AN ELECTORAL COMPETITION MODEL WITH FLIP-FLOPPING

by

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This is to certify that I have examined this copy of a master's thesis by
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and have found that it is complete and satisfactory in all respects,
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ABSTRACT

We introduce a two-candidate electoral competition model in which the candidates are free to deviate from their past announcements during the campaign stage, i.e., “flip-flop", and implement policies different from campaign promises when in office. In our model the candidates have ideal positions as well as past political announcements or behavior. They only care about winning the election but it is costly for them to flip-flop or implement policies different from their campaign promises. Voters are completely informed about the candidates' preferences and vote for the candidate whom they think will implement the policy that is closest to their ideal position. We analyze the equilibrium of this model and show that there is platform divergence even though the candidates are only motivated by office. We also analyze the factors determining the extent of flip-flopping. We extend this model to allow for incomplete information so that the voters do not know whether the candidate is a commitment type who never flip-flops and always keeps her campaign promises or an opportunist who masquerades as a commitment type by not flip-flopping. We analyze this model under the assumption that candidates' cost of implementing a policy different from campaign promises is very high and obtain results similar to those in the model with complete information.

ÖZET

Politikacıların seçim kampanyası sırasında geçmişte yapmış oldukları duyurularını değiştirmekte ve seçildiklerinde de seçim sırasında vermiş oldukları sözlerinden farklı politikalar uygulamakta özgür oldukları bir seçim rekabeti modeli tanıtıyoruz. Modelimizde adayların geçmiş duyuru ve davranışlarının yanında ideal pozisyonları var. Adaylar sadece kazanmayı önemsiyorlar fakat sık sık tavır değiştirmek veya seçim sözlerinden farklı politikalar uygulamak adaylar için maliyetli. Seçmenler, adayların tercihleri konusunda tam anlamıyla bilgilendirilmiş durumdalar ve kendi ideal pozisyonlarına en yakın politikayı uygulayacağını düşündükleri adaya oy veriyorlar. Bu modelin denge durumunu analiz ediyoruz ve adayların sadece seçimi kazanmayı önemsemelerine rağmen denge durumunda seçtikleri politikaların farklı olduğunu gösteriyoruz. Ayrıca tavır değişikliklerinin miktarını belirleyen faktörleri inceliyoruz. Bu modeli eksik bilgiye izin vererek genişletiyoruz ve böylece seçmenler, adayların her zaman sözünü tutan bir tip mi yoksa davranışını değiştirmeyerek sözünü tutmuş gibi davranan çıkarcı bir tip mi olduklarını bilmiyorlar. Bu modeli seçim kampanyası sırasında verilen sözlerden farklı bir politika uygulamanın maliyeti çok fazla varsayımı altında inceliyoruz ve bunun sonucunda elde ettiğimiz sonuçların tam bilgi altındaki modelde bulunan sonuçlarla benzer olduğunu gösteriyoruz.

CONTENTS

CHAPTER 1

INTRODUCTION

During an electoral campaign, candidates commonly deviate from their past announcements or policies, i.e., they “flip-flop." One of the last and most famous examples is Mitt Romney’s position on health-care during the 2011 electoral campaign. In 2007, he said the following about the health-care reform he implemented while he was governor: “I’m proud of what we’ve done. If Massachusetts succeeds in implementing [Romneycare], then that will be a model for the nation." During the 2011 Republican presidential primary debate he said “At the time I crafted the plan in the last campaign I was asked is [Romneycare] something that you would have the whole nation do, and I said no. This is something that was crafted for Massachusetts. It would be wrong to adopt this as a nation."

There may be various reasons for flip-flops. The candidates may have seen from the past experiences that their policies are not ideal for the public, or they may want to be attuned to the changing world, or they just may be unprincipled. Another possible reason is their desire to get the majority of votes and win the election, which is what we focus on this thesis as the main motivation. We allow for the possibility that once they are elected to office, they may deviate from their election promises and implement their ideal policy.

In two-candidate elections with majority rule, it is the median voter’s position which determines the winner. If a candidate commits to a position that is different from the ideal position of the median, the other candidate could win the election by committing to the median’s position. Thus politicians tend to move to the median voter’s position and make more centrist election promises rather than their past statements during the campaign.

However, flip-flops are costly to candidates. Candidates abuse the voters' trust when they flip-flop and this causes bad reputation. Shifting too much to the middle to win the median voter may mean losing partisan supporters. On the other hand, the voters may not believe a candidate's moderate election promises, because he may implement a policy closer to his own ideal position once elected.

