Selective and Periodic Inventory Routing Problem for Collection of End-of-Life Products

by

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to my family ...

ABSTRACT

Our study is motivated from a biodiesel production facility in Istanbul that collects waste vegetable oils from source points such as restaurants and hospitals that generate waste in large amounts and are dispersed throughout the city. The production facility uses the collected waste oil as raw material for biodiesel production. The manager of this facility needs to decide which of the present source points to include in the collection program, which of them to visit on each day, which periodic routing schedule to repeat over an infinite horizon and how many vehicles to operate such that the total collection, inventory and purchasing costs are minimized while the production requirements and operational constraints are met. For this selective and periodic inventory routing problem: First, we propose a flow-based mixed integer linear programming (MILP) formulation and test it on a real-world problem with 36 scenarios. We generate lower bounds using a partial linear relaxation model, and observe that the solutions obtained through our model are within 3.28% of optimality on the average. Several insights regarding the customer selection, routing and production decisions are acquired with further sensitivity analysis. Secondly, we compare alternative formulations and test them on six scenarios. Here, we compare three alternatives to optimize the visiting schedule and observe that our first proposed MILP model yields the best solutions. Thirdly, we propose a Lagrangian Relaxation approach for the solution of single vehicle problems. The relaxed model decomposes into two mixed integer programming models that optimize the visit schedule and the collection route in each period separately. We test the performance of this solution approach and compare the lower bounds obtained by the Lagrangian relaxation method to the ones obtained by solving the proposed MILP model within a pre-specified time limit.

ÖZETÇE

Bu çalışmada, restoran ve hastane gibi büyük miktarda atık üreten ve şehre yayılmış kaynak noktalarından atık bitkisel yağ toplayan İstanbul'daki bir biyodizel üretim tesisini inceliyoruz. Bu üretim tesisi, toplanan atık yağları biyodizel üretiminde hammadde olarak kullanmakta. Üretim tesisinin yöneticisi mevcut kaynak noktalarından hangilerini atık toplama programına dahil etmesi gerektiğine; hangilerinin her gün ziyaret edilmesi gerektiğine; sonsuz süre zarfında hangi periyodik rotalama çizelgesinin tekrarlanması gerektiğine, üretim gereksinimleri ile operasyonel kısıtlar altında toplama, envanter ve satın alma maliyetlerin toplamını minimize etmek için kaç tane araç kullanılması gerektiğine karar vermelidir. Bu seçici ve periyodik envanter rotalama problemi için ilk olarak akış tabanlı bir doğrusal tamsayılı programlama (DTP) modeli geliştirdik ve bu modeli 36 gerçek problem senaryosu ile test ettik. Kısmi doğrusal gevşetme modeli kullanarak alt limitleri oluşturduk ve modelin çözümlerine baktığımızda ortalamada 3.28% uygunluk düzeyi sağladığını gözlemledik. Duyarlılık analizleri ile müşteri seçimi, rotalama ve üretim kararları ile ilgili çeşitli gözlemler ile elde ettik. İkinci olarak, değişik formülasyonlar içeren alternatif modeller geliştirdik ve bu modelleri seçilen 6 senaryo üzerinde test ettik. Burada müşteri envanteri tutatn (DTP), ziyaret çizelgesini eniyilemek için üç alternatifi karşılaştırdık ve birinci olarak sunmuş olduğumuz DTP modelinin en iyi sonuçları sağladığını gözlemledik. Üçüncü ve son olarak da, tek araçlı problemler için Lagrange Gevşetme yaklaşımını uyguladık. Gevşetme modeli, biri ziyaret çizelgesini, diğeri ise her bir periyot için toplama rotasını eniyileyen olmak üzere iki adet karışık tamsayı programlama modeline ayrılır. Bu çözüm yaklaşımının performansını test ettik ve önceden belirlenen bir zaman limiti dahilinde, Lagrange gevşetme metodu ile elde edilen alt limitler ile önerilen DTP model ile elde edilen alt limitleri karşlaştırıldı ve metodun az miktarda geliştirme elde ettiğini gözlemledik.

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NOMENCLATURE

MILP	Mixed Integer Linear Programming
PIDRP	Production, Inventory, Distribution, and Routing Problem
SPIRP	Selective and Periodic Inventory Routing Problem
PLR	Partial Linear Relaxation
PCIMP	Periodic Collection-Inventory Management Problem
PVRP	Periodic Vehicle Routing Problem
IRP	Inventory Routing Problem
VRP	Vehicle Routing Problem
VMI	Vendor Managed Inventory
CARP	Capacitated Arc Routing Problem
TSP	Traveling Salesman Problem
ASP	Asymmetric Traveling Salesmen Problem
LP	Linear Programming
VSS	Visit Schedules Subproblem
VRS	Vehicle Routing Subproblem

Chapter 1

INTRODUCTION

Our study is motivated from a biodiesel production facility (the company) in Istanbul that collects waste vegetable oils from source points at different locations throughout the city to utilize as input material in biodiesel production. Typically, the source points include businesses that consume cooking oil in large volumes, such as restaurants, hotels and hospitals. The company makes an agreement with each source point and specifies on which days of the week it will collect the accumulated waste oil from that point. The company neither pays nor receives any money from the source points for collecting the waste oils since it is mandatory for the source points to give their waste oils to recovery facilities by law. Waste vegetable oils accumulate at different rates at the source points and uncollected amounts at any day are carried on to the next day. Thus, the company may prefer to wait for several days in order to allow a high enough amount to accumulate at that location before visiting it.

The company has a predetermined production plan and needs to obtain the input materials to follow this production plan. Thus, the production plan dictates the daily input requirements for vegetable oil. The company can satisfy the vegetable oil needed for biodiesel production either by the waste vegetable oil it collects or by purchasing virgin oil, but the latter is considered to be more costly in general. However, collection also has a certain cost due to the requirements of vehicles, drivers, fuel, etc. Thus, the manager needs to decide on how much waste vegetable oil to collect, if possible, from source points and how much to purchase on each day, depending on the available inventory at hand, in order to satisfy the input requirements for production. The manager also needs to decide on the route of each vehicle in order to make the collection at the minimum possible cost. Moreover, the amount of waste vegetable oil accumulated at the source points might be more than the amount needed for production or more than the production capacity. In such cases visiting all the source points will not be necessary. Hence, the manager has to decide which of the source points should be made a collection agreement with. In addition, some of the source points might be too far away from the biodiesel facility or their accumulation amounts might be too low which makes it uneconomical for the company to collect from such source points and purchasing virgin oil for some of the production requirements might be a better option in these situations. The company can also keep an inventory at its production facility if the available amount of waste vegetable oil is more than the production requirement in that day. In this study, we analyze the decisions considering which of the potential source points to include in the collection program, which of them to visit on each day, which periodic routing schedule to repeat over an infinite horizon and how many vehicles to operate such that the total collection, inventory and purchasing costs are minimized while the production requirements and operational constraints are met. We name this routing and scheduling problem as the selective and periodic inventory routing problem.

Our problem has some differences from the literature: (*i*) There is no need to visit all source points, (*ii*) In our models schedules are structured on the days rather than the source points, (*iii*) Differently from PVRP, there is no fixed visit frequency requirements from customers and our models consider inventory management issues, (*iv*) Differently from IRP, the company collects to satisfy its production requirements rather than deliver to avoid customer stock-outs in IRP models.

We review the related literature in Chapter 2. In Chapter 3 we give the problem definition and describe the variations of the problem. In Chapter 4 we focus on the Selective and Periodic Inventory Routing Problem with a cyclic schedule and multiple heterogenous vehicles. We develop a mixed integer programming formulation and analyze it by computational tests on data based on a real life case with 25 source points and a weekly planning cycle. Then, we compare three formulations for the problem with an acyclic planning horizon in Chapter 5. We implement a Lagrangian Relaxation approach in Chapter 6 for the acyclic problem with a single vehicle and test it with the three formulations given in Chapter 5. Finally, in Chapter 7 we give our concluding remarks and discuss briefly directions for future work.

Chapter 2

LITERATURE REVIEW

Reverse logistics, recycling and remanufacturing received considerable attention in recent years due to increasing environmental and ecological concerns as well as economical benefits. In addition to saving from direct material costs, companies can also save from disposal and energy costs through reverse logistics and remanufacturing. Studies have shown that the unit cost of remanufacturing can be about 40-60% of the unit manufacturing cost of an original product in some industries [1], [2]. Biodiesel production from waste vegetable oils is such an example. While the cost of virgin oil used in the production of biodiesel constitutes 85% of the total production cost, Gonzalez et al. [3] and Predojevic [4] state that collecting and using waste vegetable oil costs almost half the price of using virgin vegetable oil in biodiesel production.

Recovery of waste vegetable oil plays an essential role in both the environmental and economic sustainability of biodiesel [5]. It is estimated that waste cooking oils in the USA amount to somewhere between 4.5 billion to 11.3 billion liters a year; in Japan nearly 400 to 600 thousand tons of waste cooking oils are generated annually [6]. A total of 108 billion liters of waste vegetable oil is estimated to be generated in the world every year, but still, out of this quantity only 6 billion liters are collected and used in biodiesel production [7]. Utlu [8] states that about 390 thousand tons of used cooking oil is wasted per year in Turkey. This much of waste cooking oil could be used in the production of about 390 thousand tons of biodiesel which would meet 5% of total diesel fuel consumption in Turkey, and save 300 million dollars per year. In addition to the economical savings, collecting waste vegetable oil has also significant benefits to the environment by decreasing the contamination of rivers, lakes or oceans. It is stated that one liter of waste oil poured down the drain can contaminate one million liters of water and cause serious damage to the environment and the ecological life [7].

Reverse logistics and collection of recoverable products are widely studied problems in the literature [9]. Fleischmann [10] analyzes the effect of product recovery on network design issues. Teixeira [11] analyzes a case study planning vehicle routes for the collection of urban recyclable waste and develop heuristics to design vehicle routes for every day of the month, to be repeated every month. Repoussis [12] presents a web-based decision support system for efficiently and effectively managing waste lube oils collection and recycling operations. They apply their system to an industrial environment and show that improved productivity and competitiveness can be achieved.

Periodic vehicle routing problems (PVRP) (Christofides and Beasley [13] and Ball [14]) and inventory routing problems (IRP) (Dror and Ball [15], Campbell [16]) are two of the research streams related to our study. In the PVRP, routes are designed for a fixed fleet of capacitated vehicles on each day of a planning horizon to visit customers exactly a preset number of times. Beltrami and Bodin [17] and Russell and Igo [18] use PVRP for the modeling of waste collection and Golden and Wasil [19] consider beverage distribution. Since PVRP is an *NP*-hard problem, generally, heuristics or metaheuristics are used to solve instances of realistic sizes [20], [21]. Tan and Beasley [22], Russell and Gribbin [23], Gaudioso and Palletta [24], Chao [25], Alonso [26] and Coene [27] implement different heuristic ideas for PVRP. Mathematical programming based methods are also developed in the literature by some authors such as Francis and Smilowitz [28] and Mourgaya and Vanderbeck [29]. Francis [30] widen PVRP by defining visit frequency as a decision variable in the formulation of the problem, which is called PVRP with Service Choice (PVRP-SC). In most of the PVRP models, a feasible set of visit schedules are generated beforehand for each customer and a part of the problem is assigning one schedule to each customer. However, in this study, we propose a flexible model without fixed schedules that are defined a priori. Different from the previous models in the PVRP literature, we reckon with the production requirements and accumulation rates at the customer sites in forming the visit schedules and routes. This leads to a periodic routing problem integrated with the production and inventory decisions similar to the models in the IRP literature.

IRP combines the periodic routing problem with inventory control and has many applications in different areas such as the gas distribution industry, suppliers of supermarkets, department store chains, parts distribution in the automotive industry, etc (Campbell [16] and Lee [31]). In IRP, customers have a daily usage rate of a product and the product must be supplied before depletion of the stock (Moin and Salhi [32] for a review). The initial research related to IRP includes single period models where optimization is carried out over a single period (Campbell and Savelsbergh [33], Beltrami and Bodin [17], Federgruen and Zipkin [34], Federgruen [35] and Chien [36]. Because of the inefficiencies of single period models, multi period models, which are computationally more complex but tend to give better quality solutions, are developed (e.g., Dror [37] and Dror and Ball [38], Dror and Levy [39]). Anily and Federgruen [40] and Raa and Aghezzaf [41] consider infinite horizon models to create a multiple day schedule that can be repeated indefinitely. Mostly, heuristics or metaheuristics are used for the solutions of IRP (e.g., Trudeau and Dror [42], Dror and Trudeau [43], Bard [44], Jaillet et al. [45], Campbell and Savelsbergh [33], and Lee [31]). IRP combined with production decisions is referred to in the literature as the production, inventory, distribution and routing problem (PIDRP) (see Lei [46], Boudia [47], Savelsbergh and Song [48] and Bard and Nananukul [44, 49]). Again, heuristics are the main tool used to solve PIDRP since the full PIDRP has so far proven to be beyond the capability of exact methods as stated by Bard and Nananukul [49].

In this paper, we have several variations from the studies in the IRP literature. First of all, we have customer selection in our model. That is, some of the source points may not be visited at all if that is profitable to do so, whereas in the classical IRP literature all customers must be visited such that they will not be out of stock. In addition, in the classical IRP literature the need for routing stems from the stock-out constraints at the customers. However, in our model, visits to source points are due to the production requirements at the production facility. Therefore, our study also includes lot-sizing considerations as in PIDRP, which makes the problem much harder. In addition, we allow the option of purchasing the required material as a substitute to the collection activity. This option adds another dimension to the problem making the feasible solution space much larger. Note that both PVRP and IRP are known to be notoriously difficult, thus it is not realistic to expect large instances of our problem to be solved to optimality. Beside the complexity of routing, additional customer selection, lot-sizing and inventory management considerations in our problem make it much harder.

We give the related problems from the literature:

2.1 VRP

The Vehicle Routing Problem (VRP) is a known combinatorial optimization problem seeking to service a number of customers with a fleet of vehicles depart and end from one or several depots. The Vehicle Routing Problem (VRP) is introduced by Dantzig and Ramser in 1959 and the Vehicle Routing Problem (VRP) still an important problem in the fields of transportation, distribution and logistics [50].

The objective is to find a set of delivery routes satisfying some requirements or constraints result minimal total cost. Because of its essential role in logistics and distribution systems, over the last decades the VRP has drawn enormous interests by many researchers. In many sectors such as garbage collection, mail delivery, task sequencing, collection of household waste, gasoline delivery, goods distribution and snow plough the VRP has wide application area.

The VRP plays a vital role in distribution and logistics. According to the investigations of Maffioli [51], Toth and Vigo [52] reported that the use of computerized methods in distribution processes generally concludes with a savings among 5% to 20% in transportation costs.

The classical Vehicle Routing Problem (VRP) is defined on an undirected graph $G = (V; E)$ where $V = \{v_0; v_1; ...; v_n\}$ is a vertex set and $E = \{(v_i; v_j) : v_i; v_j \in V; i < j\}$ is an edge set. The depot, represented by v_0 and it houses a fleet of vehicles with capacity of Q, the rest of the vertices represent the customers. Positive distance or travel time matrix is described with $C = (c_{ij})$. C matrix can be used as transportation costs between each couple of vertices on E. Also each customer has a positive demand and service time.

The VRP consists of determining a set of vehicle routes, minimum total cost; starting and ending at the depot; and such that each customer is visited exactly once by exactly one vehicle; the total demand of any route does not exceed the vehicle capacity; the total duration or length of any route does exceed a preset bound.

The number of vehicles can either be defined in advance or be a decision variable in the model. If the vehicle numbers are a decision variable, in this case fixed-costs are sometimes incorporated in the objective function.

