2b}\left(2a - \frac{4(a-bw)(w-c-\sigma B)}{3(w-c)}\right) \end{aligned}$$

If the above value of $K$ satisfies the condition that $K \leq (a-bw)$, then it is the optimal solution. So, if $K = \frac{16(a-bw)(w-c-\sigma B)^2}{9(w-c)^2} \leq a-bw \Rightarrow 4(w-c-\sigma B) \leq 3(w-c) \Rightarrow w-c \leq 4\sigma B$, the optimal solution is$K = \frac{16(a-bw)(w-c-\sigma B)^2}{9(w-c)^2}$ and $p^* = \frac{1}{2b}\left(2a - \frac{4(a-bw)(w-c-\sigma B)}{3(w-c)}\right)$.Otherwise, the constraint will be binding and the optimal $K^* = (a-bw)$ and the optimal $p^* = \frac{(a+b(2B(1-\sigma)+w))}{2b}$. ■

**Proposition 34** *Manufacturer will set the optimal contract parameters from the solution of Problem*

$$
\begin{aligned}
\max_{w,\sigma} E[\pi_m] &= \frac{16(a-bw)(w-c-\sigma B)^2(w-c+2\sigma B)(a-cb)}{27(w-c)^3 b} \\
&\quad - \frac{32(a-bw)^2(w-c-\sigma B)^3(w-c+2\sigma B)}{81(w-c)^4 b} - \frac{16(a-bw)(w-c-\sigma B)^2 B}{9(w-c)^2} \\
-\pi_{sres} & \\
s.t\ w-c &\leq 4\sigma B \qquad (5.12)
\end{aligned}
$$

**Proof.** From the previous proposition it is found that if $w-c \leq 4\sigma B$, the optimal invested capacity for the supplier is $K^* = \frac{16(a-bw)(w-c-B)^2}{9(w-c)^2}$ and the optimal market price is $p^* = \frac{1}{2b}\left(2a - \frac{4(a-bw)(w-c-B)}{3(w-c)}\right)$. According to these parameters the manufacturer decides to the contract parameters also taking in to account the supplier's reservation profit. Thus, his problem is:

$$
\begin{aligned}
\max_{\sigma,w}\ E[\pi_m] &= (p-w)\left(K - \frac{K^2}{4(a-bp)}\right) - (1-\sigma)BK \\
s.t\ E[\pi_s] &= (w-c)\left(K - \frac{K^2}{4(a-bp)}\right) - \sigma BK \geq \pi_{sres} \\
w-c &\leq 4\sigma B \\
where\ K &= \frac{16(a-bw)(w-c-\sigma B)^2}{9(w-c)^2} \text{ and } p = \frac{1}{2b}\left(2a - \frac{4(a-bw)(w-c-\sigma B)}{3(w-c)}\right)
\end{aligned}
$$

as $\sigma$ increases the manufacturer's objective increases so the supplier's individual rationality constraint should hold as equality. Then, $\sigma BK = (w-c)\left(K - \frac{K^2}{4(a-bp)}\right) - \pi_{sres}$. ■

*Revenue Sharing Contract*

In this part we consider a revenue sharing contract offered by the manufacturer having parameters $(w, \rho)$. With a revenue sharing contract the manufacturer proposes a wholesale price and shares $(1-\rho)$ portion of his revenue.

**Proposition 35** *For given contract parameters $(w, \rho)$ and $K$ the manufacturer sets his retail price $p^* = \frac{1}{b}\left(a - \frac{1}{2}\frac{\sqrt{K(a-a\rho-bw)}}{\sqrt{1-\rho}}\right)$ if $K \leq (a - \frac{bw}{(1-\rho)})$ and $p^* = \frac{(a(1-\rho)+bw)}{2b(1-\rho)}$ if $K \geq (a - \frac{bw}{(1-\rho)})$*

**Proof.** According to Proposition 25 we know that $K \leq 2(a - bp)$ should hold for the optimal solution.

For given contract parameters $(w, \sigma)$ and $K$ the manufacturer sets his retail price as follows For a given contract parameters and $K$ the manufacturer's problem becomes

$$\begin{aligned} \max_p \ E[\pi_m] &= ((1-\rho)p - w)E[\min(D, K)] \\ s.t\ K &\leq 2(a - bp) \end{aligned}$$

$$\frac{\partial E[\pi_m]}{\partial p} = (1-\rho)K - \frac{K^2((1-\rho)a - bw)}{4\,(a-bp)^2}$$

$\frac{\partial^2 E[\pi_m]}{\partial p^2} < 0$ should hold for this function to be concave with respect to $p$ and this condition is satisfied if $\frac{a-bw}{a} > \rho$ holds .

From this equation $p$ is found as $\frac{1}{b}\left(a - \frac{1}{2}\frac{\sqrt{K(a-a\rho-bw)}}{\sqrt{1-\rho}}\right)$ or$\frac{1}{b}\left(a + \frac{1}{2}\frac{\sqrt{K(a-a\rho-bw)}}{\sqrt{1-\rho}}\right)$.If $p = \frac{1}{b}\left(a - \frac{1}{2}\frac{\sqrt{K(a-a\rho-bw)}}{\sqrt{1-\rho}}\right)$ then $\frac{a-bw}{a} > \rho$ should hold since $(a - bp) > 0$. If $p = \frac{1}{b}\left(a + \frac{1}{2}\frac{\sqrt{K(a-a\rho-bw)}}{\sqrt{1-\rho}}\right)$ then$\frac{\sqrt{K(a-a\rho-bw)}}{\sqrt{1-\rho}} > 0 \Longrightarrow \frac{a-bw}{a} > \rho$ however $(a - bp) > 0$ should also hold which will give us the constraint $\frac{a-bw}{a} \leq \rho$ and this creates a contradiction. Thus, the optimal $p^*$ is $p = \frac{1}{b}\left(a - \frac{1}{2}\frac{\sqrt{K(a-a\rho-bw)}}{\sqrt{1-\rho}}\right)$ if $K \leq 2(a-bp) = \frac{1}{\sqrt{1-\rho}}\sqrt{Ka - Ka\rho - Kbw} \Longrightarrow K \leq (a - \frac{bw}{(1-\rho)})$ is satisfied. Otherwise if $K \geq (a - \frac{bw}{(1-\rho)})$ this constraint binds and $K^* = (a - \frac{bw}{(1-\rho)})$ and $p^* = \frac{(a(1-\rho)+bw)}{2b(1-\rho)}$ ■

**Proposition 36** *For given contract parameters $(w, \rho)$ the supplier's optimal capacity is found from the solution of Problem*

$$\begin{aligned} \max_K \ E[\pi_s] &= (w - c + \rho p^*)\left(K - \frac{K^2}{4(a - bp^*)}\right) - BK \\ s.t\ K &\leq (a - \frac{bw}{(1-\rho)}) \\ p^* &= \frac{1}{b}\left(a - \frac{1}{2}\frac{\sqrt{K(a-a\rho-bw)}}{\sqrt{1-\rho}}\right) \end{aligned} \tag{5.13}$$

**Proof.** For given contract parameters $(w, \rho)$ , from proposition 35 we know that for any given capacity level $K$, the optimal market price for the manufacturer is $p^* = \frac{1}{b}\left(a - \frac{1}{2}\frac{\sqrt{K(a-a\rho-bw)}}{\sqrt{1-\rho}}\right)$ if $K \leq (a - \frac{bw}{(1-\rho)})$ and $p^* = \frac{(a(1-\rho)+bw)}{2b(1-\rho)}$ if $K \geq (a - \frac{bw}{(1-\rho)})$.Considering the optimal value of $p$ for the manufacturer as a function of $K$, the supplier solves her own problem to find the optimal amount of capacity and the optimal wholesale price for her.

In the region $K \leq (a - \frac{bw}{(1-\rho)})$ , the supplier's problem is as Problem 5.13.The optimal value of $K$ is the solution of 5.13 ■

**Proposition 37** *Manufacturer will set the optimal contract parameters from the solution of Problem*

$$\begin{aligned}
\max_{w,\rho}\ E[\pi_m] &= ((1-\rho)p - w)\left(K^* - \frac{(K^*)^2}{4(a-bp)}\right) \qquad (5.14)\\
s.t\ E[\pi_s] &= (w - c + \rho p)\left(K^* - \frac{(K^*)^2}{4(a-bp)}\right) - BK^* \geq \pi_{sres}\\
K^* &\leq (a - \frac{bw}{(1-\rho)})\\
0 &< \rho < 1\\
where\ p &= \frac{1}{b}\left(a - \frac{1}{2}\frac{\sqrt{K^*(a-a\rho-bw)}}{\sqrt{1-\rho}}\right)\\
&\quad \textit{and } K^* \textit{ is the solution of Problem 5.13}
\end{aligned}$$

**Proof.** Manufacturer decides to the contract parameters considering $K^*$ and $p^*$ and his problem will be as in Problem 5.14. ■

Chapter 6

# NUMERICAL STUDY

In the previous chapters we consider a two-party supply chain composed of a single manufacturer and a single supplier. We analyze the model in three different scenarios in terms of product demand; price dependent deterministic demand, stochastic demand and price dependent stochastic demand. We consider five types of well-known contracts in our analysis: simple wholesale price contract, bonus contract, linear two part tariff contract, revenue sharing contract and cost sharing contract and determine whether they can achieve channel coordination or not.

