INFORMATION AGGREGATION AND THE ORDER OF SPEECH IN DEBATES

by

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Abstract

This study aims to investigate the importance of the order of speech in debates in terms of acquisition of relevant information. We analyze a simple model with binary state, binary signal, binary decision, two periods and two privately informed experts who have heterogenous expertise levels and are motivated by only their career concerns. We first compare the common anti-seniority rule whereby experts speak in order of increasing expertise with the seniority rule and show that, in general, seniority rule aggregates relevant information better than the anti-seniority rule. Then we compare these fixed orders with a mechanism in which experts endogenously decide when to speak. We conclude that relevant information aggregates better and hence the decision maker is better off under this endogenous order mechanism than any fixed order mechanism.

Keywords: Cheap talk, information aggregation, debate, order, career concerns

Özet

Bu tezde, uzmanların müzakereler esnasındaki konuşma sırasının önem arzeden bilginin açığa çıkmasındaki önemi incelenmektedir. Model olarak iki elemanlı durum, sinyal ve karar uzayı ele alınmakta olup kariyer beklentileri tarafından motive edilmiş, durum hakkında özel bilgiye sahip, farklı uzmanlık seviyesindeki iki uzmanın iki zaman diliminde konuşması beklenmektedir. İlk olarak uzmanların, uzmanlık seviyeleri her konuşmacıda artacak şekilde konuştukları durum ile her konuşmacıda azalacak şekilde konuştukları durum karşılaştırılıyor ve genel olarak önce uzmanlık seviyesi daha fazla olan uzmanın konuştuğu durumlarda yararlı bilginin daha fazla öğrenildiği görülmektedir. Daha sonra bu konuşmadan önce uzmanlara dışarıdan bildirilmiş konuşma kuralları, uzmanların kendi kendilerine ne zaman konuşacaklarına karar verebildikleri endojen bir konuşma kuralıyla karşılaştırılıyor. Elde edilen sonuç bu endojen konuşma kuralında yararlı bilginin daha fazla öğrenildiği ve bu yüzden karar verici merci açısından endojen modeli uygulamanın dışarıdan bildirilmiş herhangi bir konuşma kuralını uygulamaktan daha faydalı olduğudur.

Anahtar Kelimeler: Bedelsiz konuşma, bilgi birikimi, müzakere, kural, kariyer beklentileri

Contents

1 Introduction

Consider a decision maker who is uninformed about a state of the world that is relevant to the decision and can consult a group of experts with varying levels of expertise. Assume that the experts are only concerned about their motivation, i.e., they would like their recommendations to match the state of the world, which would be realized only after the decision is made. In absence of monetary transfers, what kind of mechanism should the decision maker use? Is there a particular order with which she should consult the experts? Or is it better still to let them speak when they like?

These questions are important in any environment where a debate takes place before a decision is made. For example, consider a committee that is responsible to evaluate the candidates for an award. Assume that there are two candidates one of whom is more deserving of the award. Committee members have independent private information about the quality of the candidates and are better informed than the decision maker. They have to send their evaluation to the other members and the decision maker via email within a fixed period of time. After all the committee members send their evaluation, a decision maker makes the decision regarding the fellowship. Assume that the committee members only care about their reputation, i.e., they wish to be regarded as well-informed. Also assume that their information is not verifiable, so that their messages need not accurately follow their information. In other words, even if a member's private information indicates that candidate A is better, she may still send a message saying that candidate B is better. How much of the private information of the committee members would be reflected in the final decision? Which factors determine the quality of the decision? Should the decision maker consult better informed members or vice versa? Would he be better of if he let the members send their messages whenever they want, or is a fixed order better?

We analyze these questions in a simple model with binary state, binary decision, two experts, and two periods. Each expert receives an independent signal about the state of the world and sends a binary report, which need not be equal to the signal they receive. In other words, experts' reports are "cheap-talk". We assume that one of the experts is better informed about the state of the world, and call that expert the “senior”. We analyze two basic mechanisms through which the experts may express their opinions: (1) fixed order mechanism, in which either the senior (“seniority rule”) or the junior speaks first (“anti-seniority rule”); (2) endogenous mechanism, in which each experts chooses independently when to speak.[1]

We first show that, in general, seniority rule is better than the anti-seniority rule for the decision maker. The reason is that it is possible to learn the private signal of junior whenever it is useful by applying seniority rule for some specific values of the public prior belief whereas junior would have herd and sent the message supporting the public prior belief if anti-seniority rule had been applied. For any other value of the public prior belief except that certain interval, the information possessed by the junior does not have an impact enough to change the decision or it is impossible to learn it no matter which fixed order mechanism is applied.

Perhaps more importantly, we show that endogenous order does better than any fixed order mechanism. In other words, the decision maker is better off by letting the experts speak whenever they want. The main intuition behind the optimality of the endogenous order mechanism is as follows. Under any fixed order mechanism experts cannot condition on when to speak on their signal, whereas under the endogenous order mechanism they can. This gives them an extra tool to reveal their signal and hence a potentially extra piece of information. Of course this would be the case only if there is an equilibrium in which experts speak at different times depending on their signal, which we show to be the case under certain condition.

After reviewing the literature in Section 2, we present the model in Section 3. In Section 4 and Section 5 we analyze the equilibria of different mechanisms and in Section 6 we compare these mechanisms in terms of the ex ante payoff of the decision maker. We conclude with some remarks and suggestions for future research in Section 7.

[1]Ottaviani and Sorensen (2001) was the first to study this problem assuming fixed order and the terminology is due to them.

2 Literature Review

The main strand of the literature that our study is related to is on strategic information transmission which was pioneered by Crawford and Sobel(1982). Although Crawford and Sobel consider a cheap-talk model where there is only one sender, there are two experts who simultaneously or sequentially send their messages endogenously in our model. Furthermore, the payoff of senders which we call experts depends on only the messages sent by expert and the true state of the world. In Crawford and Sobel, payoff of the sender depends on the action taken by the receiver and the true state of the world. In this sense, it is possible to say that our experts have career concerns.

The literature on experts motivated by career was initiated by the seminal contribution of Holmstrom(1982). This paper studies how a person's career concerns may influence his incentives to put in effort on the job. It is somewhat more related to literature on moral hazard. Scharfstein and Stein(1990) is another well-known paper who investigates the behavior of agents who have career concerns and need to make an investment decision sequentially. They get some private signals which are informative about the state of the world and they will make profit or loss depending on the state. They conclude that the agent who decides later pays too much attention to what first agent has done and too little to his private signal. By concluding so, actually they are making somehow the definition of the "herd behavior". Two years later, Banerjee(1992) constructs a simple model of herd behavior.

In our model, we endogenize the order of speaking. This is one of the extensions considered by Banerjee(1992) about their model. Banerjee admits that choosing when to move taking into account the fact that waiting is costly is a more natural assumption than assuming that the order of choice is exogenously fixed. However, we have not come across the extension of his result so far.

Another attempt to make the situation more realistic is done by Swank and Visser (2006). They endogenize the nature of information rather than the order of speech. In other words, not all experts get a private signal about the state but only experts who decide to collect information and exert effort to do so. Communication process with the decision maker is again sequential. They concludes that endogenizing information replaces the herding problem by a “free-rider problem”.

Ottaviani and Sorensen(2001) proposes a variant model of Scharfstein and Stein (1990). They keep the binary spaces of Scharfstein and Stein’s model but assume that private signals of the experts are independently drawn. They let these privately informed individuals speak sequentially and publicly. They conclude that for any order of speech, experts may herd supporting the result of Bikhchandani et al.(1992). By optimizing over the order of the speech, they argue that the amount of information aggregated could be improved. This result makes Ottaviani and Sorensen(2001) the closest paper to ours. In contrast to Ottaviani and Sorensen, we do not assume any ability types for the experts. In our model, there are two experts with heterogeneous expertise levels and their expertise levels (precisions) are common knowledge. Under these circumstances, we show that “endogenous order mechanism” we develop does better than any fixed order mechanism, especially than anti-seniority Rule, in terms of information acquisition and ex-ante payoff of the decision maker. This result coincides with the result of Ottaviani and Sorensen(2001) which states that the anti-seniority Rule does not necessarily implement the first best, even in situations where other exogenously given speaking rules may implement it. In addition, Chamley(2003) also considers the case where there are only two experts and concludes that Anti-Seniority Rule is weakly dominated by the Seniority Rule. Moreover, a more recent paper Hahn(2011) also provides a result supporting our findings concluding with “anti-seniority” rule may be inferior even in a two-member committee. In his model, information is verifiable unlike our model. Another result of Hahn(2011) suggests that adding an individual expert to the committee leads to a weakly higher probability of the decision maker choosing the correct action. According to Hahn, as in the Condorcet(1785) Jury Theorem, this probability goes to 1 as the size of the committee goes to infinity.

With two experts as the source of information, this study is also related to literature on multiple senders. Krishna and Morgan(2001) studies a model in which perfectly informed experts offer advice to a decision maker. In contrast to previous papers we analyse, the welfare of the experts is also affected by the action taken by the decision maker. Information acquisition depends on the biases of the experts. If experts have conflicting interests, it is possible to get more information for the decision maker. The model where agents have conflicting interests promises an appropriate environment for the political economists. Gilligan and Krehbiel(1989) and Austen-Smith(1990) are examples from these strands of the literature.

Our paper and almost all of the above papers study a model in which the state of the world is one dimensional. On the other hand, Battaglini(2002) studies a multi-dimensional cheap talk model with multiple senders. He concludes that information can be fully revealed generically in equilibrium communication in contrast to one dimensional cheap talk models with one sender. Mcgee and Yang(2013) considers a model with two senders having partial and non-overlapping private information regarding the state of the world. Partial information implies that experts are only able to realize the related component of the state with his own expertise. This is the main departure of Mcgee and Yang from Battaglini.

Another attempt to find the optimal mechanism for the decision maker who try to maximize the probability of taking the correct decision is made by Glazer and Rubinstein(1997). The state of the world is a 5-tuple in their model and each debater is aware of all of the components of the state. These components take two possible values and decision maker has to take an action favouring one of these possible values. Before the decision has taken, debaters who have conflicting interest regarding the decision raise their arguments and counterarguments. Glazer and Rubinstein conclude that the amount of information elicited from an argument depends on the argument it counterargues in the optimal design of the debate rule. The main difference between our model and theirs is that experts does not observe the state in our model but have some private information.

