MINIMUM ENERGY CHANNEL AND NETWORK CODING WITH APPLICATIONS IN NANOSCALE COMMUNICATIONS

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MURAT KOCAOĞLU

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This is to certify that I have examined this copy of a master's thesis by

Murat Kocaoğlu

and have found that it is complete and satisfactory in all respects,

and that any and all revisions required by the final

examining committee have been made.

Committee Members:

Özgür B. Akan, Ph. D. (Advisor)

A. Murat Tekalp, Ph. D.

Fatih Alagöz, Ph. D.

Date: ______________________

Babama ve Anneme,

Cengiz Kocaoğlu ve Lütfiye Kocaoğlu.

ABSTRACT

Channel coding, as indicated by Claude E. Shannon, is mostly used to achieve the capacity, by means of introducing redundancy to combat the effects of noise. In this thesis, an energy minimizing novel channel code (MEC) with controllable reliability is proposed for the first time. The motivation stems from the severe energy constraints of nano communications. Due to their tiny size, nanodevices can only supply a limited amount of energy. The need to optimize communication techniques for minimum energy includes the development of a novel channel code that provides reliability, while keeping the transmitter energy at minimum. We develop minimum energy channel code (MEC), which uses the idea of increasing the frequency of the modulation state that requires less energy. On-off keying (OOK) modulation is the simplest of such techniques and assumed to be the underlying modulation throughout the thesis. MEC provides the desired reliability with varying delay, when source set cardinality is less than the inverse of symbol error probability. The proposed channel code is shown useful in nanosensor networks. The performance of MEC is also evaluated considering the random interference due to multiple uncoordinated users in the ad-hoc nanonetworks. The theoretical maximum node density limit in an ad-hoc nanonetwork with reliable communications between the nodes is derived. Investigating delay via varying codeword length, rate-delay-energy tradeoffs with MEC are analyzed. Lastly, for the first time in the literature, network codes that minimize energy is developed in network coding nodes with two incoming edges. To minimize the overall network energy in in-two networks, each node is assumed to employ MEC with OOK. Therefore, MENC provides the best mapping between input edges and output edge of the coding node, that minimizes the average energy at the output of the coding node, by minimizing the average weight.

ÖZET

Kanal kodlaması, bilgi kuramının kurucusu olan Claude E. Shannon'ın işaret ettiği şekilde, kanal kapasitesine erişmek ve güvenilir haberleşme sağlamak için, gönderilen bilgide yedeklilik kullanılmasına verilen isimdir. Bu tezde, literatürde ilk defa, öncelikli amacı harcanan enerjiyi en aza indirmek olan güvenilir bir kanal kodlaması (MEC) geliştirilmiştir. MEC, modülasyon yöntemi olarak dur-başla anahtarlamanın kullanıldığı varsayılarak, en az enerjiyi sağlamak için, kod ağırlığını en aza indirmektedir. Araştırmadaki temel motivasyon, nano boyutlardaki cihazlar arasındaki haberleşmeyi sağlayacak, enerjiyi verimli kullanan haberleşme teknikleri geliştirilmesi ihtiyacı olmuştur. Boyutlarından dolayı, haberleşebilen nano cihazlar çok küçük miktarlarda enerji bütçesine sahiptir. Bu da kullanılan tekniklerin enerjiyi en aza indirecek şekilde tasarlanmasını gerektirmektedir. Önerilen kanal kodunun, verici düğümdeki kaynak küme büyüklüğünün, kanalın sembol hata olasılığından küçük olması şartıyla, gecikmesi ayarlanarak istenen güvenilirliği sağladığı gösterilmiştir. Önerilen kanal kodunun en az enerji harcanmasını sağladığı ve güvenilir olduğu ispatlanarak, nano algılayıcı ağlarında etkin bir şekilde kullanılabileceği gösterilmiştir. Ayrıca bu kanal kodunun dağınık nano ağlarda, koordinesiz düğümlerden kaynaklanan girişime karşı güvenilirliği incelenmiş, kanal kodunun desteklediği en fazla nano düğüm yoğunluğu bulunmuş, ve düzensiz ağlardaki veri hızı, gecikme ve enerji ödünleşmesi incelenmiştir. Son olarak, literatürde ilk defa, en az enerjiyi sağlayan ağ kodlaması (MENC) geliştirilmiştir. Her kanaldaki enerjiyi en aza indirmek amacıyla düğümlerin MEC uyguladığı kabul edilmiştir. Buna göre MENC, ağ kodlaması yapan iki gelen ve bir giden bağlantıya sahip düğümlerde, bütün işlevler, yani ağ kodları, arasında en az enerjiyi sağlayan olarak seçilmiştir. MEC'te olduğu gibi, MENC de ağ kodlaması yapan düğümlerde en az kod ağırlığını sağlayarak enerjiyi en aza indirmektedir.

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TABLE OF CONTENTS

LIST OF FIGURES

FIGURES

ABBREVIATIONS

WNSN	Wireless Nanosensor Network
MEC	Minimum Energy Channel Coding
THz	Terahertz
OOK	On-off Keying
CNT	Carbon Nanotube
TDMA	Time Division Multiple Access
FDMA	Frequency Division Multiple Access
RS	Reed-Solomon Code
ARQ	Automatic Repeat Request
EM	Electromagnetic
CDMA	Code Division Multiple Access
MENC	Minimum Energy Network Coding
LENC	Low Energy Network Code

CHAPTER 1

INTRODUCTION

Nanoscale communications is a novel research area, which is mainly developing the communication techniques suitable for tiny nanomachines. We address the need for a novel channel code suitable for energy minimization in nano communications. We also develop energy minimizing network codes which can be used to minimize total network energy together with minimum enegy channel codes. We present preliminaries in this section, and then present the research objectives and solutions.

1.1 Nanoscale Communications

As technology evolves, the devices become smaller and smaller. As we approach the limits of the top-down device designs, nanoscale devices capable of accomplishing certain tasks are believed to be useful in many areas ranging from medicine to sensing [3]. In order to perform operations, these nanodevices need to be able to communicate with each other. However, the incredibly tiny size of nanodevices impose certain limits on their capabilities. The most important two factors to limit the performance is limited amount of energy and complexity at the nanodevices. Therefore, communication techniques employed in these tiny machines, such as channel coding and modulation, should maintain energy efficiency and complexity as the main metric to minimize.

1.2 Channel Coding

Channel coding is simply defined as adding redundancy to the transmitted data to combat the effects of noise. Block codes provide a set of *n-tuples* called the *channel codewords.*

For block codes, a codebook is defined as any selection of fixed length codewords, mapped to source symbols. For unique decodability, this mapping should be one-to-one. Hamming weight is the number of non-zero entries in the codeword. As we deal with binary codes only, weight is equivalent to the number of $1s$ in the codeword. Weight enumerator of a code is the polynomial $W_{\mathbb{C}}(z) = \sum_i c_i z^i$, where c_i is the number of codewords with weight i. Additionally, the Hamming distance between two codewords is defined as the number of bits in which they differ. In minimum distance decoding, which is the presumed decoding strategy, the received n-tuple is mapped to the closest codeword in terms of Hamming distance. Codes with distance d can correct $\lfloor \frac{d-1}{2} \rfloor$ errors. Codes with larger distance are more reliable, since more error patterns can be corrected.

1.3 Network Coding

Network coding is the processing of information flow at the relay node, before being forwarded to the next hop. Network coding is shown to be necessary to achieve capacity in general networks, where simply forwarding the data is not sufficient [1]. For a network coding node u, if the incoming channel symbols are $u_1, u_2, ..., u_N$, for all the outgoing edge i of the node u, network code is the mapping between the incoming and outgoing symbols, i.e.,

$$f_{u,i} = f(u_1, u_2, ..., u_N). \tag{1.1}$$

In the most generalized definition, there is no constraint on this mapping. However, in order the information to be preserved, such that destinations with multiple coded flows can decode the desired channel symbols, require certain conditions to be satisfied by these mappings. We will describe and deal with these conditions in detail in Chapter 4.

1.4 Research Objectives and Solutions

The objectives of our research and the solution approaches are explained in this section.

1.4.1 Minimum Energy Coding for Wireless NanoSensor Networks

Nano communication is a developing research area with the goal to design novel communication techniques for future nanodevices. Wireless nanosensor networks (WNSNs), which are collections of nanosensors with communication capabilities, can be used for sensing with extremely high resolution and low power consumption. If successfully implemented, WNSNs are believed to have revolutionary effects on our daily lives [2]. The development of novel communication techniques suitable for nanodevice characteristics is essential for WNSN designs. It is essential to develop energy-efficient communication techniques for WNSNs, since nanonodes are severely energy-constrained.

First, we propose a new OOK-based multi-carrier modulation scheme suitable for nano communications. The proposed modulation uses the available frequency bands at the Terahertz (THz) channel. Exploiting the allowable frequency bands, nanonode does not need to deal with the severe molecular noise and path loss. With the assumption of OOK modulation, a novel channel code (MEC), that minimizes the average codeword weight to minimize energy is developed. As delay is not critical, codeword length is increased with increased source set cardinality and Hamming distance. Hence, MEC minimizes the average codeword weight for all source cardinality and Hamming distance values. It turns out that the minimum average codeword weight does not depend on the probability distribution of the source symbols, but only the probability of the most probable source outcome.

1.4.2 Performance of Minimum Energy Coding in Ad-Hoc Nanonetworks

Ad-hoc nanonetworks, different from the nanosensor network scenarios, are not synchronized and uncontrollable interference degrades the system performance. Due to their flexible architecture, ad-hoc nanonetworks are important for future applications of nanodevices.

Combating noise and significant interference while satisfying the constraints imposed by nanoscale communications, i.e., keeping the dissipated energy at minimum, in ad-hoc networks is a challenging task. MEC can be used for energy minimization purposes as for WNSNs, to provide reliability against severe interference, which depends on the nanonode density. We investigate the maximum nanonode density that MEC supports in ad-hoc nanonetworks. Hence, if node density is below a certain threshold, code distance, hence the delay, can be increased to provide the desired level of reliability in ad-hoc nanonetworks. The tradeoff between rate, delay and energy is also investigated, where delay is proportional to the minimum codeword length, rate depends on the reliability of the code and energy depends on the codeword weight and the probability of the most probable source outcome.

1.4.3 Energy Minimizing Network Codes

Although there is a vast literature on network coding, the main focus is on developing solutions to achieve the network capacity. From an energy efficiency perspective, researchers have developed algorithms to obtain minimum energy paths and solve optimization problems of scheduling in network coding scenarios. However, network coding is not directly used as a tool to minimize the dissipated energy.

Selecting the network code that yields the minimum energy dissipation at the coding node with two incoming and one outgoing symbols, among all the input-output mappings is conducted in this chapter for the first time in the literature, termed as MENC. Hence in certain networks MENC can be used to capacity with minimum network energy. It turns out that the minimum average code weight at the output of the coding node only depends on the $\sum_i p_i q_i$, where p_i and q_i are the probability that the first and second incoming edge contains the symbol i respectively. MEC cannot be simply extended to general nodes with N incoming and single outgoing channel symbols, since the minimum average code weight at the relay output depends on probability distribution of incoming symbols p_i.

1.5 Thesis Outline

This thesis is organized as follows: In Chapter 2, we derive MEC and obtain its performance, with the achievable rate in nanosensor nodes and in WNSNs. In Chapter 3, the performance of MEC in ad-hoc nanonetwork scenarios is investigated. The maximum node density that MEC supports is derived and tradeoffs between the performance parameters rate, delay and energy, is presented. In Chapter 4 we derive MENC for networks composed of nodes with two incoming edges. We also develop a low energy network code (LENC) for the general network coding nodes with N incoming and one outgoing channel symbols. The thesis is concluded in Chapter 5 underlining the importing points together with the discussion of future issues.

CHAPTER 2

MINIMUM ENERGY CODING FOR WIRELESS NANOSENSOR NETWORKS

In this chapter, a new modulation and a novel minimum energy coding scheme (MEC) are proposed to achieve energy efficiency in WNSNs. Unlike the existing studies, MEC maintains the desired minimum Hamming distance, while minimizing energy, to provide reliability. It is analytically shown that, with MEC, codewords can be decoded perfectly for large code distance, if source set cardinality, M is less than inverse of symbol error probability, $1/p_s$. Performance evaluations show that MEC outperforms popular codes such as Hamming, Reed-Solomon and Golay in the average codeword energy sense at the transmitter.

The remainder of this chapter is organized as follows: In Section 2.1, we present an introduction to the nanoscale communications to present the motivation behind this work. In Section 2.2, the existing work on WNSNs is presented together with the existing minimum energy codes, and open issues that will be addressed in this chapter. In Section 2.3, low-complexity medium access techniques suitable for nanosensor networks, and WNSN architecture are discussed in detail. We develop MEC for the proposed system in Section 2.4.1 and derive the analytical expressions related to MEC performance in Section 2.4.2. In Section 2.5, MEC performance evaluations and comparison with popular block codes are presented. Additionally, effects of cell radius and coverage ratio on the maximum number of source quantization levels in a cell-based WNSN using MEC are investigated.

2.1 Motivation

Nanosensors are devices of nano dimensions with accurate sensing capabilities. One of the most promising building blocks for future nanodevices and hence nanosensor nodes are carbon nanotubes (CNT). CNTs are rolled up graphene sheets with nano dimensions that can be used as nanoantennas, nano sensing units and even nanobatteries [12], [4]. Many studies on the antenna properties of CNTs exist in the literature [12], [27]. The resonant frequency of CNT antennas is shown to be in the Terahertz band of the spectrum (0.1-10 THz). This band is not utilized by macro applications and is a candidate for communications between nanodevices [2]. The main challenge of using the THz band is the absorption of EM waves by water vapour molecules, which make communication impractical by causing severe path loss and molecular noise [14].

Potential nanosensors have significantly different performance metrics than the macro sensor nodes. Even though no complete nanonode architecture has yet been implemented, it is anticipated that power and energy efficiency are of the most critical measures, since nanosensors can only provide a limited amount of energy with their extremely small size. Hence, it is essential to develop energy-efficient communication techniques.

Employing channel coding at the nanoscale is critical to assure reliable communication between the nanodevices. The classical channel codes have various design considerations such as the efficient use of code space, as in perfect codes, bounded decoding complexity as the Shannon channel capacity is approached, as in Turbo or LDPC codes, or low encoding and decoding complexity as in cyclic and convolutional codes. However, the coding scheme for nano wireless communications should consider the *energy dissipation at the transmitter* as the main metric, since nanonodes run on a strict energy budget. Therefore, classical codes are not suitable for use in nanoscale. Different from most of the classical codes, minimum energy coding minimizes the average codeword energy if OOK is the underlying modulation technique [10]. However, the existing minimum energy codes are not reliable, since error probability can not be controlled.

In order to address this need, we develop a novel minimum energy channel code (MEC), that is reliable and suitable for nanocommunications [18, 19]. Proposed code provides the minimum average codeword energy of all the block codes, given that OOK is used as the modulation scheme. With OOK, average codeword energy is the symbol energy times average codeword weight; therefore, average energy is minimized by minimizing the average code weight. For this, codeword weights and sourceword-codeword mappings are chosen such that the expected code weight is minimized at the cost of increased codeword length, hence increased delay. Lengthy codewords could increase the energy dissipation at the transmitter due to energy dissipation of the nanosensor circuitry. To minimize the total energy dissipation at the nanosensor, code weights of MEC could be increased to allow shorter codewords. This implies a tradeoff between the transmission and processing energies and a discrete optimization problem could arise. However, such an analysis is not feasible today, since it is inaccurate to estimate the energy dissipation at the nano processing units, as no complete nanonode architecture is yet available. The suitability of MEC for nanoscale communications is shown by obtaining the achievable rate at the nanonode.

2.2 Related Work

WNSNs can be used for sensing and data collection with extremely high resolution and low power consumption in various applications [2]. Current literature shows considerable research effort on WNSNs. In [4], the authors introduce carbon nanotube sensor networks and present major challenges to be addressed for their realization. The authors in [2] provide a detailed survey on the state-of-the-art in nanosensors and emphasize potential applications and design challenges of WNSNs. In [14], the THz channel absorption and noise characteristics and THz channel capacity are investigated. Despite these studies, channel coding in nano wireless communications is still a barren field. Only recently, the authors in [15] propose using low-weight channel codes with femtosecond-long OOK pulses not only to reduce energy, but also to mitigate interference in nanonetworks. However, to the best of our knowledge, the need for developing channel codes to achieve energy-efficient and reliable nano communications has not been addressed so far.

The idea of using low-weight channel codes together with OOK modulation to reduce energy consumption is first proposed in [10] for sensor networks. Choosing codewords for each source outcome such that mean codeword energy is less than any other choice of codeword mappings is called minimum energy coding. The authors show that, for a given codebook, sorting codewords in increasing code weight order and assigning source symbols in decreasing probability order, such that the most probable source symbol is mapped to the codeword with the smallest weight yields the optimum average code weight. Later, the authors in [29] propose using codewords with maximum weight of 1. Such a mapping corresponds to minimum energy coding, if the all-zero codeword is mapped to the most probable source outcome. However, there is an important drawback to the existing minimum energy code. This code is not reliable since its Hamming distance is 1, and any bit error pattern at the receiver due to noise yields uncorrectable words. Therefore, providing reliable minimum energy codes has been an open issue.

In this chapter, first, we present a new modulation scheme suitable for nanocommunications in the THz band. Contrary to the existing nanocommunication schemes in which the whole THz band is utilized, our scheme alleviates the need to deal with the performance degradation due to molecular absorption lines and molecular noise. Later, we address the need for reliable minimum energy codes. We develop minimum energy codes with controllable reliability. The reliability of the code is controlled by changing its Hamming distance. Lastly, we show the suitability of MEC for nanosensor networks by investigating the achievable information rate of a nanosensor node and interference limited source set cardinality in WNSNs with MEC.

2.3 Wireless NanoSensor Network Architecture

Realization of WNSNs require various challenges to be addressed in a nanonode. Energy-efficiency and suitability for the THz channel are prior concerns. Complexity of the nanosensor must also be kept as low as possible. In this section, we explain the communication techniques we develop for nanosensors and discuss a feasible extension to WNSNs.