Also in price dependent deterministic and in stochastic demand scenarios we introduce a information asymmetry between the parties in a way that the supplier's capacity investment cost is her private information.

In this chapter we perform a numerical analysis to compare the performances of the coordinating and non-coordinating contracts in both symmetric and asymmetric information settings. Also, in the case of information asymmetry we investigate the value of information.

Finally, we compare the performance of a certain contract in different demand settings and investigate how the market variables will be affected with different contracts by the system parameters.

To compare the performances of different contracts we use the term efficiency of a contract which is defined as the ratio of the total channel profit with the contract to the profit of the integrated channel.

$$Eff = \frac{E[\pi_s] + E[\pi_s]}{E[\Pi]}$$

To determine the value of information in asymmetric information scenarios we consider the contracts that can coordinate the channel under symmetric information. We compare the total profit of the channel and the integrated channel profit and define the efficiency of the decentralized channel in the case of information asymmetry. Since, corresponding contracts can coordinate the channel under symmetric information, the efficiency of the

| | Demand Scenarios | | | | |
|---|---|---|---|---|---|
| | Deterministic | | Stoch. | | Stoch. Price D |
| | $c(K) = BK$ | $c(K) = BK^2$ | $c(K) = BK$ | $c(K) = BK^2$ | $c(K) = BK$ |
| Bonus for exo. w | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9648 |
| WholesalePrice | 0.7500 | 0.9996 | 0.8754 | 0.7502 | 0.9527 |
| Linear | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9797 |
| Cost Sharing | 0.7500 | 0.9996 | 1.0000 | 1.0000 | 0.9958 |
| Rev. Sharing | 1.0000 | 1.0000 | 0.9669 | 0.7508 | 0.9527 |

Table 6.1: Performance comparison of the contracts under symmetric information.

channel is directly related with the value of information such that, as the efficiency increases the value of information decreases.

Table 6.1 summarizes the resutls that are found in the previous chapters and shows the efficiencies of the bonus, the simple wholesale price, the linear, the cost sharing and the revenue sharing contracts in different demand scenarios.In Table 6.1 the benchmark that we use is $a = 1000$, $c = 5$, $B = 10$. For exogenous wholesale price setting cases we assume that $w = 35$, for price dependent demand cases we assumed $b = 5$ and for stochastic fixed retail price cases we assume that $p = 70$. It is seen that for price dependent deterministic demand bonus contract can coordinate the channel for exogenous wholesale price cases and linear and revenue sharing contracts can coordinate the channel when wholesale price is endogenous. The efficiency of the cost sharing contract is the same as the wholesale price contract in the price dependent deterministic demand case. From Table 6.1 we can also see that bonus contract, linear contract and cost sharing contract can achieve channel coordination for stochastic demand model when retail price is fixed. Revenue sharing cannot coordinate the channel in this case however it can improve the channel efficiency. Finally we see that for stochastic price dependent demand channel coordination cannot be achieved with the proposed contracts. In this case cost sharing contract is the most efficient contract among the other contracts.

### 6.1 Price Dependent Deterministic Demand

In Chapter 3 we consider deterministic price dependent demand and we determine non-coordinating and coordinating contracts. We find that for exogenous wholesale price case a bonus contract can coordinate the channel. Later, we introduce an information asymmetry between the parties and we determine the optimal bonus contract for the manufacturer to propose when the capacity investment cost of the supplier is her private information.

In Table 6.2 by using a benchmark $a = 1000$, $b = 5$, $c = 5$, $B = 10$, $w = 35$ and $\gamma - \delta = 0.1 - 25$ we compare the retail prices and profits of the parties in the centralized and decentralized channels for different parameters and we determine the effect of the system parameters to the efficiency of decentralized channel when wholesale price is fixed and the supplier's capacity investment cost function is $c(K) = BK$.

Table 6.2 also represents efficiency of the bonus contract under information asymmetry and the gain of the manufacturer (MG) and the gain of the supplier (SG) by adapting bonus contract. We define gain of the manufacturer (M) and the supplier (S) as follows:

$$MG = \frac{\text{Profit of M with contract} - \text{Profit of M without contracting}}{\text{Profit of M with contract}} \times 100$$

$$SG = \frac{\text{Profit of S with contract} - \text{Profit of S without contracting}}{\text{Profit of S with contract}} \times 100$$

Let us focus on the efficiency of the decentralized channel at first. In Table 6.2 we see that as demand, $a$, increases centralized channel profit, the profits of the manufacturer and the supplier in the decentralized case and the efficiency of the decentralized channel increase. On the contrary as the production cost, $c$, or the capacity investment cost, $B$, increase the centralized channel profit, the profits of the manufacturer and the supplier in the decentralized case and the efficiency of the decentralized channel decrease. Also, as price dependency of the demand, $b$, or the wholesale price,$w$, increase again the efficiency of the decentralized channel decrease, however compared to increase in $B$ or $c$ ,the decrease in the efficiency is more significant in this case. The inefficiency of the decentralized channel is directly related to the manufacturer's gain by proposing a coordinating contract since we assumed that the reservation profit of the supplier is equal to her profit in the decentralized case. Thus, we may conclude that it is more beneficial for the manufacturer to offer a bonus contract when the price dependency of the demand and the wholesale price are high when he faces a deterministic price dependent demand and the supplier's capacity investment cost function is $c(K) = BK$.

In Table 6.2 we see that although a bonus contract can coordinate the decentralized channel under symmetric information, it cannot achieve channel coordination under information asymmetry. We see that when supplier's capacity investment cost is her private information, for high $c$ and $B$ and for low $w$ values supplier does not accept bonus contract since her profit would be greater without a contract. We also see that the efficiency of the bonus contract for higher price dependent demand is relatively low thus, we can conclude that the value of information is higher for more price dependent demand cases. Table 6.2 also demonstrates that as the range of the supplier's capacity investment cost decreases the efficiency of the bonus contract under information asymmetry increases. Also, if the value of the supplier's capacity investment cost is closer to the upper limit of the manufacturer's knowledge of $B$ the efficiency of the bonus contract also increases, thus the value of information decreases.

| $Par.$ | Centralized | | Decentralized | | | | Bonus contract with private B | | |
|---|---|---|---|---|---|---|---|---|---|
| | $p_c$ | $\Pi_c$ | $p_d$ | $\pi_m$ | $\pi_s$ | $Eff$ | $MG(\%)$ | $SG(\%)$ | $Eff$ |
| $a$ | | | | | | | | | |
| 2000 | 207.5 | 185280 | 217.5 | 166530 | 18250 | 0.9973 | 0.1192 | 0.0182 | 0.9984 |
| 1000 | 107.5 | 42781 | 117.5 | 34031 | 8250 | 0.9883 | 0.5808 | 0.0403 | 0.9930 |
| 500 | 57.5 | 9031.3 | 67.5 | 5281.3 | 3250 | 0.9446 | 3.6277 | 0.1021 | 0.9670 |
| $b$ | | | | | | | | | |
| 20 | 32.5 | 6125 | 42.5 | 1125 | 3000 | 0.6735 | 41.4126 | 0.4410 | 0.8055 |
| 5 | 107.5 | 42781 | 117.5 | 34031 | 8250 | 0.9883 | 0.5808 | 0.0403 | 0.9930 |
| 0.5 | 1007.5 | 492530 | 1017.5 | 482650 | 9825 | 0.9999 | 0.0041 | 0.0034 | 0.9999 |
| $c$ | | | | | | | | | |
| 20 | 115 | 36125 | 117.5 | 34031 | 2062.5 | 0.9991 | NA | NA | NA |
| 5 | 107.5 | 42781 | 117.5 | 34031 | 8250 | 0.9883 | 0.5808 | 0.0403 | 0.9930 |
| 0.5 | 105.25 | 44888 | 117.5 | 34031 | 10106 | 0.9833 | 0.8818 | 0.8809 | 0.9920 |
| $B$ | | | | | | | | | |
| 20 | 112.5 | 38281 | 117.5 | 34031 | 4125 | 0.9967 | NA | NA | NA |
| 10 | 107.5 | 42781 | 117.5 | 34031 | 8250 | 0.9883 | 0.5808 | 0.0403 | 0.9930 |
| 0.1 | 102.55 | 47483 | 117.5 | 34031 | 12334 | 0.9765 | 0.5808 | 3.8711 | 0.9911 |
| $w$ | | | | | | | | | |
| 100 | 107.5 | 42781 | 150 | 12500 | 21250 | 0.7889 | 28.5851 | 13.9240 | 0.9862 |
| 35 | 107.5 | 42781 | 117.5 | 34031 | 8250 | 0.9883 | 0.5808 | 0.0403 | 0.9930 |
| 16 | 107.5 | 42781 | 108 | 42320 | 460 | 0.9999 | NA | NA | NA |
| $\gamma - \delta$ | | | | | | | | | |
| 0.1 $-10$ | 107.5 | 42781 | 117.5 | 34031 | 8250 | 0.9883 | 1.4385 | 0.0403 | 0.9999 |
| $0.1 - 25$ | 107.5 | 42781 | 117.5 | 34031 | 8250 | 0.9883 | 0.5808 | 0.0403 | 0.9930 |
| $10 - 25$ | 107.5 | 42781 | 117.5 | 34031 | 8250 | 0.9883 | 0.2894 | 2.623 | 0.9958 |

Table 6.2: Effect of system parameters to the decentralized channel efficiency and efficiency of bonus contract under asymmetric information on B for price dependent deterministic demand when w is fixed and c(K)=BK.

