

DYNAMICS of OPTICAL SOLITON - SURFACE  
PLASMON in DIELECTRIC - METAL and METAL -  
DIELECTRIC - METAL INTERFACES

by

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## ABSTRACT

Plasmonics is a research field on the physics and applications of the interaction of light with surface-plasmons on mostly- metal surfaces. From a naive point of view, there is electronics on one side, which can perform well on nano-scale structures but the data process and transfer is very slow compared to optical frequencies. On the other side, optical data transfer is ultrafast e.g. in fibers but their size is down-limited by the wavelength of light which is usually much larger than the nanoscale. Thus, plasmonics establishes a link between electronics and photonics by coupling light and surface plasmons at very small scales, at optical frequencies. This has the potential to develop surface-plasmon-integrated optoelectronic devices at nanoscales. In this respect, the excitation and control of surface-plasmons via coupling to controllable optical sources is highly desirable. In this thesis work, we investigate the dynamical properties of the interaction between an optical soliton in a nonlinear dielectric waveguide and co-propagating surface-plasmons along a metal surface. Due to the nonlinear nature of the optical soliton, the coupling parameter depends on the soliton amplitude, and thus become an inherently dynamical parameter rather than being a coupling constant. We first revisit the dynamics of the parallel system which is formulated as a Josephson junction by introducing fractional population imbalance and the relative phase variables. Then, we consider the interaction when the soliton is propagating on a parabolic trajectory with respect to the flat metal surface. Nearly full population conversion from optical soliton to surface plasmon is achievable under certain parameter values. We next investigate the dynamics in a metal/dielectric/Kerr/dielectric/metal multilayer system, in which the optical soliton is essentially coupled to two surface plasmons on either side across dielectric spacers. We found that for certain (asymmetric) spacing parameters the spatial profile of the



soliton is almost constant along the propagation. In a different multilayer configuration, we take the metal surfaces to form a parallel channel but with a parabolic opening with respect to the soliton axis. In this case, depending on the model parameters the oscillations in the spatial profile of the soliton may be induced or suppressed as it enters through the parabolic channel.

## ÖZETÇE

Plazmonik, fizikte ışın çoğunlukla metal yüzeylerde yüzey plazmonlarıyla etkileşim uygulamalarını içeren bir araştırma dalıdır. Genel bir bakış açısıyla bakarsak, bir tarafta nano-ölçeklerde iyi sonuçlar veren fakat veri akışı ve iletiminin optik frekanslara kıyasla çok yavaş olduğu elektronik, diğer tarafta ise optik verinin çok hızlı olduğu fakat boyutlarının nano-ölçekten daha büyük olan ışığın dalga boyu ile sınırlandığı fotonik bulunmaktadır. Plazmonik ise ışığı ve yüzey plazmonlarını optik frekanslar gibi çok küçük ölçeklerde çiftleştirerek elektronik ve fotonik arasında bir bağ kurar. Bu da yüzey plazmonlarıyla donanımlı nano-ölçekli optik cihazlar geliştirilmesine olanak sağlar. Bu açıdan bakıldığında yüzey plazmonlarının kontrol edilebilir optik kaynaklarla çiftleştirilerek uyarılması ve kontrol edilmesi amaçlanmaktadır. Bu tez çalışmasında nonlinear dielektrik dalga kılavuzunda yayılan optik soliton ile metal yüzeylerde yayılan yüzey plazmonlarının etkileşiminin dinamik özelliklerini inceliyoruz. Optik solitonun nonlinear doğasından kaynaklanan nedenlerle bağlama parametresi soliton genliği ile değişmektedir ve bunun bir sonucu olarak sabit bir etkileşim değerinden çok dinamik bir parametredir. Öncelikle fraksiyonel popülasyon oransızlığı ve bağlı faz terimlerini kullanarak paralel sistemin dinamiğini Josephson eklemi olarak inceledik. Daha sonra da düz metal bir yüzeye göre parabolik bir yörüngede yayılan solitonun etkileşimini göz önüne aldık. Bazı parametre değerlerinde optik solitondan yüzey plazmonuna neredeyse tam bir popülasyon transferi gerçekleştirdik. Daha sonra optik solitonun dielektriğin iki tarafındaki metal yüzeylerde yayılan yüzey plazmonlarıyla etkileştiği metal/dielektrik/Kerr/dielektrik/metal çok katmanlı sistemin dinamiğini inceledik. Bazı uzaklık parametresi değerlerinde solitonun uzaysal profilinin yayılım sırasında neredeyse sabit olduğunu bulduk. Farklı bir yapılandırmada da metal yüzeyleri ile soliton eksenindeki uzaklığı parabolik bir

tünel olarak oluşturduk ve bu durumda parametre değerlerine göre solitonun uzaysal profiline tünele girdikten sonra azaltılıp arttırılabildiğini gösterdik.

## Chapter 1

### INTRODUCTION

Surface plasmons are of interest to a wide spectrum of scientists, ranging from physicists, chemists and materials scientists to biologists. Renewed interest in surface plasmons comes from recent advances that allow metals to be structured and characterized on the nanometre scale. This has enabled us to control surface plasmon properties to reveal new aspects of their underlying science and some specific applications. For instance, surface plasmons are being explored for their potential in optics, magneto-optic data storage, microscopy and solar cells, as well as being used to construct sensors for detecting biologically interesting molecules. For researchers in the field of optics, one of the most attractive aspects of surface plasmons is the way in which they help us to concentrate and channel light using subwavelength structures. This could lead to miniaturized photonic circuits with length scales much smaller than those currently achieved. Such a circuit would first convert light into surface plasmons, which would then propagate and be processed by logic elements, before being converted back into light. Concentrating light in this way leads to an electric field enhancement that can be used to manipulate light-matter interactions and boost non-linear phenomena. The enhanced field associated with surface plasmons makes them suitable for use as sensors, and commercial systems have already been developed for sensing biomolecules. The interaction between the surface charges and the electromagnetic field that constitutes the surface plasmons has two consequences. First, the interaction between the surface charge density and the electromagnetic field results in

the momentum of the surface plasmon mode,  $\hbar k_{SP}$ , being greater than that of a free-space photon of the same frequency,  $\hbar k_0$ . ( $k_0 = \omega/c$ ) is the free-space wavevector.) The second consequence of the interaction between the surface charges and the electromagnetic field is that the field perpendicular to the surface decays exponentially with distance from the surface. The field in this perpendicular direction is said to be evanescent or near field in nature and is a consequence of the bound, non-radiative nature of surface plasmons [3], [2].

An optical soliton is a pulse that travels without distortion due to dispersion or other effects. They are a nonlinear phenomenon caused by self-phase modulation, which means that the electric field of the wave changes the index of refraction seen by the wave (Kerr effect). Self-phase modulation causes a red shift at the leading edge of the pulse. Solitons occur when this shift is canceled due to the blue shift at the leading edge of a pulse in a region of anomalous dispersion, resulting in a pulse that maintains its shape while propagating. Solitons are therefore an important development in the field of optical communications [1]. The word 'soliton' refers to highly stable localized solutions of certain nonlinear partial differential equations. The soliton concept has deeply penetrated into almost all branches of science wherever nonlinear partial differential equations are being used. This concept was first discovered in hydrodynamics in the 19th century [3]. Solitons have been extensively studied in many branches of physics such as optics, plasmas, condensed matter physics, fluid mechanics, particle physics and even astrophysics. Interestingly, over the past two decades, the field of solitons has been substantially advanced and enriched by research and discoveries in nonlinear optics. While optical solitons have been strongly investigated in both spatial and temporal domains, it can be said that much soliton research has been made on optical spatial solitons. This is partly due to the fact that although temporal solitons are fundamentally one-dimensional entities, the high dimensionality of their spatial counterparts has opened new scientific possibilities in soliton research. Another rea-

son is related to the nonlinearity. Unlike temporal optical solitons, spatial solitons employ a variety of noninstantaneous nonlinearities, ranging from the nonlinearities in photorefractive materials and liquid crystals to the nonlinearities mediated by the thermal effect, thermophoresis and the gradient force in colloidal suspensions. Such a diversity of nonlinear effects has given rise to numerous soliton phenomena, because for decades scientists were thinking that solitons must strictly be the exact solutions of the cubic nonlinear Schrodinger equation as established for ideal Kerr nonlinear media [4].

In this thesis work, we investigate the dynamical properties of the interaction between an optical soliton in a nonlinear dielectric waveguide and co-propagating surface-plasmons along a metal surface. Due to the nonlinear nature of the optical soliton, the coupling parameter depends on the soliton amplitude, and thus become an inherently dynamical parameter rather than being a coupling constant. We first revisit the dynamics of the parallel system which is formulated as a Josephson junction by introducing fractional population imbalance and the relative phase variables. Then, we consider the interaction when the soliton is propagating on a parabolic trajectory with respect to the flat metal surface. Nearly full population conversion from optical soliton to surface plasmon is achievable under certain parameter values. We next investigate the dynamics in a metal/dielectric/Kerr/dielectric/metal multilayer system, in which the optical soliton is essentially coupled to two surface plasmons on either side across dielectric spacers. We found that for certain (asymmetric) spacing parameters the spatial profile of the soliton is almost constant along the propagation. In a different multilayer configuration, we take the metal surfaces to form a parallel channel but with a parabolic opening with respect to the soliton axis. In this case, depending on the model parameters the oscillations in the spatial profile of the soliton may be induced or suppressed as it enters through the parabolic channel.

## Chapter 2

**SURFACE PLASMON-SOLITON INTERACTION IN A  
METAL DIELECTRIC INTERFACE****2.1 Surface Plasmons: Brief Theoretical Background***2.1.1 Dispersion Relation*

Surface plasmon polaritons are electromagnetic excitations propagating at the interface between a dielectric and a conductor, evanescently confined in the perpendicular direction. This strong confinement enables applications as surface plasmon resonance sensors or highly miniaturized photonic circuitry which represent a great interest for telecommunication, computing, and information processing. To study the plasmon propagation in nonlinear media, one should analyze nonlinear Maxwell's equations for the transverse magnetic waves in the presence of a metal-dielectric interface. The electromagnetic surface waves arise via the coupling of the electromagnetic fields to oscillations of the conductor's electron plasma. The wave equation should be taken as a starting point to describe the fundamentals of surface plasmon polaritons both at single, flat interfaces and in metal-dielectric multilayer structures. In the following, we proceed through the derivation of the dispersion relation of surface-plasmons starting from Maxwell's equations. This derivation can be found in many text books [5], [6].

$$\nabla \cdot D = \rho_{ext} \quad (2.1)$$

$$\nabla \cdot B = 0 \quad (2.2)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (2.3)$$

$$\nabla \times B = J_{ext} + \frac{\partial D}{\partial t} \quad (2.4)$$

Maxwell's equations have to be applied to the flat interface between a conductor and a dielectric. The curl equations of Eq.(1) can be combined, and the result is

$$\nabla \times \nabla \times E = -\mu_0 \frac{\partial^2 D}{\partial t^2} \quad (2.5)$$

Using identities  $\nabla \times \nabla \times E \equiv \nabla(\nabla \cdot E) - \nabla^2 E$  and  $\nabla \cdot (\epsilon E) \equiv E \cdot \nabla \epsilon + \epsilon \nabla \cdot E$  Eq.(2) can be rewritten as

$$\nabla \left( -\frac{1}{\epsilon} E \cdot \nabla \epsilon \right) - \nabla^2 E = -\mu_0 \epsilon_0 \epsilon \frac{\partial^2 E}{\partial t^2} \quad (2.6)$$

The first term of the left-hand side can be neglected due to the variation of the dielectric profile  $\epsilon = \epsilon(r)$  over distances on the order of one optical wavelength and the result is the wave equation itself:

$$\nabla^2 E - \frac{\epsilon}{c^2} \frac{\partial^2 E}{\partial t^2} = 0. \quad (2.7)$$

This equation has to be solved separately in regions and the results have to be matched using boundary conditions. Starting with an electric field with a harmonic time dependence  $E(r, t) = E(r)e^{-i\omega t}$ , the wave equation becomes the Helmholtz equation

$$\nabla^2 E + k_0^2 \epsilon E = 0 \quad (2.8)$$

To solve this equation a proper propagation geometry has to be defined. For the geometry in Fig.2.1 the electric field is  $E(x, y, z) = E(z)e^{i\beta x}$  and  $\epsilon = \epsilon(z)$ .

