Mean-Variance Approach to the Newsvendor Problem with Random Supply and Financial Hedging

by

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ABSTRACT

The newsvendor model is one of the basic models in inventory management. Due to the randomness in demand and supply, this model includes uncertainty which causes risk to the managers. The related literature mostly assumes that the inventory manager is risk-neutral. So, the classical newsvendor problem aims to maximize expected profit or minimize expected cost, and the risk caused by uncertainty is disregarded. However, in real life, decision makers are sensitive to risk. Thus, the risk-neutrality assumption has limitations in practice. Recently, there has been increasing interest in dealing with this issue using risk management tools. The inventory managers may prefer less gain but more stable cash flows. They are often conservative toward uncertainty in profit due to high demand and supply volatility in the market, and they may be risk-averse. In this thesis, we follow a mean-variance approach to the single-period, single-item stochastic inventory problem where the manager considers both the mean and variance of the cash flow. We also incorporate supply uncertainty based on random yield, random capacity and both random yield and random capacity. We further suppose that the randomness in demand and supply is correlated with the financial markets. The newsvendor hedges demand and supply risks investing in a portfolio composed of various financial instruments. The problem therefore includes both the determination of the optimal ordering policy and the selection of the optimal portfolio. Our aim is to maximize the hedged mean-variance cash flow. We analyze the problem using the cases based on demand and supply randomness as before. We provide explicit characterizations on the structure of the optimal policy. Finally, we present numerical examples to illustrate the effects of the degree of risk-aversion on the optimal order quantity and the effects of mean-variance approach and financial hedging on variance, or risk reduction.

Keywords : Newsvendor model, mean-variance approach, risk hedging

ÖZETÇE

Envanter yönetiminde çalışılan ana modellerden biri gazete satıcısı modelidir. Rassallığın sadece müşteri talebi ile sınırlı olmadığı arzın da rassal olduğu, verilen siparişin hepsinin teslim alınamadığı modeller yönetici için risk oluşturmaktadır. Klasik modellerde riske karşı duyarsız olan insanlar ele alınmakta ve beklenen son nakit akışını enbüyütme ya da maliyeti enküçültme amaçlanmaktadır. Bu durumda rassallığın oluşturduğu risk gözardı edilmektedir. Günümüzde insanlar riske karşı duyarlı hareket etmektedir. Bu nedenle riske duyarsız envanter modelleri pratikte yetersiz kalmaktadır. Son zamanlarda, risk yönetimi araçlarını kullanarak riske duyarlı modeller geliştirmeye artan bir ilgi vardır. Envanter yöneticileri daha az kazancı ama akışın varyasyonunun daha az olmasını tercih edebilirler. Riske duyarlı yöneticiler müşteri talebi ve arzda olan belirsizlik nedeniyle nakit akışındaki rassallığa temkinli yaklaşır. Bu tezde, biz tek dönem ve tek ürün içeren rassal envanter modellerine ortalama-varyans yaklaşımını izlemekteyiz. Tezin ilerleyen kısımlarında, rassallığı oluşturan müşteri talebinin ve arzın, finansal bazı endeksler ya da varlıklar ile korelasyonu olduğu durumlar tartışılacaktır. Gazete satıcısı bu varlıkların vadeli işlemler ve türev piyasalarında pozisyon alarak, bu korelasyondan yararlanacak ve dönem sonu nakit akışının riskini daha azaltacaktır. Böylece, karar problemi hem sipariş miktarını belirlemek hem de aynı zamanda riski azaltacak en iyi portföyü oluşturmaktır. Bu modeller ayrıntısıyla incelenip envanter yönetimine riske duyarlı bir yaklaşım sergilenmiştir. Son olarak, riske olan duyarlılığın eniyi sipariş miktarına, ortalama-varyans yaklaşımının ve finansal portföyün riske olan etkisi incelenmiştir.

Anahtar kelimeler : Gazete satıcısı problemi, ortalama-varyans yaklaşımı, risk yönetimi.

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NOMENCLATURE

D	:	Random demand
U	:	Random yield
K	:	Random capacity
y	:	Order quantity
$Q(y)$	:	Random supply if order quantity is y
c	:	Purhcasing cost of one unit of inventory
p	:	Selling price of one unit of inventory
u	:	Emergency ordering cost for one unit of inventory
s	:	Net salvage value for one unit of inventory
$\hat{p}$	:	Critical ratio or the probablity of satisfying the demand on time
θ	:	Risk-aversion parameter
S_0	:	Price of a tradable asset in the market at time 0
S	:	Price of a tradable asset in the market at time T
F_X	:	Cumulative distribution function of X
f_X	:	Probability density function corresponding to F_X
$\bar{F}_X$	:	Survival probability function of X

Chapter 1

INTRODUCTION

Inventory management is crucial for any business practice that has to keep some form of inventory. Many companies including manufacturers, wholesalers and retailers are trying to find the most efficient strategy to manage inventory. For this reason, inventory management is at the center of interest in operations research and industrial engineering. The single-period, single-item inventory management problem known as the newsvendor problem has received considerable attention due to shortening product life cycles. The newsvendor deals with perishable products that have a limited lifetime to keep in inventory. The products might be fresh fruits, vegetables, fashion goods or seasonal items such as air conditioners, woolen apparel and Christmas gifts. During the life cycle of these products the newsvendor faced with stochastic demand has to decide on a suitable order quantity that balances overage cost against underage cost. If the realized demand is greater than the quantity ordered, the newsvendor has the option to purchase the units that are short at an emergency purchase price that is higher than the regular price. In this case, an opportunity of additional profit is lost whereas if the demand is less than the order quantity, the unsold amount is salvaged at a price lower than the purchasing cost.

The classical newsvendor model is based upon risk-neutrality assumption. Thus, the aim is to maximize expected profit or minimize expected cost, yet this approach is questioned as the resulting policy is optimal on average. In this case since the problem includes uncertainty the variability of benefits (or costs) is ignored. Thus, risk exposure is inevitable, and so in practice most of the inventory managers are risk-sensitive, being conservative towards risk. Simply put, they are risk-averse and prefer a certain profit to a risky profit whose expected value equals the certain one. The literature holds many risk models used in stochastic inventory models. They can be categorized mainly as utility models, mean-variance (MV) models, satisficing probability maximization and Value-at-Risk (VaR). For a decision maker, utility is a measure of satisfaction and the aim is to maximize the decision maker's satisfaction which is the expected utility of the cash flow. The MV approach is the treatment of inventory managers' conflicting objectives of high return versus low risk. Alternatively, satisficing probability maximization refers to maximizing the probability of achieving a "target" level of profit. Finally, in the literature of finance, VaR is defined as the maximum loss on a portfolio of financial assets for a given risk level. The focus of this study is the MV approach.

Expected utility theory and the MV approach are closely related to each other. In decision sciences the utility approach and the MV approach are two common methodologies to model risk-sensitivity. The application of the utility approach is limited since it is practically difficult to assess the utility function of each decision maker. Thus, some researchers use the MV analysis. In the context of inventory management, mean measures the expected value of the cash flow while variance captures the variability of the cash flow. This problem is a type of multi-objective problem. The difficulty here lies in the conflict between the two objectives: maximizing the expected value of the cash flow and minimizing the variance of the cash flow. Therefore, we search for a compromise solution to these conflicting objectives.

The literature of the newsvendor model mostly assumes that the demand is the only source of randomness. Although demand constitutes a major source of randomness, in real life, supply randomness also exists. Recently, in the literature there has been an increased interest in modelling supply randomness as well. Because of the unforeseen events during the production and transportation of the products, the quantity received may not be equal to the quantity ordered. Specifically, during the production stage there might be long machine downtimes due to unplanned maintenance, strikes, seconds, scraps, lack of raw material, rework. During the transportation stage there might be accidents, deficiencies in the quality and various environmental factors as the possible causes of uncertainty. As a result, the quantity received may be some proportion of the quantity ordered or the capacity of the supplier may be limited by a random number. The combined randomness of demand and supply increases the risk of the decision maker. Therefore, as we consider risk-averse decision makers in this study, it is of crucial importance to include supply uncertainty in our model as well.

For most business practices, it is possible to find a correlation between the random factors of inventory like demand and supply and the financial market. For example, many financial instruments such as forwards, calls and puts exist to enable the inventory manager to hedge the inventory risk. Therefore, the inventory manager has the option to invest in a portfolio of financial assets. This results in a hedged cash flow for which the variance may be substantially less than the variance of the unhedged cash flow. Moreover, the impact of financial hedging depends on the degree of correlation between the randomness in demand, supply and the financial variables. In this thesis, we make our analysis based on the arbitrage-free complete market. Recently, there has been a huge interest in hedging operational risks using financial instruments. Yet, our study of the MV newsvendor problem combining inventory management and financial hedging is a novel and interesting one.

The motivation of this thesis is to follow the MV approach to the newsvendor problem under random demand and supply. This study consists of two main parts depending on the existence of the financial market. In the first part, we consider the MV problem without financial hedging. Firstly, we analyze the problem when the demand is the only source of uncertainty. Then, we include supply uncertainty into our problem with the assumption that demand and supply random variables are not necessarily independent. We analyze the MV problem with random yield, random capacity and both random yield and random capacity, respectively. Secondly, we consider the MV newsvendor problem with random demand and supply being correlated with the financial market. Again, we analyze the problem initially with no supply uncertainty and then include random yield, random capacity and both random yield and capacity models. For each one, as a special case we first analyze a single asset model and then models with multiple assets.

This thesis is organized as follows. In the next chapter, we review the literature of the relevant inventory models. In Chapter 3, we analyze the MV newsvendor problem with random supply without financial hedging. In Chapter 4, we characterize the optimal policy for the MV problems with random supply and financial hedging opportunity. In Chapter 5, we illustrate the models discussed in Chapter 3 and Chapter 4 with a simple numeric example and then we use a simulation-based approach for additional numerical results. To conduct Monte Carlo simulation we use Matlab as a simulation tool and provide the results. Finally in Chapter 6, we give a summary of the thesis and suggest several directions for future research.

Chapter 2

LITERATURE REVIEW

Newsvendor models are one of the basic models in inventory management. The related literature mostly assumes that the inventory manager is risk-neutral. So, the classical newsvendor problem aims to maximize expected profit or minimize expected cost, and the risk caused by demand uncertainty is disregarded. However, in real life, the decision makers are sensitive to risk. Thus, the risk-neutrality assumption has limitations in practice. Recently, there has been increasing interest in dealing with this issue using risk management tools. Inventory managers may prefer less gain but more stable cash flows. They are often conservative to uncertainty in profit due to high demand volatility in the market, and they may be risk-averse. Another group is risk-seeking decision makers, who can be viewed as gamblers. In fact, risk preferences can be categorized into three groups as risk-neutral, risk-averse and risk-seeking. In this thesis, we focus on risk-averse decision makers. There are several approaches to taking risk into account. The most common risk management models involve expected utility theory, MV analysis, satisficing probability maximization and value-at-risk (VaR). We discuss these models in the related literature in Section 2.1. The correlation between random demand and financial markets enables risk-sensitive decision makers to use financial hedging. The literature is given in Section 2.2. Finally, supply uncertainty is another important issue in inventory management, the literature of which is discussed in Section 2.3.

2.1 Risk Management Models

2.1.1 Expected Utility Theory

The expected utility model was initiated by Bernoulli [1738], but von Neumann and Morgenstern [1944] developed the modern theory of expected utility. They showed the existence of a nondecreasing utility function for rational decision makers. Since then, the expected utility theory has been widely used by risk-sensitive decision makers. First, Lau [1980] initiated the approach of utility functions for the newsvendor problem. After that, Bouakiz and Sobel [1992] used the exponential utility criterion to optimize the present value of net profit for the multi-period newsvendor problem over a finite planning horizon and infinite horizon. They show that the base-stock policy is optimal with the assumption of linear ordering cost. Eeckhoudt et al. [1995] study risk-averse newsvendors with the objective of expected utility maximization using Pratt [1964]'s argument that an increase in risk-aversion equals the concave transformation of the utility function. They show that as risk-aversion increases, the optimal order quantity decreases. Moreover, they analyze the effect of cost and price changes on the optimal order quantity. They also examine two types of changes in the degree of risk: an introduction of risky background wealth and an increase in demand uncertainty. Agrawal and Seshadri [2000a] also consider the risk-averse newsvendor in a single-period. They decide on both ordering quantity and selling price to maximize the expected utility. In their model, the demand distribution is affected by selling price which is determined by the retailer. They analyze two different cases. In the first case, price affects the scale of the demand distribution. In the second case, price affects only the location of the demand distribution. They compare the results with the risk-neutral newsvendor. They find that in the first case a risk-averse newsvendor will charge a higher price and give fewer orders. In the second case, they state that a risk-averse newsvendor will charge a lower price but they are inconclusive about order quantity. Agrawal and Seshadri [2000b] consider risk-averse retailers maximizing their expected utility and demonstrate the importance of intermediaries in supply chains. They conclude that the risk of retailers is reduced when the risk-neutral distributor offers mutually beneficial risk sharing contracts. Moreover, the inefficiency due to risk-aversion on the part of the retailers can be avoided. Schweitzer and Cachon [2000] investigate the decision bias in managers' decisions. They analyze each preference that leads to different order quantity from the risk-neutral ordering amount. Additionally, the experiments they performed reveal that for high-profit products the optimal order quantity is less than the optimal order amount of risk-neutral decision makers, and the opposite is true for low-profit products. Chen et al. [2007] consider risk-averse decision makers in multi-period inventory models. They deal with two related problems. In the first one, price is not a decision variable while in the second demand depends on price, which is also a decision variable. Moreover, they examine the theory of expected utility in multi-period inventory models. Finally, they incorporate a complete or partially complete financial market into their model. Keren and Pliskin [2006] derive the optimality conditions for a risk-averse expected utility maximizer newsvendor. They illustrate their work for a uniformly distributed demand. Wang et al. [2008] question the effect of selling price on the order quantity of a risk-averse newsvendor. They show that for most of the classes of risk-averse utility functions, as the selling price gets larger (provided that it is higher than a threshold value) the order quantity decreases. Wang and Webster [2009] consider the loss-averse single-period newsvendor. They conclude that, depending on the shortage cost, the loss-averse newsvendor may order more or less than the risk-neutral newsvendor.

2.1.2 MV Analysis

This approach originates from finance. Nobel laureate Markowitz [1959] used the MV approach for the portfolio management problem and constructed the efficient frontier. For a given value of mean return, the portfolio weights are decided so that the variance of return is minimized. The MV model is applicable and implementable since only expectation and variance of the objective function are calculated. Van Mieghem [2003] shows that MV optimization problem is equivalent to maximizing a utility function with a constant coefficient of risk-aversion or a quadratic concave utility function.

In inventory management literature, the MV approach has also received significant attention. Pioneered by Lau [1980], the newsvendor model has been studied in some detail. He considers the trade-off between the profit's expected value and its standard deviation. He concludes that the optimal order quantity is located between zero and the optimal order quantity of the risk-neutral newsvendor. Berman and Schnabel [1986] study the MV newsvendor problem for both risk-averse and risk-lover vendors. They show that under risk-aversion the amount ordered is less than the amount ordered under risk-neutrality. However, when the manager is risk-lover the ordered amount is greater than the risk-neutral one. Moreover, they also consider the problem with a fixed cost of ordering in addition to the variable purchase cost and show the optimality of (s, Q) policy where s denotes the reorder point and Q denotes the order quantity. Chen and Federgruen [2000] use a quadratic utility function for the newsvendor and then construct an efficient frontier via a numerical study. They show that without stockout cost the variance function of stochastic profit is a monotone increasing function of order quantity and a risk-averse newsvendor with the objective of maximizing expected utility should always order less than the risk-neutral newsvendor. However, if stockout cost is considered the variance function loses the monotonicity property and the optimal solution, depending on the demand distribution, can be larger or smaller than the risk-neutral newsvendor solution. In the second part, they study a single-product periodic review inventory model when customers arrive according to a Poisson process. For this model, the base-stock policy is known to be optimal. They consider both expected waiting time until demand is satisfied and its variability. Another performance measure they take into account is the steady-state holding costs incurred in the system. They make a trade-off analysis between these two performance measures. Due to managerial insights there are also other policies such as the (R, nQ) policy used in practice. This policy is such that whenever the inventory position is at or below a reorder point R, a minimum integer multiple of Q is ordered to raise the inventory position above R. They study the single-product, periodic review model with independent and identically distributed demands when stockouts are fully backlogged. The standard formulation for the (R, nQ) model is again to minimize long-run average costs. However, they make a trade-off analysis between long-run average costs and the variance of on-hand inventory as well as the variance of costs incurred in an arbitrary period. Choi et al. [2008] study MV analysis for decision makers with all kinds of risk attitudes. They study the problem with and without the stockout penalty cost. Wu et al. [2009] use power distributed demand and study the risk-averse newsvendor with the MV objective. They show that in the presence of stockout cost the risk-averse newsvendor may order more than the risk-neutral newsvendor. Choi and Chiu [2012] model the newsvendor problem with both the MV and the mean-downside-risk objectives for two cases: the first one is that the retail price is not a decision variable so it is exogenously given, the second is endogenous retail pricing case where the retail price is considered as a decision variable. For both cases, they show that the respective optimal order quantities for MV and mean-downside-risk are the same. Then, they perform sustainability measures such as the expected amount of leftover, the ratio of the expected sales to expected products leftover, the rate of return on investment and the probability of achieving the profit target. They conclude that the newsvendor adopting either the MV or the mean-downside-risk objective is more sustainable than the risk-neutral newsvendor. Choi et al. [2011] investigate a solution scheme to solve a periodic review multi-period inventory problem under an MV framework. Since it is impossible to separate variance in the sense of dynamic programming, they develop a primal-dual solution approach and show that base-stock policy is optimal.

2.1.3 Satisficing Probability Maximization

Satisficing probability maximization refers to maximizing the probability of exceeding a certain level of profit. Lau [1980] provides analytical solutions to maximize the probability of achieving a certain level of profit for some demand densities. Sankarasubramanian and Kumaraswamy [1983] determine the order quantity that maximizes the probability of exceeding a given profit. The two-product newsvendor problem is studied by Lau and Lau [1988] and numerical results are obtained. Li et al. [1990] and Li et al. [1991] extend Lau and Lau [1988] by dealing with uniformly and exponentially distributed demands. Parlar and Weng [2003] consider two conflicting objectives in the newsvendor problem namely to maximize the expected profit and maximize the probability of exceeding the expected profit.

2.1.4 VaR

Simons [1996], Jorion [1997], Dowd [1998] contribute to the VaR literature with their reviews. Luciano et al. [2003] use VaR as a risk measure in the context of a single-product multi-period economic order quantity (EOQ) inventory model and establish useful bounds. Gan et al. [2004] discuss the issue of coordinating in supply chains and develop coordinating contracts considering three cases: downside risk constraint, MV trade-off and maximization of expected utility. Tapiero [2005] provides an explanation for the VaR criterion when it is used as a tool for VaR efficiency design and demonstrates applications to single-period, multi-period and multi-product inventory problems. Ahmed et al. [2007] analyze a single-item, multi-period inventory model for a risk-averse newsvendor using coherent risk measures such as conditional value-at-risk and mean-absolute semi-deviation. They show that the optimal policy has a structure similar to the classical expected value problem. Özler et al. [2009] study single-period, multi-product inventory problem with a VaR constraint. They derive an exact total profit distribution function for the two-product case and they develop an approximation method for the multi-product case.

2.2 Financial Hedging

Financial markets enable inventory managers to diversify the risk in inventory systems by using the correlation between the demand of a product and the price of a financial asset.

Earlier work by Anvari [1987] presents a one-period newsboy problem with no set-up costs by using the capital asset pricing model (CAPM). He computes the optimal inventory level in the case of a normally distributed demand and states that, depending on the sign of the covariance, the optimal order quantity with CAPM framework can be larger or smaller than the optimal standard newsvendor solution. Then, Chung [1990] provides an alternative solution methodology to Anvari [1987]. A more recent work by Caldentey and Haugh [2006] considers the operations of a nonfinancial corporation that chooses an optimal operating policy and an optimal trading strategy in the financial markets. The corporation is risk-averse with an MV objective function. When the profits are correlated with returns in the financial markets, they deal with the problem of dynamically hedging its profits. Moreover, they also analyze the effect of different informational assumptions on the types of hedging and solution techniques. Gaur and Seshadri [2005] consider the problem of hedging inventory risk for the newsvendor problem when the demand is correlated with the price of a financial asset. The statistical evidence that an inventory index (Redbook), that represents average sales, is highly correlated with a financial index (SP500), that represents average asset prices provides an opportunity to construct static hedging strategies using both MV and utility-maximization frameworks. When there is a linear dependence between demand forecast and the price of the asset, they derive the hedged-cash flow for a perfectly-correlated arbitrage-free complete market. Thus, they show that by using financial instruments it is possible to obtain a deterministic hedged-cash flow. However, in practice perfect correlation is not realistic, so they extend their work to partially correlated markets. They conclude that the risk of inventory carrying can be replicated as a financial portfolio by using simple instruments like bonds, futures and options and a risk-averse decision maker orders more inventory when hedging is applied. Chu et al. [2009] develop a continuously reviewed single-product inventory model with uncertain demand. To mitigate inventory risk, a financial hedging approach is established and an MV criterion is used. Ding et al. [2007] combine operational and financial hedging. They deal with a global firm that sells to both domestic and foreign markets. Thus, the firm faces demand and exchange-rate uncertainties. They use an MV utility function to consider the firm's risk-aversion when there are multiple products and suppliers. From the operations point of view they suggest that the firm exploit the capacity allocation by delaying the commitment until demand and exchange rate uncertainties are realized. From the financial point of view, they suggest that the firm use call and put options to hedge exchange rate uncertainty. They conclude that with the financial hedge, the firm invests more than without the financial hedge. By using the operational hedge, the expected profit can be increased and by using the financial hedge, the variance of profit can be decreased.

2.3 Random Supply

In inventory systems, apart from the randomness in demand, supply of the products may also be random. During production or procurement of products, the supply process may be disrupted due to limitations or unforeseen events. Thus, the received amount may be less than the ordered amount. As mentioned in Chopra and Sodhi [2004] supply uncertainty may result from natural disasters, machine breakdowns, labour disputes, war, terrorism and many other causes. The study by Norrman and Jansson [2004] gives an example of such an accident. In 2000, a fire occurred at an supplier plant of Ericsson. This plant was responsible for producing radio-frequency chips. As a result, fire caused a loss of almost $400 million and eventually Ericsson decided to withdraw from the mobile phone industry. Another example is the UK chassis manufacturer UPF Thompson, which had financial problems in 2001 and so the production was suspended at Land Rover (Juttner [2005]). Additionally, Kharif [2003] writes about Motorola's camera phones. The shipping of phones failed as a result of component shortages during the holiday season in 2003. All these tragic examples demonstrate the importance of including random supply into inventory models. In the literature, also, there is a growing interest to include supply unreliability into the inventory models.

The earliest paper modeling the yield uncertainty belongs to Karlin [1958]. He argues that if the holding and shortage costs are convex increasing functions, then a critical level of initial inventory exists, below which an order should be issued. Shih [1980] considers the EOQ inventory model and stochastic single-period model under the assumption that some percentage of received items is defective. The percentage of defective items is random with a known probability distribution. The necessary optimality conditions are derived and compared to the traditional ones. Noori and Keller [1986] study a lot-size inventory problem where the quantity received as well as the demand are random. They extend Hadley and Whitin [1963] and analyze the affect of parameter changes on the optimal inventory policy and provide numerical examples for uniform, normal and exponential demand distributions. Gerchak et al. [1988] deal with a periodic review model with random yield and conclude that the optimal policy is not order-up-to anymore. Henig and Gerchak [1990] analyze periodic review inventory model with random yield. They prove that for a single-period model an optimal order point exists whose value does not depend on replenishment randomness. When yield is a random multiple of lot size, the non-order-up-to optimal policy is shown to be optimal for a finite-horizon model. Parlar and Wang [1993] consider a single-period random yield model where there are two suppliers. They conclude that diversification enables a reduction in overall yield variability. Yano and Lee [1995] review the related literature about lot sizing when the yields are random. Ciarallo et al. [1994] analyze the problem when the capacity is random with a known distribution for single-period, multiple-period and infinite-horizon scenarios. They show that for single-period model the optimal policy does not change. For multiple-period and infinite-horizon models order-up-to policies that are dependent on the distribution of capacity are optimal. Jain and Silver [1995] also study the random capacity model when a given level of capacity can be guaranteed to be available by paying a premium. Özekici and Parlar [1999] consider a special case of the random yield model where the order is either fully satisfied or remains completely unfulfilled. Erdem and Özekici [2002] is an extension of Özekici and Parlar [1999]. They also consider random capacity as the source of random supply. Gallego and Hu [2004] study the finite capacity inventory model when the system is subject to random yields. Arifoğlu and Özekici [2007] extend Gallego and Hu [2004]'s model by considering the case when the environment is only partially observable.

Okyay et al. [2010] categorize the random supply models under three groups: random yield, random capacity, random yield and random capacity. Let y be the units ordered and $Q(y)$ be the received amount.

- Random Yield: Due to unforeseen events only a fraction of ordered amount is received and

  $$Q\left(y\right) = Uy$$

  where U represents the proportion of nondefective items received.

- Random Capacity: The supplier's capacity is a random variable denoted by K so that

  $$Q\left(y\right) = \min\left\{K, y\right\}.$$

  Once y units are ordered, the supplier will ship y if its capacity K is greater than y, or K units will be shipped.

- Random Yield and Capacity: This model combines the previous two so that

  $$Q\,(y) = U \min\{K, y\}\,.$$

  When y units are ordered, at most K units can be shipped by the supplier and only a proportion U is received in good shape.

Our work is closely related to Okyay et al. [2010], Okyay et al. [2011] and Sayın [2011]. In Chapter 3, we consider the MV newsvendor model where there are risks associated with the uncertainty in demand as well as supply. Although all the three works study the newsvendor model with the same risk uncertainty categories, Okyay et al. [2010] aim expected cash flow maximization and Sayın [2011] use the expected utility maximization framework while our goal is to maximize the MV value of the cash flow. Then in Chapter 4, we consider the MV model where the randomness in demand and supply is correlated with the financial markets like Okyay et al. [2011] and Sayın [2011]. However, Okyay et al. [2011] and Sayın [2011] both use minimum-variance approach in this part. This part also differs from their works in a way that we choose the optimal order quantity and hedging portfolio in only one step following MV hedging framework.

Chapter 3

MV MODELS WITH RANDOM DEMAND AND SUPPLY

We take the MV approach to the newsvendor problem. The MV problem is a parametric optimization problem where mean measures the expected value of the cash flow while variance captures the variability of the cash flow. Considering risk-averse newsvendors, there are two ways to model the optimization problems under the MV formulation: by choosing an order quantity y, the newsvendor maximizes the mean of the cash flow subject to an upper bound on the variance of the cash flow, or vice versa, the newsvendor minimizes the variance of the cash flow subject to a lower bound on the mean of the cash flow. By systematic variation of the lower/upper bound, we can generate the efficient frontier which will be discussed further later on in this chapter. As a general idea note that this frontier consists of order quantities whose expected profit can not be increased without a simultaneous increase of its corresponding variance of profit.

Let $CF(\mathbf{X}, y)$ be the random cash flow when the order quantity is y where $\mathbf{X}$ is a vector of random variables representing demand and supply uncertainty. The MV optimization problem is

$$\max_{y \geq 0} E[CF(\mathbf{X}, y)] - \theta Var[CF(\mathbf{X}, y)] \tag{3.1}$$

where $\theta \geq 0$ denotes the relative weight of the variance criterion. It can be regarded as risk-aversion parameter. As θ increases, the newsvendor becomes more risk-averse.

In Section (3.1), we first discuss the results for the MV newsvendor where the only randomness comes from demand. Then, we include supply uncertainty into the model. In Section (3.2), we discuss random yield models when only the fraction of ordered amount can enter the stockpile. In Section (3.3), we discuss random capacity models when the supplier's capacity is random. Lastly, in Section (3.4), we combine random yield and capacity and analyze the results.

3.1 MV Model

In this section, we suppose that the newsvendor deals with stochastic demand where $\mathbf{X} = \{D\}$. Let c be the unit purchase cost, p be the unit sale price, u be the emergency ordering cost for a shortage of products after demand is realized and s be the net unit salvage value, which is obtained by subtracting inventory holding costs from salvage values. It is assumed that holding cost is charged at the end of the period. To avoid trivial cases, $p \geq u > c > 0$ and $c > s$. We also assume that D is strictly greater than 0. Let $CF(D, y)$ denote the random cash flow, that is

$$\begin{aligned} CF(D,y) &= -cy + pD + s(y-D)^+ - u(D-y)^+ \\ &= -cy + pD + s(y - \min\{D,y\}) - u(D - \min\{D,y\}) \\ &= (s-c)y + (p-u)D + (u-s)\min\{D,y\} \end{aligned} \tag{3.2}$$

where $(y-D)^+ = y - \min\{D, y\}$ and $(D-y)^+ = D - \min\{D, y\}$.

From here onwards, we will let $m(y)$ and $v(y)$ denote, respectively, the mean of the cash flow and the variance of the cash flow. For the MV model they are

$$m(y) = E[CF(D,y)] = (s-c)y + (p-u)E[D] + (u-s)E[\min\{D,y\}] \tag{3.3}$$

and

$$\begin{aligned} v(y) &= Var[CF(D,y)] \\ &= (p-u)^2 Var[D] + (u-s)^2 Var[\min\{D,y\}] \\ &\qquad +2(p-u)(u-s)Cov\left[D, \min\{D,y\}\right]. \end{aligned} \tag{3.4}$$

Our optimization problem can be written as

$$\max_{y \geq 0} H(y,\theta) = E[CF(D,y)] - \theta Var[CF(D,y)] \tag{3.5}$$

where

$$\begin{aligned} H(y,\theta) &= (s-c)y + (p-u)E[D] + (u-s)E[\min\{D,y\}] \\ &\quad -\theta \begin{pmatrix} (p-u)^2 Var[D] + (u-s)^2 Var[\min\{D,y\}] \\ +2(p-u)(u-s)Cov\left[D, \min\{D,y\}\right] \end{pmatrix} \end{aligned} \tag{3.6}$$

is the objective function for any fixed $\theta \geq 0$.

Throughout this thesis, we will let F_X, f_X, $\bar{F}_X = 1 - F_X$ denote the cumulative distribution function, the probability density function and the survival probability function respectively for any random variable X. To obtain the derivative of the mean and variance of the cash flow, note that for any random variable X we can write

$$E[\min\{X,y\}] = \int_0^y x f_X(x)dx + y\int_y^{+\infty} f_X(x)dx.$$

The derivative with respect to y is

$$\frac{dE[\min\{X,y\}]}{dy} = \int_y^{+\infty} f_X(x)dx = P\{X>y\} = E[1_{\{X>y\}}] = 1-F_X(y) = \bar{F}_X(y). \quad (3.7)$$

Similarly,

$$\begin{aligned} Var[\min\{X,y\}] &= E[(\min\{X,y\})^2] - E[\min\{X,y\}]^2 \\ &= \int_0^y x^2 f_X(x)dx + y^2\int_y^{+\infty} f_X(x)dx - E[\min\{X,y\}]^2 \quad (3.8) \end{aligned}$$

and

$$\begin{aligned} Cov[X,\min\{X,y\}] &= E[X(\min\{X,y\})] - E[X]E[\min\{X,y\}] \\ &= \int_0^y x^2 f_X(x)dx + y\int_y^{\infty} x f_X(x)dx - E[X]E[\min\{X,y\}]. \quad (3.9) \end{aligned}$$

Then, it follows from (3.7) and (3.8) that

$$\begin{aligned} \frac{dVar[\min\{X,y\}]}{dy} &= 2y\bar{F}_X(y) - 2E[\min\{X,y\}]\bar{F}_X(y) \\ &= 2\bar{F}_X(y)(y - E[\min\{X,y\}]) \quad (3.10) \\ &= 2Cov[\min\{X,y\}, 1_{\{X>y\}}]. \end{aligned}$$

In this section, we will use the second equality form of (3.10) for the sake of simplicity. Moreover, from (3.7) and (3.9), one can show that

$$\begin{aligned} \frac{dCov[X,\min\{X,y\}]}{dy} &= \int_y^{\infty} x f_X(x)dx - E[X]\int_y^{\infty} f_X(x)dx \\ &= E[X1_{\{X>y\}}] - E[X]E[1_{\{X>y\}}] \\ &= Cov[X, 1_{\{X>y\}}]. \quad (3.11) \end{aligned}$$

Particularly, to solve (3.5), we take the derivative of (3.6) with respect to y and set it equal to zero. By using (3.7), (3.10) and (3.11) where X is D, the first order condition is obtained as

$$\begin{aligned} \frac{dH(y,\theta)}{dy} &= (s-c)+(u-s)\bar{F}_D(y) \\ &\quad -2\theta\left[(u-s)(u-s)\bar{F}_D(y)S(y)+(p-u)Cov[D,1_{\{D>y\}}]\right] \\ &= 0 \end{aligned} \tag{3.12}$$

where $S(y) = y - E[\min\{D,y\}]$ denotes the expected unsold amount. It is clear that $S(y) \geq 0$. The shape of $S(y)$ will be important in later stages of our analysis. It is a convex increasing function because

$$S'(y) = 1 - \bar{F}_D(y) = F_D(y) \geq 0$$

and

$$S''(y) = f_D(y) \geq 0.$$

Moreover, $S(0) = 0$ and as the order quantity y increases to $+\infty$, the slope of $S(y)$ increases to 1.

