The Importance of Foreign Shocks in a Small Open Economy: The Case of Turkey

by

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A thesis submitted to the
Graduate School of Social Sciences & Humanities
in partial fulfillment of the requirements for the
degree of
Master of Arts
in
Economics

May 2011

Department of Economics

KOÇ UNIVERSITY

Abstract

I explain the analytically tractable optimizing small open economy model with Calvo-type staggered price setting in Gali and Monacelli (2005) and replicate the macroeconomic implications of three alternative rule-based policy regimes for the small open economy: Domestic inflation and CPI-based Taylor rules, and an exchange rate peg. The key difference between these regimes is the relative amount of exchange rate volatility that they entail. Additionally, I estimate a small open economy VAR model to analyze the interactions between the Turkish and Euro area economies using block-exogeneity restriction. I find that domestic shocks are the major sources of variability for Turkish price level, interest rates and TRY/EUR exchange rate, whereas Turkish output gap is driven mainly by the foreign shocks.

Keywords: Small open economy, Optimal monetary policy, Sticky prices, Taylor rule, Foreign shocks, Turkey, Euro area.

JEL Classification: E58, E 52, F41, F42

Özet

Gali and Monacelli (2005) çalışmasındaki analitik çözümü mümkün olan ve fiyatların Calvo yapışkan fiyatlar teorisine göre belirlendiği küçük açık ekonomi modelini anlatarak üç farklı para politikası rejiminin makroekonomik sonuçlarını tekrar ettim: Yurtiçi enflasyon ve Tüketici Fiyat Endeksi baz alınan Taylor kuralları ve sabit döviz kuru politikası. Elde edilen sonuçlar, bu farklı rejimlerin arasındaki temel farkın döviz kurunda hangi seviyede oynaklığa izin verdikleri olduğunu gösteriyor. Ek olarak, bir küçük açık ekonomi VAR modeli hesaplayarak Türkiye ekonomisi ve Avro bölgesi ekonomisi arasındaki ilişkileri blok dışsallık kısıtlamasını kullanarak analiz ettim. Türkiye'deki fiyat seviyeleri, faiz oranları ve döviz kurundaki dalgalanmaların temel kaynağının yurtiçi şoklar olduğunu, fakat Türkiye üretim açığındaki dalgalanmaların çoğunlukla dış şoklar ile açıklanabildiğini buldum.

Anahtar Sözcükler: Küçük açık ekonomi, Optimal para politikası, Yapışkan fiyatlar, Taylor kuralı, Dış Şoklar, Türkiye, Avro bölgesi.

Acknowledgements

I would like to express my deep gratitude to my supervisor Professor Sumru Altuğ for not only her continuous support throughout my thesis and course work, but also changing my perception of the science of economics, broadening my vision of life and making me a more courageous person with her knowledge, wisdom and friendly personality. I believe that the way of thinking that I learned from her is an invaluable gift that will help me in every stage of my future life.

I am also deeply grateful to Professor Levent Koçkesen for his continuous support and invaluable time that he devoted to me for guidance and by being a member of my thesis committee. I feel very lucky for being his student, being influenced by his teaching skills, intellectual virtues and high work ethics.

Also, I would like to thank Professor Evrim Akdoğu for being in my thesis committee and all my professors at Koç University Economics Department, especially to Professor Selva Demiralp, Professor Kamil Yılmaz and Professor Murat Usman, for their guidance and support during my two years at Koç University.

I also would like to thank to my friends Ramazan Bora, İlhan Güner and Metin Uyanık for their friendship and supports throughout the course work and especially during my research.

Finally, I am indebted to TÜBİTAK and Vehbi Koç Foundation for financial assistance.

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Abbreviations

TRL	**T**urkish **L**ira
EUR	**E**uro
ECB	**E**uropean **C**entral **B**ank
CBRT	**C**entral **B**ank of the **R**epublic of **T**urkey
CPI	**C**onsumer **P**rice **I**ndex
DITR	**D**omestic **I**nflation **T**argeting
CITR	CPI **I**nflation **T**argeting
PEG	**E**xchange **R**ate **P**eg
GDP	**G**ross **D**omestic **P**roduction

To my loving family…

Chapter 1

Introduction

New Keynesian economics has been built on the foundations by Fischer (1977) and Taylor and Phelps (1977), which both used stochastic rational expectations models in an environment with wage and price rigidities, respectively, to show that the proper use of a feedback monetary policy rule can reduce the fluctuations in the economic activity. Gordon (1990) gives a discussion of the properties and the early history of the field. New Keynesian approach is based on the idea that temporary nominal price rigidities are the key frictions that give rise to non neutral effects of monetary policy. As Clarida et al. (1999) state, other strands of literature either reject the idea of nominal price rigidities as in real business cycle theory or focus on other nominal rigidities such as money demand.

When there exists nominal rigidities, monetary policy can effectively change the short term real interest rate by varying the nominal interest rate. Through this mechanism, it may play a significant role over the fluctuations in the domestic variables of the economy. In contrast to the traditional mechanism, since both the firms and the households are forward-looking, beliefs about how the central bank will set the interest rate in the future also matter. Therefore, how monetary policy should respond in the short run to the disturbances hitting the economy becomes a non-trivial decision. Galí and Gertler (2007) describe the main futures of the new macroeconomic models used for monetary policy evaluation. They emphasize the significant role of expectations of future policy actions in the monetary transmission mechanism and the importance of keeping the output and the real interest rate at their natural levels.

Woodford (2003) explains the New Keynesian dynamic stochastic general equilibrium modeling methodology in a closed economy setting. In a model with both wage and price rigidity, Erceg et al. (2000) show that the monetary policy cannot replicate the Pareto optimal equilibrium that would occur in an environment absent from nominal rigidities because of the tradeoff between stabilizing the output gap, price inflation and the wage inflation. Schmitt-Grohé and Uribe (2004) studies optimal fiscal and monetary policy under a model with sticky prices where the government imposes distortionary income taxes, prints money and issues riskless bonds. They find that the optimal volatility of inflation is near zero and small deviations from full price flexibility induce near random walk behavior in government debt and tax rates.

Openness complicates the monetary policy decision problem to the extent the central bank must take into account the impact of the exchange rate on real activity and inflation. Therefore, including the exchange rate into the design of monetary policy is another crucial consideration in an open economy setting. Obstfeld and Rogoff (1995) is marked as the seminal paper in new open economy literature. Sarno (2001) and Bowman and Doyle (2005) give a summary of the work in this field and emphasize its implications for monetary policy. Lane and Ganelli (2003) discusses the key issues in open economy modeling by focusing on the currency denomination of sticky prices, the role of the current account and net foreign assets and the impact of fiscal policy.

In a small open economy setting, Svensson (2000) compare CPI and domestic inflation targeting with both strict and flexible cases, inflation targeting reaction functions and the Taylor rule. The author examines optimal monetary policy response to several different shocks and finds that flexible CPI inflation targeting stands out as successful in limiting not only the variability of CPI inflation but also the variability of the output gap and the real exchange rate.

Under the assumption of perfect exchange rate pass-through, Clarida et al. (2001) shows that the monetary policy design problem for the small open economy is equivalent to the problem of the closed economy for some certain conditions, except that the degree of openness affects how aggressively a central bank should adjust the interest rate in response to the disturbances. On the other hand, Monacelli (2006) finds that incomplete pass-through of exchange rates to prices renders the analysis of monetary policy of an open economy fundamentally different from the one of a closed economy. Moreover, Clarida et al. (2001) finds that central bank should target domestic inflation and allow the exchange rate to float despite the impact of the resulting exchange rate variability on inflation, whereas Monacelli (2006) shows that incomplete pass-through generates a short-run tradeoff between the stabilization of the output gap and inflation.

Schmitt-Grohé and Uribe (2001) lay out an optimizing model of a small open economy with price rigidities and compares the welfare costs of a dollarized economy with economies in which monetary policy takes the form of inflation targeting, money growth rate pegs, or devaluation rate rules. They find that the dollarization is the least successful of the monetary policies considered.

Corsetti and Pesenti (2001) develops a model for analyzing the welfare of the international transmission of monetary and fiscal policies. They find positive externalities of foreign monetary expansions and foreign fiscal contractions on domestic welfare.

Devereux and Engel (2003) focus on the extent to which monetary policy should aim maintaining the exchange rate. He finds that under local currency pricing optimal monetary policy in response to real shocks should be keeping the exchange rates fixed. On the other hand, when real country-specific shocks are not important and when a country's monetary sector is stable, the benefits from a freely floating exchange rate increases. Modifying the model in Devereux and Engel (2003) to produce nonsynchronous consumption movements, Duarte and Obstfeld (2008) upset the fixed exchange rate prescription even in the expenditure-switching role of exchange rate changes.

Schmitt-Grohé and Uribe (2004) studies optimal fiscal and monetary policies under sticky product prices. They find that for a small degree of price stickiness the optimal volatility of inflation is near zero. In addition, they show that small deviations from full price flexibility induce near random walk behavior in government debt and tax rates.

Leitemo and Söderström (2005) analyze the performance of common simple monetary policy rules in a small open economy model. They find that adding the exchange rate to an optimized Taylor rule gives only small improvements in terms of economic stability. Their results show that Taylor rule may be sufficient to stabilize a small open economy.

Gali and Monacelli (2005) lay out a small open economy version of a model with Calvo-type price setting with an endogenous monetary policy. The latter property allows the authors to analyze the macroeconomic implications and relative welfare rankings of three different monetary regimes. They find that these regimes can be ranked according to their implied volatility for the terms of trade and exchange rate volatility. Under a particular parametrization, they find that stabilizing domestic inflation is the optimal policy. Hence, a policy of strict domestic inflation targeting which successfully achieves a simultaneous stabilization of the output gap and domestic inflation implies a substantially greater volatility in the nominal exchange rate and terms of trade. By developing a generalized loss function for the small open economy, De Paoli (2009) also demonstrate that the exchange rate volatility has important effects on the welfare in a small open economy.

Schmitt-Grohe and Uribe (2007) analyzes optimal monetary and fiscal policy rules in a model with sticky prices, money and distortionary income taxation. They find that the inflation coefficient in the interest-rate rule plays a minor role for welfare. Moreover, optimal monetary policy does not respond to output excessively. Interest-rate rules that feature a positive response to output can lead to significant welfare losses. Additionally, optimal fiscal policy stands as a passive one in their model.

Faia and Monacelli (2008) analyze optimal monetary policy in a small open economy characterized by home bias in consumption. They show that this bias is a sufficient condition for inducing monetary policy authorities to deviate from a strategy of strict markup stabilization and contemplate some degree of exchange rate stabilization.

In a tractable model for a currency union which includes country-specific shocks and nominal rigidities, Gali and Monacelli (2008) focus on the optimal design of fiscal and monetary policies. They find that the optimal policy requires stabilizing inflation at the union level. On the other hand, the relinquishment of an independent monetary policy, coupled with nominal price rigidities, generates a stabilization role for fiscal policy from the viewpoints of individual countries and the union as a whole.

The open economy New Keynesian models provide a theoretical motivation for the empirical assessment of the importance of domestic and foreign shocks on the variability of various macroeconomic variables in small open economies. Utilizing the block exogeneity restriction, which exploits the assumption that the small open economy cannot influence significantly the developments in the rest world is another property being used in most of these models.

Cushman and Zha (1997) consider the interactions between the U.S. and the Canadian economy for the identification of the Canadian monetary policy using the block exogeneity restriction. The authors argue that the previous literature was unable to identify the monetary policy shock accurately, as it did not control for external factors explicitly.

Kim (2001) studies the effects of US monetary policy shocks on non-US G7 countries. The author finds that US monetary expansion has a positive spillover effect on output in these countries. In addition, expansion leads to a short run deterioration of the trade balance, but the balance improves persistently in the medium to long run. Contrary to previous literature which suggested that non-US G7 countries' monetary policy substantially follow the US monetary policy, the author shows that after controlling for inflationary or supply shocks, the non-US monetary authorities does not seem to respond extensively to the U.S. monetary policy shocks, except for Canada.

Muscatelli et al. (2004) examines the interaction of monetary and fiscal policies using an estimated New Keynesian dynamic general equilibrium model for the U.S. They show that the strategic complementarity or substitutability of fiscal and monetary shocks depends on the types of shocks hitting the economy, and on the assumptions made about the underlying structural model.

Giordani (2004b) focuses on the responses of a small open economy to foreign rather than to domestic shocks. He estimates a structural theoretical model from a class of New-Keynesian models and compares it with Bayesian VAR. The author finds that the U.S. shocks are a very important source of variation in all Canadian variables.

Canova (2005) examines the importance of the effects of the U.S. monetary policy shock on the Latin America economies. He does not find a major difference in terms of the transmission of the shocks between the countries with fixed and flexible exchange regimes.

Mackowiak (2006a) focuses on the role of external shocks on the macroeconomic variation in Czech Republic, Poland and Hungary by using Germany as a proxy for the external shocks. He finds that the price level and the real output in these small open economies are mainly driven by the foreign shocks.

Mackowiak (2006b) investigates the impact of Japanese monetary shocks on macroeconomic variation in East Asia economies, Hong Kong, Malaysia, Korea, Philippines, Singapore and Thailand. Using Bayesian VAR he finds that Japanese monetary shocks have only a small share on the variance in real output, trade balances and exchange rates in East Asia. In particular, he does not find evidence that expansionary Japanese monetary policy contributed to the East Asian crisis.

Mackowiak (2007) estimates the structural VAR models with block exogeneity for 10 emerging markets from Latin America and East Asia. The author finds that in a typical emerging market, external shocks account for approximately 50% of the variation in exchange rate and the price level and 40% and 33% for the variation in real output and short term interest rate, respectively. Additionally, he shows that U.S. monetary policy shocks are less important for emerging markets than other external shocks. On the other hand, the U.S. price level and real output reacts to U.S. monetary policy tightening more than the emerging economies.

Lubik and Schorfheide (2007) focuses on the conduct of monetary policy in Australia, Canada, New Zealand and the U.K. by estimating a structural general equilibrium model of a small open economy using Bayesian methods. They find that only the monetary authorities in Canada among these countries respond to exchange rate changes.

In this thesis, first, I explain the model of Gali and Monacelli (2005) and display their numerical results which shows the comparisons in terms of the macroeconomic and welfare implications of four alternative monetary policy rules: An optimal policy which successfully achieves stabilizing the domestic inflation and the output gap, a domestic inflation targeting Taylor-type rule, a CPI inflation targeting Taylor-type rule and an exchange rate peg. I replicated these results using Dynare.

Next, following Horvath and Rusnák (2009) I estimate a VAR model which can be thought of as the reduced form of the New Keynesian model in Gali and Monacelli (2005). In the VAR analysis, I focus on the interactions between the Turkish and the Euro area. I utilize the block exogeneity restriction, assuming Turkey as a small open economy and the Euro zone as the close world economy. My results are mostly in line with Giordani (2004b). I find that the domestic shocks are the dominant shocks of Turkish inflation, interest rate and the $TRY/EURexchangerate$, whereas most of the variance of the Turkish output is attributable to foreign shocks. In particular, the results show that the ECB monetary shock have a significant effect on Turkish output gap which lasts for 10 quarters after the shock. Given that the ECB monetary shock does not significantly influence the bilateral exchange rate, this effect is not likely to arise from an increase in net exports. I suggest that one possible explanation of this relationship may be the integration of the Euro area and the Turkish financial markets. Since Turkish interest rates are much higher than the ECB interest rates, when ECB implements a monetary tightening, the European banks may invest their increased liabilities in Turkish financial market which, in turn, would rise the available credits in the Turkish financial market, ending up in a boost of domestic consumption and investment levels.