The complex parameter  $\beta = k_x$  is called the propagation constant of the traveling waves and corresponds the component of the wave vector in the direction of propa-



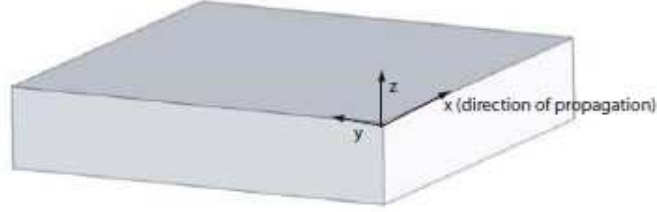


Figure 2.1: Definition of a planar waveguide geometry. The waves propagate along the x-direction in a cartesian coordinate system. [5]

gation. Using this equation one can obtain the desired form of the wave equation:

$$\frac{\partial^2 E(z)}{\partial z^2} + (k_0^2 \epsilon - \beta^2) E = 0. \quad (2.9)$$

This is the starting point for the general analysis of guided electromagnetic modes in waveguides and this equation can be expanded using curl equations. For harmonic time dependence ( $\frac{\partial}{\partial t} = -i\omega$ ), for propagation along the x-direction ( $\frac{\partial}{\partial x} = i\beta$ ) and for the homogeneity in the y-direction ( $\frac{\partial}{\partial y} = 0$ ) these identities is used.

For TM modes, the system reduces to

$$E_x = -i \frac{1}{\omega \epsilon_0 \epsilon} \frac{\partial H_y}{\partial z} \quad (2.10)$$

$$E_z = -\frac{\beta}{\omega \epsilon_0 \epsilon} H_y \quad (2.11)$$

and the wave equation for TM modes is

$$\frac{\partial^2 H_y}{\partial z^2} + (k_0^2 \epsilon - \beta^2) H_y = 0 \quad (2.12)$$

For TE modes, the system reduces to

$$H_x = -i \frac{1}{\omega \mu_0} \frac{\partial E_y}{\partial z} \quad (2.13)$$

$$H_z = -\frac{\beta}{\omega \mu_0} E_y \quad (2.14)$$

and the wave equation for TE modes is

$$\frac{\partial^2 E_y}{\partial z^2} + (k_0^2 \epsilon - \beta^2) E_y = 0 \quad (2.15)$$

This is the most simple geometry sustaining surface plasmon polaritons.  $z > 0$  region has a positive real dielectric constant  $\epsilon_2$  and  $z < 0$  region has a dielectric function  $\epsilon_1(\omega)$ . The metallic character of the lower region requires  $Re[\epsilon_1] < 0$ , and this condition is fulfilled at frequencies below the bulk plasmon frequency  $\omega_p$  where  $\omega_p^2 = \frac{ne^2}{\epsilon_0 m}$ . Here  $n$  is the number of electrons,  $e$  is the charge and  $m$  is the mass of a single electron.

The propagating TM wave solutions confined the this interface can be found by using the set of equations for both regions separately. For  $z > 0$  region

$$H_y(z) = A_2 e^{i\beta x} e^{-k_2 z} \quad (2.16)$$

$$E_x(z) = iA_2 \frac{1}{\omega \epsilon_0 \epsilon_2} k_2 e^{i\beta x} e^{-k_2 z} \quad (2.17)$$

$$E_z = -A_1 \frac{\beta}{\omega \epsilon_0 \epsilon_2} e^{i\beta x} e^{-k_2 z} \quad (2.18)$$

and for  $z < 0$  region

$$H_y(z) = A_1 e^{i\beta x} e^{k_1 z} \quad (2.19)$$

$$E_x(z) = -iA_1 \frac{1}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta x} e^{k_1 z} \quad (2.20)$$

$$E_z = -A_1 \frac{\beta}{\omega \epsilon_0 \epsilon_1} e^{i\beta x} e^{k_1 z} \quad (2.21)$$

$k_i \equiv k_{z,i}$  ( $i = 1, 2$ ) is the wave vector component for two regions, and its reciprocal value,  $\hat{z} = 1/|k_z|$ , defines the evanescent decay length of the fields perpendicular to the interface. Continuity of  $H_y$  and  $\epsilon_i E_z$  at the interface requires that  $A_1 = A_2$  and

$$\frac{k_2}{k_1} = -\frac{\epsilon_2}{\epsilon_1} \quad (2.22)$$

The surface waves exist only at interfaces between materials with opposite signs of the real part of their dielectric permittivities. Regarding this expression,  $H_y$  has to fulfill the wave equation, and this yields

$$k_1^2 = \beta^2 - k_0^2 \epsilon_1 \quad (2.23)$$

$$k_2^2 = \beta^2 - k_0^2 \epsilon_2 \quad (2.24)$$

Combining these equations the dispersion relation of surface plasmon polaritons propagating at the interface between the two half spaces can be obtained

$$\beta = k_0 \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}. \quad (2.25)$$

The TE wave solutions should also be discussed. Using Eq.(13), Eq.(14) and Eq.(15) the following can be obtain for  $z > 0$  region

$$E_y(z) = A_2 e^{i\beta x} e^{-k_2 z} \quad (2.26)$$

$$H_x(z) = -iA_2 \frac{1}{\omega_0} k_2 e^{i\beta x} e^{-k_2 z} \quad (2.27)$$

$$H_z = A_2 \frac{\beta}{\omega \mu_0} e^{i\beta x} e^{-k_2 z} \quad (2.28)$$

and for  $z < 0$  region

$$E_y(z) = A_1 e^{i\beta x} e^{k_1 z} \quad (2.29)$$

$$H_x(z) = iA_1 \frac{1}{\omega\mu_0} k_1 e^{i\beta x} e^{k_1 z} \quad (2.30)$$

$$E_z = A_1 \frac{\beta}{\omega\mu_0} e^{i\beta x} e^{k_1 z} \quad (2.31)$$

Continuity of  $E_y$  and  $H_x$  at the interface requires that  $A_1(k_1 + k_2) = 0$ . Since  $Re[k_1] > 0$  and  $Re[k_2] > 0$ , this condition is only fulfilled if  $A_1 = 0$ , so that  $A_1 = A_2 = 0$ , so for TE polarization surface modes cannot exist. In other words, surface plasmon polaritons only exist for TM polarization.

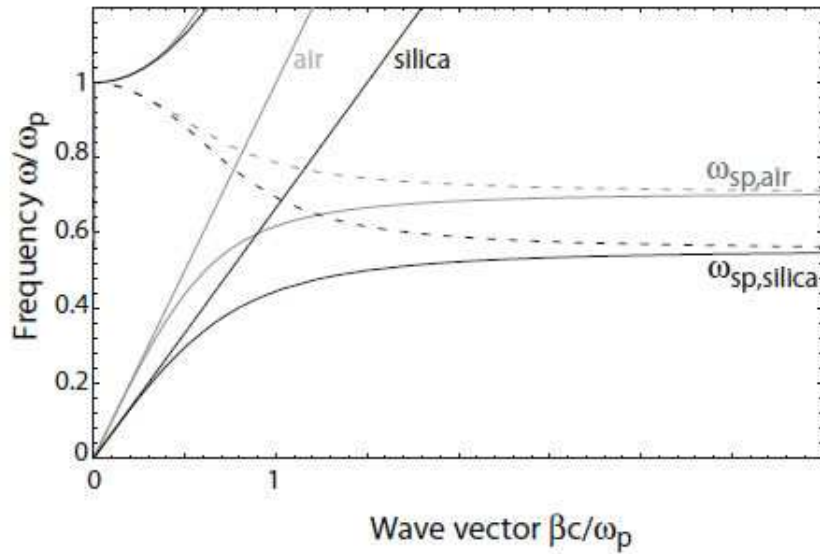


Figure 2.2: Dispersion relation  $\beta = k_0 \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}$  Eq.(2.25) of surface plasmon polaritons at the interface between a Drude metal with negligible collision frequency and air(gray curves) and silica(black curves) [5].

Fig.2.2 show the plot of the dispersion relation for a metal with negligible damping described by the dielectric function  $\epsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$  for an air ( $\epsilon_2 = 1$ ) and a fused silica ( $\epsilon_2 = 2.25$ ) interface. The frequency  $\omega$  is normalized to the plasma frequency  $\omega_p$ , and both the real and imaginary part of the wave vector  $\beta$  are shown. The surface

plasmon polariton excitations correspond to the part of the dispersion curves lying to the right of the respective light lines of air and silica.

In the regime of large wave vectors, the frequency of the surface plasmon polaritons approaches the characteristic surface plasmon frequency, and can be found by inserting  $\epsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$  into the dispersion relation

$$\omega_{sp} = \frac{\omega_p}{1 + \epsilon_2} \quad (2.32)$$

In the limit of negligible damping ( $Im[\epsilon_1] = 0$ ), the wave vector  $\beta$  goes to infinity as the frequency approaches  $\omega_{sp}$ . This mode is called the surface plasmon, which is indeed the limiting form of a surface plasmon polariton as  $\beta \rightarrow \infty$ .

## 2.2 Optical Solitons: Brief Theoretical Background

### 2.2.1 Spatial Solitons

Spatial solitons are optical beams that propagate in a nonlinear medium without diffraction, their beam diameter remains invariant during propagation. This invariance is one of the most important features of nonlinear optics, because the best-known characteristic of wave propagation is that beams which are finite in space tend to broaden due to diffraction effects. Breaking this paradigm requires a strong nonlinear interaction between the wave and the medium through which the beam is propagating. When this requirement is fulfilled, a self-trapped beam or a spatial soliton can form. A spatial soliton represents an exact balance between diffraction and nonlinearity induced self-focusing effects as shown in the Fig.2.3 [7].