3 The Model

There are two experts and a decision maker(DM). The state of the world is $\theta \in \{0,1\}$ which is not observed by any player until the end of the game. Public prior belief about θ is denoted by $p_0 = Prob(\theta = 1)$. Each expert $i = 1,2$ privately observes a signal $s_i \in \{0,1\}$ which is informative about the state. The precision of the signal of the expert i is given by $Prob(s_i = \theta \mid \theta) = q_i$. We assume that $\frac{1}{2} < q_1 < q_2$,i.e., expert 2 has more precise information. There are three periods in the game, $t = 0,1,2$. In period $t = 0$, both of the experts independently decide whether to send a message $m \in \{0,1\}$ to the decision maker or not. If an expert does not send a message in period $t = 0$, he has to do so in period $t = 1$. Experts must speak once and only once. The decision maker chooses an action $a \in \{0,1\}$ in period $t = 2$ after observing the messages of the experts but not the state, θ.

3.1 Payoffs and Strategies for the Experts

The space for the state of the world, θ, is denoted by Θ. S_i, M_i, and A denote the set of private signals for the experts, messages for the experts and actions for the decision maker respectively. Therefore, at the end of the period $t = 2$, a generic outcome is given by $z = \big(\theta, (m_1^0, m_2^0), (m_1^1, m_2^1), a\big)$ where $\theta \in \Theta$, $m_i^t \in M_i$, and $a \in A$.

Let $\Theta = \{0,1\}$, $S_i = \{0,1\}$, $M_i = \{0,1,\varnothing\}$ and $A = \{0,1\}$. Here the dimension of the message space is 3 and $m_i^t = \varnothing$ implies that expert i chooses not to speak at period t. We assume that experts care only about their reputation. More precisely, we assume that they just want their messages to match the state of the world. We also assume that there is discounting, i.e., experts prefer to speak early as long as they send the right message. In other words, the payoff function for experts is

$$u_i(z) = \begin{cases} 1, & \text{if } m_i^0 = \theta \\ \delta, & \text{if } m_i^1 = \theta \\ 0, & \text{otherwise} \end{cases}$$

where $\delta \in (0,1)$ is the discounting factor. Strategy of the expert i in period t is a function μ_i^t that determines m_i^t, the message of the expert i at period t, for any given history in the related information set. More specifically, we have the following strategies for the experts:

$$\mu_i^0 : S_i \longrightarrow M_i$$

and

$$\mu_i^1 : S_i \times M_1 \times M_2 \longrightarrow M_i$$

such that $\mu_i^1(s_i,(m_1^0,m_2^0)) \in \{0,1\}$ if $\mu_i^0(s_i) = \emptyset$ and $\mu_i^1(s_i,(m_1^0,m_2^0)) = \emptyset$ otherwise.

Therefore, expert i's strategy is a pair of function $\mu_i = (\mu_i^0,\mu_i^1)$. Let $\mu = (\mu_1,\mu_2)$ be the corresponding tuple for the experts' strategies.

There are two information sets for each expert;I_0 and I_1, the information sets at the beginning of period $t = 0$ and $t = 1$. I_0 consists of possible private signals received by the expert, i.e, $s_i = 0$ and $s_i = 1$. The elements of I_1 are the histories of the form of $(s_i,(m_1^0,m_2^0))$.

3.2 Payoffs and Strategies for the Decision Maker

We assume that the DM's payoff function is $v(z) = (\theta - \frac{1}{2})a$ where $\theta \in \Theta$ and $a \in A$. In other words, she wants her action to match the state.

Strategy of the decision maker is a function α that determines the action taken by the decision maker for any given history in the related information set. More specifically, strategy of the decision maker is the following:

$$\alpha : M_1 \times M_2 \longrightarrow A$$

Decision maker's information set, say I_2, consists of the histories of the form of (m^0, m^1) where $m^0 = (m_1^0, m_2^0)$ and $m^1 = (m_1^1, m_2^1)$.

EQUILIBRIUM CONCEPT

Our equilibrium concept will be Perfect Bayesian Equilibrium(PBE). An assessment $((\mu, \alpha), p)$ is a PBE if it satisfies sequential rationality and consistency, i.e, if the experts and the decision maker who play according to μ and α respectively are sequentially rational and the belief system p is consistent. In other words, beliefs must be derived from strategies using Bayes' Rule whenever possible. Here a belief system p is a collection of probability distributions such that $p(h) \in \Delta(\Theta)\ \forall h \in I_j$ where $j = 0, 1, 2$.

4 Equilibrium Analysis

THE DECISION MAKER

First of all, we consider the situation from the perspective of the decision maker. Since the decision maker wants her action to match the state and she is assumed to be expected-utility maximizer, her action is just conditional on the public belief about the state. The following lemma states the relation between the posterior belief, $p(m^0, m^1)$, and the action of the decision maker, a.

Lemma 1. *In any equilibrium,*

$$\alpha(m^0, m^1) = \begin{cases} 1, & p(m^0, m^1) \geq \frac{1}{2} \\ 0, & otherwise \end{cases}$$

If $p(m^0, m^1) = \frac{1}{2}$ the decision maker is indifferent between $a = 0$ and $a = 1$. For convenience we assume that $a = 1$ in this case.

Proof. Let $p(m^0, m^1) = p^*$. If the decision maker takes the action $a = 1$, her expected payoff will be $\frac{1}{2}p^* + (1 - p^*)(-\frac{1}{2})$. Otherwise it is equal to 0. Hence, taking action $a = 1$ is optimal iff $\frac{1}{2}p^* + (1 - p^*)(-\frac{1}{2}) \geq 0$. The result follows. □

THE EXPERTS

Now, we need to find which expert chooses when to speak and which message is sent. We solve the whole endogenous order model step by step by starting with finding the optimal message sent by an expert who is supposed to send one. Then we consider the case where he is supposed to send his message either at period $t = 0$ or $t = 1$. Finally we add the other expert to the model and consider the whole endogenous model. The intuition behind this approach is the fact that strategic interaction between the experts is limited in the two period-two expert model since the expected payoff of expert i depends on only m_i, and m_j is irrelevant. Hence, it is possible to use the results in the one period-one expert model, and two period-one expert model.

4.1 One period-One Expert

Since there is only one period, there is no discounting in this case. Therefore, expected payoff of an expert who receives the private signal $s_i \in S_i$ is simply $Prob(\theta = 1 \mid s_i)$ if he sends the message $m_i = 1$ and $Prob(\theta = 0 \mid s_i)$ if he sends the message $m_i = 0$. During the equilibrium search and analysis, the belief on the state is updated by experts according to their private signals and by the decision maker according to the messages sent by experts. For the purpose of our analysis, it is convenient to express Bayesian updating in log-likelihood ratio of beliefs.

Definition 1. $\ell(\lambda_0, q, s_i)$ is defined to be the log likelihood ratio(LLR) of beliefs after receiving signal s_i with precision q where $\lambda_0 = \log \frac{p_0}{1-p_0}$. In other words; $\ell(\lambda_0, q, s_i) = \log \frac{Prob(\theta=1|s_i)}{Prob(\theta=0|s_i)}$.

After defining the function ℓ we can determine the values of $\ell(\lambda_0, q, s_i = 1)$ and $\ell(\lambda_0, q, s_i = 0)$ by using the Bayes' Rule. Bayes' Rule implies the following result:

$$\ell(\lambda_0, q, s_i) = \begin{cases} \lambda_0 + e_i, & \text{if } s_i = 1 \\ \lambda_0 - e_i, & \text{if } s_i = 0 \end{cases}$$

where $e_i = \log \frac{q}{1-q}$ can be described as the influence of the expert i of precision q on the public belief.

Now the decision-making process of the expert is simpler to picture. He just wants his message to match the state at the end. Therefore, his message depends on only $\ell(\lambda_0, q, s_i)$. The following lemma states the strategy of the expert ,and the relation between $\ell(\lambda_0, q, s_i)$ and the message sent by the expert in the equilibrium.

Lemma 2. *If there is only one period and one expert, in the equilibrium we have*

$$\mu_i^0(s_i) = \begin{cases} 1, & \ell(\lambda_0, q, s_i) \geq 0 \\ 0, & \ell(\lambda_0, q, s_i) < 0 \end{cases}$$

Proof. First of all, note that Expected payoff of an expert after he gets a private signal s_i is

$$Prob(\theta = 1|s_i) = \frac{Prob(s_i|\theta = 1)Prob(\theta = 1)}{Prob(s_i|\theta = 1)Prob(\theta = 1) + Prob(s_i|\theta = 0)Prob(\theta = 0)}, \text{ if he sends } m_i = 1$$

$$Prob(\theta = 0|s_i) = \frac{Prob(s_i|\theta = 0)Prob(\theta = 0)}{Prob(s_i|\theta = 1)Prob(\theta = 1) + Prob(s_i|\theta = 0)Prob(\theta = 0)}, \text{ if he sends } m_i = 0$$

by Bayes' Rule. Hence sending $m = 1$ is optimal iff $Prob(\theta = 1|s_i) \geq Prob(\theta = 0|s_i)$. By dividing both sides by $Prob(\theta = 0|s_i)$, we get the likelihood ratio(LR) after receiving signal s_i. In other words,sending $m_i = 1$ is optimal iff $\frac{Prob(s_i|\theta=1)Prob(\theta=1)}{Prob(s_i|\theta=0)Prob(\theta=0)} \geq 1$. This is true iff $\ell(\lambda_0, q, s_i) \geq 0$ by Definition 1. The result follows. □

4.2 Two period-One Expert

There are two periods. We suppose that it is possible to get a signal about the state at the beginning of each period. When will the expert speak? Why?

Let s be the private signal received with precision q at the beginning of $t = 0$ and s' be the private signal received with a possibly different precision q' at the beginning of $t = 1$. Consider the expected payoff obtained if the agent speaks at $t = 0$. It is either

$Prob(\theta = 1 \mid s_i)$ or $Prob(\theta = 0 \mid s_i)$ depending on the message sent by the expert

Note that $Prob(\theta = 1 \mid s_i)$ could also be represented as $p(s)$ by using the belief system p. Then $Prob(\theta = 0 \mid s_i) = 1 - p(s)$. Hence $\ell(\lambda_0, q, s) = \log \frac{p(s)}{1-p(s)}$. Note that sending $m = 1$ is optimal if and only if $\ell(\lambda_0, q, s) \geq 0$ iff $p(s) \geq \frac{1}{2}$.

Is it possible to determine under what conditions the expert speaks at period $t = 0$ and does not take the advantage of an extra signal at the beginning of period $t = 1$? Intuitively, the expert speaks at period $t = 0$ if he is sure that he will not change his mind about which state is more likely no matter what the future signal is. And because of the discounting speaking at $t = 0$ is strictly preferred.

Proposition 1. *There exists* $\underline{\delta} \in (0,1)$ *such that* $\forall \delta > \underline{\delta}$, *expert sends a message at* $t = 0$ *iff* $\ell(\lambda_0, q, s)$ *has the same sign with* $\ell(\lambda(s), q', s')$ $\forall s' \in \{0,1\}$ *where* $\lambda(s) = \ell(\lambda_0, q, s)$.

