AGRICULTURAL PLANNING PROBLEMS WITH HARVEST, YIELD AND DEMAND UNCERTAINTY

by

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ABSTRACT

In this study, we focus on an agricultural planning problem under demand, yield and harvest uncertainties. We present a model that captures variations of the harvest period, the yield, and the demand. By using the proposed model, we consider finding both the optimal seeding areas and seeding times for annual plants and only the optimal seeding areas for perennial plants.

We study two versions of the problem for annual and perennial plants, depending on the number of periods in the planning horizon: single-period case, and multi-period case. We establish optimal solution for both annual and perennial plants for single-period case. When the number of suppliers increases, the computational complexity of the objective function also increases, and finding the optimal solution becomes computationally demanding. Hence, we propose a normal approximation for the supply. The approximation provides results very close to the optimal solution and the deviation from the optimal solution becomes negligible as the number of suppliers increases. In the multi-period case the optimal solution is found for the planning problem of perennial plants. However, like in the single period problem, the computational complexity of objective function increases as the number of suppliers increases. Thus, we develop efficient solution procedures to solve large-sized problem instances. Numerical experiments show that these procedures are quite accurate. For solving the multi-period planning problem of annual plants, we proposed several heuristics. We present numerical analysis that compares different approaches with each other and with the optimal solution. The best performing two approaches are selected and compared based on different criteria.

Finally, we study a case from the industry. The problem is modeled by using the presented model and solved by using two introduced approaches. We observed that the proposed solution methodology yields significant improvements in the objective function compared to the case

where a deterministic planning approach is used. We compare results of both approaches and explore the effect of system parameters in detail.

ÖZETÇE

Bu çalışmada talep, hasat ve verimin belirsiz olduğu durumlarda tarımsal planlama problemini ele aldık ve bu riskleri göz önüne alan bir model oluşturduk. Önerilen modeli kullanarak yıllık bitkiler için en iyi dikim alanları ve dikim zamanlarını, çok yıllık bitkiler için ise sadece en iyi dikim alanlarını bulmayı hedefledik.

Planlama periyodunun uzunluğuna bağlı olarak problemleri tek periyotluk problemler ve çok periyotluk olarak iki grupta inceledik. Tek periyotluk problemlerde tek yıllık ve çok yıllık bitkiler için en iyi çözümü bulduk. Tedarikçi sayısının arttığı durumlarda, hedef fonksiyonun hesaplanması çok uzun zaman alabilmektedir. Bu sebeple, toplam arz miktarını normal dağılıma sahip olduğunu varsayan bir yaklaşım geliştirdik. Bu yaklaşım en iyi sonuça oldukça yakın sonuçlar vermektedir, en iyi sonuçtan olan sapma tedarikçi sayısı artıkça yok sayılabilecek bir değere düşmektedir. ok periyotluk problemlerde ise, çok yıllık bitkilerin planlama probleminde en iyi sonuc bulunabilmektedir. Ancak, tek periyotlu problemlerde de olduğu gibi hedef fonksiyonun hesaplama zorluğu tedarikçi sayısına bağlı olarak artmaktadır. Bu nedenle, tedarikçi saysn büyük olduğu problemler için etkili çözüm yaklaşımları geliştirdik. İncelediğimiz sayısal örnekler çözüm yaklaşımlarının doğruluğunu göstermektedir. Yıllık bitkilerin çok periyotluk probleminin çözümü için de çeşitli yaklaşımlar önerildi. Bu yaklaşımları birbirleriyle ve en iyi sonuçla karşılaştıran sayısal analizlere yer verildi. Çözüm yaklaşımlarından en iyi sonuç veren iki tanesi seçildi ve farlı kriterlerine göre kıyaslandı.

Son olarak, kiraz üretimi yapan bir üretici ile ilgili bir vaka çalışması incelendi. Problemi önerilen model kullanılarak modelledik ve sunulan çözüm yaklaşımlarından ikisi ile çözdük. Bu iki yaklaşımın sonuçları karşılaştırıldı ve önerilen çözüm yaklaşmnn rassallığı göz önüne almayan çözümlere göre hedef fonksiyonu önemli ölçüde iyileştirdiği gözlemlendi. Ayrıca her iki yaklaşım için de sistem parametrelerinin hedef fonksiyondaki etkileri araştırıldı.

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To my family

Chapter 1

INTRODUCTION

The agricultural supply chain includes all the activities from production to distribution that bring the agricultural products from farm to the table. It is a network of the organizations working together to fulfill a customer request, as any other supply chain. The agricultural supply chain differs from other supply chains due to the uncertainties caused by weather, yield, harvest and demand and the importance of food quality and safety.

Matching supply and demand is an important problem in all industries. In agricultural supply chains, this problem gets complicated by the biological nature of the production process. Production planning decisions are made months before the actual demand is realized, with almost no flexibility to change the production system. In addition, the quantity produced is affected by the environmental conditions like weather and properties of soil, etc.. The weather is one of the factors that influence the growth of the seed, thereby the length of maturation. The weather conditions also affect the harvest period, during which crops are gathered. For instance, the frost or the rain during the maturation period or the harvest period can destroy all crops. Likewise undesirable weather conditions can draw out the growing season and cause the harvest season to start late. Furthermore, all through the harvest the production quantity is random due to the random yield. The yield of a farm highly depends on the weather and farm characteristics.

In this thesis we are going to work on the production planning in a premium fresh produce supply chain. We consider the planning problem of a company, that offers the fresh product with consistent high quality, superior taste and full traceability, and supplies the goods through contract farming. Providing a specific product enables the company to compete effectively in commodity markets. On the other hand, the product has limited demand, due to the premium price or unique properties that appeal to only selected segment of customers. Furthermore, excess production can not be sold in alternate channels, the oversupply should go to waste, to protect the product brand image. In premium product supply chains, high product value and limited market demand create greater incentives to avoid under or over-supply situations.

The production planning problem is challenging considering the uncertainties in agriculture. The supply is random due to uncertain yields, maturation and harvest durations. In addition, the demand is also random and unsatisfied demand will be lost. This thesis suggests a production planning approach that will take yield, harvest and demand uncertainties into account.

1.1 Background

1.1.1 Contract Farming

The advanced agro-food sector consists of the interrelated activities of planting, harvesting, storing and distributing the goods while the traditional view of agribusiness considers only activities in the farms. This change brings the need of vertical integration of the supply chains with allied industries; it is not enough to organize only the production anymore, processing and distribution activities should also be considered. For the agricultural system, one of the ways to obtain well-integrated supply chain is ‘contract farming’.

Contract farming has been defined as an agreement between farmers and firms for the production and the supply of the agricultural products under forward agreements, frequently at predetermined prices [1]. Under the contract system, the farmer agrees to supply products according to the specifications of the contract in terms of quantity, quality, price, and time.

The use of contracts improves farmers’ access to the markets, especially for small farmers. In addition, it decreases the risk of farmers by supplying input and market outlet, thereby favors the income stability. The agribusiness firms can offer technological and management assistance to the farmers or can give impetus to the long term production plans, which support the production of higher valued crops. The contract farming also provides more reliable production in terms of quality, quantity and timing which is in favor of the firms [1], [2].

Contract farming has been in existence for many years; first contracts were employed for sugar production in Taiwan after 1885 and for banana production in Central America in the early 1920s. It has become an important part of food and fiber production in Western Europe by the late twentieth century and since 1930s it has been widely used in vegetable canning industry in North America and in seed industry in Western Europe [3].

Although, contracts have been used for a long time widely in agricultural environment, the interest of policy makers, researchers and development planners in contract farming has increased considerably in latest years. The recent technological developments, changing consumer preferences, trade liberalization, financial capital mobility, food safety issues and regulations, and the advances in biotechnology have provided the drive to usage of contract farming [2].

1.1.2 Agriculture in Turkey

Turkey has 270 000 sq. km agricultural area, with around 4 million farms. 23.67% of the population work in agriculture in 2008[4].

The modern food industry in Turkey has begun with the establishment of the first sugar factory in Afyon in 1926. Although considerable progress has been achieved in agriculture with the annual programs in 1960s and with structural adjustment programs after 1980, the desired level has not been achieved in this industry. The share of food supplied by processing is around 20%, in comparison to the 60% share in the developed countries.

The first contracts in Turkey were used in sugar beet production with the start of food industry. Since the establishment of the first sugar factory, all the sugar beat production has been done under contracts [3].

The second major use of contract farming in Turkey is growing tomatos. After China and the USA, Turkey is the third biggest tomato producing country in the world, between 35,000 and 40,000 farm families produced about 6.8 million tons of tomato in 2006 and the biggest part of the production was supplied from Marmara Region. Most of the tomato production for the industry is supplied through contract farming; and according to a research, the productivity has increased and net profit of the farms has raised by 19% with the contract farming in Çanakkale province [5].

Some current example companies that use contract farming in Turkey are Tukas Food Company, Tat Food Company, Anadolu Efes Brewing Company. Figure 1.1 shows the companies that employ contract farming, and its products.

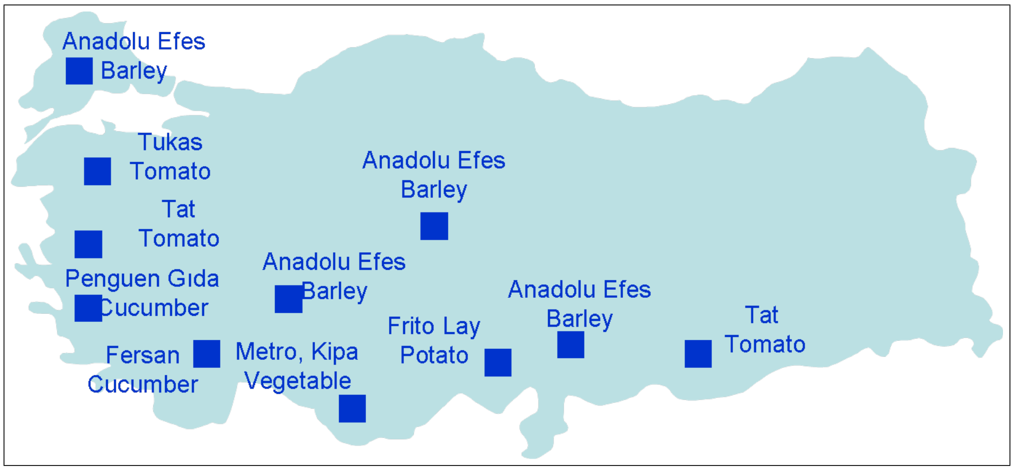

Figure 1.1: Contract Farming Practices in Turkey.

In our specific case, we consider the planing problem of one company that works with multiple farms. The firm is the supply chain integrator, it gives the proprietary seeds to the growers for free and pays for the production. According to the contract, the company pays for all the goods produced in the contracted farms. Once the products leave the field, the company takes their ownership till they are sold to the retailer. After the fruits are harvested, they are sent to the retailer distribution centers from where they are shipped to markets. In the case of excess production, since the product is perishable, the company can not keep it as an inventory. They are wasted before sending them to distribution centers, which saves the transportion and packing costs.

The company aims to supply fresh, high quality goods to its customers throughout its planning period. Due to the quality constraints, the demand can not be satisfied by outsourcing. Since the company keeps no inventory, the demand for the period should be satisfied with the fruits harvested for that period. The unsatisfied demand will be lost. The production quantity for a period consists of the total production in all farms harvested in that period. The length of the harvest period of a farm is limited and varies for each farm. Also, the time of harvest season differs based on the location of the farm. The company needs to use different farms in different locations to provide supply to its customers during the whole planning period. However, while planning the production period, the uncertainties in starting and ending times of the harvest period should be also considered. For instance, depending on the weather, the harvest period can start late or end early. This would cause the company have no supply or less supply then desired and consequently the demand can be lost.

Another issue taken into account in the planning problem, is the randomness of the yield. The yield is highly dependent on weather and growing conditions of the seeds. When the realization of the yield is lower then expected, it causes the company to lose demand. On the other hand, if the yield realization is higher then expected, the excess production is salvaged, thereby the company loses money.

The objective of this study is to develop a method that matches the total supply and the total demand such that the total profit is maximized. The demand and the supply are random, so the exact match of the demand and supply is not possible at all times. Instead, we try to find the optimal production decisions that maximizes the expected profit by considering all relevant costs and uncertanities. We study methods for two different type of plants. First type is referred as annual plants. They survive for only one growing season. In the planning problem of this type of plants, the decision variables are area of each farm and the seeding times of crops in these farms. Second type of plants is referred as the perennial plants. They live more then two years. In the case of perennial plants, the only decision variable is seeding area of utilized farms that must be contracted for the planning period. We analyze both single period and multi period planning problem for these two types of plants.

The remainder of the thesis is organized in the following order: In the following chapter the related literature is reviewed. In Chapter 3 the detailed description of the problem is given and the model for production planning of multiple farms under yield, demand and harvest uncertainty is presented. The planning problem in a single period and the solution of this problem is in Section 4. Section 5 presents multi period planning problem and the several approaches to solve this problem. In Section 6, a case in cherry farming is studied. Conclusions are presented in Section 7 along with future research directions.

Chapter 2

LITERATURE SURVEY

There are two streams of literature relevant for this thesis: planning problems of the agricultural supply chain and random yield problems. In this chapter, first we review the production planning models in agri-food supply chains and then we study the production planning problems in the existence of random yield.

2.1 Production Planning in Agricultural Supply Chains

Production planning problem in agricultural supply chains has received a great deal of attention lately. Ahumada and Villalobas [6] provide comprehensive review of studies conducted in this area. They review the models and organize them based on different factors. For instance, from the perspective of storability of the products, the papers are grouped into two; papers focus on the perishable products and papers focus on nonperishable products. From the perspective of modeling approaches the papers are divided into two: deterministic and stochastic. They also classify the papers according to the scopes: strategic, tactical, and operational planning. Models for strategic planning study the problems such as financial planning, selection of farming technology and equipment, design of supply networks, and crop rotation strategies. On the other hand tactical models focus on the short to medium term decisions such as crop planning, harvesting and planting policies and operational models concern with the harvesting plans, equipment scheduling. Figure 2.1 presents the factors used to organize the review.

The scope of our work is tactical, and we study the medium-term planning problem of perishable products by using a stochastic modeling approach. We first review the models that deals with the planning decisions of the perishable products.

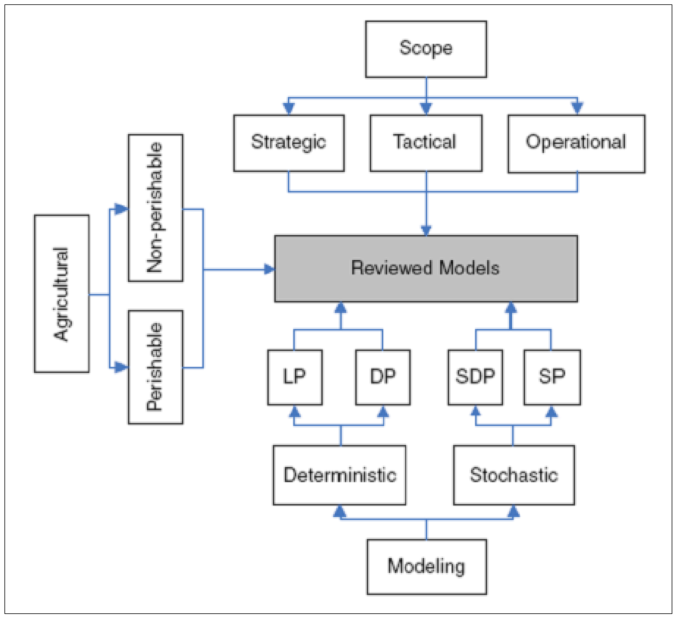

Figure 2.1: Factors used to organize the review of Ahumada and Villalobas

Hamer[7] studies the single period planting problem of a vegetable crop, assuming the demand is known in advance. An LP-based approach is used to determine the best planting and scheduling decisions. Darby-Dowman et al.[8] study the same problem and used a two-stage stochastic programming with a revenue maximizing objective. In order to control the uncertainty caused by weather, a sample of 31 weather patterns and yield profiles are considered. In the first stage a planting plan is found which is common to all scenarios, that includes the decisions as the area, spacing and timing of planting. In the second part harvest plan is studied for each scenario. The objective function of the model presented is composite function of expected profit over scenarios and a risk term representing the variation in profits between scenarios as measured by mean absolute deviation. The weights of these two terms are determined by the risk aversion coefficient.

Kazaz[9] considers production planning problem of a company that produces olive oil in Turkey in the case of random yield and demand. The problem is modeled as a single period problem and a two stage stochastic model with objective of maximizing the expected profit is developed. The company determines the number of trees to lease in the first stage, and determines the amount of olive oil to produce and olives to buy from farmers considering the realized yield and the prices of olives in the second stage. The paper differs form previous studies of random yield in the sense that it defines the sale price and purchasing cost as a function of yield.

Allen and Schuster [10] develop a mathematical model to control the harvest risk in a case study of grape by deciding the harvest scheduling. The objective is to minimize the losses in crops caused by weather and thereby, reduce the investment costs of installing excess capacity. The model includes a joint probability distribution to represent risks associated with the harvest length and crop size. The proposed model provides lower cost solutions compared with the current policies in cases which involve variability in the length of the harvest season and crop size. The main contribution of the paper is the use of nonlinear programming to reduce the risk of uncertainties caused by weather.

Production planning problems include the problems of crop planning. Itoh [11] studies crop planning where the profit coefficients are random and developed a LP model with the objective of maximizing minimum value of revenue. Romero[12] examines the crop planning problem using an approach similar to Markowitz's mean-variance approach.

Most of the studies in the perishable goods focus on the harvesting decisions; such as how to allocate transportation equipment, scheduling of packing and processing plants, or amount of harvest per period for single picking plants such as flowers, whet, potatoes and etc. Some works in this area are; Widodo et. al. [13], Caixeta and Filho [14], Ferrer et. al. [15]

The number of articles that focus on the nonperishable products are significantly higher than those that focus on perishable products. Jones et al.[16] study the production planning problem of Syngenta Seeds, and develop a production-planning model, that is used currently for 80 percent of its product. The company has two planning periods; first one refers to the North American growing season, and second period refers to the growing season in South America. The demand and the yield are taken as random through both periods, and discrete approximations are generated to the distribution functions of both. To decide on the number of acres for each period, linear programming is used and parameters for the second period are updated based on the results of first period.

Most of the work on the perishable products focus on the crop choice models. One of the interesting works on this are Kobzar [17] that develops a risk programming model with mean-variance objective function to capture joint stochastic distributions. Also, Maatman et. al. [18] who use a SP to model the farmers' response to uncertain rain, trying to minimize the food shortages for the farmer. Other works in this area are; Schilizzi and Kingwell [19], Nevo et. al. [20], Recio et. al.[21], Biswas and Pal [22].

This study considers the planning problem of a perishable product over a multi-period planning horizon in the existence of random yield and random demand. The most relevant works to ours are Jones et al.[16] and Kazaz[9]. Both consider a planning problem with two periods and the decision variable is how much to produce. In both problems, the decision maker can decide just before each period, updating the information according to the previous period. In our study we consider a planning horizon where the decision maker needs to decide before the first period starts.

2.2 Random Yield

The literature in the area of random yield is sparse. Yano and Lee[23] present an extensive literature review of lot sizing models with random yield. The inventory planning problem with random supply and random demand was first addressed by Karlin[24]. Karlin considers an inventory model where the inventory holding and shortage cost functions are convex increasing functions and the number of good units in a batch is random with a known distribution. The only decision is whether to order. He shows that there exists a single critical initial on-hand inventory below which an order should be placed.

Silver [25] presents a single-period inventory problem with a constant demand rate and considers two cases; in the first case the standard deviation of the supply quantity is independent of the lot size, in the second case it is proportional to the lot size. For both cases, the optimal lot size is a modification of EOQ. Shih [26]considers the same problem with linear inventory holding and shortage costs where the demand is random. He assumes that the yield uncertainty is caused by defective units and invariant with the batch size. He finds the optimal lot size with a simple modification in the economic order quantity formula, EOQ, as in Silver's paper. Noori and Keller[27] study the problem and provides closed form solutions for uniform and exponential demand distributions and for various distributions of the quantity received. Ehrhardt and Taube [28] generalize Shih's model by using general forms for inventory holding and shortage costs and derive the necessary optimal conditions. Gerchak et al.[29] consider the problem when there is an initial stock.

Regarding the supply uncertainty, many studies are conducted in a newsvendor setting. A comprehensive review of the newsvendor literature was provided by Khouja[30].

Anupindi and Akella [31] focus on the strategy of supplying from two suppliers for three cases. In the first model, they discuss a single-delivery contract where both suppliers supplies the whole order quantity with a given probability. In the second case, each supplier delivers a random fraction and the rest is canceled, while in the third case the order not delivered is transported in next period. They derive the optimal sourcing policies and proposed solution algorithms for three cases.

Dada et al.[32] consider the procurement problem of a newsvendor when the suppliers are unreliable. Each supplier either delivers the complete order or with some probability delivers the amount less then desired. They figure out that, in an optimal solution, a supplier is active only if all less expensive suppliers are active regardless of reliability level and offer an algorithm for determining optimal solution.

Yang et al.[33] examine a supplier selection problem, where decision maker orders from a set of suppliers with different yields while facing a random demand. They provide a formulation based on the newsvendor problem and developed solution methodology, consists of the active set method and the Newton search procedures.

Rekik et al.[34] study the newsvendor problem with unreliable supply and investigate the optimal order quantities for different cases. They derive the closed from formulation for optimal order quantity when the error in the quantity received and demand is uniformly distributed. Also an analysis is provided for normally distributed demand and error.

Our work contributes to random yield, random demand problems by providing the closed form solution of newsvendor problem when the demand and the supply are normal random variables. Rekik et al.[34] has provided analytic solutions for the cases where the standard deviation of supply can be written as an additive function of the mean. We generalize the solution for the cases where both the mean and the standard deviation are written as a function of a variable.

Most of the work in the literature focus on the random yield problems when the number of supplier is less then 3. We study the general case of the problem, where the supplier number is not limited. Dada et al.[32] also study the problem with multiple suppliers. Dada et al.[32] consider suppliers with different reliabilities and updates the critical ratio of newsvendor according to the suppliers' reliability. In our work, we assume that the yields of all suppliers are normally distributed. In that sense Yang's work is more relevant to ours [33]. They also consider multiple suppliers with normally distributed yields. Different than our work, they use a Manto-Carlo sampling method to evaluate the derivatives.

We make four contributions in this study. First, a model that captures harvest, maturation, yield and demand uncertainty in the same planning problem is presented. Second, we find an exact analytical solution for the single-period planning problem for the farm areas as well as the seeding times. Third , we propose computationally efficient heuristic solutions for the multi-period problem and analyzed their accuracy in a number of different cases. Finally, we analyzed a case study from the industry and used these heuristics to show the effectiveness of the proposed method.

Chapter 3

AGRICULTURAL PLANNING PROBLEM

In this thesis, the main objective is analyzing the multi-period planning problem of a single product produced by multiple farms of an agricultural supply chain system. The demand of the product at time t is random and denoted with D_t. The production quantity, Q_t, is the total amount of crop supplied from all farms, and the output of each farm depends on the area of seeding and the crop yield in that farm. The planting area of each farm is a decision variable whereas the yield of the farm, which means the amount of the output per unit area of land under cultivation, is a random variable.

The other decision variable of the problem for the case of annual plants is the seeding time of crops in farms. After the seeds are planted at a given time in farm i, the harvest starts after a random maturation time. When the plants are perennial, the seeding times are no longer decision variables. Instead, they are parameters that indicate the start of the maturation process. The crops are available to be picked only during the harvest period and the length of harvest period at the farm is random. Throughout this period, the target plant can be reaped more than once, and the crop should be gathered as soon as it matures; so we assume that the crops are gathered at the end of each certain time unit. Furthermore, the crops harvested in one period can only be used to satisfy the demand of the same period since no inventory can be kept due to the perishability of the product.

Considering agricultural processes are random by nature, there is always a risk in the availability of the crop in each period for each farm. The lengths of the maturation and the harvest period are the two important factors for the output availability. Although accurate estimations can be done for both, they are highly dependent on weather, and farm conditions which makes it difficult to assume those durations as deterministic variables.

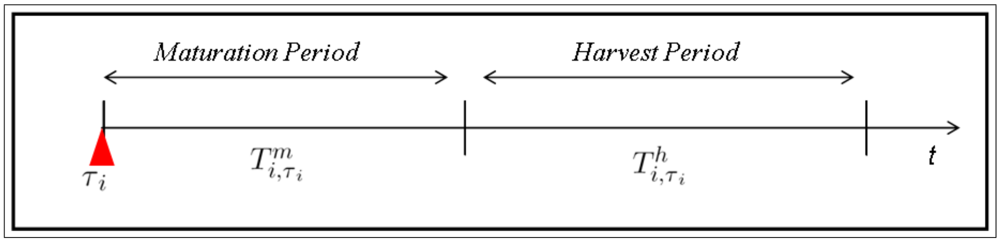

Figure 3.1: The relation between the seeding time, the maturation and the harvest period

In each period, after the crops are harvested, the company pays the supplier for all harvested products and earns revenue for the satisfied demand. The excess production is salvaged at the end of each period. The decision maker is risk neutral and aims to maximize the expected profit over the planning period.

3.1 Model

The system is modeled as a discrete time-discrete state space stochastic process observed at time $t = 1, ..., \mathcal{T}$.

Contract

We assume, the decision maker, the firm, buys the whole production from the contracted farms. If excess production occurs, the surplus is salvaged. We define r_t as the revenue , s_t as the salvage value, and c_t as the total production and distribution cost of supplying from farm i at time t where $r_t > c_t > s_t$.

Farms

There are N different farms located in different regions. The area seeded in farm i is a_i (acres). In the planning problem, a_i is a decision variable.

Demand

Demand in period t is a random variable denoted with D_t (pounds). The mean and the standard deviation of the demand are $E[D_t]$ and $Stdev[D_t]$ respectively.

Parameters	**Description**
r_t:	Unit selling price of a product at time t
s_t:	Unit salvage price of a product at time t
c_t .	Unit variable cost of a product at time t
Q_t:	Supply quantity at time t
D_t:	Demand at time t
$\pi(\mathbf{a}, \tau)$:	Profit function
ϕ :	The standard normal pdf
Φ :	The standard normal cdf
$p_{i,\tau_i}(t)$:	The probability that the output will be available from farm i in period t given that seeding time is τ_i
Y_i	The yield of farm i(pounds/acre/time)
τ_i	The seeding time of farm i
a_i	The seeding area of farm i
T^m_{i,τ_i}:	The length maturation period in farm i given that the seeding time is τ_i
T^h_{i,τ_i}:	The length of harvest period in farm i given that the seeding time is τ_i
$F_{T^h_{i,\tau_i}}(t)$:	The cumulative density function of T^h_{i,τ_i}

Table 3.1: Problem parameters

Profit Function

The total profit during the planning period is;

$$E\left[\pi\left(\mathbf{a}, \tau\right)\right] = \sum_{t=1}^{\mathcal{T}} r_t E[\min\left(Q_t, D_t\right)] + s_t E[(Q_t - D_t)^+] - c_t E[Q_t] \tag{3.1}$$

where $\mathbf{a} = (a_1, a_2, ..., a_N)$ and $\tau = (\tau_1, \tau_2, ..., \tau_N)$.

Formally, we choose the decision variables that are the farm area, a_i and the seeding time τ_i, $i = 1, ..., N$ for each farm such the total expected profit would be maximized. The problem is written as follows for annual plants;

$$\underset{\mathbf{a},\tau}{\text{Max}} \quad E[\pi\left(\mathbf{a}, \tau\right)] \tag{3.2}$$

and as following for perennial plants;

$$\underset{\mathbf{a}}{\text{Max}} \quad E[\pi\,(\mathbf{a},\tau)]$$

Harvest

The length of the maturation and the harvest period are discrete random variables, denoted with T^m_{i,τ_i} and T^h_{i,τ_i} respectively. The cumulative density function of T^h_{i,τ_i} is $F_{T^h_{i,\tau_i}}(t)$. The probability function for the maturation time is also given, $P[T^m_{i,\tau_i} = t]$ for all t,$t \in \mathcal{T}$. Combining the uncertainty in the maturity time and also in the harvest time, we can define an indicator random variable $I_{i,\tau_i}(t)$ that is 1 if the output from farm i is available at time t when the seeds are planted in period τ_i.

$$I_{i,\tau_i}(t) = \begin{cases} 1 & t - \tau_i - T^m_{i,\tau_i} \geq 0 \;,\; t - \tau_i < T^h_{i,t'} \\ 0 & otherwise \end{cases}$$

Let $p_{i,\tau_i}(t)$ be the probability that the output from farm i will be available in period t given that the seeds are planted in period τ_i. With this definition $E[I_{i,\tau_i}(t)] = p_{i,\tau_i}(t)$ and $p_{i,\tau_i}(t)$ is given as

$$\begin{aligned} p_{i,\tau_i}(t) &= \sum_{t'=\tau_i+1}^{t} P[t - t' < T^h_{i,t'}]P[T^m_{i,\tau_i} = t' - \tau_i] \qquad (3.3) \\ &= \sum_{t'=\tau_i+1}^{t} (1 - F_{T^h_{i,t'}}(t - t'))P[T^m_{i,\tau_i} = t' - \tau_i]. \end{aligned}$$

Equation (3.3) yields $p_{i,\tau_i}(t)$ based on distributions of the lengths of the harvest time and the maturity time. Let us consider the case, where T^m_{i,τ_i} and T^h_{i,τ_i} are uniformly distributed . Figure 3.2 shows $p_{i,\tau_i}(t)$ values for three different cases.

