COORDINATION OF PRODUCTION AND DISPATCHING DECISIONS
IN SUPPLY CHAIN MODELS

by

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A Thesis Submitted to the
Graduate School of Engineering
in Partial Fulfillment of the Requirements for
the Degree of

Master of Science

in

Industrial Engineering

Koç University

June, 2010

Koç University

Graduate School of Sciences and Engineering

This is to certify that I have examined this copy of a master's thesis by

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ABSTRACT

This study is focused on the coordination of transportation and production policies between a supplier and a retailer in an inventory system. Both a deterministic model, in which the production and the demand rates of a specific product are known and constant, and a stochastic model, in which the production and the demand rates are random, are considered. In these models, the supplier makes the production, holds inventory and ships the products to the retailer to satisfy the external demand that arrives to the retailer. We analyze two versions of this problem. In the first model, we assume that the retailer also holds inventory and satisfies the customer orders immediately and in the second model, we assume that the retailer does not hold inventory but accumulates the customer orders and satisfies them at a later time, where the customers are willing to wait at the expense of a waiting cost. We investigate both a vendor-managed inventory (VMI) setting and a decentralized model (non-VMI setting). In the non-VMI model, the retailer manages its own inventory and sends orders to the supplier and the supplier determines its own production process and the amount to produce in an inventory replenishment cycle considering the order quantity of the retailer. However, in the vendor-managed inventory setting, the supplier makes all the decisions and determines the length of the production and transportation cycles and the shipment quantities to the retailer. In this study, for the deterministic case, we determine the length of the optimal production and transportation cycles in both the VMI and non-VMI models and compare the costs of these models. Then, we analyze the stochastic case in which the demand and the production rates are random. We use an infinite-horizon dynamic programming approach and we show that the optimal production and transportation decisions in the stochastic case are complex and non-monotonic. Therefore, we also consider simpler policies such as time-based and quantity-based transportation policies,

which are widely used in the industry, and we analyze the efficiency of these policies compared to the optimal solution.

ÖZETÇE

Bu çalışma, bir üretim sisteminde tedarikçi-satıcı arasındaki üretim ve dağıtım kordinasyonunun sağlanması üzerine odaklanmıştır. İlk olarak, üretim ve talep oranlarının bilinen ve sabit olduğu deterministik model kullanılmıştır. Satıcı karşılaştığı dış talebi karşılamak için tedarikçiye siparişte bulunur. Tedarikçi satıcıdan gelen bu talepleri karşılayabilmek için üretimini yaptığı ürünleri stokta tutar. Bu problemin iki farklı modeli incelenmiştir. Bu iki modelin birinde satıcı stok tutarken diğerinde stok tutmayıp müşterilerin taleplerini bekletmekte ve bunun karşılığında bekletme ücreti ödemektedir. Deterministic modelde hem tedarikçi-yönetimli stok (VMI) hem de tedarikçi-yönetimli olmayan stok (non-VMI) politikaları incelenmiştir. Tedarikçi-yönetimli stok sisteminde, üretici üretim ve sipariş aralıkları ile satıcının sipariş adedini belirlemektedir. Diğer modelde ise (non-VMI) satıcı kendi stoğunu takip ederken üretici de üretim düzenini satıcının sipariş adetleri doğrultusunda kendi yönetir. VMI ve non-VMI model için en uygun üretim ve taşıma aralıkları hesaplanıp sunulmuştur. Ardından üretim ve talebin rastgele olduğu stokastik modeller geliştirilmiştir. Çözümde dinamik programlama yaklaşımı kullanılmış ve sonsuz-ufuk modeli analiz edilmiştir. Stokastik modellerde optimum üretim ve dağıtım kararlarının komplike ve düzensiz olduğu gösterilmiştir. Bu sebeple daha genel politika ve stratejiler geliştirmek adına, piyasada da sıkça kullanılan, zaman-tabanlı ve miktar-tabanlı taşıma politikaları incelenmiştir.

ACKNOWLEDGMENTS

I would like to take this opportunity to express my deepest gratitude to my advisors Onur Kaya and Lerzan Örmeci for their insightful discussions and never ending support during this work. I have learned and achieved more than I ever dreamed of in two-years by their precious guidance and help. I feel myself very lucky and proud of being one of their students.

I am grateful to Fikri Karaesmen, Yalçın Akçay and Evrim Didem Güneş for taking part in my thesis committee, for critical reading of this thesis and for their valuable suggestions and comments.

I also thank to TUBITAK (The Scientific and Technological Research Council of Turkey) for their generous financial support.

I would like to thank to my friends Aysegul Ozturk and Derya Kunduzcu for being such good friends. Thank you for all the help and fun.

I would like to thank to Nihan Comden for all the great time we spent together and for supporting and encouraging me in the last two years.

Special thanks to Bekir Yenilmez for his friendship. I am lucky to have a friend like him.

And the last but not the least, I would like to thank to my parents for believing me and trusting me at every step I have taken so far and for their endless and supporting love.

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Chapter 1

INTRODUCTION

A supply chain is composed of different entities such as raw material supplier, manufacturer, transporter, retailer, etc. whose designated aim is to convert the raw material into finished product to satisfy the customer's demand in time with least possible cost. Numerous academic studies presented that the cost savings may be realized from collaborative initiatives such as vendor-managed inventory (VMI) in a supply chain. Under a typical vendor managed inventory agreement, the supplier decides the order quantities to be sent to the retailer and manages the inventory levels at both facilities. There are a number of studies which show that the VMI can improve the supply chain's performance by reducing inventory holding costs and increasing service levels. The problem considered in this thesis concerns a single supplier (manufacturer) supplying a single retailer with a product which faces external demand. Our aim is to minimize the total cost of supply chain and improve the performance of the system using the VMI Model.

The characteristics of the inventory management system is mainly determined by the structure of the demand and the supply processes. Replenishment and dispatch policies are directly effected by the uncertainty of the demand and supply. First, we consider the deterministic model which provides a basis for the analysis for the stochastic demand and supply model. In practice, arrival probabilities of demand may not be known with certainty. For some cases, such as random capacity of the supplier or lack of raw material, supply may also be stochastic. In the stochastic models, we aim to minimize the expected average total cost of the system, since the outcome of a decision may vary and can only be predicted to some extent due to the randomness in demand and supply.

We consider two different problems throughout this thesis; in the first problem, the retailer does not hold any inventory but accumulates the demand which he faces externally. The problem of interest can be stated as follows. Consider a product such as a luxury watch or an expensive sports car that is obviously disadvantageous for the retailer to hold in stock. In this case, inventory holding cost for the retailer is high, and customer waiting cost is relatively low for reasonable time intervals so that the retailer does not carry inventory. Sales agents and stores making catalog sales would be the examples for this case. In this scenario, both the supplier and the retailer is engaged in setting an efficient integrated inventory/outbound dispatch policy. [9], [10] and [8] are the instances in the literature in this context.

In practice, responsibility of the inventory ownership is not given to the third party provider (retailer in our case), and all the real warehousing costs are paid by the supplier eventually. In technology products industry, optimal shipment consolidation policies are generally initiated under integrated warehousing/transportation contracts where both the supplier and the third party provider are interested in saving of transportation costs.

On the other hand, in the second problem, the retailer also holds inventory like the supplier and satisfies the external demand from its own inventory. All the other parameters are the same with the first problem, except there is no customer waiting cost but the inventory holding cost in the second problem in which the retailer holds inventory.

In the previous studies, Lu [26], Goyal [16] and Hill [21] examined vendor managed single supplier single retailer inventory models. In their models, the supplier produces a product and dispatches to the retailer according to some dispatch policy. The retailer holds inventory in order to satisfy the external demand which is constant and known. The supplier and the retailer cooperate to set an integrated policy in order to minimize the total cost per unit time. These models can be used for the stores that directly buy from the producer and carry inventory in real life situations. On the other hand, Çetinkaya and Lee [9] studied a vendor managed inventory, VMI, model where the supplier does not produce a product but acts as a third party warehouse, TPW. In this model, the retailer does not hold inventory but accumulates the orders and the customers agree to wait for a reasonable time. This model is suitable for the products which are unreasonable to keep in stock such as expensive laptop computers, large items like photocopy machines, luxury items etc. The main contribution of our deterministic VMI model with shipment consolidation is to combine these two concepts described above in one model. In our model, the supplier produces the product and satisfies the demand through retailer which does not hold any inventory. Our model can be used for valuable products that are supplied to customer via retailers directly from the factory.

In the stochastic models of this thesis, the supplier produces a product with a stochastic production rate. Also, the retailer faces a stochastic demand and accumulates the orders similar to the deterministic model. The supplier and the retailer cooperate in order to find an optimal production and shipment policy. To the best of our knowledge, there is no similar study with this configuration in the literature.

The remainder of the thesis is organized as follows. In Chapter 2, the literature on production and demand coordination, vendor-managed inventory system and consolidation shipment is reviewed. In Chapter 3, deterministic inventory models with shipment consolidation for both VMI and non-VMI models are presented. Two different cases in which the retailer holds and does not hold inventory are considered within this chapter. Chapter 4 studies stochastic inventory models with shipment consolidation and different dispatch policies which use dynamic programming. We investigate the dispatch policies reflecting interesting characteristics. In Chapter 5, numerical studies that compare the performances of the models studied in the thesis are given. In addition to that, the effects of different parameters, such as production set up cost, customer waiting cost, arrival and processing rates, on the models are evaluated numerically. Finally, the thesis is concluded with a short summary of the performed study and main results in Chapter 6.

Chapter 2

LITERATURE REVIEW

In this chapter, we provide details and references on the advances in the areas related to the different aspects of this thesis. We provide the literature review in two parts: In the first part, we consider the literature about production and demand coordination in inventory management problems and the second part discusses consolidation shipment policies such as time and quantity based shipment.

2.1 Literature on Production and Demand Coordination

To the best of our knowledge, [13] and [14] were two of the first papers, in which an integrated single supplier-single customer inventory model is analyzed. In the model set by Goyal [13], the supplier produces a product and dispatches the product to a retailer which faces an external demand. Goyal assumes infinite rate of production and zero lead time for the supplier and the retailer. The supplier and the retailer cooperate to set an integrated inventory policy and minimize the average total cost per unit time. Goyal concludes that, although the total cost of the supply chain decreases, the individual cost of the retailer or the supplier can increase by the integrated model. Hence, Goyal determines a method to allocate the variable cost to the retailer and the supplier.

Joglekar [25] and Banerjee [2] examine the same model with a finite production rate. Joglekar specifies a model in which a batch is dispatched in a number of separate shipments. Banerjee considers a joint economic-lot-size model in which a vendor produces for a purchaser facing a constant and known demand. Vendor produces each of the buyer's orders as a separate batch with a finite production rate. Goyal [15] suggests a more general joint economic-lot-size model for the same problem which gives a solution with a lower cost. In this paper, it is illustrated that, when the order quantity of the purchaser is Q, the production quantity of the vendor should be an integer multiple of Q. Goyal assumes that the whole batch must be finished before any dispatches.

Goyal and Gupta [18] make a review of published work on buyer-vendor coordination models up to 1988. In their paper, a scheme to classify these models is presented and some future research areas are identified.

Lu [26] examines the heuristics for the same problem, again with the assumption of producing an integer number of equal shipments, and relaxes the assumption of Goyal [15] about finishing the whole batch before dispatching. Therefore, in this model, shipments are allowed during production. Goyal [16] extends this policy to a new shipment policy involving unequal shipment quantities. According to this new policy, successive shipment sizes increase by a factor equal to the ratio of the vendor's production rate and the demand rate of the buyer.

Hill [21] illustrates that, neither of the policies of Lu and Goyal [16] are optimal. Hill, sets a λ parameter which denotes the proportional increase in the size of the successive shipments in a production setting. He concludes that, optimal λ must be in the range of $[1, P/D]$, where P is the production rate and D is the constant demand rate. It is illustrated that, Goyal's policy leads to lower costs than the equal shipment policy, only when the holding cost of the supplier is close to the holding cost of the retailer.

Hill [22] sets out an algorithm to obtain a globally-optimal solution by combining the Goyal's policy and the equal shipment size policy of Lu. It is shown that, it is optimal to dispatch the first m shipments in sizes increasing by a fixed factor of k and the remaining $n-m$ shipments in equal sizes. The numerical examples illustrate that this policy performs better than Goyal's policy and the equal shipment policy for all holding cost ratios of retailer and supplier.

Goyal and Nebebe [17] consider an alternative policy to the problem that is stated in [21]. It is assumed that the batch quantity is dispatched in several shipments to the buyer while the first shipment quantity is less then the remaining equal sized shipments. According to the numerical results, this policy generates lower costs than the previous policies in the literature.

The most common assumption in the literature is that the unit inventory costs for a product increase as stock moves down the supply chain. However, Hill and Omar [23] discuss that the opposite may sometimes hold in integrated systems. Since they consider an integrated system, the difference in holding costs between the vendor and the buyer only depends on the physical storage costs rather than the financial costs. If the vendor is a small specialized manufacturer and the buyer is a large manufacturer with low-cost bulk storage facilities, than it would be expected for the inventory holding costs of the vendor to be greater than the buyer's inventory holding costs. In their paper, Hill and Omar extend the work of Hill [22] under the assumption of larger holding costs for the vendor than for the retailer, and conclude that the shipment sizes to the buyer may vary.

Bichescu and Fry [4] examine the channel power on supply chain and compare supply chain performance under a VMI contract with the centralized supply chain and the traditional retailer-managed inventory supply chain to derive the benefits of VMI. They assume that the customer demand is normally distributed and the supplier produces with a deterministic constant rate. There is a positive lead time for the orders and the supplier can outsource as required. They analyze a VMI agreement in which the retailer chooses the service level and supplier decides the reorder point. They show that the VMI leads to supply chain savings in many cases, independent of the channel power contract. However, the amount of savings can be greatly affected by the channel power relationships.

In their paper, Yao et al. [31] compare two-level supply chains consisting of a supplier and a retailer using VMI and non-VMI systems to determine the benefits of VMI. As in our deterministic model, they assume that the demand is deterministic and known by the retailer which is also known by the supplier under VMI. Contrary to our model, the supplier does not produce but order the product from its own upstream supplier. They assume that the ordering cost of the retailer is lower under the VMI. They develop an analytical model to determine how ordering and holding costs affect the benefits of the VMI and the distribution of the benefit between the retailer and the supplier. They present their results in a numerical example and conclude that when the retailer's inventory holding cost is large relative to the supplier, the total benefit is higher. However, the total benefit of VMI may be negative in some extreme cases.

Vlist et al. [30] extend the model in the paper of Yao et al [31] with the transportation costs. They calculate the minimum average cost of the supply chain for different supplier stock patterns and compare their costs. They conclude that inventory level at the buyer increases and inventory at the supplier decreases with the VMI setting, although Yao et al. claim the opposite.

For extensive surveys on the areas of coordinating the order and the production policy in the single-supplier single-buyer supply chains, see the papers by Bhatnagar [3], Thomas [29], Sharafali [27], Eric [28]. The main difference between the above models and our model is that the retailer does not hold inventory but accumulates the orders in our model and this difference leads to significant discrepancies in the optimal production and shipment policies.

2.2 Literature on Shipment Consolidation

In a shipment consolidation strategy, orders/shipments are combined together to dispatch a larger quantity on the same vehicle [19]. In literature, there are three different types of temporal consolidation policies. These are time-based dispatch policies, quantity-based dispatch policies and hybrid dispatch policies. In a time-based policy, orders are dispatched in every pre-determined time intervals. On the other hand, under a quantity-based dispatch policy, all orders are shipped when a pre-determined consolidation quantity is reached. Lastly, under a hybrid policy, all orders are consolidated until the earliest of a pre-determined shipping date, or a minimum pre-determined shipment quantity is reached [20, 11, 12, 24].

In their paper, Blumenfeld et al. [5] determine optimal shipping strategies on freight (direct shipping, shipping via a consolidation terminal, and a combination of terminal and direct shipping) networks by analyzing the trade-offs between the transportation, inventory, and production set-up costs. They provide an interface between the transportation and the production set-up costs and their effects on inventory. They develop an optimization method that simultaneously determines optimal routes and shipment sizes for the networks with a consolidation terminal and concave cost functions.

Çetinkaya and Lee [9] study a coordination problem between a third party warehouse, TPW, and its retailer. The warehouse serves a single market area where the aggregate market demand is constant and known. In their paper, TPW makes decisions regarding the order quantity and the optimal length of the replenishment cycles in the context of VMI. They examine a shipment consolidation policy where the TPW has the autonomy to consolidate orders until an economical dispatch quantity accumulates. The retailer/distributer does not hold inventory and the customers wait to receive their orders with a waiting penalty cost per unit per unit time. Since the demand is deterministic, temporal consolidation policies are equal in this case. That is, once the optimal quantity is computed, then the optimal cycle time, T, is known or vice versa. The only difference from our deterministic model, in which the retailer does not hold inventory, is that there is not a TPW but the supplier which is also the producer in our model. Hence, contrary to their model, we consider the production set up cost and the inventory holding cost of the supplier which depends on the production schedule. Çetinkaya and Lee [9] prove that the optimal shipment release timing policy is non-stationary, which means that the transportation cycle lengths are not necessarily equal in a replenishment cycle. Their results provide a basis for the case of a stochastic demand.