In Table 6.3 by using a benchmark $a = 1000$, $b = 5$, $c = 5$, $B = 10$ we analyze the effect of system parameters on efficiency of the decentralized channel for price dependent deterministic demand and fixed w when supplier's capacity investment cost function is $c(K) = BK^2$. In Table 6.3 we see that the efficiency of the decentralized channel is relatively low compared to the case where supplier's capacity investment cost function is $c(K) = BK$. Thus, we may conclude that proposing a coordinating contract is more beneficial for the manufacturer in this case. We also see from Table 6.3 that as demand and production cost of the supplier decrease the efficiency of the channel increases. Table 6.3 also represents that when the wholesale price increase and the demand becomes more sensitive to retail price the efficiency of the decentralized channel also increases.

| | **Centralized** | | **Decentralized Channel** | | | |
|---|---|---|---|---|---|---|
| *Parameter* | $p_c$ | $\Pi_c$ | $p_d$ | $\pi_m$ | $\pi_s$ | $Eff$ |
| $a$ | | | | | | |
| 2000 | 396.13 | 3824.10 | 399.70 | 547.05 | 22.50 | 0.1489 |
| 1000 | 198.09 | 931.98 | 199.70 | 247.05 | 22.50 | 0.2892 |
| 500 | 99.07 | 221.20 | 99.70 | 97.05 | 22.50 | 0.5405 |
| $b$ | | | | | | |
| 20 | 49.89 | 50.37 | 49.93 | 22.39 | 22.50 | 0.8911 |
| 5 | 198.09 | 931.98 | 199.70 | 247.05 | 22.50 | 0.2892 |
| 0.5 | 1833.8 | 82917 | 1997 | 2943 | 22.50 | 0.0358 |
| $c$ | | | | | | |
| 20 | 198.24 | 794.12 | 199.85 | 123.64 | 5.6250 | 0.1628 |
| 5 | 198.09 | 931.98 | 199.70 | 247.05 | 22.50 | 0.2892 |
| 0.5 | 198.04 | 975.50 | 199.66 | 284.03 | 29.76 | 0.3217 |
| $B$ | | | | | | |
| 20 | 199.04 | 470.61 | 199.85 | 123.64 | 11.25 | 0.2866 |
| 10 | 198.09 | 931.98 | 199.70 | 247.05 | 22.50 | 0.2892 |
| 0.1 | 135 | 31688 | 170 | 20250 | 2250 | 0.7101 |
| $w$ | | | | | | |
| 100 | 198.09 | 931.99 | 199.05 | 470.49 | 225.63 | 0.7469 |
| 35 | 198.09 | 931.98 | 199.70 | 247.05 | 22.50 | 0.2892 |
| 16 | 198.09 | 931.99 | 199.89 | 101.14 | 3.03 | 0.1118 |

Table 6.3: Effect of system parameters to the decentralized channel efficiency for price dependent deterministic demand when w is exogenous and $c(K) = BK^2$

For price dependent deterministic demand scenario we also consider the case when wholesale price is also a decision variable. We find that coordination in the channel can be achieved with a linear and revenue sharing contracts, whereas wholesale and cost sharing contracts failed to achieve coordination.

For a model where the supplier's capacity investment cost function is $c(K) = BK$, we determine the manufacturer's and the supplier's profits with optimal wholesale price contract as

$$\pi_m^* = \frac{(a - b(B + c))^2}{16b}, \ \pi_s^* = \frac{(a - b(B + c))^2}{8b} \tag{6.1}$$

From Equation 6.1 we see that as $a$ increases both $\pi_m^*$ and $\pi_s^*$ increase and as $B$ and $b$ increase both $\pi_m^*$ and $\pi_s^*$ decrease since $\frac{\partial \pi_m^*}{\partial B}, \frac{\partial \pi_m^*}{\partial b}, \frac{\partial \pi_s^*}{\partial B}, \frac{\partial \pi_s^*}{\partial b} < 0$.

The profit of the integrated channel is $\Pi_{cent}^* = \frac{(a-b(B+c)^2}{4b}$ .Then, the efficiency of the decentralized channel with the wholesale price contract is

$$Eff = \frac{\pi_m^* + \pi_s^*}{\Pi_{cent}^*} = 3/4$$

which is not affected by the change in system parameters.

Similarly we consider the model where the supplier's capacity investment cost function is $c(K) = BK^2$. For the optimal wholesale price contract the manufacturer's and the supplier's profits are

$$\pi_m^* = \frac{(a - bc)^2}{4b(Bb + 2)^2}, \ \pi_s^* = \frac{(a - bc)^2}{4b(Bb + 2)} \tag{6.2}$$

From Equation 6.2, using the same argument as in the case $c(K) = BK$ we can say that

$a$ increases both $\pi_m^*$ and $\pi_s^*$ increase and as $B$ and $b$ increase both $\pi_m^*$ and $\pi_s^*$ decrease. The profit of the integrated channel is $\Pi_{cent}^* = \frac{(a-cb)^2}{4b(Bb+1)}$. Then, the efficiency of the decentralized channel with the wholesale price contract is

$$Eff = \frac{\pi_m^* + \pi_s^*}{\Pi_{cent}^*} = \frac{(Bb + 1)(Bb + 3)}{(Bb + 2)^2} \tag{6.3}$$

Considering Equation 6.3 we can say that the efficiency of the wholesale price contract does not depend on the demand but it depends on the capacity investment cost of the supplier and price dependency of the demand. Since the first order derivatives of the efficiency function w.r.t. both $B$ and $b$ are positive, as $B$ or $b$ increase the efficiency of the channel will increase.

We also perform a numerical analysis to find the optimal cost sharing contract offered by the manufacturer by setting the supplier's reservation profit to her profit with the wholesale price. We see that the optimal cost sharing contract gives the same profit to both the manufacturer and the supplier as in the wholesale price contract case. Thus, the efficiency of the cost sharing contract is same as the wholesale price contract. As a result, the effect of the system parameters will be the same on the efficiency of the cost sharing contract and the profits of the manufacturer and the supplier.

## 6.2 Stochastic Demand

In Chapter 4 we assumed that the demand is stochastic and retail price is fixed. For our numerical analysis we assume a uniformly distributed demand between 0 and $a$. Thus, $F(x) = \frac{x}{a}$ and $f(x) = \frac{1}{a}$. In the symmetric information setting initially we analyze the model with exogenous wholesale price and we find that a bonus contract can coordinate the channel.

In Table 6.4 using the benchmark $a = 100$, $p = 70$, $c = 5$, $B = 10$, $w = 40$ we analyze the effect of system parameters on the decision variables for stochastic demand setting where wholesale price is fixed and supplier's capacity investment cost function is $c(K) = BK$. From Table 6.4 we see that as demand increases the optimal capacities and expected profits of both the integrated and decentralized channel increase proportionally. Due to this simultaneous increase the efficiency of the decentralized channel does not change. Table 6.4 also represents that as the retail price increases the optimal capacity and the expected profit of both the integrated channel and the manufacturer in the decentralized case increase. However, the optimal invested capacity for the supplier and her profit in the decentralized channel remain constant as $p$ changes since they do not depend on the retail price. We also see that as $p$ decreases the efficiency of the decentralized channel increases since the effect of double marginalization is mitigated as $p$ gets closer to $w$.

It is seen that as the production cost, $c$, decreases the optimal invested capacities and the expected profits increase for both the integrated and decentralized channel. Also, the efficiency of the channel increases as $c$ decreases. Table 6.4 also demonstrates that as capacity investment cost, $B$, decreases the optimal capacities and the expected profits increase for both the integrated and decentralized channel and the efficiency of the decentralized channel rapidly increase. This is due to the fact that the risk of over capacity gets lower for the supplier and she has a tendency to increase her capacity. Finally we analyze the effect of wholesale price for the optimal capacities for the decentralized channel, it is obvious that the change in $w$ does not effect the integrated channel. In the decentralized channel we see that as $w$ decreases the optimal capacity, the efficiency and the expected profits of the manufacturer and supplier also decrease since the difference between the retail price and the wholesale price will increase and the effect of double marginalization gets more significant.