Proof. Assume $\ell(\lambda_0, q, s)$ has the same sign with $\ell(\lambda(s), q', s')$ $\forall s' \in \{0,1\}$.

Suppose first that $\ell(\lambda_0, q, s) \geq 0$ and $\ell(\lambda(s), q', s') \geq 0$. Above assumptions imply that if the expert sends a message at $t = 0$ he sends $m = 1$. Similarly, if he waits to send a message at $t = 1$, his message would be $m = 1$ for any value of s'. So his expected payoff is $Prob(\theta = 1 \mid s_i)$ which is actually equal to $p(s)$. Expected payoff to waiting to receive a signal s' is:

$$\delta[Prob(s' = 1 \mid s)Prob(\theta = 1 \mid s, s' = 1) + Prob(s' = 0 \mid s)Prob(\theta = 1 \mid s, s' = 0)] = \delta p(s) < p(s)$$

Result follows from *the law of iterated expectations* and by the fact that $\delta < 1$. The other case where $\ell(\lambda_0, q, s) < 0$ and $\ell(\lambda(s), q', s') < 0$ is also very similar. Hence, speaking now, at $t = 0$ is strictly preferred.

To prove the converse, assume first that δ is sufficiently large enough, i.e $\underline{\delta} < \delta < 1$ where $\underline{\delta}$ is defined as:

$$inf\{\delta : \delta[p(s) + Prob(s' = 0 \mid s)(1 - 2Prob(\theta = 1 \mid s, s' = 0))] > p(s)\} = \frac{p(s)}{p(s) + \epsilon}$$

where $\epsilon = Prob(s' = 0 \mid s)(1 - 2Prob(\theta = 1 \mid s, s' = 0))$. Assume first that $\ell(\lambda_0, q, s) \geq 0$ and $\ell(\lambda(s), q', s') < 0$. These imply that $p(s) \geq \frac{1}{2}$ and $p(s, s') < \frac{1}{2}$. Then since $Prob(\theta = 1 \mid s, s' = 0) < \frac{1}{2}$ by the assumption and $Prob(s' = 0 \mid s) > 0$ irrespective of s, we get that

$\epsilon > 0$ and clearly $\underline{\delta} < 1$. Note that the assumption that $p(s,s') < \frac{1}{2}$ is only valid if $s' = 0$ since we know that $p(s,0) < p(s) < p(s,1)$ by Bayes' Rule and $p(s) \geq \frac{1}{2}$.

Expected payoff of speaking now is again $p(s)$ since it is optimal to send $m = 1$ for the expert. And the following is the expected payoff to waiting to receive a signal s':

$$\begin{aligned}
&\delta[Prob(s'=1 \mid s)Prob(\theta=1 \mid s,s'=1) + Prob(s'=0 \mid s)(1-Prob(\theta=1 \mid s,s'=0))] \\
&= \delta[Prob(\theta=1 \mid s) - Prob(s'=0 \mid s)(Prob(\theta=1 \mid s,s'=0)) + Prob(s'=0 \mid s)(1-Prob(\theta=1 \mid s,s'=0))] \\
&= \delta[p(s) + Prob(s'=0 \mid s)(1-2Prob(\theta=1 \mid s,s'=0))] \\
&> p(s)
\end{aligned}$$

If $\delta > \underline{\delta}$, speaking at $t = 1$ is optimal. The other case is also similar. Therefore, the proof of Proposition 1 is complete.

□

4.3 Two period-Two Expert

Suppose that there are two experts with their own private information. Ottaviani and Sorensen(2001) have constrained the experts to speak in a fixed order. Since there is only *two* experts in our model, the number of possible fixed order mechanism to examine is just 2. We will compare our results with the results of Ottaviani and Sorensen(2001). Furthermore, we will consider the case where the experts possibly choose when to speak conditional on their private information and this analysis will constitute the core of this paper.

FIXED ORDER

If the experts are not allowed to speak simultaneously, one of them should speak at $t = 0$ and the other one should speak at $t = 1$. The decision maker may design the debate such that experts speak in order of increasing expertise or the senior expert takes the lead and speak first. The former design of the order is called *Anti-Seniority Rule* whereas the latter is called *Seniority Rule*. In the case that expert i speaks first, the strategies of the experts becomes $\mu_i^0 : S_i \longrightarrow M_i$ and $\mu_j^1 : S_j \times M_i \longrightarrow M_j$ since the experts are not allowed to speak simultaneously

Is it possible for the decision maker to learn the private signals of both experts by applying one of the above rules? If it is possible, does the decision of the decision maker depend on both of the signals? The following lemma answers these questions for the Anti-Seniority Rule:

Lemma 3. *If anti-seniority rule is applied, there is a set of PBE in which both experts always tell the truth iff* $max\{-e_1, e_1 - e_2\} \leq \lambda_0 < min\{e_1, e_2 - e_1\}$.

Proof. Assume both experts tell the truth when they are supposed to speak. Since the junior speaks first and tells the truth we have $\mu_1^0(s_1 = 1) = 1$ and $\mu_1^0(s_1 = 0) = 0$. Then by Lemma 2 we have $\lambda_0 + e_1 \geq 0$ and $\lambda_0 - e_1 < 0$. Since the senior tells the truth in any case we have $\mu_2^1(s_2, m_1^0) = s_2 \; \forall m_1^0 \in \{0, 1\}$. Hence again by Lemma 2, we have $\lambda_0 + e_1 + e_2 \geq 0$, $\lambda_0 + e_1 - e_2 < 0$ and $\lambda_0 - e_1 + e_2 \geq 0$, $\lambda_0 - e_1 - e_2 < 0$. Now we need to specify the region where both experts tell the truth by solving the system of inequalities. The solution set for the above 6 inequalities is :$max\{-e_1, e_1 - e_2\} \leq \lambda_0 < min\{e_1, e_2 - e_1\}$. Suppose $2e_1 < e_2$. Then we have $-e_1 < \lambda_0 < e_1$ as a solution set. If $e_2 < 2e_1$ then the solution set is $e_1 - e_2 < \lambda_0 < e_2 - e_1$. We do not include the borders in the solution set because the probability that $\lambda_0 = -e_1$ is simply 0. If $\lambda_0 \in [-e_1, e_1]$, then truth-telling strategy is sequentially rational for both experts. □

Corollary 1. *If the difference between the precisions of the experts is not enough so that* $e_2 < 2e_1$ *then the region where both experts tell the truth shrinks when the anti-seniority rule is applied.*

Proof. Since $-e_1 < e_1 - e_2 < \lambda_0 < e_2 - e_1 < e_1$, result follows by Lemma 3. □

However, note that the decision of the decision maker does not depend on the signal of the junior expert for the region $\lambda_0 \in [-e_1, e_1]$ since the decision maker takes action $a = 1$ iff $m_2^1 = 1$ and $a = 0$ iff $m_2^1 = 0$. Hence, junior is overruled in this case. Whenever the decision maker is able to learn the private signal of the junior she is also able to learn the private signal of the senior. Hence Anti-Seniority Rule is not much helpful for the decision maker although information is transmitted perfectly for the region $\lambda_0 \in [-e_1, e_1]$. If $\lambda_0 \in [e_1, e_2]$ or $\lambda_0 \in [1 - e_2, 1 - e_1]$ the decision maker is able to learn the private message of the senior although the junior just pools when he speaks at $t = 0$. In any other case there is no information transmitted to the decision maker through the Anti-Seniority Rule.

Now consider the seniority rule. Is there any extra benefit that it offers? The following lemma is related to the information transmission through the seniority rule.

Lemma 4. *There is no PBE where both experts always tell the truth if Seniority Rule is applied.*

Proof. To get a contradiction suppose both experts always tell the truth whenever they are supposed to speak. So $\mu_2^0(s_2) = s_2$ and $\mu_1^1(s_1, m_2^0) = s_1\ \forall m_2^0 \in \{0,1\}$. Then by Lemma 2 we have $\lambda_0 + e_2 \geq 0$ and $\lambda_0 - e_2 < 0$ for the senior. And $\lambda_0 + e_2 + e_1 \geq 0$, $\lambda_0 + e_2 - e_1 < 0$ and $\lambda_0 - e_2 + e_1 \geq 0$, $\lambda_0 - e_2 - e_1 < 0$ for the junior. Since $0 < e_2 - e_1 \leq \lambda_0 < e_1 - e_2 < 0$, we get a contradiction. □

However, it might be possible to find a set of PBE where the senior always tell the truth and also junior tells the truth depending on the message of the senior. The following lemma is about the existence of this kind of PBE.

Lemma 5. *There is a set of PBE where the senior always tell the truth and the junior can also tell the truth depending on the message of the senior iff* $\lambda_0 \in (e_2 - e_1, e_2)$ *or* $\lambda_0 \in (-e_2, e_1 - e_2)$.

Proof. Suppose $\lambda_0 \geq 0$.If $\mu_2^0(s_2 = 1) = 1$ then junior always pools since $e_1 < \lambda_0 + e_2$. Hence it is not possible to learn the private signal of the junior if $s_2 = 1$ when $\lambda_0 \geq 0$. Assume $s_2 = 0$. Also assume junior also tells the truth. Hence $\mu_1^1(s_1, m_2^0 = 0) = s_1$. Then by Lemma 2, we have $\lambda_0 - e_2 + e_1 \geq 0$ and $\lambda_0 - e_2 - e_1 < 0$. By solving the following system of inequalities :$\lambda_0 \geq 0$, $\lambda_0 - e_2 < 0$, $\lambda_0 + e_2 \geq 0$, $\lambda_0 - e_2 + e_1 \geq 0$,and $\lambda_0 - e_2 - e_1 < 0$. We get the solution set : $\lambda_0 \in (e_2 - e_1, e_2)$.

If $\lambda_0 < 0$ proof is similar and the solution set is $\lambda_0 \in (-e_2, e_1 - e_2)$.

It is easy to show that it is indeed a PBE if $\lambda_0 \in (e_2 - e_1, e_2)$ or $\lambda_0 \in (-e_2, e_1 - e_2)$. □

Hence if $\lambda_0 \in (e_2 - e_1, e_2)$ or $\lambda_0 \in (-e_2, e_1 - e_2)$ then decision maker could learn the private signals of both experts with a positive probability. Furthermore, if the senior sends a message contrary to public prior belief then the signal of the junior has also importance. Hence both of the signals affect the decision of the decision maker.

Although information is not always transmitted perfectly, the seniority rule provides the decision maker with more relevant information. For any other case both seniority rule and anti-seniority rule produce the same outcome. The following proposition states that.