The structure of the nanosensor node is shown in Fig. 2.1. The main block functionalities can be found in [4]. We propose using a number of CNT antennas instead of one to utilize the multiple available frequency windows in the THz band. Required energy can be provided by the battery block via nano energy harvesting systems [33]. Sensing is also CNT based. Nanosensor readings are quantized to M levels. No source coding is employed in the proposed system so as to not increase complexity. Each source signal level is mapped to $length-n$ channel codewords with a combinatorial nano-circuit. Realization of such a processing is not clear today. However, studies on CNT-based logic gate applications [13] increase hope. The

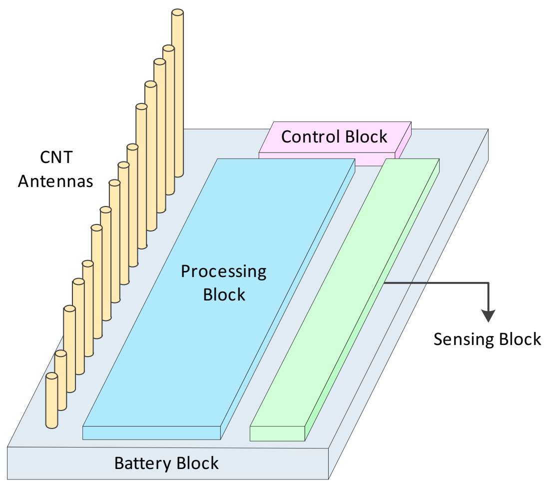


Figure 2.1: Proposed nanosensor node architecture.

processing block is also responsible for carrier generation. Even though carrier generation in nano domain is not clear, it is shown that, with their unique properties such as slowing down surface EM waves, CNTs can also be used to generate THz waves much easier than the classical techniques [17]. Applying voltage at its ends, CNTs can be used for wave generation from THz to optical frequencies. Control block contains a separate antenna for the control of the nanonode from a central unit. Nanonode activates and then transmits only when this antenna is excited. This functionality is required to provide low complexity multiple access in WNSNs, as will be explained later.

2.3.1 Multi-carrier OOK Modulation

Motivated with the THz channel characteristics, we propose a *multi-carrier modulation scheme for nano communications*. In the proposed scheme, each codeword is transmitted in parallel over different carriers. Our frequency choice considers carriers' suitability for transmission in the THz channel. As previously mentioned, the THz channel consists of several frequency windows with low absorption and low molecular noise, termed as *available windows*. Carrier frequencies are chosen among these available windows in the THz channel. Available windows vary depending on the transmission distance and water vapour amount on the transmission path [14]. CNTs are used as nanoantennas to radiate each carrier, as shown in Fig. 2.1. Each frequency window is utilized separately. Bandwidth increase is prohibited by the molecular absorption lines. Decreasing the bandwidth results in increased energy consumption per symbol, since symbol time increases. Hence, we select bandwidth as the same as the width of the available frequency windows. Therefore, picoseconds long sinusoidal pulses are used, which span a frequency band of 100-200 GHz, corresponding to the width of most of the windows in the THz channel [14].

Channel codes with minimum average weight are utilized, together with OOK modulation at each carrier to reduce the energy consumption. Proposed coding achieves the minimum codeword energy and guarantees a minimum Hamming distance at the price of lengthy codewords. Multi-carrier modulation mitigates delays due to lengthy codewords of MEC in WNSN node. The number of multi-carrier signals can be chosen to satisfy a certain delay requirement.

2.3.2 WNSN Cell Architecture

In our analysis, we consider a cell-based wireless nanosensor network. In the nanocommunications literature, a cell-based WNSN has not been considered before. The proposed WNSN architecture is also utilized to evaluate the performance of MEC in Section 2.5.

A cell is composed of a micro node, and nanosensor nodes scattered around it. In order to reduce the interference, nanonodes are deployed within a radius of αr, where r is the cell radius and α is the coverage ratio satisfying $0 \leq \alpha \leq 1$. To keep the complexity of the nanonodes low, all the control and scheduling issues are left to the micro node within the cells. A nanonode starts transmission only when an activation signal is sent by the micro node. As suggested in [3], kHz band can be used for this activation signal, with vibrating CNTs. The central micro node provides not only control, but also synchronization among the nanosensors. It is assumed that the micro node is capable of receiving the THz waves. In the current literature, many studies exist on CNT based THz receivers and it has been demonstrated that CNT bundles can be used for efficient THz detection at room temperature [34]. With their employment, multi-wavelength THz receivers with micro dimensions will be available hopefully in the near future.

Let N be the number of nodes in a WNSN cell, n the codeword length and l the number of channels for multi-carrier modulation within the cell. Assume that all the nanonodes are within a range to directly communicate with the micro node. There are two reliable medium access techniques, keeping complexity at the micro node:

Single Control Signal

Nanonodes start transmission simultaneously through different sets of channels (frequencies). In order to keep complexity at the micro node, different sets of frequencies must be used by each nanonode, and a common synchronization signal must be broadcast from the micro node for signalling the transmission of the nanonodes. Nl different THz frequency windows, and a single kHz band are allocated to a single cell. This is an FDMA-based scheme, as separate frequency windows are allocated to each nanosensor node.

Multiple Control Signals

Nanonodes use the same set of frequencies for transmission. The micro node uses control signals at different frequencies for each nanonode sequentially, since nanonodes utilize the same THz channels. Allocation of l THz and N kHz bands are needed. This is similar to the TDMA, since all the nodes use the channel in different time intervals.

In the following, we assume that the micro node uses multiple control signals, since the number of frequency windows in the THz channel is limited. Additionally, demodulating a large number of different THz signals significantly increases the complexity at the micro node.

2.4 Minimum Energy Coding with Hamming Distance Constraint

In this section, we develop a novel minimum energy coding scheme for energy-efficient nano communications. We propose new channel codes with minimum average code weight. Such codes are equivalent to the codes minimizing transmission energy for the systems employing OOK modulation. This is because, no energy is dissipated when 0 symbol is transmitted and no ARQ scheme is employed in nanocommunications for retransmissions.

Codewords with lower weight results in less energy dissipation, when transmission of 0 symbol requires less energy than the transmission of 1 symbol. OOK is an example of such modulation schemes in which transmission of 0s require no energy. As pointed out in Sec.

2.2, there has been a need to develop reliable minimum energy codes. To address this issue, we develop minimum energy channel codes with any Hamming distance d to guarantee reliability in communications between nanonodes and micro node with OOK. Proposed minimum energy coding minimizes the expected codeword weight, depending on the source probability distribution. First, we derive the construction of MEC and obtain the minimum average code weight, then, develop analytical results related with the performance metrics of MEC.

2.4.1 MEC and Minimum Average Code Weight

In the nanonode, each codeword has the same probability of occurrence as the source outcomes that they are mapped to, since no source coding mechanism is employed, as previously stated. This brings a new problem into the picture: *What is the codebook selection that minimizes the average code weight for any input probability distribution?* This problem can be emphasized as finding the weight enumerator and mapping between codewords and sourcewords such that the expected codeword weight for a given input probability mass function is minimized. It is trivial that for no Hamming distance constraint, i.e., $d = 1$, assigning codewords of maximum weight 1 yields minimum energy, as proposed in [29]. In order to obtain an analytical solution, we modify the minimum energy code problem such that code length n is kept unconstrained. Hence, we construct MEC by extending codeword length as much as we need. Later, we develop the minimum required code length for different cases in Section 2.4.2.

Let M, d, p_{max} represent number of codewords, minimum Hamming distance and maximum probability in any discrete probability distribution, respectively and x be the source random variable. We first present some lemmas with which we will prove our main theorem.

Lemma 2.4.1 *For any finite M, there exists a finite n_0 such that a constant weight code $\mathbb{C}$ of length-n_0 containing the codeword c can be constructed with minimum Hamming distance d, if $weight(c) \geq \lceil \frac{d}{2} \rceil$:*

$$\exists \mathbb{C} : dist(\mathbb{C}) \geq d \text{ for } c \in \mathbb{C} \text{ if } \mathit{weight(c)} \geq \left\lceil \frac{d}{2} \right\rceil .$$

Lemma 2.4.2 *Any codebook with Hamming distance of d contains at most a single codeword* *with weight less than* $\lceil \frac{d}{2} \rceil$.

Lemma 2.4.3 *In a codebook with Hamming distance d, any two codeword c_i and c_j should satisfy the inequality $weight(c_i) + weight(c_j) \geq d$.*

Let $\mathbb{C}_i$ be the code with weight enumerator

$$W_{\mathbb{C}_i}(z) = z^{\lfloor \frac{d}{2} \rfloor - i} + (M-1) z^{\lceil \frac{d}{2} \rceil + i}. \tag{2.1}$$

Therefore, the code $\mathbb{C}$ contains a single codeword with weight $\lfloor \frac{d}{2} \rfloor - i$ and all the other codewords have weight $\lceil \frac{d}{2} \rceil + i$. Let codeword with weight $\lfloor \frac{d}{2} \rfloor - i$ be assigned to the source symbol with maximum probability, i.e., p_{max}. Let $E_{\mathbb{C}_i}$ represent expected code weight for code $\mathbb{C}_i$.

Lemma 2.4.4

$$E_{\mathbb{C}_{i+k}} < E_{\mathbb{C}_i} \textit{ if } p_{max} > 0.5, \forall k > 0$$

Proof. Let β represent $\lfloor \frac{d}{2} \rfloor$. Then

$$\begin{aligned} E_{\mathbb{C}_i} &= p_{max}(\beta - i) + (1 - p_{max})(d - \beta + i) \\ &= p_{max}(2\beta - 2i - d) + d - \beta + i. \\ \Rightarrow E_{\mathbb{C}_i} - E_{\mathbb{C}_{i+k}} &= k(2p_{max} - 1). \end{aligned}$$

Hence, since k is positive, $E_{\mathbb{C}_{i+k}} < E_{\mathbb{C}_i}$ if $p_{max} > 0.5$. ■

Theorem 2.4.5 *Let x_i be distributed with $p_i \in \{p_1, p_2, ..., p_M\}$ and p_{max} be $\max(p_i)$. For a desired minimum Hamming distance d, the minimum expected codeword weight, $E(w)$ is*

$$\min(E[w]) = \begin{cases} (1 - p_{max})d, & p_{max} > \frac{1}{2}, \\ \frac{d}{2}, & p_{max} < \frac{1}{2}, \textit{ if d even }, \\ \lceil \frac{d}{2} \rceil - p_{max}, & p_{max} < \frac{1}{2}, \textit{ if d odd} \end{cases} \tag{2.2}$$

Proof. Let c_i be the codeword assigned to the event i with probability p_i, w_i represent $weight(c_i)$, d be the minimum Hamming distance and M be the number of codewords.

From Lemma 1, we know that, a $weight - \lceil \frac{d}{2} \rceil$ code can be constructed with finite code length for any M. Therefore, $\min(E(w)) \leq \lceil \frac{d}{2} \rceil$. From Lemma 2, we know that we can decrease the weight of only a single codeword below $\lceil \frac{d}{2} \rceil$. Then the bound can safely be improved by switching the code weight of the most probable outcome to $\lfloor \frac{d}{2} \rfloor$, since the resulted code will still satisfy the distance condition. This leads to a bound valid for any source probability distribution:

$$\min(E[w]) \leq p_{max} \left\lfloor \frac{d}{2} \right\rfloor + (1 - p_{max})(d - \left\lfloor \frac{d}{2} \right\rfloor) \tag{2.3}$$

We know from Lemma 3 that, if we want to further reduce the weight of the most probable codeword, we should increase the weight of all the other codewords to satisfy $weight(c_i) + weight(c_j) = d$ for any i, j. Lemma 4 shows that increasing i in (2.1), i.e., reducing the weight of the most probable codeword results in lower expected code weight, if $p_{max} > 0.5$. Therefore, minimum average weight is obtained when $i = \lfloor \frac{d}{2} \rfloor$ for $p_{max} > 0.5$. This corresponds to the code $\mathbb{C}_d$ with weight enumerator of $W_{\mathbb{C}_d}(z) = z^0 + (M-1)z^d$, giving the average weight of

$$E[w] = (1 - p_{max})d. \tag{2.4}$$

Note that by assigning the codeword of weight $\lfloor \frac{d}{2} \rfloor$ to any source symbol other than the most probable one, the obtained bound cannot be reached. After such a step, decreasing the weight of the chosen codeword does not decrease the expected weight since $p < p_{max}$ forces $p < 0.5$. Furthermore, no other weight decreasing scheme can be applied after such an assignment. Therefore, the best bound is obtained by decreasing the weight of the codeword with probability p_{max} and leads to bound given in (2.4). If p_{max} is less than 0.5, expected weight cannot be decreased any further than (2.3) by Lemma 4. After simple manipulations, (2.2) can be easily obtained. ■

A more realistic problem definition is as follows: *What is the minimum expected code weight for code distance d and maximum codeword weight k*, where k represents the maximum high symbols in a codeword that the nanonode can supply power for? If $k < \lceil d/2 \rceil$, there is no way to satisfy distance constraint. Hence, we assume $k \geq \lceil d/2 \rceil$.

Theorem 2.4.6 *Let x_i be distributed with $p_i \in \{p_1, p_2, ..., p_M\}$ and p_{max} be $\max(p_i)$. For a desired minimum Hamming distance d and maximum codeword weight k, if $\lceil d/2 \rceil \leq k < d$ is* *satisfied, minimum expected codeword weight, $E(w)$ is given by*

$$\min(E[w]) = \begin{cases} p_{max}(d-2k)+k, & p_{max} > \frac{1}{2}, \\ \frac{d}{2}, & p_{max} < \frac{1}{2}, \textit{ if } d \textit{ even }, \\ \left\lceil \frac{d}{2} \right\rceil - p_{max}, & p_{max} < \frac{1}{2}, \textit{ if } d \textit{ odd} \end{cases} \tag{2.5}$$

Proof. It is clear that, if $p_{max} < 0.5$, bound given in Theorem 1 can be achieved, since $k \geq \lceil d/2 \rceil$. However, if $p_{max} > 0.5$, by Lemma 4, i in (2.1) should be increased to reduce the average code weight, and could at most be $i = k - \left\lceil \frac{d}{2} \right\rceil$ due to maximum code weight constraint. Hence, for $\lceil d/2 \rceil \leq k < d$, it is easily seen that

$$\min(E[w]) = p_{max}(d-k) + (1-p_{max})k.$$

Therefore, combining both cases, theorem can be obtained in a few simple steps. ■

Note that if the maximum allowable codeword weight is greater than or equal to d, Theorem 2 reduces to Theorem 1, which shows that Theorem 2 is a generalization of Theorem 1.

Another point to consider is that, if we use all zero codeword to represent the most likely source outcome (the case when $p_{max} > 0.5$ and $k \geq d$), we cannot distinguish if the transmitter sent data or remained silent, since both yield the same output unless synchronization exists. To provide reliability, we can put a minimum distance of d with silence case also for all the codewords. This forces us to choose $weight - d$ codewords for all the input symbols to obtain the minimum expected code weight, resulting in an expected codeword weight of d. However, as explained in Section 2.3, a micro node provides the synchronization signals for the nanosensors. Hence, we assume that all zero codeword can be distinguished from the silence.

Note that MEC only determines the weight enumerator, not the codebook. Hence, minimum energy codes are not unique, since multiple codebooks satisfying the desired distance exist.

2.4.2 Analytical Results and Performance Parameters

In this section, we analytically investigate MEC using MATLAB. Let p_i be the probability of event i and w_i be the weight of the corresponding codeword c_i. Power dissipated for i^{th} event is $P_i = w_i P_{sym}$, where P_{sym} is the symbol power. The average transmission power is

$$E(P) = \sum_{i=1}^{M} w_i p_i P_{sym} = E(w) P_{sym}. \tag{2.6}$$

Note that (2.6) also represents the average power per codeword and average power per $\log(M)$ bits, since each codeword carries $\log(M)$ bits of information. It is important to stress that for different source distributions, the amount of information per codeword will be different from an information theoretic point of view. However, for simplicity, we assume each codeword carries $\log(M)$ bits of information, leaving the information theoretic analysis to a future study.

We have developed MEC by keeping the codeword length unconstrained. Let us first investigate the required codeword length for MEC.

Minimum Codeword Length

n_{min} is the minimum codeword length required to satisfy the MEC weight enumerator for given M and d values. n_{min} is important since it yields the minimum delay required to transmit a codeword. $A(n,d,w)$ is the maximum number of codewords in a code with codeword length of n, code distance of d, and constant codeword weight of w.

1. *$p_{max} < 0.5$, d even:* Weight enumerator of MEC reduces to $W_{\mathbb{C}}(z) = Mz^{d/2}$. Therefore, $n_{min} = \min\{n : A(n,d,d/2) \geq M\}$. Since $1s$ in each codeword are disjoint, $n_{min} = \frac{Md}{2}$.

2. *$p_{max} < 0.5$, d odd:* From Theorem 1, we know that the corresponding weight enumerator is $W_{\mathbb{C}}(z) = z^{\lfloor \frac{d}{2} \rfloor} + (M-1)z^{\lceil \frac{d}{2} \rceil}$. $1s$ in all the codewords should be disjoint with the $1s$ in the most probable codeword, i.e., the codeword with weight $\lfloor \frac{d}{2} \rfloor$. Therefore, $n_{min} = \lfloor \frac{d}{2} \rfloor + \min\{\tilde{n} : A(\tilde{n}, 2m+1, m+1) \geq M-1\}$, where $d = 2m+1$. The following property is helpful [26]:

$$A(n,2m-1,w) = A(n,2m,w) \Rightarrow A(\tilde{n},2m+1,m+1) = A(\tilde{n},2m+2,m+1). \tag{2.7}$$

Therefore, $\min\{\tilde{n}\} = (m+1)(M-1)$. Hence, $n_{min} = m + (m+1)(M-1) = \lceil \frac{d}{2} \rceil M - 1$.