2.1.1 Variants of Vehicle Routing Problem (VRP)

Different variant of Vehicle Routing/Scheduling Problem:

- Capacitated VRP (CVRP)
- Distance-Constrained-VRP (DVRP)
- Distance-Constrained CVRP (DCVRP)
- VRP with Time Windows (VRPTW)
- VRP with Backhauls (VRPB):
- VRP with Pickup and Delivery (VRPPD)
- Multi-compartment Vehicle Routing Problem (MCVRP)
- Periodic Vehicle Routing Problem (PVRP)
- Stochastic VRP
- Arc Routing Problems

2.1.2 Common Properties of Vehicle Routing Problems (VRPs)

Routing and Scheduling problems including all these VRP variants have some common characteristics such as:

- Size of available fleet; one or multiple,
- Type of available fleet; homogenous or heterogeneous,
- Housing of vehicles; single depot or multiple depots,
- Nature of demands ; deterministic or stochastic or partial satisfaction of demands allowed,
- Underlying network; directed or undirected or mixed or Euclidean,
- Vehicle capacity restrictions; limited or unlimited,
- Maximum route times; same for all routes or different for all routes or not imposed,
- Operations; pickups only or deliveries only or mixed (pickups & deliveries) or split deliveries (allowed or disallowed),
- Planning Horizon; single period or multiple periods,
- Time windows; one sided or two sided or soft windows or hard windows,
- Costs; variable/routing cost or fixed operating/variable acquisition costs or common carrier cost (for unserviced demands),
- Objectives; single-objective or multi-objective.

2.1.3 Solution Methods for Vehicle Routing Problem

Many researchers worked on the VRP, therefore different solution methods developed by these researchers. The proposed solution methods are:

i. Exact Solution Methods: Some methods developed and widely used as exact solution methods are Branch-and-Bound [53], [54], [55], [52], Branch-and-Cut [56], [57], [58], and Branch-and-Cut-and-Price [59], [60].

ii. Classical Heuristics: Algorithms Some classical heuristics algorithms are local search (simulated annealing, deterministic annealing, tabu search), population search (adaptive memory procedures, genetic search) and Learning mechanisms (neural networks, ant colony systems).

iii. Metaheuristics: Tabu Search, Simulated Annealing, Population Search, Genetic Algorithm, Ant system optimization and some variations or hybridizations of these metaheuristics.

2.2 PVRP

Most of the real problems which need pick-up and/or delivery operations, customers generally require frequent visits over a planning horizon. This creates demand to development of Periodic Vehicle Routing Problem (PVRP) also called allocation/routing problems. In classical VRPs the planning horizon is most of the time limited with very short time horizon, such as 8-10 hours or a day. On the other hand, PVRP one of the main additional consideration is longer planning horizons in which vehicle routes are constructed for a period (for example, one week or more than a week, also different units of times may be used).

In the periodic vehicle routing problems, deliveries are made to a set of customers over multiple time units during the period and optimizing these iterative operations can resulted significant cost savings. During each time unit within the planning period, a fleet of capacitated vehicles travels along routes which begin and end at a single depot. According to the underlying complete graph $G = (N, A)$ we can find the distances among all arcs; therefore, by using these distances we can calculate the travel costs. All the nodes N, including depot and customers are visited with predetermined frequencies over the planning period. In most of the PVRP models, researchers propose a set of schedules which are a collection of time units within the planning period in which customers receive service. During the planning period by choosing one of these schedules customers can be visited several times and the visiting frequencies k for each customer may be in a predetermined interval $1 \leq k \leq M$.

In general, after creating set of schedules, the PVRP is viewed as a multi-stage combinatorial optimization problem combining two defined problems: the assignment problem and the vehicle routing problem. Eventually, the PVRP includes three synchronous decisions: *i.* Choosing a schedule for each customer from predefined schedule set, *ii.* Assigning a group of customers to each vehicle on each day, and *iii.* Routing the vehicles for each time unit of the planning horizon.

In the classical VRP, only (ii) and (iii) decisions are made only over a single time unit. In the PVRP, each customer needs to be visited several times with a frequency of f_i during the planning period.

The PVRP has many application areas such as courier services, elevator maintenance and repair, vending machine replenishment and the delivery of interlibrary loan material. Also some applications of the PVRP can be found in environments such as fuel, oil and industrial gas distribution and waste collection.

The first PVRP model was introduced in 1974 by Beltamin and Bodin for assigning hoist compactor trucks in municipal waste collection [17]. They proposed heuristics to solve the PVRP, but did not present any model, just enlightened its complexity in comparison of the Classical VRP.

After Beltamin and Bodin, Russel and Igo [18] and Christofides and Beasley [61] put formal definitions and improved heuristic solution methods.

Russel and Igo [18] gave the formal definition of Assignment Routing Problem and they draw attention to the difficulties of choosing a schedule for each customer beside solving the routing problem. They also did not give any formulation of the problem but they offer heuristics instead.

Christofides and Beasely [61] proposed the first formulation of the PVRP. They developed a model which gives the routes at each time units in the planning period to meet the customer visit frequency requirements. They presented an integer programming problem which considers both assignments of schedules to the customers and routing of a vehicle at each time unit.

2.2.1 Variants of the Periodic Vehicle Routing Problem

Mainly the literature includes three variants related to Periodic Vehicle Routing Problems:

i. Multi-Depot PVRP

ii. PVRP with Time Windows

iii. PVRP with Service Choice

In the Multi-Depot Vehicle Routing Problem (MDVRP), period deliveries are fulfilled by using a fleet of vehicles that are holding on a number of depots. Hadjiconstantinou and Baldacci [62] combine the ideas of periodicity and multiple-depots, carrying on the PVRP to include multiple depots. This causes an increase on the complexity of the problem because besides assigning customers to depots, it includes extra decisions of assigning vehicles to depots. Also, in this problem a set of routes are designing for each day of a given D- day planning period. A fleet of vehicles based at one of the depots complete the routes for each time unit in the planning period, and vehicles must begin and end the route at its appointed depot.

The PVRP with Intermediate Facilities (PVRPIF) is close to the MDPVRP. Angelelli and Speranza [63] instead of promising multiple vehicle depots, used the idea of ”drop-off points”, or intermediate facilities, and at these points/facilities vehicle can stand along their vehicle routes to replenish their capacities. Vehicles begin and end their routes at their own depots, but visit these intermediate facilities along the way. The authors used Tabu Search method to solve the extended PVRP problem.

The Multi-Depot Vehicle Routing Problem (MDVRP) and the PVRP with Intermediate Facilities (PVRPIF) have some application areas such as waste collection recycling facilities or goods collection with warehouse facilities.

The Periodic Vehicle Routing Problem with Time Windows (PVRPTW) is also a variant which K different vehicle routes are designed to visit all customers with their allowed service frequency over the planning period, and each visit should be within a predefined time interval.

By adding time-windows, Cordeau [64] improved their previous work [20]. Their method includes a Tabu Search method for the PVRPTW, which can be used to solve the VRPTW and MDVRPTW as special cases. The improvement into the heuristic is extra penalty term included to the objective function for violations of time window constraints.

The authors also generated a set of new instances for the PVRPTW and MDVRPTW, and gave numerical results, although the quality of the solutions cannot be specifically measured in the absence of optimal solutions or lower bounds. The authors contributed a comparison of the performance of their heuristic on the Solomon VRPTW test instances [65], where it performs favorably when compared to the best known solution.

PVRP with Service Choice (PVRP-SC) is also a variant of PVRP which concerns customers who have a minimum requirement for visits over the period but are willing to accept higher visit frequency as well. This property changes the problem in terms of arrangement of visit frequencies for each customer in a flexible way and this may decrease in the routing costs.

2.2.2 Common Properties of the Periodic Vehicle Routing Problems

In general the objective of PVRP is to find a set of tours on each time unit over the planning period for each vehicle which minimizes total travel cost while satisfying operational constraints (some requirements and capacities).

Inputs, variables and aims for the PVRP can be summarized:

Given: A complete network graph $G = (N, A)$ with known arc costs c_{ij}, $\forall(i, j) \in A$, a planning period of D days indexed by d; a depot node indexed $i = 0$; a set of customer nodes $N' = N\backslash\{0\}$ with each node $i \in N'$ having a total demand of W_i over the planning period, and requiring a fixed number of visits f_i; a set of vehicles K each with capacity C; a set of schedules S.

Find: An allocation of customer nodes to schedules such that each node is visited the required number of times; a routing of vehicles for each day to visit the selected nodes during that day; with,

Objective: Minimum cost of visiting the nodes. [66]

2.2.3 Solution Methods for Periodic Vehicle Routing Problem

In particular, there exist three main solution methods which are implemented to solve PVRPs. These methods are classical heuristics, metaheuristics and mathematical programming based models.

Classical heuristics are implemented by some authors such as Beasley and Tan, Russell, and Gaudioso [22] , [23] and [24].

Beasley and Tan [22] proposed a two stage method to solve PVRP. In the first stage the authors defined the allocation of customers into predetermined schedules by solving assignment problem. Then they determined the identical VRPs for each time unit in the planning period. The most important performance criterion for their method is a cost measurement on which customer assigned to which time unit routes.

Russel and Gribbin [23] presented a multi-phase approach to the period routing problem. The first phase of analysis consists of a generalized network approximation to achieve an efficient initial solution. The second phase involves an interchange heuristic that reduces distribution costs by solving a surrogate Traveling Salesman Problem. The third phase consists of an interchange heuristic that further reduces the distribution costs by addressing the actual vehicle routes of the Period Routing Problem. A fourth phase utilizes a binary integer model to attempt further improvements.

Gaudiosio and Paletta [24] introduced a different heuristic for the PVRP which minimizes fleet size instead of transportation costs. They enforced maximum route duration and vehicle capacity constraints. Rather than enforcing a schedule set from which to choose time unite combinations, they put some upper and lower bounds on number of time units between visits for each customer. Because their algorithm's objective function is minimizing fleet size rather than transportation costs, the distance cost is generally worse than other PVRP solution methods.

Metaheuristics are also implemented by some authors such as Chao, Cordeau, and Drummond [25], [20] and [21].

Chao et al. [25] developed a method that generate an initial feasible solution to the PVRP and then iteratively use improvement steps to progress towards the optimal solution to the problem. The initial feasible solution is obtained from previously developed by Christofides and Beasley [61]. They solved a linear relaxation of the assignment problem of allocating nodes to delivery days, while minimizing the maximum load carried in any given day. While the resulting solution may not be capacity feasible, it is still useful as an initial starting point.

Cordeau et al. [20] developed a Tabu search specific insertion and route improvement techniques.

Drummond et al. [21] proposed a metaheuristic based on a combination of genetic algorithm concepts and local search heuristics. This metaheuristic is a parallel-thread population mechanism heuristic (Cordeau et al. [20]). Their method is an implementation of genetic algorithms on a parallel computing framework together with modified local search methods.

Furthermore, mathematical programming based models are implemented by some authors such as [67],[68] and [29].

Francis et al. [67],[68] widened the PVRP by defining visit frequency as a decision variable in the formulation of the problem. This newly launched problem is called the PVRP with Service Choice (PVRP-SC). This increases the difficulty of solving the problem in two ways: first, there is the added complexity of determining the service frequency; second, the vehicle capacity requirement when visiting a node also becomes a decision of the model. In the formulation of the PVRP-SC from Francis et al. [67],[68] each schedule has a monetary benefit. A positive weight converts vehicle travel and stopping time into comparable costs in the objective function. However the demand accumulated between visits, depends on the demand of the node $i \in N$ and the frequency of schedule $s \in S$, in this formulation it is approximated by the maximum accumulation between visits.

Fancis et al. [67],[68] solved this problem using the Lagrangian relaxation method, by combining branch and bound method. Instances which have at most 50 nodes can be solved with 2% of optimality gap. The conclusion from Fancis et al. [67],[68] is the largeness of the savings gained by adding service choice in the PVRP for a given instance depends on geographic distribution of nodes (especially, nodes of highest visit requirements).

Mourgaya and Vanderbeck [29] proposed a model which schedules visits and assigns these visits to vehicles but they disregarded sequencing customers will be visited within each time unit for each vehicle. In the model they had two objectives; one is regionalization which is clustering customers geographically for tour lengths and the other one is workload balancing among vehicles. The authors used truncated column generation method with rounding heuristic to solve the model. With this model they could solve the instances 50-80 customers with 5 day planning period and this range of instances are solved by using metaheuristics in most of the PVRP literature.

2.3 IRP

The inventory routing problem (IRP) is an integration of two supply chain operations, namely inventory control and vehicle routing. Inventory allocation and distribution (vehicle routing) are interrelated operations one entails information from the other. In order to decide which customers must be visited and the amount to supply each selected customer, the routing cost information is needed thus the marginal cost for each customer can be properly computed. On the other side, the transportation cost for each customer depends on the vehicle routes, which in turn requires information about customer selection and the amount of inventory allocated for each customer. Although these two issues have been examined separately, the integration of both issues can have a vital impression on entire system performance.

The IRP has received much interest, especially in the context of Vendor Managed Inventory (VMI) in a supply chain. VMI applies due to an agreement between a vendor and his customers in which the vendor has the right to choose the timing and size of the deliveries to the customers while ensuring that the customers do not run out of products.

Inventory routing problem (IRP) is a variation of well-studied vehicle routing problem (VRP) where orders are given by the customers and the aim of the supplier is to satisfy these customer orders while minimizing its total distribution cost and the planning horizon is just a single day. On the other hand, in the IRP there are no customer orders, delivery company decides how much to deliver to which customer at each day. Another difference is the planning horizon, instead of single day planning like in VRP, IRP considers a longer horizon. All the decisions taken on a day have impact on the future decisions in the IRP. One more difference is the objective functions in the IRP the objective function is to minimize total costs incurred over the planning period such as transportation costs and inventory holding costs while ensuring no customers run out of product, but VRP just consider minimizing total transportation costs.

The flexibility increases by integrating the inventory allocating problem and routing problem over the planning horizon and this may conclude with an important decrease on the distribution costs. However, increasing flexibility and the planning horizon makes the problem computationally more complex. IRP models are more realistic in the long term planning, their results are more valuable and cost efficient rather than VRP models, thus the high computational complexity can be acceptable.

There are many applications of IRP in several industries such as: The gas distribution industry [69], The petrochemical industry, Suppliers of supermarkets [70], Radice [71], Department store chains, including Walmart [72], Home products, such as Rubbermaid [73], The clothing industry, where vendor managed resupply (VMR) is encouraged by the American Apparel Manufacturers Association [74], and The automotive industry (parts distribution) [75].

2.3.1 Problem Definition

More specifically, the IRP is concerned with the repeated distribution of a single product from a single facility to a set of N customers over a given planning horizon of length T, possibly infinity. Customer i consumes the product at a given rate U_i (volume per day) and has the capability to maintain a local inventory of the product up to a maximum of C_i. The inventory at customer i is I_i^0 at time 0. A fleet of M homogeneous vehicles, with capacity Q, is available for the distribution of the product.

The objective is to minimize the average distribution costs during the planning period without causing stock-outs at any of the customers.

Three decisions have to be made: (i) When to serve a customer, (ii) How much to deliver to a customer when served, and (iii) Which delivery routes to use.

There are various characteristics of inventory routing problem: (i) The planning horizon can be finite or infinite, (ii) Inventory holding costs may or may not be considered, (iii) Inventory holding costs may be charged at the supplier only, at the supplier and the customer, or at the customers only, (iv) The production and the consumption rates can be deterministic and stochastic, (v) Production and consumption take place at discrete time instants or take place continuously, (vi) Production and consumption rates are constant over time or vary over time, and (vii) The optimal delivery policy can be chosen from among all possible policies or has to be chosen from among a specific class of policies.

2.3.2 Solution Methods

Solution techniques in the IRP can be classified into two groups which are theoretical approach; creation of the lower bounds to the problem and practical approach; heuristics to obtain near-optimal solutions.

Large majority of the papers deal with the theoretical approach use some tactics that lead to separate the IRP into two subproblems which are inventory allocation and traveling salesman problem (TSP). The inventory allocation problem is solved to decide the replenishment times and the replenishment quantities for each customer and the traveling salesman problem is solved to determine the sequences of customers will be visited on each day.

Most papers in this category adopt a two-stage solution approach:

They either find the routes first and then solve the IRP formulation, which is a simple linear programming-based inventory control problem, or solve the inventory control problem first (sometimes with approximated transportation cost), aggregate (cluster) the customers with the same replenishment time instants and then construct the routes for each cluster. As the modification of routes entails resolving a new inventory allocation problem and vice versa, most algorithms iterate between obtaining a new set of routes and resolving the inventory problem until a suitable stopping criterion is satisfied.