Çetinkaya et al. [10] consider a VMI setting in which the vendor decides the quantity and the timing of resupply to the retailer. Retailer faces a general stochastic demand process with bulk arrivals and does not carry inventory. Vendor uses an (s, S) policy for replenishment, and a quantity-based policy for transportation timings. More specifically, each demand on the retailer is sent to the vendor as an order, and shipment is carried out when an economical shipment quantity is accumulated. If the vendor does not have enough inventory when it is time to dispatch, then it can be supplied from an ample supplier. They set a renewal theoretical model, which considers the outbound transportation costs and the effects of the quantity-based consolidation policy. Their numerical results show that, cost savings can be achieved by the shipment consolidation.

Çetinkaya and Lee [8] develop a renewal theoretic model to compute the optimal replenishment quantity and dispatch frequency under a time-based shipment consolidation policy in a VMI setting. The vendor faces a sequence of random demands with identical sizes from a group of retailers in a given geographical region. Retailers does not hold inventory and agree to wait until the latest possible dispatch time. Vendor uses the (s, S) policy for replenishing its inventory and sends the accumulated orders to the retailers in every T time units. In this problem, the shipment frequency T is a decision variable and the transportation costs are important for the trade off between the scale of economies and customer waiting costs for the orders. Under the assumption of poisson demands, they obtain a closed form solution to the problem and present analytical results.

Axsater [1] considers the same problem as Çetinkaya and Lee [8]. Differently from their model, Axsater ignores the unit procurement and transportation costs since these costs are not related to the decision variables. Axsater provides an algorithm for the exact optimization of the Çetinkaya and Lee's model.

Çetinkaya and Bookbinder [6] analyze quantity and time-based policies for the private carriage and the common carriage cases. In the private carriage case, shipper owns the truck, hence the transportation costs are mostly defined by the fixed transportation cost per shipment. Shipment economies are the result of portioning out that fixed cost to the large shipment quantity. On the other hand, in the common carriage case, an outside trucking company is hired. The company applies a discount to the transportation cost per unit weight when the minimum weight to obtain the quantity discount is reached. They apply renewal theory and present numerical results.

In their paper, Çetinkaya et al. [7] make analytical comparisons between the quantity and time based policies and the hybrid policy. Using the model in [8], they build a quantity-based dispatch policy model and develop an exact optimization procedure. By comparing the models, they conclude that the quantity-based dispatch policy results in smaller expected long run average costs per unit time than both the time-based and the hybrid models. However, according to the numerical results, hybrid policies are better than the quantity-based policies in terms of service and average waiting times of the customers.

The main contribution of this thesis is the following; we combined the inventory management model studied by Lu [27], Goyal [17] and Hill [22], and the shipment consolidation policies studied by Çetinkaya and Lee [9] in one concept of which the supplier produces the product and the retailer does not hold any inventory. To the best of our knowledge, this thesis is considerably different from the existing inventory-transportation models since the two different concepts are studied in the same model. The studied concepts are the models where the supplier produces at a finite rate and there is a shipment consolidation, and also the model that the retailer does not hold any inventory in a VMI system.

Chapter 3

DETERMINISTIC MODELS FOR COORDINATED PRODUCTION AND SHIPMENT DECISIONS

In this chapter, we consider a single-supplier single-retailer supply chain system. Retailer faces an external demand and sends orders to the supplier. Retailer does not hold inventory but accumulates the orders to satisfy the customer's demand where customers are willing to wait at the expense of waiting costs. For instance, consider an expensive technology product that is not reasonable for the retailer to keep in stock. In this case, inventory holding cost for the retailer is high however waiting costs can be less for reasonable time intervals. This type of sales is common for retail stores making catalog sales.

In our study, the supplier produces and holds inventory. She satisfies the orders from the retailer according to the selected dispatch policies. When the supplier decides on the timing and the quantity of the orders, one cost saving can be realized through shipment consolidation, in which the orders are not sent immediately but they are accumulated for a while in order to satisfy the scale of economies. There are two basic considerations of shipment dispatch scheduling, which are: i) when to dispatch a vehicle in order to meet the service requirements and ii) in what quantity to dispatch so that scale of economies are satisfied.

First we examine the case in which the supplier decides on the quantity and the timing of the shipment dispatches to the retailer under the deterministic demand and production assumption. This model minimizes the total cost of an integrated inventory/shipment consolidation policy for the supplier and the retailer. In the second model, we consider a non-VMI model, in which the retailer manages its own inventory and the inventory/shipment costs are assumed by the retailer directly.

3.1 Deterministic VMI Model with Shipment Consolidation

In this model, we introduce the problem of production and transportation policy of a supplier in the case of a single supplier and a single retailer for a specific product in a vendor managed inventory model. The production and the demand rate of the product is assumed to be constant and known, denoted by μ and λ respectively.

Supplier produces the product and carries inventory to satisfy demand orders from the retailer. Retailer faces an external demand from customers and receives the product from the supplier. Retailer does not hold inventory and customers are willing to wait for their orders to be satisfied for a reasonable waiting time. Waiting cost per unit time, denoted by w, is taken as a penalty associated with delayed shipment. We assume that the transportation time between the supplier and the retailer is negligible.

As we consider the case of using VMI, the supplier has full knowledge on demand and is responsible for managing the inventory and determining the transportation amount Q, to the retailer. According to our policy, supplier produces until an optimal amount of inventory level is reached. During the production, the supplier continues to dispatch orders to the retailer. The time length between the two successive starting production decisions is called as a replenishment cycle, and the length of the time between two successive dispatches is denoted as a transportation cycle. As it can be observed in Figure 3.1, T is the length of an inventory replenishment cycle and there is a fixed number of transportation cycles within T. The length of the replenishment cycle is important because of the production set up cost, which is paid by the supplier whenever a production process starts. K_p is the production set up cost for the supplier and K_t is the fixed cost of transportation of the product per shipment. The supplier's cost of carrying one unit of product per unit time is denoted by h, and customer waiting costs on retailer per unit per unit time is denoted by w. We assume that the holding cost of the product is less than or equal to the waiting cost per unit per unit time, i.e., $h \leq w$.

The inventory transportation cycles within a replenishment cycle are illustrated in Figure 3.1. We assume that the supplier produces until a predetermined inventory level, $Qmax$, and then stops producing until the end of the replenishment cycle. The retailer accumulates the demand within a transportation cycle and satisfies all the demand at the end of the transportation cycle. At the end of each transportation cycle, the supplier dispatches an amount equal to the demand accumulated at the retailer since the previous dispatch decision.

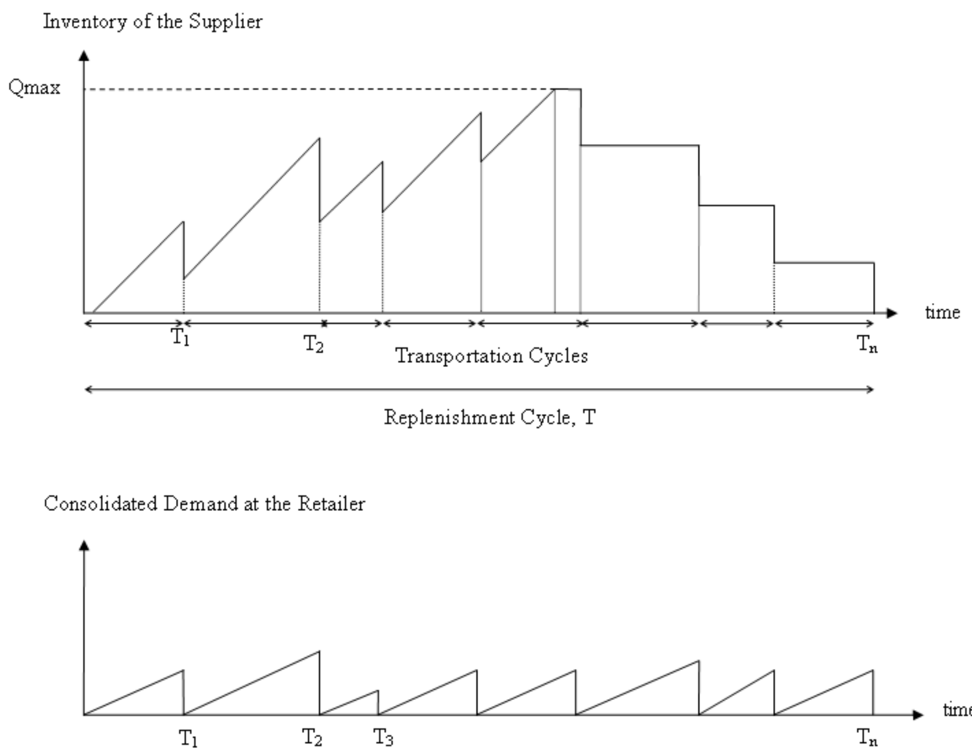

Figure 3.1: Deterministic VMI Model.

Lemma 1 *In the first cycle, the supplier produces an amount that is exactly equal to the demand accumulated in that cycle.*

Proof:

Consider a solution S, in which the supplier produces more than the demand accumulated at the retailer within the first transportation cycle and then continues the production until the inventory level at the supplier becomes Q. Consider another solution S' which is exactly equal

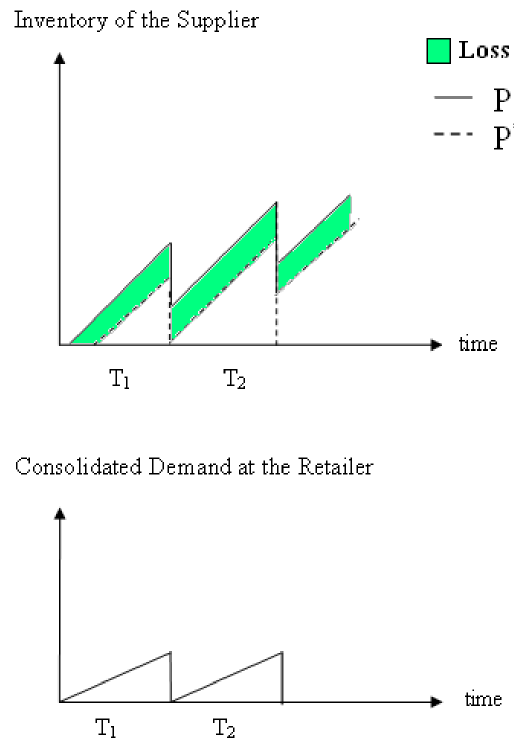

Figure 3.2: Deterministic VMI Model Lemma 1.

to S except that we start the production at a later time such that the supplier produces an amount that is exactly equal to the demand accumulated in the first cycle and then continues the production until the inventory level at the supplier becomes Q. We observe from Figure 3.2 that the supplier needs to pay a larger inventory holding cost with S compared to S' because of the excess inventory that the supplier needs to carry while everything else remains the same. Thus, in the optimal solution, the supplier must start the production at time t within T_1, such that she produces an amount that is exactly equal to the demand accumulated in the first cycle.

∎

Lemma 2 *Under an optimal policy, if a replenishment cycle consists of more than two transportation cycles, then all of the transportation cycles except the first one will be of equal length.*

Proof: Let P be a policy such that there are two successive transportation cycles, T_i and T_j, neither of which is the first transportation cycle, such that $T_i > T_j$. Also, suppose that $P^{'}$ is an identical policy to P with the exception that the lengths of these two successive transportation cycles are $T_i^{'} = T_i - \epsilon$ and $T_j^{'} = T_j + \epsilon$ where epsilon is a small constant. Hence, the total length of the two cycles are of equal length in these two policies. We consider the inventory and demand vs. time graphs, and calculate the cost differences between the policies P and $P^{'}$. The difference in the inventory holding cost which is shown as the hatched areas in the corresponding figures between the policies P and $P^{'}$ are denoted as either loss or gain. The additional area caused by $P^{'}$ compared to P is called as gain and the opposite is called as loss. In order to prove that the policy $P^{'}$ is optimal, we have to show that the net area, (net area=loss-gain), which is the difference between the loss and the gain must be positive. For this purpose, we analyze 4 cases depending on the location of the successive cycles T_i and T_j. In Case 1, we assume that both of these cycles are during the period when the supplier is making the production. In Case 2, we analyze the case when T_i is during the production and the production stops during T_j. In Case 3, we consider the case that the production stops during T_i and no production is done during T_j and finally in Case 4, we analyze the case when both T_i and T_j are in a no production period.

Case 1: In this case, we analyze the case when the two successive transportation cycles are during the production phase as seen in Figure 3.3.

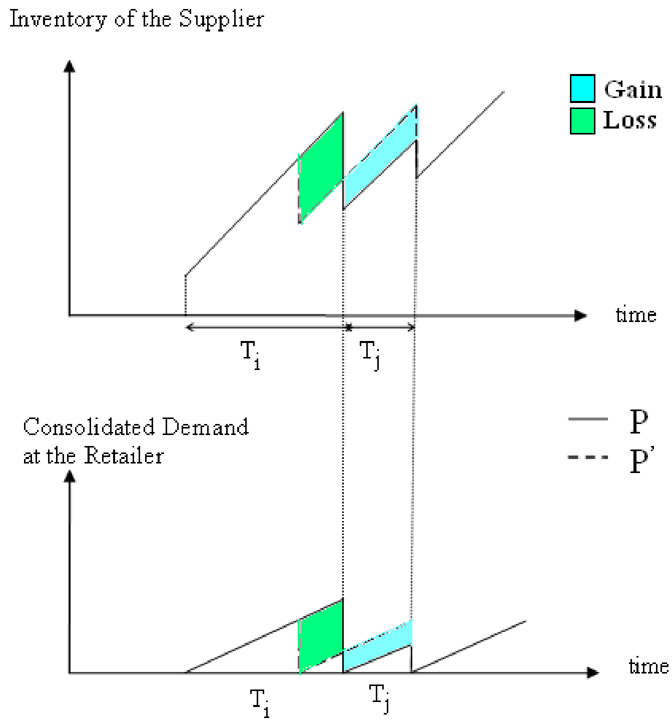

Figure 3.3: Deterministic VMI Model Lemma 2 Case 1.

The cost decrease and increase caused by $P^{'}$ will be:

The decrease in cost caused by $P^{'}$: $(T_i - \epsilon)\lambda\epsilon h + (T_i - \epsilon)\lambda\epsilon w = (T_i - \epsilon)\lambda\epsilon(h + w)$.

The increase in cost caused by $P^{'}$: $\lambda\epsilon T_j h + \lambda\epsilon T_j h = \lambda\epsilon T_j(h + w)$.

In order to prove that the policy of $P^{'}$ is better than the P, we have to show that the net cost difference (net area=loss-gain) is positive.

Net difference in the cost: $\lambda\epsilon(h + w)(T_i - \epsilon - T_j) > 0$.

Case 2: There are two successive transportation cycles in which there is a production process during the transportation cycle and in the second one supplier stops production after reaching $Qmax$. Case 2 is illustrated in Figure 3.4.

The cost decrease and increase caused by $P^{'}$ will be:

The decrease in cost by $P^{'}$: $(T_i - \epsilon)\lambda\epsilon h + (T_i - \epsilon)\lambda\epsilon w = (T_i - \epsilon)\lambda\epsilon(h + w)$.

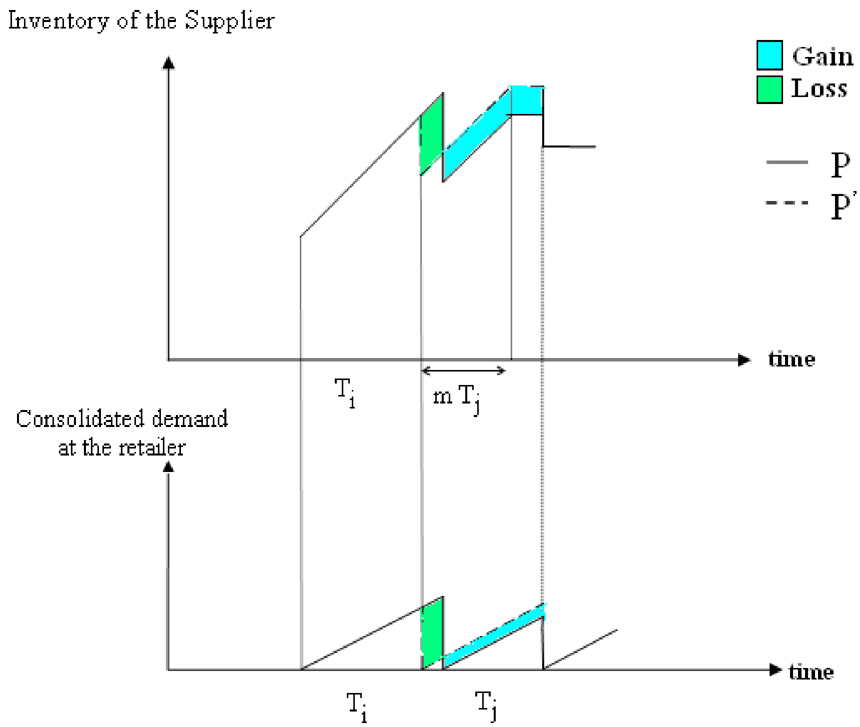

Figure 3.4: Deterministic VMI Model Lemma 2 Case 2.