Proposition 2. *If $\lambda_0 \in (e_2 - e_1, e_2)$ or $\lambda_0 \in (-e_2, e_1 - e_2)$, Seniority Rule is strictly preferred to the Anti-Seniority Rule by the decision maker. DM is indifferent between Seniority Rule and Anti-Seniority Rule for any other value of λ_0.*

Ottaviani and Sorensen (2001) compares Anti-Seniority Rule with any other speaking rules. There are $n!$ possible speaking order in their model where $n \in \mathbb{Z}^+$ is the number of experts with distinct expertise levels in contrast to our model where we fixed $n = 2$. Ottaviani and Sorensen (2001) concludes that the fact that any other speaking order implements the first best, revealing the information fully, does not mean that also Anti-seniority Rule necessarily implements the first best. We extend this result and conclude that if there are only 2 experts then Anti-Seniority Rule is weakly dominated by Seniority Rule in terms of the amount of valuable information transmitted to the DM. Also, this is the same result concluded by Chamley(2003). Although we fixed $n = 2$ in our model, the number of possible speaking order mechanism is not 2 since we propose an alternative mechanism that we call *Endogenous Order* mechanism.

ENDOGENOUS ORDER

Now, the experts are allowed to choose which period they want to speak. This allows them to speak at $t = 0$ after receiving a certain signal and wait otherwise. The outcome of this endogenously determined order of speaking may be the one where experts speak simultaneously at $t = 0$ or at $t = 1$ no matter what their private information is. Possibly the outcome may be very similar to the one of the orders determined by the anti-seniority rule or seniority rule. Alternatively, experts may simply take advantage of this endogenous mechanism and decide whether to send a message at $t = 0$ or wait according to private information they received.

If the public prior belief about state is not too extreme, it may be wise to wait to see the other expert's message. Does the message of the other expert reveal any information about the private signal of the other expert? In other words, does the other expert tell the truth? The message is valuable only if it reveals the type of the other expert. First of all, consider the case where both experts speak at $t = 0$ after both signals. In this case both of the experts are not credible because they choose to give up any possible future gain resulting from waiting even if they received a private signal contrary to the prior belief. When an expert receives a signal contrary to the public prior belief he infers that the signal received is most probably in error or the public prior belief is too extreme so that it is not a good idea to send a message proposed by that signal at $t = 0$ or $t = 1$ even if there is no error. Hence we have the following lemma.

Lemma 6. *If both experts speak at period $t = 0$ after both signals in the equilibrium then none of them tells the truth.*

Proof. Suppose in equilibrium both of the experts speak at $t = 0$ after both signals. In other words; $\mu_i^0(s_j) \in \{0, 1\}$ for each expert $i \in \{1, 2\}$ and each signal $s_j \in \{0, 1\}$. To get a contradiction suppose expert i tells the truth. Then $\mu_i^0(0) = 0$ and $\mu_i^0(1) = 1$. Sending these messages imply that $\ell(\lambda_0, q_i, 0) < 0$ and $\ell(\lambda_0, q_i, 1) \geq 0$ by lemma 2.

Consider expert j:

<u>*Case1:*</u> Assume $\ell(\lambda_0, q_j, s_j) \geq 0 \; \forall s_j$.

Expert j speaks at $t = 0$ when it is possible to learn s_i by waiting since expert i tells the truth. Then by Proposition 1 and the fact that expert j speaks at $t = 0$ after both signals, $\ell(\lambda(s_i), q_j, s_j) \geq 0. \; \forall s_i, s_j$. However for $s_i = s_j = 0$, we have $\ell(\lambda(0), q_j, 0) = \lambda(0) - a_j = \lambda_0 - a_i - a_j < \lambda_0 - a_i < 0$. Since $a_j > 0$, by $q_j > \frac{1}{2}$. So we get a contradiction.

<u>*Case2:*</u> Assume $\ell(\lambda_0, q_j, s_j) < 0 \; \forall s_j$.

Expert j speaks at $t = 0$ when it is possible to learn s_i by waiting since expert 1 tells the truth. Then by Proposition 1 and the fact that expert j speaks at $t = 0$ after both signals, $\ell(\lambda(s_i), q_j, s_j) < 0. \; \forall s_i, s_j$. We need to have indeed $\ell(\lambda(1), q_j, 1) < 0$. However $\ell(\lambda(1), q_j, 1) = \lambda(1) + a_j = \lambda_0 + a_i + a_j > \lambda_0 + a_i \geq 0$. So we get a contradiction.

<u>*Case3:*</u> Assume $\ell(\lambda_0, q_j, s_j) < 0$ if $s_j = 0$ and $\ell(\lambda_0, q_j, s_j) \geq 0$ if $s_j = 1$.

So $\lambda_0 + a_j \geq 0$ and $\lambda_0 - a_j < 0$. Then expert j also tells the truth if he speaks at $t = 0$. Then, by Proposition 1, since both experts speak at $t = 0$ when it is possible to learn the other one's signal we have the followings:

$$\lambda_0 + a_j - a_i \geq 0 \text{ and } \lambda_0 - a_j + a_i < 0 \text{ for expert } j$$

$$\lambda_0 + a_i - a_j \geq 0 \text{ and } \lambda_0 - a_i + a_j < 0 \text{ for expert } i$$

So we have a contradiction, again.

□

Hence, if both experts speak at period $t = 0$ after both signals then none of them tells the truth about their private information in the equilibrium. In this case the private signals of the experts are suppressed by the public prior belief. We have the following definition:

Definition 2 (Herding)**.** If an expert's private signal is suppressed by the public prior belief and his message does not contain any information, he is said to be *herding*.

We now show that there is a set of pure strategy pooling PBE where both experts speak at $t = 0$ after both signals and they simply herd. There is such an equilibrium because waiting is costly but there is no benefit as experts are unable to learn the other's signal.

Proposition 3. *There is a PBE in which both experts speak at* $t = 0$ *after receiving both signals and they simply herd iff* $\lambda_0 < -e_2$ *or* $\lambda_0 > e_2$*.*

Proof. Assume first that $\lambda_0 < 0$. Then clearly $\lambda_0 - e_i < 0$. Hence, only possible pure strategy pooling PBE is the one where expert i sends the message $m_i^0 = 0$ after both signal. So we need to have $\lambda_0 + e_i < 0$, too. Since $q_1 < q_2$, we have $e_1 < e_2$. So $-e_2 < -e_1$. Note that $\lambda_0 < -e_2$ iff $p_0 < 1 - q_2$ by the definition. Hence, we have the necessary condition for the existence of a PBE :$\lambda_0 < -e_2$.

Now suppose that $\lambda_0 < -e_2$. Then, $\ell(\lambda_0, q_i, s_i) < 0$ $\forall i \in \{1,2\}$ and $\forall s_i \in \{0,1\}$. Hence, sequential rationality of experts imply that $\mu_i^0(s_i) = 0$. Since experts just herd and no information is transmitted to the DM, we have $p((m_1^0 = 0, m_1^1 = \emptyset), (m_2^0 =$

$0, m_2^1 = \emptyset)) = p_0$. Also sequential rationality of the decision maker is satisfied iff $p((m_1^0, m_1^1), (m_2^0, m_2^1)) < \frac{1}{2}$, for any other history. We have the following strategies for the experts and belief system that constitutes relevant PBE:

$$\mu_1^0(s_1) = 0 \text{ and } \mu_2^0(s_2) = 0 \ \forall s_1, s_2 \in \{0,1\}$$

$$\mu_1^1(s_1, m_1 = 0, m_2) = \emptyset \text{ and } \mu_2^1(s_2, m_1, m_2 = 0) = \emptyset \ \forall m_1, m_2 \in \{0,1\}$$

$$p((m_1^0 = 0, m_1^1 = \emptyset), (m_2^0 = 0, m_2^1 = \emptyset)) = p_0 \text{ and } p((m_1^0, m_1^1), (m_2^0, m_2^1)) < \frac{1}{2}, \text{ for any other history}$$

Also, similarly, for $e_2 < \lambda_0$, or equivalently $q_2 < p_0$, there is a pure strategy pooling PBE iff

$$\mu_1^0(s_1) = 1 \text{ and } \mu_2^0(s_2) = 1 \ \forall s_1, s_2 \in \{0,1\}$$

$$\mu_1^1(s_1, m_1 = 0, m_2) = \emptyset \text{ and } \mu_2^1(s_2, m_1, m_2 = 0) = \emptyset \ \forall m_1, m_2 \in \{0,1\}$$

$$p((m_1^0 = 1, m_1^1 = \emptyset), (m_2^0 = 1, m_2^1 = \emptyset)) = p_0 \text{ and } p((m_1^0, m_1^1), (m_2^0, m_2^1)) \geq \frac{1}{2}, \text{ for any other history}$$

□

Is there any PBE in which one of the experts choose to wait and speak at $t = 1$ no matter which private signal he receives? If one of the experts speaks absolutely in the period $t = 1$ then what is the meaning of waiting for the other expert? There is a cost of waiting and there will be no benefit since it is impossible to learn something valuable from the expert who speaks at $t = 1$ no matter what happens. It is wise to speak as soon as possible then. However this implies that he also tells the truth in the equilibrium otherwise the expert who is speaking at $t = 1$ deviates and speaks at $t = 0$. Formally, we have the following proposition :

Proposition 4 (Sequentiality)**.** *If an expert chooses to speak later, at* $t = 1$*, after both signals, then the other one speaks at* $t = 0$ *after both signals and tells the truth in the equilibrium.*

Proof. Assume expert i speaks at $t = 1$ after both signals. Suppose on the contrary that expert $j \neq i$ prefers to speak at $t = 1$ after at least one signal, say the signal s^*, in the equilibrium.

Assume first that $\ell(\lambda_0, q_j, s^*) \geq 0$. Define the posterior public belief about the state after the period $t = 0$ as $p(s_i, m^0)$ where $m^0 = (m_1^0, m_2^0)$. Then, $p(s^*, (\emptyset, \emptyset)) \geq \frac{1}{2}$ and so $\mu_j^1(s^*, (\emptyset, \emptyset)) = 1$ and expected payoff of the expert j is : $\delta p(s^*)$. However, since $p(s^*, (\emptyset, \emptyset)) = p(s^*)$, speaking at $t = 0$ results in an expected payoff of $p(s^*)$. Hence, speaking at $t = 0$ is more profitable, so expert j chooses to speak earlier.

Now, suppose that expert j does not tell the truth when he speaks at $t = 0$. Then, $p(s_i, (\emptyset, m_j)) = p(s_i)$. Hence it is more profitable to speak at $t = 0$ rather than $t = 1$ for the expert i with signal s_i because of the discounting parameter δ.