2.3.3 Review of the Previous Studies on IRP

Most of the research on the IRP can be categorized into four groups as follows: (*i*) Single period models (short term), (*ii*) Multi period models (long term), (*iii*) Infinite horizon models (permanent), (*iv*) Stochastic variation of the models.

We next review studies according to these four groups.

Single Period Models

The initial studies related to the IRP include single period models where the authors focused on optimizing the problems over a horizon of one day. Beltrami and Bodin [17], Federgruen and Zipkin [34] were among the pioneers of integrating the inventory allocation and routing problems with single period models. Federgruen et al. [35] and Chien et al. [36] present some other papers employ single period models to the IRP.

A non-linear mixed integer programming model developed by Federgruen and Zipkin [34] and they used Bender's decomposition approach which decomposes the problem into a nonlinear inventory allocation problem and a TSP for each vehicle. The idea is to construct an initial feasible solution and iteratively improve the solution by exchanging customers between routes. The idea is extended for two product classes by Federgruen et al. [35]. A mixed integer programming model stated by Chien et al. [36] and they proposed a Lagrangian based procedure to generate good upper bounds.

Later on, single period models was found to be very myopic, delaying all deliveries but those necessary today. According to Campbell and Savelsbergh [33] single day approaches simplify the problem but it increase inefficiency of overall system and even create infeasibilities. Although the single period models may not project the long term planning, the models are still of some relation as they are sometimes used as the basis in the study of multi period models.

Multi Period Models

Because of inefficiencies of single period models, multi period models become popular among researchers. The multi period models are computationally more complex but they tend to give better quality solutions. The first study is implemented by Dror et al. [37] and Dror and Ball [38]. They analyzed the effect of the short term on the long term planning, they proposed single period models as subproblems and also proposed a mixed integer programming model where effects of current decisions are counted on using penalty and incentive factors. Then a similar analysis are used in Dror and Levy [39] with weekly schedule, they performed a heuristic using node and arc exchanges to decrease costs in the planning period. Same ideas enlarged in Trudeau and Dror [43] while considering stochastic demands and they developed heuristics based on linear mixed integer programming submodels to solve their problems. In the Dror and Trudeau [43] also the same idea extended and both deterministic and stochastic demands are considered. Their concentration was on the maximization of operational efficiency (average number of units delivered in one hour of operation) and the minimization of the average number of stock out in one period. Jaillet et al. [45], Bard et al. [76] and Jaillet et al. [77] extended the idea of Dror et al. [37] by considering long term. They took a rolling horizon approach to the problem by determining a schedule for two weeks, while implementing only the first week. The problem is solved iteratively in the following week for the next two weeks horizon. They considered the customers which are in optimal replenishment days within the next two weeks. An allocation problem is solved, by making little changes for customers in their replenishment days (from optimal solution) one day to another to prevent too large of a demand on any day in the two week horizon. After the customers are allocated to days, the result is a set of daily vehicle routing problems where the delivery quantity used is a quantity that approximates what will be necessary to fill each customer on that day.

Campbell and Savelsbergh [33] proposed a model, using a vendor-managed resupply policies, which considers routing customers together on a day where stock-out is not occured, but if combined they can make a full or near-full truckload delivery route. The authors introduced a two-phase solution approach implemented in a rolling horizon framework. In phase I, an approximation of the problem is constructed based on a k days planning using integer programming to find the customers to serve each day and how much to serve them. The obtained solution is then used as information for phase II where the daily scheduling plan takes place. In phase I, a large set of possible clusters are generated and the cost of serving each cluster is estimated. In the second phase, the departure times and customer sequence for the different vehicles are carried out using an insertion heuristic for solving the vehicle routing with time windows.

Lee et al. [75] worked on the inventory routing problem in an automotive part supply chain that includes several suppliers and an assembly plant. The problem is based on a finite horizon, multi-period, multi-supplier and a single assembly plant part supply network where a fleet of capacitated identical vehicles transport parts from the suppliers to satisfy the demand specified by the assembly plant for each period. This problem shows an in-bound logistic problem of type *many-to-one* network and is equivalent to the *one-to-many* under certain conditions. The authors proposed a mixed integer programming model to minimize the overall transportation cost and the inventory costs. This mixed integer programming model is decomposed into two subproblems, namely the VRP and the inventory control. A heuristic based on simulated annealing is improved to generate and evaluate alternative vehicle route sets while a linear program determine the optimum inventory levels for a given set of routes. After that, a route perturbation routine is implemented to modify a set of vehicle routes based in some information obtained from the optimal solution to the linear program. The modification of routes requires solving the linear program again to get new inventory levels. This part is implemented iteratively until a stopping criterion is reached, namely the specified maximum number of iterations. They also observed an important property that the optimal solution is dominated by the transportation cost regardless of the magnitude of the unit inventory carrying cost. Then they proved this argument analytically for a simpler version of the problem based on an infinite planning horizon and stationary demand with a single supplier providing either a single-part type or multiple-part types.

The multi-period models are useful, because they offer a more realistic trade-off between the strategic and the operational nature of the IRP models. High-quality solutions are produced by these approaches but they require significantly larger computing effort. Furthermore, they allow the effect of the long-term cost on the current schedules to be studied. Because of the increase complexity of the problem, most multi-product and multi-period models consider deterministic demand at the retailers and heuristic methods to find solutions for the multi-period models.

Infinite Horizon Models

Infinite horizon models are designed for creating a p-day schedule that can be repeated indefinitely. Christofides and Beasley [61] and Gaudioso and Paletta [24] showed that infinite horizon models are good for making strategic decisions such as determining fleet size instead of short term planning. Anily and Federgruen [40], [78] studied minimization of long-run average transportation and inventory costs in the existence of deterministic demand. Using ideas similar to those of Anily and Federgruen, Gallego and Simchi-Levi [79] showed the long-run effectiveness of separate loads to each customer by direct shipping. They presented that direct shipping is at least 94% effective over all inventory routing strategies whenever minimal economic lot size is at least 71% of truck capacity. The effectiveness worsens as economic lot size gets smaller. In most of the infinite horizon models, the demand rate is assumed to be constant and deterministic. However, when stochastic demand is incorporated into the problem, infinite time horizon approaches are unsuccessful and inefficient in the short term.

Stochastic Models

Recently, interest on stochastic models increasing because in real life product usages and consequently customer demands are stochastic rather than deterministic. Therefore an important variant of the IRP is the stochastic inventory routing problem (SIRP). These models assume that the probability distribution for customer demand is known. Kleywegt et al. [80],[81] formulated the SIRP as a Markov decision process and proposed approximate dynamic programming approach to find good solutions with reasonable computational effort. Some examples of this approach are by Minkoff [82], Bassok and Ernst [83], Barnes-Schuster and Bassok [83], Berman and Larson [84], Cetinkaya and Lee [85], Furnero and Vercellis [86]. Although the results of these researches are promising, it is hard to implement this solution approach in real life because of its computational time for realistic instances of the IRP and still inappropriate for real applications, since it is not easy to obtain probability distributions of customer demands.

2.4 Collection Problems

The information stated in this section are from a review paper proposed by Beullens and Wassenove and Oudhusden [87].

Main objectives of the reverse logistics are both the collection and movement of recoverable products. Inefficient transportation activities limit the economic success of reprocessing products. So some models developed to cope with this inefficiency and in general these models can be stated with two types: normative models and descriptive models.

The normative models search to find the optimal solution for a given problem instance. These models are combinatorial optimization problems and generally NP-hard. Greedy heuristics, various local improvement heuristics, and mathematical programming are some solution methods for normative models.

The descriptive models study the general behavior of complex systems. Detailed instance data are replaced by concise summaries, and numerical methods are often replaced by analytic methods or simulation runs. These models lead to define wide features of solutions close to the optimal. These features are then used to formulate guidelines for the design of implementable solutions.

In reverse logistics there are several application areas for implementing collection and transportation approaches: (*i*) Refuse collection from households. (*ii*) Collecting hazardous materials from industrial firms. (*iii*) White goods collection. (*iv*) Combining deliveries and collections.

All the applications implemented on these areas have common objective which is obtaining the returning goods at the lowest possible cost. In every application area there should be taken two decisions; (*i*) the decisions relevant the design of the system, such as the customer service policy according to the available degrees of freedom, and (*ii*) the decisions of vehicle routing problems which are solved taking into account the specific design and relevant additional constraints.

There are different available system design options and categorized according to four aspects; the collection infrastructure, the collection policy, the combination level of the collection, and the characteristics of the collection vehicles.

The collection infrastructure is related to places that the used products turn back to the collector. (*i*) On site collection: collecting directly from the generators, (*ii*) Unmanned drop-off sites: the generators leave the used products to previously defined sites, (*iii*) Staffed and smart drop-ff: staff supervision exists (second-hand shop or smart glass collection machine), and (*iv*) Ad hoc and mobile drop-off sites: the generator leaves the used products to a site on a given time interval, vehicles can make short stops nearby to some proper locations.

The collection policy decides the moments at which a collection point is serviced and the volume collected per visit. There are some ways to define the collection policy: (*i*) Periodic schedules: according to fixed frequencies, (*ii*) Call services (ad hoc visit): by a call from collection point, (*iii*) Triggered by a distribution schedule: if the integration of collection and delivery is allowed.

The combinational level of the collection is differs according to the different classes of goods. The combinational level may change in these ways: (*i*) Separate routing of independent resources: dedicated single compartment vehicle, (*ii*) Separate routing of shared resources: two or more classes of flows are collected by a set of vehicles (not mixing), (*iii*) Co-collecting source separated flows of goods: two or more classes are collected simultaneously, and (*iv*) Integrated collection and delivery tasks: mixing, backhauling, and partial mixing.

The characteristics of the collection vehicles have to conform to the collection infrastructure, policy and combinational level. They can either have single compartment or multi compartments or relative size compartments or different rate compartments or dual-compartment (co-collect two classes).

Some properties that may be faced with in the reverse logistics can be counted like; (*i*) Node, arc and general routing, (*ii*) Low time pressure, (*iii*) Low value of the goods, (*iv*) Standardized collection policies, (*v*) Allowing split collection, (*vi*) Sector solution, (*vii*) Minimizing the fixed cost first, (*viii*) Combining multiple inbound and outbound flows, (*ix*) Multiple vehicle types, and (*x*) Supply uncertainty.

2.4.1 Vehicle Routing Models

There are some basic models described related to vehicle routing problems (VRP), these are *Single period models* concerning a single collection tasks; (*i*) Node routing: Capacitated Vehicle Routing Problem (CVRP) with non-negative edge lengths/costs and non-negative demands for service on every vertex [50], (*ii*) Arc routing: Capacitated Arc Routing Problem (CARP) with non-negative lengths/costs and non-negative demand for service on every edge [88], and (*iii*) General routing: some necessary edges associates with non-negative length/costs and some necessary vertices are used [89].

Also there exist some *Multi period vehicle routing problems* where the planning horizon consists of several periods (days). Solution methods are; (*i*) Node routing: Periodic Vehicle Routing Problem where each vertex specifies a service frequency and considers a set of allowable combinations of service days [17], (*ii*) Arc and general routing.

Furthermore there are *Co-collection models* which classes have to be collected in separate vehicle compartments [90],[91], [92] and Integration models dealing with both collection and distribution some variants are (*i*)Backhauling [93], (*ii*) Mixing [94], (*iii*) Partial mixing [95], which are also represented as the specification of the vehicle routing problems.

2.4.2 System Design Models

Economic order quantity (EOQ) models; a famous approach to find optimal periodic schedules. With EOQ models multiple stops per vehicle generally concludes to lower collection cots than direct shipping. However this method has seen inappropriate for reverse logistics [96].

Sector Design models, aim to find a set of sectors with corresponding periodic schedules to achieve the investment cost in vehicles and crew, and weekly routing costs are minimized while giving a certain level of service to the generators [92], [91].

Co-collection models, the hope is to reduce the collection costs with combining collection of different flows of goods. [97].

Integration models; combining several flows of goods to integrate collection and distribution activities [98].

2.5 Our Contribution to the Literature

The problem we study differs the literature in the following ways:

(i) There is no need to visit all source points; (ii) In our models schedules are structured on the days: a schedule presents the visit days of a source point. On the other hand, in the literature schedules are structured on the source points: a schedule presents the source point which will be visited on that day; (iii) There is no fixed visit frequency requirements from customers as in the PVRP. In our models the decision of when to visit each customer is taken among all possible day combinations rather than taking among predefined schedule sets; (iv) Considers inventory management issues. Customers accumulate used vegetable oils and they do not require fixed visit frequency by collection company. On the other hand, the collection company has to decide the quantity that he needs to collect according to the production requirements to produce bio-diesel; (v) In our models, the company collects to satisfy its production requirements. On the other hand, in comparison IRP delivers to avoid customer stock outs.

We define and introduce SPIRP to the literature. We provide a novel mixed integer programming formulation that is effective in solving realistic sized instances to near optimality. We investigate various versions of SPIRP and formulate alternative models for them. We compare the models computationally and conduct further sensitivity analysis using the best performing model. We also implement a Lagrangian relaxation approach for a special case of the problem, namely the single vehicle case. We test this approach using three alternative models. We introduce new and realistic instances of the problem that can be used for computational testing purposes.

Chapter 3

A SELECTIVE AND PERIODIC INVENTORY ROUTING PROBLEM FOR WASTE VEGETABLE OIL COLLECTION

In this chapter, we define the waste vegetable collection problem formally as a selective and periodic inventory routing problem and discuss problem characteristics.

We are given a set of n source nodes (waste oil accumulation points) and a depot (the biodiesel production facility). A complete directed graph is defined on these nodes, with real road distances d_{ij} for each arc (i, j) in the graph. The planning horizon is cyclic and each cycle consists of a fixed number of periods, e.g., seven days. Each source node i has a fixed accumulation rate a_{it} in period t. If a source node is visited in any period, all of the waste oil accumulated so far is collected by the company. In other words, partial collection is not allowed. The biodiesel production facility has requirements r_t of used vegetable oil for each period t in the planning horizon. The requirements are satisfied from: 1) the collected waste oil, 2) the purchased virgin oil, 3) the inventory on hand, or any combination of these options. Correspondingly, the following costs are incurred: a traveling cost c per unit distance traveled, a purchasing cost p per liter of virgin vegetable oil, a holding cost h per liter of waste oil per period, and a vehicle operating cost v per vehicle per period. Each vehicle has a fixed capacity Q.

The Selective and Periodic Inventory Routing Problem (SPIRP) is to find a periodic collection schedule that repeats itself in every cycle. This schedule identifies the set of source nodes to be visited and the associated vehicle routes in each period. The objective is to minimize the sum of total travel cost, vehicle operating cost, inventory holding cost and purchasing cost while satisfying the production requirements and vehicle capacity constraints.

SPIRP is *NP*-hard as it generalizes several well known *NP*-hard problems related to routing and lot-sizing. In the case of a single period, the problem reduces to a variant of a vehicle routing problem in which customer visits are selective and the required amount should be satisfied from collection and/or purchasing. When multiple periods exist in the planning horizon but only a single customer (source node) is available, then the problem is a variant of the capacitated lot-sizing problem since the main decision is on which days to visit the customer, while considering the trade-offs among the transportation cost (which is a step function due to vehicle costs), the inventory holding cost, and the purchasing cost (which is analogous to the shortage cost).

We formulated SPIRP models with different properties and these models are presented in Chapters 4-6. These models differs depending on the planning horizon types (cyclic/acyclic) and depending on the vehicles used (single/multiple homogenous vehicles). Formulations of the models are differs according to these characteristics.

i. Cyclic Planning Horizon vs. Acyclic Planning Horizon

Cyclic planning horizon indicates infinite horizon (long-term) and acyclic planning horizon denotes finite horizon (short-term). If the model is formulated according to a cyclic planning horizon it means the results (collection program) of the model can be used each consecutive period. Ending inventories at the source points and the depot in a period are the beginning inventories for the next period for the cyclic schedule models. Results gathered from acyclic planning horizon models are valid only for the next period, because the input parameters change for the following periods. This type of models have to be executed at the beginning of each period.