Excess inventory holding cost by $P^{'}$: $\lambda\epsilon T_j h + \lambda\epsilon T_j h = \lambda\epsilon T_j(h + w)$.

In order to prove that the policy of $P^{'}$ is better than the P, we have to show that the net cost difference (net area=loss-gain) is positive.

Net cost difference by $P^{'}$: $\lambda\epsilon(h + w)(T_i - \epsilon - T_j) > 0$.

Which is the same result with case 1.

Case 3: There are two successive transportation cycles in which the supplier stops producing after reaching $Qmax$ and in the second one there is no production process as can be observed in Figure 3.5.

The cost decrease and increase caused by $P^{'}$ will be:

Loss: $(T_i - \epsilon)\lambda\epsilon h + (T_i - \epsilon)\lambda\epsilon w = (T_i - \epsilon)\lambda\epsilon(h + w)$.

Gain: $\lambda\epsilon T_j h + \lambda\epsilon T_j h = \lambda\epsilon T_j(h + w)$.

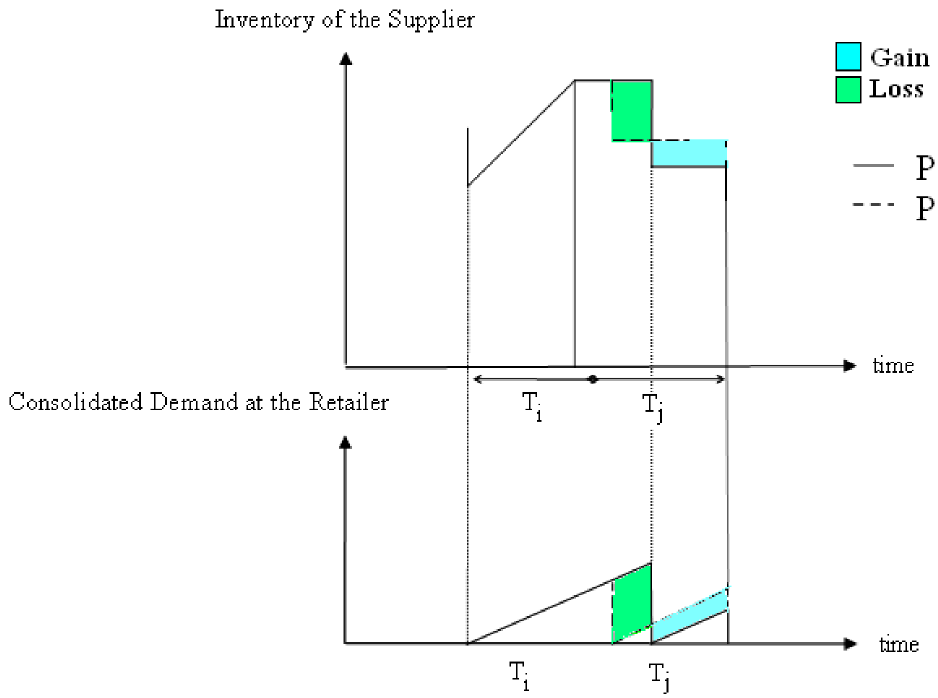

Figure 3.5: Deterministic VMI Model Lemma 2 Case 3.

In order to prove that the policy of P' is better than the P, we have to show that the net area (net area=loss-gain) is positive.

Net: $\lambda\epsilon(h+w)(T_i-\epsilon-T_j)>0$.

Which is the same result with case 1.

Case 4: There are two successive transportation cycles in which the supplier does not produce as it is illustrated in Figure 3.6.

The cost decrease and increase caused by P' will be:

Loss: $(T_i-\epsilon)\lambda\epsilon h+(T_i-\epsilon)\lambda\epsilon w=(T_i-\epsilon)\lambda\epsilon(h+w)$.

Gain: $\lambda\epsilon T_j h+\lambda\epsilon T_j h=\lambda\epsilon T_j(h+w)$.

In order to prove that the policy of P' is better than the P, we have to show that the net area (net area=loss-gain) is positive.

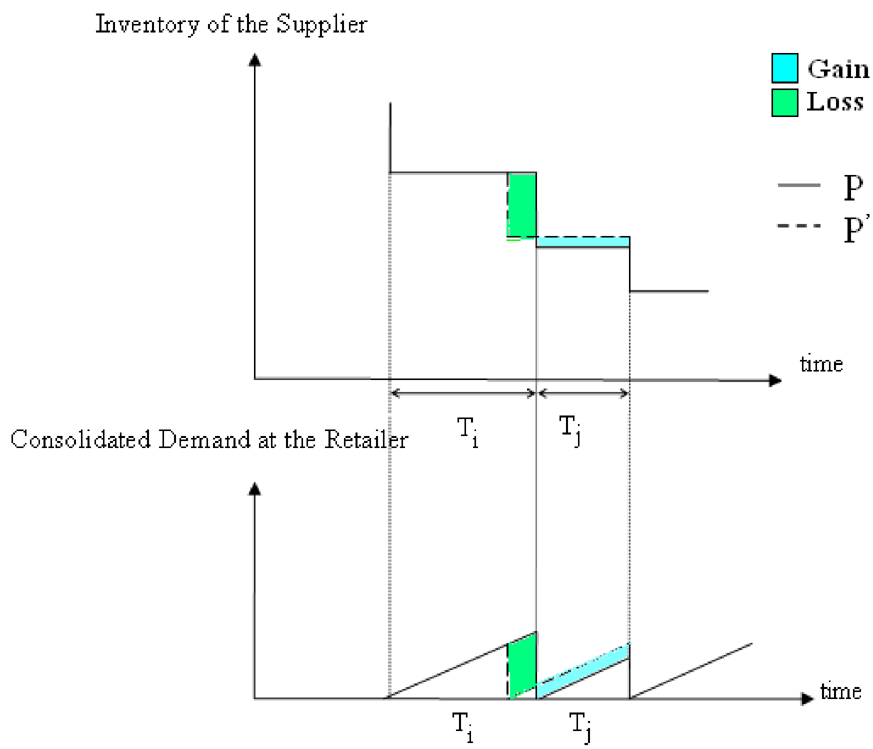

Figure 3.6: Deterministic VMI Model Lemma 2 Case 4.

Net: $\lambda\epsilon(h + w)(T_i - \epsilon - T_j) > 0$.

Since the net area is positive when $0 < \epsilon < T_i - T_j$, we can decrease the costs by decreasing T_i and increasing T_j proving that a solution in which $T_i > T_j$ can not be optimal. In the similar manner, we can also show that a solution in which $T_i < T_j$ can not be optimal either. Thus, in the optimal solution, two successive transportation cycles, except the first one, should be of equal length. ∎

Lemma 3 *Under the optimal policy, if there is more than one transportation cycle within a replenishment cycle, then the first transportation cycle is at least as long as the other cycles.*

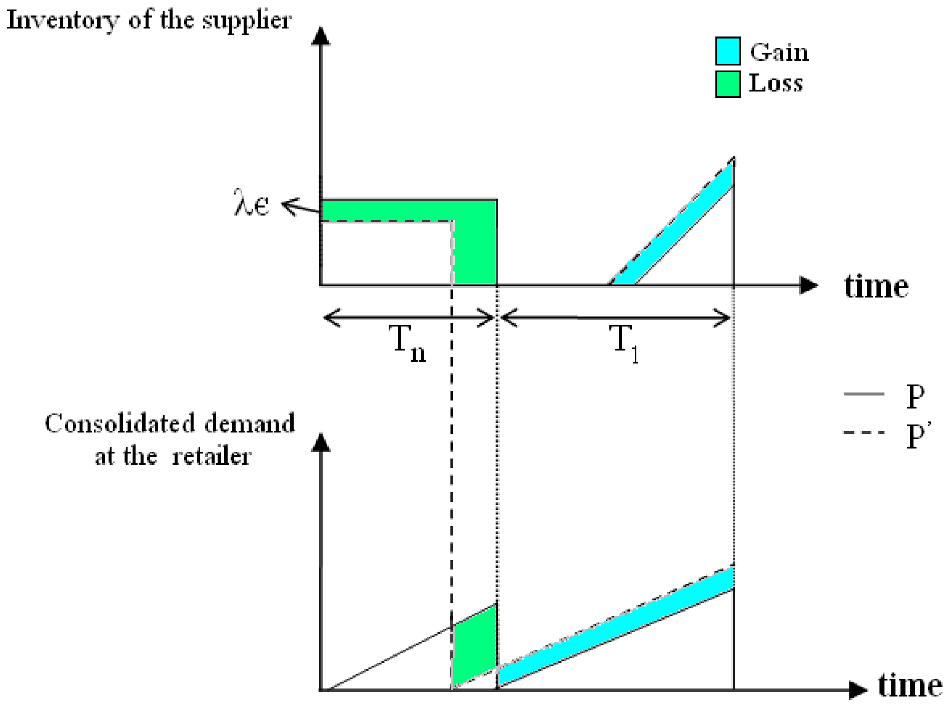

Figure 3.7: Deterministic VMI Model Lemma 3.

Proof: Let P be a policy such that the first transportation cycle, T_1, is shorter than its predecessor, T_n, such that $T_1 < T_n$. Also, suppose that $P^{'}$ is an identical policy to P with the exception that the first transportation cycle $T_1^{'} = T_1 + \epsilon$ and $T_n^{'} = T_n - \epsilon$ where epsilon is a small constant. Hence, the total length of the two cycles are of equal length in two policies. We consider the inventory and demand vs. time graphs, and calculate the cost differences between the policies P and $P^{'}$.

Note that when T_n becomes shorter by ϵ, there will be less orders accumulated at the retailer during the previous replenishment cycle. Hence, with $P^{'}$, the supplier carries less inventory in order to satisfy the demand at the retailer during the previous replenishment cycle compared to the policy P. Let $L > 0$ denote the difference in inventory holding costs between P and $P^{'}$, which is coming from the previous cycle.

Then, we can write the decrease in the inventory holding cost by $P^{'}$ as:

$$T_n\lambda\epsilon h + (T_n - \epsilon)\lambda\epsilon h + \lambda(T_n - \epsilon)\epsilon w + L = \lambda\epsilon T_n h + (T_n - \epsilon)\lambda\epsilon(h + w) + L$$

The increase in the inventory holding cost by $P^{'}$ will be:

$$\left[\frac{\lambda^2(T_1 + \epsilon)^2}{2\mu} - \frac{\lambda^2 T_1^2}{2\mu}\right]h + \lambda\epsilon T_1 w$$

Net cost difference between P and $P^{'}$ will be:

$$T_n\lambda\epsilon h + (h + w)(\epsilon\lambda T_n - \epsilon^2\lambda) - \lambda\epsilon T_1 w - \frac{\lambda^2 h\epsilon^2}{2\mu} - \frac{\lambda^2 h T_1\epsilon}{\mu} + L > 0$$

Thus, we can decrease the cost by increasing T_1 by ϵ when $T_1 < T_n$. Hence, using lemma 2, the first transportation cycle, denoted as T_1 in Figure 3.7 should be at least as long as the other cycles in the optimal solution. ∎

The optimal production and transportation schedule is illustrated in Figure 3.8. Using the Lemmas 1, 2 and 3, the supplier starts the production at a time t such that the inventory level of the supplier drops to zero at the end of the first transportation cycle. T_1 is the first transportation cycle in a replenishment cycle and length of the first transportation cycle is at least as long as the other transportation cycles. $T = T_1 + nT_c$ is the length of a replenishment cycle and the replenishment cycle consists of the first transportation cycle and n transportation cycles which have the same time length.

Observe that, in Figure 3.7, $l = \dfrac{n\lambda}{\mu}$, and lT_c is the length of the production time after the first cycle within the replenishment cycle; where $k = \left\lfloor \dfrac{n\lambda}{\mu} \right\rfloor$ is the number of T_c's within the production time period, and $d = \dfrac{l\mu - (k + \lceil m \rceil)\lambda}{\lambda} = n - (k + \lceil m \rceil)$ denotes the number of T_c's without production, where mT_c is the production time within the last transportation cycle in which there is a production process, $m = l - k$.

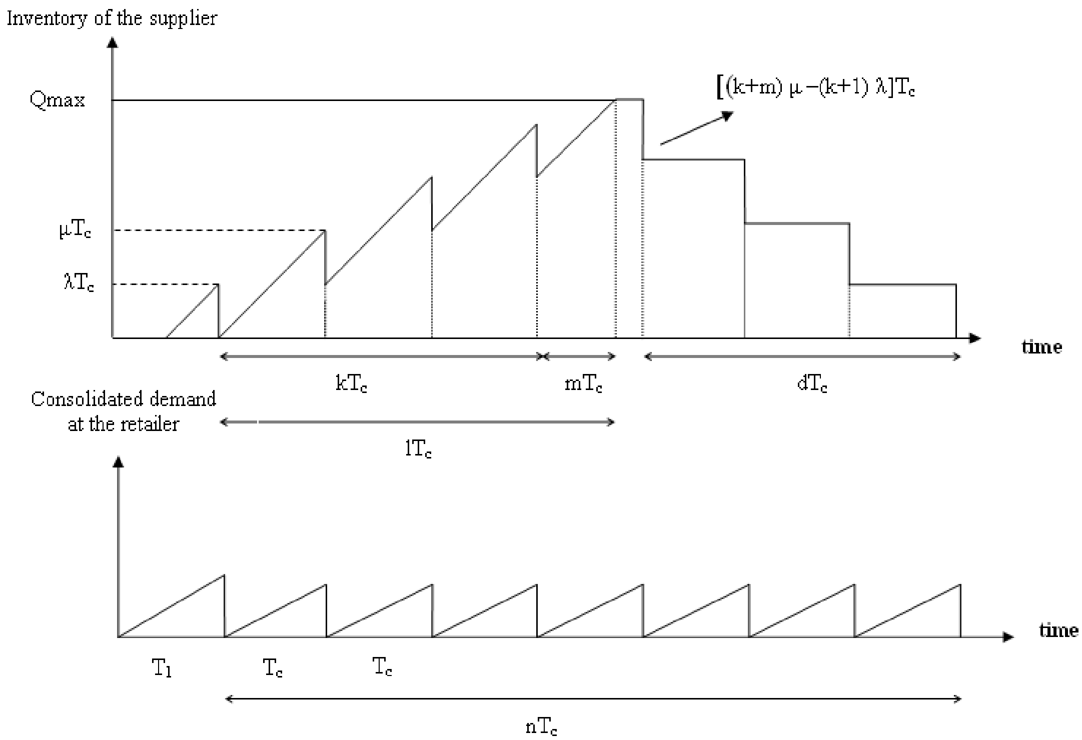

Figure 3.8: Deterministic VMI Model.

Average total cost per unit time, denoted by $G(n, T_1, T_c)$, is given by

$$G(n, T_1, T_c) = \frac{cost\ of\ an\ inventory\ replenishment\ cycle}{length\ of\ an\ inventory\ replenishment\ cycle},$$

$$\begin{aligned} G(n, T_1, T_c) &= \frac{K_p}{T} + \frac{(n+1)K_t}{T} + \frac{\lambda^2 T_1^2 + A\mu T_c^2}{2\mu T} h + \frac{\lambda(T_1^2 + nT_c^2)}{2T} w \\ &= \frac{K_p}{T_1 + nT_c} + \frac{(n+1)K_t}{T_1 + nT_c} + \frac{\lambda^2 T_1^2 + A\mu T_c^2}{2\mu(T_1 + nT_c)} h + \frac{\lambda(T_1^2 + nT_c^2)}{2(T_1 + nT_c)} w, \end{aligned}$$

where

$$A = k^2\mu - k(k-1)\lambda + [(k+l)\mu - 2k\lambda]m + 2[l\mu - k\lambda](1-m) + 2dl\mu - [2dk + d(d+1)]\lambda.$$

Note that the first term in the average total cost function is the supplier's production cost and the second term is the retailer's transportation cost. The third term is the inventory holding cost per unit time for the supplier and the last term is the customer waiting cost for the retailer.

In all the problems studied in this thesis, the objective is to minimize the total cost. Hence, our problem can be presented as

min $G(n, T_1, T_c)$

s.t. $T_1 \geq 0$, $T_c \geq 0$ and n positive integer.

Lemma 4 *For a given n, G is jointly convex in T_1 and T_c.*

Proof Let H be the Hessian matrix of G with two variables of T_1 and T_c. The determinant of the first element of H, H_{11}, and H itself will be:

$$|H_{11}| = \frac{2\mu[K_p + (n+1)K_t] + T_c^2 h(\lambda^2 n^2 + A\mu) + \mu\lambda T_c^2 wn(n+1)}{\mu(T_1 + nT_c)^3}$$

$$|H| = \frac{2[K_p + (n+1)K_t][\lambda^2 n^2 h + \mu\lambda wn(n+1) + A\mu h]}{\mu(T_1 + nT_c)^4}$$

Since $|H_{11}| \geq 0$ and $|H| \geq 0$, H is positive semidefinite. Thus G is jointly convex in T_1 and T_c.