The other case is similar. □

Corollary 2. *There is no PBE in which both experts speak at $t = 1$ simultaneously after both signals.*

It may be the case that one of the experts speaks first and the other one speaks later no matter which private signal they receive. This is exactly the same scenario where order of the speech is fixed by some designer before the debate. In a fixed order speech, who should speak first; senior or junior? Since precisions are common knowledge junior knows that any possible information revealed by the senior is more reliable than his own private information. So he prefers to wait whereas senior prefers to speak earlier because of the discounting. The following proposition states that.

Proposition 5 (Seniority)**.** *If an expert chooses to speak later, at $t = 1$, after both signals, then the other one speaks at $t = 0$ after both signals and tells the truth in the equilibrium. Moreover, senior (the one with higher precision; $q_2 > q_1$) is the one who speaks first in the equilibrium.*

Proof. The first part of the statement is actually Proposition 4 and it is proved above. Now, assume that the expert who speaks at $t = 0$ is not the senior, but the junior (the one with precision q_1). Then by Proposition 4, we know that junior tells the truth: $\ell(\lambda_0, q_1, 1) \geq 0$ and $\ell(\lambda_0, q_1, 0) < 0$. In other words:

$$\lambda_0 + e_1 \geq 0 \text{ and } \lambda_0 - e_1 < 0$$

Above results imply that $\lambda_0 + e_2 \geq 0$ and $\lambda_0 - e_2 < 0$ also hold since $e_1 < e_2$. Then by Proposition 1, since senior prefers to speak later after both signals, we have:

$$\lambda_0 + e_2 - e_1 < 0 \text{ and } \lambda_0 - e_2 + e_1 \geq 0$$

which implies $0 < e_2 - e_1 \leq \lambda_0 < e_1 - e_2 < 0$. So we get a contradiction. Hence it must be the senior who speaks first,at $t = 0$, after both signals in this kind of equilibrium.

$\square$

Now we have the following equilibrium which produces the seniority rule in a debate as an outcome of the endogenous order mechanism:

Proposition 6. *There is a PBE in which the senior speaks first at* $t = 0$ *and tells the truth, and the junior speaks second and herds when he is supposed to speak iff* $\lambda_0 \in [e_1 - e_2, e_2 - e_1]$.

Proof. We need to find the necessary conditions such that the situation where senior speaks first after both signals and tells the truth while junior speaks at $t = 1$ after both signals and herds is indeed an equilibrium. Assume that senior speaks at $t = 0$ and tells the truth, so $\ell(\lambda_0, q_2, 1) \geq 0$ and $\ell(\lambda_0, q_2, 0) < 0$. In other words:

$$\lambda_0 + e_2 \geq 0 \text{ and } \lambda_0 - e_2 < 0$$

Now we have two possibilities for the junior: $\lambda_0 + e_1 < 0$ or $\lambda_0 + e_1 \geq 0$.

<u>*Case1:*</u> Assume $\lambda_0 + e_1 < 0$. Then $\lambda_0 - e_1 < 0$ also holds. Then by Proposition 1, since junior prefers to speak later after both signals, we have:

$$\lambda_0 + e_1 + e_2 \geq 0 \text{ and } \lambda_0 - e_1 + e_2 \geq 0$$

Now we have a system of inequalities such that :

$$\lambda_0 + e_1 < 0 \; , \; \lambda_0 - e_1 < 0 \; , \; \lambda_0 - e_2 < 0$$

$$\lambda_0 + e_2 \geq 0 \; , \; \lambda_0 + e_1 + e_2 \geq 0 \; , \; \lambda_0 - e_1 + e_2 \geq 0$$

And provided that $2e_1 < e_2$, we have the following solution set for λ_0:

$$e_1 - e_2 \leq \lambda_0 < -e_1$$

Case2: Assume $\lambda_0 + e_1 \geq 0$. Then we have two possibilities for the bad signal, $s_1 = 0$.
Case2i: Assume $\lambda_0 - e_1 \geq 0$. Then by Proposition 1, since junior prefers to speak later after both signals, we have:

$$\lambda_0 + e_1 - e_2 < 0 \text{ and } \lambda_0 - e_1 - e_2 < 0$$

Now we have a system of inequalities such that :

$$\lambda_0 + e_1 - e_2 < 0 \text{ , } \lambda_0 - e_1 - e_2 < 0 \text{ , } \lambda_0 - e_2 < 0$$

$$\lambda_0 + e_2 \geq 0 \text{ , } \lambda_0 + e_1 \geq 0 \text{ , } \lambda_0 - e_1 \geq 0$$

And provided that $2e_1 < e_2$, we have the following solution set for λ_0:

$$e_1 \leq \lambda_0 < e_2 - e_1$$

Case2ii: Assume $\lambda_0 - e_1 < 0$. Then by Proposition 1, since junior prefers to speak later after both signals, we have:

$$\lambda_0 + e_1 - e_2 < 0 \text{ and } \lambda_0 - e_1 + e_2 \geq 0$$

Now we have a system of inequalities such that :

$$\lambda_0 + e_1 - e_2 < 0 \text{ , } \lambda_0 - e_1 < 0 \text{ , } \lambda_0 - e_2 < 0$$

$$\lambda_0 + e_2 \geq 0 \text{ , } \lambda_0 + e_1 \geq 0 \text{ , } \lambda_0 - e_1 + e_2 \geq 0$$

and we have the following solution set for λ_0:

$$max\{-e_1, e_1 - e_2\} \leq \lambda_0 < min\{e_2 - e_1, e_1\}$$

Now assume $2e_1 < e_2$. Then by ***Case1*** for $e_1 - e_2 \leq \lambda_0 < -e_1$, or by ***Case2i*** for $e_1 \leq \lambda_0 < e_2 - e_1$, or by ***Case2ii*** for $-e_1 \leq \lambda_0 < e_1$ we can specify a PBE such that senior speaks first after both signals and tells the truth, and junior speaks last after both signals. More specifically it is true for the union of these intervals, $e_1 - e_2 \leq \lambda_0 < e_2 - e_1$

Assume $e_2 \geq 2e_1$ Then by ***Case2ii*** we have $e_1 - e_2 \leq \lambda_0 < e_2 - e_1$. Therefore it is always a necessary condition that $e_1 - e_2 \leq \lambda_0 < e_2 - e_1$ to have a PBE in which the senior speaks first and tells the truth and the junior speaks at $t = 1$ and herds. Given that $\lambda_0 \in [e_1 - e_2, e_2 - e_1]$,the following beliefs are consistent with the related strategies of the experts and decision maker. Also both experts are sequentially rational given those strategies:

$$\mu_1^0(s_1) = \emptyset \text{ and } \mu_2^0(s_2) = s_2 \ \forall s_1, s_2 \in \{0, 1\}$$

$$\mu_2^1(s_2, (m_1^0, m_2^0)) = \emptyset \ \forall m_1^0 \in \{0, 1, \emptyset\} \text{ and } \forall m_2^0 \in \{0, 1\}$$

$$\mu_1^1(s_1, (\emptyset, m_2^0)) = \begin{cases} 1 & \text{, if } m_2^0 = 1 \\ 0 & \text{, otherwise} \end{cases}$$

$$p(s_1, (m_1^0, m_2^0)) = \begin{cases} p(s_1)(s_2 = m_2^0) & \text{,if } m_2^0 \in \{0, 1\} \\ p(s_1) & \text{, otherwise} \end{cases}$$

$$p(m^0, m^1) = p(s_2 = m_2^0)$$

where $m^t = (m_1^t, m_2^t)$ and $p(s_i)$ is simply Bayesian updated form of the public prior p_0, and $p(s_i)(s_j)$ is the Bayesian updated form of the posterior belief $p(s_i)$. □

Ottaviani and Sorensen(2001) is the closest paper to ours in terms of the way that the model is constructed. The main difference of this paper from Ottaviani and

Sorensen(2001) is that our model allows experts to decide the order of speaking endogenously. Experts may choose to speak immediately after receiving a certain private signal and it might be profitable to wait otherwise. The signal that an expert choose to speak immediately is naturally related to public prior belief. Hence if both experts choose to speak immediately after some signals there is somehow a relation between these signals. The following lemma states that in a formal way.

Lemma 7. *If one of the experts speaks at $t = 0$ after one signal, and speaks at $t = 1$ after the other signal in equilibrium then the other expert also speaks at $t = 0$ after one signal, and speaks at $t = 1$ after the other signal. Let s_i^* and s_j^* be the signals after which they speak at $t = 0$. Generically $\ell(\lambda_0, q_i, s_i^*)$ and $\ell(\lambda_0, q_j, s_j^*)$ must have the same sign and $s_i^* = s_j^*$.*

Proof. We assume expert i speaks at $t = 0$ after s_i^*, and speaks at $t = 1$ after the other signal in equilibrium. Then by Proposition 4 expert $j \neq i$ does not speak at $t = 1$ after both signals. To get a contradiction, assume that expert $j \neq i$ speaks at $t = 0$ after both signals in equilibrium.

Note that expert i waits after the signal other than s_i^* and speaks at $t = 1$ only if expert j reveals his signal at $t = 0$. Otherwise expert i should have chosen to speak at $t = 0$ because of the discounting. So expert j must tell the truth in the equilibrium : $\lambda_0 + e_j \geq 0$ and $\lambda_0 - e_j < 0$.

Also expert j is able to infer that which signal the agent i have accurately if he waits since agent i speaks at $t = 0$ iff his signal is s_i^*. So we should have $\lambda_0 + e_j - e_i \geq 0$ and $\lambda_0 - e_j + e_i < 0$ by Proposition 1.

We have two possibilities for $\ell(\lambda_0, q_i, s_i^*)$:

<u>**Case1:**</u> Assume $\ell(\lambda_0, q_i, s_i^*) \geq 0$.

<u>**Case1i:**</u> Suppose $s_i^* = 1$.Then $\lambda_0 + e_i \geq 0$.Since he speaks at $t = 0$ after the signal $s_i^* = 1$, we should have $\lambda_0 + e_i - e_j \geq 0$ by Proposition 1. But we have $\lambda_0 - e_j + e_i < 0$, so a contradiction.

<u>Case1ii:</u> Suppose $s_i^* = 0$.Then $\lambda_0 - e_i \geq 0$.Since he speaks at $t = 0$ after the signal $s_i^* = 0$, we should have $\lambda_0 - e_i - e_j \geq 0$ by Proposition 1. But we have $\lambda_0 - e_i - e_j < \lambda_0 - e_j + e_i < 0$, so a contradiction.

<u>Case2:</u> Assume $\ell(\lambda_0, q_i, s_i^*) < 0$.

<u>Case2i:</u> Suppose $s_i^* = 1$.Then $\lambda_0 + e_i < 0$.Since he speaks at $t = 0$ after the signal $s_i^* = 1$, we should have $\lambda_0 + e_i + e_j < 0$ by Proposition 1. But we have $\lambda_0 + e_j + e_i > \lambda_0 + e_j - e_i \geq 0$, so a contradiction.

