MODELING TIME ALLOCATION IN FAMILY PRACTICE IN TURKEY

by

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This is to certify that I have examined this copy of a master's thesis by

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and have found that it is complete and satisfactory in all respects,
and that any and all revisions required by the final
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To my mom

ABSTRACT

Turkey went through a major change in its primary health care system and implemented family practice between 2005 and 2010. In this study, we provide a general overview of Turkish family practice through the lens of a survey conducted among family doctors. We analyze a specific case to explore behavior of patient arrivals over time in presence of preventive services.

Further, we model the time allocation problem of a family doctor as a non-linear optimization problem. Two approaches are explored. First, preventive service and acute care services are assumed to be provided in isolation. Patients who demand these services wait in parallel queues where we assumed all demand has to be satisfied. Optimal percent of time allocated to preventive services is characterized. Second, all patients wait in a single line with a need of acute care service. Family doctor decides whether or not to offer a preventive service as an add-on for each patient, where there is no obligation to satisfy the demand for prevention service. Conditions to obtain boundary solutions, where all patients receive the same type of service, are characterized. For both models, a feedback effect of that preventive activity on the acute care demand is taken into consideration and numerical examples are provided. Finally, we looked at how we can model feedback effect on arrivals by a Markov chain approach.

ÖZETÇE

Türkiye 2005-2010 yılları arasında birinci basamak sağlık hizmetlerinde değişiklik yaptı ve Aile Hekimliği uygulamasını getirdi. Bu çalışmada Türk Aile Hekimliği uygulamasına aile hekimleri arasında yapılan bir anket çalışmasıyla genel bir bakış sağlanmıştır. Koruyucu sağlık hizmetlerinin hasta ziyaretlerine etkisini belirlemek için vaka analizi yapılmıştır.

Aynı zamanda, aile hekiminin görevler arası zaman paylaşımı problemi lineer olmayan optimizasyon problemi olarak modellenmiştir. İki farklı yaklaşım incelenmiştir. İlk yaklaşımda koruyucu sağlık hizmetlerinin ve akut hizmetlerin ayrı ayrı sağlandığı varsayılmıştır. Bu servisleri talep eden hastalar iki paralel kuyruk oluşturmakta ve talebin hepsi karşılanmak zorundadır. Koruyucu sağlık hizmetlerine sağlanması gereken optimum zaman karakterize edilmiştir. İkinci yaklaşımda bütün hastalar akut gereksinimle tek bir sırada beklemektedirler. Aile hekimi her hasta için akut servisi sağladıktan sonra koruyucu sağlık hizmeti verip vermeyeceğine karar vermektedir. Koruyucu sağlık hizmetlerine olan talebi karşılama zorunluluğu yoktur. Her hastanın aynı tip servis aldıgı şartlar karakterize edilmiştir. Her iki model için de koruyucu sağlık hizmetinin akut ihtiyaçlara olan talep üzerine geribildirim etkisi göz önünde bulundurulmuş ve sayısal örnekler sağlanmısıtır. Son olarak hasta ziyaret talebi üzerindeki geribildirim etkisi Markov zinciri yaklaşımıyla modellenmiştir.

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Love you all...

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NOMENCLATURE

DTS	Dedicated Time Slots Model
DTSE	Dedicated Time Slots with Exponential Feedback Model
DTSL	Dedicated Time Slots Model with Linear Feedback
GP	General Practitioner, family doctor
p	Percentage
PAS	Prevention as an Add-on Service
PHC	Primary Health Care

Chapter 1

INTRODUCTION

Health care industry is one of the most important and expensive industries around the world. Nearly 15% of GDP of U.S. is dedicated to health care industry. It is followed by France with 11.2%, by Switzerland with 10.7%, and by Germany with 10.5%. In Turkey this percentage is 6.2% [1]. Since the budget devoted is very high, in order to increase cost effectiveness, operational studies as well as the leading clinical trials in health care are important.

Primary Health Care (PHC) is the first degree health service that is provided to society. In 1978 Alma-Ata Declaration of World Health Organization (WHO)[1] stated what services should be involved and what should be the aim in PHC : *'protect and promote'*[3]. As a formal definition, in 1988 Australian Health Ministers' Council defined PHC as

> *... seeks to extend the first level of the health system from sick care to the development of health. It seeks to protect and promote the health of defined communities and to address individual problems and populates health at an early stage. Primary health care services involve continuity of care, health promotion and education, integration of prevention with sick care, a*

[1]WHO is the directing and coordinating authority for health within the United Nations system. It is responsible for providing leadership on global health matters, shaping the health research agenda, setting norms and standards, articulating evidence-based policy options, providing technical support to countries and monitoring and assessing health trends [2]

> *concern for population as well as individual health, community involvement and the use of appropriate technology.*

Family practice, as a part of PHC, has gained importance since 1978, when WHO introduced Health for All program. Around the world, the family practice began to rise up since then, and governments started to gave place to it in their plans. With the increase in importance and responsibility in family practice around the world, Turkey went through a transformation in 2004 in its PHC. The major change was implementing family practice into the system. This thesis is motivated with those changes in Turkey.

In 2005, Düzce was selected as pilot city to implement family practice and the practice was implemented in all cities by the end of 2010 [4]. The purpose of this reform in Turkish health system is to lower the burden on hospitals so that the hospitals can focus on serious illnesses and advanced treatments. According to WHO, the providers of PHC are family doctors, nurses or other health professionals. General Practitioners (family doctors) are key elements of primary health care systems. A GP should be capable of recognition and identification of diseases. In this purpose, a GP's career education or specialization training takes 3 years. People see qualified GPs first, if the problem cannot be handled then they are referred to a higher degree health facility [5]. With this system, people would first consult to family doctors for general, simple needs instead of going to a hospital. This would lower the congestion in hospitals. Since family practice is new in Turkey, how it works should be well understood.

The services offered by a family doctor are diagnosis & treatment, vaccination, mother-child care, management of chronic diseases, scanning of risk factors, family planning, promotion of health/prevention and administrative affairs. Preventive service, as a part of GP practice, must be given much more importance as well as given much more time, to improve society's health status in the long run. The benefits of preventive health care, and, in general, establishing more healthy life can be realized as reduced demand for health care and lower deaths in the long run. Therefore, GPs should devote a portion of their time consistently for health promotion and preventive activities. Other than administrative affairs, the services provided by a family doctor can be classified into two categories i) prevention service, ii) acute care service. The administrative tasks are usually handled by the nurse or by the doctor in after-hours. In this thesis, we focused on optimizing time allocation of a GP between prevention and acute care.

The rest of the study is structured as follows: Chapter 2 gives a brief literature related to our study. Chapter 3 presents survey results conducted among family doctors in Turkey, which provides a deeper understanding of the Turkish family practice. A specific case was analyzed to determine the patient behavior. The findings constructed the basis of our modeling approaches. Chapter 4 introduces two different queueing models to time allocation problem of a GP in capacity allocation framework, assuming a GP provides two different services: preventive or acute care. Only 6% of GPs in Turkey work with appointment, hence to be in line with practice appointment scheduling was not considered. In the first approach (Dedicated Time Slots, DTS), we consider a model where demands of two different services constitute two different parallel queues served by a single server, and each demand should be satisfied. Further, we included the decreasing effect of preventive service in the long run on patient arrivals. We analyzed different forms of feedback function. In the second approach (Prevention as an Add-on Service, PAS), we considered all patients wait in a single line, and the family doctor decided whether or not to provide preventive service after each acute care service as an add-on. Here, there is no obligation to satisfy the demand for preventive service. Again, decreasing effect of preventive service is modeled by a feedback function on arrivals. In both models, optimal time allocation of a GP between these services is explored and numerical examples are provided. Moreover, we used a Markov chain approach to model the feedback mechanism on arrivals. The process is based on one patient's probability of preventive service ensured, and long run (limiting) probabilities are found, the behavior is analyzed. Chapter 5 summarizes the findings and directs to further research.

Chapter 2

LITERATURE REVIEW

In this chapter we will briefly review the literature related with the family practice in Turkey and modeling approaches related to primary health care. We will introduce literature for preventive activity as a part of family practice.

As mentioned in Chapter 1, Turkey went through a transformation in its PHC in 2004, and implemented family practice. Although the implementation is relatively new, Akdeniz et al. (2009) [6] states that family medicine as a discipline was first introduced in 1983. The postgraduate training was started in 1985 when family medicine was included as a clinical specialty. By then, family practice residency programs was provided only in state hospitals, and by 1995, departments of family practice was funded in universities [7]. (For further information about development of family medicine residency in Turkey see [5, 8, 9]). Those papers suggest that, on the doctor side, the infrastructure was started to be constructed long ago, before implementation of family practice to primary care in Turkey. Around the world, postgraduate trainings for family physicians are established in U.S., U.K. Ireland, Spain and etc. over the last few decades [10]. Like other countries, Turkey adopted the changes too. Güneş and Yaman (2008) [11] analyzed the current system and implications of family practice in Turkey. They discussed the challenges of meeting needs.

There are also research articles about family practice residents in Turkey. Those articles show survey results which bring light on how the system works, what the needs are, and the difficulties confronted as well as how satisfied they are while training family practice [12, 13, 14]. These studies are similar to the survey that we will provide in Chapter 3, which are important while analyzing primary care system in Turkey.

Other than case studies, there are some operational studies related to primary health care. In this thesis we will focus on the time management of a GP.

One stream of literature focuses on appointment scheduling which is very important for effectiveness and patient satisfaction. However, most of the models that are used in manufacturing and logistics are not applicable to health care systems [15]. In health care problems; the questions regarding to appointment rule, patient classification and the adjustments of noises such as no-shows, urgent patients etc. should be answered. The related literature up to 2003 and modeling of appointment systems in health care systems can be seen in Veral and Çayırlı (2003) [16] more widely.

Another approach to solve scheduling problems in primary care systems can be considered as practical application of the theory; the scheduling of the parallel queues served by a single server in order to minimize the waiting time or the waiting cost of the system where there exists a switchover time or cost. 'The common property of these systems is the need to efficiently share a single resource/server among many queues/ stations' [17]. These systems are applicable in many communication, production, and health care systems. The system is known as a polling system, and there are known results for the optimal scheduling of these systems depending on the service discipline, polling order type (static or dynamic) etc. However, these problems are very difficult to solve. There is a literature review on polling systems and mathematical models to solve these systems in Vishnevskii and Semenova (2006) [18].

Despite the fact that polling systems are in use in the areas of public health care, transportation, communication, and computer systems, little work has been done about the operational efficiency of these systems. Boxma, Levy and Weststrate (1991) [17] achieved finding an optimal pre-determined polling table by using an approximate approach to the problem of minimizing mean waiting time of the system. They found two different rules for determining the efficient visit order numbers to each queue under different types of service disciplines for constructing polling tables, which are efficiently operating by using two independent analysis based on mean delay approximation and lower bound. Two rules are similar and give results very close to original problem's optimal point in most of the cases.

If the scheduling of parallel queues served by a single server satisfies the following properties; Poisson arrivals and general service time distribution (M/G/1 system), and has no switchover time or switchover costs; then optimization of scheduling problem is reached by the well known $c\mu$ rule. In other words, if there is no switchover time, the waiting cost associated with queue i is c_i and is processed at rate μ_i then the optimal policy for minimization of average waiting cost is the $c\mu$ rule which gives higher priority to the queues with larger values of $c_i\mu_i$, stated in Duenyas and Van Oyen (1996) [19],and Iravani and Kolfal (2005) [20]. For further detail and literature review on $c\mu$ rule, see Büyükkoç et al. (1985) [21] and Mieghem (1995)[22]. In this thesis, we do not consider an appointment system. According to family physicians in Turkey, appointment system is very difficult to implement because of cultural reasons and habits of patients. In addition, according to survey results in Chapter 3 only 6% of GPs in Turkey use appointment scheduling. It would be more appropriate to model the system by capacity allocation approach that is used in health care systems.

A stream of literature considers the capacity allocation problem in health care settings. Smith-Daniels et al. (1988) [23] states that there are three types of resources that should be allocated in primary care: work-force concerning doctors and nurses etc, the equipments and tests in health care facilities and facilities itself. Moreover, there are costs to be minimized such as waiting and operating costs . Bretthauer and Cote (1998) [24] considered health care systems as a queueing network which includes patients and work-force and used optimization methods while including the related costs to find the optimal capacity allocation decisions. The costs related to this system are mainly wages, waiting and operating costs whereas the decision variables are the number of doctors and nurses that should be hired and the model solves for the targeted customer service while minimizing the cost.

A GP has to accomplish different tasks and consider different costs related to these different tasks. If we consider the demand for each service constitutes a different queue, then we have a Jackson network[1]. Wein (1989) [26] solved generalized Jackson network capacity allocation problem. He found optimal service rates with general distribution for k single server with infinite capacity waiting room. Customers arrive to the system according to Poisson process. These optimal service rates are found by minimizing delays subject to linear budget constraints on capacities using Brownian approximation. The results found are the generalization of the square root capacity allocation for Jackson networks which is found by Kleinrock in 1964 [27]. The solution first satisfies the effective arrival rates then allocates excess capacity proportional to the square root of their effective arrival rates. When service time distributions are exponential, the solution reduces to square root capacity allocation found by Kleinrock [26]. Hasija et al. (2005) [28] analyzed capacity allocation problem by a queueing model for call routing in two-tier call centers. The calls arrive to the system according to Markov process, gets a service for diagnosis from gatekeeper and

[1]In a network with k interconnected queues if the customer arrivals follow Poisson process and service time distributions follow exponential distribution, the utilization of queues is less than one and a customer after leaving one queue, either moves to another queue with probability p_i or leave the system with $1 - p_i$, then the network is a Jackson Network. When service time distribution is generalized, the Jackson network is called generalized Jackson network. In the literature of health care systems exponential service times and Poisson arrivals are commonly used [25]

then they are either referred to a gatekeeper or an expert for treatment with corresponding probabilities. This model is close to our PAS model in queueing approach; however, it differs in the following senses: they assumed that after diagnosis, calls form another queue to get treatment service. In addition, the probabilistic feedback effect of treatment was not considered.

Prevention, as a part of family physician's work, is becoming increasingly important. There exists a need for preventive service especially in the fields of cancer screening, management of chronic diseases such as diabetes and hypertension etc.[29]. The impact of preventive counsels and health promotion program on doctor visits can be monitored by controlled trials. Lorig et al. (1993) [30] run a 4-month randomized trial to explore the effect of self-management educational programs in prevention. The study is done among 343 arthritis patients, and they estimated the decrease in doctor visits by 40%. Montgomery et al. (1994) [31] estimated the decrease in doctor visits as 24% among 290 Parkinson's disease patients by a 6-month randomized trial. Moreover, Vickery et al. (1983) [32] stated that the doctor visits for minor illnesses decreased by 31% as a result of self-management educational programs in prevention. Fries et al. (1998) [33] presents a summary of results of clinical trials in prevention service. The decrease in patient arrivals cannot be disregarded, and should be taken into consideration when modeling primary care.

Although preventive service is very important in the sense we mentioned above, Anderson and May (1995) [34] and Elizabeth et al. (2003) [35] pointed out that the time allocated to this service is very low. The reasons are mainly lack of time and expertise of the doctor, the insurance problems and patient refusal [36]. Focusing on the GP's lack of time for preventive service, Pollak et al. (2008) [37] estimated the time spent on preventive services in United States. They classified preventive activities according to their priority (an A type has highest priority) and estimated time spent on each type. For example, although the national smoking rate is approximately 20% percent, only 4% percent of patients discuss smoking issues (A type priority). However, there is a need for satisfaction of this type for prevention to avoid high death rates. Furthermore, even though the recommended time to spend on this preventive service is at least 3 minutes, the research shown that doctor can only spend approximately 1.4 minutes. As the study suggests, neither the prevention need of patients nor the recommended time devotion to each patient is achieved. Hence, keeping in mind that prevention is an important service to be provided, it lowers the patient arrival, modeling and optimizing time allocation gains importance.

There are studies related to the resource and time allocation problems for prevention. Rauner (2002) [38] modeled strategies for policy makers to determine cost effective HIV prevention programs. They analyzed a dynamic system for resource allocation of disease control which is illustrated by the case of advanced drug therapies. Moreover, Kaplan and Pollack (1998) [39] formulated the budget allocation problem to HIV prevention activities while maximizing the number of people prevented from infection. Güneş, Chick and Akşin (2004) [40] investigated the breast cancer screening problem by including capacity constraints. The aim was to decrease the rate of deaths due to breast cancer by matching the supply of screening tests and the demand for them. Güneş (2009) [41] modeled the time allocation problem to preventive services by Markovian queueing approach. She assumed each and every patient is provided by preventive service and considered the decreasing effect of prevention on expected service time and patient arrivals in the long run. In addition, Kunduzcu (2009) [42] used event-based dynamic programming to find the optimal admission control and scheduling policies in a facility that offers only two services: screening (less urgent need) and diagnosis (urgent need). The model accounts for the future benefits of screening activity (otherwise, $c\mu$ rule is optimal). She characterized the situations where less urgent needs gain priority over urgent needs.

In this thesis, we focused on family physician's capacity allocation problem by differentiating prevention activity from other tasks. Modeling the system with the support of Turkish family practice data, by considering the decrease in arrival rates and service time, modeling the patient arrivals in accordance with their preventive service reception, forms the uniqueness of this thesis.

Chapter 3

FAMILY PRACTICE IN TURKEY

Family practice is a new implementation in Turkish primary health care. In this chapter, we will provide information about how the system works which forms the basis of our modeling approaches. Some issues in family practice discussed in the context of Turkey, with an emphasis on the time allocation of GPs and different tasks they perform. We will present survey results[1] to give an overview of the family practice in Turkey and a specific case to have a deeper understanding.

3.1 Observations from the Family Practice of Turkey

3.1.1 Family Doctors

The survey is conducted among 384 Turkish family doctors, among pilot cities Adıyaman, Denizli, Düzce, Edirne, Elazığ, Eskişehir, Gümüşhane, Isparta, İzmir, Osmaniye and Samsun are selected. The ages of those GPs varies between 24 and 55. 297 of them are men and 87 of them are women (approximately 23% are women, 87% are men). They have been working as a GP for 18 months on the average.

64 % of GPs say that in five years possibility of quitting GP is between 0 and 25 percent. Also, 75 % of GPs are satisfied with their jobs Table 3.1 and Table 3.2).

[1]The survey was prepared and distributed in 2008 by Evrim Didem Güneş, PhD; Hakan Yaman MD,MS : Aile Hekimleri Pilot Uygulama. The data that is collected by this survey is analyzed and used in this thesis as an overview. The results used are shown in appendix in Table B.7

Table 3.1: Willingness of quitting family practice

Response	# of respondents	%
0-25 percent	246	64
25-50 percent	34	9
50-75 percent	60	16
75-100 percent	45	12

Table 3.2: Satisfaction Degree

Response	# of respondents	%
satisfied	279	75
not sure	56	15
not at all satisfied	36	10

Table 3.3: Willingness of specialization

Response	# of respondents	%
yes	171	46
no	154	42
don't know	43	12

Table 3.4: Willingness of specialization in family practice

Response	# of respondents	%
yes	115	67
no	42	25
don't know	14	8

46% of them want to specialize and among those 67% want to specialize in family practice (Table 3.3 and Table 3.4). These are the signs of satisfaction of the new PHC implementation in Turkey from the GP point of view.

3.1.2 Patients and Working Principles

Approximately 1802 female, 1780 male and a total of 3582 patients are registered to each GP. If we look at age distribution of patients, we see that nearly 50 % is between 19 and 50. On the average, family doctors work at the clinic approximately 37 hours and spend approximately 6 hours for home visits per week (86.75% of time clinic duty, 13.25% of time home visits). A consultation takes 11 minutes and 57 patients consult to each GP per day. GPs refer approximately 5.5% of the patients to a higher-level health care facility. Main reasons for referral are consulting a specialist to be sure about diagnosis and treatment, need of surgical or specialist intervention, lack of medical equipment, and insistence of the patient.

A GP works alone or usually with a nurse. They have many responsibilities such as acute care, disease prevention, and promotion of health, management of chronic diseases, mother-child health, family planning, administrative affairs etc. They have to accomplish these tasks in a limited amount of time. In Table 3.5 time allocation of a family doctor among different tasks are shown. It can be understood that they do not spend much time for promotion of health (3% of their time), most of the time they provide diagnosis and treatment services. Moreover, according to GPs, average importance level of these services are shown in Table 3.6 (5 being most important, 1 being least important). Again, we see promotion of health is the least important service among GPs.

The results in both Table 3.5 and Table 3.6 show that preventive activity has lower

Table 3.5: Time Allocation Among Tasks

Task	%
Diagnosis	24
Treatment	21
Vaccination	10
Mother-child care	8
Administrative affairs	8
Management of chronic diseases	7
Scanning of risk factors	5
Family planning	5
Other duties	4
Periodic examinations	3
Promotion of health/prevention	3
Seminars	3

importance than the other tasks or is given less time devotion; however, it is one of the most important services that a family doctor should provide in the sense that it maintains and improves the health quality of people. In Chapter 2, the importance of preventive activity is presented. The results of clinical trials suggest that it lowers the patient arrivals and improves society's health status in the long run. As a result, much more time and importance should be allocated to that service. In addition, according to our survey results, 80% of GPs think that they do not have enough time to provide all the services that is defined in GP directives, one of which is being preventive service. Since the percentage of dissatisfaction in lack of time to provide services is high, planing modeling and time allocation of a GP among different tasks gains importance. The benefits of preventive health care, and ,in general, establish-

Table 3.6: Importance Level

Vaccination	4.72
Mother-child health	4.55
Family planning	4.20
Diagnosis and treatment	4.04
Management of chronic diseases	3.94
Periodic examinations	3.88
Promotion of health	3.80

ing more healthy life is realized as reduced demand for health care as well as lower deaths in the long run. Therefore, GPs should devote a portion of of their time consistently for health promotion and preventive activities. In this thesis, we assume that GP provides two types of services; acute care and preventive activity. We will focus on optimizing the portion of time that is allocated to preventive service in our models.

Majority of GPs work without appointment (6% with appointment, 94 % without appointment). In line with practice, we do not use appointment scheduling systems to model time allocation of GP, and instead we take a capacity planning approach.

3.2 A specific case

Dr. X is a General Practitioner (GP) in Düzce, a city in Turkey. He works for 8 hours a day. The number of patients that are registered to him is 2630. We used the data that we gathered from him in order to estimate model parameters as well as model construction. Descriptive statistics are below.

45% of the patients are male and 55% of the patients are female. By number, these percentages represents 1183 and 1447 patients respectively. We can say that these percentages are nearly equal, so for further analysis gender effect is ignored.