In the canonical (Downsian) electoral competition model, candidates choose their strategies to maximize their chance of winning and their positions converge on the median voter's position, known as the median voter theorem (Black [1]). We show that introducing the possibility of flip-flopping leads to platform divergence, i.e., candidates announce policies different from what they believe to be the median position. In other words, this thesis develops an alternative theory of divergent platforms. We also analyze the determinants of the extent of the flip-flopping under both complete and incomplete information.

In our model there are two candidates who have ideal positions as well as past political announcements or behavior. They only care about winning the election but it is costly for them to flip-flop or implement policies different from their campaign promises. Voters are completely informed about the candidates' preferences and vote for the candidate whom they think will implement the policy that is closest to their ideal position. We analyze the equilibrium of this model and show that there is platform divergence even though the candidates are only motivated by office. We also analyze the factors determining the extent of flip-flopping: Not surprisingly there is more flip-flopping as its cost decreases and the utility from winning increases. We also show that as the cost of implementing a position different from the campaign promises increases, i.e., the candidates become more credible during the campaign, there is more flip-flopping. The intuition behind this last result is as follows. The reason why a candidate flip-flops is to increase the chance of being elected by convincing the voters that he will implement a policy that is closer to their ideal positions. However, if the candidate is not credible, then the voters would think that the candidate will implement a policy that is close to the candidate's own ideal position, i.e., they will not find these flip-flops convincing and this will reduce the expected benefit from flip-flopping.

We extend this model to allow for incomplete information so that the voters do not know whether the candidate is a commitment type who never flip-flops and always keeps her past announcements and campaign promises or an opportunist who masquerades as a commitment type by not flip-flopping. Equilibrium analysis of this model turns out to be too difficult in its full-fledged form and we analyze it under the assumption that candidates' cost of implementing a policy different from campaign promises is very high. Under this assumption, we obtain results that are similar to those under complete information.

The rest of the thesis is organized as follows. In Chapter 2, we make a literature review, give information about the standard election model and the other relevant models and discuss their results. In Chapter 3, we present our model and Chapter 4 investigates and interprets the main results including equilibrium strategies of the candidates. We introduce an extension of the main model and characterize the equilibrium under incomplete information in Chapter 5. Lastly, Chapter 6 concludes the thesis and discusses the future work.

CHAPTER 2

LITERATURE REVIEW

Our model is closely related to two strands of literature. In what follows we are going to review them and examine how they relate to our model. One of these is known as the Downsian model which is the standard model of the electoral competition and it originally belongs to Hotelling [2] who first suggested the spatial competition model of elections. His location model interprets how two firms choose their locations and the prices of their products on a line segment under perfect information. He assumes that the consumers are uniformly distributed over an interval and each one buy from the closest seller. His conclusion is that competing candidates' locations converge on the position of the median consumer since each candidate can increase her market share by getting closer to the other one and they come together at the center of the interval.

Downs [3] applies this approach to the political competition and used Hotelling's model to understand the politicians' equilibrium behavior. He assumes that candidates and electorate have complete information about the candidate positions and citizen preferences. The main assumption of his model is that voters have single-peaked and symmetric preferences. Single-peakedness implies that if there are two candidates with the ideal policies x and y in a closed interval and $y > x$, then the voters whose ideal policies are smaller than x chooses candidate with the policy x and the voters with ideal policies greater than y votes for the candidate at y. Symmetry implies that the voters whose ideal points are less than $(x+y)/2$ votes for the candidate at x. Another basic assumption of the model is that each candidate only cares about winning, i.e., they are not concerned about the policy of the winner. Sincere voting is the other strong assumption of his model. That is, each voter supports the candidate whose policy is closest to her ideal policy. Under these assumptions the election takes place as follows: Candidates simultaneously announce their policies then voters vote and the candidate who gets a majority of the votes wins the election.

Since each candidate prefers to win the election to a tie for the first place, and prefers a tie to lose the election; in equilibrium candidates converge on the median position by median voter theorem. This shows that candidates not only choose the same position but also adopt the median voter's policy.

Our model with complete information differs from the Downs' model in that candidates have prior positions and lying costs originating from the changes of these positions and campaign promises. As opposed to the convergence result of Downs' model, our model generates divergence, i.e., the candidates adopt different positions in equilibrium because of the flip-flopping.

We mainly investigate two related papers that model flip-flopping behavior. One of them is Banks' [4] and the other one is Callender-Wilkie's [5]. These papers are closely related to our model of incomplete information.

Banks introduce a two-candidate election model with incomplete information in which candidates do not have to announce their ideal positions during the campaign, i.e., politicians can promise to different policies from their true intentions. Before the election, each candidate develops her optimal policy that they will implement if elected. However, the voters and the other candidate have no idea about this policy and do not know whether she lies or not during the election. Once the winning candidate gets the office then she enacts her true policy. The main assumption in Banks' model is that there is an equal lying cost for the all types of candidates and it is proportional to the difference between the ideal point and the implemented policy.[1] He finds that if the lying cost is high enough, voters can infer candidates' true positions and vote for the right candidate. Also that, candidates with the extremist position make extremist campaign promises and reveal their true positions when moderate candidates make centrist announcements.