The thesis is organized as follows. In chapter 2, I present the small open economy model of Gali and Monacelli (2005) and present their numerical results. In chapter 3, I present the theoretical motivation and the methodology for the VAR model I estimate. I conclude Chapter 3 by discussing the results of the estimated VAR and comparing the effects of domestic and foreign shocks on Turkish variables. The results are summarized in Chapter 4.

Chapter 2

The Model

A century ago, Swedish economist Knut Wicksell argued that the essential monetary instrument is interest rates (Wicksell (1907)). According to his view, price stability depends on keeping the interest rate in line with the natural interest rate which is determined by real factors such as marginal productivity of capital.

Theoretical literature of monetary policy almost entirely characterized the policy instrument as money supply and considered money-growth rules in different types until recently (Woodford (2003), Chapter 1).

Reviving the earlier approach of Knut Wicksell, Woodford (2003) considered systematic monetary policies that are described in terms of rules for setting a nominal interest rate. In a model where the concepts of money supply and demand become inapplicable, Michael Woodford provides two possible explanations for the determination of the equilibrium level of prices. First, past prices may determine current equilibrium prices due to either wage or price stickiness. Second, Central Bank's interest rates may determine the equilibrium prices, which is called the Wicksellian view.

Considering the second explanation, Woodford (2003) builds a closed-economy general equilibrium model where the monetary policy is specified in terms of an interest-rate feedback rule, without any reference to the fluctuations in the money supply. The resulting framework, which is called *canonical Calvo structure* consists of an IS-block, an aggregate supply-AS block and an interest-rate feedback rule. In this modeling, inflation and output can entirely be explained in Wicksellian terms: The relation between the natural interest rate determined by real factors and the central bank's rule for adjusting the short-term interest rate determines the evolution of these variables. The discrepancy between the actual and the natural interest rate is a result of the Calvo-type staggered price setting of the firms. In other words, the failure of adjusting the prices sufficiently rapidly causes the actual interest rate to be different from the natural one.

Gali and Monacelli (2005) extends the canonical Calvo model to a two-bloc model by assuming the world economy as a continuum of small open economies represented by the unit interval. The different economies are assumed to share identical preferences, technology, and market structure. Since each economy is of measure zero, its domestic policy decisions do not affect the rest of the world. Then, the authors describe three alternative policy rules for the small open economy and compares their implication for welfare.

In this section, I will present the model in Gali and Monacelli (2005). Next, I will present the results of the comparison between the alternative policy rules which I replicated using Dynare. The variables without an i index refer to the small open economy being modeled while the variables with an $i \epsilon [0, 1]$ subscript refer to economy i, one among the continuum of economies making up the world economy. Variables with a star superscript correspond to the world economy as a whole. The lower-case variables represent the log levels

2.1 Households

The representative household in the small open economy seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{2.1}$$

where N_t denotes the hours of labor, and C_t is a Dixit-Stiglitz composite index of consumption defined by

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.2}$$

where $C_{H,t}$ and $C_{F,t}$ are consumption indices for domestic and imported goods, respectively, and $\eta > 0$ is the elasticity of substitution between home and imported foreign goods. $C_{H,t}$ and $C_{F,t}$, in turn, are given by the following CES aggregators of the quantities consumed of each type of the good:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.3}$$

$$C_{F,t} = \left(\int_0^1 C_{i,t}(i)^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}} \tag{2.4}$$

where $i\epsilon[0,1]$ represents the economy i, $j\epsilon[0,1]$ represents the good variety, the parameters $\varepsilon > 1$ and γ denote the elasticity of substitutions between varieties produced within any given country and between goods produced in different foreign countries, respectively.

$C_{i,t}$ is, in turn, a CES aggregator of the quantity of goods imported from country i and consumed by domestic households. It is given by

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.5}$$

Since parameter $\alpha\epsilon[0,1]$ is inversely related to the degree of home bias in preferences, it can be used as an index of openness.

The representative household maximizes his utility subject to a sequence of budget constraints of the form:

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)dj di + E_t[Q_{t,t+1}D_{t,t+1}]] \leq D_t + W_t N_t + T_t \tag{2.6}$$

for $t = 0, 1, 2, \ldots$, where $P_{i,t}(j)$ is the price of variety j imported from country i. D_{t+1}, which also includes the shares in firms, is the nominal payoff in period $t+1$ of the portfolio held at the end of period t. W_t is the nominal wage, and T_t denotes lump-sum transfers/taxes. All the previous variables are expressed in units of domestic currency. $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal-payoffs relevant to the domestic household.

Households are assumed to have access to a complete set of contingent claims, traded internationally. That is, available financial assets completely span the relevant uncertainty that the households face. Therefore, each household faces a single intertemporal budget constraint. Since the monetary policy is specified in terms of an interest rate rule, it is not necessary to model the money explicitly. A cashless economy is assumed so that money does not enter into the household's budget constraint.

The demand functions are obtained by solving the problem of optimal allocation for each category of goods:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \tag{2.7}$$

$$C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} C_{i,t} \tag{2.8}$$

for all $i, j \epsilon [0,1]$, where

$$P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \tag{2.9}$$

is the domestic price index and

$$P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \tag{2.10}$$

is a price index for goods imported from country i for all $i \epsilon [0,1]$ It follows from equations 2.7 and 2.8 that:

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj = P_{H,t} C_{H,t} \tag{2.11}$$

$$\int_0^1 P_{i,t}(j) C_{i,t}(j) dj = P_{i,t} C_{i,t} \tag{2.12}$$

Furthermore, the optimal allocation of expenditures on imported goods implies

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t} \tag{2.13}$$

for all $i \epsilon [0,1]$ and where

$$P_{F,t} \equiv \left(\int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \tag{2.14}$$

is the price index for imported goods, also expressed in domestic currency. The above equation implies that we can write total expenditures on imported goods as

$$\int_0^1 P_{i,t} C_{i,t} di = P_{F,t} C_{F,t} \tag{2.15}$$

Finally, the optimal allocation of expenditures between domestic and imported goods is:

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \tag{2.16}$$

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \tag{2.17}$$

where

$$P_t \equiv [(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{2.18}$$

is the consumer price index (CPI).[1] In Appendix A, I show that the price indexes for domestic and foreign goods are equal. Thus, parameter α corresponds to the share of domestic consumption allocated to imported goods. Therefore, α can be considered as a natural index of openness also in this sense.

Accordingly, total consumption expenditures by domestic households are defined as

$$P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = P_t C_t \tag{2.19}$$

Therefore, the period budget constraint can be rewritten as:

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{2.20}$$

which is the intertemporal optimality condition, and

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1} \tag{2.21}$$

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[1] For future reference, note that when $\eta = 1$, the CPI can be written as $P_t = (P_{H,t})^{1-\alpha} (P_{F,t})^{\alpha}$, while the consumption index is given by $C_t = \frac{1}{(1-\alpha)^{(1-\alpha)}} C_{H,t}^{1-\alpha} C_{F,t}^{\alpha}$.

The stochastic Euler equation is obtained by taking conditional expectations on both sides of the above equation and rearranging the terms:

$$\beta R_t E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \tag{2.22}$$

where $R_t = \frac{1}{E_t\{Q_{t,t+1}\}}$ is the gross return on a riskless one-period discount bond paying off one unit of domestic currency in $t+1$, whereh $\{Q_{t,t+1}\}$ is the price of the bond. The two optimality conditions 2.20 and 2.22 can be respectively written in log-linearized form as

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{2.23}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\} - \rho) \tag{2.24}$$

where lower cases denote the logs of the respective variables, $\rho \equiv \beta^{-1} - 1$ is the time discount rate, and $\pi_t \equiv p_t - p_{t-1}$ is CPI inflation (with $p_t \equiv \log P_t$).

2.1.1 Domestic Inflation, CPI Inflation, the Real Exchange Rate, and the Terms of Trade

The bilateral terms of trade between the domestic economy and country i is defined as $S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$,i.e. the price of country i's goods in terms of home goods. Thus, the effective terms of trade are given by:

$$\begin{aligned} S_t &\equiv \frac{P_{F,t}}{P_{H,t}} \\ &= \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \end{aligned} \tag{2.25}$$

which can be approximated by the log-linear expression up to first order:

$$s_t = \int_0^1 s_{i,t} di \tag{2.26}$$

Log-linearization of the CPI formula around a symmetric steady-state satisfying the purchasing power parity (PPP) condition $P_{H,t} = P_{F,t}$ yields [2]

$$\begin{aligned} p_t &= (1-\alpha)p_{H,t} + \alpha p_{F,t} \\ &= p_{H,t} + \alpha s_t \end{aligned} \tag{2.27}$$

where $s_t = p_{F,t} - p_{H,t}$ denotes the (log) effective terms of trade, i.e. the price of foreign goods in terms of home goods. For future reference, note that the above equations for s_t and p_t hold exactly when $\gamma = 1$ and $\eta = 1$, respectively. Domestic inflation is the rate of change in the index of domestic goods prices:

$$\pi_{H,t} = p_{H,t} - p_{H,t-1} \tag{2.28}$$

It follows that domestic inflation and CPI inflation are linked according to:

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{2.29}$$

This equation shows that the gap between two measures of inflation are proportional to the per cent change in terms of trade, where the coefficient of proportionality is the parameter of openness α.

The assumption of law of one price for individual goods at all times (both for import and export prices) implies that $P_{i,t}(j) = \xi_{i,t} P^i_{i,t}(j)$ for all $i, j \epsilon [0,1]$ where $\xi_{i,t}$ is the bilateral nominal exchange rate (the price of country i's currency in terms of domestic currency), and $P^i_{i,t}(j)$ is the price of country i's good j expressed in the producer country's currency. As stated in Lane and Ganelli (2003), such producer currency pricing(PCP) implies an active expenditure-switching role for the nominal exchange rate: If the dollar depreciates, this implies a reduction in the Turkish Lira price of US exports which in turn should raise the demand for these goods from Turkish consumers. [3]

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[2] Conditions under which PPP holds will be discussed later.

[3] Betts and Devereux (2000) and others used local currency pricing(LCP), in which the firms set prices in the currency of the purchaser. In this specification, the exchange rate does not have an expenditure-switching role. Exchange rate movements only have income affects by changing the rate at which given foreign currency profits convert into domestic currency. As a digression, Obstfeld and Rogoff (2000) shows that a floating exchange rate is optimal under PCP, whereas Devereux and Engel (1998) shows that LCP reduces the gains from a floating exchange rate, since the exchange rate does not facilitate stabilization.

Plugging the law of one price assumption into the definition of $P_{i,t}$ yields $P_{i,t} = \xi_{i,t} P^i_{i,t}$, where $P^i_{i,t} \equiv \left(\int_0^1 P^i_{i,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$. Substituting $P_{i,t}$ into the definition of $P_{F,t}$ and log-linearizing around the symmetric steady-state yields

$$\begin{aligned} P_{F,t} &= \int_0^1 (e_{i,t} + p^i_{i,t}) di \\ &= e_t + p^*_t \end{aligned} \tag{2.30}$$

where $e_t \equiv \int_0^1 e_{i,t} di$ is the (log) nominal effective exchange rate, $p^i_{i,t} \equiv \int_0^1 p^i_{i,t}(j) dj$ is the (log) domestic price index for country i (expressed in terms of its currency), and $p^*_t \equiv \int_0^1 p^i_{i,t} di$ is the (log) world price index. Observe that for the world economy there is no difference between CPI and domestic price level.

Combining equation 2.30 with the definiton of the terms of trade yields

$$s_t = e_t + p^*_t - p_{H,t} \tag{2.31}$$

Next, the bilateral real exchange rate with country i is defined as $\vartheta_{i,t} \equiv \frac{\xi_{i,t} P^i_t}{P_t}$, i.e. the ratio of the two countries' CPI's, both expressed in domestic currency. Let $q_t \equiv \int_0^1 q_{i,t} di$ be the (log) effective real exchange rate, where $q_{i,t} \equiv log \vartheta_{i,t}$. It follows that

$$\begin{aligned} q_t &= \int_0^1 (e_{i,t} + p^i_t - p_t) di \\ &= e_t + p^*_t - p_t \\ &= s_t + p_{H,t} - p_t \\ &= (1-\alpha) s_t \end{aligned} \tag{2.32}$$

where the last equality is derived by log-linearizing $\frac{P_t}{P_{H,t}} = \left[(1-\alpha) + \alpha S_t^{1-\eta} \right]^{\frac{1}{1-\eta}}$ around a symmetric steady-state, which yields $p_t - p_{H,t} = \alpha s_t$. This equality holds only up to a first order approximation when $\eta \neq 1$.

2.1.2 International Risk Sharing

Under the assumption of complete securities markets, the first order conditions of our small open economy's representative household must also hold for the representative household in any other country, say country i:

$$\beta \left(\frac{C^i_{t+1}}{C^i_t} \right)^{-\sigma} \left(\frac{P^i_t}{P^i_{t+1}} \right) \left(\frac{\varepsilon^i_t}{\varepsilon^i_{t+1}} \right) = Q_{t,t+1} \tag{2.33}$$

Combining equations 2.22 and 2.33 with the definition of real exchange rate yields:

$$C_t = \nu_i C^i_t \vartheta^{\frac{1}{\sigma}}_{i,t} \tag{2.34}$$

for all t, where ν_i is a constant which generally depends on initial conditions regarding relative net asset positions which are assumed to be identical, i.e. we have $\nu_i = \nu = 1$ for all i. In Appendix A, I show that in the symmetric steady state we also have that $C = C^i = C^*$ and $\vartheta_i = S_i = 1$, i.e. purchasing power parity holds for all i.

Taking logs on the both sides of equation 2.34 and integrating over i gives the risk sharing condition:

$$\begin{aligned} c_t &= c^*_t + \frac{1}{\sigma} q_t \\ &= c^*_t + \left(\frac{1-\alpha}{\alpha} \right) s_t \end{aligned} \tag{2.35}$$

where $c^*_t = \int_0^1 c^i_t di$ is the index of (log) world consumption. The second equality holds only up to a first order approximation when $\eta \neq 1$. This relationship linking domestic consumption with world consumption and the terms of trade is a consequence of the assumption of complete asset markets at the international level.

2.2 Firms

2.2.1 Technology

At the supply-side of the economy, each firm produces a differentiated good with a linear technology represented by the production function

$$Y_t(j) = A_t N_t(j) \tag{2.36}$$

where $a_t \equiv \log A_t$ follows an AR(1) process, i.e., $a_t = \rho_a a_{t-1} + \varepsilon_t$. Hence, the real marginal cost will be common across firms and will be given by

$$mc_t^n = -\varsigma + w_t - a_t \tag{2.37}$$

which is expressed in domestic currency and where $\varsigma \equiv -\log(1-\tau)$. τ is the employment subsidy, which will be discussed later for deriving an analytically tractable optimal rule.