Table 6.5 represents the effect of change in parameters for the case $c(K) = BK^2$ where demand is stochastic and wholesale price is fixed. We used the benchmark $a = 100$, $p = 70$, $c = 5$, $B = 10, w = 40$ for in Table 6.5 .The effect of change in $p$, $c$, $B$ and $w$ in this case is same as the effect of change in those parameters in the case $c(K) = BK$. Compared to the case $c(K) = BK$ the optimal invested capacities and expected profits of the integrated and decentralized channel are relatively low in this case. We also see that as demand increases the efficiency of the decentralized channel will decrease since the change in optimal capacity and expected profits are not proportional to the change in $a$. In Table 6.5 it can be also seen that the effect of change in $B$ is not as significant as in the case $c(K) = BK$. Additionally, for relatively high $B$ values the efficiency of the decentralized channel is higher in the case of $c(K) = BK$ compared to the case of $c(K) = BK^2$.

Table 6.6 represents the effect of change in parameters to the bonus contract offered by the manufacturer when he does not know the exact value of supplier's capacity investment cost, $B$. While analyzing this effect we use the benchmark $\phi = 0.5$, $a = 100$, $p = 70$, $c = 45$, $B_H - B_L = 8 - 5$. The first two columns represents the amount of bonus payment for the low and high-capacity cost supplier respectively. We assumed that the reservation profits of the high-capacity cost and low-capacity cost suppliers are equal to each other and its the low capacity cost supplier's profit in the decentralized channel. From Table 6.6 it can be seen that except for the cases $c = 25$ and $w = 20$ it is optimal for the manufacturer not to offer any menu of contracts. This can be due to relatively high reservation profits and manufacturer prefers not to screen the suppliers since he should give also information rent

| | *Centralized* | | *Decentralized* | | | |
|---|---|---|---|---|---|---|
| *Parameter* | $K_c$ | $E[\Pi_c]$ | $K_d$ | $E[\pi_m]$ | $E[\pi_s]$ | $Eff$ |
| $a$ | | | | | | |
| 1000 | 846.15 | 23269 | 714.29 | 13776 | 8928.60 | 0.9757 |
| 100 | 84.62 | 2327 | 71.43 | 1378 | 892.86 | 0.9757 |
| 10 | 8.46 | 233 | 7.14 | 138 | 89.29 | 0.9757 |
| $p$ | | | | | | |
| 100 | 89.47 | 3803 | 71.43 | 2755 | 892.86 | 0.9593 |
| 70 | 84.62 | 2327 | 71.43 | 1378 | 892.86 | 0.9757 |
| 45 | 75 | 1125 | 71.43 | 230 | 892.86 | 0.9977 |
| $c$ | | | | | | |
| 25 | 77.78 | 1361 | 33.33 | 833 | 83.33 | 0.6735 |
| 5 | 84.62 | 2327 | 71.43 | 1378 | 892.86 | 0.9757 |
| 1 | 85.51 | 2523 | 74.36 | 1401 | 1078.20 | 0.9830 |
| $B$ | | | | | | |
| 30 | 53.85 | 942 | 14.29 | 398 | 35.71 | 0.4602 |
| 10 | 84.62 | 2327 | 71.43 | 1378 | 892.86 | 0.9757 |
| 1 | 98.46 | 3151 | 97.14 | 1498 | 1651.40 | 0.9998 |
| $w$ | | | | | | |
| 60 | 84.62 | 2327 | 81.82 | 483 | 1840.90 | 0.9989 |
| 40 | 84.62 | 2327 | 71.43 | 1378 | 892.86 | 0.9757 |
| 20 | 84.61 | 2327 | 33.33 | 1.389 | 83.33 | 0.6327 |

Table 6.4: Effect of system parameters to the decentralized channel efficiency for stochastic demand when w is exogenous and $c(K) = BK$

| | *Centralized* | | *Decentralized* | | | |
|---|---|---|---|---|---|---|
| *Parameter* | $K_c$ | $E[\Pi_c]$ | $K_d$ | $E[\pi_m]$ | $E[\pi_s]$ | $Eff$ |
| $a$ | | | | | | |
| 1000 | 3.24 | 105.28 | 1.75 | 52.36 | 30.57 | 0.7877 |
| 100 | 3.15 | 102.30 | 1.72 | 51.15 | 30.10 | 0.7942 |
| 10 | 2.45 | 79.72 | 1.49 | 41.35 | 26.06 | 0.8457 |
| $p$ | | | | | | |
| 100 | 4.53 | 215.39 | 1.72 | 102.31 | 30.10 | 0.6147 |
| 70 | 3.15 | 102.30 | 1.72 | 51.15 | 30.10 | 0.7942 |
| 45 | 1.96 | 39.22 | 1.72 | 8.53 | 30.10 | 0.9849 |
| $c$ | | | | | | |
| 25 | 2.20 | 49.51 | 0.75 | 22.25 | 5.58 | 0.5621 |
| 5 | 3.15 | 102.30 | 1.72 | 51.15 | 30.10 | 0.7942 |
| 1 | 3.34 | 115.06 | 1.91 | 56.83 | 37.30 | 0.8181 |
| $B$ | | | | | | |
| 30 | 1.07 | 34.83 | 0.58 | 17.35 | 10.15 | 0.7894 |
| 10 | 3.15 | 102.30 | 1.72 | 51.15 | 30.10 | 0.7942 |
| 1 | 24.53 | 797.17 | 14.89 | 413.54 | 260.64 | 0.8457 |
| $w$ | | | | | | |
| 60 | 3.15 | 102.30 | 2.68 | 26.41 | 73.60 | 0.9776 |
| 40 | 3.15 | 102.30 | 1.72 | 51.15 | 30.10 | 0.7942 |
| 20 | 3.15 | 102.30 | 0.74 | 37.08 | 5.58 | 0.4171 |

Table 6.5: Effect of system parameters to the decentralized channel efficiency for stochatic demand when w is exogenous and $c(K) = BK^2$

to the low cost supplier in addition to her reservation profit. However, for high production cost and low wholesale price values the reservation profits of the suppliers are low thus it is optimal for the manufacturer to offer a menu of contracts to screen the type of the supplier. In Table 6.6 $VI$ stands for the value of information.

$$VI = 1 - \frac{\text{Manufacturer's profit under asymmetric information}}{\text{Manufacturer's profit under symmetric information}}$$

We can claim that when it is more beneficial for the manufacturer to offer a menu of bonus contract to the suppliers (i.e. when $w$ is low or $c$ is high), the value of information increases, when he prefers to offer a menu of bonus contract. This is because manufacturer gives information rent to the low-capacity-cost supplier. It is also seen that value of information is higher when the difference between $B_H$ and $B_L$ is high.

For stochastic demand scenario we also consider the case where wholesale price is also a decision parameter. For endogenous wholesale price cases we determine that linear contract and cost sharing contract can coordinate the decentralized channel, however revenue sharing contract cannot achieve coordination in the channel.

In Table 6.7 we consider the decentralized channel and compare the optimal wholesale price contract and optimal revenue sharing contract offered by the manufacturer, and analyze the effect of change in system parameters to the decision variables for $c(K) = BK$. Table 6.7 shows that the efficiency of the revenue sharing contract is higher than the efficiency of the wholesale price contract in all cases, except for high $B$ values. This is due to the fact that with the optimal revenue sharing contract the supplier has an incentive to increase her capacity up to the maximum amount of demand and this will increase the expected amount of sales consequently the expected profits of the manufacturer and the supplier also increase for low capacity investment costs. However, for significant $B$ values capacity investment costs more, thus the profits of the manufacturer and the supplier decrease. Moreover, due to supplier's reservation profit the manufacturer's profit with the revenue sharing contract for high B values is dramatically lower than his profit with the wholesale price contract. Thus, it is not reasonable for the manufacturer to offer a revenue sharing contract when supplier has high capacity investment cost. Table 6.7 also shows the effect of change in parameters to the system variables and it is seen that this effect is same as in the exogenous wholesale price setting , however in this case that effect is not that significant. For instance, in this case as $B$ increases the decrease in the efficiency of the

