Money and Output in a Heterogeneous Cash-in-Advance Economy

by

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Abstract

This thesis formulates a cash-in-advance model with two agents differing in terms of productivity levels, time-preferences and availability of credit production technology. The heterogeneity of the agents allows them to engage in trade in the factor and the goods markets, and a monetary equilibrium is achieved for deflation rates within a range determined by agents' time-preferences. We examine how the changes in the monetary transfers made by the government to the agents under various degrees of cash-constraints affect the equilibrium. We show that the equilibrium levels are only affected by the changes in the monetary transfers made to the less productive agent. Both the direction and the size of this effect are shown to depend on the degrees of cash-constraints. Additionally, the analysis yields different results depending on the range from which monetary transfer levels are chosen.

Keywords: Cash constraint, cash-in-advance, inflation, monetary transfer.

Özet

Bu tezde, birbirlerinden üretkenlik düzeyi, zaman tercihi ve kredi üretim teknolojisinin bulunurluğu açılarından farklılık gösteren iki ajanın yer aldığı bir peşin para modeli formüle edilmiştir. Ajanların heterojenliği, faktör ve mal piyasalarında birbirleriyle alışverişte bulunmalarına olanak sağlamaktadır. Ajanların zaman tercihleri tarafından belirlenen bir aralıktaki deflasyon oranları için parasal denge kurulmuştur. Hükümet tarafından ajanlara yapılan parasal aktarımların çeşitli para kısıtı dereceleri altında dengeyi ne şekilde etkilediği incelenmiştir. Denge düzeylerinin sadece daha az üretken olan ajana yapılan parasal aktarımlardan etkilendiği; bu etkinin hem yönünün, hem de büyüklüğünün para kısıtlarının derecesine bağlı olduğu gösterilmiştir. Ek olarak, parasal aktarım düzeylerinin seçildiği aralığa bağlı olarak, yapılan analizin sonuçları farklılıklar göstermektedir.

Anahtar Kelimeler: Enflasyon, para kısıtı, parasal transfer, peşin para.

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Abbreviations

MIU	**M**oney **i**n the **U**tility
CIA	**C**ash **i**n **A**dvance
OLG	**O**verlapping **G**enerations
IES	**I**ntertemporal **E**lasticity of **S**ubstitution

Chapter 1

Introduction

Economic models are abstractions, and hence they might exclude important aspects of the real economy. How to incorporate money, an important element of our daily lives and economic activities, into economic models has been an important topic for economists. The number and extent of the studies within macroeconomics which deal with monetary phenomena have led to a subfield named "monetary economics".

Monetary economists have mainly made use of four different kinds of models in order to examine the role and effects of money. Sidrauski (1967) formulated "money-in-the-utility function" (as known as "MIU function" or "Sidrauski") model, in which an agent would derive utility not only from the consumption she makes, but also from the real money she holds. By utilizing MIU function models, Fischer (1983) studied the government regulation of banks, Sargent and Wallace (1981) investigated the coordination of monetary and fiscal policy, and Wallace (1983) examined inferior rates of return on government supplied currencies. Despite the growing popularity of attributing utility to real money balances in order to incorporate money into economic models, money did not play any of its traditional roles such as being medium of exchange, store of value or unit of account in these models.

Clower (1967) proposed an alternative method addressing the microfoundations of a monetary economy. Unlike traditional non-monetary models, which allow agents in the economy to trade with each other "quid pro quo" (in-kind payments for previous purchases made after production takes place), he introduced money as the medium of exchange. As in the real life, an agent who would like to make a purchase would pay the necessary amount with money, hence her transactions were subject to financial constraints determined by the amount of money she held. Such constraints were named “cash-in-advance” (CIA) or “Clower” constraints, and such models are referred as “cash-in-advance” (CIA) models as the agents in the economy are required to make their purchases of factors of production in cash before the production takes place. Lucas (1980, 1982) and Stockman (1981) pioneered the use of CIA models for understanding the monetary phenomena. Cooley and Hansen (1989) initiated a vein of studies which made use of such models for quantitative analysis. A shortcoming of this class of models is the fact that they are prone to get complicated by the inclusion of additional cash-in-advance constraints as additional transactions are incorporated into the models.

An alternative environment for studying monetary economics stemmed from an article written by Samuelson (1956). Unlike prevalent micro-founded macro models, instead of laboring infinitely-lived agents, Samuelson modeled finitely-lived economic agents, and allowed agents at different ages to live at the same time. Due to the coexistence of different generations, such models were given the name “Overlapping Generations” (OLG) models. In Samuelson’s economy, young and middle-aged agents were working, but they were not able to carry goods over into their retirement years. By the introduction of money into the system, young and middle-aged agents were able to convert their products into money, which they could convert back into goods in their retirement years. As Jevons (1875) stated, trade between agents is a “double-coincidence of wants problem”. Since the desire of younger generations for selling goods and gaining money coincides with older generations’ desire for giving money away in return for obtaining goods, intergenerational trade, which would not be possible without money, normally takes place in the OLG models.

Although the roots of monetary OLG models can be traced back to Samuelson (1956), it was mostly Lucas (1972) and Wallace (1980) who pioneered elaborate examination of them. Lucas (1972) studied the systemic relation between the rate of change in nominal prices and the level of real output in a business cycle context. Wallace (1980), on the other hand, investigated fiat-money related topics such as the efficiency of fiat and commodity money systems, fiat money financed deficits, and country-specific fiat money. One criticism which has arisen for OLG models is that money basically serves as the store-of-value in such models and is modeled in a way in which its medium-of-exchange property is clouded by its store-of-value property.

The last method used by economists in order to model monetary economies borrows from the search theory, which was popularized by labor economists, and focuses on randomly meeting agents. Once again, if "double-coincidence of wants" is present, economic activity takes place. For monetary economies, economic activity corresponds to selling/purchasing of goods in return for money. Similar to cash-in-advance models, search-theoretic approach emphasizes the medium-of-exchange role of money. Kiyotaki and Wright (1989, 1993) worked on such search-theoretic models, examined the equilibria in which fiat currency circulates as the general medium-of-exchange, and showed the welfare improvements of introducing fiat currency into a commodity money economy. The basic search-theoretic model of money was extended by Trejos and Wright (1995) and by Shi (1995) to allow for divisible commodities. Further studies were conducted by Molico (1997) to allow for divisible money and by Williamson and Wright (1994) to examine the effects of informational frictions.

Quantity theory of money tells us that inflation is a monetary phenomenon, and a change in money supply would be reflected in inflation. Therefore, once a monetary economy is modeled, a natural question for an economist to ask is what the effects of inflation on output and growth are. Many empirical studies have been conducted in order to answer this question, but the evidence found is rather ambiguous. On the one hand, Gillman et al. (2004) conducting an international panel data study and Fountas et al. (2006) working with international G7 time series found that inflation has a negative effect on output. On the other hand, Bruno and Easterly (1998) working on a 5-year average or annual data across countries could not find a robust relationship. Furthermore, working with developed countries, Mankiw (1989) found out a positive relationship at cyclical frequencies. In the same strand of literature, Boyd and Smith (1998) pointed to a problem that having less developed financial systems and less opportunities for obtaining credits, developing countries are more vulnerable to the effects of inflation. Additionally, since the degrees of liquidity constraints are affected by credits, Boyd and Smith (1998) claim that not only the existence of credits in the economy, but also the difference between firms and consumers in terms of obtaining credits play a role on determining the effects of inflation.

As for theoretical modeling, among the four aforementioned classes of monetary economies, the focus of this paper will be on cash-in-advance models. As stated previously, cash-in-advance models emphasize the most important role of money, serving as a medium of exchange, and provide a convenient environment for adding new features into the economy in order to make it more realistic and rich enough to study various problems.

One of the pioneering works in this literature, Stockman (1981), modeled the economy with equal CIA constraints for both consumption and investment, and reached the conclusion that there is a negative long-run relationship between output and inflation as advocated by Friedman (1977). This result opposes the results of an earlier CIA study done by Lucas (1980), in which with CIA constraint being only applied to consumption, the long-run neutrality of inflation was shown. Another study assuming CIA constraints both on consumption and investment, Abel (1985), further found out that as a result of this double implementation of CIA constraints, money is effective on the speed of adjustment between steady states as well.

Both Stockman (1981) and Abel (1985) assumed that the degree of CIA constraints on investment relative consumption equals to 'one'. However in reality, one of these activities might have tighter liquidity constraints than the other. Wang and Yip (1992), Palivos et al. (1993) and Chang and Tsai (2003) worked with models in which the degree of CIA constraints on investment relative consumption is less than 'one'. However, Lu et al. (2011) advocates that the degree of CIA constraints on investment relative consumption might also be greater than 'one'. Especially, the availability of credit production, and the specifics of who can attain credit, play a role in the determination of relative tightness of liquidity constraints of different activities.

The approach taken by Lu et al. (2011), namely allowing the degree of CIA constraints on investment relative to consumption to be greater than 'one' and endogenizing the degree of CIA constraint on consumption by incorporating a credit production function into the model, leads them to unorthodox results. By examining the effects of a permanent rise in the growth rate of money, they show that depending on whether the degree of CIA constraints on investment relative consumption is greater or less than the unity, the relationship between inflation and output might change direction.

According to Lu et al. (2011), the increase in the growth rate of money affects the output through three channels. Firstly, due to the increase in inflation, the agent of consideration will increase the production of credit, which will relax the CIA constraint on consumption. As a result, real balances available for investment increases, hence the capital stock increases (positive capital stock effect). Large (small) degrees of CIA constraint on investment result in larger (smaller) increases in the capital stock. Secondly, increasing demand for credit causes more capital to be allocated to credit production (negative capital reallocation effect). Lastly, labor displays an ambiguous behavior. Since it complements capital, large degrees of CIA constraint on investment result in increases in the labor supply as well as the capital stock in the goods sector. On the other hand, small degrees of CIA constraint on investment might decrease labor supply in goods sector as more capital is allocated to credit production and capital stock allocated to goods production decreases. Therefore, a large degree of CIA constraint on investment will create a large positive capital stock effect, and an accompanying positive labor supply effect. Additionally, a large negative capital reallocation effect on output will be observed. The final result on output depends on the interaction of these three channels.

The positive capital stock effect and labor supply effect on output are dominated by the negative capital reallocation effect when the degree of CIA constraints on investment is smaller than the degree of CIA constraints on consumption, resulting in lower output due to the increase in money growth rate. On the other hand, if the degree of CIA constraints on investment is greater than the degree of CIA constraints on consumption, the positive capital stock effect and positive labor supply effect dominate the negative capital reallocation effect, resulting in higher output due to the increase in money growth rate.

As modern macroeconomics is based on micro foundations, in all these CIA models, the functioning of the economy is studied through examining the behaviors of homogeneous households represented by a single agent. Fuerst (1992) described these homogeneous households as entities formed by household members engaging in different economic activities, such as consumption, investment or labor supply. Yet, this homogeneity assumption is an unrealistic one, and it is possible to obtain a more realistic setup by incorporating heterogeneity into the models. In such a route, Başçı and Sağlam (2005) study the optimal money growth policy in a cash-in-advance setting in which two agents differing in patience levels and know-how interact with each other. In their model, the only factor of production is labor. Since the agents are heterogeneous in terms of know-how, hence in terms of productivity, it is more profitable for both agents if less productive agent sells its labor to the more productive agent, and buys goods from her at the end of the production process instead of engaging in goods production by herself. In equilibrium, all the money in the economy is held by the more productive type. As a result of their study, Başçı and Sağlam (2005) find that some of the conventional results in monetary economics are subject to restrictions when agents in the economy are heterogeneous.