In order to understand how spatial soliton can exist, some considerations about a simple convex lens should be made. As shown in the Fig.2.4, an optical field approaches the lens and then it is focused [9], [11].

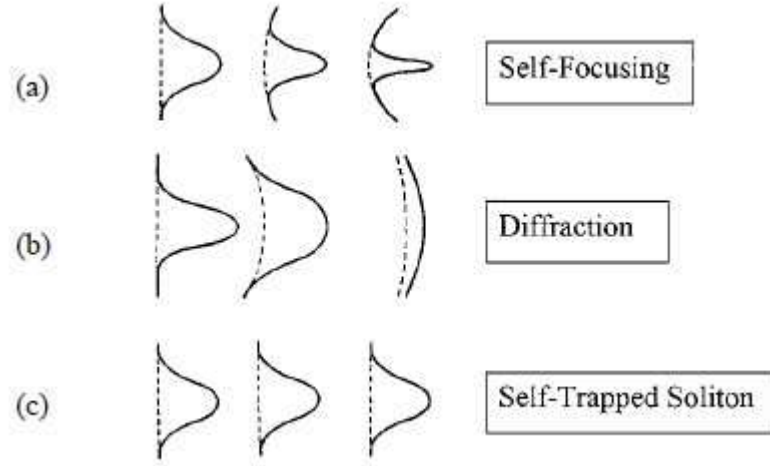


Figure 2.3: Schematic showing the spatial beam profiles (solid line) and phase fronts (dashed line) for (a) beam self-focusing, (b) normal beam diffraction, and (c) soliton propagation [7].

The effect of the lens is to introduce a non-uniform phase change that causes focusing. This phase change is a function of the space and can be represented with

$$\phi(x) = k_0 n L(x) \quad (2.33)$$

where  $L(x)$  is the width of the lens, changing in each point with a shape that is the same of  $\phi(x)$  because  $k_0$  and  $n$  are constants. In other words, in order to get a focusing effect a phase change of such a shape have to be introduced, but the width does not change [10]. If the width  $L$  is fixed in each point, but the value of the refractive index  $n(x)$  exactly the same effect with a completely different approach will occur. The change in the refractive index introduces a focusing effect that can balance the natural diffraction of the field. If the two effects balance each other perfectly, then a confined field propagating within the fiber occurs. That's the way graded-index fibers work, and spatial solitons are based on the same principle: the Kerr effect introduces a self-phase modulation that changes the refractive index according to the intensity

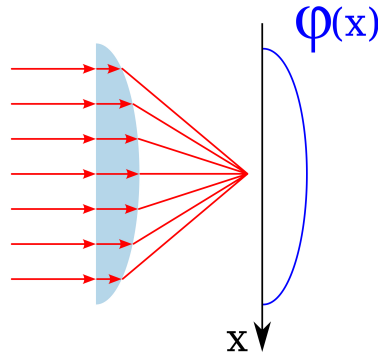


Figure 2.4: The focusing effect of a simple convex lens. The lens introduce a non-uniform phase change  $\phi$  that causes focusing [12].

[10]:

$$\phi(x) = k_0 n(x) L = k_0 L [n + n_2 I(x)] \quad (2.34)$$

The field creates a fiber-like guiding structure while propagating. If the field creates a fiber and it is the mode of such a fiber at the same time, it means that the focusing nonlinear and diffractive linear effect are perfectly balanced and the field will propagate forever without changing its shape. In order to have a self-focusing effect,  $n_2$  must be positive, otherwise nonlinear behavior will not be noticed. The optical waveguide the soliton creates while propagating is not only a mathematical model, but it actually exist and can be used to guide other waves at different frequencies. This way it is possible to let light interact with light at different frequencies as it is impossible in linear media.

An electric field is propagating in a medium showing optical Kerr effect, so the refractive index is given by

$$n(I) = n + n_2 I \quad (2.35)$$

where  $I = \frac{|E|^2}{2\eta}$  and  $\eta = \eta_0/\eta$  and  $\eta_0 = \sqrt{\mu_0/\epsilon_0} \approx 377\Omega$ . The field is propagating in the  $z$  direction with a phase constant  $k_0 n$ . Since the field is infinite in  $y$  direction, we

will ignore any dependence on  $y$  axis, and the field can be expressed as

$$E(x, z, t) = A_m a(x, z) e^{i(k_0 n z - \omega t)} \quad (2.36)$$

where  $A_m$  is the maximum amplitude of the field and  $a(x, z)$  is a dimensionless normalized function that represents the shape of the electric field along  $x$  axis [8]. Now we have to solve the Helmholtz equation  $\nabla^2 E + k_0^2 n^2(I) E = 0$ , and we assume that  $a(x, z)$  changes slowly while propagating, i.e.

$$\left| \frac{\partial^2 a(x, z)}{\partial z^2} \right| \ll \left| k_0 \frac{\partial a(x, z)}{\partial z} \right| \quad (2.37)$$

and the following equation is obtained:

$$\frac{\partial^2 a}{\partial x^2} + i 2 k_0 n \frac{\partial a}{\partial z} + k_0^2 [n^2(I) - n^2] a = 0 \quad (2.38)$$

After introducing an approximation that is valid because the nonlinear effects are always much smaller than linear ones:

$$[n^2(I) - n^2] = [n(I) - n][n(I) + n] = n_2 I (2n + n_2 I) \approx 2nn_2 I \quad (2.39)$$

then we express the intensity in terms of the electric field:

$$[n^2(I) - n^2] \approx 2nn_2 \frac{|A_m|^2 |a(x, z)|^2}{2\eta_0/n} = n^2 n_2 \frac{|A_m|^2 |a(x, z)|^2}{\eta_0} \quad (2.40)$$

the equation becomes:

$$\frac{1}{2} \frac{\partial^2 a}{\partial \xi^2} + i \frac{\partial a}{\partial \zeta} + N^2 |a|^2 a = 0 \quad (2.41)$$

We will now assume  $n_2 > 0$  so that the nonlinear effect will cause self-focusing. In order to make this evident, we will write in the equation  $n_2 = |n_2|$ . Let us now define



some parameters and replace them in the Eq.(2.41):

$\xi = \frac{x}{X_0}$ , so we can express the dependence on the x axis with a dimensionless parameter;  $X_0$  is a length.

$L_d = X_0^2 k_0 n$ , after the electric field has propagated across z for this length, the linear effects of diffraction can not be neglected anymore.

$\zeta = \frac{z}{L_d}$ , for studying the z-dependence with a dimensionless variable.

$L_{nl} = \frac{2\eta_0}{k_0 n |n_2| |A_m|^2}$ , after the electric field has propagated across z for this length, the nonlinear effects can not be neglected anymore. This parameter depends upon the intensity of the electric field, that's typical for nonlinear parameters.

$$N^2 = \frac{L_d}{L_{nl}}$$

After the replacements, Eq.(2.41) becomes

$$\frac{1}{2} \frac{\partial^2 a}{\partial \xi^2} + i \frac{\partial a}{\partial \zeta} + N^2 |a|^2 a = 0 \quad (2.42)$$

This is the nonlinear Schrödinger equation.

For  $N \ll 1$ , then the nonlinear part can be neglected.

For  $N \gg 1$ , then the nonlinear effect will be more evident than diffraction, and the field will tend to focus.

For  $N \approx 1$ , then the two effects balance each other and the equation can be solved.

For  $N = 1$ , the solution of the equation is the fundamental soliton [10]:

$$a(\xi, \zeta) = \text{sech}(\xi) e^{i\zeta/2} \quad (2.43)$$

It still depends on z, but only in phase, so the shape of the field will not change during propagation as shown in the Fig.2.5. For soliton solutions, N must be an integer and it is said to be the order or the soliton. For higher values of N, there are no closed form expressions, but the solitons exist and they are all periodic with different periods.

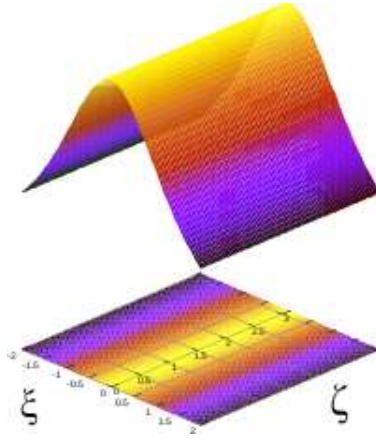


Figure 2.5: The shape of the soliton while propagating with  $N=1$  [12].

Their shape can easily be expressed only immediately after generation:

$$a(\xi, \zeta = 0) = N \operatorname{sech}(\xi) \quad (2.44)$$

### 2.2.2 Temporal Solitons

An actual picture of the contrast between soliton propagation and normal diffraction for a beam in a photorefractive material is shown in the Fig.2.6. Such spatial solitons belong to the same family of phenomena, the temporal soliton. A temporal soliton forms when group velocity dispersion is totally counteracted by temporal self-focusing or self-phase modulation effects [7].

All solitons require that a strong enough nonlinear interaction takes place between themselves and the material in which they propagate. This interaction typically requires that the so-called diffraction length for the spatial case or the dispersion length for the temporal case is comparable to a nonlinear length that characterizes self-focusing in the medium. What sets spatial solitons apart from their temporal counterparts is their dimensionality. Temporal solitons are described by a (1+1)-dimensional space-time evolution equation, whereas spatial solitons are by

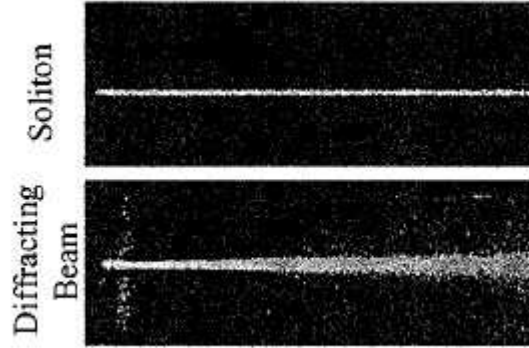


Figure 2.6: Top view photograph of a  $10\text{-}\mu\text{m}$ -wide spatial soliton propagating in a strontium barium niobate photorefractive crystal (top), and for comparison, the same beam diffracting when the nonlinearity is turned off (bottom) [7].

nature (2+1)-dimensional beams. This difference in the dimensions causes interesting processes in spatial case such as full three-dimensional interaction between solitons, soliton spiraling, vortex solitons, angular momentum effects, rotating dipole vector solitons, etc.