In order to get a rough idea about the MV problem, we analyze two extreme cases. When $\theta = 0$, the newsvendor is risk-neutral and so the expected profit is maximized, or the problem is

$$\max_{y\geq 0} m(y) = E[CF(D,y)].$$

The first order condition is

$$\begin{aligned} m'(y) &= (s-c)+(u-s)\bar{F}_D(y) \\ &= 0. \end{aligned} \tag{3.13}$$

The mean of the cash flow is concave since

$$m''(y) = -(u-s)f_D(y) \leq 0. \tag{3.14}$$

This also implies that $m'(y)$ is decreasing in y. It follows from (3.13) that the optimal order quantity satisfies

$$P\{D \leq y^*_{RN}\} = \frac{u-c}{u-s} = \hat{p} \tag{3.15}$$

where y^*_{RN} is the optimal order quantity for the risk-neutral newsvendor and $\hat{p}$ denotes a critical ratio which satisfies $0 < \hat{p} < 1$. Note that this is equivalent to the classical newsvendor solution. Therefore, (3.15) gives the optimality condition provided that $m'(0) > 0$ and $m'(+\infty) < 0$.

The optimal solution is $y^*_{RN} = 0$ if $m'(0) \leq 0$; or

$$m'(0) = (s - c) + (u - s)(1 - P\{D = 0\}) \leq 0.$$

Equivalently, we can state that if

$$P\{D = 0\} \geq \hat{p} \tag{3.16}$$

then $y^*_{RN} = 0$. Note that the ratio in the right hand side of (3.16) is between 0 and 1. Additionally, in this thesis, we assumed D to be strictly greater than 0. However, without this assumption (3.16) may hold. In this case, we can conclude that if $P\{D = 0\} = 1$, the newsvendor clearly orders nothing.

Moreover, the optimal solution is $y^*_{RN} = +\infty$ if $m'(+\infty) \geq 0$; or

$$m'(+\infty) = (s - c) + (u - s)P\{D = +\infty\} \geq 0.$$

Equivalently, we can state that if

$$P\{D = +\infty\} \geq 1 - \hat{p} \tag{3.17}$$

then $y^*_{RN} = +\infty$. The ratio in the right hand side of (3.17) is again between 0 and 1. If demand is not finite, or $P\{D = +\infty\} = 1$, we have $y^*_{RN} = +\infty$. From now on, in our analysis we assume $m'(0) > 0$ and $m'(+\infty) < 0$ to avoid trivial cases.

As θ increases to $+\infty$, the newsvendor becomes extremely risk-averse. The objective contribution due to expected return becomes negligible and the problem turns into minimizing the variance of the cash flow, or

$$\min_{y \geq 0} v(y) = Var[CF(D, y)]. \tag{3.18}$$

We take the first derivative of (3.4) with respect to y and see that the variance of the cash flow is increasing in y since

$$v'(y) = 2(u - s)\left[(u - s)\bar{F}_D(y)S(y) + (p - u)Cov[D, 1_{\{D>y\}}]\right] \geq 0. \tag{3.19}$$

Here, (3.19) follows from the fact that $k(y) = Cov\left[D, 1_{\{D>y\}}\right] \geq 0$. To see this, note that

$$\begin{aligned} k(y) &= E[D1_{\{D>y\}}] - E[D]E[1_{\{D>y\}}] \\ &= \left(\int_y^{+\infty} x f_D(x)dx - E[D] \int_y^{+\infty} f_D(x)dx \right) \\ &= \int_y^{+\infty} (x - E[D]) f_D(x)dx. \end{aligned} \tag{3.20}$$

It can be seen that $k(0) = 0$. The derivative of (3.20) with respect to y is

$$k'(y) = -(y - E[D]) f_D(y)$$

so that

$$\text{sign } (k'(y)) = \begin{cases} + & y \leq E[D] \\ - & y > E[D] \end{cases} .$$

Therefore, starting from 0, until y becomes $E[D]$, $k(y)$ increases. Then, when $y > E[D]$, $k(y)$ is decreasing in y. However, in the region $y > E[D]$, $k(y)$ never becomes negative since

$$k(y) = \int_y^{\infty} (x - E[D]) f_D(x)dx \geq \int_y^{\infty} (y - E[D]) f_D(x)dx = (y - E[D]) P\{D \geq y\} \geq 0.$$

Put another way, $Cov\left[D, 1_{\{D>y\}}\right] \geq 0$ because $1_{\{D>y\}}$ is an increasing function of D.

Thus, the optimal order quantity in (3.18) is 0. For this case, the expected cash flow (3.3) and variance of the cash flow (3.4) are

$$E[CF(D, 0)] = (p - u)E[D]$$

and

$$Var[CF(D, 0)] = (p - u)^2 Var[D] \tag{3.21}$$

since $CF(D, 0) = (p - u)D$.

Lemma 3.1.1 *(a) $E[CF(D, y)]$ is concave in y; it is increasing on $[0, y^*_{RN}]$ and decreasing on $(y^*_{RN}, +\infty)$. (b) $Var[CF(D, y)]$ is a nondecreasing function of y. Moreover,*

$$(p - u)^2 Var[D] \leq Var[CF(D, y)] \leq (p - s)^2 Var[D]$$

for all y.

Proof. It is seen from (3.14) that $E[CF(D,y)]$ is concave and the maximum is attained at y^*_{RN} by (3.15). Since the derivative of $Var[CF(D,y)]$ with respect to order quantity (3.19) is positive, $Var[CF(D,y)]$ is nondecreasing. Moreover,

$$\begin{aligned}\lim_{y\to+\infty} Var[CF(D,y)] &= (p-u)^2 Var[D] + (u-s)^2 \lim_{y\to+\infty} Var[\min\{D,y\}] \\ &\quad +2(p-u)(u-s) \lim_{y\to+\infty} Cov\left[D, \min\{D,y\}\right] \\ &= (p-s)^2 Var[D]\end{aligned}$$

and the lower bound for $Var[CF(D,y)]$ is given in (3.21). ∎

It follows from part (a) of Lemma 3.1.1 and (3.15) that $E[CF(D,y)]$ is maximized at a finite, positive point. To obtain it, θ is set to zero in the MV formulation. This point corresponds to the classical newsvendor solution y^*_{RN} that satisfies (3.15). However, to maximize the MV objective we must consider both the mean and variance of the cash flow. For the MV optimization problem, the difficulty of identifying an optimal solution is the conflict among the two objectives. For instance, the best solution for minimizing the variance of the cash flow is the worst for maximizing the mean of the cash flow. Therefore, a good compromise between the objectives acceptable to the newsvendor is sought. For the multi-objective problems a feasible solution is called efficient (Pareto optimal or non-dominated) if there is no other feasible solution where all the objectives get a better value. We state that y is dominated if and only if there exists y' that satisfies $E[CF(D,y')] \geq E[CF(D,y)]$ and $Var[CF(D,y')] \leq Var[CF(D,y)]$ where at least one of the inequalities is strict.

Let $y(\theta)$ be an optimal solution to (3.5). Then it should satisfy

$$E[CF(D,y(\theta))] - \theta Var[CF(D,y(\theta))] \geq E[CF(D,y')] - \theta Var[CF(D,y')] \tag{3.22}$$

for all y'. Suppose that there exists y' such that

$$E[CF(D,y')] \geq E[CF(D,y(\theta))] \tag{3.23}$$

and

$$Var[CF(D,y')] \leq Var[CF(D,y(\theta))]. \tag{3.24}$$

Multiplying (3.24) by $\theta \geq 0$ and subtracting it from (3.23) we get

$$E[CF(D,y')] - \theta Var[CF(D,y')] \geq E[CF(D,y(\theta))] - \theta Var[CF(D,y(\theta))]$$

which contradicts (3.22). Clearly, the solution to the MV problem consists of non-dominated order quantities. From Lemma 3.1.1, it can be easily verified that the order quantities in the region $[0, y^*_{RN}]$ are all non-dominated. Moreover, order quantities in $(y^*_{RN}, +\infty)$ are all dominated, since y^*_{RN} dominates all $y > y^*_{RN}$. This also implies that $y(\theta) \leq y^*_{RN}$ for all $\theta \geq 0$.

Proposition 3.1.2 *The optimal order quantity $y(\theta)$ that maximizes the MV objective is less than or equal to the classical newsvendor solution y^*_{RN} for any $\theta \geq 0$.*

Proof. Suppose that there exists an optimal order quantity that satisfies $y(\theta) > y^*_{RN}$. Then by concavity of the expected value of the cash flow we know that

$$E[CF(D, y(\theta))] < E[CF(D, y^*_{RN})]$$

and since variance of the cash flow is increasing

$$Var[CF(D, y(\theta))] \geq Var[CF(D, y^*_{RN})].$$

From the above arguments we can state that $y(\theta)$ is dominated by y^*_{RN} and this is a contradiction. Therefore, for our analysis we only need to consider the order quantities that lie in the region $[0, y^*_{RN}]$. ■

Theorem 3.1.3 *The optimal order quantity $y(\theta)$ that maximizes the MV objective (3.6) is obtained from (3.12) by solving*

$$F_D(y(\theta)) + 2\theta\left[(u-s)\bar{F}_D(y(\theta))S(y(\theta)) + (p-u)Cov[D, 1_{\{D>y(\theta)\}}]\right] = \hat{p}. \tag{3.25}$$

Moreover, $y(\theta)$ decreases as θ increases.

Proof. For any fixed y, let $\theta(y)$ satisfy the optimality condition (3.25) which can be written as

$$F_D(y) + 2\theta(y)\left[(u-s)\bar{F}_D(y)S(y) + (p-u)Cov[D, 1_{\{D>y\}}]\right] = \hat{p}$$

or

$$\theta(y) = \frac{\hat{p} - F_D(y)}{2\left[(u-s)\bar{F}_D(y)S(y) + (p-u)Cov[D, 1_{\{D>y\}}]\right]}. \tag{3.26}$$

In later parts of our analysis we use the integral form of $k(y) = Cov[D, 1_{\{D>y\}}]$ in (3.20) to simplify the calculations. Thus, (3.26) can be written as

$$\theta(y) = \frac{\hat{p} - F_D(y)}{2\left\{(u-s)\bar{F}_D(y)S(y) + (p-u)\int_y^\infty (x - E[D])f_D(x)dx\right\}}.$$

For all y in the region $[0, y^*_{RN}]$, $\hat{p} \geq F_D(y)$ is valid and in this non-dominated region we can show that $\theta(y)$ is decreasing in y. The derivative of (3.26) is

$$\theta'(y) = \frac{\left\{\begin{array}{c} -f_D(y)\left[(u-s)\bar{F}_D(y)S(y) + (p-u)\int\limits_y^\infty (x-E[D])f_D(x)dx\right] \\ -[(u-s)\left(-f_D(y)S(y) + \bar{F}_D(y)S'(y)\right) \\ +(p-u)\left(-(y-E[D])f_D(y)\right)](\hat{p}-F_D(y)) \end{array}\right\}}{2\left\{(u-s)\bar{F}_D(y)S(y) + (p-u)\int\limits_y^\infty (x-E[D])f_D(x)dx\right\}^2}. \tag{3.27}$$

If $y \leq E[D]$, then since $\bar{F}_D(y) \geq \hat{p} - F_D(y)$, (3.27) gives

$$\begin{aligned} \theta'(y) &\leq \frac{\left\{\begin{array}{c} -(u-s)f_D(y)\bar{F}_D(y)S(y) - (p-u)f_D(y)\int\limits_y^\infty (x-E[D])f_D(x)dx \\ +(u-s)f_D(y)S(y)\bar{F}_D(y) - (u-s)\bar{F}_D(y)F_D(y)(\hat{p}-F_D(y)) \\ +(p-u)f_D(y)(y-E[D])(\hat{p}-F_D(y)) \end{array}\right\}}{2\left\{(u-s)\bar{F}_D(y)S(y) + (p-u)\int\limits_y^\infty (x-E[D])f_D(x)dx\right\}^2} \\ &\leq 0. \end{aligned} \tag{3.28}$$

If $y > E[D]$, then since $x \geq y > E[D]$ and $\bar{F}_D(y) \geq \hat{p} - F_D(y)$, (3.27) leads to

$$\begin{aligned} \theta'(y) &\leq \frac{\left\{\begin{array}{c} -(u-s)f_D(y)\bar{F}_D(y)S(y) - (p-u)f_D(y)\int\limits_y^\infty (y-E[D])f_D(x)dx \\ +(u-s)f_D(y)S(y)\bar{F}_D(y) - (u-s)\bar{F}_D(y)F_D(y)(\hat{p}-F_D(y)) \\ +(p-u)f_D(y)(y-E[D])(\hat{p}-F_D(y)) \end{array}\right\}}{2\left\{(u-s)\bar{F}_D(y)S(y) + (p-u)\int\limits_y^\infty (x-E[D])f_D(x)dx\right\}^2} \\ &\leq -\frac{(u-s)\bar{F}_D(y)F_D(y)(\hat{p}-F_D(y))}{2\left\{(u-s)\bar{F}_D(y)S(y) + (p-u)\int\limits_y^\infty (x-E[D])f_D(x)dx\right\}^2} \\ &\leq 0 \end{aligned} \tag{3.29}$$

where the second inequality follows from

$$\int\limits_y^\infty (y-E[D])f_D(x)dx = (y-E[D])\bar{F}_D(y) \geq (y-E[D])(\hat{p}-F_D(y)).$$

Moreover, note that $\theta(0) = +\infty$ and $\theta(y^*_{RN}) = 0$. Therefore, $\theta(y)$ decreases from $+\infty$ to 0 as y increases from 0 to y^*_{RN}. Up to now, by showing $\theta(y)$ is a decreasing function of y, we establish the existence of an order quantity for each risk-aversion level θ. Note that on $(y^*_{RN}, +\infty)$, $\theta(y) \leq 0$ so that this region is dominated. Additionally, the first derivative of the MV objective function (3.12) evaluated at $y = 0$ and $y = y^*_{RN}$ are

$$\begin{aligned} \left.\frac{dH(y,\theta)}{dy}\right|_{y=0} &= (s-c) + (u-s)\bar{F}_D(0) \\ &\quad -2\theta(u-s)\left[(u-s)\bar{F}_D(0)S(0) + (p-u)Cov[D, 1_{\{D>0\}}]\right] \\ &= u - c \geq 0 \end{aligned} \tag{3.30}$$

and

$$\begin{aligned} \left.\frac{dH(y,\theta)}{dy}\right|_{y=y^*_{RN}} &= -2\theta(u-s)\left[(u-s)\bar{F}_D(y^*_{RN})S(y^*_{RN}) \right. \\ &\quad \left. +(p-u)Cov[D, 1_{\{D>y^*_{RN}\}}]\right] \\ &\leq 0. \end{aligned} \tag{3.31}$$

Therefore, the MV objective function is quasi-concave and the order quantity that lies between 0 and y^*_{RN} maximizes (3.5). For any $\theta \geq 0$, by taking the inverse Θ^{-1} of $\theta(y)$, we can obtain the optimal order quantity corresponding to that θ value so that

$$y(\theta) = \Theta^{-1}(\theta). \tag{3.32}$$

The objective function in (3.6) is not necessarily concave. However, it follows from (3.30) and (3.31) and the fact $\theta(y)$ is decreasing in y in the region $[0, y^*_{RN}]$ that there exists an optimal order quantity $0 \leq y(\theta) \leq y^*_{RN}$ which satisfies (3.25) for any $\theta \geq 0$. According to the newsvendor's preference of risk given by θ, the optimal order quantity is chosen between 0 and y^*_{RN}. Since $\theta'(y) \leq 0$, we know that $\theta(y)$ is decreasing in y. Therefore, the inverse given in (3.32) is also decreasing and this leads to the conclusion that as risk-aversion θ increases, the optimal order quantity $y(\theta)$ decreases. ■

As a general remark throughout this thesis we state that the uniqueness of the optimal order quantity is ensured if and only if $\theta(y)$ is a strictly decreasing function of y. Otherwise, the optimal order quantity, $y(\theta)$, is the minimum y satisfying (3.25). Also, note that we can write (3.25) as

$$m'(y(\theta)) - \theta v'(y(\theta)) = 0$$

where $m'(y)$ and $v'(y)$ are given in (3.13) and (3.19). When $\theta = 0$, (3.25) becomes (3.15). When $\theta > 0$, the only difference is an additional term that comes from including the variance of the cash flow into the objective function.

In this section, we have investigated optimal order quantities and corresponding expected profits and variance of profits when demand constitutes the only source of randomness. Consideration of variance in the objective function is important since it captures the stochastic nature of the problem and at the same time it enables us to include the newsvendor's risk attitude. Therefore, in our analysis we find tailor-fit optimal order quantity for the newsvendor with each risk-aversion level. We observe that a risk-averse newsvendor orders lower than the classical newsvendor solution. Another conclusion is that as the newsvendor becomes more risk-averse the optimal order quantity decreases.

3.1.1 MV Model with Exponential Demand

Up to this point, any demand distribution is considered. We now illustrate a numerical example where demand is exponentially distributed with rate $\lambda = 0.1$. The newsvendor can buy each item with purchase cost $c = 0.5$, sell it at a price $p = 1$, and the leftovers can be salvaged at $s = 0.1$. For simplicity, we assume that emergency cost is equal to sale price; that is $u = 1$. For this setting to calculate the mean and variance of the cash flow, we obtain the following explicit expressions.

Mean of sales is

$$\begin{aligned} E[\min\{D, y\}] &= \int_0^y \lambda e^{-\lambda x} x dx + \int_y^\infty \lambda e^{-\lambda x} y dx \\ &= \frac{1 - e^{-\lambda y}}{\lambda} - y e^{-\lambda y} + y e^{-\lambda y} \\ &= \frac{1 - e^{-\lambda y}}{\lambda} \end{aligned} \tag{3.33}$$

second moment of sales is

$$\begin{aligned} E[(\min\{D, y\})^2] &= \int_0^y \lambda e^{-\lambda x} x^2 dx + \int_y^\infty \lambda e^{-\lambda x} y^2 dx \\ &= \frac{2}{\lambda^2} - 2\left(\frac{y e^{-\lambda y}}{\lambda} + \frac{e^{-\lambda y}}{\lambda^2}\right) + y^2 e^{-\lambda y} \end{aligned}$$

and, lastly, variance of the sales is

$$\begin{aligned} Var[\min\{D,y\}] &= E[(\min\{D,y\})^2] - E[\min\{D,y\}]^2 \\ &= \frac{2}{\lambda^2} - 2\left(\frac{ye^{-\lambda y}}{\lambda} + \frac{e^{-\lambda y}}{\lambda^2}\right) + y^2 e^{-\lambda y} - \left(\frac{1-e^{-\lambda y}}{\lambda}\right)^2. \end{aligned} \tag{3.34}$$

The salvage amount can be obtained easily as

$$\begin{aligned} S(y) &= y - E[\min\{D,y\}] \\ &= \frac{y\lambda - 1 + e^{-\lambda y}}{\lambda}. \end{aligned}$$

The mean and variance of the cash flow are

$$E[CF(D,y)] = (s-c)y + (p-s)E[\min\{D,y\}] \tag{3.35}$$

and

$$Var[CF(D,y)] = (p-s)^2 Var[\min\{D,y\}]. \tag{3.36}$$

For the exponential demand distribution, by substituting (3.33) into (3.35), the mean of the cash flow is obtained as

$$E[CF(D,y)] = (s-c)y + (p-s)\left(\frac{1-e^{-\lambda y}}{\lambda}\right)$$

and by substituting (3.34) to (3.36), the variance of the cash flow becomes

$$Var[CF(D,y)] = (p-s)^2 \left[\frac{2}{\lambda^2} - 2\left(\frac{ye^{-\lambda y}}{\lambda} + \frac{e^{-\lambda y}}{\lambda^2}\right) + y^2 e^{-\lambda y} - \left(\frac{1-e^{-\lambda y}}{\lambda}\right)^2\right].$$

The optimality condition in (3.25) can be updated as

$$1 - e^{-\lambda y(\theta)} + 2\theta(p-s)e^{-\lambda y(\theta)}\left(\frac{y(\theta)\lambda - 1 + e^{-\lambda y(\theta)}}{\lambda}\right) = \hat{p}$$

or

$$\theta(y) = \frac{\hat{p} - (1-e^{-\lambda y})}{2(p-s)e^{-\lambda y}\left(\frac{y\lambda - 1 + e^{-\lambda y}}{\lambda}\right)}.$$

The graphical illustrations of the relationships between $\theta(y)$ versus y and $E[CF(D,y(\theta))]$ versus $Var[CF(D,y(\theta))]$ as θ changes from 0 to 5 with 0.01 increments are shown in Figure 3.1 and Figure 3.2 respectively.

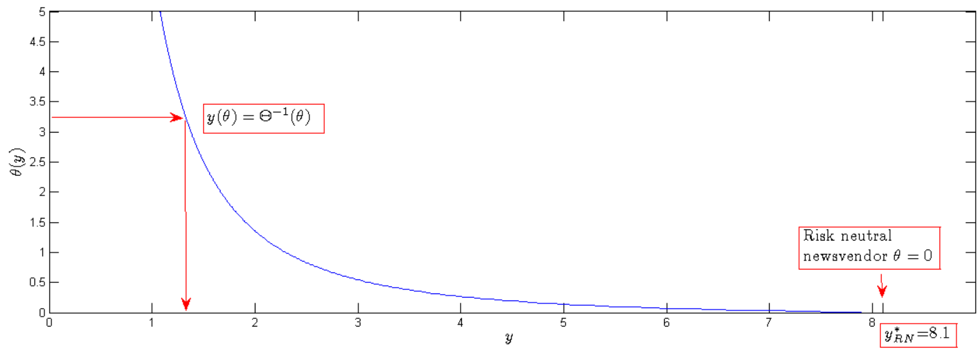

Figure 3.1: $\theta(y)$ as a Function of y

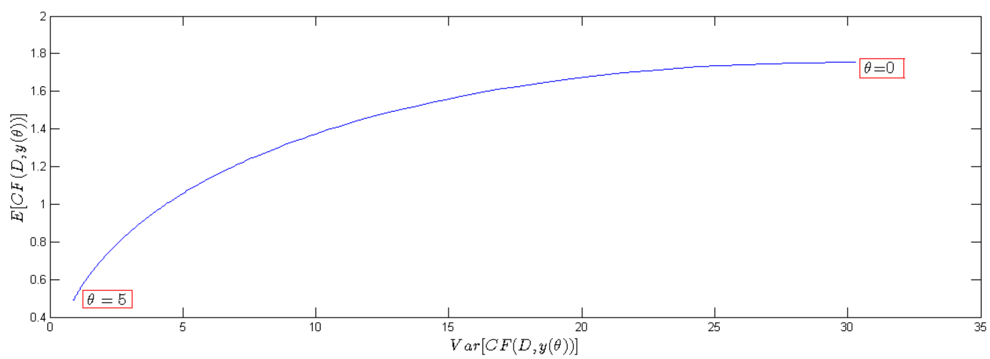

Figure 3.2: Efficient Frontier

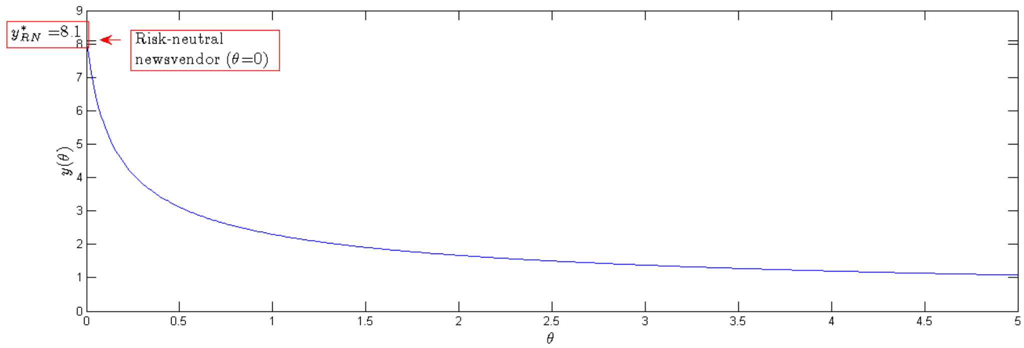

Figure 3.3: The Effect of Risk-Aversion Parameter on Optimal Order Quantity

For every risk-aversion parameter θ, the optimal order quantity is obtained by taking the inverse

$$y(\theta) = \Theta^{-1}(\theta).$$

Figure 3.3 shows the inverse of $\theta(y)$. It is seen that as risk-aversion increases the optimal order quantity decreases. When $\theta = 0$, the newsvendor is risk-neutral and the problem becomes the classical newsvendor problem. From (3.15) the optimal order quantity y^*_{RN} satisfies

$$1 - e^{-\lambda y^*_{RN}} = \frac{0.5}{0.9}$$

and by solving this equation for $\lambda = 0.1$, we obtain the newsvendor order quantity as $y^*_{RN} = 8.1$.

When $\theta = 5$, the newsvendor becomes risk-averse, the optimal order quantity $y(\theta)$ decreases to 1.07. At this point, the mean and variance of the cash flow are

$$E[CF(D, 1.07)] = 0.4887$$

and

$$Var[CF(D, 1.07)] = 0.8760.$$

Similar to the efficient frontier obtained by the Markowitz model in portfolio theory; for this example, mean of the cash flow and variance of the cash flow constitute the efficient frontier. They are the best mean-variance combinations for risk-averse investors. Therefore, we limit our study to this frontier. From Figure 3.2, it can be seen that as the newsvendor wants to increase the gain, the risk he/she is taking also increases.

3.1.2 Efficient Frontier

We stated previously that for risk-averse newsvendors, the order quantities in the region $[0, y^*_{RN}]$ are all non-dominated. When we choose each order quantity in the region $[0, y^*_{RN}]$, and map it to the MV space by plotting the corresponding mean of the cash flow versus the variance of the cash flow, the resulting image of the order quantities in $[0, y^*_{RN}]$ is called the efficient frontier that shows the trade-offs between the mean and variance of the cash flow. The slope of a point on this curve can be obtained by

$$\theta(y) = \frac{dE[CF(D, y)]/dy}{dVar[CF(D, y)]/dy}.$$

To observe the change in the slope as the order quantity increases from 0 to y^*_{RN}, we simply take the derivative of $\theta(y)$ with respect to y. It follows from (3.28) and (3.29) that $\theta'(y) \leq 0$. Therefore, as the order quantity increases the slope of a point on the efficient frontier decreases. We can generate the efficient frontier numerically. When $y = 0$, $\theta(0) = +\infty$. When $y = y^*_{RN}$, $\theta(y^*_{RN}) = 0$. Thus, the efficient frontier that has a shape depicted in Figure 3.2 is observed to be convex to the left.

Up to this point, we analyzed the MV newsvendor problem and the effect of risk-aversion on the optimal order quantity when demand is the only source of uncertainty. As we stated before, supply uncertainty is another significant source of randomness for inventory management. Because of some unforeseen events during production or transportation processes, suppliers may not meet the quantity ordered. Starting from the following section, we analyze supply randomness in three categories: random yield, random capacity and random yield and capacity. Remarkably, in literature there is no example that discusses the MV model with random supply. Therefore, the analysis in the remainder of this chapter is new.

3.2 MV Model with Random Yield

In this section, we also consider the supply uncertainty when it is caused by random yield in the sense that the received amount is a random proportion of order quantity due to defects, errors during production process or transportation problems, etc.. Let $0 \leq U \leq 1$ be a random variable representing the proportion of non-defective items received, thus when y units are ordered Uy amount is received. For generality, we assume that U and D are not necessarily independent and the conditional density function of demand given $U = v$ is $f_{D|v}$. For the random yield model the cash flow in (3.2) can be updated as

$$CF(D, U, y) = (s - c)Uy + (p - u)D + (u - s)\min\{D, Uy\}.$$

The mean and variance of the cash flow are

$$m(y) = E[CF(D, U, y)] = (s - c)yE[U] + (p - u)E[D] + (u - s)E[\min\{D, Uy\}]$$

and

$$\begin{aligned} v(y) &= Var[CF(D, U, y)] \\ &= (s - c)^2 y^2 Var[U] + (p - u)^2 Var[D] + (u - s)^2 Var[\min\{D, Uy\}] \\ &\quad +2(s - c)(p - u)yCov[U, D] + 2(p - u)(u - s)Cov[D, \min\{D, Uy\}] \\ &\quad\quad +2(s - c)(u - s)yCov[U, \min\{D, Uy\}] \\ &\geq 0. \end{aligned} \tag{3.37}$$

The aim of the risk-averse newsvendor now is

$$\max_{y \geq 0} H(y, \theta) = E[CF(D, U, y)] - \theta Var[CF(D, U, y)]$$

for any $\theta \geq 0$. Equivalently, the objective function $H(y, \theta)$ for any any fixed $\theta \geq 0$ can be written as

$$\begin{aligned} H(y, \theta) &= (s - c)yE[U] + (p - u)E[D] + (u - s)E[\min\{D, Uy\}] \qquad (3.38) \\ &\quad -\theta \left\{ \begin{array}{c} (s - c)^2 y^2 Var[U] + (p - u)^2 Var[D] \\ +(u - s)^2 Var[\min\{D, Uy\}] + 2(s - c)(p - u)yCov[U, D] \\ +2(p - u)(u - s)Cov[D, \min\{D, Uy\}] \\ +2(s - c)(u - s)yCov[U, \min\{D, Uy\}] \end{array} \right\}. \end{aligned}$$

To obtain the optimality condition, note that for random variables X and V we can write

$$E[\min\{X, Vy\}] = \int_0^1 f_V(v)dv \left(\int_0^{vy} x f_{X|v}(x)dx + vy \int_{vy}^{+\infty} f_{X|v}(x)dx \right).$$

The derivative with respect to y is

$$\begin{aligned} \frac{dE[\min\{X, Vy\}]}{dy} &= \int_0^1 v f_V(v)dv \int_{vy}^{+\infty} f_{X|v}(x)dx = E[V1_{\{X > Vy\}}] \\ &= E[V] - E[V1_{\{X \leq Vy\}}] \end{aligned} \tag{3.39}$$

by using the fact that $1_{\{X>Vy\}} = 1 - 1_{\{X\leq Vy\}}$. Similarly,

$$\begin{aligned} Var[\min\{X,Vy\}] &= E[(\min\{X,Vy\})^2] - E[\min\{X,Vy\}]^2 \\ &= \int_0^1 f_V(v)dv \left(\int_0^{vy} x^2 f_{X|v}(x)dx + v^2y^2 \int_{vy}^{+\infty} f_{X|v}(x)dx \right) \\ &\quad -E[\min\{X,Vy\}]^2. \end{aligned} \tag{3.40}$$

Then, it follows from (3.39) and (3.40) that

$$\begin{aligned} \frac{dVar[\min\{X,Vy\}]}{dy} &= 2y \int_0^1 v^2 f_V(v)dv \int_{vy}^{+\infty} f_{X|v}(x)dx \\ &\quad -2E[\min\{X,Vy\}]E[V1_{\{X>Vy\}}] \\ &= 2(yE[V^2 1_{\{X>Vy\}}] - E[\min\{X,Vy\}]E[V1_{\{X>Vy\}}]) \\ &= 2Cov[\min\{X,Vy\}, V1_{\{X>Vy\}}]. \end{aligned} \tag{3.41}$$

Moreover, the covariance terms in (3.37) are

$$\begin{aligned} Cov[X,\min\{X,Vy\}] &= E[X\min\{X,Vy\}] - E[X]E[\min\{X,Vy\}] \\ &= \int_0^1 f_V(v)dv \left(\int_0^{vy} x^2 f_{X|v}(x)dx + vy \int_{vy}^{+\infty} x f_{X|v}(x)dx \right) \\ &\quad -E[X]E[\min\{X,Vy\}] \end{aligned} \tag{3.42}$$

and

$$\begin{aligned} Cov\,[V,\min\{X,Vy\}] &= E[V\min\{X,Vy\}] - E[V]E[\min\{X,Vy\}] \\ &= \int_0^1 f_V(v)dv \left(v\int_0^{vy} x f_{X|v}(x)dx + v^2y \int_{vy}^{+\infty} f_{X|v}(x)dx \right) \\ &\quad -E[V]E[\min\{X,Vy\}]. \end{aligned} \tag{3.43}$$

Then, using (3.39) and (3.42) one can obtain

$$\begin{aligned} \frac{dCov[X,\min\{X,Vy\}]}{dy} &= \int_0^1 v f_V(v)dv \int_{vy}^{+\infty} x f_{X|v}(x)dx - E[X]E[V1_{\{X>Vy\}}] \\ &= E[XV1_{\{X>Vy\}}] - E[X]E[V1_{\{X>Vy\}}] \\ &= Cov[X, V1_{\{X>Vy\}}]. \end{aligned} \tag{3.44}$$

Lastly, it follows from (3.39) and (3.43) that

$$\begin{aligned}
\frac{dCov\left[V, \min\{X, Vy\}\right]}{dy} &= \int_0^1 v^2 f_V(v)dv \int_{vy}^{+\infty} f_{X|v}(x)dx - E[V]E[V1_{\{X>Vy\}}] \\
&= E[V^2 1_{\{X>Vy\}}] - E[V]E[V1_{\{X>Vy\}}] \\
&= Cov[V, V1_{\{X>Vy\}}]. \qquad (3.45)
\end{aligned}$$

Again, initially we analyze two extreme cases. The case $\theta = 0$ corresponds to the risk-neutral newsvendor and so the expected profit is maximized, or

$$\max_{y\geq 0} m(y) = E[CF(D, U, y)].$$

Using (3.39) where X is D and V is U, the first order condition is

$$\begin{aligned}
m'(y) &= (u-c)E[U] - (u-s)E[U1_{\{D\leq Uy\}}] \\
&= 0. \qquad (3.46)
\end{aligned}$$

It is clear that $1_{\{D\leq Uy\}}$ is increasing in y so that

$$m''(y) = -(u-s)\int_0^1 v^2 f_U(v) f_{D|v}(vy)dv \leq 0 \qquad (3.47)$$

and the mean of the cash flow is concave. This also implies that $m'(y)$ is decreasing in y. Thus, it follows from (3.46) that the optimal order quantity y^*_{RN} satisfies

$$\frac{E[U1_{\{D\leq Uy^*_{RN}\}}]}{E[U]} = \hat{p} \qquad (3.48)$$

where $\hat{p}$ denotes the same critical ratio as in (3.15). Note that (3.48) is equivalent to the classical newsvendor solution with random yield model given in Okyay et al. [2010]. Therefore, (3.48) gives the optimality condition provided that $m'(0) > 0$ and $m'(+\infty) < 0$ and there exists an optimal order quantity y^*_{RN} satisfying (3.48).