Setting the first derivative of $G(n, T_1, T_c)$ equal to zero and solving for T_1, we obtain an expression of T_1 as a function of T_c and n:

$$T_1 = (\lambda^2 h + \mu\lambda w)\sqrt{2\mu[K_p + (n+1)K_t] + \mu T_c^2(AT_1 + \lambda n\mu) + (\lambda^2 T_1 + \mu\lambda w)n^2 T_c^2} - nT_c.$$

By substituting this expression of T_1 into the cost function and setting the first derivative of $G(n, T_c)$ equal to zero and solving for T_c, we obtain T_c as a function of n,

$$T_c = \sqrt{\frac{2n^2\lambda(\lambda h + \mu w)[K_p + (n+1)K_t]}{(Ah + \lambda nw)[\lambda n^2(\lambda h + \mu w) + \mu(Ah + \lambda nw)]}}.$$

We obtain the cost function as a function of n and make a numerical search for the optimal n which gives the smallest cost function value. Our cost function to make numerical search is as follows:

$$G[n, T_1(n), T_c(n)] = \sqrt{\frac{2\lambda(\lambda h + \mu w)[K_p + (n+1)K_t](Ah + \lambda nw)}{\lambda n^2(\lambda h + \mu w) + \mu(Ah + \lambda nw)}}.$$

■

3.2 Deterministic non-VMI Model with Shipment Consolidation

In this model, we consider a non vendor managed inventory model such that the retailer buys the product from the supplier and the retailer decides on the timings and the quantities of the orders. The main difference from using VMI is that the order quantity of the retailer is not determined by the supplier in a non-VMI system. Since the demand is observed by the retailer, the retailer determines its own order quantity. When a retailer decides on the order quantity under deterministic conditions, he will place orders based on his economic order quantity. Once the retailer reaches its reorder point, a replenishment request is sent to the supplier, and the order quantity Q is immediately shipped to the retailer. In this model, the supplier needs to decide on his own production quantities. Observe that, for a given order quantity Q determined by the retailer, it is optimal for the supplier to produce an amount equal to nQ in a production cycle where n is an integer. Otherwise, there will be an unnecessary excess inventory left over at the end of the production cycle and it will increase the inventory costs of the supplier. In this case, the supplier should decide on the value of n to minimize his own costs, considering the value of Q which is determined by the retailer.

All the parameters used in this model are the same as the VMI model in the previous section. Then the cost function of the retailer is as follows:

$$C_{ret} = \frac{2\lambda K_t + Q^2 w}{2Q}.$$

Setting the first derivative of retailer's cost function equal to zero and solving for Q, we obtain the optimum order quantity Q for the retailer:

$$Q_{ret} = \sqrt{\frac{2K_t\lambda}{w}} \qquad and \qquad T_c = \frac{Q_{ret}}{\lambda}.$$

According to Q_{ret} and T_c, determined by the retailer, the supplier determines the number of transportation cycles in a production cycle, which is denoted by n. Then the average total cost of the supplier is calculated as follows:

$$C_{sup} = \frac{K_p}{nT_c} + \frac{AT_c}{2n}h,$$

where

$$\begin{aligned} A &= \frac{\lambda^2}{\mu} + k^2\mu - k(k-1)\lambda + [(2k+m)\mu - 2k\lambda]m + 2[(k+m)\mu - k\lambda](1-m) \\ &+ 2d(k+m)\mu - [2dk + d(d+1)]\lambda \\ &= k^2(\mu - \lambda) - m^2\mu - k\lambda + 2\mu l, \end{aligned}$$

and all the other parameters are the same as the previous section.

Then, the total cost for the supply chain without VMI is:

$$C_{total} = \frac{K_p + nK_t}{nT_c} + \frac{(Ah + n\lambda w)T_c}{2n}.$$

3.3 Deterministic Models When The Retailer Holds Inventory

In the previous sections, we assume that the retailer does not carry any inventory. However, in this section the retailer holds inventory and the external demand of the customer is immediately satisfied. We assume that the supplier's cost of carrying one unit of a product per unit-time, h_1, is less than the retailer's cost of carrying one unit of a product per unit-time, h_2. As a result, both the supplier and the retailer would like to take advantage of cost saving opportunities at the supplier's location. All the other parameters are the same with the models studied in the previous chapter, except that h is replaced by h_1 and, w is replaced by h_2.

In this section, production and demand rates are assumed to be constant and known. Supplier produces the product and both the retailer and the supplier carry inventory to satisfy the demand. Retailer satisfies the external demand from its inventory which is replenished by the supplier when inventory of the retailer drops to zero.

3.3.1 Deterministic VMI Model When The Retailer Holds Inventory

Goyal [16] considers an integrated single supplier (vendor) single retailer (buyer) problem, in which the supplier manufactures at a finite rate. Demand of the retailer is assumed to be deterministic and the supplier dispatches shipments to the retailer per increasing quantities by a factor which equals to the production rate divided by the demand rate. In his study Goyal uses the same numerical examples as Lu [26], who considers the same problem based on the integral number of equal shipments assumption. Numerical results of the Goyal gives better solutions than the solutions of the study of Lu.

Hill [22] considers the same problem and concludes that the optimal shipment policy involves a predetermined number, m, of shipments in a production cycle which increases by a fixed factor and followed by fixed (number of transportation cycles-m) number of equal sized shipments. The optimal sequence of shipments can therefore be written in the form $(q, kq, k^2q, ..., k^{m-1}q, q^c, ..., q^c)$ where $q^c \geq k^{m-1}q$. Figure 3.9 shows the graph of the supplier, retailer and the system inventory levels for this case. As the goal is to minimize the stock held by the retailer, the deliveries are made when the retailer is about to be out of stock. This is done to maintain the buffer stock x which is the stock that is needed by the retailer to satisfy the demand during the suppliers manufacture of the first shipment. Hence the total stock in the system is held at the bare minimum level.

Since this model was researched by Goyal, Lu and Hill [16, 26, 22], we are not going to discuss the model in detail.

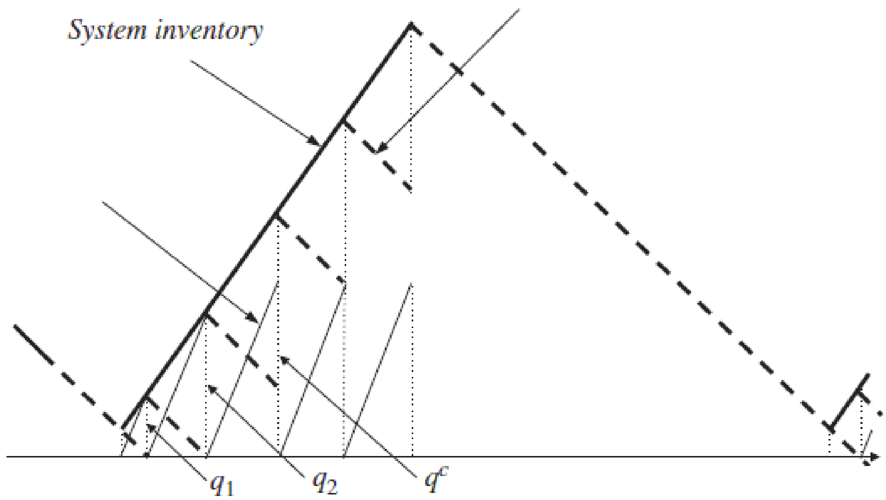

Figure 3.9: VMI Model with Deterministic Production and Demand.

3.3.2 Deterministic non-VMI Model When The Retailer Holds Inventory

In the previous VMI model, demand is assumed to be deterministic and known by both the retailer and the supplier. On the contrary, in a supply chain without VMI, the supplier observes demand only indirectly through the retailer's ordering policy.

In this section, since the demand is known by the retailer, we assume that he determines his order quantity and so the time length between the transportation cycles, T_c. Once the retailer's inventory level drops to 0, a replenishment request is sent to the supplier, and the order quantity is immediately shipped to the retailer. Supplier determines its own production process and the length of the replenishment cycles. The main difference between not using and using VMI is that the retailer's order quantity is not determined by the supplier in a non-VMI system.

In case of not using VMI, the inventory model of the retailer is the same as the economic order quantity model in inventory management. Observe that this model will be exactly the same as the model analyzed in section 3.2. The length of T_c and the average total cost of the retailer are calculated as follows:

$G_{ret}(T_c) = \frac{K_t}{T_c} + \frac{\lambda T_c h_2}{2}$, where $T_c = \sqrt{\frac{2K_t}{\lambda h_2}}$.

According to T_c which is determined by the retailer, supplier determines the number of transportation cycles in a replenishment cycle, denoted by n. Then the average total cost of the supplier is calculated as follows:

$G_{sup}(n) = \frac{K_p}{nT_c} + \frac{AT_c h_1}{2n}$,

where

$A = \frac{\lambda^2}{\mu} + \mu + (k^2 - 1)\mu - k(k-1)\lambda + [(2k+m)\mu - 2k\lambda]m + 2[(k+m)\mu - k\lambda](1-m) + 2d(k+m)\mu - [2dk + d(d+1)]\lambda$.

Then, the total cost for the supply chain without VMI is:

$$G_{noVMI} = G_{ret}(T_c) + G_{sup}(n) = \frac{nK_t + K_p}{nT_c} + \frac{n\lambda T_c h_2 + AT_c h_1}{2n} = \frac{(2nK_t + K_p)\lambda h_2 + AK_t h_1}{n\sqrt{2K_t \lambda h_2}}.$$

Chapter 4

STOCHASTIC MODELS FOR COORDINATED PRODUCTION AND SHIPMENT DECISIONS

In this chapter, we analyze the coordinated inventory and shipment models under stochastic demand and stochastic production. In the inventory system literature, the main source of the system uncertainty is assumed to be randomness of the demand. However, in some cases, the uncertainty in an inventory system can be caused by the random capacity of the supplier. Lack of raw material, equipment failures and scraps in a production run may cause the randomness in production. Additionally, classical literature assumes that demand should be satisfied as they arrive. However, for some cases, a shipment consolidation policy can be implemented at the expense of the inventory holding costs and the customer waiting costs. The main motivation of the supplier and the retailer for a shipment consolidation is to dispatch larger quantities satisfying the economies of scale in transportation. Shipment consolidation, that is implemented on its own without coordination with the inventory decisions is denoted as a pure consolidation policy. However, if the shipment consolidation is coordinated with inventory decisions, than this approach is called as integrated inventory shipment consolidation policy. There are three different shipment consolidation policies in the logistics literature. These are quantity-based, time-based and hybrid policies. Under a quantity-based policy, shipments are dispatched when a pre-determined consolidation quantity, Q, is reached. However, under a time-based policy, shipments are dispatched in every pre-determined T periods, hence the dispatch quantity is a random variable, and the transportation scale of economies may not be satisfied for some instances. Hybrid policy is also called as time-and-quantity based policy. Under this policy, a dispatch decision is at made at either when the size of a consolidated load exceeds Q (predetermined dispatch quantity), or when the time from the last dispatch exceeds T (predetermined dispatch time), $min\{T(Q), Q\}$.

In the deterministic demand model explained in the previous chapter, these shipment consolidation policies are equivalent. That is, once the optimal order quantity is obtained than the optimal transportation cycle length is known, and vice versa. However, for the case of stochastic demand, the type of the shipment consolidation has an impact on the cost saving of the vendor managed inventory system. In this chapter, we examine the cases in which the supplier decides on the quantity and timing of the shipment dispatches according to the different shipment policies by receiving demand information at the retailer under a VMI model setting. We use Markov Decision Processes (MDP) to model this problem. First, we present a general model using dynamic programming to obtain the optimal dispatching and replenishment policies under general conditions. Then, we also examine the time-based and quantity-based dispatch policies and provide models using dynamic programming approach in order to compare their cost savings.

We use the same notation as in the deterministic model in the previous chapter. Because of the stochastic rates of the demand and the production, the supplier may not satisfy all demand orders from its own inventory. We assume that there is an ample supplier where the supplier can buy with a cost of s per unit per unit time. We assume that the retailer is facing a random demand which follows a poisson process with an arrival rate of λ and the production times are assumed to have an exponential distribution with mean $1/\mu$. Cost parameters taken into account are as follows:

K_p : fixed cost of production set up

K_t : fixed cost of dispatching shipment from the supplier to the retailer

h : inventory holding cost per unit per unit-time

w : customer waiting cost per unit per unit-time

s : supplier's unit purchase price from an ample supplier

4.1 VMI Model with Stochastic Production and Demand

In this model, the supplier manages a stochastic inventory system for a product that is undesirable for the retailer to keep in stock. Retailer does not satisfy the demand from its inventory but accumulates the orders. The supplier satisfies the orders from its own inventory or by buying from an ample supplier with a cost of s. The production rate of the supplier is μ, where the production amount is a random variable distributed with Poisson distribution. The demand also has a Poisson distribution with rate λ. The inventory level of the supplier is denoted by I_1 and the quantity of the demand orders at the retailer, which are waiting to be satisfied, is denoted by I_2. I_1 and I_2 are bounded by the capacity of the supplier and the retailer.

We use a dynamic programming approach and analyze an infinite-horizon model in order to obtain optimal production and dispatch policies. Every time a product is produced by the supplier or a demand arrives for the retailer, supplier must take decisions such as dispatching and stock replenishing. Decisions of the supplier on start/continue production or dispatching the product depend on the current inventory of the supplier and number of the orders accumulated at the retailer. Whenever the supplier decides to dispatch, she needs to determine the dispatch quantity Q, which is $0 \leq Q \leq I_2$, where I_2 is the total unsatisfied demand accumulated at the retailer. After each decision and the action, the inventory on hand and the accumulated unsatisfied order quantity are recorded. The state of the system can be defined as (j, I_1, I_2), where j represents the state of the production and I_1 and I_2 denote the inventory level of the supplier, and the accumulated demand at the retailer, respectively. Let S be the state space of the system and be defined as follows: $S = \{(j, I_1, I_2) : (I_1, I_2) \geq 0, j \in \{0, 1\}\}$ where $j = 0$ means there is no production at the supplier and $j = 1$ denotes that the production process is ongoing at the supplier at that time. This model can be formulated as a continuous time Markov decision process with the objective of minimizing expected total discounted cost over an infinite time horizon, with a discount rate β. We use uniformization and normalization to analyze the system. We rescale the parameters as $\lambda' + \mu' + \beta' = 1$, so that the system will be observed at exponentially distributed intervals with mean 1. In the remainder of the thesis, the tildes will be omitted to simplify the appearance. Let $V(j, I_1, I_2)$ be the expected minimum total cost when the production is in state j, inventory level of the supplier is I_1 and there are I_2 customers waiting at the retailer.

The mathematical model for state $\{0, I_1, I_2\}$ is as follows:

$$\begin{aligned} V(0, I_1, I_2) &= \min_{0 \leq Q < I_2+1} \{\lambda V(1, max(0, I_1 - Q), I_2 + 1 - Q) \\ &+ \mu V(1, max(1, I_1 + 1 - Q), I_2 - Q) + s(Q - I_1)^+ + K_p + I_{Q>0} K_t, \\ &\quad \lambda V(0, max(0, I_1 - Q), I_2 + 1 - Q) + \mu V(0, I_1, I_2) + s(Q - I_1)^+ + I_{Q>0} K_t\} \\ &+ I_1 h + I_2 w. \end{aligned} \tag{4.1}$$

In the equation 4.1, $V(0, I_1, I_2)$ denotes that there is no production process at the supplier, supplier's inventory level equals to I_1, and there are I_2 customers at the retailer. After each production or on a new demand arrival, the supplier decides on the dispatch quantity minimizing the total cost, which is $0 \leq Q \leq I_2$. If the supplier decides to dispatch more products than there are in her inventory, then she buys from an ample supplier with a cost of s per unit per unit time, $s(Q - I_1)^+$. Note that, s can be chosen high enough if outsourcing is not allowed. Whenever a shipment is dispatched, $Q > 0$, the supplier must pay the transportation cost K_t, leading to the term $I_{Q>0} K_t$.

Supplier may decide to start the production and whenever supplier starts producing, the state of the production changes from $j = 0$ to $j = 1$. After the decision of starting production, the inventory level of the supplier increases by 1 with the probability of μ, leading to the term $\mu V(1, max(1, I_1 + 1 - Q), I_2 - Q)$, or a new demand arrival may occur with probability of λ, which leads to the term $\lambda V(1, max(0, I_1 - Q), I_2 + 1 - Q)$. In the case of new demand arrival, the number of customers waiting at the retailer, I_2, increases by 1. Each time production starts, the supplier must pay production set up cost, K_p. If the supplier decides to dispatch more than her own inventory, then she has to buy $Q - I_1$ units of the product from an ample supplier with the cost of $s(Q-I_1)^+$. For every dispatch decision, the supplier has to pay the transportation cost K_t.