Let $Y_t \equiv \left[\int_0 1 Y_t(i)^{1-\frac{1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ represent an CES index for aggregate output. For future reference, note the approximate aggregate production function relating the previous index to aggregate employment:

$$N_t \equiv \int_0^1 N_t(j)dj = \frac{Y_t Z_t}{A_t} \tag{2.38}$$

where $Z_t \equiv \int_0 1 \frac{Y_t(j)}{Y_t} dj$. The equilibrium variations in $z_t \equiv \log Z_t$ around the perfect foresight steady state are of second order (See Appendix C). Thus, and up to a first order approximation, we have

$$y_t = n_t + a_t \tag{2.39}$$

Similarly, firms in the rest of the world are assumed to have access to an identical technology, with (log) productivity following an exogenous process $a_t^* = \rho_a^* a_{t-1}^* + \varepsilon_t^*$ where $\{\varepsilon_t^*\}$ is white noise, possibly correlated with $\{\varepsilon_t\}$. An approximate aggregate relationship between output and employment identical to the above equation also holds for the world economy as a whole.

2.2.2 Price Setting

Firms set prices in a Calvo-type staggered fashion (Calvo (1983)). In this setting, a measure $1-\theta$ of randomly selected firms set new prices each period. An individual firm's probability of reoptimizing in any given period is independent of the time it last reset its price. The derivation of optimal price-setting strategy of a typical firm resetting its price in period t is given in Appendix B:

$$p^r_{H,t} = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{mc^n_{t+k}\} \tag{2.40}$$

where p^r denotes the (log) of resetted domestic prices, and $\mu \equiv \log \frac{\varepsilon}{\varepsilon - 1}$, which corresponds to the log of the gross markup in the steady state (or, equivalently, the optimal markup in the flexible economy). Firms in the rest of the world have an analogous price setting rule. Assuming the same degree of price stickiness in the rest of the world, θ^*, with that of the small open economy, θ, simplifies the calculations.

The firms' price-setting decision is a forward-looking one. This is because the firms which are adjusting prices in any given period consider that the price they set will remain effective for a random number of periods. As a result, they set the price as a markup over a weighted average of expected future marginal costs. [4].

2.3 Equilibrium

2.3.1 Aggregate Demand and Output Relations

2.3.1.1 World Consumption and Output

Since the world economy consists of infinitely small open economies, a representative small open economy has no influence on the rest of the world. This creates the only difference between the preferences of the representative households in the world economy and that in the small open economy: World economy's household puts a negligible weight on the goods imported from the small open economy. Combining the log-linearized Euler equation with the market clearing condition $y^*_t = c^*_t$, implies:

$$y^*_t = E_t\{y^*_{t+1}\} - \frac{1}{\sigma}(r^*_t - E_t\{\pi^*_{t+1}\} - \rho) \tag{2.41}$$

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[4] In the flexible price limit (i.e., as $\lim_{\theta \to 0}$, the conventional markup rule $p^r_{H,t} = \mu + mc_t + p_{H,t}$ is obtained.

The previous equation, which is known as the new IS equation, differs from the traditional one since current world output depends on anticipated world output and real interest rates. Since households want to smooth consumption, if expected future output rises, current consumption increases, which in turn rises the current output. If expected future interest rate increases, current consumption decreases due to intertemporal substitution of consumption, which in turn decreases the current output. Next, we derive the IS equation for the small open economy.

2.3.1.2 Consumption and Output in the Small Open Economy

Let $C^*_{H,t}(j)$ denote the world demand for the domestic good j. Then, market clearing in the small economy requires

$$\begin{aligned} Y_t(j) &= C_{H,t}(j) + C^*_{H,t}(j) \\ &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}(1-\alpha)C_t + \left(\frac{P_{H,t}}{\varepsilon_t P^*_t}\right)^{-\eta}\alpha^* Y^*_t\right] \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}\nu Y^*_t\left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}(1-\alpha)\vartheta_t^{\frac{1}{\sigma}} + \left(\frac{P_{H,t}}{\varepsilon_t P^*_t}\right)^{-\eta}\alpha\right] \end{aligned} \tag{2.42}$$

for all $j\epsilon[0,1]$ and all t, where the second equality follows from equations 2.3, 2.4, 2.7, and 2.8 together with the analogous expressions for the rest of the world. The third equality is derived by using 2.34 and the condition $\frac{\alpha^*}{\nu} = \alpha$ required for a zero trade balance in the steady state (see appendix A).

Plugging equation 2.42 into the definition of aggregate output $Y_t \equiv \left[\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ yields

$$Y_t = \nu Y^*_t S^\eta_t\left[(1-\alpha)\vartheta_t^{\frac{1}{\sigma}-\eta} + \alpha\right] \tag{2.43}$$

which, up to a first order approximation, can be written in log-linearized form as:

$$y_t = y^*_t + \frac{\omega_\alpha}{\sigma}s_t \tag{2.44}$$

where $\omega_\alpha \equiv 1 + \alpha(2-\alpha)(\sigma\eta - 1) > 0$, and where the subscript α emphasizes the dependence of ω_α on the degree of openness of the economy. Note that in the particular case of $\sigma\eta = 1$ the exact log-linear relationship is obtained:

$$y_t = y_t^* + \eta s_t \tag{2.45}$$

Alternatively, using equation 2.35 to substitute out for s_t in equation 2.44, one can derive an expression for domestic consumption as a weighted average of domestic and world output, which will hold up to a first order approximation:

$$c_t = \Phi_\alpha y_t + (1 - \Phi_{\alpha)} y_t^* \tag{2.46}$$

where $\Phi_\alpha \equiv \frac{1-\alpha}{\omega_\alpha} > 0$. Note that in the particular case of $\alpha = 0$, which corresponds to a closed economy, we have $\omega_0 = 1$, $\Phi_0 = 1$, and hence $c_t = y_t$ for all t. Furthermore, in the particular case of $\sigma\eta = 1$, one can combine the exact relationships in equations 2.35 and 2.45 to obtain

$$c_t = (1 - \alpha)y_t + \alpha y_t^* \tag{2.47}$$

Finally, combining equations 2.27, 2.45 and 2.47 with the consumer's log-linear Euler equation yields a difference equation for domestic output in terms of domestic real interest rates and world output:

$$y_t = E_t\{y_{t+1}\} - \frac{\omega_\alpha}{\sigma}(r_t - E_t\{\pi_{H,t+1}\} - \rho) + (\omega_\alpha - 1)E_t\{\Delta y_{t+1}^*\} \tag{2.48}$$

Solving the above equation forward, it is clear that the level of output in the small open economy is negatively related to current and anticipated domestic real interest rates. It is also related to anticipated world output growth, which in turn depends on expected future word real interest rates, with a coefficient $(\omega_\alpha - 1)$ whose sign is positive (negative) if $\sigma\eta > 1 (< 1)$.

2.3.1.3 The Trade Balance

Let $nx_t \equiv \left(\frac{1}{Y}\right)\left(Y_t - \frac{P_t}{P_{H,t}}C_t\right)$ denote net exports in terms of domestic output, expressed as a fraction of steady state output Y. In the particular case of $\sigma = \eta = 1$, it follows from equations 2.43 and 2.34 that $P_{H,t}Y_t = P_t C_t$ for all t, which means a balanced trade at all times. More generally, a first-order approximation yields $nx_t \simeq y_t - c_t - \alpha s_t$.

Combining this result with equations 2.44 and 2.46 implies

$$\begin{aligned} nx_t &= (1-\Phi_\alpha)(y_t - y_t^*) - \alpha s_t && (2.49) \\ &= \frac{\alpha\Lambda}{\omega_\alpha}(y_t - y_t^*) && (2.50) \end{aligned}$$

where $\Lambda = (2-\alpha)(\sigma\eta - 1) + (1-\sigma)$.

Again, in the special case of $\sigma = \eta = 1$, $nx_t = 0$ for all t. However, this property also holds for any configuration of parameters such that $\Lambda = 0$ This property is an important issue in the choice of an interest rate rule that will result in a determinate equilibrium and will be discussed later.

2.3.2 The Supply Side: Marginal Cost and Inflation Dynamics

2.3.2.1 Marginal Cost and Inflation Dynamics in the Rest of the World

Since the model assumes a continuum of small open economies making up the world economy, fluctuations in the small open economy do not influence the world economy. Therefore, the dynamics of inflation in world economy corresponds to those of a closed economy characterized by staggered price setting à la Calvo, as in Woodford (2003), Chapter 4. Combining the optimal price setting equation in 2.40 corresponding to the world economy with the log-linear version of the equation describing the evolution of the aggregate price level, one can derive the difference equation:

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \lambda mcr_t^* \tag{2.51}$$

where $mcr_t^* \equiv mc_t^* + \mu$ denotes the (log) real marginal cost, expressed as a deviation from its steady state value $(-\mu)$, while the slope coefficient is given by $\lambda \equiv \frac{(1-\theta)(1-\beta\theta}{\theta}$.

Note that, under our assumptions the (log) real marginal cost is given by

$$\begin{aligned} mc_t^* &= -\varsigma^* + (w_t^* + p_t^*) - a_t^* \\ &= -\varsigma^* + \sigma c_t^* + \varphi n_t^* - a_t^* \\ &= -\varsigma^* + (\sigma + \varphi) y_t^* - (1+\varphi) a_t^* && (2.52) \end{aligned}$$

where $\varsigma^* \equiv -log(1-\tau^*)$. τ^* denotes a constant employment subsidy whose role is discussed later.

2.3.2.2 Marginal Cost and Inflation Dynamics in the Small Open Economy

In the small open economy, the dynamics of domestic inflation in terms of real marginal cost are described by an equation analogous to the (closed) world economy counterpart. Hence,

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda mcn_t \tag{2.53}$$

However, the determination of the real marginal cost as a function of domestic output in the small open economy differs from that in the closed economy, due to the existence of a wedge between output and consumption, and between domestic and consumer prices. We indeed have

$$\begin{aligned} mc_t &= -\varsigma + q_t - a_t - p_{H,t} \\ &= -\varsigma + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= -\varsigma + \sigma c_t + \varphi n_t + \alpha s_t - a_t \\ &= -\varsigma + \sigma y_t^* + \varphi y_t + s_t - (1+\varphi)a_t \end{aligned} \tag{2.54}$$

where $\varsigma \equiv -log(1-\tau)$, and the last equality makes use of equation 2.35. Therefore, marginal cost is increasing in the terms of trade and world output. Both variables affect the real wage, through the wealth effect on labor supply resulting from their impact on domestic consumption. In addition, changes in the terms of trade have a direct effect on the product wage, for any given real wage. The influence of technology (through its direct effect on labor productivity) and of domestic output (through its effect on employment and, hence, the real wage) is analogous to that observed in the closed economy.

Finally, using equation 2.44 to substitute for s_t, one can rewrite the previous expression for the real marginal cost in terms of domestic output, productivity and world output:

$$mc_t = -\varsigma + \left(\frac{\sigma}{\omega_\alpha}\right) y_t + \sigma(1 - \frac{1}{\omega_\alpha})y_t^* - (1+\varphi)a_t \tag{2.55}$$

Next, I will describe the model's equilibrium dynamics in a canonical representation in terms of output gap and inflation.

2.3.3 Equilibrium Dynamics: A Canonical Representation

The linearized equilibrium dynamics for the small open economy have a representation in terms of output gap and domestic inflation analogous to that of its closed economy counterpart. This canonical representation, provides the basis for the analysis and evaluation of alternative policy rules. First, define the output gap x_t as the deviation of (log) output yn, from its natural level y^f, where the latter is in turn defined as the equilibrium level of output in the flexible price equilibrium:

$$x_t \equiv y_t - yn_t \tag{2.56}$$

2.3.3.1 World Equilibrium Dynamics

Under flexible prices, real marginal costs (and hence markups) in the world economy will be constant over time, and given by $mc^* \equiv -\mu$, the level that would yield under flexible prices. Evaluating equation 2.52 at the flexible price equilibrium, one can derive the natural level of output as follows:

$$yn_t^* = \Omega_0 + \Gamma_0 a_t^* \tag{2.57}$$

where $\Omega_0 \equiv \frac{\varsigma^* - \mu}{\sigma + \varphi}$, and $\Gamma_0 \equiv \frac{1+\varphi}{\sigma+\varphi}$. In addition, one can derive a simple relationship between real marginal cost (in terms of deviations from its steady state value) and the output gap:

$$mcr_t^* = (\sigma + \varphi) x_t^* \tag{2.58}$$

By combining the latter result with equation 2.51, the New Keynesian Phillips curve (or NKPC for short) is derived:

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \kappa_0 x_t^* \tag{2.59}$$

where $\kappa_0 = \lambda(\sigma + \varphi)$.

The NKPC is simply a log-linear approximation about the steady-state of the aggregation of the individual firm pricing decisions. In contrast to the traditional Phillips Curve, there is no inertia or lagged dependence in inflation. Rather, inflation entirely depends on current and anticipated economic conditions.

One can also rewrite equation 4 in terms of the world output gap:

$$x_t^* = E_t\{x_{t+1}^*\} - \frac{1}{\sigma}(r_t^* - E_t\{\pi_{t+1}^*\} - rr_t^*) \tag{2.60}$$

where $rr_t^* \equiv -\sigma(1 - \rho_a^*)\Gamma_0 a_t^* + \rho$ is the natural (or Wicksellian) expected real rate of interest, i.e., the one that would prevail in a flexible price equilibrium.

Equations 2.51 and 2.60, combined with a monetary policy rule determining the world interest rate fully describe the equilibrium dynamics of the world inflation and output gap.

2.3.3.2 Equilibrium Dynamics for the Small Open Economy

The natural level of output in the small open economy can be found after imposing the flexible price level of the real marginal cost, $mc_t = -\mu$ for all t and solving for domestic output from equation 2.55:

$$y_t^f = \Omega_\alpha + \Gamma_\alpha a_t + \Theta_\alpha y_t^* \tag{2.61}$$

where $\Omega_\alpha \equiv \frac{\omega_\alpha(\varsigma - \mu)}{\sigma + \omega_\alpha \varphi}$, $\Gamma_\alpha \equiv \frac{\omega_\alpha(1+\varphi)}{\sigma + \varphi\omega_\alpha}$, and $\Theta_\alpha \equiv \frac{\sigma(1-\omega_\alpha)}{\sigma+\varphi\omega_\alpha}$ Equation 2.61 also shows the relation between the output gap and the real marginal cost

$$mcr_t = \left(\frac{\sigma}{\omega_\alpha} + \varphi\right) x_t \tag{2.62}$$

which gives a NKPC for the small open economy when combined with equation 2.53

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_\alpha x_t \tag{2.63}$$

where $\kappa_\alpha \equiv \lambda\left(\frac{\sigma}{\omega_\alpha} + \varphi\right)$. Note that for $\alpha = 0$ the slope coefficient is given by $\kappa_0 \equiv \lambda(\sigma + \varphi)$ and the above equation corresponds to the closed world economy NKPC. The same is true for the $\sigma\eta = 1$ case, which implies that $\omega_\alpha = 1$. More generally, this shows that the form of the Phillips equation for the small open economy corresponds to that of the closed economy, at least as far as domestic inflation is concerned. The degree of openness affects the dynamics of inflation only through its influence on the size of the slope of the Phillips curve, i.e., the size of the inflation response to any given variation in the output gap. In the open economy, a change in domestic output has an effect on marginal cost through its impact on employment (captured by φ), and the terms of trade (captured by $\frac{\sigma}{\omega_\alpha}$, which is a function of the degree of openness and the substitutability between domestic and foreign goods). In particular, under the assumption that $\sigma\eta > 1$, an increase in openness lowers the size of the adjustment in the terms of trade necessary to absorb a change in domestic output (relative to world output), thus dampening the impact of the latter on marginal cost and inflation.