Alternatively, the form and definition of $p_{i,\tau_i}(t)$ allows us to incorporate expert opinion. Furthermore, specifying $p_{i,\tau_i}(t)$ directly makes it easier to capture harvest risk dependencies among different farms possibly located in the same region.

Yield

In this formulation, yield is defined as the amount of output gained from one acre of seeded farm. When the harvest starts at farm i, at the end of each time period, an output of $Y_i a_i$ is obtained until the end of the harvest period. The yield is denoted with Y_i. We assume that Y_i is a normal random variable with mean, $E[Y_i]$, and variance, $Var[Y_i]$.

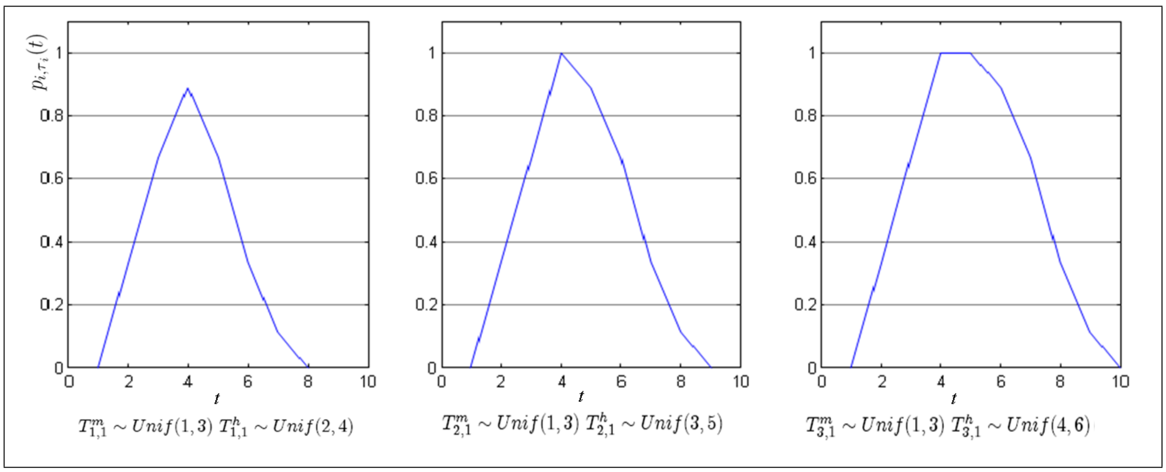

Figure 3.2: The harvest probabilities with different harvest and maturation period length distributions

Supply

The supply from farm i at time t is $q_{i,t} = I_{i,\tau_i}(t) Y_i a_i$(pounds). The total supply from all the farms at time t is $Q_t = \sum_{i=1}^{N} q_{i,t}$. Figure 3.3 depicts a sample realization of the output from 3 different farms and the total output in each period.

Since $I_{i,\tau_i}(t)$ and Y_i are random variables Q_t is also a random variable. The first two moments of Q_t are

$$E[Q_t] = \sum_{i=1}^{N} E[q_{i,t}] = \sum_{i=1}^{N} E[I_{i,\tau_i}(t) Y_i a_i] = \sum_{i=1}^{N} p_{i,\tau_i}(t) E[Y_i] a_i \tag{3.4}$$

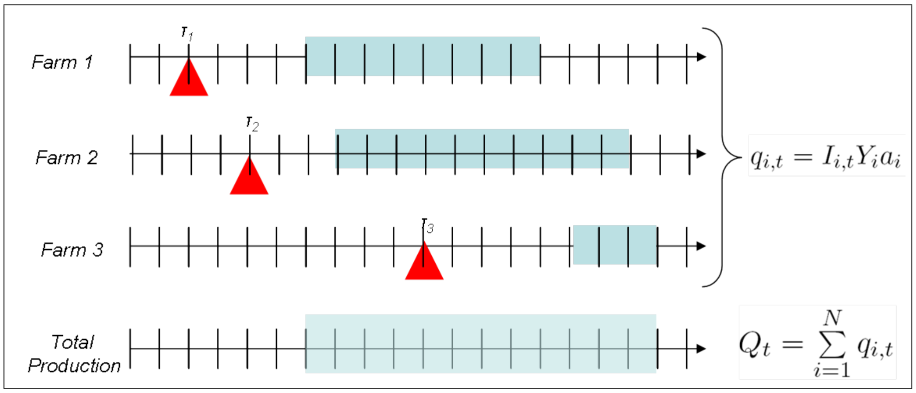

Figure 3.3: A sample realization for the total production Q_t for a specific case

and

$$\begin{aligned} E[Q_t^2] &= E\left[\left(\sum_{i=1}^{N} q_{i,t}\right)^2\right] = E\left[\left(\sum_{i=1}^{N} I_{i,\tau_i} Y_i a_i\right)^2\right] \\ &= \sum_{i=1}^{N} p_{i,\tau_i}(t) E[Y_i^2] a_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} p_{i,\tau_i(t)} p_{j,\tau_j(t)} E[Y_i] E[Y_j] a_i a_j. \end{aligned} \tag{3.5}$$

By using the first two moments, the variance of the total supply can be calculated:

$$Var[Q_t] = \sum_{i=1}^{N} \left(p_{i,\tau_i}(t) E[Y_i^2] a_i^2 - p_{i,\tau_i}^2(t) E[Y_i]^2 a_i^2 \right) \tag{3.6}$$

3.1.1 Modeling of the Problem when Variabilities are Ignored

The maximization problem given Equation (3.2) is complex due to the randomness in the problem parameters. If variabilities of problem parameters are ignored, the problem can be modeled as a linear integer model. We model the problem for the annual plants. The model can be used for the annual plants when the seeding time variables are taken as parameters. The problem is modeled with using only the expected values of the random variables; yield, demand, maturation and harvest lengths. Then the indicator variable $I_{i,\tau_i}(t)$ becomes a definite parameter, $I'_{i,\tau_i}(t)$, and is written as follows;

$$I'_{i,\tau_i}(t) = \begin{cases} 1 & t - \tau_i - E\left[T^m_{i,\tau_i}\right] \geq 0 \ , \ t - \tau_i < E\left[T^h_{i,t'}\right] \\ 0 & otherwise \end{cases} \tag{3.7}$$

Then, the multi-farm, multi-period problem can be modeled as follows;

$$\underset{\mathbf{a},\tau}{\text{Max}} \quad \sum_{t=1}^{\mathcal{T}} r_t \min(E[Q_t] - E[D_t]) + s_t(E[Q_t] - E[D_t])^+ - c_t E\left[Q_t\right] \tag{3.8}$$

subject to

$$E[Q_t] = \sum_{i=1}^{N} I'_{i,\tau_i}(t) E\left[Y_i\right] a_i, \quad t = 1, ...\mathcal{T} \tag{3.9}$$

$$\tau_i = \sum_{t=1}^{\mathcal{T}} t St_{i,t} \quad i = 1, ..., N \tag{3.10}$$

$$\sum_{t=1}^{\mathcal{T}} St_{i,t} \leq 1, \quad i = 1, ..., N \tag{3.11}$$

$$St_{i,t} \in \{0, 1\}, \quad i = 1, ..., N; t = 1, ...\mathcal{T} \tag{3.12}$$

The above model is a nonlinear integer maximization problem. By rewriting some variables and some constraints it can be written as a linear model. Assigning positive variables for the difference between the demand and the supply quantities shown below

$$E\left[Q_t\right] - E\left[D_t\right] = \Delta_t^+ - \Delta_t^-,$$

and rewriting the objective function to avoid minimum and positive functions yield,

$$\breve{\pi}\left(\mathbf{a}, \tau\right) = \sum_{t=1}^{\mathcal{T}} r_t \left(E\left[D_t\right] - \Delta_t^-\right) + s_t \Delta_t^+ - c_t E\left[Q_t\right]$$

where $\breve{\pi}$ is the profit obtained when all the random variables are replaced with their expected values.

Another nonlinearity in the model is in the constraint (3.9). It can be replaced by three constraints to avoid the nonlinearity. Then the model is written as an mixed-integer linear model:

$Model_0$

$$\underset{\mathbf{a},\tau}{\text{Max}} \quad \sum_{t=1}^{\mathcal{T}} r_t \left(E\left[D_t\right] - \Delta_t^- \right) + s_t \Delta_t^+ - c_t E\left[Q_t\right] \tag{3.13}$$

subject to

$$E[Q_t] \leq M I'_{i,\tau_i}(t), \quad i = 1, ..., N; t = 1, ...\mathcal{T}$$

$$E\left[Y_i\right] a_i \leq E[Q_t] + M\left(1 - I'_{i,\tau_i}(t)\right), \quad i = 1, ..., N; t = 1, ...\mathcal{T}$$

$$E[Q_t] \leq E\left[Y_i\right] a_i, \quad i = 1, ..., N; t = 1, ...\mathcal{T}$$

$$E\left[Q_t\right] - E\left[D_t\right] = \Delta_t^+ - \Delta_t^-, \quad t = 1, ...\mathcal{T}$$

$$\tau_i = \sum_{t=1}^{\mathcal{T}} t St_{i,t} \quad i = 1, ..., N$$

$$\sum_{t=1}^{\mathcal{T}} St_{i,t} \leq 1, \quad i = 1, ..., N$$

$$St_{i,t} \in \{0, 1\}, \quad i = 1, ..., N; t = 1, ...\mathcal{T}$$

where M is a large number.

Chapter 4

SINGLE PERIOD PROBLEM

In this chapter, we examine the planning problem for a single period. The problem is the same as the main problem described in the previous chapter, except for the fact that the demand only exists for a single period $t^{\dagger}$. This problem is of interest for planning problems for seasonal products.

4.1 Single Farm, Single Period Problem

In this section we consider the case where there is only one farm available for seeding, $N = 1$. In the beginning of the planning period, the seeding area and , if necessary, the seeding time decisions should be given for the farm. Throughout this section, the planning problem of annual plants is taken as the main problem since the solution of this problem includes the solution of the planning problem of perennial plants.

Let the harvest period start at t' and end at t'' based on the seeding time τ. Throughout all the periods t ,where $t' < t < t''$, the expected output from the farm is as follows;

$$E[Q_t] = E[I_{1,\tau_1}(t)E[Y_1]a_1] = p_{1,\tau_1}(t)E[Y_1]a_1 \tag{4.1}$$

We want to maximize the expected profit of the period $t^{\dagger}$, which is

$$E[\pi\,(a_1,\tau)] = r_{t^{\dagger}}E[\min\,(Q_{t^{\dagger}},D_{t^{\dagger}})] + s_{t^{\dagger}}E[(Q_{t^{\dagger}}-D_{t^{\dagger}})^{+}] - c_{t^{\dagger}}E[Q_{t^{\dagger}}] \tag{4.2}$$

The aim is to find the optimal decisions for the seeding time and the area for the seeding process. The decision variables are the farm area of farms and seeding times for the annual plants and only the seeding areas for the perennial plants.

The problem is similar to the newsvendor problem in the sense that a single decision must be made before observing the demand when there are overage and underage costs, however in this case the supply is also random.

4.1.1 Newsvendor Problem with Random Supply

In this section, newsvendor problem with random supply is investigated when supply and demand are normally distributed variables. Let Q and D be normal. The mean and standard deviation of Q and D are denoted with (μ_Q, σ_Q) and (μ_D, σ_D) respectively. The subscripts i and t is dropped from the notations, listed in Table 3.1. The single period stochastic inventory model, referred as the newsvendor model, is a well-known problem in the literature. The expected profit for the classical newsvendor problem is written as;

$$E[\pi'(Q)] = rE[\min(Q, D)] + sE[(Q - D)^+] - cE[Q].$$

Replacing $E[(Q - D)^+]$ with $Q - E[\min(Q, D)]$, the expected profit becomes;

$$E[\pi'(Q)] = (r - s)E[\min(Q, D)] - (c - s)E[Q]. \tag{4.3}$$

One of the main assumptions in classical newsvendor problem is that there is no variability in the supplied quantity. In this part we will investigate the one-period inventory model with normally distributed demand and supply. The expected value of the minimum of bivariate two normal random variables can be written in the following way [35];

$$E[\min(Q, D)] = \mu_Q \Phi\left(\frac{\mu_D - \mu_Q}{\theta}\right) + \mu_D \Phi\left(\frac{\mu_Q - \mu_D}{\theta}\right) - \theta\phi\left(\frac{\mu_D - \mu_Q}{\theta}\right). \tag{4.4}$$

where $\theta = \sqrt{\sigma_Q^2 - 2\rho\sigma_Q\sigma_Q + \sigma_D^2}$ and ρ is the correlation coefficient between Q and D.
In our problem, Q and D are independent variables, thus the correlation coefficient ρ is zero. Using Equation (4.4) Equation (4.3) can be written as;

$$E[\pi'(Q)] = (r-s)\left[\mu_Q\Phi\left(\frac{\mu_D-\mu_Q}{\theta}\right)+\mu_D\Phi\left(\frac{\mu_Q-\mu_D}{\theta}\right)-\theta\phi\left(\frac{\mu_D-\mu_Q}{\theta}\right)\right]-(c-s)\,\mu_Q. \tag{4.5}$$

By using the above closed-form expression for the expected total profit, the decision variables that maximize the expected total profit can be determined. In order to provide a general solution we consider the case where the mean and the standard deviation of Q are functions of a single decision variable x. The following theorem shows that the optimal x can be determined by solving a nonlinear equation.

Theorem 1 *Let the mean and the standard deviation of the supply be functions of parameter x, $\mu_Q = f(x)$ and $\sigma_Q = g(x)$. If $f(x)$ is linear and $g(x)$ is concave , then the optimal x should satisfy the following,*

$$f(x) = \mu_D - \theta(x)\Phi^{-1}\left[\frac{(c-s)}{(r-s)} + g(x)\frac{\partial g(x)}{\partial x}\theta(x)^{-1}\phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\frac{\partial f(x)}{\partial x}^{-1}\right] \tag{4.6}$$

Proof 1 *See Appendix.*

By using the above results, we first analyze specialized cases for additive, multiplicative, and additive-multiplicative random yield cases and then apply this result to the single period agricultural planning problem.

4.1.1.1 Specialized Cases

Let us consider the case when the quantity can be written as ; $Q = \gamma x + \varepsilon$, where γ and ε are normal random variables with $(\mu_\gamma, \sigma_\gamma)$ and $(\mu_\varepsilon, \sigma_\varepsilon)$. For this instance the mean and the standard deviation of the supply are $f(x) = \mu_\gamma x +\ \mu_\varepsilon$ and $g(x) = \sqrt{\sigma_\gamma^2 x^2 + \sigma_\varepsilon^2}$,respectively. Using Equation (4.6), the following condition can be found for optimal x.

$$f(x) = \mu_D - \theta\left(x\right)\Phi^{-1}\left[\frac{(c-s)}{(r-s)} - \sigma_\gamma^2 x\theta\left(x\right)^{-1}\phi\left(\frac{\mu_D - \mu_\tau x - \mu_\tau}{\theta\left(x\right)}\right)\mu_\tau^{-1}\right] \tag{4.7}$$

where $\theta(x) = \sqrt{\sigma_\gamma^2 x^2 + \sigma_\varepsilon^2 + \sigma_D^2}$. In the following sections, we investigate two cases; additive and multiplicative random yield problems, those can be derived from this model.

Additive Case

In the additive case the standard deviation of the supply quantity is independent of Q; Q can be written as $Q = x + \varepsilon$. Since ε is normal with $(\mu_\varepsilon, \sigma_\varepsilon)$, the supply is normal with the mean, $f(x) = x+ \mu_\varepsilon$ and the standard deviation, $g(x) = \sigma_\varepsilon$. Without the loss of generality assume $\mu_\varepsilon = 0$. If we place $f(x) = x$ and $g(x) = \sigma_\varepsilon$ into Equation (4.6), we can find a closed form expression for the optimal quantity for this case.

$$\mu_Q = \mu_D - \theta\left(x\right)\Phi^{-1}\left(\frac{c-s}{r-s}\right) \tag{4.8}$$

Note that $\Phi^{-1}\left(\frac{c-s}{r-s}\right)$ can be written as -$\Phi^{-1}\left(1-\frac{c-s}{r-s}\right)$, the optimal quantity becomes,

$$\mu_Q = \mu_D + \theta\left(x\right)\Phi^{-1}\left(\frac{r-s}{r-s}\right).$$

where $\theta(x) = \sqrt{\sigma_\varepsilon^2 + \sigma_D^2}$. This result is the same as the optimal quantity presented in the analysis of Rekik et al.[34].

Multiplicative Case

In the multiplicative case, the deviation is proportional to the expected value of supply , $Q = \gamma x$. Let γ be normal with $(\mu_\varepsilon, \sigma_\varepsilon)$, then Q is normal with $(f(x), g(x))$, where $f(x) = \mu_\gamma x$ and $g(x) = \sigma_\gamma x$.Using Equation (4.6), the optimal x, should satisfy;

$$f(x) = \mu_D - \theta\left(x\right)\Phi^{-1}\left[\frac{(c-s)}{(r-s)} - \sigma_\gamma^2 x \theta\left(x\right)^{-1}\phi\left(\frac{\mu_D - \mu_\gamma x}{\theta\left(x\right)}\right)\mu_\gamma^{-1}\right] \tag{4.9}$$

where $\theta = \sqrt{\sigma_\gamma^2 x^2 + \sigma_D^2}$. The optimal quantities in Rekik et al.[34] satisfy Equation (4.9).

4.1.2 Solution of Single-Farm, Single-Period Problem

In the previous section, we examined the newsvendor problem with random supply, and determined the optimal decision for the supply quantity when both the demand and the supply are normally distributed. Through this section we study the solution of single farm, single period problem, described in Section 4.1, and we aim to maximize the expected profit given in Equation (4.2), by finding the optimal decisions for the farm area and the seeding time. The problem differs from the newsvendor problem described in the previous section, since the distribution of the supply quantity is unknown.

The supply quantity depends on two random variables; one is the binary indicator that shows the availibilty of harvest in the related period, the other is the random variable yield. From section (3) , it is known that in period t with probability $p_{i,\tau_i}(t)$ the output quantity from farm i is equal to $Y_i a_i$, otherwise it is zero.

$$Q_t = \sum_{i=1}^{1} q_{i,t} = \begin{cases} 0 & \text{with probability} \quad 1 - p_{1,\tau_1}(t) \\ Y_1 a_1 & \text{with probability} \quad p_{1,\tau_1}(t) \end{cases}$$

Although from Equations (3.4) and (3.6), the mean and variance of the variable Q_t can be obtained, we do not know the distribution. Thereby, $E\left[\min(Q_t, D_t)\right]$ can not be found analytically. On the other hand, we know Y_i and consequently $Y_i a_i$ are normal. Let us call $Y_i a_i$ as $q'_{i,t}$ and $\sum_{i=1}^{1} q'_{i,t}$ as Q'_t in period t, then $min(Q_t, D_t)$ and $E\left[\min(Q_t, D_t)\right]$ can be written as follows;

$$\min(Q_t, D_t) = \sum_{i=1}^{1} q_{i,t} = \begin{cases} 0 & \text{with probability} \quad 1 - p_{1,\tau_1}(t) \\ \min(Q'_t, D_t) & \text{with probability} \quad p_{1,\tau_1}(t) \end{cases}$$

Using Equation (4.4), $E\left[\min(Q_t, D_t)\right]$ is written as,

$$\begin{aligned} E\left[\min(Q_t, D_t)\right] \quad = \quad & p_{1,\tau_1}(t)\Bigg(E\left[Q'_t\right] \Phi \left(\frac{E\left[D_t\right] - E\left[Q'_t\right]}{\theta'_t} \right) + E\left[D_t\right] \Phi \left(\frac{E\left[Q'_t\right] - E\left[D_t\right]}{\theta'_t} \right) \\ & - \theta'_{t^\dagger} \phi \left(\frac{E\left[D_t\right] - E\left[Q'_t\right]}{\theta'_t} \right) \Bigg). \end{aligned}$$

where $E\left[Q_t'\right] = E\left[Y\right]a$, $Var\left[Q_t'\right] = E\left[Y^2\right]a^2 - E\left[Y\right]a^2$ and $\theta_t' = \sqrt{Var\left[Q_t'\right] + Var\left[D_t\right]}$. Then the expected profit for period $t^\dagger$ given in Equation (4.2), can be written as;

$$\begin{aligned} E\left[\pi\left(\mathbf{a},\tau\right)\right] \quad = \quad & (r_{t^\dagger} - s_{t^\dagger})p_{1,\tau_1}(t^\dagger)\Bigg(E\left[Q_{t^\dagger}'\right]\Phi\left(\frac{E\left[D_{t^\dagger}\right] - E\left[Q_{t^\dagger}'\right]}{\theta_{t^\dagger}'}\right) \\ & + E\left[D_{t^\dagger}\right]\Phi\left(\frac{E\left[Q_{t^\dagger}'\right] - E\left[D_{t^\dagger}\right]}{\theta_{t^\dagger}'}\right) - \theta_{t^\dagger}'\phi\left(\frac{E\left[D_{t^\dagger}\right] - E\left[Q_{t^\dagger}'\right]}{\theta_{t^\dagger}'}\right)\Bigg) \\ & -(c_{t^\dagger} - s_{t^\dagger})E[Q_{t^\dagger}]. \end{aligned} \tag{4.10}$$

There are two decision variables in the problem; **a** and τ. First we discuss the optimal **a**.

Theorem 2 *Let the mean and standard deviation of Q_t' be functions of parameter a , $E\left[Q_t'\right] = f'(a)$, and $Stdev\left[Q_t'\right] = g'(a)$. If $f'(a)$ is linear and $g'(a)$ is concave , then the optimal a should satisfy the following,*

$$f'(a) = E\left[D_t\right] - \theta_t'(a)\Phi^{-1}\left[\frac{(c_t - s_t)}{(r_t - s_t)} + g'(a)\frac{\partial g'(a)}{\partial a}\theta_t'(a)^{-1}\phi\left(\frac{E\left[D_t\right] - f'(a)}{\theta_t'(a)}\right)\frac{\partial f'(a)}{\partial a}^{-1}\right] \tag{4.11}$$

Proof 2 *See Appendix.*

After replacing the $f'(a_1)$, $g'(a_1)$,θ_t', $\frac{\partial g'(a_1)}{\partial a_1}$ and $\frac{\partial' f(a_1)}{\partial a_1}$ in Equation (4.11), the optimality condition for a_1 in Theorem 2 becomes;

$$a_1 = E\left[Y_1\right]^{-1}\Bigg(E[D_t] - \theta_t'(a)\Phi^{-1}\Bigg(\frac{(c_t - s_t)}{(r_t - s_t)} + \\ \frac{a_1\left(Var\left[Y_1\right]\right)}{\sqrt{\left(E\left[Y_1^2\right]a_1^2 - E\left[Y_1\right]a_1^2\right) + E[D_t]^2}}\phi\left(\frac{E[D_t] - E\left[Y_1\right]a_1}{\sqrt{\left(E\left[Y_1^2\right]a_1^2 - E\left[Y_1\right]a_1^2\right) + \sigma_{D_t}^2}}\right)E\left[Y_1\right]^{-1}\Bigg)\Bigg).$$

From the equation above, the optimal decision for the farm area can be found independent of the seeding time decision. Also, Theorem 2 provides the optimal solution for the planning problem of perennial plants.

Theorem 3 *Let τ_1^k denote the seeding time. The seeding time τ_1^k is optimal as long as there exists no $\tau_1^{k\prime}$ values that has bigger $p_{1,\tau_1^{k\prime}(t)}$ where $k \neq k\prime$.*

Proof 3 *See Appendix.*

Theorem 2 and Theorem 3 provide the optimal solution for the single farm, single period problem.

Let us study the optimal solution in a numerical example with parameters given in Table 4.1. We aim to maximize the profit for the period 8. By using Theorem 2 and 3, the optimal decisions for seeding time and the planting area can be found as $a_1^\star = 0.311$,$\tau_1^\star = 5$. The harvest probabilities for the seeding time $\tau_1 = 5$ can be seen in the following figure;

Parameters	**Values**
$r_{t^\dagger}$:	5
$c_{t^\dagger}$:	2
$s_{t^\dagger}$:	0.5
$D_\dagger$:	$Norm(2000, 500)$
Y_1:	$Norm(6000, 1000)$
${T_{i,t^\dagger}}^h$:	$U(2, 3)$
${T_{i,t^\dagger}}^m$:	$U(1, 5)$

Table 4.1: Parameters of Numeric Example

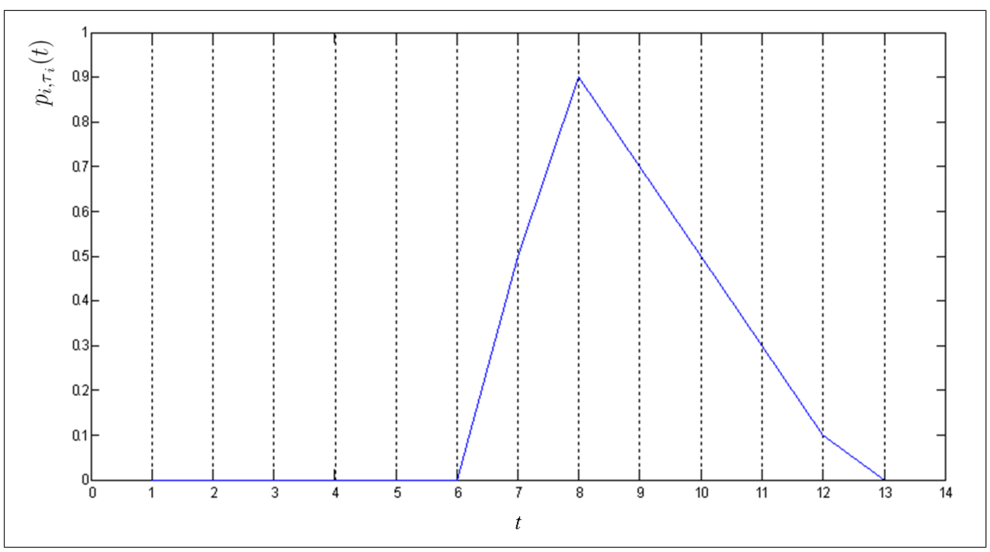

Figure 4.1: Harvest probabilities when seeding time, τ_1 is 5

The change in the profit function as a_1 and τ_1 change can be seen in Figure 4.2 and 4.3.

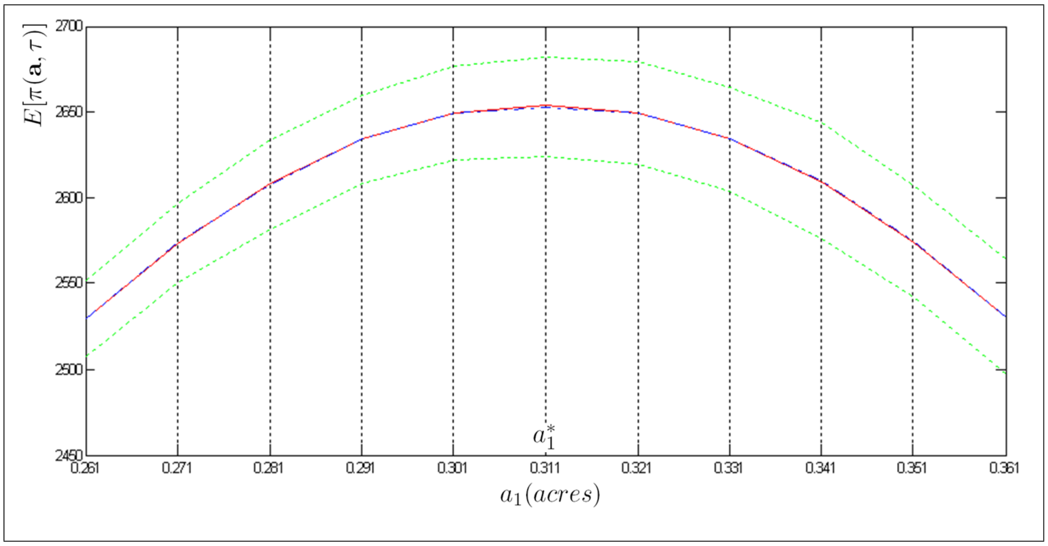

Figure 4.2: The relation between seeding area and expected profit

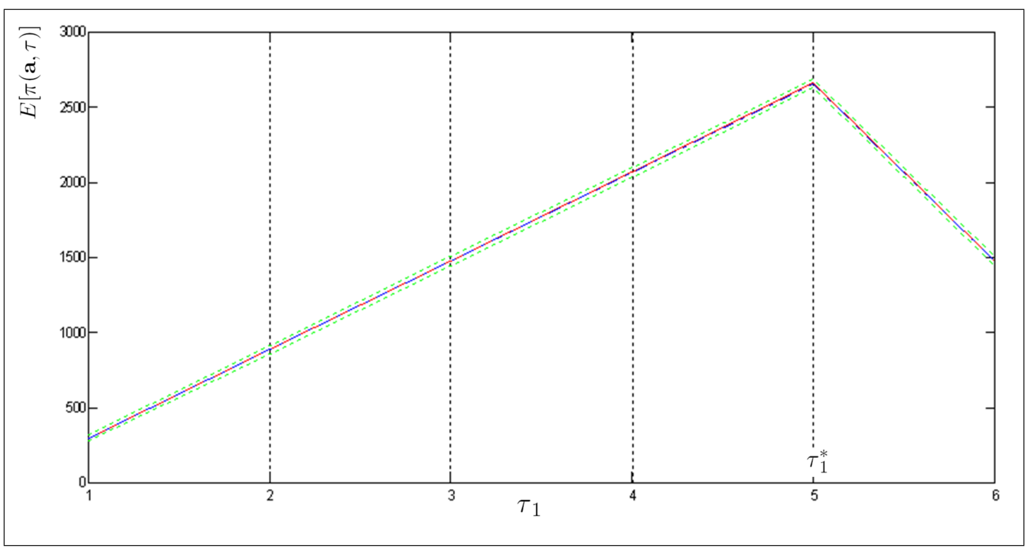

Figure 4.3: The relation between seeding time and expected profit

In Figure 4.2 and Figure 4.3, we can see the changing profit with respect to different a and τ values. Also, in both graphs the expected profit, and 95% confidence interval for the expected profit, based on the simulation results, can be seen. It can be seen that the expected profit, $E[\pi(\mathbf{a}, \tau)]$ value given in Equation(4.2), is very close to simulation results.