On the other hand, the supplier may decide not to start production and there may be a new demand at the retailer with probability λ. Arrival of a new customer increases I_2 by 1 and it leads to the term $\lambda V(0, max(0, I_1-Q), I_2+1-Q)$.

In every period, the supplier must pay the inventory holding cost, I_1h and the retailer pays customer waiting cost I_2w. We also include the term $\mu V(0, I_1, I_2)$ to account for the dummy production in state $j=0$.

For state $\{1, I_1, I_2\}$ the mathematical model is:

$$\begin{aligned} V(1, I_1, I_2) &= \min_{0 \le Q < I_2+1} \{\lambda V(1, max(0, I_1-Q), I_2-Q+1) \\ &+ \mu V(1, max(1, I_1-Q+1), I_2-Q) + s(Q-I_1)^+ + I_{Q>0}K_t, \\ &\quad \lambda V(0, max(0, I_1-Q), I_2-Q+1) + \mu V(0, I_1, I_2) + s(Q-I_1)^+ + I_{Q>0}K_t\} \\ &+ I_1h + I_2w. \end{aligned} \tag{4.2}$$

In equation 4.2, $V(1, I_1, I_2)$ denotes that there is a production at the supplier, the inventory level of the supplier and the number of customers at the retailer are I_1 and I_2, respectively. The supplier may decide to continue the production or stop.

If the supplier continues production, the new product may be produced with probability μ before any demand arrival, or a demand arrival may occur with probability λ before the production. Similar to equation 4.1, after each production of an item or a new demand arrival, the supplier decides on the dispatch quantity minimizing the total cost, which is $0 \le Q \le I_2$, and whenever $Q > 0$, the supplier must pay the transportation cost K_t, $I_{Q>0}K_t$.

In this case, if the supplier decides to stop production, then j changes from 1 to 0. The rest of the terms in the equation 4.2 are the same as in the equation 4.1.

In this section, we develop a dynamic programming model for a single supplier single retailer inventory control and shipment planning problem, aiming to optimize the expected minimum total discounted cost. Since our objective is to understand the behavior of the optimal policies, we study a general model. However, because of the non monotonicity and the complex results depending on the states of the system, it is difficult to obtain a general policy on the shipment and inventory management. Also, in practice it is hard to implement such policies since the decisions that need to be taken depend on the states and there are no general rules, which causes the companies invest money and time while making decisions. Thus, managers would prefer more applicable policies in order to adapt the fast pace of the market. Hence, in the next section, we study the temporal shipment dispatch policies to obtain more general policies and strategies.

4.2 VMI Model with Time-Based Dispatch Policy

In a time-based dispatch policy, the accumulated load, which satisfy all outstanding demands, is dispatched in every T time lengths. Under a time-based dispatch policy, the dispatch quantity is a random variable in the stochastic demand case. Therefore, for some demand quantities dispatch load may not realize the economies of scale. However, a time-based policy assures that each demand is dispatched at a predetermined shipment date. Hence, a time-based policy is more appropriate for satisfying customer service requirements because of the pre-determined due dates of the shipments. Moreover, in practical applications, it may be easier to schedule dispatches so that a shipment is realized on a periodic basis.

In the transportation contracts between the supply chain members, time-based shipment policies are also known as time definite delivery (TDD) agreements. These kind of contracts are common between the manufacturers and their third party logistics service provider partners. According to the TDD agreements, the third party logistics provider supplies warehousing and transportation for a manufacturer and guarantees the timely delivery to the customers of the manufacturer. TDD agreements may also be adopted for the inbound dispatches to the manufacturer itself.

In this model, similar to the previous models, the retailer faces an external demand from customers and accumulates these orders to be satisfied at a later time when a shipment is received from the supplier. The retailer does not hold any inventory. The supplier produces the product and she is authorized to manage her inventory and determine the dispatch frequency to the retailer. The supplier adopts a time-based policy and ships an amount equal to the amount of the accumulated orders at the retailer during a predetermined time length T. Along with the optimal maximum inventory level, Q_{max} for the supplier, the transportation cycle length, T, is also a decision variable. As shipments are consolidated, the trade off between scale economies associated with the transportation and customer waiting costs play an important role in this model.

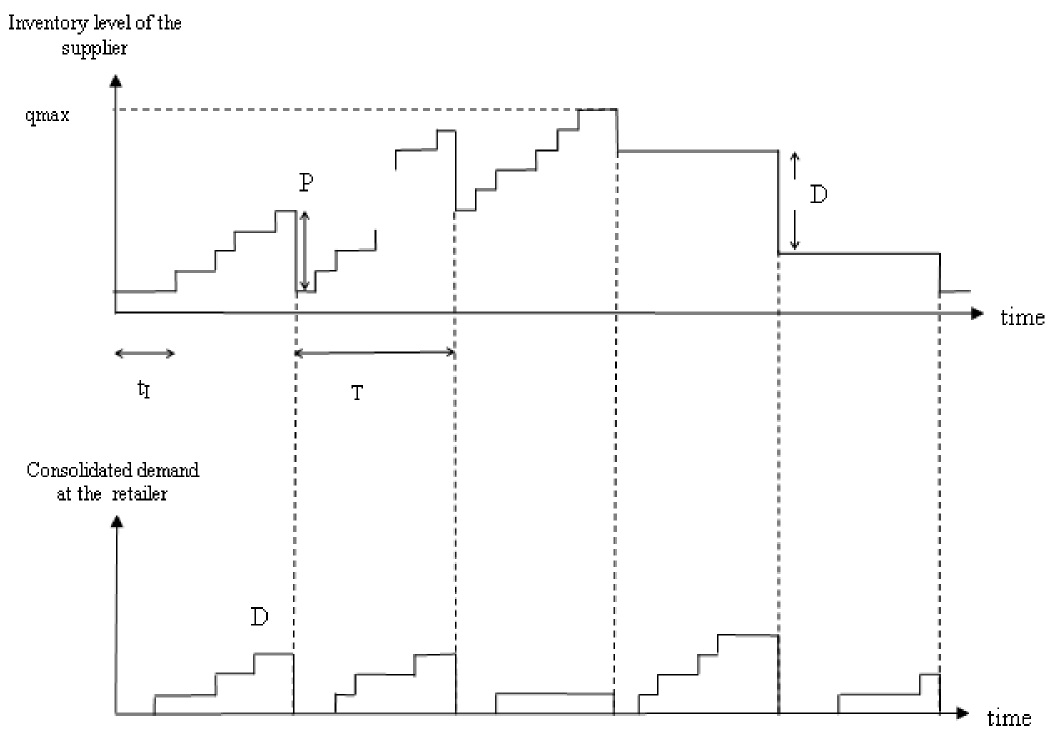

Figure 4.1: Time-Based Stochastic Production Model.

In this model, production and demand quantities are stochastic and assumed to have a poisson distribution with rate μ and λ respectively. In this model, we aim to determine the optimal values of the cycle length, T, the maximum inventory level that the supplier should set for production, Q_{max} and when to start the production if he is in a non-production state. As seen in figure 4.1, when the supplier is in a non-production state and has an inventory level of I just after making a shipment to the retailer, he waits for a certain time, t_I and then starts the production until the predetermined maximum inventory level Q_{max} is achieved. The supplier dispatches the shipments to the retailer in every T time periods. Note that if $t_I > T$, then the supplier would not produce anything in that transportation cycle. Let P the be amount of the products that the supplier produces and D be the demand accumulated at the retailer during a transportation cycle period, T. The supplier dispatches D units of the product to the retailer from its own inventory or by buying from an ample supplier at the end of each T time periods. Once the inventory level of the supplier reaches Q_{max}, then the supplier stops production for some time and whenever the supplier decides to start production again the new replenishment cycle begins.

Let P_1 be the quantity of the products produced in time length T. The probability that $P_1 = x$ is:

$$P(P_1 = x) = P_{P_1} = \frac{e^{-\mu T}(\mu T)^x}{x!}, \quad x \geq 0.$$

Let t_I be the waiting time before starting the production, whenever the inventory level on the supplier equals to I and there is no production at the supplier and P_2 denote the quantity of the products produced in time length $T - t_I$ which has the probability distribution as:

$$P(P_2 = x) = P_{P_2} = \frac{e^{-\mu(T-t_I)}[\mu(T - t_I)]^x}{x!}, \quad x \geq 0.$$

Let $V_i(j, I)$ be the total β-discounted minimal cost of the system when it is currently in state $(j; I)$ at stage i where j represents the state of the production process and I is the inventory level of the supplier. $j = 0$ denotes that there is no production currently at the supplier and,

$j = 1$ denotes ongoing producing at the supplier. Let S be the state space of the system and define it as follows: $S = \{f(j; I) : I \geq 0; j \epsilon \{0, 1\}\}$.

We look at the system at every T time units, after a shipment is made, and write a dynamic programming model for this system. For a given T and Q_{max}, the dynamic programming model for state $(1, I)$ is as follows:

$$\begin{aligned} V_i(1,I) &= E_{D,P_1}\Big\{e^{-\beta T}V_{i+1}[1_{I+P_1<Q_{max}}, max(0, min\{Q_{max}, I+P_1\} - D)] \\ &+ e^{-\beta T}s[D - (min\{Q_{max}, I+P_1\})]^+ + \frac{DT}{2}w \\ &+ \Big[IT + \frac{T}{P_1+1}\frac{(min\{Q_{max}-I, P_1+1\} - 1)(min\{Q_{max}-I, P_1+1\})}{2} \\ &+ min\{Q_{max}-I, P_{1+1}\}\Big(T - \frac{T}{P_1+1}min\{Q_{max}-I, P_1+1\}\Big)\Big]h\Big\} \\ &+ K_t. \end{aligned} \tag{4.3}$$

$$\begin{aligned} V_i(1,I) &= \sum_{P_1=0}^{Q_{max}-I-1}\Big\{\sum_{D=0}^{\infty}\Big[e^{-\beta T}V_{i+1}[1, max(0, I+P_1-D)] + e^{-\beta T}s[D-(I+P_1)]^+ + \frac{DT}{2}w\Big]p_D \\ &+ \Big(IT + \frac{P_1T}{2}\Big)h\Big\}P_{P_1} \\ &+ \sum_{P_1=Q_{max}-I}^{\infty}\Big\{\sum_{D=0}^{\infty}\Big[e^{-\beta T}V_{i+1}[0, max(0, Q_{max}-D)] + e^{-\beta T}s(D-Q_{max})^+ + \frac{DT}{2}w\Big]p_D \\ &+ \Big[IT + \frac{T}{P_1+1}\frac{(Q_{max}-I-1)(Q_{max}-I)}{2} + (Q_{max}-I)\Big(T - \frac{T}{P_1+1}(Q_{max}-I)\Big)\Big]h\Big\}P_{P_1} \\ &+ K_t. \end{aligned} \tag{4.4}$$

In the equation 4.4, $V_i(1, I)$ denotes that the production is ongoing and there are I products at the supplier. According to our assumption for this model, whenever a production starts, the supplier must produce till the inventory level reaches Q_{max}. Hence, the supplier does not have to decide on producing or not since the production has already been started. Therefore, in this equation, there is no minimization, but we sum the costs over the demand and the production probabilities.

First two lines of the equation 4.4, are the total cost of producing less than Q_{max} in the time length T with probability P_{P_1}. In the case of production quantity of P_1, the inventory level of the supplier increases by P_1 and decreases by the quantity of the demand D with probability p_D, where $0 \leq D \leq \infty$. If the accumulated demand within T is higher than the inventory level of the supplier at the end of T, then the supplier must buy the difference from an ample supplier with the cost of s per unit, leading to a cost of $s[D - (I + P_1)]^+$. Then, $\frac{DT}{2}w$ is the customer waiting cost during T depending on the demand quantity D and the probability p_D, and $\left(IT + \frac{P_1 T}{2}\right)h$ is the inventory holding cost of the supplier, which depends on the quantity of P_1 with probability distribution P_{P_1}.

The third and fourth lines of the equation 4.4 are the total cost in the case of stopping the production within the transportation cycle T. According to our assumption, the supplier produces till Q_{max}, and when this level is achieved, we stop the production. At the end of the time length of T, just after a shipment is made, the inventory level of the supplier equals to $max(0, Q_{max} - D)$.

At the end of the each transportation cycle, the supplier must pay the transportation cost of K_t.

Similarly, for given T, Q_{max} and t_I values, the dynamic programming model for state $(0, I)$ is as follows:

$$
\begin{aligned}
V_i(0,I) &= E_{D,P_1}\Big\{e^{-\beta T}V_{i+1}[1_{I+P_2<Q_{max}}, max(0, min\{Q_{max}, I+P_2\}-D)] \\
&+ e^{-\beta T}s[D-min\{Q_{max}, I+P_2\}]^+ + \frac{DT}{2}w \\
&+ \left[IT + \frac{(T-t_I)min\{Q_{max}-I, P_2+1\}min\{Q_{max}-I-1, P_2\}}{2(P_2+1)}\right] \\
&+ (1_{I+P_2<Q_{max}})(Q_{max}-I)\left(T - \frac{(T-t_I)(Q_{max}-I-1)}{P_2+1}\right)h\Big\} + K_p + K_t, \\
&\sum_{D=0}^{\infty}\left[e^{-\beta T}V_{i+1}[0, max(0, I-D)] + e^{-\beta T}s(D-I)^+ + \frac{DT}{2}w\right]p_D + ITh + K_t.
\end{aligned} \tag{4.5}
$$

$$
\begin{aligned}
V_i(0,I) &= min\Big\{\sum_{P_2=0}^{Q_{max}-I-1}\Big\{\sum_{D=0}^{\infty}\left[e^{-\beta T}V_{i+1}[1, max(0, I+P_2-D)] + e^{-\beta T}s[D-(I+P_2)]^+ + \frac{DT}{2}w\right]p_D \\
&+ \left[IT + \frac{P_2(T-t_I)}{2}\right]h\Big\}P_{P_2} \\
&+ \sum_{P_2=Q_{max}-I-1}^{inf}\Big\{\sum_{D=0}^{\infty}\left[e^{-\beta T}V_{i+1}[0, max(0, Q_{max}-D)] + e^{-\beta T}s(D-Q_{max})^+ + \frac{DT}{2}w\right]p_D \\
&+ \left[IT + \frac{T-t_I}{P_2+1}\frac{(Q_{max}-I-1)(Q_{max}-I)}{2} + (Q_{max}-I)\left(T - \frac{(T-t_I)(Q_{max}-I-1)}{P_2+1}\right)\right]h\Big\}P_{P_2} \\
&+ K_p, \\
&\sum_{D=0}^{\infty}\left[e^{-\beta T}V_{i+1}[0, max(0, I-D)] + e^{-\beta T}s(D-I)^+ + \frac{DT}{2}w\right]p_D + ITh\Big\} + K_t.
\end{aligned} \tag{4.6}
$$

In the equation 4.6, $V_i(0,I)$ denotes that there is no production at the supplier and the inventory level of the supplier is I at the beginning of stage i. The supplier has to decide whether to start production during this transportation cycle or not. In case of starting production, the supplier has to chose the production starting time, which is denoted as t_I, to minimize the total cost. P_2 is the number of units produced during the time length $T - t_I$. Whenever the production starts $j = 0$ becomes $j = 1$.

The first two lines of equation 4.6 show the cost when the supplier starts production at time t_I and the total inventory of the supplier is less than Q_{max} at the end of T. In this case, the inventory of the supplier equals to $max(0, I + P_2 - D)$ at the end of the transportation cycle depending on the demand quantity D. Similar to Equation 4.3, the supplier must buy the excess demand from an ample supplier with a cost of $s[D - (I + P_2)]^+$. Then, $\sum_{D=0}^{\infty} \left(\frac{DT}{2} w \right) p_D$ is the customer waiting cost depending on D and $\sum_{P_2=0}^{Q_{max}-I-1} \left\{ \left[IT + \frac{P_2(T - t_I)}{2} \right] h \right\} P_{P_2}$ is the inventory holding cost of the supplier depending on the quantity of P_2.

The third and fourth lines of equation 4.6 denote the cost of the supplier when he produces $Q_{max} - I$ units during $T - t_I$ time units. In this case, the inventory level of the supplier is $max(0, Q_{max} - D)$ at the end of the transportation cycle, and $j = 0$ since the inventory level reaches to Q_{max} within T and the production is stopped. Thus, the cost of the next state will be $V_{i+1}[0, max(0, Q_{max} - D)]$. Since buying from ample supplier, customer waiting and the inventory holding cost terms are similar to the terms in the previous equation, we are not going to discuss them further. When the supplier starts production, he must pay the production set up cost K_p.

The last line of the equation 4.6, is the cost when the supplier decides not to produce during stage i. Since there is no production, the inventory of the supplier decreases by the demand quantity D at the end of the transportation cycle, $V_{i+1}[0, max(0, I - D)]$. If the demand is more than the inventory of the supplier, then the supplier must buy from an ample supplier with a cost of $s(D - I)^+$. In both cases the transportation cost must be paid in every T time intervals. ITh is the inventory holding cost of the supplier during the time length T. The supplier has to choose between starting production or not according to their costs. Hence, there is a minimization problem between the sum of the first five lines and the last line of the equation 4.6. In both cases, the supplier must pay the transportation cost K_t at the end of T.