The optimal order quantity is $y^*_{RN} = 0$ if $m'(0) \leq 0$; or

$$m'(0) = (u-c)E[U] - (u-s)E[U1_{\{D=0\}}] \leq 0. \qquad (3.49)$$

Equivalently, we can conclude that if

$$\frac{E[U1_{\{D=0\}}]}{E[U]} \geq \hat{p} \qquad (3.50)$$

then $y^*_{RN} = 0$. Clearly, the ratio in the right hand side of (3.49) is between 0 and 1. Here, the same remark of the previous section is valid. By our assumption D is strictly greater than 0 in which case (3.50) can not be true. However, we aim to show the result when this assumption does not hold.

Moreover, the optimal solution is $y^*_{RN} = +\infty$ if $m'(+\infty) \geq 0$; or

$$m'(+\infty) = (u-c)E[U] - (u-s)E[U1_{\{D<+\infty\}}] \geq 0.$$

Equivalently if

$$\frac{E[U1_{\{D=+\infty\}}]}{E[U]} \geq 1-\hat{p} \tag{3.51}$$

then $y^*_{RN} = +\infty$. The ratio in the right hand side of (3.51) is again between 0 and 1. To avoid these trivial cases in our analysis we assume $m'(0) > 0$ and $m'(+\infty) < 0$.

As θ increases to $+\infty$, the newsvendor becomes extremely risk-averse and the problem becomes minimizing the variance of the cash flow, or

$$\min_{y\geq 0} v(y) = Var[CF(D,U,y)]. \tag{3.52}$$

To carry out the analysis as in the previous section, we make the following assumption throughout the remainder of this thesis.

Assumption 3.2.1 *The function $v(y) = Var[CF(D,U,y)]$ is nondecreasing in y and convex on $[0, y^*_{RN}]$.*

This assumption implies that

$$\begin{aligned} v'(y) &= 2[(s-c)^2 y Var[U] + (u-s)^2 Cov[\min\{D,Uy\}, U1_{\{D>Uy\}}] \\ &\quad +(s-c)(p-u)Cov[U,D] + (p-u)(u-s)Cov[D, U1_{\{D>Uy\}}] \\ &\quad +(s-c)(u-s)\left(Cov[U, \min\{D,Uy\}] + yCov[U, U1_{\{D>Uy\}}]\right)] \\ &\geq 0 \end{aligned} \tag{3.53}$$

for all y. This is obtained by using (3.41), (3.44) and (3.45) where X is D and V is U. Moreover, $v''(y) \geq 0$ on $[0, y^*_{RN}]$. Assumption 3.2.1 also implies that

$$\begin{aligned} v'(0) &= 2[(s-c)(p-u)Cov[U,D] + (p-u)(u-s)Cov[D, U1_{\{D>0\}}]] \\ &= 2(p-u)(u-c)Cov[U,D] \\ &\geq 0 \end{aligned}$$

and, as a consequence, $Cov[U, D] \geq 0$ so that U and D are positively correlated. The reverse is not necessarily true and positive correlation between U and D does not necessarily imply that $Var[CF(D, U, y)]$ is increasing.

Under this assumption, the optimal order quantity in (3.52) is 0. At this point, the corresponding mean and variance of the cash flow are respectively

$$E[CF(D, U, 0)] = (p - u)E[D]$$

and

$$Var[CF(D, U, 0)] = (p - u)^2 Var[D] \tag{3.54}$$

since $CF(D, U, 0) = (p - u)D$.

Lemma 3.2.2 *(a)* $E[CF(D, U, y)]$ *is concave in* y; *it is increasing on* $[0, y^*_{RN}]$ *and decreasing on* $(y^*_{RN}, +\infty)$. *(b) Moreover,*

$$(p - u)^2 Var[D] \leq Var[CF(D, U, y)]$$

for all y.

Proof. $E[CF(D, U, y)]$ is concave since (3.47) holds and the maximum is attained at y^*_{RN} by (3.48). Moreover, $Var[CF(D, U, y)]$ is bounded below by $(p - u)^2 Var[D]$ as given in (3.54). ■

It follows from part (a) of Lemma 3.2.2 and (3.48) that $E[CF(D, U, y)]$ is maximized at a finite, positive point. We can obtain this point when θ is equal to zero in the MV formulation. It corresponds to classical newsvendor solution with random yield that satisfies (3.48). However, to maximize the MV objective we must take into account both the mean and variance of the cash flow. In the MV analysis, we need to consider the order quantities that lie in the region $[0, y^*_{RN}]$ which is the non-dominated region. Moreover, the order quantities in $(y^*_{RN}, +\infty)$ are all dominated. This argument implies that $y(\theta) \leq y^*_{RN}$ for all $\theta \geq 0$.

Proposition 3.2.3 *The optimal order quantity* $y(\theta)$ *that maximizes the MV objective is less than or equal to the classical newsvendor solution* y^*_{RN} *for all* $\theta \geq 0$.

Proof. The argument is similar to the discussion of Proposition 3.1.2 in Section 3.1. ■

Using (3.39), (3.41), (3.44) and (3.45) where X is D and V is U, we differentiate the objective function (3.38) with respect to y and obtain the first order condition as

$$g(y,\theta) = \frac{dH(y,\theta)}{dy} = m'(y) - \theta v'(y) = 0 \tag{3.55}$$

where $m'(y)$ and $v'(y)$ are given in (3.46) and (3.53).

Theorem 3.2.4 *The optimal order quantity $y(\theta)$ that maximizes the MV objective (3.38) is obtained from (3.55) by solving*

$$m'(y(\theta)) - \theta v'(y(\theta)) = 0. \tag{3.56}$$

Moreover, $y(\theta)$ decreases as θ increases.

Proof. For any fixed y, let $\theta(y)$ satisfy the optimality condition (3.56) which can be written as

$$\theta(y) = \frac{m'(y)}{v'(y)}. \tag{3.57}$$

For all y in the non-dominated region $[0, y^*_{RN}]$, we can show that $\theta(y)$ is decreasing in y. The derivative of (3.57) is

$$\theta'(y) = \frac{m''(y)v'(y) - v''(y)m'(y)}{(v'(y))^2} \leq 0. \tag{3.58}$$

We already know by Lemma 3.2.2 that $E[CF(D,U,y)]$ is concave so that $m'(y) \geq 0$ on $[0, y^*_{RN}]$ and $m''(y) \leq 0$. By our assumption $Var[CF(D,U,y)]$ is nondecreasing so that $v'(y) \geq 0$. Moreover, $v''(y) \geq 0$ on $[0, y^*_{RN}]$; that is, $Var[CF(D,U,y)]$ is convex along the non-dominated region so that (3.58) follows. Note that

$$\theta(0) = \frac{E[U]}{2(p-u)Cov[U,D]} \geq 0$$

and

$$\theta(y^*_{RN}) = 0$$

since $m'(y^*_{RN}) = 0$ by concavity of $E[CF(D,U,y)]$. Therefore, $\theta(y)$ decreases from $\theta(0)$ to 0 as y increases from 0 to y^*_{RN}. Up to now, by showing $\theta(y)$ is a decreasing function of y, we establish the existence of an order quantity for each risk-aversion level θ that is between $0 \leq \theta \leq \theta(0)$. Note that on $(y^*_{RN}, +\infty)$, $\theta(y) \leq 0$ and so this is the dominated region. Additionally, along the non-dominated region $[0, y^*_{RN}]$ the second order condition is obtained as

$$\frac{d^2 H(y,\theta)}{dy^2} = m''(y) - \theta v''(y) \leq 0.$$

Since the second order condition is satisfied, the objective function is concave on $[0, y^*_{RN}]$. Moreover, the first derivative of the MV objective function (3.55) evaluated at $y = 0$ is

$$\begin{aligned} \left.\frac{dH(y,\theta)}{dy}\right|_{y=0} &= (u-c)E[U] - (u-s)E[U1_{\{D\leq 0\}}] - 2\theta(p-u)(u-c)Cov[U,D] \\ &= (u-c)E[U] - 2\theta(p-u)(u-c)Cov[U,D] \\ &\geq 0 \end{aligned}$$

and (3.55) is nonpositive on $(y^*_{RN}, +\infty)$ because $m(y)$ is decreasing while $v(y)$ is increasing on $(y^*_{RN}, +\infty)$. This implies that the MV objective function is decreasing along the non-dominated region. Therefore, it is quasi-concave and the order quantity that is between 0 and y^*_{RN} is a maximizer of (3.38). For any $0 \leq \theta < \theta(0)$, by taking the inverse Θ^{-1} of $\theta(y)$, we can obtain the optimal order quantity corresponding to that θ value so that

$$y(\theta) = \Theta^{-1}(\theta). \tag{3.59}$$

According to the newsvendor's risk-aversion level given by θ, the optimal order quantity is chosen between 0 and y^*_{RN}. Since $\theta'(y) \leq 0$, we know that $\theta(y)$ is decreasing in y, so the inverse given in (3.59) is also decreasing. Therefore, similar to the previous section, we state that as the level of risk-aversion θ increases the optimal order quantity $y(\theta)$ decreases. ■

Note that we can also write (3.56) as

$$\frac{E[U1_{\{D\leq Uy(\theta)\}}]}{E[U]} + \theta\bar{v}(y(\theta)) = \hat{p} \tag{3.60}$$

where

$$\bar{v}(y) = \frac{2\left\{\begin{array}{c} (s-c)^2 yVar[U] + (u-s)^2 Cov\left[\min\{D,Uy\}, U1_{\{D>Uy\}}\right] \\ +(s-c)(p-u)Cov\left[U,D\right] + (p-u)(u-s)Cov\left[D, U1_{\{D>Uy\}}\right] \\ +(s-c)(u-s)\left(Cov\left[U, \min\{D,Uy\}\right] + yCov\left[U, U1_{\{D>Uy\}}\right]\right) \end{array}\right\}}{(u-s)E[U]}.$$

When $\theta = 0$, (3.60) becomes the same as (3.48). When $\theta > 0$, the only difference is an additional term given by $\bar{v}(y)$ that comes from the variance of the cash flow.

The optimal order quantity is $y(\theta) = 0$ if $g(0, \theta) \leq 0$; that is

$$g(0, \theta) = (u - c)E[U] - (u - s)E[U1_{\{D=0\}}] - 2\theta(p - u)(u - c)Cov[U, D] \leq 0.$$

Equivalently, we can conclude that if

$$\theta \geq \frac{E[U]}{2(p - u)Cov[U, D]} = \theta(0)$$

then $y(\theta) = 0$.

As a special case, when $U = 1$ which means that there is no randomness in yield, the optimality condition is (3.25).

This section examined the optimal order quantities for the random yield model. The arguments that a risk-averse newsvendor orders less than the classical newsvendor and as the risk-aversion level increases the optimal order quantity decreases are valid under Assumption 3.2.1.

As a general remark, throughout this thesis the MV analysis is based on Assumption 3.2.1. If we only assume that the variance of the cash flow is increasing in y, then we can still argue that the non-dominated region lies on $[0, y^*_{RN}]$ and the MV newsvendor orders less than the risk-neutral one. Nevertheless, the variance of the cash flow being convex along the non-dominated region ensures that as the MV newsvendor becomes more risk-averse, the order quantity decreases. Suppose for a moment that the variance of the cash flow is decreasing in y. Although we could not find such an example to illustrate this case, if it were, then the non-dominated region would be $(y^*_{RN}, +\infty)$ and the MV newsvendor would order more than the risk-neutral one. Moreover, if we further assume that the variance of the cash flow is convex along the non-dominated region then as the newsvendor would be more risk-averse, the order quantity would increase.

3.3 MV Model with Random Capacity

In this section, in addition to the demand uncertainty we consider the effects of supply uncertainty when it is caused by random capacity. Due to limited capacity the received amount of order quantity may be less than the ordered amount. Thus, we express the received amount from ordering y units as $min\{K, y\}$ where $K \geq 0$ is the maximum number of units the supplier can ship. We further assume that D and K may be dependent and the conditional density function of D given $K = z$ is $f_{D|z}$. Moreover, it is assumed that

$P\{K > z\} > 0$ for all z, therefore there is always some probability that our order will be satisfied in full. The cash flow in (3.2) can be updated for the random capacity model as

$$CF(D, K, y) = (s - c)\min\{K, y\} + (p - u)D + (u - s)\min\{D, K, y\}.$$

The mean and variance of the cash flow are

$$\begin{aligned} m(y) &= E[CF(D, K, y)] \\ &= (s - c)E[\min\{K, y\}] + (p - u)E[D] + (u - s)E[\min\{D, K, y\}] \end{aligned}$$

and

$$\begin{aligned} v(y) &= Var[CF(D, K, y)] \\ &= (s - c)^2 Var[\min\{K, y\}] + (p - u)^2 Var[D] + (u - s)^2 Var[\min\{D, K, y\}] \\ &\quad +2(s - c)(p - u)Cov[D, \min\{K, y\}] + 2(p - u)(u - s)Cov[D, \min\{D, K, y\}] \\ &\quad +2(s - c)(u - s)Cov[\min\{K, y\}, \min\{D, K, y\}] \\ &\geq 0. \end{aligned}$$

The MV formulation of the risk-averse newsvendor in (3.1) can be updated as

$$\max_{y \geq 0} H(y, \theta) = E[CF(D, K, y)] - \theta Var[CF(D, K, y)]$$

or in open form the objective function is expressed as

$$\begin{aligned} H(y, \theta) &= (s - c)E[\min\{K, y\}] + (p - u)E[D] + (u - s)E[\min\{D, K, y\}] \qquad (3.61) \\ &\quad -\theta \left\{ \begin{array}{c} (s - c)^2 Var[\min\{K, y\}] + (p - u)^2 Var[D] \\ +(u - s)^2 Var[\min\{D, K, y\}] + 2(s - c)(p - u)Cov[D, \min\{K, y\}] \\ +2(p - u)(u - s)Cov[D, \min\{D, K, y\}] \\ +2(s - c)(u - s)Cov[\min\{K, y\}, \min\{D, K, y\}] \end{array} \right\} \end{aligned}$$

for any $\theta \geq 0$.

Before analyzing the optimality condition note that for any random variables X and Z, we can write that

$$\frac{dE[\min\{X, Z, y\}]}{dy} = P\{X > y, Z > y\}. \qquad (3.62)$$

This follows from (3.7) by putting $min\{X, Z\}$ as the random variable X. Similar to (3.10), we can write

$$\begin{aligned} \frac{dVar[\min\{Z,y\}]}{dy} &= 2y\bar{F}_Z(y) - 2E[\min\{Z,y\}]\bar{F}_Z(y) \\ &= 2Cov[\min\{Z,y\}, 1_{\{Z>y\}}]. \end{aligned} \tag{3.63}$$

Moreover, the derivative of the other variance term in (3.61) can be expressed as

$$\frac{dVar[\min\{X,Z,y\}]}{dy} = 2Cov[\min\{X,Z,y\}, 1_{\{X>y,Z>y\}}] \tag{3.64}$$

which follows from (3.10) where $min\{X, Z\}$ is the random variable X. Similarly, the covariance terms in (3.61) are

$$\begin{aligned} Cov[X, \min\{Z,y\}] &= E[X\min\{Z,y\}] - E[X]E[\min\{Z,y\}] \\ &= \int_0^{+\infty} \min\{z,y\} f_Z(z)dz \int_0^{+\infty} x f_{X|z}(x)dx \\ &\quad -E[X]E[\min\{Z,y\}] \\ &= \int_0^{y} z f_Z(z)dz \int_0^{+\infty} x f_{X|z}(x)dx \\ &\quad +y\int_y^{+\infty} f_Z(z)dz \int_0^{+\infty} x f_{X|z}(x)dx - E[X]E[\min\{Z,y\}], \end{aligned}$$

$$\begin{aligned} Cov[X, \min\{X,Z,y\}] &= E[X\min\{X,Z,y\}] - E[X]E[\min\{X,Z,y\}] \\ &= \int_0^{+\infty} f_Z(z)dz \left(\int_0^{\min\{z,y\}} x^2 f_{X|z}(x)dx + \min\{z,y\} \int_{\min\{z,y\}}^{+\infty} x f_{X|z}(x)dx \right) \\ &\quad -E[X]E[\min\{X,Z,y\}] \\ &= \int_0^{y} f_Z(z)dz \left(\int_0^{z} x^2 f_{X|z}(x)dx + z\int_z^{+\infty} x f_{X|z}(x)dx \right) \\ &\quad + \int_y^{+\infty} f_Z(z)dz \left(\int_0^{y} x^2 f_{X|z}(x)dx + y\int_y^{+\infty} x f_{X|z}(x)dx \right) \\ &\quad -E[X]E[\min\{X,Z,y\}] \end{aligned} \tag{3.65}$$

and

$$
\begin{aligned}
Cov[\min\{Z,y\},\min\{X,Z,y\}] &= E[\min\{Z,y\}\min\{X,Z,y\}] \\
&\quad -E[\min\{Z,y\}]E[\min\{X,Z,y\}] \\
&= \int_0^{+\infty} f_Z(z)dz \left(\min\{z,y\} \int_0^{\min\{z,y\}} x f_{X|z}(x)dx \right. \\
&\quad \left. + (\min\{z,y\})^2 \int_{\min\{z,y\}}^{+\infty} f_{X|z}(x)dx \right) \\
&\quad -E[\min\{Z,y\}]E[\min\{X,Z,y\}] \\
&= \int_0^{y} f_Z(z)dz \left(z \int_0^{z} x f_{X|z}(x)dx + z^2 \int_z^{+\infty} f_{X|z}(x)dx \right) \\
&\quad + \int_y^{+\infty} f_Z(z)dz \left(y \int_0^{y} x f_{X|z}(x)dx + y^2 \int_y^{+\infty} f_{X|z}(x)dx \right) \\
&\quad -E[\min\{Z,y\}]E[\min\{X,Z,y\}]. \qquad (3.66)
\end{aligned}
$$

Then, we can obtain their derivatives as

$$
\begin{aligned}
\frac{dCov[X,\min\{Z,y\}]}{dy} &= \int_y^{+\infty} f_Z(z)dz \int_0^{+\infty} x f_{X|z}(x)dx - E[X]P\{Z>y\} \\
&= E[X1_{\{Z>y\}}] - E[X]P\{Z>y\} \\
&= Cov[X, 1_{\{Z>y\}}], \qquad (3.67)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{dCov[X,\min\{X,Z,y\}]}{dy} &= \int_y^{+\infty} f_Z(z)dz \int_y^{+\infty} x f_{X|z}(x)dx \\
&\quad -E[X]P\{X>y,Z>y\} \\
&= E[X1_{\{X>y,Z>y\}}] - E[X]P\{X>y,Z>y\} \\
&= Cov[X, 1_{\{X>y,Z>y\}}] \qquad (3.68)
\end{aligned}
$$

which follows from (3.62) and (3.65). Lastly,

$$\begin{aligned}
\frac{dCov[\min\{Z,y\},\min\{X,Z,y\}]}{dy} &= \int_y^{+\infty} f_Z(z)dz \left(\int_0^y x f_{X|z}(x)dx \right. \\
&\quad \left. +2y \int_y^{+\infty} f_{X|z}(x)dx \right) - P\{Z>y\}E[\min\{X,Z,y\}] \\
&\quad -E[\min\{Z,y\}]P\{X>y, Z>y\} \\
&= E[\min\{X,Z,y\}1_{\{Z>y\}}] - P\{Z>y\}E[\min\{X,Z,y\}] \\
&\quad +E[\min\{Z,y\}1_{\{X>y,Z>y\}}] \\
&\quad -E[\min\{Z,y\}]P\{X>y,Z>y\} \\
&= Cov[\min\{X,Z,y\},1_{\{Z>y\}}] \\
&\quad +Cov[\min\{Z,y\},1_{\{X>y,Z>y\}}] \qquad (3.69)
\end{aligned}$$

which is obtained by using (3.62) and (3.66).

The two extreme cases are as follows; $\theta = 0$ corresponds to the risk-neutral newsvendor and so the expected profit is maximized, or

$$\max_{y \geq 0} m(y) = E[CF(D,K,y)].$$

By using (3.62) where X is D and Z is K, we obtain the first order condition

$$\begin{aligned}
m'(y) &= (s-c)P\{K>y\} + (u-s)P\{D>y, K>y\} \\
&= 0. \qquad (3.70)
\end{aligned}$$

This can be written as

$$\begin{aligned}
m'(y) &= P\{K>y\}((s-c) + (u-s)P\{D>y \mid K>y\}) \\
&= 0. \qquad (3.71)
\end{aligned}$$

Noting that $P\{K>y\} > 0$ for all y by our assumption, we can write (3.71) as

$$(s-c) + (u-s)P\{D>y \mid K>y\} = 0.$$

For this model, we do not necessarily end up with a concave objective function since $m'(y)$ is not necessarily decreasing. To carry out the analysis, let

$$h(y) = P\{D \leq y \mid K > y\}. \tag{3.72}$$

Assumption 3.3.1 *The conditional probability $h(y)$ is strictly increasing in y.*

Then, we obtain the optimality condition

$$P\{D \leq y^*_{RN} \mid K > y^*_{RN}\} = \hat{p} \tag{3.73}$$

which is equivalent to the classical newsvendor solution with random capacity given in Okyay et al. [2010]. It follows from (3.71) and (3.72) that $m'(y)$ is nonnegative and decreasing on $[0, y^*_{RN}]$ and nonpositive on $(y^*_{RN}, +\infty)$. Thus, the objective function is concave increasing on $[0, y^*_{RN}]$ and decreasing on $(y^*_{RN}, +\infty)$. This implies that the objective function is quasi-concave and the solution y^*_{RN} is the optimal solution. If $h(0) < \hat{p} < h(+\infty)$, then there exists a unique $0 < y^*_{RN} < +\infty$ satisfying (3.73) so that $h(y^*_{RN}) = \hat{p}$ (or $m'(y^*_{RN}) = 0$). We also argue that the optimal order quantity is trivially $y^*_{RN} = 0$ if $h(0) \geq \hat{p}$; that is

$$h(0) = P\{D = 0 \mid K > 0\} \geq \hat{p}. \tag{3.74}$$

Again since we assumed D to be strictly greater than 0, (3.74) can not hold. However, without this assumption the case given in (3.74) may happen. Moreover, we argue that $y^*_{RN} = +\infty$ if $h(+\infty) \leq \hat{p}$; that is

$$\begin{aligned} h(+\infty) &= P\{D < +\infty \mid K = +\infty\} \\ &= \frac{P\{D < +\infty, K = +\infty\}}{P\{K = +\infty\}} \\ &= \frac{P\{K = +\infty\} - P\{D = +\infty, K = +\infty\}}{P\{K = +\infty\}} \\ &= 1 - P\{D = +\infty \mid K = +\infty\} \leq \hat{p}. \end{aligned}$$

Equivalently, if

$$P\{D = +\infty \mid K = +\infty\} \geq 1 - \hat{p}$$

then $y^*_{RN} = +\infty$. We can conclude that if demand is finite $P\{D = +\infty\} = 0$, then the optimal order quantity is also finite. To avoid trivial cases, we assume $h(0) < \hat{p}$ (or $m'(0) > 0$) and $h(+\infty) > \hat{p}$ (or $m'(+\infty) < 0$).

When we increase θ to $+\infty$, the newsvendor's level of risk-aversion increases extremely and the problem turns into minimizing the variance of the cash flow, or

$$\min_{y\geq 0} v(y) = Var[CF(D,K,y)]. \tag{3.75}$$

To carry out the analysis as in Section 3.1, we consider the random capacity model with the following assumption.

Assumption 3.3.2 *The function $v(y) = Var[CF(D,K,y)]$ is nondecreasing in y and convex on $[0, y^*_{RN}]$.*

This assumption implies that

$$\begin{aligned} v'(y) &= 2\Big[(s-c)^2 Cov[\min\{K,y\}, 1_{\{K>y\}}] + (u-s)^2 Cov[\min\{D,K,y\}, 1_{\{D>y,K>y\}}] \\ &\quad + (s-c)(p-u)Cov[D, 1_{\{K>y\}}] + (p-u)(u-s)Cov[D, 1_{\{D>y,K>y\}}] \\ &\quad + (s-c)(u-s)\begin{pmatrix} Cov[\min\{D,K,y\}, 1_{\{K>y\}}] \\ +Cov[\min\{K,y\}, 1_{\{D>y,K>y\}}] \end{pmatrix}\Big] \\ &\geq 0 \end{aligned} \tag{3.76}$$

for all y. This is obtained by using (3.63), (3.64), (3.67), (3.68) and (3.69) where X is D and Z is K, and $v''(y) \geq 0$ on $[0, y^*_{RN}]$.

The optimal order quantity to the problem in (3.75) is 0. At this order quantity, the corresponding expected value of the cash flow and variance of the cash flow are respectively

$$E[CF(D,K,0)] = (p-u)E[D]$$

and

$$Var[CF(D,K,0)] = (p-u)^2 Var[D] \tag{3.77}$$

since $CF(D,K,0) = (p-u)D$.

Lemma 3.3.3 *(a) $E[CF(D,K,y)]$ is quasi-concave in y; it is increasing on $[0, y^*_{RN}]$ and decreasing on $(y^*_{RN}, +\infty)$. (b) Moreover,*

$$(p-u)^2 Var[D] \leq Var[CF(D,K,y)] \leq Var[(s-c)K + (u-s)\min\{D,K\} + (p-u)D]$$

for all y.

Proof. $E[CF(D,K,y)]$ is quasi-concave since (3.72) is strictly increasing in y by Assumption 3.3.1. The maximum is attained at y^*_{RN} by (3.73). Moreover,

$$\begin{aligned}\lim_{y\to+\infty} Var[CF(D,K,y)] &= \lim_{y\to+\infty}(s-c)^2Var[\min\{K,y\}] + \lim_{y\to+\infty}(p-u)^2Var[D] \\ &\quad + \lim_{y\to+\infty}(u-s)^2Var[\min\{D,K,y\}]] \\ &\quad + \lim_{y\to+\infty} 2(s-c)(u-s)Cov\left[\min\{K,y\},\min\{D,K,y\}\right] \\ &\quad + \lim_{y\to+\infty} 2(p-u)(u-s)Cov[D,\min\{D,K,y\}] \\ &\quad + \lim_{y\to+\infty} 2(s-c)(p-u)Cov[D,\min\{K,y\}] \\ &= Var[(s-c)K + (u-s)\min\{D,K\} + (p-u)D]\end{aligned}$$

for all y and the lower bound for $Var[CF(D,K,y)]$ is $(p-u)^2Var[D]$ as given in (3.77). ■

It follows from part (a) of Lemma 3.3.3 and (3.73) that $E[CF(D,K,y)]$ is maximized at a finite, positive point. We obtain this point by setting $\theta = 0$ in the MV formulation. It is the classical newsvendor solution y^*_{RN} that satisfies (3.73). However, we must also consider the variance of the cash flow. In the MV analysis, we need to consider the order quantities that lie in the non-dominated region, $[0, y^*_{RN}]$. Moreover, the order quantities in $(y^*_{RN}, +\infty)$ are all dominated. This implies that $y(\theta) \le y^*_{RN}$ for all $\theta \ge 0$.

Proposition 3.3.4 *The optimal order quantity $y(\theta)$ that maximizes the MV objective is less than or equal to the classical newsvendor solution y^*_{RN} for all $\theta \ge 0$.*

Proof. This is similar to the proof of Proposition 3.1.2 given in Section 3.1. ■

Using (3.62), (3.63), (3.64), (3.67), (3.68) and (3.69) where X is D and Z is K, we obtain the first order condition

$$g(y,\theta) = \frac{dH(y,\theta)}{dy} = m'(y) - \theta v'(y) = 0 \tag{3.78}$$

where $m'(y)$ and $v'(y)$ are given in (3.70) and (3.76).

Theorem 3.3.5 *The optimal order quantity $y(\theta)$ that maximizes the MV objective (3.61) is obtained from (3.78) by solving*

$$m'(y(\theta)) - \theta v'(y(\theta)) = 0. \tag{3.79}$$

Moreover, $y(\theta)$ is decreasing in θ.

Proof. For any fixed y, let $\theta(y)$ satisfy the optimality condition (3.79) which can be written as

$$\theta(y) = \frac{m'(y)}{v'(y)}. \tag{3.80}$$

For all y in the non-dominated region $[0, y^*_{RN}]$, we can show that $\theta(y)$ is decreasing in y. The functional form of $\theta'(y)$ is given in (3.58). As our discussion about the behavior of the function in (3.72) suggests, in the region $[0, y^*_{RN}]$, $m'(y) \geq 0$ and $m''(y) \leq 0$. As for $Var[CF(D, K, y)]$, via our assumption it is nondecreasing so that $v'(y) \geq 0$. Moreover, in the non-dominated region $[0, y^*_{RN}]$, $v''(y) \geq 0$ that is $Var[CF(D, K, y)]$ is convex. So, under the assumptions imposed on $m(y)$ and $v(y)$, $\theta'(y) \leq 0$ so that $\theta(y)$ is decreasing in y. Moreover, note that

$$\theta(0) = \frac{P\{K > 0\}((s-c) + (u-s)P\{D > 0 \mid K > 0\})}{2\left[(s-c)(p-u)Cov[D, 1_{\{K>0\}}] + (p-u)(u-s)Cov[D, 1_{\{D>0,K>0\}}]\right]} = +\infty$$

and

$$\theta(y^*_{RN}) = 0$$

since $m'(y^*_{RN}) = 0$ by quasi-concavity of $E[CF(D, K, y)]$. Therefore, $\theta(y)$ decreases from $+\infty$ to 0 as y increases from 0 to y^*_{RN}. Up to this point, by showing $\theta(y)$ is a decreasing function of y, we establish the existence of an order quantity for each risk-aversion level $\theta \geq 0$. Note that on $(y^*_{RN}, +\infty)$, $\theta(y) \leq 0$ and this implies that the region $(y^*_{RN}, +\infty)$ is dominated. Additionally, in the non-dominated region $[0, y^*_{RN}]$ the second order condition is obtained as

$$\frac{d^2 H(y, \theta)}{dy^2} = m''(y) - \theta v''(y) \leq 0.$$

Since the second order condition is satisfied, the objective function is concave on $[0, y^*_{RN}]$. Moreover, the first derivative of the MV objective function (3.78) evaluated at $y = 0$ is

$$\begin{aligned}
\left.\frac{dH(y,\theta)}{dy}\right|_{y=0} &= P\{K > 0\}((s-c) + (u-s)P\{D > 0 \mid K > 0\}) \\
&\quad -2\theta \begin{bmatrix} (s-c)(p-u)Cov[D, 1_{\{K>0\}}] \\ +(p-u)(u-s)Cov[D, 1_{\{D>0,K>0\}}] \end{bmatrix} \\
&= (u-c) \geq 0
\end{aligned}$$

and (3.78) is nonpositive on $(y^*_{RN}, +\infty)$ since $m(y)$ is decreasing and $v(y)$ is increasing along this region. Therefore, the MV objective function is decreasing on $(y^*_{RN}, +\infty)$ so that it is quasi-concave and the order quantity that is between 0 and y^*_{RN} is a maximizer of (3.61).

For any $\theta \geq 0$, by taking the inverse Θ^{-1} of $\theta(y)$, we can obtain the optimal order quantity corresponding to that θ value so that

$$y(\theta) = \Theta^{-1}(\theta).$$

According to the newsvendor's level of risk-aversion given by θ, the optimal order quantity changes between 0 and y^*_{RN}. It can be seen that $\theta(y)$ is decreasing in y. Similar to the previous section, we state that as the level of risk-aversion θ increases the optimal order quantity $y(\theta)$ decreases. ■

Also, note that we can write (3.79) as

$$P\{D \leq y(\theta) \mid K > y(\theta)\} + \theta\bar{v}(y(\theta)) = \hat{p} \tag{3.81}$$

where

$$\bar{v}(y) = \frac{2\left\{\begin{array}{c} (s-c)^2 Cov\left[\min\{K,y\}, 1_{\{K>y\}}\right] \\ +(u-s)^2 Cov\left[\min\{D,K,y\}, 1_{\{D>y,K>y\}}\right] \\ +(s-c)(p-u)Cov\left[D, 1_{\{K>y\}}\right] \\ +(p-u)(u-s)Cov\left[D, 1_{\{D>y,K>y\}}\right] \\ +(s-c)(u-s)\left(\begin{array}{c} Cov\left[\min\{D,K,y\}, 1_{\{K>y\}}\right] \\ +Cov\left[\min\{K,y\}, 1_{\{D>y,K>y\}}\right] \end{array}\right) \end{array}\right\}}{(u-s)}.$$

Also, remark that (3.81) is similar to the characterization in (3.73). The only difference is that we have an additional term given by $\bar{v}(y)$. When $\theta = 0$, (3.81) becomes the same as (3.73) .