Callander-Wilkie extend Banks' model and change his assumption that the lying cost is same for all types of candidates by allowing candidates to have heterogeneous lying costs. They categorize candidates as costly liars and cost-free liars. The presence of cost-free liars causes voter inference problem, that is, voters cannot properly distinguish moderate candidates from extremists. As a result of this, divergent platforms are discussed in this model and the most centrist candidate might not always win the election. Furthermore, they conclude that in despite of the general opinion that liars are favored in the elections, the honest candidates are not always defeated, i.e., lying does not always guarantee to win the election.

In our model with incomplete information, the winning candidate cannot change her campaign promise as opposed to the Banks' and Callender-Wilkie's models where the candidates have their freedom to apply their true policy once they are elected. In other words, the cost of implementing a policy different from the promise given in the campaign is too high in our model. In addition to that, there is also a lying cost if candidates change their past announcements before the election campaign. These lying costs cause divergence in equilibrium.

Kartik-McAfee [6] also analyzes a related model in which some candidate types have a character, that is, they commit to their campaign promises and incur an infinite cost. Bernhardt-Ingberman [7] provides a model in which there is a cost of lying depending on the candidates' statuses (e.g incumbent, challenger).

[1] In Downs' model, this lying cost is considered to be infinite. Thus, the candidates do not change their campaign platforms and they implement them after the election.

In the election models, the candidates generally have perfect information about the preferences of the electorate but Ledyard [8] raises up the issue that in real world, politicians cannot obtain full information about voters. He presents an election model in which each candidate cares only about winning and has private information about voters. He analyzes the consequences of the order of candidates' choices, public polls and sequences of elections with several examples of electoral competition models. Chan [9] also studies a Downsian electoral competition model in which he assumes that the candidates are privately-informed. Under this assumption, he finds that candidates choose positions which are different from the median voter's position when they receive uninformative signals and their policies tend to converge when they have strong signals.

There are numerous papers extending Downs' model to various ways. One of these works is Hinich [10] which generalizes Downs' model to multiple-issue politics and shows that candidates may adopt different positions in equilibrium.

Wittman [11] introduces an electoral competition model which assumes that each candidate cares both about winning the election and how close the policy of the winner is to her ideal position. In his model, candidates increase their probability of winning by shifting to the median voter's position but they move away from their own ideal policies. Based on this assumption, Wittman shows that in equilibrium, candidates adopt the median voter's policy under certainty. However if they are uncertain about the median voter's position then they announce divergent policies.

Calvert [12] also develops an election model in which candidates are policy-motivated, i.e. they care about policy outcomes. He shows that in equilibrium candidates choose separate platforms.

CHAPTER 3

THE MODEL

There are two political candidates, A and B, whose ideal policies are t_1 and t_2, respectively. The policy space P is the closed interval $[0, 1]$.[1] In our model, different from the other models, the candidates have announced the policies, x_1 and x_2, in the past. During the campaign, they choose their election promises y_1 and y_2. At this stage, the parties may promise to implement their previously announced policies or flip-flop. The voters elect the candidates upon observing these announced positions. After the election, the winning candidate implements p_i which may be different from her campaign promise y_i where $i = 1, 2$.

There is a continuum of voters distributed over the set $\Theta = [0, 1]$. Each voter j has single peaked preferences $\succsim_j$over the policy space $P = [0, 1]$ such that

$$p_1 \succsim_j p_2 \text{ if and only if } |p_1 - \mu| \leq |p_2 - \mu| \tag{3.1}$$

for each $(p_1, p_2) \in P^2$ and $\mu \in \Theta$ is the ideal position of voter j. Each voter votes sincerely for the candidate whose policy is closest to her ideal point. If she is indifferent between the two candidates then she votes for each with probability one half. The candidate who receives the majority of the votes wins the election.

There is a cost of announcing a position other than x_i (announcing $y_i \neq x_i$) during the campaign and it is given by

$$-\alpha(x_i - y_i)^2$$

[1] We can identify the set of all political positions with the closed interval $[0, 1]$.We may think that 0 is the most leftist position and 1 is the most rightist position.

where $\alpha > 0$. The candidates obtain utility from winning the election but encounter a cost of lying. So the expected payoff of candidate i is

$$u_i(y_i, p_j) = -\alpha(x_i - y_i)^2 + \pi_i(y_i, p_j)k \tag{3.2}$$

where $k > 0$ is constant and $\pi_i(y_i, p_j)$ is the probability of winning for the candidate i, where $i = 1, 2$. The candidates do not know the actual distribution of the voters but believe that the median position is distributed uniformly over $[0, 1]$.