<u>Case2ii:</u> Suppose $s_i^* = 0$.Then $\lambda_0 - e_i < 0$.Since he speaks at $t = 0$ after the signal $s_i^* = 0$, we should have $\lambda_0 - e_i + e_j < 0$ by Proposition 1. But we have $\lambda_0 - e_i + e_j \geq 0$, so a contradiction.

Hence if one of the experts speaks at $t = 0$ after one signal, and speaks at $t = 1$ after the other signal in equilibrium then the other expert also speaks at $t = 0$ after one signal, and speaks at $t = 1$ after the other signal.

Let s_i^* and s_j^* be the signals that they speak at $t = 0$ after experts receive. We want to show that $\ell(\lambda_0, q_i, s_i^*)$ and $\ell(\lambda_0, q_j, s_j^*)$ must have the same sign, generically.

Suppose that $\ell(\lambda_0, q_i, s_i^*) \geq 0$ and $\ell(\lambda_0, q_j, s_j^*) < 0$ to get a contradiction. Then $\ell(\lambda_0, q_i, s_i^*) - a_j \geq 0$ and $\ell(\lambda_0, q_j, s_j^*) + a_i < 0$ by Proposition 1 and the fact that experts speak at $t = 0$ after s_i^* and s_j^*. Consider the four different cases for s_i^* and s_j^*.

<u>Casei:</u> $s_i^* = s_j^* = 1$. Then $\lambda_0 + e_i - e_j \geq 0$ and $\lambda_0 + e_j + e_i < 0$ by above results. So we have a contradiction since $\lambda_0 + e_i - e_j < \lambda_0 + e_j + e_i$.

<u>Caseii:</u> $s_i^* = s_j^* = 0$. Then $\lambda_0 - e_i - e_j \geq 0$ and $\lambda_0 - e_j + e_i < 0$ by above results. So we have a contradiction since $\lambda_0 - e_i - e_j < \lambda_0 - e_j + e_i$.

<u>Caseiii:</u> $s_i^* = 0$ and $= s_j^* = 1$. Then $\lambda_0 - e_i - e_j \geq 0$ and $\lambda_0 + e_j + e_i < 0$ by above results. So we have a contradiction since $\lambda_0 - e_i - e_j < \lambda_0 + e_j + e_i$.

Caseiv: $s_i^* = 1$ and $s_j^* = 0$. Then $\lambda_0 + e_i - e_j \geq 0$ and $\lambda_0 - e_j + e_i < 0$ by above results. So we have a contradiction.

Now we want to prove that $s_i^* = s_j^*$. To get a contradiction, assume that $s_i^* \neq s_j^*$, but $s_i^* = 0$ and $= s_j^* = 1$. Note that $\ell(\lambda_0, q_i, s_i^*)$ and $\ell(\lambda_0, q_j, s_j^*)$ must have the same sign.So there are two possibilities:

Case1: $\ell(\lambda_0, q_i, 0) \geq 0$ and $\ell(\lambda_0, q_j, 1) \geq 0$. First of all, $\ell(\lambda_0, q_i, 1) > \ell(\lambda_0, q_i, 0) \geq 0$. Then by the fact that expert i speaks at $t = 0$ given $s_i = 0$, and speaks at $t = 1$ given $s_i = 1$ and by Proposition 1 we have the following results: $\lambda_0 - e_i - e_j \geq 0$ and $\lambda_0 + e_i - e_j < 0$, since $\lambda_0 + e_i \geq 0$.

Case2: $\ell(\lambda_0, q_i, 0) < 0$ and $\ell(\lambda_0, q_j, 1) < 0$. First of all, $\ell(\lambda_0, q_j, 0) < \ell(\lambda_0, q_j, 1) < 0$. Then by the fact that expert j speaks at $t = 0$ given $s_j = 1$, and speaks at $t = 1$ given $s_j = 0$ and by Proposition 1 we have the following results: $\lambda_0 + e_j + e_i < 0$ and $\lambda_0 - e_j + e_i \geq 0$, since $\lambda_0 - e_j < 0$.

So must have $s_i^* = s_j^*$.

□

Now by Lemma 7, we have two possible candidates left for a PBE. We need to find under what conditions there is a PBE in which both experts speak at $t = 0$ after receiving the same signal, say $s = 0$, and speak at $t = 1$ after receiving $s = 1$, and vice versa.

The following proposition constitutes these equilibrium

Proposition 7. *There is a PBE in which experts choose to wait if they receive a private signal contrary to public prior belief and speak immediately if the received private signal is in favour of the public prior belief iff* $\lambda_0 \in (-(e_1 + e_2), e_1 - e_2)$ *or* $\lambda_0 \in (e_2 - e_1, e_1 + e_2)$.

Proof. Assume first the $\lambda_0 < 0$. Suppose each expert speaks at $t = 0$ after receiving $s = 0$, and speak at $t = 1$ otherwise. Then by Lemma 7 we have two possibilities:
*Case1:*Assume $\ell(\lambda_0, q_1, 0) \geq 0$ and $\ell(\lambda_0, q_2, 0) \geq 0$.

So $\lambda_0 - e_1 \geq 0$ and $\lambda_0 - e_2 \geq 0$. Speaking at $t = 0$ is optimal only if the followings are also true by Proposition 1: $\lambda_0 - e_1 - e_2 \geq 0$ and $\lambda_0 - e_2 - e_1 \geq 0$.

Note that $\ell(\lambda_0, q_1, 1) > \ell(\lambda_0, q_1, 0) \geq 0$. Speaking at $t = 1$ is only preferred if $\ell(\lambda_0, q_1, 1) - e_2 < 0$. But $\lambda_0 + e_1 - e_2 > \lambda_0 - e_1 - e_2 \geq 0$. So we get a contradiction.

Case2: Assume $\ell(\lambda_0, q_1, 0) < 0$ and $\ell(\lambda_0, q_2, 0) < 0$.

So $\lambda_0 - e_1 < 0$ and $\lambda_0 - e_2 < 0$. Speaking at $t = 0$ is optimal only if the followings are also true by Proposition 1: $\lambda_0 - e_1 + e_2 < 0$ and $\lambda_0 - e_2 + e_1 < 0$.

We have four different cases for the signs of the LLR when experts receive the signal $s = 1$.

*Case2i:*Suppose $\ell(\lambda_0, q_1, 1) < 0$ and $\ell(\lambda_0, q_2, 1) < 0$

So $\lambda_0 + e_1 < 0$ and $\lambda_0 + e_2 < 0$.

Since experts speak at $t = 1$ when they receive $s = 1$, we should have the following results by Proposition 1: $\lambda_0 + e_1 + e_2 \geq 0$ and $\lambda_0 + e_2 + e_1 \geq 0$.

Now we have the following system of inequalities to solve:

$$\lambda_0 - e_1 < 0 \, , \, \lambda_0 - e_2 < 0 \, , \, \lambda_0 + e_1 < 0 \, , \, \lambda_0 + e_2 < 0$$

$$\lambda_0 - e_1 + e_2 < 0 \, , \, \lambda_0 - e_2 + e_1 < 0 \text{ ,and } \lambda_0 + e_1 + e_2 \geq 0$$

and solution set for λ_0 is: $-(e_1 + e_2) \leq \lambda_0 < -e_2$.

*Case2ii:*Suppose $\ell(\lambda_0, q_1, 1) < 0$ and $\ell(\lambda_0, q_2, 1) \geq 0$. So $\lambda_0 + e_1 < 0$ and $\lambda_0 + e_2 \geq 0$.

Since experts speak at $t = 1$ when they receive $s = 1$, we should have the following results by Proposition 1: $\lambda_0 + e_1 + e_2 \geq 0$and $\lambda_0 + e_2 - e_1 < 0$.

Now we have the following system of inequalities to solve:

$$\lambda_0 - e_1 < 0 \, , \, \lambda_0 - e_2 < 0 \, , \, \lambda_0 + e_1 < 0 \, , \, \lambda_0 + e_2 \geq 0$$

$$\lambda_0 - e_1 + e_2 < 0 \, , \, \lambda_0 - e_2 + e_1 < 0 \text{ ,and } \lambda_0 + e_1 + e_2 \geq 0$$

and solution set for λ_0 is: $-e_2 \leq \lambda_0 < min\{e_1 - e_2, -e_1\}$.

Case2iii: Suppose $\ell(\lambda_0, q_1, 1) \geq 0$ and $\ell(\lambda_0, q_2, 1) < 0$

Above assumption is not satisfied since we have assumed that $q_1 < q_2$ WLOG, so

$e_1 < e_2$.

<u>*Case2iv:*</u> Suppose $\ell(\lambda_0, q_1, 1) \geq 0$ and $\ell(\lambda_0, q_2, 1) \geq 0$. So $\lambda_0 + e_1 \geq 0$ and $\lambda_0 + e_2 \geq 0$. Since experts speak at $t = 1$ when they receive $s = 1$, we should have the following results by Proposition 1: $\lambda_0 + e_1 - e_2 < 0$and $\lambda_0 + e_2 - e_1 < 0$.

Now we have the following system of inequalities to solve:

$$\lambda_0 - e_1 < 0 \, , \, \lambda_0 - e_2 < 0 \, , \, \lambda_0 + e_1 \geq 0 \, , \, \lambda_0 + e_2 \geq 0$$

$$\lambda_0 - e_1 + e_2 < 0 \text{ ,and } \lambda_0 - e_2 + e_1 < 0$$

and solution set for λ_0 is: $-e_1 \leq \lambda_0 < e_1 - e_2$.

Assume that $e_2 < 2e_1$. Then $min\{e_1 - e_2, -e_1\} = -e_1$. So if $-(e_1 + e_2) \leq \lambda_0 < -e_2$(by Case2i) or $-e_2 \leq \lambda_0 < -e_1$(by Case2ii) or $-e_1 \leq \lambda_0 < e_1 - e_2$(by Case2iv), then there is a PBE where expert speak at $t = 0$ if they receive the signal $s = 0$, and speak at $t = 1$ otherwise. Consequently, $-(e_1 + e_2) \leq \lambda_0 < e_1 - e_2$ is a necessary condition for that kind of PBE.

Now assume $2e_1 < e_2$. Then $min\{e_1 - e_2, -e_1\} = e_1 - e_2$. So if $-(e_1 + e_2) \leq \lambda_0 < -e_2$(by Case2i) or $-e_2 \leq \lambda_0 < e_1 - e_2$(by Case2ii) then there is a PBE where expert speak at $t = 0$ if they receive the signal $s = 0$, and speak at $t = 1$ otherwise. Consequently, $-(e_1 + e_2) \leq \lambda_0 < e_1 - e_2$ is a necessary condition for that kind of PBE.