As a special case, suppose that $K = +\infty$ to guarantee there is no capacity restriction, then the optimality condition (3.79) becomes (3.25).

In this section, we studied the optimal order quantities considering the MV model where there is capacity randomness. We conclude that under Assumption 3.3.1 and 3.3.2 a risk-averse newsvendor orders less than the classical newsvendor and a more risk-averse newsvendor will order even less.

3.4 MV Model with Random Yield and Capacity

This section incorporates both supply uncertainties as random yield and random capacity into the MV newsvendor model. When y units are ordered, $Umin\{K, y\}$ amount is received.

Here, U and K are random variables representing the proportion of non-defective items received and the capacity of the supplier, respectively. We suppose that D, U and K are not necessarily independent. The conditional density functions $f_{K|U=v}$ and $f_{D|K=z,U=v}$ exist. Then, the cash flow in (3.2) is

$$CF(D,U,K,y) = (s-c)U\min\{K,y\} + (p-u)D + (u-s)\min\{D, U\min\{K,y\}\}.$$

The mean and variance of the cash flow are respectively

$$m(y) = E[CF(D,U,K,y)] = (s-c)E[U\min\{K,y\}]+(p-u)E[D]+(u-s)E[\min\{D,UK,Uy\}] \tag{3.82}$$

and

$$\begin{aligned} v(y) &= Var[CF(D,U,K,y)] \\ &= (s-c)^2 Var\left[U\min\{K,y\}\right] + (p-u)^2 Var\left[D\right] + (u-s)^2 Var\left[\min\{D,UK,Uy\}\right] \\ &\qquad +2(s-c)(p-u)Cov\left[D, U\min\{K,y\}\right] \\ &\qquad\quad +2(p-u)(u-s)Cov\left[D, \min\{D,UK,Uy\}\right] \\ &\qquad\qquad +2(s-c)(u-s)Cov\left[U\min\{K,y\}, \min\{D,UK,Uy\}\right] \\ &\geq 0. \end{aligned}$$

The aim of the risk-averse newsvendor is

$$\max_{y\geq 0} H(y,\theta) = E[CF(D,U,K,y)] - \theta Var[CF(D,U,K,y)] \tag{3.83}$$

where

$$\begin{aligned} H(y,\theta) &= (s-c)E[U\min\{K,y\}] + (p-u)E[D] + (u-s)E[\min\{D,UK,Uy\}] \\ &\quad -\theta \left\{ \begin{array}{c} (s-c)^2 Var\left[U\min\{K,y\}\right] + (u-s)^2 Var\left[\min\{D,UK,Uy\}\right] \\ +(p-u)^2 Var\left[D\right] + 2(s-c)(p-u)Cov\left[D, U\min\{K,y\}\right] \\ +2(p-u)(u-s)Cov\left[D, \min\{D,UK,Uy\}\right] \\ +2(s-c)(u-s)Cov\left[U\min\{K,y\}, \min\{D,UK,Uy\}\right] \end{array} \right\} \end{aligned} \tag{3.84}$$

is the objective function for any fixed $\theta \geq 0$.

Note that for any random variables X, V and Z we can write

$$E[V\min\{Z,y\}] = \int_0^1 v f_V(v)dv \left(\int_0^y z f_{Z|v}(z)dz + y\int_y^{+\infty} f_{Z|v}(z)dz \right) \tag{3.85}$$

and the derivative of (3.85) is

$$\frac{dE[V\min\{Z,y\}]}{dy} = \int_0^1 v f_V(v)dv \int_y^{+\infty} f_{Z|v}(z)dz = E[V1_{\{Z>y\}}]. \tag{3.86}$$

Moreover, one can show that

$$\begin{aligned} E[\min\{X,VZ,Vy\}] &= \int_0^1 f_V(v)dv \left(\int_0^{+\infty} f_{Z|v}(z)dz \left(\int_0^{v\min\{z,y\}} x f_{X|vz}(x)dx \right.\right. \\ &\quad \left.\left. + v\min\{z,y\} \int_{v\min\{z,y\}}^{+\infty} f_{X|vz}(x)dx \right)\right) \\ &= \int_0^1 f_V(v)dv \left(\int_0^{y} f_{Z|v}(z)dz \left(\int_0^{vz} x f_{X|vz}(x)dx + vz \int_{vz}^{+\infty} f_{X|vz}(x)dx \right) \right. \\ &\quad \left. + \int_y^{+\infty} f_{Z|v}(z)dz \left(\int_0^{vy} x f_{X|vz}(x)dx + vy \int_{vy}^{+\infty} f_{X|vz}(x)dx \right)\right) \end{aligned}$$

where $f_{Z|v}(z)$ is the conditional density function of Z given $V = v$ and $f_{X|vz}(x)$ is the conditional density of X given $V = v$ and $Z = z$ and its derivative is

$$\frac{dE[\min\{X,VZ,Vy\}]}{dy} = \int_0^1 v f_V(v)dv \int_y^{+\infty} f_{Z|v}(z)dz \int_{vy}^{+\infty} f_{X|vz}(x)dx = E[V1_{\{X>Vy,Z>y\}}]. \tag{3.87}$$

The variance terms in (3.84) are

$$\begin{aligned} Var[V\min\{Z,y\}] &= E[(V\min\{Z,y\})^2] - E[V\min\{Z,y\}]^2 \\ &= \int_0^1 v^2 f_V(v)dv \left(\int_0^y z^2 f_{Z|v}(z)dz + y^2 \int_y^{+\infty} f_{Z|v}(z)dz \right) \\ &\quad -E[V\min\{Z,y\}]^2 \end{aligned} \tag{3.88}$$

and

$$
\begin{aligned}
Var[\min\{X,VZ,Vy\}] &= E[(\min\{X,VZ,Vy\})^2] - E[\min\{X,VZ,Vy\}]^2 \\
&= \int_0^1 f_V(v)dv \int_0^{+\infty} f_{Z|v}(z)dz \left(\int_0^{v\min\{z,y\}} x^2 f_{X|vz}(x)dx \right. \\
&\quad \left. +v^2 \min\{z,y\}^2 \int_{v\min\{z,y\}}^{+\infty} f_{X|vz}(x)dx \right) - E[\min\{X,VZ,Vy\}]^2 \\
&= \int_0^1 f_V(v)dv \left(\int_0^y f_{Z|v}(z)dz \left(\int_0^{vz} x^2 f_{X|vz}(x)dx \right.\right. \\
&\quad \left. +v^2 z^2 \int_{vz}^{+\infty} f_{X|vz}(x)dx \right) \\
&\quad + \int_y^{+\infty} f_{Z|v}(z)dz \left(\int_0^{vy} x^2 f_{X|vz}(x)dx \right. \\
&\quad \left.\left. +v^2 y^2 \int_{vy}^{+\infty} f_{X|vz}(x)dx \right)\right) \\
&\quad -E[\min\{X,VZ,Vy\}]^2. \qquad (3.89)
\end{aligned}
$$

Then, their derivatives can be obtained as

$$
\begin{aligned}
\frac{dVar[V\min\{Z,y\}]}{dy} &= 2y \int_0^1 v^2 f_V(v)dv \int_y^{+\infty} f_{Z|v}(z)dz \\
&\quad -2E[V\min\{Z,y\}]E[V1_{\{Z>y\}}] \\
&= 2(yE[V^2 1_{\{Z>y\}}] - E[V\min\{Z,y\}]E[V1_{\{Z>y\}}]) \\
&= 2Cov[V\min\{Z,y\}, V1_{\{Z>y\}}] \qquad (3.90)
\end{aligned}
$$

and

$$\begin{aligned}
\frac{dVar[\min\{X,VZ,Vy\}]}{dy} &= 2y\int_0^1 v^2 f_V(v)dv \int_y^{+\infty} f_{Z|v}(z)dz \int_{vy}^{+\infty} f_{X|vz}(x)dx \\
&\quad -2E[\min\{X,VZ,Vy\}]E[V1_{\{X>Vy,Z>y\}}] \\
&= 2\left(yE[V^2 1_{\{X>Vy,Z>y\}}]\right. \\
&\quad \left.-E[\min\{X,VZ,Vy\}]E[V1_{\{X>Vy,Z>y\}}]\right) \\
&= 2Cov[\min\{X,VZ,Vy\},V1_{\{X>Vy,Z>y\}}]. \qquad (3.91)
\end{aligned}$$

Here, (3.90) follows from (3.86) and (3.88) while (3.91) is obtained by using (3.87) and (3.89). Similarly, the covariance terms in (3.84) are

$$\begin{aligned}
Cov\,[X,\min\{X,VZ,Vy\}] &= E[X\min\{X,VZ,Vy\}] - E[X]E[\min\{X,VZ,Vy\}] \\
&= \int_0^1 f_V(v)dv \int_0^{+\infty} f_{Z|v}(z)dz \left(\int_0^{v\min\{z,y\}} x^2 f_{X|vz}(x)dx\right. \\
&\quad \left. +v\min\{z,y\}\int_{v\min\{z,y\}}^{+\infty} xf_{X|vz}(x)dx\right) \\
&\quad -E[X]E[\min\{X,VZ,Vy\}] \\
&= \int_0^1 f_V(v)dv\left(\int_0^y f_{Z|v}(z)dz\left(\int_0^{vz} x^2 f_{X|vz}(x)dx\right.\right. \\
&\quad \left. +vz\int_{vz}^{+\infty} xf_{X|vz}(x)dx\right) + \int_y^{+\infty} f_{Z|v}(z)dz\left(\int_0^{vy} x^2 f_{X|vz}(x)dx\right. \\
&\quad \left.\left. +vy\int_{vy}^{+\infty} xf_{X|vz}(x)dx\right)\right), \qquad (3.92)
\end{aligned}$$

$$\begin{aligned}
Cov[X, V\min\{Z,y\}] &= E[XV\min\{Z,y\}] - E[X]E[V\min\{Z,y\}] \\
&= \int_0^1 vf_V(v)dv \int_0^{+\infty} \min\{z,y\} f_{Z|v}(z)dz \int_0^{+\infty} x f_{X|vz}(x)dx \\
&\quad -E[X]E[V\min\{Z,y\}] \\
&= \int_0^1 vf_V(v)dv \left(\int_0^y z f_{Z|v}(z)dz + \int_0^{+\infty} y f_{Z|v}(z)dz \right) \int_0^{+\infty} x f_{X|vz}(x)dx \\
&\quad -E[X]E[V\min\{Z,y\}] \qquad (3.93)
\end{aligned}$$

and

$$\begin{aligned}
Cov\left[V\min\{Z,y\}, \min\{X,VZ,Vy\}\right] &= E[V\min\{Z,y\}\min\{X,VZ,Vy\}] \\
&\quad -E[V\min\{Z,y\}]E[\min\{X,VZ,Vy\}] \\
&= \int_0^1 vf_V(v)dv \int_0^{+\infty} \min\{z,y\} f_{Z|v}(z)dz \\
&\quad \left(\int_0^{v\min\{z,y\}} x f_{X|vz}(x)dx \right. \\
&\quad \left. +v\min\{z,y\} \int_{v\min\{z,y\}}^{+\infty} f_{X|vz}(x)dx \right) \\
&\quad -E[V\min\{Z,y\}]E[\min\{X,VZ,Vy\}] \\
&= \int_0^1 vf_V(v)dv \left(\int_0^y z f_{Z|v}(z)dz \left(\int_0^{vz} x f_{X|vz}(x)dx \right.\right. \\
&\quad \left. +vz \int_{vz}^{+\infty} f_{X|vz}(x)dx \right) \\
&\quad + \int_y^{+\infty} y f_{Z|v}(z)dz \left(\int_0^{vy} x f_{X|vz}(x)dx \right. \\
&\quad \left.\left. +vy \int_{vy}^{+\infty} f_{X|vz}(x)dx \right)\right). \qquad (3.94)
\end{aligned}$$

One can obtain the derivatives of the covariance terms as

$$\begin{aligned}
\frac{dCov\left[X, \min\{X, VZ, Vy\}\right]}{dy} &= \int_0^1 v f_V(v)dv \int_y^{+\infty} f_{Z|v}(z)dz \int_{vy}^{+\infty} x f_{X|vz}(x)dx \\
&\quad -E[X]E[V1_{\{X>Vy,Z>y\}}] \\
&= E[XV1_{\{X>Vy,Z>y\}}] - E[X]E[V1_{\{X>Vy,Z>y\}}] \\
&= Cov\left[X, V1_{\{X>Vy,Z>y\}}\right] \qquad (3.95)
\end{aligned}$$

using (3.87) and (3.92). Moreover,

$$\begin{aligned}
\frac{dCov\left[X, V\min\{Z, y\}\right]}{dy} &= \int_0^1 v f_V(v)dv \int_y^{+\infty} f_{Z|v}(z)dz \int_0^{+\infty} x f_{X|vz}(x)dx \\
&\quad -E[X]E[V1_{\{Z>y\}}] \\
&= E[XV1_{\{Z>y\}}] - E[X]E[V1_{\{Z>y\}}] \\
&= Cov\left[X, V1_{\{Z>y\}}\right] \qquad (3.96)
\end{aligned}$$

follows from (3.86)and (3.93). Lastly,

$$\begin{aligned}
\frac{dCov\left[V\min\{Z, y\}, \min\{X, VZ, Vy\}\right]}{dy} &= \int_0^1 v f_V(v)dv \int_y^{+\infty} f_{Z|v}(z)dz \left(\int_0^{vy} x f_{X|vz}(x)dx\right. \\
&\quad \left. +vy\int_{vy}^{+\infty} f_{X|vz}(x)dx + yv\int_{vy}^{+\infty} f_{X|vz}(x)dx\right) \\
&\quad -E[V1_{\{Z>y\}}]E[\min\{X, VZ, Vy\}] \\
&\quad -E[V\min\{Z, y\}]E[V1_{\{X>Vy,Z>y\}}] \\
&= E[V1_{\{Z>y\}}\min\{X, VZ, Vy\}] \\
&\quad -E[V1_{\{Z>y\}}]E[\min\{X, VZ, Vy\}] \\
&\quad +E[V^2\min\{Z, y\}1_{\{X>Vy,Z>y\}}] \\
&\quad -E[V\min\{Z, y\}]E[V1_{\{X>Vy,Z>y\}}] \\
&= Cov\left[\min\{X, VZ, Vy\}, V1_{\{Z>y\}}\right] \\
&\quad +Cov\left[V\min\{Z, y\}, V1_{\{X>Vy,Z>y\}}\right] \qquad (3.97)
\end{aligned}$$

is obtained by using (3.86), (3.87) and (3.94).

As a special case, suppose that $\theta = 0$ so that the newsvendor is risk-neutral and the aim is to maximize the expected profit, or

$$\max_{y \geq 0} m(y) = E[CF(D, U, K, y)]. \tag{3.98}$$

Using (3.86) and (3.87), we take the derivative of (3.98) and set it equal to zero to obtain the first order condition

$$\begin{aligned} m'(y) &= (s-c)]E[U1_{\{K>y\}}] + (u-s)E[U1_{\{D>Uy,K>y\}}] \\ &= 0 \end{aligned}$$

which can be written as

$$\begin{aligned} m'(y) &= E[U1_{\{K>y\}}]\left((s-c) + (u-s)\left(\frac{E[U1_{\{D>Uy,K>y\}}]}{E[U1_{\{K>y\}}]}\right)\right) \\ &= 0. \end{aligned} \tag{3.99}$$

Since $P\{K > y\} > 0$ for all y by our assumption and further supposing that $U \neq 0$, we can conclude that $E[U1_{\{K>y\}}] > 0$ so that we can write

$$(s-c) + (u-s)\left(\frac{E[U1_{\{D>Uy,K>y\}}]}{E[U1_{\{K>y\}}]}\right) = 0.$$

For this model, the objective function (3.82) is not necessarily concave since $m'(y)$ is not necessarily decreasing. To carry out the analysis, let

$$h(y) = \frac{E[U1_{\{D\leq Uy,K>y\}}]}{E[U1_{\{K>y\}}]} = 1 - \frac{E[U1_{\{D>Uy,K>y\}}]}{E[U1_{\{K>y\}}]}. \tag{3.100}$$

Assumption 3.4.1 *The conditional probability $h(y)$ is strictly increasing in y.*

Then, we can obtain the optimality condition as

$$\frac{E[U1_{\{D\leq Uy^*_{RN},K>y^*_{RN}\}}]}{E[U1_{\{K>y^*_{RN}\}}]} = \hat{p} \tag{3.101}$$

since $E[U1_{\{D>Uy,K>y\}}] = E[U1_{\{K>y\}}] - E[U1_{\{D\leq Uy,K>y\}}]$. Note that (3.101) is equivalent to the classical newsvendor solution with random yield and capacity given in Okyay et al. [2010]. Furthermore, since $h(y)$ is increasing in y, it follows from (3.99) and (3.100) that the derivative $m'(y)$ is nonnegative and decreasing on $[0, y^*_{RN}]$ and nonpositive on $(y^*_{RN}, +\infty)$. Thus, the objective function is concave increasing on $[0, y^*_{RN}]$ and decreasing on $(y^*_{RN}, +\infty)$

so that the objective function is quasi-concave and the solution y^*_{RN} satisfying (3.101) is indeed the optimal solution.

Provided that $h(0) < \hat{p} < h(+\infty)$, there exists $0 < y^*_{RN} < +\infty$ that satisfies $h(y^*_{RN}) = \hat{p}$, or $m'(y^*_{RN}) = 0$. Moreover, we also claim that $y^*_{RN} = 0$ if $h(0) \geq \hat{p}$; that is

$$\begin{aligned} h(0) &= \frac{E[U1_{\{D\leq 0, K>0\}}]}{E[U1_{\{K>0\}}]} \\ &= \frac{E[U1_{\{D=0, K>0\}}]}{E[U1_{\{K>0\}}]} \geq \hat{p}. \end{aligned} \tag{3.102}$$

Note that since we assumed D to be strictly greater than 0, (3.102) can not hold. However, we consider the case given in (3.102) without this assumption. Similarly, $y^*_{RN} = +\infty$ if $h(+\infty) \leq \hat{p}$; that is

$$\begin{aligned} h(+\infty) &= \frac{E[U1_{\{D<+\infty, K=+\infty\}}]}{E[U1_{\{K=+\infty\}}]} \\ &= \frac{E[U1_{\{K=+\infty\}}] - E[U1_{\{D=+\infty, K=+\infty\}}]}{E[U1_{\{K=+\infty\}}]} \\ &= 1 - P\{D = +\infty \mid K = +\infty\} \leq \hat{p}. \end{aligned}$$

Equivalently, we can conclude that if

$$P\{D = +\infty \mid K = +\infty\} \geq 1 - \hat{p}$$

then $y^*_{RN} = +\infty$. Therefore, if the demand is finite the optimal order quantity is also finite. From now on, we assume $h(0) < \hat{p}$ (or $m'(0) > 0$) and $h(+\infty) > \hat{p}$ (or $m'(+\infty) < 0$) to avoid trivial cases.

Another special case is when $\theta = +\infty$ so that the newsvendor becomes extremely risk-averse and the aim is to minimize the variance of the cash flow, or

$$\min_{y\geq 0} v(y) = Var[CF(D, U, K, y)]. \tag{3.103}$$

To carry out the analysis as in Section 3.1, we consider the random yield and capacity model with the following assumption.

Assumption 3.4.2 *The function* $v(y) = Var[CF(D, U, K, y)]$ *is nondecreasing in* y *and convex on* $[0, y^*_{RN}]$.

This implies that

$$\begin{aligned} v'(y) &= 2\left((s-c)^2 Cov\left[U\min\{K,y\}, U1_{\{K>y\}}\right]\right. \\ &\qquad +(u-s)^2 Cov\left[\min\{D,UK,Uy\}, U1_{\{D>Uy,K>y\}}\right] \\ &\qquad +(s-c)(u-s)\left(\begin{array}{c} Cov\left[U1_{\{K>y\}}, \min\{D,UK,Uy\}\right] \\ +Cov\left[U\min\{K,y\}, U1_{\{D>Uy,K>y\}}\right] \end{array}\right) \\ &\qquad +(s-c)(p-u)Cov\left[D, U1_{\{K>y\}}\right] \\ &\qquad \left. +(p-u)(u-s)Cov\left[D, U1_{\{D>Uy,K>y\}}\right]\right) \\ &\geq 0 \end{aligned} \tag{3.104}$$

for all y which is obtained by using (3.90), (3.91), (3.96), (3.95) and (3.97) where X is D, V is U and Z is K, and $v''(y) \geq 0$ on $[0, y^*_{RN}]$.

Under this assumption, the optimal order quantity to the problem in (3.103) is 0. At this order quantity, the corresponding expected value of the cash flow and variance of the cash flow are respectively

$$E[CF(D,U,K,0)] = (p-u)E[D]$$

and

$$Var[CF(D,U,K,0)] = (p-u)^2 Var[D] \tag{3.105}$$

since $CF(D,U,K,0) = (p-u)D$.

Lemma 3.4.3 *(a) $E[CF(D,U,K,y)]$ is quasi-concave in y; it is increasing on $[0, y^*_{RN}]$ and decreasing on $(y^*_{RN}, +\infty)$. (b) Moreover,*

$$(p-u)^2 Var[D] \leq Var[CF(D,U,K,y)] \leq Var\left[(s-c)UK + (p-u)D + (u-s)\min\{D,UK\}\right]$$

for all y.

Proof. $E[CF(D,U,K,y)]$ is quasi-concave since by our assumption (3.100) is strictly increasing in y. The maximum is attained at y^*_{RN} by (3.101). Moreover,

$$\begin{aligned}\lim_{y\to+\infty} Var[CF(D,U,K,y)] &= \lim_{y\to+\infty} 2(s-c)(u-s)Cov\left[U\min\{K,y\},\min\{D,UK,Uy\}\right] \\ &\quad + \lim_{y\to+\infty}\left((s-c)^2 Var\left[U\min\{K,y\}\right]+(p-u)^2 Var[D]\right) \\ &\quad + \lim_{y\to+\infty} 2(p-u)(u-s)Cov\left[D,\min\{D,UK,Uy\}\right] \\ &\quad + \lim_{y\to+\infty} 2(s-c)(p-u)Cov\left[D,U\min\{K,y\}\right] \\ &\quad + \lim_{y\to+\infty}(u-s)^2 Var\left[\min\{D,UK,Uy\}\right] \\ &= Var\left[(s-c)UK+(p-u)D+(u-s)\min\{D,UK\}\right]\end{aligned}$$

for all y and the lower bound for $Var[CF(D,U,K,y)]$ is $(p-u)^2Var[D]$ as given in (3.105). ■

It follows from part (a) of Lemma 3.4.3 and (3.101) that $E[CF(D,U,K,y)]$ is maximized at a finite, positive point. We can obtain this point by setting $\theta = 0$. It is the classical newsvendor solution that satisfies (3.101). However, in the MV problem we must consider both the mean and variance of the cash flow. We focus on the order quantities that lie in the non-dominated region, $[0, y^*_{RN}]$. Moreover, the order quantities in $(y^*_{RN}, +\infty)$ are all dominated. This implies that $y(\theta) \le y^*_{RN}$ for all $\theta \ge 0$.

Proposition 3.4.4 *The optimal order quantity $y(\theta)$ that maximizes the MV objective is less than or equal to the classical newsvendor solution y^*_{RN} for all $\theta \ge 0$.*

Proof. The reader may refer to Proposition 3.1.2 in Section 3.1, similar arguments apply here as well. ■

Using (3.86), (3.87), (3.90), (3.91), (3.96), (3.95) and (3.97) where X is D, V is U and Z is K, we differentiate the objective function (3.84) and set it equal to zero so that the first order condition is

$$g(y,\theta) = \frac{dH(y,\theta)}{dy} = m'(y) - \theta v'(y) = 0 \tag{3.106}$$

where $m'(y)$ and $v'(y)$ are given in (3.99) and (3.104).

Theorem 3.4.5 *The optimal order quantity $y(\theta)$ that maximizes the MV objective (3.84) is obtained from (3.106) by solving*

$$m'(y(\theta)) - \theta v'(y(\theta)) = 0. \tag{3.107}$$

Moreover, $y(\theta)$ is decreasing in θ.

Proof. For any fixed y, let $\theta(y)$ satisfy the optimality condition (3.106) which can be written as

$$\theta(y) = \frac{m'(y)}{v'(y)}.$$

For all y in the non-dominated region $[0, y^*_{RN}]$, we can show that $\theta(y)$ is decreasing in y. The functional form of $\theta'(y)$ is given in (3.58). As our discussion about the behavior of the function in (3.100) suggests $h(y)$ is increasing in y so that in the region $[0, y^*_{RN}]$, $m'(y) \geq 0$ and $m''(y) \leq 0$. As for $Var[CF(D,U,K,y)]$, via our assumption it is nondecreasing so that $v'(y)$. Moreover, in the non-dominated region $[0, y^*_{RN}]$, $v''(y) \geq 0$ that is $Var[CF(D,U,K,y)]$ is convex. Therefore $\theta'(y) \leq 0$ and we argue that $\theta(y)$ is decreasing in y. Moreover, note that

$$\begin{aligned}
\theta(0) &= \frac{(s-c)E[U1_{\{K>0\}}] + (u-s)E[U1_{\{D>0,K>0\}}]}{2\left[(s-c)(p-u)Cov\left[D, U1_{\{K>0\}}\right] + (p-u)(u-s)Cov\left[D, U1_{\{D>0,K>0\}}\right]\right]} \\
&= \frac{E[U]}{2(p-u)Cov[D,U]}
\end{aligned}$$

and

$$\theta(y^*_{RN}) = 0$$

since $m'(y^*_{RN}) = 0$ by quasi-concavity of $E[CF(D,U,K,y)]$. Therefore, $\theta(y)$ decreases from $\theta(0)$ to 0 as y increases from 0 to y^*_{RN}. Up to now, by showing $\theta(y)$ is a decreasing function of y, we establish the existence of an order quantity for each risk-aversion level θ that is between $0 \leq \theta \leq \theta(0)$. Note that on $(y^*_{RN}, +\infty)$, $\theta(y) \leq 0$ and therefore this region is dominated. Additionally, along the non-dominated region $[0, y^*_{RN}]$, the second order condition is obtained as

$$\frac{d^2H(y,\theta)}{dy^2} = m''(y) - \theta v''(y) \leq 0.$$

Since the second order condition is satisfied, the objective function is concave on $[0, y^*_{RN}]$. Moreover, the first derivative of the MV objective function (3.106) evaluated at $y = 0$ is

$$\begin{aligned}
\left.\frac{dH(y,\theta)}{dy}\right|_{y=0} &= (s-c)E[U1_{\{K>0\}}] + (u-s)E[U1_{\{D>0,K>0\}}] \\
&\quad -2\theta\left((s-c)(p-u)Cov[D, U1_{\{K>0\}}]\right. \\
&\quad \left. +(p-u)(u-s)Cov[D, U1_{\{D>0,K>0\}}]\right) \\
&= (u-c)E[U] - 2\theta(p-u)(u-c)Cov[D,U] \\
&\geq 0
\end{aligned}$$

and (3.106) is nonpositive on $(y^*_{RN}, +\infty)$ because $m(y)$ is decreasing while $v(y)$ is increasing along this region. This implies that the MV objective function is decreasing on $(y^*_{RN}, +\infty)$.

Therefore, it is quasi-concave and the order quantity between 0 and y^*_{RN} is optimal for (3.83). For any $0 \leq \theta < \theta(0)$, by taking the inverse Θ^{-1} of $\theta(y)$, we can obtain the optimal order quantity corresponding to that θ value so that

$$y(\theta) = \Theta^{-1}(\theta).$$

According to the newsvendor's level of risk-aversion given by θ, the optimal order quantity is chosen between 0 and y^*_{RN}. Since $\theta(y)$ is decreasing in y, the inverse function is also decreasing. Therefore, similar to the previous section, we state that as the level of risk-aversion θ increases the optimal order quantity $y(\theta)$ decreases. ■

Note that we can also write (3.107) as

$$\frac{E[U1_{\{D \leq Uy(\theta), K > y(\theta)\}}]}{E[U1_{\{K > y(\theta)\}}]} + \theta \bar{v}(y(\theta)) = \hat{p} \tag{3.108}$$

where

$$\bar{v}(y) = \frac{2\left\{\begin{array}{c} (s-c)^2 Cov\left[U\min\{K,y\}, U1_{\{K>y\}}\right] \\ +(u-s)^2 Cov\left[\min\{D,UK,Uy\}, U1_{\{D>Uy,K>y\}}\right] \\ +(s-c)(p-u)Cov\left[D, U1_{\{K>y\}}\right] \\ +(p-u)(u-s)Cov\left[D, U1_{\{D>Uy,K>y\}}\right] \\ +(s-c)(u-s)\left(\begin{array}{c} Cov\left[U1_{\{K>y\}}, \min\{D,UK,Uy\}\right] \\ +Cov\left[U\min\{K,y\}, U1_{\{D>Uy,K>y\}}\right] \end{array}\right) \end{array}\right\}}{(u-s)}.$$

Also, remark that (3.108) is similar to the characterization in (3.101). The only difference comes from $\bar{v}(y)$. Moreover, if $\theta = 0$, (3.108) is same as (3.101).

The optimal order quantity is $y(\theta) = 0$ if $g(0,\theta) \leq 0$; or

$$g(0,\theta) = (u-c)E[U] - 2\theta(p-u)(u-c)Cov[D,U] \leq 0.$$

Equivalently, we can conclude that if

$$\theta \geq \frac{E[U]}{2(p-u)Cov[D,U]} = \theta(0)$$

then $y(\theta) = 0$.

As a special case, we suppose that $K = +\infty$ so that the model turns into the MV model with random yield and the optimality condition is (3.60). Moreover, when $U = 1$ that is the model becomes the MV model with random capacity, the optimality condition is (3.81).

Finally, when $U = 1$ and $K = +\infty$ so that the model is the MV model, the optimality condition becomes (3.25).

In this section, we investigated the optimal order quantities considering the MV model where there exists yield and capacity randomness. We conclude that under Assumption 3.4.1 and 3.4.2, the arguments that a risk-averse newsvendor orders less than the classical newsvendor and as the level of risk-aversion increases the optimal order quantity decreases are valid.

Up to this point, we considered risk-averse newsvendors adopting MV approach. First, in Section 3.1 we investigated the case that randomness exists only in demand, then we included the models with random supply in Sections 3.2-3.4. In the following chapter, we will study MV approach when there exists a financial hedging opportunity.

Chapter 4

MV MODELS WITH HEDGING

In the previous chapter, we consider the newsvendor problem with the MV objective where the cash flow is random due to the stochastic nature of demand and supply. We implicitly assumed that demand and supply are not correlated with the financial markets. However, Gaur and Seshadri [2005] suggest that in real life the sales amount is in fact correlated with the financial markets. Therefore, in this chapter, we suppose that there is a financial market in which there are financial securities correlated with demand and supply. The inventory manager decides on both the order quantity and the amount of investment on a portfolio of financial securities.

Okyay et al. [2011] analyze the newsvendor problem with hedging and develop a risk-sensitive solution approach to the problem by considering both the mean and the variance of the cash flow. They follow a two-step approach. At the first step, they aim to find an optimal portfolio of financial securities that minimizes the variance of the hedged cash flow for any possible order quantity. Then, at the second step, with this optimal portfolio they decide on an optimal order quantity which maximizes the mean of the hedged cash flow. Moreover, Sayın [2011] uses the same risk-sensitive, two-step solution approach. Although the first step remains the same as Okyay et al. [2011], as a second step, she aims to maximize the expected utility of the hedged cash flow. Our work is different from them in two ways; firstly, our aim is to maximize the hedged MV cash flow and, secondly, we jointly determine the optimal order quantity and hedging portfolio in one step.

We assume that the length of the period is T during which the risk-free interest rate is r. The newsvendor buys the items at c, sells them at p, salvages the unsold items at s and compensates the stockouts at u which satisfy $p \geq u > ce^{rT} > 0$ and $ce^{rT} > s$ to avoid trivial situations. Except for the cash payment made at time 0 to buy the items, all cash flows occur at time T. Let $\mathbf{X} = (D, U, K)$ be a vector of random variables corresponding to demand and supply uncertainty, S be the price of a primary asset in the market at time T. Suppose that there exist at least one derivative security ($n \geq 1$) in the market where $f_i(S)$ is the net payoff of the ith derivative security at the end of the period. Moreover, let α_i be the amount of security i in the portfolio and $CF(\mathbf{X}, y)$ be the unhedged cash flow. Throughout this chapter, we assume that the random vector $\mathbf{X}$ is correlated with the financial variable S. The total hedged cash flow at time T is given by

$$CF_{\boldsymbol{\alpha}}(\mathbf{X}, S, y) = CF(\mathbf{X}, y) + \sum_{i=1}^{n} \alpha_i f_i(S).$$

The net payoff of the ith derivative security $f_i(S)$ is the payoff $\hat{f}_i(S)$ received at time T minus its investment cost f_i^0 compounded to period T so that $f_i(S) = \hat{f}_i(S) - e^{rT} f_i^0$. For instance, if the derivative is a call option with strike price K, then $\hat{f}_i(S) = (S-K)^+ = \max\{S-K, 0\}$ so that the net payoff is $f_i(S) = \max\{S-\kappa, 0\} - e^{rT} f_i^0$. In this chapter, we impose the condition that there is a complete arbitrage-free market with some risk-neutral probability measure Q. In that case, the price of the ith derivative security will be $f_i^0 = e^{-rT} E_Q[\hat{f}_i(S)]$ and this will lead to $E_Q[f_i(S)] = E_Q\left[\hat{f}_i(S) - f_i^T\right] = 0$ where $f_i^T = e^{rT} f_i^0$ is the price of the derivative security compounded to time T.