Using equation 2.48, one can derive a version of the new IS Equation for the open economy in terms of the output gap:

$$x_t = E_T\{x_{t+1}\} - \frac{\omega_\alpha}{\sigma}(r_t - E_t\{\pi_{H,t+1}\} - rr_t) \tag{2.64}$$

where

$$rr_t \equiv \rho - \frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma + \varphi\omega_\alpha} a_t - \varphi\Theta_\alpha E_t\{\Delta y^*_{t+1}\} \tag{2.65}$$

is the small open economy's natural rate of interest.

The IS-type equation characterizing the small open economy's equilibrium differs from that of closed world economy in two respects: First, the degree of openness influences the sensitivity of the output gap to interest rate changes. In particular, if $\sigma\eta > 1$, an increase in openness raises that sensitivity. Second, the natural interest rate depends on the expected world output growth due to openness, in addition to domestic productivity.

2.4 Simple Monetary Policy Rules for The Small Open Economy

In this section I will describe the three alternative monetary rules for the small open economy defined in Gali and Monacelli (2005) and examine their macroeconomic implications. Under domestic inflation-based Taylor rule (DITR) the monetary authority aims to fully stabilize domestic inflation. Similarly, under CPI inflation-based Taylor rule (CITR) the monetary authority seeks to fully stabilize CPI inflation. Finally, the exchange rate peg (PEG) is a policy that pegs the exchange rate to the world currency. In all three rules, the world monetary authority is assumed to be successfully stabilizing the world prices and output gap, i.e., $x_t^* = \pi_t^* = 0$ for all t. I will also examine the assumptions which makes this the optimal policy for the closed economy.

2.4.1 Domestic Inflation-Based Taylor Rule (DITR)

In this section, the equilibrium processes for the different variables are characterized for the small open economy under the assumption that the domestic monetary authority pursues a domestic inflation-based Taylor rule (DITR), i.e. $r_t = \rho + \phi_\pi \pi_{H,t}$, which implies:

$$x_t = \pi_{H,t} = 0 \tag{2.66}$$

all t. This, in turn, implies $y_t = x_t$ and $r_t = rr_t$ for all t, with all the remaining variables matching their natural level all the times.

Equation 2.61 shows that under DITR, output in the small open economy is positively correlated with a domestic technology shock. However, output's response to a rise in world output is ambiguous. That response is negative if $\omega_\alpha > 1$. If this condition is satisfied, the real appreciation associated with a lower world interest rate results in expenditure-switching which dominates the positive direct demand effect.

Since under DITR both domestic and the world inflation are zero, it follows that $e_t = s_t$, i.e., the nominal exchange rate moves in perfect correlation with the terms of trade.

Therefore, the nominal exchange rate inherits all the statistical properties of the terms of trade, including its stationarity. More specifically, by combining equations 2.44, 2.57 and 2.61,the equilibrium behavior of the nominal exchange can be derived:

$$e_t = \frac{\sigma}{\omega_\alpha}(yn_t - yn_t^*) \tag{2.67}$$

$$= \frac{\sigma(1+\varphi)}{\sigma + \varphi\omega_\alpha}(a_t - a_t^*) \tag{2.68}$$

where the second equality holds up to a constant term. Thus, we see that the nominal exchange rate varies with the productivity differential, depreciating (appreciating) in response to a relative increase in domestic (world) productivity.

The variance of the nominal exchange rate under flexible prices will be proportional to

$$(\sigma_a - \sigma_{a^*})^2 + 2\sigma_a\sigma_{a^*}(1 - \rho_{a,a^*}) \tag{2.69}$$

where σ_a and σ_{a^*} denote the standard deviation of domestic and world productivities, relatively, and ρ_{a,a^*} denotes their correlation. Hence, we see that the required volatility of the nominal exchange rate under DITR is increasing with the extent of the asymmetry in terms of the magnitude and the comovement between the two shocks.

In addition we can also derive the implied equilibrium process for the CPI level. Given the constancy of domestic and world prices it is given by:

$$p_t = \alpha e_t \tag{2.70}$$

$$= \frac{\alpha\sigma(1+\varphi}{\sigma + \varphi\omega_\alpha}(a_t - a_t^*) \tag{2.71}$$

where the second equality follows from equation 2.68. Therefore under DITR, the CPI level varies with the productivity differential and inherits its statistical properties. The same is true for the real exchange rate, which is proportional to the nominal exchange rate, with the coefficient of proportionality being $(1 - \alpha)$.

2.4.2 CPI Inflation-Based Taylor Rule (CITR)

Under CITR, the monetary authority implements an interest rate rule of form $r_t = \rho + \phi_\pi \pi_t$. Such a policy requires

$$\pi_t = 0 \tag{2.72}$$

for all t.Under the assumption that the world economy pursues an optimal policy, i.e. $\pi^* = 0$, one can set $p_t = p_t^* = 0$ for all t without loss of generality, which implies

$$p_{H,t} = -\alpha s_t \tag{2.73}$$

Therefore the domestic price level and the terms of trade have common dynamics. Using the previous expression to substitute for s_t in equation 2.44, and plugging the resulting equation into equation 2.55 yields

$$mc_t = -\frac{1}{\alpha}\left(1 + \frac{\varphi\omega_\alpha}{\sigma}\right)p_{H,t} - (1+\varphi)(a_t - a_t^*) \tag{2.74}$$

By substituting the latter equality in equation 2.44, plugging the resulting equation into equation 2.55 and combining with equation 2.53 one can derive the following stochastic second order difference equation describing the equilibrium behavior of the domestic price level:

$$\gamma_c p_{H,t} = p_{H,t-1} + \beta E_t\{p_{H,t+1}\} - \lambda(1+\varphi)(a_t - a_t^*) \tag{2.75}$$

where $\gamma_c \equiv 1 + \beta + \frac{\lambda}{\alpha}\left(1 + \frac{\varphi\omega_\alpha}{\sigma}\right)$. Under the simplifying assumption that $\rho_a = \rho_a^*$ this equation has a unique stationary representation given by

$$p_{H,t} = \xi_c p_{H,t-1} - \zeta_c(a_t - a_t^*) \tag{2.76}$$

where $\xi_c \equiv \frac{1}{2\beta}\left(\gamma_c - \sqrt{\gamma_c^2 - 4\beta}\right)\epsilon(0,1)$, and $\zeta_c \equiv \frac{\lambda\varepsilon_c(1+\varphi)}{1-\xi_c\beta\rho_a)} > 0$
Observe that under CITR because an increase in domestic productivity leads to a real depreciation of the terms trade and given domestic prices to an increase in CPI inflation, a change in relative productivity has a negative effect on the domestic prices. Therefore, stabilization of CPI inflation requires inducing a decline in domestic prices and a smaller real depreciation, both of which can be attained by means of a negative output gap.

The equilibrium processes for the terms of trade, the nominal and real exchange rates can be derived by combining equations 2.73 and 2.75, it is possible to derive. Note that under CITR $q_t = e_t$ all t. In particular,

$$e_t = q_t = -\frac{1-\alpha}{\alpha} p_{H,t} \tag{2.77}$$

Thus, under CIT, the equilibrium nominal exchange rate must be stationary and, given the evolution of the domestic price level, it must depreciate in response to a rise in productivity. Furthermore, and in contrast with the DITR regime, both the nominal and the real exchange rates will display some endogenous persistence, beyond that inherited from the productivity differential.

2.4.3 Exchange Rate Peg (PEG)

Under PEG, the monetary authority in the small open economy implements $e_t = 0$ for all t by adopting the world currency. This monetary integration equalizes the domestic interest rate to the world interest rate. Furthermore, constancy of the nominal exchange rate and world prices implies $s_t = -p_{H,t}$ and $q_t = -p_t$ for all t.
Following the same way as in CITR targeting case, one can derive a second order difference equation for the domestic price level:

$$\gamma_c p_{H,t} = p_{H,t-1} + \beta E_t\{p_{H,t+1}\} - \lambda(1+\varphi)(a_t - a_t^*) \tag{2.78}$$

where $\gamma_c \equiv 1 + \beta + \lambda\left(1 + \frac{\varphi\omega_\alpha}{\sigma}\right)$. The unique stationary representation for $p_{H,t}$ in this case is given by:

$$p_{H,t} = \xi_e p_{H,t-1} - \zeta_e(a_t - a_t^*) \tag{2.79}$$

where $\xi_e \equiv \frac{1}{2\beta}\left(\gamma_e - \sqrt{\gamma_e^2 - 4\beta}\right) \epsilon(0,1)$, and $\zeta_e \equiv \frac{\lambda\xi_e(1+\varphi)}{1-\xi_e\beta\rho_a} > 0$. The stationarity of the domestic price level is again a direct implication of the stationarity of the terms of trade, given the constancy of the nominal exchange rate and the world price level. Notice that the sign and the qualitative pattern of the response of domestic prices under a PEG is identical to the one derived for a CITR regime. It displays the same endogenous persistence beyond that inherited from variations in the productivity differential. The difference between the two responses arises from the fact that $\gamma_e < \gamma_c$, which, in turn, implies $\xi_e > \xi_c$ and $\zeta_e > \zeta_c$. Therefore PEG regime leads to a stronger and more persistent adjustment of domestic prices (and, hence, of the output gap) in response to a shock in the productivity differential, relative to a CITR regime.

Furthermore, and given $e_t = p_t^* = 0$ for all t, it follows that the CPI level is proportional to the domestic price level:

$$p_t = (1 - \alpha)p_{H,t} \tag{2.80}$$

Hence, CPI level shares the statistical properties of the domestic price level. In particular, and in contrast with a DITR regime, after a rise in the productivity differential, CITR must fall under a PEG.

The common property between the three alternative policy rules is that they all imply a stationary price level and nominal exchange rate, which is in contrast with the non-stationarity of both variables observed in the data. Gali and Monacelli (2005) states that this contrast may suggest that non of these regimes is a good proxy for the policies pursued by actual economies.

2.5 Optimal Monetary Policy

2.5.1 Optimal Monetary Policy in the World Economy

Following Rotemberg and Woodford (1998), Gali and Monacelli (2005) assume that the fiscal authority in the (closed) world economy fully neutralizes the distortions associated with firms' market power by means of a constant employment subsidy τ^*. This assumption enables the authors to show that the flexible price equilibrium allocation is the optimal allocation in the world economy.

In order to see this, assume that the utility function of the representative household in the small open economy takes the form of:

$$U(C_t, N_t) = U(C_t) - V(N_t) \tag{2.81}$$

Then, the optimal allocation must maximize $U(C_t^*) - V(N_t^*)$ subject to $C_t^* = A_t^* N_t^*$, for all t. The associated first order condition is given by $V'(N_t^*) = U'(C_t^*) A_t^*$.

On the other hand, the flexible price equilibrium satisfies:

$$\begin{aligned} 1 - \frac{1}{\varepsilon} &= MC_t^* \\ &= \frac{(1-\tau^*)}{A_t^*} \frac{V'(N_t*)}{U'(C_t^*)} \end{aligned} \tag{2.82}$$

Hence, by setting $\tau^* = \frac{1}{\varepsilon}$ (or, equivalently, $\nu^* = \mu$) the world policymaker guarantees the optimality of the flexible price equilibrium allocation. In such an environment, the optimal monetary policy is the one that succeeds in maintaining the output at its natural level and fully stabilizing prices, i.e. $x_t^* = \pi_t^* = 0$ at $t = 0$.

Given 2.60, one can derive an expression for the interest rate that supports the optimal allocation in the world economy:

$$\begin{aligned} r_t^* &= rr_t^* \\ &= \rho - \sigma(1 - \rho_a^*) \end{aligned} \tag{2.83}$$

In order to interpret equation 2.83 as an optimal rule, one could add an extra term such as$\varphi_\pi \pi_t^*$ with $\varphi_\pi > 1$, which could eliminate the indeterminacy that would otherwise be associated with an interest rate that depends on exogenous variables only. Woodford (2003) states that such a term will be zero in equilibrium.

2.5.2 Optimal Monetary Policy in the Small Open Economy: A Special Case

Corsetti and Pesenti (2001) states that in an open economy the possibility of influencing the terms of trade in a way beneficial to domestic consumer is an additional factor to market power that distorts the incentives of the monetary authority. This possibility is a consequence of the imperfect substitutability between domestic and foreign goods, combined with sticky prices, which renders monetary policy non-neutral.

Gali and Monacelli (2005) assume the presence of an employment subsidy that exactly offsets the combined effects of market power and the terms of trade distortions in the steady state. This assumption rules out the existence of an average inflation (deflation) bias, and allows the authors to focus on the policies consistent with a zero average inflation, in a way analogous to the world economy.

In order to characterize the optimal allocation, one need to solve the social planner's problem who faces the resource constraints of the small open economy in equilibrium, which is:

$$maxU(C_t) - V(N_t) \tag{2.84}$$

subject to

$$C_t = f(Y_t, Y_t^*) \tag{2.85}$$

The resource constraint includes the risk sharing condition 2.35 and the equilibrium relationship between the terms of trade, and domestic and foreign output given in equation 2.43, and takes the world output as given.

The social planner's problem is solved as:

$$V'(N_t)N_t = (1-\alpha)U'(C_t)C_t \tag{2.86}$$

The derivation of a tractable, analytical solution is possible in the special case of $\sigma = \eta = 1$. As shown above, in this special case, the exact relationship between the domestic consumption, and domestic and foreign output levels is given by $C_t = Y_t^{(1-\alpha)}(\vartheta Y_t^*)^\alpha$. Under this preferences, the optimal allocation implies a constant employment:

$$N = (1-\alpha)^{(\frac{1}{1+\varphi})} \tag{2.87}$$

On the other hand, the flexible price equilibrium in the small open economy satisfies:

$$\begin{aligned} 1 - \frac{1}{\varepsilon} &= MC_t \\ &= \frac{1-\tau}{A_t}\left(\frac{Y_t}{\vartheta Y_t^*}\right)^\alpha \frac{C'(N_t)}{U'(C_t)} \\ &= (1-\tau)N_t^{1+\varphi} \end{aligned} \tag{2.88}$$

Therefore, the social planner can guarantee the optimal flexible price equilibrium allocation by setting τ such that $(1-\tau)(1-\alpha) = 1 - \frac{1}{\varepsilon}$ or, equivalently $\nu = \mu + log(1-\alpha)$ is satisfied. As in the closed economy case, the optimal monetary policy requires stabilizing the output gap (i.e., $x_t = 0$ for all t),which implies that domestic prices are also stabilized under optimal policy (i.e., $\pi_{H,t} = 0$ for all t). Thus, in the special case under consideration, domestic inflation targeting is indeed the optimal policy.

2.5.3 A Numerical Analysis: Dynamic Effects of a Domestic Productivity Shock

In this section, I present the quantitative results based on a calibrated version of the model in Gali and Monacelli (2005) which I replicated using Dynare.