3. $p_{max} > 0.5$ In this case, MEC requires mapping the all-zero codeword to the most probable source event. Then we have, $W_{\mathbb{C}}(z) = z^0 + (M-1)z^d$. Minimum codeword length is found as $n_{min} = \min\{n : A(n,d,d) \geq M-1\}$. In the literature, there is no explicit formulation for

$A(n,d,d)$. We can use the existing lower bounds on the code size. From [26], we know that,

$$A(n,2m,w) = A(n,2m-1,w) \geq \frac{1}{q^{m-1}}\binom{n}{w} \Rightarrow A(n,d,d) \geq \frac{1}{q^{\lceil \frac{d}{2} \rceil - 1}}\binom{n}{d}, \tag{2.8}$$

where q is a prime power such that $q \geq n$.

The codewords for $p_{max} < 0.5$ and $d - even$ case can be constructed by cyclic shifting of a $d/2 - length$ block of $1s$ by an amount of $d/2$. Based on this cyclic shifting idea, we have developed a code construction scheme. In this approach, blocks of $1s$ are shifted by proper amounts to satisfy the Hamming distance with the previous codeword. The obtained minimum codeword length under such a construction is found as

$$n_{min} = d + (M-2)\left\lceil \frac{d}{2} \right\rceil. \tag{2.9}$$

Sample codebooks generated by this scheme can be found in Appendix. This construction achieves the minimum code length for $p_{max} < 0.5$ and $d - even$ since $1s$ should be disjoint. Unexpectedly, this scheme also achieves minimum codeword length for $p_{max} < 0.5$ and $d - odd$, since (2.9) reduces to n_{min} obtained for this case. However, obtained codeword length with this scheme is significantly greater than minimum codeword length for $p_{max} > 0.5$ case. For example, for $M = 112$ and $d = 8$, minimum codeword length of 27 is sufficient from (2.8), instead of $n = 448$, obtained from (2.9). However, to be able to numerically analyze the error performance of the code, and obtain results valid for all the p_{max} and d values, we use (2.9) in our analysis.

If the Hamming distance between the codewords is increased, more codeword errors can be corrected with minimum distance decoding. However, the codeword length of MEC also increases with the Hamming distance, which result in a larger number of error patterns. Therefore, increasing code distance does not necessarily increase reliability of MEC. Hence, analysis of error correcting capability of MEC for large Hamming distance is worth considering.

Error Resilience

The received n-tuples are mapped to the codeword to which they are closest in terms of Hamming distance. Then the probability that codeword is correctly decoded is

$$\xi_d = \sum_{i=0}^{\lfloor \frac{d-1}{2} \rfloor} \binom{n_{min}}{i} p_s^i (1-p_s)^{n_{min}-i}. \tag{2.10}$$

We want to observe the limiting behavior of probability of decoding the received codeword correctly as d tends to infinity. Thus, assuming very large and even d, without loss of generality,

$$\xi = \lim_{d\to\infty} \sum_{i=0}^{\frac{d}{2}-1} \binom{dM/2}{i} p_s^i (1-p_s)^{dM/2-i} = \lim_{n\to\infty} \sum_{i=0}^{\frac{n}{M}-1} \binom{n}{i} p_s^i (1-p_s)^{n-i}, \tag{2.11}$$

$$= \lim_{n\to\infty} 0.5 \left(1 - erf\left(\frac{n/M - 1 - np_s}{\sqrt{2np_s(1-p_s)}}\right)\right) \tag{2.12}$$

$$= \begin{cases} 1, & p_s < 1/M \\ 0, & p_s > 1/M \end{cases}. \tag{2.13}$$

Expression in the limit in (2.12) is the cumulative distribution function of Gaussian distribution with mean np and variance $np(1-p)$ and erf is the standard error function. Equality in (2.12) follows from that, for large n, binomial distribution can be approximated by Gaussian distribution with aforementioned mean and variance. Note that perfect communication can be achieved among nanosensor nodes and micro node, if source set cardinality is less than the inverse of symbol error probability, by keeping the Hamming distance sufficiently large. Hence, if symbol error probability is decreased, nanosensor readings can be quantized with smaller quantization steps.

The micro node utilizes coherent detection and hard decoding to detect the transmitted symbol. Therefore, symbol error probability is given as $p_s = 0.5\left[1 - erf\left((A^2/8\sigma_n^2)^{0.5}\right)\right]$, where A is the received signal level when symbol 1 is transmitted, and σ_n^2 is the noise power at the receiver. It is sufficient to consider the spreading loss only, since carrier frequencies are at the available frequency windows in the THz band, where molecular absorption is low. Interference created by other cells due to frequency reuse should be considered in the noise power calculation. Let S be the set of nodes interfering with node i. Then the received signal and noise power are

$$P_r = \frac{P_{sym}}{A(f,d)} = A^2/2 \text{ and } \sigma_n^2 = k_b TB + P_{sym} \sum_{i \epsilon S} \frac{1}{A(f_i,d_i)}, \tag{2.14}$$

where k_b, T, B are Boltzmann constant, temperature and bandwidth, respectively. $A(f,d) = (4\pi f d/c)^2$ is the frequency and distance-dependent loss term, where c is the speed of light.

Energy per Information Bit

Next, we obtain energy per information bit to demonstrate the energy efficiency of our coding scheme. Probability that a codeword is correctly decoded is

$$\xi_d \approx \frac{\text{\# of codewords correctly decoded}}{\text{\# of codewords received}}, \tag{2.15}$$

for a large number of transmitted codewords, for a code with distance d. If Q codewords are transmitted, then $\log(M)Q\xi$ bits of information is received. Average energy transmitted per codeword is $E_C = P_{sym}E(w)T$ joules, where T is the symbol duration. Then, the total energy dissipated for Q transmissions is $E_C Q$. Therefore, the average energy per bit is expressed as

$$\eta = \frac{E(w)P_{sym}T}{\log(M)\xi} \; \textit{joules/bit}. \tag{2.16}$$

Spectral Efficiency

Finally, we investigate spectral efficiency, which is one of the important parameters in a communication system. It is defined as the ratio of data rate to the bandwidth and yields how efficiently channel bandwidth is utilized. Spectral efficiency of MEC is

$$\nu = \frac{\xi \log(M)}{lTB} \approx \frac{\log(M)}{2l} \; \textit{bps/Hz}. \tag{2.17}$$

For single THz window, i.e., $l = 1$, the spectral efficiency is $\log(M)/2$. For approximation to be valid, correct decoding probability of MEC should be close to 1, implying a certain amount of delay depending on M, since $\xi \to 1$ for large code distance, hence, large codeword lengths.

Sample Codebooks of MEC

From Theorem 2.4.6, the minimum expected weight codes have the following weight enumerators.

$$W_{\mathbb{C}}(z) = \begin{cases} z^0 + (M-1)z^d, & p_{max} > 0.5 \\ z^{\lfloor \frac{d}{2} \rfloor} + (M-1)z^{\lceil \frac{d}{2} \rceil}, & p_{max} < 0.5. \end{cases}$$

Accordingly, sample codebooks for $d = 4$ can be generated as

$$\left\{\begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\cdots 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0\cdots 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0\cdots 0 & 0 & 0 & 0 \\ \vdots & \vdots & & & & & & & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\cdots 0 & 0 & 1 & 1 \end{matrix}\right\} \tag{2.18}$$

with $p_{max} < 0.5$ and

$$\left\{\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\cdots 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0\cdots 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0\cdots 0 & 0 & 0 & 0 \\ \vdots & \vdots & & & & & & & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\cdots 1 & 1 & 1 & 1 \end{matrix}\right\} \tag{2.19}$$

with $p_{max} > 0.5$. Similarly, for $d = 5$, sample codebooks are

$$\left\{\begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\cdots 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0\cdots 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1\cdots 0 & 0 & 0 & 0 \\ \vdots & \vdots & & & & & & & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\cdots 0 & 1 & 1 & 1 \end{matrix}\right\} \tag{2.20}$$

with $p_{max} < 0.5$, and

$$\left\{\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\cdots 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0\cdots 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1\cdots 0 & 0 & 0 & 0 \\ \vdots & \vdots & & & & & & & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\cdots 1 & 1 & 1 & 1 \end{matrix}\right\} \tag{2.21}$$

with $p_{max} > 0.5$. Number of rows in the codebooks gives the number of codewords, and first row is the codeword mapped to the outcome with probability p_{max}.

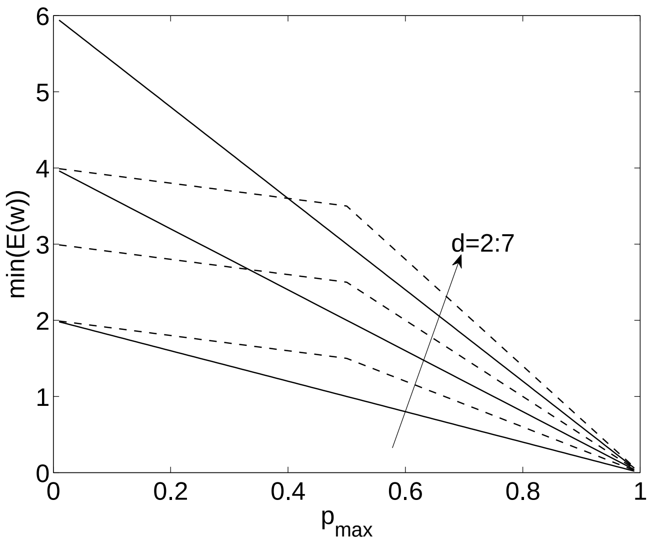


Figure 2.2: p_{max} vs. $\min(E(w))$ for d=2:7 (even: solid, odd: dashed).

2.5 Performance Evaluation and Discussions

In this section, we investigate error correction capability and energy-efficiency of MEC via numerical evaluations in MATLAB. Currently, a prototype nanosensor node is not available, hence we evaluate performance via numerical evaluation of analytical parameters. An (n,k) code maps 2^k sourcewords into $length-n$ channel codewords. For comparison, we use MEC with $M = 2^k$. MEC is compared with the (7,4), (15,11) Hamming, (21,6) binary Reed-Solomon and (23,12) Golay codes. As the Hamming and Golay codes are compared with MEC in terms of average code weight and decoding probability, comparison with the Reed-Solomon code is limited to the expected code weight due to space requirements. The Hamming codes are distance-3 codes, and can correct 1 bit error whereas the Golay code is distance-7 and can correct 3 bit errors. The minimum distance of (21,6) binary Reed-Solomon code is known to be 6. The achievable rate of the nanosensor node is shown to be currently limited by the state-of-the-art power and energy limitations of CNT antennas and nano energy harvesting systems. Lastly, interference limited maximum number of quantization levels in the nanosensor node is obtained, in a cell-based WNSN with MEC. It is shown through simulations that coverage ratio of $\alpha < 1$ allows MEC to support a large number of quantization levels at the WNSN node.

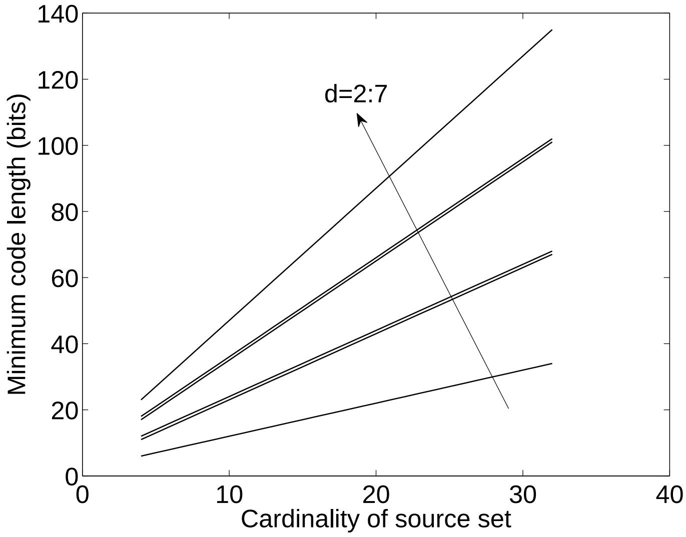


Figure 2.3: Minimum required codeword length - n, for d from 2 to 7.

2.5.1 Performance of Minimum Energy Coding

2.5.1.1 Minimum weight and code length of MEC

Variation of $\min(E(w))$ as a function of p_{max}, as given in (2.2), is shown in Fig. 2.2. As observed, increasing d might decrease the expected code weight for different intervals of p_{max}. Therefore, since energy-efficiency is the primary concern in nano communications, a minimum distance of $d+1$ can be chosen, instead of d to reduce the average code weight, depending on the value of p_{max}.

Code length variation with related parameters is illustrated in Fig. 2.3. It is shown that, increasing the distance, i.e., d by 1 does not increase required code length much for odd values of d. The corresponding delay overhead can be obtained by simply multiplying the codeword length with the symbol time, since no ARQ scheme is employed. For $T = 10$ psec., delay is on the order of nanoseconds for moderate M and d values.

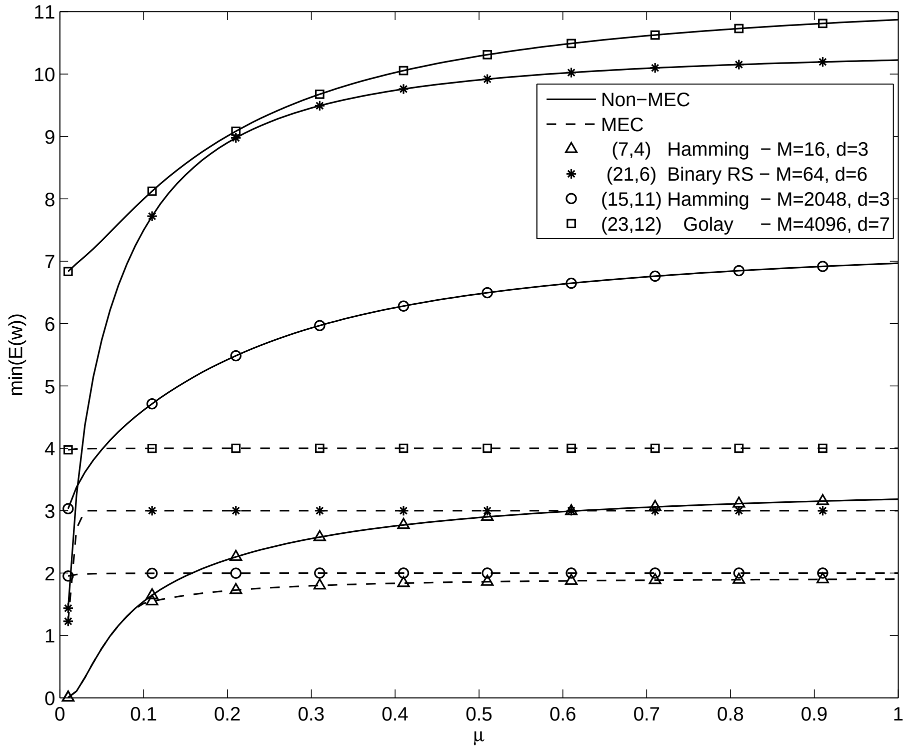
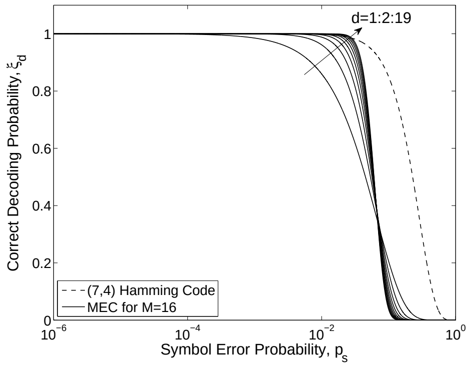


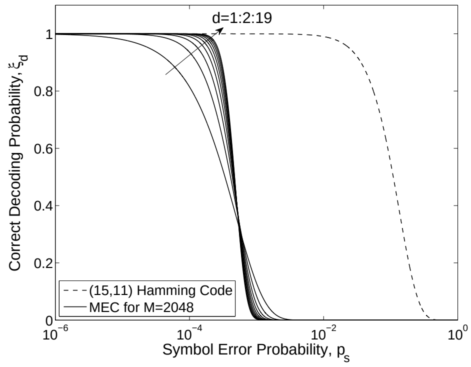
Figure 2.4: Minimum code weight vs. source mean for (7,4), (15,11) Hamming, (21,6) Binary Reed-Solomon and (23,12) Golay code and corresponding MEC

2.5.1.2 Average code weight vs. source distribution

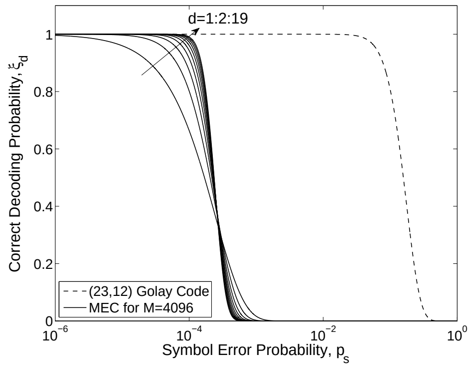
Performance of MEC is compared with the (7,4), (15,11) Hamming, (21,6) binary Reed Solomon and (23,12) Golay codes in Fig. 2.4 in terms of expected code weight. In order to reach minimum code weight for the Hamming, Reed-Solomon and Golay codes, more probable source symbols are assigned to codewords with less weight, using weight enumerators of these codes. We have utilized the binary expansion of 8-ary (7,2) Reed Solomon code for which a sample weight enumerator is given in [28]. We use normalized discrete samples of an exponential pdf with varying mean - μ in a fixed interval, to generate the discrete distributions with different variances. It is clear from Fig. 2.4 that MEC is superior; in other words, codes compared with MEC are not as efficient in terms of mean energy per codeword. Performance gap of the codes closes as μ, i.e., variance of discrete distribution is decreased, which increase p_{max}. This is expected, since all the codes contain the all-zero codeword,



(a)



(b)



(c)

Figure 2.5: Codeword decoding probability at the receiver for (7,4), (15,11) Hamming and (23,12) Golay codes and MEC with odd distances from 1 to 19.

which mainly determines minimum average weight for large values of p_max. As observed, if μ exceeds a threshold, corresponding to $p_{max} < 0.5$ region, MEC clearly outperforms the existing schemes due to the abrupt change of weight distribution of MEC.