An electric field is propagating in a medium showing optical Kerr effect through a guiding structure such as an optical fiber that limits the power on the  $xy$  plane. If the field is propagating towards  $z$  with a phase constant  $\beta_0$ , then it can be expressed in the following form:

$$E(r, t) = A_m a(t, z) f(x, y) e^{i(\beta_0 z - \omega_0 t)} \quad (2.45)$$

where  $A_m$  is the maximum amplitude of the field,  $a(t, z)$  is the envelope that shapes the impulse in the time domain,  $f(x, y)$  represents the shape of the field on the  $xy$  plane which does not change during propagation [10]. Since in the medium there is a dispersion we can not neglect, the relationship between the electric field and its

polarization is given by a convolution integral

$$\tilde{E}(r, \omega - \omega_0) = \int_{-\infty}^{\infty} E(r, t) e^{-i(\omega - \omega_0)t} dt \quad (2.46)$$

The complete expression of the field in the frequency domain is

$$\tilde{E}(r, \omega - \omega_0) = A_m \tilde{a}(\omega - \omega_0, z) f(x, y) e^{i\beta_0 z} \quad (2.47)$$

Now Helmholtz equation can be solved in the frequency domain  $\nabla^2 \tilde{E} + n^2(\omega) k_0^2 \tilde{E} = 0$ , and following is obtained

$$2i\beta_0 \frac{\partial \tilde{a}}{\partial z} + [\beta^2(\omega) - \beta_0^2] \tilde{a} = 0 \quad (2.48)$$

Here the phase constant is expressed with the following notation:

$$n(\omega) k_0 = \beta(\omega) = \beta_0 + \beta_l(\omega) + \beta_{nl} = \beta_0 + \delta\beta(\omega) \quad (2.49)$$

where  $\beta_0$  is the linear non dispersive component,  $\beta_l(\omega)$  is the linear dispersive component, and  $\beta_{nl}$  is the nonlinear component. After introducing the same approximation that has been made in spatial soliton case, the equation has its final form:

$$\frac{1}{2} \frac{\partial^2 a}{\partial \tau^2} + i \frac{\partial a}{\partial \zeta} + N^2 |a|^2 a = 0 \quad (2.50)$$

where  $\tau = T/T_0$ ,  $\zeta = z/L_d$ ,  $N^2 = L_d/L_{nl}$ . The first order soliton is given by [8], [10]

$$a(\tau, \zeta) = \text{sech}(\tau) e^{i\zeta/2} \quad (2.51)$$

In the next section the interaction between surface plasmon and spatial soliton will be achieved by a heuristic coupling mechanism.

### 2.3 Surface Plasmon-Spatial Soliton Interaction

There are two ways for excitations of plasmons. The first is via the evanescent wave generated at the total internal reflection, and the second is via a periodic structure producing evanescent modes. Considering the first method, if there is a dielectric waveguiding layer on the interface, propagating modes of the waveguide have evanescent tails outside the waveguide which interact resonantly with plasmons at the metal surface. In the Ref. [13] another way of resonant interaction of plasmons with electromagnetic waves is proposed. They introduce a nonlinear dielectric medium, which admits spatial solitons which may propagate parallel to the metal-dielectric interface and similar to the modes of the dielectric waveguide. As a result, effective linear and nonlinear waveguides are coupled by the plasmon-soliton configuration, and an interaction between plasmon and soliton may be possible at certain resonant parameters [14].

#### 2.3.1 Model Description

To create a system that consists of optical soliton and surface-plasmon propagating along the metal-dielectric interface, we adopt the following model in Ref. [13]:

The system shown in Fig.2.7 is formed by two interfaces: a metal/linear-dielectric interface and a linear-dielectric/nonlinear-dielectric interface. The distance from the surface plasmon propagation axis to the soliton center axis is  $d$ . The metal possesses a negative dielectric constant  $\epsilon_1$  whereas the dielectric constant of the dielectric medium is  $\epsilon_2$  and it has the same value for the linear and nonlinear media. It is assumed that the nonlinearity is self-focusing. The interaction between a surface plasmon guided wave at the metal/dielectric interface that propagates along the  $z$  direction and a spatial soliton in the nonlinear medium propagating along the same direction will be analyzed. The nonlinear behavior of the soliton is able to provide a propagation con-

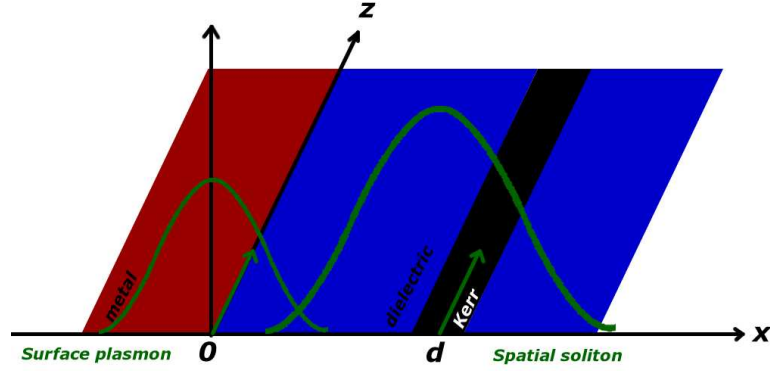


Figure 2.7: Plasmon and soliton in a metal-dielectric-Kerr nonlinearity system

stant satisfying  $\beta_s > (\omega/c)\sqrt{\epsilon_2}$  that grows with the soliton peak amplitude. On the other hand, the surface plasmon is formed by two evanescent wave tails both in the metal ( $e^{-\kappa_1|x|}$ ) and in the dielectric ( $e^{-\kappa_2|x|}$ ). For this reason the plasmon propagation constant is given in the dielectric by  $\beta_p = \sqrt{\kappa_2^2 + (\omega/c)^2\epsilon_2}$ , which automatically fulfills the same condition as the soliton:  $\beta_p > (\omega/c)\sqrt{\epsilon_2}$ . It is expected that under appropriate conditions controlled by the soliton power, it is possible to achieve the matching of the propagation constants of plasmon and soliton. This phase-matching condition should give rise to a mechanism of nonlinear resonant transfer of energy from soliton to plasmon.

In this model  $\Psi_p$  and  $\Psi_s$  are uncoupled plasmon and soliton fields propagating in the direction  $z$ . The total wave function is represented via the ansatz

$$\Psi(x, z) = \Psi_p(x, z) + \Psi_s(x, z) \quad (2.52)$$

where transverse profiles of plasmon and soliton are

$$\Psi_p(x, z) = c_p(z)\psi_p(x)\Psi_s(x, z) = c_s(z)\psi_s(x) \quad (2.53)$$

Here  $\psi_p(x) = e^{-\kappa_p x}$  with  $\kappa_p = \sqrt{k_p^2 - k^2}$  and  $\psi_s(x, |c_s|) = \text{sech}[\kappa_s(x - d)]$  with  $\kappa_s = k\sqrt{\gamma/2}|c_s|$ . Here  $c_p$  and  $c_s$  are the plasmon and soliton amplitudes,  $k$  is the wavevector of light and  $k_p$  is the surface-plasmon wavevector.  $\gamma$  is the nonlinearity parameter of the medium. We emphasize that the transverse profile of the soliton depends on its amplitude  $|c_s|$  which is the driving parameter.

Due to the spatial overlapping of the plasmon and soliton transverse profiles, the plasmon and soliton fields couple and obey the linear and nonlinear oscillator equations:

$$c_p'' + \beta_p^2 c_p = q(|c_s|)c_s, \quad (2.54)$$

$$c_s'' + \beta_s^2(|c_s|)c_s = q(|c_s|)c_p. \quad (2.55)$$

### 2.3.2 The Nonlinear Coupling Function

The strategy to excite plasmon-polariton mode consists of the generation of evanescent waves in the dielectric medium. Evanescent exponentially decaying tails behave very much alike those corresponding to the plasmon-polariton mode close to the resonant condition  $\beta \approx \beta_p$ . This property allows significant coupling between the modes due to the non-negligible overlap of their corresponding field amplitudes. [The physical reason behind the possibility of the existence of interaction between a soliton and a plasmon-polariton is the peculiarity of a soliton as a guided mode.] The soliton propagation constant depends on the peak soliton amplitude.

In Eq.(2.54) and (Eq.2.55) the derivatives are with respect to the dimensionless

coordinate  $kz$ ,  $\beta_p = \frac{k_p}{k} > 1$ ,  $\beta_s = \frac{k_s}{k} \cong 1 + \gamma|c_s|^2/4$  and  $q$  is the coupling function:

$$q(|C_s|) \approx \exp(-kd\sqrt{\gamma/2}|C_s|). \quad (2.56)$$

The coupling function is the overlapping of the tails of the plasmon and soliton waves in the area between metal and dielectric, and equal to the soliton field at the metal surface. Here  $d$  is the distance between metal and the nonlinearity in the dielectric.

The initial motivation is to obtain an effective plasmon-soliton transition, and it occurs only near the resonance  $\beta_p = \beta_s$ . Eq.(2.54) and Eq.(2.55) can be simplified by neglecting the second order derivatives and making the substitution  $c_{p,s}(kz) = C_{p,s}(kz)e^{ikz}$ . We obtain:

$$-iC'_p = \nu_p C_p - \frac{q(|C_s|)}{2} C_s \quad (2.57)$$

$$-iC'_s = -\frac{q(|C_s|)}{2} C_p + \nu_s(|C_s|) C_s \quad (2.58)$$

where  $\nu_p = \beta_p - 1 \ll 1$ ,  $\nu_s = \beta_s - 1 = \gamma|C_s|^2/4 \ll 1$ .

### 2.3.3 Surface Plasmon-Spatial Soliton Coupling with Parabolic Distance

We modify this model such that the distance between metal and nonlinearity is a parabolic function of  $z$  as shown in Fig.2.8, and the new coupling coefficient is

$$q(|C_s|) \approx \exp(-kd(1 + \alpha\xi^2)\sqrt{\gamma/2}|C_s|) \quad (2.59)$$

where  $\xi = kz$ .