The main assumptions underlying the calibration are $\sigma = \eta = \gamma = 1$, as consistent with the special case given above. Other parameters are assumed as follows: $\varphi = 3$, which implies a labor supply elasticity of 1/3. $\mu = 1.2$ for the steady-state mark-up, which implies that ε, the elasticity of substitution between differentiated goods of the same origin is 6. $\theta = 0.75$, which is consistent with an average period of one year between price adjustments. $\beta = 0.99$, which implies a riskless annual return of about %4 in the steady state. $\alpha = 0.4$, which is the import/GDP ratio for Canada, which the authors take as a prototype small open economy. In the calibration of the interest rate rules the original Taylor estimate is followed by setting $\phi_\pi = 1.5$, while $\rho = (0.99)^{-1} - 1$.

In order to calibrate the stochastic properties of the exogenous driving forces, Gali and Monacelli (2005) fit AR(1) processes to log labor productivity in Canada, which is their proxy for domestic productivity and log U.S. GDP as a proxy for world output, using quarterly, HP-filtered data over the sample period 1963:1-2002:4. They obtain the following estimates:

$$a_t = \underset{(0.06)}{0.66} \quad a_{t-1} + \varepsilon_t^a, \sigma_a = 0.0071 \tag{2.89}$$

$$\begin{aligned} y_t^* = \quad & \underset{(0.04)}{0.66} \quad y_{t-1}^* + \varepsilon_t^*, \sigma_{y^*} = 0.0078 \end{aligned} \tag{2.90}$$

with $(\varepsilon_t^a, \varepsilon_t^*) = 0.3$.

2.5.3.1 Impulse Responses

Figures 1 - 4 gives the impulse responses to a unit innovation in a_t under DITR, CITR, optimal and PEG regimes, respectively. By construction, under the optimal policy regime, domestic inflation and the output gap remains unchanged. In addition, the shock leads to a persistent reduction in the domestic interest rate as it is needed in order to support the transitory expansion in consumption and output consistent with the flexible price equilibrium allocation. Since the world nominal interest rates are constant, the UIP condition implies an initial nominal depreciation followed by the expectations of a future appreciation, as reflected in the response of the nominal exchange rate. Relative to all other regimes, the constancy of domestic prices accounts for a larger real depreciation and therefore for a further expansion in demand and output through a rise in net exports. Given constant world prices and the stationarity of the terms of trade, the constancy of domestic prices implies a mean-reverting response of the nominal exchange rate.

In contrast to the optimal policy, both CITR and DITR regimes generate a permanent fall in both domestic and CPI prices. Behavior of the terms of trade is the most important difference between the two regimes: Under CITR the initial response of the terms of trade is more muted, and is followed by a hump-shaped pattern.

Finally, Under PEG regime, the response of output gap and inflation are qualitatively similar to the CITR case. However, since the domestic currency cannot depreciate to support the expansion in consumption and output of the flexible price allocation terms of trade has only a small fluctuation, which causes an amplification of the responses of the domestic inflation and the output gap.

As discussed below, the different dynamics of the terms of trade are unambiguously associated with a welfare loss, relative to the optimal policy.

2.6 The Welfare Costs of Alternative Simple Rules

2.6.1 The Welfare Loss Function

In this section I present the second order approximation of representative consumer's utility about the flexible price equilibrium allocation. Starting with the general case, I show the final results for the special case $\sigma = \eta = 1$, for which there exists an analytically tractable solution for the optimal monetary policy as discussed above.

Below, I make frequent use of the following second order approximation of percent deviations in terms of log deviations.

$$\frac{Y_t - Y}{Y} = y_t + \frac{1}{2} y_t^2 + u(||a||^3) \tag{2.91}$$

where $u(||a||)^3$ represents terms that are of third or higher order, in the bound $||a||$ on the amplitude of the relevant shocks.
The approximation of $U(C_T) = logC_t$ about the flexible price equilibrium yields:

$$\begin{aligned} U(C_t) &= c_t + c_t^g + u(||a||^3) \\ &= c_t + (1-\alpha) y_t^g + u(||a||^3) \end{aligned} \tag{2.92}$$

where the upper script g indicates the growth rate of the relevant variable.

Similarly, letting $V(N_t) \equiv V(N_t)$ yields

$$V(N_t) = V_t + V_t' N_t \left(n_t^g + \frac{1}{2}(1+\varphi)(n_t^g)^2 \right) + u(||a||^3) \tag{2.93}$$

Next, the previous expression should be written in terms of the output gap. Using the fact that $N_t = \left(\frac{Y_t}{A_t}\right)\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di$, we have:

$$n_t^g = y_t^g + z_t \tag{2.94}$$

where $z_t \equiv log \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di$

The following lemma shows that z_t is proportional to the cross-sectional distribution of relative prices, hence, of second order:

Lemma 1: $z_t = \frac{\varepsilon}{2} var_i \left\{p_{H,t}(i)\right\} + u(||a||^3)$.
Proof: Let $p_{H,t}^d(i)$. Notice that

$$\begin{aligned}\left(\frac{P_{H,t(i)}}{P_{H,t}}\right)^{1-\varepsilon} &= exp[(1-\varepsilon)p_{H,t}^d(i)] \\ &= 1 + (1-\varepsilon)p_{H,t}^d(i) + \frac{(1-\varepsilon)^2}{2}(p_{H,t}^d)^2 + u(||a||^3)\end{aligned} \tag{2.95}$$

Furthermore, from the definition of $P_{H,t}$, we have $1 = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\varepsilon} di$. Hence, it follows that

$$E_i\{p_{H,t}^d(i)\} = \frac{\varepsilon - 1}{2} E_i\{(p_{H,t}(i))^2\} \tag{2.96}$$

In addition, a second order approximation to $\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}$ yields:

$$\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} = 1 - \varepsilon p_{H,t}^d(i) + \frac{\varepsilon^2}{2}(p_{H,t}^d(i))^2 + u(||a||^3) \tag{2.97}$$

Combining the previous results, it follows that:

$$\begin{aligned}\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di &= 1 + \frac{\varepsilon}{2} E_i(\{p_{H,t}^d(i))^2\} \\ &= 1 + \frac{\varepsilon}{2} var_i\{p_{H,t}(i)\}\end{aligned} \tag{2.98}$$

from which follows that $z_t = \frac{\varepsilon}{2} var_i\{p_{H,t}(i)\} + u(||a||^3)$

Therefore, the second order approximation to the disutility of the labor can be rewritten as:

$$V(N_t) = V_t + V_t' N_t \left[(y_t^g) + z_t + \frac{1}{2}(1+\varphi)(y_t^g)^2\right] 4u(||a||^3) \tag{2.99}$$

Under the optimal subsidy scheme assumed, the optimality condition $V_t' N_t = (1-\alpha)$ holds for all t, allowing us to rewrite the period utility as:

$$U(C_t) - V(N_t) = -(1-\alpha)\left[z_t + \frac{1}{2}(1+\varphi)(y_t^g)^2\right] + t.i.p. + u(||a||^3) \tag{2.100}$$

where $t.i.p.$ denotes terms independent of policy.
Lemma 2: $\sum_{t=0}^{\infty}\beta^t var_i\{p_{H,t}(i)\} = \frac{1}{\lambda}\Sigma_{t=0}^{\infty}\beta^t\pi_{H,t}^2$, where $\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta}$.

The proof is given in Woodford (2003).

Collecting all the previous results, we can write the second order approximation to the small open economy's consumer's utility function as follows

$$W = -\frac{(1-\alpha)}{2}\sum_{T=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\lambda}\pi_{H,t}^2 + (1+\varphi)(y_t^g)^2\right] + t.i.p. + u(||a||^3) \tag{2.101}$$

which is expressed as a fraction of steady-state consumption.

Taking unconditional expectation of 2.101 and letting $\beta \rightarrow 1$, the expected welfare losses of any policy that deviated from strict inflation targeting can be written in terms of the variances of inflation and the output gap:

$$V = \frac{(1-\alpha)}{2}\left[\frac{\varepsilon}{\lambda}var(\pi_{H,t}) + (1+\varphi)var(y_t^g)\right] \tag{2.102}$$

2.6.2 Second Moments and Welfare Losses

In order to complement the quantitative analysis, Gali and Monacelli (2005) reports business cycle properties of several key variables under alternative policy regimes. I present their results in Tables 1– 5, which I replicated using Dynare. The numbers confirm that the critical element that distinguishes each simple rule relative to the optimal policy is the excess smoothness of both the terms of trade and the (first differenced) nominal exchange rate. [5]. This is also reflected in the too high volatility of the output gap and the domestic inflation under the simple rules. In particular, under PEG regime

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[5] The statistics are reported for the nominal depreciation rate, as opposed to the level, given that both DITR and CITR imply a unit root in the nominal exchange rate

both output gap and inflation volatility is amplified to the largest extent, with the CITR regime lying somewhere in between. In addition, the terms of trade are more stable under PEG regime than under any other policy regime. That finding, which is consistent with the evidence of Mussa (1986), points to the existence of excess smoothness in real exchange rates under fixed exchange rates. That feature is a consequence of the inability of prices (which are sticky) to compensate for the constancy of the nominal exchange rate[6]

In Tables 1-4, I report the welfare losses associated with the three simple rules DITR, CITR, and PEG. Table 1 reports welfare losses in the case of the benchmark parametrization. Table 2 shows the results by decreasing the steady-state mark-up to 1.1 by setting $\varepsilon = 11$. This implies a larger penalization of inflation variability in the loss function. In Table 3, I show the results by decreasing the elasticity of labor supply to 0.1. This implies a larger penalization of output gap variability. Finally, in Table 4 I report the results with lowered effects of both the steady-state mark-up and the elasticity of labor supply compared to the benchmark case. Under this parametrization, the PEG regime leads to non-trivial welfare losses relative to the optimum.

All entries must be read as percentage units of steady-state consumption, and in deviation from the first-best represented by DITR.

Under the benchmark parametrization, all rules are suboptimal since they involve non-trivial deviations from full domestic price stability. Another prominent result is that in each case an exchange rate peg implies a substantially larger deviation from the first best than DITR and CITR. However the implied welfare losses are quantitatively small for all policy regimes.

Lowering the elasticity of labor has a general effect of generating a substantial magnification of the welfare losses relative to the benchmark case. When both the mark-up and the elasticity of labor supply is lowered simultaneously, this loss gets larger. In the

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[6] Monacelli (2004) gives a more detailed analysis of the implications of fixed exchange rates. He finds that with complete exchange rate pass through, their results are consistent with Mussa (1986) finding that industrial countries moving from fixed to floating exchange rate regimes experience dramatic rises in the variability of the real exchange rate.

latter case, the PEG regime leads to non-trivial welfare losses relative to the optimum.

Another interesting result is that under all scenarios considered here the two stylized Taylor rules, DITR and CITR, imply very similar welfare losses. Gali and Monacelli (2005) emphasizes that while this points to a substantial irrelevance in the specification of the inflation index in the monetary authority's interest-rate rule, the same result may once again be sensitive to the assumption of complete exchange rate pass-through specified in our context. In the context of a different model, with both tradable and non-tradable goods and capital accumulation, Schmitt-Grohé and Uribe (2001) points out that the welfare ranking between domestic and CPI targeting may be sensitive to the specification of other distortions in the economy, for instance, the adoption of a transaction role for money.

Chapter 3

A Small Open Economy VAR

In this section, I estimate a seven-variable VAR system to model the interactions between the Euro area and Turkish economies. The EU and Turkey are linked by a Customs Union Agreement, which aims promoting trade and economic relations. Turkey and the EU has deep trade relationships. Indeed, the EU ranks by far as number one in both Turkey's imports and exports while Turkey ranks 7th in the EU's top import and 5th in export markets[1], which provides me the motivation to assess and compare the Euro zone and the Turkish shocks on Turkish economy.

The VAR model I estimate can be thought of as the reduced form of the structural forms of the open economy New Keynesian models. The choice variables for this empirical exercise is consistent with Svensson (2000), Giordani (2004b), Gali and Monacelli (2005) and Galí and Gertler (2007), among others.

3.1 Theoretical Motivation

Basing on the model of Gali and Monacelli (2005) which I presented in Chapter 2, the equations characterizing a small open economy such as Turkey can be summarized as follows:

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[1] The statistics are take from European Commission's official website http://ec.europa.eu/trade/creating-opportunities/bilateral-relations/countries/turkey/

- A partially forward-looking New Keynesian Phillips Curve (NKPC)

$$\pi^*_{t+1} = \alpha_\pi \pi_t + (1-\alpha_\pi)E_t\{\pi_{t+2}\} + \alpha_x x_{t+1} + \alpha_q(q_t - q_{t-1}) + \varepsilon^{AP}_{t+1} \qquad (3.1)$$

  where π_t denotes inflation, x_t is output gap defined as $x_t = y_t - y^N_t$, y_t is log real GDP and y^N_t is the log real potential output, q_t is the log of real exchange rate and $\varepsilon^{CP}_t \sim nid(0, \sigma^2_{CP})$ is a cost-push shock. The coefficients are assumed to be non-negative.

- An IS/AD equation

$$x_{t+1} = \beta_x x_t + (1-\beta_x)E_t\{x_{t+2}\} - \beta_r(r_t - E_t\{\pi_{t+1}\} + \beta_{x^*} x^*_{t+1} + \beta_q E_t\{q_{t+1}\} + \varepsilon^{AD}_{t+1} \qquad (3.2)$$

  where r_t is the short-term interest rate and x^*_t is the foreign output gap and $\varepsilon_{AD} \sim nid(0, \sigma^2_{AD})$ is representing the aggregate demand shock. All coefficients are expected to be positive.

- An uncovered interest parity (UIP) condition

$$(r_t - E_t\{\pi_{t+1}\}) - (r^*_t - E_t\{\pi^*_{t+1}\}) = E_t\{q_{t+1}\} - q_t \qquad (3.3)$$

  where π^*_t denotes foreign inflation rate.

- An equation characterizing the monetary policy as a Taylor-type policy rule:

$$r_{t+1} = \rho_r r_t + (1-\rho_r)(\gamma_x x_{t+1} + \gamma_\pi \pi_{t+1} + \gamma_{r^*} r^*_{t+1} + \gamma_{x^*} x^*_{t+1} + \gamma_{\pi^*} \pi^*_{t+1}) + \varepsilon^{M}_{t+1}P \qquad (3.4)$$

As in general in New Keynesian open economy models, it is assumed that the small open economy has no direct influence on the closed world economy. Differing in this assumption compared to the small open economy, the equations characterizing the rest of the world as a closed economy can be summarized as follows:

- A New Keynesian Phillips Curve (NKPC)

$$\pi^*_{t+1} = \alpha^*_\pi \pi^*_t + (1-\alpha^*_\pi)E_t\{\pi^*_{t+2}\} + \alpha^*_x x^*_{t+1} + \varepsilon^{CP^*}_{t+1} \qquad (3.5)$$

- An IS/AD equation

$$x^*_{t+1} = \beta^*_x x^*_t + (1 - \beta^*_x)E_t\{x^*_{t+2}\} - \beta^*_r(r^*_t - E_t\{\pi^*_{t+1}\}) + \varepsilon^{AD^*}_{t+1} \tag{3.6}$$

- An equation characterizing the monetary policy as a Taylor-type policy rule:

$$r^*_{t+1} = \rho^*_r r^*_t + (1 - \rho^*_r)(\gamma^*_x x^*_{t+1} + \gamma^*_\pi \pi_{t+1}) + \varepsilon^{MP^*}_{t+1} \tag{3.7}$$

  where r^*_t denotes foreign short-term interest rate as the tool for monetary policy.