In this paper, we build a new environment to study an old topic, namely the role of monetary transfers, hence inflation, on output and on other key elements of the economy. The model to be constructed is a monetary model with cash-in-advance constraints, and includes crucial elements of Başçı and Sağlam (2005) and Lu et al. (2011). We follow the distinctive approach of Başçı and Sağlam (2005) by making use of two heterogeneous agents, instead of a single representative agent, in order to obtain a representative economy. In addition to the labor and the goods markets in Başçı and Sağlam (2005), our model also accommodates a capital market. Due to their heterogeneity in productivity, agents engage in trade in these three markets: First in the capital market, then in the labor market, and lastly in the goods market. Başçı and Sağlam (2003), utilizing a similar environment with the exception of the capital market, showed that the sequencing of markets affects the equilibrium prices and allocations. Therefore, our choice of sequencing of markets are also likely to have an effect on our results. After setting up our model of a two-agent cash-constrained economy, we examine the effect of monetary transfers on the economy for varying degrees of cash-constraints as Lu et al. (2011) does in a single representative agent economy context. In Lu et al. (2011), the monetary transfer to the representative agent is the sole determinant of the inflation level. In our model, on the other hand, there are two agents, and the two separate levels of monetary transfers made to these agents together determine the inflation level.

Following chapters will first describe the model economy, and then will state the agents' problems and equilibrium conditions. Since the model contains monetary terms, and these terms follow the behavior of inflation rather than staying constant in the equilibrium, we need to transform nominal variables into real variables by detrending them in order to obtain a steady-state equilibrium. After getting the conditions for a steady-state equilibrium, we will analyze the features of the model. However, the nature of the economy we study makes analytical examination of the equilibrium quite cumbersome. Nonetheless, numerical analysis techniques allow us to find numerical answers to our questions. After reporting the results of numerical analyses, we conclude the paper by highlighting our main findings and discussing topics which might be interesting for future work.

Chapter 2

The Model

2.1 The Environment

The economy we model involves two inifinitely-lived agents indexed by the subscript $i = 1, 2$. Time is indexed by $t = 0, 1, ...$ There are two factors of production, labor and capital. Produced goods can be either consumed or invested in capital stock. In addition to consumption, both agents also value leisure. Therefore, agents try to maximize their life-time discounted utilities represented by $\sum_{t=0}^{\infty} \beta_i^t u_i(c_{i,t}, x_{i,t})$ where $\beta_i \in (0, 1)$ is the discount factor, $c_{i,t}$ is period-t consumption, $x_{i,t}$ is period-t leisure, and $u_i(c_{i,t}, x_{i,t})$ is the instantaneous utility function of each type i agent. The utility is assumed to be strictly increasing and strictly concave in consumption and leisure; i.e., $\partial^2 u_i(c_i, x_i)/\partial c_i^2 < 0 < \partial u_i(c_i, x_i)/\partial c_i$ and $\partial^2 u_i(c_i, x_i)/\partial x_i^2 < 0 < \partial u_i(c_i, x_i)/\partial x_i$ for all $c_i, x_i > 0$. In order to simplify the analysis, we will assume the following form for the utility function in which utility is assumed to be separable in consumption and leisure: $u_i(c_{i,t}, x_{i,t}) = (c_{i,t}^{1-\sigma} - 1)/(1 - \sigma) + \chi(x_{i,t}^{1-\varepsilon} - 1)/(1 - \varepsilon)$ where $1/\sigma$ is intertemporal elasticity of substitution (IES) for consumption, $1/\varepsilon$ is the IES for leisure, and χ is the coefficient serving to differentiate the utilities derived from leisure and consumption.

Each agent is endowed with an initial level of capital, $k_{i,0}$, and with one unit of time. While both agents have goods production technology and spend the fraction l of their time on goods production, unlike agent 2, agent 1 also has credit production technology and spends the fraction n of her time on credit production. The remaining parts of their time endowment, $x_{1,t} = 1 - l_{1,t} - n_{1,t}$ and $x_{2,t} = 1 - l_{2,t}$, denote the fraction of time spent on leisure. In addition to deciding how to allocate labor between different activities, agent 1 should also decide how to allocate her capital as she produces in two sectors. Let us denote the capital used by agent i in goods production as $k_{y_{i,t}}$, and the capital used in credit production as $k_{d_{i,t}}$.

The nature of the two-agent model allows the agents to trade factors of production among each other. $k^{\tau}_{y_{i,t}}$ and $l^{\tau}_{i,t}$ denote the amount of capital and labor traded respectively by agent i at time t. A negative number indicates that agent i is a seller whereas a positive number indicates that she is a buyer of the respective factor of production. Therefore, the amounts of capital and labor which will be used for each agent's goods production are determined only after the trade in the capital and the labor markets takes place.

The goods production function, $y_{i,t} = f(k_{y_{i,t}}, k^{\tau}_{y_{i,t}}, l_{i,t}, l^{\tau}_{i,t})$, is strictly increasing and strictly concave in its arguments. In order to simplify our analysis, we will assume that the production function is of the following Cobb-Douglas form $f(k_{y_{i,t}}, k^{\tau}_{y_{i,t}}, l_{i,t}, l^{\tau}_{i,t}) = A_i(k_{y_{i,t}} + k^{\tau}_{y_{i,t}})^{\alpha_1}(l_{i,t} + l^{\tau}_{i,t})^{\alpha_2}$, where $A_i > 0$ and $0 < \alpha_1, \alpha_2 < 1$.

Once the production takes place, the agents can also trade goods, and determine the amount of goods they have at the end of the period. The amount of goods traded are denoted by $q_{i,t}$, and as before, a negative number means that agent i is a seller whereas a positive number indicates that she is a buyer of goods. After the goods trade takes place and final amount of goods each agent holds is determined, the agents decide how to split the goods they have between consumption and investment.

All three markets in which trade takes place, namely, labor, capital and goods markets, are subject to cash-in-advance constraints. Φ_l and Φ_k denote the degree of CIA constraints in the labor and the capital markets, respectively. When a purchase is made in the labor (capital) market, the fraction Φ_l (Φ_k) of the purchase must be paid in money whereas the remaining fraction $1 - \Phi_l$ ($1 - \Phi_k$) is paid in-kind after the goods production takes place. On the other hand, the degree of CIA constraint on the goods market, Φ_q, is endogenously determined in the system due to the existence of credit production technology. Similar to the labor and the capital markets, Φ_q, denotes the fraction of a purchase made in the goods market, which must be paid in money. However, $1 - \Phi_q$ of the purchase is paid in credit. We can therefore define credit as the fraction of good purchases which is not paid in money; i.e., $d_t \equiv (1 - \Phi_{q_{1,t}})q_{1,t}$. We model the credit production function as Gillman and Kejak (2005, 2009) suggest:

$$d_t = q_{1,t} B \left(k_{d_{1,t}}/q_{1,t}\right)^{\gamma_1} (n_{1,t}/q_{1,t})^{\gamma_2}, \;\; B > 0, \;\; 0 < \gamma_1, \gamma_2 < 1, \;\; \gamma_1 + \gamma_2 < 1.$$

The credit production function exhibits constant returns to scale in its three factors, capital and labor used for credit production and the amount of goods traded. Due to the condition $\gamma_1 + \gamma_2 < 1$, the marginal cost of the credit supply per unit of traded good increases, meaning that credit production becomes more costly as more goods are traded between agents. Combining $d_t \equiv (1 - \Phi_{q_{1,t}})q_{1,t}$ and $d_t = q_{1,t} B \left(k_{d_{1,t}}/q_{1,t}\right)^{\gamma_1} (n_{1,t}/q_{1,t})^{\gamma_2}$ results in $\Phi_{q_{1,t}} = 1 - B \left(k_{d_{1,t}}/q_{1,t}\right)^{\gamma_1} (n_{1,t}/q_{1,t})^{\gamma_2}$, which means that endogenously determined degree of CIA constraint on the goods market is increasing in the amount of goods traded while decreasing in capital and labor used for credit production.

2.2 Money and Government

$M_{1,t}$ and $M_{2,t}$ denote the amount of money held by agent 1 and agent 2 at the beginning of period t. The total amount of money in the economy is equal to the sum of the amounts held by agents, $M_t = M_{1,t} + M_{2,t}$. The total money stock evolves, for all $t \geq 0$, according to

$$M_{t+1} = (1 + \xi)M_t, \quad \text{with } \xi > -1,$$

which is fully anticipated by both type of agents in the economy. The money growth targeted by the government is achieved by lump-sum monetary transfers to the agents. If a negative money growth rate is targeted, then a money tax is imposed on agents. For the sake of simplicity, we will use the term "monetary transfer", but there will be times in which we actually refer to "money tax" as it will be the agent transferring her money to the government rather than the opposite. $\tau_1 M_t$ and $\tau_2 M_t$ denote the amount of monetary transfers at period t made to agent 1 and agent 2, respectively, and the sum of these transfers is equal to the change in the total money supply, ξM_t, at this period. Therefore, the evolution of money stock depends on two components, the amount of transfers to agent 1 and agent 2, as expressed by the equation $\tau_1 + \tau_2 = \xi$.

In our model, we will study a special case in which both agents are subject to monetary transfers during the period, but as a result of agents' transactions, all money is held by agent 2 at the end of the period. In consistence with this setting, initial money level of agent 1 is set equal to $M_{1,1} = 0$, and initial money level of agent 2 is what determines the initial money stock in the economy, $M_{2,1} = M_1$.

2.3 Transactions

As the environment, and the evolution of money in this environment is specified, we can describe the transactions the agents engage in.

- Endowed with certain amounts of money and capital and one unit of time, agents first decide on the amount of time they will allocate to leisure, $x_{i,t}$.
- Agent 1 decides on the amounts of capital and labor that she will spending on credit production, $k_{d_{1,t}}$ and $n_{1,t}$ respectively. Here, $k_{d_{1,t}}$ is defined as a fraction of the total capital agent 1 holds, $k_{d_{1,t}} = s_{1,t}k_{1,t}$. Therefore, in order to determine the amount of capital that will be used in the credit sector, agent 1 decides on the value of the fraction s. As agent 1 uses the rest of the capital she holds on goods production, the amount of capital which will be used on goods production is once again defined by making use of the fraction s and agent 1's total capital k, as $k_{y_{1,t}} = (1 - s_{1,t})k_{1,t}$. However, unlike the amounts of capital and labor which will be used on credit production, the amount of credit to be produced is not yet determined since the credit amount also depends on the amount of goods traded.
- Agent 2 does not have the technology to produce credit. Therefore, all the capital she holds is used in goods production. The one unit of time she is endowed with is split between goods production and leisure. As the amount of time spent on leisure is determined, the amount of time which will be spent on goods production is determined, too.
- Before the markets for factors of productions open, the government makes the monetary transfer to agent 2. Afterwards, the capital market opens, and trade in the capital market takes place. This is followed by the monetary transfer made

to agent 1, and the opening of the labor market. Both the capital and the labor markets are subject to CIA constraints. Therefore, $k^{\tau}_{y_{i,t}}$ denoting the amount of capital traded for goods production purposes, a monetary amount of $\Phi_k k^{\tau}_{y_{i,t}}$ is paid by the capital buyer to capital seller. The remaining part of $(1-\Phi_k)k^{\tau}_{y_{i,t}}$ is paid in-kind to the seller by the buyer after the production takes place. Similarly, $l^{\tau}_{i,t}$ denoting the amount of labor traded for goods production purposes, a monetary amount of $\Phi_l l^{\tau}_{i,t}$ is paid by the labor buyer to labor seller. Once again, the remaining part of $(1-\Phi_l)l^{\tau}_{i,t}$ is paid in-kind to the seller by the buyer after the production takes place.