4.3 VMI Model with Quantity-Based Dispatch Policy

A quantity-based dispatch policy makes a shipment whenever the outstanding customer orders are accumulated to a predetermined quantity, Q. Under the quantity-based policy, the dispatch quantity assures transportation scale of economies, but a specific dispatch time cannot be guaranteed. In the literature, it is presented that the quantity-based policy is always superior to the time-based dispatch policy in terms of the resulting average supply chain costs. On the other hand, because the quantity-based shipment requires continuous updating of the information on customer orders, it may incur higher auditing cost than the time-based shipment, where the system is periodically reviewed. Moreover, it is not practical to make dispatches whenever the dispatch quantity Q is realized, compared to the periodical shipment of the time based policy.

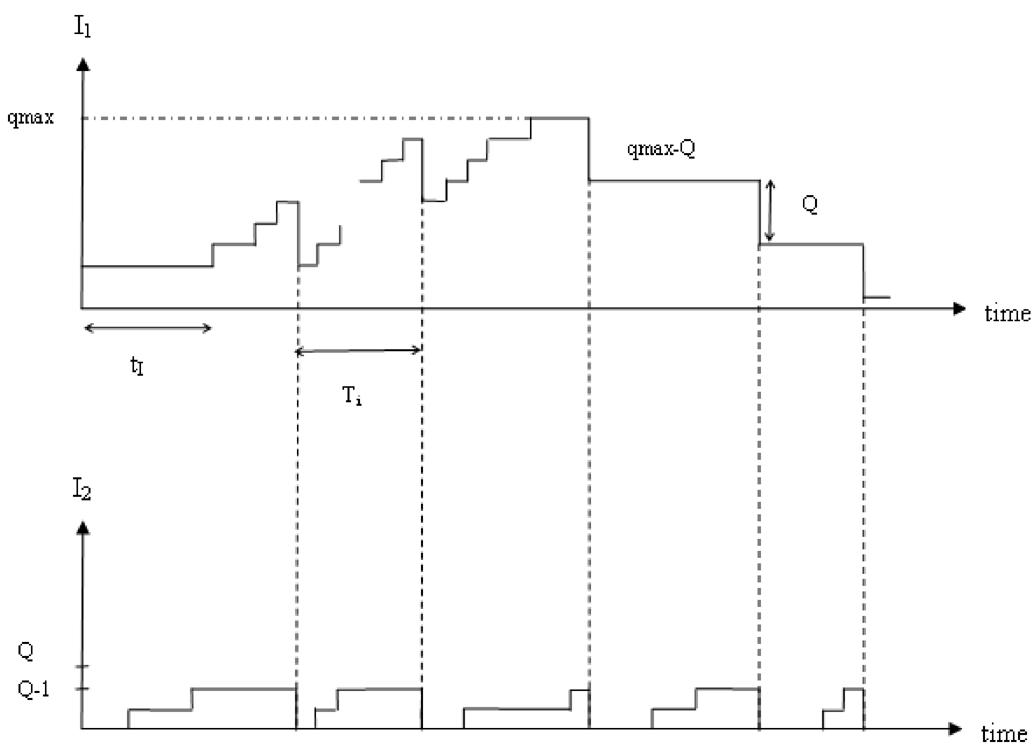

Figure 4.2: Quantity-Based Stochastic Production Model.

In this model, production of the supplier and the demand are stochastic and have a Poisson distribution with rate μ and λ respectively. The supplier must dispatch a quantity of Q to the retailer whenever there are Q orders at the retailer. Let t_I denote the waiting time before starting the production, whenever the inventory level on the supplier equals to I just after a shipment is made, and the supplier is not currently in a production state. As seen in figure 4.2, the supplier starts the production after t_I time units in the first transportation cycle within a replenishment cycle and he continues the production till the inventory level of Q_{max} is reached and then stops production. Let T denote the time length between successive dispatches. Note that T is a random variable in this case and has an Erlang distribution with the parameters Q and λ:

$$f(T; Q, \lambda) = \frac{\lambda^Q T^{Q-1} e^{-\lambda T}}{(Q-1)!}, \quad T, \lambda \geq 0, \; Q \textit{ is integer.}$$

The amount of the products produced during T time units is denoted as P_1 and it is a random value depending on the length of the time length T and has probability distribution $P_{P_1}(x)$, where

$$P(P_1 = x) = P_{P_1} = \frac{e^{-\mu T}(\mu T)^x}{x!}, \quad x \geq 0.$$

Also let P_2 denote the amount of the products produced in time length $T - t_I$ which has the probability:

$$P(P_2 = x) = P_{P_2} = \frac{e^{-\mu(T-t_I)}[\mu(T - t_I)]^x}{x!}, \quad x \geq 0.$$

$V_i(j, I)$ is the total β-discounted minimum cost of the system. The states of the system and the actions are the same as in the time-based dispatch policy model discussed in the previous section.

We look at the system at the times of the shipments and write a dynamic programming model for this system. For a given Q and Q_{max}, the dynamic programming model for state $(1, I)$ is as follows:

$$\begin{aligned} V_i(1,I) &= E_{T,P_1}\Big\{e^{-\beta T}V_{i+1}[1_{I+P_1<Q_{max}}, max(0, min\{Q_{max}, I+P_1\}-Q)] \\ &+ e^{-\beta T}s[Q-(min\{Q_{max}, I+P_1\})]^+ \\ &+ \Big[IT + \frac{T}{2}\frac{min\{Q_{max}-I, P_1+1\}min\{Q_{max}-I-1, P_1\}}{P_1+1} \\ &+ (Q_{max}-I)\Big(T - \frac{T}{P_1+1}min\{Q_{max}-I-1, P_1+1\}\Big)\Big]h + \frac{(Q-1)T}{2}w\Big\} \\ &+ K_t. \end{aligned} \tag{4.7}$$

$$\begin{aligned} V_i(1,I) &= \int_{T=0}^{\infty}\Big\{\sum_{P_1=0}^{Q_{max}-I-1}\Big[e^{-\beta T}V_{i+1}[1, max(0, I+P_1-Q)] + e^{-\beta T}s[Q-(I+P_1)]^+ \\ &+ \Big(IT + \frac{P_1 T}{2}\Big)h\Big]P_{P_1} \\ &+ \sum_{Q_{max}-I}^{inf}\Big[e^{-\beta T}V_{i+1}[0, max(0, Q_{max}-Q)] + e^{-\beta T}s(Q-Q_{max})^+ \\ &+ \Big[IT + \frac{T}{P_1+1}\frac{(Q_{max}-I-1)(Q_{max}-I)}{2} + (Q_{max}-I)\Big(T - \frac{T(Q_{max}-I-1)}{P_1+1}\Big)\Big]h\Big]P_{P_1} \\ &+ \frac{(Q-1)T}{2}w\Big\}f(T) \\ &+ K_t. \end{aligned} \tag{4.8}$$

In the equation 4.8, $V_i(1, I)$ denotes that the system is in stage i, the supplier is in a production state and there are I units of product at the supplier. According to our assumptions, after the production begins, the supplier has to continue the production until her inventory reaches

Q_{max}.

The first two lines of the equation 4.8 denote the cost when the inventory level of the supplier does not reach to Q_{max} in that cycle, when the demand accumulated at the retailer is equal to Q. At the end of the period, inventory level of the supplier increases by the amount that is produced within T minus the dispatch quantity Q, leading to the value function $V_{i+1}[1, max(0, I+P_1-Q)]$. Whenever the dispatch quantity Q is more than the inventory of the supplier, the supplier has to buy $Q-(I+P_1)$ units of the product from an ample supplier with the unit cost of s. $\left(IT + \frac{P_1 T}{2}\right)h$ is the inventory holding cost of the supplier within time period T.

The third and fourth lines of the equation 4.8 denote the cost for the case of producing $Q_{max} - I$ units within T. The inventory level of the supplier is equal to $Q_{max} - Q$ at the end of the period and the supplier has to buy from an ample supplier if Q is bigger than Q_{max} with a cost of $s(Q - Q_{max})^+$. The fourth line of the equation 4.8 is the inventory holding cost of the supplier.

Customer waiting cost of the retailer within T is $\frac{(Q-1)T}{2}w$, and the supplier has to pay the transportation cost K_t at the end of each stage.

$$
\begin{aligned}
V_i(0, I) &= min\Bigg\{ \int_{T=0}^{\infty} \left[e^{-\beta T} V_{i+1}[0, max(0, I-Q)] + e^{-\beta T} s(Q-I)^+ + ITh + \frac{(Q-1)T}{2} w \right] f(T), \\
& \int_{T=0}^{t_I} \left[e^{-\beta T} V_{i+1}[0, max(0, I-Q)] + e^{-\beta T} s(Q-I)^+ + ITh + \frac{(Q-1)T}{2} w \right] f(T) \\
&+ E_{D, T>t_I} \Bigg\{ e^{-\beta T} V_{i+1}[1_{I+P_2<Q_{max}}, max(0, min\{Q_{max}, I+P_2\} - Q)] \\
&+ e^{-\beta T} s[Q - min\{Q_{max}, I+P_2\}]^+ \\
&+ \Bigg[IT + (T - t_I) \frac{min\{Q_{max} - I, P_2+1\} min\{Q_{max} - I - 1, P_2\}}{2(P_2+1)} \\
&+ (Q_{max} - I)\left((T - t_I) - \frac{T - t_I}{P_2+1} min\{Q_{max} - I - 1, P_2 + 1\} \right) \Bigg] h + \frac{(Q-1)T}{2} w + K_p \Bigg\} \Bigg\} \\
&+ K_t.
\end{aligned} \tag{4.9}
$$

$$
\begin{aligned}
V_i(0,I) &= min\Bigg\{ \int_{T=0}^{\infty} \left[e^{-\beta T} V_{i+1}[0, max(0, I-Q)] + e^{-\beta T} s(Q-I)^{+} + ITh + \frac{(Q-1)T}{2} w\right] f(T), \\
& \int_{T=0}^{t_I} \left[e^{-\beta T} V_{i+1}[0, max(0, I-Q)] + e^{-\beta T} s(Q-I)^{+} + ITh + \frac{(Q-1)T}{2} w\right] f(T) \quad (4.10) \\
&+ \int_{T=t_I}^{\infty} \Bigg\{ \sum_{P_2=0}^{Q_{max}-I-1} \Bigg[e^{-\beta T} V_{i+1}[1, max(0, I+P_2-Q)] + e^{-\beta T} s[Q-(I+P_2)]^{+} \quad (4.11) \\
&+ \left(IT + \frac{P_2(T-t_I)}{2}\right) h \Bigg] P_{P_2} \quad (4.12) \\
&+ \sum_{P_2=Q_{max}-I}^{\infty} \Bigg[e^{-\beta T} V_{i+1}[0, max(0, Q_{max}-Q)] + e^{-\beta T} s(Q-Q_{max})^{+} \quad (4.13) \\
&+ \Bigg[IT + \frac{(Q_{max}-I-1)(Q_{max}-I)(T-t_I)}{2(P_2+1)} \quad (4.14) \\
&+ (Q_{max}-I)\left((T-t_I) - \frac{(Q_{max}-I-1)(T-t_I)}{P_2+1}\right)\Bigg] h \Bigg] P_{P_2} \quad (4.15) \\
&+ K_P + \frac{(Q-1)T}{2} w \Bigg\} f(T) \Bigg\} \quad (4.16) \\
&+ K_t. \quad (4.17)
\end{aligned}
$$

In the equation 4.9, $V_i(0,I)$ denotes that the system is at stage i, there is no production at the supplier, and the inventory level of the supplier equals to I. Since the production has not started yet, the supplier has to decide on whether to start the production or not. If the supplier decides to commence production then she has to determine the time to start production, which is denoted by t_I in the above equation.

The first line of equation 4.9 denotes the cost for the case when the supplier does not start production within T. The state of the production remains the same, $j = 0$, and the inventory level of the supplier is $I - Q$ or 0 at the end of the period. If the dispatch quantity Q is more than the inventory of the supplier, then the supplier buys $Q - I$ units of the product from an ample supplier. ITh and $\frac{(Q-1)T}{2}w$ is the inventory holding and the customer waiting costs of the supplier and the retailer respectively.

Equation 4.10 is the cost that the supplier decides to start production, but the length of the time period T is shorter than the waiting time before production t_I. Hence, there is no production within T and the production state j remains 0. Inventory level of the supplier is $I - Q$ at the end of the transportation cycle, the supplier holds inventory level of I during the cycle, and if Q is more than the amount of the inventory at the supplier, the supplier buys the difference from an ample supplier.

4.11 and 4.12 denote the cost when the supplier decides to start production and produces less than or equal to $Q_{max} - I - 1$ during the cycle. P_2 denotes the amount that the supplier produces during the transportation cycle. Since the production is ongoing, j changes from 0 to 1.

4.13, 4.14 and 4.15 denote the cost in the case that the supplier decides to start production and the supplier produces $Q_{max} - I$ units during this cycle. After reaching the inventory level of Q_{max}, the supplier stops production, thus j equals to zero at the end of the cycle.

In the equation 4.17, the first line is the cost when the supplier decides not to start production, where the sum of the equations from 4.10 to 4.16 denotes the cost of the decision of starting production. K_P is the production set up cost of the supplier and $\frac{(Q-1)T}{2}w$ is the customer waiting cost of the retailer. In both decisions, the supplier has to pay the transportation cost at the end of the transportation cycle in order to dispatch Q units of the product to the retailer.

4.4 Stochastic Inventory Models When The Retailer Holds Inventory

In this section, we consider the stochastic demand and production for the retailer and the supplier respectively in a model where the retailer also holds inventory and satisfies the customer demand immediately from the inventory. Similarly to the previous chapter, we analyze the time-based and quantity-based dispatch policies for this model.

4.4.1 VMI Model with Time-Based Dispatch Policy

In time based dispatch policy, the supplier sends product to the retailer in every predetermined T time units. In every dispatch, supplier increases the retailer's inventory to the level of Q. If the inventory of the retailer drops to zero before the dispatch of the supplier, the retailer can not satisfy all the external demand within T and incurs a lost sales cost of b per unit per unit time. In this case, the supplier sends Q units to the retailer at the time of shipment. In the case of the demand in T being less than Q, the supplier dispatches to the retailer in the amount of D, where D denotes the quantity of the external demand. At the dispatch time, if the inventory of the supplier is not sufficient to satisfy the order of the retailer, the supplier supplements from an ample supplier with a cost of s per unit per unit time.

In this section, production of the supplier is also stochastic as the demand, and P_1 denotes the quantity of the products produced in time length T. The probability that $P_1 = x$ is:

$$P(P_1 = x) = P_{P_1} = \frac{e^{-\mu T}(\mu T)^x}{x!}, \qquad x \geq 0.$$

Similar to the previous section, $V_i(j, I)$ is the expected minimum total cost at stage i when the inventory level of the supplier is I. $j = 0$ denotes that there is no production and $j = 1$ means that the supplier is in production in that stage.

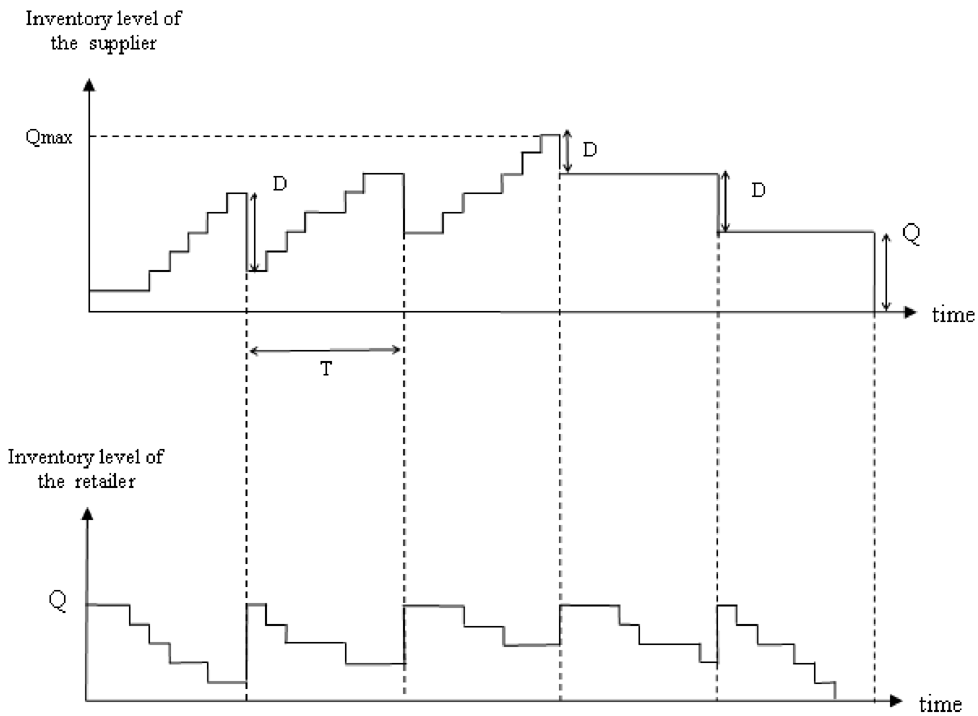

Figure 4.3: Time-Based Stochastic Production Model.