Similar to the chapter with no hedging opportunity there exist two ways to model the hedged MV optimization problems. The newsvendor decides on an order quantity y and hedging portfolio $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \cdots, \alpha_n)$ either to maximize the mean of the hedged cash flow such that the variance of the hedged cash flow can not exceed a threshold level or to minimize the variance of the hedged cash flow such that the mean of the hedged cash flow should be greater than a threshold level. We can combine them into a single MV optimization problem such that the problem becomes

$$\begin{aligned}
\max_{y \geq 0, \boldsymbol{\alpha}} H(y, \boldsymbol{\alpha}, \theta) &= E\left[CF_{\boldsymbol{\alpha}}(\mathbf{X}, S, y)\right] - \theta Var\left[CF_{\boldsymbol{\alpha}}(\mathbf{X}, S, y)\right] \\
&= E\left[CF(\mathbf{X}, y) + \sum_{i=1}^{n} \alpha_i f_i(S)\right] - \theta Var\left[CF(\mathbf{X}, y) + \sum_{i=1}^{n} \alpha_i f_i(S)\right] \\
&= E\left[CF(\mathbf{X}, y)\right] - \theta Var\left[CF(\mathbf{X}, y) + \sum_{i=1}^{n} \alpha_i f_i(S)\right] \qquad (4.1)
\end{aligned}$$

since $E[f_i(S)] = 0$ by the arbitrage-free market assumption. Note that shortselling is possible since we do not impose any nonnegativity restrictions on the portfolio $\boldsymbol{\alpha}$. We will first analyze the general case and then take a look at the special models.

The objective function in (4.1) can be explicitly written as

$$\begin{aligned} H(y, \boldsymbol{\alpha}, \theta) &= E[CF(\mathbf{X}, y)] \\ &\quad -\theta \left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j Cov(f_i(S), f_j(S)) \right. \\ &\quad \left. +2 \sum_{i=1}^{n} \alpha_i Cov\left(f_i(S), CF(\mathbf{X}, y)\right) + Var(CF(\mathbf{X}, y)) \right\} \end{aligned} \tag{4.2}$$

for any fixed $\theta \geq 0$. We can rewrite (4.2) in compact matrix notation as

$$\begin{aligned} H(y, \boldsymbol{\alpha}, \theta) &= E\left[CF(\mathbf{X}, y)\right] - \theta \left[Var(CF(\mathbf{X}, y) + \boldsymbol{\alpha}^{\mathbf{T}} \mathbf{f}(S))\right] \\ &= E[CF(\mathbf{X}, y)] - \theta \left[\boldsymbol{\alpha}^{\mathbf{T}} \mathbf{C} \boldsymbol{\alpha} + 2\boldsymbol{\alpha}^{\mathbf{T}} \boldsymbol{\mu}(y) + Var(CF(\mathbf{X}, y))\right] \end{aligned} \tag{4.3}$$

In (4.3), $\boldsymbol{\alpha}^{\mathbf{T}}$ is the transpose of $\boldsymbol{\alpha}$, $\mathbf{f}(\mathbf{S})$ is the column vector with entries

$$\mathbf{f}(\mathbf{S}) = \left(f_1(S), f_2(S), \cdots, f_n(S)\right),$$

$\mathbf{C}$ is the covariance matrix of the securities; that is

$$C_{ij} = Cov(f_i(S), f_j(S))$$

and $\boldsymbol{\mu}(y)$ is a vector with entries

$$\mu_i(y) = Cov(f_i(S), CF(\mathbf{X}, y)).$$

The optimal portfolio can be expressed in a compact formula as stated in the following proposition.

Theorem 4.0.6 *The optimal financial portfolio is given by*

$$\boldsymbol{\alpha}^*(y) = -\mathbf{C}^{-1} \boldsymbol{\mu}(y) \tag{4.4}$$

for any order quantity y.

Proof. The gradient of the objective function with respect to $\boldsymbol{\alpha}$ is

$$\frac{dH(y, \boldsymbol{\alpha}, \theta)}{d\boldsymbol{\alpha}} = -2\theta\left(\mathbf{C}\boldsymbol{\alpha} + \boldsymbol{\mu}(y)\right) \tag{4.5}$$

and the Hessian is

$$\frac{d^2 H(y, \boldsymbol{\alpha}, \theta)}{d\boldsymbol{\alpha}^2} = -2\theta \mathbf{C} \leq 0.$$

As the covariance matrix is always positive definite, the second order condition is satisfied and by setting (4.5) equal to zero, we obtain (4.4). ■

Now, we focus on the optimal order quantity that maximizes the MV hedged cash flow.

Theorem 4.0.7 *The optimal order quantity $y(\theta)$ satisfies*

$$\frac{dE[CF(\mathbf{X},y)]}{dy} - \theta\left(-2\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\frac{d\boldsymbol{\mu}(y)}{dy} + \frac{dVar[CF(\mathbf{X},y)]}{dy}\right) = 0 \tag{4.6}$$

for any $\theta \geq 0$.

Proof. We take the gradient of the objective function with respect to y and set it equal to zero

$$\frac{dH(y,\boldsymbol{\alpha},\theta)}{dy} = \frac{dE[CF(\mathbf{X},y)]}{dy} - \theta\left(2\boldsymbol{\alpha}^{\mathbf{T}}\frac{d\boldsymbol{\mu}(y)}{dy} + \frac{dVar[CF(\mathbf{X},y)]}{dy}\right) = 0. \tag{4.7}$$

Since we obtained the optimal portfolio for multiple securities in (4.4), we substitute it into (4.7) which results in (4.6). ■

Note that, one can also express (4.6) as

$$\frac{dE[CF(\mathbf{X},y)]}{dy} - \theta\frac{dVar[CF_{\alpha^*}(\mathbf{X},S,y)]}{dy} = 0.$$

Therefore, we obtain the optimal order quantity $y(\theta)$ and the optimal portfolio $\alpha(\theta) = \alpha^*(y(\theta))$. However, note that to guarantee optimality, we need to make some assumptions that will be discussed further for each model later on in this chapter.

When there is only one security (*i.e.*, $n = 1$) used for financial hedging, there is further simplification in our results given in the following corollary.

Corollary 4.0.8 *Suppose that only one security is used, then the optimal portfolio consists of*

$$\alpha^*(y) = -\frac{Cov(f(S),CF(\mathbf{X},y))}{Var(f(S))}. \tag{4.8}$$

Moreover, the optimal order quantity $y(\theta)$ satisfies

$$\frac{dE[CF(\mathbf{X},y)]}{dy} - \theta\left[\begin{array}{c} -2\left(\frac{Cov(f(S),CF(\mathbf{X},y))}{Var(f(S))}\right)\frac{dCov(f(S),CF(\mathbf{X},y))}{dy} \\ +\frac{dVar[CF(\mathbf{X},y)]}{dy} \end{array}\right] = 0 \tag{4.9}$$

for any $\theta \geq 0$.

Proof. The results follow from Proposition 4.0.6 and 4.0.7 by noting that if there is only one derivative security for hedging with payoff $f(S)$, then $C = Cov(f(S), f(S)) = Var\,(f(S))$ and $\mu(y) = Cov(f(S), CF(\mathbf{X}, y))$. More explicitly, to solve (4.1), we first take the gradient of the objective function with respect to α

$$\frac{dH(y,\alpha,\theta)}{d\alpha} = -2\theta[\alpha Var(f(S)) + Cov(f(S),CF(\mathbf{X},y))] \tag{4.10}$$

and the second derivative is

$$\frac{d^2H(y,\alpha,\theta)}{d\alpha^2} = -2\theta Var(f(S)) \leq 0.$$

Since the second order condition is satisfied, we conclude that (4.10) gives the optimal solution. Then, we take the gradient of the objective function with respect to y and set it equal to 0. This leads to

$$\begin{aligned} \frac{dH(y,\alpha,\theta)}{dy} &= \frac{dE[CF(\mathbf{X},y)]}{dy} - \theta\left(2\alpha\frac{dCov(f(S),CF(\mathbf{X},y))}{dy} + \frac{dVar[CF(\mathbf{X},y)]}{dy}\right) \\ &= 0. \end{aligned} \tag{4.11}$$

Substituting $\alpha^*(y)$ into (4.11), we obtain (4.9). ∎

Next, we investigate special models to have insights for the optimal hedging portfolio and the optimal order quantity. In Section 4.1, we first consider the MV model where demand is the only source of randomness. Then, in Sections 4.2, 4.3 and 4.4 we include the random supply models based on random yield, random capacity and random yield and random capacity, respectively.

4.1 MV Model

In this section we assume that there is no randomness in supply. We deal with random demand D which is correlated with the financial variable S. The total hedged cash flow at the end of the period is

$$\begin{aligned} CF_{\boldsymbol{\alpha}}(\mathbf{X},S,y) &= CF(D,y) + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S) \\ &= (s-ce^{rT})y + (p-u)D + (u-s)\min\{D,y\} + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S) \end{aligned}$$

where $\mathbf{X} = \{D\}$. We analyze the problem for multiple securities and then for a single security.

The optimization problem is

$$\max_{y\geq 0,\boldsymbol{\alpha}} H(y,\boldsymbol{\alpha},\theta) = E\left[CF(D,y)\right] - \theta\left[Var(CF(D,y) + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S))\right] \tag{4.12}$$

The optimal portfolio is same as (4.4) where $\boldsymbol{\mu}(y)$ can be updated as

$$\mu_i(y) = Cov(f_i(S), CF(D,y)).$$

From (4.7), the first order condition with respect to the order quantity becomes

$$\begin{aligned}\frac{dH(y,\boldsymbol{\alpha},\theta)}{dy} &= (s-ce^{rT})+(u-s)\bar{F}_D(y) \\ &\quad -2\theta(u-s)\left[\boldsymbol{\alpha}^{\mathbf{T}}\hat{\boldsymbol{\mu}}(y)+(u-s)\bar{F}_D(y)S(y)+(p-u)Cov(D,1_{\{D>y\}})\right] \\ &= 0 \qquad (4.13)\end{aligned}$$

where $\hat{\mu}_i(y) = Cov(f_i(S),1_{\{D>y\}})$. This follows by noting that

$$\mu_i(y)=(p-u)Cov(f_i(S),D)+(u-s)Cov(f_i(S),\min\{D,y\})$$

so that

$$\mu_i'(y)=\frac{d\mu_i(y)}{dy}=(u-s)Cov(f_i(S),1_{\{D>y\}}).$$

Since we have obtained the optimal portfolio, we substitute it into (4.13) to obtain the first order condition

$$\frac{dH(y,\theta)}{dy}=m'(y)-\theta v_{\alpha^*}'(y)=0. \qquad (4.14)$$

Note that

$$m(y)=E\left[CF(D,y)\right]$$

and

$$v_{\boldsymbol{\alpha}^*}(y)=Var\left[CF_{\alpha^*}(D,S,y)\right].$$

Moreover, their derivatives are

$$m'(y)=(s-ce^{rT})+(u-s)\bar{F}_D(y)$$

and

$$v_{\boldsymbol{\alpha}^*}'(y)=2(u-s)\left\{\begin{array}{c}-\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\hat{\boldsymbol{\mu}}(y)\\ +(u-s)\bar{F}_D(y)S(y)+(p-u)Cov(D,1_{\{D>y\}})\end{array}\right\}.$$

To guarantee the existence of the optimal order quantity corresponding to each risk-aversion level we need to impose some conditions on the structure of $v_{\alpha^*}(y)$. The following assumption is made throughout the remainder of this chapter.

Assumption 4.1.1 *The function $v_{\boldsymbol{\alpha}^*}(y)=Var\left[CF_{\boldsymbol{\alpha}^*}(D,S,y)\right]$ is nondecreasing in y and convex on $[0,y^*_{RN}]$.*

Proposition 4.1.2 *The optimal order quantity $y(\theta)$ that maximizes the hedged MV objective is less than or equal to the classical newsvendor solution y^*_{RN}.*

Proof. Suppose that there exists an optimal order quantity that satisfies $y(\theta) > y^*_{RN}$. Then from Lemma 3.1.1, we know that the expected value of the cash is concave so that

$$m(y) = E\left[CF(D, y(\theta))\right] < E\left[CF(D, y^*_{RN})\right]$$

Moreover, since $v'_{\boldsymbol{\alpha}^*}(y) \geq 0$

$$Var\left[CF_{\boldsymbol{\alpha}}(D, S, y(\theta))\right] \geq Var\left[CF_{\boldsymbol{\alpha}}(D, S, y^*_{RN})\right].$$

From the above arguments we can state that $y(\theta)$ is dominated by y^*_{RN} and this is a contradiction. Therefore, for our analysis we only need to consider the order quantities that lie in the region $[0, y^*_{RN}]$. ■

Theorem 4.1.3 *The optimal order quantity $y(\theta)$ that maximizes the hedged MV objective is obtained from (4.14) by solving*

$$m'(y(\theta)) - \theta v'_{\boldsymbol{\alpha}^*}(y(\theta)) = 0. \tag{4.15}$$

Moreover, $y(\theta)$ decreases as θ increases.

Proof. For any fixed y, let $\theta(y)$ satisfy the optimality condition (4.15) which can be written as

$$\theta(y) = \frac{m'(y)}{v'_{\alpha^*}(y)}. \tag{4.16}$$

For all y in the non-dominated region $[0, y^*_{RN}]$, one can show that $\theta(y)$ is decreasing in y since the derivative of (4.16) is

$$\theta'(y) = \frac{m''(y)v'_{\alpha^*}(y(\theta)) - m'(y)v''_{\alpha^*}(y(\theta))}{\left(v'_{\alpha^*}(y(\theta))\right)^2} \leq 0. \tag{4.17}$$

We already know by Lemma 3.1.1 that $m'(y) \geq 0$ on $[0, y^*_{RN}]$ and $m''(y) \leq 0$ since $E[CF(D, y)]$ is concave. Moreover, by our assumption $v'_{\boldsymbol{\alpha}^*}(y(\theta)) \geq 0$ for all y and $v''_{\alpha^*}(y(\theta)) \geq 0$ on $[0, y^*_{RN}]$ so that (4.17) follows. Note that

$$\theta(0) = \frac{(s - ce^{rT}) + (u - s)\bar{F}_D(0)}{2(u-s)\left\{\begin{array}{c} -\boldsymbol{\mu}(0)^{\mathbf{T}}\mathbf{C}^{-1}\hat{\boldsymbol{\mu}}(0) \\ +\left[(u-s)\bar{F}_D(0)S(0) + (p-u)Cov(D, 1_{\{D>0\}})\right] \end{array}\right\}} = \frac{(u - ce^{rT})}{0} = +\infty$$

and

$$\theta(y^*_{RN}) = 0$$

since $m'(y^*_{RN}) = 0$ by concavity of $E[CF(D,y)]$. Therefore, $\theta(y)$ decreases from $+\infty$ to 0 as y increases from 0 to y^*_{RN}. Up to now, by showing $\theta(y)$ is a decreasing function of y, we establish the existence of an order quantity for each risk-aversion level $\theta \geq 0$. Note that on $(y^*_{RN}, +\infty)$, $\theta(y) \leq 0$ and so this is the dominated region. Additionally, along the non-dominated region $[0, y^*_{RN}]$ the second order condition is obtained as

$$\frac{d^2 H(y,\theta)}{dy^2} = m''(y) - \theta v''_{\boldsymbol{\alpha}^*}(y) \leq 0.$$

Since the second order condition is satisfied, the objective function is concave on $[0, y^*_{RN}]$. Moreover, the first derivative of the MV objective function (4.14) evaluated at $y = 0$ is

$$\begin{aligned} \left.\frac{dH(y,\theta)}{dy}\right|_{y=0} &= (s - ce^{rT}) + (u-s)\bar{F}_D(0) \\ &\quad -2(u-s)\theta \left\{ \begin{array}{c} -\boldsymbol{\mu}(0)^{\mathbf{T}}\mathbf{C}^{-1}\hat{\boldsymbol{\mu}}(0) \\ +\left[(u-s)\bar{F}_D(0)S(0) + (p-u)Cov(D, 1_{\{D>0\}})\right] \end{array} \right\} \\ &= u - ce^{rT} \geq 0. \end{aligned}$$

and (4.14) is nonpositive on $(y^*_{RN}, +\infty)$ since $m(y)$ is decreasing while $v_\alpha(y)$ is increasing. This implies that the MV objective function is decreasing on $(y^*_{RN}, +\infty)$. Hence, the objective function is quasi-concave and the order quantity that is between 0 and y^*_{RN} is a maximizer of (4.12). For any $\theta \geq 0$, by taking the inverse Θ^{-1} of $\theta(y)$, we can obtain the optimal order quantity corresponding to that θ value so that

$$y(\theta) = \Theta^{-1}(\theta).$$

According to the newsvendor's level of risk-aversion given by θ, the optimal order quantity changes between 0 and y^*_{RN}. As (4.17) indicates, $\theta(y)$ is decreasing in y, so the inverse is also decreasing. Therefore, we again state that as the level of risk-aversion θ increases the optimal order quantity $y(\theta)$ decreases. ■

If there is a single hedging asset ($n = 1$), the optimal portfolio in (4.8) can be updated as

$$\alpha^*(y) = -\frac{Cov(f(S), CF(D,y))}{Var(f(S))}.$$

We can rewrite the optimal portfolio as

$$\alpha^*(y) = -(p-u)\beta_D(\infty) - (u-s)\beta_D(y) \tag{4.18}$$

for any given y where

$$\beta_D(y) = \frac{Cov(f(S), \min\{D, y\})}{Var(f(S))}$$

and

$$\beta_D(\infty) = \frac{Cov(f(S), D)}{Var(f(S))}.$$

The same assumptions for $v_{\alpha^*}(y)$ are also required for this special case. Under this assumption, the optimal order quantity now satisfies (4.15) where $m'(y)$ is exactly the same. The only difference is $v'_{\boldsymbol{\alpha}^*}(y)$ which is in this case

$$v'_{\boldsymbol{\alpha}^*}(y) = 2(u-s)\left\{ \begin{array}{c} -\left((p-u)\beta_D(\infty) + (u-s)\beta_D(y)\right)\hat{\mu}(y) \\ +(u-s)\bar{F}_D(y)S(y) + (p-u)Cov(D, 1_{\{D>y\}}) \end{array} \right\}.$$

Consequently, the non-dominated order quantities lie on $[0, y^*_{RN}]$, the objective function is quasi-concave and the conclusion that as the level of risk-aversion increases the optimal order quantity decreases are valid, too.

As a special case, suppose there is no correlation between the random factors in inventory system (i.e. demand, supply) and financial markets, then $\boldsymbol{\alpha}^* = 0$ and the model reverts back to the MV problem with no hedging opportunity. Thus, the optimality condition becomes (3.25).

4.2 MV Model with Random Yield

In this section we analyze the supply uncertainty when it is subject to random yield. The amount received from ordering y units is Uy where $0 \leq U \leq 1$. For generality, we further assume that D and U are not necessarily independent and the conditional density function of D given $U = v$ is $f_{D|v}$. Moreover, D and U are correlated with S. For the random yield model the total hedged cash flow at the end of the period can be written as

$$\begin{aligned} CF_{\boldsymbol{\alpha}}(\mathbf{X}, S, y) &= CF(D, U, y) + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S) \\ &= (s - ce^{rT})Uy + (p-u)D + (u-s)\min\{D, Uy\} + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S) \end{aligned}$$

where $\mathbf{X} = \{D, U\}$. We analyze the problem for multiple securities and then for a single security, in turn.

We want to solve the following optimization problem

$$\max_{y \geq 0, \boldsymbol{\alpha}} H(y, \boldsymbol{\alpha}, \theta) = E\left[CF(D, U, y)\right] - \theta Var\left[CF(D, U, y) + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S)\right]. \tag{4.19}$$

The optimal portfolio is (4.4) where $\boldsymbol{\mu}(y)$ can be updated as

$$\mu_i(y) = Cov(f_i(S), CF(D, U, y)).$$

The first order condition with respect to order quantity can be updated as

$$\begin{aligned} \frac{dH(y, \boldsymbol{\alpha}, \theta)}{dy} &= (u - ce^{rT})E[U] - (u - s)E[U1_{\{D \leq Uy\}}] \\ &\quad -2\theta \left\{ \begin{array}{c} \boldsymbol{\alpha}^{\mathbf{T}}\boldsymbol{\mu}'(y) + (s - ce^{rT})^2 yVar[U] \\ +(u - s)^2 Cov[\min\{D, Uy\}, U1_{\{D > Uy\}}] \\ +(s - ce^{rT})(p - u)Cov[U, D] \\ +(p - u)(u - s)Cov[D, U1_{\{D > Uy\}}] \\ +(s - ce^{rT})(u - s) \left(\begin{array}{c} Cov[U, \min\{D, Uy\}] \\ +yCov[U, U1_{\{D > Uy\}}] \end{array} \right) \end{array} \right\} \\ &= 0 \end{aligned} \tag{4.20}$$

where

$$\mu_i(y) = (s - ce^{rT})yCov(f_i(S), U) + (p - u)Cov(f_i(S), D) + (u - s)Cov(f_i(S), \min\{D, Uy\})$$

and its derivative is

$$\mu_i'(y) = (s - ce^{rT})Cov(f_i(S), U) + (u - s)Cov(f_i(S), U1_{\{D > Uy\}}).$$

By substituting the optimal portfolio into (4.20) we obtain the first order condition

$$g(y, \theta) = \frac{dH(y, \theta)}{dy} = m'(y) - \theta v'_{\alpha^*}(y) = 0 \tag{4.21}$$

where

$$m(y) = E\left[CF(D, U, y)\right]$$

and

$$v_{\boldsymbol{\alpha}^*}(y) = Var\left[CF_{\boldsymbol{\alpha}^*}(D, U, S, y)\right].$$

Their derivatives are

$$m'(y) = (u - ce^{rT})E[U] - (u - s)E[U1_{\{D \leq Uy\}}]$$

and

$$v'_{\boldsymbol{\alpha}^*}(y) = 2 \left(\begin{array}{c} -\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\boldsymbol{\mu}'(y) + (s - ce^{rT})^2 yVar[U] \\ +(u - s)^2 Cov[\min\{D, Uy\}, U1_{\{D > Uy\}}] + (s - ce^{rT})(p - u)Cov[U, D] \\ +(p - u)(u - s)Cov[D, U1_{\{D > Uy\}}] \\ +(s - ce^{rT})(u - s)\left(Cov[U, \min\{D, Uy\}] + yCov[U, U1_{\{D > Uy\}}]\right) \end{array} \right).$$

To guarantee that there exists an optimal order quantity corresponding to each risk-aversion level, we consider the random yield model with the following assumption.

Assumption 4.2.1 *The function* $v_{\boldsymbol{\alpha}^*}(y) = Var\left[CF_{\boldsymbol{\alpha}^*}(D, U, S, y)\right]$ *is nondecreasing in* y *and convex on* $[0, y^*_{RN}]$.

This assumption implies that

$$\begin{aligned} v'_{\boldsymbol{\alpha}^*}(0) &= 2(p-u)(u-ce^{rT})\left(-Cov\left(f_i(S), D\right)^{\mathbf{T}}\mathbf{C}^{-1}Cov\left(f_i(S), U\right) + Cov[U, D]\right) \\ &\geq 0. \end{aligned}$$

Proposition 4.2.2 *The optimal order quantity* $y(\theta)$ *that maximizes the hedged MV objective is less than or equal to the classical newsvendor solution* y^*_{RN}.

Proof. Similar reasoning discussed in Proposition 4.1.2 is valid. ■

Theorem 4.2.3 *The optimal order quantity* $y(\theta)$ *that maximizes the hedged MV objective is obtained from (4.21) by solving*

$$m'(y(\theta)) - \theta v'_{\boldsymbol{\alpha}^*}(y(\theta)) = 0. \tag{4.22}$$

Moreover, $y(\theta)$ *decreases as* θ *increases.*

Proof. For any fixed y, let $\theta(y)$ satisfy the optimality condition (4.22) which can be written as

$$\theta(y) = \frac{m'(y)}{v'_{\alpha^*}(y)}.$$

For all y in the non-dominated region $[0, y^*_{RN}]$, one can show that $\theta(y)$ is decreasing in y. The argument is the same as the previous section. Moreover, note that

$$\begin{aligned} \theta(0) &= \frac{(u-ce^{rT})E[U] - (u-s)E[U1_{\{D=0\}}]}{2(p-u)(u-ce^{rT})\left(-Cov\left(f_i(S), D\right)^{\mathbf{T}}\mathbf{C}^{-1}Cov\left(f_i(S), U\right) + Cov\left[U, D\right]\right)} \\ &= \frac{E[U]}{2(p-u)\left(-Cov\left(f_i(S), D\right)^{\mathbf{T}}\mathbf{C}^{-1}Cov\left(f_i(S), U\right) + Cov\left[U, D\right]\right)} \end{aligned}$$

and

$$\theta(y^*_{RN}) = 0$$

since $m'(y^*_{RN}) = 0$ by concavity of $E[CF(D,U,y)]$. Therefore, $\theta(y)$ decreases from $\theta(0)$ to 0 as y increases from 0 to y^*_{RN}. Up to now, by showing $\theta(y)$ is a decreasing function of y, we establish the existence of an order quantity $y(\theta)$ for each risk-aversion level $0 \le \theta \le \theta(0)$. Note that on $(y^*_{RN}, +\infty)$, $\theta(y) \le 0$ and this guarantees that this is the dominated region. Additionally, along the non-dominated region $[0, y^*_{RN}]$ the second order condition is obtained as

$$\frac{d^2 H(y,\theta)}{dy^2} = m''(y) - \theta v''_{\boldsymbol{\alpha}^*}(y) \le 0.$$

Since the second order condition is satisfied, the objective function is concave on $[0, y^*_{RN}]$. Moreover, the first derivative of the MV objective function (4.21) evaluated at $y = 0$ is

$$\begin{aligned} \left.\frac{dH(y,\theta)}{dy}\right|_{y=0} &= (u - ce^{rT})E[U] \\ &\quad -2\theta(p-u)(u-ce^{rT}) \begin{pmatrix} -Cov\left(f_i(S), D\right)^{\mathbf{T}} \mathbf{C}^{-1} Cov\left(f_i(S), U\right) \\ +Cov[U,D] \end{pmatrix} \\ &\ge 0 \end{aligned}$$

and (4.21) is nonpositive on $(y^*_{RN}, +\infty)$ since $m(y)$ is decreasing while $v_\alpha(y)$ is increasing. Therefore, the MV objective function is decreasing on $(y^*_{RN}, +\infty)$. This implies that the objective function is quasi-concave and the order quantity that is between 0 and y^*_{RN} is a maximizer of (4.19). For any $0 \le \theta \le \theta(0)$, by taking the inverse Θ^{-1} of $\theta(y)$, we can obtain the optimal order quantity corresponding to that θ value so that

$$y(\theta) = \Theta^{-1}(\theta).$$

According to the newsvendor's level of risk-aversion given by θ, the optimal order quantity changes between 0 and y^*_{RN}. We conclude that $\theta(y)$ is decreasing in y, so the inverse is also decreasing so that as the level of risk-aversion θ increases the optimal order quantity $y(\theta)$ decreases. ■

Note that, the optimal order quantity is $y(\theta) = 0$ if $g(0,\theta) < 0$; or

$$\begin{aligned} g(0,\theta) &= (u - ce^{rT})E[U] \\ &\quad -2\theta(p-u)(u-ce^{rT}) \left(-Cov\left(f_i(S), D\right)^{\mathbf{T}} \mathbf{C}^{-1} Cov\left(f_i(S), U\right) + Cov[U,D]\right) \\ &< 0. \end{aligned}$$

Equivalently, we can conclude that if

$$\theta > \frac{E[U]}{2(p-u)\left(-Cov\left(f_i(S), D\right)^{\mathbf{T}} \mathbf{C}^{-1} Cov\left(f_i(S), U\right) + Cov[U,D]\right)} = \theta(0)$$

then $y(\theta) = 0$.

If there is a single hedging asset ($n = 1$), the optimal portfolio in (4.8) can be updated as

$$\alpha^*(y) = -\frac{Cov(f(S), CF(D,U,y))}{Var(f(S))}.$$

This can be explicitly written as

$$\alpha^*(y) = -(s - ce^{rT})y\beta_U - (p-u)\beta_D(\infty) - (u-s)\beta_{D,U}(y) \tag{4.23}$$

for any given y where

$$\beta_U = \frac{Cov(f(S), U)}{Var(f(S))},$$

$$\beta_D(y) = \frac{Cov(f(S), \min\{D, y\})}{Var(f(S))}$$

and

$$\beta_{D,U}(y) = \frac{Cov(f(S), \min\{D, Uy\})}{Var(f(S))}.$$

Under the same assumptions for $v_{\alpha^*}(y)$, the optimal order quantity now satisfies (4.22) where $m'(y)$ is exactly the same. The only difference is in $v'_{\boldsymbol{\alpha}^*}(y)$ which is in this case

$$v'_{\boldsymbol{\alpha}^*}(y) = 2\left\{\begin{array}{c} -\left((s-ce^{rT})y\beta_U + (p-u)\beta_D(\infty) + (u-s)\beta_{D,U}(y)\right)\mu'(y) \\ +(s-ce^{rT})^2 yVar[U] + (u-s)^2 Cov[\min\{D,Uy\}, U1_{\{D>Uy\}}] \\ +(s-ce^{rT})(p-u)Cov[U,D] + (p-u)(u-s)Cov[D, U1_{\{D>Uy\}}] \\ +(s-ce^{rT})(u-s)\left(Cov[U,\min\{D,Uy\}] + yCov[U, U1_{\{D>Uy\}}]\right) \end{array}\right\}.$$

Consequently, the non-dominated order quantities lie on $[0, y^*_{RN}]$, the objective function is quasi-concave and the conclusion that as the level of risk-aversion increases the optimal order quantity decreases are valid, too.

Lastly, consider a special case that there exists no correlation between the random factors in inventory system (i.e. demand, supply) and financial markets, then $\boldsymbol{\alpha}^* = 0$ and the model reverts back to the random yield model (3.56).

4.3 MV Model with Random Capacity

In this section we analyze the MV newsvendor problem including supply uncertainty when it is caused by random capacity. The amount received from ordering y units is $\min\{K, y\}$ where K is a random variable. For generality, we further assume that D and K are not necessarily independent and the conditional density function of D given $K = z$ is $f_{D|z}$. Moreover, both D and K are correlated with S. The total hedged cash flow at the end of the period equals

$$\begin{aligned} CF_{\boldsymbol{\alpha}}(\mathbf{X}, S, y) &= CF(D, K, y) + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S) \\ &= (s - ce^{rT}) \min\{K, y\} + (p - u)D + (u - s) \min\{D, K, y\} + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S) \end{aligned}$$

where $X = \{D, K\}$. We again analyze the problem for multiple securities and then for a single security, respectively.

We want to decide on the best values of y and $\boldsymbol{\alpha}$ for the following MV optimization problem

$$\max_{y \geq 0, \boldsymbol{\alpha}} H(y, \boldsymbol{\alpha}, \theta) = E\left[CF(D, K, y)\right] - \theta Var\left[CF(D, K, y) + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S)\right]. \tag{4.24}$$

The optimal portfolio is (4.4) where $\boldsymbol{\mu}(y)$ can be updated as

$$\mu_i(y) = Cov(f_i(S), CF(D, K, y)).$$

From (4.7), the first order condition with respect to order quantity becomes

$$\begin{aligned} \frac{dH(y, \boldsymbol{\alpha}, \theta)}{dy} &= P\{K > y\}\left((s - ce^{rT}) + (u - s)P\{D > y \mid K > y\}\right) \\ &\quad -2\theta \left\{ \begin{array}{c} \boldsymbol{\alpha}^{\mathbf{T}}\boldsymbol{\mu}'(y) + (s - ce^{rT})^2 Cov[\min\{K, y\}, 1_{\{K>y\}}] \\ +(u - s)^2 Cov[\min\{D, K, y\}, 1_{\{D>y, K>y\}}] \\ +(s - ce^{rT})(p - u)Cov[D, 1_{\{K>y\}}] \\ +(p - u)(u - s)Cov[D, 1_{\{D>y, K>y\}}] \\ +(s - ce^{rT})(u - s)\left(\begin{array}{c} Cov[\min\{D, K, y\}, 1_{\{K>y\}}] \\ +Cov[\min\{K, y\}, 1_{\{D>y, K>y\}}] \end{array} \right) \end{array} \right\} \\ &= 0 \end{aligned} \tag{4.25}$$

where

$$\begin{aligned} \mu_i(y) &= (s - ce^{rT})Cov(f_i(S), \min\{K, y\}) + (p - u)Cov(f_i(S), D) \\ &\quad +(u - s)Cov(f_i(S), \min\{D, K, y\}) \end{aligned}$$

and the derivative of $\mu_i(y)$ is

$$\mu_i'(y) = (s - ce^{rT})Cov(f_i(S), 1_{\{K>y\}}) + (u - s)Cov(f_i(S), 1_{\{D>y, K>y\}}).$$

By substituting the optimal portfolio for multiple assets into (4.25) we obtain the first order condition

$$\frac{dH(y,\theta)}{dy} = m'(y) - \theta v'_{\alpha^*}(y) = 0 \tag{4.26}$$

where

$$m(y) = E\left[CF(D,K,y)\right]$$

and

$$v_{\boldsymbol{\alpha}^*}(y) = Var\left[CF_{\alpha^*}(D,K,S,y)\right].$$

The derivatives can be obtained as

$$m'(y) = P\{K > y\}\left((s - ce^{rT}) + (u-s)P\{D > y \mid K > y\}\right)$$

and

$$v'_{\boldsymbol{\alpha}^*}(y) = 2\left\{\begin{array}{c} -\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\boldsymbol{\mu}'(y) + (s-ce^{rT})^2 Cov[\min\{K,y\}, 1_{\{K>y\}}] \\ +(u-s)^2 Cov[\min\{D,K,y\}, 1_{\{D>y,K>y\}}] \\ +(s-ce^{rT})(p-u)Cov[D, 1_{\{K>y\}}] \\ +(p-u)(u-s)Cov[D, 1_{\{D>y,K>y\}}] \\ +(s-ce^{rT})(u-s)\left(\begin{array}{c} Cov[\min\{D,K,y\}, 1_{\{K>y\}}] \\ +Cov[\min\{K,y\}, 1_{\{D>y,K>y\}}] \end{array}\right) \end{array}\right\}.$$

To guarantee that there exists an optimal order quantity corresponding to each risk-aversion level, we consider the random capacity model with the following assumption.