- Once the trade takes place in factor markets, the amount of factors of production each agent will be using, $k_{y_{i,t}}+k^{\tau}_{y_{i,t}}$ and $l_{i,t}+l^{\tau}_{i,t}$, are determined. Goods production takes place, and is followed by opening of the goods market. As the amount of goods which will be traded, $q_{i,t}$, is determined, the amount of credit, $d_{i,t}$, and the degree of CIA constraint on the goods market, $\Phi_{q_{i,t}}$, are also determined. The monetary amount of $\Phi_{q_{i,t}}q_{i,t}$ is paid by agent 1 to agent 2. The remaining part, $(1-\Phi_{q_{i,t}})q_{i,t}$ is paid to agent 2 by the credit produced by agent 1.

- Having made production and traded goods, the final amounts of goods held by the agents at the end of period t are determined. Finally, agents decide on how much of the final goods at hand to consume, $c_{i,t}$, and how much of it to invest in capital stock, $I_{i,t}$, in order to make use of it in future production.

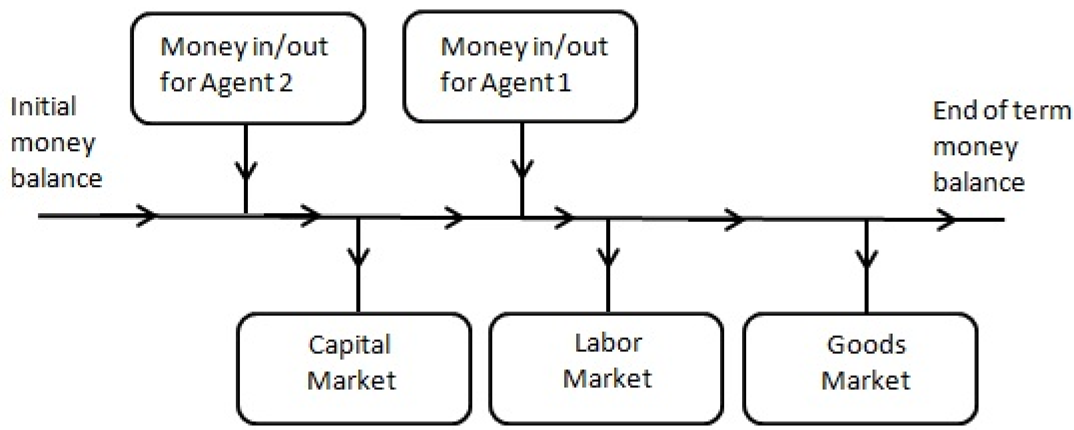

Fig. 1. The sequence of transactions in each period

2.4 Agents' Problems

Based on the described environment and the transactions that take place in this environment, agent 1's problem is as follows:

$$\max \sum_{t=0}^{\infty} \beta_1^t (\frac{c_{1,t}^{1-\sigma} - 1}{1-\sigma} + \chi \frac{x_{1,t}^{1-\varepsilon} - 1}{1-\varepsilon}) \tag{2.1}$$

subject to

$$c_{1,t} + I_{1,t} = A_1 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2} + q_{1,t} - (1 - \Phi_l) l_{1,t}^{\tau} \frac{w_t}{p_t} - (1 - \Phi_k) k_{y_{1,t}}^{\tau} \tag{2.2}$$

$$k_{1,t+1} = (1-\delta) k_{1,t} + I_{1,t} \tag{2.3}$$

$$k_{d_{1,t}} = s_{1,t} k_{1,t} \tag{2.4}$$

$$k_{y_{1,t}} = (1 - s_{1,t}) k_{1,t} \tag{2.5}$$

$$0 \leq s_{1,t} \leq 1 \tag{2.6}$$

$$x_{1,t} = 1 - l_{1,t} - n_{1,t} \tag{2.7}$$

$$- k_{y_{1,t}} \leq k_{y_{1,t}}^{\tau} \leq \frac{M_{1,t}}{\Phi_k p_t} \tag{2.8}$$

$$- l_{1,t} \leq l_{1,t}^{\tau} \leq \frac{M_{1,t} + \tau_1 M_t - k_{y_{1,t}}^{\tau} \Phi_k p_t}{\Phi_l w_t} \tag{2.9}$$

$$0 \leq q_{1,t} \leq \frac{M_{1,t} + \tau_1 M_t - l_{1,t}^{\tau} \Phi_l w_t - k_{y_{1,t}}^{\tau} \Phi_k p_t}{\Phi_{q_{1,t}} p_t} \tag{2.10}$$

$$M_{1,t} - k_{y_{1,t}}^{\tau} \Phi_k p_t + \tau_1 M_t \geq 0 \tag{2.11}$$

$$M_{1,t+1} = M_{1,t} + \tau_1 M_t - l_{1,t}^{\tau} \Phi_l w_t - k_{y_{1,t}}^{\tau} \Phi_k p_t - q_{1,t} \Phi_{q_{1,t}} p_t \tag{2.12}$$

$$M_{1,0} \geq 0 \tag{2.13}$$

$$k_{1,0} \geq 0 \tag{2.14}$$

$$\Phi_{q_{1,t}} = 1 - B \left(\frac{k_{d_{1,t}}}{q_{1,t}} \right)^{\gamma_1} \left(\frac{n_{1,t}}{q_{1,t}} \right)^{\gamma_2} \tag{2.15}$$

Term (1) contains agent 1's maximized life-time utility function. Equation (2) is agent 1's budget constraint in terms of real goods. Equation (3) is the law of motion for capital stock. Equation (4) and equation (5) describe how agent 1 allocates her capital stock between credit and goods sectors by using the choice variable s. Inequality (6) denotes that the choice variable should be a positive number less than 1 as it stands for the fraction of total capital used in the credit sector. Equation (7) denotes that agent 1 has one unit of time, which can be split between leisure and goods or credit production. Inequality (8) denotes that agent 1 cannot sell more capital than she allocated to goods production nor she can buy more capital than her monetary budget allows. Similarly, inequality (9) denotes that agent 1 cannot sell more labor than she allocated to goods production nor she can buy more labor than her after-capital-market monetary budget allows. As we built our model in a way that agent 1 is the buyer in the goods market, inequality (10) denotes that the amount of goods agent 1 can buy is restricted by her monetary budget. Inequality (11) denotes that agent 1 might be subject to money taxes, but not to the extent in which her monetary balance falls to a negative level. Equation (12) is agent 1's monetary budget constraint. Inequalities (14) and (15) indicate that initial money and capital levels of agent 1 should be positive. Finally, equation (15) shows how the degree of cash constraint on the goods market is determined for agent 1.

Similarly, agent 2's problem is as follows:

$$\max \sum_{t=0}^{\infty} \beta_2^t (u_2(c_{2,t}, x_{2,t}) = \frac{c_{2,t}^{1-\sigma} - 1}{1-\sigma} + \chi \frac{x_{2,t}^{1-\varepsilon} - 1}{1-\varepsilon}) \tag{2.16}$$

subject to

$$c_{2,t} + I_{2,t} = A_2 (k_{y_{2,t}} + k_{y_{2,t}}^{\tau})^{\alpha_1} (l_{2,t} + l_{2,t}^{\tau})^{\alpha_2} + \Phi_{q_{2,t}} q_{2,t} - (1-\Phi_l) l_{2,t}^{\tau} \frac{w_t}{p_t} - (1-\Phi_k) k_{y_{2,t}}^{\tau} \tag{2.17}$$

$$k_{2,t+1} = (1-\delta) k_{2,t} + I_{2,t} \tag{2.18}$$

$$k_{y_{2,t}} = k_{2,t} \tag{2.19}$$

$$x_{2,t} = 1 - l_{2,t} \tag{2.20}$$

$$-k_{y_{2,t}} \leq k_{y_{2,t}}^{\tau} \leq \frac{M_{2,t} + \tau_2 M_t}{\Phi_k p_t} \tag{2.21}$$

$$-l_{2,t} \leq l_{2,t}^{\tau} \leq \frac{M_{2,t} + \tau_2 M_t - k_{y_{2,t}}^{\tau} \Phi_k p_t}{\Phi_l w_t} \tag{2.22}$$

$$0 \leq -q_{2,t} \leq f(k_{y_{2,t}}, k^{\tau}_{y_{2,t}}, l_{2,t}, l^{\tau}_{2,t}) - (1-\Phi_l)\frac{w_t}{p_t} l^{\tau}_{2,t} - (1-\Phi_k) k^{\tau}_{y_{2,t}} \tag{2.23}$$

$$M_{2,t} + \tau_2 M_t \geq 0 \tag{2.24}$$

$$M_{2,t+1} = M_{2,t} + \tau_2 M_t - l^{\tau}_{2,t}\Phi_l w_t - k^{\tau}_{y_{2,t}}\Phi_k p_t - q_{2,t}\Phi_{q_{2,t}} p_t \tag{2.25}$$

$$M_{2,0} \geq 0 \tag{2.26}$$

$$k_{2,0} \geq 0 \tag{2.27}$$

$$\Phi_{q_{2,t}} = 1 - B\left(\frac{k_{d_{1,t}}}{-q_{2,t}}\right)^{\gamma_1}\left(\frac{n_{1,t}}{-q_{2,t}}\right)^{\gamma_2} \tag{2.28}$$

Term (16) contains agent 2's maximized life-time utility function. Equation (17) is agent 2's budget constraint in terms of real goods. Equation (18) is the law of motion for capital stock. Since agent 2 only produces goods, she uses all her capital stock in goods production, and equation (19) indicates that. Equation (20) denotes that agent 2 has one unit of time, which can be split between leisure and goods production. Inequalities (21), (22) and (24) correspond to inequalities (8), (9) and (11) in agent 1's problem, and describe the respective restrictions on the amounts of capital and labor that agent 2 can buy or sell and on the amount of money tax agent 2 can be subject to. As we built our model in a way that agent 2 is the seller in the goods market, inequality (23) denotes that the amount of goods agent 1 can sell is restricted by her production and her transactions in the goods and labor markets. Equation (25) is agent 2's monetary budget constraint. Inequalities (26) and (27) indicate that initial money and capital levels of agent 2 should be positive. Lastly, equation (28) shows how the degree of cash constraint on the goods market is determined for agent 2.