2.5.1.3 Correct codeword decoding vs. symbol error

Codeword decoding performances of the MEC, Golay and Hamming codes are illustrated in Fig. 2.5. The proposed code is not as effective as the other codes in error correction. This is due to the vast difference in codeword lengths. Lengthy codes have more uncorrectable error patterns, which decreases the error correction probability for the same Hamming distance. It is observed that, as code distance is increased, probability of correct decoding tends to 1, if symbol error probability, p_s, is less than the inverse of source set cardinality, $1/M$, and tends to 0 otherwise, verifying (2.13). Intuitively, increasing M increases the amount of information

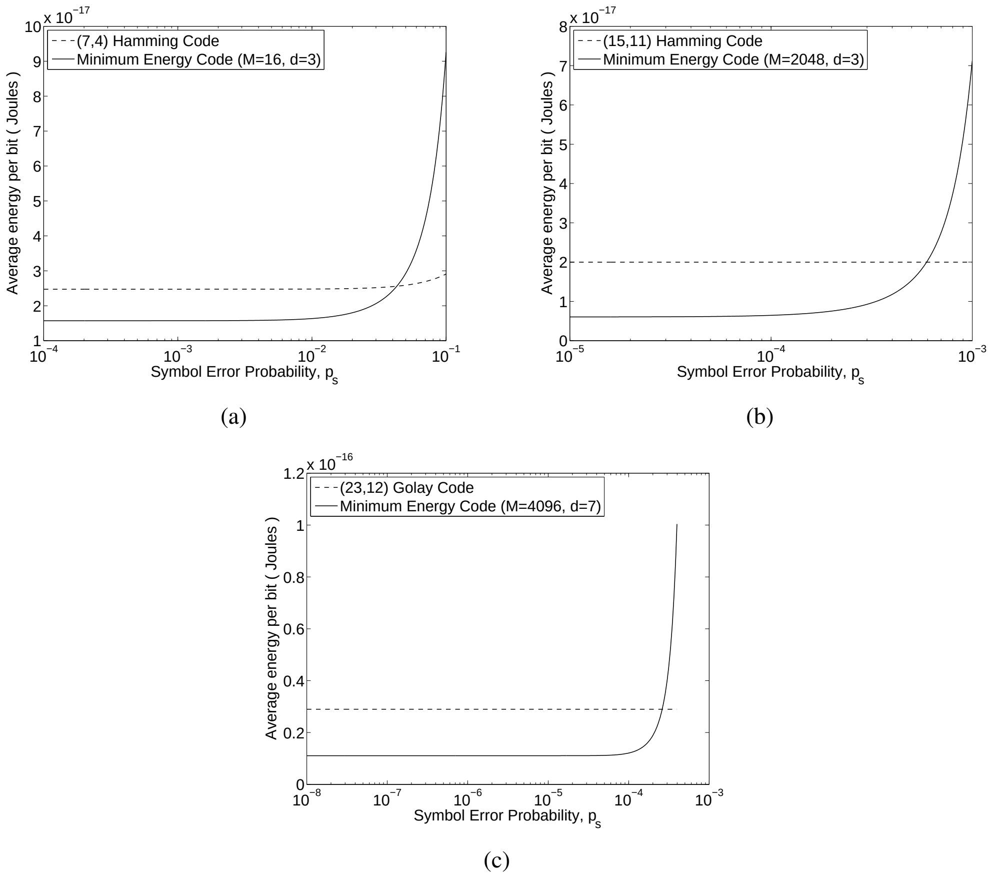


Figure 2.6: Average energy per bit comparison between (7,4), (15,11) Hamming codes, (23,12) Golay code and MEC.

to be transmitted, which requires more reliable channels.

2.5.1.4 Energy efficiency vs. symbol error

Lastly, we demonstrate the energy efficiency of MEC as given in (2.16). The average energy per received bit, i.e., η, is shown in Fig. 2.6 for a symbol energy of 10^{-5} pJ, which is justified in Section 2.5.2. Samples of a Gaussian distribution with $\sigma = 0.5$ are taken and normalized. η is calculated for each case separately using (2.16). MEC is better in terms of average energy per bit for symbol error probabilities less than a threshold. As p_s exceeds the threshold, average energy per bit exponentially increases, since correct codeword decoding is unlikely. Note that the observed behavior is dominated by $1/\xi$ factor in (2.16).

2.5.2 Achievable Rate of WNSN Nodes

In this section, we investigate the feasibility of MEC for WNSN nodes, using state-of-the-art power and energy limits in the nano-domain. It is theoretically calculated in [31] that a CNT antenna can radiate EM waves with power up to $5\mu W$. We allocate the available power equally to each CNT antenna. In [16], the authors investigate an ultra-nano capacitor to store energy obtained from piezoelectric nano-generator energy harvesting system. Up to 800 pJ of energy can be stored in the capacitor. Charging time for the capacitor depends on the frequency of vibration that the nanonode is exposed to. In order to charge nano capacitor with 100 pJ of energy, 160 cycles are required. If nodes gather energy from a 50 Hz source, such as a vent, 160 cycles correspond to 3.2 seconds. T, i.e., symbol time is 10 picoseconds due to the proposed modulation. Therefore, symbol energy is constant and equals to $\varepsilon_{sym} = P_{sym}T = 10^{-5}pJ$.

Therefore, a nanosensor node can transmit 10^7 high symbols in 3.2 seconds. Finding the amount of information that can be carried with 10^7 high symbols, we can calculate the achievable transmission bit rate of a nanosensor node. Let $p_{max} < 0.5$ and d be even for simplicity. Then, using (2.2) and (2.6), $\log(M)$ bits of information is carried with codewords of average energy $d\varepsilon_{sym}/2$ on the average. Hence, average transmission rate is limited by

$$R = \frac{2\times 10^7 \log(M)}{3.2d} = 6.25 \times \frac{\log(M)}{d} Mbps. \tag{2.22}$$

Note that it takes n/l symbol times to transmit a single codeword where l is number of carriers. This sets another limit on the transmission rate, since

$$R < \frac{\log(M)l}{n_{min}T} = \frac{\log(M)l}{(d+(M-2)\lceil d/2\rceil)T}. \tag{2.23}$$

This bound is illustrated in Fig. 2.7 for $l = 5$. Comparing the results in Fig. 2.7 and (2.22) shows that energy budget currently available at the nanonode limits its rate, rather than the codeword length. As a result, codeword length, which is the major drawback of MEC, does not limit available transmission rate. Moreover, since the rate is limited by available energy, MEC provides the maximum information rate, as it minimizes the energy per codeword. As illustrated in Fig. 2.7, code length allows transmission rates up to 10s of Gbps.

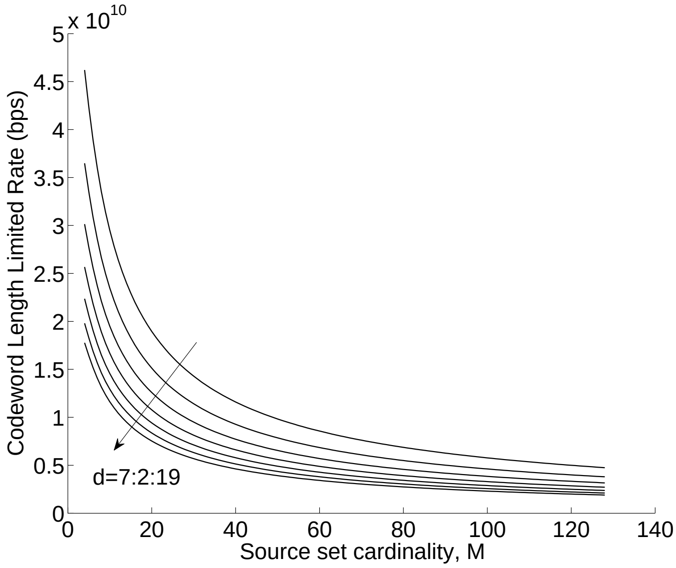


Figure 2.7: Code length limited rate of nanonode in bps.

2.5.3 Effect of Interference on Quantization at WNSN Node

The cell-based WNSN discussed in Sec. 2.3.2 is used to investigate the effects of interference on the nanosensor node's maximum number of quantization levels, i.e., M in WNSNs. We assume a frequency reuse ratio of 1/4. As explained, a TDMA-based scheme, in which channel use times are allocated by the central micro node, is assumed within each cell. As a result, at most one nanonode transmits at any time instant inside a cell, mitigating the intra-cell interference. Additionally, interference from the other cells using the same set of frequencies is only due to a single nanonode, which is active at the time of transmission. This leads to an analysis, independent from either the size of the network or the nanonode density. An approximate formula for the interference at the micro node is utilized. The effects of 50 closest cells using the same set of frequency bands are considered, which is sufficient, since interference power is inversely proportional with the square of the distance. Noise is the thermal noise as in (2.14), however, its effect is negligible compared to the interference. Channel loss is spreading loss only, since available frequency windows are utilized with low absorption. We assume $l = 5$ with frequencies 0.1, 0.3, 1, 1.5 and 2 THz. All the channels contribute to average symbol error probability equally.

We consider the worst case scenario by assuming that the transmitting node in each interfering

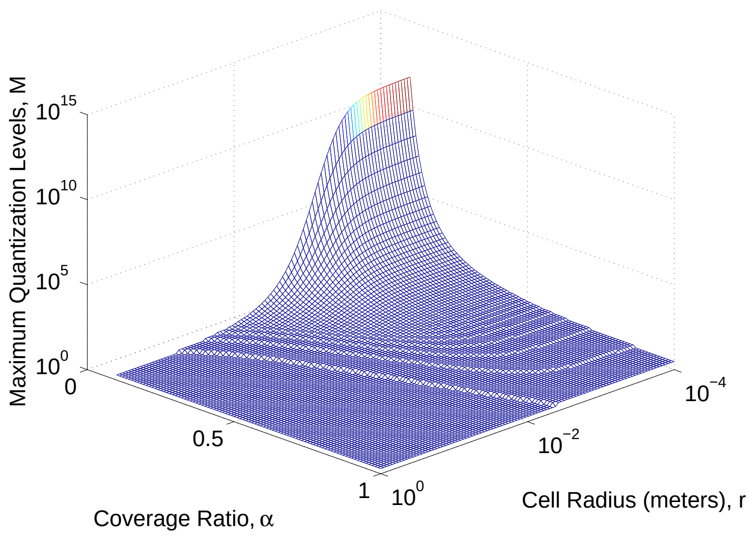


Figure 2.8: Maximum number of quantization levels at the nanonode for MEC.

cell is as close as possible to the node in consideration. A cell coverage ratio of α is utilized to decrease the interference. It is assumed that the decoder at the micro node conducts coherent detection of the received signal with hard decoding, since the time instant that nanonode initializes its transmission is declared by the micro node. As shown, perfect transmission can be achieved with MEC, if $M < 1/p_s$. Interference limited maximum of M, obtained from the symbol error probability is shown in Fig. 2.8. As observed, employing a coverage ratio of $\alpha < 1$, a large number of quantization levels can be achieved. With more optimistic assumptions, like cooperation between micro nodes to reduce the interference, improved results could be obtained. Therefore, MEC can be used to achieve perfect communications in a cell-based WNSN employing cell coverage with cell radius up to several millimetres.

CHAPTER 3

ON THE NODE DENSITY LIMITS AND RATE DELAY ENERGY TRADEOFFS IN AD-HOC NANONETWORKS

Ad-hoc nanonetworks are collections of nanonodes without central controller units, and are the most promising network architectures in nano communications. Derivation of maximum nanonode density can pave the way for determining the capacity of ad-hoc nanonetworks. We consider ad-hoc nanonetworks with minimum energy coding (MEC). Maximum nanonode density for reliable communication in an ad-hoc nanonetwork without any medium access control is derived, and density dependent reliability analysis is conducted. Rate-delay-energy tradeoffs are also investigated with achievable rates, with constant codebook size and constant Hamming distance, separately.

The rest of this chapter is organized as follows. In Sec. 3.1, motivation and related work are presented. In Sec. 3.2, a review of minimum energy coding (MEC) is provided and important results are underlined. In Sec. 3.3, the ad-hoc nanonetwork description is provided with the assumptions on the nanonodes and the network. Later, we analyze the upper limit of node density for the nanonetwork together with its effects on reliability in Sec. 3.4. Besides, rate-delay-energy tradeoffs for constant code distance and constant source set cardinality are investigated.

3.1 Motivation and Related Work

Nanonetworks, composed of a large number of nanonodes are believed to find application in various fields, from healthcare to industrial applications [2]. In nano-EM communications, electromagnetic waves are used as information carrier. Hence, it is similar to classical communications used in our daily lives. Due to extremely small size, new materials specific to nano-domain, such as carbon nanotubes (CNTs), should be used. In order to address novel characteristics of nanodevices, development of new communication techniques is required.

In the literature, there exist several studies on nanonetworks [23, 18, 5, 15]. The first work on ad-hoc nanonetworks is presented in [5]. Authors investigated the challenges to be addressed for the realization of ad-hoc nanonetworks. In [15], authors showed that without any medium access control mechanism, low weight channel codes can be used for communication in nanonetworks, together with OOK modulation, without considering any specific coding scheme. To address the severe energy efficiency requirement, we proposed novel minimum energy channel codes (MEC), for reliable nanoscale communications in [18]. We have considered a nanosensor network, in which time or frequency resources are orthogonally allocated, and investigated its feasibility. However, nanonetworks with central controller units might not always be feasible. Therefore, investigation of ad-hoc nanonetworks is an open research problem requiring further investigation. To the best of authors' knowledge, no channel coding scheme, guaranteeing reliability is proposed for ad-hoc nanonetworks.

In this chapter, we propose using MEC for reliable communication in ad-hoc nanonetworks [20]. A probabilistic analysis is conducted to show that, reliable communication is possible in ad-hoc nanonetworks without any medium access scheme. We investigate the maximum node density, that allows perfectly reliable communication, with increasing code distance. Relation between nanonode density of ad-hoc network and reliability is revealed through simulations. We also investigate rate-delay-energy tradeoffs of ad-hoc nanonetworks with MEC. It is shown that, for maximally dense networks, increasing delay using larger source set cardinality increases the rate. Achievable rates for sufficiently large code distance and source set cardinality are also analytically derived.

3.2 Minimum Energy Codes

In Chapter 2, we have proposed novel minimum energy codes with controllable reliability to be used for nano communications. In this section, we review the basic results obtained in that chapter and highlight the parts that will be used in this chapter.

Similar to Chapter 2, p_{max}, d and M represent the probability of most likely source outcome, code distance, and source set cardinality, respectively. As a precondition of the proposed minimum energy codes, an OOK modulation scheme with l carriers, each located at allowable frequency bands in the THz channel is proposed. In this chapter, we assume $l = 1$ for simplicity. The analysis can be easily generalized to any l.

3.2.1 Minimum Expected Weight

In classical communication scenarios, both source coding and channel coding schemes are employed in general. Source coding reduces redundancies in the source symbols, whereas channel codes are used to add redundancy to combat channel noise and other detrimental factors. To keep complexity low, our proposed technique do not employ any source coding mechanism. Another advantage of not using source coding is that, codewords to be used are not equiprobable in general, which can be exploited to reduce energy consumption on the average, using minimum energy codes. Our proposed minimum energy codes, MEC, yields the minimum average code weight, depending on the source distribution and desired reliability via Hamming distance. From Chapter 2, we know that

$$\min(E(w)) = \begin{cases} (1-p_{max})d, & p_{max} > \frac{1}{2}, \\ \frac{d}{2}, & p_{max} < \frac{1}{2}, \text{ if } d \text{ even} \\ \lceil \frac{d}{2} \rceil - p_{max}, & p_{max} < \frac{1}{2}, \text{ if } d \text{ odd.} \end{cases} \tag{3.1}$$

3.2.2 Codebook Generation

Although different approaches exist to generate a minimum energy codebook, a simple and tractable scheme is proposed. With this approach, minimum codeword length is obtained as

$$n_{min} = d + (M-2)\left\lceil \frac{d}{2} \right\rceil. \tag{3.2}$$

To ease the analysis, we stick with the assumptions in the previous chapter, and assume $p_{max} > 0.5$ and d is even.

A sample codebook generation can be given as follows: If p_{max} is greater than 0.5, all 0 codeword should exist in the codebook and be mapped to the most probable source outcome.

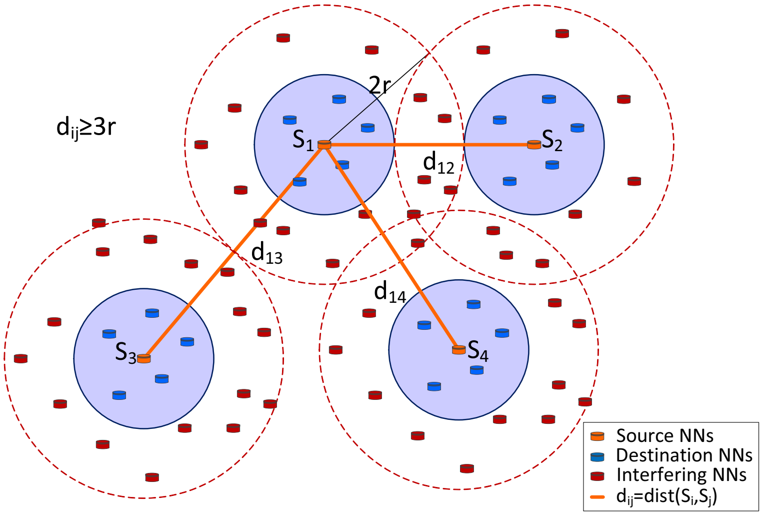


Figure 3.1: Ad-hoc Nanonetwork with potential destinations and potential interfering nanonodes

In order to assure minimum Hamming distance, other codewords should be $weight-d$, since code distance is desired to be d. Codeword selection is more straightforward if $p_{max} < 0.5$. Let $\mathbf{1}_k$ and $\mathbf{0}_k$ represent a $length-k$ block of ones, and a $length-k$ block of zeros, respectively. Then the rows of the following matrix are codewords generated by the proposed mechanism for $p_{max} < 0.5$ and right hand side of the equation yields the case for $d = 4$:

$$\begin{Bmatrix} \mathbf{0}_0 & \mathbf{1}_{\frac{d}{2}} & \mathbf{0}_{n-\frac{d}{2}} \\ \mathbf{0}_{\frac{d}{2}} & \mathbf{1}_{\frac{d}{2}} & \mathbf{0}_{n-d} \\ \mathbf{0}_d & \mathbf{1}_{\frac{d}{2}} & \mathbf{0}_{n-\frac{3d}{2}} \\ \vdots & \vdots & \vdots \end{Bmatrix} = \begin{Bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & & & & & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 1 \end{Bmatrix}$$

Note that $\mathbf{0}_0$ represents null vector with dimension 0.