Assumption 4.3.1 *The function $v_{\boldsymbol{\alpha}^*}(y) = Var\left[CF_{\boldsymbol{\alpha}^*}(D,K,S,y)\right]$ is nondecreasing in y and convex on $[0, y^*_{RN}]$.*

Proposition 4.3.2 *The optimal order quantity $y(\theta)$ that maximizes the hedged MV objective is less than or equal to the classical newsvendor solution y^*_{RN}.*

Proof. Similar reasoning discussed in Proposition 4.1.2 is valid. ■

Theorem 4.3.3 *The optimal order quantity $y(\theta)$ that maximizes the hedged MV objective is obtained from (4.26) by solving*

$$m'(y(\theta)) - \theta v'_{\boldsymbol{\alpha}^*}(y(\theta)) = 0. \tag{4.27}$$

Moreover, $y(\theta)$ decreases as θ increases.

Proof. For any fixed y, let $\theta(y)$ satisfy the optimality condition (4.27) which can be written as

$$\theta(y) = \frac{m'(y)}{v'_{\alpha^*}(y)}.$$

For all y in the non-dominated region $[0, y^*_{RN}]$, one can show that $\theta(y)$ is decreasing in y. We already know from our discussion about the behavior of the function in (3.72) that in the region $[0, y^*_{RN}]$, $m'(y) \geq 0$ and $m''(y) \leq 0$. Moreover, via Assumption 4.3.1 $v'_{\boldsymbol{\alpha}^*}(y) \geq 0$ for all y and $v''_{\boldsymbol{\alpha}^*}(y) \geq 0$ on $[0, y^*_{RN}]$ so that $\theta(y)$ is decreasing in y. Moreover, note that

$$\theta(0) = \frac{P\{K>0\}\left((s-ce^{rT}) + (u-s)P\{D>0 \mid K>0\}\right)}{2\left\{\begin{array}{c} -\boldsymbol{\mu}(0)^{\mathbf{T}}\mathbf{C}^{-1}\boldsymbol{\mu}'(0) \\ +(s-ce^{rT})^2 Cov[\min\{K,0\}, 1_{\{K>0\}}] \\ +(u-s)^2 Cov[\min\{D,K,0\}, 1_{\{D>0,K>0\}}] \\ +(s-ce^{rT})(p-u) Cov[D, 1_{\{K>0\}}] \\ +(p-u)(u-s) Cov[D, 1_{\{D>0,K>0\}}] \\ +(s-ce^{rT})(u-s)\left(\begin{array}{c} Cov[\min\{D,K,0\}, 1_{\{K>0\}}] \\ +Cov[\min\{K,0\}, 1_{\{D>0,K>0\}}] \end{array}\right) \end{array}\right\}} = +\infty$$

and

$$\theta(y^*_{RN}) = 0$$

since $m'(y^*_{RN}) = 0$ by quasi-concavity of $E[CF(D,K,y)]$. Therefore, $\theta(y)$ decreases from $+\infty$ to 0 as y increases from 0 to y^*_{RN}. Up to now, by showing $\theta(y)$ is a decreasing function of y, we establish the existence of an order quantity for each risk-aversion level $\theta \geq 0$. Note that on $(y^*_{RN}, +\infty)$, $\theta(y) \leq 0$ and this is the dominated region. Additionally, along the non-dominated region $[0, y^*_{RN}]$ the second order condition is obtained as

$$\frac{d^2H(y,\theta)}{dy^2} = m''(y) - \theta v''_{\boldsymbol{\alpha}^*}(y) \leq 0.$$

Since the second order condition is satisfied, the objective function is concave on $[0, y^*_{RN}]$. Moreover, the first derivative of the MV objective function (4.26) evaluated at $y = 0$ is

$$\begin{aligned} \left.\frac{dH(y,\theta)}{dy}\right|_{y=0} &= P\{K>0\}\left((s-ce^{rT}) + (u-s)P\{D>0 \mid K>0\}\right) \\ &= (u-ce^{rT}) \geq 0 \end{aligned}$$

and (4.26) is nonpositive on $(y^*_{RN}, +\infty)$ because $m(y)$ is decreasing while $v_\alpha(y)$ is increasing. Thus, the MV objective function is decreasing on $(y^*_{RN}, +\infty)$. This implies that the objective function is quasi-concave and the order quantity that is between 0 and y^*_{RN} is a maximizer of (4.24). For any $\theta \geq 0$, by taking the inverse Θ^{-1} of $\theta(y)$, we can obtain the optimal order quantity corresponding to that θ value so that

$$y(\theta) = \Theta^{-1}(\theta).$$

According to the newsvendor's level of risk-aversion given by θ, the optimal order quantity changes between 0 and y^*_{RN}. $\theta(y)$ is decreasing in y, so the inverse is also decreasing. Similar to the previous section, we state that as the level of risk-aversion increases the optimal order quantity decreases. ■

If there is a single hedging asset ($n = 1$), the optimal portfolio in (4.8) can be updated as

$$\alpha^*(y) = -\frac{Cov(f(S), CF(D,K,y))}{Var(f(S))}.$$

This can be explicitly written as

$$\alpha^*(y) = -(s - ce^{rT})\beta_K(y) - (p-u)\beta_D(\infty) - (u-s)\beta_{D,K}(y) \tag{4.28}$$

for any given y where

$$\beta_K(y) = \frac{Cov(f(S), \min\{K,y\})}{Var(f(S))},$$

$$\beta_D(y) = \frac{Cov(f(S), \min\{D,y\})}{Var(f(S))}$$

and

$$\beta_{D,K}(y) = \frac{Cov(f(S), \min\{D,K,y\})}{Var(f(S))}.$$

The same assumptions for $v_{\alpha^*}(y)$, are also held for this special case so that the optimal order quantity now satisfies (4.27) where $m'(y)$ is exactly the same. The only difference is $v'_{\boldsymbol{\alpha}^*}(y)$ which is in this case

$$v'_{\boldsymbol{\alpha}^*}(y) = 2\left\{\begin{array}{c} -\left((s-ce^{rT})\beta_K(y) + (p-u)\beta_D(\infty) + (u-s)\beta_{D,K}(y)\right)\mu'(y) \\ +(s-ce^{rT})^2 Cov[\min\{K,y\}, 1_{\{K>y\}}] \\ +(u-s)^2 Cov[\min\{D,K,y\}, 1_{\{D>y,K>y\}}] \\ +(s-ce^{rT})(p-u)Cov[D, 1_{\{K>y\}}] \\ +(p-u)(u-s)Cov[D, 1_{\{D>y,K>y\}}] \\ +(s-ce^{rT})(u-s)\left(\begin{array}{c} Cov[\min\{D,K,y\}, 1_{\{K>y\}}] \\ +Cov[\min\{K,y\}, 1_{\{D>y,K>y\}}] \end{array}\right) \end{array}\right\}.$$

Consequently, the non-dominated order quantities lie on $[0, y^*_{RN}]$, the objective function is quasi-concave and the conclusion that as the level of risk-aversion increases the optimal order quantity decreases are valid, too.

As a special case suppose that $\boldsymbol{\alpha}^* = 0$ then the model reverts back to the random capacity model (3.79).

4.4 MV Model with Random Yield and Capacity

In this section we work on the MV newsvendor problem when supply randomness results from both yield and capacity. Therefore, the amount received from ordering y units is $U \min\{K, y\}$ where U and K are random variables. Moreover, we suppose that D, U and K are not necessarily independent and have a joint distribution function, $F_{DKU}(x, z, v) = P\{D \leq x, K \leq z, U \leq v\}$. The conditional distribution function of D for given $K = z$ and $U = v$ is $f_{D|zv}$ and the conditional distribution function of K for a given $U = v$ is $f_{K|v}$. We also suppose that D, U and K are correlated with S. The total hedged cash flow equals to

$$\begin{aligned} CF_{\boldsymbol{\alpha}}(\mathbf{X}, S, y) &= CF(D, U, K, y) + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S) \\ &= \left(s - ce^{rT}\right) U \min\{K, y\} + (p-u)D + (u-s)\min\{D, UK, Uy\} + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S) \end{aligned}$$

where $\mathbf{X} = (D, U, K)$. We again analyze the problem for multiple securities and then for a single security, in turn.

We want to solve the following MV optimization problem

$$\max_{y \geq 0, \boldsymbol{\alpha}} H(y, \boldsymbol{\alpha}, \theta) = E\left[CF(D, U, K, y)\right] - \theta Var\left[CF(D, U, K, y) + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S)\right]. \tag{4.29}$$

The optimal portfolio is (4.4) where $\boldsymbol{\mu}(y)$ can be updated as

$$\mu_i(y) = Cov(f_i(S), CF(D, U, K, y)).$$

From (4.7), the first order condition with respect to order quantity becomes

$$\begin{aligned}\frac{dH(y,\boldsymbol{\alpha},\theta)}{dy} &= (s-ce^{rT})E[U1_{\{K>y\}}]+(u-s)E[U1_{\{D>Uy,K>y\}}]\\ &\quad -2\theta\left\{\begin{array}{c}\boldsymbol{\alpha}^{\mathbf{T}}\boldsymbol{\mu}'(y)+(s-ce^{rT})^2Cov\left[U\min\{K,y\},U1_{\{K>y\}}\right]\\ +(u-s)^2Cov\left[\min\{D,UK,Uy\},U1_{\{D>Uy,K>y\}}\right]\\ +(s-ce^{rT})(p-u)Cov\left[D,U1_{\{K>y\}}\right]\\ +(p-u)(u-s)Cov\left[D,U1_{\{D>Uy,K>y\}}\right]\\ +(s-ce^{rT})(u-s)\left(\begin{array}{c}Cov\left[U1_{\{K>y\}},\min\{D,UK,Uy\}\right]\\ +Cov\left[U\min\{K,y\},U1_{\{D>Uy,K>y\}}\right]\end{array}\right)\end{array}\right\}\\ &= 0\end{aligned}$$

where

$$\begin{aligned}\mu_i(y) &= (s-ce^{rT})Cov(f_i(S),U\min\{K,y\})+(p-u)Cov(f_i(S),D)\\ &\quad +(u-s)Cov(f_i(S),\min\{D,UK,Uy\})\end{aligned}$$

and the derivative of $\mu_i(y)$ is

$$\mu_i'(y)=(s-ce^{rT})Cov(f_i(S),U1_{\{K>y\}})+(u-s)Cov(f_i(S),U1_{\{D>Uy,K>y\}}).$$

By substituting the optimal portfolio for multiple assets into (4.25) we obtain the first order condition

$$g(y,\theta)=\frac{dH(y,\theta)}{dy}=m'(y)-\theta v'_{\alpha^*}(y)=0 \tag{4.30}$$

where

$$m(y)=E\left[CF(D,U,K,y)\right]$$

and

$$v_{\boldsymbol{\alpha}^*}(y)=Var\left[CF_{\alpha^*}(D,U,S,y)\right].$$

Their derivatives are

$$m'(y)=(s-ce^{rT})E[U1_{\{K>y\}}]+(u-s)E[U1_{\{D>Uy,K>y\}}]$$

and

$$v'_{\boldsymbol{\alpha}^*}(y)=2\left\{\begin{array}{c}-\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\boldsymbol{\mu}'(y)+(s-ce^{rT})^2Cov\left[U\min\{K,y\},U1_{\{K>y\}}\right]\\ +(u-s)^2Cov\left[\min\{D,UK,Uy\},U1_{\{D>Uy,K>y\}}\right]\\ +(s-ce^{rT})(p-u)Cov\left[D,U1_{\{K>y\}}\right]\\ +(p-u)(u-s)Cov\left[D,U1_{\{D>Uy,K>y\}}\right]\\ +(s-ce^{rT})(u-s)\left(\begin{array}{c}Cov\left[U1_{\{K>y\}},\min\{D,UK,Uy\}\right]\\ +Cov\left[U\min\{K,y\},U1_{\{D>Uy,K>y\}}\right]\end{array}\right)\end{array}\right\}.$$

To guarantee that there exists an optimal order quantity corresponding to each risk-aversion level, we consider this model with the following assumption.

Assumption 4.4.1 *The function $v_{\boldsymbol{\alpha}^*}(y) = Var\left[CF_{\boldsymbol{\alpha}^*}(D, U, K, S, y)\right]$ is nondecreasing in y and convex on $[0, y^*_{RN}]$.*

This assumption implies that

$$\begin{aligned} v'_{\boldsymbol{\alpha}^*}(0) &= 2(p-u)(u-ce^{rT})\left(-Cov(f_i(S), D)^{\mathbf{T}}\mathbf{C}^{-1}Cov\left[f_i(S), U\right] + Cov[U, D]\right) \\ &\geq 0. \end{aligned}$$

Proposition 4.4.2 *The optimal order quantity $y(\theta)$ that maximizes the hedged MV objective is less than or equal to the classical newsvendor solution y^*_{RN}.*

Proof. Similar reasoning discussed in Lemma 3.1.1 is valid. ■

Theorem 4.4.3 *The optimal order quantity $y(\theta)$ that maximizes the hedged MV objective is obtained from (4.30) by solving*

$$m'(y(\theta)) - \theta v'_{\boldsymbol{\alpha}^*}(y(\theta)) = 0. \tag{4.31}$$

Moreover, $y(\theta)$ decreases as θ increases.

Proof. For any fixed y, let $\theta(y)$ satisfy the optimality condition (4.31) which can be written as

$$\theta(y) = \frac{m'(y)}{v'_{\alpha^*}(y)}.$$

For all y in the non-dominated region $[0, y^*_{RN}]$, one can show that $\theta(y)$ is decreasing in y. The argument follows from Section (4.3) Moreover, note that

$$\begin{aligned} \theta(0) &= \frac{(s-ce^{rT})E[U1_{\{K>0\}}] + (u-s)E[U1_{\{D>0,K>0\}}]}{2(p-u)(u-ce^{rT})\left(-Cov(f_i(S), D)^{\mathbf{T}}\mathbf{C}^{-1}Cov(f_i(S), U) + Cov[U, D]\right)} \\ &= \frac{E[U]}{2(p-u)\left(-Cov(f_i(S), D)^{\mathbf{T}}\mathbf{C}^{-1}Cov(f_i(S), U) + Cov[U, D]\right)} \end{aligned}$$

and

$$\theta(y^*_{RN}) = 0$$

since $m'(y^*_{RN}) = 0$ by quasi-concavity of $E[CF(D, U, K, y)]$. Therefore, $\theta(y)$ decreases from $\theta(0)$ to 0 as y increases from 0 to y^*_{RN}. Up to now, by showing $\theta(y)$ is a decreasing function of y, we establish the existence of an order quantity for each risk-aversion level $0 \leq \theta \leq \theta(0)$. Note that on $(y^*_{RN}, +\infty)$, $\theta(y) \leq 0$ which ensures that this is the dominated region. Additionally, along the non-dominated region $[0, y^*_{RN}]$ the second order condition is obtained as

$$\frac{d^2 H(y,\theta)}{dy^2} = m''(y) - \theta v''_{\boldsymbol{\alpha}^*}(y) \leq 0.$$

Since the second order condition is satisfied, the objective function is concave on $[0, y^*_{RN}]$. Moreover, the first derivative of the MV objective function (4.30) evaluated at $y = 0$ is

$$\begin{aligned} \left.\frac{dH(y,\theta)}{dy}\right|_{y=0} &= (u - ce^{rT})E[U] \\ & \quad -2\theta(p-u)(u - ce^{rT}) \left(\begin{array}{c} -Cov(f_i(S), D)^{\mathbf{T}} \mathbf{C}^{-1} Cov(f_i(S), U) \\ +Cov[U, D] \end{array} \right) \\ &\geq 0 \end{aligned}$$

and (4.30) is nonpositive on $(y^*_{RN}, +\infty)$ because $m(y)$ is decreasing while $v_\alpha(y)$ is increasing. Therefore, the MV objective function is decreasing on $(y^*_{RN}, +\infty)$. This implies that the objective function is quasi-concave and the order quantity that is between 0 and y^*_{RN} is a maximizer of (4.29). For any $0 \leq \theta \leq \theta(0)$, by taking the inverse Θ^{-1} of $\theta(y)$, we can obtain the optimal order quantity corresponding to that θ value so that

$$y(\theta) = \Theta^{-1}(\theta).$$

According to the newsvendor's level of risk-aversion given by θ, the optimal order quantity changes between 0 and y^*_{RN}. We show that $\theta(y)$ is decreasing in y so that the inverse is also decreasing. Thus, similar to the previous section, we state that as the level of risk-aversion θ increases the optimal order quantity $y(\theta)$ decreases. ■

Note that, the optimal order quantity is $y(\theta) = 0$ if $g(0, \theta) < 0$; or

$$\begin{aligned} g(0,\theta) &= (u - ce^{rT})E[U] \\ & \quad -2\theta(p-u)(u - ce^{rT}) \left(-Cov\left(f_i(S), D\right)^{\mathbf{T}} \mathbf{C}^{-1} Cov\left(f_i(S), U\right) + Cov[U, D]\right) \\ &< 0. \end{aligned}$$

Equivalently, we can conclude that if

$$\theta > \frac{E[U]}{2(p-u)\left(-Cov\left(f_i(S), D\right)^{\mathbf{T}} \mathbf{C}^{-1} Cov\left(f_i(S), U\right) + Cov[U, D]\right)} = \theta(0)$$

then $y(\theta) = 0$.

If there is a single hedging asset ($n = 1$), the optimal portfolio in (4.8) can be updated as

$$\alpha^*(y) = -\frac{Cov\left(f(S), CF(D,U,K,y)\right)}{Var\left(f(S)\right)}.$$

This can be explicitly written as

$$\alpha^*(y) = -(s - ce^{rT})\beta_{U,K}(y) - (p-u)\beta_D(\infty) - (u-s)\beta_{D,U,K}(y)$$

for any given y where

$$\beta_{U,K}(y) = \frac{Cov\left(f(S), U\min\{K,y\}\right)}{Var(f(S))},$$

$$\beta_D(y) = \frac{Cov(f(S), \min\{D,y\})}{Var(f(S))}$$

and

$$\beta_{D,U,K}(y) = \frac{Cov(f(S), \min\{D, UK, Uy\})}{Var(f(S))}.$$

The same assumptions for $v_{\alpha^*}(y)$ are also held for this special case. The optimal order quantity now satisfies (4.31) where $m'(y)$ is exactly the same. The only difference is $v'_{\boldsymbol{\alpha}^*}(y)$ which is in this case

$$v'_{\boldsymbol{\alpha}^*}(y) = 2\left\{\begin{array}{c} -\left((s-ce^{rT})\beta_{U,K}(y) + (p-u)\beta_D(\infty) + (u-s)\beta_{D,U,K}(y)\right)\mu'(y) \\ +(s-ce^{rT})^2 Cov\left[U\min\{K,y\}, U1_{\{K>y\}}\right] \\ +(u-s)^2 Cov\left[\min\{D,UK,Uy\}, U1_{\{D>Uy,K>y\}}\right] \\ +(s-ce^{rT})(p-u)Cov\left[D, U1_{\{K>y\}}\right] \\ +(p-u)(u-s)Cov\left[D, U1_{\{D>Uy,K>y\}}\right] \\ +(s-ce^{rT})(u-s)\left(\begin{array}{c} Cov\left[U1_{\{K>y\}}, \min\{D,UK,Uy\}\right] \\ +Cov\left[U\min\{K,y\}, U1_{\{D>Uy,K>y\}}\right] \end{array}\right) \end{array}\right\}.$$

Consequently, the non-dominated order quantities lie on $[0, y^*_{RN}]$, the objective function is quasi-concave and the conclusion that as the level of risk-aversion increases the optimal order quantity decreases are valid, too.

As a special case, suppose that there exists no correlation between demand, supply and financial markets, then $\boldsymbol{\alpha}^* = 0$ and the model becomes the random yield and capacity model (3.108).

Chapter 5

NUMERICAL ILLUSTRATIONS

Up to this point, we discuss the newsvendor problem within the MV framework. In Chapter 3, we analyze the MV newsvendor problem where the source of risk comes from demand as well as supply. Then, in Chapter 4, we consider the same problem when the risks associated with the cash flow in the inventory system can be hedged by investing in a portfolio of instruments in the financial markets. This chapter demonstrates the results of Chapter 3 and Chapter 4 by some illustrative numerical examples. First, we construct a simple example to investigate the effects of parameters on the decision variables. Then, we use the Monte Carlo method to simulate our models and comment on the effect of MV approach and hedging on the optimal decisions.

5.1 A Simple Example

The aim of this section is to illustrate how some important parameters affect the optimal order quantity. We assume that there exists no salvage cost, so that $s = 0$. The newsvendor purchases each item at a purchase cost c and sells it at sale price p. For simplicity, we assume that, unless otherwise stated, emergency cost is equal to the sale price, that is $p = u$. We first solve the problem when there is no hedging option, and then we consider the case when there exists some correlation between random variables of inventory system and financial markets.

5.1.1 MV Models Without Hedging

For this example, we assume that hedging option does not exist and the source of randomness comes only from the demand. Then, we also consider the case when there exists supply uncertainty caused by random yield, random capacity and, lastly, both random yield and random capacity. Eeckhoudt et al. [1995] discuss a similar example where the aim is to maximize the expected utility of the cash flow where utility function is exponential. We use the same setting of the example, yet our aim is to optimize the MV objective.

MV Model

We first consider the case when the randomness results only from the demand. Let demand take two values as $D \in \{0, M\}$, with probabilities p_1 and $p_2 = 1 - p_1$, respectively. For this example, the MV objective function in (3.6) can be updated as

$$\begin{aligned} H(y,\theta) &= -cy + pE[\min\{D,y\}] - \theta p^2 Var[\min\{D,y\}] \\ &= -cy + pp_2 \min\{M,y\} - \theta p^2 p_2(1-p_2)\min\{M,y\}^2. \end{aligned} \quad (5.1)$$

Before going into the analysis of the MV order quantity, we first simplify the problem by taking $\theta = 0$ and analyze the case for the classical newsvendor. The optimal order quantity is

$$y^*_{RN} = \begin{cases} 0 & p_1 \geq \hat{p} \\ M & p_1 < \hat{p} \end{cases} \quad (5.2)$$

where $\hat{p} = (p - c)/p$.

When the MV objective is considered, we already know that the non-dominated order quantities lie in $[0, y^*_{RN}]$. This means that the order quantity for the MV problem can not exceed M. This decision is logical since we know for sure that demand can not exceed M units. Note that

$$g(y,\theta) = \frac{dH(y,\theta)}{dy} = \begin{cases} -c + pp_2 - 2\theta p^2 p_2(1-p_2)y & 0 \leq y \leq M \\ -c & y > M \end{cases} \quad (5.3)$$

and

$$\frac{dg(y,\theta)}{dy} = \begin{cases} -2\theta p^2 p_2(1-p_2)y & 0 \leq y \leq M \\ 0 & y > M \end{cases} .$$

In the non-dominated region $[0, M]$, $dg(y,\theta)/dy \leq 0$ so that the objective function in (5.1) is concave. This also implies that $g(y,\theta)$ is decreasing in y.

The optimal order quantity can be easily obtained from (5.3) by setting $g(y,\theta) = 0$

$$y(\theta) = \frac{\hat{p} - p_1}{2pp_1(1-p_1)\theta}.$$

The optimal solution is $y(\theta) = 0$ if $g(0,\theta) \leq 0$; or

$$g(0,\theta) = -c + pp_2 \leq 0.$$

Equivalently, we can conclude that if

$$p_1 \geq \hat{p}$$

then $y(\theta) = 0$ for all $\theta \geq 0$. Moreover, the optimal solution is $y(\theta) = M$ if $g(M,\theta) \geq 0$; or

$$g(M,\theta) = -c + pp_2 - 2\theta p^2 p_2(1-p_2)M \geq 0.$$

Equivalently, we can conclude that if

$$\theta \leq \frac{\hat{p} - p_1}{2pp_1(1-p_1)M}$$

then $y(\theta) = M$.

In conclusion, we can write

$$y(\theta) = \begin{cases} 0 & p_1 \geq \hat{p} \\ \frac{\hat{p}-p_1}{2pp_1(1-p_1)\theta} & p_1 < \hat{p} \text{ and } \frac{\hat{p}-p_1}{2pp_1(1-p_1)M} < \theta < +\infty \\ M & p_1 < \hat{p} \text{ and } \theta \leq \frac{\hat{p}-p_1}{2pp_1(1-p_1)M} \end{cases}.$$

The optimal order quantity depends on both the demand and the risk-aversion parameter θ. When $P\{D = 0\}$ is more than $\hat{p}$, the newsvendor orders nothing. When $P\{D = 0\}$ is less than $\hat{p}$, if θ is less than $(\hat{p} - p_1)/2pp_1(1-p_1)M$, θ is so small that the newsvendor behaves like a risk-neutral newsvendor and orders M units; if θ is more than $(\hat{p} - p_1)/2pp_1(1-p_1)M$, the newsvendor orders $(\hat{p} - p_1)/2pp_1(1-p_1)\theta$ units. We can see that as risk-aversion increases, the optimal order quantity decreases. Moreover, by comparing the optimal policies for the classical newsvendor problem and the MV problem, we can conclude that the risk-averse newsvendor may give an order amount between 0 and M unlike the risk-neutral newsvendor since the risk-averse newsvendor also considers risk in the MV approach.

MV Model with Shortage Cost

The MV newsvendor problem we considered in this thesis does not include an explicit shortage penalty cost for stockout. Nevertheless, as we know when there is insufficient stock to meet the demand during a period, the newsvendor misses a chance to earn a marginal profit of $u - c$. Here, we also want to see the effect of shortage cost in our numerical analysis and on the optimal order quantity. We study the same example of random demand model yet this time we take $u \geq p$ to represent the unit stockout cost. In this case, the cash flow in (3.2) can be updated as

$$CF(D,y) = -cy + (p-u)D + u\min\{D,y\}.$$

The mean and variance of the cash flow are derived as follows

$$\begin{aligned} E[CF(D,y)] &= -cy + (p-u)E[D] + uE[\min\{D,y\}] \\ &= -cy + (p-u)p_2M + up_2\min\{M,y\} \end{aligned}$$

and

$$\begin{aligned} Var[CF(D,y)] &= (p-u)^2Var(D) + u^2Var(\min\{D,y\}) \\ &\quad +2u(p-u)Cov(D,\min\{D,y\}) \\ &= p_2(1-p_2)\left((p-u)^2M^2 + u^2\min\{M,y\}^2 + 2u(p-u)M\min\{M,y\}\right) \\ &\geq 0. \end{aligned} \tag{5.4}$$

The MV objective function in (3.6) can be written as

$$\begin{aligned} H(y,\theta) &= -cy + (p-u)p_2M + up_2\min\{M,y\} \\ &\quad -\theta p_2(1-p_2)\left((p-u)^2M^2 + u^2\min\{M,y\}^2 + 2u(p-u)M\min\{M,y\}\right). \end{aligned} \tag{5.5}$$

As a special case, $\theta = 0$ corresponds to the classical newsvendor. The optimal order quantity is

$$y^*_{RN} = \begin{cases} 0 & p_1 \geq \hat{p} \\ M & p_1 < \hat{p} \end{cases} \tag{5.6}$$

where $\hat{p} = (u-c)/u$. Comparing (5.6) to (5.2), we observe that for the risk-neutral newsvendor, shortage cost has no effect on the optimal ordering policy. Because, similar to the case without shortage cost, the mean of the cash flow is concave.

Then, we consider the MV problem and take the derivative of (5.4) to obtain

$$\frac{dVar[CF(D,y)]}{dy} = \begin{cases} 2p_2(1-p_2)\left(u^2y + u(p-u)M\right) & 0 \leq y \leq M \\ 0 & y > M \end{cases}.$$

Therefore, $Var[CF(D,y)]$ is not a monotone increasing function of y. More precisely, $Var[CF(D,y)]$ is decreasing on $[0, ((u-p)/u)M]$, increasing on $[((u-p)/u)M, M]$ and constant after M. This implies that the non-dominated region is $[((u-p)/u)M, M]$.

Along the non-dominated region, the derivative of (5.5) is

$$g(y,\theta) = \frac{dH(y,\theta)}{dy} = -c + up_2 - 2\theta p_2(1-p_2)\left(u^2 y + u(p-u)M\right) \tag{5.7}$$

for $((u-p)/u)\,M \le y \le M$. Also, the second derivative of (5.5) is

$$\frac{dg(y,\theta)}{dy} = -2\theta p_2(1-p_2)u^2 \le 0 \tag{5.8}$$

for $((u-p)/u)\,M \le y \le M$. Therefore, $g(y,\theta)$ is decreasing in y as (5.8) implies and the objective function in (5.5) is concave.

The optimal order quantity can be obtained from (5.7) by setting $g(y,\theta) = 0$ so that

$$\begin{aligned} y(\theta) &= \frac{-c + up_2 - 2\theta p_2(1-p_2)u(p-u)M}{2\theta p_2(1-p_2)u^2} \\ &= \frac{\hat{p} - p_1}{2\theta p_1(1-p_1)u} + \left(\frac{u-p}{u}\right)M. \end{aligned}$$

The optimal solution is $y(\theta) = ((u-p)/u)\,M$ when $g\left(((u-p)/u)\,M, \theta\right) \le 0$; or

$$g\left(\left(\frac{u-p}{u}\right)M, \theta\right) = -c + up_2 \le 0.$$

Equivalently, we can state that if

$$p_1 \ge \hat{p}$$

then $y(\theta) = ((u-p)/u)\,M$ for all $\theta \ge 0$. Moreover, the optimal solution is $y(\theta) = M$ if $g(M,\theta) \ge 0$; or

$$\begin{aligned} g(M,\theta) &= -c + up_2 - 2\theta p_2(1-p_2)Mup \\ &\ge 0. \end{aligned}$$

Equivalently, we can state that if

$$\theta \le \frac{\hat{p} - p_1}{2pp_1(1-p_1)M}$$

then $y(\theta) = M$.

In conclusion, we can write

$$y(\theta) = \begin{cases} \left(\frac{u-p}{u}\right)M & p_1 \ge \hat{p} \\ \frac{\hat{p}-p_1}{2\theta p_1(1-p_1)u} + \left(\frac{u-p}{u}\right)M & p_1 < \hat{p} \text{ and } \frac{\hat{p}-p_1}{2pp_1(1-p_1)M} < \theta < +\infty \\ M & p_1 < \hat{p} \text{ and } \theta \le \frac{\hat{p}-p_1}{2pp_1(1-p_1)M} \end{cases}. \tag{5.9}$$

We can see from (5.9) that, when $p_1 \ge \hat{p}$, the optimal order quantity under shortage cost, $((u-p)/u)\,M$, goes beyond $y^*_{RN} = 0$.