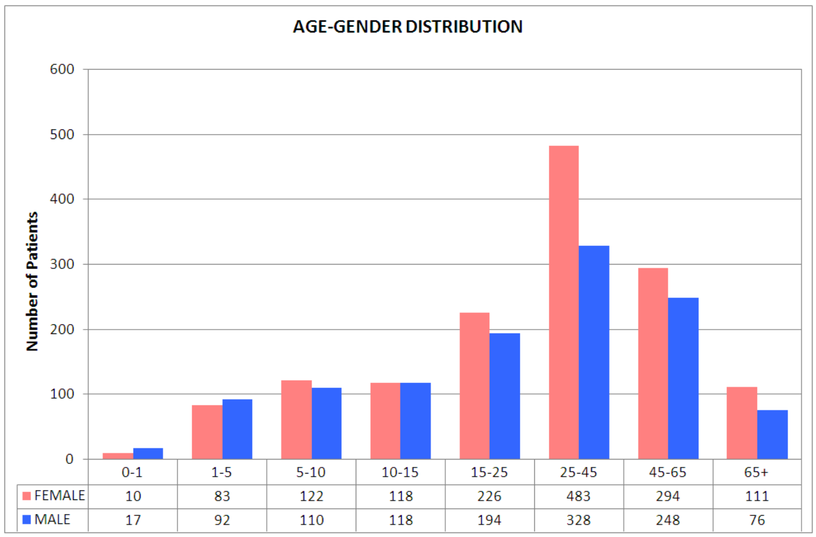

Figure 3.1: Age-Gender Distribution

Figure 3.1 shows age-gender distribution of his patients. The increase-decrease pattern in different genders among age groups is similar. This parallelism will eliminate the age caused differences within female and male groups.

Figure 3.2 shows average consultation numbers in one year per days of the week. Dr.X carries out a mobile preventive and diagnostic service in the villages of Düzce on Wednesdays as a requirement of being a GP and therefore accepts patients in his office only before noon on Wednesdays. The decrease in the number of patients that is seen at the clinic on Wednesday is explained by this mobile prevention activity. The pattern is typical, it shows seasonality within a week.

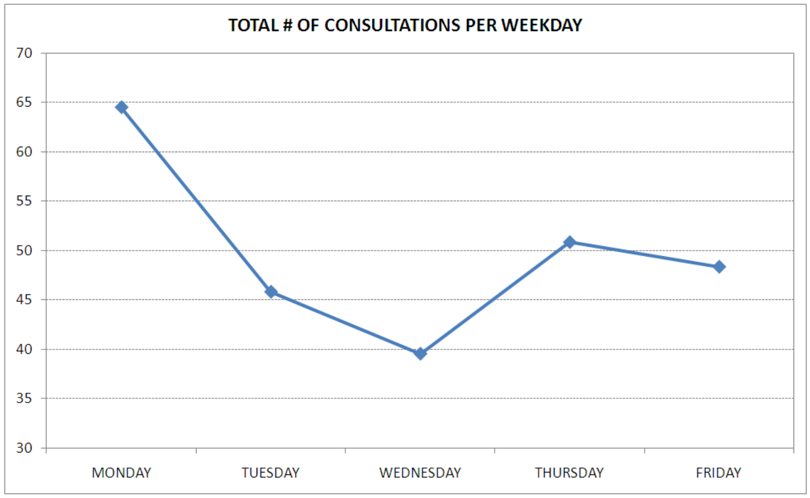

Figure 3.2: Consultation distribution per weekdays

3.2.1 Patient Segmentation

As mentioned in Chapter 1, we will focus on two types of services that is provided by a GP: prevention (mobile or not) and acute care. Dr. X gives importance to preventive service very much and devotes his time to preventive activity as much as possible. However, the data available did not specify patients who received prevention services and therefore, we could not exactly separate patients who took prevention and who did not. Luckily, we were able to classify patients under certain conditions who are provided preventive service accordingly. The classification of those patients are discussed deeply with Dr. X and will be explained.

According to their objectives preventive service can be divided into three groups; primary, secondary and tertiary. Primary preventive service deals with people with no symptoms and targets to decrease the odds of catching a disease. Secondary preventive service deals with people who are at an early stage of disease and targets to treat. Tertiary preventive service deals with people who have chronic diseases and targets to keep illness under control [43]. In other words, preventive service is the actions that are taken for preventing diseases before they show up, early diagnosis and treatment of diseases, and keeping under control of symptomatic illnesses. For this reason, we decided to take obesity, anxiety and hypertension patients as a proxy for the patients with preventive service needs. Obesity and anxiety patients were taking secondary preventive service whereas hypertension patients took tertiary prevention. Those patients are called type 2 patients and going to form the second queue in Section 4.1. Other than those patients are type 1 patients (patients that are provided acute care service) who are going to form the first queue in Section 4.1

For obesity patients prevention activity can be done by offering a special diet and monitoring the progress on health and also by doing this preventing from further diseases. For anxiety patients, the prevention activity can be considered as relaxing the patient by talking about their fears by preventing deep impacts that can be caused by their behaviors and sometimes by guiding them in their life. This will improve their social relations and prevent them from future harms. For hypertension patients, there are tests should be run to trace their health-status. These tests are also done periodically but would result in early diagnosis of other diseases that can be caused by hypertension. By using these type of patients we are willing to monitor the treatment progress. We believe that as prevention activity increases the number of times that they consult to doctor will decrease since they are in treatment progress.

Next, we looked at the arrivals of those patients in order to see if to group those patients as one is reasonable or not. In figure 3.3 patients' total monthly consultations per patient per day that belong to obesity, anxiety, and hypertension groups are shown. Data is from August 2009 to July 2010. The number of patients that are assigned to these three different groups are different than each other hence we normalized total monthly consultations by dividing each total by the number of patients that belong to each group. Thus the graph shows average number of consultations

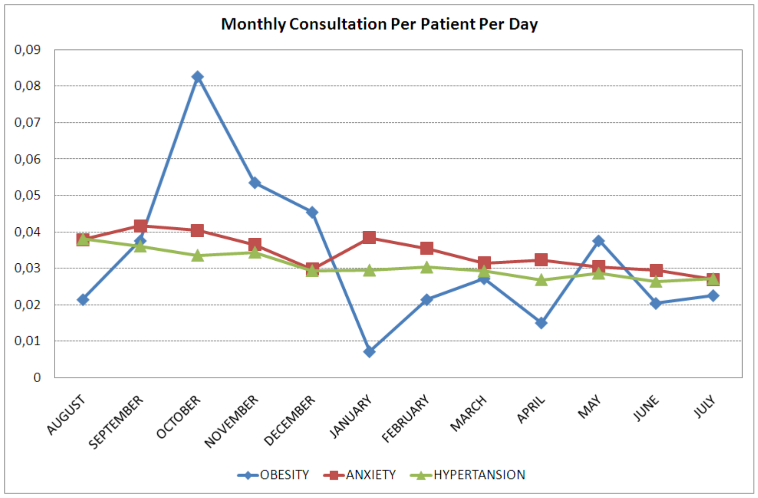

Figure 3.3: Obesity, anxiety, and hypertension groups' monthly consultations per patient per day

per month.

In Figure 3.3 we see close numbers for anxiety and hypertension groups but very different numbers for obesity group. This difference is most probably caused by the difference of numbers of patients that are assigned to obesity group. The patients that are assigned to obesity group are 7 people where anxiety group consists of 168 and hypertension group consists of 414 people. For the obesity group, since the number of people that are registered to this group is very low, the results are not very accurate. In the light of this information, we disregard obesity group and deal with anxiety and hypertension group which is shown in Figure 3.4.

It can be seen from Figure 3.4 that the consultation numbers per patient per day are very close for anxiety and hypertension group. We tested whether there exists a statistical difference between the average consultation numbers related to those groups or not, by performing independent samples t-test for each month. Null hypothesis

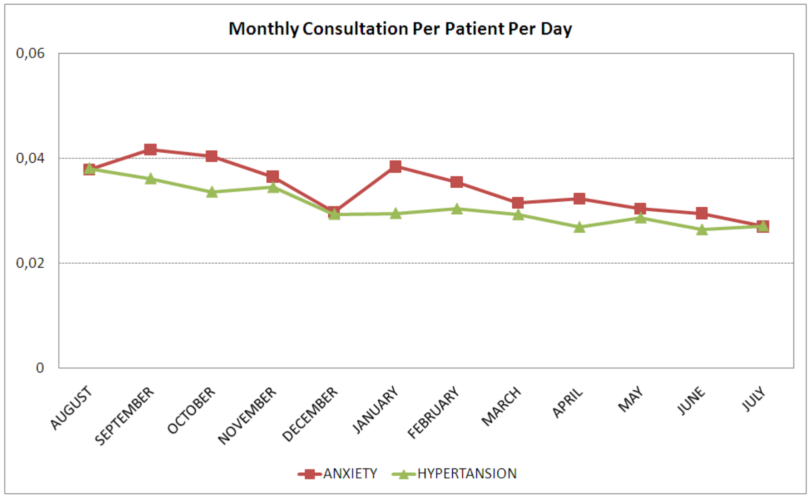

Figure 3.4: Anxiety and hypertension patients' consultation per patient per day

suggests their average consultations are equal. We compared the two graphs month by month. In Table B.2 the results are shown. According to test results, we can say that there is not enough evidence to reject the null hypothesis that the means are equal. So we can group those patients as one and treat them as type 2 patients who need and get preventive activity. From now on we refer to anxiety and hypertension patients as type 2 patients and make inference according to their behavior.

Figure 3.5 shows the average consultation per patient over months for total (all of the patients registered to doctor), type 1 (acute care), and type 2 (prevention) patients. Type 1 patients does not include type 2 patients and we cannot see same pattern in the arrivals of these two different kinds of patients. Hence, it is can be inferred that it is a fair assumption that we separate those patients. In addition, we tested whether the differences in the arrival rates are statistically significant or not. We used independent samples t-test and concluded the differences are significant. The results are shown in Table B.4. Moreover, 'total' reflects the high arrival rates of type 2 patients, the arrival rate for total and type 1 significantly different due to these high

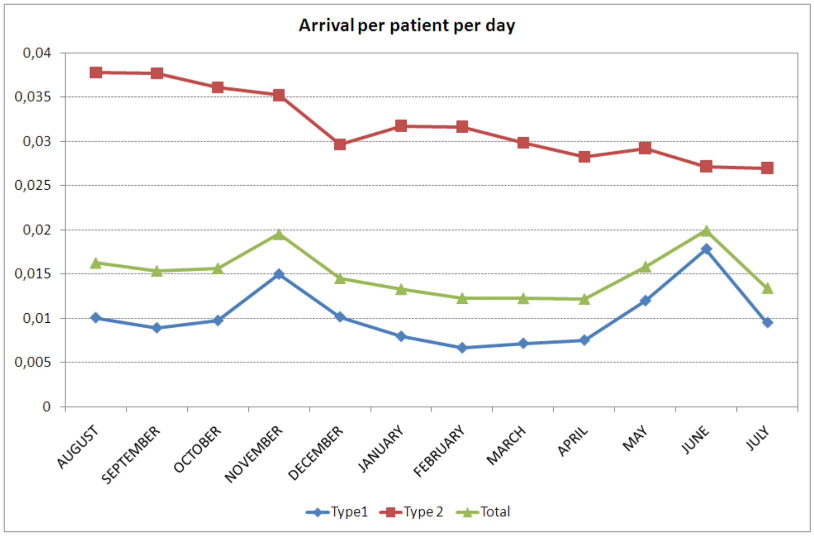

Figure 3.5: Consultations

averages. We can say that total arrival is affected by type 2 patient arrival rate and consider these effects in the models. In conclusion, type 1 and type 2 patients differ in their arrival rates and their arrival patterns. We will focus on the behavior of type 2 patients in the following section, assuming it will affect whole population. We will then use this information when introducing feedback effect in the models in Chapter 4.

3.2.2 Feedback effect

In Figure 3.6 type 2 patients' consultation per month per day numbers are shown. Monthly totals are divided by the work days that belong to that month. Here, there exists a decrease in the monthly consultation. This decrease can be explained by seasonality, going out of town and other kinds of environmental reasons as well as feedback effect due to the preventive service they get. We tested whether the decrease is statistically significant or not. We compared the means of the first (August 2009) and the last (July 2010) averages. We used paired samples t-test and Wilcoxon

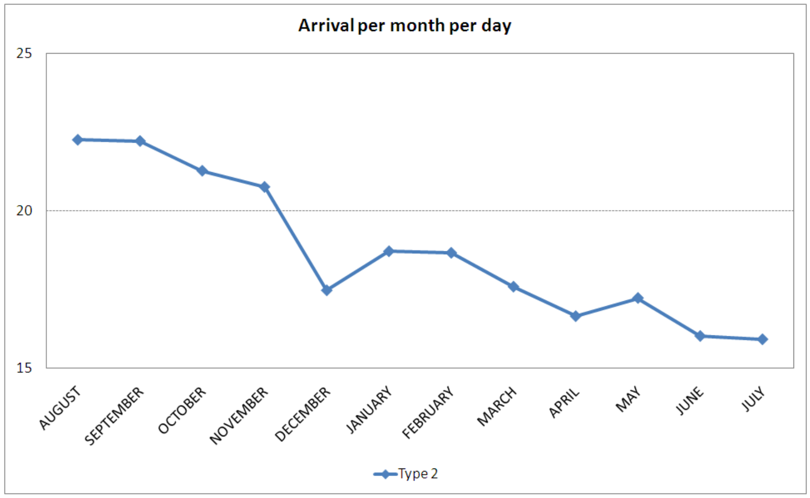

Figure 3.6: Prevention given patients' arrival rates per day over months

signed rank test to analyze before-after comparison. In Table B.5 and Table B.6 the results of the tests are shown. Both tests give the same result that the difference is statistically significant. We can conclude that prevention patients' per month per day consultations have decreased.

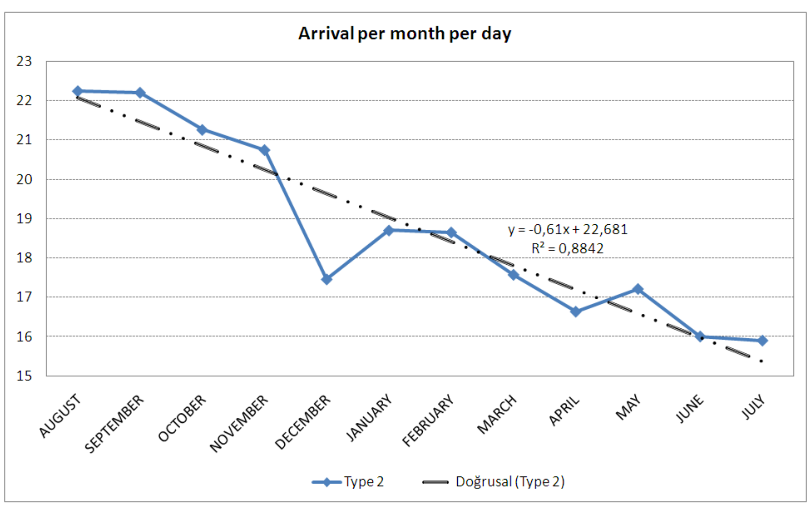

Figure 3.7: Linear Relation

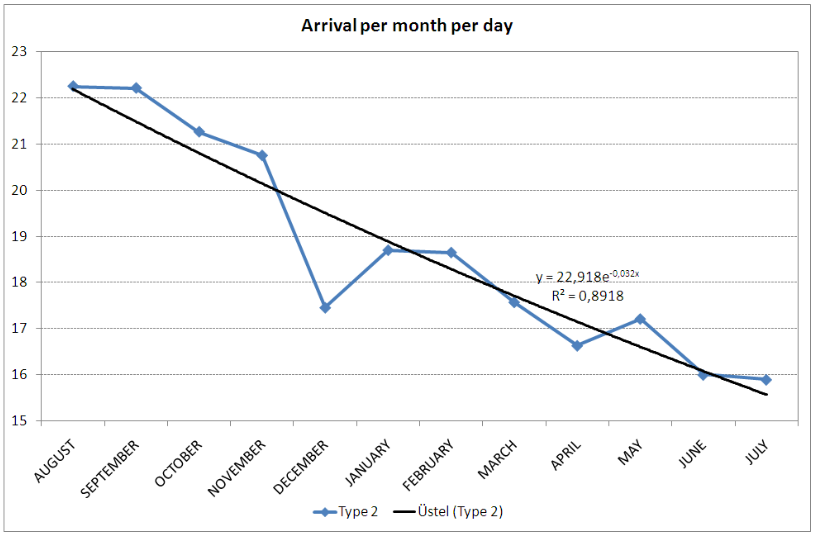

Figure 3.8: Exponential Relation

In this case if we assume the reason of the decrease is feedback effect, it is a fair assumption that the decrease will reflect to whole system. In Dr.X case, it could be said two things; the relationship is nearly linear or is nearly exponential. The relationships are shown in Figure 3.7 and in Figure 3.8.

These figures show very strong relationship between the time amount and the decrease in arrival with very high R-squares (0,8842 and 0,8918 respectively). Here, in Figures 3.7 and 3.8 x represents x-axis which is defined as months. Every month has a number starting from 1. As time goes by, as number of months increases, the arrival rate decreases. It can be considered as, as the cumulative time spent for preventive service increases, arrival rate will decrease. We will use this information in Sections 4.2.1, 4.2.2 and 4.4 when we are modeling feedback function.

Chapter 4

CAPACITY ALLOCATION MODELS

Family doctors have limited time and in that limited time they have to provide many services. Among these tasks; we focus on acute care and prevention in this study. The survey results, which was introduced in Chapter 3, provide an overview of family practice in Turkey and showed that only 6% of GPs use appointment system in practice. Since the percentage is very low, instead of appointment scheduling, we used time/capacity allocation framework to be in line with practice. The time allocation problem of a GP is characterized by queueing approach.

We studied two different queueing approaches. First one is Dedicated Time Slots model which assumes that the acute care and prevention services are provided at different times, and patients arrive specifically for that service. We have two separate queues served by a single server, GP. The demand for each service should be satisfied.

Second one is Prevention as an Add-on Service which assumes that all patients wait in a single queue served by GP. Whether to offer a preventive service or not after each acute care service as an add-on depends on the choice of GP. There is no obligation to satisfy the demand for preventive activity.

We try to find optimal time allocation between preventive activity and acute care in steady-state. In reality, arrival rate of patients could be higher than the service rate of the doctor which means the system is instable, doctor cannot serve all of the patients who are waiting. However, we assume that this is not the case, the system is stable in our models. In addition, when we look from doctor's point of view, provision of preventive service by doctor can depend on the number of patients who are waiting in line. The doctor can choose not to provide prevention if the number of waiting patients is high. Nevertheless, we assume the doctor's choice is independent from the number of waiting patients.

4.1 Dedicated Time Slots Model

In DTS, the demand for acute care and prevention services constitute different queues, the patients of these two different types of service arrive with different rates. They are assumed to be provided in isolation and each demand should be satisfied. This approach is motivated by the results in Section 3.1.2, when you can differentiate the patient arrivals of each type. It is stated that 13.25% of time a GP performs home visits, mobile preventive activity. Moreover, the specific case in Section 3.2 showed that the GP provides preventive activity on Wednesday afternoons. In this context, acute care and preventive activity constitutes different queues and the services are provided at different times.

Moreover, their service times may also be different, and there is a high possibility that their waiting cost per patient is also different. We assume that arrivals follow Poisson process, and service times follow exponential distribution. Wein (1989) [26] model a problem close to ours discussed widely in Chapter 2. It is different in the sense that it optimizes the service rates with a strict constraint on budget.

In DTS model, we analyze optimal capacity allocation decision between two types of queues. First queue represents the patients waiting for acute care whereas the second queue represents the patients waiting for preventive services. The GP, serves acute care patients $100p\%$ of time and in the remaining time serves for prevention.

Table 4.1: DTS Model Parameters and Decision Variable

Model parameters	
λ_1	Arrival rate for patients demanding acute care
λ_2	Arrival rate for patients demanding prevention
$\frac{1}{\mu_1}$	Expected service time for acute care
$\frac{1}{\mu_2}$	Expected service time for prevention
c_1	Waiting cost of acute care patients (per patient, per unit time)
c_2	Waiting cost of prevention patients (per patient, per unit time)
Decision Variable	
p	Percentage of service capacity allocated for acute care

(p represents probability hence percentage is defined as $100p$. Throughout this thesis p will be referred to percentage. To illustrate $p = 0.01$ stands for 1%).

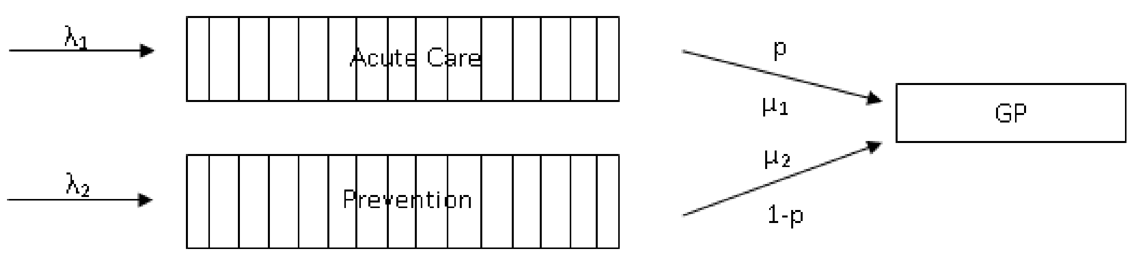

Figure 4.1: General system representation for GP's office arrivals

In a traditional queueing model, service takes place continuously 24 hours a day and 7 days a week. Hence service rate μ and expected service rate $\frac{1}{\mu}$ are defined accordingly. However, a GP works for generally 8 hours in a day and 5 days in a week. Consequently; we need to rescale service rates with a rescaling factor to maintain time continuum [44]. In our model, service rates are inputs, so when we assign values to service rates this rescaling is taken into consideration. However, here we have a GP (a single server) who serves for two different queues. We model the allocation of time into two dedicated portions as splitting the available service capacity into two parallel servers. Again, to maintain time continuum we rescaled service rates with rescaling factors p and $1 - p$ for μ_1 and μ_2 respectively. By doing so, we assure that a GP cannot give two services at the same time. We get average rescaled effective service rates as $p\mu_1$ and $(1-p)\mu_2$ and rescaled effective service times as $\frac{1}{p\mu_1}$ and $\frac{1}{(1-p)\mu_2}$ [44].

p is the portion of time allocated to acute care service as a decision variable. Assuming exponential service and Poisson arrivals, expected total waiting cost (expected number of people in the system in steady state $\times$ cost per patient) can be found as;

$$\frac{c_1\lambda_1}{\mu_1 p - \lambda_1} + \frac{c_2\lambda_2}{\mu_2\left(1-p\right) - \lambda_2}$$

The objective of the GP is to finish the work with minimum waiting for patients. The mathematical representation of the model is:

$$\min_p \frac{c_1\lambda_1}{\mu_1 p - \lambda_1} + \frac{c_2\lambda_2}{\mu_2\left(1-p\right) - \lambda_2}$$

$s.t.$

$$\mu_1 p \geq \lambda_1 + \epsilon_1$$

$$\mu_2(1-p) \geq \lambda_2 + \epsilon_2$$

$$0 < p < 1$$

In above optimization model constraints are the stability conditions for the GP, guarantee that the demand rate should be less than service rate. In addition, it implies that there is an obligation to satisfy each demand. This model is appropriate when there is a need for satisfaction of prevention, like A type prevention which is mentioned in Pollak et al. (2008) [37].