Chapter 3

Equilibrium

3.1 Equilibrium Conditions

Equilibrium is obtained when both agents solve their optimization problems and markets clear at the same time. We have already mentioned that we will focus on the equilibrium in which agent 1 does not hold any money, $M_{1,t+1} = 0$, and agent 2 holds all the money, $M_{2,t+1} = M_{t+1}$ at the end of the period. The derivation of the agents' optimization conditions are provided in the Appendix A. Appendix B examines the conditions for obtaining a monetary equilibrium as desired, and hence complements Appendix A. The optimization conditions for agent 1 are as follows:

$$\chi x_{1,t}^{-\varepsilon} = c_{1,t}^{-\sigma} A_1 \alpha_2 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2 - 1} \tag{3.1}$$

$$c_{1,t}^{-\sigma} A_1 \alpha_1 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1 - 1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2} = \chi x_{1,t}^{-\varepsilon} \frac{\gamma_1}{\gamma_2} \left(\frac{n_{1,t}}{k_{d_{1,t}}} \right) \tag{3.2}$$

$$\begin{aligned} c_{1,t}^{-\sigma} & \left(A_1 \alpha_2 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2 - 1} - (1 - \Phi_l) \frac{w_t}{p_t} \right) \\ & \times \left(p_t B \left(\frac{k_{d_{1,t}}}{q_{1,t}} \right)^{\gamma_1} \gamma_2 \left(\frac{n_{1,t}}{q_{1,t}} \right)^{\gamma_2 - 1} \right) = \chi x_{1,t}^{-\varepsilon} \Phi_l w_t \end{aligned} \tag{3.3}$$

$$c_{1,t}^{-\sigma}\left(A_1\alpha_1(k_{y_{1,t}}+k_{y_{1,t}}^{\tau})^{\alpha_1-1}(l_{1,t}+l_{1,t}^{\tau})^{\alpha_2}-(1-\Phi_k)\right) \times\left(B\left(\frac{k_{d_{1,t}}}{q_{1,t}}\right)^{\gamma_1}\gamma_2\left(\frac{n_{1,t}}{q_{1,t}}\right)^{\gamma_2-1}\right)=\chi x_{1,t}^{-\varepsilon}\Phi_k \tag{3.4}$$

$$c_{1,t}^{-\sigma}\left(B\left(\frac{k_{d_{1,t}}}{q_{1,t}}\right)^{\gamma_1}\gamma_2\left(\frac{n_{1,t}}{q_{1,t}}\right)^{\gamma_2-1}\right)=\chi x_{1,t}^{-\varepsilon}\left(\Phi_{q_{1,t}}+B\left(\gamma_1+\gamma_2\right)\left(\frac{k_{d_{1,t}}}{q_{1,t}}\right)^{\gamma_1}\left(\frac{n_{1,t}}{q_{1,t}}\right)^{\gamma_2}\right) \tag{3.5}$$

$$\frac{c_{1,t}^{-\sigma}}{\beta_1}=c_{1,t+1}^{-\sigma}\left(A_1\alpha_1(k_{y_{1,t+1}}+k_{y_{1,t+1}}^{\tau})^{\alpha_1-1}(l_{1,t+1}+l_{1,t+1}^{\tau})^{\alpha_2}(1-s_{1,t+1})\right) +\chi x_{1,t+1}^{-\varepsilon}\left(\frac{\gamma_1}{\gamma_2}\left(\frac{n_{1,t+1}}{k_{d_{1,t+1}}}\right)s_{1,t+1}\right) \tag{3.6}$$

The optimization conditions for agent 2 are as follows:

$$\chi x_{2,t}^{-\varepsilon}=c_{2,t}^{-\sigma}A_2\alpha_2(k_{y_{2,t}}+k_{y_{2,t}}^{\tau})^{\alpha_1}(l_{2,t}+l_{2,t}^{\tau})^{\alpha_2-1} \tag{3.7}$$

$$A_2\alpha_2(k_{y_{2,t}}+k_{y_{2,t}}^{\tau})^{\alpha_1}(l_{2,t}+l_{2,t}^{\tau})^{\alpha_2-1}=\frac{w_t}{p_t} \tag{3.8}$$

$$A_2\alpha_1(k_{y_{2,t}}+k_{y_{2,t}}^{\tau})^{\alpha_1-1}(l_{2,t}+l_{2,t}^{\tau})^{\alpha_2}=1 \tag{3.9}$$

Note that agent 2 does not make any investment in the given setting, resulting in having zero capital at the end of each period in equilibrium (See Appendix A). Therefore, the capital she needs for production purposes can only be obtained from agent 1 through trade.

All three markets, namely the capital, labor and goods markets, should clear at the equilibrium. These conditions are given by the following equations:

$$l_{1,t}^{\tau}+l_{2,t}^{\tau}=0$$
$$k_{y_{1,t}}^{\tau}+k_{y_{2,t}}^{\tau}=0$$
$$q_{1,t}+q_{2,t}=0$$

Walras' Law tells us that if $n-1$ markets clear, then the n^{th} market also clears. Therefore, it is enough to include the following two equations into the set of equations describing the equilibrium.

$$q_{1,t} + q_{2,t} = 0 \tag{3.10}$$

$$k^{\tau}_{y_{1,t}} + k^{\tau}_{y_{2,t}} = 0 \tag{3.11}$$

Appendix C shows how the labor market clearing condition is derived through other equilibrium conditions.

3.2 Detrending

Our model contains nominal variables which grow with the inflation. In order to have an equilibrium at a steady state, we need to detrend nominal variables so that all variables are constant in the equilibrium. Following Juillard (2009), we define the ratio of end of period money stock to beginning of period money stock as $g_t \equiv M_{t+1}/M_t$, which means that $g_t \equiv 1 + \xi_t$. This expression can be written as $g \equiv 1 + \xi$ in case of having a constant rate of inflation. In order to make a nominal variable X_t stationary, we can divide it to M_t. Here, $\widehat{X}_t$ being the stationary version of the nominal variable X_t, we can express the nominal variable as $X_t = \widehat{X}_t M_t$.

The nominal variables which need to be detrended in our model are: $M_{1,t}, M_{2,t}, w_t, p_t, M_{1,t+1},$ and $M_{2,t+1}$. We can replace them with the following terms on the right hand side in order to have equilibrium conditions with stationary variables:

$$M_{1,t} = \widehat{M}_{1,t} M_t$$
$$M_{2,t} = \widehat{M}_{2,t} M_t$$
$$w_t = \widehat{w}_t M_t$$
$$p_t = \widehat{p}_t M_t$$
$$M_{1,t+1} = \widehat{M}_{1,t+1} M_{t+1}$$
$$M_{2,t+1} = \widehat{M}_{2,t+1} M_{t+1}$$

As a result, the equilibrium at the steady state should be characterized by equations (3)-(5), (7), (15), (18)-(20), (28), (29)-(30), (32)-(35), (37)-(39), and the following equations from (40) to (46):

$$c_{1,t} + I_{1,t} = A_1(k_{y_{1,t}} + k^{\tau}_{y_{1,t}})^{\alpha_1}(l_{1,t} + l^{\tau}_{1,t})^{\alpha_2} + q_{1,t} - (1-\Phi_l)l^{\tau}_{1,t}\frac{\widehat{w}_t}{\widehat{p}_t} - (1-\Phi_k)k^{\tau}_{y_{1,t}} \quad (3.12)$$

$$\widehat{M}_{1,t+1}g = \widehat{M}_{1,t} + \tau_1 - l^{\tau}_{1,t}\Phi_l\widehat{w}_t - k^{\tau}_{y_{1,t}}\Phi_k\widehat{p}_t - q_{1,t}\Phi_{q_{1,t}}\widehat{p}_t \quad (3.13)$$

$$\widehat{M}_{1,t+1} = 0 \quad (3.14)$$

$$c^{-\sigma}_{1,t}\left(A_1\alpha_2(k_{y_{1,t}} + k^{\tau}_{y_{1,t}})^{\alpha_1}(l_{1,t} + l^{\tau}_{1,t})^{\alpha_2-1} - (1-\Phi_l)\frac{\widehat{w}_t}{\widehat{p}_t}\right) = \chi x^{-\varepsilon}_{1,t}\left(\widehat{p}_t B\left(\frac{k_{d_{1,t}}}{q_{1,t}}\right)^{\gamma_1}\gamma_2\left(\frac{n_{1,t}}{q_{1,t}}\right)^{\gamma_2-1}\right)^{-1}\Phi_l\widehat{w}_t \quad (3.15)$$

$$c_{2,t} + I_{2,t} = A_2(k_{y_{2,t}} + k^{\tau}_{y_{2,t}})^{\alpha_1}(l_{2,t} + l^{\tau}_{2,t})^{\alpha_2} + \Phi_{q_{2,t}}q_{2,t} - (1-\Phi_l)l^{\tau}_{2,t}\frac{\widehat{w}_t}{\widehat{p}_t} - (1-\Phi_k)k^{\tau}_{y_{2,t}} \quad (3.16)$$

$$\widehat{M}_{2,t+1}g_t = \widehat{M}_{2,t} + \tau_2 - l^{\tau}_{2,t}\Phi_l\widehat{w}_t - k^{\tau}_{y_{2,t}}\Phi_k\widehat{p}_t - q_{2,t}\Phi_{q_{2,t}}\widehat{p}_t \quad (3.17)$$

$$A_2\alpha_2(k_{y_{2,t}} + k^{\tau}_{y_{2,t}})^{\alpha_1}(l_{2,t} + l^{\tau}_{2,t})^{\alpha_2-1} = \frac{\widehat{w}_t}{\widehat{p}_t} \quad (3.18)$$

Once we determine the theoretical conditions for a stationary equilibrium, we can make numerical analysis by setting numerical values to the parameters and simulating the model on the computer.

Chapter 4

Numerical Analysis

4.1 Parameter Values

As Blanchard and Fischer (1993) indicated, models based on cash-in-advance constraints quickly become cumbersome for tracking analytically. However, we can examine these models numerically and develop insights about them. Numerical examination requires appointing numbers to parameters in models. This is done based on theoretical assumptions of the model, previous calibration work made on the pertaining parameters, and examining the reactions of the variables to the appointed parameter values.

The model we are examining assumes that the agents differ with respect to their productivity in goods production. Therefore, we set $A_1 = 0.5$ and $A_2 = 5$ since agent 1 is treated as the less productive agent in the model. We also assume that the goods production technology of agents exhibits decreasing returns to scale, which means $\alpha_1 + \alpha_2 < 1$. Accordingly, we set $\alpha_1 = 0.4$ and $\alpha_2 = 0.4$.

Unlike Lu et al. (2011), which sets $B = 1.5457$, our model requires high productivity for the credit production technology. Therefore, we assign $B = 4$. The remaining parameters belonging to the credit production, namely γ_1 and γ_2, are assigned similar values to those in Lu et al. (2011), 0.25 and 1 respectively. This is also the case for the risk aversion, which is set at $\sigma = 2$. We set the risk aversion parameter for leisure as $\varepsilon = 2.25$, a number close to the risk aversion parameter for consumption.

The model used by Lu et al. (2011) requires them to assign a value for χ, favoring leisure greatly over consumption. We set $\chi = 0.7$ such that the agent favors consumption over leisure albeit not with a dramatic difference. The depreciation parameter δ is set to be equal to 0.1, and the time discount ratio for agents 1 and 2 (β_1 and β_2) are set to be equal to 0.91 and 0.98 respectively in accordance with the model's ordering of agents' time discount parameters.

The net inflation in the economy, ξ, has two components as already mentioned, the government transfer ratios to agent 1, τ_1, and agent 2, τ_2. The ratio belonging to second agent does not appear in equilibrium conditions, and hence does not directly affect the equilibrium outcomes. Whereas, τ_1 directly shows up in the equations describing the equilibrium. Hence, τ_1 will be one of the key parameters while conducting our study as we will vary its value in order to see the effects of this variation on the economy. Meanwhile, as shown in Appendix B, τ_2 plays an important role in order to sustain a monetary equilibrium as desired, and hence, its value should be adjusted as we change τ_1.

The other key parameters of our study are Φ_k and Φ_k. As we said, we will examine the effects of varying τ_1 on the equilibrium outcomes, but we will do this under different environments, namely under different cash-in-advance constraints for capital, Φ_k, and for labor, Φ_l.

4.2 Results

The model we built provides quite a rich environment which enables us to run several numerical exercises. We examined the impact of the variation in τ_1 on other variables for five cases. The variables whose behavior we examined are the degree of cash constraint on the goods market (Φ_q), total output (y), agent 1's output (y_1), agent 2's output (y_2), the amount of time agent 1 spends on leisure (x_1), the amount of time agent 2 spends on leisure (x_2), the amount of goods traded (q_1), the amount of labor traded (l_1^τ), the amount of capital traded ($k_{y_2}^\tau$), and the division of capital between sectors for agent 1 (s_1).