We introduce 'fractional population imbalance'  $Z = \frac{|C_s|^2 - |C_p|^2}{|C_s|^2 + |C_p|^2}$ , and 'relative phase difference'  $\phi = \phi_s - \phi_p$  between the soliton and the surface plasmon where  $C_{s,p} = A_{s,p}e^{i\phi_{s,p}}$  [15]. Using this substitution and the fact that  $|C_s|^2 + |C_p|^2 = N$  ( $N$  is



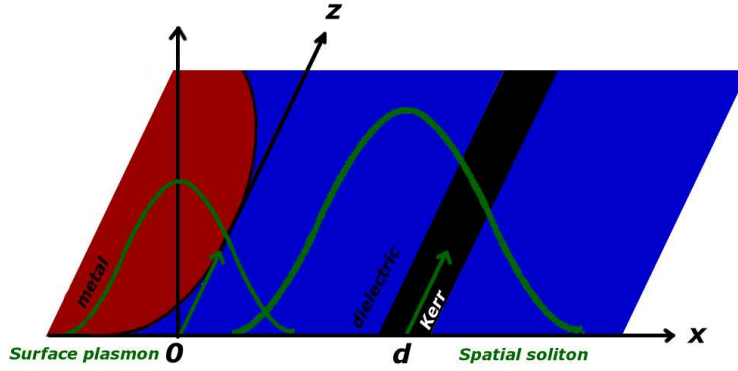


Figure 2.8: Modified plasmon-soliton system. The distance between metal surface and the Kerr nonlinearity is a parabolic function of  $z$ .

a normalized constant for isolated system and equal to 1) Eq.(2.57) and Eq.(2.58) become

$$\dot{Z} = -q(Z)\sqrt{1-Z^2} \sin \phi, \quad (2.60)$$

$$\dot{\Phi} = \Lambda(1+Z) - \nu_p + \frac{q(Z)Z \cos \phi}{\sqrt{1-Z^2}}. \quad (2.61)$$

## 2.4 Numerical Analysis

### 2.4.1 Parallel System

In the model, [13] the distance between metal and the nonlinearity in the dielectric is constant, so two layers are parallel to each other. In this case in which there are no dissipative effects, a decrease in the total population  $|C_s|^2 + |C_p|^2$  is not expected and this quantity is conserved. However, there are mutual and continuous transitions between surface plasmon and soliton, also the population imbalance ' $Z$ ' is not constant and undergoes mutual and continuous transitions as well as shown in the Fig.2.9

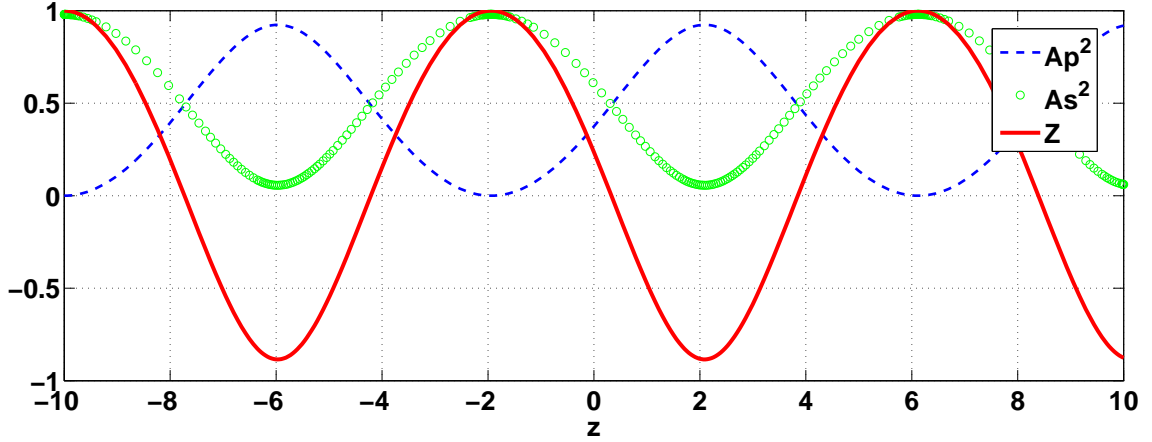


Figure 2.9: Surface plasmon-soliton interaction in a parallel metal/dielectric system (above),  $kd = 1.8$ ,  $\alpha = 0$ .

#### 2.4.2 Parabolic System

This section is the heart of this research, because the goal of the research in [13] is to obtain a full transition from soliton to surface plasmon amplitude. It has been tried to obtain by adding a dissipation, Eq.(2.62), to the system, in which the total energy decreases in time.

$$\nu_{p,s} = \nu_{p,s} + i\sigma_{p,s} \quad (2.62)$$

However, and both the soliton and surface plasmon amplitudes decrease as shown in Fig.2.10 and Fig.2.11.

In this thesis, we implement this through the parabolic modification of the coupling parameter as discussed in the previous section. In the system, the distance between metal surface and the nonlinearity in the dielectric is a parabolic function of  $z$ , and  $kd$  is the minimum distance between layers. The transition is observed to occur in the neighbourhood of this minimum distance. Since the distance is increasing parabolically, outside of this neighbourhood the coupling between soliton and surface plasmon is negligible.

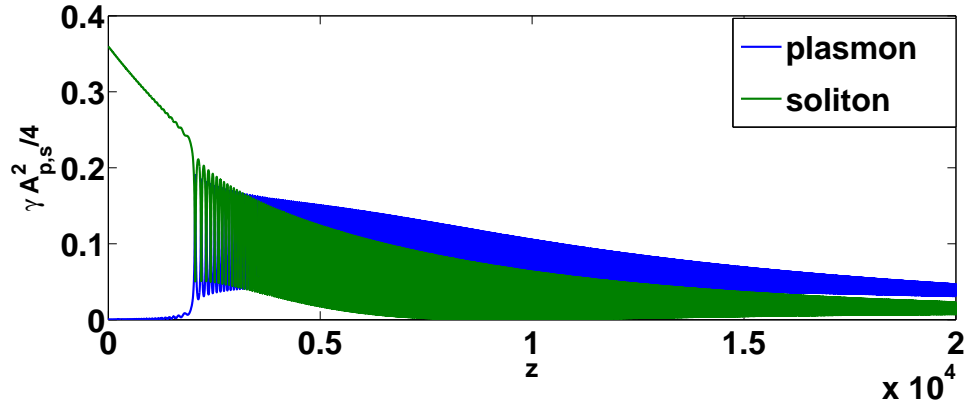


Figure 2.10: Dynamics in plasmon-soliton system with small dissipation.  $\sigma_p = 0, \sigma_s = 10^{-4}$ .

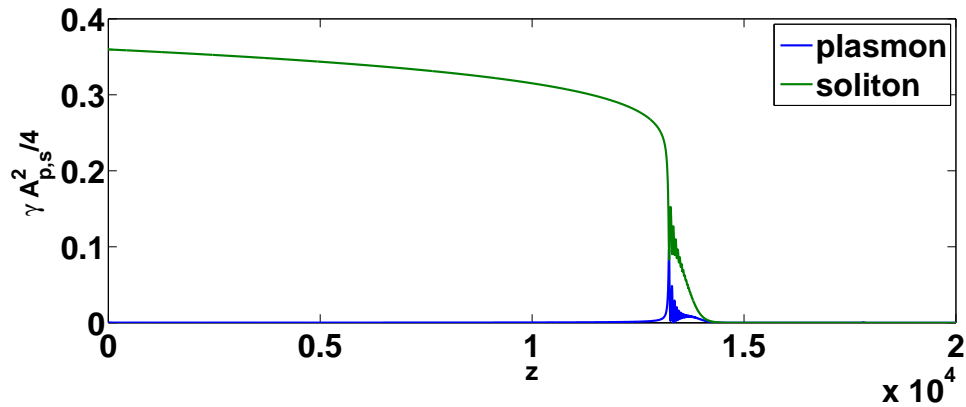


Figure 2.11: Dynamics in plasmon-soliton system with small dissipation.  $\sigma_p = 10^{-2}, \sigma_s = 0$ .

Fig.2.12, Fig.2.13, and Fig.2.14 show the output value of  $Z$  as a function of  $kd$  and  $\alpha$ . From these figures we try to determine feasible  $\alpha$  and  $kd$  values which yield maximum transition.

Here  $\nu_p$  is taken 0.2, and  $\gamma$  is taken 0.1. According to the results, the minimum  $Z$  value is obtained when  $kd$  is 1.8. In the light of these values, Fig.2.15 show the optimal  $\alpha$  value, which is 0.84.

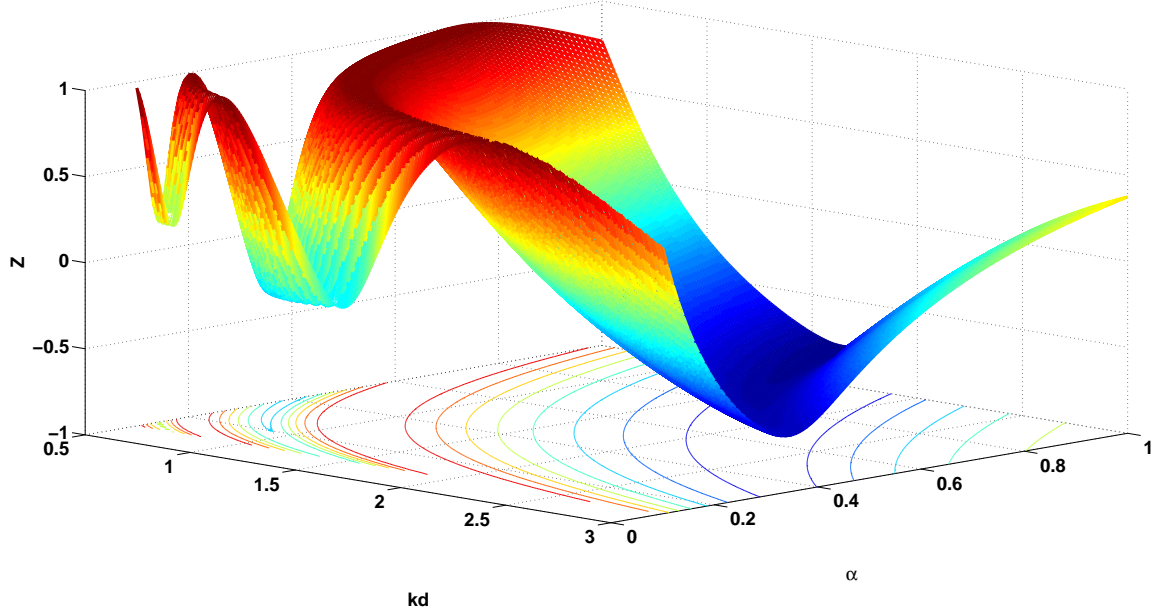
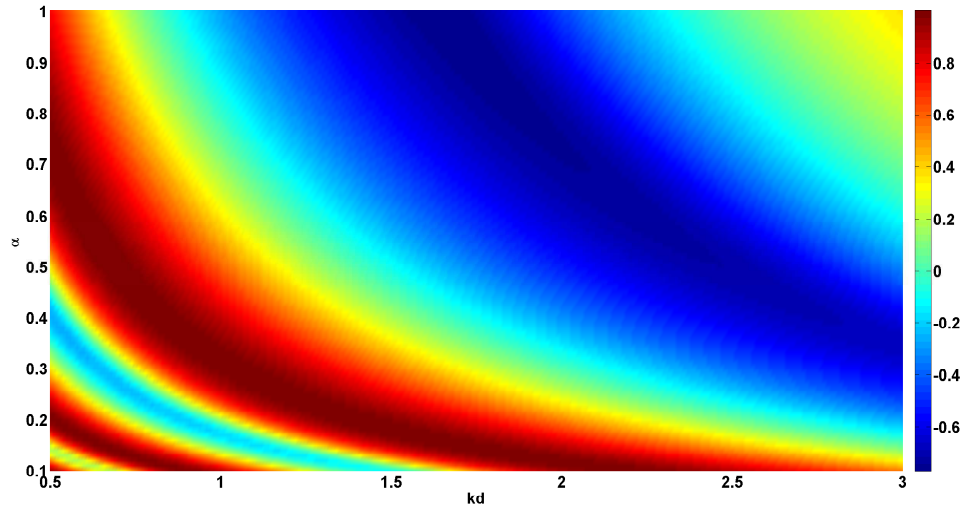
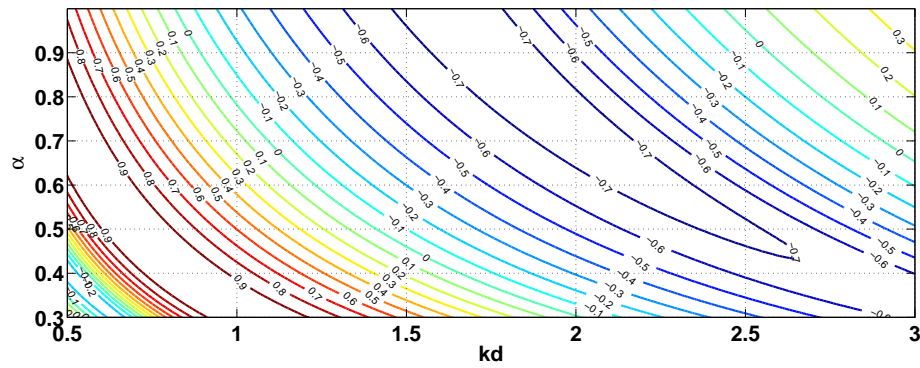


Figure 2.12: The minimum  $Z_{out}$  value corresponds to  $1.5 < kd < 2$

For the optimal  $\alpha$  and  $kd$  values we obtain nearly ideal transition from plasmon amplitude to soliton amplitude. The population imbalance  $Z$  which is initially 1, becomes -0,7669. Following figures show transitions for different  $\alpha$  values.