4.2 Multi Farm, Single Period Problem

In this section the planning problem for multi farm and single period is studied. The problem is similar to the single farm, single period problem. The only difference is in the total production variable. In this case, the supply quantity depends on the production of multiple farms. The expected profit for period $t^\dagger$ is written as follows;

$$E[\pi(\mathbf{a}, \tau)] = (r_{t^\dagger} - s_{t^\dagger})E[\min(Q_{t^\dagger}, D_{t^\dagger})] - (c_{t^\dagger} - s_{t^\dagger})E[Q_{t^\dagger}] \tag{4.12}$$

Like in the previous section the aim is to maximize the profit in period $t^\dagger$. There are multiple farms available and before the planning period starts, the decision maker needs to determine the seeding areas and seeding times for those farms.

$$\underset{\mathbf{a},\tau}{\text{Max}} \quad E[\pi(\mathbf{a}, \tau)] \tag{4.13}$$

4.2.1 Solution of the Multi Farm, Single Period Problem

In this part, to provide better understanding of the solution, we first investigate the solution for the case of two farms available and then generalize it for the multiple farm problem.

Let us consider the supply quantity when there are two farms. Seeding times and planting areas are given. Based on the seeding times, the harvest period can be calculated for each farm by using the probability function defined in Equation(3.3). The farms has maturation and harvest period distributions that leads harvest probabilities shown in Figure 3.2. The harvest probabilities for the first farm can be seen in the first graph of the figure. For the second farm, the harvest probabilities are given in the third graph.

Assume that the demand exists for only the fourth period, $t^\dagger = 4$. Both of the farms are seeded in period 1. From Figure 3.2, the probabilities for the fourth period can be seen; the harvest probability for the first farm is 0.9 while it equals to 1 for the second farm.

In Figure 4.4, the histogram of output quantity of two farms are given. The total output in period t, Q_t, shown in the third row, is equal to the sum of the supply quantities of the farms in period $t^{\dagger}$. The histogram of $q_{i,t}$s and Q_t obtained by simulation can be seen in the following figure.

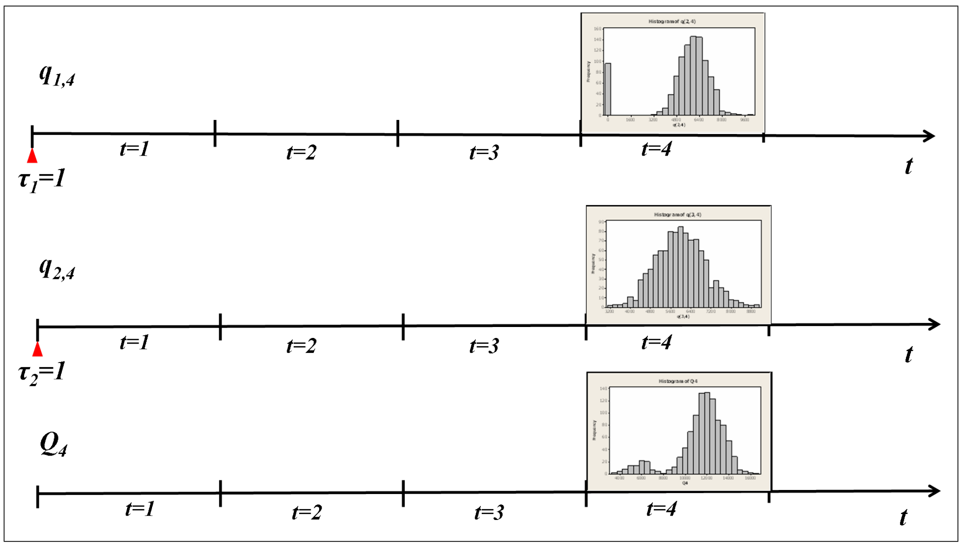

Figure 4.4: Histogram of $q_{i,t}$s and Q_t

As it can be seen from Figure 4.4, the production quantity has a different pattern. The total supply quantity gathered from all two farms is written as follows;

$$Q_t = \begin{cases} 0 & \text{with probability} \quad (1 - p_{1,\tau_1}(t))(1 - p_{2,\tau_2}(t)) \\ q'_{1,t} & \text{with probability} \quad p_{1,\tau_1}(t)(1 - p_{2,\tau_2}(t)) \\ q'_{2,t} & \text{with probability} \quad (1 - p_{1,\tau_1}(t))p_{2,\tau_2}(t) \\ \sum_{i=1}^{N} q'_{i,t} & \text{with probability} \quad p_{1,\tau_1}(t)p_{2,\tau_2}(t) \end{cases}$$

By using the Q_t given above, and the fact that $q'_{i,t}$s are normal, the $\min(Q_t, D_t)$ can be written as;

$$\min(Q_t, D_t) = \begin{cases} 0 & \text{with probability} \quad (1-p_{1,\tau_1}(t))(1-p_{2,\tau_2}(t)) \\ \min\left(q'_{1,t}, D_t\right) & \text{with probability} \quad p_{1,\tau_1}(t)(1-p_{2,\tau_2}(t)) \\ \min\left(q'_{2,t}, D_t\right) & \text{with probability} \quad (1-p_{1,\tau_1}(t))p_{2,\tau_2}(t) \\ \min\left(\sum_{i=1}^{N} q'_{i,t}, D_t\right) & \text{with probability} \quad p_{1,\tau_1}(t)p_{2,\tau_2}(t) \end{cases}$$

Then the expected minimum of Q_t and D_t and the the profit function is written as follows;

$$\begin{aligned} E\left[\min(Q_t, D_t)\right] &= p_{1,\tau_1}(t)\left(1-p_{2,\tau_2}(t)\right)\min\left(q'_{1,t}, D_t\right) + \left(1-p_{1,\tau_1}(t)\right)p_{2,\tau_2}(t)\min\left(q'_{2,t}, D_t\right) \\ &\quad + p_{1,\tau_1}(t)p_{2,\tau_2}(t)\ \min\left(\sum_{i=1}^{N} q'_{i,t}, D_t\right) \end{aligned} \tag{4.14}$$

and

$$\begin{aligned} E[\pi(\mathbf{a},\tau)] &= (r_{t^\dagger} - s_{t^\dagger})\Bigg(p_{1,\tau_1}(t^\dagger)\left(1-p_{2,\tau_2}(t^\dagger)\right)\min\left(q'_{1,t^\dagger}, D_{t^\dagger}\right) \\ &\quad + \left(1-p_{1,\tau_1}(t^\dagger)\right)p_{2,\tau_2}(t^\dagger)\min\left(q'_{2,t^\dagger}, D_{t^\dagger}\right) \\ &\quad + p_{1,\tau_1}(t)p_{2,\tau_2}(t^\dagger)\min\left(\sum_{i=1}^{N} q'_{i,t}, D_t\right)\Bigg) - (c_t - s_t)E\left[Q_t\right] \end{aligned} \tag{4.15}$$

For the general case, where there are N farms available, the expected Q_t is written as,

$$E\left[Q_{t^\dagger}\right] = \sum_{\alpha_i=0}^{1} \cdots \sum_{\alpha_N=0}^{1} \left(\prod_{i=1}^{N} p_{i,\tau_i}^{\alpha_i}(t^\dagger)\left(1 - p_{i,\tau_i}^{1-\alpha_i}(t^\dagger)\right)\right)\left(\sum_{i=1}^{N} q'_{i,t^\dagger}\alpha_i\right).$$

Then, the minimum of $Q_{t^\dagger}$ and $D_{t^\dagger}$ and the the profit function is written as,

$$E\left[\min(Q_{t^\dagger}, D_{t^\dagger})\right] = \sum_{\alpha_i=0}^{1} \cdots \sum_{\alpha_N=0}^{1} \left(\prod_{i=1}^{N} p_{i,\tau_i}^{\alpha_i}(t^\dagger)\left(1 - p_{i,\tau_i}^{1-\alpha_i}(t^\dagger)\right)\right)\min\left(\left(\sum_{i=1}^{N} q'_{i,t^\dagger}\alpha_i\right), D_{t^\dagger}\right), \tag{4.16}$$

and

$$E[\pi(\mathbf{a},\tau)] = (r_{t^\dagger} - s_{t^\dagger}) \sum_{\alpha_i=0}^{1} ... \sum_{\alpha_N=0}^{1} \left(\prod_{i=1}^{N} p_{i,\tau_i}^{\alpha_i}(t^\dagger) \left(1 - p_{i,\tau_i}^{1-\alpha_i}(t^\dagger)\right)\right) \min \left(\left(\sum_{i=1}^{N} q'_{i,t^\dagger} \alpha_i \right), D_{t^\dagger} \right) - (c_{t^\dagger} - s_{t^\dagger}) E\left[Q_{t^\dagger}\right]. \quad (4.17)$$

Theorem 4 *$E[\pi(\mathbf{a},\tau)]$ is a concave function of $\mathbf{a}$. For all N values, $N, \in R^+$, there exist a unique $\mathbf{a} = (a_1, ..., a_N)$, that maximizes the expected profit function, $E[\pi(\mathbf{a},\tau)]$.*

Proof 4 *See Appendix.*

In numerical experiments, we observed that Theorem 3 holds for the optimal seeding time for each farm in the multi-farm problem. However, we could not prove this result in this thesis. Therefore it is stated as a conjecture.

Conjecture 1 *Let τ_i^k denote the seeding time,where $i \in N$. The seeding time τ_i^k is optimal as long as there exists no $\tau_i^{k'}$ values that has bigger $p_{i,\tau_i^{k'}}(t)$.*

4.3 Normal Approximation for The Total Supply in The Single Period Problem

The profit function in Equation(4.18) has computational complexity $O(2^N)$ which makes it difficult to evaluate for big instances of N. Motivated by the solution of newsvendor problem in section 4.1.1, let us approximate the supply as a normally distributed random variable in each period t. Using Equations (3.4) and (3.6),the mean and the standard deviation of supply quantity for period $t^\dagger$ are written as,

$$E[Q_{t^\dagger}] = \sum_{i=1}^{N} p_{i,\tau_i}(t^\dagger) E[Y_i] a_i,$$

and

$$Stdev\left[Q_{t^\dagger}\right] = \sqrt{\sum_{i=1}^{N} \left(p_{i,\tau_i}(t^\dagger) E[Y_i^2] a_i^2 - p_{i,\tau_i}^2(t^\dagger) E[Y_i]^2 a_i^2 \right)}.$$

Under the approximation that the total supply is normally distributed, the profit function can be approximated as;

$$\tilde{\pi}(\mathbf{a},\tau) = (r_{t^\dagger} - s_{t^\dagger})\Bigg(E[Q_{t^\dagger}]\,\Phi\left(\frac{E[D_{t^\dagger}] - E[Q_{t^\dagger}]}{\theta_{t^\dagger}}\right) + E[D_{t^\dagger}]\,\Phi\left(\frac{E[Q_{t^\dagger}] - E[D_{t^\dagger}]}{\theta_{t^\dagger}}\right) - \theta_{t^\dagger}\phi\left(\frac{E[D_{t^\dagger}] - E[Q_{t^\dagger}]}{\theta_{t^\dagger}}\right)\Bigg) - (c_{t^\dagger} - s_{t^\dagger})E[Q_{t^\dagger}]. \quad (4.18)$$

where $\tilde{\pi}$ is the expected profit obtained when Q_t is approximated with a random variable and D_t is a normal variable. From Section 4.1.1 we know that when both the demand and the supply are normally distributed, in the optimal solution all $a_i \in \mathbf{a}$, satisfy Equation (4.6) for the given seeding times, $\tau_i \in \tau$. If we place $E[Q_t]$ and $E[Q_t]$ into Equation (4.6), then we can write the optimal $\tilde{a}_i^*$ as following,

$$\tilde{a}_i^* = p_{i,\tau_i}^{-1}(t)E[Y_i]^{-1}\Bigg(E[D_t] - E[Y_i]^{-1}\theta_t(a_i,\tau_i)\Phi^{-1}\Big(\frac{(c_t - s_t)}{(r_t - s_t)} - \frac{(E[Y_i^2]a_i^2 - p_{i,\tau_i}(t)E[Y_i]^2a_i^2)}{\theta_t}\phi\Big(\frac{E[D_t] - E[Q_t]}{\theta_t}\Big)\Big)\Bigg). \quad (4.19)$$

Although the single period planing problem is very similar to the newsvendor problem with random supply, it differs in the sense that there exist two decision variables, **a** and τ while the other has only one variable that corresponds to **a**. Thus, not all $\mathbf{a},\tau$ pairs that satisfy Equation (4.6) are optimal.

Theorem 5 *Let a_i^k be the value that satisfies the Equation (4.19) given that the seeding time is τ_i^k, such that $\tau_i^k = k$. The seeding time τ_i^k and the seeding area a_i^k are optimal as long as there exists no τ_i^k values that has bigger $p_{i,\tau_i^k}(t)$.*

Proof 5 *See Appendix.*

Theorem 3 and Theorem 5 suggest that both for the exact solution and for the approximated solution, the optimal time decisions provide the maximum harvest probability for the target period, thus in both cases, the optimal time decisions are the same.

Let us study the performance of the normality approximation for the single period, multi farm problem. We compare the results obtained by the solution approach with normality assumption to the optimal result for eight different cases where the number of available farms varies from 1 to 8. All farms are assumed to be identical. For each case, we generate 60 scenarios with different expected yield, expected demand, production cost and harvest probability values. The parameters of scenarios can be seen in the Table 4.3.

While generating harvest probability values, only the maximum of harvest probabilities are considered. The optimal solution uses the maximum value, thus scenarios are created for the maximum value of harvest probability, not for the maturation and harvest probability distribution functions. The maximum value of harvest probability depends on the distributions of harvest and maturation lengths. Let T^m_{i,τ_i} and T^h_{i,τ_i} be uniformly distributed with (a_m, b_m) and (a_h, b_h) respectively. In the following table the maximum harvest probabilities are given for different distributions of harvest and maturation lenghts. In all cases $E[T^h_{i,\tau_i}]$ and $E[T^m_{i,\tau_i}]$ are constant.

$\mathbf{b_m - a_m}$	$\mathbf{b_h - a_h}$			
	2	3	4	5
2	1	1	0.95	0.89
3	0.93	0.88	0.83	0.78
4	0.71	0.71	0.69	0.67

Table 4.2: The maximum value of harvest probability as distribution parameters of maturation and harvest length change

Table 4.2 shows that the maximum harvest probability decreases as the variation in lengths of harvest and maturation decreases. This time, we keep $b_m - a_m$ and $b_h - a_h$ constant and change $E[T^h_{i,\tau_i}]$ and $E[T^m_{i,\tau_i}]$. The change of $E[T^m_{i,\tau_i}]$ does not affect the maximum harvest probability, while increase of $E[T^h_{i,\tau_i}]$ increases the maximum value of the harvest probabilities. The seeding times of the optimal solution are found by using Conjecture 1, and the seeding times of the solution approach are found by using Theorem 5. Both provide the same solution, thus the optimal seeding times are the same for both the optimal solution and the approximated

Parameters	**Values**
N_t :	$2, 3, 4$
$r_{t^\dagger}$:	5
$c_{t^\dagger}$:	$2, 3$
$s_{t^\dagger}$:	0.5
$E[D_{t^\dagger}]$:	2000
$Cv[D_{t^\dagger}]$:	$0.1, 0.2, 0.3$
$max(p_{i,\tau_i}(t^\dagger))$:	$1, 0.9, 0.8, 0.7, 0.6$
$E[Y_{t^\dagger}]$:	6000
$Cv[Y_{t^\dagger}]$:	$0.1, 0.2$

Table 4.3: Parameters of Scenarios

solution. On the other hand, the seeding areas are different for the two solutions. The optimal seeding area decisions are described in section 4.2.1, and can be found by Theorem 4 . However, in the second case, the total supply quantity is assumed as normal and the seeding areas are found by Theorem 1.

We investigate the performance of the approximation as the number of the available farms increases. In Figure 4.5, the percent difference between the optimal solution and the approximated solution in the base 10 logarithmic scale can be seen for different N values.

Figure 4.5 shows that as the number of the available farms increases, the approximation provides better results. The difference between the optimal solution and the approximate solution is 2.3% when the number of farms is one and it decreases to 0.001% as the number of farms increases to eight. Also, from the central limit theorem it is known that the sum of random variables approaches to the normal distribution as the number of random variables increases. It can be stated that the solution approach performs better as the number of farms increases.

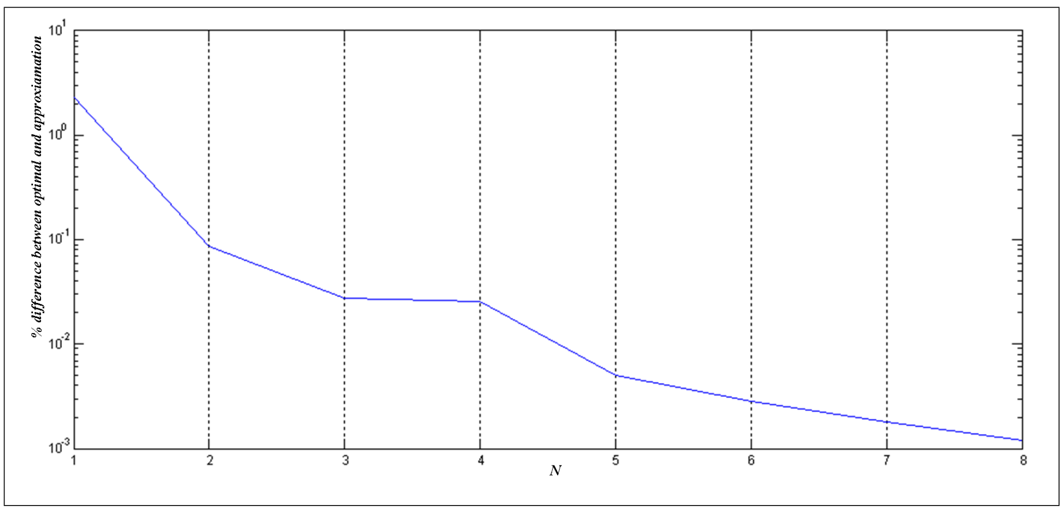

Figure 4.5: The relationship between the number of farms and the percentage error between the optimal solution and the approximation

Chapter 5

MULTI PERIOD PROBLEM

In this chapter, we consider the planning problem when the planning period consist of $\mathcal{T}$ periods and the number of available farms is N, where $N \geq 2$. The demand exists for all periods and the aim is to maximize the sum of profit through the planning period. In the beginning of the planning period, the decision maker needs to decide the farm areas to be seeded and the seeding times if necessary.

Although the problem is similar to the multi farm single period problem, discussed in the previous section, it differs from that problem due to the number of periods considered in the planning period. In the multi period problem, the production through all periods should be taken into account, while in the single period problem only the production in a certain period, $t_{\dagger}$, is considered. In the multiperiod problem, the production quantity of a farm in all periods is dependent to each other. Deciding about the seeding time and the seeding area of the farm, determines the expected production through all the harvest period. The expected output quantity is given in Equation(3.4).

Let us consider the supply quantity when there are three farms with the given seeding times and planting areas. The yield of each farm is random with normal distribution. Based on the seeding times, the harvest period can be calculated for each farm by using the probability function defined in Equation(3.3). Farms have maturation and harvest period distributions that are shown in Figure 3.2.

In Figure 5.1, the histogram of output quantity of all three farms are given. The seeding time for the first farm $\tau_1 = 4$. By looking at the Figure 3.2, it is easy to see that the harvest period can start after one period of the seeding time and can last for six periods. Namely it is possible to harvest crops in periods between the fifth and tenth period. Both the second and the third farm are seeded in period 1, leading possible harvest periods between second period and seventh period for the second farm, and between second period and seventh period for the third one.

Throughout the periods discussed above, it is possible to have an output. The total output in period t, Q_t, shown in the fourth row, is equal to the sum of the supply quatitites of all farms that are available in that period t. For instance, in period 3, the total quantity is equal to the production in Farm 2 and in Farm 3, and in period 6, the total supply is equal to the the sum of the production all three farms while in period 10, since there is only one farm available, the total production quatitity is equal to the production of farm 3. The histogram of $q_{i,t}$ and Q_t obtained by simulation can be seen in the following figure.

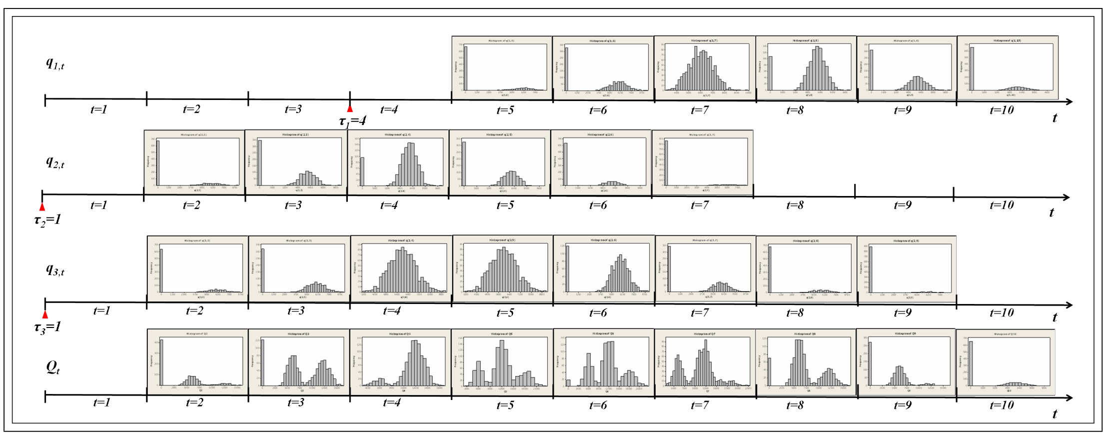

Figure 5.1: The histogram of $q_{i,t}$s and Q_t

In the previous section, the expected profit in period $t_\dagger$ is given in Equation (4.18). The total profit of all periods is equal to the sum of profits in each period and is written as follows;

$$E[\pi(\mathbf{a},\tau)] = \sum_{t}^{\mathcal{T}}(r_t - s_t)\sum_{\alpha_i=0}^{1}\dots\sum_{\alpha_N=0}^{1}(\prod_{i=1}^{N}p_{i,\tau_i}^{\alpha_i}(t)(1-p_{i,\tau_i}^{1-\alpha_i}(t)))\min((\sum_{i=1}^{N}q'_{i,t}\alpha_i),D_t) - (c_t - s_t)E[Q_t] \quad (5.1)$$

We examine the multi farm, multi period problem for two cases. In the first case, the planning problem of perennial plants, where seeding times are not decision variables, is studied. In the second case, the planning problem of annual plants is considered. In this case, the decision maker needs to decide both for seeding areas and seeding times.

5.1 The Planning Problem of Perennial Plants

In the planning problem of perennial plants, the aim is to maximize the total profit by deciding how many acres to seed in each farm.

$$\underset{\mathbf{a}}{\text{Max}} \quad E[\pi\left(\mathbf{a},\tau\right)] \tag{5.2}$$

where $E[\pi\left(\mathbf{a},\tau\right)]$ is given in Equation (5.1).

Theorem 6 *$E[\pi(\mathbf{a},\tau)]$ is a concave function of $\mathbf{a}$. For all N and T values,where $N, T \in R^+$, there exist a unique $\mathbf{a} = (a_1, ..., a_N)$, that maximizes the expected profit function, $E[\pi(\mathbf{a},\tau)]$ for given τ.*

Proof 6 *See Appendix A.*

Theorem 6, states that the optimal $\mathbf{a}$ can be found if the seeding times are given. However the profit function has an order of 2^n time complexity, $O(2^n)$, to evaluate which makes Theorem 6 computationally inefficient when N is big. To overcome that problem, we propose several heuristics to solve the problem effectively when N is big.

5.1.1 Approaches for the Determination of Farm Areas

Throughout this section we examine two approaches for finding the seeding areas. Proposed approaches and the exact solution are summarized and the time complexity of objective function is given in Figure 5.2. A_{exct} represents the optimal solution found by using Theorem 6. A_{norm} and A_{exp} are two approaches presented in this section. A_{norm} applies the approximation introduced for single period case in section 4.3 to multi period case. A_{exp} solves the problem replacing all the random variables with their expected values. First, both approaches are introduced, then they are compared with the optimal solution for 288 different scenarios. From Theorem 6, it is known that the optimal **a** can be found for the given seeding times. To find the optimal solution, we found the optimal **a** for each possible τ. The $\tau, \mathbf{a}^*$ pair that provides the maximum profit is taken as the optimal solution. The parameters of scenarios are given in the Table 5.1.

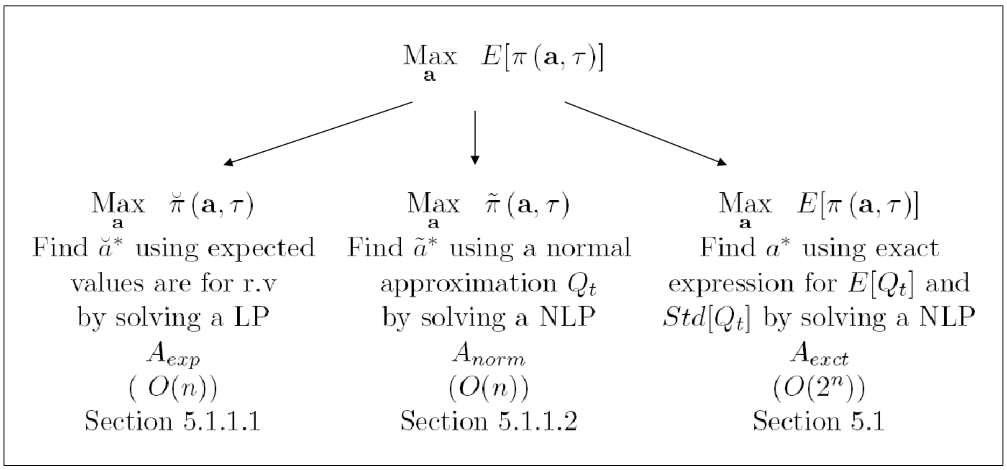

Figure 5.2: The approaches for the determination of Farm Areas

5.1.1.1 *Normality Approximation for The Total Supply Quantity,* A_{norm}

In Section 4.3, we study solution of the single period problem with normality approximation for the total supply quantity in that period. This time we approximate the supply quantity in each period t, as normally distributed with mean $E[Q_t]$, and standard deviation $Std[Q_t]$, given in Equations (3.4) and (3.6). By using Equation (4.5), the expected profit can be written as;

$$\begin{aligned} \tilde{\pi}(\mathbf{a}, \tau) &= \sum_{t=1}^{\mathcal{T}} (r_t - s_t)(E[Q_t]\Phi(\frac{E[D_t] - E[Q_t]}{\theta_t}) + E[D_t]\Phi(\frac{E[D_t] - E[Q_t]}{\theta_t}) \\ & -\theta\phi(\frac{E[D_t] - E[Q_t]}{\theta_t})) - (c_t - s_t)E[Q_t] \end{aligned} \tag{5.3}$$

We can obtain an approximate solution to the problem given in Equation (5.2) by maximizing $\tilde{\pi}(\mathbf{a}, \tau)$ given in Equation (5.3);

$$\underset{\mathbf{a}}{\text{Max}} \quad \tilde{\pi}(\mathbf{a}, \tau) \tag{5.4}$$

The solution is given in the following theorem.