Since we would like to examine the effects of cash-in-advance constraints on the relationship between τ_1 and the aforementioned variables, we repeated the same exercise for five different values of Φ_k and Φ_l. We started by setting Φ_k and Φ_l to be equal to 0.1, and continued by setting them to be equal to 0.2, 0.3, 0.4, and lastly to 0.5. The results corresponding to these cases are reported under case 1 through case 5 in Appendix D.

Although their model is quite different than ours, while examining the effects of changing τ_1, Lu et al. (2011) sets 0.05 as the initial value of τ_1, and then increases it to 0.055. In our analysis, we did not focus on a one particular change between two τ_1 values, such as 0.05 and 0.055. Instead we covered a wide range of τ_1 values between 0.055 and 0.100, which allowed us to compare economy's reactions to the changes in the rate of monetary transfers made to the less productive agent when the rate is at the lower or upper end of this range.

The model contains three markets in which trade occurs. All of them are subject to cash-in-advance constraints, but only the degree of CIA constraint on the goods market, Φ_q, is endogenously determined. Table 1 in Appendix D allows us to track the behavior of Φ_q as τ_1 increases for five cases. For lower Φ_k and Φ_l values such as 0.1 and 0.2, higher τ_1 values lead to higher Φ_q values. For $\Phi_k = \Phi_l = 0.3$, the effect of τ_1 on Φ_q is less robust. For $\Phi_k = \Phi_l = 0.4$, changes between lower τ_1 values do not affect Φ_q, whereas in the upper end of the τ_1 values, Φ_q actually decreases. Once again, for $\Phi_k = \Phi_l = 0.5$, the effects of τ_1 on Φ_q become less clear. In the lower end, Φ_q exhibits some concave behavior, and in the upper end, it increases with τ_1.

Arguably the most noteworthy variable of the economy is the total output. The results of the numerical exercises for the total output are reported in Table 2 of Appendix D. Except for case 2 in which $\Phi_k = \Phi_l = 0.2$ and case 4 in which $\Phi_k = \Phi_l = 0.4$, the total output seems to initially rise with τ_1, making a peak and then starting to decline with higher values of τ_1. In case 2, the output fluctuates more than other cases, and in case 4, it does not react to the changes in τ_1 until the higher values of τ_1. With higher values of τ_1, we see that a positive relationship between τ_1 and the total output is established. Note that for a given τ_1 value, the increase in Φ_k and Φ_l values almost always causes the total output to increase.

Although we have not identified the mechanism lying behind the relationship between the monetary transfer made to the less productive agent and the total output, as in Lu et al. (2011), this relationship is likely to be determined by the interaction of various effects. Monetary transfers made to the less productive agent increases agent's monetary holdings. Due to that rise, there is a positive effect on the amounts of capital and labor the less productive agent can buy. However, depending on the relative degrees of cash constraints on the factor and the goods markets, the less productive agent might choose to allocate more factors on credit production, which would adversely affect goods production. Therefore, the relative degrees of cash constraints on the factor and the goods markets determine whether "the positive capital and labor stock effect" or "the negative capital and labor allocation effect" prevails.

In addition to the changes in the total output, changes in individual agents' outputs might be of interest. Table 3 of Appendix D reports the effects of τ_1 on agent 1's output, y_1. Agent 1's output has a pattern similar to total output's. Except for case 4, y_1 increases with the rise in τ_1, but after a certain point, further increasing τ_1 causes y_1 to decrease. In case 4, y_1 seems to be staying constant or even decreasing with the increase in τ_1, but as we get closer to the highest values of τ_1, y_1 increases. Not surprisingly, as the increase in Φ_k and Φ_l values causes the total output to increase for a given τ_1 value, it also causes agent 1's output to increase in general.

Agent 2's output, reported on Table 4 of Appendix D, has a very similar pattern to agent 1's. Namely, except for case 4, it increases with τ_1 initially, and starts to decrease with τ_1 after a certain level. In case 4, it preserves the flat shape for a while, and begins to rise with τ_1 only for the upper end values of τ_1 range. However, unlike agent 1's output (and total output), the effect of increasing Φ_k and Φ_l values on agent 2's output for a given τ_1 value is less clear.

Tables 5 and 6 of Appendix D report how the amount of time allocated to leisure by agents 1 and 2 respectively react to the changes in τ_1 for the five cases. Contrary to our expectations, the amount of time agent 1 allocates to leisure behaves similar to agent 1's output. Except for case 4, the amount of time allocated to leisure by agent 1 increases with τ_1 initially, but after a certain level it starts to decrease. In case 4, similar to agent 1's output behavior, it remains flat until the upper end of τ_1 range where it starts to increase with τ_1. On the other hand, the amount of time allocated to leisure by agent 2 shows little variation. Its value is already low, and τ_1 seems not to have any dramatic impact on it.

Tables 7, 8 and 9 of Appendix D focus on the markets in which trade takes place. From Table 7, we can see that the initial increase in τ_1 generally results in an increase in the amount of goods traded. However, further advancing the value of τ_1 causes the amount of goods traded to decrease after a certain level of transfer payment to agent 1. As usual, our variable of interest, q_1, exhibits a different pattern in cases 2 and 4. In case 2, q_1 fluctuates widely, which makes it difficult to diagnose the pattern it follows. In case 4, it remains almost constant until the upper end of τ_1 range and from thereafter, it starts to increase with τ_1. The relation between Φ_k and Φ_l values, and q_1 for a given τ_1 value is not robust. However, for lower τ_1 values, q_1 initially decreases, then starts to increase in general as Φ_k and Φ_l values increase. Whereas for higher τ_1 values, Φ_k and Φ_l have an adverse effect on q_1.

From Table 8, we see that the amount of labor traded generally decreases with τ_1 until a certain level, and afterwards it starts to increase. An exception is case 4, in which the amount of labor remains more or less constant initially, and starts to decrease towards the upper end of τ_1 range.

The amount of capital traded, whose behavior is reported in Table 9, increases with τ_1 initially, and starts to decrease with τ_1 after a certain level in case 1, 3 and 5. Similar to other variables, the fluctuations in the amount of capital traded in case 2 prevent us to have a robust view on the impact of τ_1 on $k^{\tau}_{y_2}$. As usual, in case 4, the variable of interest remains constant as τ_1 increases, and starts to increase towards the upper end of τ_1 range. Increasing Φ_k and Φ_l for a given τ_1 does not seem to have a robust effect either on l^{τ}_1 or on $k^{\tau}_{y_2}$.

Lastly, Table 9 reports the effect of τ_1 on the fraction of capital, s_1, spent on credit sector by agent 1 for the five cases. In case 1, s_1 initially decreases with τ_1, and starts to increase with τ_1 after a certain level. For cases 2, 3, and 4, the increase in τ_1 does not seem to have much of an effect on s_1. In case 5, opposite of the effect in case 1 is observed, namely s_1 initially decreases with τ_1, and starts to increase afterwards.

Increasing Φ_k and Φ_l for a given τ_1 has a clear effect on s_1. For the τ_1 values at the lower range, increasing Φ_k and Φ_l for a given τ_1 increases s_1. On the other hand, for the τ_1 values at the upper range, increasing Φ_k and Φ_l for a given τ_1 decreases it.

Chapter 5

Conclusion

We have built an economic model with cash-in-advance constraints, and studied the effects of monetary transfers in this economy. Our model differs from more traditional CIA models in terms of incorporating heterogeneity into the economy as in Başçı and Sağlam (2005) and examining the effects of monetary transfers under various rates of degrees of cash constraints instead of picking one specific degree for each constraint.

Başçı and Sağlam (2005) show that the cash-in-advance constraint on the labor market results in inefficient use of resources, and hence output distortion. This ineffiency can be eliminated by a deflation level determined by agents' patience levels as long as more productive agents are also more patient ones. We built upon this setting, and set higher discount rates for agents with higher productivity as suggested. Incorporating capital market and credit production technology into the model enrich the setting in Başçı and Sağlam (2005). Furthermore, we use the model to study the relation between money and output growth, a topic Başçı and Sağlam (2005) does not investigate.

The heterogeneity assumption in our model leads the inflation to have two components, namely the amount of monetary transfers made to the less productive agent, τ_1, and to the more productive agent, τ_2. In our model, it is the τ_1 component of inflation, which actually affects variables, including output, in equilibrium. τ_2 component, on the other hand, is important in terms of not letting agent 2 violate her CIA constraints while engaging in monetary transactions and not letting the monetary equilibrium break down. Since the particular monetary equilibrium we studied requires a deflation rate in a range determined by agents' time preferences, increasing τ_1 in order to study the consequences of this increase might cause the deflation rate to fall out of the acceptable range. Therefore, τ_2 should be used in order to balance the change in τ_1, and to keep the deflation at an acceptable level.

Lu et al. (2011) stresses the importance of the degrees of cash constraints while studying the relation between monetary transfers and output. They show that increasing the amount of monetary transfers results in higher output when the degree of cash constraint on investment is greater than the degree of cash constraint on consumption. An increase in monetary transfers decreases the total output otherwise. However, in our model, changing the degrees of CIA constraints on the capital and the labor markets seems to have little effect on the relationship between the amount of monetary transfers made to the less productive agent and variables such as output or leisure. On the other hand, the relationship between the amount of monetary transfers made to the less productive agent and variables such as the degree of cash constraint on the goods market, the amount of goods traded, the amount of capital traded or the distribution of capital among different sectors are all affected by changes in the chosen degrees of cash constraints. Therefore, we reach to the same conclusion as Lu et al. (2011) that the chosen degrees of cash constraints play a role in determining both the direction and the size of the effect of monetary transfers on other variables. Thus, studies conducted for particular degrees of cash constraints should be subject to scrutiny since their findings might alter as a result of changing the degrees of cash constraints.

Lu et al. (2011) shows the monotonous relationship between the amount of monetary transfers and other variables, which changes size and direction depending on the relative degrees of cash constraints, by increasing the amount of monetary transfers from one certain value to another. We, on the other hand, find that the range chosen for monetary transfers also affects the results. Unlike the monotonous relationship Lu et al. (2011) suggests, we observe a non-monotonous relationship between the amount of monetary transfers made to the less productive agent and other variables. For a given degree of cash constraints, increasing the monetary transfer of the less productive agent initially increases the total output, the amount of time allocated to leisure by agents, the amount of goods traded, and the amount of capital traded. Furthermore, increasing τ_1 initially decreases the amount of labor traded or the amount of capital allocated to goods sector by agent 1. However, for higher τ_1 values, these affects are all reversed. Increasing τ_1 starts to decrease the total output, the amount of time allocated to leisure by agents, the amount of goods traded, and the amount of capital traded. Similarly, the amount of labor traded and the amount of capital allocated to goods sector by agent 1 increase when already high τ_1 value further rises. Therefore, the effect of τ_1 on the variables in the economy does not only depend on the chosen degrees of cash constraints, but also on the current level of the monetary transfers made to the less productive agent.