The game is as follows:

Stage I: Candidate 1 and 2 independently choose y_1, y_2.

Stage II: After observing candidates' policies voters vote sincerely and then the winner (say i) implements p_i.

This defines an extensive form game. However, it is more convenient to work with an equivalent reduced strategic form game.

Now we will determine the equilibrium policy p_i of the winning candidate. Each candidate knows her optimal behavior which she will implement at the office. Announcing policies different from their true positions or campaign promises have a cost for the candidates. This cost increases both with the distance between the true position and the implemented policy, and between the campaign promises and the implemented policy. Thus the candidate who wins the election finds her optimal policy by minimizing the cost

$$-\left\{(1-c)(p_i - t_i)^2 + c(p_i - y_i)^2\right\}$$

and it is minimized at

$$p_i = cy_i + (1-c)t_i \tag{3.3}$$

where $i = 1, 2$ and $c \in (0, 1]$. Here c parametrizes the lying cost arising from implementing a policy different from the campaign promise and the ideal point. This cost is common knowledge.

Therefore, for each y_i there is a unique p_i, for each $i = 1, 2$. Hence we can work exclusively with p_i's by defining the following payoff function

$$u_i(p_1, p_2) = -\frac{\alpha}{c^2}(p_i - (cx_i + (1-c)t_i))^2 + \pi_i(p_1, p_2)k \tag{3.4}$$

where

$$\pi_i(p_1, p_2) = \begin{cases} \frac{p_1 + p_2}{2} & \text{if } p_i < p_j \\ \frac{1}{2} & \text{if } p_i = p_j \\ 1 - \frac{p_1 + p_2}{2} & \text{if } p_i > p_j \end{cases} \qquad (i \neq j)$$

$u_i(p_1, p_2)$ is obtained by substituting y_i using (3.3). Thus our reduced form game is defined as the following strategic form game $G = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ where

- The set of players: $N = \{1, 2\}$,
- The set of strategies: $S_i = P$
- The payoff function: $u_i : P \times P \to \mathbb{R}$ which is as defined in (3.4).

CHAPTER 4

MAIN RESULTS

This chapter provides a characterization for the Nash equilibria of the election model first under certainty and then under uncertainty.

4.1 The Election Model with Complete Information

To find the Nash equilibria of the game, first we construct and analyze the players' best response correspondences. For simplicity we use the following notations:

Notation 1 $\hat{p}_i := cx_i + (1-c)t_i$ *and* $a := \frac{\alpha}{c^2}$ *where* $i = 1, 2$

Assumption 2 $\hat{p}_1 + \frac{k}{4a} < \frac{1}{2} < \hat{p}_2 - \frac{k}{4a}$

Proposition 3 *Under Assumption 2, the best response correspondence of candidate* 1 *is given by*

$$BR_1(p_2) = \begin{cases} \hat{p}_1 + \frac{k}{4a}, & p_2 \geq \frac{1}{2} \\ \hat{p}_1 + \frac{k}{4a}, & \hat{p}_1 + \frac{k}{4a} < p_2 < \frac{1}{2} \ \ \& \ \left(p_2 - \hat{p}_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(2p_2 - 1) \geq 0 \\ \emptyset, & \hat{p}_1 + \frac{k}{4a} < p_2 < \frac{1}{2} \ \ \& \ \left(p_2 - \hat{p}_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(2p_2 - 1) < 0 \\ \emptyset, & \hat{p}_1 - \frac{k}{4a} \leq p_2 \leq \hat{p}_1 + \frac{k}{4a} \\ \hat{p}_1 - \frac{k}{4a}, & p_2 < \hat{p}_1 - \frac{k}{4a} \end{cases}$$

and the best response correspondence of candidate 2 *is given by*

$$BR_2(p_1) = \begin{cases} \hat{p}_2 - \frac{k}{4a}, & p_1 \leq \frac{1}{2} \\ \hat{p}_2 - \frac{k}{4a}, & \frac{1}{2} < p_1 < \hat{p}_2 - \frac{k}{4a} \ \& \ \left(p_1 - \hat{p}_2 - \frac{k}{4a}\right)^2 + \frac{k}{a}(1 - 2p_1) \geq 0 \\ \emptyset, & \frac{1}{2} < p_1 < \hat{p}_2 - \frac{k}{4a} \ \& \ \left(p_2 - \hat{p}_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(1 - 2p_1) < 0 \\ \emptyset, & \hat{p}_2 - \frac{k}{4a} \leq p_1 \leq \hat{p}_2 + \frac{k}{4a} \\ \hat{p}_2 + \frac{k}{4a}, & p_1 > \hat{p}_2 + \frac{k}{4a} \end{cases}$$

Proof. In a strategic form game, the best response correspondence of player i is the correspondence $B_i : S_{-i} \rightrightarrows S_i$ given by

$$B_i(p_{-i}) = \{p_i \in S_i : u_i(p_i, p_{-i}) \geq u_i(p_i^{'}, p_{-i}) \text{ for all } p_i^{'} \in S_i\}.$$

So, first consider candidate 1. Fix a p_2 and let p_1^* be a best response to p_2.