ϵ_1 and ϵ_2 are very small positive numbers that are used to ensure the system stability. The decision variable p, or the percentage of time spent for acute care is defined to be between 0 and 1.

4.1.1 Analysis

This problem is a non-linear optimization problem with linear constraints. Second derivative of total cost function with respect to p is;

$$\frac{d^2\mathrm{TC}}{dp^2} = \frac{2c_1\lambda_1\mu_1^2}{(-\lambda_1 + p\mu_1)^3} + \frac{2c_2\lambda_2\mu_2^2}{(-\lambda_2 + (1-p)\mu_2)^3}$$

This derivative is positive under the assumptions

$$\lambda_1 < \mu_1,\ \lambda_2 < \mu_2,\ \lambda_1,\, \lambda_2,\, \mu_1,\, \mu_2,\, c_1,\, c_2 > 0$$

together with stability constraints

$$(\mu_1 p - \lambda_1 - \epsilon_1) > 0 \textit{ and } \mu_2(1-p) - \lambda_2 - \epsilon_2) > 0$$

It implies total cost function is convex. Since total cost function is convex, we can find unique solution to this non-linear optimization problem which is going to be globally optimal according to KKT conditions. This unique solution should satisfy the following stability constraints:

$$\mu_1 p \geq \lambda_1 + \epsilon_1$$

$$p \geq \frac{\lambda_1 + \epsilon_1}{\mu_1} > 0 \tag{4.1.1}$$

$$\mu_2(1-p) \geq \lambda_2 + \epsilon_2$$

$$p \geq 1 - \frac{\lambda_2 + \epsilon_2}{\mu_2} < 1 \tag{4.1.2}$$

4.1.1 and 4.1.2 are equivalent to the following;

$$\frac{\lambda_1}{\mu_1} < p < 1 - \frac{\lambda_2}{\mu_2} \tag{4.1.3}$$

4.1.3 gives a system stability condition on the model parameters;

$$\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} < 1 \, or \tag{4.1.4}$$

$$\mu_1\mu_2 - \lambda_1\mu_2 - \lambda_2\mu_1 > 0 \tag{4.1.5}$$

To solve this non-linear optimization problem, we used Lagrangian Relaxation. The boundaries of p are disregarded since constraints 4.1.1 and 4.1.2 already satisfy the boundary conditions on p. Hence, the Lagrangian function becomes,

$$L(p, u_1, u_2) = \frac{c_1\lambda_1}{\mu_1 p - \lambda_1} + \frac{c_2\lambda_2}{\mu_2(1-p) - \lambda_2} - u_1(\mu_1 p - \lambda_1 - \epsilon_1) - u_2(\mu_2(1-p) - \lambda_2 - \epsilon_2) \tag{4.1.6}$$

The optimal solution should satisfy

$$\begin{aligned} \frac{\partial L(p, u_1, u_2)}{\partial p} &= 0, \\ u_1 \left(\mu_1 p - \lambda_1 - \epsilon_1\right) &= 0, \\ u_2 \left(\mu_2 \left(1-p\right) - \lambda_2 - \epsilon_2\right) &= 0 \end{aligned}$$

The following proposition describes the solution to the DTS model.

Proposition 4.1. *The optimal solution of the Dedicated Time Slots optimization problem can be one of the following;*

- *Interior Solution*

  If the two queues are identical (i.e, $c_1 = c_2, \mu_1 = \mu_2, \lambda_1 = \lambda_2$) then optimal solution is an interior one and is equal to 1/2. Otherwise, we will assume $c_1\lambda_1\mu_2 \neq c_2\lambda_2\mu_1$, and interior solution is given as;

  $$u_1 = 0, u_2 = 0, p = \frac{\lambda_1\mu_1\mu_2\left(\left(c_1+c_2\right)\lambda_2 - c_1\mu_2\right) + \sqrt{c_1c_2\lambda_1\lambda_2\mu_1\mu_2\left(\lambda_2\mu_1 + \left(\lambda_1-\mu_1\right)\mu_2\right)^2}}{\mu_1\mu_2\left(c_2\lambda_2\mu_1 - c_1\lambda_1\mu_2\right)}$$

- *Boundary Solution 1*

  If only the first constraint is binding, and $\frac{c_1\lambda_1\mu_1}{\epsilon_1^2} < \frac{c_2\lambda_2\mu_2}{\left(\lambda_2 - \mu_2 + \frac{\mu_2(\lambda_1+\epsilon_1)}{\mu_1}\right)^2}$; then the optimal solution is given by

  $$u_2 = 0, u_1 = \frac{-\frac{c_1\lambda_1\mu_1}{\epsilon_1^2} + \frac{c_2\lambda_2\mu_2}{\left(\lambda_2-\mu_2+\frac{\mu_2(\lambda_1+\epsilon_1)}{\mu_1}\right)^2}}{\mu_1}, p = \frac{\lambda_1+\epsilon_1}{\mu_1}$$

- *Boundary Solution 2*

  If only the second constraint is binding, and $\frac{c_2\lambda_2\mu_2}{\epsilon_2^2} < \frac{c_1\lambda_1\mu_1}{\left(\lambda_1 - \frac{\mu_1(-\lambda_2+\mu_2-\epsilon_2)}{\mu_2}\right)^2}$*; then the optimal solution is given by*

$$u_1 = 0, u_2 = \frac{-\frac{c_2\lambda_2\mu_2}{\epsilon_2^2} + \frac{c_1\lambda_1\mu_1}{\left(\lambda_1 - \frac{\mu_1(-\lambda_2+\mu_2-\epsilon_2)}{\mu_2}\right)^2}}{\mu_2}, p = \frac{-\lambda_2 + \mu_2 - \epsilon_2}{\mu_2}$$

According to KKT conditions, for a boundary solution to be optimal, it should satisfy the positivity condition on the Lagrange multipliers u_1 and u_2. In this case if u_1 of boundary solution 1 and u_2 of boundary solution 2 are both negative, the optimal solution is the interior solution. Then, we can write a condition for the solution to be interior:

$$\frac{\epsilon^2\lambda_2\mu_1\mu_2}{\lambda_1\left(\lambda_2\mu_1 + (\epsilon + \lambda_1 - \mu_1)\,\mu_2\right)^2} < \frac{c_1}{c_2} < \frac{\lambda_2\left(\mu_1\left(\epsilon + \lambda_2 - \mu_2\right) + \lambda_1\mu_2\right)^2}{\epsilon^2\lambda_1\mu_1\mu_2}$$

If above condition is satisfied (implying $u_1 < 0$ and $u_2 < 0$), the optimal solution is interior. Since ϵ is a very small number, it makes left hand side of the inequality nearly zero and right hand side of the inequality a very large number. In that case the interval of $\frac{c_1}{c_2}$ becomes very large and in reasonable cost assignments the ratio stays in that interval. As a result; the optimal solution becomes an interior one. For the remaining analysis, we will focus on the interior solution.

4.1.2 Sensitivity Analysis

In this section we will analyze how optimal values of p change with respect to model parameters. We will assume the optimal solution is the interior solution, where the constraints are non-binding. Since the solution is defined as the solution to the first order derivative of the Lagrange function, we use implicit differentiation to find the reaction of p to the changes in parameters. Then we investigate the sign of the derivative that is found by implicit differentiation. The following proposition summarizes the results:

Proposition 4.2. *Comparative Statics for DTS model;*

$$\frac{\partial p^*}{\partial \lambda_1} > 0,\ \frac{\partial p^*}{\partial \lambda_2} < 0,\ \frac{\partial p^*}{\partial \mu_1} < 0,\ \frac{\partial p^*}{\partial \mu_2} > 0,\ \frac{\partial p^*}{\partial c_1} > 0 \text{ and } \frac{\partial p^*}{\partial c_2} < 0$$

p represents the portion of time that should be spent on acute care. As arrival rate or waiting cost of acute care patients increase, ceteris paribus, the portion of time that should be spent on their treatment should also increase to satisfy demand and to lower the waiting cost of them respectively. Whereas as service rate for this patients increases, the portion should decrease. Similarly, as arrival rate or waiting cost of prevention and chronic diseases increase, p should decrease (in that case $(1-p)$, the portion of time that should be spent on prevention and management of chronic diseases increases) to satisfy demand and to lower the waiting cost respectively. Whereas if the service rate for that kind of patients increases ceteris paribus, p should increase too (in that case $(1-p)$, the portion of time that should be spent on prevention and management of chronic diseases decreases). Analytical results are consistent with intuition.

4.2 Dedicated Time Slots Model with Feedback

In the previous model, we assumed that demand from acute care patients follow Poisson distribution with parameter λ_1. Now we consider a case where mean arrival rate rate for acute care depends on the time that is spent on the prevention service. As prevention service increases, we expect a decrease in the arrival rate of acute care patients. There are clinical trials that suggest the arrival rate of patients decrease drastically when prevention is provided [33] (for detailed information see Chapter 2). Furthermore, in Chapter 3 in Figure 3.5 we see a decrease in the arrival rates of prevention patients (type 2 patients) in time.

All of these suggest, as patients get certain prevention, the odds of getting sick should decrease. We call it the feedback effect. Again, in Figure 3.5 we see that the prevention patients' high arrival rates affect whole patients' arrival rates (represented by Total). In that sense, we include the effect in the total arrival rate.

λ_1 is the representation of total arrival rate, assuming prevention patients will join this queue in time for acute needs, after preventive service provided to them. In our DTS with Feedback model, the portion of time that is spent on prevention services is shown by $(1-p)$. In this context, as $(1-p)$ increases λ_1 should decrease. In other words, λ_1 is a decreasing function of $(1-p)$. Equivalently, we model λ_1 as an increasing function of p. In the following, we will analyze different functional forms of λ_1, other model parameters and the decision variable is the same as defined before.

4.2.1 Model Definition: Linear λ_1

In Figure 3.7 we have shown that the decrease in the arrival rate of prevention patients (type 2 patients) can be modeled as a linear function. With the data provided, linear model fit gives $R^2 = 0,8842$. Here, λ_1 is a linear function of p, $\lambda_1 = \lambda_0(1+p)$. We call this model as Dedicated Time Slots Model with Linear Feedback (DTSL). The mathematical representation of this model is:

$$\min_p \frac{c_1\lambda_0(1+p)}{\mu_1 p - \lambda_0(1+p)} + \frac{c_2\lambda_2}{\mu_2\,(1-p) - \lambda_2}$$

$$s.t.$$

$$\mu_1 p \geq \lambda_0(1+p) + \epsilon_1 \tag{4.2.1}$$

$$\mu_2(1-p) \geq \lambda_2 + \epsilon_2 \tag{4.2.2}$$

$$0 < p < 1$$

Here λ_0 is a threshold value which is equivalent to $\frac{\lambda_1}{2}$ in the model without feedback (DTS). So that if $p = 1$ (meaning no prevention) $\lambda_1 = \lambda_1$. Constraints are the stability conditions for the GP, guarantee that the demand rate should be less than service rate. ϵ_1 and ϵ_2 are very small positive numbers that are used to make the system stability and utilization less than 1 certain. The decision variable p, or the percentage of time spent for acute care is defined to be between 0 and 1.

4.2.1.1 Analysis

This problem is again a non-linear optimization problem with linear constraints; similar to our DTS model, hence solution technique is similar too. The boundaries of p are disregarded since constraint 4.2.1 and 4.2.2 already satisfy the boundary conditions on p. Then the optimization problem is defined as;

$$\min_{p} \frac{c_1\lambda_0(1+p)}{\mu_1 p-\lambda_0(1+p)}+\frac{c_2\lambda_2}{\mu_2\left(1-p\right)-\lambda_2}$$

s.t.

$$\mu_1 p \geq \lambda_0(1+p)+\epsilon_1$$

$$\mu_2(1-p) \geq \lambda_2+\epsilon_2$$

Second derivative of TC with respect to p is;

$$\frac{d^2\mathrm{TC}}{dp^2}=\frac{2c_1\lambda_0\left(\lambda_0-\mu_1\right)\mu_1}{\left(\left(1+p\right)\lambda_0-p\mu_1\right)^3}-\frac{2c_2\lambda_2\mu_2^2}{\left(\lambda_2+\left(-1+p\right)\mu_2\right)^3}>0$$

This derivative is positive under the assumptions

$$\lambda_0<\mu_1,\ \lambda_2<\mu_2\ \lambda_0,\ \lambda_2,\ \mu_1,\ \mu_2,\ c_1,\ c_2>0$$

together with stability constraints:

$$\mu_1 p-\lambda_0(1+p)-\epsilon_1 \geq 0,\ \mu_2(1-p)-\lambda_2-\epsilon_2 \geq 0$$

It implies that total cost function is convex. Since total cost function is convex, we can find the unique solution to this non-linear optimization problem which is globally optimal according to KKT conditions. This unique solution should satisfy the following stability constraints:

$$\mu_1 p>\lambda_0\left(1+p\right) \tag{4.2.3}$$

$$\mu_2\left(1-p\right)>\lambda_2 \tag{4.2.4}$$

4.2.3 and 4.2.4 are together equivalent to the following;

$$\frac{\lambda_0}{\mu_1 - \lambda_0} < p < \frac{\mu_2 - \lambda_2}{\mu_2}$$

This also gives a system stability condition on the model parameters;

$$\frac{\lambda_0}{\mu_1 - \lambda_0} < 1 - \frac{\lambda_2}{\mu_2}$$

equivalently;

$$\mu_2\mu_1 - \lambda_2\mu_1 - 2\lambda_0\mu_2 + \lambda_2\lambda_0 > 0 \tag{4.2.5}$$

To solve the non-linear optimization problem, we used Lagrangian Relaxation. The Lagrange function becomes,

$$L(p, u_1, u_2) = \frac{c_1\lambda_0(1+p)}{\mu_1 p - \lambda_0(1+p)} + \frac{c_2\lambda_2}{\mu_2(1-p) - \lambda_2} - u_1(\mu_1 p - \lambda_0(1+p) - \epsilon_1) - u_2(\mu_2(1-p) - \lambda_2 - \epsilon_2) \tag{4.2.6}$$

The optimal solution should satisfy

$$\begin{aligned} \frac{\partial L(p, u_1, u_2)}{\partial p} &= 0, \\ u_1 \left(\mu_1 p - \lambda_0(1+p) - \epsilon_1\right) &= 0, \\ u_2 \left(\mu_2 \left(1-p\right) - \lambda_2 - \epsilon_2\right) &= 0 \end{aligned}$$

The following proposition describes the optimal solution to the Dedicated Time Slots Model with Linear Feedback model.

Proposition 4.3. *The optimal solution of the DTSL optimization problem can be one of the following;*

- *Interior Solution:*
  Solution to the first order derivative of Lagrange function

$$u_1 = 0,\ u_2 = 0,$$
$$p = \frac{\lambda_0\mu_2\left(c_2\lambda_2\left(\mu_1 - \lambda_0\right) - c_1\mu_1\left(\mu_2 - \lambda_2\right)\right) + \sqrt{c_1c_2\lambda_0\lambda_2\mu_1\mu_2\left(\lambda_0\left(\lambda_2 - 2\mu_2\right) + \mu_1\left(-\lambda_2 + \mu_2\right)\right)^2}}{\mu_2\left(c_2\lambda_2\left(\lambda_0 - \mu_1\right)^2 - c_1\lambda_0\mu_1\mu_2\right)}$$

- *Boundary solution 1:*

  If only the first constraint is binding, and

$$\frac{c_2\lambda_2\mu_2}{\left(\lambda_2-\mu_2+\frac{\mu_2(-\lambda_0-\epsilon_1)}{\lambda_0-\mu_1}\right)^2} > \frac{c_1\lambda_0\mu_1}{\left(\lambda_0+\frac{\lambda_0(-\lambda_0-\epsilon_1)}{\lambda_0-\mu_1}-\frac{\mu_1(-\lambda_0-\epsilon_1)}{\lambda_0-\mu_1}\right)^2}$$

  (the condition that $u_i's \geq 0$); then the optimal solution is given by

$$u_2 = 0,\ u_1 = \frac{-\frac{c_1\lambda_0\mu_1}{\left(\lambda_0+\frac{\lambda_0(-\lambda_0-\epsilon_1)}{\lambda_0-\mu_1}-\frac{\mu_1(-\lambda_0-\epsilon_1)}{\lambda_0-\mu_1}\right)^2}+\frac{c_2\lambda_2\mu_2}{\left(\lambda_2-\mu_2+\frac{\mu_2(-\lambda_0-\epsilon_1)}{\lambda_0-\mu_1}\right)^2}}{-\lambda_0+\mu_1},$$

$$p = \frac{-\lambda_0-\epsilon_1}{\lambda_0-\mu_1}$$

- *Boundary solution 2:*

  If only the second constraint is binding, and

$$\frac{c_1\lambda_0\mu_1}{\left(\lambda_0+\frac{\lambda_0(-\lambda_2+\mu_2-\epsilon_2)}{\mu_2}-\frac{\mu_1(-\lambda_2+\mu_2-\epsilon_2)}{\mu_2}\right)^2} > \frac{c_2\lambda_2\mu_2}{\epsilon_2^2}$$

  (the condition that $u_i's \geq 0$); then the optimal solution is given by

$$u_1 = 0,\ u_2 = \frac{\frac{c_1\lambda_0\mu_1}{\left(\lambda_0+\frac{\lambda_0(-\lambda_2+\mu_2-\epsilon_2)}{\mu_2}-\frac{\mu_1(-\lambda_2+\mu_2-\epsilon_2)}{\mu_2}\right)^2}-\frac{c_2\lambda_2\mu_2}{\epsilon_2^2}}{\mu_2},$$

$$p = \frac{-\lambda_2+\mu_2-\epsilon_2}{\mu_2}$$

According to KKT conditions a boundary solution to be optimal it should satisfy the positivity condition on the Lagrange multipliers u_1 and u_2. In this case if u_1 of boundary solution 1 and u_2 of boundary solution 2 are both negative, the optimal solution is an interior solution. This implies if

$$\frac{\epsilon^2\lambda_2(\lambda_0-\mu_1)^2\mu_2}{\lambda_0\mu_1(\lambda_2(-\lambda_0+\mu_1)+(\epsilon+2\lambda_0-\mu_1)\mu_2)^2} < c_1/c_2 < \frac{\lambda_2((\epsilon+\lambda_2)(\lambda_0-\mu_1)+(-2\lambda_0+\mu_1)\mu_2)^2}{\epsilon^2\lambda_0\mu_1\mu_2}$$

condition on costs hold, the optimal solution is interior. For very small ϵ the solution is interior. For the remaining we will focus only on interior solution.

4.2.1.2 Sensitivity Analysis

In this section we will analyze how optimal solution p^* of the DTS with Linear Feedback model change with respect to model parameters [1]. Similar to DTS, interior solution is the root of first order derivative of Lagrange function related to the expected total waiting cost function. As a result, implicit differentiation is used. The following proposition summarizes the results:

Proposition 4.4. *Comparative Statics for DTSL;*

$$\frac{\partial p^*}{\partial \lambda_0} > 0,\ \frac{\partial p^*}{\partial \lambda_2} < 0,\ \frac{\partial p^*}{\partial \mu_1} < 0,\ \frac{\partial p^*}{\partial \mu_2} > 0,\ \frac{\partial p^*}{\partial c_1} > 0\, and\, \frac{\partial p^*}{\partial c_2} < 0$$

We expected to find parallel results with DTS and our expectations are met. Linear λ_1 is a reasonable approach in the sense that it does not violate convexity assumption of total cost function and the sensitivity results are logical.

4.2.2 Model definition: Exponential λ_1

In figure 3.8, we have shown that the decrease in the arrival rate of prevention patients (type 2 patients) can be modeled as an exponential. With the data provided, linear model fit gives $R^2 = 0,8918$. Here, we define $\lambda_1 = \frac{\lambda_0}{e^{1-p}}$ which is increasing in p, percentage of time allocated to acute care service and decreasing in $(1-p)$, percentage of time allocated to preventive service. We call this model as Dedicated Time Slots model with Exponential Feedback (DTSE)

In the case of linear λ_1 in DTSL, we were able to find closed form solutions to our non-linear cost minimization optimization problem. Unlike linear model, we were not able to find closed form solutions since we have non-linear objective function and non-linear constraints. However, if we do numerical analysis we are able to find an

[1]Model constraints and assumption on model parameters are taken into consideration when analyzing the sensitivity of decision variable p

optimal solution by search algorithms if cost function is convex. Then the problem becomes convex optimization with convex constraints.The mathematical representation of DTSE is:

$$\min_{p} \frac{c_1 \frac{\lambda_0}{e^{1-p}}}{\mu_1 p - \frac{\lambda_0}{e^{1-p}}} + \frac{c_2 \lambda_2}{\mu_2 (1-p) - \lambda_2}$$

$$s.t.$$

$$\mu_1 p - \frac{\lambda_0}{e^{1-p}} - \epsilon_1 > 0$$

$$\mu_2(1-p) - \lambda_2 - \epsilon_2 > 0$$

$$0 < p < 1$$

Here λ_0 is a threshold value which is equivalent to λ_1 in the model without feedback.

4.2.2.1 Convexity of the total cost function

If the second derivative of the cost function with respect to p is positive then the cost function is convex and we can find a minimizer.

$$\frac{d^2\mathrm{TC}}{dp^2} = \frac{e^{1+p} c_1 \lambda_0 \mu_1 \left(e^p (-2+p) \lambda_0 + e (2 + (-2+p) p) \mu_1\right)}{\left(ep\mu_1 - e^p \lambda_0\right)^3} + \frac{2 c_2 \lambda_2 \mu_2^2}{\left((1-p) \mu_2 - \lambda_2\right)^3} > 0$$

Second derivative of the cost function in the case $\lambda_1 = \frac{\lambda_0}{e^{1-p}}$ is always positive implying there exists a minimum point.

4.2.3 Model Definition: General case, λ_1 as a convex increasing function of p

We saw that defining λ_1 as a linear increasing function of p is reasonable. We can generalize this result to specific function. Any function that is monotonically increasing and convex, would satisfy our expectations under some conditions. As long as total cost function remains convex we can find optimal solutions using non-linear optimization techniques. The solution would be harder to find for non-linear functions of λ_1 since stability constraints would become non-linear too. In that case, we are not able to solve the optimization problem as we solved before. For simple non-linear constraints, we may be able to find optimal solutions. Otherwise, we will use search algorithms to find the optimal solution.