3.3 Nanoscale Ad-hoc Networks

The term ad-hoc network is in general used to represent networks with no central controllers, opposed to cellular networks. These networks are studied in the literature for many years, under different assumptions such as mobile nodes, or multi hop communications. Ad-hoc nanonetworks are first considered in [5]. In that work, authors illustrate the challenges for the realization of CNT-based nanoscale ad-hoc networks.

The ad-hoc network considered in this work is given in Fig. 3.1. Nanonodes do not forward codewords, different from classical ad-hoc networks, due to complexity considerations. A source nanonode attempts communication with a nanonode within its range of operation, r. Transmission range is used to obtain the maximum node density in the ad-hoc nanonetwork. Destination nodes attempt to decode as many codewords as possible, without considering the source nanonode. Hence, no link is established between source and destination nanonodes. Such a scenario applies to nanosensor networks, in which a destination nanonode collects as much data as possible.

Errors are assumed to be due to collisions only, which is justified by keeping symbol error probability below 10^{-9}, by choosing the proper transmission distance for fixed transmission power. Similar to our previous work [18], a CNT antenna is set to dissipate a power of $5\mu W$, which is currently the largest power level a CNT antenna can support [31]. In the symbol error probability calculation, only path loss and thermal noise are included as detrimental factors. Molecular absorption and molecular noise is ignored, since the selected modulation scheme uses allowable frequency windows in the THz band, in which molecular effects are negligible. Nanonodes in the interference range are assumed to be distributed within an area of $2\pi(2r)^2$, where r is the maximum distance, at which error probability of OOK modulated symbols does not exceed 10^{-9}. Since the transmission range of a nanonode is r, a destination within range can be compromised by nodes within $2r$ range. Hence, maximum number of nanonodes supplied by MEC should be distributed within a range of $2r$. Choosing the band of operation as 1 THz and bandwidth as 10 GHz, r is easily found to be equal to 10^{-3} meters.

3.4 Limits and Tradeoffs in Ad-hoc Nanonetworks with MEC

Nanonetworking is a promising field of research. Feasibility of nanonetworks depends on energy efficiency, robustness against highly dynamic conditions and self sustainability. It might not always be possible to implement central controller units in certain nanonetworking scenarios. In such cases, the communication scheme should provide reliable access to the channel. However, using complex medium access techniques is not feasible in nanonodes due to the limited complexity. Moreover, popular spread spectrum multiple access techniques such as CDMA cannot be used, since the THz channel shows frequency selective characteristics, which would result severe distortion of the signal, when passed through the channel. It is proposed in [15] that, using low weight channel codes might drop the necessity of a medium access scheme. In this chapter, the same idea is employed for the analysis of ad-hoc nanonetworks using MEC. It is expected that, as more and more nodes communicate with each other within the transmission range of a source node, successful communication probability decreases. For nanonetworks using MEC, the following question is answered: *"What is the maximum node density an ad-hoc nanonetwork with MEC can supply without compromising reliability?"*. Discovering the maximum nanonode density is important, since it indicates the communication and computation capability of a nanonetwork within a given amount of area. It is shown that, reliable communication can be achieved in ad-hoc nanonetworks satisfying $s < 1/p$, where p is the transmission probability and s is the number of nanonodes within the interference range of source. Furthermore, rate-delay tradeoffs for ad-hoc nanonetworks are investigated. The question, *"What is the achievable rate in ad-hoc nanonetworks for sufficiently large delay?"*, is answered. It is shown that, different rate expressions emerge for constant M or d.

3.4.1 Maximum Node Density vs. Reliability

First, we analyze the successful codeword decoding probability at the destination nanonode. It is assumed that a nanonode attempts transmission with probability of p. Instead of having nanonodes transmitting continuously, nodes transmitting only when they require, not only reduces interference, but also the energy consumption. s is the number of nodes within the interference range of the source nanonode. It is assumed that s neighbor nanonodes exist within a distance of $2r$, when nanonodes are uniformly distributed with density of

$$\rho = \frac{s+1}{2\pi(2r)^2}. \tag{3.3}$$

To assure reliability, the correct codeword decoding probability is derived. Probability that the transmitted bit is 1 is

$$\eta_1 = \sum_{i=1}^{M} p(1|c_i)p(c_i) \tag{3.4}$$

$$= \sum_{i=1}^{M} \frac{w_i}{n_{min}} p_i = \frac{E(w)}{n_{min}}, \tag{3.5}$$

using the law of total probability, where w_i is the weight of the codeword c_i that is mapped to source with probability p_i. In order the transmitted bit to be received by the destination properly, interfering nodes should either transmit 0 bit or not transmit at all. The probability that there is at least single node transmitting bit 1 within the interference range, i.e., p_x is

$$p_x = 1 - \sum_{i=0}^{s} \binom{s}{i} (p\eta_0)^i (1-p)^{(s-i)} \tag{3.6}$$

$$= 1 - (1 - p(1-\eta_0))^s, \tag{3.7}$$

where $\eta_0 = 1 - \eta_1$, i.e., the transmission probability of bit 0. Collision probability is non-zero only if 0 is transmitted by the source. If 1 is transmitted, as interference caused by other nodes cannot flip this bit, collision cannot occur. As a result, collision probability of the nanonode can be calculated as

$$p_c = \eta_1 0 + \eta_0 p_x = \eta_0 - \eta_0 (1 - p(1-\eta_0))^s. \tag{3.8}$$

As previously discussed, symbol error probability is assured to be less than 10^{-9}, by choosing the transmission range. Hence, collision can be considered as the only error source, leading to simplified analysis of correct decoding probability. Eventually, since a maximum of $\lfloor \frac{d-1}{2} \rfloor$ collisions can be corrected, correct decoding probability at the destination, ξ_d, can be written as

$$\xi_d = \sum_{i=0}^{\lfloor \frac{d-1}{2} \rfloor} \binom{n_{min}}{i} p_c^i (1-p_c)^{n_{min}-i}. \tag{3.9}$$

A similar expression as (3.9) is obtained in our previous work [18], and using a similar analysis, it can be concluded that, ξ_d converges to 1 with increasing Hamming distance, if $p_c < 1/M$. In other words, reliable communication can be achieved if collision probability is less than the inverse of source set cardinality. For simplicity, assume $p_{max} < 0.5$ and d even. Using (3.1), (3.2), (3.5) and (3.8), we reach to the condition

$$1 - \left[1 - \frac{p}{M}\right]^s < \frac{1}{M-1} \tag{3.10}$$

$$p < M \left[1 - \left(\frac{M-2}{M-1}\right)^{1/s}\right]. \tag{3.11}$$

(3.11) gives the relation between p, M, and s, i.e. node transmission probability, source set cardinality, and number of neighboring nodes, for which reliable communication is possible with MEC. It is easily seen that, expression on the right hand side of (3.11) decreases with increasing M. Hence, we can take the limit of (3.11) to obtain an upper bound for p, valid for any M, to achieve reliable communication.

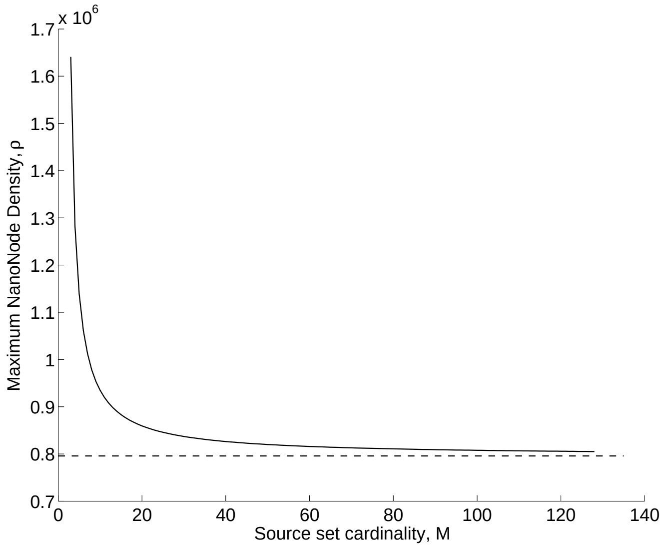


Figure 3.2: Maximum allowed node density vs. source set cardinality for p=0.05. Limiting density is also indicated at $\rho_{lim} = 1/(pA_{tr})$

$$
\begin{aligned}
p_{lim} &< \lim_{M\to\infty} M\left(1-\left(1-\frac{1}{M-1}\right)^{\frac{1}{s}}\right) \\
&= \lim_{M\to\infty} M\left(1-\sum_{k=0}^{\infty} C\left(\frac{1}{s},k\right)\left(\frac{-1}{M-1}\right)^{k}\right) \\
&= \lim_{M\to\infty} \sum_{k=1}^{\infty} C\left(\frac{1}{s},k\right)\frac{(-1)^{k+1}M}{M-1^{k}} = \frac{1}{s},
\end{aligned}
\tag{3.12}
$$

where the general binomial coefficient for $k > 0$ is

$$
C\left(\frac{1}{s},k\right) = \frac{1}{k!}\left(\frac{1}{s}\right)\left(\frac{1}{s}-1\right)\cdots\left(\frac{1}{s}-k+1\right). \tag{3.13}
$$

It is concluded that $p < 1/s$ is the required condition, satisfying (3.11) for any M. Hence, MEC supplies an ad-hoc nanonetwork having s neighbor nodes, only if nodes transmit with probability less than $1/s$. Interpreting the result from the other side leads to the desired bound for maximum node density. Maximum number of nodes within the interference range should be less than $1/p$. This corresponds to the density

$$
\rho_{max} = \frac{\lfloor 1/p \rfloor + 1}{A_{tr}} = \frac{\lfloor 1/p \rfloor + 1}{8\pi} \times 10^{6} m^{-2}, \tag{3.14}
$$

where A_{tr} is the transmission area. The result is verified by simulation results as illustrated in Fig. 3.2.

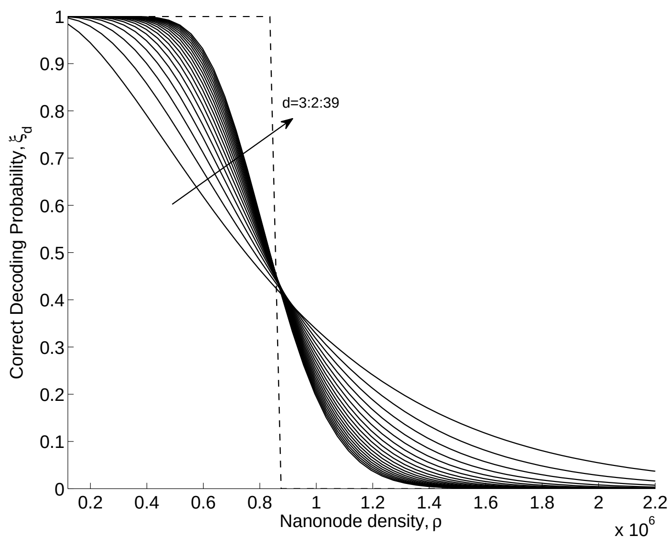


Figure 3.3: Correct decoding probability vs number of source nodes for p=0.05

Variation of reliability with respect to node density is shown in Fig. 3.3. As claimed in the analytical evaluations, reliability can be increased by increasing Hamming distance of the channel code if node density is below a threshold, corresponding to $1/p$ number of neighboring nodes. This shows that, MEC with large delays can compensate the absence of a medium access control scheme up to nanonode density of ρ_{max}.

3.4.2 Rate Delay Energy Tradeoffs in Ad-Hoc Nanonetworks

Rate-delay tradeoff of networks is vastly investigated in the classical communication scenarios. In most communication scenarios, capacity is achieved with infinite delay. This makes the rate-delay tradeoffs worthy of investigation, to uncover the rate achievable with finite delay. The same idea applies in nanonetworks using MEC. Different from classical scenarios, energy dissipation should also exist in this picture, as it is one of the most important metrics for nano communications. As no retransmission or channel contention exists, delay is solely due to codeword length, ignoring propagation delay, since transmission range of nanonodes is very small. Hence, delay is given as

$$\delta = n_{min} T, \tag{3.15}$$

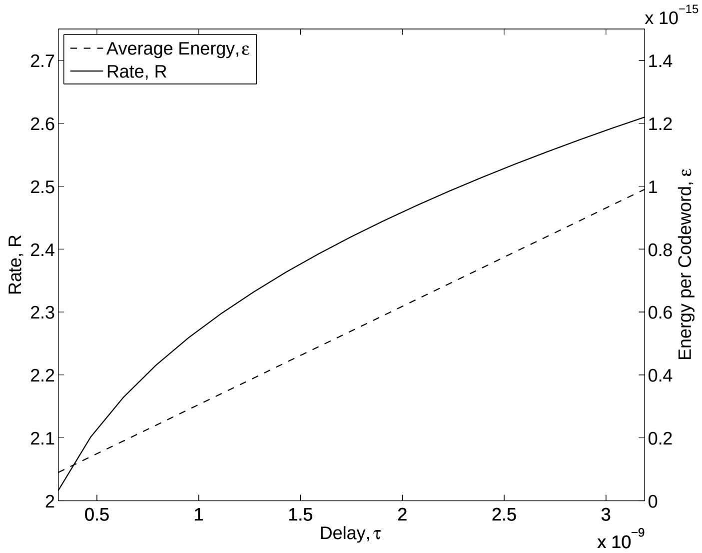


Figure 3.4: Rate-delay-energy tradeoffs for $M = 16$, $p = 0.01$.

where T is the symbol duration. Due to the used modulation technique, $T = 10\ ps$. Additionally, we provide a different rate definition as

$$R = \xi_d \log M. \tag{3.16}$$

(3.16) is the expected amount of information correctly decoded at the destination nanonode. Without an information theoretic approach, this definition provides a simple and insightful way to investigate rate-delay tradeoffs in ad-hoc nanonetworks. Average energy per codeword depends on average weight, since OOK-based modulation at available THz windows is used. It is formulated as,

$$\varepsilon = E(w)PT, \tag{3.17}$$

where P is the symbol power. Due to the power limits of CNTs, $P = 5\ \mu W$.

As given in (3.2), codeword length depends on d and M. We use two different approaches to reveal rate-delay tradeoffs. In both approaches, $p = 1/s$ is assumed.

3.4.2.1 Rate-Delay-Energy Tradeoffs, Constant M

Increasing code distance, d, increases the delay via codeword length according to (3.2). Since increasing d makes communication more reliable, rate is expected to increase with delay. The

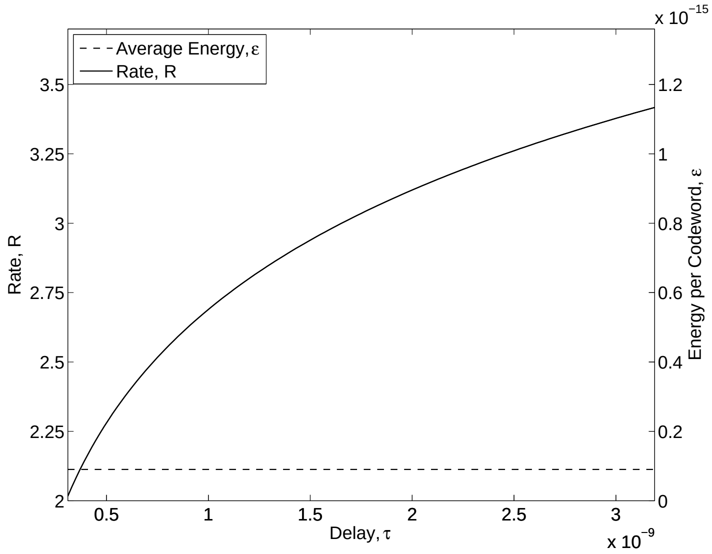


Figure 3.5: Rate-delay-energy tradeoffs for $d = 3$, $p = 0.01$.

variation of rate delay and energy with increasing Hamming distance is observed in Fig. 3.4. Convergence of rate-delay curve to $\log M$ is expected, since

$$\lim_{d \to \infty} R = \lim_{d \to \infty} \xi_d \log M = \log M. \tag{3.18}$$

As expected code weight increases with d, average energy per codeword also increases. This result indicates that, larger delays via increasing Hamming distance, can make communication reliable for a maximum rate of $\log(M)$.

3.4.2.2 Rate-Delay-Energy Tradeoffs, Constant d

Increasing source cardinality, M, increases the delay via codeword length according to (3.2). The question that whether increasing M provides more reliable communication or not is more ambiguous. Under some simplifying assumptions, we can reach an approximate expression for collision probability, which can later be used to approximate correct decoding probability. Under the condition that d is even and $p_{max} < 0.5$, using the fact that $\eta_1 = 1/M$, for sufficiently large s,

$$p_x = 1 - (1 - \frac{p}{M})^s \approx 1 - e^{-1/M}, \tag{3.19}$$

This leads to,

$$p_c \approx \frac{M-1}{M}(1-e^{-1/M}). \tag{3.20}$$

Since, $n \gg \left\lfloor \frac{d-1}{2} \right\rfloor$, from (3.9), we can write

$$\begin{aligned} \xi_d &\approx (1-p_c)^n \sum_{i=0}^{\left\lfloor \frac{d-1}{2} \right\rfloor} \left(\frac{np_c}{1-p_c} \right)^i \frac{1}{i!} && (3.21) \\ &\approx e^{-\frac{d}{2}} \sum_{i=0}^{\left\lfloor \frac{d-1}{2} \right\rfloor} \left[\frac{d}{2}(M-1)(e^{\frac{1}{M}}-1) \right]^i \frac{1}{i!}. && (3.22) \end{aligned}$$

(3.22) is obtained using the expression in (3.20). In the limit,

$$\lim_{M\to\infty} \xi_d \approx e^{-\frac{d}{2}} \sum_{i=0}^{\left\lfloor \frac{d-1}{2} \right\rfloor} \left(\frac{d}{2} \right)^i \frac{1}{i!} = \xi'. \tag{3.23}$$

From (3.23), as M increases, ξ_d converges to a nonzero probability value. Therefore, for very large M,

$$R \approx \xi' \log M. \tag{3.24}$$

This result reveals that, by increasing the source set cardinality at the source nanonode, one can increase communication rate and still satisfy reliable communication with high probability, for large delays. The variation of rate delay and energy with increasing source set cardinality is observed in Fig. 3.5, agreeing with theoretical results. Average energy does not increase with delay, since $E(w)$ does not depend on M.