Our findings have important policy implications. First of all, our research shows that in order to change the equilibrium outcomes, the policy makers should change the level of monetary transfers made to the less productive agents in the economy. Furthermore, the non-monotonous relationship, which we generally observe between the monetary transfer made to the less productive agent and the total output, suggests that there is an optimal level of monetary transfer for maximizing the total output. Hence, the government should target this optimal level if it aims to maximize the total production. Lastly, by changing the degrees of cash constraints on various markets, the government might prefer to change the size and the direction of the relationship between the monetary transfers and the output or other variables. Although, the degrees of cash constraints on factor markets are exogenously given and changed in our model, in reality, those degrees are determined by institutions which can be transformed by government actions. Further research can be made on the institutional determinants of the degrees of cash constraints so that the government can pursue policies which would establish a monetary transfer - output relationship as it desires.

The rich nature of the environment in which we built our model allows us to track various paths for future work. Opening sequence of factor markets was an important decision we made while modeling the economy. We can examine how changing the opening sequence of factor markets affects our results. Secondly, while studying the effects of CIA constraints and monetary transfers on equilibrium levels of variables, we ignored the transition paths of these variables and focused on long-run outcomes. Examining the transition paths of variables can be a worthwhile effort as variables' short-run behaviors might be different than their long-run behaviors. Thirdly, we studied the case in which all money in the economy is held by the more patient agent in the equilibrium. This is consistent with Becker (1978) as he showed that additively separable preferences asymptotically lead to corner solutions, and wealth of less patient consumers approaches to zero. Lucas and Stokey (1984), on the other hand, showed that if agents have increasing marginal impatience, which requires their preferences not to be additively separable, then all agents have positive wealth levels. Therefore, we can remodel the economy in a way that the agents would have increasing marginal impatience levels to examine whether this change would lead both agents to hold positive amounts of money in the equilibrium. If the monetary equilibrium changes as suggested by Lucas and Stokey (1984), this might have an effect on the findings of our analysis as well. Lastly, government has the direct control of the money stock in our economy. However, in an attempt to have a more realistic setting, we can model the economy in a way that the government would need to engage in open market operations, as in Wallace (1981), for increasing or decreasing the total money stock.

Chapter 6

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Appendix A

Agents' Optimization Problems

Both agents' functions to be optimized are subject to inequality constraints. Nonetheless, numerical examinations show that given the parameter values are set to be equal to the values specified in Section 4, those inequality constraints are not violated when the agents solve their optimization problems as if they are only subject to equality constraints. Therefore, we can make use of the method of Lagrange multipliers in order to determine the optimizing conditions to agents' problems.

The function to be optimized and its equality constraints for agent 1 lead to the following Lagrangian equation :

$$\mathcal{L}_1 = \sum_{t=0}^{\infty} \beta_1^t \left[\frac{c_{1,t}^{1-\sigma} - 1}{1-\sigma} + \chi \frac{x_{1,t}^{1-\varepsilon} - 1}{1-\varepsilon} + \right.$$
$$\lambda_t^1 \left(A_1 (k_{y_{1,t}} + k_{y_{1,t}}^\tau)^{\alpha_1} (l_{1,t} + l_{1,t}^\tau)^{\alpha_2} + q_{1,t} - (1-\Phi_l) l_{1,t}^\tau \frac{w_t}{p_t} - (1-\Phi_k) k_{y_{1,t}}^\tau - c_{1,t} - I_{1,t} \right)$$
$$\left. + \lambda_t^2 (M_{1,t} + \tau_1 M_t - l_{1,t}^\tau \Phi_l w_t - k_{y_{1,t}}^\tau \Phi_k p_t - q_{1,t} \Phi_{q_{1,t}} p_t - M_{1,t+1}) \right]$$

Agent 1 has the following variables as control variables: $n_{1,t}, l_{1,t}, s_{1,t}, l_{1,t}^\tau, k_{y_{1,t}}^\tau, q_{1,t}, c_{1,t}, I_{1,t}$ and $M_{1,t+1}$. An interior solution for a variable would require the Lagrangian function to be differentiated with respect to that variable, and to be set equal to zero. We follow this procedure for all the control variables of agent 1 except for $M_{1,t+1}$, and obtain the following equalities:

$\frac{\partial \mathcal{L}_1}{\partial n_{1,t}} = 0$:

$$\chi x_{1,t}^{-\varepsilon} = \lambda_t^2 p_t B \left(\frac{k_{d_{1,t}}}{q_{1,t}}\right)^{\gamma_1} \gamma_2 \left(\frac{n_{1,t}}{q_{1,t}}\right)^{\gamma_2 - 1} \tag{A.1}$$

$\frac{\partial \mathcal{L}_1}{\partial l_{1,t}} = 0$:

$$\chi x_{1,t}^{-\varepsilon} = \lambda_t^1 A_1 \alpha_2 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2 - 1} \tag{A.2}$$

$\frac{\partial \mathcal{L}_1}{\partial s_{1,t}} = 0$:

$$\lambda_t^1 A_1 \alpha_1 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1 - 1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2} = \lambda_t^2 p_t B \gamma_1 \left(\frac{k_{d_{1,t}}}{q_{1,t}}\right)^{\gamma_1 - 1} \left(\frac{n_{1,t}}{q_{1,t}}\right)^{\gamma_2} \tag{A.3}$$

$\frac{\partial \mathcal{L}_1}{\partial l_{1,t}^{\tau}} = 0$:

$$\lambda_t^1 \left(A_1 \alpha_2 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2 - 1} - (1 - \Phi_l) \frac{w_t}{p_t} \right) = \lambda_t^2 \Phi_l w_t \tag{A.4}$$

$\frac{\partial \mathcal{L}_1}{\partial k_{y_{1,t}}^{\tau}} = 0$:

$$\lambda_t^1 \left(A_1 \alpha_1 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1 - 1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2} - (1 - \Phi_k) \right) = \lambda_t^2 \Phi_k p_t \tag{A.5}$$

$\frac{\partial \mathcal{L}_1}{\partial q_{1,t}} = 0$:

$$\lambda_t^1 = \lambda_t^2 \left(\Phi_{q_{1,t}} p_t + p_t B \left(\frac{k_{d_{1,t}}}{q_{1,t}}\right)^{\gamma_1} (\gamma_1 + \gamma_2) \left(\frac{n_{1,t}}{q_{1,t}}\right)^{\gamma_2} \right) \tag{A.6}$$

$\frac{\partial \mathcal{L}_1}{\partial c_{1,t}} = 0$:

$$c_{1,t}^{-\sigma} = \lambda_t^1 \tag{A.7}$$

$\frac{\partial \mathcal{L}_1}{\partial I_{1,t}} = 0$:

$$\begin{aligned} \frac{\lambda_t^1}{\beta_1} = \lambda_{t+1}^1 & \left(A_1 \alpha_1 (k_{y_{1,t+1}} + k_{y_{1,t+1}}^{\tau})^{\alpha_1 - 1} (l_{1,t+1} + l_{1,t+1}^{\tau})^{\alpha_2} (1 - s_{1,t+1}) \right) \\ & + \lambda_{t+1}^2 \left(p_{t+1} B \gamma_1 \left(\frac{k_{d_{1,t+1}}}{q_{1,t+1}}\right)^{\gamma_1 - 1} \left(\frac{n_{1,t+1}}{q_{1,t+1}}\right)^{\gamma_2} s_{1,t+1} \right) \end{aligned} \tag{A.8}$$

As we previously indicated, we are interested in a specific equilibrium, in which agent 1 does not hold any money at the end of a period. This requires $\partial \mathcal{L}_1 / \partial M_{1,t+1} \leq 0$ and by differentiating the Lagrangian function with respect to $M_{1,t+1}$, we obtain the following condition:

$$-\lambda_t^2 + \beta_1 \lambda_{t+1}^2 \leq 0 \tag{A.9}$$

From the equations we obtained, we can make use of $\lambda_t^1 = c_{1,t}^{-\sigma}$ and $\lambda_t^2 = \chi x_{1,t}^{-\varepsilon} \left(p_t B \left(k_{d_{1,t}}/q_{1,t} \right)^{\gamma_1} \gamma_2 \left(n_{1,t}/q_{1,t} \right)^{\gamma_2 - 1} \right)^{-1}$ in order to eliminate Lagrange multipliers from optimizing conditions. Then, the optimizing conditions we previously found for agent 1 take the following forms:

$$\chi x_{1,t}^{-\varepsilon} = c_{1,t}^{-\sigma} A_1 \alpha_2 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2 - 1} \tag{A.10}$$

$$c_{1,t}^{-\sigma} A_1 \alpha_1 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1 - 1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2} = \chi x_{1,t}^{-\varepsilon} \frac{\gamma_1}{\gamma_2} \left(\frac{n_{1,t}}{k_{d_{1,t}}} \right) \tag{A.11}$$

$$\begin{aligned} c_{1,t}^{-\sigma} & \left(A_1 \alpha_2 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2 - 1} - (1 - \Phi_l) \frac{w_t}{p_t} \right) \\ & \times \left(p_t B \left(\frac{k_{d_{1,t}}}{q_{1,t}} \right)^{\gamma_1} \gamma_2 \left(\frac{n_{1,t}}{q_{1,t}} \right)^{\gamma_2 - 1} \right) = \chi x_{1,t}^{-\varepsilon} \Phi_l w_t \end{aligned} \tag{A.12}$$

$$\begin{aligned} c_{1,t}^{-\sigma} & \left(A_1 \alpha_1 (k_{y_{1,t}} + k_{y_{1,t}}^{\tau})^{\alpha_1 - 1} (l_{1,t} + l_{1,t}^{\tau})^{\alpha_2} - (1 - \Phi_k) \right) \\ & \times \left(B \left(\frac{k_{d_{1,t}}}{q_{1,t}} \right)^{\gamma_1} \gamma_2 \left(\frac{n_{1,t}}{q_{1,t}} \right)^{\gamma_2 - 1} \right) = \chi x_{1,t}^{-\varepsilon} \Phi_k \end{aligned} \tag{A.13}$$

$$c_{1,t}^{-\sigma} \left(B \left(\frac{k_{d_{1,t}}}{q_{1,t}} \right)^{\gamma_1} \gamma_2 \left(\frac{n_{1,t}}{q_{1,t}} \right)^{\gamma_2 - 1} \right) = \chi x_{1,t}^{-\varepsilon} \left(\Phi_{q_{1,t}} + B \left(\gamma_1 + \gamma_2 \right) \left(\frac{k_{d_{1,t}}}{q_{1,t}} \right)^{\gamma_1} \left(\frac{n_{1,t}}{q_{1,t}} \right)^{\gamma_2} \right) \tag{A.14}$$

$$\begin{aligned} \frac{c_{1,t}^{-\sigma}}{\beta_1} = c_{1,t+1}^{-\sigma} & \left(A_1 \alpha_1 (k_{y_{1,t+1}} + k_{y_{1,t+1}}^{\tau})^{\alpha_1 - 1} (l_{1,t+1} + l_{1,t+1}^{\tau})^{\alpha_2} (1 - s_{1,t+1}) \right) \\ & + \chi x_{1,t+1}^{-\varepsilon} \left(\frac{\gamma_1}{\gamma_2} \left(\frac{n_{1,t+1}}{k_{d_{1,t+1}}} \right) s_{1,t+1} \right) \end{aligned} \tag{A.15}$$

Similarly, the function to be optimized for agent 2 and the equality constraints it is subject to lead to the following Lagrangian equation:

$$\mathcal{L}_2 = \sum_{t=0}^{\infty} \beta_1^t \Bigg[\frac{c_{2,t}^{1-\sigma} - 1}{1-\sigma} + \chi \frac{x_{2,t}^{1-\varepsilon} - 1}{1-\varepsilon}$$
$$+\mu_t^1 \left(A_2 (k_{y2,t} + k_{y2,t}^{\tau})^{\alpha_1} (l_{2,t} + l_{2,t}^{\tau})^{\alpha_2} + \Phi_{q2,t} q_{2,t} - (1-\Phi_l) l_{2,t}^{\tau} \frac{w_t}{p_t} - (1-\Phi_k) k_{y2,t}^{\tau} - c_{2,t} - I_{2,t} \right)$$
$$+ \mu_t^2 (M_{2,t} + \tau_2 M_t - l_{2,t}^{\tau} \Phi_l w_t - k_{y2,t}^{\tau} \Phi_k p_t - q_{2,t} \Phi_{q2,t} p_t - M_{2,t+1}) \Bigg]$$