4.2.3.1 Convexity of the total cost function

Assume λ_1 is an increasing function of p which is shown by $\lambda_1(p)$. Also assume $\lambda_1(p)$ is convex. These implies $\lambda_1'(p) > 0$ and $\lambda_1''(p) > 0$. Under these assumptions total cost function becomes;

$$\text{TC}=\frac{c_1\lambda_1(p)}{\mu_1 p - \lambda_1(p)} + \frac{c_2\lambda_2}{\mu_2\,(1-p) - \lambda_2}$$

If total cost function above is convex, then we can find a minimizer. The second derivative of the total cost function is;

$$\frac{d^2\text{TC}}{dp^2} = \frac{c_1\mu_1\left(-2\lambda_1'(p)^2 p - 2\lambda_1(p)\mu_1 + \lambda_1''(p)p\left(\lambda_1(p) - p\mu_1\right) + 2\lambda_1'(p)\left(\lambda_1(p) + p\mu_1\right)\right)}{\left(\lambda_1(p) - p\mu_1\right)^3} - \frac{2c_2\lambda_2\mu_2^2}{\left(\lambda_2 + (-1+p)\,\mu_2\right)^3}$$

The sufficient condition for convexity of expected total cost function is;

$$\frac{c_1\mu_1(2\lambda_1'(p)^2 p + 2\lambda_1(p)\mu_1 + \lambda_1''(p)p(p\mu_1 - \lambda_1(p))}{(p\mu_1 - \lambda_1(p))^3} + \frac{2c_2\lambda_2\mu_2^2}{((1-p)\mu_2 - \lambda_2)^3} > \frac{2\lambda_1'(p)(\lambda_1(p) + p\mu_1))}{(p\mu_1 - \lambda_1(p))^3} \tag{4.2.7}$$

If the condition in equation 4.2.7 is satisfied, the total cost function is convex and we will be able to find a minimizer (optimal minimum) to non-linear optimization problem by using suitable search algorithms or non-linear optimization techniques.

4.3 Numerical Analysis for DTS, DTSL, and DTSE

In this section, we are going to compare numerical results of three models. First model has no feedback effect (DTS), second model has linear feedback effect (DTSL), and the third model has exponential feedback effect (DTSE). Analysis are done for 43150 scenarios with different parameters, which are feasible for all three models. In

Table 4.2 the range of the parameters are shown. For DTSL, λ_0 is set to $\frac{\lambda_1}{2}$ in order to make a fair comparison with other models. The percentage of time that should be spent on acute care and prevention are compared as well as the costs. In addition, sensitivity of DTSE can be investigated regarding different model parameters.

Table 4.2: Range of Parameters

Parameter	Range	Parameter	Range
λ_1	11-30	μ_2	76-100
λ_2	20-40	c_1	1-20
μ_1	50-77	c_2	1-5

Sensitivity analysis of DTS and DTSL were done in Sections 4.1.2 and 4.2.1.2 respectively. However, we could not find a closed form solution for DTSE. We were not able to determine the reaction to changes in model parameters. The graphs below will help to gain an intuition about the third model's response.

Figures 4.2 and 4.3 represents the behavior of p, percentage of time that should be allocated to acute care service, to the changes in waiting costs. c_1 and c_2 are waiting costs assigned to acute care patients and prevention patients respectively. For all three models as c_1, the waiting cost of acute care patients, increases; p increases and $1-p$ decreases accordingly to lower the expected total waiting cost. As c_2, the waiting cost of prevention patients, increases; p decreases and $1-p$ increases accordingly to lower the expected total waiting cost.

Figure 4.4 and 4.5 shows the response of p to the changes in service rates. μ_1 and μ_2 are the pre-determined service rates for acute care and prevention respectively. They can be considered as average service rates when only one task is performed. For

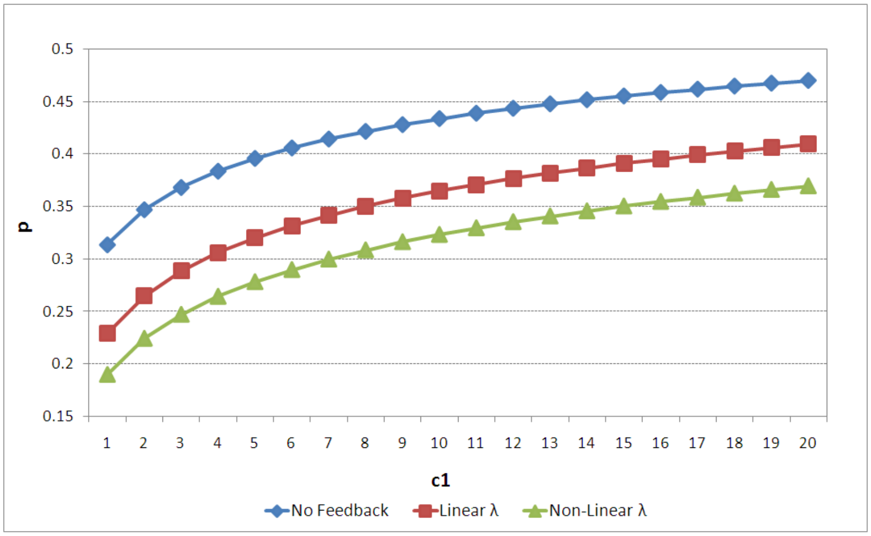

Figure 4.2: Reaction of p to the changes in waiting cost of acute care patients $\lambda_1 = 14$, $\lambda_2 = 30$, $\mu_1 = 75$, $\mu_2 = 80$, $c_2 = 3$

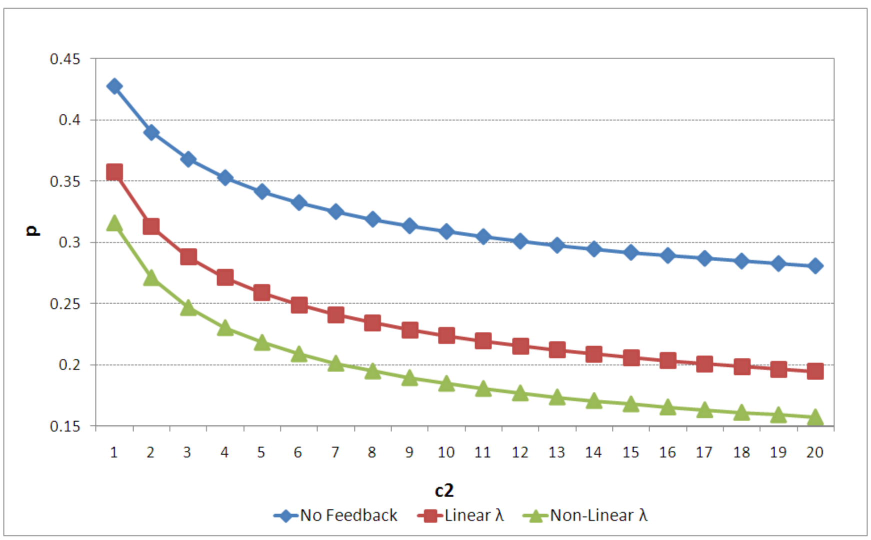

Figure 4.3: Reaction of p to the changes in waiting cost of prevention patients $\lambda_1 = 14$, $\lambda_2 = 30$, $\mu_1 = 75$, $\mu_2 = 80$, $c_1 = 3$

all models, as μ_1 increases p decreases and as μ_2 increases p increases.

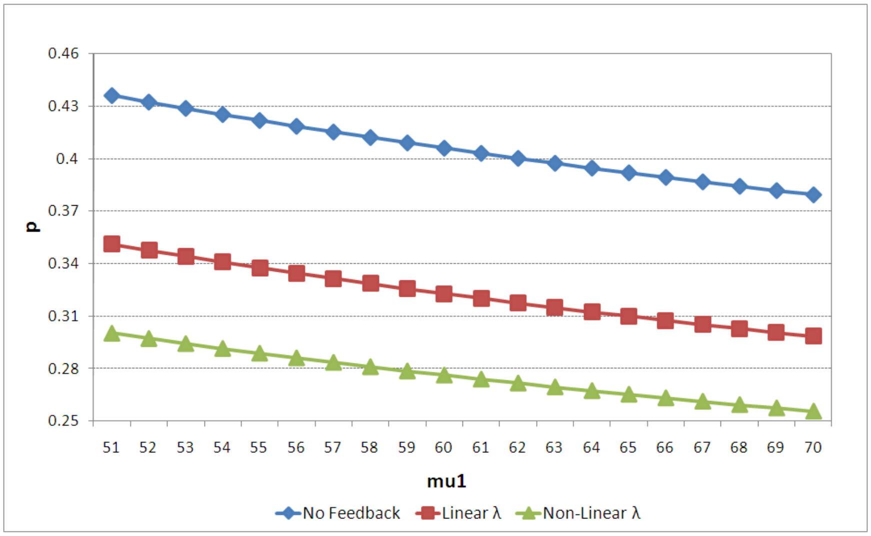

Figure 4.4: Reaction of p to the changes in service rate of acute care patients $\lambda_1 = 14$,$\lambda_2 = 30$, $\mu_2 = 80$, $c_1 = 1$, $c_2 = 1$

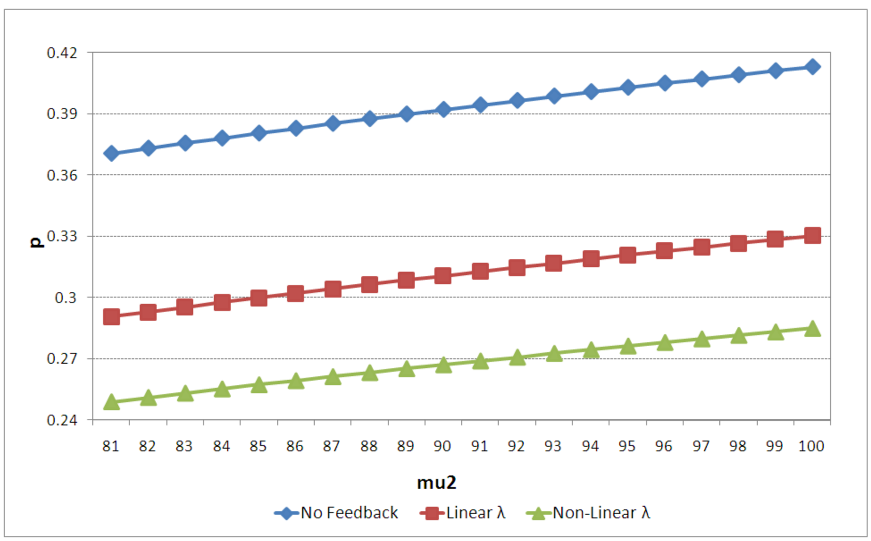

Figure 4.5: Reaction of p to the changes in service rate of prevention patients $\lambda_1 = 14$, $\lambda_2 = 30$, $\mu_1 = 75$, $c_1 = 1$, $c_2 = 13$

Figure 4.6 and 4.7 shows the changes in p due to changes in demand. As λ_1, arrival rate for acute care patients, increases ceteris paribus; p increases to satisfy demand.

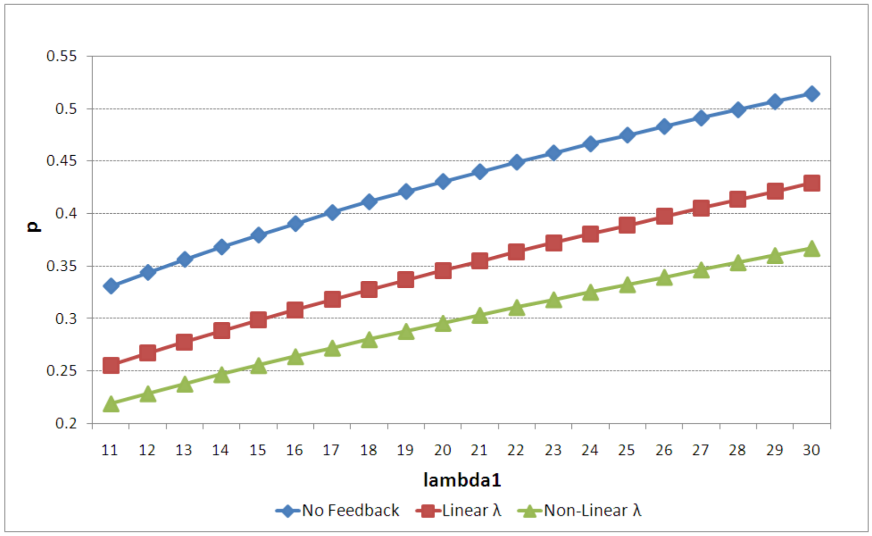

Figure 4.6: Reaction of p to the changes in arrival rate of acute care patients $\lambda_2 = 30$, $\mu_1 = 75$, $\mu_2 = 80$, $c_1 = 1$, $c_2 = 1$

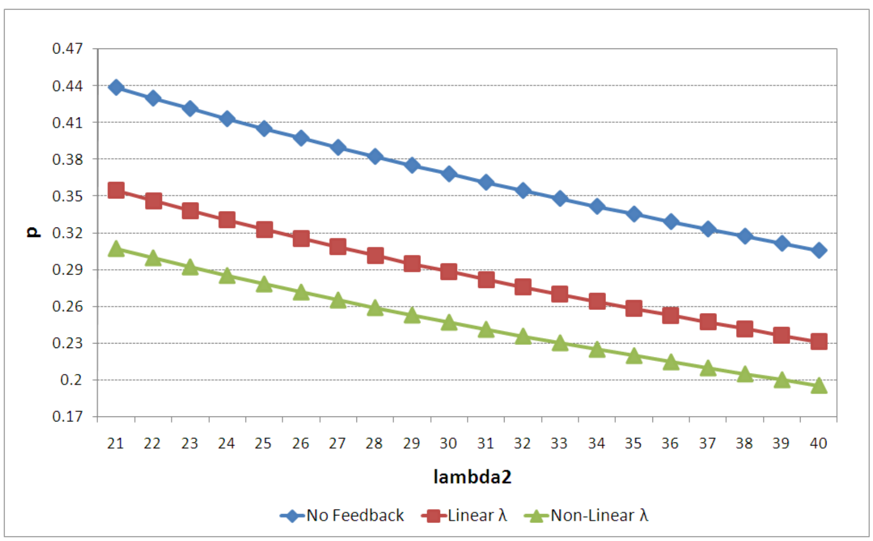

Figure 4.7: Reaction of p to the changes in arrival rate of prevention patients $\lambda_1 = 14$, $\mu_1 = 75$, $\mu_2 = 80$, $c_1 = 1$, $c_2 = 1$

$1 - p$ decreases accordingly. As λ_2, arrival rate for prevention patients, increases ceteris paribus; p decreases and $1 - p$ increases accordingly to satisfy demand from prevention patients.

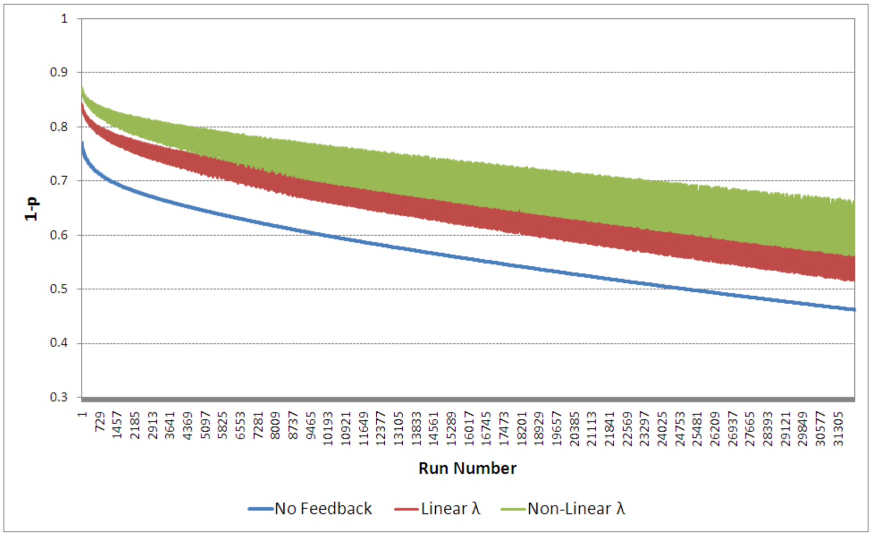

Figure 4.8: Percentage of time that should be allocated to prevention service for three models

Figure 4.8 shows the comparison of prevention service between the three models. The percentage of time that should be allocated to prevention service is lower for model with no feedback than the model with linear feedback and it is lower for the model with linear feedback than the model with exponential feedback. In feedback effect context, we can say that exponential feedback is more effective on arrival rate than the linear feedback effect, it decreases more fast. In other words, the percentage of time that should be allocated to prevention service is more responsive to exponential feedback. Strongest feedback effect is related to third model, consequently; the trade of between service time and arrival rate is higher. We can say that, as feedback effect becomes stronger, more time is spent on prevention.

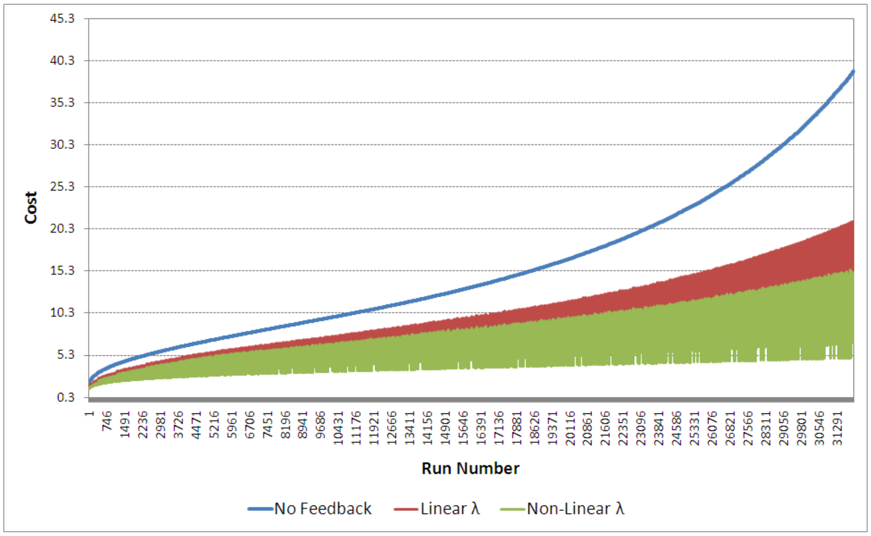

Figure 4.9: Waiting cost comparison for three models

Figure 4.9 shows the comparison of costs between three models. Cost is higher for model without feedback than the model with linear feedback and it is higher for the model with linear feedback than the model with exponential feedback. We can conclude that, when feedback effect on the arrival rates gets stronger the system will devote more time to that service. When more time is devoted to that service, the operational costs will lower in the long run.

4.4 Prevention as an Add-on Service

In first approach, DTS, we defined two types of patients joining two separate queues. First queue is for acute care patients and second queue is for prevention patients. We found how much time should be allocated for those two different kinds of queues. In addition, we assume prevention will reduce sickness rate. Overall, total arrival rate will decrease.

For mobile preventive activity of GP, assuming there exists another queue is applicable. However, prevention can be provided in the clinic too. Keeping in mind that on the average, only 13.25% of time GP serves for mobile preventive activity, we need to build a model which includes time dedicated to prevention at the clinic. In reality, such a model is more applicable according to the service provision process in practice.

Our second approach, Prevention as an Add-on service (PAS), assumes prevention service is provided to some patients. Every patient joins the same queue and the doctor decides whether to fulfill only the acute care need or to offer some preventive service after the acute care as an add-on, resulting in no obligation for demand satisfaction for prevention. In this case, model is a single queue single server model in which the service provider offers two types of services. Due to time allocation to prevention service, feedback is included in the model.

In [41] there exists a similar model which assumes all patients join to a single queue but all of them gets preventive service. Moreover, a feedback mechanism on both arrival rate and service time is developed; however, they assumed that service time distribution is exponential. In PAS, we assumed all patients join a single queue, arriving according to a Poisson process to the system and the doctor offers two types of service; short or long. In that case, service time distribution is no longer exponential. Short service stands for the service provided to satisfy the coming need of the patient, long service stands for the service if prevention as an add-on is provided.

As percentage of time that is allocated to long service increases, meaning time devoted to prevention service increases, arrival rate to the system will decrease considering the feedback effect; i.e arrival rate is a decreasing function of the time allocated to it. Doctor has to decide what percentage of time he should offer prevention. The objective is to minimize waiting cost.

4.4.1 Model Definition

Patients arrive to the system according to Poisson process. Doctor serves with a service rate μ_1 with probability $1-p$ referring to short service or serves with a service rate μ_2 with probability p referring to long service. Assume both service time distributions follow exponential distribution. As decision variable p, the portion of time that is allocated to prevention service, increases; arrival rate decreases as a consequence of the feedback effect. In other words $\lambda(p)$ is decreasing in p.

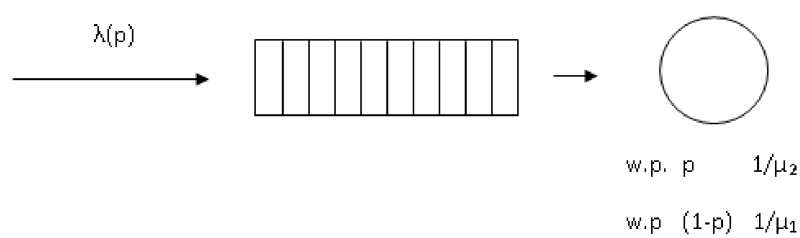

Figure 4.10: Model Representation

Assume; $X_1 \sim EXPO(\mu_1)$ and $X_2 \sim EXPO(\mu_2)$ represents short and long service time distributions respectively. This implies the condition $\mu_1 > \mu_2$ on service rates. Total service time distribution takes the value X_2 with probability p and X_1 with

Table 4.3: Model Parameters and The Decision Variable

Model Parameters	
$\lambda(p)$	Arrival rate of patients (decreasing function of the decision variable)
$\frac{1}{\mu_1}$	Average service time for short service
$\frac{1}{\mu_2}$	Average service time for long service
c	Waiting cost (per patient per unit time)
Decision variable	
p	Percentage of patients who are offered prevention service

probability $(1-p)$. In other words if we define a new random variable (Y) for the total service time of the system, its probability density function is represented by

$$f_Y(y) = (p)f_{X_2}(x_2) + (1-p)f_{X_1}(x_1) \tag{4.4.1}$$

where X_1, X_2 are exponentially distributed random variables with rates μ_1 and μ_2, and $1-p$ and p are the probabilities that Y will take on the form of the exponential distribution with rate μ_1 or μ_2. Random variable Y is hyper-exponentially distributed. The system becomes M/G/1, and we can use Pollaczek-Khinchin formula to find average number of people in the system. According to P-K formula;

$$L = \frac{\lambda(p)^2 E(Y^2)}{2(1-\lambda(p)E(Y))} + \lambda(p)E(Y)$$

where

$$E(Y) = \frac{(1-p)}{\mu_1} + \frac{p}{\mu_2} \tag{4.4.2}$$

$$= \frac{\mu_2(1-p) + p\mu_1}{\mu_1\mu_2} \tag{4.4.3}$$

$$E(Y^2) = \frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2} \tag{4.4.4}$$

$E(Y)$ and $E(Y^2)$ are calculated by using properties of the hyperexponential distribution. Objective function is to minimize the expected total cost which is found by substituting $E(Y)$ and $E(Y^2)$ values in P-K formula and multiplying it by c, waiting cost per patient per unit time. The mathematical representation of the model is:

$$\min_p \frac{c\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)}{2\left(1-\lambda(p)\left(\frac{(1-p)}{\mu_1}+\frac{p}{\mu_2}\right)\right)}+c\lambda(p)\left(\frac{(1-p)}{\mu_1}+\frac{p}{\mu_2}\right)$$

$$s.t.$$

$$1-\lambda(p)\left(\frac{(1-p)}{\mu_1}+\frac{p}{\mu_2}\right)-\epsilon\geq 0$$

$$0\leq p\leq 1$$

where $\lambda(p)$ is a decreasing function of the decision variable p.