These results show that nearly full transition from spatial soliton to surface plasmon can be achieved by a heuristic coupling mechanism. We've modified the model in [13] such that the distance between the metal surface and the nonlinearity in the dielectric is a parabolic function of  $z$ . However, in this modification  $\alpha$  should be small, i.e.  $\alpha < 1$ , because the coupled oscillator equations, Eq.(2.54) and Eq.(2.55), which are obtained from the Maxwell equations are for one dimensional case. Since our modification makes the system two dimensional, we assume that  $\alpha$  is small enough to keep the system in one dimensional form.

Figure 2.13: Contour plot of  $Z_{out}$  values with respect to  $kd$  and  $\alpha$ Figure 2.14: More detailed contour plot of  $Z_{out}$  values with respect to  $kd$  and  $\alpha$

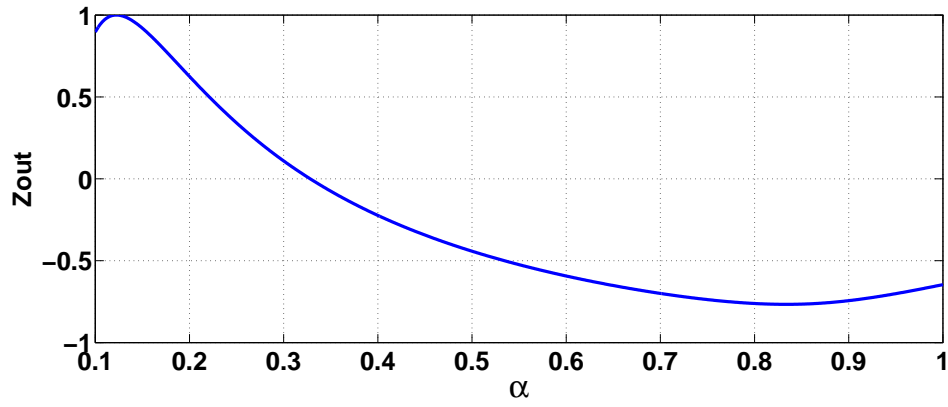


Figure 2.15: The minimum  $Z_{out}$  value corresponds  $\alpha = 0.84$

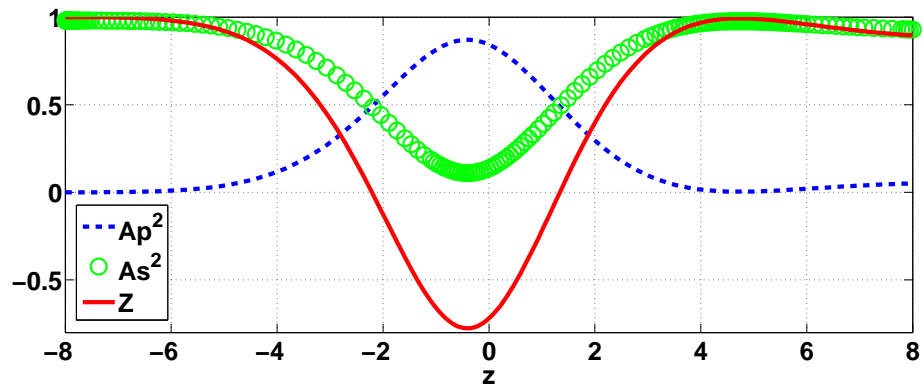


Figure 2.16:  $\alpha = 0.1, kd = 1.8, \nu_p = 0.2$

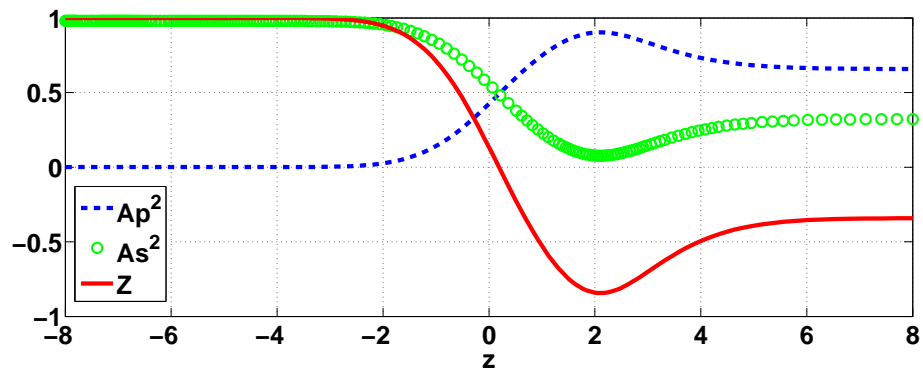


Figure 2.17:  $\alpha = 0.1, kd = 1.8, \nu_p = 0.2$

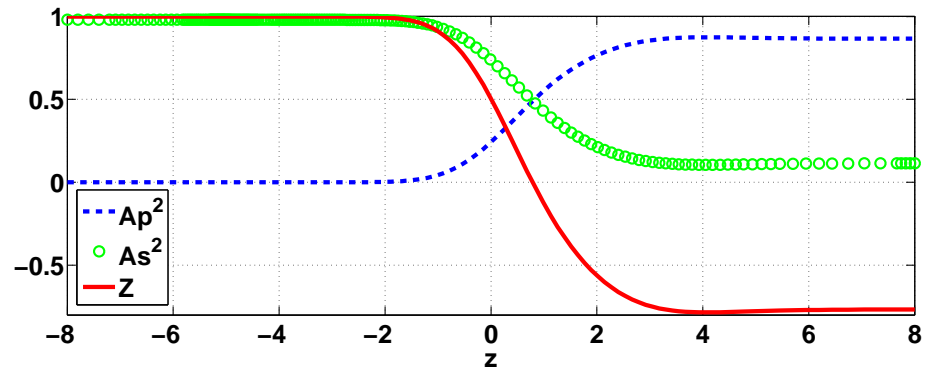


Figure 2.18: The optimum  $\alpha$  value 0.84, which enables nearly ideal transition between soliton and surface-plasmon,  $kd = 1.8, \nu_p = 0.2$

## Chapter 3

## SURFACE PLASMON-SOLITON INTERACTION IN METAL-DIELECTRIC-METAL INTERFACE

### 3.1 Surface Plasmons in Multilayers: Brief Theoretical Background

In a multilayer system, each single interface can sustain bound surface plasmon polaritons. When the separation between adjacent interfaces is comparable to the decay length of the interface mode, interactions between surface plasmon polaritons give rise to coupled modes. To illuminate the general properties of coupled surface plasmon polaritons, two specific systems in Fig. 3.1 should be analyzed: Insulator/metal/insulator heterostructure, and metal/insulator/metal heterostructure [5], [6].

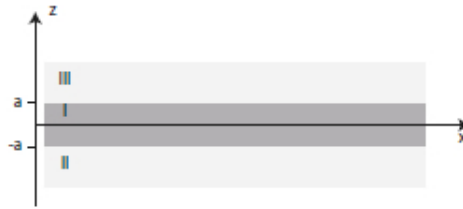


Figure 3.1: Geometry of a three-layer system consisting of a thin layer I sandwiched between two infinite half spaces II and III.

The field components of TM modes for  $z > a$  region are

$$H_y(z) = A e^{i\beta x} e^{-k_3 z} \quad (3.1)$$

$$E_x(z) = iA \frac{1}{\omega \epsilon_0 \epsilon_3} k_3 e^{i\beta x} e^{-k_3 z} \quad (3.2)$$



$$E_z = -A \frac{\beta}{\omega \epsilon_0 \epsilon_3} e^{i\beta x} e^{-k_3 z} \quad (3.3)$$

for  $z < -a$  region

$$H_y(z) = B e^{i\beta x} e^{k_2 z} \quad (3.4)$$

$$E_x(z) = -iB \frac{1}{\omega \epsilon_0 \epsilon_2} k_2 e^{i\beta x} e^{k_2 z} \quad (3.5)$$

$$E_z = -B \frac{\beta}{\omega \epsilon_0 \epsilon_2} e^{i\beta x} e^{k_2 z} \quad (3.6)$$

In the core region  $-a < z < a$ , the modes localized at the bottom and top interface couple, yielding

$$H_y = C e^{i\beta x} e^{k_1 z} + D e^{i\beta x} e^{-k_1 z} \quad (3.7)$$

$$E_x = -iC \frac{1}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta x} e^{k_1 z} + iD \frac{1}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta x} e^{-k_1 z} \quad (3.8)$$

$$E_z = C \frac{\beta}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta x} e^{k_1 z} + D \frac{\beta}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta x} e^{-k_1 z} \quad (3.9)$$

The requirement of continuity of  $H_y$  and  $E_x$  at  $z = a$  leads to

$$A e^{-k_3 a} = C e^{k_1 a} + D e^{-k_1 a} \quad (3.10)$$

$$\frac{A}{\epsilon_3} k_3 e^{-k_3 a} = -\frac{C}{\epsilon_1} k_1 e^{k_1 a} + \frac{D}{\epsilon_1} k_1 e^{-k_1 a}$$

and at  $z = -a$

$$B e^{-k_2 a} = C e^{-k_1 a} + D e^{k_1 a} \quad (3.12)$$

$$\frac{B}{\epsilon_2} k_2 e^{-k_2 a} = -\frac{C}{\epsilon_1} k_1 e^{-k_1 a} + \frac{D}{\epsilon_1} k_1 e^{k_1 a}$$

This is a linear system of four coupled equations.  $H_y$  further has to fulfill the wave equation in the three distinct regions, via  $k_i^2 = \beta^2 - k_0^2 \epsilon_i$  (3.13) for  $i = 1, 2, 3$ . Solving this system of linear equations results in an implicit expression for the dispersion relation linking  $\beta$  and  $\omega$  via

$$e^{-4k_1 a} = \frac{k_1/\epsilon_1 + k_2/\epsilon_2}{k_1/\epsilon_1 - k_2/\epsilon_2} \frac{k_1/\epsilon_1 + k_3/\epsilon_3}{k_1/\epsilon_1 - k_3/\epsilon_3} \quad (3.14)$$

In the special case of  $\epsilon_2 = \epsilon_3$  and  $k_2 = k_3$ , the dispersion relation can be split into a pair of equations

$$\tanh k_1 a = -\frac{k_2 \epsilon_1}{k_1 \epsilon_2} \quad (3.15)$$

$$\tanh k_1 a = -\frac{k_1 \epsilon_2}{k_2 \epsilon_1} \quad (3.16)$$

The dispersion relations can now be applied to dielectric/metal/dielectric and metal/dielectric/metal structures to investigate the properties of the coupled surface plasmon polariton modes in these two systems.