The mathematical model for the state $(1, I)$ is as follows:

$$\begin{aligned} V_i(1,I) = & \sum_{P_1=0}^{Q_{max}-I} \Bigg\{ \sum_{D=0}^{\infty} \Bigg[e^{-\beta T} V_{i+1}[1, max(0, I+P_1-min(Q,D))] + b(D-Q)^+ \\ + & s[min(Q,D)-(I+P_1)]^+ + \Bigg[Q[min(Q,D)+1] - \frac{min(Q,D)[min(Q,D)+1]}{2} \Bigg] \frac{T}{D+1} h_2 \Bigg] p_D \\ + & \left(IT + \frac{P_1 T}{2} \right) h_1 \Bigg\} P_{P_1} \\ + & \sum_{P_1=Q_{max}-I+1}^{\infty} \Bigg\{ \sum_{D=0}^{\infty} \Bigg[e^{-\beta T} V_{i+1}[0, max(0, Q_{max}-min(Q,D))] + b(D-Q)^+ + s[min(Q,D)-Q_{max}]^+ \\ + & \Bigg[Q[min(Q,D)+1] - \frac{min(Q,D)[min(Q,D)+1]}{2} \Bigg] \frac{T}{D+1} h_2 \Bigg] p_D \\ + & \Bigg[IT + \frac{T}{P_1+1} \frac{(Q_{max}-I-1)(Q_{max}-I)}{2} + (Q_{max}-I)\Big(T - \frac{T}{P_1+1}(Q_{max}-I) \Big) \Bigg] h_1 \Bigg\} P_{P_1} + K_t \end{aligned} \quad (4.18)$$

In equation 4.18, $V_i(1, I)$ is the expected minimum total cost when the system is in stage i, the supplier is in production and the inventory level of the supplier is I. First three lines denote the total cost when the produced quantity within T is less than Q_{max}. Inventory of the supplier increases by P_1 and decreases by the minimum of the predetermined amount Q or the total demand at the retailer within T. Whenever the demand is more than Q, the inventory of the retailer, there is a backlog cost of b per unit per unit time. If the inventory of the supplier is not sufficient to satisfy the dispatch quantity, $min(Q, D)$, then she has to supply from an ample supplier with the cost of s per unit per unit time. h_2 is the inventory holding cost of the retailer and the total inventory holding cost of the retailer depends on the demand during the time length T. Inventory holding cost of the supplier is h_1. Last three lines of the equation 4.18 denote the total cost when the supplier produces until Q_{max} and stops production. In this case, j changes to 0 from 1. At the end of the each dispatch time T, the supplier must dispatch the minimum of (Q, D) and pay the transportation cost of K_t.

In this model, t_I denotes the waiting time before starting the production, whenever the inventory level on the supplier equals to I and there is no production at the supplier. Let P_2 denote the quantity of the products produced in time length $T - t_I$ which has the probability:

$$P(P_2 = x) = P_{P_2} = \frac{e^{-\mu(T-t_I)}[\mu(T - t_I)]^x}{x!}, \quad x \geq 0.$$

The mathematical model for the state $(0, I)$ is as follows:

$$
\begin{aligned}
V_i(0,I) = min\Bigg\{ & \sum_{P_2=0}^{Q_{max}-I} \Bigg\{ \sum_{D=0}^{inf} \Bigg[e^{-\beta T} V_{i+1}[1, max(0, I+P_2-min(Q,D))] + b(D-Q)^+ \\
+ \quad & s[min(Q,D)-(I+P_2)]^+ + \Bigg[Q[min(Q,D)+1] - \frac{min(Q,D)[min(Q,D)+1]}{2} \Bigg] \frac{T}{D+1} h_2 \Bigg] p_D \\
+ \quad & \Bigg[IT + \frac{P_2(T-t_I)}{2} \Bigg] h_1 \Bigg\} P_{P_2} \\
+ \quad & \sum_{P_2=Q_{max}-I+1}^{inf} \Bigg\{ \sum_{D=0}^{inf} \Bigg[e^{-\beta T} V_{i+1}[0, max(0, Q_{max}-min(Q,D))] + b(D-Q)^+ + s[min(Q,D)-Q_{max}]^+ \\
+ \quad & \Bigg[Q[min(Q,D)+1] - \frac{min(Q,D)[min(Q,D)+1]}{2} \Bigg] \frac{T}{D+1} h_2 \Bigg] p_D \\
+ \quad & \Bigg[IT + \frac{T-t_I}{P_2+1} \frac{(Q_{max}-I-1)(Q_{max}-I)}{2} + (Q_{max}-I)\Bigg(T - \frac{(T-t_I)(Q_{max}-I-1)}{P_2+1} \Bigg) \Bigg] h_1 \Bigg\} P_{P_2} \\
+ \quad & K_t + K_p, \\
& \sum_{D=0}^{inf} \Bigg[e^{-\beta T} V_{i+1}[0, max(0, I-min(Q,D))] + b(D-Q)^+ + s[min(Q,D)-I]^+ \\
+ \quad & \Bigg[Q[min(Q,D)+1] - \frac{min(Q,D)[min(Q,D)+1]}{2} \Bigg] \frac{T}{D+1} h_2 \Bigg\} p_D + ITh_1 + K_t \Bigg\}
\end{aligned}
\tag{4.19}
$$

In equation 4.19, $V_i(0, I)$ denotes that the system is at stage i, there is no production at the supplier and the inventory level of the supplier is I. Sum of the first seven lines is the cost of the system when the supplier decides to start production, thus there is a production set up cost K_p at the end of line seven, and the last two lines are the cost in the case of deciding not to produce in that stage. The first three lines denote the cost when the supplier produces less than Q_{max} within T. P_2 differs from P_1 in the equation 4.18, since there is a waiting time before starting production in equation 4.19 and production has been already started in the previous stage in equation 4.18. In the last two lines of the equation, the supplier does not produce but dispatches the minimum of the Q or demand to the retailer. Since all the terms are similar to the terms in 4.18, we do not discuss them in detail.

4.4.2 VMI Model with Quantity-Based Dispatch Policy

In this model, we assume that when the inventory level of the retailer drops to 0 and a demand arrives to the retailer, the supplier makes a shipment of amount Q to the retailer. We assume that the lead time is negligible, there is no need to make a shipment before the inventory of the retailer is 0 and a demand comes to the retailer. If the inventory level of the supplier is not sufficient to ship Q units, then the supplier buys the necessary quantity from an ample supplier. Therefore, the supplier always sends Q units to the retailer.

All the formulations are identical to the quantity-based dispatch policy which is studied in the previous chapter thus we shall not discuss the models again. As stated before, the parameter h is replaced by h_1 and w by h_2.

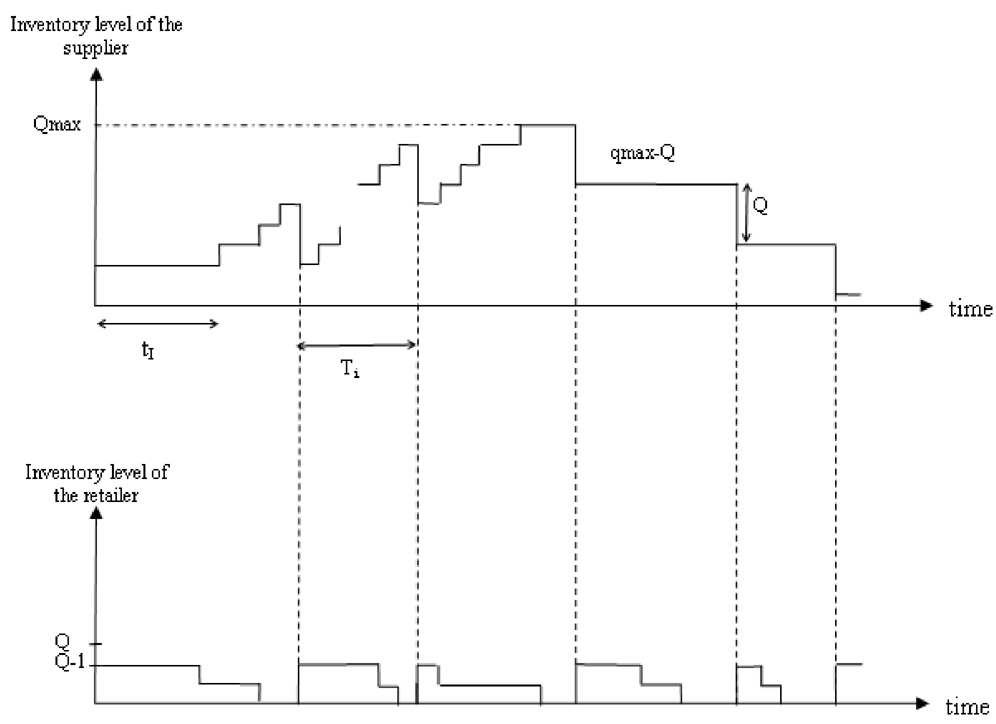

Figure 4.4: Quantity-Based Stochastic Production Model.

The mathematical model for state $(1, I)$ is as follows:

$$
\begin{aligned}
V_i(1,I) &= \int_{T=0}^{\infty}\Bigg\{\sum_{P_1=0}^{Q_{max}-I-1}\Big[e^{-\beta T}V_{i+1}[1,max(0,I+P_1-Q)]+e^{-\beta T}s[Q-(I+P_1)]^{+} \\
&+ \left(IT+\frac{P_1T}{2}\right)h_1\Big]P_{P_1}+\sum_{Q_{max}-I}^{inf}\Big[e^{-\beta T}V_{i+1}[0,max(0,Q_{max}-Q)]+e^{-\beta T}s(Q-Q_{max})^{+} \\
&+ \Big[IT+\frac{T}{P_1+1}\frac{(Q_{max}-I-1)(Q_{max}-I)}{2}+(Q_{max}-I)\left(T-\frac{T(Q_{max}-I-1)}{P_1+1}\right)\Big]h_1\Big]P_{P_1} \\
&+ \frac{(Q-1)T}{2}h_2\Bigg\}f(T)+K_t.
\end{aligned}
\tag{4.20}
$$

The mathematical model for state $(0, I)$ is as follows:

$$
\begin{aligned}
V_i(0,I) &= min\Bigg\{\int_{T=0}^{\infty}\Big[e^{-\beta T}V_{i+1}[0,max(0,I-Q)]+e^{-\beta T}s(Q-I)^{+}+ITh_1+\frac{(Q-1)T}{2}h_2\Big]f(T), \\
&\int_{T=0}^{t_I}\Big[e^{-\beta T}V_{i+1}[0,max(0,I-Q)]+e^{-\beta T}s(Q-I)^{+}+ITh_1+\frac{(Q-1)T}{2}h_2\Big]f(T) \\
&+ \int_{T=t_I}^{\infty}\Bigg\{\sum_{P_2=0}^{Q_{max}-I-1}\Big[e^{-\beta T}V_{i+1}[1,max(0,I+P_2-Q)]+e^{-\beta T}s[Q-(I+P_2)]^{+} \\
&+ \left(IT+\frac{P_2(T-t_I)}{2}\right)h_1\Big]P_{P_2} \\
&+ \sum_{P_2=Q_{max}-I}^{\infty}\Big[e^{-\beta T}V_{i+1}[0,max(0,Q_{max}-Q)]+e^{-\beta T}s(Q-Q_{max})^{+} \\
&+ \Big[IT+\frac{(Q_{max}-I-1)(Q_{max}-I)(T-t_I)}{2(P_2+1)} \\
&+ (Q_{max}-I)\left((T-t_I)-\frac{(Q_{max}-I-1)(T-t_I)}{P_2+1}\right)\Big]h_1\Big]P_{P_2}+K_P+\frac{(Q-1)T}{2}h_2\Bigg\}f(T)\Bigg\}+K_t.
\end{aligned}
\tag{4.21}
$$

Chapter 5

NUMERICAL ANALYSIS

In this chapter, we present the results of our numerical study. The purpose of our numerical study is to explain the impact of various factors such as production and demand rates, production, transportation, inventory holding and customer waiting costs. In Section 5.1, the two deterministic models referred in Chapter 3 are compared. Savings using VMI model is tabulated and the effect of the parameters are discussed. In Section 5.2, the results of the VMI model with stochastic production and demand are presented while changing the input parameters. We also compare the time-based and quantity-based models with the general model with serves as a lower bound on the costs of the system.

5.1 Deterministic VMI and non-VMI Models

In this section, we analyze the cost savings obtained by using a VMI model as opposed to a non-VMI model and also analyze the sensitivity of the cost performances of the deterministic VMI and non-VMI models depending on the parameters of the system.

We use the parameters $\lambda = 0.3$, $\mu = 0.7$ and $h = 2$. We also use different values for the other parameters such as: $K_p = 400, 600, 800$, $K_t = 100, 150, 200$ and $w = 4, 8, 16$. In Table 5.1, we present the results for these parameter settings.

Parameters Varied					Deterministic VMI Model				Det. non-VMI Model			Savings
Kp	Kt	w	w/h	Kp/Kt	n	T1	Tc	Cost	n	Tc	Cost	%
400	100	4	2	4	3	20.77	10.70	30.26	4	12.9	36.0	15.8%
400	100	8	4	4	5	14.04	7.91	37.31	5	9.1	41.2	9.5%
400	100	16	8	4	7	9.26	6.00	46.81	8	6.5	49.5	5.5%
400	150	4	2	3	2	23.17	13.58	33.77	3	15.8	40.3	16.2%
400	150	8	4	3	4	15.88	9.65	42.20	4	11.2	46.8	9.8%
400	150	16	8	3	5	10.62	7.55	53.73	6	7.9	56.9	5.5%
400	200	4	2	2	2	25.14	14.73	36.63	3	18.3	44.1	16.9%
400	200	8	4	2	3	17.41	11.49	46.27	4	12.9	51.5	10.1%
400	200	16	8	2	4	11.78	8.80	59.59	5	9.1	63.1	5.6%
600	100	4	2	6	4	23.07	10.60	33.61	5	12.9	39.7	15.4%
600	100	8	4	6	6	15.35	8.07	40.79	6	9.1	45.0	9.3%
600	100	16	8	6	8	9.97	6.19	50.42	9	6.5	53.2	5.2%
600	150	4	2	4	3	25.44	13.10	37.07	4	15.8	44.0	15.8%
600	150	8	4	4	5	17.20	9.69	45.69	5	11.2	50.5	9.5%
600	150	16	8	4	7	11.34	7.35	57.33	8	7.9	60.6	5.5%
600	200	4	2	3	3	27.48	14.15	40.04	3	18.3	47.7	16.1%
600	200	8	4	3	4	18.73	11.39	49.78	5	12.9	55.2	9.8%
600	200	16	8	3	6	12.49	8.47	63.17	6	9.1	66.9	5.6%
800	100	4	2	8	5	25.01	10.37	36.44	5	12.9	42.8	14.9%
800	100	8	4	8	7	16.46	8.10	43.74	8	9.1	48.1	9.1%
800	100	16	8	8	10	10.57	6.05	53.44	11	6.5	56.4	5.2%
800	150	4	2	5	4	27.38	12.58	39.90	4	15.8	47.2	15.5%
800	150	8	4	5	5	18.31	10.32	48.64	6	11.2	53.6	9.3%
800	150	16	8	5	8	11.94	7.41	60.36	9	7.9	63.8	5.3%
800	200	4	2	4	3	29.37	15.13	42.80	4	18.3	50.9	15.8%
800	200	8	4	4	5	19.86	11.19	52.76	5	12.9	58.3	9.5%
800	200	16	8	4	7	13.09	8.49	66.20	8	9.1	70.0	5.5%

Table 5.1: Numerical Results of Deterministic Models

We observe that, in each scenario, the cost of the VMI model resulted to be less than the cost of the non-VMI model. The cost saving percentages from using VMI are tabulated in Table 5.1 in the "Savings" column.

Setting the customer arrival rate $\lambda = 0.3$, production rate of the supplier $\mu = 0.7$ and inventory holding cost of the supplier $h = 2$, we have observed that the maximum cost saving is achieved when both the K_p/K_t and the w/h ratios are at their minimum values. In general, cost savings in using VMI model decrease as w increases, w/h increases, K_p decreases, K_t increases or K_p/K_t decreases.

In VMI setting, the overall cost is decreased by optimizing the cost considering the expenses of both the supplier and the retailer simultaneously. However, in non-VMI setting the retailer minimizes his costs by considering only his own expenses. As customer waiting cost, w increases, the overall supply chain cost becomes more dependent to the cost of the retailer. Hence the saving due to the VMI setting decreases as w and w/h increases. In the same manner, when the transportation costs of the product K_t increases or the K_p/K_t ratio decreases, the saving due to the VMI setting decreases.

In addition, we observe that the ratio of T_1/T_c decreases as w increases, w/h increases, K_p decreases, K_t increases or K_p/K_t decreases.

Finally, the cost savings seem less sensitive to the changes in the ratio of K_p/K_t than changes in the ratio of w/h, which means the production and the transportation cost is less dominant in cost saving.