MV Model with Random Yield

In this part, in addition to demand, we consider the case when supply yield is also random. Thus, when y units are ordered, Uy amount is received. Similar to the example where the randomness only exists in demand, here U also takes two values $U \in \{0, N\}$ where $0 < N \leq 1$. Moreover, U and D have the following joint probability mass function

$D = 0$ and $U = 0$	with probability p_1
$D = 0$ and $U = N$	with probability p_2
$D = M$ and $U = 0$	with probability p_3
$D = M$ and $U = N$	with probability p_4

For this example, the mean of the cash flow is

$$\begin{aligned} E[CF(D,U,y)] &= -cyE[U] + pE[\min\{D, Uy\}] \\ &= -cy(p_2 + p_4)N + pp_4 \min\{M, Ny\} \end{aligned} \tag{5.10}$$

and the variance of the cash flow is

$$\begin{aligned} Var[CF(D,U,y)] &= c^2y^2Var[U] + p^2Var[\min\{D, Uy\}] \\ &\quad -2pcyCov[U, \min\{D, Uy\}] \\ &= c^2y^2N^2(p_2 + p_4)(1 - p_2 - p_4) + p^2p_4(1 - p_4)\min\{M, Ny\}^2 \\ &\quad -2pcyNp_4(1 - p_2 - p_4)\min\{M, Ny\} \\ &\geq 0. \end{aligned} \tag{5.11}$$

Moreover, the MV objective function in (3.38) can be updated as

$$\begin{aligned} H(y,\theta) &= -cy(p_2 + p_4)N + pp_4 \min\{M, Ny\} \\ &\quad -\theta \begin{pmatrix} c^2y^2N^2(p_2 + p_4)(1 - p_2 - p_4) + p^2p_4(1 - p_4)\min\{M, Ny\}^2 \\ -2pcyNp_4(1 - p_2 - p_4)\min\{M, Ny\} \end{pmatrix}. \end{aligned} \tag{5.12}$$

Before analyzing the MV order quantity, we first simplify the problem by taking $\theta = 0$ and analyze the case for the classical newsvendor. The first derivative of (5.10) is

$$\frac{dE[CF(D,U,y)]}{dy} = \begin{cases} -c(p_2 + p_4)N + pp_4N & 0 \leq y \leq \frac{M}{N} \\ -c(p_2 + p_4)N & y > \frac{M}{N} \end{cases}$$

and the optimal order quantity becomes

$$y^*_{RN} = \begin{cases} 0 & \frac{p_2}{p_2+p_4} \geq \hat{p} \\ \frac{M}{N} & \frac{p_2}{p_2+p_4} < \hat{p} \end{cases} .$$

Note that, we carry out the analysis with the assumption that $p_2/(p_2+p_4) < \hat{p}$. Then, for the MV analysis we take the first derivative of (5.11) to obtain

$$\frac{dVar[CF(D,U,y)]}{dy} = \begin{cases} 2yN^2\left(c^2(p_2+p_4)(1-p_2-p_4)\right. & 0 \leq y \leq \frac{M}{N} \\ \qquad \left. + p^2p_4(1-p_4) - 2pcp_4(1-p_2-p_4)\right) & \\ 2(1-p_2-p_4)cN\left(ycN(p_2+p_4) - pMp_4\right) & y > \frac{M}{N} \end{cases} \tag{5.13}$$

and we observe the sign of (5.13) as follows

$$sign\left(\frac{dVar[CF(D,U,y)]}{dy}\right) = \begin{cases} + & 0 \leq y \leq \frac{M}{N} \\ - & \frac{M}{N} < y \leq \frac{pM}{cN}\left(\frac{p_4}{p_2+p_4}\right) \\ + & y > \frac{pM}{cN}\left(\frac{p_4}{p_2+p_4}\right) \end{cases} .$$

Therefore, the non-dominated region is $[0, (pMp_4)/cN(p_2+p_4)]$. In this case, the order quantity corresponding to each risk-aversion level $\theta \geq 0$ is not unique anymore. Therefore, we continue the analysis with a specific example discussed later in this chapter.

MV Model with Random Capacity

For this example, the capacity of the supplier is also random. In other words, when y units are ordered $min\{K, y\}$ is received. Both demand and capacity take two values as $D \in \{0, M\}$ and $K \in \{0, M\}$ and their joint probability distribution function is given by

$D = 0$ and $K = 0$	with probability p_1
$D = 0$ and $K = M$	with probability p_2
$D = M$ and $K = 0$	with probability p_3
$D = M$ and $K = M$	with probability p_4

The mean and variance of the cash flow are, respectively

$$\begin{aligned} E[CF(D,K,y)] &= -cE[\min\{K,y\}] + pE[\min\{D,K,y\}] \qquad (5.14) \\ &= -c(p_2+p_4)\min\{M,y\} + pp_4\min\{M,y\} \end{aligned}$$

and

$$\begin{aligned} Var[CF(D,K,y)] &= c^2 Var(\min\{K,y\}) + p^2 Var(\min\{D,K,y\}) \\ &\quad -2pcCov(\min\{K,y\}, \min\{D,K,y\}) \\ &= c^2(p_2+p_4)(1-p_2-p_4)\min\{M,y\}^2 + p^2 p_4(1-p_4)\min\{M,y\}^2 \\ &\quad -2pcp_4(1-p_2-p_4)\min\{M,y\}\min\{M,y\} \\ &= \min\{M,y\}^2 d \\ &\geq 0 \end{aligned} \tag{5.15}$$

where

$$d = c^2(p_2+p_4)(1-p_2-p_4) + p^2 p_4(1-p_4) - 2pcp_4(1-p_2-p_4).$$

The MV objective function in (3.61) can be written as

$$H(y,\theta) = -c(p_2+p_4)\min\{M,y\} + pp_4\min\{M,y\} - \theta\min\{M,y\}^2 d.$$

Before analyzing the MV problem, as a special case, we take $\theta = 0$ and the problem turns into the classical newsvendor problem. The first derivative of (5.14) is

$$\frac{dE[CF(D,K,y)]}{dy} = \begin{cases} -c(p_2+p_4) + pp_4 & 0 \leq y \leq M \\ 0 & y > M \end{cases} \tag{5.16}$$

and the optimal order quantity is

$$y^*_{RN} = \begin{cases} 0 & \frac{p_2}{p_2+p_4} \geq \hat{p} \\ M & \frac{p_2}{p_2+p_4} < \hat{p} \end{cases}.$$

Note that, we carry out the analysis with the assumption that $p_2/(p_2+p_4) < \hat{p}$. Then, to analyze the MV problem we take the first and second derivative of (5.15) as follows

$$\frac{dVar[CF(D,K,y)]}{dy} = \begin{cases} 2yd \geq 0 & 0 \leq y \leq M \\ 0 & y > M \end{cases} \tag{5.17}$$

and

$$\frac{d^2Var[CF(D,K,y)]}{dy^2} = \begin{cases} 2d \geq 0 & 0 \leq y \leq M \\ 0 & y > M \end{cases}. \tag{5.18}$$

Therefore, as (5.16) indicates, $E[CF(D,K,y)]$ is quasi-concave, more explicitly $E[CF(D,K,y)]$ is linearly increasing on $0 \leq y \leq M$ and constant afterwards, whereas as (5.17) and (5.18) points out $Var[CF(D,K,y)]$ is quasi-convex. It is convex increasing on $0 \leq y \leq M$ and constant afterwards. We also see that this example supports Assumption 3.3.2. Therefore, we state that the non-dominated order quantities lie in $[0, M]$ and we restrict our analysis to this region. The first derivative of the MV problem is

$$g(y,\theta) = \frac{dH(y,\theta)}{dy} = \begin{cases} -c(p_2+p_4) + pp_4 - 2\theta yd & 0 \leq y \leq M \\ 0 & y > M \end{cases}. \tag{5.19}$$

The optimal order quantity can be obtained from (5.19) by setting $g(y,\theta) = 0$ so that

$$y(\theta) = \frac{p(p_2+p_4)\left(\hat{p} - \frac{p_2}{p_2+p_4}\right)}{2\theta d}.$$

The optimal solution is $y(\theta) = 0$ when $g(0,\theta) \leq 0$; that is

$$g(0,\theta) = -c(p_2+p_4) + pp_4 \leq 0.$$

Equivalently, we can state that if

$$\frac{p_2}{p_2+p_4} \geq \hat{p}$$

then $y(\theta) = 0$ for all $\theta \geq 0$. Moreover, the optimal solution is $y(\theta) = M$ if $g(M,\theta) \geq 0$; that is

$$\begin{aligned} g(M,\theta) &= -c(p_2+p_4) + pp_4 - 2\theta Md \\ &\geq 0. \end{aligned}$$

Equivalently, we can declare that if

$$\theta \leq \frac{p(p_2+p_4)\left(\hat{p} - \frac{p_2}{p_2+p_4}\right)}{2Md}$$

then $y(\theta) = M$.

In conclusion, we can write

$$y(\theta) = \begin{cases} 0 & \frac{p_2}{p_2+p_4} \geq \hat{p} \\ \frac{p(p_2+p_4)\left(\hat{p} - \frac{p_2}{p_2+p_4}\right)}{2\theta d} & \frac{p_2}{p_2+p_4} < \hat{p} \text{ and } \frac{p(p_2+p_4)\left(\hat{p} - \frac{p_2}{p_2+p_4}\right)}{2Md} < \theta < +\infty \\ M & \frac{p_2}{p_2+p_4} < \hat{p} \text{ and } \theta \leq \frac{p(p_2+p_4)\left(\hat{p} - \frac{p_2}{p_2+p_4}\right)}{2Md} \end{cases}.$$

The optimal order quantity depends on both the demand and capacity randomness and also the risk-aversion parameter θ. If $P\{D = 0 \mid K = M\} \geq \hat{p}$, the risk-averse newsvendor orders nothing. When $P\{D = 0 \mid K = M\} < \hat{p}$, if θ is greater than $(p\hat{p}(p_2 + p_4) - pp_2)/2Md$, the risk-averse newsvendor orders $(p\hat{p}(p_2 + p_4) - pp_2)/2\theta d$ units; and if θ is less than $(p\hat{p}(p_2 + p_4) - pp_2)/2Md$, the newsvendor orders M units. Thus, we can declare that the optimal order quantity $y(\theta)$ decreases as θ increases. Moreover, the decision that at most M units are ordered is logical since demand and capacity can be at most M.

MV Model with Random Yield and Capacity

Finally, we suppose that randomness results from demand, yield and capacity. Thus, when y units are ordered, $U\min\{K, y\}$ is received. For this example, demand, yield and capacity take two values as $D \in \{0, M\}$, $U \in \{0, N\}$ and $K \in \{0, M\}$. Their joint discrete probability distribution function is

$D = 0, U = 0, K = 0$	with probability p_1
$D = 0, U = 0, K = M$	with probability p_2
$D = 0, U = N, K = 0$	with probability p_3
$D = 0, U = N, K = M$	with probability p_4
$D = M, U = 0, K = 0$	with probability p_5
$D = M, U = 0, K = M$	with probability p_6
$D = M, U = N, K = 0$	with probability p_7
$D = M, U = N, K = M$	with probability p_8

.

For this example, the mean and variance of the cash flow are, respectively

$$\begin{aligned} E[CF(D,U,K,y)] &= -cE[U\min\{K,y\}] + pE[\min\{D,UK,Uy\}] \\ &= -c(p_4 + p_8)N\min\{M,y\} + pp_8\min\{M,NM,Ny\} \end{aligned} \tag{5.20}$$

and

$$\begin{aligned} Var[CF(D,U,K,y)] &= c^2Var(U\min\{K,y\}) + p^2Var(\min\{D,UK,Uy\}) \\ &\quad -2pcCov(U\min\{K,y\}, \min\{D,UK,Uy\}) \\ &= c^2(p_4+p_8)(1-p_4-p_8)N^2\min\{M,y\}^2 \\ &\quad +p^2p_8(1-p_8)\min\{M,NM,Ny\}^2 \\ &\qquad -2pcp_8(1-p_4-p_8)N\min\{M,y\}\min\{M,NM,Ny\} \\ &= N^2\min\{M,y\}^2d \\ &\geq 0. \end{aligned} \tag{5.21}$$

where

$$d = c^2(p_4 + p_8)(1 - p_4 - p_8) + p^2 p_8(1 - p_8) - 2pcp_8(1 - p_4 - p_8).$$

The MV objective function in (3.84) can be written as

$$H(y, \theta) = -c(p_2 + p_4)\min\{M, y\} + pp_4 \min\{M, M, y\} - \theta N^2 \min\{M, y\}^2 d.$$

Before analyzing the MV problem, as a special case, when we set $\theta = 0$, the problem becomes the classical newsvendor problem with random yield and capacity. The first derivative of (5.20) is

$$\frac{dE[CF(D,U,K,y)]}{dy} = \begin{cases} (-c(p_4 + p_8) + pp_8)\,N & 0 \le y \le M \\ 0 & y > M \end{cases} \tag{5.22}$$

and the optimal order quantity is

$$y^*_{RN} = \begin{cases} 0 & \frac{p_4}{p_4+p_8} \ge \hat{p} \\ M & \frac{p_4}{p_4+p_8} < \hat{p} \end{cases}.$$

Note that we carry out the analysis with the assumption that $p_2/\,(p_2 + p_4) < \hat{p}$. Then, we consider the MV problem, the first and second derivative of (5.21) are

$$\frac{dVar[CF(D,U,K,y)]}{dy} = \begin{cases} 2yN^2 d \ge 0 & 0 \le y \le M \\ 0 & y > M \end{cases}$$

and

$$\frac{d^2Var[CF(D,U,K,y)]}{dy^2} = \begin{cases} 2N^2 d \ge 0 & 0 \le y \le M \\ 0 & y > M \end{cases}.$$

Therefore, as (5.22) shows, $E[CF(D,U,K,y)]$ is quasi-concave, more explicitly it is linearly increasing on $0 \le y \le M$ and constant afterwards M whereas as (5.17) indicates $Var[CF(D,U,K,y)]$ is quasi-convex. It is convex increasing on $0 \le y \le M$ and constant after M. Notice that this example supports Assumption 3.4.2. Therefore, we state that the non-dominated order quantities lie in $[0, M]$ and we restrict our analysis to this region. Moreover, the first derivative of the MV problem is

$$g(y,\theta) = \frac{dH(y,\theta)}{dy} = \begin{cases} -c(p_4 + p_8)N + pp_8 N - 2\theta N^2 yd & 0 \le y \le M \\ 0 & y > M \end{cases}. \tag{5.23}$$

The optimal order quantity can be obtained from (5.23) by setting $g(y, \theta) = 0$ so that

$$y(\theta) = \frac{p(p_4 + p_8)\left(\hat{p} - \frac{p_4}{p_4+p_8}\right)}{2\theta N d}.$$

The optimal solution is $y(\theta)=0$ when $g(0,\theta)\le 0$; that is

$$g(0,\theta)=N\left(-c(p_4+p_8)+pp_8\right)\le 0.$$

Equivalently, we can state that if

$$\frac{p_4}{p_4+p_8}\ge\hat{p}$$

then $y(\theta)=0$, for all $\theta\ge 0$. Moreover, the optimal solution is $y(\theta)=M$ if $g(M,\theta)\ge 0$; that is

$$\begin{aligned} g(M,\theta) &= -c(p_4+p_8)N+pp_8N \;-2\theta N^2Md \\ &\ge 0. \end{aligned}$$

Equivalently, we can state that if

$$\theta\le\frac{p(p_4+p_8)\left(\hat{p}-\frac{p_4}{p_4+p_8}\right)}{2NMd}$$

then $y(\theta)=M$.

In conclusion, we can write

$$y(\theta)=\begin{cases} 0 & \frac{p_4}{p_4+p_8}\ge\hat{p} \\ \frac{p(p_4+p_8)\left(\hat{p}-\frac{p_4}{p_4+p_8}\right)}{2\theta Nd} & \frac{p_4}{p_4+p_8}<\hat{p}\text{ and }\frac{p(p_4+p_8)\left(\hat{p}-\frac{p_4}{p_4+p_8}\right)}{2NMd}<\theta<+\infty \\ M & \frac{p_2}{p_2+p_4}<\hat{p}\text{ and }\theta\le\frac{p(p_4+p_8)\left(\hat{p}-\frac{p_4}{p_4+p_8}\right)}{2NMd} \end{cases}.$$

We can also make the same remarks for the optimal order quantity characterization. The optimal order quantity depends on the probability

$$P\{D=0\mid U=N,K=M\}=\frac{p_4}{p_4+p_8}$$

and increases from 0 to M as θ decreases from $+\infty$ to $\left(p\hat{p}(p_4+p_8)-pp_4\right)/\left(2NMd\right)$. We again argue that the decision of ordering at most M units is logical.

In summary, we illustrate all results obtained up to this point with a numerical example. Suppose that M is 100 and N is 0.5. Moreover, the probabilities for a MV model, MV model with random capacity, and MV model with random yield and capacity are respectively

$\boldsymbol{p}_1=[0.25,\ 0.75]$,

$\boldsymbol{p}_2=[0.09,\ 0.16,\ 0.15,\ 0.60]$,

$\boldsymbol{p}_3=[0.01,\ 0.09,\ 0.05,\ 0.10,\ 0.10,\ 0.25,\ 0.10,\ 0.30]$.

We want to analyze the effect of the risk-aversion parameter. Suppose that the newsvendor sells newspapers at $p = 28\ TL$ and buys them at $c = 20\ TL$. Up to now, for each case, we show that the optimal order quantity depends on both the probabilities and the level of risk-aversion. Now, we redefine that condition depending on only the level of risk-aversion. We obtain a critical value of risk-aversion, $\widehat{\theta}$ and show the optimal order quantity is of the form

$$y(\theta) = \begin{cases} \frac{C}{\theta} & \theta > \widehat{\theta} \\ y_{\max} & \theta \leq \widehat{\theta} \end{cases} \tag{5.24}$$

where C is a constant term depending on probabilities and cost parameters. When the risk-aversion level is less than $\widehat{\theta}$, the optimal order quantity takes the largest value that is $y_{\max}$. For instance, when supply randomness results from both random yield and capacity, C is

$$C = \frac{p\hat{p}(p_4 + p_8) - pp_4}{2Nd} = 0.00678$$

and while the level of risk-aversion is larger than

$$\widehat{\theta} = \frac{p\hat{p}(p_4 + p_8) - pp_4}{2NMd} = 0.00007,$$

the optimal order quantity has the structure, $\frac{C}{\theta}$. When the level of risk-aversion is less than or equal to $\widehat{\theta}$, the optimal order quantity equals

$$y_{\max} = M = 100.$$

Table 5.1 shows the values of $C, \widehat{\theta}$ and $y_{\max}$ for all cases where Model 1-3 represent MV Model, MV Model with Random Capacity and MV Model with Random Yield and Capacity, respectively. Note that for all cases, as θ increases from 0 to $+\infty$, the optimal order quantity decreases from the largest value $y_{\max}$ to 0.

	Model 1	**Model 2**	**Model 3**
C	0.00709	0.00117	0.00678
$\widehat{\theta}$	0.00007	0.00001	0.00007
$y_{\max}$	100	100	100

Table 5.1: The constant term, the critical risk-aversion level, and the maximium order quantity

For the MV model with shortage cost the optimal policy has a different form than other cases. Numerically, we take $p = 28$, $c = 20$ and $u = 32$ with the same probabilities as MV model, $\boldsymbol{p}_1 = [0.25,\ 0.75]$. For this case, we find the optimal order policy in (5.24) such that

$$y(\theta) = \begin{cases} \frac{0.010417}{\theta} + 12.5 & \theta > 0.000119 \\ 100 & \theta \leq 0.000119 \end{cases}$$

where the critical value of θ is $\hat{\theta} = 0.000119$.

As for the MV model with random yield we did not go through a complete analysis. Yet, we aim to illustrate with a specific example, for instance, if we take $\theta = 0.01$ and the probabilities as $\boldsymbol{p}_4 = [0.10,\ 0.15,\ 0.35,\ 0.40]$, then the order quantities that make the first derivative equal to zero are either 0.233 or 203.525. However, the second one is the maximizer of (5.12).

Up to here, we demonsrate a numerical example for the MV model without hedging opportunity. Now, we repeat the similar example for the MV model with financial hedging opportunity.

5.1.2 MV Models with Financial Hedging

In this section, we suppose that the randomness in demand and supply is correlated with the financial markets. First, we illustrate the example where the randomness only results from the demand. Then, we include random supply.

MV Model

For this example, both D and $f(S)$ take two values as $D \in \{0, M\}$ and $f(S) \in \{-L, L\}$ for computational simplicity. So, their joint distribution function is

$f(S) = -L,\ D = 0$	with probability p_1
$f(S) = -L,\ D = M$	with probability p_2
$f(S) = L,\ D = 0$	with probability p_3
$f(S) = L,\ D = M$	with probability p_4

.

To make sure that there is no arbitrage opportunity, we assume that

$$E[f(S)] = -(p_1 + p_2)L + (p_3 + p_4)L = 0 \tag{5.25}$$

so that

$$Var\left(f\left(S\right)\right) = L^2. \tag{5.26}$$

Note that (5.25) and (5.26) are valid in the remainder of this section. Moreover, throughout this example we make our calculations based on the fact that the newsvendor orders at most M units which is logical because demand can be at most M units.

The optimal portfolio for a single asset with random demand in (4.18) becomes

$$\alpha^*\left(y\right) = -p\beta_D(y)$$

for any y where

$$\beta_D(y) = \frac{Cov(f(S), \min\{D, y\})}{Var(f(S))}.$$

It can be calculated that

$$\begin{aligned} Cov\left(f\left(S\right), \min\left\{D, y\right\}\right) &= E\left[f\left(S\right)\min\left\{D, y\right\}\right] - E\left[f\left(S\right)\right]E\left[\min\left\{D, y\right\}\right] \\ &= E\left[f\left(S\right)\min\left\{D, y\right\}\right] \\ &= p_1\left(-L\min\left\{0, y\right\}\right) + p_2\left(-L\min\left\{M, y\right\}\right) \\ &\qquad + p_3\left(L\min\left\{0, y\right\}\right) + p_4\left(L\min\left\{M, y\right\}\right) \\ &= \left(p_4 - p_2\right)Ly \end{aligned}$$

and its derivative is

$$\hat{\mu}(y) = \frac{dCov\left(f\left(S\right), \min\left\{D, y\right\}\right)}{dy} = Cov(f(S), 1_{\{D>y\}}) = \left(p_4 - p_2\right)L.$$

Hence, the optimal portfolio is

$$\alpha^*\left(y\right) = p\frac{\left(p_2 - p_4\right)}{L}y. \tag{5.27}$$

Here, we observe that the sign of (5.27) depends on the sign of $\left(p_2 - p_4\right)$. Moreover, since

$$\begin{aligned} Cov\left(f\left(S\right), D\right) &= E\left[f\left(S\right)D\right] - E\left[f\left(S\right)\right]E\left[D\right] \\ &= E\left[f\left(S\right)D\right] \\ &= \left(p_4 - p_2\right)LM \end{aligned}$$

if $\left(p_4 - p_2\right)$ is positive, $f\left(S\right)$ and D are positively correlated in which case the sign of the optimal portfolio is negative that indicates shortselling the portfolio and if $\left(p_4 - p_2\right)$

is negative, $f(S)$ and D are negatively correlated and the sign of (5.27) becomes positive which means the portfolio is bought.

The optimality condition in (4.15) can be updated as

$$g(y,\theta) = \frac{dH(y,\theta)}{dy} = -ce^{rT} + p(p_2 + p_4) - 2\theta p^2 yd = 0 \tag{5.28}$$

where $d = \Big(-(p_4 - p_2)^2 + (p_2 + p_4)(1 - p_2 - p_4)\Big)$. The derivative of (5.28) is

$$\frac{dg(y,\theta)}{dy} = -2\theta p^2 d.$$

Provided that $d \geq 0$, the second order condition is satisfied and the optimal order quantity can be obtained from (5.28) so that

$$\begin{aligned} y(\theta) &= \frac{-ce^{rT} + p(p_2 + p_4)}{2\theta p^2 d} \\ &= \frac{\hat{p} - (p_1 + p_3)}{2\theta p d}. \end{aligned}$$

The optimal solution is $y(\theta) = 0$ if $g(0,\theta) \leq 0$; or

$$g(0,\theta) = -ce^{rT} + p(p_2 + p_4) \leq 0.$$

Equivalently, we can conclude that if

$$p_1 + p_3 \geq \hat{p}$$

then $y(\theta) = 0$ for all $\theta \geq 0$. Moreover, the optimal solution is $y(\theta) = M$ if $g(M,\theta) \geq 0$; or

$$g(M,\theta) = -ce^{rT} + p(p_2 + p_4) - 2\theta p^2 Md \geq 0.$$

Equivalently, we can conclude that if

$$\theta \leq \frac{\hat{p} - (p_1 + p_3)}{2pMd}$$

then $y(\theta) = M$.

In conclusion, we can write

$$y(\theta) = \begin{cases} 0 & p_1 + p_3 \geq \hat{p} \\ \frac{\hat{p} - (p_1+p_3)}{2\theta p d} & p_1 + p_3 < \hat{p} \text{ and } \frac{\hat{p}-(p_1+p_3)}{2pMd} < \theta < +\infty \\ M & p_1 + p_3 < \hat{p} \text{ and } \theta \leq \frac{\hat{p}-(p_1+p_3)}{2pMd} \end{cases}.$$

The optimal order quantity depends on the probability distribution of demand and the risk-aversion parameter θ. If $P\{D = 0\}$ is more than $\hat{p}$, the newsvendor orders nothing. When $P\{D = 0\}$ is less than $\hat{p}$, if θ is more than $(\hat{p} - (p_1 + p_3))/2pMd$, the newsvendor orders $(\hat{p} - (p_1 + p_3))/2\theta pd$ units; if θ is less than $(\hat{p} - (p_1 + p_3))/2pMd$, θ is so small that the newsvendor behaves like a risk-neutral newsvendor and orders M units. We can see that as risk-aversion increases, the optimal order quantity decreases.

MV Model with Random Yield

Now, we include yield randomness into our model. For this example, D, $f(S)$ and U all take two possible values as $D \in \{0, M\}$, $f(S) \in \{-L, L\}$ and $U \in \{0, N\}$. Their joint distribution is given as

$f(S) = -L,\ D = 0,\ U = 0$	with probability p_1
$f(S) = -L,\ D = 0,\ U = N$	with probability p_2
$f(S) = -L,\ D = M,\ U = 0$	with probability p_3
$f(S) = -L,\ D = M,\ U = N$	with probability p_4
$f(S) = L,\ D = 0,\ U = 0$	with probability p_5
$f(S) = L,\ D = 0,\ U = N$	with probability p_6
$f(S) = L,\ D = M,\ U = 0$	with probability p_7
$f(S) = L,\ D = M,\ U = N$	with probability p_8

Note that we make our calculations based on the fact that the newsvendor orders at most M/N units which is logical because demand can be at most M units. The optimal portfolio for a single asset with random yield in (4.23) is updated as

$$\alpha^*(y) = ce^{rT}y\beta_U - p\beta_{D,U}(y)$$

for any y where

$$\beta_U = \frac{Cov(f(S), U)}{Var(f(S))}$$

and

$$\beta_{D,U}(y) = \frac{Cov(f(S), \min\{D, Uy\})}{Var(f(S))}.$$

One can show that

$$\begin{aligned} Cov\left(f\left(S\right),U\right) &= E\left[f\left(S\right)U\right]-E\left[f\left(S\right)\right]E\left[U\right] \\ &= E\left[f\left(S\right)U\right] \\ &= p_2\left(-L\right)N+p_4\left(-L\right)N+p_6\left(L\right)N+p_8\left(L\right)N \\ &= \left(p_6+p_8-p_2-p_4\right)LN \end{aligned}$$

and

$$\begin{aligned} Cov\left(f\left(S\right),\min\left\{D,Uy\right\}\right) &= E\left[f\left(S\right)\min\left\{D,Uy\right\}\right] \\ &\quad -E\left[f\left(S\right)\right]E\left[\min\left\{D,Uy\right\}\right] \\ &= E\left[f\left(S\right)\min\left\{D,Uy\right\}\right] \\ &= \left(p_8-p_4\right)LNy \end{aligned} \tag{5.29}$$

and the derivative of (5.29) equals

$$\frac{dCov\left(f\left(S\right),\min\left\{D,Uy\right\}\right)}{dy}=\left(p_8-p_4\right)LN.$$

Thus, the optimal portfolio is

$$\alpha^*\left(y\right)=\left(ce^{rT}\left(p_6+p_8-p_2-p_4\right)-p\left(p_8-p_4\right)\right)\left(\frac{N}{L}\right)y.$$

The optimality condition in (4.22) can be updated as

$$g(y,\theta)=\left(-ce^{rT}(p_2+p_4+p_6+p_8)+p(p_4+p_8)\right)N-2\theta yd=0 \tag{5.30}$$

where

$$d=N^2\left\{\begin{array}{c} \left(ce^{rT}\left(p_6+p_8-p_2-p_4\right)-p\left(p_8-p_4\right)\right)^2 \\ +(ce^{rT})^2(p_2+p_4+p_6+p_8)(1-p_2-p_4-p_6-p_8) \\ +p^2(p_4+p_8)(1-p_4-p_8) \\ -2ce^{rT}p(p_4+p_8)(1-p_2-p_4-p_6-p_8) \end{array}\right\}.$$

The derivative of (5.30) is

$$\frac{dg(y,\theta)}{dy}=-2\theta d.$$

Provided that $d\geq 0$, the second order condition is satisfied and the optimal order quantity can be obtained from (5.30) which is

$$\begin{aligned} y(\theta) &= \frac{\left(-ce^{rT}(p_2+p_4+p_6+p_8)+p(p_4+p_8)\right)N}{2\theta d} \\ &= \frac{\left(\hat{p}\left(p_2+p_4+p_6+p_8\right)-\left(p_2+p_6\right)\right)pN}{2\theta d}. \end{aligned}$$

The optimal solution is $y(\theta) = 0$ if $g(0, \theta) \leq 0$; or

$$g(0, \theta) = \left(-ce^{rT}(p_2 + p_4 + p_6 + p_8) + p(p_4 + p_8)\right) N \leq 0.$$

Equivalently, we can conclude that if

$$\frac{p_2 + p_6}{p_2 + p_4 + p_6 + p_8} \geq \hat{p}$$

then $y(\theta) = 0$ for all $\theta \geq 0$. Moreover, the optimal solution is $y(\theta) = M$ if $g(M, \theta) \geq 0$; or

$$g(M, \theta) = \left(-ce^{rT}(p_2 + p_4 + p_6 + p_8) + p(p_4 + p_8)\right) N - 2\theta M d \geq 0.$$

Equivalently, we can state that if

$$\theta \leq \frac{\left(\hat{p}\left(p_2 + p_4 + p_6 + p_8\right) - \left(p_2 + p_6\right)\right) pN}{2Md}$$

then $y(\theta) = M$.

In conclusion, we can write

$$y(\theta) = \begin{cases} 0 & \frac{p_2+p_6}{p_2+p_4+p_6+p_8} \geq \hat{p} \\ \frac{(\hat{p}(p_2+p_4+p_6+p_8)-(p_2+p_6))pN}{2\theta d} & \frac{p_2+p_6}{p_2+p_4+p_6+p_8} < \hat{p} \text{ and} \\ & \frac{(\hat{p}(p_2+p_4+p_6+p_8)-(p_2+p_6))pN}{2Md} < \theta < +\infty \\ M/N & \frac{p_2+p_6}{p_2+p_4+p_6+p_8} < \hat{p} \text{ and } \theta \leq \frac{(\hat{p}(p_2+p_4+p_6+p_8)-(p_2+p_6))pN}{2Md} \end{cases}.$$

The optimal order quantity depends on the probability distribution of demand and yield and the risk-aversion parameter θ. The optimal order quantity depends on the probability

$$P\{D = 0 \mid U = N\} = \frac{p_2 + p_6}{p_2 + p_4 + p_6 + p_8}$$

and increases from 0 to M/N as θ decreases from $+\infty$ to

$$\frac{\left(\hat{p}\left(p_2 + p_4 + p_6 + p_8\right) - \left(p_2 + p_6\right)\right) pN}{2Md}.$$

MV Model with Random Capacity

Now, we consider supply randomness in capacity. For our example, demand, $f(S)$ and capacity take two values, $D \in \{0, M\}$, $f(S) \in \{-L, L\}$ and $K \in \{0, M\}$. The joint distribution function is given by

$f(S) = -L,\ D = 0,\ K = 0$	with probability p_1
$f(S) = -L,\ D = 0,\ K = M$	with probability p_2
$f(S) = -L,\ D = M,\ K = 0$	with probability p_3
$f(S) = -L,\ D = M,\ K = M$	with probability p_4
$f(S) = L,\ D = 0,\ K = 0$	with probability p_5
$f(S) = L,\ D = 0,\ K = M$	with probability p_6
$f(S) = L,\ D = M,\ K = 0$	with probability p_7
$f(S) = L,\ D = M,\ K = M$	with probability p_8

Note that we make our calculations based on the fact that the newsvendor orders at most M units which is logical because demand can be at most M units. The optimal portfolio for a single asset with random capacity in (4.28) is updated as

$$\alpha^*(y) = ce^{rT}\beta_K(y) - p\beta_{D,K}(y)$$

for any y where

$$\beta_K(y) = \frac{Cov(f(S), \min\{K, y\})}{Var(f(S))}$$

and

$$\beta_{D,K}(y) = \frac{Cov(f(S), \min\{D, K, y\})}{Var(f(S))}.$$

One can show that

$$\begin{aligned} Cov\left(f\left(S\right), \min\{K, y\}\right) &= E\left[f\left(S\right)\min\{K, y\}\right] - E\left[f\left(S\right)\right]E\left[\min\{K, y\}\right] \\ &= E\left[f\left(S\right)\min\{K, y\}\right] \\ &= p_2\left(-L\right)y + p_4\left(-L\right)y + p_6\left(L\right)y + p_8\left(L\right)y \\ &= \left(p_6 + p_8 - p_2 - p_4\right)Ly \end{aligned} \quad (5.31)$$

and the derivative of (5.31) is

$$\frac{dCov\left(f\left(S\right), \min\{K, y\}\right)}{dy} = Cov\left(f\left(S\right), 1_{\{K>y\}}\right) = \left(p_6 + p_8 - p_2 - p_4\right)L.$$

Moreover,

$$\begin{aligned} Cov\left(f\left(S\right), \min\left\{D, K, y\right\}\right) &= E\left[f\left(S\right)\min\left\{D, K, y\right\}\right] \\ &\quad - E\left[f\left(S\right)\right]E\left[\min\left\{D, K, y\right\}\right] \\ &= E\left[f\left(S\right)\min\left\{D, K, y\right\}\right] \\ &= \left(p_8 - p_4\right)Ly. \end{aligned} \quad (5.32)$$

and the derivative of (5.32) equals

$$\frac{dCov\left(f\left(S\right),\min\left\{D,K,y\right\}\right)}{dy}=Cov\left(f\left(S\right),1_{\{D>y,K>y\}}\right)=\left(p_8-p_4\right)L.$$

Hence, the optimal portfolio is

$$\alpha^*\left(y\right)=\left(ce^{rT}\left(p_6+p_8-p_2-p_4\right)-p\left(p_8-p_4\right)\right)\left(\frac{y}{L}\right).$$

The optimality condition in (4.27) can be updated as

$$g(y,\theta)=\left(p_2+p_4+p_6+p_8\right)\left(-ce^{rT}+p(p_4+p_8)\right)-2\theta yd=0 \tag{5.33}$$

where

$$d=\left\{\begin{array}{c}\left(ce^{rT}\left(p_6+p_8-p_2-p_4\right)-p\left(p_8-p_4\right)\right)^2\\ +(ce^{rT})^2(p_2+p_4+p_6+p_8)(1-p_2-p_4-p_6-p_8)\\ +p^2(p_4+p_8)(1-p_4-p_8)\\ -2ce^{rT}p(p_4+p_8)(1-p_2-p_4-p_6-p_8)\end{array}\right\}.$$

The derivative of (5.33) is

$$\frac{dg(y,\theta)}{dy}=-2\theta d$$

Provided that $d\geq 0$, the second order condition is satisfied and the optimal order quantity can be obtained from (5.33) which is

$$\begin{aligned}y(\theta)&=\frac{\left(p_2+p_4+p_6+p_8\right)\left(-ce^{rT}+p(p_4+p_8)\right)}{2\theta d}\\&=\frac{\left(\hat{p}-\left(1-p_4-p_8\right)\right)p(p_2+p_4+p_6+p_8)}{2\theta d}.\end{aligned}$$

The optimal solution is $y(\theta)=0$ if $g(0,\theta)\leq 0$; or

$$g(0,\theta)=\left(p_2+p_4+p_6+p_8\right)\left(-ce^{rT}+p(p_4+p_8)\right)\leq 0.$$

Equivalently, we can state that if

$$1-p_4-p_8\geq\hat{p}$$

then $y(\theta)=0$ for all $\theta\geq 0$. Moreover, the optimal solution is $y(\theta)=M$ if $g(M,\theta)\geq 0$; or

$$g(M,\theta)=\left(p_2+p_4+p_6+p_8\right)\left(-ce^{rT}+p(p_4+p_8)\right)-2\theta Md\geq 0.$$

Equivalently, we can state that if

$$\theta\leq\frac{\left(\hat{p}-\left(1-p_4-p_8\right)\right)p(p_2+p_4+p_6+p_8)}{2Md}$$

then $y(\theta) = M$.