Theorem 7 *Let $\tilde{\mathbf{a}}^*$ be the $\boldsymbol{a}$ value that maximizes $\tilde{\pi}(\mathbf{a}, \tau)$. All $a_i \in \tilde{\mathbf{a}}^*$, should satisfy the following;*

$$\sum_{t=1}^{\mathcal{T}} p_{i,\tau}(t)\Bigg(E[Y_i]\left((r_t - c_t)\,\Phi\left(\frac{E[D_t]-E[Q_t]}{\theta_t}\right) - (c_t - s_t)\right) - (r_t - c_t)\,\frac{a_i(E[Y_i^2] - p_{i,\tau}(t)E[Y_i]^2)}{\theta_t}\phi\left(\frac{E[D_t]-E[Q_t]}{\theta_t}\right)\Bigg) = 0 \tag{5.5}$$

Proof 7 *See Appendix A.*

By solving the set of equations stated above, optimal values of $\tilde{a}$ can be found. Using $\tilde{\mathbf{a}}^*$ found by this approach, the expected profit for each scenario is found and compared with the optimal profit. Let us define $OG()$ as the optimality gap which shows the percentage deviation from the optimal solution. The optimality gap with respect to the number of farms is given in Figure 5.3.

The average optimality gap is 0.64%, and we observe that the optimality gap decreases as the number of farms increases. The results of this approach is given in Appendix B, Table B.1.

where $OG(A_{norm}) = 100 * E[\pi(\mathbf{a}^*, \tau)] - E[\pi(\tilde{\mathbf{a}}^*, \tau)])/E[\pi(\mathbf{a}^*, \tau)]$,

$Cv()$ denotes coefficient of variation,

$E[D_t^1]$=(2000 2000 2000 2000 2000 2000 2000 2000 2000 2000)

$E[D_t^2]$=(2000 2000 3000 3000 2000 2000 2000 2000 2000 2000)

$E[D_t^3]$=(2000 4000 1000 3000 2000 3000 1000 4000 2500 1500)

and DD_m and DD_h are general discrete distributions.

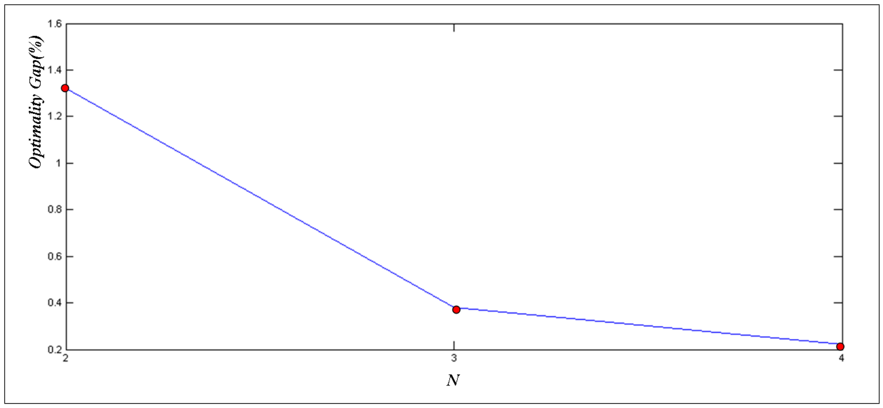

Figure 5.3: The optimality gap of A_{norm} with respect to the changing number of farms

Parameters	**Values**
N :	$2, 3, 4$
T :	10
r_t :	5
c_t :	$2, 4$
s_t :	0.5
$E[D_t]$	$E[D_t^1], E[D_t^2], E[D_t^3]$
$Cv[D]_t$	$0.1, 0.3$
$E[Y_i]$:	6000
$Cv[Y_i]$:	0.1
T^m_{i,τ_i} :	$U(2,4), U(1,5), DD^1_m(t_1, t_2, ..., t_{\mathcal{T}} : p_1^{m_1}, p_2^{m_1}, .., p_{\mathcal{T}}^{m_1}),$ $\mathrm{DD}^2_m(t_1, t_2, ..., t_{\mathcal{T}} : p_1^{m_2}, p_2^{m_2}, .., p_{\mathcal{T}}^{m_2})$
T^h_{i,τ_i} :	$U(3,5), U(2,6), DD^1_h(t_1, t_2, ..., t_{\mathcal{T}} : p_1^{h_1}, p_2^{h_1}, .., p_{\mathcal{T}}^{h_1}),$ $\mathrm{DD}^2_h(t_1, t_2, ..., t_{\mathcal{T}} : p_1^{h_2}, p_2^{h_2}, .., p_{\mathcal{T}}^{h_2})$

Table 5.1: Parameters of Scenarios

$DD^1_m(1, 2, 3, 4, 5, ..., \mathcal{T} : 0, 0.2, 0.6, 0.2, 0, ..., 0)$

$DD^2_m(1, 2, 3, 4, 5, ..., \mathcal{T} : 0.1, 0.1, 0.6, 0.1, 0.1, ..., 0)$

$DD^1_h(1, 2, 3, 4, 5, 6, ..., \mathcal{T} : 0, 0, 0.2, 0.6, 0.2, 0, ..., 0)$

$DD^2_h(1, 2, 3, 4, 5, 6, ..., \mathcal{T} : 0, 0, 0.2, 0.6, 0.2, 0, ..., 0)$

5.1.1.2 Deterministic Approach

The problem is modeled by using the expected values of random variables in Section 3.1.1, $Model_0$. In that model, both the seeding area, $\mathbf{a}$, and the seeding times, τ, are decision variables. When the seeding times are no longer decision variables, the model is written as follows;

$$\operatorname*{Max}_{\mathbf{a}} \quad \breve{\pi}(\mathbf{a}, \tau) \tag{5.6}$$

subject to

$$E[Q_t] \leq MI'_{i,\tau_i}(t), \quad i = 1, ..., N; t = 1, ...\mathcal{T} \tag{5.7}$$

$$E[Y_i] a_i \leq E[Q_t] + M\left(1 - I'_{i,\tau_i}(t)\right), \quad i = 1, ..., N; t = 1, ...\mathcal{T} \tag{5.8}$$

$$E[Q_t] \leq E[Y_i] a_i, \quad i = 1, ..., N; t = 1, ...\mathcal{T} \tag{5.9}$$

$$E[Q_t] - E[D_t] = \Delta_t^+ - \Delta_t^-, \quad t = 1, ...\mathcal{T}$$

where $\breve{\pi}(\mathbf{a}, \tau)$ is given in Equation(3.13).

The model is solved for 288 scenarios given in Table 5.1, using CPLEX solver in GAMS. Results of all scenarios are given in Appendix B, Table B.2. The optimality gap with respect to the number of available farms is given in the following figure.

The average optimality gap of this approach is 10.26% and it increases as the number of farms increases. Results show that the previous approach, A_{norm} performs better.

The optimal solution is not found for bigger N values due to the high computational time. For bigger instances of N, two approaches are compared with each other. We generated 96 scenarios for each N with parameters given in Table 5.1. The results are given in Appendix B, Table B.3. Approach A_{norm} provides higher objective value then approach A_{exp} by 15.43%

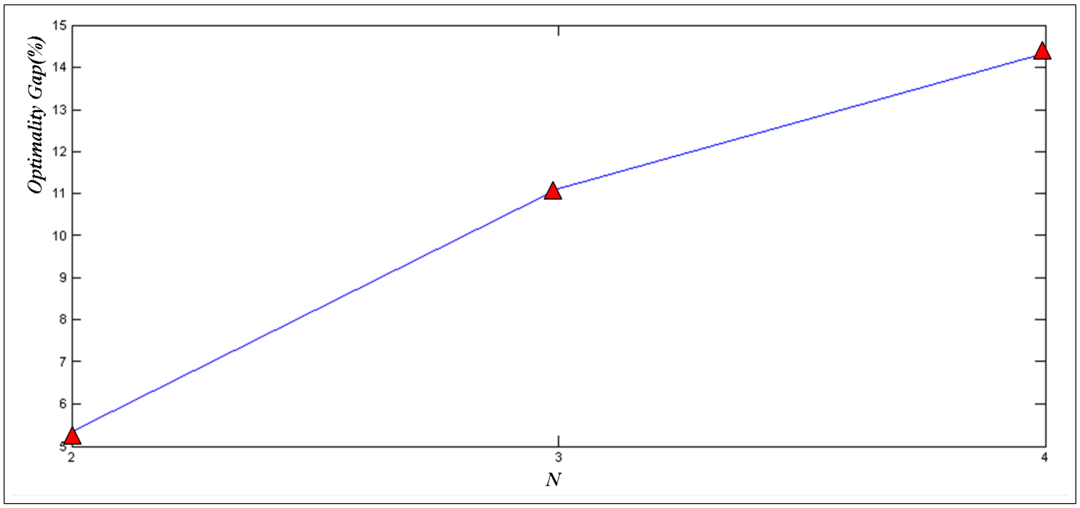

Figure 5.4: The optimality gap of A_{exp} with respect to the changing number of farms

on average. Let $\Delta_{norm-exp}$ donate the percent difference between objective values of A_{norm} and A_{exp}. $\Delta_{norm-exp} = (E[\pi(\tilde{\mathbf{a}}^*, \tau)] - E[\pi(\breve{\mathbf{a}}^*, \tau)])/E[\pi(\tilde{\mathbf{a}}^*, \tau)]$. The following figure shows $\Delta_{norm-exp}$ on a logarithmic scale.

It can be observed that A_{norm} performs better then A_{exp} and the difference increases as the number of farms increases. A_{exp} fails to employ multi farms to decrease risk in the total supply, leading to bigger variations from the optimal. On the other hand, the results of A_{norm} get better as the number of available farms increases. This can be explained by two facts. The real optimal profit tends to increase, when there are more available farms. The decision maker gets the opportunity to decrease the risks by employing more farms. Second, the normality approximation works better as the number employed increases.

5.2 The Planning Problem of Annual Plants

In the planning problem of annual plants, we need to find optimal decisions not only for seeding areas but also for seeding times. Although Thereom 6 provides the optimal values for a, the optimal τ can not be found analytically. For the multi farm, multi period problem where the

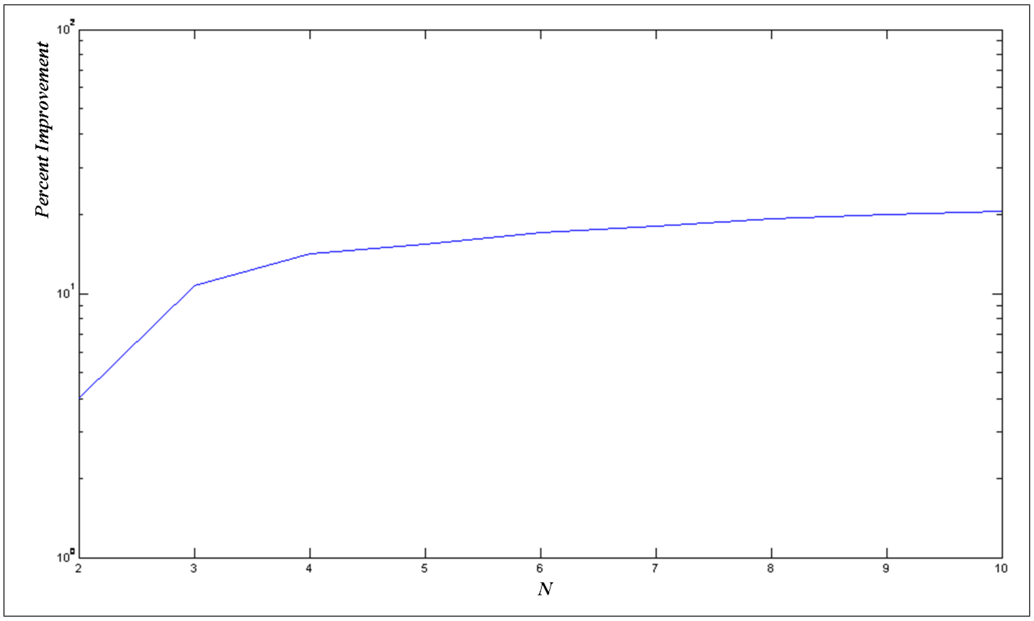

Figure 5.5: The difference between A_{norm} and A_{exp}, $\Delta_{norm-exp}$ with respect to the changing number of farms

decision maker also needs to decide on seeding times of the farms, the optimal solution can be found by solving the following model.

$Model_1$

$$\underset{\mathbf{a},\mathcal{T}}{\text{Max}} \quad E\left[\pi\left(\mathbf{a},\tau\right)\right] \tag{5.10}$$

subject to

$$\tau_i = \sum_{t=1}^{\mathcal{T}} tSt_{i,t}, \quad i = 1, ..., N \tag{5.11}$$

$$\sum_{t=1}^{\mathcal{T}} St_{i,t} \leq 1, \quad i = 1, ..., N \tag{5.12}$$

$$St_{i,t} \in \{0, 1\}, \quad i = 1, ..., N; t = 1, ...\mathcal{T} \tag{5.13}$$

where $E[\pi(\mathbf{a}, \tau)]$ is given in Equation (5.1).

The above problem is a mixed-integer non-linear optimization problem and expected profit function has evaluation complexity of order 2^n, $O(2^n)$. In order to find an approximate solution to the above problem, we propose several approaches.

5.2.1 Approaches for the Determination of Farm Areas and Seeding Times

In this section we propose eight approaches, grouped into three. In the first part, the problem is modeled by using the normality approximation for the total supply quantity and to solve the model two methods are discussed. In the second part, we analyze the results of $Model_0$, introduced in Section 3.1.1 and study on the improvements. Finally, we add a new constraint to $Model_0$, by analyzing the weakness in the second part and apply the improvements proposed in the previous part. All proposed approaches are summarized and the time complexity of objective function evaluations are given in Figure 5.6. Like in the previous section, the results of the heuristics are compared with the optimal solution for 288 different scenarios, parameters of which are given in Table 5.1. To find the optimal solution total enumeration and Theorem 6 are used. The optimal $\mathbf{a}$ is found for each possible seeding time. Then a, τ values that provide the highest profit is taken as optimal solution.

5.2.1.1 Normality Assumption for The Total Supply Quantity

In section 5.1.1.1, the function in Equation (5.3) is maximized when seeding times are known. In order to find optimal $\tilde{\tau}$ as well as optimal $\tilde{a}$, the following model should be solved.

$Model_2$

$$\operatorname*{Max}_{\mathbf{a},\tau} \quad \tilde{\pi}(\mathbf{a}, \tau) \tag{5.14}$$

subject to

$$\tau_i = \sum_{t=1}^{\mathcal{T}} t St_{i,t}, \quad i = 1, ..., N \tag{5.15}$$

$$\sum_{t=1}^{\mathcal{T}} St_{i,t} \leq 1, \quad i = 1, ..., N \tag{5.16}$$

$$St_{i,t} \in \{0, 1\}, \quad i = 1, ..., N; t = 1, ...\mathcal{T} \tag{5.17}$$

The above model is a nonlinear integer model. It is solved for 288 scenarios by using DICOPT solver in GAMS. The parameters of the scenarios are given in Table 5.1. The problem is nonlinear, thus we can not guarantee the optimal solution. The optimality gaps are given in Appendix B, Table B.4 and the average optimality gap with respect to number of farms is shown in Table 5.2.

Iterative Approach

We propose a two-stage approach where in the first stage the farm sizes are determined for given seeding time and in the second stage, the best seeding times are determined for given seeding areas . By solving these two problems iteratively, we obtain an approximate solution to $Model_2$.

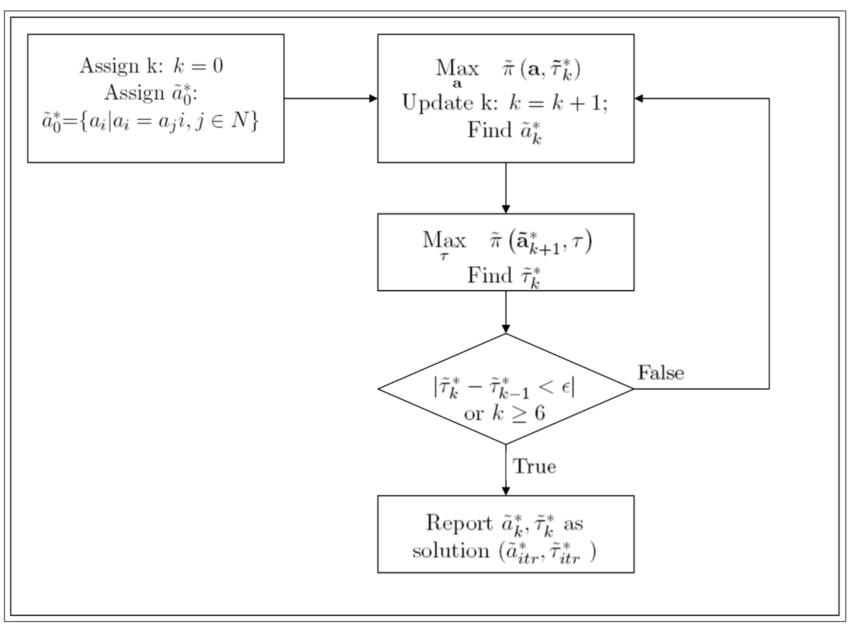

Figure 5.6: The iterative approach, $AT_{norm-iter}$

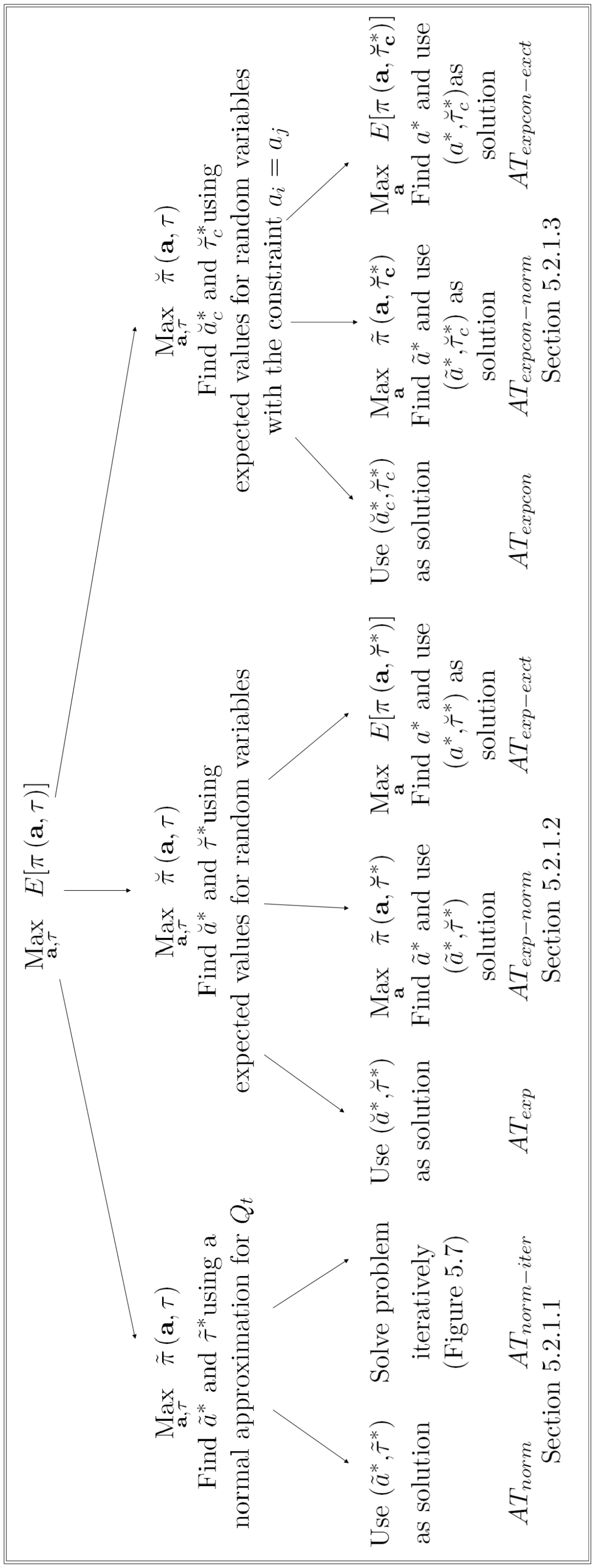

Figure 5.7: Approaches for the Determination of Farm Areas and Seeding Times

In order to determine seeding areas, **a**, we solve the problem given in the previous section for given seeding times. The problem becomes the same as the problem described in Section 5.1.1.1, of which solution is given in Theorem 7. The starting τ is taken as, the optimal τ, when **a** is an a random vector where all $a_i \in \mathbf{a}$ are equal to each other.

After seeding areas are found, $Model_2$ is solved again with provided seeding areas. This time seeding times are decision variables while seeding areas are parameters. When the seeding areas taken as parameters, the problem can be formulated as a mixed-integer linear programming program.

The two problems are solved iteratively for 288 scenarios described in previous sections, until the same result is given successively or the number of iterations exceeds 6. 244 of 288 scenarios, the iterative method converged to a solution. In cases where the maximum number of iterations exceeded, the last solution is taken as the optimal. The results are provided in Appendix B, Table B.5. The optimality gap is given in the following table;

N	**Optimality Gap(%)**	
	AT_{norm}	$AT_{norm-iter}$
2	6.20	2.76
3	4.37	4.34
4	6.56	3.29
Average	5.71	3.46

Table 5.2: The optimality gap of AT_{norm} and $AT_{norm-iter}$

Table 5.2 shows that solving $Model_2$ iteratively rather then directly provides better results. We also observe that the performance of both approaches are not affected by the number of available farms.

5.2.1.2 Deterministic Approach

It is known that $Model_0$, presented in Section 3.1, is linear so it can be solved precisely. In this section we analyze the results of that model and propose several heuristics to improve the solution. First the approaches will be discussed, then results for all approaches will be compared together. The problem is solved as it is modeled in Section 3.1 by using Conopt solver in GAMS.

The first approach,AT_{exp}, is solving $Model_0$ and using $\breve{\mathbf{a}}^*$ and $\breve{\tau}^*$ values provided by the model as solution.

In the previous parts of this section, it is given that the optimal values for **a** can be found when seeding times are given. Using $\breve{\tau}^*$ given by the model above, the optimal values for **a** are found. This method is referred as $AT_{exp-exct}$.

As a third approach, we use the approximation given in the Section 5.1.1.1. By using Theorem 7, $\tilde{\mathbf{a}}^*$ are found for the seeding times provided by the model, $\breve{\tau}^*$. This approach is referred as $AT_{exp-norm}$. The results of these approaches for all scenarios are given in Appendix B, Table B.6,Table B.7 and Table B.8 respectively.

N	**Optimality Gap (%)**		
	AT_{exp}	$AT_{exp-exct}$	$AT_{exp-norm}$
2	3.50	0.19	1.53
3	10.05	7.51	8.84
4	13.37	10.94	12.21
Average	8.97	6.21	7.53

Table 5.3: The optimality gap of AT_{exp},$AT_{exp-exct}$, and $AT_{exp-norm}$

We expect $AT_{exp-exct}$ performs best while it uses the optimal seeding areas with given $\breve{\tau}^*$. But since this approach has high computational time, we don not prefer to use it. Comparing other approaches, we can say $AT_{exp-norm}$ performs better then AT_{exp}. However, $AT_{exp-norm}$ is not employing multi farms like AT_{exp} and for that reason the optimality gap increases as the number of available farms increases.

5.2.1.3 Deterministic Approach with Additional Constraint

In the previous section, it is observed that the results of the model is not utilizing all available farms although they are identical. It is known that for sum of random variables, increasing the number of variables in the sum, decreases the variation. Based on this idea, we add the following constraint to $Model_0$.

$$a_{i+1} - a_i = 0 \quad i = 1, ..., N-1 \tag{5.18}$$

After the model is solved, the solution provided by the model, $\breve{a}_c^*$ and $\breve{\tau}_c^*$ is used as an approach for the problem and referred as AT_{expcon}. Using $\breve{\tau}_c^*$ values as parameters, we maximize $E[\pi(\mathbf{a}, \tau)]$ to find $\mathbf{a}^*$ and refer this approach as $AT_{expcon-exct}$. Next, with $\breve{a}_c^*$ values as parameters, $\tilde{\pi}(\mathbf{a}, \tau)$ is maximized to find $\tilde{\mathbf{a}}^*$, and results are reported as $AT_{expcon-exct}$. The optimality gaps of these approaches for all scenarios are given in Appendix B, Table B.6,Table B.7 and Table B.8 respectively. The average optimality gaps are shown in the following table.

N	**Optimality Gap (%)**		
	AT_{expcon}	$AT_{expcon-exct}$	$AT_{expcon-norm}$
2	4.69	0.41	1.78
3	13.93	6.27	7.36
4	12.98	7.00	7.78
Average	10.54	4.56	5.64

Table 5.4: The optimality gap of AT_{expcon}, $AT_{expcon-exct}$ and $AT_{expcon-norm}$

Although $AT_{expcon-exct}$ performs best among those three approaches, we do not select $AT_{expcon-exct}$ because of the computational time problem.

The optimal solution is not provided for bigger N values due to the high computational time. The average optimality gaps of all proposed approaches are given in Table 5.5.

Best two approaches introduced in this section are $AT_{norm-iter}$ in Section 5.2.1.1 and $AT_{expcon-norm}$ in Section 5.2.1.3 with optimality gaps 7.63% and 5.64% respectively. Assuming AT_{exp} as the base solution, we compared the improvement of those two approaches for different N values, where $N \in 2, ...10$. 98 scenarios are generated for each N value.

Let Δ denotes improvement of approaches according to base solution. $\Delta_{normiter-exp}$, Δ_{n-e} in short , and $\Delta_{expconnorm-exp}$, Δ_{e-e} in short , are compared in the following table. Results for all scenarios can be seen in Appendix B, Table B.9 and B.10 respectively.

SolutionApproach	**Average Optimality Gap (%)**
AT_{norm}	5.71
$AT_{norm-iter}$	3.46
AT_{exp}	8.97
$AT_{exp-exct}$	6.21
$AT_{exp-norm}$	7.54
AT_{expcon}	10.54
$AT_{expcon-exct}$	4.56
$AT_{expcon-norm}$	5.64

Table 5.5: The optimality gaps of the approaches for the determination of Farm areas and Seeding times

$AT_{norm-iter}$				$AT_{expcon-norm}$			
N	$\bar{\Delta}_{n-e}$	$f[\Delta_{n-e} \geq 0]$	$f[\Delta_{n-e} \geq x] = 0.95$	N	$\bar{\Delta}_{e-e}$	$f[\Delta_{e-e} \geq 0]$	$f[\Delta_{e-e} \geq x] = 0.95$
2	0.87	62.50	−5.05	2	1.93	58.33	−2.94
3	6.60	83.33	−18.54	3	3.05	82.29	−1.64
4	12.57	91.67	−9.97	4	6.46	87.50	−1.00
5	14.42	89.58	−18.19	5	7.51	87.50	−0.70
6	18.99	92.71	−15.38	6	9.16	86.46	−1.53
7	19.89	92.71	−5.51	7	11.12	91.67	−0.51
8	22.95	98.96	10.00	8	11.56	91.67	−0.49
9	25.37	98.96	11.73	9	13.36	92.71	−0.48
10	21.94	91.67	−17.97	10	12.83	92.71	−0.48
Average	15.96	89.12	−7.70	*Average*	8.56	85.65	−1.09

Table 5.6: Percent Improvement of $AT_{norm-iter}$ and $AT_{expcon-norm}$ according to base solution AT_{exp}.

where $\bar{\Delta}$ denotes the mean, and $f[\Delta]$ denotes the frequency.

The approach $AT_{norm-iter}$ provides better results then AT_{exp} for 89.12% of time and improves results by 15.96% on average. On the other hand, 5% of time, it may provide solutions 7.7% lower then AT_{exp}. Other approach,$AT_{expcon-norm}$ performs better then AT_{exp} for 85.65% of time and gives results, that are 8.56% better then base approach. The result provided by $AT_{expcon-norm}$ is lower then AT_{exp} on average 1.09% for 5% percent of time. So, the decision maker should prefer $AT_{norm-iter}$ procedure under the assumption that he is risk neutral.

In all cases, normal approximation to total supply quantity provide close results to the optimal solution. However, the problem is nonlinear so the problem can not be solved precisely.

On the other hand,the deterministic model can be solved easily, but it fails to consider the variability in supply.

As a result, we propose using approach A_{norm} described in Section 5.1.1.1 to solve planning problems of pereannual plants and $AT_{norm-iter}$ to solve planning problems of annual plants.

Chapter 6

THE PLANNING PROBLEM OF ALARA AGRI BUSINESS

In this section, we present the planning problem of Alara Agri Business and analyze the possible improvements over the deterministic approach by using the proposed solution approach for annual plants.

6.1 Alara Agri Business

Alara Agri Business produces and exports fresh cherries and figs to 22 countries across 5 continents. It has established in 1986 in Bursa, and now it is the world's largest producer and exporter of fresh cherries and figs.

The company has 300ha orchards and 10000 contracted growers. Throughout the harvest season, every day the products are gathered according to quality specifications. Mobile hydrocoolers are used in order to cool the fruits right after the harvest in each region, and then they are shipped directly to the central pack house in Bursa. The fruits are kept at the ideal temperature through the whole chain. Once the fruits arrive at the packing facility, the quality assurance department checks the product and then the fruits are transferred to the packing line. The fruits are sorted and packed according to the customer demand. Alara implements a traceability system from the orchard to the supermarket shelf. Also, the company assures the quality of the fruits.