| $Parameter$ | $c_{bL}$ | $c_{bH}$ | $E[\pi_s]_L$ | $E[\pi_s]_H$ | $E[\pi_M]$ | $Eff_m$ | |
|---|---|---|---|---|---|---|---|
| $\phi$ | | | | | | | |
| 0.9 | – | – | 1026 | 806.6 | 1625 | 0.059 | DC |
| 0.5 | – | – | 1026 | 806.6 | 1625 | 0.059 | DC |
| 0.2 | – | – | 1026 | 806.6 | 1625 | 0.059 | DC |
| $a$ | | | | | | | |
| 1000 | – | – | 10260 | 8066 | 16250 | 0.059 | DC |
| 100 | – | – | 1026 | 806.6 | 1625 | 0.059 | DC |
| 10 | – | – | 102.6 | 80.66 | 162.5 | 0.059 | DC |
| $p$ | | | | | | | |
| 100 | – | – | 1026 | 806 | 3018.9 | 0.0629 | DC |
| 70 | – | – | 1026 | 806.6 | 1625 | 0.059 | DC |
| 45 | – | – | 1026 | 806.6 | 464 | 0.098 | DC |
| $c$ | | | | | | | |
| 25 | 35 | 24 | 368 | 125 | 1399 | 0.153 | (H,L) |
| 5 | – | – | 1026 | 806.6 | 1625 | 0.059 | DC |
| 1 | – | – | 1223 | 994 | 1653 | 0.045 | DC |
| $B_H$–$B_L$ | | | | | | | |
| $10, 1$ | – | – | 1266 | 666 | 1555 | 0.111 | DC |
| $8, 5$ | – | – | 1026 | 806.6 | 1625 | 0.059 | DC |
| $5, 1$ | – | | 1375 | 1041 | 1701 | 0.027 | DC |
| $w$ | | | | | | | |
| 60 | | | 2264 | 2008 | 489 | 0.014 | DC |
| 35 | – | – | 1026 | 806.6 | 1625 | 0.059 | DC |
| 20 | 50 | 34 | 594 | 333 | 2168 | 0.168 | (H,L) |

Table 6.6: Effect of change in parameters to the bonus contract under under asymmetric information on B for stochastic demand when w is fixed

| | Simple Wholesale Price Contract | | | | | Revenue Sharing Contract | | | |
|---|---|---|---|---|---|---|---|---|---|
| *Parameter* | $w$ | $K_d$ | $E[\pi_m]$ | $E[\pi_s]$ | $Eff$ | $K_d$ | $E[\pi_m]$ | $E[\pi_s]$ | $Eff$ |
| $a$ | | | | | | | | | |
| 1000 | 27.10 | 547.46 | 1706 | 3312 | 0.8754 | 1000 | 19188 | 3312.5 | 0.9669 |
| 100 | 27.10 | 54.75 | 1706 | 331.15 | 0.8754 | 100 | 1918.8 | 331.25 | 0.9669 |
| 10 | 27.10 | 5.47 | 171 | 33.12 | 0.8754 | 10 | 191.9 | 33.1 | 0.9669 |
| $p$ | | | | | | | | | |
| 100 | 30.44 | 60.69 | 2941 | 466.36 | 0.8965 | 100 | 3283 | 466.9 | 0.9862 |
| 70 | 27.10 | 54.75 | 1705 | 331.15 | 0.8754 | 100 | 1918.8 | 331.25 | 0.9669 |
| 45 | 23.34 | 45.47 | 761 | 187.54 | 0.8449 | 100 | 811.75 | 188.25 | 0.8889 |
| $c$ | | | | | | | | | |
| 25 | 44.20 | 47.92 | 940 | 220.48 | 0.8526 | 100 | 1029 | 220.45 | 0.9184 |
| 5 | 27.10 | 54.75 | 1705 | 331.15 | 0.8754 | 100 | 1918.8 | 331.25 | 0.9669 |
| 1 | 23.60 | 55.75 | 1866 | 351.16 | 0.8789 | 100 | 2098.8 | 351.25 | 0.9713 |
| $B$ | | | | | | | | | |
| 30 | 47.81 | 29.93 | 564.65 | 190.71 | 0.8027 | 29.93 | 564.65 | 190.71 | 0.8027 |
| 10 | 27.10 | 54.75 | 1705 | 331.15 | 0.8754 | 100 | 1918.8 | 331.25 | 0.9669 |
| 1 | 10 | 80 | 2880 | 160 | 0.9648 | 100 | 2990 | 160 | 0.9998 |

Table 6.7: Comparison of the wholesale price and revenue sharing contracts for stochastic demand case when c(K)=BK.

decentralized channel is not that noticeable since the manufacturer proposes the wholesale price and he will increase $w$ as $B$ increases.

In Table 6.8 we consider the decentralized channel and compare the optimal wholesale price contract and optimal revenue sharing contract offered by the manufacturer, and analyze the effect of change in system parameters to the decision variables for $c(K) = BK^2$.Table 6.8 shows that the efficiency of the wholesale price contract and revenue sharing contract are almost the same. Revenue sharing contract cannot improve the channel efficiency as in the case $c(K) = BK$ since capacity investment is costly in this case and the

| | Wholesale Price Contract | | | | | Revenue Sharing Contract | | | |
|---|---|---|---|---|---|---|---|---|---|
| $Parameter$ | $w$ | $K_d$ | $E[\pi_m]$ | $E[\pi_s]$ | $Eff$ | $K_d$ | $E[\pi_m]$ | $E[\pi_s]$ | $Eff$ |
| $a$ | | | | | | | | | |
| 1000 | 37.46 | 1.62 | 52.68 | 26.30 | 0.7502 | 1.621 | 52.68 | 26.31 | 0.7503 |
| 100 | 37.11 | 1.58 | 51.56 | 25.37 | 0.7520 | 1.58 | 51.58 | 25.38 | 0.7521 |
| 10 | 34.16 | 1.27 | 42.7 | 18.55 | 0.7684 | 1.27 | 42.7 | 18.55 | 0.7684 |
| $p$ | | | | | | | | | |
| 100 | 51.68 | 2.28 | 108.95 | 53.23 | 0.7530 | 2.28 | 108.95 | 53.23 | 0.7530 |
| 70 | 37.11 | 1.58 | 51.56 | 25.37 | 0.7520 | 1.58 | 51.58 | 25.38 | 0.7521 |
| 45 | 24.85 | 0.98 | 19.71 | 9.76 | 0.7512 | 0.983 | 19.77 | 9.78 | 0.7514 |
| $c$ | | | | | | | | | |
| 25 | 47.31 | 1.10 | 24.89 | 12.31 | 0.7514 | 1.10 | 24.89 | 12.31 | 0.7514 |
| 5 | 37.11 | 1.58 | 51.56 | 25.37 | 0.7520 | 1.58 | 51.58 | 25.38 | 0.7521 |
| 1 | 35.06 | 1.67 | 58.02 | 28.52 | 0.7521 | 1.67 | 58.02 | 28.52 | 0.7521 |
| $B$ | | | | | | | | | |
| 30 | 37.37 | 0.54 | 17.46 | 8.68 | 0.7507 | 0.54 | 17.47 | 8.69 | 0.7508 |
| 10 | 37.11 | 1.58 | 51.56 | 25.37 | 0.7520 | 1.58 | 51.58 | 25.38 | 0.7521 |
| 1 | 34.16 | 12.72 | 427.04 | 185.49 | 0.7684 | 12.72 | 427.04 | 185.49 | 0.7684 |

Table 6.8: Comparison of the wholesale price and revenue sharing contracts for stochastic demand case when $c(K) = BK^2$

supplier does not have an incentive to increase her capacity with a revenue sharing contract. The effect of change in parameters to the system variables and it is seen that this effect is same as in the exogenous wholesale price setting , however in this case that effect is not that significant. For instance, in this case as $B$ increases the decrease in the efficiency of the decentralized channel is not that noticeable since the manufacturer proposes the wholesale price and he will increase $w$ as $B$ increases.

In Table 6.9 we compare the linear contract and cost sharing contract in the case of

| | **Linear Contract** | **Cost SharingContract** |
|---|---|---|
| $\mathbf{E}[\pi_m]$ | 2116.4 | 2460 |
| $\mathbf{E}[\pi_s]_L$ | 930.9 | 233 |
| $\mathbf{E}[\pi_s]_H$ | 219.1 | 219.1 |
| **Eff** | 0.8771 | 0.8806 |
| $\mathbf{Eff}_m$ | 0.7468 | 0.8679 |

Table 6.9: Comparison of Linear and Cost Sharing Contracts in the case of Information Asymmetry.

information asymmetry when $a = 100$, $p = 70$, $c = 5$, $B_H = 10$, $B_L = 2$, $\phi = 0.5$.

From Table 6.9 we see that proposing a menu of cost sharing contract is more beneficial for the manufacturer in this case since $VI$ value is smaller in cost sharing contract. Also, it is seen that low-capacity-cost supplier's profit, $\mathbf{E}[\pi_s]_L$, is significantly lower and high-capacity-cost supplier's profit $\mathbf{E}[\pi_s]_H$ is a little when manufacturer offers a cost sharing contract compared the case when he offers a linear contract.

Table 6.10 gives us the effect of change of parameters on the optimal solution of the asymmetric capacity investment cost case. As a benchmark we assumed $a = 100$, $p = 70$, $c = 5$, $B_H = 10$, $B_L = 2$, $\phi = 0.5$. The first two columns are the wholesale prices for the low-capacity-cost and high-capacity-cost supplier.