- Suppose $p_2 \geq \frac{1}{2}$. Candidate 1 chooses her campaign policy to maximize her expected payoff given the other candidate's strategy. First consider candidate 1 chooses $p_1 < p_2$ then her expected payoff is given by

  $$u_1(p_1, p_2) = -a(p_1 - \hat{p}_1)^2 + \frac{p_1 + p_2}{2}k$$

  Candidate 1's best response to p_2 can be obtained by solving

  $$\arg\max_{p_1^* < p_2} \left\{ -a(p_1 - \hat{p}_1)^2 + \frac{p_1 + p_2}{2}k \right\}$$

  Thus, the following first order condition must hold:

  $$-2a(p_1^* - \hat{p}_1) + \frac{k}{2} = 0$$

  and it is solved at

  $$p_1^* = \hat{p}_1 + \frac{k}{4a} < p_2$$

which gives the payoff

$$u_1(p_1^*, p_2) = \frac{k^2}{16a} + \frac{\hat{p}_1 + p_2}{2}k$$

Now we will show that any other choice of p_1 leads to a strictly smaller payoff. By choosing $p_1 = p_2$ candidate 1 gets

$$u_1(p_2) = -a(p_2 - \hat{p}_1)^2 + \frac{k}{2}$$

Since the payoff function is not continuous, we calculate the left-hand limit of the payoff function at the point $p_1 = p_2$.

$$\lim_{p_1 \to p_2^-} -a(p_1 - \hat{p}_1)^2 + \frac{p_1 + p_2}{2}k = -a(p_2 - \hat{p}_1)^2 + p_2 k \tag{4.1}$$

As $p_2 \geq \frac{1}{2}$,

$$-a(p_2 - \hat{p}_1)^2 + p_2 k \geq -a(p_2 - \hat{p}_1)^2 + \frac{k}{2}$$

and since when $p_1 < p_2$ the maximum payoff is $\frac{k^2}{16a} + \frac{\hat{p}_1 + p_2}{2}k$, we have

$$\frac{k^2}{16a} + \frac{\hat{p}_1 + p_2}{2}k > -a(p_2 - \hat{p}_1)^2 + p_2 k \geq -a(p_2 - \hat{p}_1)^2 + \frac{k}{2} \tag{4.2}$$

Thus, $p_1 = p_2$ leads to a smaller payoff. By choosing $p_1 > p_2$, she gets

$$u_1(p_1, p_2) = -a(p_1 - \hat{p}_1)^2 + \left(1 - \frac{p_1 + p_2}{2}\right)k$$

The right-hand limit of the function is

$$\lim_{p_1 \to p_2^+} -a(p_1 - \hat{p}_1)^2 + \left(1 - \frac{p_1 + p_2}{2}\right)k = -a(p_2 - \hat{p}_1)^2 + (1 - p_2)k. \tag{4.3}$$

Again as $p_2 \geq \frac{1}{2}$,we have

$$\frac{k^2}{16a} + \frac{\hat{p}_1 + p_2}{2}k > -a(p_2 - \hat{p}_1)^2 + \frac{k}{2} \geq -a(p_2 - \hat{p}_1)^2 + (1 - p_2)k.$$

It follows that $p_1 > p_2$ yield a smaller payoff, too. Therefore if $p_2 \geq \frac{1}{2}$ then the unique best response of candidate 1 is $p_1^* = \hat{p}_1 + \frac{k}{4a}$.

- Suppose $p_2 < \frac{1}{2}$ then there are four cases.

  (*i*) If $\hat{p}_1 + \frac{k}{4a} < p_2 < \frac{1}{2}$ and $\left(p_2 - \hat{p}_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(2p_2 - 1) \geq 0$ then $p_1^* = \hat{p}_1 + \frac{k}{4a}$.