The fig season starts in the middle of August and finishes in the middle of October. The figs are supplied through contracted growers around Bursa. On the other hand, the company supplies the cherry for eight months, four months in Turkey, and four months in Argentina. In Argentina, again the company supplies all the fruits from local growers but in Turkey the company also produces cherry as well as buying from the contracted growers. The harvest period of cherry is from May to August, in different regions across Turkey. The harvest availabilities can be seen in the following Figure 6.1.

	May				June				July				August			
Region	W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4	W1	W2	W3	W4
Manisa			●	●	●	●										
Izmir				●	●	●										
Bursa					●	●	●	●	●							
Sakarya					●	●	●	●								
Yalova					●	●	●	●								
Gaziantep						●	●									
Mardin						●	●									
Çanakkale						●	●	●								
Aydın						●	●	●								
Denizli						●	●	●								
Burdur						●	●	●								
Amasya						●	●	●								
Balıkesir						●	●	●								
Tekirdağ						●	●	●								
Tokat							●	●								
Isparta							●	●								
Afyon							●	●	●	●						
Uşak							●	●	●	●						
Çankırı							●	●	●	●						
Bilecik							●	●	●	●						
Karaman							●	●	●	●						
Kütahya							●	●	●	●						
K.Maraş							●	●	●	●	●					
Konya							●	●	●	●						
Icel							●	●	●	●						
Malatya							●	●	●	●						
Adana							●	●	●	●						
Niğde										●	●	●				
Kayseri										●	●	●				
Eskişehir											●	●	●	●		

Figure 6.1: The harvest availabilities of cherry in Turkey

The company has four cherry orchards, total 300ha; in Manisa, Çanakkale, Bursa and Eskişehir. The aim of having orchards in those regions is to assure the cherry supply in early and late periods. Through the main period, the company does not face any difficulties in supplying cherry from the growers that Alara established a contractual agreement with. The forecast about the cherry production is done based on the weather conditions through the year. The weather is compared to the previous years, and the yield estimation is done depending on the year that seems similar to the current year.

6.2 Planning Problem

We assume there is an available farm in each of the locations given in Figure 6.1 and all the farms are identical except their harvest availabilities that can be seen in the same figure. Since cherry trees are perennial plants, the planning problem does not involve the decision of seeding times. However, there exists different maturation period for each farm. In order to capture the randomness caused by maturation time, we assume that all farms are seeded in the beginning of May.

We incorporate an expert opinion[1] to decide the probability function for the harvest and maturation lengths. We use two types of distribution for the lengths of harvest and maturation period. Both distributions have the same mean but different variations. The time between theoretical seeding times and expected start of the harvest, are taken as the expected length of maturation. Also, based on Figure 6.1, the length of available periods are taken as the expected length of harvest. The distributions are summarized in Table 6.1.

T^m_{i,τ_i}
$DDa_{m_0}(0,...,\xi_m,...\mathcal{T}+1:0,...,0,1,0,...,0)$
$DDa_{m_1}(0,...,\xi_m-1,\xi_m,\xi_m+1,...\mathcal{T}+1:0,...,,0,0.2,0.6,0.2,0,...,0)$
$DDa_{m_2}(0,...,\xi_m-2,\xi_m-1,\xi_m,\xi_m+1,\xi_m+2,...\mathcal{T}+1:0,...,0,0.1,0.2,0.4,0.2,0.1,0,...,0)$

T^h_{i,τ_i}
$DDa_{h_0}(0,...,\xi_h,...\mathcal{T}+1:0,...,0,1,0,...,0)$
$DDa_{h_1}(0,...,\xi_h-1,\xi_h,\xi_h+1,...\mathcal{T}+1:0,...,0,0.2,0.6,0.2,0,...,0)$
$DDa_{h_2}(0,...,\xi_h-2,\xi_h-1,\xi_h,\xi_h+1,\xi_h+2,...\mathcal{T}+1:0,...,0,0.1,0.2,0.4,0.2,0.1,0,...,0)$

Table 6.1: The harvest and maturation lenght distributions in the planning problem of Alara Agri Business

[1] Belit Balcı, Orchard Operational Director of Alara Agri Business

where ξ_m and ξ_h equal to the expected values of maturation and harvest lengths in each case.

The problem is modeled and solved in two different methods. First, A_{exp} approach, given in section 5.1.1.2, is used to solve the problem. As a second method, A_{norm} approach, described in section 5.1.1.1 is used. We solve the problem with both approaches for 162 scenarios. The parameters of scenarios are given in Table 6.2.

The results of all scenarios is given in Appendix C, Table C.1. The average expected profit of A_{norm} is 643119, while it is 546551 for A_{exp}. The service levels for A_{norm} and A_{exp} are 71.45% and 73.34% respectively. The increase in the profit is 17.67% when A_{norm} is used instead of A_{exp} while service level does not change significantly. In Table 6.3, we present some results that we want to analyze further.

Parameters	**Values**
N :	30
T :	16
r_t :	5
c_t :	$2, 4$
s_t :	0.5
$E[D_t]$	(0 1000 2000 3000 4000 4000 4000 4000 4000 4000 4000 4000 3000 2000 1000 0)
$Cv[D_t]$	$0, 0.1, 0.3$
$E[Y]$:	30
$Cv[Y_t]$:	$0, 0.1, 0.3$
T^m_{i,τ_i}	$DDa_{m_0}, DDa_{m_1}, DDa_{m_2}$
T^h_{i,τ_i}	$DDa_{h_0}, DDa_{h_1}, DDa_{h_2}$

Table 6.2: Parameters of Scenarios

From Table 6.3, it can be observed that the improvement in the profit changes considerably. For instanstance the improvement in the profit function in the first two scenarios is 0 while it is 877.51% for scenario 162. Note that the variation of lenght of both maturation and harvest periods increase becomes 1.68 in scenario 162 while the expected lenght of those periods are 6.6 and 3.5 respectively. By analyzing Table C.1, it can be claimed that approach A_{norm} performs better than approach A_{exp} when the randomness in the problem increases.

$Cv[D_t]$	$Cv[Y_i]$	T^h_{i,τ_i}	T^m_{i,τ_i}	c_t	$Sc.No$	Expected Profit		$\Delta_{n-e}(\%)$	Expected S.Level	
						A_{norm}	A_{exp}		A_{norm}	A_{exp}
0	0	DDa_{h_0}	DDa_{m_0}	2	1	1184995,65	1185000,00	0,00	0,84	0,86
				4	2	389999,99	390000,00	0,00	0,82	0,82
			DDa_{m_1}	2	3	1092251,08	1059329,40	3,11	0,81	0,80
				4	4	312204,20	282351,90	10,57	0,69	0,77
			DDa_{m_2}	2	5	1045023,16	947744,25	10,26	0,78	0,74
				4	6	287718,94	190174,80	51,29	0,64	0,73
0.3	0.3	DDa_{h_2}	DDa_{m_0}	2	157	932106,99	858369,84	8,59	0,78	0,71
				4	158	210157,98	77931,10	169,67	0,60	0,71
			DDa_{m_1}	2	159	926872,89	830395,12	11,62	0,76	0,69
				4	160	206196,51	49940,00	312,89	0,57	0,69
			DDa_{m_2}	2	161	920321,56	798231,75	15,30	0,76	0,67
				4	162	199861,17	20445,99	877,51	0,55	0,67
Average						642309,18	557492,85	122,57	0,72	0,74

Table 6.3: The results of some scenarios

We want to study the effects of randomness in detail. Thus, we study the effects of change in variability of demand, yield ,maturation length and harvest length in the proposed solution. The change in the average profit function,$\Delta_{norm-exp}$, with respect to changing coefficinst variations of demand, and yield, is given in the following figure.

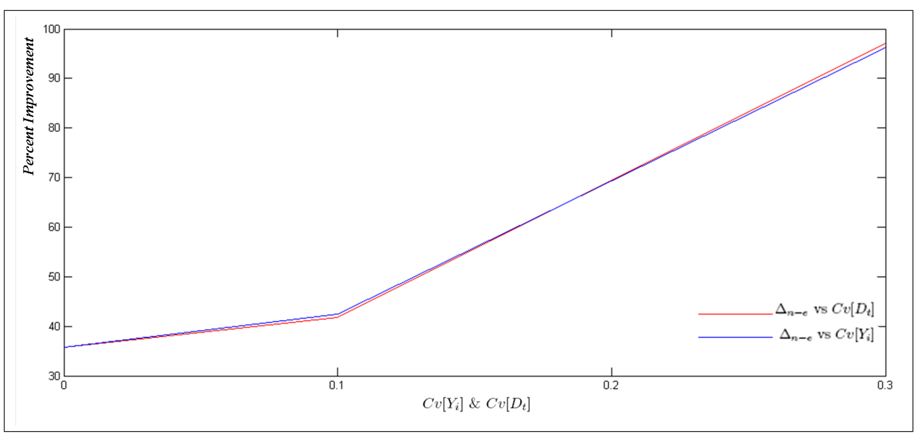

Figure 6.2: The change in Δ as coefficient variations of demand and yield increase

Figure 6.2 shows that as the variations of demand and yield increase, the difference between A_{exp} and A_{norm} increases as expected. In addition, it can be claimed that the change in variation of demand and yield has the same effect in the $\Delta_{norm-exp}$. Next, we analyze the effect of change in variation of maturation and harvest periods in the profit function in Figure 6.3.

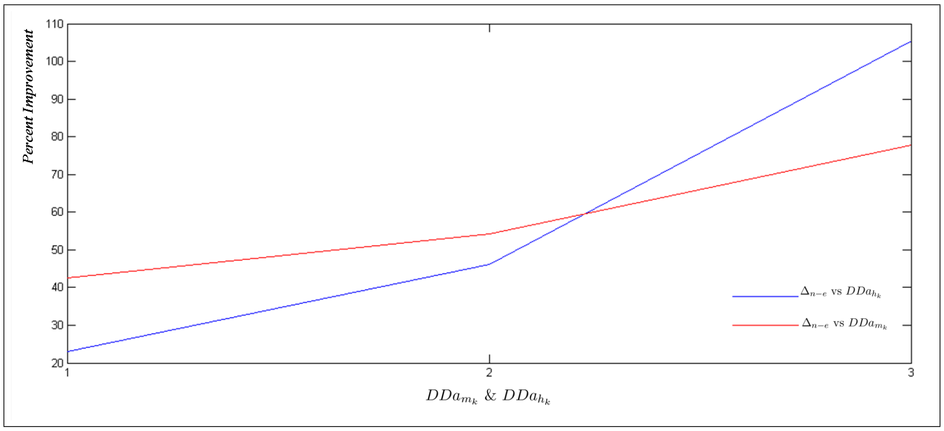

Figure 6.3: The change in Δ as variations of harvest length and demand length increase

Figure 6.3 shows the change in the variation of maturation length effects $\Delta_{norm-exp}$ more.

We know A_{exp} fails to employ multiple farms to decrease the variation in the supply quantity. To illustrate, the pooling effect in the problem, we analyzed ten alternate solutions of A_{exp}. At first, just one of ten identical farms in Figure 6.1 is used. Then, the number of farms employed is increased while the total seeded area remains the same. The results are as follows;

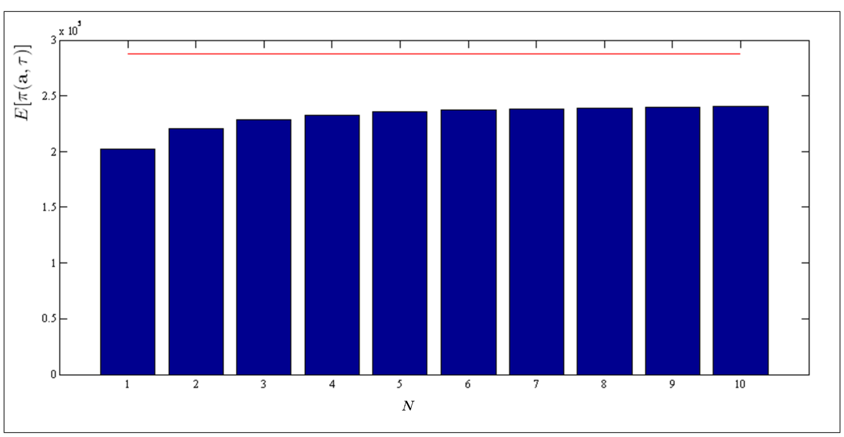

Figure 6.4: The expected profit of as the number of farms employed increases

The blocks show the expected profit of the different solutions while the line above them illustrates the optimal profit. As the figure shows, employing multiple farms, increases the the expected profit. However, although the number of farms increases, A_{norm} still performs better.

In the next two figures, we see the total supply quantity provided by the solutions of two approaches.

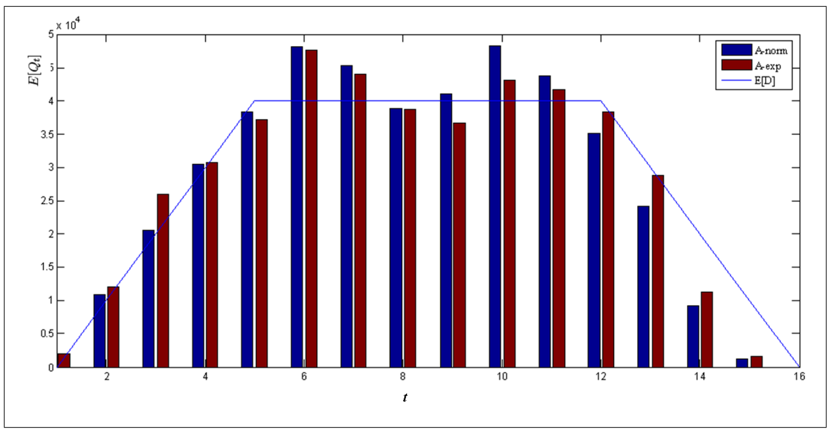

Figure 6.5: The total supply quantity of A_{norm} and A_{exp} when $c_t = 2$

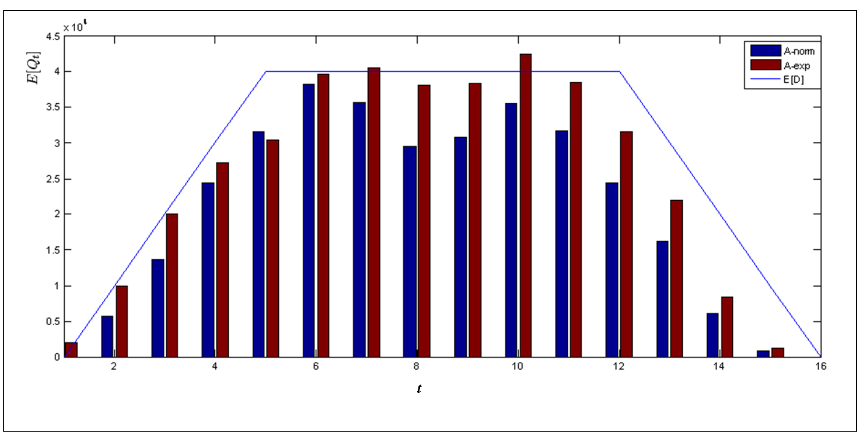

Figure 6.6: The total supply quantity of A_{norm} and A_{exp} when $c_t = 4$

In the first figure c_t equals to 2, while it is 4 in the second one. The figures show that A_{norm}

is more sensitive to the changes in costs. When c_t equals to 2, supply quantities are almost same. However, as c_t increases to 4, the supply quantity of solution of A_{norm} becomes significantly lower then one of A_{exp}.

This case study shows that the proposed method that captures the demand, harvest, and yield uncertanities jointly allows the decision maker increase their profit substantially. The expected benefit increases with the uncertanities present in the model. As a result, we propose this model and the solution procedure as an effective planning tool to be used in agricultural supply chain.

Chapter 7

CONCLUSION

In this thesis, we consider an agricultural planning problem when both demand and supply are random. In the literature, there are some works similar to this problem but none of them offers an approach considering all variability caused by harvest, yield and demand.

We study two versions of the problem: single period and multiperiod. We first analyze the problem when the demand exist for one period and only one supplier is available. This version of the problem is similar to the newsvendor problem. In the literature, there are some studies that tackle the newsvendor problem with random supply. However, none of them could solve the problem when both are normally distributed. We provide the analytical solution to the newsvendor problem for that case, then utilize this solution in our solution procedure. For the single period problem, we then study the case when there are multiple identical suppliers. When the number of the suppliers increases, it becomes difficult to find the optimal solution due to the computational complexity of the objective function. Therefore we propose a normal approximation for the supply. The normality approximation provides results that deviate from the optimal optimal solution by 2.3% on average for single farm case. The results of approximation improves as the number of farms increases, the optimality gap decreases to 0.001% as number of supplier increases to 8.

In the second part we first study the solution of the multiperiod, multi supplier problem for perennial plants where the seeding time is not a decision variable. As in the single period, multi supplier case, exact evaluation of the profit function is computationally demanding. Hence efficient solution procedures are required to solve large-sized problem instances. Recognizing this fact, we develop some efficient solution procedures for both problems. For the planing problem of perennial plants, we propose a normal approximation to the supply quantity and use the solution of the newsvendor problem with random supply. The problem is also solved with a deterministic approach. Both solutions are compared with the optimal solution for a number of cases when the number of farms is small so an optimal solution can be found numerically. The proposed approach yields solutions that deviate from the optimal solutions by no more than 1% on average. When the number of farms is large, the two solution approaches are compared with each other. The proposed approach provides higher profit than the deterministic approach by 15% on average.

Finally, for the multiperiod planning problem of annual plants where both farm areas and seeding times are decision variables, we proposed several heuristics for that problem and compared them with the optimal solution for small instances of N. The optimal solution of the planning problem of annual plants is found by total enumeration and nonlinear optimization. Taking the optimality gap as an performance criteria, we identified two best heuristics. First one uses the normal approximation for the total supply, and solves a nonlinear-integer model iteratively. The second one finds the seeding times using a deterministic model where all seeding areas are forced to be equal to each other, and then finds the seeding areas based on those seeding times using the normal approximation to the total supply. Both of the proposed methods provide better objective results then deterministic approach. The first one provides better results on average 15.96% while the second one improves the results by 8.56%.

The main contributions of this study are:

- Presenting a model that captures variations of the harvest period, the yield and the demand.

- Finding an exact solution of seeding areas and seeding time for single period case.

- Developing computationally efficient heuristic solutions for the multi-period problem.

- Analyzing a case study from the industry and showing the effectiveness of the proposed method.

This work can be extended in several ways. Some noteworthy of future research directions are analyzing the problem under different type of contracts between the firm and the suppliers including profit sharing, total rent, etc., solving the problem for different decision makers with different risk aversion and finding more efficient solution procedures for the multiperiod problem. Also, the dependency of harvest probabilities between the farms in the same region can be explored. A possible way of incorporating the dependency is using the same distribution of maturation period and harvest period for the farms located in the same region. More effective handing requires further investment.

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Appendix A

PROOFS FOR THEOREMS

A.1 Proofs for Chapter 4

A.1.1 Proof of Theorem 1

The first derivative of $E[\pi'[Q]]$(4.5) is as follows;

$$\frac{\partial E[\pi]}{\partial x} = (r-s)\left(\frac{\partial f(x)}{\partial x}\Phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right) + (f(x)-\mu_D)\,\phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\frac{\partial}{\partial x}\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\right.$$
$$\left. - \frac{\partial \theta(x)}{\partial x}\phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right) + \theta(x)\phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\frac{\partial}{\partial x}\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\right) - (c-s)\,\frac{\partial f(x)}{\partial x} = 0$$

where $\theta(x) = \sqrt{g^2(x) + \sigma_D^2}$ and

$$\frac{\partial \theta(x)}{\partial x} = g(x)\frac{\partial g(x)}{\partial x}\theta(x)^{-1}$$

then, the derivative becomes;

$$\frac{\partial E[\pi]}{\partial x} = (r-s)\left(\frac{\partial f(x)}{\partial x}\Phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right) - g(x)\frac{\partial g(x)}{\partial x}\theta(x)^{-1}\phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\right) - (c-s)\,\frac{\partial f(x)}{\partial x}$$

The second derivative of the profit function is as following;

$$\frac{\partial^2 E[\pi]}{\partial x^2} = (r-s)\left(\frac{\partial^2 f(x)}{\partial x^2}\Phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right) + \frac{\partial f(x)}{\partial x}\phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\frac{\partial}{\partial x}\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\right.$$
$$- \left(\left(\frac{\partial g(x)}{\partial x}\frac{\partial^2 g(x)}{\partial x^2}\right)\theta(x)^{-1} - g^2(x)\frac{\partial g(x)}{\partial x}^2\theta(x)^{-3}\right)\phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right)$$
$$\left. + g(x)\frac{\partial g(x)}{\partial x}\theta(x)^{-1}\phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\frac{\partial}{\partial x}\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\right) - (c-s)\frac{\partial^2 f(x)}{\partial^2 x} = 0$$

where $\theta(x) = \sqrt{g^2(x) + \sigma_D^2}$ and

$$\frac{\partial}{\partial x}\left(\frac{\mu_D - f(x)}{\theta(x)}\right) = -\frac{\mu_D - f(x)}{\theta^2(x)}g(x)\frac{\partial g(x)}{\partial x}\theta(x)^{-1} - \frac{\partial f(x)}{\partial x}\theta(x)^{-1}$$

Replacing $\frac{\partial}{\partial x}\left(\frac{\mu_D - f(x)}{\theta(x)}\right)$ with the function given above and replacing $\theta(x)$ with $g^2(x) + \sigma_D^2$ yield;

$$\frac{\partial^2 E[\pi]}{\partial^2 x} = \frac{\partial^2 f(x)}{\partial^2 x}\left((r-s)\Phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right) - (c-s)\right)$$
$$- \theta(x)^{-3}\phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\left[\begin{array}{c}\left(\frac{\partial f(x)}{\partial x}\theta(x) + g(x)\frac{\partial g(x)}{\partial x}\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\right)^2 \\ +\sigma_D^2\left(g(x)\frac{\partial^2 g(x)}{\partial^2 x} + \frac{\partial g(x)}{\partial x}^2\right) + g(x)^3\frac{\partial^2 g(x)}{\partial^2 x}\end{array}\right]$$

Let φ_1 denote
$\left((r-s)\Phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right) - (c-s)\right)$
and φ_2 denote
$\left(\frac{\partial f(x)}{\partial x}\theta(x) + g(x)\frac{\partial g(x)}{\partial x}\left(\frac{\mu_D - f(x)}{\theta(x)}\right)\right)^2 + \sigma_D^2\left(g(x)\frac{\partial^2 g(x)}{\partial^2 x} + \frac{\partial g(x)}{\partial x}^2\right) + g(x)^3\frac{\partial^2 g(x)}{\partial^2 x}$.
Then the second derivative, is written as;

$$\frac{\partial^2 E[\pi]}{\partial^2 x} = \frac{\partial^2 f(x)}{\partial x}\varphi_1 - \theta(x)^{-3}\varphi_2\phi\left(\frac{\mu_D - f(x)}{\theta(x)}\right)$$

Since Φ and ϕ donate the standard normal cumulative and probability distribution function respectively, the terms $\Phi\left(\frac{\mu_D - \mu_Q}{\theta}\right)$ and $\phi\left(\frac{\mu_D - \mu_Q}{\theta}\right)$ are always non-negative. Also $\theta(x)$, which is equal to $\sqrt{\sigma_Q^2 - 2\rho\sigma_Q\sigma_Q + \sigma_D^2}$, and $g(x)$, that is the standard deviation supply, are non-negative. Although we know (r-s) is bigger then (c-s) by definition, since Φ changes from 0 to 1 , the sign of φ_1 can not be known. On the other hand, we know φ_2 is positive when the second derivative of g(x) is non-negative. The second derivative profit function is non-positive as long as $\frac{\partial^2 g(x)}{\partial^2 x}$ is non-negative and $\frac{\partial^2 f(x)}{\partial x}$ is equal to zero.

When second derivative non-negative,x value that satisfies the first order optimality condition is optimal x.

A.1.2 Proof of Theorem 2

We want to find the optimal **a** to maximize $E[\pi(\mathbf{a},\tau)]$, given in Equation (4.11). The first order optimality condition to find a is:

$$\frac{\partial E[\pi(\mathbf{a},\tau)]}{\partial a} = 0$$

$$\begin{aligned} E\left[\pi\left(\mathbf{a},\tau\right)\right] \;&=\; (r_{t^\dagger} - s_{t^\dagger})p_{1,\tau_1}(t^\dagger)\Bigg(E\left[Q'_{t^\dagger}\right]\Phi\left(\frac{E\left[D_{t^\dagger}\right] - E\left[Q'_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right) \\ &+E\left[D_{t^\dagger}\right]\Phi\left(\frac{E\left[Q'_{t^\dagger}\right] - E\left[D_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right) - \theta'_{t^\dagger}\phi\left(\frac{E\left[D_{t^\dagger}\right] - E\left[Q'_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right)\Bigg) \\ &-(c_{t^\dagger} - s_{t^\dagger})E[Q_{t^\dagger}] \end{aligned}$$

where $\theta_t = \sqrt{\sigma_{Q_t}^2 + \sigma_{D_t}^2}$.
Replacing $(r_{t^\dagger} - s_{t^\dagger})p_{1,\tau_1}(t^\dagger)$ with $(r_{t^\dagger} - s_{t^\dagger})'$, expected profit is written as follows;

$$\begin{aligned} E\left[\pi\left(\mathbf{a},\tau\right)\right] \;&=\; (r_{t^\dagger} - s_{t^\dagger})'\Bigg(E\left[Q'_{t^\dagger}\right]\Phi\left(\frac{E\left[D_{t^\dagger}\right] - E\left[Q'_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right) \\ &+E\left[D_{t^\dagger}\right]\Phi\left(\frac{E\left[Q'_{t^\dagger}\right] - E\left[D_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right) - \theta'_{t^\dagger}\phi\left(\frac{E\left[D_{t^\dagger}\right] - E\left[Q'_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right)\Bigg) \\ &-(c_{t^\dagger} - s_{t^\dagger})E[Q_{t^\dagger}] \end{aligned}$$

Let Q' be functions of parameter a , $\mu_{Q'_t} = f'_t(a)$, and $\sigma_{Q'_t} = g'_t(a)$. If $f'(a)$ is linear and

$g'(a)$ is concave, then from Theorem 1, we know optimal a should satisfy Equation (4.6).

A.1.3 Proof of Theorem 3

In Theorem 3 , we need to prove that Equation (4.11) is an increasing function of $p_{1,\tau_1}(t^\dagger)$. The equation (4.11) is as follows;

$$\begin{aligned} E\left[\pi\left(\mathbf{a},\tau\right)\right] &= (r_{t^\dagger}-s_{t^\dagger})p_{1,\tau_1}(t^\dagger)\Bigg(E\left[Q'_{t^\dagger}\right]\Phi\left(\frac{E\left[D_{t^\dagger}\right]-E\left[Q'_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right) \\ &\quad +E\left[D_{t^\dagger}\right]\Phi\left(\frac{E\left[Q'_{t^\dagger}\right]-E\left[D_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right)-\theta'_{t^\dagger}\phi\left(\frac{E\left[D_{t^\dagger}\right]-E\left[Q'_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right)\Bigg) \\ &\quad -(c_{t^\dagger}-s_{t^\dagger})E[Q_{t^\dagger}] \end{aligned}$$

The derivative of the function is written as;

$$\begin{aligned} \frac{\partial E[\pi\left(a,\tau\right)]}{\partial p_{1,\tau_1}(t^\dagger)} &= (r_{t^\dagger}-s_{t^\dagger})p_{1,\tau_1}(t^\dagger)\Bigg(E\left[Q'_{t^\dagger}\right]\Phi\left(\frac{E\left[D_{t^\dagger}\right]-E\left[Q'_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right) \\ &\quad +E\left[D_{t^\dagger}\right]\Phi\left(\frac{E\left[Q'_{t^\dagger}\right]-E\left[D_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right)-\theta'_{t^\dagger}\phi\left(\frac{E\left[D_{t^\dagger}\right]-E\left[Q'_{t^\dagger}\right]}{\theta'_{t^\dagger}}\right)\Bigg) \\ &\quad -(c_{t^\dagger}-s_{t^\dagger})E[Q_{t^\dagger}] \end{aligned}$$

which can be rewritten as a function of expected profit.

$$\frac{\partial E[\pi\left(a,\tau\right)]}{\partial p_{1,\tau_1}(t^\dagger)}=\frac{E[\pi\left(a,\tau\right)]}{p_{1,\tau_1}(t^\dagger)}$$

It is known that in an optimal solution $E[\pi\left(a,\tau\right)]$ is always greater or equal to zero, then the derivative of $E[\pi\left(\mathbf{a},\tau\right)]$ over $p_{1,\tau_1}(t^\dagger)$ is always greater or equal to zero.

A.1.4 Proof of Theorem 4

We need to show $E[\pi(a,\tau)]$, Equation (4.18), is concave function of a_i where $a_i \in a$. We take the derivative of function for two cases; for $\alpha_i = 0$ and $\alpha_i = 1$.