Considering Table 6.10 we can claim that as the portion of the high capacity cost supplier's significantly low it is optimal for the manufacturer to offer only low capacity cost supplier, since information rent given to low capacity cost supplier increases significantly as portion of the high capacity cost supplier decreases rapidly.

Similarly, as the difference between the capacity investment costs increases it becomes optimal to offer only low capacity cost supplier because of the same argument of decreasing $\phi$.From Table 6.10 we also see that as $\phi$ gets higher i.e. the portion of the high cost type supplier increases it becomes optimal for the manufacturer to offer a menu of contracts instead of a single contract designed for the low capacity cost supplier. This is due to the fact that manufacturer does not want to miss the portion of high capacity cost suppliers.

It is also seen that as the portion of the high capacity cost supplier increases the efficiency of the menu of contracts decrease since the information rent given to the low capacity cost supplier will increase. Additionally from Table 6.10 we see that the efficiency of the channel rapidly decreases when manufacturer decides to offer only the contract designed for the low capacity cost supplier.

Table 6.10 also illustrates that offering to high capacity cost supplier is never optimal. Also, we can see that manufacturer offers less amount of wholesale price to the high capacity cost supplier. Thus, we can conclude that high capacity cost supplier does not get the efficient value of $w$ i.e. there is distortion in the optimal solution for the screening problem. And as the portion of the high capacity cost supplier decreases this distortion increases. We see that for significantly high retail prices the difference between the capacity investment costs of the high and low cost supplier's becomes negligible thus, the efficiency of the contract is relatively higher. By making the same argument we can claim that as the values of $B_H$ and $B_L$ get closer to each other the efficiency of the menu of contract will increase.

We know that in the symmetric information case coordination in the channel can be achieved with a linear contract. However in the case of information asymmetry linear contract cannot achieve the channel coordination. By looking at $VI$, we can claim that knowing the capacity investment cost is relatively more valuable when the portion of the high capacity cost supplier is high, the demand for the product and the retail price are significantly low and the production cost and the difference between the capacity investment costs of the supplier's are high.

In Table 6.11 we determine the effect of change in parameters in the cost sharing contract under information asymmetry. $w_L$ and $w_H$ are the wholesale prices offered for the low and high capacity cost suppliers. In addition to wholesale price manufacturer also offers to pay $(1-\sigma)$ portion of the supplier's capacity investment cost. Table 6.11 shows that it is optimal for the manufacturer to pay all capacity investment costs of the high-capacity-cost supplier while offering her low wholesale price. Comparing the efficiencies of the linear and cost sharing contracts under information asymmetry we can conclude that it is beneficial for the manufacturer to offer cost sharing contract when he does not know the exact value of the supplier's capacity investment cost. Table 6.11 shows that it is always optimal for the manufacturer to offer a menu of contracts.

| $Parameter$ | $w_L$ | $w_H$ | $E[\pi_s]_L$ | $E[\pi_s]_H$ | $E[\pi_M]$ | $Eff$ | $VI$ |
|---|---|---|---|---|---|---|---|
| $\phi$ | | | | | | | |
| 0.9 | 70 | 66 | 940.7 | 219.1 | $2107.9_{(H,L)}$ | 0.7859 | 0.2562 |
| 0.5 | 70 | 46 | 930.9 | 219.1 | $2116.4_{(H,L)}$ | 0.8771 | 0.2562 |
| 0.2 | 70 | - | 219.1 | - | $2267.1_{(L)}$ | 0.8000 | 0.2000 |
| $a$ | | | | | | | |
| 1000 | 70 | 46 | 9309 | 2191 | $21164_{(H,L)}$ | 0.8771 | 0.2562 |
| 100 | 70 | 46 | 930.9 | 219.1 | $2116.4_{(H,L)}$ | 0.8771 | 0.2562 |
| 10 | 70 | 46 | 93.09 | 21.91 | $211.64_{(H,L)}$ | 0.8771 | 0.2562 |
| $p$ | | | | | | | |
| 100 | 100 | 65 | 922 | 271 | $3523_{(H,L)}$ | 0.9159 | 0.1737 |
| 70 | 70 | 46 | 930.9 | 219.1 | $2116.4_{(H,L)}$ | 0.8771 | 0.2562 |
| 25 | 25 | – | 97.75 | – | $356_{(L)}$ | 0.5000 | 0.5000 |
| $c$ | | | | | | | |
| 25 | 70 | 53 | 808 | 175 | $1198_{(H,L)}$ | 0.8228 | 0.3623 |
| 5 | 70 | 46 | 930.9 | 219.1 | $2116.4_{(H,L)}$ | 0.8771 | 0.2562 |
| 1 | 70 | 45 | 918 | 226 | $2303_{(H,L)}$ | 0.8842 | 0.2387 |
| $B_H{-}B_L$ | | | | | | | |
| $25-2$ | 70 | – | 219.1 | – | $1416_{(L)}$ | 0.5000 | 0.5000 |
| $10-2$ | 70 | 46 | 930.9 | 219.1 | $2116.4_{(H,L)}$ | 0.8771 | 0.2562 |
| $5-2$ | 70 | 42 | 491 | 219.1 | $2550_{(H,L)}$ | 0.9518 | 0.1000 |

Table 6.10: Effect of change in parameters for linear contract under asymmetric information on B for stochastic demand

| $Parameter$ | $w_L$ | $w_H$ | $\sigma_L$ | $\sigma_H$ | $E[\pi_s]_L$ | $E[\pi_s]_H$ | $E[\pi_M]$ | $Eff$ | $VI$ |
|---|---|---|---|---|---|---|---|---|---|
| $\phi$ | | | | | | | | | |
| 0.9 | 11 | 9 | 0.1 | 0 | 225 | 219.1 | 2167 | 0.7858 | 0.2352 |
| 0.5 | 11 | 9 | 0.1 | 0 | 225 | 219.1 | 2460 | 0.8806 | 0.1321 |
| 0.2 | 11 | 9 | 0.1 | 0 | 225 | 219.1 | 2678 | 0.9516 | 0.0547 |
| $a$ | | | | | | | | | |
| 1000 | 11 | 9 | 0.1 | 0 | 2250 | 2191 | 24596 | 0.8806 | 0.1321 |
| 100 | 11 | 9 | 0.1 | 0 | 225 | 219.1 | 2460 | 0.8806 | 0.1321 |
| 10 | 11 | 9 | 0.1 | 0 | 23 | 21.91 | 246 | 0.8806 | 0.1321 |
| $p$ | | | | | | | | | |
| 100 | 12 | 10 | 0.1 | 0 | 275 | 271 | 3896 | 0.9170 | 0.0899 |
| 70 | 11 | 9 | 0.1 | 0 | 225 | 219.1 | 2460 | 0.8806 | 0.1321 |
| 25 | 13 | 7 | 0.4 | 0.1 | 106 | 100 | 427 | 0.6541 | 0.4006 |
| $c$ | | | | | | | | | |
| 25 | 32 | 29 | 0.2 | 0.1 | 180 | 175 | 1523 | 0.8290 | 0.1892 |
| 5 | 11 | 9 | 0.1 | 0 | 225 | 219.1 | 2460 | 0.8806 | 0.1321 |
| 1 | 7 | 6 | 0.1 | 0.1 | 230 | 226 | 2656 | 0.8876 | 0.1223 |
| $B_H - B_L$ | | | | | | | | | |
| $25-2$ | 17 | 9 | 0 | 0.2 | 225 | 219.1 | 1915 | 0.7009 | 0.3243 |
| $10-2$ | 11 | 9 | 0.1 | 0 | 225 | 219.1 | 2460 | 0.8806 | 0.1321 |
| $5-2$ | 10 | 9 | 0.1 | 0 | 225 | 219.1 | 2683 | 0.9529 | 0.0532 |

Table 6.11: Effect of system parameters to the efficiency of cost sharing contract under asymmetric information on B for stochastic demand

### 6.3 Price Dependent Stochastic Demand

In Chapter 5 we consider price dependent stochastic demand scenario when supplier's capacity investment cost function is $c(K) = BK$. Similar to the previous chapters we first consider the exogenous wholesale price case. In Table 6.12 we compare centralized and decentralized channel without contract and decentralized channel with bonus contract using the benchmark $a = 1000$, $b = 5$, $c = 5$, $B = 10$, $w = 35$. From Table 6.12 we see that bonus contract cannot coordinate the channel in this case although it can coordinate the channel in deterministic and stochastic demand with fixed retail price scenarios. Table 6.12 also demonstrates that when demand is low, price dependency of the demand is high, production cost and capacity investment cost of the supplier is low and wholesale price is high, the optimal bonus contract cannot be determined since in those cases the supplier's reservation profit is high and manufacturer manufacturer prefers to stay in the decentralized channel. It can be also determined that proposing a bonus contract is beneficial for the manufacturer when price dependency of demand is low, production cost and capacity investment cost of the supplier is high and wholesale price is low.