In conclusion, we can write

$$y(\theta) = \begin{cases} 0 & 1 - p_4 - p_8 \geq \hat{p} \\ \frac{(\hat{p}-(1-p_4-p_8))p(p_2+p_4+p_6+p_8)}{2\theta d} & 1 - p_4 - p_8 < \hat{p} \text{ and} \\ & \quad \frac{(\hat{p}-(1-p_4-p_8))p(p_2+p_4+p_6+p_8)}{2Md} < \theta < +\infty \\ M & 1 - p_4 - p_8 < \hat{p} \text{ and } \theta \leq \frac{(\hat{p}-(1-p_4-p_8))p(p_2+p_4+p_6+p_8)}{2Md} \end{cases}.$$

The similar remarks of the previous example are valid. The optimal order quantity depends on the probability distribution of demand and capacity, net payoff of asset and the risk-aversion parameter θ. The optimal order quantity depends on the probability

$$1 - P\{D = M, K = M\} = 1 - p_4 - p_8$$

and increases from 0 to M as θ decreases from $+\infty$ to

$$\frac{(\hat{p} - (1 - p_4 - p_8))\,(p_2 + p_4 + p_6 + p_8)p}{2Md}.$$

MV Model with Random Yield and Capacity

Last, we consider supply randomness in both yield and capacity. For our example, demand, $f(S)$, yield and capacity take the following values, $D \in \{0, M\}$, $f(S) \in \{-L, L\}$, $U \in \{0, N\}$ and $K \in \{0, M\}$. Their joint distribution function is

$f(S) = -L,\ D = 0,\ U = 0,\ K = 0$	with probability p_1
$f(S) = -L,\ D = 0,\ U = 0,\ K = M$	with probability p_2
$f(S) = -L,\ D = 0,\ U = N,\ K = 0$	with probability p_3
$f(S) = -L,\ D = 0,\ U = N,\ K = M$	with probability p_4
$f(S) = -L,\ D = M,\ U = 0,\ K = 0$	with probability p_5
$f(S) = -L,\ D = M,\ U = 0,\ K = M$	with probability p_6
$f(S) = -L,\ D = 0,\ U = N,\ K = 0$	with probability p_7
$f(S) = -L,\ D = 0,\ U = N,\ K = M$	with probability p_8
$f(S) = L,\ D = 0,\ U = 0,\ K = 0$	with probability p_9
$f(S) = L,\ D = 0,\ U = 0,\ K = M$	with probability p_{10}
$f(S) = L,\ D = 0,\ U = N,\ K = 0$	with probability p_{11}
$f(S) = L,\ D = 0,\ U = N,\ K = M$	with probability p_{12}
$f(S) = L,\ D = M,\ U = 0,\ K = 0$	with probability p_{13}
$f(S) = L,\ D = M,\ U = 0,\ K = M$	with probability p_{14}
$f(S) = L,\ D = M,\ U = N,\ K = 0$	with probability p_{15}
$f(S) = L,\ D = M,\ U = N,\ K = M$	with probability p_{16}

Note that we make our calculations based on the fact that the newsvendor orders at most M units which is logical because demand can be at most M units. The optimal portfolio for a single asset with random yield and capacity in (4.28) is updated as

$$\alpha^*(y) = ce^{rT}\beta_{U,K}(y) - p\beta_{D,U,K}(y)$$

for any y where

$$\beta_{U,K}(y) = \frac{Cov\left(f(S), U\min\{K,y\}\right)}{Var(f(S))},$$

$$\beta_{D,U,K}(y) = \frac{Cov(f(S), \min\{D, UK, Uy\})}{Var(f(S))}.$$

It follows that

$$\begin{aligned} Cov\left(f\left(S\right), U\min\left\{K,y\right\}\right) &= E\left[f\left(S\right)U\min\left\{K,y\right\}\right] \\ &= p_4\left(-L\right)Ny + p_8\left(-L\right)Ny + p_{12}\left(L\right)Ny + p_{16}\left(L\right)Ny \\ &= \left(p_{12}+p_{16}-p_4-p_8\right)LNy \end{aligned}$$

and

$$\begin{aligned} Cov\left(f\left(S\right), \min\left\{D, UK, Uy\right\}\right) &= E\left[f\left(S\right)\min\left\{D, K, y\right\}\right] \\ &= p_8\left(-L\right)Ny + p_{16}\left(L\right)Ny \\ &= \left(p_{16} - p_8\right)LNy. \end{aligned}$$

Hence, the optimal portfolio is

$$\alpha^*\left(y\right) = \left(ce^{rT}\left(p_{12} + p_{16} - p_4 - p_8\right) - p\left(p_{16} - p_8\right)\right)\left(\frac{N}{L}\right)y.$$

The optimality condition in (4.31) can be updated as

$$g(y,\theta) = \left(-ce^{rT}(p_4 + p_8 + p_{12} + p_{16}) + pp_{16}\right)N - 2\theta yd = 0 \tag{5.34}$$

where

$$d = N^2\left\{\begin{array}{c} \left(ce^{rT}\left(p_6 + p_8 - p_2 - p_4\right) - p\left(p_8 - p_4\right)\right)^2 \\ +(ce^{rT})^2(p_4 + p_8 + p_{12} + p_{16})(1 - p_4 - p_8 - p_{12} - p_{16}) \\ +p^2 p_{16}(1 - p_{16})N \\ -2ce^{rT}pp_{16}(1 - p_{16}) \end{array}\right\}.$$

The derivative of (5.34) is

$$\frac{dg(y,\theta)}{dy} = -2\theta d$$

Provided that $d \geq 0$, the second order condition is satisfied and the optimal order quantity can be obtained from (5.34) which is

$$\begin{aligned} y(\theta) &= \frac{\left(-ce^{rT}(p_4 + p_8 + p_{12} + p_{16}) + pp_{16}\right)N}{2\theta d} \\ &= \frac{\left(\hat{p}\left(p_4 + p_8 + p_{12} + p_{16}\right) - \left(p_4 + p_8 + p_{12}\right)\right)pN}{2\theta d}. \end{aligned}$$

The optimal solution is $y(\theta) = 0$ if $g(0,\theta) \leq 0$; or

$$g(0,\theta) = \left(-ce^{rT}(p_4 + p_8 + p_{12} + p_{16}) + pp_{16}\right)N \leq 0.$$

Equivalently, we can conclude that if

$$\frac{p_4 + p_8 + p_{12}}{p_4 + p_8 + p_{12} + p_{16}} \geq \hat{p}$$

then $y(\theta) = 0$ for all $\theta \geq 0$. Moreover, the optimal solution is $y(\theta) = M$ if $g(M,\theta) \geq 0$; or

$$g(M,\theta) = \left(-ce^{rT}(p_4+p_8+p_{12}+p_{16}) + pp_{16}\right)N - 2\theta Md \geq 0.$$

Equivalently, we can conclude that if

$$\theta \leq \frac{\left(\hat{p}\left(p_4+p_8+p_{12}+p_{16}\right) - \left(p_4+p_8+p_{12}\right)\right)pN}{2Md}$$

then $y(\theta) = M$.

In conclusion, we can write

$$y(\theta) = \begin{cases} 0 & \frac{p_4+p_8+p_{12}}{p_4+p_8+p_{12}+p_{16}} \geq \hat{p} \\ \frac{(\hat{p}(p_4+p_8+p_{12}+p_{16})-(p_4+p_8+p_{12}))pN}{2\theta d} & \frac{p_4+p_8+p_{12}}{p_4+p_8+p_{12}+p_{16}} < \hat{p} \text{ and} \\ & \frac{(\hat{p}(p_4+p_8+p_{12}+p_{16})-(p_4+p_8+p_{12}))pN}{2Md} < \theta < +\infty \\ M & \frac{p_4+p_8+p_{12}}{p_4+p_8+p_{12}+p_{16}} < \hat{p} \text{ and} \\ & \theta \leq \frac{(\hat{p}(p_4+p_8+p_{12}+p_{16})-(p_4+p_8+p_{12}))pN}{2Md} \end{cases}.$$

We again state that the optimal order quantity depends on the probability distribution of demand, yield and capacity, net payoff of asset and the risk-aversion parameter θ. The optimal order quantity depends on the probability

$$P\{D=0 \mid U=N, K=M\} = \frac{p_4+p_8+p_{12}}{p_4+p_8+p_{12}+p_{16}}$$

and increases from 0 to M as θ decreases from $+\infty$ to

$$\frac{\left(\hat{p}\left(p_4+p_8+p_{12}+p_{16}\right) - \left(p_4+p_8+p_{12}\right)\right)pN}{2Md}.$$

Up to this point, we consider a simple example to analyze the effect of some parameters on the optimal order quantity. In the next section, we use the Monte Carlo method to simulate the models.

5.2 *Simulation*

In this section, we present numerical examples to quantify the effects of the MV framework and financial hedging to compensate for demand and supply risks. As the base scenario, we take the example in Gaur and Seshadri [2005] where they use a stock to hedge the demand risk. Let the initial stock price S_0 be \$660 and the interest rate be $r = 10\%$ per year. We assume that $T = 6$ months and the return S_T/S_0 has a lognormal distribution under the risk-neutral measure with mean $\left(r - \frac{\sigma^2}{2}\right)T$ and standard deviation $\sigma\sqrt{T}$ where $\sigma = 20\%$ per year, that is

$$\ln\left(\frac{S_T}{S_0}\right) \sim N\left(\left(r - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right) = N\left(0.04, 0.14142\right). \tag{5.35}$$

Let the demand be $D = b\,S + \epsilon$ where $b = 10$, $S = S_T$ and ϵ has a normal distribution with mean zero and standard deviation σ_ϵ. Therefore, random demand is linearly correlated with the financial market. The financial parameters are: $p = 1$, $u = 0.7$, $c = 0.6$ and $s = 0.1$.

In our analysis, we consider three types of financial portfolios. The first portfolio consists of futures only and has the net payoff $f_1(S)$, the second portfolio consists of the call option with some strike price κ only and has the net payoff $f_2(S)$. Finally, the third portfolio uses both instruments jointly and has the net payoffs $f_1(S)$ and $f_2(S)$. The payoff of these derivative securities are

$$f_1\left(S\right) = S - e^{rT}S_0$$

and

$$f_2\left(S\right) = \max\left\{S - \kappa, 0\right\} - e^{rT}C$$

where the strike price is $\kappa = y/b$ and the price of the call option at time 0 is C. Under the assumption that there exists no arbitrage opportunity in the market which means that the expected gain from the financial portfolio is 0 we have

$$C = E_Q\left[e^{-rT}\max\left\{S - \kappa, 0\right\}\right]$$

and

$$E_Q\left[f_1\left(S\right)\right] = E_Q\left[f_2\left(S\right)\right] = 0$$

when Q is the risk-neutral probability measure given in (5.35).

Throughout the remainder of this chapter all of the numerical calculations are done using Monte Carlo simulations via Matlab. The cash flows are generated by using the simulated values of S, D, U, and K whenever needed. We consider four different models: first the MV model, second we include random yield, third we include random capacity and last we include both random yield and capacity. For each model, we will compare the following eight different scenarios:

Scenario 1:	Newsvendor does not use any portfolio and aims to maximize expected cash flow $(\theta = 0)$,
Scenario 2:	Newsvendor uses the first portfolio (future) and aims to maximize expected cash flow $(\theta = 0)$,
Scenario 3:	Newsvendor uses the second portfolio (call option) and aims to maximize expected cash flow $(\theta = 0)$,
Scenario 4:	Newsvendor uses the third portfolio (future and call option) and aims to maximize expected cash flow $(\theta = 0)$,
Scenario 5:	MV newsvendor does not use any portfolio and aims to maximize the MV cash flow,
Scenario 6:	MV newsvendor uses the first portfolio (future) and aims to maximize the MV cash flow,
Scenario 7:	MV newsvendor uses the second portfolio (call option) and aims to maximize the MV cash flow,
Scenario 8:	MV newsvendor uses the third portfolio (future and call option) and aims to maximize the MV cash flow.

5.2.1 MV Model

In this subsection, we analyze the case where demand is the only source of uncertainty. The unhedged cash flow at time T is written as

$$CF(D,y) = -ce^{rT}y + pD + s\max\{y-D,0\} - u\max\{D-y,0\}$$

and the hedged cash flow at time T is

$$CF_{\boldsymbol{\alpha}^*(y)}(D,S,y) = CF(D,y) + \boldsymbol{\alpha}^*(y)\,\mathbf{f}(S).$$

We first suppose that the standard deviation of demand error is $\sigma_\epsilon = 0$ so that a perfect correlation between demand and the stock price exits and the risk-aversion parameter is $\theta = 0.01$. We run our simulation for different order quantities and generate $50,000$ instances to calculate the stock prices, demand quantities and payoff of the derivative securities. Then, we use the formulas obtained in Theorem 4.0.6 and Corollary 4.0.8 of Chapter 4 to calculate the optimal portfolios. Finally, we generate another $50,000$ instances to obtain profits. For each scenario, we calculate the mean, the variance, and the MV value of the cash flow for each optimal order quantity.

$\sigma_\epsilon = 0$	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
S1	5804	2457.38	125412.62	—	—
S2	5802	2456.68	7150.09	—	-3.4823
S3	5800	2455.86	18470.47	—	-3.5538
S4	5804	2458.14	0	—	$-9.00, 6.00$
S5	4839	2416.35	88988.72	1526.46	—
S6	5263	2440.85	1252.58	2428.32	-3.1106
S7	5070	2429.72	1235.52	2417.36	-3.0688
S8	5804	2458.14	0	2458.14	$-9.00, 6.00$

Table 5.2: The means and variances of the cash flows, MV values and the optimal portfolios for random demand model when the standard deviation of demand error is 0

For scenarios 1-4, we decide on the optimal order quantity values based on the mean of the cash flows. Therefore, for these scenarios we consider the risk-neutral newsvendor. For scenarios 5-9, we find the optimal order quantities of a risk-averse newsvendor adopting MV strategy with $\theta = 0.01$. The results of our analysis are depicted in Table 5.2. For scenarios 1-4, since the expected values of the cash flows are almost the same, we can make fair comparisons on risk reduction enabled by financial hedging looking at the variances. For scenarios 5-8, we should compare the MV values. We see that when the standard deviation of the demand error is zero, hedging with a portfolio of future and option eliminate the variance of the cash flow totally for both the risk-neutral and MV newsvendor. Moreover, financial hedging enables considerable increases in the value of the MV objective. For example, consider the case when both portfolios are used. The financial hedging provides increase in the MV value of the cash flow by 61% for the MV newsvendor. Then, we analyze the effect of the MV objective by comparing scenario 1 and scenario 5. The risk-averse newsvendor orders less and so the expected profit is also less. However, the variance of the expected cash flow is reduced by 29%.

Moreover, we analyze the problem for different demand variations. We run the simulation by changing order quantity values when σ_ϵ is 300 and 600. The results are summarized in Table 5.3 when $\sigma_\epsilon = 300$ in which case a high degree of correlation between demand and the stock price exits, and later in Table 5.4 when $\sigma_\epsilon = 600$ in which case a lower degree of a correlation between demand and the stock price exists.

$\sigma_\epsilon = 300$	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
S1	5742	2451.26	139461.28	—	—
S2	5740	2450.55	20552.52	—	-3.4916
S3	5736	2449.74	29809.55	—	-3.5601
S4	5744	2451.94	14854.75	—	$-8.9135, 5.8281$
S5	4666	2404.47	97418.62	1430.29	—
S6	5027	2426.23	9731.94	2328.91	-3.0764
S7	4922	2419.64	9580.85	2323.82	-3.0621
S8	5179	2435.42	9850.49	2336.91	$-9.3145, 6.2787$

Table 5.3: The means and variances of the cash flows, MV values and the optimal portfolios for random demand model when the standard deviation of demand error is 300

When the standard deviation of the demand error is small ($\sigma_\epsilon = 300$) indicating a high degree of correlation between demand and the stock price, a decrease of 89.3% in the variance of the cash flow in the classical model and an increase of 63.4% in the MV value is achieved. For $\sigma_\epsilon = 600$, the variance of the cash flow can be lowered by 68.6% for the classical model while the MV value can be increased by 77.7%. Therefore, we conclude that the variance reductions decrease when the degree of correlation between the random demand and financial variables decrease.

We also analyze the effect of risk-aversion parameter θ on the optimal order quantity. We again take the same example where $\sigma_\epsilon = 600$. Table 5.5 depicts the optimal order quantities, mean, variance and MV value of the cash flows and the optimal portfolios. From Table 5.5, we conclude that as risk-aversion increases, the optimal order quantity decreases. Moreover, from the variances of the cash flows, we can state that hedging always reduces the variation and increases the MV value of the cash flow.

As for the optimal portfolio structure, it is always optimal to sell the future since in our example demand and stock price are taken to be positively correlated. However, in the optimal portfolio, the call option is bought when used as the second instrument along with the future, but is sold when it is used as the only instrument. Comparing the variance values for scenarios 1-4 and MV values for scenarios 5-8, it is also interesting to note that using a portfolio consisting only of the future is much more effective than the call option itself. Hence, we conclude that the call option serves to fine tune the portfolio along with

$\sigma_\epsilon = 600$	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
S1	5588	2435.22	181262.37	—	—
S2	5587	2434.51	60560.87	—	-3.5191
S3	5583	2433.72	66289.60	—	-3.5761
S4	5587	2435.82	56965.91	—	$-8.9780, 5.7328$
S5	4225	2374.08	122123.91	1152.84	—
S6	4493	2390.76	34399.51	2046.77	-3.046
S7	4477	2389.35	34341.32	2045.94	-3.0447
S8	4493	2392.61	34310.28	2049.51	$-16.8198, 13.7806$

Table 5.4: The means and variances of the cash flows, MV values and the optimal portfolios for random demand model when the standard deviation of demand error is 600

the future but is not as effective when used alone.

5.2.2 MV Model with Random Yield

This subsection presents an example for the random yield model. There can be many functional forms of constructing a relation between the stock price and the yield and in our example we take

$$U = 1 - e^{-(1/S_0)(\gamma+S_T)}$$

where γ is a normally distributed random variable with mean zero and standard deviation σ_γ. We apply the same base scenario and use identical portfolio options. We fix σ_ϵ to 600 and θ to 0.01 and for different values of σ_γ $(0, 200, 400)$, we find the optimal order quantities for each scenario and analyze the effect of the MV approach and financial hedging. We present the means, variations, MV values and the optimal portfolios in Table 5.6, 5.7 and 5.8.

The classical newsvendor using both financial instruments achieves considerable variance reductions: 68.7% when $\sigma_\gamma = 0$, 68.1% when $\sigma_\gamma = 200$ and 64.5% when $\sigma_\gamma = 400$. For the MV newsvendor the increases in the MV values are: 100% when $\sigma_\gamma = 0$, 105% when $\sigma_\gamma = 200$, 107% when $\sigma_\gamma = 400$. Again, we observe that for the classical model, the reductions in variance terms decreases as σ_γ increases.

Then, for the same example, we vary the standard deviations of yield and demand errors, σ_γ and σ_ϵ, together. Tables 5.9 and 5.10 report the results of this experiment. We

	θ	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
S5	0.002	4796	2409.32	130766.35	2147.78	—
	0.010	4225	2374.08	122123.91	1152.84	—
	0.015	4094	2365.34	121409.15	544.20	—
S6	0.002	5046	2421.10	41524.70	2338.05	-3.1785
	0.010	4493	2390.76	34399.51	2046.77	-3.046
	0.015	4357	2382.11	33683.70	1876.86	-3.0315
S7	0.002	5002	2418.66	41219.73	2336.22	-3.1719
	0.010	4477	2389.35	34341.32	2045.94	-3.0447
	0.015	4345	2380.93	33648.57	1876.20	-3.0308
S8	0.002	5092	2424.16	41754.74	2340.65	$-10.1498, 7.0222$
	0.010	4493	2392.61	34310.28	2049.51	$-16.8198, 13.7806$
	0.015	4350	2384.16	33606.50	1880.06	$-21.6067, 18.5793$

Table 5.5: The means and variances of the cash flows, MV values and the optimal portfolios for different degrees of risk-aversion when the standard deviation of demand error is 600

conclude that for the classical model using both instruments when the standard deviations are smaller, ($\sigma_\gamma = \sigma_\epsilon = 200$) the variance reduction is 89.6%, when we increase the standard deviation, ($\sigma_\gamma = \sigma_\epsilon = 400$) we again obtain variance reductions but less than before, 74%. For the MV newsvendor, MV value can be improved by 82.5% and 89.6%, respectively. As for the optimal portfolio structure, the same remarks of the previous subsection are valid.

5.2.3 MV Model with Random Capacity

In this subsection, we again apply the same base example, in addition, we assume the following relationship between the strike price and the capacity, $K = k\, S_T + \eta$ where $k = 9$ and η has a normal distribution with mean zero and standard deviation σ_η. We fix θ to 0.01. Firstly, we consider the case that there is ample demand in the market with respect to the capacity. (i.e., $P\{D > K\} = 1$) and there exists perfect correlation between the demand and the stock also the capacity and stock ($\sigma_\epsilon = \sigma_\eta = 0$). From Table 5.11, it can be seen that a portfolio consisting of the future is extremely effective and removes all the variance.

$\sigma_\gamma = 0$	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
S1	9358	2473.81	184519.25	—	—
S2	9357	2473.51	57652.53	—	-3.6167
S3	9370	2473.45	175667.82	—	-11.9629
S4	9357	2473.54	57579	—	$-3.6443, 1.0804$
S5	7331	2409.53	136736.08	1042.16	—
S6	7924	2434.52	34360.08	2090.92	-3.3120
S7	5442	2332.02	33935.90	1992.66	-3.3222
S8	7925	2434.59	3434.88	2091.10	$-3.3473, 0.1634$

Table 5.6: The means and variances of the cash flows, MV values and the optimal portfolios when the standard deviation of demand error is 600 and yield error is 0

Next, we consider imperfect correlations with the market and take $\sigma_\epsilon = \sigma_\eta = 600$. The results are depicted in Table 5.12. Hedging enables 66.7% reduction in the variance for the classical newsvendor and 77.8% increase in the MV value for the risk-averse newsvendor. We observe that when the correlations between the demand and the market, and the capacity and the market weaken the variance reduction is less but still considerable.

5.2.4 MV Model with Random Yield and Capacity

Lastly, we analyze the combination of random yield and capacity models. The same base scenario and the identical portfolio options are taken as before. We fix the standard deviations as $\sigma_\epsilon = 600$, $\sigma_\gamma = 400$ and $\sigma_\eta = 600$ and $\theta = 0.01$. Then, we calculate the optimal order quantities and optimal portfolios for each scenario. The resulting means, variances and MV values are summarized in Table 5.13. Again, we conclude that the effect of financial hedging is significant. For example, for the classical newsvendor 72,9% reduction in the variance of the cash flow and 108% increase in the MV objective is achieved. For the optimal portfolio structure, interestingly, the risk-neutral newsvendor using both instruments sells both the future and call option. Other than that for the optimal portfolio the same remarks of the previous subsection are valid.

$\sigma_\gamma = 200$	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
S1	8534	2421.38	190072.73	—	—
S2	8534	2421.08	60676.46	—	-3.6536
S3	8534	2420.33	162224.14	—	-8.89
S4	8535	2421.17	60503.69	—	$-3.7373, 0.8777$
S5	6046	2345.15	136633.23	978.82	—
S6	6850	2378.39	36992.50	2008.46	-3.2902
S7	5518	2321.63	35604.23	1965.59	-3.3066
S8	6859	2378.77	36995.36	2008.82	$-3.4080, 0.1979$

Table 5.7: The means and variances of the cash flows, MV values and the optimal portfolios when the standard deviation of demand error is 600 and yield error is 200

$\sigma_\gamma = 400$	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
S1	7635	2356.65	201542.92	—	—
S2	7634	2356.35	71829.76	—	-3.6572
S3	7635	2355.60	138718.91	—	-6.2641
S4	7635	2355.60	71420.40	—	$-3.8882, 0.7718$
S5	1758	2151.68	125712.11	894.55	—
S6	4928	2277.68	41894.44	1858.74	-3.2221
S7	4737	2269.67	41206.62	1857.60	-3.2188
S8	4942	2278.35	41944.76	1858.90	$-4.0119, 0.7931$

Table 5.8: The expected values of the means and variances of the cash flows, MV values and the optimal portfolios when the standard deviation of demand error is 600 and yield error is 400

$\sigma_\gamma = \sigma_\epsilon$	Scenario	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
200	S1	8776	2437.59	143961.61	—	—
	S2	8775	2437.29	15037.13	—	-3.6463
	S3	8782	2436.69	123472.26	—	-9.7065
	S4	8776	2437.37	14880.48	—	$-3.7108, 0.97$
	S5	6809	2378.56	109860.26	1279.96	—
	S6	7871	2420.13	8414.12	2335.99	-3.3717
	S7	5541	2322.78	6695.38	2255.83	-3.3112
	S8	7884	2420.60	8403.88	2336.56	$-3.4417, 0.2992$

Table 5.9: The means and variances of the cash flows, MV values and the optimal portfolios when the standard deviations of demand error and yield error vary together

$\sigma_\gamma = \sigma_\epsilon$	Scenario	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
400	S1	7746	2363.68	175525.54	—	—
	S2	7745	2363.38	45920.84	—	-3.6558
	S3	7740	2362.59	117400.16	—	-6.5040
	S4	7746	2363.49	45502.87	—	$-3.8692, 0.8043$
	S5	1759	2151.77	107587.25	1075.89	—
	S6	5126	2285.67	24551.80	2040.15	-3.2289
	S7	4796	2272.10	23344.38	2038.66	-3.2215
	S8	5115	2285.29	24504.13	2040.25	$-3.6892, 0.4655$

Table 5.10: The means and variances of the cash flows, MV values and the optimal portfolios when the standard deviations of demand error and yield error vary together

$\sigma_\epsilon = \sigma_\eta$	Scenario	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
0	S1	11481	2514.17	127241.73	—	—
	S2	11481	2513.87	0	—	-3.6230
	S3	11006	2514.19	126406.07	—	-19.5297
	S4	11006	2514.19	0	—	$-3.6231, 0.0721$
	S5	5127	2434.74	89508.94	1539.65	—
	S6	11481	2513.87	0	2513.87	-3.6230
	S7	5445	2458.91	1740.30	2441.51	-3.1903
	S8	11481	2513.87	0	2513.87	$-3.6231, 0$

Table 5.11: The means and variances of the cash flows, MV values and the optimal portfolios when demand is ample (D>K) and capacity is perfectly correlated with the stock

$\sigma_\epsilon = \sigma_\eta$	Scenario	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
600	S1	6763	2459.45	196364.74	—	—
	S2	6763	2459.14	65799.67	—	-3.67
	S3	6751	2458.57	92421.42	—	-4.6963
	S4	6751	2458.57	65330.44	—	$-4.0452, 0.5955$
	S5	4235	2373.52	122076.93	1152.75	—
	S6	4651	2398.97	35002.39	2048.94	-3.0655
	S7	4651	2398.96	35001.73	2048.95	-3.0737
	S8	4675	2400.39	35071.04	2050.52	$-6.1532, 3.0841$

Table 5.12: The means and variances of the cash flows, MV values and the optimal portfolios when the standard deviations of demand error and capacity error are 600

Scenario	$y(\theta)$	Mean	Variance	MV	Portfolio(α)
S1	11986	2334.22	177934.12	—	—
S2	11986	2334.17	48078.53	—	-3.6490
S3	11562	2334.25	177684.12	—	-27.6521
S4	10796	2334.18	48076.41	—	$-3.6478, -0.9086$
S5	1653	2148.33	125658.85	891.74	—
S6	5171	2285.74	42676.47	1858.97	-3.2657
S7	4710	2268.64	41116.54	1857.47	-3.23
S8	5192	2286.54	42739.68	1859.15	$-3.8325, 0.5727$

Table 5.13: The means and variances of the cash flows, MV values and the optimal portfolios when the standard deviations of demand error is 600, capacity error is 600 and yield error is 400

Chapter 6

CONCLUSIONS

This thesis focuses on the single-period, single-item inventory problem when the decision-maker (newsvendor) is risk-averse. We use the MV framework to model risk-aversion. In our models, we deal with the risks or uncertainties that result from both random demand and random supply. Our thesis consists of two parts. In the first part, we deal with the newsvendor models with random supply when the aim is to maximize the MV objective. In the second part, we study the same model when the randomness in demand and supply is correlated with the financial markets.

In the first part, we consider the MV newsvendor problem considering random demand as well as random supply based on random yield and random capacity. In all cases, we find the optimal order quantity for each risk-aversion level. Therefore, our results present tailor-fit solutions to every newsvendor with different risk-attitudes. However, the characterizations require certain properties and assumptions on the structure of the objective function. For the random demand case, the objective function is quasi-concave, so we find explicit characterizations for the optimal order quantity. For other cases, the existence of the solution require certain assumptions. For random yield case, we need the variance function to be nondecreasing and convex on the non-dominated region. For random capacity, and random yield and capacity the assumption on the variance function is the same. Additionally for these cases, we establish the quasi-concavity of the mean function with certain assumptions. Then, we can state the quasi-concavity of the MV objective function. For all, we discuss the effect of risk-aversion on the optimal order quantity.

In the second part, we further suppose that the randomness in demand and supply is correlated with the financial markets. The newsvendor hedges demand and supply risks investing in a portfolio composed of various financial instruments to mitigate inventory risks. In this problem, not only the optimal order quantity but also the optimal financial portfolio is decided in only one step. We analyze two types of portfolios: one consisting of a single asset and another one with multiple assets, for random demand and supply cases. For all cases, we establish quasi-concavity of the objective function and then we find explicit characterizations for the optimal order quantities and optimal portfolios. However, the existence of the optimal order quantities in these cases require certain assumptions.

In the numerical part, we provide some illustrative examples. The effects of risk-aversion on the optimal order quantities are examined. Moreover, we also analyze the effect of MV approach and the financial hedging on the variance function. We show that the optimal order quantity to the MV problem is less than the optimal order quantity of the classical newsvendor problem. Furthermore, the more risk-averse the newsvendor is, the smaller his order. We also observe that the optimal order quantity depends on both the demand and supply uncertainties. We further conclude that the financial hedging reduces the variance of the problem significantly and increases MV value.

This research can be extended in several directions. Firstly, it is theoretically possible to extend the MV hedging model using a vector of financial securities. For example, the demand can be correlated with one derivative security and the capacity can be correlated with another derivative security etc.. Secondly, although we consider the newsvendor's cash flow, the MV approach we studied in this thesis is applicable for any problem where the mean function is concave and the variance function is increasing. Other possible extensions are multi-period, infinite-period, continous time inventory models or multi-product models adopting MV strategy. In addition, Bayesian models, random environment models and hidden Markov models are other suitable areas for extensions.

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