The effect of demand arrival rate of the retailer and the production rate of the supplier on deterministic models are studied and the results are tabulated in Table 5.2.

In order to investigate the effects of the demand arrival and production rate on the cost and also the cost saving using VMI model, λ/μ ratio is changed while keeping K_p, K_t, h and w constant, which are 600, 150, 8 and 4 respectively. According to the results as shown in Table 5.2, the decrease on λ decreases the cost. Also the saving due to using VMI model becomes more significant as λ decreases. We also observe that as the ratio of λ/μ increases, the lengths of the transportation cycles decrease while the number of transportation cycles in a replenishment

Parameters Varied	Deterministic VMI Model				Determ. non-VMI Model			Savings
Lambda/Mu	n	T1	Tc	Cost	n	Tc	Cost	%
0.18	4	26.74	13.46	33.50	4	15.8	38.6	13.3%
0.25	4	22.54	11.98	38.32	5	13.7	43.7	12.3%
0.34	4	19.55	11.05	42.36	5	12.3	47.6	10.9%
0.43	5	17.20	9.69	45.69	5	11.2	50.5	9.5%
0.55	5	15.17	9.42	48.18	6	10.4	52.3	7.9%
0.68	7	13.30	8.46	49.64	7	9.7	52.8	5.9%
0.83	9	11.34	8.23	49.17	9	9.1	51.0	3.6%

Table 5.2: The Effect of Demand Arrival/Production Rate on Deterministic Models

cycle increases.

5.2 VMI Model with Stochastic Production and Demand

In this section, we study the effect of the parameters such as K_p, K_t and w on the cost of the stochastic production and demand model. The demand arrival rate $\lambda = 0.3$, production rate of the supplier $\mu = 0.69$, discount rate $\beta = 0.01$ and the inventory holding cost of the supplier $h = 2$ are taken as constant parameters with indicated values. The numerical results are tabulated in Table 5.3.

Parameters Varied					4.1 Stoc. Model
Kp	Kt	w	Kp/Kt	w/h	Avg. Cost
400	100	4	4	2	3231
400	100	8	4	4	4117
400	100	16	4	8	5085
400	150	4	3	2	3502
400	150	8	3	4	4412
400	150	16	3	8	5830
400	200	4	2	2	3910
400	200	8	2	4	4813
400	200	16	2	8	6385

Table 5.3: VMI Model with Stochastic Production and Demand

According to the results, the average cost of the supply chain decreases as the customer waiting cost w and w/h ratio decrease. In addition to that, when the transportation cost per shipment, K_t, increases and the ratio of $\frac{K_p}{K_t}$ decreases, average cost of the supply chain increases.

In the Figure 5.2 below, the optimal dispatch quantities according to the level of the inventory of the supplier and the number of the customers accumulated at the retailer are illustrated.

We observe that the supplier has a tendency to ship more units to the retailer when there is a production ongoing at the supplier. In addition, we observe that the supplier does not ship anything to the retailer if his inventory level or the number of accumulated orders at the retailer is below some level. However, as the inventory level of the supplier and the accumulated orders at the retailer increase, more units are shipped from the supplier to the retailer. But, we also observe that there are irregularities in the optimal shipment pattern.

In Table 5.4, we present the average costs under the general model as well as the time-based

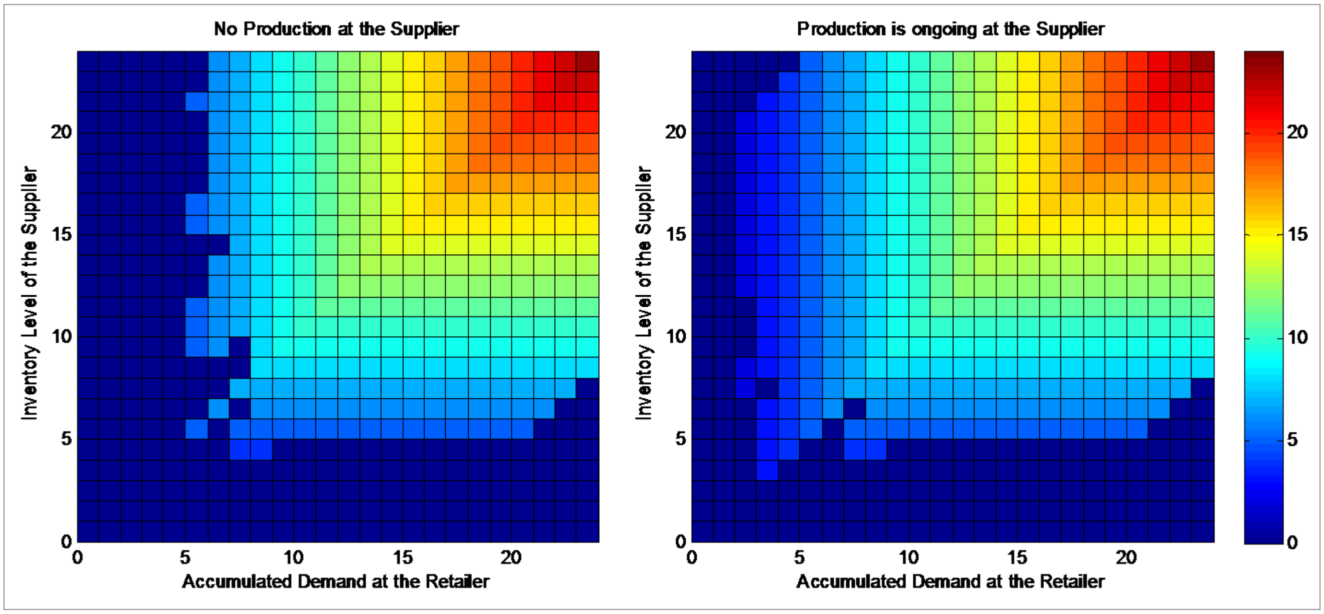

Figure 5.1: Dispatch Quantities in VMI Model with Stochastic Production and Demand.

and quantity based models for different K_p, K_t and w values. We also present the optimal T, Q and Q_{max} values in the time-based and quantity based models. In all the scenarios, the inventory holding cost of the supplier, h; demand arrival rate, λ; the production rate, μ; discount rate, β; and the supplier's product buying cost from an ample supplier, s are taken as 2, 0.3, 0.0.69, 0.01 and 350 respectively. The percentage increase in costs of the time-based and the quantity-based models relative to stochastic VMI model are shown in the "Cost Inc." columns.

Parameters Varied					4.1 Stoc. Model	4.2 Time-Based Policy			4.3 Quantity-Based Policy		
Kp	Kt	w	Kp/Kt	w/h	Avg. Cost	T	Q_{max}	Cost Inc.	Q	Q_{max}	Cost Inc.
400	100	4	4	2	3231	10.5	12	27.9%	3	10	16.8 %
400	100	8	4	4	4117	7.6	11	13.9%	2	11	2.1%
400	100	16	4	8	5085	5.9	11	8.5%	2	11	0.1%
400	150	4	3	2	3502	11.7	12	32.4%	3	10	23.0%
400	150	8	3	4	4412	9.6	12	20.2%	3	10	7.2%
400	150	16	3	8	5830	7.1	11	8.2%	2	11	0.1%
400	200	4	2	2	3910	14.0	13	29.5%	5	11	19.4%
400	200	8	2	4	4813	10.5	12	20.5%	3	10	9.4%
400	200	16	2	8	6385	8.1	12	9.5%	2	11	1.1%
600	100	4	6	2	3291	9.6	13	36.2%	3	10	27.0%
600	100	8	6	4	4214	7.7	13	19.1%	2	11	7.8%
600	100	16	6	8	5183	5.9	13	12.9%	2	11	0.1%
600	150	4	4	2	3514	12.1	14	41.3%	3	10	34.1%
600	150	8	4	4	4509	9.3	13	24.9%	3	10	14.0%
600	150	16	4	8	5928	7.2	13	12.0%	2	11	0.3%
600	200	4	3	2	3616	13.6	14	48.8%	3	10	45.1%
600	200	8	3	4	4909	10.5	14	25.1%	3	10	15.6%
600	200	16	3	8	6483	8.2	13	12.9%	2	11	0.8%
800	100	4	8	2	4587	9.8	15	3.8%	3	10	0.1%
800	100	8	8	4	4311	7.8	14	23.1%	2	11	13.3%
800	100	16	8	8	5282	5.8	14	16.1%	2	11	0.4%
800	150	4	5	2	3514	12.2	15	49.4%	3	10	45.7%
800	150	8	5	4	4605	9.4	15	28.4%	3	10	20.5%
800	150	16	5	8	6026	7.2	14	15.0%	2	11	1.0%
800	200	4	4	2	3617	13.8	15	56.5%	3	10	56.3%
800	200	8	4	4	5004	10.5	15	28.2%	3	10	21.5%
800	200	16	4	8	6580	8.2	14	15.6%	2	11	4.5%

Table 5.4: Stochastic VMI Model vs. Stochastic VMI Models with Shipment Consolidation

We observe that, in the time-based dispatch policy, when w and w/h increases the time length of the transportation cycles, T, and the maximum level of the inventory at the supplier, Q_{max} decreases. As the customer waiting costs become higher, the system has a tendency to release the shipments in shorter time arrivals which means a decrease in T. In the same manner, with an increase in w, there are less orders accumulated at the retailer and less inventory carried at the supplier, hence the Q_{max} decreases. As the production set up cost of the supplier K_p increases, the supplier produces more during a replenishment cycle and so the the maximum level of the inventory at the supplier, Q_{max} increases. Also T and Q_{max} increase when the product shipment cost K_t increases since the supplier tends to dispatch the product rarely and with higher quantities of the product.

With the quantity-based dispatch policy, when the transportation cost of the product per shipment, K_t decreases, the quantity of the product dispatched, Q and the maximum level of the inventory at the supplier, Q_{max} decreases in order to satisfy the lower inventory holding and customer waiting costs. Hence, when K_p/K_t increases, Q and Q_{max} decrease. When the customer waiting cost increases, retailer accumulates less orders and satisfies the demand more frequently. Hence, the shipment quantity Q decreases as w increases.

Cost increases in the time-based and the quantity-based models relative to the general stochastic model decrease as w and w/h increase.

Considering the percent cost increases in the "Cost Inc" column, in every scenario the quantity-based dispatch policy outperforms the time-based dispatch policy. However, in practical applications, it is important to take into account the simplicity and periodic delivery advantages of time-based dispatch policies in evaluating the cost improvements obtained through quantity-based policy. That is, in practice, it may be easier to schedule shipments so that a shipment is released on a periodic-basis, rather than as needed. It is worth noting that, both time-based and quantity-based dispatch policies are popular in practice, and they are incorporated in VMI contracts for the purposes of achieving timely delivery and load optimization, respectively. Typically, time-based policies are used for lower volume, higher value products, such as expensive hardware in the computer industry, to guarantee timely delivery. Quantity-based policies are used for higher volume, lower value items, such as peripheral computer equipment. On the other hand, our numerical results show that quantity-based policies also offer large savings for lower volume, lower value items, i.e., those items where the inventory holding cost h are smaller. Note that h is typically a percentage of the per unit procurement cost/value.

In table 5.4, the effect of the $\frac{Demand\ \ arrival\ \ rate}{Production\ \ rate}$ ratio, λ/μ, is studied and tabulated with the constant parameters taken as, $K_p = 600$, $K_t = 150$, $h = 2$, $w = 8$, $\lambda = 0.3$, $\mu = 0.69$, $s = 350$. According to the results, when λ/μ ratio increases, average costs of all the three stochastic models

Param. varied	4.1 Stoc. Model	4.2 Time-Based Policy				4.3 Quantity-Based Policy			
Lambda/Mu	Avg. Cost	T	Q_{max}	Avg. Cost	Cost Inc.	Q	Q_{max}	Avg. Cost	Cost Inc.
0.25	4040	10.5	11	4754	18%	3	10	4310	7%
0.34	4299	10.5	12	5220	21%	3	10	4762	11%
0.43	4509	9.3	13	5631	25%	3	10	5140	14%
0.55	4875	8.8	14	5938	22%	3	11	5475	12%
0.68	5548	8.4	15	6202	12%	3	14	5681	2%
0.83	6049	8.0	17	6490	7%	4	16	6178	2%

Table 5.5: The Effect of Demand Arrival/Production Rate on Stochastic Models

increase. We observe that, as the ratio λ/μ increases, the products are shipped more often to the retailer and more products are produced in a production cycle. We also observe that the time-based and quantity-based models perform close to the optimal solution when the ratio λ/μ is either very large or very small, however, as λ/μ ratio gets close to 0.5, the performances of the time-based and quantity-based models worsen.

Chapter 6

CONCLUSION

As a final note, additional savings may also be achievable by implementing non-stationary time-based, quantity-based, and hybrid policies, and analyses of such dispatch policies, as well as the structure of optimal dispatch policies, remain worthwhile areas for future research."

In this study, optimal outbound dispatch policies in supply chain models are investigated. Our aim was to minimize the total cost of supply chain and improve the performance of the systems with using various models. These models included deterministic model with and without VMI, a stochastic VMI model and VMI models with time-based and quantity-based dispatch policies.

We have considered two different context in this study. In the first context, the retailer does not hold any inventory but accumulates the external demand. In the second context, the retailer also holds inventory and satisfies the external demand.

The difference of this study from the previous studies, and also the main contribution of this thesis, is that the two different concepts, which are the models where the supplier produces at a finite rate and there is a shipment consolidation, and also the model that the retailer does not hold any inventory in a VMI system, are studied in the same model.

We have started with the literature review on the vendor-managed inventory system and consolidation shipment. Later on, deterministic inventory models with shipment consolidation for both VMI and non-VMI models are presented. In this chapter, two different cases in which the retailer holds and does not hold inventory are considered. The study continued on with stochastic inventory models including shipment consolidation and various dispatch policies. Solution of these models are acquired with dynamic programming. In the proceeding chapter, numerical studies are carried on to compare the performances of the models that are aforementioned. Additionally, the effects of the parameters on the total cost, including production set up cost and customer waiting cost, are evaluated numerically.

In the deterministic inventory models, a single-supplier single-retailer supply chain system is considered. In this system, retailer faced an external demand and sent orders to the supplier. In one of the models, retailer did not hold inventory whereas in the other retailer hold inventory. VMI and non-VMI systems are considered for both of the two cases. The main difference between not using and using VMI is that the order quantity of the retailer is not determined by the supplier in a non-VMI system. In a vendor managed inventory setting, that the supplier decides on the timing and the quantity of the orders, it is shown that cost saving can be realized through shipment consolidation.

According to the results of the deterministic models, it is proved that in the first transportation cycle, the supplier produces an amount that is exactly equal to the demand accumulated in that cycle for the optimal policy. Moreover, the first transportation cycle is at least as long as the other cycles whereas all the other transportation cycles was equal in length. Finally the optimal cost function is obtained for all of the models.

A general model is examined in order to set the main principles of the dispatching and replenishment policies. In the stochastic models with shipment consolidation, the aim was to minimize the expected average total cost of the system. For the case of stochastic demand, the type of the shipment consolidation that is chosen, have an impact on the cost saving of the vendor managed inventory system. Two types of shipment consolidation were studied in the VMI setting. The first one is time-based dispatch policy, and the second one is quantity-based dispatch policy.

In a time-based dispatch policy, the accumulated load satisfying all outstanding demands, is dispatched in every predetermined T time lengths so the dispatch quantity is a random variable. This model assures that each demand is dispatched at a predetermined shipment date therefore it is more appropriate for satisfying customer service requirements. A quantity-based dispatch policy ships a dispatch whenever the outstanding customer orders (demands) are accumulated to a predetermined quantity, Q. The dispatch quantity assures transportation scale of economies. It is shown that the quantity-based policy is always superior to the time-based dispatch policy in terms of the resulting average supply chain costs.

In the last chapter, we present the results of our numerical study. The two deterministic models are numerically solved and the cost savings due to the VMI setting are examined. The stochastic models are solved using dynamic programming approach with an infinite horizon, and their numerical results are compared and tabulated.

In the deterministic models it is shown that the minimum cost is achieved when the K_p/K_t ratio is at its highest value whereas the w/h ratio is at its smallest value. Also, the decrease on λ resulted in a decrease in the cost.

In the VMI model with stochastic production and demand, the average cost of the supply chain was decreased as, w and w/h decreased. Also, the supplier has a tendency to dispatch a shipment to the retailer when there is a production ongoing at the supplier. Moreover, the supplier dispatches the same amount independent of the production, after a determined level of inventory is reached.

In the time-based and the quantity-based models, it is presented that "the relative costs increase" decrease as w and w/h increase, compared to the VMI model with stochastic production and demand model. Also, the average costs of all the three stochastic models increased with increasing λ.

In this study, two concepts are merged together and the improvements are shown and supported with numerical results. As a future work, hybrid dispatch policies may be included in order to decrease the overall cost and increase the performance. Other possible extensions may consider (i) variable unit costs of transportation for shipments, (ii) cargo capacity for dispatch quantities, and (iii) the production capacity of the manufacturer for stochastic demand processes.

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