As pointed in Horvath and Rusnák (2009), the VAR model I estimate can be thought of as a reduced form of the New Keynesian model of Gali and Monacelli (2005). Therefore, the variables I include in the model are output gap, aggregate price level, interest rate and their foreign counterparts, plus the bilateral nominal exchange rate.

3.2 A Small Open Economy VAR

In this section, I present a seven variable VAR system to model the interactions between the Euro area and Turkish economies. Sims et al. (1990) claims that when it appears likely that the data are integrated, transforming the models to stationary form by difference or cointegrating operators is often unnecessary. Following Sims et al. (1990), I estimate the VAR model in levels for the following reasons. As stated in Horvath and Rusnák (2009), first, it is sometimes difficult in small samples to determine whether a cointegrating relationship is present. Second, imposing the cointegrating restriction inappropriately could possibly lead to incorrect inference. Following Horvath and Rusnák (2009), I begin by assuming that the economy is described by a structural form equation, which is of linear, stochastic dynamic form:

$$A(L)y(t) = \varepsilon(t) \tag{3.8}$$

where $A(L)$ is an kxk matrix polynomial in the log operator (with non-negative powers), $y(t)$ is an $kx1$ vector of observations, and $\varepsilon(t)$ is a $kx1$ vector of structural disturbances which are assumed to be mutually uncorrelated. This assumption requires $\varepsilon(t)$ to be serially uncorrelated and $var(\varepsilon(t)) = \Lambda$ where Λ is a diagonal matrix with diagonal elements being the variances of structural disturbances.

Next, I divide the model into the Euro area and Turkey blocks by rewriting the matrices $A(L)$, $y(t)$ and $\varepsilon(t)$ as:

$$A(L) = \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix} \tag{3.9}$$

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} \tag{3.10}$$

$$y(t) = \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \end{bmatrix} \tag{3.11}$$

The model contains k_1 domestic variables in Turkish economy vector $y_1(t)$ and k_2 Euro area variables in the Euro area vector $y_2(t)$, which are exogenous to the small open economy. Therefore, $A_ij(L)$ is a k_ixk_j matrix, whereas $\varepsilon_i(t)$ and $y_i(t)$ are k_ix1 matrices.

The vector of Turkish variables consists of output gap (x_t^{TR}), consumer price level (p_t^{TR}), the short-term interest rate as monetary policy tool (r_t^{TR}) and the exchange rate $(e_t^{TRL/EU})$:

$$y_1(t)' = (x_t^{TR} \quad p_t^{TR} \quad r_t^{TR} \quad e_t^{TRL/EU}) \tag{3.12}$$

Basing on the model in Gali and Monacelli (2005), the monetary authority in the closed world economy is assumed to be successful at fully stabilizing its inflation and output gap. Therefore, the closed economy is at its natural output level. Hence, the vector of foreign variables consists of the Euro area real GDP (y_t^EU), Euro area aggregate price level $p_t^EU)$ and the Euro area short term interest rate (r_t^EU):

$$y_2(t)' = (x_t^{EU} \quad p_t^{EU} \quad r_t^{EU}) \tag{3.13}$$

where all variables except for the output gap and the interest rate are in log levels.

As stated before, since Turkey is being modeled as a small open economy, Turkish shocks are unlikely to have significant effect on the Euro area economy. Accordingly, the restriction $A_21(L) = 0$ is appropriate. Cushman and Zha (1997)) and Zha (1999) also utilized this block exogeneity restriction in their studies of small open economies .

In order to be able to carry out the estimation, I consider the corresponding reduced form following Horvath and Rusnák (2009):

$$y(t) = B(L)y(t-1) + u(t) \tag{3.14}$$

where $B(L)$ is a polynomial matrix in the lag operator and $var(u(t)) = \Sigma$. The structural innovations are recovered as follows: Rewrite the matrix $A(L)$ as $A(L) = A_0 + A^0(L)$, where A_0 is the coefficient matrix on L^0 in $A(L)$ which shows the contemporaneous coefficient matrix in the structural form. $A^0(L)$ is the coefficient matrix in $A(L)$ without the contemporaneous coefficient A_0. Also, the structural equation can be rewritten as $A_0 y(t) + A^0(L)y(t) = \varepsilon(t)$. Premultiplying the structural equation by A_0^-1 and rearranging the variables yields $y(t) = -A_0^-1A^0(L)y(t) + A_0^-1\varepsilon(t)$. Therefore, the relationship between the reduced form residuals and the structural shocks is given by $u(t) = A_0^-1\varepsilon(t)$.

Compared to the structural form, there are less parameters estimated in the reduced form VAR. In order to solve this identification problem, I need to impose $n(n-1)/2$ restrictions. Following Horvath and Rusnák (2009), I obtain identification by Choleski recursive scheme. In this scheme, matrix A_0^-1 is a lower triangular.

Following Mojon and Peersman (2001), the variables in each block are ordered as follows: output gap, price level, interest rate. I add the exchange rate only for Turkish block. Therefore, the recursive scheme is:

$$\begin{bmatrix} u_t^1 \\ u_t^2 \\ u_t^3 \\ u_t^4 \\ u_t^5 \\ u_t^6 \\ u_t^7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ d_{21} & 1 & 0 & 0 & 0 & 0 & 0 \\ d_{31} & d_{32} & 1 & 0 & 0 & 0 & 0 \\ d_{41} & d_{42} & d_{43} & 1 & 0 & 0 & 0 \\ d_{51} & d_{52} & d_{53} & d_{54} & 1 & 0 & 0 \\ d_{61} & d_{62} & d_{63} & d_{64} & d_{65} & 1 & 0 \\ d_{71} & d_{72} & d_{73} & d_{74} & d_{75} & d_{76} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^{x(EU)} \\ \varepsilon_t^{p(EU)} \\ \varepsilon_t^{i(EU)} \\ \varepsilon_t^{x(TR)} \\ \varepsilon_t^{p(TR)} \\ \varepsilon_t^{i(TR)} \\ \varepsilon_t^{e(TR/EU)} \end{bmatrix} \tag{3.15}$$

The third and the sixth equations can be viewed as the reaction functions of the European Central Bank (ECB) and the Central Bank of the Republic of Turkey (CBRT), respectively. Therefore, these equations can be interpreted as the monetary policy shocks of the corresponding bloc. By inspection of the sixth equation, one sees that the CBRT reacts to domestic and Euro area output and prices and the ECB interest rate. In this baseline specification, contemporaneous exchange rate is missing in the reaction function of the CBRT. However, as stated in Calvo and Reinhart (2002), the monetary authorities in open economies are often very sensitive to exchange rate developments. Therefore, this issue must be addressed in the future.

3.3 Data Description

I use the quarterly data for Turkish seasonally adjusted real GDP, Turkish consumer price index (CPI), the CBRT interest rate, TRL/EUR exchange rate, Euro zone seasonally adjusted real GDP, consumer price level and the ECB interest rate from quarter 1, 2002 to quarter 2, 2010 (34 observations). The Euro area came into force in 1^{st} January 1999 when Euro was introduced and the ECB took the responsibility for the monetary policy of the member states. However, the sample begins in 2002, since Turkey adopted the floating exchange rate regime in 22^{nd} February 2001 and the CBRT became independent, announcing the price stability as the main goal of the monetary policy in 2002.

The source of the datas for Turkish real GDP and CPI, the CBRT interest rate and the TRL/EUR exchange rate is the CBRT Electronic Data Delivery System (EDDS). The output gap, which is defined as the difference between seasonally adjusted real GDP and potential output, is estimated by the Hodrick-Prescott filter with a smoothing parameter of 1600. I used the quarterly average of interbank money market overnight simple interest rates as the indicator of the CBRT's monetary policy.

The datas for Euro zone are seasonally adjusted real GDP, Euro area harmonized index of consumer prices (HICP) and the ECB 3-month money market interest rate, which are downloaded from Eurostat.

3.4 Results

In this section I will interpret the results of the VAR analysis. In Appendix F, I present the impulse responses assessing the magnitude and the persistence of the reaction to the domestic and foreign monetary shocks. In Appendix G, I present the variance decomposition of the domestic variables in order to make a comparison of the magnitude and the importance between domestic and foreign shocks. As suggested by the Schwarz information criterion, I set the lag to 1 in my VAR model.

Figure 5 reports the estimates on the response of domestic monetary policy shock. After a domestic monetary policy shock of one standard deviation, the response of prices, output gap and the exchange rate seems to be insignificant. This is interesting since it implies that Turkish monetary policy does not play an important role for the domestic output, price and exchange rate developments.

Figure 6 reports the estimates on the effect of a Euro area monetary policy shock. The responses of Turkish price level, interest rate and the exchange rate seems to be insignificant to a Euro area monetary policy shock of one standard deviation. This means that the Turkish monetary authority is not reacting to the ECB monetary tightening extensively. Also, ECB monetary policy plays a little role for the Turkish price level.

However, the Turkish output gap seems to respond to the ECB monetary shock quite extensively. This positive response remains significant for ten quarters following the shock. Since the exchange rate is not affected by the shock, the positive response of Turkish output is not likely to reflect an increase in net exports. One possible explanation may be the integration of the Euro area and the Turkish financial sectors. After an ECB montary shock, since Turkish interest rates are higher than the ECB interest rates, the Euro area banks may invest their liabilities in Turkish banks, which, in turn, eases the access to borrowing in the Turkish financial market, and end up boosting domestic consumption and investment. Nevertheless, the estimates are surrounded by a certain degree of uncertainty.

In Table 6, I report the results on the variance decomposition of the Turkish price level. The results show that the role of external shocks increases as a source of price fluctuations over time, however the domestic shocks remain the dominant source of the variance in the price level. Interestingly, after 4 years, %11 of the Turkish price level variations can be attributed to the Euro area monetary policy disturbances. This result contrasts when compared with the importance of domestic monetary policy shock, which accounts only for %1 of the fluctuations in the price level. My results are generally consistent with Giordani (2004b) which estimates that %40 of variation in Canadian price level is due to foreign (U.S.) shocks in the long term.

Table 7 presents my results on the variance decomposition of Turkish output gap. Interestingly, most of the variance of the Turkish output gap is driven by the foreign shocks. Initially, %83 of the variation of domestic output is attributable to foreign shocks, where %16 of total variation is due to ECB monetary tightening. In the long run, the share of domestic shocks rises to %30, where only %3 of the variance is due to domestic monetary shock. Surprisingly, ECB monetary shock is a major factor in the variations of the Turkish output gap, which comprises %50 of the total variation after 9 quarters. As I stated earlier, this may be due to the increase in foreign exchange in the Turkish economy, as a result of financial integration between the Turkish and Euro area economies.This result is consistent with Giordani (2004b), who finds that approximately %70 of the fluctuations in Canadian output are due to foreign shocks in the long run. Similarly, the corresponding estimates of Cushman and Zha (1997) is %75. Also, Del Negro and Obiols-Homs (2001) finds that $\%75-85$ of the variance in the Mexican output is due to external shocks. However, my results are inconsistent with Mackowiak (2006a) who finds that the external shocks explain %30 of long run variation in economic activity for Czech Republic and Poland and only %13 for Hungary. Their finding may support my explanation since the interest rate levels is much lower in Poland and Hungary, compared to Turkey.

Table 8 reports my results on the variance decomposition of Turkish interest rate. I find that the most of the variance is explained by domestic factors initially, whereas the effect of the foreign shocks rises to %64 in the long run. This is in lie with most of the empirical evidence on other emerging economies. Mackowiak (2007) finds that the contribution of external shocks rises in the long run for Malaysia, Philippines, Singapore and Mexico. However the same study finds thatthe contribution of the external shocks are decreasing for Chile, Hongkong and Korea over time.

As pointed earlier, another interesting result is that the ECB monetary shock has no significant effect on Turkish interest rates, only %1 of the variation in Turkish interest rate is attributable to the monetary tightening of the ECB.

Finally, Table 9 gives my results on the variance decomposition of the exchange rate. I find that external shocks explain only %7 of the variation in the exchange rate and this share does not significantly increase in the long-run. Exchange rate variations are mainly driven by the domestic factors. Surprisingly, both Turkish and Euro area monetary authorities do not seem to have a significant role in exchange rate developments. However, ECB monetary shocks has a bigger impact compared to the domestic ones.

Chapter 4

Summary and Concluding Remarks

In this thesis, I explained the model in Gali and Monacelli (2005) and reported their numerical results, which I replicated using Dynare. Additionally, following Horvath and Rusnák (2009) I estimated a seven variable VAR Model which can be thought of as the reduced form of a class of open economy New Keynesian models such as Svensson (2000), Giordani (2004a), Gali and Monacelli (2005) and Galí and Gertler (2007). In this empirical analysis, I focused on the interactions between Turkey and the Euro area, utilizing the block exogeneity restriction by assuming that the Turkish variables do not have a direct effect on the Euro area.

Gali and Monacelli (2005) develops an analytically tractable optimizing model of a small open economy with Calvo-type staggered price setting. The equilibrium dynamics of this model have a canonical representation in terms of the domestic inflation and the output gap which is analogous to that of its closed economy counterpart presented in Woodford (2003). However, there are two differences. First, in small open economy's canonical representation some equilibrium coefficients depend on the degree of openness and the substitutability across goods produced in different countries. Second, In the small open economy, the natural levels of output and interest rate generally depends on the foreign disturbances, whereas the small open economy has no direct influence on the closed world economy.

After a specific parameterization, the paper derives the welfare loss function by a second order approximation to the utility of the representative household in the small open economy. Next, four different monetary policy rules are assessed in terms of their welfare ranking. The numerical analysis shows that there is a trade-off between the stabilization of both the nominal exchange rate and the terms of trade, on the one hand, and the stabilization of domestic inflation and the output gap on the other hand.

My findings from the estimated VAR model is consistent with Giordani (2004b). I find that the domestic shocks are the dominant sources of variability of Turkish inflation, whereas most of the variance of the Turkish output is attributable to foreign shocks. In particular, the results show that the ECB monetary shock has a significant effect only on output gap among the Turkish variables, which lasts for 10 quarters after the shock. I suggest that one possible explanation of this relationship may be the integration of the Euro area and the Turkish financial markets. Since Turkish interest rates are much higher than the ECB interest rates, when ECB implements a monetary tightening, the European banks may invest their increased liabilities in the Turkish financial market which, in turn, would rise the available credits in the Turkish financial market, ending up in a boost of domestic consumption and investment levels. Finally, I find that the Turkish interest rate and the TRY/EUR exchange rate are mainly driven by the domestic factors.

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A. The Perfect Foresight Steady State

In order to show how the home economy's terms of trade are uniquely pinned down in the perfect foresight steady state, Gali and Monacelli (2005) invoke symmetry among all countries other than the home country, and then show how the terms of trade and output in the home economy are determined. Without loss of generality, we assume a unit value for productivity in all foreign countries, and a productivity level A in the home economy. As shown below, in the symmetric case (when $A = 1$) the terms of trade for the home economy must necessarily be equal to unity in the steady state, whereas output in the home economy coincides with that in the rest of the world.