According to the nature of the problems both cyclic and acyclic planning horizon models have advantages. For example if a long-term collection plan needed and the production requirement is stationary in that case cyclic planning horizon models will be more useful. On the other hand, if the production requirement is not stationary and differs period by period in this case acyclic will be more valuable.

ii. Single Vehicle vs. Multiple Vehicles

Using multiple vehicles enables collecting larger amounts at any day of the cycle, but it also increases the total vehicle operating costs, which is charged per each vehicle used.

The objective function also changes depending on the single and multiple vehicle cases. To minimize the total cost, in case of multiple vehicles, we include the Vehicle Operating cost to the objective function. new paragraph:

In the multiple vehicle case, we assume a homogeneous fleet with fixed capacity and operating cost.

Chapter 4

SPIRP WITH CYCLIC SCHEDULE AND MULTIPLE VEHICLES: A MIP MODEL AND A CASE STUDY

In this chapter, we solve the collection logistics problem of a biodiesel production facility in Istanbul, Turkey and model it using mixed integer linear programming (MILP). We solve problems of size 25 source points for a 7-day cyclic planning period. We generate lower bounds with a partial linear relaxation model and observe that the solutions obtained through our MILP model are within 3.28% of optimality on the average. We extract managerial insights regarding the customer selection, routing and production decisions with further sensitivity analysis. We give the formulation of the MILP model in Section (4.1). In Section (4.2), we describe the data set and present the computing platform. Finally, Section (4.3) includes the computational results of this chapter.

4.1 Mathematical Model Formulation

In this section, we formulate a MILP model to solve SPIRP. A solution to SPIRP consists of mainly two components: 1) a visiting schedule that reveals which nodes are visited, and 2) a set of vehicle routes in each period of the planning cycle. The proposed MILP formulation determines the visiting schedule using binary variables that select sources to visit in each period. In addition, it employs variables to account for the collected amounts and maintains inventory variables to update the amount of accumulated waste at the sources according to the visiting schedule. The requirements at the biodiesel production facility are satisfied via an inventory balance constraint. For the vehicle routing decisions, our MILP model uses a single commodity flow formulation to ensure connectivity and subtour elimination. That is, we define continuous variables to represent the flow of the commodities along the arcs traveled by the vehicles and incorporate the binary node selection variables into the flow balance constraints.

In the literature, to solve the vehicle routing problem (VRP) and its variants, vehicle flow models that utilize one of the numerous types of subtour elimination constraints, and set-partitioning models that require a very large number of variables have been used extensively (see [99] for an in-depth discussion on the basic models proposed for the VRP). Recently, the use of the commodity flow variables have been observed to be computationally advantageous for some VRP variants [100]. Motivated by these results, we adapted a single commodity flow formulation for SPIRP. In our computational tests, we observed that our commodity flow formulation presented in Section 3.1 outperforms the vehicle flow formulation that uses lifted Miller-Tucker-Zemlin (MTZ) constraints [101]. Furthermore, we tested a multi-commodity flow model by adapting the Gouveia-Pires [102] formulation for the asymmetric traveling salesman problem (ATSP). The linear programming (LP) relaxation of such formulations has been shown to be stronger than using the lifted MTZ constraints for the ATSP (see Öncan et al. [103]). However, the presence of a very large number of variables in the resulting MILP model of SPIRP slows down the branch and bound algorithm dramatically. Hence, we propose the single commodity flow MILP model formulated in Section 3.1.

We also note that our MILP formulation for SPIRP does not impose any restrictions on the schedule of customer visits. It does not assume fixed visit frequencies or a limited number of predetermined schedules as most studies in the literature do (e.g., Francis and Smilowitz [28]). If we generate all possible schedules a priori and assign a schedule to each source node using binary variables, then there would be $O(n2^{\tau})$ binary variables in our problem, where τ is the number of periods in the cycle. Instead, in our proposed MILP formulation we keep the number of binary variables in the order of $O(n^2\tau)$.

4.1.1 MILP model

The index sets, parameters, and decision variables of the model are defined below.

Index Sets

$I = 0, 1, ..., n$: the set of n source nodes and the depot 0,

$IC = 1, ..., n$: the set of n source nodes only (a subset of I),

$T = 1, ..., \tau$: the set of τ periods in the cyclic planning horizon.

Parameters

c : traveling cost per unit distance.

d_{ij} : distance from node i to node j, $(i, j \in I, d_{ij} \neq d_{ji})$.

a_{it} : waste vegetable oil accumulation amount in period t at node i, $(i \in IC, t \in T)$.

r_t : waste oil requirement of the company per period, $t \in T$.

h : inventory holding cost per period for storing one liter oil at the depot.

v : operating cost per vehicle.

p : virgin vegetable oil purchasing price per liter.

Q : vehicle capacity in liters.

A_i : total weekly accumulation of waste oil at node i, $(i \in IC)$. This number serves as the Big-M number in our model. It is given by the formula $A_i = \sum_{t \in T} a_{it}$.

Decision variables

X_{ijt} : binary variable indicating if arc (i, j) is traversed by a vehicle in period t, $(i, j \in I, t \in T)$.

Y_{it} : binary variable indicating if node i has been visited in period t, $(i \in IC, t \in T)$.

Z_i : binary variable indicating if node i has been visited at least once during a cycle $(i \in IC)$. It attains the value 0, if node i is not visited at all.

F_{ijt} : the amount of waste oil flow from node i to node j in period t, $(i, j \in I, t \in T)$.

W_{it} : the amount of waste oil collected from node i in period t, $(i \in IC, t \in T)$.

I_{it} : ending inventory of waste oil by the end of period t at node i, $(i \in I, t \in T)$.

I_{i0} : initial inventory of waste oil at the beginning of the cycle at node i, $(i \in I)$.

S_t : the amount of waste oil purchased by the collecting company in period t, $(t \in T)$.

MILP formulation

$$\min TC = c \sum_{i \in I} \sum_{j \in I, (j \neq i)} \sum_{t \in T} d_{ij} X_{ijt} + v \sum_{i \in IC} \sum_{t \in T} X_{0it} + h \sum_{t \in T} I_{0t} + p \sum_{t \in T} S_t \tag{4.1}$$

s.t.

$$\sum_{j \in I, j \neq i} F_{ijt} - \sum_{j \in I, i \neq j} F_{jit} = W_{it}, \forall i \in IC, \forall t \in T \tag{4.2}$$

$$F_{ijt} \leq (Q - a_{jt}) X_{ijt}, \forall i \in I, \forall j \in I, \forall t \in T, \ \ i \neq j \tag{4.3}$$

$$F_{ijt} \leq Q - W_{jt}, \forall i \in I, \forall j \in IC, \forall t \in T, \ \ i \neq j \tag{4.4}$$

$$F_{ijt} \geq W_{it} - A_i(1 - X_{ijt}), \forall i \in IC, \forall j \in I, \forall t \in T, \ \ i \neq j \tag{4.5}$$

$$\sum_{j \in I,\, j \neq i} X_{jit} = Y_{it},\ \forall i \in IC,\ \forall t \in T \tag{4.6}$$

$$\sum_{j \in I,\, j \neq i} X_{ijt} = Y_{it},\ \forall i \in IC,\ \forall t \in T \tag{4.7}$$

$$\sum_{i \in IC} X_{i0t} = \sum_{i \in IC} X_{0it},\ \forall t \in T \tag{4.8}$$

$$W_{it} \leq A_i Y_{it},\ \forall i \in IC,\ \forall t \in T \tag{4.9}$$

$$I_{it} \leq A_i(1 - Y_{it}),\ \forall i \in IC,\ \forall t \in T \tag{4.10}$$

$$I_{it} = I_{it-1} + a_{it} Z_i - W_{it},\ \forall i \in IC,\ \forall t \in T \tag{4.11}$$

$$I_{i0} = I_{i\tau},\ \forall i \in I \tag{4.12}$$

$$I_{0t} = I_{0t-1} + \sum_{i \in IC} W_{it} + S_t - r_t,\ \forall t \in T \tag{4.13}$$

$$Z_i \leq \sum_{t \in T} Y_{it},\ \forall i \in IC \tag{4.14}$$

$$Z_i \geq Y_{it},\ \forall i \in IC,\ \forall t \in T \tag{4.15}$$

$$X_{ijt} \in \{0,1\},\ \forall i \in I,\ \forall j \in I,\ \forall t \in T,\ \ i \neq j \tag{4.16}$$

$$Y_{it} \in \{0,1\},\ \forall i \in IC,\ \forall t \in T \tag{4.17}$$

$$Z_i \in \{0,1\},\ \forall i \in IC \tag{4.18}$$

$$F_{ijt} \geq 0,\ \forall i \in I,\ \forall j \in I,\ \forall t \in T,\ \ i \neq j \tag{4.19}$$

$$W_{it} \geq 0,\ \forall i \in IC,\ \forall t \in T \tag{4.20}$$

$$I_{it} \geq 0,\ \forall i \in I,\ \forall t \in T \tag{4.21}$$

$$I_{i0} \geq 0,\ \forall i \in I \tag{4.22}$$

$$S_t \geq 0,\ \forall t \in T \tag{4.23}$$

The objective function of the model is the total cost TC, comprised of the transportation costs, vehicle operating costs, inventory holding costs, and purchasing costs incurred by the collection company during a cycle.

Constraints (4.2) represent the flow balance at each source node i. Constraints (4.3) and (4.4) provide upper bounds on the flow variables F_{ijt} by taking into account the vehicle capacity and the waste oil quantity collected from node j when a vehicle travels from i to

j in period t. Here, the vehicle capacity is adjusted by the amount to be collected at node j to strengthen the formulation. Lower bounds on the flow variables in Constraints (4.5) ensure that if a vehicle travels from i to j in period t, all of the accumulated amount at node i should be collected. Incoming and outgoing degree balance constraints are given in (4.6) and (4.7) for each source node i, ensuring that the incoming/outgoing degree of node i must be equal to 1, if node i is visited in period t; and equal to 0, otherwise. These constraints couple the binary X_{ijt} and Y_{it} variables. The depot's degree balance constraints, which enforce the incoming and outgoing degrees to be the same, are provided in (4.8).

Constraints (4.9) ensure that the collection amount at node i in period t must be zero unless it is visited in that period. Constraints (4.10)-(4.15) are used to calculate the inventory at the source nodes and the depot. To prevent partial collection of waste oil at a source node i, Constraints (4.10) make sure that the inventory at node i must be zero at the end of period t, if it is visited in that period. Constraints (4.11) update the ending inventory at a source node i in period t by incorporating the daily accumulated waste amount at i, namely a_{it}, and the amount of waste collected from node i in period t, W_{it}. If node i is not visited in period t, then W_{it} will be zero and the inventory increases by a_{it}. However, when node i is not visited at all within the cycle, i.e., $Z_i = 0$, its inventory remains unchanged. We impose that the beginning and ending inventories of the cycle be equal for each source node in (4.12). Inventory balance constraints (4.13) for the depot take into account the total collected amount, the purchased amount and the requirement in each period t in the cycle. Finally, Constraints (4.14) and (4.15) relate the binary decision variables Z_i to Y_{it} so that if node i is visited in any period within the cycle, then Z_i should take the value 1; and 0, otherwise.

4.1.2 Valid inequalities

We add the following valid inequalities to strengthen the MILP model:

$$Q \sum_{i \in IC} X_{0it} \geq \sum_{i \in IC} W_{it}, \ \forall t \in T, \tag{4.24}$$

$$Q(\sum_{i \in IC} X_{0it} - 1) + 1 \leq \sum_{i \in IC} W_{it}, \ \forall t \in T. \tag{4.25}$$

Inequalities (4.24) require that the number of vehicles dispatched in a period must be sufficient to carry the collected amount by considering both the total vehicle capacity and the collected amounts. Furthermore, we enforce by means of inequalities (4.25) that dispatching one less vehicle will not be sufficient to transport all of the collected amount in a period. To model the strict inequality required for this purpose, the number one is added to the left-hand side of the inequality (4.25) under the assumption of integer waste oil requirements.

In addition, we include the subtour elimination constraints (4.26) to break subtours of size two, and constraints (4.27) and (4.28) to avoid visits to a node which is not in the schedule.

$$X_{ijt} + X_{jit} \leq Y_{it}, \forall i \in IC, \forall j \in IC, \ \ i \neq j, \forall t \in T, \tag{4.26}$$

$$X_{i0t} \leq Y_{it}, \forall i \in IC, \forall t \in T, \tag{4.27}$$

$$X_{0it} \leq Y_{it}, \forall i \in IC, \forall t \in T. \tag{4.28}$$

4.1.3 A partial relaxation

Since the LP relaxation of the MILP model gives weak lower bounds, we benefit from a partial linear relaxation of the MILP model to generate stronger bounds as follows. First, we eliminate the binary constraints (5.11) and add the following additional constraints.

$$X_{ijt} \geq 0, \forall i \in I, \forall j \in I, \forall t \in T, \ \ i \neq j \tag{4.29}$$

$$V_t = \sum_{j \in IC} X_{0jt}, \forall t \in T \tag{4.30}$$

$$V_t \in Z^+, \forall t \in T \tag{4.31}$$

This way, the binary sequencing variables X_{ijt} are relaxed, but the formulation is tightened by enforcing integrality on the sum of X_{0jt} variables, namely V_t, using equations (4.30) and (4.31). The integer variable V_t represents the number of vehicles dispatched in period t.

Furthermore, instead of using equations (4.26)-(4.28) as valid inequalities, equations (4.32) and (4.33) are used in the partial relaxation model.

$$X_{ijt} \leq Y_{it}, \forall i \in IC, \forall j \in I, \forall t \in T, \ \ i \neq j \tag{4.32}$$

$$X_{jit} \leq Y_{it}, \forall i \in IC, \forall j \in I, \forall t \in T, \ \ i \neq j \tag{4.33}$$

In our experiments, we observed that this partial linear relaxation (PLR) model provides quite strong lower bounds when solved within a time limit of one hour, as seen in the next section.

4.2 Experimentation with real-world data

In this section we analyze the characteristics of the solutions obtained from the SPIRP model through numerical experiments with the biodiesel producer's data. The MIP model and the corresponding PLR model have been solved with real distance and traveling cost data for a planning horizon of seven days.

4.2.1 Acquisition of the problem data

For waste vegetable oil collection, we include 25 hospitals in the company's collection program. The hospitals and the recycling facility operated by the company constitute a complete collection network. In the remainder of the paper, we refer to the recycling facility as the depot where all vehicle routes originate and terminate. The asymmetric shortest path distances between each origin and destination pair on this complete network have been obtained from Google Maps. These distances multiplied by the unit traveling cost correspond to the asymmetric arc costs of the complete network under consideration. All hospitals are located on the Asian side of Istanbul, while the depot is situated in Gebze, about 50 km east of Istanbul on the northern shore of the Sea of Marmara. Figure 4.1 shows the hospitals' geographical distribution on the eastern side of the Bosphorus.

Besides the distances, there are several other input parameters such as the costs of inventory holding, transportation, purchasing, and vehicle operating; the vehicle capacity, the daily waste oil accumulation rates at each hospital, and the daily waste oil requirement of the company. We obtained the values of these parameters, where possible, from various information sources on the web and through private communication with the company. For the ones which remain hypothetical, we conducted systematic scenario analysis to comprehend their effect on the vehicle schedules and routes obtained.