When $\alpha_i = 0$;

$$\frac{\partial E[\pi\left(a,\tau\right)]}{\partial a_i} = 0$$

When $\alpha_i = 1$;

$$\begin{aligned}\frac{\partial E[\pi(a,\tau)]}{\partial a_i} &= (r_{t^\dagger} - s_{t^\dagger})\sum_{\alpha_1=0}^{1}\dots\sum_{\alpha_{i-1}=0}^{1}\sum_{\alpha_{i+1}=0}^{1}\dots\sum_{\alpha_N=0}^{1}(\prod_{i=1}^{N}p_{i,\tau_i}(t^\dagger)) \\ &\left(\frac{\partial \min\left(\left(\sum_{i=1}^{N}q'_{i,t^\dagger}\alpha_i\right), D_{t^\dagger}\right) - (c_{t^\dagger} - s_{t^\dagger})E\left[Q_{t^\dagger}\right]}{\partial a_i}\right)\end{aligned}$$

From Theorem 2, we know $\left(\frac{\partial \min\left(\left(\sum_{i=1}^{N}q'_{i,t^\dagger}\alpha_i\right), D_{t^\dagger}\right) - (c_{t^\dagger} - s_{t^\dagger})E\left[Q_{t^\dagger}\right]}{\partial a_i}\right)$ is concave. Then Equation (4.18) is also concave.

A.1.5 Proof of Theorem 5

Assume τ_i^k and $\tau_i^{k'}$ are two seeding times, such that the probability of harvesting in period t given that seeding time is $\tau_i^k, p_{i,\tau_i^k}(t)$, greater then the same probability when the seeding time is $\tau_i^{k'}, p_{i,\tau_i^{k'}}(t)$. Let $\mathbf{a}_i^{k*}$ denote the optimal $\mathbf{a}$ for the seeding time τ_i^k. We need to show that for all $\tau_i^k, \tau_i^{k'} \in \mathcal{T}$, $\tilde{\pi}\left(\mathbf{a}_i^{k*}, \tau_i^k\right)$ is greater or equal to $\tilde{\pi}\left(\mathbf{a}_i^{*}, \tau_i^{k'}\right)$. In order to show $\tilde{\pi}\left(\mathbf{a}_i^{1*}, \tau_i^1\right) >= \tilde{\pi}\left(\mathbf{a}_i^{1*}, \tau_i^2\right)$ when $p_{i,\tau_i^1}(t) >= p_{i,\tau_i^2}(t)$, check the first derivative of $\tilde{\pi}$ with respect to p_{i,τ_i} at point $\left(\mathbf{a}_i^{1*}, \tau_i^1\right)$.

$$\begin{aligned}\frac{\partial E[\tilde{\pi}(a,\tau_i)]}{\partial p_{i,\tau_i}} &= a_i\Bigg(E[Y_i]((r_t - c_t)\Phi(\tfrac{E[D_t]-E[Q_t]}{\theta_t}) - (c_t - s_t)) - \\ &-(r_t - c_t)\tfrac{a_i(E[Y^i]-2p_{i,\tau_i}(t)E[Y]^i)}{2\theta_t}\phi(\tfrac{E[D_t]-E[Q_t]}{\theta_t})\Bigg)\end{aligned}$$

It is known that for $\left(a_i^{1*}, \tau_i^1\right)$ the derivative of expected profit with respect to a_i is zero, by re-organizing Equation (4.19),$\frac{\partial E[\tilde{\pi}(a,\tau)]}{\partial p_{i,\tau_i}}$ can be written as follows;

$$\frac{\partial E[\tilde{\pi}(a,\tau)]}{\partial a_i} = p_{i,\tau_i}\Bigg(E[Y_i]((r_t-c_t)\Phi(\tfrac{E[D_t]-E[Q_t]}{\theta_t})\theta_t)-(c_t-s_t))- (r_t-c_t)\tfrac{a_i(E[Y_i^2]-p_{i,\tau_i}(t)E[Y_i]^2)}{\theta_t}\phi(\tfrac{E[D_t]-E[Q_t]}{\theta_t})\Bigg)$$

If the terms in parenthesis are examined, it can be seen that the second term of both equations are greater or equal to zero: $(r_t - c_t)$, a_i, θ_t and $\phi\left(\frac{E[D_t]-E[Q_t]}{\theta_t}\right)$ are positive by definition, and it is known that expected value of the square of a variable is greater then equal to square of the expected value of the variable so both $(E[Y_i^2] - 2p_{i,\tau_i}(t)E[Y_i]^2)$ and $(E[Y_i^2] - p_{i,\tau_i}(t)E[Y_i]^2)$ are positive . Since the first terms are the same in the two equations,to show $\frac{\partial E[\tilde{\pi}(a,\tau)]}{\partial p_{i,\tau_i}}$ is greater or equal to zero while $\frac{\partial E[\tilde{\pi}(a,\tau)]}{\partial a_i}$ equals to zero, it is sufficient to prove $(r_t - c_t)\frac{a_i(E[Y_i^2]-p_{i,\tau_i}(t)E[Y_i]^2)}{\theta_t}\phi(\frac{E[D_t]-E[Q_t]}{\theta_t}) \geq (r_t - c_t)\frac{a_i(E[Y_i^2]-2p_{i,\tau_i}(t)E[Y_i]^2)}{2\theta_t}\phi(\frac{E[D_t]-E[Q_t]}{\theta_t})$ The difference of these two terms can be written as; $\frac{a_i(r_t-c_t)}{2\theta_t}\phi(\frac{E[D_t]-E[Q_t]}{\theta_t})(E[Y_i^2])$

Since all multiplier factors are greater or equal to zero, the term is non-negative. This proves that when $\frac{\partial\tilde{\pi}}{\partial a_i}$ is zero $\frac{\partial\tilde{\pi}}{\partial p_{i,\tau_i}}$ is non-negative.

A.2 Proofs for Chapter 5

A.2.1 Proof of Theorem 6

We need to show $E[\pi(a,\tau)]$(5.1) is a concave function of a_i.

$$E[\pi(\mathbf{a},\tau)] = \sum_{t}^{\mathcal{T}}(r_t-s_t)\sum_{\alpha_i=1}^{1}...\sum_{\alpha_N=1}^{1}\left(\prod_{i=1}^{N}p_{i,\tau_i}^{\alpha_i}(t)\left(1-p_{i,\tau_i}^{1-\alpha_i}(t)\right)\right)\min\left(\left(\sum_{i=1}^{N}q'_{i,t}\alpha_i\right),D_t\right) -(c_t-s_t)E\left[Q_t\right]$$

From Theorem 4, we know

$$(r_t-s_t)\sum_{\alpha_i=1}^{1}...\sum_{\alpha_N=1}^{1}\left(\prod_{i=1}^{N}p_{i,\tau_i}^{\alpha_i}(t)\left(1-p_{i,\tau_i}^{1-\alpha_i}(t)\right)\right)\min\left(\left(\sum_{i=1}^{N}q'_{i,t}\alpha_i\right),D_t\right)-(c_t-s_t)E\left[Q_t\right]$$

is a concave function of a_i for $i \in N$. Since the sum of concave functions is also concave, equation (5.1) is also concave.

A.2.2 Proof of Theorem 7

We need to show $E[\tilde{\pi}(a, \tau)]$(5.3) is a concave function of a_i.

$$\begin{aligned}\tilde{\pi}(\mathbf{a}, \tau) &= \sum_{t=1}^{\mathcal{T}}(r_t - s_t)(E[Q_t]\Phi(\frac{E[D_t] - E[Q_t]}{\theta_t}) + E[D_t]\Phi(\frac{E[D_t] - E[Q_t]}{\theta_t}) \\ &\quad -\theta\phi(\frac{E[D_t] - E[Q_t]}{\theta_t})) - (c_t - s_t)E[Q_t]\end{aligned}$$

From Theorem 4, we know
$(r_t - s_t)(E[Q_t]\Phi(\frac{E[D_t]-E[Q_t]}{\theta_t}) + E[D_t]\Phi(\frac{E[D_t]-E[Q_t]}{\theta_t}) - \theta\phi(\frac{E[D_t]-E[Q_t]}{\theta_t})) - (c_t - s_t)E[Q_t]$
is a concave function of a_i. Then equation (5.3) is also concave function of a.

Appendix B

RESULTS OF COMPUTATIONAL STUDIES

The List of Abbreviations used in Appendix B.

N :Number of Farms

D.S :Demand Scenarios

1:[2000 2000 2000 2000 2000 2000 2000 2000 2000 2000]

2:[2000 2000 3000 3000 2000 2000 2000 2000 2000 2000]

3:[2000 4000 1000 3000 2000 3000 1000 4000 2500 1500]]

Cv() :Coefficient of variation

c_t :Cost of a unit product at time t

P.S :Harvest Probability Scenarios

1: T^m_{i,τ_i}~$U(2,4)$, T^h_{i,τ_i}~$U(3,5)$

2: T^m_{i,τ_i}~$U(2,4)$, T^h_{i,τ_i}~$U(2,6)$

3: T^m_{i,τ_i}~$U(1,5)$, T^h_{i,τ_i}~$U(3,5)$

4: T^m_{i,τ_i}~$U(1,5)$, T^h_{i,τ_i}~$U(2,6)$

5: T^m_{i,τ_i}~$DD^1_m(t_1, t_2, ..., t_{\mathcal{T}} : p^{m_1}_1, p^{m_1}_2, .., p^{m_1}_{\mathcal{T}})$, T^h_{i,τ_i}~$DD^1_h(t_1, t_2, ..., t_{\mathcal{T}} : p^{m_1}_1, p^{m_1}_2, .., p^{m_1}_{\mathcal{T}})$

6: T^m_{i,τ_i}~$DD^1_m(t_1, t_2, ..., t_{\mathcal{T}} : p^{m_1}_1, p^{m_1}_2, .., p^{m_1}_{\mathcal{T}})$, T^h_{i,τ_i}~$DD^2_h(t_1, t_2, ..., t_{\mathcal{T}} : p^{m_1}_1, p^{m_1}_2, .., p^{m_1}_{\mathcal{T}})$

7: T^m_{i,τ_i}~$DD^2_m(t_1, t_2, ..., t_{\mathcal{T}} : p^{m_1}_1, p^{m_1}_2, .., p^{m_1}_{\mathcal{T}})$, T^h_{i,τ_i}~$DD^1_h(t_1, t_2, ..., t_{\mathcal{T}} : p^{m_1}_1, p^{m_1}_2, .., p^{m_1}_{\mathcal{T}})$

8: T^m_{i,τ_i}~$DD^2_m(t_1, t_2, ..., t_{\mathcal{T}} : p^{m_1}_1, p^{m_1}_2, .., p^{m_1}_{\mathcal{T}})$, T^h_{i,τ_i}~$DD^2_h(t_1, t_2, ..., t_{\mathcal{T}} : p^{m_1}_1, p^{m_1}_2, .., p^{m_1}_{\mathcal{T}})$

			N-D.S								
			2			3			4		
Cv(D)	c	P.S	1	2	3	1	2	3	1	2	3
0.1	1	1	2,99	2,70	0,01	1,11	0,89	0,07	0,51	0,55	0,19
		2	4,46	3,65	0,12	1,40	1,07	0,12	0,55	0,53	0,33
		3	4,50	3,52	1,00	0,74	0,83	0,28	0,96	0,46	0,25
		4	4,49	3,43	0,31	0,56	0,57	0,15	0,88	0,21	0,17
		5	1,90	1,60	0,13	0,51	0,41	0,43	0,20	0,26	0,07
		6	3,21	2,66	0,55	1,12	0,88	0,38	0,51	0,59	0,11
		7	4,86	3,95	1,16	1,05	1,10	0,10	0,84	0,80	0,18
		8	4,29	3,56	1,27	1,08	1,16	0,03	0,82	0,80	0,17
	3	1	1,61	1,69	0,05	0,51	0,57	0,14	0,34	0,06	0,15
		2	2,98	1,56	0,06	0,47	0,71	0,15	0,46	0,67	0,02
		3	1,76	2,39	0,01	0,76	1,06	0,18	0,75	0,01	0,19
		4	1,93	2,77	0,39	1,08	1,12	0,29	0,09	0,00	0,23
		5	0,66	0,68	0,00	0,21	0,24	0,01	0,12	0,13	0,01
		6	1,88	1,87	0,01	0,61	0,76	0,05	0,38	0,47	0,04
		7	2,55	2,34	0,03	0,88	1,31	0,11	0,35	0,39	0,14
		8	2,95	2,79	0,04	0,85	1,37	0,19	0,45	0,58	0,15
0.3	1	1	1,26	1,28	0,04	0,23	0,28	0,06	0,13	0,16	0,01
		2	2,20	2,04	0,12	0,42	0,41	0,08	0,20	0,19	0,01
		3	2,02	1,66	0,52	0,24	0,30	0,14	0,14	0,15	0,13
		4	2,01	0,51	0,15	0,14	0,18	0,04	0,08	0,07	0,10
		5	0,75	0,71	0,08	0,08	0,11	0,08	0,05	0,06	0,04
		6	1,43	1,37	0,17	0,25	0,31	0,12	0,16	0,18	0,09
		7	2,51	2,21	0,55	0,35	0,46	0,14	0,26	0,31	0,15
		8	1,90	2,48	0,62	0,38	0,49	0,12	0,27	0,31	0,14
	3	1	0,17	0,26	0,05	0,02	0,03	0,00	0,01	0,02	0,00
		2	0,30	0,44	0,05	0,07	0,09	0,03	0,03	0,04	0,03
		3	0,22	0,43	0,08	0,12	0,19	0,06	0,06	0,06	0,04
		4	0,23	0,31	0,23	0,14	0,20	0,09	0,06	0,06	0,04
		5	0,09	0,14	0,05	0,01	0,01	0,02	0,00	0,01	0,02
		6	0,24	0,33	0,03	0,04	0,06	0,01	0,02	0,02	0,01
		7	0,29	0,40	0,04	0,09	0,14	0,03	0,02	0,09	0,02
		8	0,33	0,47	0,05	0,10	0,16	0,03	0,06	0,09	0,02
Average			1,32			0,38			0,22		

Table B.1: The optimality gap of A_{norm} for different scenarios

			N-D.S								
			2			3			4		
Cv(D)	c	P.S	1	2	3	1	2	3	1	2	3
0.1	1	1	2,53	0,94	3,02	8,54	7,47	7,66	10,22	9,91	8,14
		2	2,32	0,79	2,62	11,00	10,20	8,51	12,81	13,05	12,99
		3	3,12	1,50	3,45	15,64	15,08	14,26	18,78	19,57	18,67
		4	4,01	2,36	1,29	17,24	16,70	16,41	21,13	21,80	21,06
		5	2,71	1,05	0,29	5,89	4,89	3,94	7,58	6,50	6,28
		6	2,60	0,97	0,38	6,94	6,11	4,76	8,78	7,90	7,82
		7	1,89	0,96	1,30	9,27	8,65	6,40	10,96	10,92	9,81
		8	1,80	0,74	6,38	10,28	9,69	7,75	12,00	12,17	10,91
	3	1	2,40	1,58	5,47	5,95	4,62	15,39	9,46	8,11	17,76
		2	1,29	0,53	4,09	6,59	5,19	16,13	10,68	9,47	19,12
		3	4,90	4,06	12,56	8,27	7,04	23,57	13,09	12,89	28,08
		4	16,77	15,59	18,15	10,38	9,32	23,32	15,81	16,22	28,44
		5	1,13	0,55	3,73	3,83	2,83	11,72	6,48	5,41	13,39
		6	1,44	2,93	4,14	4,74	3,68	13,05	7,46	6,35	15,23
		7	3,58	2,71	5,21	5,43	4,45	14,57	8,75	9,35	17,74
		8	3,05	2,09	29,60	6,25	5,24	15,59	9,77	9,79	18,99
0.3	1	1	6,22	2,52	3,50	12,65	9,61	8,54	14,87	12,09	12,14
		2	5,75	2,16	0,45	14,49	11,99	10,29	17,33	15,10	14,40
		3	6,19	2,55	1,36	18,40	16,49	15,46	22,37	21,20	20,17
		4	4,10	3,65	1,68	19,83	17,95	17,57	24,35	23,26	22,53
		5	6,64	2,79	0,90	10,85	7,39	5,59	12,41	9,01	7,85
		6	6,39	2,60	1,09	11,52	8,41	6,61	13,42	10,32	9,27
		7	4,80	2,51	2,09	13,21	10,65	8,66	15,59	13,17	11,62
		8	4,51	32,32	6,33	13,98	11,55	9,97	16,60	14,33	12,67
	3	1	4,11	2,45	5,44	9,83	7,93	14,89	12,74	10,41	17,52
		2	5,80	2,02	4,16	13,98	10,30	15,73	17,70	14,05	19,34
		3	7,93	5,25	14,89	17,72	13,63	23,13	20,40	18,77	27,37
		4	17,38	17,40	21,12	20,00	15,94	23,10	23,24	21,68	27,94
		5	2,49	1,28	5,20	6,88	5,24	12,83	9,15	7,38	14,92
		6	3,24	1,76	5,81	8,70	6,99	14,35	11,17	9,43	16,85
		7	4,01	2,29	6,68	10,26	8,64	15,84	13,23	12,80	18,76
		8	4,65	25,82	32,07	11,75	10,11	16,90	15,01	14,07	20,09
Average			5,34			11,11			14,35		

Table B.2: The optimality gap of A_{exp} for different scenarios

				N								
D.S	Cv(D)	C	P.S	2	3	4	5	6	7	8	9	10
1	0.1	1	1	-0,48	7,51	9,76	10,12	12,43	12,87	13,12	14,20	14,36
			2	-2,24	9,73	12,32	14,47	16,28	17,19	18,01	18,63	19,06
			3	-1,44	15,01	17,99	19,86	22,37	23,76	24,58	25,32	25,93
			4	-0,50	16,78	20,43	21,94	24,57	26,31	27,00	27,78	28,51
			5	0,82	5,41	7,39	8,06	8,58	8,90	10,13	10,35	10,85
			6	-0,63	5,89	8,31	9,18	9,83	10,21	11,70	11,96	12,65
			7	-3,12	8,31	10,21	11,58	12,17	12,58	14,43	14,69	15,54
			8	-2,60	9,30	11,28	12,90	13,55	14,02	16,01	16,31	17,21
		3	1	0,80	5,47	9,15	8,02	10,47	11,06	11,44	13,01	13,28
			2	-1,74	6,15	10,26	11,80	14,43	16,11	17,32	18,45	19,28
			3	3,20	7,56	12,44	16,26	19,79	21,80	23,45	24,81	25,84
			4	15,13	9,40	15,73	18,52	22,49	24,69	26,11	27,68	28,84
			5	0,47	3,63	6,37	7,26	9,10	9,67	10,05	11,17	11,45
			6	-0,45	4,15	7,11	8,27	10,65	11,32	11,60	13,07	13,23
			7	1,06	4,59	8,42	9,90	12,47	13,36	13,81	15,45	15,75
			8	0,10	5,45	9,36	10,92	12,44	13,52	16,35	17,09	18,48
	0.3	1	1	5,03	12,45	14,77	15,20	17,26	17,61	17,83	18,88	19,01
			2	3,63	14,13	17,16	19,30	20,77	21,65	22,36	22,93	23,33
			3	4,26	18,20	22,27	24,46	26,47	27,69	28,42	29,18	29,72
			4	2,13	19,72	24,29	26,31	28,56	29,94	30,60	31,47	32,09
			5	5,94	10,78	12,37	13,14	13,61	13,92	15,12	15,33	15,76
			6	5,03	11,29	13,28	14,25	14,82	15,19	16,69	16,93	17,50
			7	2,34	12,91	15,37	16,54	17,21	17,63	19,45	19,72	20,39
			8	2,65	13,65	16,38	17,70	18,46	18,94	20,89	21,20	21,91
		3	1	3,95	9,81	12,73	13,30	16,03	17,07	17,35	18,81	19,25
			2	5,51	13,92	17,68	20,28	22,23	23,47	24,49	25,34	25,96
			3	7,73	17,62	20,36	23,24	25,98	27,73	29,12	30,28	31,14
			4	17,19	19,89	23,20	25,68	28,74	30,70	31,90	33,21	34,17
			5	2,40	6,88	9,14	10,04	11,96	12,44	12,74	13,79	14,00
			6	3,00	8,66	11,16	12,53	13,39	13,99	16,18	16,61	17,54
			7	3,73	10,18	13,21	14,64	15,65	16,34	18,96	19,43	20,62
			8	4,34	11,66	14,96	16,79	17,96	18,77	21,31	21,86	23,03

				N								
D.S	Cv(D)	C	P.S	2	3	4	5	6	7	8	9	10
2	0.1	1	1	-1,81	6,63	9,41	8,76	11,42	11,79	11,99	13,11	13,23
			2	-2,97	9,23	12,59	13,98	15,64	16,82	17,39	17,98	18,46
			3	-2,09	14,37	19,19	20,51	22,79	24,24	24,95	25,95	26,58
			4	-1,10	16,22	21,63	22,63	25,20	26,80	27,35	28,50	29,25
			5	-0,57	4,50	6,25	7,26	7,74	8,11	9,06	9,31	9,67
			6	-1,73	5,28	7,36	8,57	9,14	9,58	10,76	11,03	11,55
			7	-3,12	7,63	10,20	11,40	12,17	12,59	14,12	14,40	15,07
			8	-2,92	8,62	11,46	12,79	13,66	14,12	15,75	16,06	16,76
		3	1	-0,11	4,07	8,05	7,10	9,25	9,85	10,22	11,61	11,88
			2	-1,05	4,52	8,86	10,98	13,53	15,08	16,25	17,29	18,05
			3	1,71	6,04	12,88	16,16	19,31	21,14	22,64	23,92	24,87
			4	13,19	8,29	16,22	18,64	22,07	24,21	25,43	26,94	28,00
			5	-0,13	2,59	5,29	5,94	7,76	8,44	8,66	9,72	9,84
			6	1,08	2,94	5,91	7,17	9,50	10,14	10,45	11,88	12,06
			7	0,38	3,18	8,99	8,22	9,56	10,44	13,18	13,78	15,00
			8	-0,72	3,93	9,27	10,03	11,60	12,64	15,25	15,96	17,15
	0.1	1	1	1,26	9,36	11,95	11,64	13,96	14,25	14,43	15,60	15,70
			2	0,12	11,62	14,93	16,59	18,19	19,15	19,69	20,30	20,73
			3	0,91	16,23	21,08	22,77	25,00	26,39	26,94	27,83	28,48
			4	3,16	17,80	23,21	24,71	27,19	28,76	29,23	30,24	30,97
			5	2,10	7,29	8,95	9,81	10,33	10,66	11,63	11,85	12,18
			6	1,25	8,13	10,16	11,19	11,79	12,19	13,43	13,68	14,13
			7	0,31	10,24	12,90	14,18	14,91	15,38	16,91	17,21	17,76
			8	30,60	11,11	14,06	15,50	16,31	16,83	18,47	18,79	19,38
		3	1	2,20	7,90	10,40	11,28	13,79	14,71	15,02	16,36	16,76
			2	1,59	10,22	14,02	16,02	18,18	19,55	20,53	21,46	22,14
			3	4,84	13,47	18,72	21,64	24,39	26,12	27,38	28,52	29,36
			4	17,14	15,77	21,63	24,13	27,23	29,18	30,24	31,54	32,49
			5	1,15	5,23	7,37	8,22	10,02	10,49	10,77	11,76	11,96
			6	1,44	6,94	9,40	10,75	11,60	12,18	14,26	14,68	15,56
			7	1,90	8,52	12,73	12,52	13,61	14,35	16,91	17,43	18,60
			8	25,47	9,97	13,99	14,58	15,86	16,74	19,11	19,72	20,79

				N								
D.S	Cv(D)	C	P.S	2	3	4	5	6	7	8	9	10
3	0.3	1	1	3,02	7,59	7,97	9,12	11,20	11,77	12,05	13,22	13,40
			2	2,51	8,41	12,71	15,26	17,03	18,15	18,73	19,37	19,87
			3	2,47	14,02	18,46	21,09	23,27	24,50	25,15	25,90	26,37
			4	0,99	16,28	20,92	23,33	25,77	27,16	27,73	28,60	29,16
			5	0,15	3,52	6,21	6,32	8,14	8,57	8,75	9,85	9,97
			6	-0,16	4,40	7,72	9,34	10,06	10,55	12,19	12,60	13,08
			7	0,14	6,31	9,65	11,19	12,02	12,48	14,78	15,15	15,89
			8	5,18	7,72	10,76	12,41	13,41	13,93	16,54	16,93	17,66
		3	1	5,42	15,27	17,64	15,09	16,60	17,49	17,63	18,11	18,55
			2	4,03	16,01	19,11	18,44	20,41	21,96	22,38	23,16	23,80
			3	12,55	23,43	27,95	26,54	28,76	30,69	32,13	33,39	34,15
			4	17,83	23,10	28,27	28,13	30,61	32,73	33,95	35,35	36,23
			5	3,73	11,71	13,38	12,10	13,79	13,85	14,39	15,11	15,45
			6	4,13	13,00	15,19	16,45	17,31	17,77	18,75	19,09	19,79
			7	5,18	14,48	17,62	19,26	20,41	21,05	22,60	23,04	24,08
			8	29,57	15,42	18,87	20,64	21,89	22,61	24,42	24,92	26,13
	2	1	1	3,46	8,48	12,13	12,25	14,61	15,26	15,36	16,49	16,74
			2	0,34	10,22	14,39	16,54	18,28	19,27	19,86	20,50	20,97
			3	0,85	15,34	20,07	22,63	24,69	25,88	26,54	27,25	27,75
			4	1,53	17,54	22,46	24,82	27,12	28,48	29,09	29,90	30,47
			5	0,83	5,51	7,81	7,84	9,52	9,87	10,06	10,94	11,05
			6	0,92	6,50	9,19	9,42	11,34	11,77	12,02	13,01	13,17
			7	1,56	8,53	11,49	12,97	13,78	14,29	16,51	16,86	17,49
			8	5,74	9,86	12,55	14,21	15,13	15,70	18,19	18,57	19,23
		2	1	5,39	14,89	17,52	16,29	17,95	18,97	19,31	20,00	20,49
			2	4,11	15,71	19,32	19,64	21,88	23,28	23,83	24,78	25,47
			3	14,82	23,08	27,34	26,54	29,35	31,13	32,40	33,56	34,40
			4	20,93	23,03	27,91	28,17	31,23	33,18	34,31	35,57	36,49
			5	5,15	12,82	14,91	16,00	16,65	17,09	17,84	18,12	18,48
			6	5,78	14,34	16,84	18,17	18,99	19,53	20,66	21,02	21,59
			7	6,64	15,81	18,75	20,34	21,32	21,98	23,66	24,10	24,91
			8	32,04	16,87	20,08	21,82	22,90	23,64	25,58	26,06	27,01
Average				4,04	10,76	14,16	15,32	17,07	18,00	19,11	19,89	20,51

Table B.3: The difference between A_{norm} and A_{exp},$\Delta_{norm-exp}$ for different scenarios

			N-D.S								
			2			3			4		
Cv(D)	c	P.S	1	2	3	1	2	3	1	2	3
0.1	1	1	0,00	0,63	26,91	0,00	4,04	26,88	1,32	6,98	27,29
		2	0,06	0,19	18,38	0,00	0,00	0,00	0,55	0,00	18,16
		3	0,00	4,55	7,97	0,00	0,00	3,37	0,00	0,00	2,99
		4	3,25	0,51	4,01	0,00	0,00	1,84	2,18	0,00	1,87
		5	0,00	20,90	34,38	2,04	0,00	0,82	1,80	2,57	0,77
		6	0,00	0,00	30,08	2,06	0,00	1,09	1,98	2,09	0,53
		7	0,04	0,18	25,55	0,00	0,00	8,38	0,65	2,48	0,65
		8	0,00	0,00	21,97	0,00	0,00	7,77	0,54	1,60	1,63
	3	1	1,76	0,00	22,17	3,64	3,09	26,34	8,86	35,78	26,64
		2	0,19	0,00	12,56	5,55	4,69	0,00	9,69	8,99	23,54
		3	0,00	0,00	9,28	6,77	6,25	3,12	11,68	12,16	2,31
		4	0,00	0,00	0,00	8,61	8,35	2,91	14,15	15,32	1,94
		5	0,00	24,27	0,00	2,73	2,29	0,00	5,41	4,89	0,98
		6	0,00	0,00	27,24	3,34	2,98	0,00	9,02	8,17	1,13
		7	0,00	0,00	24,34	3,74	3,57	0,00	7,11	6,83	1,37
		8	2,13	1,66	21,84	6,36	5,79	0,00	9,87	9,40	1,42
0.3	1	1	0,66	0,86	25,53	0,00	0,00	25,58	0,52	0,00	26,47
		2	0,17	0,39	16,68	0,00	0,00	17,01	0,84	0,00	17,72
		3	0,00	3,65	7,78	0,00	0,00	3,07	1,75	0,00	3,01
		4	1,13	0,00	3,92	0,66	0,85	1,62	0,00	0,00	2,00
		5	0,92	0,00	10,05	2,95	0,00	1,21	1,27	2,04	0,72
		6	0,59	0,80	28,75	0,00	0,00	1,49	0,89	0,65	0,82
		7	0,17	0,41	28,87	0,00	0,00	2,00	0,55	0,00	0,69
		8	0,00	0,03	21,11	0,00	0,00	8,03	0,56	1,05	0,69
	3	1	2,78	1,87	21,73	6,28	7,39	25,80	9,31	10,17	25,95
		2	1,80	0,81	16,95	10,33	9,19	22,22	14,21	12,99	22,86
		3	2,05	0,98	2,02	12,46	11,82	0,00	17,38	17,06	1,96
		4	2,25	1,62	3,97	14,05	13,63	0,98	19,66	19,53	1,56
		5	0,00	0,00	0,00	4,51	4,01	0,00	6,83	6,18	1,23
		6	0,00	0,00	0,00	5,64	5,32	0,00	8,20	7,80	1,21
		7	0,00	0,00	31,27	6,51	6,50	0,00	9,61	9,49	0,94
		8	3,31	2,41	0,00	10,51	9,79	0,00	13,81	13,22	0,87
Average			6,20			4,37			6,56		