First, the goods market condition, when evaluated in the steady state, implies:

$$\begin{aligned} Y &= (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C + \alpha \int_0^1 \left(\frac{P_H}{\xi_i P_F^i}\right)^{-\eta} C^i di \\ &= \left(\frac{P_H}{P}\right)^{-\eta} \left[(1-\alpha)C + \alpha \int_0^1 \left(\frac{\xi_i P_F^i}{P_H}\right)^{\gamma-\eta} Q_i^{\eta} C^i di\right] \\ &= h(S)^{\eta} C \left[(1-\alpha) + \alpha \int_0^1 (S^i S_i)(\gamma-\eta) Q_i^{\eta-\frac{1}{\sigma}} di\right] \\ &= h(S)^{\eta} C \left[(1-\alpha) + \alpha S^{\gamma-\eta} q(S)^{\eta-\frac{1}{\sigma}}\right] \qquad (1) \end{aligned}$$

where we have made use of the international risk sharing condition, i.e., $C_t = \vartheta_i C_t^i Q_{i,t}^{\frac{1}{\sigma}}$ and the relationship:

$$\begin{aligned} \frac{P}{P_H} &= \left[(1-\alpha) + \alpha \int_0^1 (S_i)^(1-\eta) di\right]^{\frac{1}{1-\eta}} \\ &= \left[(1-\alpha) + \alpha (S)^{1-\eta}\right]^{\frac{1}{1-\eta}} \\ &\equiv h(S) \qquad (2) \end{aligned}$$

and we have made the substitution $Q = \frac{S}{h(S)} \equiv q(S)$. Note that $q(S)$ is strictly increasing in S.

Under the assumptions above, the international risk condition implies that the relationship

$$\begin{aligned} C &= C^* Q^{\frac{1}{\sigma}} \\ &= C^* q(S)^{\frac{1}{\sigma}} \end{aligned} \tag{3}$$

must also hold in the steady state.

Hence, combining the two relations above, and imposing the world market clearing condition $C^* = Y^*$, we obtain

$$\begin{aligned} Y &= \left[(1-\alpha)h(S)^{\eta} q(S)^{\frac{1}{\sigma}} + \alpha S^{\gamma-\eta} h(S)^{\eta} q(S)^{\eta}\right] Y^* \\ &= \left[(1-\alpha)h(S)^{\eta} q(S)^{\frac{1}{\sigma}} + \alpha h(S)^{\gamma} q(S)^{\gamma}\right] Y^* \\ &= v(S) Y^* \end{aligned} \tag{4}$$

where $v(S) > 0$, $v'(S) > 0$, and $v(1) = 1$.

Furthermore, at the steady state labor market clearing condition implies:

$$\begin{aligned} C^{\sigma}\left(\frac{Y}{A}\right)^{\varphi} &= \frac{W}{P} \\ &= A\frac{1-\frac{1}{\varepsilon}}{(1-\tau)}\frac{P_H}{P} \\ &= A\frac{1-\frac{1}{\varepsilon}}{(1-\tau)}\frac{1}{h(S)} \end{aligned} \tag{5}$$

which, combined with the risk sharing condition, yields:

$$Y = A^{\frac{1+\varphi}{\varphi}}\left(\frac{1-\frac{1}{\varepsilon}}{(1-\tau)(Y^*)^{\sigma} S}\right)^{\frac{1}{\varphi}} \tag{6}$$

Notice that, conditional on A and Y^*, 4 and6 constitute a system of two equations in Y and S, with the unique solution:

$$Y = Y^* = A^{\frac{1+\varphi}{\sigma+\varphi}} \left(\frac{1 - \frac{1}{\varepsilon}}{1 - \tau} \right)^{\frac{1}{\sigma+\varphi}} \tag{7}$$

and

$$S = 1 \tag{8}$$

which, in turn, implies $S_i = 1$ for all i.

B. Optimal Price Setting In The Calvo Model

Following Calvo (1983), Gali and Monacelli (2005) assume that each firm resets its price with probability $1-\theta$ each period, independently of the time elapsed since its last price adjustment. Thus, each period a measure $1-\theta$ of (randomly selected) firms reset their prices, while a fraction θ keep their prices unchanged. Let $P^r_{H,t}(j)$ denote the price set by a firm j adjusting its price in period t. Therefore, under the Calvo price-setting structure, $P_{H,t+k}(j) = P^r_{H,t}(j)$ with probability θ^k for $k = 0, 1, 2, \ldots$ Since all firms resetting prices in any given period will choose the same price, we henceforth drop the j subscript.

When setting a new price in period t, firm j seeks to maximize the current value of its dividend stream, conditional on that price being effective:

$$\underset{P^r_{H,t}}{max} \quad \sum_{t=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} [Y_{t+k} (P^r_{H,t} - MC^r_{t+k})] \right\} \tag{9}$$

subject to the sequence of demand constraints:

$$Y_{t+k}(j) \leq \left(\frac{P^r_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left(C_{H,t+k} + \int_0^1 C^i_{H,t+k} di \right) \equiv Y^d_{t+k}(P^r_{H,t}) \tag{10}$$

where $MC^n_t = \frac{(1-\tau)W_t}{A_t}$ denotes the nominal marginal cost.

Thus, $P^r_{H,t}$ must satisfy the first order condition:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t,t+k} \left(P^r_{H,t} - \frac{\varepsilon}{\varepsilon - 1} MC^n_{t+k} \right) \right\} = 0 \tag{11}$$

Using the fact that $Q_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma}(P_t/P_{t+k})$, we can rewrite the previous condition as:

$$\sum_{k=0}^{\infty}(\beta\theta)^k E_t \left\{ P_{t+k}^{-1} C_{t+k}^{-\sigma} Y_{t+k} \left(P_{H,t}^r - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k}^n \right) \right\} = 0 \tag{12}$$

or, in terms of stationary variables,

$$\sum_{k=0}^{\infty}(\beta\theta)^k E_t \left\{ C_{t+k}^{-\sigma} Y_{t+k} \frac{P_{H,t-1}}{P_{t+k}} \left(\frac{P_{H,t}^r}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon - 1} \Pi_{t-1,t+k}^H MC_{t+k} \right) \right\} = 0 \tag{13}$$

where $\Pi_{t-1,t+k}^H \equiv \frac{P_{H,t+k}}{P_{H,t-1}}$, and $MC_{t+k} = \frac{MC_{t+k}^n}{P_{H,t+k}}$. Log-linearizing the previous condition around the zero-inflation steady-state with balanced trade, we obtain:

$$p_{H,t}^r = p_{H,t-1} + \sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\pi_{H,t+k}\} + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k}^{dev}\} \tag{14}$$

where $mc_t^{dev} \equiv mc_t - mc$ is the (log) deviation of real marginal cost from its steady state value, $mc = -log\frac{\varepsilon}{\varepsilon-1} \equiv -\mu$.

Observe that we can rewrite the previous expression in more compact form as:

$$p_{H,t}^r - p_{H,t-1} = \beta\theta E_t\{p_{H,t+1}^r - p_{H,t}\} + \pi_{H,t} + (1-\beta\theta)mc_t^{dev} \tag{15}$$

Alternatively, using the relationship $mc_t^{dev} = mc_t^n - p_{H,t} + \mu$ to substitute for mc_t^{dev} in 15, and after some straightforward algebra, we obtain a version of the price-setting rule in terms of expected nominal marginal costs:

$$p_{H,t}^{dev} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t mc_{t+k}^n \tag{16}$$

Under the assumed price-setting structure, the dynamics of the domestic price index are described by the equation:

$$P_{H,t} \equiv \left[\theta P_{H,t-1}^{1-\varepsilon} + (1-\theta)(P_{H,t}^r)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{17}$$

which can be log-linearized around the zero-inflation steady state to yield:

$$\pi_{H,t} = (1-\theta)(p_{H,t}^r - p_{H,t-1}) \tag{18}$$

Finally, combining the previous expression with 15 to yield, after some algebra,

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda mc_t^{dev} \tag{19}$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

C. Optimal Policy Implementation

After setting $r_t = rr_t$ and plugging this into the IS-equation characterizing the small open economy, the equilibrium conditions can be summarized by means of the difference equation:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = A_0 \begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} \tag{20}$$

where

$$A_0 = \begin{bmatrix} 1 & \sigma^{-1} \\ \kappa & \beta + \kappa\sigma^{-1} \end{bmatrix} \tag{21}$$

Clearly, $x_t = \pi_t = 0$ for all t constitutes a solution to 20. Yet, as shown in Blanchard and Kahn (1980), a necessary and sufficient condition for the uniqueness of such a solution in a system with no predetermined variables like 20 is that the two eigenvalues of A_0 lie inside the unique circle. It is easy to check, however, that such a condition is not satisfied in our case. More precisely, while both eigenvalues of A_0 can be shown to be real and positive, the largest is always greater than one. As a result, there exists a continuum of solutions in a neighborhood of $(0, 0)$ that satisfy the equilibrium conditions (local indeterminacy) and one cannot rule out the possibility of equilibria displaying fluctuations driven by self-fulfilling revisions in expectations (stationary sunspot fluctuations).

That indeterminacy problem can be avoided, and thus the uniqueness of the equilibrium allocation restored, by having the central bank follow a rule which would imply that the interest rate should respond to inflation and/or the output gap were those variables to deviate from their (zero) target values. More precisely, suppose that the central bank commits itself to following rule:

$$r_t = rr_t + \phi_\pi \pi_t + \phi_x x_t \tag{22}$$

In that case, the equilibrium is described by a stochastic difference equation like 20, with:

$$A_T = \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_x) \end{bmatrix} \tag{23}$$

where $\Omega \equiv \frac{1}{\sigma + \phi_x + \kappa\phi_\pi}$. If we restrict ourselves to non-negative values of ϕ_π and ϕ_x, a necessary and sufficient condition for A_T to have both eigenvalues inside the unit circle (thus, implying that $(0, 0)$ is the unique non-explosive solution to 20) is given by:

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_x > 0 \tag{24}$$

Notice that, once uniqueness is restored, the term $\phi_\pi \pi_t + \phi_x x_t$ appended to the interest rate rule vanishes, implying that $r_t = rr_t$ for all t. Intuitively, stabilization of the output gap and inflation requires a credible threat by the central bank to vary the interest rate sufficiently in response to any deviations of inflation/or the output gap from target; yet the very existence of that threat makes its effective application unnecessary.