The company policy is to adopt a uniform vehicle type for its collection operations. We consider two alternative light commercial vehicles: Fiat Doblo Cargo Maxi and Fiat Fiorino Cargo, and we aim to analyse which of these two alternatives is more suitable for the company. Their fuel and operating costs (parameters c and v in the SPIRP model) are

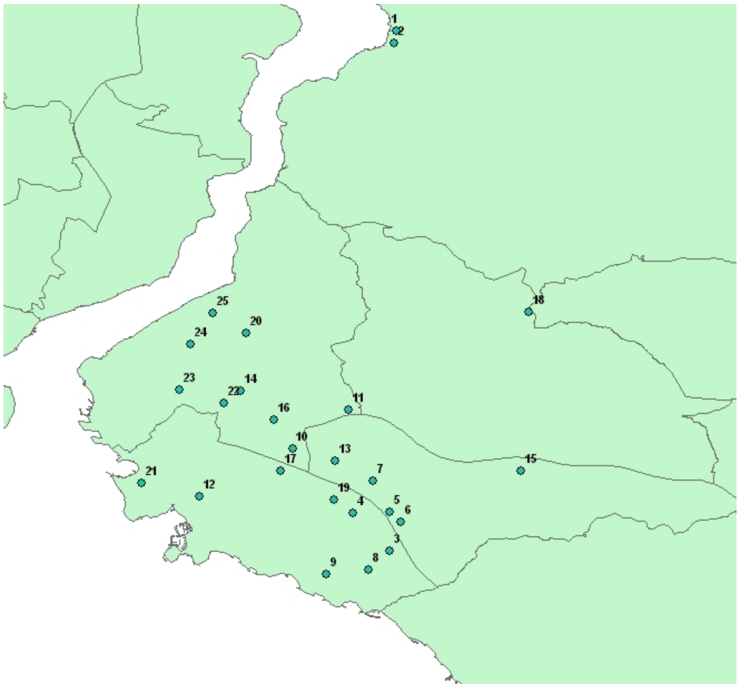

Figure 4.1: The geographical locations of the hospitals on the Asian side of Istanbul.

calculated in Table 4.1. The data on driver wages, vehicle leasing costs, and Euro 4 diesel prices were inquired in August 2010, and may show fluctuations throughout the year.

Among all parameters of the SPIRP model, four of them attain hypothetical values: purchasing price (p) and inventory holding cost (h) of one liter of oil, daily waste oil requirement (r) of the company, and waste vegetable oil accumulation rates (a_{it}) at the hospitals during the 7-day period. Since virgin oil can also be used as a raw material in biodiesel production [104], we assume that p is at most as high as the wholesale price of virgin vegetable oil, which is around 1.25 TL/lt. We have run the base SPIRP model with two other p values, namely 0.50 and 0.25 TL/lt. The cost of storing one liter of waste oil in the depot of the company, namely h, has been calculated as the daily interest rate times the highest purchasing price. This yields 0.02 TL/day for h.

The values of a_{it} have been generated in proportion to the average amounts of medical waste disposed of by the hospitals in 2009, as obtained from the Metropolitan Municipality of Istanbul. Let w_i denote the average medical waste produced by hospital i in kilograms per day, and let $\overline{w}$ denote the grand average w_i value of all 25 hospitals included in the collection

Table 4.1: Fuel and daily operating costs of the alternative light commercial vehicles

Vehicle Model	**Fiat Doblo Cargo Maxi**	**Fiat Fiorino Cargo**
Payload (except the driver)	920 kg	550 kg
Fuel Consumption (urban)	7 lt / 100 km	6 lt / 100 km
Leasing Cost	70 TL/day	50 TL/day
Vehicle Operating Cost	110 TL/day	90 TL/day
Traveling Cost	0.22 TL/km	0.19 TL/km
Common Parameters		
Wage of the drivers	40 TL/day	
Price of Euro 4 Diesel	3.08 TL/lt	

program. For each day $t \in \{1, ..., 7\}$, a_{it} has been derived from a normal distribution with mean $\mu \frac{w_i}{\overline{w}}$ and standard deviation equal to one fourth of the mean. We set μ to 30 liters to analyze the case with low rates of waste oil accumulation, and for the high rates to 60 liters. Consequently, low (high) accumulation rates vary between 1 and 240 (2 and 468) lt/day.

We tested the proposed SPIRP model with three levels of waste oil requirements: Low, medium, and high. When the accumulation rates at the hospitals are low, the daily requirements of the company are set to 600, 750, and 900 liters for low, medium, and high levels, respectively. In case of high accumulation rates, these requirements are adjusted as 1200, 1500, and 1800 liters. Since the waste oil requirements of the company are determined according to long-term production plans, we assume that the daily requirements do not vary across the 7-day production cycle.

We created a test bed of 36 SPIRP instances each of which corresponds to a unique scenario. The instances differ in vehicle related cost and capacity data, waste oil accumulation rates, daily requirement levels, and finally vegetable oil purchasing prices per liter. The problem instance names are indicative of these specifications. For example, (Fio-30-Med-050) means that the type of the collection vehicles is Fiat Fiorino Cargo, the mean of the normal distribution fitted to the waste oil accumulation rates at the hospitals is 30 lt/day, the daily waste oil requirement is at medium level, and the unit purchasing price is 0.50 TL/lt.

4.2.2 Computing platform and Cplex options

All experiments and scenario analyses were conducted on a server equipped with Intel Xeon X5460 3.16 GHz Quad-Core processor and 16 GB RAM. The operating system of this PC is 64-bit Windows Server 2003 Service Pack 2. The 64-bit version of the mathematical modeling and optimization suite GAMS 23.6 was used to create the proposed PIRP and PLR models.

Cplex 12.2 was employed with the following options turned on: `nodelim` 50000000; `threads` 0; `parallelmode` 1; `workmem` 14250; `nodefileind` 2 (GAMS/Cplex 12 Solver Manual, 2007). This way, the computing load of Cplex is distributed on to as many as four cores of the Xeon Quad-Core processor. The other Cplex options such as `cuts`, `nodesel`, `varsel`, and `bttol` did not prove beneficial in our preliminary test runs.

The time limit for each SPIRP model was set to two hours (7200 s), while each PLR model was run for one hour (3600 s).

4.3 Computational Results

4.3.1 Test results and optimality gaps

After solving the test instances within the specified time limits, we recorded the following results of interest:

- TC_{SPIRP}: The final upper bound (best feasible objective value) of each SPIRP instance.
- Gap_{SPIRP}: The final gap between the upper and lower bounds of each SPIRP instance, i.e., $\frac{UB_{SPIRP} - LB_{SPIRP}}{UB_{SPIRP}}$
- $Gap_{SPIRP-PLR}$: The final gap between the SPIRP model's best upper bound and the PLR model's best lower bound, i.e., $\frac{UB_{SPIRP} - LB_{PLR}}{UB_{SPIRP}}$
- $BestGap$: The best possible gap, i.e., $\min\{Gap_{SPIRP}, Gap_{SPIRP-PLR}\}$.
- TC_{inv}: The total inventory holding cost in the 7-day cycle.
- TC_{purch}: The total purchasing cost in the 7-day cycle.

- TC_{trans}: The total transportation cost in the 7-day cycle.
- TC_{veh}: The total vehicle operating cost in the 7-day cycle.
- $\#Veh$: The total number of vehicle routes in the 7-day cycle.
- $\%Purch$: The ratio of purchased amount to total waste oil requirement.
- I_0: The depot's beginning (as well as ending) inventory in the 7-day cycle.
- I_{max}: The depot's maximum inventory in the steady state of the 7-day cycle. It reveals the storage space needed for the viability of the company's waste oil collection operations.
- N: The number of sources visited at least once in the 7-day cycle of the collection operations.
- $Visits$: The number of hospitals visited on each day during the 7-day cycle.

The solution quality of our Cplex runs performed on the SPIRP test instances is measured by the lesser of Gap_{SPIRP} and $Gap_{SPIRP-PLR}$, since both LB_{SPIRP} and LB_{PLR} constitute a lower bound on TC^*_{SPIRP}, the actual optimal objective value of the SPIRP.

Table 4.2 shows the experimentation results. At first glance, we observe that the PLR model helps reduce the average optimality gap of the instances from 6.31% to 3.28%.

In all 36 scenarios, we looked into the inclusion of the two farthest hospitals (labeled as 1 and 2 in Figure 4.1) in the collection program of the company. When the daily waste oil requirement level is low, hospital 2 is never visited regardless of the vehicle type and accumulation rate. Hospital 1 is visited only in three of the 12 scenarios. When the requirement level rises to medium, the company includes hospital 1 in seven and hospital 2 in six scenarios. Finally, when the requirements are high, both hospitals are visited in all 12 scenarios. These results suggest that the larger the company's daily waste oil requirement, the higher the likelihood of visiting the first two hospitals that are considerably farther than the others.

The experimentation results are consolidated in Table 4.3 and Table 4.4 to discern the effects of problem specifications on the company's best decisions and associated costs. The results indicate that the biodiesel production company would be better off by using Doblo Cargo Maxi which has a slightly higher daily operating cost and worse mileage, but offers 67% more payload capacity than Fiorino Cargo. This advantage is directly reflected on the TC_{SPIRP} values. With Doblo Cargo Maxi, the company can save about 21.4% in the total recurrent cost of the weekly collection operations while meeting the daily requirement with 0.8% less purchasing from an external virgin oil source and with nine vehicle routes on average instead of 14. As can be seen in Table 4.3, the average TC_{SPIRP} rises as the purchasing price p increases from 0.25 to 1.25 TL/lt. This rise is attributable to the total cost of purchasing, which is rapidly increasing in the case of both Fiorino and Doblo. The biggest share in the total cost of waste oil collection belongs to vehicle leasing and operating whereas the smallest share belongs to inventory holding. As p doubles from 0.25 to 0.50, $\%Purch$ drops, but it remains stable when p rises to 1.25 TL/lt.

Table 4.4 displays the effects of three problem specifications on the number of dispatched vehicles denoted by $\#Veh$ and on the values of n and $\%Purch$. The consolidated results clearly point to an upward trend in the first two outputs no matter which type of vehicle is used. On the other hand, when the daily requirement level is raised from low to medium, the third output $\%Purch$ first drops if it was nonzero, but then it becomes dramatically higher than ever as the requirement level is raised further to high. This unimodal trend can be explained as follows. The total volume of waste oil accumulated daily at all 25 hospitals is 808.6 (1499.4) liters when μ equals 30(60) liters. If the daily oil requirement level is raised to high, i.e., if it becomes 900 (1800) lt/day for μ=30 (60), it exceeds the total accumulation amount, and the company has no choice but to purchase virgin oil. However, if the requirement level is medium, i.e., 750 (1500) lt/day for μ=30 (60), the company meets the requirement from the accumulated waste oil at the hospitals. Hence, it dispatches more vehicles to visit more hospitals than it was doing so at the low requirement level, which in turn leads to a decrease in the ratio of purchased virgin oil.

In Appendix 1 we represent vehicle routes during the planning horizon on a sample problem solution (Fio-30-Med-0.50).

Table 4.2: Experimentation results of interest.

Problem Instance	TC_{SPIRP}	Gap_{SPIRP}	$BestGap$	TC_{inv}	TC_{purch}	TC_{trans}	TC_{veh}	$\#Veh$	$\%Purch$	I_0	I_{max}	N	$Visits$
Fio-30acc-LOW-025	**906.61**	6.14%	6.14%	23.58	1.25	161.78	720	8	0.12%	154	387	18	3, 8, 6, 9, 3, 3, 7
Fio-30acc-MED-025	**1099.96**	3.24%	1.90%	19.16	82.5	188.3	810	9	6.29%	0	342	18	5, 10, 5, 4, 6, 4, 3
Fio-30acc-HIGH-025	**1362.98**	4.13%	4.13%	11.74	238.5	212.74	900	10	15.14%	0	297	23	5, 10, 8, 7, 4, 13, 6
Fio-60acc-LOW-025	**1763.69**	4.20%	3.60%	0	201.5	302.19	1260	14	9.60%	0	0	21	9, 10, 12, 7, 9, 10, 9
Fio-60acc-MED-025	**2239.14**	5.38%	5.00%	20.42	1	417.72	1800	20	0.04%	23	423	25	9, 11, 6, 11, 6, 5, 11
Fio-60acc-HIGH-025	**2747.38**	4.47%	4.47%	0	526	421.38	1800	20	16.70%	0	0	25	10, 15, 12, 13, 6, 6, 11
Dob-30acc-LOW-025	**704.17**	9.98%	1.78%	35.04	3.5	115.63	550	5	0.33%	596	596	15	0, 3, 3, 5, 0, 6, 6
Dob-30acc-MED-025	**854.01**	7.95%	2.04%	44.2	0.5	149.31	660	6	0.04%	409	748	19	6, 9, 0, 9, 4, 5, 7
Dob-30acc-HIGH-025	**1043.33**	5.31%	4.05%	1.42	224.25	157.66	660	6	14.24%	11	31	24	0, 5, 4, 5, 5, 11, 10
Dob-60acc-LOW-025	**1310.63**	5.43%	3.25%	37.78	57.5	225.35	990	9	2.74%	52	628	21	7, 11, 3, 7, 12, 4, 8
Dob-60acc-MED-025	**1652.91**	6.11%	3.60%	35.88	1	296.03	1320	12	0.04%	105	582	25	9, 8, 8, 10, 9, 3, 6
Dob-60acc-HIGH-025	**2133.87**	3.93%	3.93%	0.16	526	287.71	1320	12	16.70%	8	8	25	4, 5, 11, 9, 5, 8, 9
Fio-30acc-LOW-050	**902.4**	6.12%	1.01%	22.78	0	159.62	720	8	0.00%	266	360	16	4, 4, 2, 6, 5, 4, 7
Fio-30acc-MED-050	**1128.31**	5.69%	1.12%	24.68	0.5	203.13	900	10	0.02%	0	406	19	9, 5, 9, 5, 6, 7, 4
Fio-30acc-HIGH-050	**1552.29**	5.50%	5.50%	10.16	320	232.13	990	11	10.16%	131	190	25	5, 8, 4, 10, 10, 7, 11
Fio-60acc-LOW-050	**1791.48**	5.68%	3.01%	20.38	7.5	323.6	1440	16	0.18%	330	330	20	11, 8, 7, 11, 5, 7, 8
Fio-60acc-MED-050	**2340.64**	9.44%	9.03%	1.4	2	447.24	1890	21	0.04%	9	15	25	16, 12, 13, 16, 9, 10, 11
Fio-60acc-HIGH-050	**3264.39**	3.48%	3.48%	0	1052	412.39	1800	20	16.70%	0	0	25	8, 13, 9, 7, 9, 11, 8
Dob-30acc-LOW-050	**702.82**	9.78%	1.02%	36.18	0	116.64	550	5	0.00%	285	601	16	4, 0, 4, 10, 3, 0, 4
Dob-30acc-MED-050	**849.02**	7.39%	1.46%	41.76	2.5	144.76	660	6	0.10%	745	745	19	0, 3, 5, 4, 5, 5, 8
Dob-30acc-HIGH-050	**1259.48**	8.78%	5.04%	0	320	169.48	770	7	10.16%	0	0	25	6, 3, 3, 7, 5, 6, 8
Dob-60acc-LOW-050	**1371.33**	9.54%	5.34%	30	4.5	236.83	1100	10	0.11%	300	573	20	5, 6, 3, 12, 3, 3, 10
Dob-60acc-MED-050	**1765.15**	12.02%	7.71%	18	2	315.15	1430	13	0.04%	578	578	25	9, 11, 4, 11, 10, 7, 13
Dob-60acc-HIGH-050	**2658.82**	3.12%	3.12%	0.16	1052	286.66	1320	12	16.70%	0	8	25	5, 7, 7, 4, 6, 11, 5
Fio-30acc-LOW-125	**900.23**	5.94%	0.77%	22.84	0	157.39	720	8	0.00%	341	341	14	3, 3, 3, 3, 8, 4, 3
Fio-30acc-MED-125	**1134.54**	6.13%	1.65%	23.88	1.25	209.41	900	10	0.02%	10	405	19	8, 6, 9, 5, 12, 7, 3
Fio-30acc-HIGH-125	**2026.03**	3.91%	3.91%	10.02	800	226.01	990	11	10.16%	14	236	25	9, 5, 6, 9, 5, 7, 6
Fio-60acc-LOW-125	**1807.34**	6.51%	4.60%	20.46	18.75	328.13	1440	16	0.18%	0	356	20	11, 13, 7, 8, 13, 5, 11
Fio-60acc-MED-125	**2259**	6.03%	3.66%	30.36	5	423.64	1800	20	0.04%	117	455	25	13, 11, 9, 12, 10, 9, 13
Fio-60acc-HIGH-125	**4947.23**	4.42%	4.42%	0	2630	427.23	1890	21	16.70%	0	0	25	7, 9, 7, 11, 7, 9, 11
Dob-30acc-LOW-125	**710.94**	10.82%	2.15%	37.1	6.25	117.59	550	5	0.12%	28	604	18	10, 2, 0, 7, 6, 0, 4
Dob-30acc-MED-125	**847.14**	7.08%	1.24%	43.2	1.25	142.69	660	6	0.02%	0	749	19	3, 7, 3, 11, 7, 7, 0
Dob-30acc-HIGH-125	**1741.87**	6.49%	0.63%	0.08	800	171.79	770	7	10.16%	0	4	25	4, 5, 2, 7, 8, 6, 6
Dob-60acc-LOW-125	**1363.81**	9.09%	1.05%	30.28	1.25	232.28	1100	10	0.01%	321	591	15	4, 6, 2, 8, 3, 4, 6
Dob-60acc-MED-125	**1655.77**	6.02%	1.43%	38.18	5	292.59	1320	12	0.04%	320	631	25	9, 5, 7, 7, 11, 5, 5
Dob-60acc-HIGH-125	**4238.5**	1.99%	1.99%	0	2630	288.5	1320	12	16.70%	0	0	25	5, 6, 8, 6, 4, 8, 6
Averages	**1695.5**	**6.31%**	**3.28%**	**19.2**	**320.1**	**250.0**	**1106.1**	**11.3**	**0.1**	**143.1**	**339.4**	**21.5**	

Table 4.3: The effects of vehicle type and purchasing price on the average results.