4.4.2 Analysis: General $\lambda(p)$

For further analysis for simplicity, assume that cost function is characterized by the number of people in the queue and the cost per patient is unit cost, ($c=1$). In that case, expected total cost becomes;

$$L(p) = \frac{\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)}{2\left(1-\lambda(p)\left(\frac{(1-p)}{\mu_1}+\frac{p}{\mu_2}\right)\right)}$$

$$L(p) = \frac{\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)}{2\left(1-\rho(p)\right)}$$

where $\rho(p)$ is the utilization. Since we cannot determine the convexity of this cost function, we analyzed the behavior of nominator and denominator. We would like to determine the sufficient conditions where expected total cost is monotone in p which makes a boundary solution optimal.

An 'all policy' is where $p^*=1$ whereas a 'nothing policy' is where $p^*=0$. The following propositions describe the conditions of all or nothing policy for PAS.

Proposition 4.5. *If* $\frac{|\lambda'(p)|}{\lambda(p)} > \frac{\mu_1^2-\mu_2^2}{2\mu_2^2}$*; optimal solution is given by* $p = 1$*.*

Proposition 4.6. *If* $\frac{|\lambda'(p)|}{\lambda(p)} < \frac{\mu_1^2-\mu_2^2}{2\mu_1^2}$*; optimal solution is given by* $p = 0$*.*

If nominator of the total cost function is decreasing whereas the denominator is increasing for all p, total cost will always decrease. Proposition 4.5 describes these sufficient conditions for a monotone decreasing total cost. This implies the optimal solution will be achieved at maximum value of p. Increasing denominator implies decreasing $\rho(p)$. If utilization function (which is always positive by definition) is always decreasing, Equation 4.4.5, stability condition, is always satisfied for $p = 1$, which in that case is the optimal solution.

Conversely, Proposition 4.6 describes the sufficient conditions for a monotone increasing total cost with the reverse logic mentioned above, increasing nominator and decreasing denominator for all p, which implies the optimal solution will be achieved at minimum value of p. Decreasing denominator implies increasing $\rho(p)$. Assuming $p = 0$ is feasible, which means no prevention activity is allowed, Equation 4.4.6 specifies minimum value of p is 0, which in that case is the optimal solution.

In addition, Proposition 4.5 states that 'all policy' is optimal when relative change of $\lambda(p)$ with respect to p is greater than a specific ratio which is a function of the magnitudes of service rates. The smaller the ratio is, the easier the fulfillment of the condition might be. As μ_1, the expected service rate for acute care service, decreases and μ_2, the expected service rate for prevention, increases; the ratio gets smaller in which case the condition is more likely to be satisfied. This is an expected result. Decrease in μ_1 implies increase in expected service time, $E(Y)$ (Equation 4.4.2), system will assign less time devotion to that service; whereas increase in μ_2 implies decrease in expected service time, system will assign much time devotion to that service. Consequently, these effects on the system will force p to take larger values, and when the condition is satisfied p takes its maximum value which is 1.

A similar situation occurs in Proposition 4.6. 'Nothing policy' is optimal when relative change of patient arrival function is smaller than a specific ratio which is a function of μ_1 and μ_2, expected service rates of acute care and prevention services respectively. As the value of the ratio increases, the satisfaction of the condition will be easier. We can rewrite the ratio as $\frac{1}{2} - \frac{\mu_2^2}{2\mu_1^2}$. It is obvious that if μ_2 decreases and μ_1 increases the ratio will increase and the condition is more likely to be satisfied. This is again an expected result. As expected service rate of acute care patients increases; expected service time, $E(Y)$, will decrease and much time will be devoted to that service, whereas when service rate of prevention patients, μ_2, decreases; $E(Y)$ will increase and less time will be assigned. p, the portion of time that is devoted to prevention service will be forced to decrease and when the condition is satisfied, takes its minimum value which is 0.

4.4.3 Analysis: $\lambda(p) = \lambda_0(2 - kp)$

Assume total cost function as just waiting time with unit cost, $\lambda_0 < \mu_2 < \mu_1$ and $\lambda(p) = \lambda_0(2 - kp)$ where $0 < k < 2$ [2] (to make sure $\lambda(p)$ is a positive function). In that case, the cost function becomes;

$$L(p) = \frac{(2-kp)^2\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)}{2\left(1 - \frac{(2-kp)\lambda_0(p\mu_1+(1-p)\mu_2)}{\mu_1\mu_2}\right)}$$

$$L(p) = \frac{(2-kp)^2\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)}{2\left(1 - \rho(p)\right)}$$

Since we cannot determine the convexity of this cost function, we would like to determine where all or nothing policy is optimal.

[2] k is a positive constant which can be assumed as effect of prevention. This constant's derivation will be explained in Section 4.7

The following propositions specify the conditions, presented in Section 4.4.2 as Proposition 4.5 and 4.6, for pre-determined linear arrival function.

Proposition 4.7. *If* $\frac{\mu_1}{\sqrt{1+k}} \leq \mu_2 < \mu_1$*; optimal solution* $p = 1$.

Proposition 4.8. *If* $0 < \mu_2 \leq \mu_1 \sqrt{\frac{2-3k}{2-k}}$ *and* $0 < k < \frac{2}{3}$*; optimal solution is given by* $p = 0$.

Proposition 4.7 and Proposition 4.8 are the sufficient conditions for monotone decreasing and monotone increasing total cost function respectively, in which cases all or nothing policy is optimal. Here, we have pre-determined patient arrival function, $\lambda(p)$ which is linear and decreasing in p. The conditions are functions of service rates and k, the effectiveness of prevention service.

Proposition 4.7, specifies that as k, the effectiveness of prevention, increases; the the number of values that μ_2, expected service rate for long service, can take to satisfy the condition increases. In other words, the range of μ_2 gets wider and the condition gets more relaxed. Moreover, expected service rate for prevention can take lower values as k increases. This implies, as prevention becomes more effective, the effect of decrease in service rate of prevention, which increases expected service time, $E(Y)$, would become redundant. Decrease in $\lambda(p)$ would dominate increase in $E(Y)$, resulting in increase in p. When the condition satisfied, the optimal solution will be given by 1, the maximum value of p.

Likewise, Proposition 4.8, specifies the boundary of μ_2 as a function of μ_1 and k. For smaller values of k, the upper bound will increase, resulting in larger values of μ_2. As k, decreases the upper bound on μ_2 will increase, which results in higher values of expected service time of preventive service. As a result, if effectiveness of prevention is low and expected service rate of prevention is high, system will choose decreasing expected service time of the system, $E(Y)$, over increasing arrival rate resulting in lower values of p. If the condition above is satisfied, the optimal solution will be given by minimum value of p, 0.

4.4.4 Analysis: $\lambda(p) = \frac{\lambda_0}{kp}$

Assume total cost function is just waiting time with unit cost and feedback effect is represented by $\lambda(p) = \frac{\lambda_0}{kp}$.

$$L(p) = \frac{\left(\frac{\lambda_0}{kp}\right)^2 \left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)}{2\left(1 - \frac{\left(\frac{\lambda_0}{kp}\right)(p\mu_1 + (1-p)\mu_2)}{\mu_1\mu_2}\right)}$$

$$L(p) = \frac{\left(\frac{\lambda_0}{kp}\right)^2 \left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)}{2\left(1 - \rho(p)\right)}$$

Proposition 4.9. *Optimal solution is always 1.*

We can say that for this form of feedback effect, it is very strong and it is going to be more advantageous to do prevention all the time to every patient in the long run. The system will reflect the effect of prevention very well. Decrease in arrival rate will dominate the increase in expected service time, resulting in providing prevention service to each and every patient.

4.5 Numerical Analysis of PAS with different forms of $\lambda(p)$

To have a intuition about how different $\lambda(p)$ functions react to changes in p, we provide Figure 4.11. The functions that are represented, are the functions that we used in our analysis in this section.

4.5.1 $\lambda(p) = \lambda_0(2 - p)$

We ran 10080 scenarios with different model parameters. Range of parameters are shown in Table 4.4.

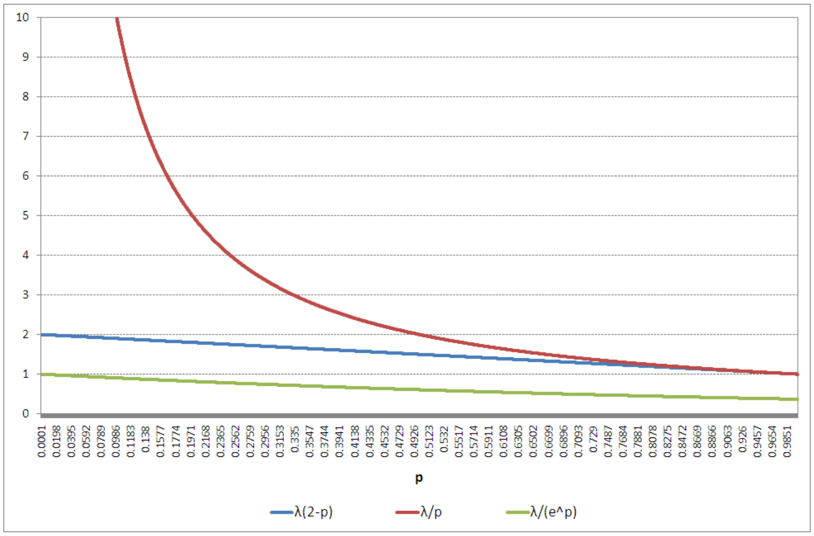

Figure 4.11: Different forms of $\lambda(p)$

Table 4.4: Range of Parameters

Parameter	Range
λ	2-29
μ_1	30-60
μ_2	10-39

For all scenarios, optimal p is achieved at 0. The sufficient condition for monotone increasing total cost function stated in Proposition 4.6 is,

$$\frac{|\lambda'(p)|}{\lambda(p)} = \frac{|-\lambda_0|}{\lambda_0(2-p)} = \frac{1}{2-p} < \frac{\mu_1^2 - \mu_2^2}{2\mu_1^2}$$

$$2\mu_1^2 < (2-p)(\mu_1^2 - \mu_2^2)$$

$$p\mu_1^2 < -(2-p)\mu_2^2$$

This is not possible for any values of p since the left hand side of the inequality is positive whereas the right hand side is negative. Even though the sufficient condition for $p = 0$ is never satisfied, the model can result in $p = 0$ optimal solution. This implies trade off between decreasing arrival rate and increasing average service time is not strong. It might be the result of selected parameters as well as the arrival function, which might not be able to reflect the effect of prevention to the system properly.

4.5.2 $\lambda(p) = \frac{\lambda_0}{p}$

We ran 17365 scenarios with different model parameters. Range of parameters are shown in Table 4.5

Table 4.5: Range of Parameters

Parameter	Range
λ	1-50
μ_1	50-70
μ_2	40-60

For all scenarios, optimal p is achieved at 1. This was the case for $k = 1$ in Section 4.4.4. It was shown in Proposition 4.9 for this form of feedback effect optimal solution is always 1.

4.5.3 $\lambda(p) = \frac{\lambda_0}{e^p}$

We ran 17365 scenarios with different model parameters. Range of parameters are shown in Table 4.4

For all scenarios, optimal p is achieved at 1. We can say that arrival function of this type reflects the effectiveness of preventive activity to the system very strongly, resulting in optimal solution $p = 1$, providing every patient prevention service.

As a reminder, for this form of feedback function the relative change in $\lambda(p)$ with respect to p in Propositions 4.5 and 4.6 becomes 1.

$$\frac{|\lambda'(p)|}{\lambda(p)} = \frac{|-e^{-p}\lambda_0|}{e^{-p}\lambda_0} = 1$$

The condition in Proposition 4.5 becomes

$$1 > \frac{\mu_1^2 - \mu_2^2}{2\mu_2^2}$$

$$3\mu_2^2 > \mu_1^2$$

As a result, for those cases, the sufficient condition for monotone decreasing total cost function is always satisfied, resulting in $p = 1$. However, the condition in Proposition 4.6 becomes

$$1 < \frac{\mu_1^2 - \mu_2^2}{2\mu_1^2}$$

$$\mu_1^2 < -\mu_2^2$$

which is impossible because of model assumptions. The sufficient conditions for monotone increasing total cost function is never satisfied. Keep in mind that, although this condition is not satisfied, $p = 0$ can also be achieved.

4.6 Model outputs with data provided from case study (section 3.2)

We are going to analyze our models' optimal solutions with the λ values that are derived from case study. λ_1 parameter for both DTS and PAS models, is taken as 'total' arrival rate mentioned in Section 3.2. λ_2 parameter in DTS model is taken as 'type 2' patients' arrival rate. They are average daily arrival rates obtained from yearly data. Costs per patient are assumed to be equal and 1 for both models. For this section, when considering optimal time allocation to preventive activity in each approach, it should kept in mind that for DTS model it is $(1-p)$, for PAS model it is p.

We will analyze two separate cases. In the first one we will assume expected service rates are equal for two approaches resulting in an assumption of different expected service times of each service, acute care and prevention. In the second one, we will assume expected service rates are different for each service; however, expected service times are equal.

Service rates are derived for DTS by assuming prevention service lasts 6.85 minutes and acute care service takes 8 minutes. For PAS it is assumed that long service takes 8 minutes whereas short one takes 6.85 minutes, implying there exists an add-on of 1.15 minutes for prevention. Values of model parameters derived from case study are shown in Table 4.6.

Table 4.6: Values

DTS		PAS	
Parameter	Value	Parameter	Value
λ_1	39.51153855	λ_1	39.51153855
λ_2	18.7155417	λ_2	-
μ_1	60	μ_1	70
μ_2	70	μ_2	60
c_1	1	c_1	1
c_2	1	c_2	-

Model results are shown in Table 4.7. For PAS, optimal solution is achieved at $p = 1$ regardless of the type of the feedback function. We see for DTS approach, p lowers as feedback gets stronger. This means as feedback function becomes more

Table 4.7: Model Results

DTS				PAS		
Model	p	$1-p$	Cost	Model	p	Cost
No Fb	0.703792	0.296208	23.8174	Non-Linear Fb	1	1.26976
Linear Fb	0.641568	0.358432	8.28425	Linear Fb	1	1.26976
Exponential Fb	0.565561	0.434439	4.66668	Exponential Fb	1	0.0774454

responsive to the unit change in time allocation to preventive service, time allocated to prevention service, $1-p$, starts to increase.

As a second case, we again assumed costs per patient are same and equal to 1. Service rates for prevention and acute care are calculated by assuming prevention takes 3 minutes and acute care service takes 8 minutes. In DTS model those services are separated and service rates are calculated accordingly, μ_1 for just acute care μ_2 for just prevention. In PAS model, short service means just acute care which takes 8 minutes and long service means acute care+prevention which takes 8+3=11 minutes, μ_1 for short service, μ_2 for long service and they are calculated accordingly. This kind of estimation for service rates is more fair. Model parameter values are shown in Table 4.8.

The results are shown in Table 4.9. Again we see for DTS model, the time dedicated to prevention increases as feedback effect gets stronger whereas for PAS model the optimal solution is always 1.

In DTS model, there is an obligation to satisfy the demand for prevention. Stability condition for prevention queue ensures that some time is dedicated to prevention. Consequently, we have no 0 or 1 solution. However, in the PAS model, there is no such

Table 4.8: Values

DTS		PAS	
Parameter	Value	Parameter	Value
λ_1	39.51153855	λ_1	39.51153855
λ_2	18.7155417	λ_2	-
μ_1	60	μ_1	60
μ_2	160	μ_2	40
c_1	1	c_1	1
c_2	1	c_2	-

Table 4.9: Model Results

DTS				PAS		
Model	p	$1-p$	Cost	Model	p	Cost
No Fb	0.816464	0.183536	5.9268	Non-Linear Fb	1	8.68816
Linear Fb	0.771034	0.228966	4.14791	Linear Fb	1	8.68816
Exponential Fb	0.696039	0.303961	2.938148	Exponential Fb	1	0.207426

obligation. In Proposition 4.5, we stated that when μ_1 decreases and μ_2 increases, the model tends to give optimal solution as 1. This means, when the gap between service rates is small, the model reaches its minimum at $p = 1$. With parameters in Table 4.6 and Table 4.8, service rates are very close the each other, hence optimal solution 1 is reasonable.

In addition, in DTS model, we can clearly see the effect of change in feedback function by comparing the results of different forms of feedback. As we mentioned before, the time allocated to prevention service increases as the feedback function gets stronger. However, in PAS model, it seems that regardless of feedback type, the system tends to provide prevention each and every patient. This might be the result of system dynamics. PAS model is affected more by the changes in arrival rate which probably dominates the change in total expected service time.

In DTS, as feedback function gets stronger the costs decrease while time allocated to preventive service increases. In PAS, for optimal solution 1, the waiting costs are the same for non-linear and linear feedback; however, the cost in exponential feedback is lower than them. This is the case because when $p = 1$, total cost function for non-linear and linear feedback in PAS are the same, and for exponential feedback it is smaller.

4.7 Markov Chain Approach for Modeling $\lambda(p)$

In this chapter notation of p is consistent with the model PAS. To be applicable to DTS approach, p and $1 - p$ should be reversed.

So far we assumed prevention activity has a negative effect on arrival rate, and did optimization for general $\lambda(p)$ or pre-determined $\lambda(p)$ according to this assumption about the feedback effect. $\lambda(p)$ is set to be a decreasing function of p. However, we have not discussed how we can model feedback effect on arrival rate, in other words how we can determine $\lambda(p)$ function.

One approach to model $\lambda(p)$ can be by using Markov chain. We define a Markov chain for one person in the patient list to explore the effect of prevention in the long run. Then we will use this approach to model total arrival function.

4.7.1 States, Transitions, and Long Run Behavior

The states of Markov chain, specified for one patient, are being in 'healthy' or 'sick' condition assuming transitions occurs on a monthly basis. It is done so since transitions should be specified by equal time intervals. Daily transition, will not be appropriate since specify the daily effect of prevention will be difficult by case studies or even meaningless, when the impact of prevention is perceived in the long run is considered. Moreover, every doctor visit was not considered as a transition step, since the time period of those visits is not supposed to be the same. Keeping in mind that, every transition occurs in one month period, in the long run, by finding stationary distribution, we can determine long run probability for being sick which can be considered as probability of arrival for one person.

First, we should understand the meanings of states. Generally, a person consults a GP if he is sick already. When one person visits the doctor due to his illness and gets a kind of prevention service regarding his sickness then the state of this person is sick but when one person visits the doctor due to his illness and gets prevention service on *any other* kind of disease, not regarding his actual illness (the reason that he visits the doctor) then the state of this person is healthy. Here we assume once the prevention service is given before the disease or illness emerges, it is more effective. In other words, the probability of moving from being healthy to healthy due to prevention is higher than moving from sick to healthy.

PAS states that with probability p doctor gives long service meaning gives prevention service and with probability $(1-p)$ doctor gives short service meaning no preventive activity. In this case we can specify the Markov chain as;

$$P = \begin{array}{c c} & \begin{array}{cc} S & H \end{array} \\ \begin{array}{c} S \\ H \end{array} & \begin{pmatrix} 1-\alpha\, p & \alpha\, p \\ 1-\beta\, p & \beta\, p \end{pmatrix} \end{array}$$

where α and β are coefficients for specifying effectiveness of preventive activity with the assumption $\beta > \alpha > 0$, prevention taken when in healthy condition is more effective than prevention taken when in sick condition.

Stationary distribution Π and steady-state probabilities of being sick Π_S or healthy Π_H can be found by;

$$\lim_{n\to\infty} P^{(n)} = \Pi$$

$$\begin{bmatrix}\Pi_S & \Pi_H\end{bmatrix} = \begin{bmatrix}\Pi_S & \Pi_H\end{bmatrix}\begin{bmatrix}1-\alpha\, p & \alpha\, p \\ 1-\beta\, p & \beta\, p\end{bmatrix}$$

$$\Pi_S + \Pi_H = 1$$
$$\Pi_S = (1-\alpha\, p)\,\Pi_S + (1-\beta\, p)\,\Pi_H$$
$$\Pi_H = \alpha\, p\,\Pi_S + \beta\, p\,\Pi_H$$

By solving above equations we find;

$$\Pi_S = \frac{1-\beta\, p}{1-\beta\, p+\alpha\, p}$$

$$\Pi_H = \frac{\alpha\, p}{1-\beta\, p+\alpha\, p}$$

The derivative of Π_S with respect to p is;

$$\frac{d\,\Pi_S}{d\,p} = -\frac{\alpha}{(1-p\,(\beta-\alpha))^2}$$

which is always negative; meaning Π_S, long run probability of being sick, is a decreasing function of prevention activity, consistent with our assumption that as prevention activity increases arrival rate decreases.

4.7.2 Total Arrival Function

Remember that Π_S is the steady-state probability of being sick for one person. According to PAS, probability of providing prevention service remains same for all individuals. In that case, Π_S remains same for all people in the patient list.

Moving one step further, we can treat that case as a binomial event which is repeated for N times, the total number of people in the patient list, and model $\lambda(p)$ by using Poisson approximation to binomial. In other words,

$$\begin{aligned} \lambda(p) &= N\,\Pi_S \\ &= N\frac{1-\beta\, p}{1-\beta\, p+\alpha\, p} \end{aligned}$$

When N is large and Π_S is small, we can use Poisson approximation to binomial for modeling $\lambda(p)$.

If we rewrite the function as

$$\begin{aligned} \lambda(p) &= \frac{1-\beta\, p}{(1-\beta\, p)(1+\frac{\alpha\, p}{1-\beta\, p})} \\ &= \frac{1}{1\,+\,f(p)} \end{aligned} \qquad (4.7.1)$$

which is similar to the feedback function $\lambda(p) = \frac{\lambda_0}{kp}$ we defined in Section 4.4.4. For that form of feedback it is always optimal to provide maximum level of prevention. For feedback form in Equation 4.7.1, it might be the same case since the effect of prevention is reflected more.

As another approach, if we analyze the nominator and the denominator of Π_S function separately, we see both are decreasing functions of p.

$$\frac{d(1-\beta)}{dp} = -\beta < 0$$

and

$$\frac{d(1-\beta\,p+\alpha\,p)}{dp}=\alpha-\beta<0$$
$$|-\beta|=\beta>\beta-\alpha=|\alpha-\beta|$$

which implies the decrease in nominator will dominate the decrease in denominator. If we consider that the decrease in total arrival function is reflected by the function in the nominator, just to have an intuition, we have

$$\lambda(p)=N(1-\beta\,p)$$

This function is similar to our pre-determined decreasing linear arrival function in Section 4.5.1 when analyzing PAS model with linear feedback effect. If we use Markov chain approach to estimate the total arrival function in the steady-state, we identify similar $\lambda(p)$ function to we analyzed before.