| | Centralized | | Decentralized | | | | Bonus Contract | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $Parameter$ | $p_c$ | $E[\Pi_c]$ | $p_d$ | $E[\pi_m]$ | $E[\pi_s]$ | $Eff$ | $p$ | $E[\pi_m]$ | $E[\pi_s]$ | $Eff$ |
| $a$ | | | | | | | | | | |
| 2000 | 210 | 176220 | 237 | 162440 | 9613 | 0.9764 | 284 | 94348 | 54071 | 0.8423 |
| 1000 | 109 | 38675 | 126 | 33196 | 4345 | 0.9707 | 122 | 30967 | 6345 | 0.9648 |
| 500 | 59 | 7351 | 71 | 5151 | 1711 | 0.9336 | - | - | - | - |
| $b$ | | | | | | | | | | |
| 20 | 33 | 3942 | 43 | 1097 | 1580 | 0.6792 | - | - | - | - |
| 5 | 109 | 38675 | 126 | 33196 | 4345 | 0.9707 | 122 | 30967 | 6345 | 0.9648 |
| 0.5 | 1010 | 487630 | 1126 | 470810 | 5175 | 0.9761 | 1066 | 478550 | 6847 | 0.9954 |
| $c$ | | | | | | | | | | |
| 20 | 111 | 32129 | 163 | 16266 | 271 | 0.5147 | 130 | 26821 | 3766 | 0.9520 |
| 5 | 109 | 38675 | 126 | 33196 | 4345 | 0.9707 | 122 | 30967 | 6345 | 0.9648 |
| 0.5 | 109 | 40688 | 121 | 33839 | 6040 | 0.9801 | – | – | – | – |
| $B$ | | | | | | | | | | |
| 20 | 117 | 31376 | 163 | 16266 | 543 | 0.5357 | 142 | 21096 | 2692 | 0.7582 |
| 10 | 109 | 38675 | 126 | 33196 | 4345 | 0.9707 | 122 | 30967 | 6345 | 0.9648 |
| 0.1 | 100 | 47416 | 117 | 34031 | 12293 | 0.9766 | – | – | – | – |
| $w$ | | | | | | | | | | |
| 100 | 109 | 38675 | 150 | 12500 | 18750 | 0.8190 | – | – | – | – |
| 35 | 109 | 38675 | 126 | 33196 | 4345 | 0.9707 | 122 | 30967 | 6345 | 0.9648 |
| 16 | 109 | 38675 | 188 | 2194 | 4.5 | 0.0569 | 122 | 32744 | 4567 | 0.9648 |

Table 6.12: Effect of system parameters for price dependent stochastic demand and fixed w

| | $WholesalePrice$ | $Linear$ | $Cost\ Sharing$ | $RevenueSharing$ |
|---|---|---|---|---|
| $p$ | 129 | 123 | 115 | 192 |
| $K$ | 593 | 732 | 808 | 38 |
| $E[\pi_m]$ | 33486 | 34531 | 35135 | 33486 |
| $E[\pi_s]$ | 3374 | 3374 | 3394 | 3374 |
| $Eff$ | 0.9527 | 0.9797 | 0.9958 | 0.9527 |

Table 6.13: Performance comparison of simple wholesale, linear and cost sharing contracts for price dependent stochastic demand.

In Chapter 5 we also assume that w is a decision variable. In Table 6.13 we compare the decision variables, the profits of the agents with wholesale price, linear, cost sharing and revenue sharing contract offered by the manufacturer using a benchmark $a = 1000, b = 5, c = 5, B = 10$. In our analysis we assumed that in the decentralized channel manufacturer offers a wholesale price contract to the supplier since he has the bargaining power. While finding the optimal contract parameters for linear, cost sharing and revenue sharing contracts we assume that supplier's reservation profit equals to her profit with wholesale price contract.

Table 6.13 shows that the most efficient contract in this case is cost sharing contract, which also gives the most profits to both the manufacturer and the supplier. From Table 6.13 we can also see that channel efficiency and the profits of the agents with revenue sharing contract is the same as their values with a wholesale price contract. However, with a revenue sharing contract manufacturer's optimal retail price is higher and corresponding to that the supplier's optimal capacity investment is lower with a revenue sharing contract compared to simple wholesale price contract. The efficiency of the linear contract is not as good as the cost sharing contract in this case, however its better compared to the efficiency of the wholesale price contract.

In Table 6.14 we compare the centralized case and the simple wholesale price contract when demand is price dependent and stochastic using the benchmark $a = 1000, b = 5, c = 5, B = 10$ . Table 6.14 demonstrates that for high $a$ values the retail price,the invested

| | Centralized | | | Decentralized (Wholesale Contract) | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $a$ | $p_c$ | $K$ | $E[\Pi_c]$ | $w$ | $p$ | $K_d$ | $E[\pi_m]$ | $E[\pi_s]$ | $Eff$ |
| 2000 | 212 | 1788 | 176230 | 36 | 235 | 1482 | 162530 | 10346 | 0.9810 |
| 1000 | 111 | 800 | 38692 | 32 | 129 | 593 | 33486 | 3374 | 0.9527 |
| 500 | 60 | 320 | 7368 | 27 | 72 | 201 | 5718 | 859 | 0.8927 |
| $b$ | | | | | | | | | |
| 20 | 35 | 400 | 4000 | 23 | 41 | 195 | 2537 | 539 | 0.7692 |
| 5 | 111 | 800 | 38692 | 32 | 129 | 593 | 33486 | 3374 | 0.9527 |
| 0.5 | 1012 | 977 | 487630 | 42 | 1046 | 928 | 478520 | 8374 | 0.9985 |
| $c$ | | | | | | | | | |
| 20 | 119 | 727 | 32404 | 46 | 136 | 530 | 27779 | 2935 | 0.9479 |
| 5 | 111 | 800 | 38692 | 32 | 129 | 593 | 33486 | 3374 | 0.9527 |
| 0.5 | 109 | 822 | 40688 | 27 | 127 | 612 | 35305 | 3507 | 0.9539 |
| $B$ | | | | | | | | | |
| 20 | 119 | 664 | 31392 | 50 | 143 | 421 | 24485 | 3633 | 0.8957 |
| 10 | 111 | 800 | 38692 | 32 | 129 | 593 | 33486 | 3374 | 0.9527 |
| 0.1 | 102 | 973 | 47434 | 5.4 | 102 | 973 | 47336 | 96 | 0.9999 |

Table 6.14: Effect of change in parameters for integrated channel and simple wholesale price contract

capacity and total chain profit in the centralized channel and the manufacturer's and supplier's profit and efficiency of the contract is also high. It is also seen that efficiency of the contract and the profits of the manufacturer and supplier is noticibly low when demand is highly price dependent.

| | $LinearContract$ | | | | | | |
|---|---|---|---|---|---|---|---|
| $a$ | $w$ | $t$ | $p$ | $K$ | $E[\pi_m]$ | $E[\pi_s]$ | $Eff$ |
| 2000 | 43 | -7403 | 223 | 1745 | 164960 | 1034 | 0.9948 |
| 1000 | 40 | -4375 | 123 | 732 | 34531 | 3374 | 0.9797 |
| 500 | 34 | -1890 | 71 | 252 | 6007 | 859 | 0.9320 |
| $b$ | | | | | | | |
| 20 | 26 | -460 | 41 | 239 | 2702 | 539 | 0.8105 |
| 5 | 40 | -4375 | 123 | 732 | 34531 | 3374 | 0.9797 |
| 0.5 | 45 | -1400 | 1022 | 976 | 479150 | 8374 | 0.9998 |
| $c$ | | | | | | | |
| 20 | 50 | -4.064 | 130 | 657 | 28707 | 2935 | 0.9765 |
| 5 | 40 | -4375 | 123 | 732 | 34531 | 3374 | 0.9797 |
| 0.5 | 36 | -4467 | 121 | 755 | 36386 | 3507 | 0.9805 |
| $B$ | | | | | | | |
| 20 | 63 | -7866 | 140 | 526 | 25707 | 3633 | 0.9347 |
| 10 | 40 | -4375 | 123 | 732 | 34531 | 3374 | 0.9797 |
| 0.1 | 5.4 | 0 | 102 | 973 | 47336 | 96 | 0.9999 |

Table 6.15: Effect of change in parameters in Linear Contract when demand is price dependent and stochastic

In Table 6.15 we analyze the effect of change in parameters for linear contract using the benchmark $a = 1000, b = 5, c = 5, B = 10$. It is seen that the efficiency of the linear contract is higher than the efficiency the wholesale price contract. Also, when price dependency of the demand is low, $b = 0.5$ the efficiency of linear contract is significantly high, which is reasonable since linear contract coordinates the channel when demand is stochastic. Table 6.15 also demonstrates that manufacturer takes $t$ from the supplier instead of giving. This is because the manufacturer proposes higher wholesale prices in this case.