CHAPTER 4

ENERGY MINIMIZATION WITH NETWORK CODING

Network coding is mostly used to achieve the capacity of communication networks. In this chapter, for the first time in the literature, we design energy minimizing network codes to achieve capacity. Assuming that the 0 bit symbol has no energy cost, average energy at each edge in the network is minimized with minimum energy channel coding (MEC). Minimum energy network code (MENC) provides the best mapping between the input and output symbols at the coding node in terms of average codeword energy. We define the class of networks composed of coding nodes with N incoming and 1 outgoing symbols as In-N networks. First, we derive the condition on the network code to minimize the average energy in In-Two networks and propose two linear MENCs. Then, we investigate the two well-known examples of In-Two networks, i.e., two-way relay and butterfly networks, and obtain total average energy expressions. Second, we investigate the network codes minimizing the energy in In-N networks. Since the minimum energy network code for In-Two networks cannot be generalized to In-N networks, we propose a low energy network code (LENC) to reduce the energy. We compare MENC and LENC with the classical XOR and random network codes in In-Two and In-Three networks, and show that the proposed network codes provide significant energy gains.

The remainder of this chapter is organized as follows: In Section 4.1 motivation behind this chapter is presented. In Section 4.2, the existing literature on the energy efficient network coding is explained together with the motivation of the chapter. In Section 4.3, we present the network model and the notations. Later, the minimum energy channel code, which is the underlying channel coding at each edge to minimize energy, is explained in Section 4.4. In Section 4.5, we discuss the networks composed of coding nodes with two edges, i.e., In-Two networks, and develop the minimum energy network code (MENC). Latin square structure of the network codes is used to prove that MENC provides the minimum energy. In Section 4.6, we investigate the networks composed of coding nodes with N incoming edges. We present a low energy network code and show that it provides reversible mapping at the relay. The performances of MENC and LENC are compared with XOR and random network codes in Section 4.7. It is shown that the proposed network code provides significant energy gains, and LENC performs similar to MENC for In-Three networks.

4.1 Motivation

Network coding, as proposed in [1], is the method of combining the information flows at the relay node to achieve the network capacity, where routing only is insufficient in general [1]. Even though the main research interest focuses on achieving the network capacity [1, 24, 22, 8, 9], there are several papers discussing the energy efficient communications with network coding [32]-[7]. However, in none of these studies, network coding is used as a tool to minimize the dissipated energy. If we can find network codes that minimize the average energy among all the input-output mappings in the coding node, we can achieve capacity with minimum network energy. The purpose of this chapter is to propose such a network coding technique, that minimizes the energy for communications.

In a communication system, if no energy is dissipated in one of the modulation states, average energy can be reduced by increasing the frequency of this state [10]. On-off keying is the most commonly used example of such modulation schemes. Throughout the chapter, we assume that the energy cost of 0 bit is zero. Therefore, the minimum energy and minimum codeword weight terms are used interchangeably, since minimizing the average weight is equivalent to minimizing the average codeword energy. In [18], we have developed a novel channel code called minimum energy coding (MEC), and showed its suitability for nano communications due to significant energy efficiency it provides compared to the classical schemes. MEC with Hamming distance constraint provides a way to minimize energy under OOK assumption, with full control on the reliability. In this work, MEC is assumed as the underlying channel code at each node to minimize energy.

In this chapter, we propose a linear network code, called the minimum energy network code (MENC), that ensures the minimum energy dissipation in In-Two networks, i.e., networks composed of coding nodes with two input and one output symbols [21]. The developed network code maps the incoming edges to the codewords of MEC such that the average codeword weight at the output is minimized. Since MEC minimizes the average energy at each individual link, total network energy is minimized with MENC. We also investigate the generalized In-N networks, i.e., networks composed of coding nodes with N input and one output edges, and propose a low energy network code (LENC). LENC can be used to achieve network capacity, while guaranteeing low energy dissipation compared to random network coding.

4.2 Related Work

Network coding is first proposed in [1], where the authors show that routing only is not sufficient to achieve the network capacity and processing of the data flows at the intermediate nodes, i.e., network coding, is required. Network coding idea has been employed in various settings. Physical layer network coding is proposed in [35], where the authors exploit the broadcast nature of the wireless nodes in ad-hoc networks to improve the throughput. The major research effort on network coding focus on the capacity of networks. In this regard, authors in [24] show that the linear network coding is sufficient to achieve the network capacity in multicast networks. An algebraic framework for linear network codes is developed in [22]. In [8], the authors show that linear network coding is insufficient in general networks, even for large finite fields and with vector extensions, with a counter example. In [9], the authors study the relation of polynomial collections and matroidal networks, and show that the linear solvability of networks in a finite field depends on the solvability of the associated polynomial in the same field.

There are several studies on energy efficient communication with network coding. In [32], a linear program that determines the path yielding the minimum energy per bit is developed in multicast networks. Each link is assumed to have an associated cost. It is shown that with network coding, not only energy per bit is reduced, but also the minimum energy solution is found in polynomial time. However, authors do not design network codes to achieve energy efficiency, rather exploit the network coding idea. Authors in [25] optimize the network resources in coded wired and wireless networks to minimize a given cost criterion, and choose the best subgraph. In [6], energy efficient communication is achieved in wireless networks, by decomposing the network coding sessions into multicast and unicast. Moreover, optimization algorithms for solving the link scheduling problem is investigated. In [11], authors minimize the total energy consumed in the network with lifetime constraints, by determining the traffic on each edge, assuming XOR network coding. Recently in [7], an optimization problem to minimize energy in two-way network coding with the rate constraint is developed. However, authors incorporate the routing and scheduling into the optimization, using the fixed XOR network coding. Therefore, the energy minimization is not obtained with network coding.

Despite the numerous attempts to reduce energy in network coding scenarios, none determines the best network code that minimizes energy, but rather uses the existing network codes. Moreover, energy minimization is not achieved by the network code, but either scheduling or resource allocation algorithms.

In this work, for the first time in the literature, we design the network code with the objective of energy minimization. In the code design, the mapping at the coded node is assured to preserve the information, while minimizing the energy. Assuming no energy cost for the zero symbol, we choose the network code that minimizes the average codeword weight at the outgoing edges of the relay node to minimize the energy. We consider the network coding with two inputs and n-inputs separately. For the node with two incoming edges, minimum energy network code (MENC) is derived and for more general setting with N incoming edges, a low energy network code is developed to reduce the energy at the relay.

4.3 Network Model

In this section, we present the network model and the assumptions on the network. We mainly employ the notations of [9]. A network is represented with a directed graph with edge set ε and node set ν. The finite set μ with cardinality M, i.e., $|\mu| = M$, is called the message set.

A set $\mathcal{A}$ with minimum of two elements is called an alphabet. The alphabet represents the channel symbols, i.e., codewords, used at the edges of the network. Therefore, the size of the alphabet $\mathcal{A}$ is equal to the source set cardinality at the source node, i.e., $|\mathcal{A}| = M$. For simplicity, $\mathcal{A} = \{0,1,...,M-1\}$. We assume that each channel symbol $i \in \mathcal{A}$ represents a *length-n* codeword c_i composed of binary digits. Hence, there is a one-to-one and onto mapping between the channel symbols and codewords, i.e., $\xi : \{c_0, c_1, ..., c_{M-1}\} \rightarrow \mathcal{A}$, where $\xi(c_i) = i$.

Input and output edge sets of node u are Γ^u_{in} and Γ^u_{out} respectively. A node u is called a source node if Γ^u_{out} is empty, and destination node if Γ^u_{in} is empty. The source and destination nodes are called the *end nodes* and nodes with both incoming and outgoing edges are called the *relay nodes*. The set of channel symbols delivered to node u on its input edges is $In(u)$, and set of symbols generated by u is $Out(u)$. Let τ be the number of incoming edges of the relay node u, i.e., $|In(u)| = \tau$. Then, for each edge $e \in \Gamma^u_{out}$, the mapping

$$f_u(e) : \mathcal{A}^\tau \to \mathcal{A}, \tag{4.1}$$

is the network code in node u. We call a network code as *reversible* if, for any relay node, given any $\tau - 1$ input messages and the message at any outgoing edge of node u, i.e., $f_u(e)$, the unknown last message at the input can be uniquely determined. This condition imposes the conservation of information. The map $\mathcal{V} : \mu \to \mathcal{A}$ is the assignment of messages to the channel symbols, hence to codewords. At this point, we list our assumptions on the network.

- The node set is composed of wireless nodes only. Therefore, only a single channel symbol is transmitted from each node. Hence, for a coding node u, $f_u(e) = f_u, \forall e \in \Gamma^u_{out}$, and $|Out(u)| = 1$, $\forall u \in \varepsilon$.
- The map $\mathcal{V}$ is determined by minimum energy channel coding (MEC) to minimize the average weight depending on the message probabilities [18].
- All the network coding nodes in the network employ *reversible* network coding.

It is important to note that, even though the reversible coding is required in general, there may be scenarios in which loss of information due to unreversible network codes can be compensated from the other edges. However, in this work, in an attempt to develop codes where network coding is required to achieve the capacity, we assume reversible network codes.

4.4 Minimum Energy Channel Coding

In this section, we explain the underlying channel code employed at the network edges. The channel codewords are binary *n-tuples* mapped to the source messages. This mapping should be one-to-one for unique decodability. The set of fixed-length codewords is called the codebook. Hamming weight or simply the weight of a binary codeword is the number of ones it contains. Weight enumerator of a codebook $\mathcal{C}$ is the polynomial $W_{\mathcal{C}} = \sum_i l_i z^i$, in which there are l_i number of weight i codewords. Hamming distance between the codewords is the number of bits that they differ in. With minimum distance decoding, the received *n-tuple* is mapped to the closest codeword in terms of Hamming distance. A code corrects t errors, if it has Hamming distance of $2t+1$. Codes with larger Hamming distance are more reliable, as they can correct more error patterns.

Minimum energy coding (MEC) maps the messages to codewords such that the average codeword energy is minimized under OOK assumption. Hence, minimizing the average code weight is equivalent to minimizing the average energy dissipated for communications. Let the message set μ be given by $\mu = \{x_0, x_1, ..., x_{M-1}\}$ with cardinality M, and x_i be the source symbol with probability p_i, mapped to the codeword c_i. The following theorem is the main result of [18]:

Theorem 4.4.1 *Let x_i has probability $p_i \in \{p_0, p_1, ..., p_{M-1}\}$ and p_{max} be $\max(p_i)$. For a desired minimum Hamming distance d, the minimum expected codeword weight, $E[w]$ is*

$$\min(E[w]) = \begin{cases} (1-p_{max})d, & p_{max} \geq 0.5, \\ \frac{d}{2}, & p_{max} < 0.5, \textit{ if d even} \\ \lceil \frac{d}{2} \rceil - p_{max}, & p_{max} < 0.5, \textit{ if d odd} \end{cases}, \tag{4.2}$$

where corresponding codebook has weight enumerator

$$W_{\mathcal{C}}(z) = \begin{cases} z^0 + (M-1)z^d, & p_{max} \geq 0.5 \\ z^{\lfloor \frac{d}{2} \rfloor} + (M-1)z^{\lceil \frac{d}{2} \rceil}, & p_{max} < 0.5. \end{cases}$$

Therefore, depending on the probability of the most probable source outcome, we either use the codeword set with all-zero codeword, i.e., $\mathcal{C}_0$, with weight enumerator $W_{\mathcal{C}_0} = z^0 + (M-1)z^d$ or set with codewords of equal weight of $d/2$ (for even d), i.e., $\mathcal{C}_1$, with weight enumerator $W_{\mathcal{C}_1} = z^{\lfloor \frac{d}{2} \rfloor} + (M-1)z^{\lceil \frac{d}{2} \rceil}$. For both cases, the codeword with less weight is mapped to the source with probability p_{max}.

When we employ MEC in the networks, the probability calculations are more tedious as we will see in the following sections. It is important to discuss if we can choose different sets of codewords for each edge e in the network. The best codebook selection, i.e., the codebook minimizing the average energy, depends on the probability of the most probable event at node

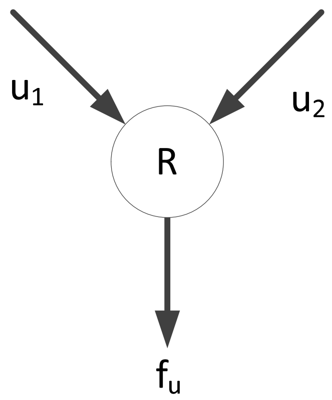


Figure 4.1: Network coding relay node in In-Two networks.

u, for each edge $e \in \Gamma^u_{out}$. This cannot always be supported in the networks, since any node u', where $e \in \Gamma^{u'}_{in}$ needs to distinguish between the selected codeword sets, which require extra bandwidth. As we aim to minimize the energy, we assume that each source selects the codeword set minimizing its average codeword energy, and codebook selection can be inferred by the destination. In practice, the choice of codeword set can be indicated with a single bit, since only two codeword sets, i.e., $\mathcal{C}_0$ and $\mathcal{C}_1$ are sufficient for all the source distributions, as the choice only depends on whether p_{max} is greater or less than 0.5. Therefore, we assume that the indication of the selected codeword set is perfectly announced to the receivers without significant change in the performance.

4.5 Minimum Energy Network Coding for In-Two Networks

In-Two networks are composed of coding nodes u with two input and one output symbols, i.e., $|In(u)| = 2$ and $|Out(u)| = 1$. Network may also consist of forwarding nodes, with $|In(u)| = |Out(u)| = 1$. In order to minimize the average energy in the overall network, we employ MEC as demonstrated in Section 4.4, to minimize the average codeword weight at the output edge of each node, depending on the message probabilities. Similar to the source nodes, network coding nodes also employ MEC at their outgoing edges. The probability distribution of the messages at the network coding node depends on the probability of the incoming channel symbols. Fig. 4.1 shows the network coding node in an In-Two network. For node u, $In(u) = \{u_1, u_2\}$ and $\Gamma^u_{in} = \{e_1, e_2\}$, where e_i contains the channel symbol u_i. We introduce the latin

	q_0	q_1	q_2	q_3
p_0	2	3	0	1
p_1	1	2	3	0
p_2	0	1	2	3
p_3	3	0	1	2

Figure 4.2: A latin square with 4 rows and columns.

squares to define the In-Two network codes.

4.5.1 Latin Squares as Network Codes

Latin squares are two dimensional mathematical objects useful to define network codes.

Definition: A latin square of order M is an $M \times M$ square matrix containing each symbol from a set of cardinality M exactly once in each row and each column.

Due to their nature, latin squares can be used to represent reversible network codes for In-Two networks. A 4×4 latin square is provided in Fig. 4.2 as an example. The origin of latin squares is assumed to be the top-left corner. Assume that the $i+1^{th}$ row (column) shows that the symbol i is received on the first (second) incoming link of node u, i.e., $u_1 = i$ ($u_2 = i$). For example, if we know that the first link contains 3, and the outgoing link contains 1, we can infer that the other incoming message should be 2, since each symbol appears exactly once in each row and each column. Similarly, the well-known XOR network code can be represented by a 2×2 latin square with zeros in the diagonal. It can be written as

$$f_u(u_1, u_2) = u_1 + u_2 \pmod 2, \tag{4.3}$$

where channel symbols are binary. Therefore when f_u and either of u_i's are known, the unknown input can be uniquely determined by $u_1 = f_u + u_2, \pmod 2$.

4.5.2 Minimum Energy Network Coding - MENC

From Section 4.4, we know that the selection between the codebooks $\mathcal{C}_0$ and $\mathcal{C}_1$ depends on the probability of the most probable source event. Let source events be sorted in decreasing probability order such that $p_i \geq p_{i+1}$. If probability of the most probable event, i.e., p_0, is greater than 0.5, codebook $\mathcal{C}_0$ is chosen, which assigns 0 weight codeword to c_0, and d weight codewords to all the other codewords, where d is the desired Hamming distance of the code. If $p_0 < 0.5$, either all codewords have $d/2$ weight if d is even, or c_0 has weight $w_0 = \lfloor d/2 \rfloor$ and all other codewords have weight $w_i = \lceil d/2 \rceil$ if d is odd, with the selection of codebook C_1. As stated previously, this coding technique is employed at the network coding node to minimize energy associated with the outgoing edges. Since either $\mathcal{C}_0$ or $\mathcal{C}_1$ is selected, without loss of generality, we can say that $weight(c_0) \leq weight(c_i), \forall i \in \{0, 1, ..., M-1\}$.

We first present two lemmas which are useful for the proof of our main theorem. The following lemma gives the condition on the completability of partially filled latin squares [30].

Lemma 4.5.1 *An $M \times M$ partial latin square with $M-1$ filled entries can always be completed to a latin square.*

The direct result of this is that, an $M \times M$ partial latin square filled with M number of a single type symbol, and satisfying latin square conditions can be completed.

Assume that the symbols are ordered in terms of their probabilities in the rows and columns of the latin square, i.e., i^{th} symbol on the first (second) incoming edge of node u is more probable than the j^{th} symbol on the first (second) incoming edge, if $i < j$. In other words, referring to Fig. 4.2, $p_i \geq p_j$ and $q_i \geq q_j$ for $i < j$.