4.7.3 α and β Determination

So far we determined total arrival function by Markov chain approach. We used α and β coefficients to reflect the effectiveness of prevention to the system. These coefficients are same with k coefficient that we used as a measure of effectiveness in Section 4.4, while analyzing different forms of $\lambda(p)$ in PAS model.

The effect of prevention can be traced by clinical trials. Fries et al. (1998) [33] provides a summary about the randomized clinical trials of self-management educational programs in prevention. The results suggest that there is a decrease in doctor visits related to prevention service. For example, consultations decreased by 5-17% percent among California employees, by 7.5% among insured Californian families and by 16% among arthritis patients. These percentages, depending on the prevention service provided, can be used to estimate the effect of prevention that is represented by α and β.

Chapter 5

CONCLUSIONS AND FUTURE WORK

Family practice has gained importance since the implementation of PHC in Turkey. The survey results have shown that 75% of the GPs are satisfied with their jobs; however, 80% of them stated that they don't have enough time to accomplish all the tasks. Prevention is one of the most important tasks among them. The studies have shown that, preventive service lowers the number of doctor visits as well as it lowers the deaths in the long run. The case that we analyzed also shown similar results. The patients that are provided prevention service regulary, are more likely to come doctor's office less often. Although this impact of prevention is crucial, in the sense that it lowers the congestion in the office as well as maintains the society's good health status in the long run, the time devoted to this service is very low all around the world.

In this study, we focused on time allocation problem of a GP among two different tasks: prevention and acute care service in a capacity allocation framework. Since survey results shown that only 6% of GPs work with appointment and also they stated that implementaiton of appointment system is very hard due to the habits of patients, to be in line with practice we didn't prefer appointment scheduling approach. Instead, we analyzed two queueing approaches. In the first approach we assumed that we can differentiate the patients according to their needs. The patients who demand acute care and preventive service constitutes two diffent queues served by a single server. Each demand should be satisfied. Steady-state performance is used and modeled as a non-linear optimization problem. Further, we considered a feedback function on arrival rate which reflects the effect of preventive service in the long run to the total arrival rate. We analyzed a linear feedback function and characterized the optimal time allocated to preventive service while minimizing total cost. We also analyzed an exponential feedback function. We presented numerical results for that model. The numerical results show that the time allocated to preventive service in exponential feedback model is higher than the time allocated to preventive service in linear feedback model, and the time allocated to preventive service is higher for linear feedback model than no feedback model. We also show that the waiting cost is higher for no feedback model than linear feedback model, and it is higher for linear feedback model than the exponential feedback model. We can conclude that, as feedback effect gets stronger the system is more likely to devote more time to preventive service. In the long run, the operational costs would also decrease. The family doctors should start to analyze the data that they collected and detect those demand changes according to prevention services they provided. They should consider to devote more time to the preventive services with the highest impact.

In the second approach, patients come to doctors office and constitute a single queue where doctor provides long or standard (short) service to each patient. Long service stands for an acute care service combined with preventive advice whereas standard service is acute care service. In practice, this model is more realistic and more applicable since it is always not possible to differentiate patients beforehand. In addition, it is more applicable that a GP decides whether a patient needs any preventive recommendation/service or not and takes action accordingly by providing long or short service. The decision variable is the portion of time that a GP should provide long service which is the percentage of patients that are offered preventive service. In the analysis, we used steady-state expected performance of the system, and modeled as a non-linear optimization problem. Moreover, in the long run the effect of prevention on total arrival rate is characterized by a general decreasing feedback function of the decision variable. We explored sufficient conditions for monotone decreasing and monotone increasing total cost function where the optimal solution is 0 or 1 respectively. If the expected service rate for acute care is low and expected service rate for prevention is high, the system is more likely to devote more time to prevention. Conversely, for high values of expected service rate of acute care and low values of expected service rate of prevention, the system is more likely to devote less time to prevention. We provided numerical examples for three different types of feedback function. For all cases the optimal solution is achieved at either 1 or 0. We explored that this model tends to work with an 'all or nothing' policy.

As an extension, we analyzed the feedback mechanism on arrivals by a Markov chain approach. One step transition is assumed to be one month time period. One patient's probability of prevention service provided, characterizes the Markov chain, and long run probabilities are found. The behavior of long run probability constituted the basis of one patient's arrival function. Assuming the number of patients that is registered to a GP is large, Poisson approximation to Binomial is used to characterize the general arrival function in the steady-state which is decreasing in the amount of prevention service provided. This is a parallel result with the clinical trials. And also parallel with our feedback function assumptions.

For further research, the data could be recorded collaboratively with the doctor in order to differentiate more effectively the patients who are offered preventive service. Then, it could be analyzed to determine the feedback effect to the whole system. For DTS approach, the number of queues could be increased and analyzed accordingly. Furthermore, for both models different distributions of both service and arrival rates could be considered. Different forms of feedback function could also be analyzed. Especially for PAS, more representative feedback functions which allows GP to devote some time to each service can be considered. This also can be achieved by a pre-determined constraint on demand satisfaction for prevention patients. Different approaches other than Markov chain can be used to model feedback function.

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Appendix A

PROOFS

A.1 Proposition 4.1

Proof. If constraints are not binding, the optimal solution is found by equating the first derivative of the Lagrange function to zero. Let the first derivative of the Lagrange function (4.1.6) with respect to p be

$$F(.) = -u_1\mu_1 - \frac{c_1\lambda_1\mu_1}{(-\lambda_1 + p\mu_1)^2} + u_2\mu_2 + \frac{c_2\lambda_2\mu_2}{(-\lambda_2 + (1-p)\,\mu_2)^2}$$

In this case, since the derivative of the Lagrange function is quadratic in p, we have 2 possible interior solutions.

Solution1:

$$u_1 = 0, u_2 = 0, p = \frac{\lambda_1\mu_1\mu_2\left((c_1 + c_2)\,\lambda_2 - c_1\mu_2\right) + \sqrt{c_1c_2\lambda_1\lambda_2\mu_1\mu_2\left(\lambda_2\mu_1 + (\lambda_1 - \mu_1)\,\mu_2\right)^2}}{\mu_1\mu_2\left(c_2\lambda_2\mu_1 - c_1\lambda_1\mu_2\right)}$$

Solution2:

$$u_1 = 0, u_2 = 0, p = \frac{\lambda_1\mu_1\mu_2\left((c_1 + c_2)\,\lambda_2 - c_1\mu_2\right) - \sqrt{c_1c_2\lambda_1\lambda_2\mu_1\mu_2\left(\lambda_2\mu_1 + (\lambda_1 - \mu_1)\,\mu_2\right)^2}}{\mu_1\mu_2\left(c_2\lambda_2\mu_1 - c_1\lambda_1\mu_2\right)}$$

Solution 1 and 2 represents the interior solution of the non-linear optimization problem of high-level approach. The derivative of Lagrange function with respect to p is in the quadratic form hence we have two roots.

A.1.1 Investigation of Solution 1 and Solution 2

Here we are going to characterize the possible interior solutions. We will analyze the solutions by investigating their two parts.

$$c_2\lambda_2\mu_1 - c_1\lambda_1\mu_2 \tag{A.1.1}$$

$$(c_1 + c_2)\lambda_2 - c_1\mu_2 \tag{A.1.2}$$

A.1.1 comes from denominator and A.1.2 comes from nominator of the solutions. We did investigation by analyzing their signs.

- Case 1: $c_2\lambda_2\mu_1 - c_1\lambda_1\mu_2 > 0$ and $(c_1 + c_2)\lambda_2 - c_1\mu_2 > 0$

  In this case, Solution 2 to be positive ($0 < p < 1$ should hold for solutions) it must satisfy

$$\lambda_1\mu_1\mu_2\left((c_1 + c_2)\lambda_2 - c_1\mu_2\right) > \sqrt{c_1c_2\lambda_1\lambda_2\mu_1\mu_2\left(\lambda_2\mu_1 + (\lambda_1 - \mu_1)\mu_2\right)^2} \tag{A.1.3}$$

  since the denominator coefficient A.1.1 is positive by case assumption. Otherwise solution would be negative and not be feasible. Again we should check whether stability conditions hold for a feasible solution.

$$\mu_1\left(\frac{\lambda_1\mu_1\mu_2\left((c_1 + c_2)\lambda_2 - c_1\mu_2\right) - \sqrt{c_1c_2\lambda_1\lambda_2\mu_1\mu_2\left(\lambda_2\mu_1 + (\lambda_1 - \mu_1)\mu_2\right)^2}}{\mu_1\mu_2\left(c_2\lambda_2\mu_1 - c_1\lambda_1\mu_2\right)}\right) - \lambda_1 > 0$$

$$\frac{\lambda_1\mu_1\mu_2\left((c_1 + c_2)\lambda_2 - c_1\mu_2\right) - \sqrt{c_1c_2\lambda_1\lambda_2\mu_1\mu_2\left(\lambda_2\mu_1 + (\lambda_1 - \mu_1)\mu_2\right)^2}}{\mu_2\left(c_2\lambda_2\mu_1 - c_1\lambda_1\mu_2\right)} - \lambda_1 > 0$$

  By case assumptions and A.1.3 every term is positive, the inequality sign does not change when multiplying the terms.

$$\lambda_1\mu_1\mu_2\left((c_1 + c_2)\lambda_2 - c_1\mu_2\right) - \sqrt{c_1c_2\lambda_1\lambda_2\mu_1\mu_2\left(\lambda_2\mu_1 + (\lambda_1 - \mu_1)\mu_2\right)^2} > \lambda_1\mu_2\left(c_2\lambda_2\mu_1 - c_1\lambda_1\mu_2\right)$$

$$\lambda_1\mu_1\mu_2\left((c_1 + c_2)\lambda_2 - c_1\mu_2\right) > \lambda_1\mu_2\left(c_2\lambda_2\mu_1 - c_1\lambda_1\mu_2\right)$$

$$\lambda_1\mu_1\mu_2\left(\left(c_1+c_2\right)\lambda_2-c_1\mu_2\right)>\lambda_1\mu_2\left(c_2\lambda_2\mu_1-c_1\lambda_1\mu_2\right)$$

$$\mu_1\left(\left(c_1+c_2\right)\lambda_2-c_1\mu_2\right)>\left(c_2\lambda_2\mu_1-c_1\lambda_1\mu_2\right)$$

$$\mu_1c_1\lambda_2-c_1\mu_2\mu_1>-c_1\lambda_1\mu_2$$

$$\mu_2\mu_1-\mu_1\lambda_2-\lambda_1\mu_2<0$$

This contradicts with the system stability condition 4.1.5. In that case, solution 2 is not feasible, hence solution 1 is optimal.

- Case 2: $c_2\lambda_2\mu_1-c_1\lambda_1\mu_2>0$ and $\left(c_1+c_2\right)\lambda_2-c_1\mu_2<0$

  In this case the denominator is positive and first part of the nominator is negative. The second part of the nominator should be positive otherwise p would be negative which is not possible. Hence, solution 1 is optimal.

- Case 3: $c_2\lambda_2\mu_1-c_1\lambda_1\mu_2<0$ and $\left(c_1+c_2\right)\lambda_2-c_1\mu_2>0$

  This case is not possible since it gives

$$\frac{\lambda_2}{\mu_2-\lambda_2}<\frac{c_1}{c_2}<\frac{\lambda_2\mu_1}{\lambda_1\mu_2}$$

This implies

$$\frac{\lambda_2}{\mu_2-\lambda_2}<\frac{\lambda_2\mu_1}{\lambda_1\mu_2}$$

In other words

$$\frac{1}{\mu_2-\lambda_2}<\frac{\mu_1}{\lambda_1\mu_2}$$

and

$$\mu_1\mu_2-\lambda_1\mu_2-\lambda_2\mu_1<0$$

This contradicts the system stability condition 4.1.5 on the model parameters hence this case is not possible.

- Case 4: $c_2\lambda_2\mu_1 - c_1\lambda_1\mu_2 < 0$ and $(c_1 + c_2)\lambda_2 - c_1\mu_2 < 0$

  In that case, there exist no problem with positivity of solution 2. We should check stability conditions for feasibility.

$$\mu_2\left(1-\left(\frac{\lambda_1\mu_1\mu_2\left((c_1+c_2)\lambda_2-c_1\mu_2\right)-\sqrt{c_1c_2\lambda_1\lambda_2\mu_1\mu_2\left(\lambda_2\mu_1+(\lambda_1-\mu_1)\mu_2\right)^2}}{\mu_1\mu_2\left(c_2\lambda_2\mu_1-c_1\lambda_1\mu_2\right)}\right)\right)-\lambda_2>0$$

$$\mu_2\left(1-\left(\frac{\lambda_1\mu_1\mu_2\left((c_1+c_2)\lambda_2-c_1\mu_2\right)-\sqrt{c_1c_2\lambda_1\lambda_2\mu_1\mu_2\left(\lambda_2\mu_1+(\lambda_1-\mu_1)\mu_2\right)^2}}{\mu_1\mu_2\left(c_2\lambda_2\mu_1-c_1\lambda_1\mu_2\right)}\right)\right)-\lambda_2>0$$

  or equivalently;

$$\frac{-c_2\lambda_2\mu_1\left(\lambda_2\mu_1+(\lambda_1-\mu_1)\mu_2\right)+\sqrt{c_1c_2\lambda_1\lambda_2\mu_1\mu_2\left(\lambda_2\mu_1+(\lambda_1-\mu_1)\mu_2\right)^2}}{\mu_1\left(c_2\lambda_2\mu_1-c_1\lambda_1\mu_2\right)}>0$$

  implying

$$c_2\lambda_2\mu_1\left(\lambda_2\mu_1+(\lambda_1-\mu_1)\mu_2\right)>0\lambda_2\mu_1+\lambda_1\mu_2-\mu_1\mu_2>0$$

  which contradicts with the system stability condition 4.1.5. Hence solution 1 is optimal.

A.1.2 Results:

It is proven that whatever the model parameters are, Solution 1 gives the optimal solution for the Lagrange function if the solution is an interior one. To put in other words unless the solution is a boundary solution, the positive root of the derivative of the Lagrange function gives the optimal solution. □

A.2 Proposition 4.2: Comparative Statics of DTS

Proof. Let the first derivative of the Lagrangian function with respect to p be

$$F(.)=-u_1\mu_1-\frac{c_1\lambda_1\mu_1}{\left(-\lambda_1+p\mu_1\right)^2}+u_2\mu_2+\frac{c_2\lambda_2\mu_2}{\left(-\lambda_2+(1-p)\mu_2\right)^2}$$

Then p^* is the solution to $F(.) = 0$. The interior solution is found by equating the first derivative of Lagrangian function to zero, the comparative statics should be analyzed by implicit function theorem.

Reaction to λ_1***:***

By implicit function theorem we can write:

$$\begin{aligned}\frac{\partial p^*}{\partial \lambda_1} &= -\frac{\partial F(.)}{\partial \lambda_1}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*} \\ &= -\frac{c_1\mu_1\left(\lambda_1 + p\mu_1\right)\left(-\frac{(\lambda_1 - p\mu_1)^3}{2c_1\lambda_1\mu_1^2} - \frac{2c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)}{\left(\lambda_1 - p\mu_1\right)^3} > 0\end{aligned}$$

Reaction to λ_2***:***

By implicit function theorem:

$$\begin{aligned}\frac{\partial p^*}{\partial \lambda_2} &= -\frac{\partial F(.)}{\partial \lambda_2}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*} \\ &= -\frac{c_2\mu_2\left(-\lambda_2 + (-1+p)\mu_2\right)\left(-\frac{(\lambda_1 - p\mu_1)^3}{2c_1\lambda_1\mu_1^2} - \frac{2c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)}{\left(\lambda_2 + (-1+p)\mu_2\right)^3} < 0\end{aligned}$$

Reaction to μ_1***:***

By implicit function theorem:

$$\begin{aligned}\frac{\partial p^*}{\partial \mu_1} &= -\frac{\partial F(.)}{\partial \mu_1}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*} \\ &= \frac{\left(u_1\left(\lambda_1 - p\mu_1\right)^3 + c_1\lambda_1\left(\lambda_1 + p\mu_1\right)\right)\left(-\frac{(\lambda_1 - p\mu_1)^3}{2c_1\lambda_1\mu_1^2} - \frac{2c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)}{\left(\lambda_1 - p\mu_1\right)^3}\end{aligned}$$

Since $u_1 = 0$ the equation becomes:

$$\frac{\partial p^*}{\partial \mu_1} = -\frac{\partial F(.)}{\partial \mu_1}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*}$$

$$= \frac{c_1\lambda_1\left(\lambda_1 + p\mu_1\right)\left(-\frac{(\lambda_1-p\mu_1)^3}{2c_1\lambda_1\mu_1^2} - \frac{2c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)}{\left(\lambda_1 - p\mu_1\right)^3} < 0$$

Reaction to μ_2***:***

By implicit function theorem:

$$\frac{\partial p^*}{\partial \mu_2} = -\frac{\partial F(.)}{\partial \mu_2}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*}$$

$$= -\left(-\frac{\left(\lambda_1 - p\mu_1\right)^3}{2c_1\lambda_1\mu_1^2} - \frac{2c_2\lambda_2\mu_2^2}{\left(\lambda_2 + (-1+p)\,\mu_2\right)^3}\right)\left(u_2 + \frac{c_2\lambda_2\left(\lambda_2 - (-1+p)\,\mu_2\right)}{\left(\lambda_2 + (-1+p)\,\mu_2\right)^3}\right)$$

Since $u_2 = 0$ the equation becomes:

$$\frac{\partial p^*}{\partial \mu_2} = -\frac{\partial F(.)}{\partial \mu_2}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*}$$

$$= -\frac{c_2\lambda_2\left(\lambda_2 - (-1+p)\,\mu_2\right)\left(-\frac{(\lambda_1-p\mu_1)^3}{2c_1\lambda_1\mu_1^2} - \frac{2c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)}{\left(\lambda_2 + (-1+p)\,\mu_2\right)^3} > 0$$

Reaction to c_1***:***

By implicit function theorem:

$$\frac{\partial p^*}{\partial c_1} = -\frac{\partial F(.)}{\partial c_1}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*}$$

$$= \frac{\lambda_1\mu_1\left(-\frac{(\lambda_1-p\mu_1)^3}{2c_1\lambda_1\mu_1^2} - \frac{2c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)}{\left(\lambda_1 - p\mu_1\right)^2} > 0$$

Reaction to c_2:

By implicit function theorem:

$$\frac{\partial p^*}{\partial c_2} = -\frac{\partial F(.)}{\partial c_2}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*}$$

$$= -\frac{\lambda_2\mu_2\left(-\frac{(\lambda_1-p\mu_1)^3}{2c_1\lambda_1\mu_1^2}-\frac{2c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)}{(\lambda_2+(-1+p)\mu_2)^2} < 0$$

□

A.3 Proposition 4.3

Proof. If constraints are not binding, then the optimal solution to high-level approach optimization problem with linear feedback effect is found by equating the first derivative of the Lagrange function to zero. Let the first derivative of the Lagrange function 4.2.6 with respect to p be

$$F(.) = -u_1(-\lambda_0+\mu_1) - \frac{c_1(1+p)\lambda_0(-\lambda_0+\mu_1)}{(-(1+p)\lambda_0+p\mu_1)^2} + \frac{c_1\lambda_0}{-(1+p)\lambda_0+p\mu_1} + u_2\mu_2 + \frac{c_2\lambda_2\mu_2}{(-\lambda_2+(1-p)\mu_2)^2}$$

In this case, since the derivative of the Lagrange function is quadratic, we have 2 possible interior solutions.

Solution1:

$$u_1 = 0,\ u_2 = 0,$$
$$p = \frac{\lambda_0\mu_2(c_2\lambda_2(\mu_1-\lambda_0)-c_1\mu_1(\mu_2-\lambda_2)) + \sqrt{c_1c_2\lambda_0\lambda_2\mu_1\mu_2(\lambda_0(\lambda_2-2\mu_2)+\mu_1(-\lambda_2+\mu_2))^2}}{\mu_2(c_2\lambda_2(\lambda_0-\mu_1)^2-c_1\lambda_0\mu_1\mu_2)}$$

Solution2:

$$u_1 = 0,\ u_2 = 0,$$
$$p = \frac{\lambda_0\mu_2(c_2\lambda_2(\mu_1-\lambda_0)-c_1\mu_1(\mu_2-\lambda_2)) - \sqrt{c_1c_2\lambda_0\lambda_2\mu_1\mu_2(\lambda_0(\lambda_2-2\mu_2)+\mu_1(-\lambda_2+\mu_2))^2}}{\mu_2(c_2\lambda_2(\lambda_0-\mu_1)^2-c_1\lambda_0\mu_1\mu_2)}$$

Solution 1 and 2 represents the interior solution of the non-linear optimization problem of high-level approach with linear feedback effect. The derivative of the Lagrange function with respect to p is in the quadratic form hence we have two roots.

A.3.1 Investigation of Solution 1 and Solution 2

Here we are going to investigate possible interior solutions. These solutions have two parts:

$$c_2\lambda_2\left(\lambda_0-\mu_1\right)^2-c_1\lambda_0\mu_1\mu_2 \tag{A.3.1}$$

$$c_2\lambda_2\left(\mu_1-\lambda_0\right)-c_1\mu_1\left(\mu_2-\lambda_2\right) \tag{A.3.2}$$

A.3.1 comes from denominator and A.3.2 comes from nominator of the solutions. We did investigation by analyzing their signs.