In Table 6.16 we investigate the effect of parameters to the decision variables when cost sharing contract is proposed. We used the benchmark $a = 1000, b = 5, c = 5, B = 10$ again.

| | *Cost Sharing Contract* | | | | | | |
|---|---|---|---|---|---|---|---|
| *Par.* | $w$ | $\sigma$ | $p$ | $K$ | $E[\pi_m]$ | $E[\pi_s]$ | $Eff$ |
| $a$ | | | | | | | |
| 2000 | 28.60 | 0.62 | 217.44 | 1794 | 165500 | 10409 | 0.9982 |
| 1000 | 22.60 | 0.50 | 115.33 | 808 | 35135 | 3394 | 0.9958 |
| 500 | 17.30 | 0.42 | 63.69 | 318 | 6447.7 | 860.74 | 0.9919 |
| $b$ | | | | | | | |
| 20 | 12.50 | 0.34 | 36.33 | 398 | 3405 | 544.56 | 0.9876 |
| 5 | 22.60 | 0.50 | 115.33 | 808 | 35135 | 3394 | 0.9958 |
| 0.5 | 39.30 | 0.86 | 1020.60 | 978 | 479170 | 8382 | 0.9999 |
| $c$ | | | | | | | |
| 20 | 37 | 0.49 | 122.65 | 734 | 29295 | 2960 | 0.9954 |
| 5 | 22.60 | 0.50 | 115.33 | 808 | 35135 | 3394 | 0.9958 |
| 0.5 | 18.40 | 0.51 | 113.42 | 825 | 37000 | 3521 | 0.9959 |
| $B$ | | | | | | | |
| 20 | 29.10 | 0.40 | 123.88 | 677 | 27506 | 3638 | 0.9921 |
| 10 | 22.60 | 0.50 | 115.33 | 808 | 35135 | 3394 | 0.9958 |
| 0.1 | 5.40 | 1.00 | 102.70 | 973 | 47336 | 97 | 0.9999 |

Table 6.16: Effect of change in parameters with cost sharing contract.

We see that the efficiency of the cost sharing contract is relatively high compared to linear and wholesale price contracts. In Table 6.17 we consider the revenue sharing contract under price dependent stochastic demand setting using the same benchmark and we see that the efficiency of the revenue sharing contract is the same as the efficiency of the wholesale price contract.

| | **Revenue Sharing Contract** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Par.** | **w** | $\rho$ | $p$ | **K** | $\mathbf{E}[\pi_m]$ | $\mathbf{E}[\pi_s]$ | $Eff$ |
| $a$ | | | | | | | |
| 2000 | 50 | 0.74 | 208.34 | 919.90 | 162530 | 10346 | 0.9810 |
| 1000 | 50 | 0.74 | 192.46 | 38.40 | 33486 | 3374 | 0.9527 |
| 500 | 50 | 0.48 | 97.33 | 19.20 | 5.718 | 860 | 0.8927 |
| $b$ | | | | | | | |
| 20 | 49 | 0 | 48.80 | 48 | 2537 | 539 | 0.7692 |
| 5 | 50 | 0.74 | 192.46 | 38.40 | 33486 | 3374 | 0.9527 |
| 0.5 | 50 | 0.90 | 500.25 | 299.90 | 478520 | 8374 | 0.9985 |
| $c$ | | | | | | | |
| 20 | 50 | 0.74 | 192.46 | 38.40 | 27779 | 2935 | 0.9479 |
| 5 | 50 | 0.74 | 192.46 | 38.40 | 33486 | 3374 | 0.9527 |
| 0.5 | 50 | 0.74 | 192.46 | 38.40 | 35305 | 3507 | 0.9539 |
| $B$ | | | | | | | |
| 20 | 50 | 0.74 | 192.46 | 38.40 | 24485 | 3633 | 0.8957 |
| 10 | 50 | 0.74 | 192.46 | 38.40 | 33486 | 3374 | 0.9527 |
| 0.1 | 50 | 0.74 | 192.46 | 38.40 | 47336 | 97 | 1.0000 |

Table 6.17: Effect of change in parameters with revenue sharing contract when demand is price dependent and stochastic.

# Chapter 7

## CONCLUSIONS

Capacity investment has a significant role in supply chains. Suppliers have a tendency for low capacity installations since they have to take the risk of over capacity and this cause shortages in the channel. In decentralized supply chains, the inefficiency caused by the fact that each party in the supply chain makes decisions in order to optimize its own objective, can be eliminated with the help of supply chain contracts which enable sharing the risk between the buyer and supplier. Information sharing between the parties also plays an important role in the supply chains. In practice, full information between the parties in the channel case is rare. This is either because of information sharing is difficult and costly or because of the party having the superior information has an incentive to keep it private. Parties in a supply chain may have asymmetric information in terms of many parameters such as demand information, production costs and capacity investment costs.

The aim of this study is to mitigate the inefficiency of the decentralized chain caused by limited capacity and double marginalization, by considering five well-known contracts, namely, simple wholesale price contract, bonus contract, linear contract, cost sharing contract and revenue sharing contract. While analyzing these contracts we do not specifically look for a coordinating contract but focus on the non-coordinating contracts, as well as the coordinating ones, find the optimal contract parameters and compare the performances of different contracts with each other. We also investigate how the market variables will be effected with different contracts by the system parameters and compare the performances of the contracts for the asymmetric information models and analyze the value of information in our models. In this thesis, we consider a two-party supply chain composed of a single manufacturer and a single supplier where supplier invests on her capacity much before the manufacturer gives his order.

At first we consider that the manufacturer faces a price dependent deterministic demand where he sets the retail price for the end product for both exogenous and endogenous wholesale price cases. For the exogenous wholesale price case we determine that the bonus contract can coordinate the channel. Also, we see that for the case $c(K) = BK$ when demand is highly sensitive to the retail price and the wholesale price of the product is high the efficiency of the decentralize channel is low. Thus, in those cases it is more beneficial for the manufacturer to offer a bonus contract. Later we introduce a information asymmetry in the channel where the manufacturer does not know the exact value of the supplier's capacity investment cost. In that case bonus contract fails in coordinating the channel. We also see that the efficiency of the bonus contract for higher price dependent demand is relatively low thus, we can conclude that the value of information is higher for more price dependent demand cases. We then look in to the case in which supplier's capacity investment function is $c(K) = BK^2$. In that case we see that the efficiency of the decentralized channel is relatively low compared to the case where supplier's capacity investment cost function is $c(K) = BK$. Thus, proposing a coordinating contract is more beneficial for the manufacturer in this case. Additionally we assume that the wholesale price is also a decision variable and we determine that linear and revenue sharing contracts can coordinate the channel however cost sharing contract cannot coordinate the channel and gives the same efficiency as the decentralized channel.

Secondly, we focus on the stochastic demand models. For the exogenous wholesale price cases we find that bonus contract can still coordinate the channel. However it fails to coordinate the channel under information asymmetry. We determine offering a menu of bonus contract is beneficial for the manufacturer when wholesale price is low or production cost is high. Otherwise it is more beneficial for the manufacturer not to offer any contract to the suppliers. For the stochastic demand model when the wholesale price is endogenous, we find that the linear contract and the cost sharing contracts can coordinate the channel, whereas the revenue sharing contract cannot achieve channel coordination. We compared the efficiency of the revenue sharing contract with the efficiency of the decentralized channel and we find that when supplier's capacity investment cost function is $c(K) = BK$ proposing a revenue sharing contract is beneficial in general except for the case of high capacity investment costs. Although the revenue sharing contract can improve the efficiency of the decentralized channel for the case $c(K) = BK$ we determine that for $c(K) = BK^2$ it cannot improve the channel efficiency.

We also consider information asymmetry for endogenous wholesale price case and we find that proposing a cost sharing contract is beneficial for the manufacturer when he does not know the exact value of the supplier's capacity investment cost. Additionally, it is never optimal for the manufacturer to offer only high-capacity -cost supplier under information asymmetry.

After considering the stochastic demand model we analyze the case when manufacturer faces a price dependent stochastic demand. We find that channel coordination in this case cannot be achieved. For the exogenous wholesale price case offering a bonus contract improves the channel efficiency when price dependency of demand is low, production cost and capacity investment cost of the supplier is high and wholesale price is low. For the adjustable wholesale price case, we determine that cost sharing contract is the most efficienct contract. We find that both linear and cost sharing contracts can almost coordinate the channel for really low price dependent demand and we find that revenue sharing contract gives the same channel efficiency and profit distribution between the agents as the decentralized case with the wholesale price contract.

In our study we assume that the manufacturer has the bargaining power and offers the contract. As a further research area, the case in which the supplier is the powerful agent can also be considered. Also a model composed of a single manufacturer and multiple suppliers that are producing a complementary products for the end product can be considered again under both symmetric and asymmetric information.

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