Let $g_{i,j}$ be the interchange operation between ith row and jth row of a latin square, where $i < j$, with 0 symbol of the i^{th} row on the m^{th} column and of the j^{th} row on the n^{th} column. Then we have the following lemma.

Lemma 4.5.2 *The interchange operation decreases the average weight on the outgoing edge of network coding node, if $m > n$.*

Proof. From MEC, $w_0 \leq w_i$,$\forall i \in \mathcal{A}$ and $w_i = w_j = w_1$,$\forall i, j$ such that $i \neq 0, j \neq 0$. Let the average weight before the interchange operation be W_b and after the operation be W_a. Then we have,

$$\begin{aligned}
W_b - W_a &= w_0(p_i q_m + p_j q_n) + w_1(p_i q_n + p_j q_m) \\
&\quad - w_0(p_j q_m + p_i q_n) - w_1(p_i q_m + p_j q_n) \\
&= w_0 p_j(q_n - q_m) + q_0 p_i(q_m - q_n) \\
&\quad + q_1 p_i(q_n - q_m) + w_1 p_j(q_m - q_n) \\
&= w_1(p_i - p_j)(q_n - q_m) - w_0(p_i - p_j)(q_n - q_m) \\
&= (w_1 - w_0)(p_i - p_j)(q_n - q_m) \qquad (4.4)
\end{aligned}$$

In (4.4), the right hand side is always positive. Hence, the interchange operation decreases the average weight. ■

Theorem 4.5.3 *A minimum energy network code for a node with two incoming links is given by a latin square with 0s on the main diagonal.*

Proof. From Lemma 4.5.1, we conclude that, a partial latin square containing all the zero symbols is completable. As a result, there is a latin square for every pattern of 0 symbols.

Due to the 2-dimensional nature of a latin square, every partial latin square of order M filled with M number of 0s can be obtained by sufficient number of row or column switching applied to a partially filled latin square with random pattern of M number of 0 symbols.

From Lemma 4.5.2 we know that, to minimize the average code weight, i.e., average energy, we should apply row switching operations until the index of the column containing the 0 symbol for i^{th} row is less than that of j^{th} row for all i, j satisfying $i < j$. Sufficiently applying this operation leads us to the latin square with diagonal entries filled with 0s. We can always complete this partial latin square to obtain a valid network code.

Therefore the minimum energy network code for the node with two incoming links is represented by a latin square with the main diagonal filled with 0 symbols. ■

The following theorem is useful to characterize the minimum energy network codes:

Theorem 4.5.4 *The linear network codes*

$$f_u = u_1 + (M-1)u_2, \ (\bmod M) \qquad (4.5)$$

$$f_u = (M-1)u_1 + u_2, \ (\bmod M) \qquad (4.6)$$

are minimum energy network codes.

Proof. Both f_u's yield network codes represented by a latin square with main diagonal filled with zero symbols. Hence, it achieves the minimum average energy. ■

Therefore, there exist linear network codes minimizing the average energy. The code is linear at the channel symbol level. Due to its linear nature, it can be implemented easily.

This network code can be used to achieve the max-flow min-cut capacity, which is not achievable with routing only, and minimize the average energy at the same time.

4.5.3 Codebook Selection at the Relay

In MEC, we determine the codeword set by inspecting if $p_0 < 0.5$ or not. Similarly, we determine the codeword set at the relay by checking the probability of 0 symbol. However, this time the probability of the channel symbols depend on the probability of the incoming codewords. Assume that the symbols at the relay inputs are independent. Let p_i and q_i be the probability that the symbol i is transmitted at the incoming edges e_1 and e_2 respectively. Since MENC is represented by an all-zero diagonal latin square, the probability of 0 symbol at the relay, i.e., p_0^R, is given by $p_0^R = \sum_{i=0}^{M-1} p_i q_i$. Then we have the following theorem yielding the minimum average codeword energy obtained with network coding at the relay:

Theorem 4.5.5 *Let p_i and q_i be the probability that the symbol i is received at the incoming edges e_1 and e_2 respectively. Then the minimum average codeword weight at the outgoing edge of relay in In-Two networks is given by*

$$\min(E_R[w]) = \begin{cases} \left(1 - \sum_i p_i q_i\right) d, & \sum_i p_i q_i \geq 0.5 \\ d/2, & \sum_i p_i q_i < 0.5, \textit{ if } d \textit{ even} \\ \lceil d/2 \rceil - \sum_i p_i q_i, & \sum_i p_i q_i < 0.5, \textit{ if } d \textit{ odd}, \end{cases} \tag{4.7}$$

where the corresponding codebook has weight enumerator

$$W_C(z) = \begin{cases} z^0 + (M-1) z^d, & \sum_i p_i q_i \geq 0.5 \\ z^{\lfloor \frac{d}{2} \rfloor} + (M-1) z^{\lceil \frac{d}{2} \rceil}, & \sum_i p_i q_i < 0.5. \end{cases}$$

Proof. Let $\tilde{p} = \sum_i p_i q_i$. Following the proof of Theorem 4.4.1 in [18], (4.7) follows immediately. ■

Therefore, relay node should know $\sum_i p_i q_i$ in order to select the codebook yielding the minimum average energy. In dynamical environments where source probabilities change in time, obtaining this term might not always be possible. In order to relieve the relay, we show that it does not need the probabilities of the incoming symbols, if $p_1, q_1 < 0.5$.

Lemma 4.5.6 *Let p_i, $i \in \{0,1,...,M-1\}$ be a probability distribution such that $0.5 \geq p_0 \geq p_1 \geq ... \geq p_{M-1} \geq 0$. Then,*

$$\arg\max_{(p_0,p_1,...,p_{M-1})} \left(\sum_i p_i^2 \right) = (0.5, 0.5, 0, ..., 0). \tag{4.8}$$

Proof. Skipped. ■

Lemma 4.5.7 *Let u be the coding node in In-Two network, and p_i and q_i be the probability that symbol i is received at the incoming edges e_1 and e_2 respectively, where $p_i \geq p_{i+1}$ and $q_i \geq q_{i+1}$, $\forall i \in \{0,1,...,M-2\}$. Then we have,*

$$\textit{If } (p_1 \leq 0.5) \wedge (q_1 \leq 0.5) \Rightarrow \sum_{i=0}^{M-1} p_i q_i \leq 0.5. \tag{4.9}$$

Proof. From Cauchy-Schwarz inequality, we have,

$$\sum_i p_i q_i \leq \sqrt{\sum_i p_i} \sqrt{\sum_i q_i}. \tag{4.10}$$

From Lemma 4.5.6, we know that $\sum_i p_i \leq 0.5$ if $p_i \leq 0.5$. Therefore, $\sum_i p_i q_i \leq 0.5$ if $p_i, q_i < 0.5$ ■

Hence, we conclude that if the most probable message of the source nodes is less that 0.5, the coding nodes do not need probability distributions of their incoming symbols to select the energy minimizing codebook, since then the probability of 0 symbol is always less than 0.5.

From a similar argument, we see that for the other cases, such a conclusion is not possible. For example, if $p_1 < 0.5$ and $q_1 > 0.5$, from Cauchy-Schwarz inequality, $\sum_i (p_i q_i) < \sqrt{0.5}$.

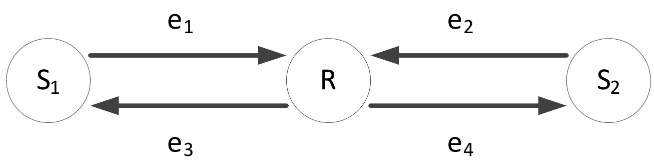


Figure 4.3: Two-way relay network.

4.5.4 Example Networks

In this section, we consider two example networks and obtain their total energy dissipation with MENC. We also give an example network to show that the proposed network code is not always sufficient to achieve the network capacity. In the following, it is assumed that the destinations know the network codes a priori.

4.5.4.1 Two-Way Relay Network

Two-way relay network is a widely known example in the network coding literature. There are two source nodes which are destinations of each other, and these sources are connected via a relay node as shown in Fig. 4.3. Sources S_1 and S_2 transmit their channel symbols, i.e., x and y, simultaneously. The relay node R combines the packets it receives and broadcasts. Representing the symbol at each edge e_i by u_i, $e_1 = e_4 = x$, $e_2 = e_5 = y$, and $u_3 = u_4$ due to the broadcast nature of the relay node. With MENC, $u_3 = u_1 + (M-1)u_2 \pmod M$. Then source nodes S_1 and S_2 can decode the symbol transmitted by each other, since they already know the symbol they transmitted, with the operations $y = (M-1)u_3 + x$ and $x = u_4 + y$, respectively where operations are in $\bmod M$. Therefore, each node receives one packet in two time units. Note that reversible network codes are required to achieve network capacity.

The total average energy dissipated in the network is the summation of average energies dissipated at each link. For simplicity, assume that $p_0, q_0, \sum_i p_i q_i \geq 0.5$. Then the total average energy of the two-way relay network is

$$\begin{aligned} J_{TW} &= ((1-p_0)d + (1-q_0)d + (1-\sum_i p_i q_i)d)P \\ &= (3 - p_0 - q_0 - \sum_i p_i q_i)dP, \end{aligned} \tag{4.11}$$

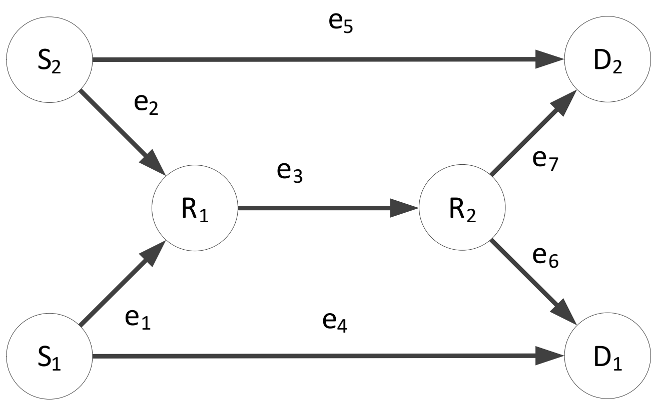


Figure 4.4: Butterfly network.

where P is the energy per high bit, i.e., the cost of single codeword weight in terms of energy. Note that as p_0 and q_0 goes to 1, average network energy goes to zero. Total energy expressions for the other cases can also be obtained easily.

4.5.4.2 Butterfly Network

Butterfly network is the most famous network in the network coding literature, as it is studied in [1]. Fig. 4.4 shows the butterfly network with two source and two destinations. With reversible network coding, destinations can decode the messages of both transmitters. As before, $u_3 = u_1 + (M-1)u_2 \pmod M$. Due to the broadcast nature of nodes $u_2 = u_5$, $u_1 = u_4$, and since the relay R_2 broadcasts u_3, $u_6 = u_7 = u_3$. Single codeword energy is sufficient to form all the outgoing edges of a node due to the broadcast nature. Assuming $p_0, q_0, \sum_i p_i q_i \geq 0.5$ similar to the two-way relay network, the total network energy with MENC is,

$$J_B = ((1-p_0)d + (1-q_0)d + 2(1-\sum_i p_i q_i)d)P$$
$$= (4 - p_0 - q_0 - 2\sum_i p_i q_i)dP. \tag{4.12}$$

In the next section, we provide a counter example, to show that MENC cannot always achieve network capacity for all In-Two networks. This results is in parallel with the findings in [8] that linear network coding is not always sufficient.

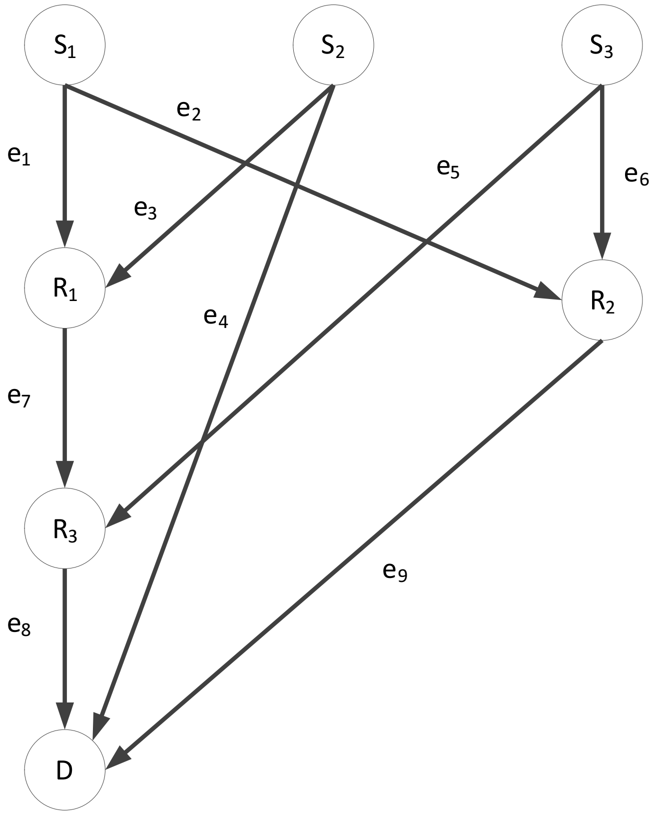


Figure 4.5: Example network in which single MENC does not achieve capacity.

4.5.4.3 Unachievability of Capacity with MENC

Consider the network in Fig. 4.5. Sources S_1, S_2, S_3 broadcast channel symbols x, y, z, respectively. Therefore, $u_1 = u_2 = x$, $u_3 = u_4 = y$, $u_5 = u_6 = z$. To minimize the network energy, MENC is employed at the nodes, R_1, R_2 and R_3. Then employing $u_7 = u_1 + (M-1)u_3$, $u_8 = u_7 + (M-1)u_5$ and $u_9 = u_2 + (M-1)u_6$ in $\mod M$, we have,

$$
\begin{aligned}
u_7 &= x + (M-1)y \ (\mod M) \\
u_8 &= x + (M-1)y + (M-1)z \ (\mod M) \\
u_9 &= x + (M-1)z \ (\mod M).
\end{aligned}
$$

As a result, destination receives the symbols, $x + (M-1)y + (M-1)z, y$ and $x + (M-1)z$. Since operations are in modulo M, these equations are linearly dependent. Hence, destination cannot decode x and z separately.

Note that we can circumvent this problem by selecting different MENCs at each node. Con-

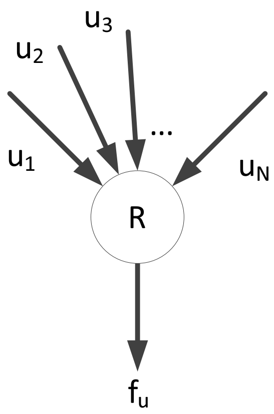


Figure 4.6: Network coding node in In-N networks.

sider the network with the operations $u_7 = (M-1)u_1 + u_3$, $u_8 = u_7 + (M-1)u_5$ and $u_9 = u_2 + (M-1)u_6$ in $\mod M$. Then,

$$
\begin{aligned}
u_4 &= y \\
u_8 &= (M-1)x + y + (M-1)z \pmod M \\
u_9 &= x + (M-1)z \pmod M.
\end{aligned}
$$

In this case, destination decodes all the symbols as

$$2x = (M-1)(u_8 + (M-1)u_4) + u_9 \pmod M \tag{4.13}$$

$$y = u_4 \tag{4.14}$$

$$2z = (M-1)(u_8 + (M-1)u_4 + u_9) \pmod M \tag{4.15}$$

Therefore, we show that MENC cannot always be used to achieve the network capacity. However, in special cases as shown above, using combination of two MENCs leads to capacity achieving network codes for In-Two networks.

4.6 Low Energy Network Coding for In-N Networks

In-N networks are defined as networks composed of coding nodes with N incoming edges only. As we investigate in Section 4.5.1, network codes in In-Two networks are equivalent to latin squares. Similarly, if there are more than two incoming edges, the mathematical object

p_0	r_0	r_1	r_2
q_0	0	1	2
q_1	2	0	1
q_2	1	2	0

p_1	r_0	r_1	r_2
q_0	2	0	1
q_1	1	2	0
q_2	0	1	2

p_2	r_0	r_1	r_2
q_0	1	2	0
q_1	0	1	2
q_2	2	0	1

Figure 4.7: LENC for In-Three network with M=3.

corresponding to the network code is an N-dimensional latin square, i.e., a latin hypercube. Next, we introduce the latin hypercube concept.

4.6.1 Latin Hypercubes as Network Codes

Consider the node shown in Fig. 4.6. Due to the reversibility condition on the network code, knowing any $N-1$ number of inputs and the output, we should be able to deduce the remaining unknown channel symbol at the input. In other words, given any set of the incoming symbols $\mathcal{U}_i$ satisfying $|\mathcal{U}_i| = N-2$, the network code mapping the unknown two incoming edge symbols to the outgoing edge symbol should be representable by a latin square. The structure satisfying this condition is called a latin hypercube.

Definition: An N dimensional latin hypercube of order M is an M^N cube satisfying that each symbol $i \in \{0,1,...,M-1\}$ occurs once through each line, where a line is the set of values of the latin hypercube when all but one dimension is fixed.

Consider the case with $N=3$. The structure corresponding to a network code becomes a latin cube with three dimensions. Apart from rows and columns, the dimension providing depth to the cube is called “file". Redefining the latin structure for cube, we say a latin cube is the cube containing all the elements of the set $\{0,1,...M-1\}$ exactly once in each row, each column and each file. It is easier to visualize the structure for three dimensions, however, difficulties in comprehending the view of hypercubes arise with higher dimensions.

Due to the high dimensional nature of latin hypercubes, hence reversible network codes, the elegant and simple findings of minimum energy network coding for In-Two networks cannot be obtained. As we see in the following sections, even in the three dimensional case, the min-

p_0

q_0	s_0	s_1	s_2
r_0	0	1	2
r_1	2	0	1
r_2	1	2	0

q_1	s_0	s_1	s_2
r_0	2	0	1
r_1	1	2	0
r_2	0	1	2

q_2	s_0	s_1	s_2
r_0	1	2	0
r_1	0	1	2
r_2	2	0	1

p_1

q_0	s_0	s_1	s_2
r_0	2	0	1
r_1	1	2	0
r_2	0	1	2

q_1	s_0	s_1	s_2
r_0	1	2	0
r_1	0	1	2
r_2	2	0	1

q_2	s_0	s_1	s_2
r_0	0	1	2
r_1	2	0	1
r_2	1	2	0

p_2

q_0	s_0	s_1	s_2
r_0	1	2	0
r_1	0	1	2
r_2	2	0	1

q_1	s_0	s_1	s_2
r_0	0	1	2
r_1	2	0	1
r_2	1	2	0

q_2	s_0	s_1	s_2
r_0	2	0	1
r_1	1	2	0
r_2	0	1	2

Figure 4.8: LENC for In-Four network with M=3.

imum energy network code depends on the conditions on the input probabilities. Therefore, we present a network code which provides low energy dissipation, if not minimum, for the general in-N networks. We develop low energy network code (LENC) by investigating the three dimensional case in detail.