Vehicle Type	Purch. Price	TC_{SPIRP}	TC_{inv}	TC_{veh}	TC_{trans}	TC_{purch}	I_0	I_{max}	N	$\%Purch$
Fiorino	p = 0.25	1687	12	1215	284	175	30	242	21.7	7.98%
Cargo	p = 0.50	1830	13	1290	296	230	123	217	21.7	4.52%
	p = 1.25	2179	18	1290	295	576	80	299	21.3	4.52%
	Grand Avg.	1899	15	1265	292	327	78	252	21.6	5.67%
Doblo	p = 0.25	1283	26	917	205	135	197	432	21.5	5.68%
Cargo Maxi	p = 0.50	1434	21	972	212	230	318	418	21.7	4.52%
	p = 1.25	1760	25	953	208	574	112	430	21.2	4.51%
	Grand Avg.	1492	24	947	208	313	209	427	21.4	4.90%

4.3.2 Sensitivity to the purchasing price p

Additional tests were conducted to explore the sensitivity of the MILP model to the purchasing price p. To this end, a problem instance (Fio-30-Med) was selected and solved for the p values varying between 0.15 and 0.50 TL/lt. The results are provided in Table 4.5. We observe that when p is as low as 0.15, the biodiesel production company does not engage in waste vegetable oil collection at all, and purchases the entire oil requirement of the 7-day planning horizon (5250 liters) from an external source. The ratio of purchasing drops rapidly as p gets higher. It becomes negligible after the value of 0.30 TL/lt. The number of vehicle dispatches increases until this p value too. Thereafter it remains stable at 10. The other results of interest in Table 4.5 exhibit a nonstationary pattern. The hospital visitation schedule is sensitive to the slightest change in p. As p increases, the company apparently collects more waste oil to rectify the shrinking amount of purchased oil. This is best confirmed by the departure of one extra vehicle during the 7-day cycle when p becomes 0.30 TL/lt or higher. The extra vehicle augments the total cost by 90 TL. The company reacts to this by not visiting the hospitals with too low accumulation rates. Instead, the one which has a relatively high accumulation rate, but was not visited before due to its significant distance from the depot is added to the collection program.

Table 4.4: The effects of vehicle type, mean accumulation rate, and daily requirement level.

		Mean Waste Oil Accumulation Rate					
	Level of Daily	$\mu = $ **30 lt/day**			$\mu = $ **60 lt/day**		
Vehicle Type	Requirement	*#Veh*	*N*	*%Purch*	*#Veh*	*N*	*%Purch*
Fiorino	**Low**	8.0	16.0	0.0%	15.3	20.3	3.3%
Cargo	**Medium**	9.7	18.7	2.1%	20.3	25.0	0.0%
	High	10.7	24.3	11.8%	20.3	25.0	16.7%
	Grand Avg.	9.4	19.7	4.7%	18.7	23.4	6.7%
Doblo	**Low**	5.0	16.3	0.2%	9.7	18.7	1.0%
Cargo Maxi	**Medium**	6.0	19.0	0.1%	12.3	25.0	0.0%
	High	6.7	24.7	11.5%	12.0	25.0	16.7%
	Grand Avg.	5.9	20.0	3.9%	11.3	22.9	5.9%

Table 4.5: The purchasing price sensitivity in problem (Fio-30-Med).

p	TC_{SPIRP}	$BestGap$	TC_{inv}	TC_{trans}	$\#Veh$	$\%Purch$	I_0	I_{max}	N	$Visits$
0.15	788	0.00%	0	0	0	100.00%	0	0	0	0, 0, 0, 0, 0, 0, 0
0.20	1049	0.60%	0	19	1	89.52%	0	0	6	0, 0, 0, 0, 0, 6, 0
0.25	1100	1.90%	19	188	9	6.29%	0	342	18	5, 10, 5, 4, 6, 4, 3
0.30	1138	3.82%	32	204	10	0.11%	279	479	20	2, 9, 6, 3, 5, 10, 5
0.35	1127	1.62%	24	202	10	0.10%	69	406	19	6, 3, 5, 9, 5, 7, 6
0.40	1127	0.97%	25	202	10	0.02%	8	418	19	7, 7, 9, 4, 4, 6, 5
0.45	1129	1.16%	24	204	10	0.02%	8	407	19	8, 5, 6, 5, 4, 8, 6
0.50	1128	1.12%	25	203	10	0.02%	0	406	19	9, 5, 9, 5, 6, 7, 4

Table 4.6: Large instances based on Dob-30-0.25.

n	*DailyReq.*	TC_{SPIRP}	*BestGap*	TC_{inv}	TC_{trans}	$\#Veh$	$\%Purch$	I_0	I_{max}	N	$Visits$
30	1000	1117	4.81%	0	186	7	9.21%	0	0	28	10, 5, 7, 11, 5, 3, 7
30	1250	1607	7.19%	2	193	8	24.32%	0	106	30	10, 6, 6, 7, 9, 9, 8
30	1500	2036	5.36%	1	186	8	36.93%	0	28	30	8, 9, 0, 7, 8, 4, 5
Avg.			**5.79%**								
35	1000	1220	13.82%	25	200	9	0.30%	81	446	22	3, 9, 2, 4, 11, 4, 3
35	1250	1507	12.14%	42	255	11	0.01%	572	591	34	7, 10, 9, 12, 14, 3, 10
35	1500	1903	10.16%	3	254	11	16.56%	0	95	35	15, 12, 11, 8, 6, 6, 7
Avg.			**12.04%**								
40	1000	1120	8.37%	50	163	8	1.53%	438	893	19	8, 5, 7, 6, 0, 8, 6
40	1250	1499	14.21%	45	241	11	0.13%	400	806	31	9, 5, 9, 7, 12, 17, 8
40	1500	1984	18.63%	39	292	13	8.51%	555	725	40	13, 9, 11, 12, 3, 15, 12
Avg.			**13.74%**								

4.3.3 Performance on larger instances

We tested the proposed MILP and PLR formulations on nine larger sized SPIRP instances, and observed the increase in optimality gaps with the problem size. The number of source nodes, n, in these instances ranges from 30 to 40. The asymmetric distance matrix associated with the new source nodes was obtained again from Google Maps. For each source node count, three daily requirement scenarios were tested. GAMS/Cplex 12.2 was run for three (one and a half) hours to solve the MILP (PLR) model of each larger instance. The results are presented in Table 4.6. While the *BestGap* values are quite good for $n = 30$, they deteriorate when n becomes 35 and 40. Again we observe a slight increase in the weekly number of vehicle routes as the daily requirement level grows. The ratio of purchasing jumps dramatically for the highest daily requirement level.

Chapter 5

SPIRP WITH ACYCLIC SCHEDULE AND MULTIPLE VEHICLES: A COMPARISON OF ALTERNATIVE MODELS

In this chapter we solve Selective and Periodic Inventory Routing Problem with acyclic planning horizon. The models consist of two parts such as Vehicle Routing part and Visit Schedule part. Vehicle Routing part is common for the three models but the Visit Schedule parts are different for each model. The first model is exactly acyclic version of SPIRP model, the second model keeping history about previous visit day in the period, and the third model is using schedules to obtain visit schedules. All the valid inequality constraints used for SPIRP model in subsection 4.1.2 are also used for these models.

Objective Function

$$\min TC = c\sum_{i\in I}\sum_{j\in I,\,(j\neq i)}\sum_{t\in T} d_{ij}X_{ijt} + v\sum_{i\in IC}\sum_{t\in T} X_{0it} + h\sum_{t\in T} I_{0t} + p\sum_{t\in T} S_t \tag{5.1}$$

Vehicle Routing

s.t.

$$\sum_{j\in I,\,j\neq i} F_{ijt} - \sum_{j\in I,\,i\neq j} F_{jit} = W_{it},\ \forall i\in IC,\ \forall t\in T \tag{5.2}$$

$$F_{ijt} \leq (Q - a_{jt})X_{ijt},\ \forall i\in I,\ \forall j\in I,\ \forall t\in T,\ \ i\neq j \tag{5.3}$$

$$F_{ijt} \leq Q - W_{jt},\ \forall i\in I,\ \forall j\in IC,\ \forall t\in T,\ \ i\neq j \tag{5.4}$$

$$F_{ijt} \geq W_{it} - A_i(1 - X_{ijt}),\ \forall i\in IC,\ \forall j\in I,\ \forall t\in T,\ \ i\neq j \tag{5.5}$$

$$\sum_{j\in I,\,j\neq i} X_{jit} = Y_{it},\ \forall i\in IC,\ \forall t\in T \tag{5.6}$$

$$\sum_{j\in I,\,j\neq i} X_{ijt} = Y_{it},\ \forall i\in IC,\ \forall t\in T \tag{5.7}$$

$$\sum_{i\in IC} X_{i0t} = \sum_{i\in IC} X_{0it},\ \forall t\in T \tag{5.8}$$

$$W_{it} \leq A_i Y_{it},\ \forall i \in IC,\ \forall t \in T \tag{5.9}$$

$$I_{0t} = I_{0t-1} + \sum_{i \in IC} W_{it} + S_t - r_t,\ \forall t \in T \tag{5.10}$$

$$X_{ijt} \in \{0,1\},\ \forall i \in I,\ \forall j \in I,\ \forall t \in T,\ \ i \neq j \tag{5.11}$$

$$Y_{it} \in \{0,1\},\ \forall i \in IC,\ \forall t \in T \tag{5.12}$$

$$F_{ijt} \geq 0,\ \forall i \in I,\ \forall j \in I,\ \forall t \in T,\ \ i \neq j \tag{5.13}$$

$$W_{it} \geq 0,\ \forall i \in IC,\ \forall t \in T \tag{5.14}$$

$$I_{i0} \geq 0,\ \forall i \in I \tag{5.15}$$

$$S_t \geq 0,\ \forall t \in T \tag{5.16}$$

5.1 Model 1

The first model calculates the Visit Schedule part by using I_{it} and Z_i variables and equations (5.17-5.21).

Constraints (5.17) make sure that the inventory at node i must be zero at the end of period t, if it is visited in that period. Constraints (5.18) update the ending inventory at a source node i in period t by incorporating the daily accumulated waste amount at i, namely a_{it}, and the amount of waste collected from node i in period t, W_{it}. If node i is not visited in period t, then W_{it} will be zero and the inventory increases by a_{it}. However, when node i is not visited at all within the cycle, i.e., $Z_i = 0$, its inventory remains unchanged. Constraints (5.19) represents the initial inventory for at node i.

Constraints (5.20) and (5.21) relate the binary decision variables Z_i to Y_{it} so that if node i is visited in any period within the planning horizon, then Z_i should take the value 1; and 0, otherwise.

Z_i : binary variable indicating if node i has been visited at least once during the period ($i \in IC$). It attains the value 0, if node i is not visited at all.

I_{it} : ending inventory of waste oil by the end of period t at node i, ($i \in I, t \in T$).

$$I_{it} \leq A_i(1 - Y_{it}),\ \forall i \in IC,\ \forall t \in T \tag{5.17}$$

$$I_{it} = I_{it-1} + a_{it} Z_i - W_{it},\ \forall i \in IC,\ \forall t \in T \tag{5.18}$$

$$I_{i0} = 0,\ \forall i \in I \tag{5.19}$$

$$Z_i \leq \sum_{t \in T} Y_{it}, \forall i \in IC \tag{5.20}$$

$$Z_i \geq Y_{it}, \forall i \in IC, \forall t \in T \tag{5.21}$$

$$Z_i \in \{0, 1\}, \forall i \in IC \tag{5.22}$$

$$I_{it} \geq 0, \forall i \in I, \forall t \in T \tag{5.23}$$

5.2 Model 2

The second model calculates the Visit Schedule part by using Z_{it_1t} variables and equations (5.24-5.26).

Constraints (5.24) ensure that collected amount from node i must be zero at the day t, if it is not visited in that day. On the other hand if node i visited at day t, by the help Z_{it_1t} variable and a_{it} parameter we can calculate the accumulated amount from day t_1 to t. Constraints (5.25) relate the binary decision variables Z_{it_1t} to Y_{it} so that if node i is visited in day t_1 and the next visit is on day t, then Z_{it_1t} should take the value 1; and 0, otherwise. Constraints (5.26) ensures that there is not a visit between day t_1 and t if Z_{it_1t} equals 1.

$T_0 = 0, ..., t = 7$: the set of the eight days of the planning horizon including previous day of the planning horizon.

Z_{it_1t} : binary variable indicating if node i has been visited in period t and the previous visit was on period t_1, $(i \in IC,\ t_1 \in T_0,\ t \in T)$.

$$W_{i,t} = \sum_{t_1 \in T_0, (t_1 \leq t)} \left(\sum_{p=t_1+1}^{t} a_{ip} \right) Z_{it_1t}, \forall i \in IC, \forall t \in T \tag{5.24}$$

$$Y_{i,t} = \sum_{t_1 \in T_0, (t_1 \leq t)} Z_{it_1t}, \forall i \in IC, \forall t \in T \tag{5.25}$$

$$\sum_{p=t_1}^{t-1} \sum_{q=p+1}^{t} (Z_{ipq}) - Z_{it_1t} \leq (t - t_1)(1 - Z_{it_1t}), \forall i \in IC, \forall t_1 \in T_0, \forall t \in T,\ (t_1 + 1 < t)$$

$$Y_{it} \geq Z_{it_1t}, \forall i \in IC,\ , \forall t \in T,\ , \forall t_1 \in T_0 \tag{5.26}$$

$$Z_{it_1t} \in \{0, 1\}, \forall i \in I, \forall t_1 \in T_0, \forall t \in T \tag{5.27}$$

5.3 Model 3

The third model calculates the Visit Schedule part by using Z_{ik} variables, $Schedule_{kt}$, $Collection_{ikt}$ parameters and equations (5.28-5.30).

Constraints (5.28)relate the binary decision variables Z_{ik} to Y_{it} so that if schedule k is chosen for node i and includes a visit at day t then Y_{it} take the value 1; and 0, otherwise. Constraints (5.29) ensure that only a schedule can be chosen for a node i. Constraints (5.30) calculates the collected amount for node i at day t.

$K = 1, ..., k$: the set of k schedules which is consist of $2^{t=7}$ planning day combinations.

Z_{ik} : binary variable indicating if schedule k is chosen for node i, $(i \in IC, k \in K)$.

$Schedule_{kt}$: binary parameter which indicates schedule k includes a visit at day t.

$Collection_{ikt}$: collection amount at day t if schedule k is chosen for node i.

$$Y_{it} = \sum_{k \in K} Z_{ik} * Schedule_{kt}, \forall i \in IC, \forall t \in T \tag{5.28}$$

$$\sum_{k \in K} Z_{ik} = 1, \forall i \in IC \tag{5.29}$$

$$W_{it} = \sum_{k \in K} Z_{ik} * Collection_{ikt}, \forall i \in IC, \forall t \in T \tag{5.30}$$

$$Z_{ik} \in \{0, 1\}, \forall i \in I, \forall k \in K \tag{5.31}$$

5.4 Computational Results

We compared these three model on 6 data instances consists of 15 hospitals and a depot. When the accumulation rates at the hospitals are low, the daily requirements of the company are set to 450, 550, and 650 liters for low, medium, and high levels, respectively. In the case of high accumulation rates, these these requirements are adjusted as 750, 1000, and 1250 liters.