Also given $\lambda_0 \in (-(e_1 + e_2), e_1 - e_2)$, the following beliefs are consistent with related strategies of the decision maker and the experts.

$$\forall i \in \{1,2\} \, , \, \mu_i^0(s_i) = \begin{cases} 0 & \text{, if } s_i = 0 \\ \emptyset & \text{, otherwise} \end{cases}$$

$$\mu_i^1(s_i, (m_i^0, m_j^0)) = \begin{cases} 1 & \text{, if } m_j = \emptyset \text{ and } s_i = 1 \\ 0 & \text{, otherwise} \end{cases}$$

$$p(s_i,(m_i^0,m_j^0)) = \begin{cases} p(s_i)(s_j=0) & \text{if } m_j^0 = 0 \\ p(s_i)(s_j=1) & \text{if } m_j^0 = \emptyset \end{cases}$$

$$\text{and } p(s_i,(m_i^0,m_j^0=1)) \geq p(s_i)(s_j=1)$$

$$p(m^0,m^1) = \begin{cases} p(s_1=1)(s_2=1) & \text{, if } m_1^1 = m_2^1 = 1 \\ p(s_i=0)(s_j=1) & \text{, if } m_i^0 = 0 \text{ , } m_j^1 = 0 \\ p(s_1=0)(s_2=0) & \text{, if } m_1^0 = m_2^0 = 0 \end{cases}$$

Furthermore given those beliefs strategies of the experts and the decision maker are sequentially rational. So this is indeed a PBE.

Similarly, if $\lambda_0 > 0$ there is a PBE in which experts choose to wait if they receive $s_i = 0$ and speak otherwise. Necessary condition for this type of equilibrium is $\lambda_0 \in (e_2 - e_1, e_1 + e_2)$ by a similar analysis to the case where $\lambda_0 < 0$. Given $\lambda_0 \in (e_2 - e_1, e_1 + e_2)$, it is straightforward to specify the following PBE:

$$\forall i \in \{1,2\} \text{ , } \mu_i^0(s_i) = \begin{cases} 1 & \text{, if } s_i = 1 \\ \emptyset & \text{, otherwise} \end{cases}$$

$$\mu_i^1(s_i,(m_i^0,m_j^0)) = \begin{cases} 0 & \text{, if } m_j = \emptyset \text{ and } s_i = 0 \\ 1 & \text{, otherwise} \end{cases}$$

$$p(s_i,(m_i^0,m_j^0)) = \begin{cases} p(s_i)(s_j=1) & \text{if } m_j^0 = 1 \\ \\ p(s_i)(s_j=0) & \text{if } m_j^0 = \emptyset \end{cases}$$

$$\text{and } p(s_i,(m_i^0,m_j^0=0)) \leq p(s_i)(s_j=0)$$

$$p(m^0,m^1) = \begin{cases} p(s_1=0)(s_2=0) & \text{, if } m_1^1 = m_2^1 = 0 \\ \\ p(s_i=1)(s_j=0) & \text{, if } m_i^0 = 1 \text{ , } m_j^1 = 1 \\ \\ p(s_1=1)(s_2=1) & \text{, if } m_1^0 = m_2^0 = 1 \end{cases}$$

□

5 Equilibria in Endogenous Order-Overview

Let $\lambda_0 < -(e_1+e_2)$ In this case the public prior belief is too extreme. Each expert is almost sure that $\theta = 0$ and any signal contrary to this belief is assumed to be a result of an error possibly occurred during the transmission of the private signal. Sending a message depending on that private signal is most likely bad for the reputation of the expert. In this region, private signals of the experts are suppressed by the public prior belief, so they herd and send message 0 in the first period. It is impossible to learn the private signals of the experts for the DM.

Let $-(e_1+e_2) < \lambda_0 < -e_2$. Then we have multiple equilibria. One of them results from the herding behaviour of the experts because the public prior belief is still too extreme and experts send message 0 in the first period. irrespective of the private signal they received. The other equilibrium is the one that is uniquely associated with the endogenous order mechanism. Experts prefer to wait if they receive a signal contrary to public prior belief, i.e if $s_i = 1$ and choose to speak immediately if the signal supports the public prior belief, i.e if $s_i = 0$. If this equilibrium is reached at the end, information is transmitted to DM perfectly and both s_1 and s_2 are valuable information.

Let $-e_2 < \lambda_0 < e_1 - e_2$. In this region, there is a unique PBE in which experts predetermine the period when they would speak before and depending on the signal they received. If they receive a private signal contrary to the public prior belief it is wise to wait to see whether other expert reveals his signal or not. In this case remaining silent at $t = 0$ also reveals the private signal of the other expert. Therefore, given a public prior belief in this region, DM infers the realization of the private signals of the experts form the period they spoke. Information is perfectly transmitted.

Let $e_1 - e_2 < \lambda_0 \leq e_2 - e_1$. In the equilibrium, experts apply the Seniority Rule without any exogenous design. Senior takes the lead and speak earlier no matter which signal he receives since this is the optimal thing to do for his reputation and also he knows that any message revealed by the junior will not change his message even if the fact that junior always tells the truth. Furthermore, it is costly to wait to see whether the junior reveals his private signal or not. On the other hand, the junior prefers to wait because he can learn valuable information by listening to the senior and the message of the senior means a lot since public prior belief is around a small neighbourhood of $p_0 = \frac{1}{2}$. However the message of the junior is not credible for the decision maker in any case because of the herding behaviour of the junior. No matter which private signal he received, he mimics that he received the same private message with the senior because of the reputational concerns.

Let $e_2 - e_1 < \lambda_0 < e_2$. Since the equilibria are symmetric around $\lambda_0 = 0$ we have again only the PBE that is uniquely associated with the endogenous order mechanism. This time most likely state is just $\theta = 1$. In other words, experts will speak immediately if they receive a private signal supporting the most likely state, $\theta = 1$, and they wait otherwise. At the end of the process, the DM learns private signals of both experts. Furthermore, both of the signals affect the action taken by the DM.

Let $e_2 < \lambda_0 < e_1 + e_2$. In this region, there are two possible PBE. One of them is again the one that is uniquely associated with the endogenous order mechanism with the fact that most likely state is $\theta = 1$ and the strategies of the experts are explained before. As an outcome of this PBE, the DM could take the action $a = 0$ iff both $s_1 = s_2 = 0$. Otherwise the posterior belief is always above $\lambda = 0$ and the DM always take the action $a = 1$. Hence both of the signals are valuable and they are revealed if this PBE is reached. Note that the public prior belief is becoming too extreme again. However, this time it is in favour of $\theta = 1$. Hence there is also a PBE in which both of the experts speak as soon as possible and send the message $m = 1$ no matter which signal they have received. If this equilibrium is reached then there is no information transmitted.

Let $e_1 + e_2 < \lambda_0$ The public prior belief is too extreme so that experts do not have any other choice but sending the message $m = 1$ to the decision maker as soon as possible. Experts just herd and no information is transmitted to the DM.

6 Fixed vs Endogenous Order

It is important to learn private signals of the experts for the DM because sometimes messages sent by experts are not so reliable. Sometimes even learning a private signal does not offer any help to the decision maker during the decision-making process. Experts send their messages according to their private information, their precision and the public belief at that moment. Does the mechanism used by the DM to elicit information also matter? Experts may be told exactly when to speak, and they speak only when they are supposed to do so (Fixed order), or they may choose when to speak within a two period long recommendation process (Endogenous order). Is there any difference between these two different mechanisms in terms of the amount of information transmitted? If so, which one is more effective to elicit information? Does the mechanism that the DM prefers differ from the most effective mechanism in terms of information transmission?

Let $\lambda_0 < -(e_1 + e_2)$ There is a unique pure strategy PBE in this region if the endogenous order mechanism is used. Since the public prior belief is too extreme, experts do not rely on their own private information and herd. They speak at $t = 0$ simultaneously since there is a discounting factor. Because of the herding behaviour of the experts, no information is aggregated. Posterior public belief is the same as the public prior belief. Similarly any fixed order mechanism also does not reveal any information. Irrespective of who speaks first, both experts herd when they are supposed to speak. Since Endogenous Order mechanism, Anti-Seniority Rule, and Seniority Rule produce the same outcome if $\lambda_0 < -(e_1 + e_2)$, the DM is indifferent between Endogenous Order mechanism and any fixed order mechanism.

Let $-(e_1 + e_2) < \lambda_0 < -e_2$. No matter which fixed order mechanism is used, experts just herd in equilibrium. They do not rely on their private information but they mimic as if they received a private message supporting the public prior belief. No information is transmitted to the DM. Therefore, ex-ante payoff of the decision maker is simply 0. On the other hand, there is another PBE in the endogenous order mechanism where experts speak at $t = 0$ if the private signal is $s = 0$, and speak at $t = 1$ otherwise. Note that DM has an incentive to take action $a = 0$ initially, since $\lambda_0 < 0$. However if experts send their messages according to their strategies in the equilibrium, DM may change his decision to $a = 1$ in case of $s_1 = s_2 = 1$ since $\lambda_0 + e_1 + e_2 \geq 0$. In other words, DM learns something valuable rather than learning just s_1 and s_2 in the equilibrium. Information is transmitted perfectly and also the information affects the decision of the decision maker if endogenous order mechanism is used. The most informative equilibrium is reached through the endogenous order mechanism because by observing the period experts speak, DM infers the private signals of experts without looking at their messages. To show that Endogenous Order Mechanism is strictly preferred by the DM, consider the ex-ante welfare of the DM when public prior belief is known and $-(e_1 + e_2) < \lambda_0 < -e_2$. Ex-ante welfare of the DM depends on the realizations of s_1, s_2 and θ and their joint probability. We define the joint probability $Prob(s_1 = s_1^*, s_2 = s_2^* \mid \theta = \theta^*)Prob(\theta = \theta^*)$ as $\xi\big((s_1^*, s_2^*), \theta^*\big)$. Therefore, ex-ante welfare of the DM is:

$$\sum_{s_1^*, s_2^*, \theta^* \in \{0,1\}} \xi\big((s_1^*, s_2^*), \theta^*\big) v(z)$$

where $z = \big(\theta, (m_1^0, m_2^0), (m_1^1, m_2^1), a\big)$ and $\theta = \theta^*$.

Remember that in the endogenous model, the DM takes action $a = 1$ iff $s_1 = s_2 = 1$. Otherwise, he takes action $a = 0$. Hence, $v(z) = 0$ for any other realizations of s_1 and s_2 but $s_1 = s_2 = 1$. Therefore, ex-ante welfare of the DM is: $\frac{1}{2}\Big(\xi\big((1,1),1\big) - \xi\big((1,1),0\big)\Big)$.