- Case 1: $c_2\lambda_2\left(\lambda_0-\mu_1\right)^2-c_1\lambda_0\mu_1\mu_2>0$ and $c_2\lambda_2\left(\mu_1-\lambda_0\right)-c_1\mu_1\left(\mu_2-\lambda_2\right)>0$

  In this case, Solution 2 to be positive ($0<p<1$ should hold for solutions) must satisfy;

  $$\lambda_0\mu_2\left(c_2\lambda_2\left(\mu_1-\lambda_0\right)-c_1\mu_1\left(\mu_2-\lambda_2\right)\right)>\sqrt{c_1c_2\lambda_0\lambda_2\mu_1\mu_2\left(\lambda_0\left(\lambda_2-2\mu_2\right)+\mu_1\left(-\lambda_2+\mu_2\right)\right)^2}$$

  since the denominator coefficient A.3.1 is positive by case assumption. Otherwise solution would be negative and not be feasible. Again we should check whether stability conditions hold for a feasible solution.

  $$\mu_1\left(\frac{\lambda_0\mu_2\left(c_2\lambda_2\left(\mu_1-\lambda_0\right)-c_1\mu_1\left(\mu_2-\lambda_2\right)\right)-\sqrt{c_1c_2\lambda_0\lambda_2\mu_1\mu_2\left(\lambda_0\left(\lambda_2-2\mu_2\right)+\mu_1\left(-\lambda_2+\mu_2\right)\right)^2}}{\mu_2\left(c_2\lambda_2\left(\lambda_0-\mu_1\right)^2-c_1\lambda_0\mu_1\mu_2\right)}\right)-$$
  $$\lambda_0\left(1+\left(\frac{\lambda_0\mu_2\left(c_2\lambda_2\left(\mu_1-\lambda_0\right)-c_1\mu_1\left(\mu_2-\lambda_2\right)\right)-\sqrt{c_1c_2\lambda_0\lambda_2\mu_1\mu_2\left(\lambda_0\left(\lambda_2-2\mu_2\right)+\mu_1\left(-\lambda_2+\mu_2\right)\right)^2}}{\mu_2\left(c_2\lambda_2\left(\lambda_0-\mu_1\right)^2-c_1\lambda_0\mu_1\mu_2\right)}\right)\right)>0$$

or equivalently;

$$\frac{c_1\lambda_0\mu_1\mu_2\left(\lambda_0\left(\lambda_2-2\mu_2\right)+\mu_1\left(-\lambda_2+\mu_2\right)\right)+\left(-\lambda_0+\mu_1\right)\sqrt{c_1c_2\lambda_0\lambda_2\mu_1\mu_2\left(\lambda_0\left(\lambda_2-2\mu_2\right)+\mu_1\left(-\lambda_2+\mu_2\right)\right)^2}}{\mu_2\left(-c_2\lambda_2\left(\lambda_0-\mu_1\right)^2+c_1\lambda_0\mu_1\mu_2\right)}>0$$

but this is never possible since the first part of the nominator is positive by stability condition 4.2.5 and the denominator is negative by case assumption, the whole expression is negative so the solution does not satisfy 4.2.2 stability condition. Hence, solution 1 gives optimal.

- Case 2: $c_2\lambda_2\left(\lambda_0-\mu_1\right)^2-c_1\lambda_0\mu_1\mu_2>0$ and $c_2\lambda_2\left(\mu_1-\lambda_0\right)-c_1\mu_1\left(\mu_2-\lambda_2\right)<0$

  In this case the denominator is positive and first part of the nominator is negative. The second part of the nominator should be positive otherwise p would be negative which is not possible. Hence, solution 1 is optimal.

- Case 3: $c_2\lambda_2\left(\lambda_0-\mu_1\right)^2-c_1\lambda_0\mu_1\mu_2<0$ and $c_2\lambda_2\left(\mu_1-\lambda_0\right)-c_1\mu_1\left(\mu_2-\lambda_2\right)>0$

  This case is not possible since it gives

$$\frac{\lambda_2(\mu_1-\lambda_0)^2}{\lambda_0\mu_1\mu_2}<\frac{c_1}{c_2}<\frac{\lambda_2(\mu 1-\lambda_0)}{\mu_1(\mu_2-\lambda_2)}$$

  This implies

$$\frac{\lambda_2(\mu_1-\lambda_0)^2}{\lambda_0\mu_1\mu_2}<\frac{\lambda_2(\mu 1-\lambda_0)}{\mu_1(\mu_2-\lambda_2)}$$

  In other words

$$\frac{\mu_1-\lambda_0}{\lambda_0\mu_2}<\frac{1}{\mu_2-\lambda_2}$$

  and

$$\mu_2\mu_1-\lambda_2\mu_1-2\lambda_0\mu_2+\lambda_2\lambda_0<0$$

  This contradicts the system stability condition 4.2.5 on the model parameters hence this case is not possible.

- Case 4: $c_2\lambda_2\left(\lambda_0-\mu_1\right)^2-c_1\lambda_0\mu_1\mu_2<0$ and $c_2\lambda_2\left(\mu_1-\lambda_0\right)-c_1\mu_1\left(\mu_2-\lambda_2\right)<0$

In that case solution 2 is never optimal since there are no model parameters that satisfying;

$$\mu_2\left(1-\left(\frac{\lambda_0\mu_2\left(c_2\lambda_2\left(\mu_1-\lambda_0\right)-c_1\mu_1\left(\mu_2-\lambda_2\right)\right)-\sqrt{c_1c_2\lambda_0\lambda_2\mu_1\mu_2\left(\lambda_0\left(\lambda_2-2\mu_2\right)+\mu_1\left(-\lambda_2+\mu_2\right)\right)^2}}{\mu_2\left(c_2\lambda_2\left(\lambda_0-\mu_1\right)^2-c_1\lambda_0\mu_1\mu_2\right)}\right)\right)-\lambda_2>0$$

equivalently;

$$\frac{c_2\lambda_2\left(\mu_1-\lambda_0\right)\left(\lambda_0\left(\lambda_2-2\mu_2\right)+\mu_1\left(-\lambda_2+\mu_2\right)\right)+\sqrt{c_1c_2\lambda_0\lambda_2\mu_1\mu_2\left(\lambda_0\left(\lambda_2-2\mu_2\right)+\mu_1\left(-\lambda_2+\mu_2\right)\right)^2}}{c_2\lambda_2\left(\lambda_0-\mu_1\right)^2-c_1\lambda_0\mu_1\mu_2}>0$$

In that case, the first part of the nominator is positive by 4.2.5 but the denominator is negative by case assumption, and the whole expression is negative so the solution does not satisfy 4.2.2 stability condition. Hence solution 1 is optimal.

A.3.2 Results :

It is proven that whatever the model parameters are, Solution 1 gives the optimal solution for the Lagrange function of high-level approach with linear feedback effect optimization problem if the solution is an interior one. To put in other words unless the solution is a boundary solution, the positive root of the derivative of the Lagrange function gives the optimal solution. □

A.4 Proposition 4.4: Comparative Statics of DTSL

Proof. Let the first derivative of Lagrangian function with respect to p be

$$F(.) = -u_1\left(-\lambda_0+\mu_1\right)-\frac{c_1\left(1+p\right)\lambda_0\left(-\lambda_0+\mu_1\right)}{\left(-\left(1+p\right)\lambda_0+p\mu_1\right)^2}+\frac{c_1\lambda_0}{-\left(1+p\right)\lambda_0+p\mu_1} +u_2\mu_2+\frac{c_2\lambda_2\mu_2}{\left(-\lambda_2+\left(1-p\right)\mu_2\right)^2}$$

Then p^* is the solution to $F(.)=0$. The interior solution is found by equating the first derivative of Lagrangian function to zero, the comparative statics should be analyzed by implicit function theorem.

Reaction to λ_0***:***

By implicit function theorem:

$$\begin{aligned}\frac{\partial p^*}{\partial \mu_1} &= -\frac{\partial F(.)}{\partial \mu_1}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*} \\ &= \frac{u_1\left((1+p)\,\lambda_0 - p\mu_1\right)^3 + c_1\mu_1\left((1+p)\,\lambda_0 + p\mu_1\right)}{\left((1+p)\,\lambda_0 - p\mu_1\right)^3}\end{aligned}$$

Since $u_1 = 0$ the equation becomes:

$$\begin{aligned}\frac{\partial p^*}{\partial \mu_1} &= -\frac{\partial F(.)}{\partial \mu_1}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*} \\ &= \frac{c_1\mu_1\left((1+p)\,\lambda_0 + p\mu_1\right)}{2\left((1+p)\,\lambda_0 - p\mu_1\right)^3\left(\frac{c_1\lambda_0\mu_1(-\lambda_0+\mu_1)}{((1+p)\lambda_0-p\mu_1)^3} + \frac{c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)} > 0\end{aligned}$$

Reaction to λ_2***:***

By implicit function theorem:

$$\begin{aligned}\frac{\partial p^*}{\partial \lambda_2} &= -\frac{\partial F(.)}{\partial \lambda_2}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*} \\ &= \frac{c_2\mu_2\left(-\lambda_2 + (-1+p)\,\mu_2\right)}{2\left(\lambda_2 + (-1+p)\,\mu_2\right)^3\left(\frac{c_1\lambda_0\mu_1(-\lambda_0+\mu_1)}{((1+p)\lambda_0-p\mu_1)^3} + \frac{c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)} < 0\end{aligned}$$

Reaction to μ_1***:***

By implicit function theorem:

$$\frac{\partial p^*}{\partial \mu_1} = -\frac{\partial F(.)}{\partial \mu_1}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*}$$
$$= -\frac{u_1\left((1+p)\lambda_0 - p\mu_1\right)^3 + c_1\lambda_0\left((1+p)\lambda_0 + p\mu_1\right)}{2\left((1+p)\lambda_0 - p\mu_1\right)^3\left(\frac{c_1\lambda_0\mu_1(-\lambda_0+\mu_1)}{((1+p)\lambda_0-p\mu_1)^3} + \frac{c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)}$$

Since $u_1 = 0$ the equation becomes:

$$\frac{\partial p^*}{\partial \mu_1} = -\frac{\partial F(.)}{\partial \mu_1}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*}$$
$$= -\frac{c_1\lambda_0\left((1+p)\lambda_0 + p\mu_1\right)}{2\left((1+p)\lambda_0 - p\mu_1\right)^3\left(\frac{c_1\lambda_0\mu_1(-\lambda_0+\mu_1)}{((1+p)\lambda_0-p\mu_1)^3} + \frac{c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)} < 0$$

Reaction to μ_2:

By implicit function theorem:

$$\frac{\partial p^*}{\partial \mu_2} = -\frac{\partial F(.)}{\partial \mu_2}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*}$$
$$= \frac{u_2 + \frac{c_2\lambda_2(\lambda_2-(-1+p)\mu_2)}{(\lambda_2+(-1+p)\mu_2)^3}}{2\left(\frac{c_1\lambda_0\mu_1(-\lambda_0+\mu_1)}{((1+p)\lambda_0-p\mu_1)^3} + \frac{c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)}$$

Since $u_2 = 0$ the equation becomes:

$$\frac{\partial p^*}{\partial \mu_2} = -\frac{\partial F(.)}{\partial \mu_2}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*}$$
$$= \frac{c_2\lambda_2\left(\lambda_2 - (-1+p)\mu_2\right)}{2\left(\lambda_2 + (-1+p)\mu_2\right)^3\left(\frac{c_1\lambda_0\mu_1(-\lambda_0+\mu_1)}{((1+p)\lambda_0-p\mu_1)^3} + \frac{c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)} > 0$$

Reaction to c_1:

By implicit function theorem:

$$\begin{aligned}\frac{\partial p^*}{\partial c_1} &= -\frac{\partial F(.)}{\partial c_1}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*} \\ &= -\frac{\lambda_0\mu_1}{2\left((1+p)\lambda_0 - p\mu_1\right)^2\left(\frac{c_1\lambda_0\mu_1(-\lambda_0+\mu_1)}{((1+p)\lambda_0-p\mu_1)^3} + \frac{c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)} > 0\end{aligned}$$

Reaction to c_2:

By implicit function theorem:

$$\begin{aligned}\frac{\partial p^*}{\partial c_2} &= -\frac{\partial F(.)}{\partial c_2}\left[\frac{\partial F(.)}{\partial p}\right]^{-1}_{p=p^*} \\ &= \frac{\lambda_2\mu_2}{2\left(\lambda_2 + (-1+p)\mu_2\right)^2\left(\frac{c_1\lambda_0\mu_1(-\lambda_0+\mu_1)}{((1+p)\lambda_0-p\mu_1)^3} + \frac{c_2\lambda_2\mu_2^2}{(\lambda_2+(-1+p)\mu_2)^3}\right)} < 0\end{aligned}$$

□

A.5 Proposition 4.5

If $\frac{|\lambda'(p)|}{\lambda(p)} > \frac{\mu_1^2-\mu_2^2}{2\mu_2^2}$; optimal solution is $p = 1$

Proof.

$$L(p) = \frac{\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)}{2\left(1 - \lambda(p)\left(\frac{(1-p)}{\mu_1} + \frac{p}{\mu_2}\right)\right)}$$

$$L(p) = \frac{\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)}{2\left(1 - \rho(p)\right)}$$

If $L(p)$ is monotone decreasing ($\frac{dL(p)}{dp} < 0$ for all p), then optimal solution is $p = 1$. Sufficient conditions for

$$\frac{dL(p)}{dp} < 0$$

are

$$\frac{d\rho(p)}{dp} < 0$$

$$\frac{d\left(\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)\right)}{dp} < 0$$

$$\frac{d\rho(p)}{dp} = \frac{\lambda(p)\left(\mu_1 - \mu_2\right)}{\mu_1\mu_2} + \frac{\lambda'(p)\left(p\mu_1 + (1-p)\mu_2\right)}{\mu_1\mu_2}$$

$$= \frac{\lambda(p)\left(p\mu_1 + (1-p)\mu_2\right)}{\mu_1\mu_2}\left(\frac{\lambda'(p)}{\lambda(p)} + \frac{\mu_1 - \mu_2}{p\mu_1 + (1-p)\mu_2}\right)$$

since $0 < p < 1$, $\lambda(p) > 0$, $\mu_1 > \mu_2 > 0$,

$$\frac{\lambda(p)\left(p\mu_1 + (1-p)\mu_2\right)}{\mu_1\mu_2} > 0$$

and since $\lambda(p)$ is decreasing in p, $\lambda'(p) < 0$; to be

$$\frac{d\rho(p)}{dp} < 0$$

the following should be true;

$$\frac{\lambda'(p)}{\lambda(p)} + \frac{\mu_1 - \mu_2}{p\mu_1 + (1-p)\mu_2} < 0$$

$$\frac{|\lambda'(p)|}{\lambda(p)} > \frac{\mu_1 - \mu_2}{p\mu_1 + (1-p)\mu_2}$$

$$\frac{d\left(\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)\right)}{dp} = \lambda(p)^2\left(-\frac{2}{\mu_1^2} + \frac{2}{\mu_2^2}\right) + 2\lambda(p)\lambda'(p)\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)$$

$$= \frac{2\lambda(p)^2(2p\mu_1^2 - 2(-1+p)\mu_2^2)}{\mu_1^2\mu_2^2}\left(\frac{\lambda'(p)}{\lambda(p)} + \frac{\mu_1^2 - \mu_2^2}{2p\mu_1^2 - 2(-1+p)\mu_2^2}\right)$$

since $0 < p < 1$, $\lambda(p) > 0$, $\mu_1 > \mu_2 > 0$;

$$\frac{2\lambda(p)^2(2p\mu_1^2 - 2(-1+p)\mu_2^2)}{\mu_1^2\mu_2^2} > 0$$

and since $\lambda(p)$ is decreasing in p, $\lambda'(p) < 0$; to be

$$\frac{d\left(\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)\right)}{dp} < 0$$

the following should be true;

$$\frac{\lambda'(p)}{\lambda(p)} + \frac{\mu_1^2-\mu_2^2}{2p\mu_1^2 - 2(-1+p)\mu_2^2} < 0$$

$$\frac{|\lambda'(p)|}{\lambda(p)} > \frac{\mu_1^2-\mu_2^2}{2(\mu_2^2(1-p)+\mu_1^2 p)}$$

We have two conditions for monotone decreasing total cost function;

$$\frac{|\lambda'(p)|}{\lambda(p)} > \frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2}$$

$$\frac{|\lambda'(p)|}{\lambda(p)} > \frac{\mu_1^2-\mu_2^2}{2(\mu_2^2(1-p)+\mu_1^2 p)}$$

If

$$\frac{|\lambda'(p)|}{\lambda(p)} > \max_p\left\{\frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2}\right\}$$

$$\frac{|\lambda'(p)|}{\lambda(p)} > \frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2}$$

for all p.

$$\frac{d\left(\frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2}\right)}{dp} = -\frac{(\mu_1-\mu_2)^2}{(p\mu_1-(-1+p)\mu_2)^2} < 0$$

implying

$$\frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2}$$

is a decreasing function in p and reaches its maximum value when $p=0$. When $p=0$;

$$\frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2} = \frac{\mu_1-\mu_2}{\mu_2}$$

Hence

$$\frac{|\lambda'(p)|}{\lambda(p)} > \frac{\mu_1-\mu_2}{\mu_2}$$

is a sufficient condition for $\frac{d\rho(p)}{dp} < 0$.

If

$$\frac{|\lambda'(p)|}{\lambda(p)} > \max_p \left\{ \frac{\mu_1^2 - \mu_2^2}{2(\mu_2^2(1-p) + \mu_1^2 p)} \right\}$$

$$\frac{|\lambda'(p)|}{\lambda(p)} > \frac{\mu_1^2 - \mu_2^2}{2(\mu_2^2(1-p) + \mu_1^2 p)}$$

for all p.

$$\frac{d\left(\frac{\mu_1^2-\mu_2^2}{2(\mu_2^2(1-p)+\mu_1^2 p)}\right)}{dp} = -\frac{(\mu_1^2 - \mu_2^2)^{\,2}}{2\,(p\mu_1^2 - (-1+p)\mu_2^2)^{\,2}} < 0$$

implying

$$\frac{\mu_1^2 - \mu_2^2}{2(\mu_2^2(1-p) + \mu_1^2 p)}$$

is a decreasing function in p and reaches its maximum value when $p = 0$. When $p = 0$;

$$\frac{\mu_1^2 - \mu_2^2}{2(\mu_2^2(1-p) + \mu_1^2 p)} = \frac{\mu_1^2 - \mu_2^2}{2\mu_2^2}$$

Hence

$$\frac{|\lambda'(p)|}{\lambda(p)} > \frac{\mu_1^2 - \mu_2^2}{2\mu_2^2}$$

is a sufficient condition for

$$\frac{d\left(\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)\right)}{dp} < 0$$

For all $\mu_1 > \mu_2$;

$$\frac{\left(\frac{\mu_1^2-\mu_2^2}{2\mu_2^2}\right)}{\left(\frac{\mu_1-\mu_2}{\mu_2}\right)} = \frac{\mu_1 + \mu_2}{2\mu_2} > 1$$

$$\frac{\mu_1^2 - \mu_2^2}{2\mu_2^2} > \frac{\mu_1 - \mu_2}{\mu_2}$$

hence;

$$\frac{|\lambda'(p)|}{\lambda(p)} > \frac{\mu_1^2 - \mu_2^2}{2\mu_2^2}$$

is a sufficient condition for

$$\frac{d\rho(p)}{dp} < 0$$

and

$$\frac{d\left(\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)\right)}{dp}<0$$

. In that case, nominator of total cost function will always decrease whereas the denominator will always increase implying total cost will always decrease. Then the optimal solution is $p = 1$. □

A.6 Proposition 4.6

If $\frac{|\lambda'(p)|}{\lambda(p)} < \frac{\mu_1^2-\mu_2^2}{2\mu_1^2}$; optimal solution is $p = 0$

Proof.

$$L(p) = \frac{\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)}{2\left(1-\lambda(p)\left(\frac{(1-p)}{\mu_1}+\frac{p}{\mu_2}\right)\right)}$$

$$L(p) = \frac{\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)}{2\left(1-\rho(p)\right)}$$

If $L(p)$ is monotone increasing ($\frac{dL(p)}{dp} > 0$ for all p), then optimal solution is $p = 0$.

$$\frac{dL(p)}{dp}>0$$

are

$$\frac{d\rho(p)}{dp}>0$$

$$\frac{d\left(\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)\right)}{dp}>0$$

$$\frac{d\rho(p)}{dp}=\frac{\lambda(p)\left(\mu_1-\mu_2\right)}{\mu_1\mu_2}+\frac{\lambda'(p)\left(p\mu_1+(1-p)\mu_2\right)}{\mu_1\mu_2}$$
$$=\frac{\lambda(p)\left(p\mu_1+(1-p)\mu_2\right)}{\mu_1\mu_2}\left(\frac{\lambda'(p)}{\lambda(p)}+\frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2}\right)$$

since $0 < p < 1$, $\lambda(p) > 0$, $\mu_1 > \mu_2 > 0$,

$$\frac{\lambda(p)\left(p\mu_1+(1-p)\mu_2\right)}{\mu_1\mu_2}>0$$

and since $\lambda(p)$ is decreasing in p, $\lambda'(p) < 0$; to be

$$\frac{d\rho(p)}{dp} > 0$$

, it should be;

$$\frac{\lambda'(p)}{\lambda(p)} + \frac{\mu_1 - \mu_2}{p\mu_1 + (1-p)\mu_2} > 0$$
$$\frac{|\lambda'(p)|}{\lambda(p)} < \frac{\mu_1 - \mu_2}{p\mu_1 + (1-p)\mu_2}$$

$$\frac{d\left(\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)\right)}{dp} = \lambda(p)^2\left(-\frac{2}{\mu_1^2} + \frac{2}{\mu_2^2}\right) + 2\lambda(p)\lambda'(p)\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)$$
$$= \frac{2\lambda(p)^2(2p\mu_1^2 - 2(-1+p)\mu_2^2)}{\mu_1^2\mu_2^2}\left(\frac{\lambda'(p)}{\lambda(p)} + \frac{\mu_1^2 - \mu_2^2}{2p\mu_1^2 - 2(-1+p)\mu_2^2}\right)$$

since $0 < p < 1$, $\lambda(p) > 0$, $\mu_1 > \mu_2 > 0$;

$$\frac{2\lambda(p)^2(2p\mu_1^2 - 2(-1+p)\mu_2^2)}{\mu_1^2\mu_2^2} > 0$$

and since $\lambda(p)$ is decreasing in p, $\lambda'(p) < 0$; to be

$$\frac{d\left(\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)\right)}{dp} > 0$$

, it should be;

$$\frac{\lambda'(p)}{\lambda(p)} + \frac{\mu_1^2 - \mu_2^2}{2p\mu_1^2 - 2(-1+p)\mu_2^2} > 0$$
$$\frac{|\lambda'(p)|}{\lambda(p)} < \frac{\mu_1^2 - \mu_2^2}{2(\mu_2^2(1-p) + \mu_1^2 p)}$$

We have two conditions for monotone increasing total cost function;

$$\frac{|\lambda'(p)|}{\lambda(p)} < \frac{\mu_1 - \mu_2}{p\mu_1 + (1-p)\mu_2}$$

$$\frac{|\lambda'(p)|}{\lambda(p)} < \frac{\mu_1^2 - \mu_2^2}{2(\mu_2^2(1-p) + \mu_1^2 p)}$$

If

$$\frac{|\lambda'(p)|}{\lambda(p)} < \min_{p}\left\{\frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2}\right\}$$

$$\frac{|\lambda'(p)|}{\lambda(p)} < \frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2}$$

for all p.

$$\frac{d\left(\frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2}\right)}{dp} = -\frac{(\mu_1-\mu_2)^2}{(p\mu_1-(-1+p)\mu_2)^2} < 0$$

implying

$$\frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2}$$

is a decreasing function in p and reaches its minimum value when $p=1$. When $p=1$;

$$\frac{\mu_1-\mu_2}{p\mu_1+(1-p)\mu_2} = \frac{\mu_1-\mu_2}{\mu_1}$$

Hence

$$\frac{|\lambda'(p)|}{\lambda(p)} < \frac{\mu_1-\mu_2}{\mu_1}$$

is a sufficient condition for $\frac{d\rho(p)}{dp} > 0$.