4.6.2 Low Energy Network Coding - LENC

Assume that the source set cardinality $M = 3$ and the network coding node has three incoming edges, i.e., $N = 3$. The reversible network code should be represented by a three dimensional latin cube of order 3. Let the probability mass function of the incoming flows be p_i, q_i and r_i, $i \in \{0, 1, ..., M-1\}$, where the incoming symbols are x, y and z respectively. An example latin cube representing the network code is provided in Fig. 4.7. Each latin square gives the mapping of the network code, for a given x value. For example, the first latin square gives the mapping of y and z to f_u, when $x = 0$.

Assume $p_0 > q_0, r_0$. From Section 4.5.2, we know that the latin square with the main diagonal

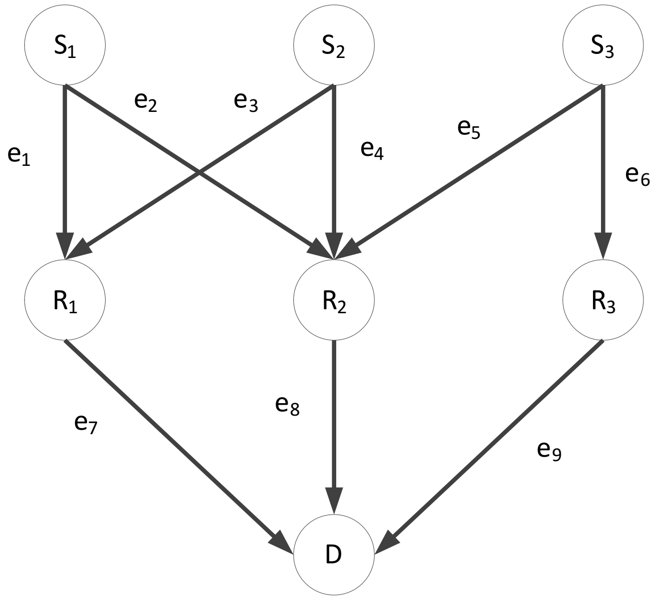


Figure 4.9: Example network with MENC and LENC.

filled with 0 symbol provides the minimum energy. Hence, we should reserve this latin square for the most probable p_i. Since p_i's are ordered such that $p_i \geq p_{i+1}$, the minimum energy latin square is reserved for p_0 as shown in Fig. 4.7. Moreover, since $p_0 > q_0, r_0$, this selection provides the largest gain in terms of average energy. However, it is not straightforward to select the other latin square assignments, since the values of p_i, q_i and r_i for $i \neq 0$ determine this mapping to minimize the average weight. As for the given example, the network code obtained by swapping the second and third latin squares provides lower average energy than the one provided, if $(q_1 - q_2)(r_0 - r_1) < (q_0 - q_1)(r_1 - r_2)$.

The above network code is obtained with cyclic shifts of rows. Note that the code value increases by 1 along z, and by 2 along x and y dimensions. This selection of increments is intentional to assure the reversibility of the network code. Moreover, the provided network code is linear. We define the generalization as low energy network codes:

Definition: For a coding node with N incoming edges with symbols $\{u_1, u_2, ...u_N\}$, and symbol set cardinality of M, if $p(u_\tau = 0) \leq p(u_i = 0), \forall i \neq \tau$, *the low energy network code* is,

$$f_u = u_\tau + \sum_{i \neq \tau} (M-1) u_i. \quad (4.16)$$

Lemma 4.6.1 *The low energy network code is reversible.*

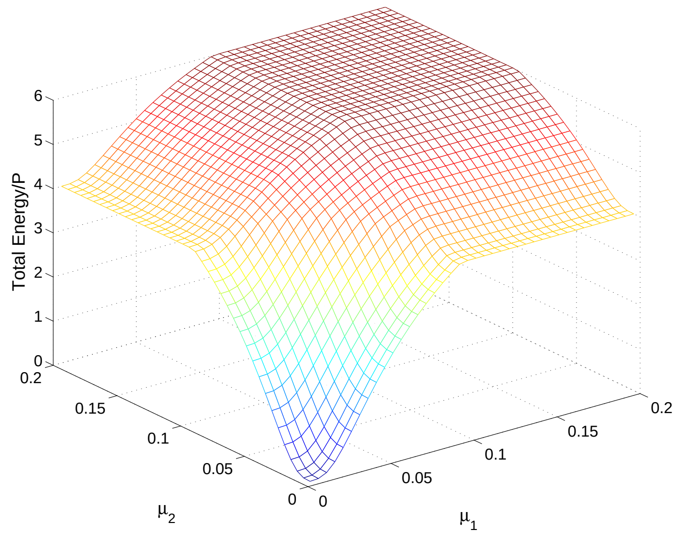


Figure 4.10: Normalized total network energy per bit energy in two-way relay network with different source distributions ($d = 4$).

Proof. A reversible network code should be representable by a latin hypercube. Therefore, given any $N-2$ inputs, the mapping should be a latin square. Let the unknown inputs be (u_i, u_j). If $i, j \neq \tau$, the mapping is given by $f_u = c + (M-1)u_i + (M-1)u_j$, where c is a known constant. This corresponds to a valid latin square, since with the multiplicative factor of $(M-1)$, moving along either dimension decreases the code value by one, which is the left shift operation of rows. If either i or j is τ, the code is the minimum energy latin square. ∎

Lemma 4.6.1 assures that the LENC is representable by a valid latin hypercube. We aim to map minimum energy latin square, i.e., the latin square with all-zero diagonal, to the more probable incoming combinations by choosing the coefficient of $(M-1)$ for all but least probable event. For example, if $p_0 > q_0, r_0$, the minimum energy latin square is assigned to the mapping of (u_2, u_3), when $u_1 = 0$, which is highly probable, leading to significant decrease in the average weight. The code provided in Fig. 4.7 is actually an LENC, for $r_0 < q_0, p_0$.

Another example LENC is shown in Fig. . Let u_1, u_2, u_3 and u_4 be the incoming edge symbols with pmf's of p_i, q_i, r_i and s_i respectively. Assume that $s_0 < p_0, q_0, r_0$. Then LENC is given by the mapping $f_u = u_4 + (M-1)(u_1 + u_2 + u_3), \mod M$. To visualize the 4 dimensional latin hypercube corresponding to the LENC, latin squares corresponding to each value of (u_1, u_2) pair is demonstrated with varying u_3 and u_4.

4.6.3 Example Network

In this section, a sample In-Three network with LENC is investigated. Fig. 4.9 shows the network, which has two coding nodes R_1, R_2 with two and three incoming edges, respectively. Due to the broadcast nature, $u_1 = u_2 = x$, $u_3 = u_4 = y$ and $u_5 = u_6 = z$. Assuming $M = 4$ and $p(u_2 = 0) > p(u_4 = 0), p(u_5 = 0)$, employing MENC at R_1 and LENC at R_2, we have $u_7 = u_1 + 3u_3$ and $u_8 = u_2 + 3u_4 + 3u_5$. Then the destination obtains $u_7 = x + 3y \mod M, u_8 = x + 3y + 3z \mod M, u_9 = z$. Since they are linearly dependent, destination cannot decode all three symbols. Even if we switch to $u_7 = 3u_1 + u_3$, the received symbols are still linearly dependant.

$$u_7 = 3x + y \mod M \tag{4.17}$$

$$u_8 = x + 3y + 3z \mod M \tag{4.18}$$

$$u_9 = z. \tag{4.19}$$

Therefore, the network capacity cannot be achieved by MENC and LENC for this network. Allowing the In-Three network code, $u_8 = 3u_2 + 3u_4 + u_5$ leads destination to decode all the packets at the cost of slight energy increase.

4.7 Performance Evaluation

In this section, we conduct numerical evaluations to show the performance of minimum energy network code and low energy network code in In-Two and In-N networks. The probabilities are obtained by sampling and normalizing the exponential pdf with parameter μ.

First, we obtain the total energy in two-way relay network with MENC, given in Fig. 4.10. μ_1 and μ_2 are the parameters of exponential distribution. (4.11) and other equations derived for all cases of probabilities are considered. As observed, as distributions become more uniform with increased μ, *weight-d/2* codewords are assigned to each node, leading to average weight of $E[w] = 3d/2 = 6$. Network energy reduces significantly, if both pmf's are far from uniform

Second, we compare the proposed minimum energy network code for In-Two networks with the classical XOR network coding. It is important to note that, XOR network coding satisfies the minimum energy conditions for the binary field. However, with the employment of channel code to provide reliability, XOR does not minimize the average energy. We consider

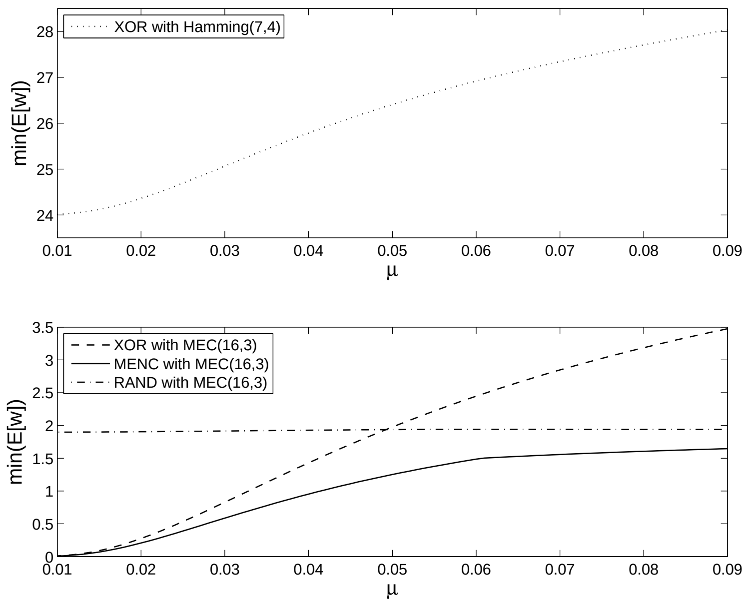


Figure 4.11: Average weight per codeword at the relay for XOR with (7,4) Hamming, XOR with MEC, random network coding with MEC and MENC with MEC for In-Two networks.

the scenario where the input probability distributions p_i, q_i satisfy the condition $p_0, q_0 > 0.5$. This is due to that, for the case $p_0, q_0 < 0.5$, as we show in Section 4.5.3, minimum average weight does not change with the source pmf.

Fig. 4.11 shows the minimum average weight comparison for XOR network code, random network code and MENC. The scenario with Hamming channel code instead of MEC is also provided for comparison. For fairness, MEC(M,d) with $M = 16$ and $d = 3$ is compared to (7,4) Hamming. As expected, employing MEC at the source nodes greatly reduces the energy per codeword at the relay, since codewords are selected to minimize the average weight.

The energy efficiency of MENC compared to XOR shows that with MENC combined with MEC, energy can further be reduced, especially for small μ values. Using MENC, network capacity can be achieved dissipating almost half the energy required with XOR network coding, depending on μ. We also compare the MENC performance with random network codes. Random network coding is employed by generating a random latin square of order M. Fig. 4.11 also shows the minimum average weight comparison between MENC and random network code. We have generated 1000 random latin squares, each representing a network code, and used the zero symbol probability to find the average codeword weight, depending on the

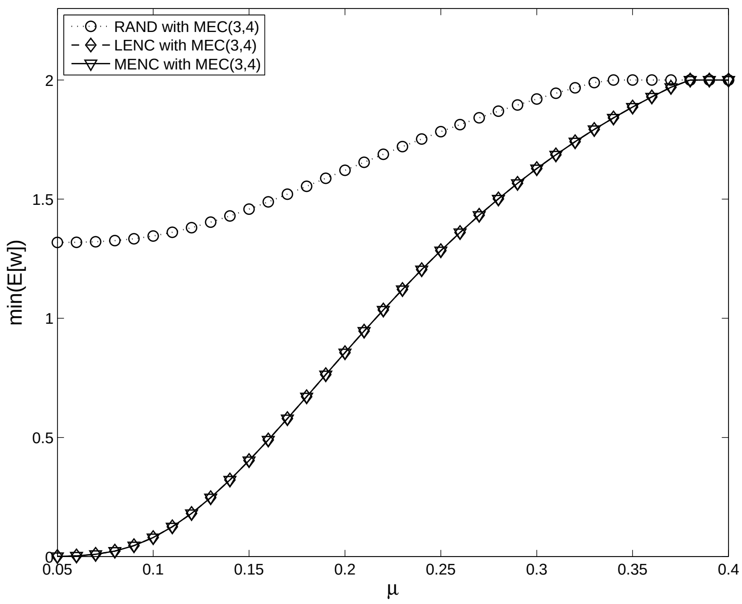


Figure 4.12: Average energy per codeword for, random network coding, MENC and LENC for In-Three networks. $p_0, q_0, r_0 > 0.5$

location of zeros in the randomly generated latin squares. The *o*-shaped markers show the average of the expected code weight over the 1000 trials at each μ value. As μ is increased, the distribution becomes more uniform. Figure shows that, MENC provides significant energy gains for small μ, leading to probability distributions with $\sum_i p_i q_i < 0.5$.

Since the probability calculations and latin cube generation process is tedious for In-N networks, we consider only the In-Three networks with $M = 3$. Fig. 4.12 shows the variation of the energy in In-Three networks. The underlying network code is LENC represented by the latin square in Fig. 4.7. Performance of LENC is compared with MENC and random network coding. The random code is run 1000 times and the resultant minimum expected weights are averaged. As observed, LENC clearly outperforms the random network coding. Moreover, LENC provides performance almost the same as MENC.

CHAPTER 5

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

In this thesis, we have developed novel minimum energy channel codes and showed that they are suitable for communications in nanosensor networks and ad-hoc nanonetworks. Unlike the existing minimum energy codes, MEC provides controllable reliability while assuring that minimum energy is dissipated at the transmitter. Based on MEC, we develop minimum energy network codes, which is the first network code in the literature, that minimizes the network energy. In the next sections, we provide the details of the contributions and point out future issues regarding minimum energy channel and network codes.

5.1 Contributions

In this section, we sum up the contributions of each chapter and underline the important results.

5.1.1 Minimum Energy Channel Coding

In this chapter, first we propose a multi-carrier OOK modulation suitable for nanocommunications, motivated with the THz channel characteristics. Later, we develop a novel minimum energy coding scheme, MEC, for nano communications in cell-based WNSNs employing OOK modulation. Different from previous studies, MEC satisfies a minimum Hamming distance to guarantee reliability. It is analytically shown that codewords can be decoded perfectly with MEC using large code distance, if the number of quantization levels in the WNSN node is less than the inverse of symbol error probability. It is demonstrated via simulations that, the proposed code is superior to popular block codes such as Hamming, Reed-Solomon and

Golay. The state-of-the-art power and energy limits in nano domain are used to obtain achievable rates of WNSN nodes. Neglecting the processing power, nanosensor nodes are expected to transmit data at rates on the order of $Mbps$. In the proposed cell-based WNSN, it is shown that with MEC, perfect communication with large number of quantization levels at the nanonode can be achieved for a cell radius of several millimetres. Numerical evaluations show that MEC can be used as an energy-efficient and reliable coding scheme in nano domain for future WNSNs.

5.1.2 Node Density and Rate Delay Reliability Tradeoffs in Ad Hoc Nanonetworks with MEC

In this chapter, limits and tradeoffs of ad-hoc nanonetworks employing MEC are investigated. It is demonstrated that reliable communication, i.e., perfect codeword decoding can be achieved using MEC, in ad-hoc nanonetworks with a bounded number of interfering nanonodes, without any medium access control. The exact expression for decoding probability is derived, and it is concluded that nanonode density, ρ, should be less than the inverse of node transmission probability, $1/p$, to achieve reliable communication. Furthermore, rate-delay-energy tradeoffs are analyzed for constant codebook size, M, with varying Hamming distance, d, and vice versa. The analyses reveal that for constant M, average energy and rate increase with delay. However, for constant d, energy dissipation remains constant, as delay increases. Additionally, it is shown that increasing source set cardinality leads to rates a constant multiple of $\log M$ for ad-hoc nanonetworks.

5.1.3 Minimum Energy Network Coding

In this chapter we developed network codes minimizing the average codeword energy for In-Two networks. The linear codes $f_u = u_1 + (M-1)u_2$ and $f_u = (M-1)u_1 + u_2$ satisfy the minimum energy conditions. Furthermore, we proposed low energy network codes for the general In-N networks and shown that it performs similar to MENC via numerical evaluations in an In-Three network.

5.2 Future Research Directions

The minimum energy code is generated with the cyclic shifts. Even though this technique provides minimum code length for $p_{max} < 0.5$, codeword length could be made much smaller for $p_{max} > 0.5$ with better techniques. Development of shorter codewords satisfying the minimum energy requirement and desired Hamming distance is an open issue for minimum energy codes.

For minimum energy network codes, the future work includes the development of the network class, for which MENC achieves capacity. Another open issue is the development of minimum energy network codes for nodes with multiple incoming and outgoing edges.

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CURRICULUM VITAE

Murat Kocaoglu received the B.S. degree in Electrical and Electronics Engineering with a minor degree in Physics from Middle East Technical University, Ankara, Turkey in 2010. He received his M.S. degree in Electrical and Computer Engineering under the supervision of Prof. Ozgur B. Akan in Electrical and Electronics Engineering from Koc University, Istanbul, Turkey in 2012. His research interests include nanoscale communications, energy efficient channel and network coding and information theory.