  We already solved maximization problem above and we obtained

$$p_1^* = \hat{p}_1 + \frac{k}{4a}$$

  when $p_1 < p_2$. First we check that whether there is a choice of p_1 that yields a higher payoff. By choosing $p_1 > p_2$, she gets the payoff

$$u_1(p_1, p_2) = -a(p_1 - \hat{p}_1)^2 + \left(1 - \frac{p_1 + p_2}{2}\right) k$$

  We have again

$$\lim_{p_1 \to p_2^+} -a(p_1 - \hat{p}_1)^2 + \left(1 - \frac{p_1 + p_2}{2}\right) k = -a(p_2 - \hat{p}_1)^2 + (1 - p_2)k$$

  and

$$\lim_{p_1 \to p_2^-} -a(p_1 - \hat{p}_1)^2 + \frac{p_1 + p_2}{2} k = -a(p_2 - \hat{p}_1)^2 + p_2 k$$

  On the other hand, candidate 1's payoff to $p_1 = p_2$ is given by

$$-a(p_2 - \hat{p}_1)^2 + \frac{k}{2}$$

  However since $p_2 < \frac{1}{2}$ in this case, the above three statements are ordered as follows:

$$-a(p_2 - \hat{p}_1)^2 + (1 - p_2)k > -a(p_2 - \hat{p}_1)^2 + \frac{k}{2} > -a(p_2 - \hat{p}_1)^2 + p_2 k \quad (4.4)$$

  This means that $\exists \epsilon > 0$ such that candidate 1 can get arbitrarily close to the payoff $-a(p_2 - \hat{p}_1)^2 + (1 - p_2)k$ by choosing $p_1 = p_2 + \epsilon$, and obtain a higher payoff than $\frac{k^2}{16a} + \frac{\hat{p}_1 + p_2}{2} k$. To eliminate this possibility and to $p_1^* = \hat{p}_1 + \frac{k}{4a}$ be the unique best response to $p_2 < \frac{1}{2}$, we must have

$$u_1(p_1^*, p_2) \geq u_1(p_1, p_2)$$

where $p_1 > p_2$, i.e.

$$-a(\hat{p}_1 + \frac{k}{4a} - \hat{p}_1)^2 + \frac{\hat{p}_1 + \frac{k}{4a} + p_2}{2}k \geq -a(p_2 - \hat{p}_1)^2 + (1 - p_2)k$$
$$\left(p_2 - \hat{p}_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(2p_2 - 1) \geq 0$$

We know that this holds by the assumption above. Therefore we conclude that if

$$\hat{p}_1 + \frac{k}{4a} < p_2 < \frac{1}{2} \text{ and } \left(p_2 - \hat{p}_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(2p_2 - 1) \geq 0$$

holds, then $p_1^* = \hat{p}_1 + \frac{k}{4a}$.

(ii) If $\hat{p}_1 + \frac{k}{4a} < p_2 < \frac{1}{2}$ and $\left(p_2 - \hat{p}_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(2p_2 - 1) < 0$ then there is no best response.

In this case, candidate 1 can obtain higher payoff by choosing $p_1 > p_2$ instead of choosing
$p_1 = \hat{p}_1 + \frac{k}{4a}$ and can get arbitrarily close to the payoff of

$$-a(p_2 - \hat{p}_1)^2 + (1 - p_2)k.$$

Candidate 1 would like to choose the smallest real number that is higher than p_2, and such number does not exist. Therefore if $\hat{p}_1 + \frac{k}{4a} < p_2 < \frac{1}{2}$ and $\left(p_2 - \hat{p}_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(2p_2 - 1) < 0$ then there is no best response.

(iii) If $p_2 < \hat{p}_1 - \frac{k}{4a}$ then the best response is $p_1^* = \hat{p}_1 - \frac{k}{4a}$

By choosing $p_1^* > p_2$, candidate 1 gets the payoff of

$$u_1(p_1, p_2) = -a(p_1 - \hat{p}_1)^2 + \left(1 - \frac{p_1 + p_2}{2}\right) k$$

We can find candidate 1's best response by solving

$$\arg\max_{p_1^* > p_2} \left\{ -a(p_1 - \hat{p}_1)^2 + \left(1 - \frac{p_1 + p_2}{2}\right) k \right\}$$

The following first order condition must hold:

$$-2a(p_1^* - \hat{p}_1) - \frac{k}{2} = 0$$

which is solved at

$$p_1^* = \hat{p}_1 - \frac{k}{4a} > p_2$$

Now since $p_2 < \frac{1}{2}$, the following is true:

$$-a(p_2 - \hat{p}_1)^2 + p_2 k < -a(p_2 - \hat{p}_1)^2 + \frac{k}{2} < -a(p_2 - \hat{p}_1)^2 + (1 - p_2)k$$

Thus neither $p_1 = p_2$ nor $p_1 < p_2$ yields a higher payoff. Therefore the unique best response is $p_1^* = \hat{p}_1 - \frac{k}{4a}$.

(iv) If $\hat{p}_1 - \frac{k}{4a} \leq p_2 < \hat{p}_1 + \frac{k}{4a}$ then there is no best response.