If

$$\frac{|\lambda'(p)|}{\lambda(p)} < \min_{p}\left\{\frac{\mu_1^2-\mu_2^2}{2(\mu_2^2(1-p)+\mu_1^2 p)}\right\}$$

$$\frac{|\lambda'(p)|}{\lambda(p)} < \frac{\mu_1^2-\mu_2^2}{2(\mu_2^2(1-p)+\mu_1^2 p)}$$

for all p.

$$\frac{d\left(\frac{\mu_1^2-\mu_2^2}{2(\mu_2^2(1-p)+\mu_1^2 p)}\right)}{dp} = -\frac{(\mu_1^2-\mu_2^2)^2}{2\,(p\mu_1^2-(-1+p)\mu_2^2)^2} < 0$$

implying

$$\frac{\mu_1^2-\mu_2^2}{2(\mu_2^2(1-p)+\mu_1^2 p)}$$

is a decreasing function in p and reaches its minimum value when $p=1$. When $p=1$;

$$\frac{\mu_1^2-\mu_2^2}{2(\mu_2^2(1-p)+\mu_1^2 p)} = \frac{\mu_1^2-\mu_2^2}{2\mu_1^2}$$

Hence

$$\frac{|\lambda'(p)|}{\lambda(p)} < \frac{\mu_1^2 - \mu_2^2}{2\mu_1^2}$$

is a sufficient condition for

$$\frac{d\left(\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)\right)}{dp} > 0$$

For all $\mu_1 > \mu_2$;

$$\frac{\left(\frac{\mu_1^2-\mu_2^2}{2\mu_1^2}\right)}{\left(\frac{\mu_1-\mu_2}{\mu_1}\right)} = \frac{\mu_1 + \mu_2}{2\mu_1} < 1$$

$$\frac{\mu_1^2 - \mu_2^2}{2\mu_1^2} < \frac{\mu_1 - \mu_2}{\mu_1}$$

hence;

$$\frac{|\lambda'(p)|}{\lambda(p)} < \frac{\mu_1^2 - \mu_2^2}{2\mu_1^2}$$

is a sufficient condition for

$$\frac{d\rho(p)}{dp} > 0$$

and

$$\frac{d\left(\lambda(p)^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)\right)}{dp} > 0$$

. In that case, nominator of total cost function will always increase whereas the denominator will always decrease implying total cost will always increase. Then the optimum solution is $p = 0$.

□

A.7 Proposition 4.7

If $\sqrt{\frac{\mu_1^2}{1+k}} \leq \mu_2 < \mu_1$; total cost is always decreasing and the optimal solution is $p = 1$.

Proof.

$$L(p) = \frac{(2-kp)^2\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)}{2\left(1-\rho(p)\right)}$$

where

$$\rho(p) = \frac{(2-kp)\lambda_0\,(p\mu_1 + (1-p)\mu_2)}{\mu_1\mu_2}$$

If $L(p)$ is monotone decreasing ($\frac{dL(p)}{dp} < 0$ for all p), then optimal solution is $p = 1$. Sufficient conditions for

$$\frac{dL(p)}{dp} < 0$$

are

$$\frac{d\rho(p)}{dp} < 0$$

$$\frac{d\left((2-kp)^2\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)\right)}{dp} < 0$$

If

$$\frac{d\rho(p)}{dp} = \frac{\lambda_0\,((2-2kp)\mu_1 - (2+k-2kp)\mu_2)}{\mu_1\mu_2} < 0$$

then since $\lambda_0, \mu_1, \mu_2 > 0$;

$$(2-2kp)\mu_1 - (2+k-2kp)\mu_2 < 0$$

$$(2-2kp)\mu_1 < (2+k-2kp)\mu_2$$

since $0 < k < 2$ and $0 < p < 1$; $2 + k - 2kp > 0$ we can divide both sides of the inequality by it without changing the direction of the inequality.

$$\frac{\mu_2}{\mu_1} > \frac{2-2kp}{2+k-2kp}$$

$$\frac{d\left(\frac{2-2kp}{2+k-2kp}\right)}{dp} = -\frac{2k^2}{(-2+k(-1+2p))^2} < 0$$

implying

$$\frac{2-2kp}{2+k-2kp}$$

is a decreasing function in p and takes its maximum value when $p = 0$. When $p = 0$,

$$\frac{\mu_2}{\mu_1} > \frac{2-2kp}{2+k-2kp} = \frac{2}{2+k}$$

If

$$\frac{d\left((2-kp)^2\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)\right)}{dp} = (2-kp)^2\lambda_0^2\left(-\frac{2}{\mu_1^2}+\frac{2}{\mu_2^2}\right) - 2k(2-kp)\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right) < 0$$

then since $\lambda_0, \mu_1, \mu_2 > 0$,$0 < p < 1$ and $0 < k < 2$, $(2-kp) > 0$;

$$2k(2-kp)\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right) > (2-kp)^2\lambda_0^2\left(-\frac{2}{\mu_1^2}+\frac{2}{\mu_2^2}\right)$$

$$k\ \left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right) > (2-kp)\ \left(-\frac{1}{\mu_1^2}+\frac{1}{\mu_2^2}\right)$$

$$k\left(2p\mu_1^2+2(1-p)\mu_2^2\right) > (2-kp)\left(\mu_1^2-\mu_2^2\right)$$

$$\mu_2^2(2-3kp+2k) > \mu_1^2(2-3kp)$$

since $0 < p < 1$ and $0 < k < 2$, $(2-3kp+2k) > 0$

$$\frac{\mu_2^2}{\mu_1^2} > \frac{2-3kp}{2+2k-3kp}$$

$$\frac{d\left(\frac{2-3kp}{2+2k-3kp}\right)}{dp} = -\frac{6k^2}{(-2+k(-2+3p))^2} < 0$$

implying

$$\frac{2-3kp}{2+2k-3kp}$$

is a decreasing function in p and takes its maximum value when $p = 0$. When $p = 0$,

$$\frac{\mu_2^2}{\mu_1^2} > \frac{2-3kp}{2+2k-3kp} = \frac{1}{1+k}$$

$$\frac{\mu_2}{\mu_1} > \frac{1}{\sqrt{1+k}}$$

together with the other condition,

$$\frac{\mu_2}{\mu_1} > \frac{1}{\sqrt{1+k}} > \frac{2}{2+k}$$

hence

$$\frac{\mu_2}{\mu_1} > \frac{1}{\sqrt{1+k}}$$

is a sufficient condition for

$$\frac{d\rho(p)}{dp} < 0$$

and

$$\frac{d\left((2-kp)^2\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)\right)}{dp} < 0$$

. In that case, nominator of total cost function will always decrease whereas the denominator will always increase implying total cost will always decrease. Then the optimum solution is $p = 1$. □

A.8 Proposition 4.8

If $0 < \mu_2 \leq \mu_1\sqrt{\frac{2-3k}{2-k}}$ and $0 < k < \frac{2}{3}$; total cost is always increasing and the optimal solution is $p = 0$.

Proof.

$$L(p) = \frac{(2-kp)^2\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)}{2\left(1-\rho(p)\right)}$$

where

$$\rho(p) = \frac{(2-kp)\lambda_0\left(p\mu_1+(1-p)\mu_2\right)}{\mu_1\mu_2}$$

If $L(p)$ is monotone increasing ($\frac{dL(p)}{dp} > 0$ for all p), then optimal solution is $p = 0$. Sufficient conditions for

$$\frac{dL(p)}{dp} > 0$$

are

$$\frac{d\rho(p)}{dp} > 0$$

$$\frac{d\left((2-kp)^2\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)\right)}{dp} > 0$$

If

$$\frac{d\rho(p)}{dp} = \frac{\lambda_0\left((2-2kp)\mu_1-(2+k-2kp)\mu_2\right)}{\mu_1\mu_2} > 0$$

then since $\lambda_0, \mu_1, \mu_2 > 0$;

$$(2-2kp)\mu_1 - (2+k-2kp)\mu_2 > 0$$

$$(2-2kp)\mu_1 > (2+k-2kp)\mu_2$$

since $0 < k < 2$ and $0 < p < 1$; $2+k-2kp > 0$ we can divide both sides of the inequality by it without changing the direction of the inequality.

$$\frac{\mu_2}{\mu_1} < \frac{2-2kp}{2+k-2kp}$$

$$\frac{d\left(\frac{2-2kp}{2+k-2kp}\right)}{dp} = -\frac{2k^2}{(-2+k(-1+2p))^2} < 0$$

implying

$$\frac{2-2kp}{2+k-2kp}$$

is a decreasing function in p and takes its minimum value when $p = 1$. When $p = 1$,

$$\frac{\mu_2}{\mu_1} < \frac{2-2kp}{2+k-2kp} = \frac{2-2k}{2-k}$$

If

$$\frac{d\left((2-kp)^2\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right)\right)}{dp} = (2-kp)^2\lambda_0^2\left(-\frac{2}{\mu_1^2}+\frac{2}{\mu_2^2}\right) - 2k(2-kp)\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right) > 0$$

then since $\lambda_0, \mu_1, \mu_2 > 0$,$0 < p < 1$ and $0 < k < 2$, $(2-kp) > 0$;

$$2k(2-kp)\lambda_0^2\left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right) < (2-kp)^2\lambda_0^2\left(-\frac{2}{\mu_1^2}+\frac{2}{\mu_2^2}\right)$$

$$k\ \left(\frac{2(1-p)}{\mu_1^2}+\frac{2p}{\mu_2^2}\right) < (2-kp)\ \left(-\frac{1}{\mu_1^2}+\frac{1}{\mu_2^2}\right)$$

$$k\left(2p\mu_1^2+2(1-p)\mu_2^2\right) < (2-kp)\left(\mu_1^2-\mu_2^2\right)$$

$$\mu_2^2(2-3kp+2k) < \mu_1^2(2-3kp)$$

since $0 < p < 1$ and $0 < k < 2$, $(2-3kp+2k) > 0$ we can divide both sides of the inequality by it without changing the direction of the inequality.

$$\frac{\mu_2^2}{\mu_1^2} < \frac{2-3kp}{2+2k-3kp}$$

$$\frac{d\left(\frac{2-3kp}{2+2k-3kp}\right)}{dp} = -\frac{6k^2}{(-2+k(-2+3p))^2} < 0$$

implying

$$\frac{2-3kp}{2+2k-3kp}$$

is a decreasing function in p and takes its minimum value when $p = 1$. When $p = 1$,

$$\frac{\mu_2^2}{\mu_1^2} < \frac{2-3kp}{2+2k-3kp} = \frac{2-3k}{2-k}$$

since $0 < k < 2$, $2-3k$ can be negative whereas $\frac{\mu_2^2}{\mu_1^2}$ can never be negative. This implies a boundary condition on k.

$$2-3k > 0$$

$$k < \frac{2}{3}$$

$$0 < k < \frac{2}{3}$$

Then;

$$\frac{\mu_2}{\mu_1} < \sqrt{\frac{2-3k}{2-k}}$$

together with the other condition if $0 < k < \frac{2}{3}$;

$$\sqrt{\frac{2-3k}{2-k}} < \frac{2-2k}{2-k}$$

Then;

$$\frac{\mu_2}{\mu_1} < \sqrt{\frac{2-3k}{2-k}} < \frac{2-2k}{2-k}$$

hence

$$\frac{\mu_2}{\mu_1} < \sqrt{\frac{2-3k}{2-k}}$$

$$0 < \mu_2 \leq \mu_1\sqrt{\frac{2-3k}{2-k}}$$

In this case total cost function is monotone increasing and optimal solution is $p = 0$. $\square$

A.9 Proposition 4.9

Optimal solution is always 1.

Proof.

$$L(p) = \frac{\left(\frac{\lambda_0}{kp}\right)^2 \left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)}{2\left(1-\rho(p)\right)}$$

where

$$\rho(p) = \frac{\left(\frac{\lambda_0}{kp}\right)\left(p\mu_1 + (1-p)\mu_2\right)}{\mu_1\mu_2}$$

If $L(p)$ is monotone decreasing ($\frac{dL(p)}{dp} < 0$ for all p), then optimal solution is given by 1.

$$\frac{d\rho(p)}{dp} = -\frac{\lambda}{kp^2\mu_1} < 0$$

implying denominator is always increasing.

$$\frac{d\left(\frac{\lambda_0}{kp}\right)^2 \left(\frac{2(1-p)}{\mu_1^2} + \frac{2p}{\mu_2^2}\right)}{dp} = \frac{2\lambda^2\left(\frac{-2+p}{\mu_1^2} - \frac{p}{\mu_2^2}\right)}{k^2p^3} < 0$$

implying nominator is always decreasing.

As a result, $L(p)$ is monotone decreasing and optimal solution is given by 1. $\square$

Appendix B

TABLES

Table B.1: Group Statistics

Group Statistics					
	anxiety,hypertension	N	Mean	Std. Deviation	Std. Error Mean
august	0	168	,76	,865	,067
	1	414	,76	,877	,043
september	0	168	,79	,972	,075
	1	414	,69	,868	,043
october	0	168	,77	,875	,068
	1	414	,64	,885	,044
november	0	168	,58	,696	,054
	1	414	,55	,727	,036
december	0	168	,65	,848	,065
	1	414	,64	,822	,040
january	0	168	,77	,882	,068
	1	414	,59	,759	,037
february	0	168	,71	,807	,062
	1	414	,61	,845	,042
march	0	168	,66	,832	,064
	1	414	,61	,829	,041
april	0	168	,61	,725	,056
	1	414	,51	,732	,036
may	0	168	,71	,829	,064
	1	414	,65	,821	,040
june	0	168	,80	,962	,074
	1	414	,69	,910	,045
july	0	168	,55	,724	,056
	1	414	,56	,802	,039

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
august	Equal variances assumed	0,021	0,885	-0,06	580	0,951	-0,005	0,08	-0,162	0,152
	Equal variances not assumed			-0,06	313,1	0,951	-0,005	0,079	-0,161	0,151
september	Equal variances assumed	1,539	0,215	1,285	580	0,199	0,106	0,082	-0,056	0,267
	Equal variances not assumed			1,225	280,7	0,222	0,106	0,086	-0,064	0,275
october	Equal variances assumed	0,001	0,974	1,613	580	0,107	0,13	0,081	-0,028	0,289
	Equal variances not assumed			1,62	312,6	0,106	0,13	0,08	-0,028	0,288
november	Equal variances assumed	0,185	0,668	0,496	580	0,62	0,033	0,066	-0,096	0,162
	Equal variances not assumed			0,506	322,3	0,613	0,033	0,064	-0,094	0,159
december	Equal variances assumed	0,045	0,832	0,13	580	0,897	0,01	0,076	-0,139	0,159
	Equal variances not assumed			0,128	300,9	0,898	0,01	0,077	-0,141	0,161
january	Equal variances assumed	3,011	0,083	2,449	580	0,015	0,178	0,073	0,0353	0,322
	Equal variances not assumed			2,299	272,6	0,022	0,178	0,078	0,0257	0,331
february	Equal variances assumed	0,332	0,565	1,338	580	0,182	0,102	0,076	-0,048	0,252
	Equal variances not assumed			1,364	322,9	0,174	0,102	0,075	-0,045	0,249
march	Equal variances assumed	0,017	0,898	0,621	580	0,535	0,047	0,076	-0,102	0,196
	Equal variances not assumed			0,62	308,5	0,535	0,047	0,076	-0,102	0,197
april	Equal variances assumed	0,019	0,891	1,548	580	0,122	0,103	0,067	-0,028	0,235
	Equal variances not assumed			1,555	312,1	0,121	0,103	0,067	-0,027	0,234
may	Equal variances assumed	0,345	0,557	0,81	580	0,418	0,061	0,075	-0,087	0,209
	Equal variances not assumed			0,807	306,9	0,421	0,061	0,076	-0,088	0,21
june	Equal variances assumed	0,883	0,348	1,304	580	0,193	0,11	0,085	-0,056	0,277
	Equal variances not assumed			1,273	294,6	0,204	0,11	0,087	-0,06	0,281
july	Equal variances assumed	0,582	0,446	-0,21	580	0,832	-0,015	0,071	-0,155	0,125
	Equal variances not assumed			-0,22	340,6	0,824	-0,015	0,068	-0,15	0,119

Table B.2: Independent Samples T-test

Table B.3: Group Statistics

Group Statistics					
	type2-type1	N	Mean	Std. Deviation	Std. Error Mean
august	1	2121	,19	,478	,010
	2	582	,76	,873	,036
september	1	2121	,16	,435	,009
	2	582	,72	,900	,037
october	1	2121	,18	,455	,010
	2	582	,68	,884	,037
november	1	2121	,17	,442	,010
	2	582	,56	,718	,030
december	1	2121	,22	,492	,011
	2	582	,65	,828	,034
january	1	2121	,36	,590	,013
	2	582	,64	,800	,033
february	1	2121	,19	,451	,010
	2	582	,64	,835	,035
march	1	2121	,23	,548	,012
	2	582	,63	,830	,034
april	1	2121	,20	,499	,011
	2	582	,54	,731	,030
may	1	2121	,22	,516	,011
	2	582	,66	,823	,034
june	1	2121	,26	,509	,011
	2	582	,73	,926	,038
july	1	2121	,25	,540	,012
	2	582	,56	,780	,032

		Independent Samples Test								
		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
august	Equal variances assumed	389,192	,000	-20,799	2701	,000	-,570	,027	-,624	-,516
	Equal variances not assumed			-15,147	679,250	,000	-,570	,038	-,644	-,496
september	Equal variances assumed	600,151	,000	-20,894	2701	,000	-,555	,027	-,607	-,503
	Equal variances not assumed			-14,434	657,140	,000	-,555	,038	-,631	-,480
october	Equal variances assumed	464,102	,000	-18,377	2701	,000	-,494	,027	-,547	-,441
	Equal variances not assumed			-13,028	667,569	,000	-,494	,038	-,569	-,420
november	Equal variances assumed	441,341	,000	-16,400	2701	,000	-,395	,024	-,442	-,347
	Equal variances not assumed			-12,622	706,307	,000	-,395	,031	-,456	-,333
december	Equal variances assumed	378,367	,000	-15,547	2701	,000	-,423	,027	-,476	-,370
	Equal variances not assumed			-11,757	697,227	,000	-,423	,036	-,493	-,352
january	Equal variances assumed	112,685	,000	-9,354	2701	,000	-,281	,030	-,340	-,222
	Equal variances not assumed			-7,894	762,869	,000	-,281	,036	-,350	-,211
february	Equal variances assumed	486,032	,000	-17,004	2701	,000	-,443	,026	-,494	-,392
	Equal variances not assumed			-12,316	676,838	,000	-,443	,036	-,514	-,372
march	Equal variances assumed	243,667	,000	-13,636	2701	,000	-,395	,029	-,452	-,338
	Equal variances not assumed			-10,858	725,340	,000	-,395	,036	-,467	-,324
april	Equal variances assumed	268,470	,000	-12,978	2701	,000	-,338	,026	-,389	-,287
	Equal variances not assumed			-10,509	735,458	,000	-,338	,032	-,401	-,275
may	Equal variances assumed	292,896	,000	-16,079	2701	,000	-,448	,028	-,503	-,393
	Equal variances not assumed			-12,477	710,733	,000	-,448	,036	-,519	-,378
june	Equal variances assumed	347,423	,000	-16,088	2701	,000	-,469	,029	-,526	-,411
	Equal variances not assumed			-11,734	679,944	,000	-,469	,040	-,547	-,390
july	Equal variances assumed	175,479	,000	-11,072	2701	,000	-,311	,028	-,366	-,256
	Equal variances not assumed			-9,038	740,576	,000	-,311	,034	-,378	-,243

Table B.4: Difference between type 1 and type 2 patients' arrivals

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	August	,76	582	,873	,036
	July	,56	582	,780	,032

Paired Samples Test

	Paired Differences					t	df	Sig(2-tailed)
				95 Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	lower	upper			
Pair 1 august-july	,201	1,028	,043	,117	,285	4,720	581	,000

Table B.5: Paired t-test

Ranks

		N	Mean Rank	Sum of Ranks
July - august	Negative Ranks	201[a]	157,64	31685,50
	Positive Ranks	112[b]	155,85	17455,50
	Ties	269[c]		
	Total	582		

a.july<august

b.july>august

c.july=august

Test Statistics[b]

	July-August
Z	$-4,724^a$
Asymp.Sig (2-tailed)	,000

a.Based on positive Ranks

b.Wilcoxon Signed Rank Test

Table B.6: Wilcoxon Signed Rank test

Descriptive Statistics					
	N	Minimum	Maximum	Mean	Std. Deviation
Age(doctor)	384	24	55	36.32	5.903
Gender(f=1,m=0)	384	0	1	.23	.419
Experience (month)	384	1	66	18.02	7.252
People registered	384	1001	5673	3582.16	506.975
Female registered	384	501	3007	1802.61	265.307
Male registered	384	500	2743	1779.55	257.345
<1	384	1	500	49.52	30.271
1-5	384	5	675	202.17	80.138
6-14	381	10	1111	518.62	165.411
15-18	381	5	850	240.75	83.982
19-50	381	30	4143	1747.41	421.434
51-64	381	11	2200	485.62	187.426
>65	381	19	1210	359.86	188.810
Working hours (in clinic)	384	6	88	37.14	7.719
Working hours (mobile)	384	0	35	5.66	6.428
Average consultation duration (min)	384	1	70	10.76	5.619
Number of assistants	384	0	4	1.22	.582
Diagnosis (percentage)	384	2.083	69.048	23.82148	11.451464
Treatment	384	.787	62.500	21.01058	11.050285
Vaccination	384	.000	36.585	9.78056	6.011333
Management of chronic diseases	384	.000	33.333	6.62349	4.307501
Periodic examinations	384	.000	30.612	3.42147	3.103974
Risk factor scanning	384	.000	19.608	5.07261	3.154727
Mother-child health	384	.000	27.027	7.58703	4.656366
Family planning	384	.000	19.231	4.99282	3.082138
Promotion of health	384	.000	14.218	2.94473	1.814306
Administrative affairs	384	.000	45.455	7.62039	5.336940
Seminars	384	.000	14.815	3.19603	2.023527
Other	384	.000	29.268	3.92879	3.031416
Appointment (yes=1,no=0)	384	0	1	.06	.238
Enough time (yes=1,no=0)	384	0	1	.20	.401
Refer higher level (percentage)	384	0	90	5.52	8.473
Daily consultation	384	3	135	57.02	21.444
Quit family practise in 5 years (percentage)	384	0	100	24.64	31.461
Diagnosis and treatment (level of importance)	384	0	5	4.04	1.157
Vaccination	384	0	5	4.72	1.021
Management of chronic diseases	384	0	5	3.94	1.182
Periodic examinations	384	0	5	3.88	1.219
Risk factor scanning	384	0	5	4.07	1.128
Mother-child health	384	0	5	4.55	1.076
Family planning	384	0	5	4.20	1.263
Promotion of health	384	0	5	3.80	1.224
Administrative affairs	384	0	5	3.08	1.287
Seminars	384	0	5	3.69	1.185
Other	384	0	5	.78	1.458
Valid N (listwise)	380				

Table B.7: Survey Results : Descriptive Statistics

VITA

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