The variables $l_{2,t}, l_{2,t}^{\tau}, q_{2,t}, c_{2,t}, k_{y2,t}^{\tau}, I_{2,t}$ and $M_{2,t+1}$ are the control variables of agent 2. Once again, in order to obtain interior solutions, the Lagrangian function needs to be differentiated with respect to the control variables, and to be set equal to zero. We follow this procedure for agent 2's control variables except for $I_{2,t}$ and $M_{2,t+1}$, and get the following equations :

$\frac{\partial \mathcal{L}_2}{\partial l_{2,t}} = 0$:

$$\chi x_{2,t}^{-\varepsilon} = \mu_t^1 A_2 \alpha_2 (k_{y2,t} + k_{y2,t}^{\tau})^{\alpha_1} (l_{2,t} + l_{2,t}^{\tau})^{\alpha_2 - 1} \tag{A.16}$$

$\frac{\partial \mathcal{L}_2}{\partial l_{2,t}^{\tau}} = 0$:

$$\mu_t^1 \left(A_2 \alpha_2 (k_{y2,t} + k_{y2,t}^{\tau})^{\alpha_1} (l_{2,t} + l_{2,t}^{\tau})^{\alpha_2 - 1} - (1-\Phi_l) \frac{w_t}{p_t} \right) = \mu_t^2 \Phi_l w_t \tag{A.17}$$

$\frac{\partial \mathcal{L}_2}{\partial q_{2,t}} = 0$:

$$\mu_t^1 \left(\Phi_{q2,t} + B(\gamma_1 + \gamma_2) \left(\frac{k_{d_{1,t}}}{-q_{2,t}} \right)^{\gamma_1} \left(\frac{n_{1,t}}{-q_{2,t}} \right)^{\gamma_2} \right)$$
$$= \mu_t^2 \left(\Phi_{q2,t} p_t + p_t B(\gamma_1 + \gamma_2) \left(\frac{k_{d_{1,t}}}{-q_{2,t}} \right)^{\gamma_1} \left(\frac{n_{1,t}}{-q_{2,t}} \right)^{\gamma_2} \right)$$

which can be expressed as

$$\mu_t^1 = \mu_t^2 p_t \tag{A.18}$$

$\frac{\partial \mathcal{L}_2}{\partial c_{2,t}} = 0$:

$$c_{2,t}^{-\sigma} = \mu_t^1 \tag{A.19}$$

$\frac{\partial \mathcal{L}_2}{\partial k_{2,t}^{\tau}} = 0$:

$$\mu_t^1 \left(A_2 \alpha_1 (k_{y2,t} + k_{y2,t}^{\tau})^{\alpha_1 - 1} (l_{2,t} + l_{2,t}^{\tau})^{\alpha_2} - (1-\Phi_k) \right) = \mu_t^2 \Phi_k p_t \tag{A.20}$$

When we differentiate the Lagrangian function with respect to $I_{2,t}$, we will not set it equal to zero. With the help of other optimizing conditions, eventually we will show that it is less than zero:

$\frac{\partial \mathcal{L}_2}{\partial I_{2,t}}$:

$$-\mu_t^1 + \beta_2 \mu_{t+1}^1 \left(A_2 \alpha_1 (k_{y2,t+1} + k_{y2,t+1}^\tau)^{\alpha_1 - 1} (l_{2,t+1} + l_{2,t+1}^\tau)^{\alpha_2} \right) \tag{A.21}$$

Similar to agent 1's optimization problem, we need to be careful with Lagrangian function for agent 2. The fact that we are interested in the equilibrium in which agent 2 holds all the money at the end of the period requires $\partial \mathcal{L}_2 / \partial M_{2,t+1} \geq 0$, which leads to the following inequality obtained from the Lagrangian function:

$$-\mu_t^2 + \beta_2 \mu_{t+1}^2 \geq 0 \tag{A.22}$$

With the help of two equations we found, $\mu_t^1 = c_{2,t}^{-\sigma}$ and $\mu_t^2 = c_{2,t}^{-\sigma}/p_t$, we can eliminate the Lagrange multipliers from the rest of agent 2's optimizing conditions:

$$\chi x_{2,t}^{-\varepsilon} = c_{2,t}^{-\sigma} A_2 \alpha_2 (k_{y2,t} + k_{y2,t}^\tau)^{\alpha_1} (l_{2,t} + l_{2,t}^\tau)^{\alpha_2 - 1} \tag{A.23}$$

$$A_2 \alpha_2 (k_{y2,t} + k_{y2,t}^\tau)^{\alpha_1} (l_{2,t} + l_{2,t}^\tau)^{\alpha_2 - 1} = \frac{w_t}{p_t} \tag{A.24}$$

$$A_2 \alpha_1 (k_{y2,t} + k_{y2,t}^\tau)^{\alpha_1 - 1} (l_{2,t} + l_{2,t}^\tau)^{\alpha_2} = 1 \tag{A.25}$$

The term obtained by differentiating the Lagrange function with respect to the investment takes the following form:

$$-c_{2,t}^{-\sigma} + \beta_2 c_{2,t+1}^{-\sigma} \left(A_2 \alpha_1 (k_{y2,t+1} + k_{y2,t+1}^\tau)^{\alpha_1 - 1} (l_{2,t+1} + l_{2,t+1}^\tau)^{\alpha_2} \right) \tag{A.26}$$

Note that all the variables in the above equation are real variables, and in a steady-state equilibrium, we expect $A_2 \alpha_1 (k_{y2,t+1} + k_{y2,t+1}^\tau)^{\alpha_1 - 1} (l_{2,t+1} + l_{2,t+1}^\tau)^{\alpha_2}$ to be equal to $A_2 \alpha_1 (k_{y2,t} + k_{y2,t}^\tau)^{\alpha_1 - 1} (l_{2,t} + l_{2,t}^\tau)^{\alpha_2}$. But the latter term is already shown to be equal to 1. Therefore, the condition found as a result of differentiating the Lagrangian with respect to the investment boils down to

$$-c_{2,t}^{-\sigma} + \beta_2 c_{2,t+1}^{-\sigma}. \tag{A.27}$$

Once again, in a steady-state equilibrium, $c_{2,t}^{-\sigma} = c_{2,t+1}^{-\sigma}$, and we get

$$-c_{2,t}^{-\sigma} + \beta_2 c_{2,t+1}^{-\sigma} < 0, \tag{A.28}$$

as $\beta_2 < 1$. What this tells to us is that the optimal level of investment for agent 2 is zero. There is a corner solution, rather than an interior solution. Since, the law of motion for capital accumulation is given by $k_{2,t+1} = (1-\delta)k_{2,t} + I_{2,t}$, in the equilibrium, zero investment means setting the level of capital equal to zero. Therefore, in the equilibrium, agent 2 must obtain all the capital she needs from agent 1, through trade.

Appendix B

Monetary Equilibrium

We need to examine the conditions under which agent 1 does not prefer to hold any money and agent 2 prefers to hold all the money at the end of the period. From Lagrangian equations we studied in Appendix A, we know that this requires two inequalities not to be violated. Firstly, we examine the inequality belonging to agent 1:

$$-\lambda_t^2 + \beta_1 \lambda_{t+1}^2 \leq 0 \tag{B.1}$$

As we know that $\lambda_t^2 = \chi x_{1,t}^{-\varepsilon} \left(p_t B \left(k_{d_{1,t}} / q_{1,t} \right)^{\gamma_1} \gamma_2 \left(n_{1,t} / q_{1,t} \right)^{\gamma_2 - 1} \right)^{-1}$, we can plug the variable λ_t^2 into the inequality and get:

$$\begin{aligned} & - \chi x_{1,t}^{-\varepsilon} \left(p_t B \left(\frac{k_{d_{1,t}}}{q_{1,t}} \right)^{\gamma_1} \gamma_2 \left(\frac{n_{1,t}}{q_{1,t}} \right)^{\gamma_2 - 1} \right)^{-1} \\ + \beta_1 \chi x_{1,t+1}^{-\varepsilon} & \left(p_{t+1} B \left(\frac{k_{d_{1,t+1}}}{q_{1,t+1}} \right)^{\gamma_1} \gamma_2 \left(\frac{n_{1,t+1}}{q_{1,t+1}} \right)^{\gamma_2 - 1} \right)^{-1} \leq 0 \end{aligned} \tag{B.2}$$

Multiplying both sides with p_t and rearranging the terms lead to the following inequality:

$$\begin{aligned} \beta_1 \chi x_{1,t+1}^{-\varepsilon} & \left((1+\xi) B \left(\frac{k_{d_{1,t+1}}}{q_{1,t+1}} \right)^{\gamma_1} \gamma_2 \left(\frac{n_{1,t+1}}{q_{1,t+1}} \right)^{\gamma_2 - 1} \right)^{-1} \\ & \leq \chi x_{1,t}^{-\varepsilon} \left(B \left(\frac{k_{d_{1,t}}}{q_{1,t}} \right)^{\gamma_1} \gamma_2 \left(\frac{n_{1,t}}{q_{1,t}} \right)^{\gamma_2 - 1} \right)^{-1} \end{aligned} \tag{B.3}$$

At the steady state equilibrium, terms cancel out each other and we obtain:

$$\beta_1 \leq 1 + \xi \tag{B.4}$$

The second inequality which should not be violated belongs to agent 2:

$$-\mu_t^2 + \beta_2 \mu_{t+1}^2 \geq 0 \tag{B.5}$$

Agent 2's optimization problem tells that $\mu_t^2 = c_{2,t}^{-\sigma}/p_t$, and by plugging this term into the above inequality and by rearranging the terms, we obtain:

$$\beta_2 \frac{c_{2,t+1}^{-\sigma}}{p_{t+1}} \geq \frac{c_{2,t}^{-\sigma}}{p_t} \tag{B.6}$$

By multiplying both sides with p_t, we get:

$$\beta_2 c_{2,t+1}^{-\sigma} \geq c_{2,t}^{-\sigma}(1+\xi) \tag{B.7}$$

At the steady state equilibrium, terms cancel out each other and we obtain:

$$\beta_2 \geq 1 + \xi \tag{B.8}$$

By combining the inequalities we obtained from agent 1 and agent 2, we get the following deflation (as both $\beta_1 \leq 1$ and $\beta_2 \leq 1$) condition in order to have a monetary equilibrium as we described before:

$$\beta_1 \leq 1 + \xi \leq \beta_2 \tag{B.9}$$

Since $\xi \equiv \tau_1 + \tau_2$, and we conduct our analysis by varying the values of τ_1, the above inequality condition puts a restriction on the values τ_2 can take. In order the monetary equilibrium not to be destroyed, τ_2 should balance the variation in τ_1 such that the deflation level is kept within the range determined by β_1 and β_2.