## 3.2 Surface Plasmon-Soliton Transition in Multilayer Systems

### 3.2.1 Parabolic System

A transition from surface-plasmon amplitude to soliton amplitude can be achieved in multilayer systems such as metal/dielectric/metal interface. The distance from the first metal to the nonlinearity in the dielectric is again a parabolic function of  $\alpha_1$ , and the minimum distance between two layers is denoted as  $kd_1$ . The distance from the second metal to the nonlinearity in the dielectric is also a parabolic function of  $\alpha_2$ , and the minimum distance between two layers is  $kd_2$ , as shown in Fig.3.2.

In this multilayer system, we have two different coupling functions since we have two different surface plasmon-spatial soliton interactions. The corresponding coupling

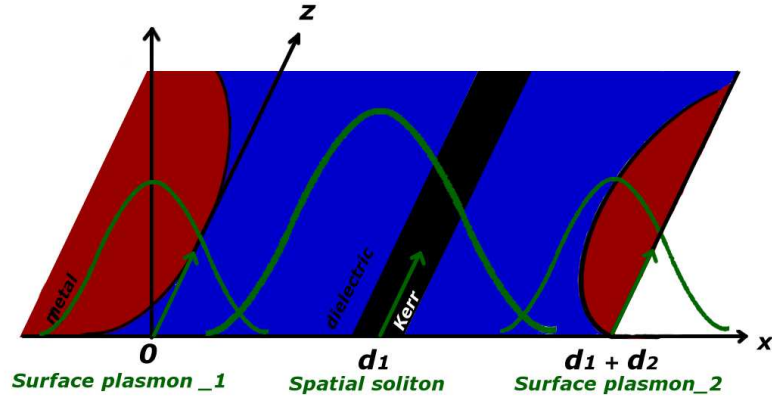


Figure 3.2: Two surface plasmons and a spatial soliton in a metal/dielectric/Kerr/dielectric/metal multilayer.

functions are as follows:

$$q_1(|C_s|) = e^{-kd_1(1+\alpha\xi^2)}\sqrt{\gamma/2}|C_s| \quad (3.17)$$

$$q_2(|C_s|) = e^{-kd_2(1+\alpha\xi^2)}\sqrt{\gamma/2}|C_s| \quad (3.18)$$

where  $\xi = kz$ . Our previous coupled oscillator equations, Eq.(2.54) and Eq.(2.55), need modifications since we are working with three-layer system. The new equations are

$$c_{p_1}'' + \beta_{p_1}^2 c_{p_1} = q_1(|c_s|)c_s \quad (3.19)$$

$$c_{p_2}'' + \beta_{p_2}^2 c_{p_2} = q_2(|c_s|)c_s \quad (3.20)$$

$$c_s'' + \beta_s^2 c_s = q_1(|c_s|)c_{p_1} + q_2(|c_s|)c_{p_2} \quad (3.21)$$

Making the substitutions  $c_{p_1} = C_{p_1}e^{i\xi}$ ,  $c_{p_2} = C_{p_2}e^{i\xi}$ , and  $c_s = C_s e^{i\xi}$  the equations become

$$-iC'_{p_1} = \nu_{p_1} C_{p_1} - \frac{q_1(|C_s|)}{2} C_s \quad (3.22)$$

$$-iC'_{p_2} = \nu_{p_2} C_{p_2} - \frac{q_2(|C_s|)}{2} C_s \quad (3.23)$$

$$-iC'_s = \nu_s C_s - \frac{q_1(|C_s|)}{2} C_{p_1} - \frac{q_2(|C_s|)}{2} C_{p_2} \quad (3.24)$$

and the following substitutions is made:

$$C_{p_1} = A_{p_1} e^{i\phi_{p_1}} \quad (3.25)$$

$$C_{p_2} = A_{p_2} e^{i\phi_{p_2}} \quad (3.26)$$

$$C_s = A_s e^{i\phi_s} \quad (3.27)$$

We used 'ode45' function in MATLAB to solve these differential equations. The initial amplitude of first surface plasmon,  $A_{p_1}$ , is 0.99 and the soliton,  $A_s$ , and the second surface plasmon amplitudes,  $A_{p_2}$ , are zero.  $\alpha$  has the same optimal value as in Chapter 2, which is 0.84, and the optimal  $kd_1$  and  $kd_2$  values are 1.8 and 10.2 respectively. The initial  $\phi_{p_1}$ ,  $\phi_{p_2}$ , and  $\phi_s$  values are taken zero. Under these circumstances the first surface plasmon transfer most of its energy to soliton and a small part to second surface plasmon as shown in the Fig.3.3.

Following figure, Fig.3.4, gives more insight to surface plasmon-spatial soliton transition in this configuration. The optimal  $kd_1$  value, 1.8, can be shown in the figure.

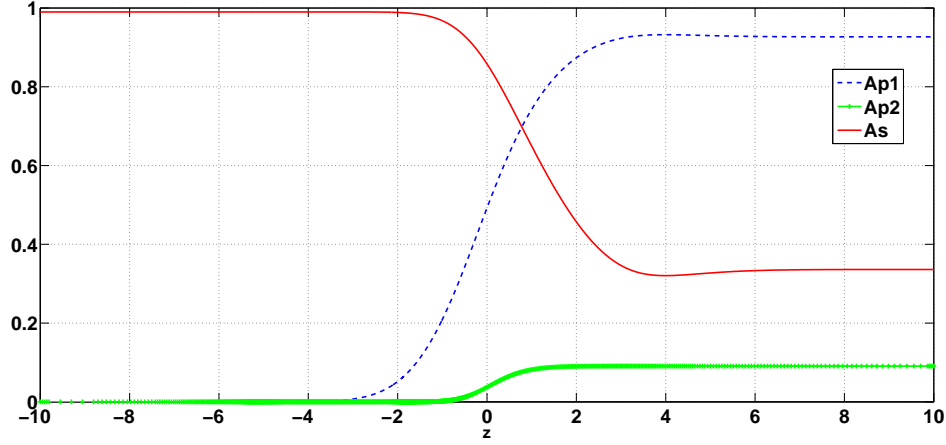


Figure 3.3: Here  $\alpha = 0.84$ ,  $kd_1 = 1.8$ , and  $kd_2 = 10.2$ . In the effective area, the second surface-plasmon forces the soliton and the first surface-plasmon to interact. A transition is achieved, but not as ideal as in the two-layer system.

### 3.3 Stable Soliton Amplitudes in Multilayer Systems

#### 3.3.1 Parallel System

In the metal/dielectric/metal interface, where the distances between metals and the nonlinearity in the dielectric are constant (Fig.3.5), a simple transition between surface plasmon and soliton amplitudes cannot be achieved since the distance is unchanged and there is no energy dissipation. Subsequent transitions occur and the energy oscillates between surface plasmon and soliton amplitudes. However, in some specific configurations no transition occurs at all, and  $A_s$  is observed to be stable in magnitude. Initial  $A_s$  and  $A_{p_1}$  values for one of these configurations are 0.4 and 0.6 respectively, where initial  $A_{p_2}$  value is zero.  $kd_1$  and  $kd_2$  values are 8 and 3 respectively. Fig.3.6 shows the plot of  $A_s$  for this case, and Fig.3.7 shows the  $A_{p_1}$  and  $A_{p_2}$  graphs which correspond to constant  $A_s$  graph.

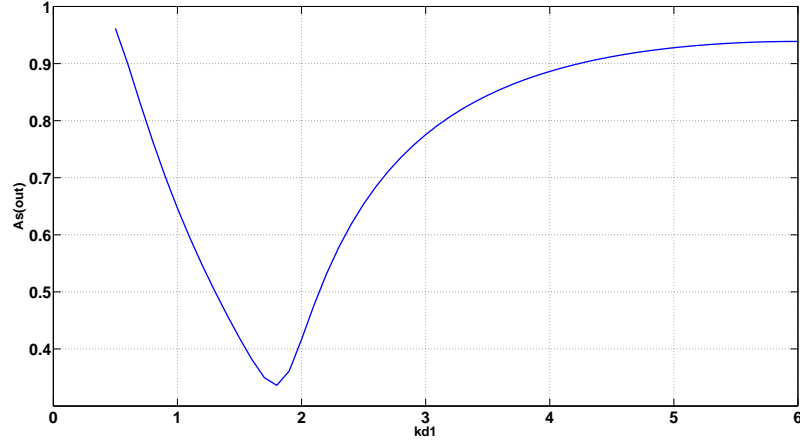


Figure 3.4: Graph of  $As_{out}$  values with respect to  $kd_1$ .  $kd_1 + kd_2$  value is constant, and equal to 12.  $\alpha$  is 0.84.

### 3.4 Transition from Stable Amplitudes to Oscillatory Amplitudes

#### 3.4.1 Half Parallel/Half Parabolic System

Another configuration in metal/dielectric/metal interface is that the distance between second metal and the nonlinearity in the dielectric is constant, and the distance between the nonlinearity and the first metal is a parabolic function of  $z$  (Fig.3.8). In this configuration, the coupling functions are

$$q_1(|C_s|) = e^{-kd_1\sqrt{\gamma/2}|C_s|} \quad (3.28)$$

$$q_2(|C_s|) = e^{-kd_2(1+\alpha\xi^2)\sqrt{\gamma/2}|C_s|} \quad (3.29)$$

It can be foreseen by the knowledge of the previous sections, that subsequent transitions occur between first two layers, and in the area of the effective transition near the minimum distance between soliton and first metal layers the transitions are affected by the first metal although there is no initial surface plasmon amplitude  $A_{p2}$  on the metal. Then, subsequent transitions shifts in amplitudes, and continue the

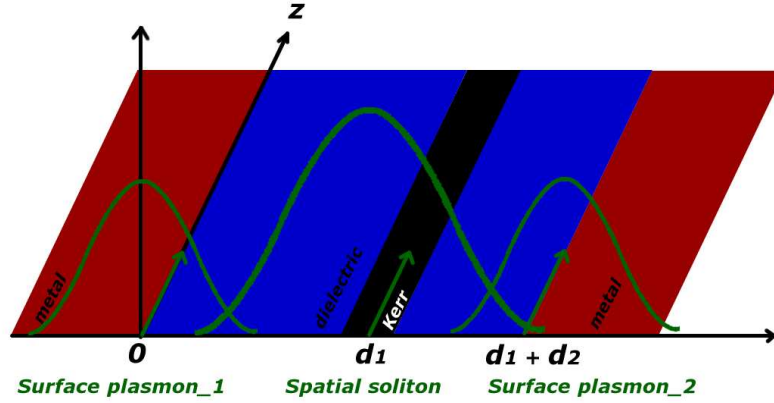


Figure 3.5: Two surface plasmons and a spatial soliton in a metal/dielectric/Kerr/dielectric/metal system.