We know that

$$-a(p_2 - \hat{p}_1)^2 + p_2 k < -a(p_2 - \hat{p}_1)^2 + \frac{k}{2} < -a(p_2 - \hat{p}_1)^2 + (1 - p_2)k$$

where $p_2 < \frac{1}{2}$. Thus choosing $p_1 = p_2$ is better than choosing a number that is smaller than p_2.

On the other hand, choosing a little bit bigger number than p_2 is better than choosing p_2. However the candidate wants to play the smallest number which is higher than p_2 and such number does not exist.Therefore, there is no best response.

Now we will find the candidate 2's best response correspondence. Fix a p_1 and let p_2^* be a best response to p_1.

- Suppose $p_1 \leq \frac{1}{2}$. First consider candidate 1 chooses $p_2 > p_1$ then her expected payoff is given by

$$u_2(p_1, p_2) = -a(p_2 - \hat{p}_2)^2 + \left(1 - \frac{p_1 + p_2}{2}\right) k.$$

Candidate 2's best response to p_1 can be obtained by solving

$$\arg\max_{p_2^* > p_1} \left\{ -a(p_2 - \hat{p}_2)^2 + \left(1 - \frac{p_1 + p_2}{2}\right) k \right\}$$

Thus, the following first order condition must hold:

$$-2a(p_2^* - \hat{p}_2) - \frac{k}{2} = 0$$

which is solved at

$$p_2^* = \hat{p}_2 - \frac{k}{4a} > p_1$$

and it gives the payoff

$$u_2(p_1, p_2^*) = \frac{k^2}{16a} + \frac{p_1 + \hat{p}_2}{2} k$$

By choosing $p_2 = p_1$ candidate 2 gets the payoff

$$u_2(p_1) = -a(p_1 - \hat{p}_2)^2 + \frac{k}{2}$$

We have

$$\lim_{p_2 \to p_1^+} -a(p_2 - \hat{p}_2)^2 + \left(1 - \frac{p_1 + p_2}{2}\right) k = -a(p_1 - \hat{p}_2)^2 + (1 - p_1)k. \quad (4.5)$$

As $p_1 \leq \frac{1}{2}$, the following is true:

$$-a(p_1 - \hat{p}_2)^2 + (1 - p_1)k \geq -a(p_1 - \hat{p}_2)^2 + \frac{k}{2}$$

Since when $p_2 > p_1$ the maximum payoff is $\frac{k^2}{16a} + \frac{p_1 + \hat{p}_2}{2}k$, we have

$$\frac{k^2}{16a} + \frac{p_1 + \hat{p}_2}{2}k > -a(p_1 - \hat{p}_2)^2 + (1 - p_1)k \geq -a(p_1 - \hat{p}_2)^2 + \frac{k}{2} \quad (4.6)$$

Thus, $p_1 = p_2$ does not yield a higher payoff.

By choosing $p_2 < p_1$, her payoff is

$$u_2(p_1, p_2) = -a(p_2 - \hat{p}_2)^2 + \frac{p_1 + p_2}{2}k$$

We have

$$\lim_{p_2 \to p_1^-} -a(p_2 - \hat{p}_2)^2 + \frac{p_1 + p_2}{2}k = -a(p_1 - \hat{p}_2)^2 + p_1 k \quad (4.7)$$

Again as $p_1 \leq \frac{1}{2}$, we have

$$\frac{k^2}{16a} + \frac{\hat{p}_1 + p_2}{2}k > -a(p_1 - \hat{p}_2)^2 + \frac{k}{2} \geq -a(p_1 - \hat{p}_2)^2 + p_1 k$$

It follows that she cannot obtain a higher payoff by choosing $p_2 < p_1$. Therefore if $p_1 \leq \frac{1}{2}$ then the unique best response of candidate 2 is

$$p_2^* = \hat{p}_2 - \frac{k}{4a}.$$

- Suppose $p_1 > \frac{1}{2}$ then there are four cases.

  (i) If $\frac{1}{2} < p_1 < \hat{p}_2 - \frac{k}{4a}$ and $\left(\hat{p}_2 - p_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(1 - 2p_1) \geq 0$ then $p_2^* = \hat{p}_2 - \frac{k}{4a}$.