Firstly, when we compare the complexities of the models on their additional decision variables to calculate Visit Schedule parts, we resulted as: *(i)* Model 1 has I_{it} and Z_i variables with sizes $(N \times T)$ and (N) respectively. The complexity of additional variables

Table 5.1: Comparison of three models over six scenarios.

		Model 1			**Model 2**			**Model 3**		
Scenario #	**Scenario Name**	**UB**	**LB**	**Gap**	**UB**	**LB**	**Gap**	**UB**	**LB**	**Gap**
1	Fio-60acc-MED-050	1622.94	1537.95	5.24%	1591.84	1519.60	4.54%	1594.19	1536.37	3.63%
2	Fio-60acc-HIGH-050	2218.17	2162.79	2.50%	2245.75	2120.01	5.60%	2229.26	2160.23	3.10%
3	Dob-60acc-LOW-050	839.63	830.32	1.11%	840.4	794.05	5.52%	842.47	798.64	5.20%
4	Fio-30acc-HIGH-025	964.76	959.92	0.50%	969.25	937.71	3.25%	991.09	947.73	4.38%
5	Fio-60acc-MED-025	1473.33	1441.78	2.14%	1473.06	1413.52	4.04%	1484.49	1440.37	2.97%
6	Dob-30acc-HIGH-025	831.57	823.08	1.02%	832.68	792.07	4.88%	831.57	781.20	6.06%
	Average	1325.067	1292.64	2.08%	1325.497	1262.827	4.64%	1328.845	1277.424	4.22%

of Model 1 is $O^{(N \times T)+(N)}$. *(ii)* Model 2 has Z_{it_1t} variables with size $(N \times (T+1) \times T)$. The complexity of additional variables of Model 2 is $O^{(N \times (T+1) \times T)}$. *(iii)* Model 3 has Z_{ik} variables with size $(N \times K)$ where $K = 2^T$. The complexity of additional variables of Model 3 is $O^{(N \times 2^T)}$. In our models $N = 15$ and $T = 7$, therefore Model 1 is the less complex model. According to the computational complexities of the additional decision variables of the models, we are expecting that Model 1 will end up with the best results.

Then, we compare three alternatives to optimize the visiting schedule and according to the Table (5.1) we observe that our first proposed MILP model yields the best solutions. Upper bounds for the models are almost same for both, but the lower bound quality determines the best model. The average optimality gap between upper bound and lower bound has the minimum values in Model 1. The optimality gap for Model 1 is 2.08%, Model 2 is 4.64% and Model 3 is 4.22% on average. Upper bounds of the both models are almost same but the lower bounds of Model 1 are stronger than the others.

Chapter 6

SPIRP WITH ACYCLIC SCHEDULE AND SINGLE VEHICLE: A LAGRANGIAN RELAXATION APPROACH

In this chapter, we consider the single vehicle version of SPIRP with acyclic schedule where waste oil collection is conducted by a single vehicle. The MILP formulation of this problem allows us to decouple the scheduling and routing decisions by relaxing the linking constraints. Therefore, we investigate a Lagrangian Relaxation approach in order to obtain strong lower bounds for the problem and generate good feasible solutions.

The relaxed problem is divided into two subproblems: the Visit Schedules Subproblem (VSS) where the model decides which customers will be visited in which day while satisfying the constraints, and the Vehicle Routing Subproblem (VRS) where the model defines the route schedules for each time unit within the planning horizon. The two resulting subproblems are solved independently and Lagrangian multipliers are associated with the relaxed constraints. In every iteration Lagrangian multipliers are updated according to subgradient optimization method and both models are solved repeatedly until an acceptable gap between upper and lower bound is found or the iteration limit is reached.

Each subproblem is a difficult problem by itself. In the visit schedule subproblem, a vehicle must be assigned to some customers for each day, such that capacity constraints are satisfied. The capacity constraint is a "knapsack" type of constraint. The vehicle routing subproblem decomposes for each day. The daily problem is a version of the traveling salesman problem (TSP) in which it is not necessary to visit all nodes (TSP with profits). The objective is to find a route with minimum travel cost, where the costs may be negative due to the Lagrangian multiplier values.

6.1 Lagrangian Relaxation Approach

The aim of the Lagrangian relaxation approach is to acquire a relaxed problem which can be solved efficiently. The approach fundamentally removes complicated constraints and adds them into the objective function by using Lagrangian multipliers. With these set of multipliers, the relaxed problem's objective function value becomes as a lower bound on the optimal cost of the original problem. In the literature numerous successful applications of Lagrangian relaxation have been stated, [105] and [106] proposed the Lagrangian relaxation approach.

We relaxed constraints (6.1) and (6.2) then incorporated them in the objective function using respective λ_{it} and μ_{jt} Lagrangian multipliers.

$$Y_{it} - \sum_{j=0,\ (j \neq i)}^{n} X_{ijt} = 0, \qquad (\times \lambda_{it}) \qquad , \forall i \in I, \forall t \in T \tag{6.1}$$

$$Y_{jt} - \sum_{i=0,\ (i \neq j)}^{n} X_{ijt} = 0, \qquad (\times \mu_{jt}) \qquad , \forall j \in I, \forall t \in T \tag{6.2}$$

$$\begin{aligned} Min \qquad & h \sum_{t=1}^{T} I_t + p \sum_{t=1}^{T} S_t + c \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{t=1}^{T} X_{ijt} d_{ij} \\ & + \sum_{i=0}^{N} \sum_{t=1}^{T} \lambda_{it} (Y_{it} - \sum_{j=0,\ (j \neq i)}^{N} X_{ijt}) + \sum_{j=0}^{N} \sum_{t=1}^{T} \mu_{jt} (Y_{jt} - \sum_{i=0,\ (i \neq j)}^{N} X_{ijt}) \end{aligned} \tag{6.3}$$

After rearranging the new objective function given in equation (6.3) we obtain objective function for the relaxed problem which is represented below with equation (6.4).

Objective Function of Relaxed Problem:

$$\begin{aligned} Min \quad & h \sum_{t=1}^{T} I_t + p \sum_{t=1}^{T} S_t + \sum_{i=0}^{N} \sum_{j=0,\ (j \neq i)}^{N} \sum_{t=1}^{T} X_{ijt} (c \times d_{ij} - \lambda_{it} - \mu_{jt}) \\ & + \sum_{i=0}^{N} \sum_{t=1}^{T} \lambda_{it} Y_{it} + \sum_{j=0}^{N} \sum_{t=1}^{T} \mu_{jt} Y_{jt} \end{aligned} \tag{6.4}$$

6.1.1 General Flow of the Lagrangian Approach

In the Lagrangian relaxation method: First we initialize the model with initial parameters, then we calculate the UB and compare the new upper bound UB' with our current upper bound UB. If the new upper bound is lower than the current upper bound we update the current UB. Then we calculate the LB and compare the new lower bound LB' with our current upper bound LB. If the new lower bound is greater than the current upper bound we update the current LB. We check our Termination Criteria are satisfied or not satisfied? If the termination criteria satisfied we end the model up with UB and LB, if they are not satisfied we update the Lagrangian multipliers by using subgradient optimization method and we continue the process until reaching the termination criteria.

6.1.2 Updating The Lagrangian Multipliers

For a set of Lagrangian multipliers λ_{it} and μ_{jt}, the objective function value of the relaxed problem gives a lower bound for the original problem. To find the best lower bound, we need to searches for the values of the multipliers. For this purposewe used subgradient optimization method.

According to the subgradient optimization method, the formula of updating Lagrangian multipliers is given below.

$$\lambda_{it}^{n+1} = \lambda_{it}^{n} + \phi^{n}(Y_{it} - \sum_{j=0,\ (j\neq i)}^{N} X_{ijt}) \tag{6.5}$$

$$\mu_{jt}^{n+1} = \mu_{jt}^{n} + \psi^{n}(Y_{jt} - \sum_{i=0,\ (i\neq j)}^{N} X_{ijt}) \tag{6.6}$$

Here ϕ^n and ψ^n are step sizes for the multipliers and the formula for calculating step sizes are given below.

$$\phi^{n} = \frac{\alpha^{n}(UB - L^{n})}{\sum_{i=0}^{N}\sum_{t=1}^{T}(Y_{it} - \sum_{j=0,\ (j\neq i)}^{N} X_{ijt})^2} \tag{6.7}$$

$$\psi^{n} = \frac{\beta^{n}(UB - L^{n})}{\sum_{j=0}^{N}\sum_{t=1}^{T}(Y_{jt} - \sum_{i=0,\ (i\neq j)}^{N} X_{ijt})^2} \tag{6.8}$$

6.1.3 Termination Criteria

The procedures are repeated until one of the termination criteria is met. In our method we have different termination criteria: (*i*) Gap between upper bound and lower bound. If the gap is less than 1% we terminate the method. (*ii*) Iteration number. We set a maximum iteration number as 150 iterations, after completing 150 iterations the model terminated. (*iii*) Iteration number without improvement. We calculate the improvement at every iteration and if the model is not improve at consecutive 5 iterations the method terminated.

6.2 Subproblems

As we mentioned before in the Lagrangian relaxation method, the assignment decisions (Y_{it}) are separated from routing decisions (X_{ijt}). Therefore we obtain two separate subproblems: Visit Schedules Subproblem (VSS) and Vehicle Routing Subproblem (VRS).

6.2.1 Visit Schedules Subproblem (VSS)

The visit schedules subproblem includes the collection quantity constraints, the requirements & inventory balance constraints, the vehicle capacity constraint. These constraints are the same constraints mention in Chapter 5.

In the VSS the main decision is to determine which customers will be visited in which day by satisfying the requirement from manufacturer and the vehicle capacity limitation. The objective function for the Assignment subproblem in Equation (6.9).

Objective Function for Visit Schedules Subproblem (VSS):

$$Min \;\; h\sum_{t=1}^{T} I_t + p\sum_{t=1}^{T} S_t + \sum_{i=0}^{N}\sum_{t=1}^{T} \lambda_{it} Y_{it} + \sum_{j=0}^{N}\sum_{t=1}^{T} \mu_{jt} Y_{jt} \tag{6.9}$$

6.2.2 Vehicle Routing Subproblem (VRS)

In the vehicle routing subproblem the decision for the model is to determine daily vehicle schedule. The objective function of the VRS is given in equation (6.10) and the constraints are only the subtour elimination constraints.

Objective Function for Vehicle Routing Subproblem (VRS):

$$Min \;\; \sum_{i=0}^{N} \sum_{j=0,\ (j\neq i)}^{N} \sum_{t=1}^{T} X_{ijt}(c * d_{ij} - \lambda_{it} - \mu_{jt}) \tag{6.10}$$

Subtour Elimination Constraints

We test several well-known alternative approaches from the Asymmetric Traveling Salesmen Problem (ATSP) literature.

In the vehicle routing subproblem constraints shown in equation (6.16) are subtour elimination constraints that prevents subtours in each time unit over the planning horizon.

Subtour elimination constraints stated in Equations (6.11) gives one of the strongest formulations but it contains an exponential number of constraints. [107] Therefore solving large instances models is getting impossible and complex.

$$\sum_{i \in Q} \sum_{j \in Q} X_{ijt} \leq \sum_{i \in Q} \sum_{j} X_{ijt} - 1,\ Q \subseteq V \setminus \{0\},\ \|Q| \geq 2 \tag{6.11}$$

The MTZ Subtour Elimination constraints stated in Equations (6.12-6.13) proposed by Miller et al. [108] and has polinomial number of equations to eliminate the subtours.

$$\begin{aligned} & u_{it} - u_{jt} + (n-1)X_{ijt} \leq (n-2), \\ & \forall i \in IC,\ \ (i \neq j),\ \forall j \in IC,\ \forall t \in T \end{aligned} \tag{6.12}$$

$$\begin{aligned} & 1 \leq u_{it} \leq n \\ & ,\ \forall i \in I,\ \forall j \in I,\ \forall t \in T \end{aligned} \tag{6.13}$$

The MTZ Subtour elimination constraints stated in Equations (6.14-6.15) are strengthened by Laporte et al. [109].

$$\begin{aligned} & u_{it} - u_{jt} + (n-1)X_{ijt} + (n-3)X_{jit} \leq (n-2) \\ & ,\ \forall i \in IC,\ \ (i \neq j),\ \forall j \in IC,\ \forall t \in T \end{aligned} \tag{6.14}$$

$$\begin{aligned} & 1 + (n-3)X_{i0t} + \sum_{j \in IC, j \neq i} X_{jit} \leq u_{it} \leq (n-1) - (n-3)X_{0it} - \sum_{j \in IC, j \neq i} X_{ijt} \\ & ,\ \forall i \in I,\ \forall j \in I,\ \forall t \in T \end{aligned} \tag{6.15}$$

According to a comparative analysis by Öncan et al. [103] Gavish and Graves (GG) constraints given in Equations (6.16-6.17) are better than MTZ constraints. According to our experiment we obtain best results with GG, therefore in our models we use Gavish and Graves Flow Conservation constraints as the subtour elimination constraints.

$$\sum_{j \in I,\ (j \neq i)} F_{jit} - \sum_{j \in I,\ (j \neq i)} F_{ijt} = \sum_{j \in I,\ (j \neq i)} X_{ijt},\ \forall i \in IC,\ \forall t \in T \tag{6.16}$$

$$0 \leq F_{ijt} \leq nX_{ijt},\ \forall i \in I,\ (i \neq j),\ \forall j \in IC,\ \forall t \in T \tag{6.17}$$

$$X_{ijt} \in \{0, 1\},\ \forall i \in I,\ \forall j \in I,\ \forall t \in T,\ \ i \neq j \tag{6.18}$$

6.2.3 A Lagrangian Based Heuristic (LBH)

Although the solution of the relaxed problem gives us a lower bound for the original problem, these solutions are not necessarily feasible solutions for the original problem. To speed up finding feasible solutions in the Lagrangian relaxation model we propose a Lagrangian base heuristic method. This method uses the results of the visit schedules subproblem as inputs and performs traveling salesman problem (TSP) for customers which are defined in the visit schedules subproblem. According to these routing results which are found in the Lagrangian Based heuristic and the assignment result that are found in the visit schedules subproblem, the objective function of the original problem calculated and this value gives us upper bounds for the original problem.

6.3 Computational Results

We propose a Lagrangian Relaxation approach for the solution of single vehicle problems. The relaxed model decomposes into two mixed integer programming models that optimize the visit schedule and the collection route in each period separately. We test the performance of this solution approach on six scenarios and compare the lower bounds obtained by the Lagrangian relaxation method to the ones obtained by solving the proposed MILP model within a pre-specified time limit.

Results of the Lagrangian Relaxation method are presented in Table (6.1), Table (6.2), and Table (6.3). According to the results, lower bounds generated by Lagrangian relaxation method are better than the ones obtained solving the MILP models by Cplex.

Lagrangian relaxation method improves the lower bounds of all models. When we solved the Model 1 with Cplex as MILP, we end up with lower bound is equal to 821.22 on average where lower bound obtained after executing Lagrangian relaxation method is equal to 823.52 on average. These improvement can also be seen for Model 2 from 807.00 to 812.95 and Model 3 from 812.87 to 814.59 on average.

As expected, improvement on lower bounds resulted in improvement on Gaps. Optimality gap is 6.63% for the Model 1 and Lagrangian relaxation method decreases the gap to 6.37%. The gap is also decreases in Model 2 from 8.00% to 7.32% and in Model 3 from 7.66% to 7.45%. Highest improvement on optimality gap is in Model 2 where the smallest gap is in Model 1.

To sum up, in our experiments Lagrangian relaxation method has minor improvements on lower bounds and therefore optimality gaps but has no affect on upper bounds. The optimality gaps for the models improves from 7.43% to 7.05% on average.

Table 6.1: Computational results of Model 1 with the Lagrangian Relaxation method.