D. Dynamic Effects of a Domestic Productivity Shock

FIGURE 1: **Impulse responses to optimal policy under DITR**

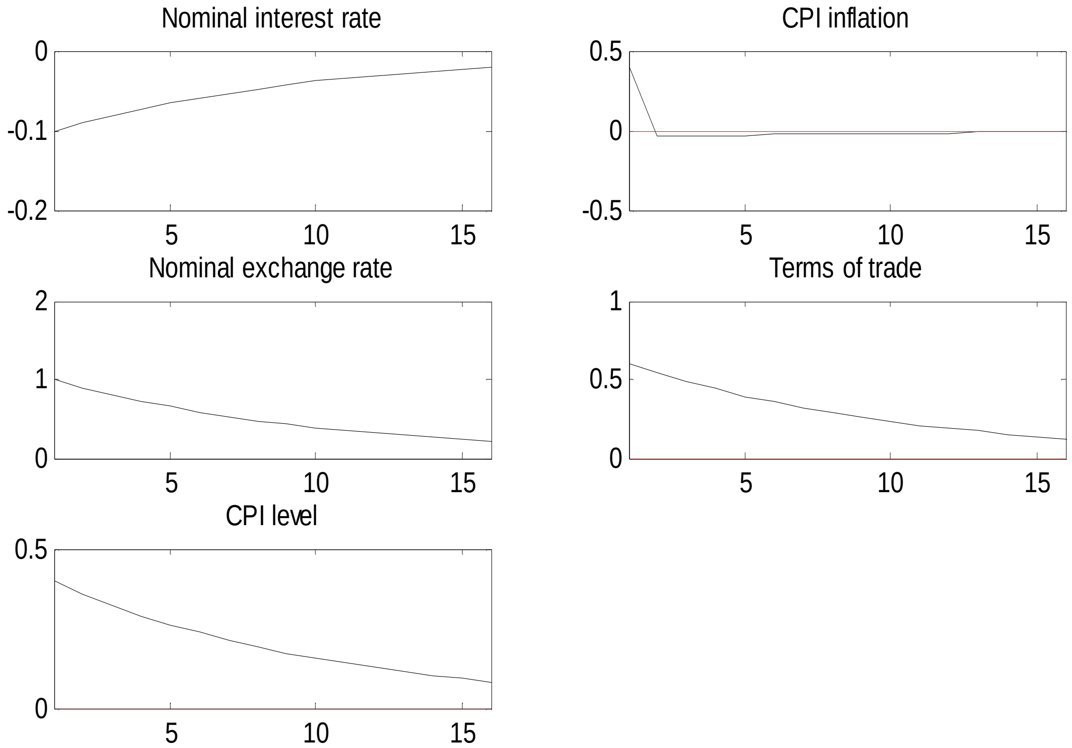


Figure 2: **Impulse responses to optimal policy under CITR**

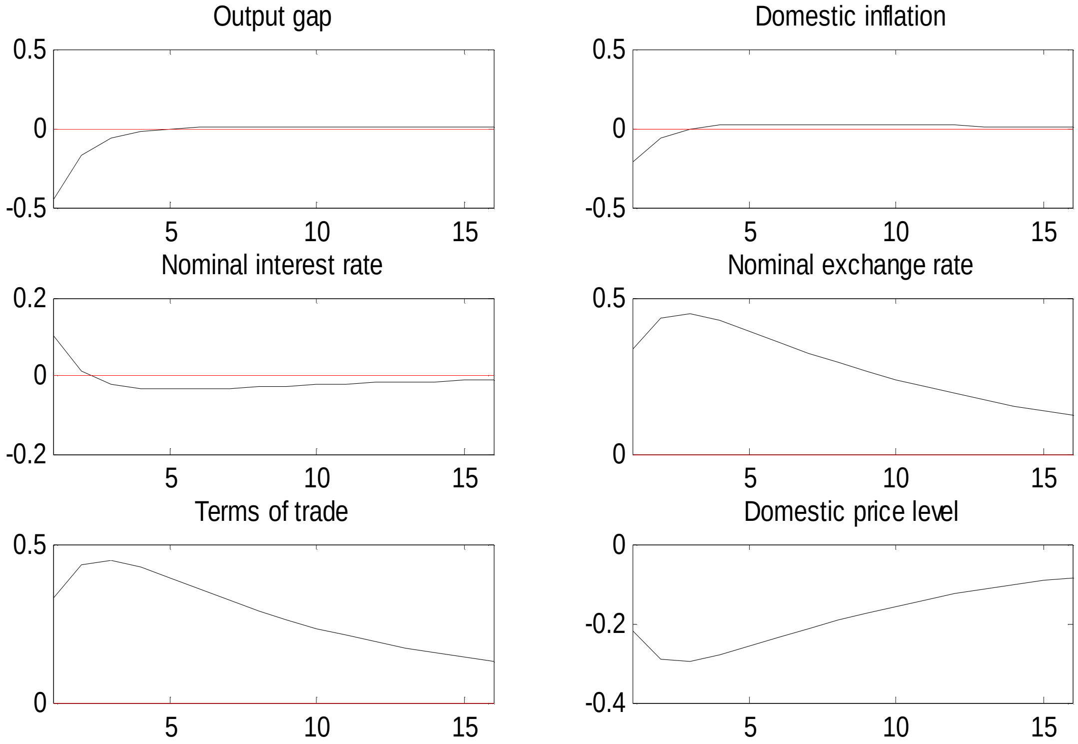


Figure 3: **Impulse responses to optimal policy under Optimal Policy Rule**

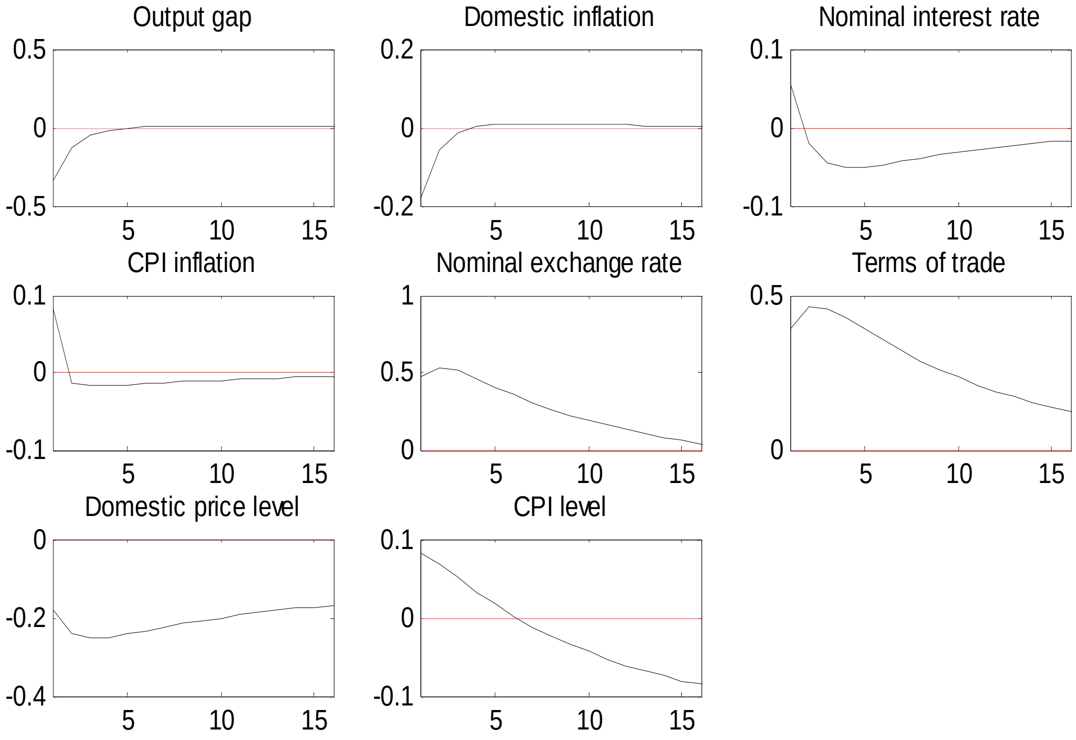
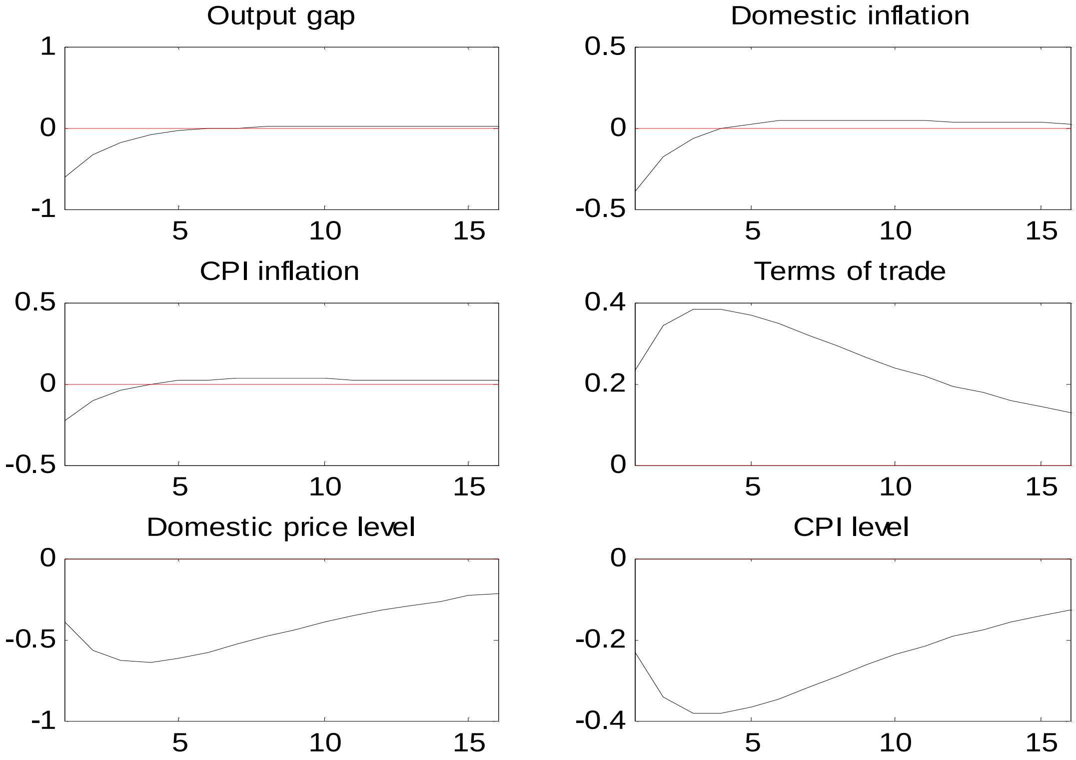


Figure 4: **Impulse responses to optimal policy under PEG egime**



E. The Welfare Costs of Alternative Policy Rules

Table 1: **Cyclical properties of alternative policy regimes**

	Optimal sd%	**DI Taylor sd%**	**CPI Taylor sd%**	**Peg sd%**
Output	0.95	0.68	0.72	0.86
Domestic inflation	0.00	0.27	0.27	0.36
CPI Inflation	0.38	0.41	0.27	0.36
Nominal Infl. Rate	0.32	0.41	0.41	0.21
Terms of Trade	1.60	1.53	1.43	1.17
Nominal Depr. Rate	0.95	0.86	0.53	0.00

Note: Entries are percentage units of steady state consumption.

Table 2: **Benchmark** ($\mu = 1.2$, $\varphi = 3$)

	DI Taylor	**CPI Taylor**	**Peg**
Var(π_H)	0.0157	0.0151	0.0268
Var(x)	0.0009	0.0019	0.0053
Total	0.0166	0.0170	0.0321

Note: Entries are percentage units of steady state consumption.

TABLE 3: **Low steady state mark-up** ($\mu = 1.1$, $\varphi = 3$)

	DI Taylor	**CPI Taylor**	**Peg**
Var(π_H)	0.0287	0.0277	0.0491
Var(x)	0.0009	0.0019	0.0053
Total	0.0297	0.0296	0.0544

Note: Entries are percentage units of steady state consumption.

TABLE 4: **Low elasticity of labor supply** ($\mu = 1.2$, $\varphi = 10$)

	DI Taylor	**CPI Taylor**	**Peg**
Var(π_H)	0.0235	0.0240	0.0565
Var(x)	0.0005	0.0020	0.0064
Total	0.0240	0.0261	0.9630

Note: Entries are percentage units of steady state consumption.

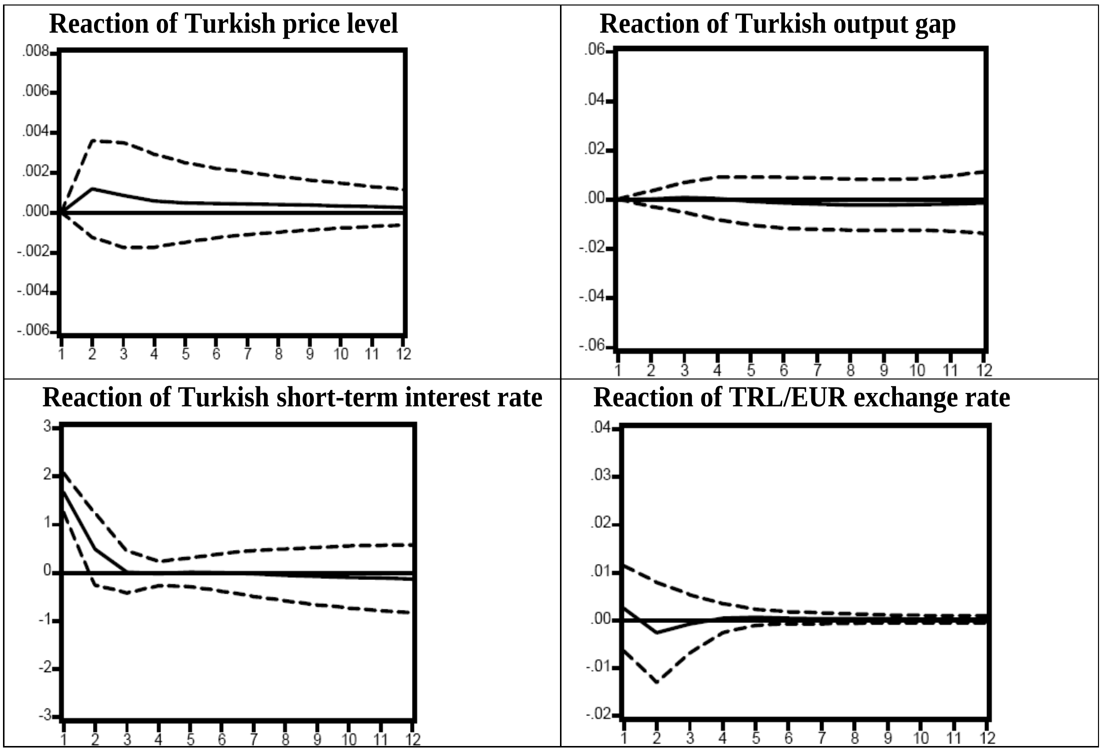
TABLE 5: **Low mark-up and elasticity of labor-supply** ($\mu = 1.1$, $\varphi = 10$)

	DI Taylor	**CPI Taylor**	**Peg**
Var(π_H)	0.0431	0.0441	0.1036
Var(x)	0.0005	0.0020	0.0064
Total	0.0436	0.0461	0.1101

Note: Entries are percentage units of steady state consumption.

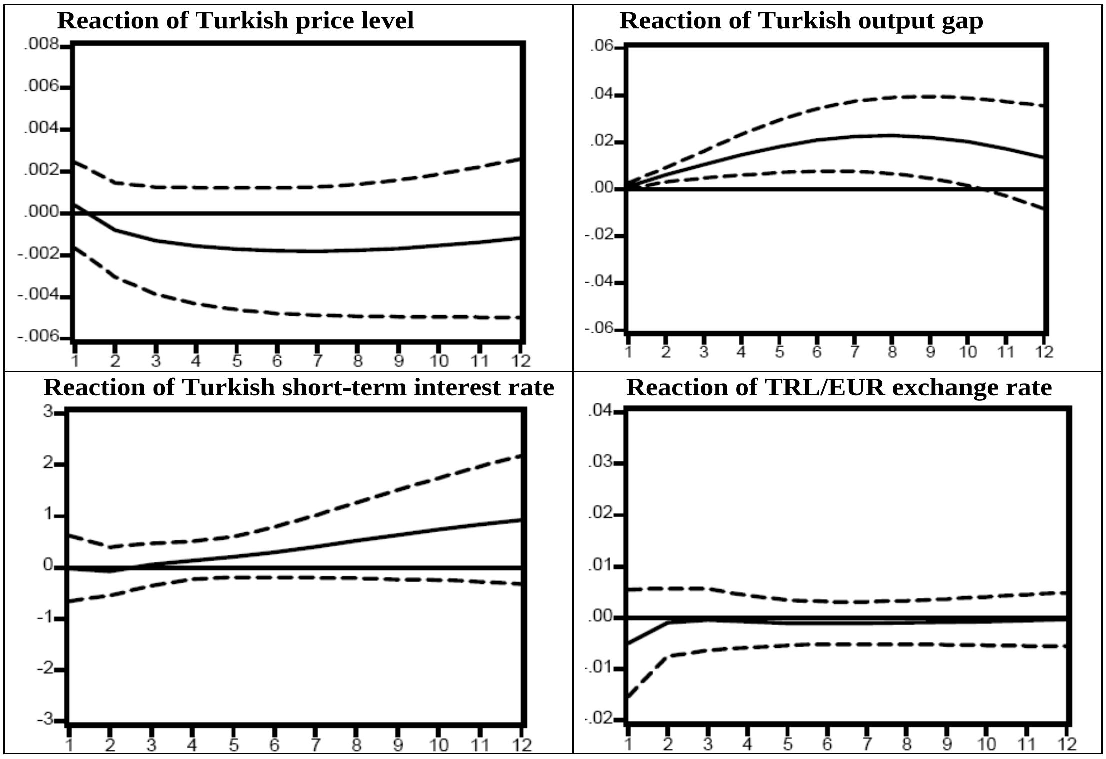
F. Impulse Responses from the VAR Analysis

Figure 5: **Domestic monetary policy shock, impulse responses**



Note: VAR Orthogonal Impulse Responses. One standard deviation Innovations

Figure 6: **Euro area monetary policy shock, impulse responses**



Reaction of Turkish price level

Reaction of Turkish output gap

Reaction of Turkish short-term interest rate

Reaction of TRL/EUR exchange rate

Note: VAR Orthogonal Impulse Responses. One standard deviation Innovations

G. Variance Decomposition from the Estimated VAR

Table 6: **Price level-domestic vs. foreign shocks, variance decomposition**

Horizon	**External shocks**	r^{ECB} **shock**	**Domestic shocks**	r^{CBRT} shock
3	0.32	0.02	0.68	0.01
6	0.38	0.06	0.62	0.01
9	0.41	0.09	0.59	0.01
12	0.42	0.11	0.58	0.01

Note: The horizon is quarterly. External and domestic shocks add to one.

Table 7: **Output gap-domestic vs. foreign shocks, variance decomposition**

Horizon	**External shocks**	r^{ECB} **shock**	**Domestic shocks**	r^{CBRT} shock
3	0.83	0.16	0.17	0.000
6	0.77	0.37	0.23	0.001
9	0.75	0.50	0.25	0.003
12	0.70	0.40	0.30	0.003

Note: The horizon is quarterly. External and domestic shocks add to one.

Table 8: **Interest rate-domestic vs. foreign shocks, variance decomposition**

Horizon	**External shocks**	r^{ECB} **shock**	**Domestic shocks**	r^{CBRT} shock
3	0.32	0.001	0.68	0.52
6	0.48	0.016	0.52	0.33
9	0.59	0.056	0.41	0.18
12	0.64	0.109	0.36	0.11

Note: The horizon is quarterly. External and domestic shocks add to one.

Table 9: **Exchange rate-domestic vs. foreign shocks, variance decomposition**

Horizon	**External shocks**	r^{ECB} **shock**	**Domestic shocks**	r^{CBRT} shock
3	0.07	0.02	0.93	0.012
6	0.07	0.02	0.93	0.012
9	0.08	0.03	0.92	0.012
12	0.08	0.03	0.92	0.012

Note: The horizon is quarterly. External and domestic shocks add to one.