Table B.4: The optimality gap of AT_{norm} for different scenarios

			N-D.S								
			2			3			4		
Cv(D)	c	P.S	1	2	3	1	2	3	1	2	3
0.1	1	1	2,99	2,70	0,01	1,11	0,89	26,96	1,29	0,55	27,30
		2	4,46	3,65	0,12	1,40	1,07	0,47	1,18	0,53	18,21
		3	4,50	5,17	1,00	0,74	0,83	0,28	0,96	0,46	0,25
		4	4,49	1,23	0,31	0,56	0,57	0,15	0,88	0,21	0,17
		5	1,90	1,60	0,59	0,51	0,41	0,75	1,14	1,24	0,56
		6	3,21	2,66	0,55	3,81	0,88	0,68	1,27	1,44	0,59
		7	4,86	3,95	1,16	1,05	1,10	0,10	1,01	0,80	0,90
		8	5,61	4,52	1,27	1,08	1,16	22,59	0,74	0,80	0,58
	3	1	26,40	1,32	22,43	26,73	0,57	26,38	4,88	1,88	26,64
		2	2,98	3,29	12,89	0,47	1,59	22,93	0,97	0,82	23,54
		3	2,83	3,45	1,81	10,93	1,06	0,18	0,66	2,11	2,45
		4	2,49	5,64	0,39	1,08	1,12	0,29	0,09	0,00	0,23
		5	0,66	0,68	0,00	32,01	2,95	6,75	1,28	3,82	1,02
		6	1,88	19,07	0,01	0,61	16,50	0,05	0,93	1,03	1,17
		7	2,55	2,34	0,03	0,88	1,31	0,11	0,35	0,63	1,52
		8	2,95	2,79	0,04	0,85	1,37	0,19	0,45	1,14	1,57
0.3	1	1	1,26	1,28	0,04	0,23	0,28	0,75	0,57	1,26	0,01
		2	2,20	2,04	0,12	0,42	0,41	1,38	0,91	0,19	0,01
		3	2,02	3,03	0,52	0,24	0,30	0,14	1,76	0,15	0,13
		4	2,01	0,51	0,15	0,14	0,18	0,52	0,08	0,07	0,10
		5	0,75	0,71	0,08	2,75	0,11	0,08	0,35	1,04	0,39
		6	1,43	1,37	0,17	3,14	0,31	0,12	0,37	0,74	0,39
		7	2,51	2,21	0,55	24,21	0,46	29,76	0,60	0,31	29,84
		8	3,02	2,48	0,62	0,38	0,49	0,12	0,60	1,12	0,87
	3	1	0,17	1,90	21,76	25,08	0,03	25,80	25,74	0,08	25,95
		2	0,30	0,44	17,00	0,07	0,09	22,22	0,03	0,04	22,86
		3	0,22	4,75	0,79	0,12	0,19	0,06	1,30	0,37	0,04
		4	0,23	0,31	2,12	0,14	0,20	6,33	0,06	0,26	0,04
		5	0,09	0,14	0,05	30,92	0,01	6,45	0,00	0,41	1,23
		6	0,24	0,33	0,03	0,04	0,06	0,01	0,02	0,29	1,21
		7	0,29	0,40	0,04	0,09	0,14	0,03	0,22	0,09	0,95
		8	0,33	0,47	0,05	5,65	0,16	0,03	0,06	0,09	0,89
Average			2,76			4,34			2,77		

Table B.5: The optimality gap of $AT_{norm-iter}$ for different scenarios

			N-D.S								
			2			3			4		
Cv(D)	c	P.S	1	2	3	1	2	3	1	2	3
0.1	1	1	2,39	0,94	0,14	8,41	7,47	7,66	10,10	9,91	10,79
		2	1,81	0,79	0,21	10,53	10,20	8,51	12,35	13,05	12,99
		3	1,65	1,50	0,97	14,36	15,08	14,26	17,55	19,57	18,67
		4	1,63	2,36	1,29	15,19	16,70	16,41	19,18	21,80	21,06
		5	2,75	1,05	0,29	5,93	4,89	4,31	7,62	6,50	6,64
		6	2,35	0,97	0,38	6,71	6,11	4,88	8,55	7,90	7,82
		7	1,89	0,92	1,30	8,71	8,65	6,40	10,40	10,92	9,81
		8	1,80	1,06	1,29	9,38	9,69	7,14	11,12	12,17	10,91
	3	1	1,67	0,86	6,53	5,24	3,92	10,49	8,78	7,43	13,01
		2	2,50	1,72	8,25	7,74	6,32	12,27	11,78	10,55	15,40
		3	3,50	2,68	12,56	10,03	8,77	10,79	14,76	14,52	16,06
		4	3,95	3,04	14,56	12,22	11,13	11,16	17,54	17,90	17,08
		5	1,13	0,55	3,73	3,83	2,83	5,99	6,48	5,41	7,77
		6	1,44	0,72	4,14	4,74	3,68	6,98	7,46	6,35	9,31
		7	1,76	0,91	5,21	5,43	4,45	6,23	8,75	7,68	9,70
		8	2,02	1,08	5,46	6,25	5,24	7,18	9,77	8,87	10,92
0.3	1	1	5,32	2,52	0,39	11,80	9,61	8,54	14,05	12,09	12,14
		2	4,54	2,16	0,45	13,40	11,99	10,29	16,27	15,10	14,40
		3	4,14	2,55	1,36	16,62	16,49	15,46	20,68	21,20	20,17
		4	4,10	3,65	1,68	17,35	17,95	17,57	22,01	23,26	22,53
		5	5,86	2,79	0,90	10,11	7,39	6,03	11,67	9,01	8,28
		6	5,36	2,60	1,09	10,54	8,41	6,80	12,47	10,32	9,46
		7	4,80	2,48	2,09	11,94	10,65	8,66	14,36	13,17	11,62
		8	4,51	2,34	2,06	12,41	11,55	9,39	15,08	14,33	12,67
	3	1	3,79	2,14	8,94	9,83	7,64	14,11	12,74	10,41	16,77
		2	5,80	3,59	10,91	13,98	11,74	15,46	17,70	15,43	19,07
		3	7,93	5,25	14,89	17,72	15,62	13,75	22,34	20,64	18,50
		4	9,02	6,59	18,18	20,00	17,98	14,22	25,22	23,58	19,61
		5	2,49	1,28	5,20	6,88	5,24	10,12	9,15	7,38	12,27
		6	3,24	1,76	5,81	8,70	6,99	11,01	11,17	9,43	13,60
		7	4,01	2,29	6,68	10,26	8,64	10,17	13,23	11,57	13,29
		8	4,65	2,76	7,17	11,75	10,11	10,96	15,01	13,54	14,38
Average			3,50			10,04			13,37		

Table B.6: The optimality gap of AT_{exp} for different scenarios

			N-D.S								
			2			3			4		
Cv(D)	c	P.S	1	2	3	1	2	3	1	2	3
0.1	1	1	0,52	0,00	0,00	6,66	6,59	7,05	8,38	9,06	10,20
		2	0,06	0,00	0,00	8,94	9,48	7,87	10,79	12,36	12,38
		3	0,00	0,80	0,00	12,93	14,48	13,10	16,17	19,00	17,57
		4	0,00	1,74	0,25	13,78	16,17	15,23	17,84	21,30	19,94
		5	0,76	0,00	0,00	4,01	3,88	3,80	5,74	5,51	6,14
		6	0,44	0,00	0,11	4,88	5,19	4,39	6,76	7,00	7,35
		7	0,04	0,00	0,69	6,98	7,80	5,76	8,71	10,09	9,19
		8	0,00	0,20	0,72	7,71	8,90	6,54	9,49	11,40	10,34
	3	1	0,00	0,00	0,00	3,64	3,09	7,42	7,23	6,63	10,02
		2	0,00	0,33	0,00	5,37	5,00	8,96	9,51	9,29	12,21
		3	0,08	0,66	0,00	6,84	6,87	6,51	11,75	12,74	12,03
		4	0,06	0,63	0,75	8,66	8,92	6,04	14,20	15,86	12,31
		5	0,00	0,00	0,00	2,73	2,29	4,27	5,41	4,89	6,08
		6	0,00	0,00	0,00	3,34	2,98	5,19	6,11	5,67	7,57
		7	0,00	0,00	0,00	3,74	3,57	4,58	7,11	6,83	8,11
		8	0,00	0,00	0,00	4,32	4,21	5,38	7,91	7,87	9,19
0.3	1	1	0,66	0,00	0,00	7,47	7,27	7,28	9,82	9,82	10,93
		2	0,17	0,00	0,00	9,43	10,05	9,05	12,44	13,22	13,21
		3	0,00	0,63	0,00	13,02	14,83	13,71	17,25	19,64	18,52
		4	0,00	1,94	0,27	13,81	16,49	15,83	18,68	21,89	20,90
		5	0,92	0,00	0,09	5,39	4,73	4,73	7,04	6,39	7,01
		6	0,59	0,00	0,31	6,03	5,97	5,58	8,06	7,93	8,27
		7	0,17	0,00	0,82	7,67	8,38	7,25	10,20	10,96	10,26
		8	0,00	0,00	0,85	8,28	9,43	8,07	11,07	12,28	11,40
	3	1	0,00	0,00	0,00	6,28	5,62	7,56	9,31	8,45	10,42
		2	0,00	0,00	0,00	8,69	8,45	8,52	12,64	12,28	12,43
		3	0,00	0,00	0,00	10,62	10,94	5,94	15,65	16,24	11,13
		4	0,00	0,40	1,83	12,07	12,56	5,31	17,81	18,53	11,26
		5	0,00	0,00	0,00	4,51	4,01	5,95	6,83	6,18	8,20
		6	0,00	0,00	0,00	5,64	5,32	6,67	8,20	7,80	9,39
		7	0,00	0,00	0,00	6,51	6,50	6,24	9,61	9,49	9,50
		8	0,00	0,00	0,00	7,44	7,56	6,69	10,86	11,08	10,27
Average			0,19			7,51			10,94		

Table B.7: The optimality gap of $AT_{exp-exct}$ for different scenarios

			N-D.S								
			2			3			4		
Cv(D)	c	P.S	1	2	3	1	2	3	1	2	3
0.1	1	1	2,83	8,82	10,50	2,70	9,11	11,51	0,01	7,62	10,76
		2	3,53	12,10	13,88	3,65	12,79	15,57	0,12	8,68	13,15
		3	4,50	16,84	19,94	5,16	18,23	22,55	1,00	14,54	18,94
		4	4,49	17,65	21,52	6,00	19,80	24,71	1,22	16,64	21,27
		5	2,57	5,76	7,45	1,60	5,43	7,02	0,13	4,24	6,57
		6	3,03	7,35	9,18	2,66	7,71	9,47	0,17	4,92	7,86
		7	4,12	10,78	12,43	3,95	11,45	13,64	1,59	6,85	10,25
		8	4,29	11,68	13,37	4,52	12,85	15,24	1,67	7,72	11,47
	3	1	1,61	5,19	8,73	1,69	4,72	8,21	0,05	7,87	10,45
		2	2,98	8,19	12,21	3,29	7,82	11,98	0,06	9,35	12,58
		3	2,83	9,41	14,18	3,45	9,49	15,20	0,01	6,83	12,33
		4	2,49	10,89	16,29	3,28	11,35	18,10	0,76	6,43	12,67
		5	0,66	3,37	6,03	0,68	2,95	5,53	0,00	4,88	6,68
		6	1,88	5,16	7,87	1,87	4,79	7,43	0,01	5,75	8,11
		7	2,55	6,19	9,49	2,34	5,82	9,00	0,03	4,94	8,46
		8	2,95	7,14	10,62	2,79	6,88	10,44	0,04	5,75	9,55
0.3	1	1	1,55	8,29	10,63	1,28	8,46	10,97	0,04	7,46	11,10
		2	1,57	10,70	13,67	2,04	11,88	14,99	0,12	9,48	13,62
		3	2,02	14,77	18,92	3,03	16,90	21,59	0,52	14,55	19,31
		4	2,01	15,54	20,31	4,25	18,46	23,74	0,78	16,64	21,66
		5	1,56	6,00	7,64	0,71	5,40	7,06	0,09	4,80	7,08
		6	1,57	6,96	8,97	1,37	7,25	9,18	0,35	5,74	8,43
		7	1,94	9,31	11,80	2,21	10,40	12,93	1,19	7,76	10,76
		8	1,90	10,02	12,77	2,48	11,67	14,45	1,26	8,65	11,95
	3	1	0,17	6,44	9,46	0,26	5,86	8,69	0,05	7,69	10,54
		2	0,30	8,96	12,90	0,44	8,85	12,67	0,05	8,61	12,52
		3	0,22	10,82	15,83	0,43	11,32	16,60	0,08	6,09	11,27
		4	0,23	12,27	18,00	0,88	12,98	18,92	1,98	5,51	11,46
		5	0,09	4,59	6,91	0,14	4,14	6,30	0,05	6,07	8,33
		6	0,24	5,87	8,42	0,33	5,64	8,11	0,03	6,76	9,48
		7	0,29	6,78	9,87	0,40	6,87	9,86	0,04	6,31	9,57
		8	0,33	7,75	11,16	0,47	8,00	11,51	0,05	6,76	10,34
Average			7,77			8,22			6,60		

Table B.8: The optimality gap of $AT_{exp-norm}$ for different scenarios

			N-D.S								
			2			3			4		
Cv(D)	c	P.S	1	2	3	1	2	3	1	2	3
0.1	1	1	2,39	2,54	3,86	5,49	1,10	7,26	6,88	5,80	2,98
		2	1,81	2,10	3,48	4,71	1,14	6,26	6,62	6,63	1,99
		3	1,65	2,35	3,07	12,19	15,53	14,31	14,46	20,32	19,58
		4	1,63	3,28	4,15	13,06	16,93	16,17	16,18	22,18	21,55
		5	2,75	2,73	1,54	6,09	0,83	5,47	6,45	4,41	2,36
		6	2,35	2,44	1,46	5,16	0,71	5,02	5,85	4,35	2,06
		7	1,89	1,46	1,12	7,96	9,47	7,41	9,16	12,10	11,42
		8	1,80	1,66	1,16	8,56	10,37	8,09	9,76	13,08	12,36
	3	1	1,67	0,86	11,58	24,17	22,63	17,48	0,70	0,65	7,34
		2	2,50	1,72	14,47	25,32	23,69	18,22	1,60	1,79	8,01
		3	3,50	2,68	19,19	19,58	18,17	12,98	34,79	34,02	25,58
		4	3,95	3,04	21,96	21,39	20,14	12,09	36,92	36,62	25,46
		5	1,13	0,55	6,62	23,20	21,94	13,42	0,53	0,45	4,01
		6	1,44	0,72	7,80	23,04	21,73	14,00	0,72	0,64	4,42
		7	1,76	0,91	8,92	10,84	9,77	11,77	19,48	18,26	18,97
		8	2,02	1,08	9,92	11,37	10,28	12,30	20,01	18,93	19,65
0.3	1	1	5,32	1,53	2,52	6,70	2,07	7,38	8,01	5,48	2,48
		2	4,54	1,10	2,23	5,65	1,93	7,37	8,19	6,58	1,70
		3	4,14	1,22	2,44	14,11	15,00	14,38	17,02	20,09	20,02
		4	4,10	2,52	3,61	14,89	16,37	16,27	18,46	21,92	22,03
		5	5,86	1,73	0,22	7,79	2,06	6,19	7,15	4,33	1,82
		6	5,36	1,44	0,25	6,65	1,80	5,96	6,70	4,40	1,59
		7	4,80	0,51	0,16	10,97	9,13	8,00	12,72	12,06	11,64
		8	4,51	0,52	0,18	11,39	10,00	8,71	13,35	13,05	12,56
	3	1	3,79	2,14	16,39	29,05	26,34	19,19	1,52	1,33	9,95
		2	5,80	3,59	19,56	31,85	29,07	19,61	3,41	3,48	10,55
		3	7,93	5,25	24,17	27,29	24,87	14,69	42,84	40,37	27,24
		4	9,02	6,59	28,28	29,29	26,95	13,89	45,17	42,77	27,21
		5	2,49	1,28	10,48	26,70	24,47	15,67	1,20	0,97	7,34
		6	3,24	1,76	11,88	27,40	25,11	16,14	1,72	1,61	7,43
		7	4,01	2,29	12,85	15,75	13,95	14,08	24,31	22,29	21,11
		8	4,65	2,76	14,11	16,96	15,14	14,42	25,56	23,72	21,64
Average			4,69			13,93			12,98		

Table B.9: The optimality gap of AT_{expcon} for different scenarios

			N-D.S								
			2			3			4		
Cv(D)	c	P.S	1	2	3	1	2	3	1	2	3
0.1	1	1	0,52	0,63	2,38	4,98	0,00	4,65	5,39	2,19	2,23
		2	0,06	0,19	1,94	4,30	0,00	4,01	5,67	2,29	1,51
		3	0,00	0,80	2,14	10,99	13,64	12,54	13,44	17,91	16,84
		4	0,00	1,74	3,24	11,88	15,09	14,46	15,18	19,85	18,90
		5	0,76	0,00	0,00	5,47	0,00	2,48	4,05	1,52	0,77
		6	0,44	0,00	0,11	4,62	0,00	2,22	4,32	1,57	0,66
		7	0,04	0,00	0,69	6,39	7,76	5,76	7,73	10,01	9,19
		8	0,00	0,20	0,72	7,05	8,73	6,46	8,38	11,15	10,22
	3	1	0,00	0,00	0,00	3,30	2,75	7,42	0,00	0,03	1,49
		2	0,00	0,33	0,00	4,80	4,45	8,96	0,41	0,64	1,75
		3	0,08	0,66	0,00	6,29	6,34	6,51	10,81	11,86	11,89
		4	0,06	0,63	0,75	8,02	8,33	6,04	13,13	14,87	12,13
		5	0,00	0,00	0,00	2,55	2,12	4,27	0,00	0,00	0,98
		6	0,00	0,00	0,00	3,06	2,71	5,19	0,00	0,00	1,13
		7	0,00	0,00	0,00	3,65	3,49	4,58	6,97	6,68	8,11
		8	0,00	0,00	0,00	4,21	4,11	5,38	7,72	7,69	9,19
0.3	1	1	0,66	0,86	2,17	5,40	0,00	4,64	5,94	1,82	2,04
		2	0,17	0,39	1,80	4,56	0,00	5,06	6,63	1,97	1,38
		3	0,00	0,63	2,12	11,07	13,98	13,17	14,47	18,41	17,78
		4	0,00	1,94	3,27	11,91	15,40	15,08	15,98	20,35	19,87
		5	0,92	0,00	0,09	6,26	0,00	3,22	4,32	1,45	0,83
		6	0,59	0,00	0,31	5,30	0,00	3,24	4,48	1,42	0,77
		7	0,17	0,00	0,82	7,07	8,29	7,25	9,20	10,80	10,26
		8	0,00	0,00	0,85	7,61	9,23	8,00	9,96	11,94	11,29
	3	1	0,00	0,00	0,00	5,57	4,91	7,56	0,00	0,00	1,16
		2	0,00	0,00	0,00	7,49	7,26	8,52	1,06	1,21	1,72
		3	0,00	0,00	0,00	9,47	9,80	5,94	13,79	14,39	10,98
		4	0,00	0,40	1,83	10,81	11,31	5,31	15,83	16,57	11,09
		5	0,00	0,00	0,00	4,15	3,66	5,95	0,00	0,00	1,23
		6	0,00	0,00	0,00	5,05	4,74	6,67	0,16	0,27	1,21
		7	0,00	0,00	0,00	6,33	6,33	6,24	9,28	9,17	9,50
		8	0,00	0,00	0,00	7,21	7,34	6,69	10,45	10,68	10,27
Average			0,41			6,27			7,00		

Table B.10: The optimality gap of $AT_{expcon-exct}$ for different scenarios

			N-D.S								
			2			3			4		
Cv(D)	c	P.S	1	2	3	1	2	3	1	2	3
0.1	1	1	2,83	3,04	2,98	6,68	0,89	5,26	6,72	2,49	2,45
		2	3,53	2,99	2,80	6,11	1,07	4,82	7,47	2,43	1,66
		3	4,50	5,16	3,77	14,27	16,99	14,10	16,39	20,64	18,30
		4	4,49	6,00	4,85	15,17	18,26	15,94	18,13	22,47	20,33
		5	2,57	2,72	0,59	6,99	0,41	2,83	4,80	1,76	0,93
		6	3,03	2,84	0,55	6,59	0,88	2,71	5,43	1,89	0,93
		7	4,12	3,95	1,16	10,03	11,28	6,85	11,12	13,33	10,25
		8	4,29	4,52	1,27	10,83	12,55	7,69	11,91	14,70	11,42
	3	1	1,61	1,69	0,05	4,91	4,48	7,87	0,34	0,40	1,54
		2	2,98	3,29	0,06	6,50	6,06	9,35	0,97	1,38	1,87
		3	2,83	3,45	0,01	7,95	7,97	6,83	11,84	12,91	12,15
		4	2,49	3,28	0,76	9,15	9,53	6,43	13,75	15,60	12,50
		5	0,66	0,68	0,00	3,60	3,25	4,88	0,12	0,13	1,02
		6	1,88	1,87	0,01	5,02	4,71	5,75	0,38	0,47	1,17
		7	2,55	2,34	0,03	6,28	5,97	4,94	9,50	9,10	8,46
		8	2,95	2,79	0,04	7,12	6,90	5,75	10,43	10,29	9,55
0.3	1	1	1,55	1,84	2,35	5,99	0,28	4,78	6,45	1,86	2,14
		2	1,57	1,69	2,27	5,20	0,41	5,33	7,23	2,01	1,50
		3	2,02	3,03	3,07	12,54	15,75	14,05	15,78	19,86	18,57
		4	2,01	4,25	4,20	13,38	17,06	15,90	17,29	21,71	20,61
		5	1,56	1,82	0,08	6,81	0,11	3,27	4,68	1,50	0,88
		6	1,57	1,81	0,17	5,99	0,31	3,34	4,98	1,50	0,88
		7	1,94	2,21	0,55	8,63	10,24	7,76	10,63	12,63	10,75
		8	1,90	2,48	0,62	9,26	11,38	8,61	11,46	13,93	11,89
	3	1	0,17	0,26	0,05	5,74	5,16	7,69	0,01	0,02	1,16
		2	0,30	0,44	0,05	7,64	7,49	8,61	1,10	1,27	1,72
		3	0,22	0,43	0,08	9,61	10,08	6,09	13,88	14,59	11,11
		4	0,23	0,88	1,98	10,94	11,60	5,51	15,92	16,79	11,27
		5	0,09	0,14	0,05	4,30	3,87	6,07	0,00	0,01	1,23
		6	0,24	0,33	0,03	5,31	5,09	6,76	0,18	0,29	1,21
		7	0,29	0,40	0,04	6,64	6,75	6,31	9,59	9,59	9,57
		8	0,33	0,47	0,05	7,55	7,81	6,76	10,76	11,13	10,34
Average			1,78			7,36			7,78		

Table B.11: The optimality gap of $AT_{expcon-norm}$ for different scenarios

				N								
D.S	Cv(D)	C	P.S	2	3	4	5	6	7	8	9	10
1	0.1	1	1	-0,62	7,97	9,79	12,80	14,50	15,47	16,48	17,14	17,82
			2	-2,70	10,20	12,74	16,30	18,81	20,24	21,50	22,24	23,25
			3	-2,90	15,91	20,12	22,92	26,90	29,20	30,57	31,90	32,67
			4	-2,90	17,26	22,64	25,00	29,37	32,43	33,67	35,37	36,49
			5	0,87	5,76	7,02	8,82	9,44	10,88	10,08	11,91	12,31
			6	-0,87	3,11	7,96	9,84	11,74	12,67	13,23	13,92	14,24
			7	-3,02	8,39	10,49	13,23	14,75	15,99	16,82	17,49	16,74
			8	-3,88	9,15	11,67	14,45	16,30	-7,00	18,63	18,29	19,91
		3	1	-25,15	-22,67	4,27	-19,91	13,86	15,61	12,11	18,37	8,61
			2	-0,49	7,88	12,26	15,75	19,87	22,18	20,53	21,41	26,52
			3	0,69	-1,00	16,55	21,54	26,99	30,39	33,18	35,61	37,42
			4	1,52	12,69	21,16	26,75	31,72	34,41	38,18	41,18	42,88
			5	0,47	-29,31	5,56	6,72	7,51	11,29	12,09	13,02	13,66
			6	-0,45	4,33	7,05	10,20	10,09	13,29	14,65	14,93	12,64
			7	-0,81	4,81	9,20	10,05	14,25	16,30	13,66	19,20	20,06
			8	-0,94	5,77	10,33	13,69	16,17	-10,80	20,05	17,58	23,45
	0.3	1	1	4,28	13,12	15,68	18,67	19,52	21,36	22,29	22,81	22,98
			2	2,45	14,99	18,34	22,36	24,62	25,88	27,35	27,97	28,95
			3	2,21	19,64	23,85	29,55	33,09	35,33	36,86	38,17	38,84
			4	2,18	20,82	28,12	31,62	35,77	38,46	40,32	41,54	42,73
			5	5,43	8,18	12,81	14,83	15,48	16,47	17,06	17,38	17,77
			6	4,14	8,27	13,82	16,25	17,15	18,35	18,74	19,50	19,33
			7	2,40	-13,93	16,07	19,28	20,45	21,95	22,36	22,77	23,07
			8	1,56	13,73	17,05	20,62	22,41	23,65	24,66	25,29	25,96
		3	1	3,76	-16,91	-14,90	18,08	17,43	21,68	18,93	19,42	16,57
			2	5,84	16,17	21,48	24,62	27,36	28,18	29,28	30,14	35,60
			3	8,38	21,38	27,09	33,21	38,56	41,33	44,60	47,02	48,74
			4	9,66	24,83	33,66	38,76	43,00	45,79	50,63	53,69	55,32
			5	2,46	-25,82	10,06	4,35	13,58	12,26	7,18	12,99	16,97
			6	3,09	9,48	12,56	15,51	15,46	18,63	16,85	20,79	17,67
			7	3,87	11,33	15,00	18,87	18,55	19,53	24,20	25,66	26,69
			8	4,53	6,92	17,59	21,63	23,11	23,10	28,10	29,35	30,12
2	0.1	1	1	-1,78	7,11	10,39	11,87	13,51	14,68	15,73	15,41	17,11
			2	-2,89	10,16	14,41	16,13	18,64	20,58	21,54	22,55	23,40
			3	-3,73	16,78	23,75	27,59	29,52	32,00	34,09	35,35	36,48
			4	1,16	19,36	27,60	32,01	33,68	36,62	39,14	40,73	41,68
			5	-0,56	4,71	5,62	7,72	8,93	9,52	9,96	10,40	10,65
			6	-1,70	5,58	7,02	9,37	-7,52	11,58	12,22	12,72	13,19
			7	-3,06	8,27	11,36	13,33	14,91	16,05	16,83	17,37	17,95
			8	-3,50	9,44	12,95	14,66	16,94	16,44	19,25	19,86	20,46
		3	1	-0,47	3,48	6,00	-6,73	8,50	9,18	9,68	15,91	10,37
			2	-1,60	5,05	10,87	13,86	18,15	17,17	18,18	23,39	24,78
			3	-0,79	8,45	14,52	21,53	24,55	29,22	31,73	33,95	35,57
			4	-2,69	11,26	21,79	26,89	30,94	33,36	36,84	39,44	41,72
			5	-0,13	-0,13	1,68	-17,61	8,41	9,61	6,72	11,18	7,19
			6	-18,48	-13,31	5,68	8,67	8,60	9,42	10,02	14,31	14,54
			7	-1,44	3,28	7,64	10,41	12,86	15,02	16,16	17,75	18,51
			8	-1,73	4,09	8,48	12,17	13,48	17,57	15,87	20,70	22,11