MODEL 1		**UB**	**LB**	**LB**	**Gap**	**Gap**
Scenario #	**Scenario Name**		**with Cplex**	**with Lagrangian LB**	**with Cplex**	**with Lagrangian LB**
1	Dob-60acc-LOW-050	837.83	770.64	**772.69**	8.02%	7.77%
2	Dob-60acc-MED-050	847.19	795.54	**799.45**	6.10%	5.64%
3	Dob-60acc-HIGH-050	983.78	927.15	**929.62**	5.76%	5.51%
4	Dob-60acc-LOW-025	837.59	**772.57**	770.92	7.76%	7.96%
5	Dob-60acc-MED-025	846.91	794.49	**798.60**	6.19%	5.70%
6	Dob-60acc-HIGH-025	922.01	866.92	**869.86**	5.98%	5.66%
	Average	**879.22**	**821.22**	**823.52**	**6.63%**	**6.37%**

Table 6.2: Computational results of Model 2 with the Lagrangian Relaxation method.

MODEL 2		**UB**	**LB**	**LB**	**Gap**	**Gap**
Scenario #	**Scenario Name**		**with Cplex**	**with Lagrangian LB**	**with Cplex**	**with Lagrangian LB**
1	Dob-60acc-LOW-050	837.24	758.31	**769.57**	9.43%	8.08%
2	Dob-60acc-MED-050	846.87	**781.65**	778.71	7.70%	8.05%
3	Dob-60acc-HIGH-050	975.73	915.81	**919.61**	6.14%	5.75%
4	Dob-60acc-LOW-025	836.14	763.98	**772.38**	8.63%	7.63%
5	Dob-60acc-MED-025	844.38	776.67	**777.58**	8.02%	7.91%
6	Dob-60acc-HIGH-025	919.90	845.58	**859.85**	8.08%	6.53%
	Average	**876.71**	**807.00**	**812.95**	**8.00%**	**7.32%**

Table 6.3: Computational results of Model 3 with the Lagrangian Relaxation method.

MODEL 3		**UB**	**LB**	**LB**	**Gap**	**Gap**
Scenario #	**Scenario Name**		**with Cplex**	**with Lagrangian LB**	**with Cplex**	**with Lagrangian LB**
1	Dob-60acc-LOW-050	831.71	754.16	**762.19**	9.32%	8.36%
2	Dob-60acc-MED-050	857.51	776.62	**777.70**	9.43%	9.31%
3	Dob-60acc-HIGH-050	986.64	915.72	**918.17**	7.19%	6.94%
4	Dob-60acc-LOW-025	832.50	765.34	**768.34**	8.07%	7.71%
5	Dob-60acc-MED-025	843.79	**795.86**	792.83	5.68%	6.04%
6	Dob-60acc-HIGH-025	927.41	**869.53**	868.31	6.24%	6.37%
	Average	**879.93**	**812.87**	**814.59**	**7.66%**	**7.45%**

Chapter 7

CONCLUSIONS

In this thesis we studied the waste vegetable oil collection problem for a real life biodiesel production facility. We developed a mixed integer linear program to model the customer selection and periodic routing problem considering the production requirements at the production facility. For this reverse logistics problem, we decide on which of the present source points to include in the collection program, which periodic routing schedule to repeat over an infinite horizon, how much virgin oil to purchase on each day and how many vehicles to operate such that the total collection, inventory and purchasing costs are minimized while the production requirements and operational constraints are met. The novelty of this model is that it can handle all possible visit schedules for all source nodes without introducing an exponential number of binary decision variables.

In this study, we solve problem instances of size $n = 25$ within 3.28% of optimality on average, however the solutions obtained for larger-sized SPIRP instances result in much higher optimality gaps. Thus, heuristics will be required in such cases.

We compare alternative formulations and test them on six scenarios to optimize the visiting schedule and observe that our first proposed MILP model yields the best solutions. Since the gap between upper and lower bounds has the minimum values in Model 1 on the average.

We propose a Lagrangian Relaxation approach for the solution of single vehicle problems. The relaxed model decomposes into two mixed integer programming models that optimize the visit schedule and the collection route in each period separately. We test the performance of this solution approach on six scenarios and compare the lower bounds obtained by the Lagrangian relaxation method to the ones obtained by solving the proposed MILP model within a pre-specified time limit. Results concluded with slight improvements on the gap. In our experiments, Lagrangian relaxation method has minor improvements on lower bounds and therefore optimality gaps but has no affect on upper bounds. The optimality gaps for the models improves from 7.43% to 7.05% on average. Therefore, Lagrangian relaxation does not seem to be an effective solution approach.

Future work on the SPIRP problem could focus on strengthening the lower bounds further by deriving some more valid inequalities. in addition for the solution of larger instances in shorter time, a metaheuristic procedure could be developed.

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VITA

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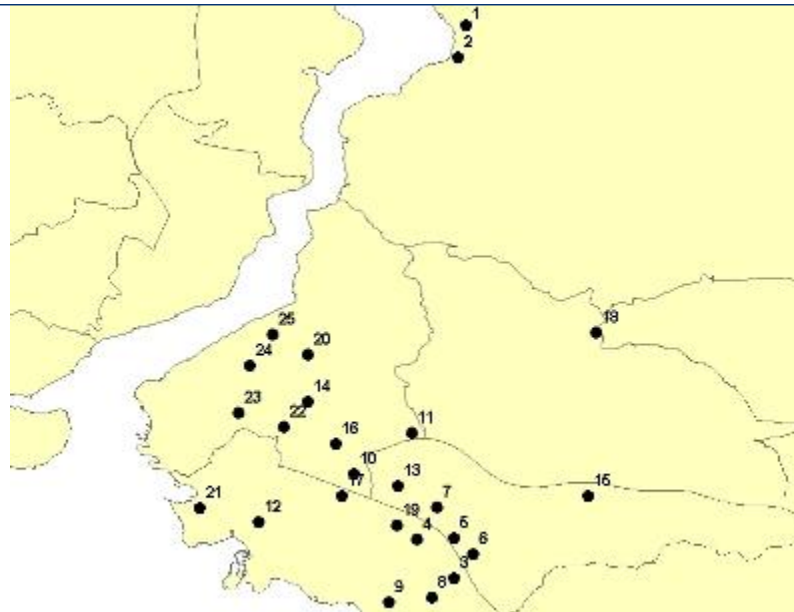
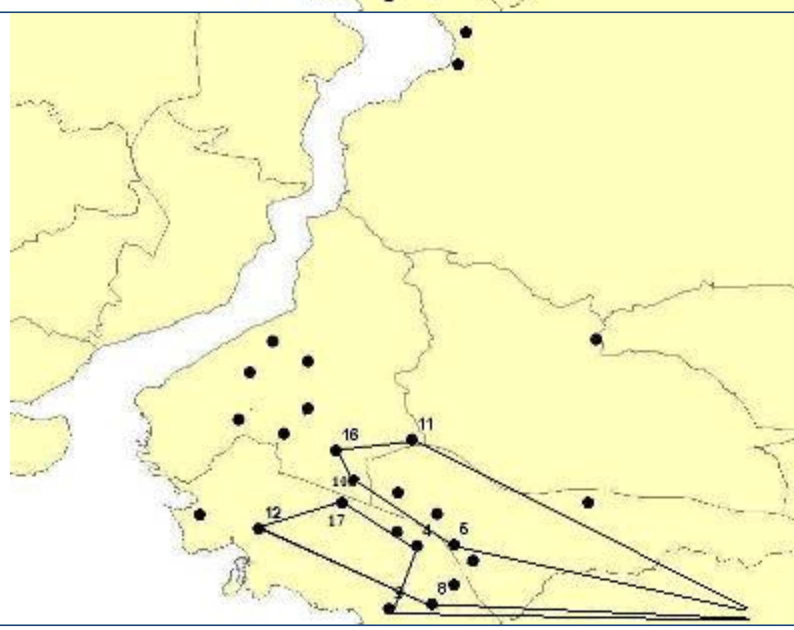
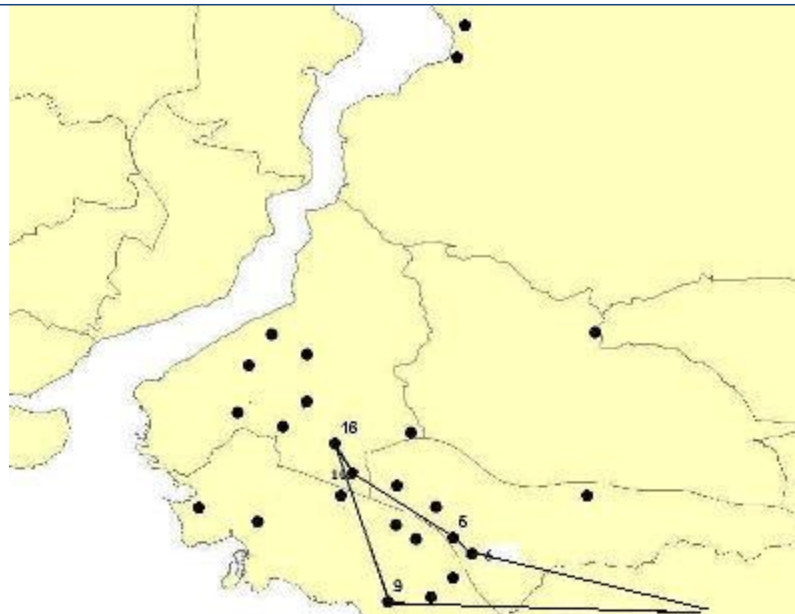
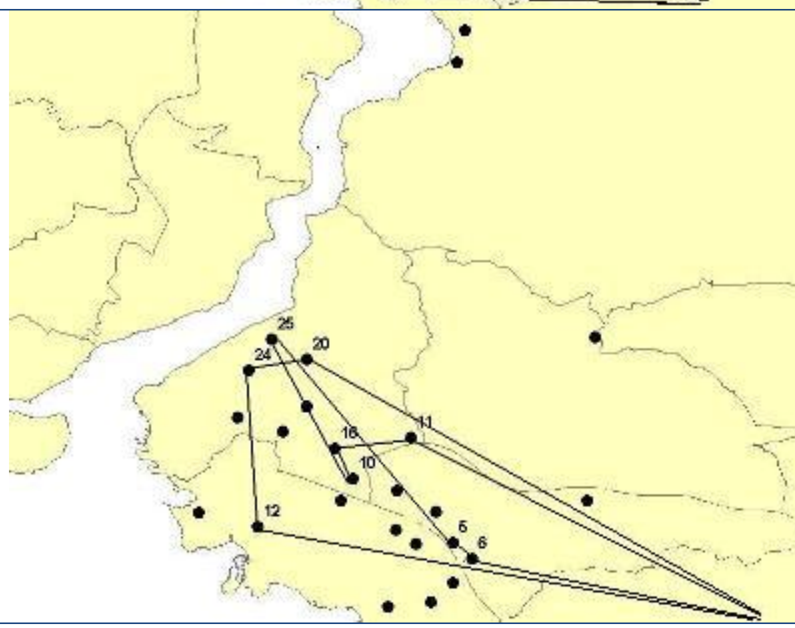
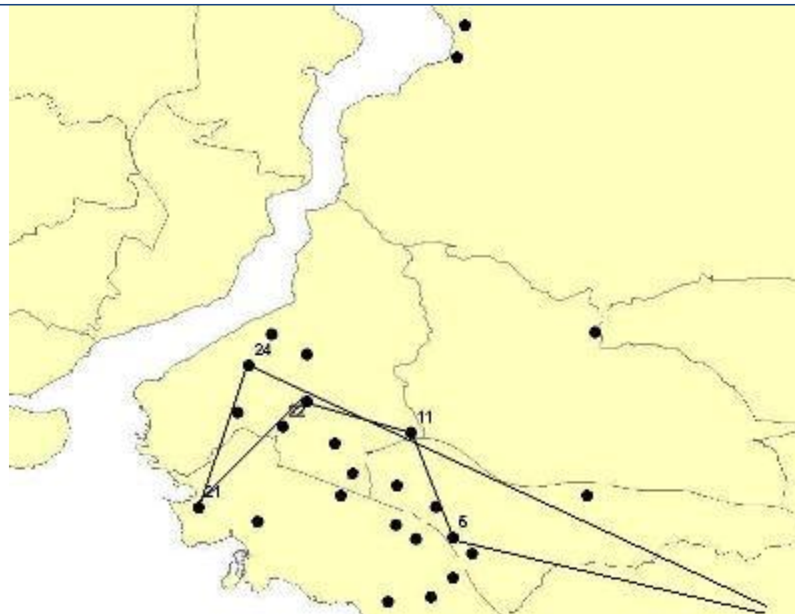
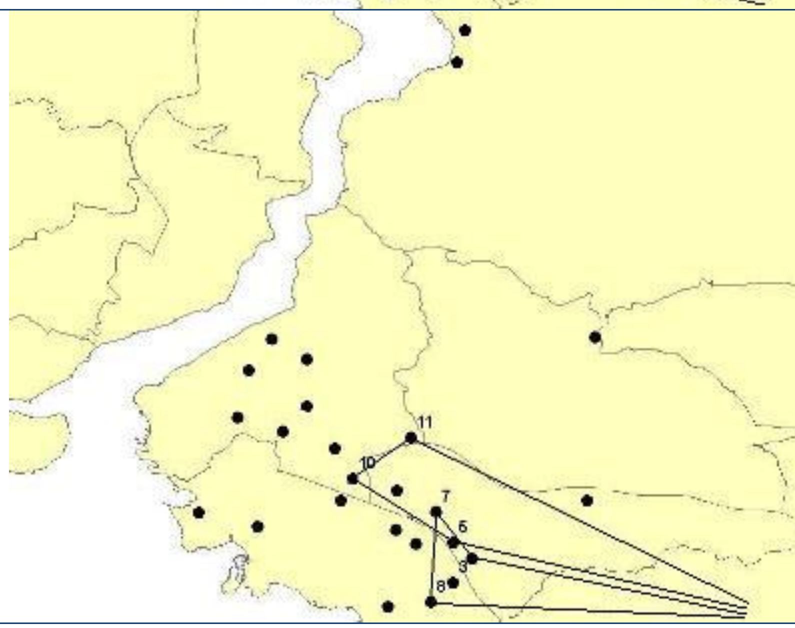
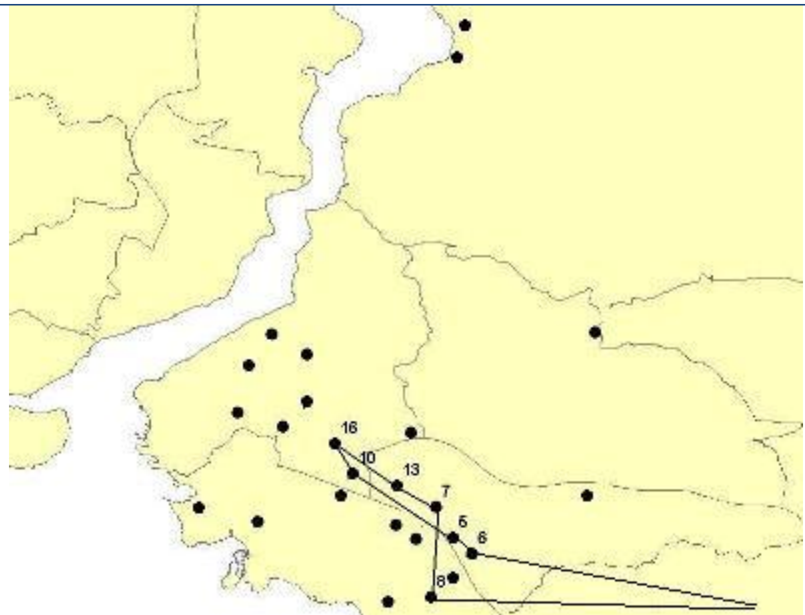
APPENDIX

Appendix 1. Illustration of the daily vehicle routes in the best integer feasible solution for the scenario Fio-30-Med-050

Locations of the source nodes	
Vehicle Routes at Day 1	

Vehicle Routes at Day 2	
Vehicle Routes at Day 3	

Vehicle Routes at Day 4	
Vehicle Routes at Day 5	

Vehicle Routes at Day 6	
Vehicle Routes at Day 7