Lemma 8. *If $-(e_1 + e_2) < \lambda_0 < -e_2$ then ex-ante welfare of the DM is strictly positive when the endogenous order mechanism will be applied.*

Proof. We need to show that $\frac{1}{2}\Big(\xi\big((1,1),1\big) - \xi\big((1,1),0\big)\Big) > 0$. Then by the definition of the function ξ and the fact that private signals of experts are independent of each other, it is enough to show that $\big(p_0 q_1 q_2 - (1-p_0)(1-q_1)(1-q_2)\big) > 0$, or equivalently $p_0 > \frac{1-q_1-q_2+q1q_2}{1+2q1q_2-q1-q_2}$. However, this is actually a result of the assumption that $-(e_1 + e_2) < \lambda_0 < -e_2$ since it is equivalent to $1 - q_2 > p_0 > \frac{1-q_1-q_2+q1q_2}{1+2q1q_2-q1-q_2}$. □

Corollary 3. *If $-(e_1 + e_2) < \lambda_0 < -e_2$ and it is common knowledge, endogenous order mechanism is strictly preferred to any fixed order mechanism by the DM.*

Let $-e_2 < \lambda_0 < e_1 - e_2$ Similarly, experts reveal their private information if $-e_2 < \lambda_0 < e_1 - e_2$ and endogenous order mechanism is applied. Hence, ex-ante welfare of the DM is again $\frac{1}{2}\big(p_0 q_1 q_2 - (1-p_0)(1-q_1)(1-q_2)\big) > 0$. On the other hand, we need to make another assumption to be able to forecast the path of the process under a fixed order mechanism. The relation between the expertise level of the junior and the difference between expertise levels of senior and junior matters in the fixed order mechanism. Assume first that $e_1 - e_2 < -e_1$. Now, suppose that Seniority Rule will be applied. Senior speaks first and reveals his private information by telling the truth. If his message is contrary to public prior belief, junior also tells the truth and reveals his private information. In other words, DM is able to learn the signal of the junior whenever it is useful since $\lambda_0 + e_2 + e_1 > 0$ and $\lambda_0 + e_2 - e_1 < 0$. Otherwise, if the senior sends a message supporting the public prior belief, junior herds and DM takes action $a = 0$. Overall, DM takes action $a = 1$ iff $s_1 = s_2 = 1$. Therefore, ex-ante welfare of the DM when Seniority-rule is applied is equal to exactly the ex-ante welfare of DM when endogenous order mechanism will be applied.

Corollary 4. *DM is indifferent between applying Seniority Rule and the Endogenous Order Mechanism if* $-e_2 < \lambda_0 < e_1 - e_2$ *and* λ_0 *is common knowledge.*

Now suppose that anti-seniority rule is applied. Junior never reveals his information. When he is supposed to speak only senior tells the truth. Therefore, posterior public belief is only affected by the message of senior. DM takes action $a = 1$ iff $s_2 = 1$. Hence, ex-ante welfare of the DM when anti-seniority rule will be applied is:

$$\frac{1}{2}\Big(\xi\big((1,1),1\big) - \xi\big((1,1),0\big)\Big) + \frac{1}{2}\Big(\xi\big((0,1),1\big) - \xi\big((0,1),0\big)\Big)$$

Lemma 9. *If* $-e_2 < \lambda_0 < e_1 - e_2$ *then ex-ante welfare of DM when Anti-Seniority Rule is applied is strictly smaller than ex-ante welfare of DM in the case of Seniority Rule.*

Proof. It is enough to show that $\xi\big((0,1),1\big) - \xi\big((0,1),0\big) < 0$, or equivalently $p_0(1 - q_1)q_2 - (1 - p_0)q_1(1 - q_2) < 0$. This inequality holds iff $p_0 < \frac{q_1 - q_1 q_2}{q_1 - 2q_1 q_2 + q_2}$ holds. Notice that the assumption $\lambda_0 < e_1 - e_2$ actually implies that $p_0 < \frac{q_1 - q_1 q_2}{q_1 - 2q_1 q_2 + q_2}$. So we have the result. □

The other case where $-e_1 < e_1 - e_2$ is also similar. The only difference is that junior also tells the truth when $\lambda_0 \in (-e_1, e_1 - e_2) \subseteq (-e_2, e_1 - e_2)$. However, DM takes action $a = 1$ iff $s_2 = s_1 = 1$. Otherwise, she takes action $a = 0$ since $\lambda_0 + e_2 - e_1 < 0$ which is the biggest value of all other realizations of s_1 and s_2. Therefore, ex-ante welfare of DM is: $\frac{1}{2}\Big(\xi\big((1,1),1\big) - \xi\big((1,1),0\big)\Big)$. We have the following corollary:

Corollary 5. *If expertise levels of the experts are close enough such that* $-e_1 < e_1 - e_2$ *holds and* $-e_1 < \lambda_0 < e_1 - e_2$ *then ex-ante welfare of Seniority Rule, Anti-Seniority Rule, and Endogenous Order Mechanism are all the same and equivalent to* $\big(p_0 q_1 q_2 - (1 - p_0)(1 - q_1)(1 - q_2)\big) > 0$.

Let $e_1 - e_2 < \lambda_0 < e_2 - e_1$:First of all, in the equilibrium senior speaks at $t = o$ and tells the truth and junior speaks at $t = 1$ and herds if endogenous order mechanism is applied. This is actually what Seniority Rule offers. Hence they are equivalent if $e_1 - e_2 < \lambda_0 < e_2 - e_1$. Now, assume first that $-e_1 < e_1 - e_2$. If the Seniority Rule will be applied, Senior tells the truth and then junior just herds. DM takes action $a = 1$ iff $s_2 = 1$. Hence, ex-ante welfare of DM is:

$$\frac{1}{2}\Big(\xi\big((1,1),1\big) - \xi\big((1,1),0\big)\Big) + \frac{1}{2}\Big(\xi\big((0,1),1\big) - \xi\big((0,1),0\big)\Big)$$

However, note that this time $\xi\big((0,1),1\big) - \xi\big((0,1),0\big) > 0$ since $e_1 - e_2 < \lambda_0 < e_2 - e_1$ and so that $p_0 > \frac{q_1 - q_1 q_2}{q_1 - 2q_1 q_2 + q_2}$. On the other hand, junior tells the truth and also senior tells the truth when he is supposed to talk when Anti-Seniority Rule is applied. However, DM takes his decision independent of the signal of junior. She takes action $a = 1$ iff $s_2 = 1$. Therefore, ex-ante welfare of DM is equivalent to $\frac{1}{2}\Big(\xi\big((1,1),1\big) - \xi\big((1,1),0\big)\Big) + \frac{1}{2}\Big(\xi\big((0,1),1\big) - \xi\big((0,1),0\big)\Big)$, again. DM is different between applying Seniority Rule, which is the outcome of Endogenous Order Mechanism in this case, and Anti-Seniority Rule. If $e_1 - e_2 < -e_1$ then everything is similar to above case except that DM is unable to learn s_1 if $\lambda_0 \in (e_1 - e_2, -e_1)$ or $\lambda_0 \in (e_1, e_2 - e_1)$. However, it is not an important issue because s_1 actually does not have a role in the decision-making process.

Proposition 8. *Assume that public prior is common knowledge. DM strictly prefers Endogenous Order Mechanism to any fixed order mechanism if $-(e_1 + e_2) < \lambda_0 < -e_2$. Otherwise, DM is indifferent between applying Endogenous Order Mechanism and Seniority Rule. Furthermore, Anti-Seniority Rule is also strictly dominated by the Seniority Rule if $-e_2 < \lambda_0 < e_1 - e_2$. Otherwise, DM is indifferent between applying Anti-Seniority and Seniority Rule.*

Since the necessary conditions we have found for PBEs are symmetric around $\lambda = 0$, the analysis and the outcomes of the fixed order mechanisms and the endogenous mechanism for the regions where $\lambda_0 > e_2 - e_1$ are quite similar to the related region where $\lambda_0 < e_1 - e_2$. We have the following theorem that constitutes the main result of this paper.

Theorem 1 (Main Theorem)**.** *Suppose that experts know the public prior belief and DM knows only the distribution of public prior belief over the interval [0,1]. If* $Prob(-(e_1+e_2) < \lambda_0 < -e_2) \neq 0$ *or* $Prob(e_2 < \lambda_0 < e_1 + e_2) \neq 0$ *then Endogenous Order Mechanism is strictly preferred by the DM.*

7 Conclusion

In this paper, we consider a situation in which an uninformed decision maker consult two experts who have private signals that are not fully conclusive. These experts are motivated by their career concerns instead of some monetary transfers. We try to maximize the amount of relevant information elicited from these experts who are supposed to speak within two discrete periods by optimizing the order of speech. We depart from Ottaviani and Sorensen(2001),the closest paper to ours, by assuming an endogenous order of speech whereas they consider only the orders exogenously given to the experts.

Ottaviani and Sorensen(2001) concludes that the amount of information transmitted to the decision maker is limited during a debate. At some point some experts, especially ones that are not too well informed, begin to suppress their private information and mimic preceding more competent experts. This behaviour is called 'herding behaviour' in the literature. It results from the reputational concerns of the experts and the fact that they know that the information possessed by senior expert is more reliable than theirs.

We introduce a discounting parameter into the model and let the experts speak whenever they want. As a result, experts motivated by career concerns reveal their private information whenever needed whereas they would have herd if they had been forced to speak within an exogenously given fixed order. In the endogenous order mechanism, experts have an additional tool to achieve their reputational goals by choosing the time they speak after receiving their private information.

Consider the hiring policy of a firm. Interviews are done sequentially by the departments of the firm and after a few interviews a decision is given about an applicant. The order of the departments make the interview is always fixed. Our main finding implies that it is possible to make more accurate decisions about the compatibility of the applicants by letting distinct departments to make interviews whenever they want.

Our model is also applicable in most of the places where debate occurs, for instance an academic admission committee or a classical jury environment. The order the juror speaks or committee members reveal their opinions matters in terms of the decision taken. Therefore, it might be possible to take more accurate actions in these environments if the endogenous order mechanism we have proposed is used.

To make the situation a little bit more realistic, it is possible to introduce a 'joy of telling the truth' into the model. People tend to tell the truth by their nature so they should receive an extra payoff when they tell the truth. Therefore, it should affect the herding behavior of the experts and it might be worthwhile to analyse this variant of our original model.

Furthermore, it is possible to consider a situation in which experts are also biased rather than just being reputationally motivated. In this case, experts are also interested in the decision taken by the decision maker. In some cases, it is more profitable to report the message favouring the unlikely state, especially if the expert is biased towards to that state. This is related to 'anti-herding' results of Levy(2004) and the nature of 'herding' and 'anti-herding' behaviour of experts under endogenous order mechanism we propose constitutes an interesting future research area for us.

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