				N								
D.S	Cv(D)	C	P.S	2	3	4	5	6	7	8	9	10
2	0.3	1	1	1,27	10,32	12,32	15,43	17,18	17,99	19,16	19,97	20,57
			2	0,13	13,15	17,55	19,78	22,42	24,09	24,92	26,07	26,91
			3	-0,49	19,38	26,71	29,48	33,33	35,86	37,63	38,80	40,17
			4	3,26	21,66	30,22	35,01	38,22	40,37	42,51	44,10	45,42
			5	2,14	7,86	8,75	11,10	12,13	12,60	13,19	13,45	13,81
			6	1,27	8,85	10,68	13,06	13,80	14,97	15,67	16,13	16,54
			7	0,28	11,40	14,81	17,15	18,79	19,74	20,59	21,18	21,66
			8	-0,14	12,50	15,41	18,34	20,86	21,95	22,65	23,71	24,20
		3	1	0,25	8,23	11,53	12,82	16,45	18,25	19,10	20,32	21,14
			2	3,27	13,20	18,20	21,71	25,21	24,39	25,40	29,72	31,42
			3	0,52	18,29	25,54	30,75	35,37	38,54	40,93	43,19	44,68
			4	6,72	21,68	30,53	36,69	40,84	44,73	44,45	49,71	51,07
			5	1,16	5,52	7,52	8,64	9,35	12,35	13,12	13,88	14,44
			6	1,46	7,46	10,08	12,05	14,69	16,24	17,39	14,84	18,97
			7	1,93	9,31	12,98	16,40	18,34	17,45	21,56	23,00	2,75
			8	2,35	11,07	15,55	19,09	21,18	21,09	24,89	26,55	27,87
3	0.1	1	1	0,13	-20,90	-18,51	14,88	15,96	17,60	18,50	19,20	19,90
			2	0,09	8,80	-6,00	-3,95	21,33	-1,56	23,88	24,64	25,52
			3	-0,03	16,30	22,64	13,39	30,32	32,46	33,61	34,95	35,81
			4	1,00	19,45	26,46	31,37	34,71	37,29	39,03	40,06	41,17
			5	-0,31	3,72	6,52	-31,45	-31,05	10,90	11,27	11,95	-29,89
			6	-0,16	4,42	7,84	10,70	11,49	13,25	14,08	14,54	15,14
			7	0,14	6,74	9,89	13,77	14,74	16,84	17,34	18,26	18,88
			8	0,03	-16,63	11,60	15,77	16,99	19,06	20,16	20,72	-12,49
		3	1	-17,01	-17,75	-15,67	17,25	27,25	21,48	22,68	31,43	21,97
			2	-5,06	-12,15	-9,62	19,68	29,24	-5,01	26,92	35,19	29,87
			3	12,29	11,89	16,21	0,24	31,53	27,87	30,78	38,54	33,75
			4	16,59	12,23	20,32	23,61	30,07	30,55	33,26	37,71	37,16
			5	3,87	-0,82	7,32	-32,74	-24,65	11,03	12,07	25,38	-31,20
			6	4,31	7,45	8,99	11,65	23,54	14,34	14,93	27,17	16,30
			7	5,46	6,53	9,07	12,90	25,93	16,76	17,71	31,11	20,00
			8	5,73	7,53	10,50	14,79	-16,18	19,22	21,20	29,62	23,40
	0.3	1	1	0,36	8,52	13,80	16,48	17,75	19,24	20,19	-12,80	-12,53
			2	0,34	9,94	16,80	-1,74	22,27	24,32	25,57	1,96	27,08
			3	0,85	18,12	25,10	29,24	32,79	34,92	36,14	37,46	38,41
			4	1,55	20,68	28,96	33,29	37,22	39,82	41,53	42,66	43,83
			5	0,84	6,33	8,60	-29,43	-28,93	12,56	12,86	13,58	-27,84
			6	0,93	7,16	10,01	12,61	13,47	-23,15	-22,83	16,17	16,77
			7	1,58	-23,11	-20,61	16,19	17,19	-17,71	20,03	20,49	-16,69
			8	1,47	10,23	13,52	18,13	19,24	21,29	21,82	22,99	23,75
		2	1	-14,08	-13,61	-11,03	22,46	32,94	25,84	27,03	0,51	27,80
			2	-6,83	-8,00	-4,68	-2,53	35,79	31,28	33,21	40,96	35,59
			3	16,56	15,86	22,66	6,26	35,11	32,30	33,70	43,09	38,32
			4	19,63	9,20	24,35	28,25	34,36	35,05	36,17	41,65	41,68
			5	5,42	4,08	12,58	-28,72	-18,22	17,19	18,03	34,67	-26,97
			6	6,14	12,35	14,34	17,61	-15,12	20,36	21,46	35,36	22,71
			7	7,12	11,28	14,23	18,14	33,41	-22,95	22,52	38,24	-21,80
			8	7,67	12,28	15,76	20,14	32,77	24,28	25,94	38,23	27,49
Average				0,87	6,60	12,58	14,42	18,99	19,89	22,95	25,37	21,94

Table B.12: Percent Improvement of $AT_{norm-iter}$ according to base solution AT_{exp}.

				N								
D.S	Cv(D)	C	P.S	2	3	4	5	6	7	8	9	10
1	0.1	1	1	-0,45	1,89	3,76	9,72	2,86	13,20	13,76	15,76	15,88
			2	-1,75	4,94	5,57	13,09	5,20	17,92	18,35	20,75	20,97
			3	-2,90	0,10	1,41	2,21	-1,44	3,16	3,38	3,58	3,74
			4	-2,90	0,02	1,30	2,08	-1,80	3,03	3,25	3,45	3,61
			5	0,19	-1,12	3,06	7,13	0,98	9,10	9,89	11,14	11,18
			6	-0,69	0,13	3,41	8,34	2,10	10,90	11,75	13,21	13,32
			7	-2,27	-1,45	-0,81	-0,27	-2,37	0,53	0,81	1,05	1,26
			8	-2,54	-1,61	-0,89	-0,31	-2,76	0,57	0,89	1,15	1,36
		3	1	0,05	0,35	9,25	9,63	13,76	14,22	15,37	15,56	17,87
			2	-0,49	1,34	12,26	13,49	18,57	19,62	21,24	21,74	24,76
			3	0,69	2,30	3,43	4,22	4,87	5,37	5,78	6,13	6,42
			4	1,52	3,50	4,60	5,43	6,07	6,57	6,98	7,31	7,59
			5	0,47	0,24	6,80	6,89	10,01	10,22	10,77	10,84	12,69
			6	-0,45	-0,30	7,66	8,06	11,66	12,24	13,06	13,28	15,38
			7	-0,81	-0,90	-0,83	-0,73	-0,62	-0,51	-0,41	-0,30	-0,21
			8	-0,94	-0,93	-0,73	-0,54	-0,36	-0,19	-0,04	0,11	0,25
	0.3	1	1	3,98	6,59	8,84	15,23	7,25	18,93	19,46	21,50	21,60
			2	3,11	9,47	10,80	18,74	9,53	23,63	24,05	26,45	26,67
			3	2,21	4,89	6,17	6,95	3,54	7,84	8,11	8,32	8,48
			4	2,18	4,80	6,06	6,83	3,09	7,72	7,99	8,19	8,35
			5	4,57	3,66	7,92	12,46	5,11	14,50	15,27	16,60	16,63
			6	4,00	5,08	8,55	13,81	6,24	16,43	17,23	18,74	18,83
			7	2,99	3,76	4,35	4,82	3,03	5,51	5,77	5,99	6,18
			8	2,73	3,60	4,26	4,78	2,64	5,54	5,82	6,06	6,26
		3	1	3,76	4,54	14,59	15,22	19,66	20,18	21,16	21,38	23,63
			2	5,84	7,37	20,17	21,27	27,20	28,12	29,66	30,11	33,03
			3	8,38	9,84	10,90	11,65	12,22	12,66	13,02	13,31	13,56
			4	9,66	11,33	12,44	13,22	13,80	14,25	14,61	14,90	15,15
			5	2,46	2,78	10,06	10,40	13,58	13,88	14,28	14,38	16,07
			6	3,09	3,71	12,38	13,04	16,79	17,40	18,02	18,27	20,20
			7	3,87	4,03	4,20	4,36	4,51	4,64	4,76	4,88	4,98
			8	4,53	4,76	5,00	5,21	5,41	5,59	5,75	5,90	6,03

				N								
D.S	Cv(D)	C	P.S	2	3	4	5	6	7	8	9	10
2	0.1	1	1	-2,12	7,11	8,23	8,61	8,05	13,23	14,59	15,95	15,84
			2	-2,22	10,16	12,22	12,89	13,09	18,70	20,12	21,54	21,59
			3	-3,71	-2,26	-1,33	-0,69	-0,27	0,02	0,24	0,41	0,54
			4	-3,72	-1,87	-0,85	-0,15	0,34	0,64	0,86	1,04	1,18
			5	-1,69	4,71	5,06	5,20	3,88	8,17	9,14	10,24	10,04
			6	-1,89	5,58	6,53	6,85	6,03	10,26	11,57	12,65	12,57
			7	-3,06	-2,88	-2,71	-2,57	-2,43	-2,32	-2,21	-2,12	-2,03
			8	-3,50	-3,17	-2,88	-2,63	-2,41	-2,22	-2,05	-1,90	-1,77
		3	1	-0,84	-0,58	7,59	8,01	11,67	12,16	14,24	14,75	16,02
			2	-1,60	0,28	10,25	11,59	16,33	17,49	20,34	21,38	23,13
			3	-0,79	0,87	1,88	2,65	3,27	3,76	4,16	4,50	4,78
			4	-0,25	1,80	2,80	3,62	4,23	4,71	5,11	5,44	5,71
			5	-0,13	-0,44	5,58	5,67	8,50	8,72	10,38	10,64	11,69
			6	-1,15	-1,06	6,28	6,70	10,08	10,70	12,59	13,28	14,36
			7	-1,44	-1,59	-1,54	-1,46	-1,37	-1,28	-1,19	-1,10	-1,01
			8	-1,73	-1,75	-1,56	-1,39	-1,23	-1,07	-0,93	-0,79	-0,66
	0.3	1	1	0,69	10,32	11,65	12,25	11,57	16,97	18,05	19,30	19,19
			2	0,47	13,15	15,41	16,52	16,58	22,50	23,61	24,93	24,99
			3	-0,49	0,89	1,70	2,23	2,60	2,86	3,06	3,20	3,32
			4	-0,62	1,08	2,01	2,61	3,01	3,30	3,52	3,68	3,81
			5	0,99	7,86	8,25	8,38	7,47	11,43	12,36	13,42	13,27
			6	0,81	8,85	9,84	10,20	9,68	13,79	14,92	15,99	15,92
			7	0,28	0,46	0,62	0,76	0,89	1,00	1,09	1,18	1,26
			8	-0,14	0,19	0,47	0,71	0,91	1,09	1,25	1,38	1,50
		3	1	1,93	2,68	11,60	12,28	16,08	16,66	18,65	19,16	20,32
			2	3,27	4,82	16,75	17,91	23,26	24,25	27,08	27,99	29,59
			3	5,09	6,56	7,62	8,38	8,95	9,40	9,76	10,06	10,31
			4	6,11	7,79	8,89	9,67	10,24	10,69	11,05	11,34	11,59
			5	1,16	1,45	7,96	8,33	11,14	11,47	12,95	13,25	14,12
			6	1,46	2,04	10,08	10,78	14,22	14,88	16,61	17,24	18,17
			7	1,93	2,07	2,23	2,38	2,52	2,66	2,78	2,89	2,99
			8	2,35	2,56	2,78	3,00	3,19	3,37	3,53	3,67	3,80

				N								
D.S	Cv(D)	C	P.S	2	3	4	5	6	7	8	9	10
3	0.1	1	1	-2,85	2,59	9,35	9,31	10,86	13,98	15,53	14,45	16,21
			2	-2,59	4,04	13,02	13,77	15,11	19,61	20,60	19,69	22,27
			3	-2,82	0,18	0,45	0,62	0,72	0,79	0,84	0,88	0,91
			4	-3,60	0,56	0,92	1,15	1,31	1,40	1,46	1,51	1,55
			5	-0,31	1,55	6,12	6,05	7,06	9,05	9,55	9,23	10,80
			6	-0,16	2,28	7,47	7,71	8,94	11,46	11,84	11,62	13,53
			7	0,14	-0,48	-0,48	-0,48	-0,48	-0,48	-0,48	-0,48	-0,48
			8	0,03	-0,59	-0,56	-0,54	-0,52	-0,51	-0,50	-0,49	-0,48
		3	1	6,93	2,93	13,18	9,66	19,67	17,37	19,14	25,34	18,64
			2	8,92	3,33	15,99	12,11	21,09	22,29	24,88	28,14	23,83
			3	14,35	4,44	4,65	12,86	20,93	24,40	21,25	30,55	28,07
			4	16,16	5,32	5,53	15,07	20,64	26,63	23,65	30,41	31,36
			5	3,87	1,18	7,32	3,72	16,37	11,21	12,07	22,56	11,64
			6	4,31	1,33	8,99	4,93	16,32	13,66	14,95	22,81	14,01
			7	5,46	1,37	1,37	5,44	16,82	14,35	11,40	24,65	16,09
			8	5,73	1,54	1,54	6,75	16,21	16,11	12,99	24,09	18,16
	2	1	1	-1,97	4,11	11,38	11,92	12,93	15,93	17,40	16,30	18,07
			2	-1,82	5,53	15,06	16,21	17,16	21,33	22,32	21,48	24,07
			3	-1,73	1,67	2,00	2,21	2,36	2,46	2,53	2,59	2,64
			4	-2,56	2,02	2,48	2,76	2,94	3,08	3,17	3,25	3,31
			5	0,84	2,93	8,07	8,34	9,04	11,13	11,52	11,27	12,63
			6	0,93	3,71	9,47	10,19	10,98	13,52	13,80	13,62	15,29
			7	1,58	0,98	0,98	0,98	0,99	0,99	0,99	0,99	0,99
			8	1,47	0,87	0,90	0,93	0,95	0,97	0,98	0,99	1,01
		0.3	1	9,76	7,48	18,75	14,47	25,22	23,51	25,46	31,49	24,40
			2	12,20	8,10	21,44	16,91	25,96	28,34	31,26	33,48	29,62
			3	17,40	8,87	9,07	16,91	24,79	28,86	25,24	34,12	32,06
			4	19,80	10,15	10,37	19,30	24,15	31,45	28,00	33,66	35,58
			5	5,42	4,50	12,58	7,50	23,15	16,04	17,30	29,75	16,12
			6	6,14	4,77	14,34	9,02	22,95	18,77	20,43	29,98	18,88
			7	7,12	4,30	4,30	8,88	22,54	18,71	15,55	30,67	20,14
			8	7,67	4,73	4,72	10,53	21,92	20,70	17,42	30,02	22,48
Average				1,93	3,05	6,46	7,51	9,16	11,12	11,56	13,36	12,83

Table B.13: Percent Improvement of $AT_{expcon-norm}$ according to base solution AT_{exp}.

Appendix C

RESULTS OF PLANNING PROBLEM OF ALARA AGRI BUSINESS

The List of Abbreviations used in Appendix B.

Cv()	:Coefficient of variation
c_t	:Cost of a unit product at time t
Sc.No	:Scenario Number
S.Level	:Service Level

T^m_{i,τ_i}
$DDa_{m_0}(0, ..., \xi_m, ...\mathcal{T}+1 : 0, ..., 0, 1, 0, ..., 0)$
$DDa_{m_1}(0, ..., \xi_m - 1, \xi_m, \xi_m + 1, ...\mathcal{T}+1 : 0, ..., , 0, 0.2, 0.6, 0.2, 0, ..., 0)$
$DDa_{m_2}(0, ..., \xi_m - 2, \xi_m - 1, \xi_m, \xi_m + 1, \xi_m + 2, ...\mathcal{T}+1 : 0, ..., 0, 0.15, 0.15, 0.4, 0.15, 0.15, 0, ..., 0)$

T^h_{i,τ_i}
$DDa_{h_0}(0, ..., \xi_h, ...\mathcal{T}+1 : 0, ..., 0, 1, 0, ..., 0)$
$DDa_{h_1}(0, ..., \xi_h - 1, \xi_h, \xi_h + 1, ...\mathcal{T}+1 : 0, ..., 0, 0.2, 0.6, 0.2, 0, ..., 0)$
$DDa_{h_2}(0, ..., \xi_h - 2, \xi_h - 1, \xi_h, \xi_h + 1, \xi_h + 2, ...\mathcal{T}+1 : 0, ..., 0, 0.15, 0.15, 0.4, 0.15, 0.15, 0, ..., 0)$

Table C.1: The harvest and maturation lenght distributions

where ξ_m and ξ_h equal to the expected values of maturation and harvest lengths in each case.

$Cv[D_t]$	$Cv[Y_i]$	DDa_h	DDa_m	c_t	$Sc.No$	Expected Profit		$\Delta_{n-e}(\%)$	Expected S.Level	
						A_{norm}	A_{exp}		A_{norm}	A_{exp}
0	0	0	1	2	1	1184995,65	1185000,00	0,00	0,84	0,86
				4	2	389999,99	390000,00	0,00	0,82	0,82
			2	2	3	1092251,08	1059329,40	3,11	0,81	0,80
				4	4	312204,20	282351,90	10,57	0,69	0,77
			3	2	5	1045023,16	947744,25	10,26	0,78	0,74
				4	6	287718,94	190174,80	51,29	0,64	0,73
		1	1	2	7	1121012,31	1113070,20	0,71	0,83	0,83
				4	8	332901,43	328640,25	1,30	0,76	0,80
			2	2	9	1072832,34	1021091,25	5,07	0,80	0,78
				4	10	302363,69	252100,85	19,94	0,67	0,76
			3	2	11	1039459,51	929049,75	11,88	0,78	0,74
				4	12	284897,42	176835,65	61,11	0,64	0,72
		2	1	2	13	1074235,34	1044184,80	2,88	0,82	0,78
				4	14	306688,23	264379,05	16,00	0,72	0,78
			2	2	15	1052746,53	992929,20	6,02	0,79	0,74
				4	16	294118,88	212419,90	38,46	0,66	0,74
			3	2	17	1032805,83	935937,60	10,35	0,78	0,71
				4	18	280471,01	157028,95	78,61	0,63	0,71
	0.1	0	1	2	19	1161663,92	1131535,45	2,66	0,84	0,83
				4	20	364042,00	343216,67	6,07	0,78	0,80
			2	2	21	1085958,53	1015487,35	6,94	0,80	0,78
				4	22	308606,63	248807,01	24,03	0,68	0,75
			3	2	23	1038087,38	912435,61	13,77	0,78	0,73
				4	24	284766,09	165135,42	72,44	0,64	0,71
		1	1	2	25	1108820,72	1064727,87	4,14	0,83	0,81
				4	26	325263,47	289375,24	12,40	0,74	0,78
			2	2	27	1066845,37	980232,17	8,84	0,80	0,77
				4	28	299332,50	221147,39	35,35	0,67	0,74
			3	2	29	1034958,85	896761,27	15,41	0,78	0,72
				4	30	282839,66	154063,18	83,59	0,63	0,71
		2	1	2	31	1064163,89	1012334,39	5,12	0,81	0,76
				4	32	302098,75	232238,97	30,08	0,70	0,76
			2	2	33	1047183,18	965934,15	8,41	0,79	0,73
				4	34	291408,54	186168,43	56,53	0,66	0,73
			3	2	35	1029521,67	915263,17	12,48	0,78	0,70
				4	36	278845,09	135740,56	105,43	0,63	0,70
	0.3	0	1	2	37	1089692,38	1011088,83	7,77	0,80	0,78
				4	38	319747,20	246602,27	29,66	0,71	0,75
			2	2	39	1042587,05	915763,00	13,85	0,77	0,73
				4	40	288233,44	171857,26	67,72	0,64	0,71
			3	2	41	1008542,79	832503,46	21,15	0,76	0,69
				4	42	271153,75	107227,73	152,88	0,61	0,69

$Cv[D_t]$	$Cv[Y_i]$	DDa_h	DDa_m	c_t	$Sc.No$	Expected Profit A_{norm}	Expected Profit A_{exp}	$\Delta_{n-e}(\%)$	Expected S.Level A_{norm}	Expected S.Level A_{exp}
0	0.3	1	1	2	43	1054386,60	956318,08	10,25	0,80	0,76
				4	44	297586,44	204658,28	45,41	0,68	0,74
			2	2	45	1028193,18	887848,21	15,81	0,77	0,72
				4	46	281955,20	152858,02	84,46	0,64	0,71
			3	2	47	1006288,08	820408,81	22,66	0,76	0,69
				4	48	269830,96	97898,70	175,62	0,61	0,68
		2	1	2	49	1025887,01	940423,91	9,09	0,79	0,73
				4	50	283215,24	160637,38	76,31	0,66	0,73
			2	2	51	1014889,97	904557,58	12,20	0,77	0,70
				4	52	276514,22	123963,95	123,06	0,63	0,70
			3	2	53	1003803,68	864366,87	16,13	0,76	0,68
				4	54	267748,25	84311,73	217,57	0,61	0,68
0.1	0	0	1	2	55	1140763,59	1127378,80	1,19	0,83	0,83
				4	56	344061,47	330520,67	4,10	0,77	0,80
			2	2	57	1073703,81	1012518,33	6,04	0,80	0,78
				4	58	297369,93	238821,20	24,52	0,67	0,75
			3	2	59	1028820,87	911277,82	12,90	0,78	0,73
				4	60	275551,31	159142,30	73,15	0,63	0,71
		1	1	2	61	1093568,01	1061443,64	3,03	0,83	0,81
				4	62	314012,56	278605,58	12,71	0,73	0,78
			2	2	63	1055878,80	978204,63	7,94	0,80	0,77
				4	64	289284,56	213651,78	35,40	0,66	0,74
			3	2	65	1025916,28	896204,08	14,47	0,78	0,72
				4	66	273858,54	148201,77	84,79	0,63	0,71
		2	1	2	67	1054741,24	1003197,61	5,14	0,81	0,76
				4	68	293595,18	223306,87	31,48	0,70	0,76
			2	2	69	1038057,76	959183,54	8,22	0,79	0,73
				4	70	281999,19	178701,94	57,80	0,65	0,73
			3	2	71	1019668,77	909687,20	12,09	0,78	0,70
				4	72	270267,04	131043,53	106,24	0,62	0,70
	0.1	0	1	2	73	1130741,07	1105465,14	2,29	0,83	0,82
				4	74	336519,70	314213,11	7,10	0,76	0,79
			2	2	75	1068324,91	994924,91	7,38	0,80	0,77
				4	76	294269,00	226596,93	29,86	0,67	0,74
			3	2	77	1024996,58	896234,60	14,37	0,78	0,72
				4	78	273562,85	150134,91	82,21	0,62	0,71
		1	1	2	79	1086308,69	1041858,58	4,27	0,82	0,80
				4	80	309634,45	264640,29	17,00	0,72	0,77
			2	2	81	1050207,59	961486,29	9,23	0,79	0,76
				4	82	286886,30	201952,08	42,06	0,66	0,74
			3	2	83	1021440,71	881217,90	15,91	0,77	0,72
				4	84	272061,47	139325,42	95,27	0,62	0,70

$Cv[D_t]$	$Cv[Y_i]$	DDa_h	DDa_m	c_t	$Sc.No$	Expected Profit		$\Delta_{n-e}(\%)$	Expected S.Level	
						A_{norm}	A_{exp}		A_{norm}	A_{exp}
0.1	0.1	2	1	2	85	1049291,25	991604,78	5,82	0,81	0,76
				4	86	290298,89	211842,06	37,04	0,69	0,76
			2	2	87	1033587,43	949660,35	8,84	0,79	0,73
				4	88	280046,84	169185,33	65,53	0,65	0,73
			3	2	89	1016729,54	902344,44	12,68	0,78	0,70
				4	90	268764,15	122584,97	119,25	0,62	0,70
0.1	0.3	0	1	2	91	1071157,78	999678,61	7,15	0,80	0,78
				4	92	304661,86	232976,99	30,77	0,69	0,75
			2	2	93	1027555,11	907415,51	13,24	0,77	0,73
				4	94	277501,59	161724,30	71,59	0,63	0,71
			3	2	95	996441,80	825351,82	20,73	0,76	0,69
				4	96	261613,50	98938,64	164,42	0,60	0,68
		1	1	2	97	1039401,99	946040,44	9,87	0,79	0,76
				4	98	286177,36	191923,78	49,11	0,67	0,74
			2	2	99	1015185,13	879044,47	15,49	0,77	0,72
				4	100	271769,10	142410,03	90,84	0,63	0,71
			3	2	101	995715,66	812208,17	22,59	0,76	0,69
				4	102	260788,39	90575,01	187,93	0,60	0,68
		2	1	2	103	1012489,47	929546,46	8,92	0,79	0,73
				4	104	273511,92	148570,14	84,10	0,65	0,73
			2	2	105	1002989,35	895160,17	12,05	0,77	0,70
				4	106	267029,73	114409,62	133,40	0,62	0,70
			3	2	107	992675,99	855543,43	16,03	0,76	0,68
				4	108	258772,09	77012,07	236,01	0,60	0,68
0.3	0	0	1	2	109	1018947,56	1003630,72	1,53	0,82	0,80
				4	110	241758,90	200289,48	20,70	0,66	0,76
			2	2	111	976584,14	911729,46	7,11	0,79	0,75
				4	112	222321,99	136475,06	62,90	0,60	0,72
			3	2	113	946159,30	830335,78	13,95	0,77	0,71
				4	114	207729,24	79156,19	162,43	0,56	0,70
		1	1	2	115	988831,72	950172,40	4,07	0,81	0,78
				4	116	230809,23	163969,94	40,76	0,64	0,75
			2	2	117	963705,46	883886,71	9,03	0,79	0,74
				4	118	218219,29	118239,39	84,56	0,60	0,72
			3	2	119	944206,94	817232,86	15,54	0,77	0,70
				4	120	207243,47	70549,43	193,76	0,56	0,69
		2	1	2	121	962691,88	904562,73	6,43	0,80	0,74
				4	122	220278,09	124387,44	77,09	0,63	0,74
			2	2	123	952616,81	872407,71	9,19	0,78	0,71
				4	124	214233,76	93322,47	129,56	0,59	0,71
			3	2	125	942292,32	838213,18	12,42	0,77	0,69
				4	126	206282,17	58488,99	252,69	0,56	0,69

$Cv[D_t]$	$Cv[Y_i]$	DDa_h	DDa_m	c_t	$Sc.No$	Expected Profit		$\Delta_{n-e}(\%)$	Expected S.Level	
						A_{norm}	A_{exp}		A_{norm}	A_{exp}
0.3	0.1	0	1	2	127	1013579,32	992484,97	2,13	0,81	0,79
				4	128	239687,08	193035,40	24,17	0,66	0,76
			2	2	129	972395,19	902688,25	7,72	0,79	0,75
				4	130	221074,19	130137,11	69,88	0,60	0,72
			3	2	131	942563,38	822027,64	14,66	0,77	0,71
				4	132	206567,42	73687,13	180,33	0,56	0,70
		1	1	2	133	984623,33	940414,99	4,70	0,81	0,78
				4	134	228807,76	157284,35	45,47	0,64	0,75
			2	2	135	960281,30	875768,25	9,65	0,78	0,74
				4	136	217340,59	112756,64	92,75	0,60	0,72
			3	2	137	941876,27	809793,78	16,31	0,77	0,70
				4	138	206458,72	65866,39	213,45	0,56	0,69
		2	1	2	139	958582,52	899237,75	6,60	0,80	0,74
				4	140	218828,03	119076,78	83,77	0,62	0,74
			2	2	141	949165,23	868299,10	9,31	0,78	0,71
				4	142	213242,74	87853,80	142,72	0,59	0,71
			3	2	143	940231,88	832865,33	12,89	0,77	0,69
				4	144	205279,43	54087,56	279,53	0,56	0,69
	0.3	0	1	2	145	974347,53	922996,17	5,56	0,79	0,76
				4	146	226090,61	144783,57	56,16	0,62	0,73
			2	2	147	943084,14	842647,96	11,92	0,77	0,72
				4	148	211927,84	89016,62	138,08	0,58	0,70
			3	2	149	921403,10	771196,00	19,48	0,75	0,68
				4	150	200550,44	37920,77	428,87	0,55	0,68
		1	1	2	151	952605,30	876550,92	8,68	0,78	0,75
				4	152	217463,27	112534,39	93,24	0,61	0,72
			2	2	153	934220,80	818419,53	14,15	0,76	0,71
				4	154	208816,00	72341,40	188,65	0,57	0,70
			3	2	155	921926,29	760972,68	21,15	0,75	0,68
				4	156	200224,95	31109,13	543,62	0,55	0,67
		2	1	2	157	932106,99	858369,84	8,59	0,78	0,71
				4	158	210157,98	77931,10	169,67	0,60	0,71
			2	2	159	926872,89	830395,12	11,62	0,76	0,69
				4	160	206196,51	49940,00	312,89	0,57	0,69
			3	2	161	920321,56	798231,75	15,30	0,76	0,67
				4	162	199861,17	20445,99	877,51	0,55	0,67
Average						643119,07	546551,09	58,21	0,71	0,73

Table C.2: The expected profit and service levels for A_{norm} and A_{exp} for Alara Agri Business Problem

VITA

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