Appendix C

Labor Market Clearing

In the equilibrium, not only both agents optimize, but also all markets clear. We have three markets in this model, labor, capital and goods markets, which clear if the following equations are satisfied:

$$l^{\tau}_{1,t} + l^{\tau}_{2,t} = 0$$
$$k^{\tau}_{y_{1,t}} + k^{\tau}_{y_{2,t}} = 0$$
$$q_{1,t} + q_{2,t} = 0$$

However, Walras' Law tells that if $n-1$ markets clear, then the n^{th} market would also clear. Therefore, when we write down the equations describing the equilibrium, it is enough to focus on the following two equations:

$$q_{1,t} + q_{2,t} = 0 \tag{C.1}$$

$$k^{\tau}_{y_{1,t}} + k^{\tau}_{y_{2,t}} = 0 \tag{C.2}$$

The following equation shows how the monetary balance of agent 1 evolves:

$$M_{1,t+1} = M_{1,t} + \tau_1 M_t - l^{\tau}_{1,t}\Phi_l w_t - k^{\tau}_{y_{1,t}}\Phi_k p_t - q_{1,t}\Phi_{q_{1,t}} p_t \tag{C.3}$$

As a result of detrending, it takes the following form:

$$\widehat{M}_{1,t+1}(1+\tau_1+\tau_2) = \widehat{M}_{1,t} + \tau_1 - l^{\tau}_{1,t}\Phi_l \widehat{w}_t - k^{\tau}_{y_{1,t}}\Phi_k \widehat{p}_t - q_{1,t}\Phi_{q_{1,t}}\widehat{p}_t \tag{C.4}$$

We know that agent 1 does not hold money at the beginning or at the end of the periods, meaning that $\widehat{M}_{1,t+1} = \widehat{M}_{1,t} = 0$. Therefore, the above equation can be expressed as:

$$\tau_1 = l^{\tau}_{1,t}\Phi_l\widehat{w}_t + k^{\tau}_{y_{1,t}}\Phi_k\widehat{p}_t + q_{1,t}\Phi_{q_{1,t}}\widehat{p}_t \tag{C.5}$$

Similarly, the monetary balance of agent 2 evolves according to following equation:

$$M_{2,t+1} = M_{2,t} + \tau_2 M_t - l^{\tau}_{2,t}\Phi_l w_t - k^{\tau}_{y_{2,t}}\Phi_k p_t - q_{2,t}\Phi_{q_{2,t}} p_t \tag{C.6}$$

Detrending this equation results in the following equation:

$$\widehat{M}_{2,t+1}(1+\tau_1+\tau_2) = \widehat{M}_{2,t} + \tau_2 - l^{\tau}_{2,t}\Phi_l\widehat{w}_t - k^{\tau}_{y_{2,t}}\Phi_k\widehat{p}_t - q_{2,t}\Phi_{q_{2,t}}\widehat{p}_t \tag{C.7}$$

Since we are examining the case in which agent 2 holds all the money at the beginning and at the end of the period, $\widehat{M}_{2,t} = \widehat{M}_{2,t+1} = 1$, we obtain the following condition:

$$\tau_1 = -l^{\tau}_{2,t}\Phi_l\widehat{w}_t - k^{\tau}_{y_{2,t}}\Phi_k\widehat{p}_t - q_{2,t}\Phi_{q_{2,t}}\widehat{p}_t \tag{C.8}$$

By using the goods market clearing condition, $q_{1,t} + q_{2,t} = 0$, we can show that $\Phi_{q_{1,t}}$ and $\Phi_{q_{2,t}}$ are equal to each other since these two coefficents are formulated as below:

$$\Phi_{q_{1,t}} = 1 - B\left(\frac{k_{d_{1,t}}}{q_{1,t}}\right)^{\gamma_1}\left(\frac{n_{1,t}}{q_{1,t}}\right)^{\gamma_2} \tag{C.9}$$

$$\Phi_{q_{2,t}} = 1 - B\left(\frac{k_{d_{1,t}}}{-q_{2,t}}\right)^{\gamma_1}\left(\frac{n_{1,t}}{-q_{2,t}}\right)^{\gamma_2} \tag{C.10}$$

If we combine agents' goods market cash-constraint equality ($\Phi_{q_{1,t}} = \Phi_{q_{2,t}}$), goods market clearing condition ($q_{1,t}+q_{2,t} = 0$), and capital market clearing condition ($k^{\tau}_{y_{1,t}}+k^{\tau}_{y_{2,t}} = 0$) with the two conditions we have found by using agents' monetary balance equations ($\tau_1 = l^{\tau}_{1,t}\Phi_l\widehat{w}_t + k^{\tau}_{y_{1,t}}\Phi_k\widehat{p}_t + q_{1,t}\Phi_{q_{1,t}}\widehat{p}_t$ and $\tau_1 = -l^{\tau}_{2,t}\Phi_l\widehat{w}_t - k^{\tau}_{y_{2,t}}\Phi_k\widehat{p}_t - q_{2,t}\Phi_{q_{2,t}}\widehat{p}_t$), we get that $l^{\tau}_{1,t} = -l^{\tau}_{2,t}$. This last equality, which can also be expressed as $l^{\tau}_{1,t} + l^{\tau}_{2,t} = 0$, confirms that the remaining market, namely the labor market, also clears.

Appendix D

Tables

TABLE D.1: Effect of Monetary Transfer on the Degree of Cash Constraint on the Goods Market

Case	τ_1	0.055	0.060	0.065	0.080	0.085	0.090	0.095	0.100
1	Φ_q	0.492	0.490	0.499	0.611	0.535	0.701	0.695	0.648
2	Φ_q	0.588	0.663	0.663	0.609	0.656	0.665	0.652	0.646
3	Φ_q	0.560	0.574	0.504	0.575	0.563	0.530	0.590	0.584
4	Φ_q	0.508	0.508	0.508	0.509	0.508	0.498	0.431	0.381
5	Φ_q	0.505	0.516	0.443	0.459	0.511	0.547	0.551	0.551

TABLE D.2: Effect of Monetary Transfer on Total Output

Case	τ_1	0.055	0.060	0.065	0.080	0.085	0.090	0.095	0.100
1	y	3.266	3.297	3.315	3.296	3.415	3.090	3.128	3.171
2	y	3.325	3.183	3.194	3.306	3.167	3.196	3.230	3.245
3	y	3.289	3.314	3.481	3.320	3.287	3.337	3.277	3.290
4	y	3.321	3.322	3.322	3.320	3.318	3.377	3.498	3.511
5	y	3.376	3.392	3.469	3.426	3.384	3.336	3.319	3.306

Table D.3: Effect of Monetary Transfer on Agent 1's Output

Case	τ_1	0.055	0.060	0.065	0.080	0.085	0.090	0.095	0.100
1	y_1	1.984	1.976	1.994	1.805	1.913	1.729	1.734	1.755
2	y_1	1.772	1.787	1.782	1.744	1.795	1.764	1.753	1.762
3	y_1	1.898	1.867	1.913	1.854	1.895	1.930	1.866	1.860
4	y_1	1.970	1.970	1.969	1.957	1.944	1.942	1.983	2.057
5	y_1	1.932	1.896	1.950	1.933	1.917	1.875	1.864	1.861

Table D.4: Effect of Monetary Transfer on Agent 2's Output

Case	τ_1	0.055	0.060	0.065	0.080	0.085	0.090	0.095	0.100
1	y_2	1.282	1.320	1.321	1.490	1.501	1.360	1.394	1.416
2	y_2	1.554	1.396	1.412	1.562	1.371	1.432	1.478	1.484
3	y_2	1.391	1.447	1.568	1.465	1.392	1.407	1.412	1.430
4	y_2	1.350	1.352	1.353	1.363	1.374	1.435	1.515	1.454
5	y_2	1.444	1.496	1.519	1.493	1.466	1.462	1.455	1.445

Table D.5: Effect of Monetary Transfer on the Time Agent 1 Spends on Leisure

Case	τ_1	0.055	0.060	0.065	0.080	0.085	0.090	0.095	0.100
1	x_1	0.670	0.686	0.690	0.745	0.737	0.668	0.690	0.705
2	x_1	0.764	0.673	0.679	0.764	0.671	0.688	0.711	0.716
3	x_1	0.682	0.700	0.768	0.711	0.684	0.699	0.679	0.687
4	x_1	0.658	0.659	0.660	0.668	0.674	0.703	0.757	0.740
5	x_1	0.682	0.691	0.729	0.713	0.682	0.673	0.671	0.669

Table D.6: Effect of Monetary Transfer on the Time Agent 2 Spends on Leisure

Case	τ_1	0.055	0.060	0.065	0.080	0.085	0.090	0.095	0.100
1	x_2	0.014	0.013	0.014	0.014	0.014	0.015	0.015	0.015
2	x_2	0.015	0.016	0.016	0.016	0.014	0.015	0.016	0.016
3	x_2	0.015	0.016	0.015	0.016	0.015	0.015	0.016	0.016
4	x_2	0.014	0.015	0.014	0.015	0.015	0.015	0.013	0.012
5	x_2	0.014	0.016	0.014	0.014	0.014	0.016	0.016	0.016

Table D.7: Effect of Monetary Transfer on the Amount of Goods Traded

Case	τ_1	0.055	0.060	0.065	0.080	0.085	0.090	0.095	0.100
1	q_1	3.061	3.107	3.192	3.083	3.206	2.554	2.571	2.745
2	q_1	2.873	2.472	2.481	2.792	2.520	2.488	2.561	2.590
3	q_1	2.660	2.669	3.121	2.702	2.683	2.791	2.563	2.586
4	q_1	2.768	2.773	2.775	2.791	2.803	2.915	3.427	3.818
5	q_1	2.739	2.711	3.145	3.004	2.664	2.527	2.494	2.503

Table D.8: Effect of Monetary Transfer on the Amount of Labor Traded

Case	τ_1	0.055	0.060	0.065	0.080	0.085	0.090	0.095	0.100
1	l_1^τ	0.779	0.771	0.782	0.748	0.731	0.760	0.762	0.728
2	l_1^τ	0.699	0.750	0.742	0.687	0.759	0.732	0.714	0.720
3	l_1^τ	0.760	0.739	0.705	0.723	0.755	0.750	0.747	0.734
4	l_1^τ	0.777	0.777	0.776	0.770	0.764	0.740	0.722	0.737
5	l_1^τ	0.736	0.714	0.702	0.712	0.736	0.738	0.742	0.746

Table D.9: Effect of Monetary Transfer on the Amount of Capital Traded

Case	τ_1	0.055	0.060	0.065	0.080	0.085	0.090	0.095	0.100
1	$k_{y_2}^\tau$	0.160	0.170	0.169	0.199	0.201	0.170	0.187	0.177
2	$k_{y_2}^\tau$	0.209	0.176	0.176	0.202	0.172	0.178	0.187	0.193
3	$k_{y_2}^\tau$	0.174	0.181	0.201	0.178	0.171	0.174	0.177	0.178
4	$k_{y_2}^\tau$	0.173	0.174	0.173	0.173	0.172	0.179	0.197	0.194
5	$k_{y_2}^\tau$	0.180	0.189	0.198	0.194	0.183	0.184	0.186	0.185

Table D.10: Effect of Monetary Transfer on the Division of Capital Between Sectors for Agent 1

Case	τ_1	0.055	0.060	0.065	0.080	0.085	0.090	0.095	0.100
1	s_1	0.051	0.051	0.045	0.024	0.033	0.063	0.066	0.065
2	s_1	0.062	0.060	0.060	0.062	0.061	0.060	0.061	0.061
3	s_1	0.058	0.058	0.059	0.059	0.059	0.060	0.060	0.060
4	s_1	0.063	0.063	0.063	0.062	0.062	0.061	0.061	0.066
5	s_1	0.061	0.061	0.064	0.064	0.059	0.058	0.059	0.060