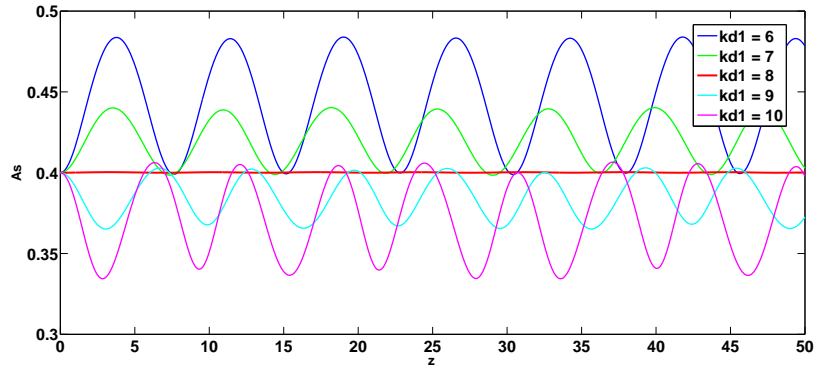


Figure 3.6: Graph of  $A_s$  with different  $kd_1$  configurations.  $kd_1 + kd_2 = 11$ . When  $kd_1 = 8$ , spatial oscillations in  $A_s$  is highly suppressed.

oscillations. Fig.3.9 shows the plot of an ordinary case.

In some specific configurations this change in  $A_{p1}, A_{p2}$ , and  $A_s$  becomes a switch between oscillatory and stable amplitudes. It can be both ways as shown in Fig.3.10 and Fig.3.11.

In this chapter, we investigated the interaction between surface plasmons and spatial solitons in metal/dielectric/Kerr nonlinearity/dielectric/metal system. We obtain results for three different case. First, metals are bent such that the distances

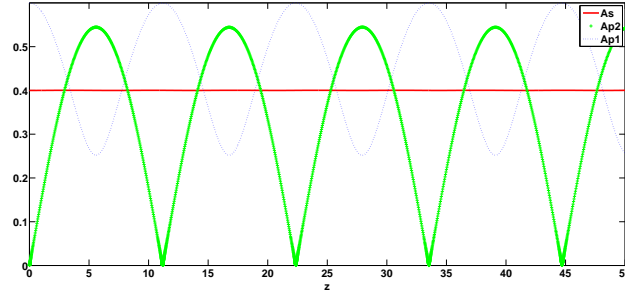


Figure 3.7:  $A_{p1}$  and  $A_{p2}$  graphs of the constant  $A_s$  value.  $kd_1 = 8$ .  $kd_1 + kd_2 = 11$ .

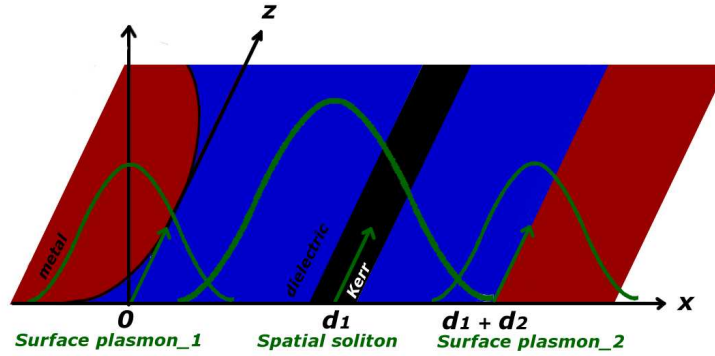


Figure 3.8: Two surface plasmons and a spatial soliton in half parabolic/half parallel system.

between metal surfaces and the nonlinearity are parabolic functions of  $z$ . In this case, we have looked for a similar transition like we have obtained in Chapter 2, but we couldn't obtain a full transition. In the second case, we keep the system unmodified, such that the metals and the nonlinearity are parallel to each other. In this case, spatial oscillations in  $A_s$  are highly suppressed. In the third and final case, one of the metals is parallel to the nonlinearity, and the other metal is bent. The effect of the bent metal shows itself only near the minimum distance between the metal and the nonlinearity, and the amplitudes of the oscillations in  $A_s$  and  $A_{p2}$  changes after surface plasmon and spatial soliton pass through this neighbourhood.



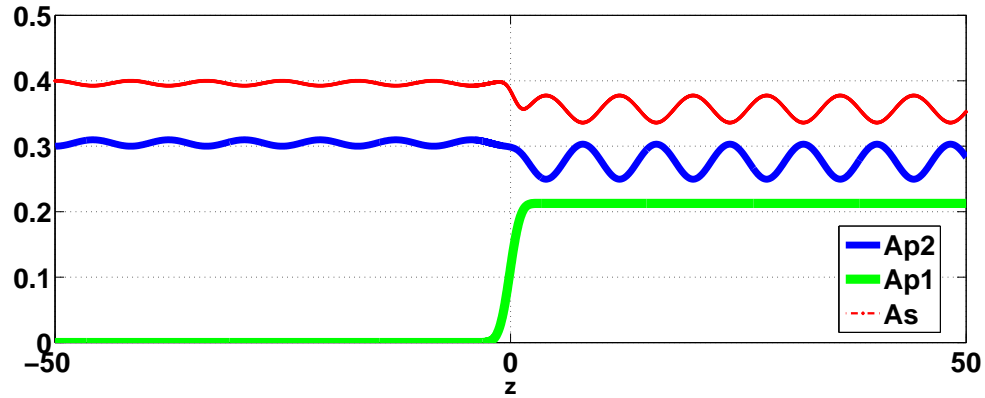


Figure 3.9: The amplitudes change after the interaction with the first metal.  $kd_1 = 3.4$ ,  $kd_2 = 7.6$ , and  $\nu_p = 0.2$

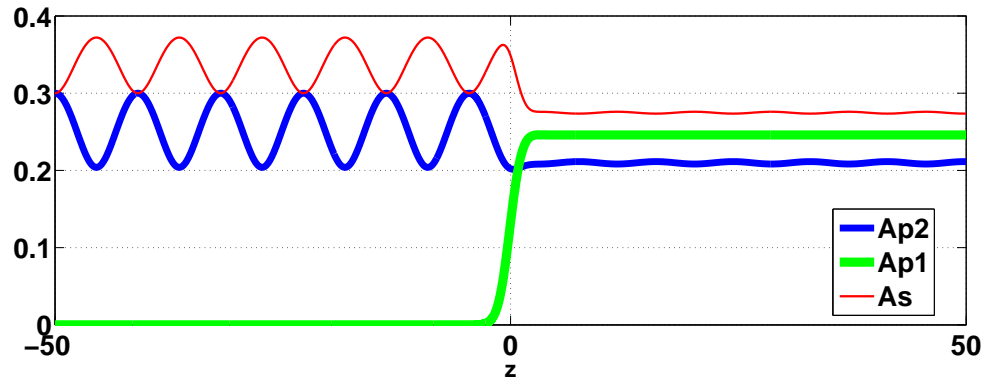


Figure 3.10: Near the minimum distance between first metal and the nonlinearity in the dielectric, second surface plasmon and soliton interacts with the second metal and the oscillations in  $A_{p2}$  and  $A_s$  are highly suppressed.  $kd_1 = 5.2$ ,  $kd_2 = 6.8$ , and  $\nu_p = 0.2$

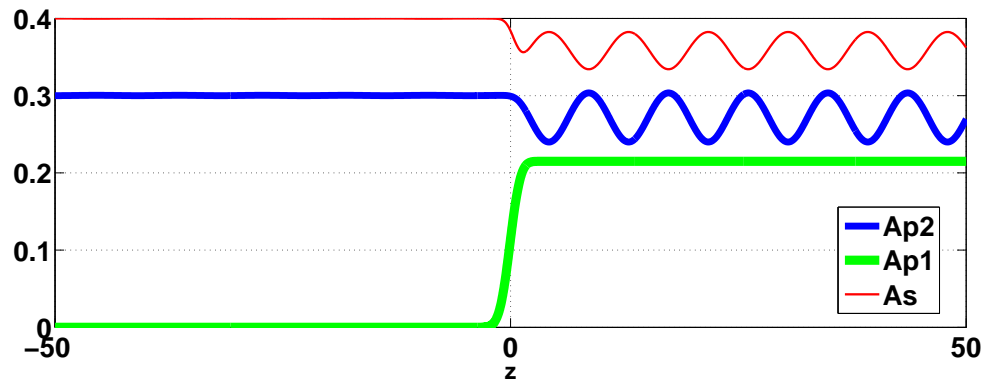


Figure 3.11: The amplitudes of second surface plasmon and soliton which are constant start oscillating after interacting with the first metal near the minimum distance between the first metal and the nonlinearity in the dielectric.  $kd_1 = 4.4$ ,  $kd_2 = 7.6$ , and  $\nu_p = 0.2$

## Chapter 4

### CONCLUSION

In the presented work, we studied the dynamical properties of the interaction between an optical soliton in a nonlinear dielectric waveguide and co-propagating surface-plasmons along a metal surface. In the metal/dielectric system the coupling parameter depends on the soliton amplitude instead of being a coupling constant. Our goal was to obtain a transition between soliton and surface-plasmon amplitudes. To achieve this goal we consider the interaction when the soliton is propagating on a parabolic trajectory with respect to the flat metal surface. After numerical calculations we have achieved nearly full population conversion from optical soliton to surface-plasmon under certain parameter values. With the success of this goal, we expand our system to a metal/dielectric/Kerr/dielectric/metal multilayer interface, in which the optical soliton is essentially coupled to two surface plasmons on either side across dielectric spacers. We found that for certain spacing parameters the spatial profile of the soliton is almost constant along the propagation. In a different multilayer configuration, we form a system in which one surface-plasmon propagates on a parallel trajectory, and the other surface-plasmon propagates on a parabolic trajectory. In this case, depending on the model parameters the oscillations in the spatial profile of the soliton may be induced or suppressed as it enters through the parabolic channel. In this way we switch the spatial profile of the soliton from oscillation to constant propagation or visa versa.

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## VITA

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