  We already solved maximization problem above when $p_2 > p_1$ and we obtained

$$p_2^* = \hat{p}_2 - \frac{k}{4a}.$$

First by choosing $p_2 < p_1$, candidate 2 gets the payoff

$$u_2(p_1, p_2) = -a(p_2 - \hat{p}_2)^2 + \frac{p_1 + p_2}{2}k$$

We have again

$$\lim_{p_2 \to p_1^+} -a(p_2 - \hat{p}_2)^2 + \left(1 - \frac{p_1 + p_2}{2}\right)k = -a(p_1 - \hat{p}_2)^2 + (1 - p_1)k$$

and

$$\lim_{p_2 \to p_1^-} -a(p_2 - \hat{p}_2)^2 + \frac{p_1 + p_2}{2}k = -a(p_1 - \hat{p}_2)^2 + p_1 k$$

On the other hand, candidate 2's payoff to $p_2 = p_1$ is given by

$$-a(p_1 - \hat{p}_2)^2 + \frac{k}{2}.$$

However since $p_1 > \frac{1}{2}$ in this case, the above three statements are ordered as follows:

$$-a(p_1 - \hat{p}_2)^2 + p_1 k > -a(p_1 - \hat{p}_2)^2 + \frac{k}{2} > -a(p_1 - \hat{p}_2)^2 + (1 - p_1)k$$

This means that $\exists \epsilon > 0$ such that candidate 1 can get arbitrarily close to the payoff $-a(p_1 - \hat{p}_2)^2 + p_1 k$ by choosing $p_2 = p_1 - \epsilon$, and obtain a higher payoff than $\frac{k^2}{16a} + \frac{p_1 + \hat{p}_2}{2}k$. To eliminate this possibility and to $p_2^* = \hat{p}_2 - \frac{k}{4a}$ be the unique best response to $p_1 > \frac{1}{2}$ we must have

$$u_2(p_1, p_2^*) \geq u_2(p_1, p_2)$$

where $p_2 < p_1$, i.e.

$$-a(\hat{p}_2 - \frac{k}{4a} - \hat{p}_2)^2 + \frac{p_1 + \hat{p}_2 - \frac{k}{4a}}{2}k \geq -a(p_1 - \hat{p}_2)^2 + p_1 k$$
$$\left(\hat{p}_2 - p_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(1 - 2p_1) \geq 0$$

We know that this holds by the assumption above. Therefore we conclude that if

$$\frac{1}{2} < p_1 < \hat{p}_2 - \frac{k}{4a} \text{ and } \left(\hat{p}_2 - p_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(1 - 2p_1) \geq 0$$

holds then $p_2^* = \hat{p}_2 - \dfrac{k}{4a}$.

(ii) If $\dfrac{1}{2} < p_1 < \hat{p}_2 - \dfrac{k}{4a}$ and $\left(\hat{p}_2 - p_1 - \dfrac{k}{4a}\right)^2 + \dfrac{k}{a}(1 - 2p_1) < 0$ then there is no best response.

In this case, candidate 2 can obtain higher payoff by choosing $p_2 < p_1$ instead of choosing
$p_2 = \hat{p}_2 - \dfrac{k}{4a}$ and can get arbitrarily close to the payoff of

$$-a(p_1 - \hat{p}_2)^2 + p_1 k.$$

Candidate 2 would like to choose the highest real number that is smaller than p_1,and such number does not exist. Therefore if $\dfrac{1}{2} <$ $p_1 < \hat{p}_2 - \dfrac{k}{4a}$ and $\left(p_2 - \hat{p}_1 - \dfrac{k}{4a}\right)^2 + \dfrac{k}{a}(2p_2 - 1) < 0$ then there is no best response.

(iii) If $p_1 > \hat{p}_2 + \dfrac{k}{4a}$ then the best response is $p_2^* = \hat{p}_2 + \dfrac{k}{4a}$
By choosing $p_2^* < p_1$ candidate 2 gets

$$u_2(p_1, p_2) = -a(p_2 - \hat{p}_2)^2 + \frac{p_1 + p_2}{2}k$$

We can find candidate 2's best response by solving

$$\arg\max_{p_2^* < p_1} \left\{ -a(p_2 - \hat{p}_2)^2 + \frac{p_1 + p_2}{2}k \right\}$$

The following first order condition must hold:

$$-2a(p_2^* - \hat{p}_2) + \frac{k}{2} = 0$$

which is solved at

$$p_2^* = \hat{p}_2 + \frac{k}{4a} < p_1$$

Now since $p_1 > \frac{1}{2}$, the following is true:

$$-a(p_1 - \hat{p}_2)^2 + p_1 k > -a(p_1 - \hat{p}_2)^2 + \frac{k}{2} > -a(p_1 - \hat{p}_2)^2 + (1 - p_1)k$$

Thus neither $p_2 = p_1$ nor $p_2 > p_1$ yields a higher payoff. Therefore the unique best response is $p_2^* = \hat{p}_2 + \frac{k}{4a}$.

(iv) If $\hat{p}_2 - \frac{k}{4a} < p_1 \leq \hat{p}_2 + \frac{k}{4a}$ then there is no best response.

We know that

$$-a(p_1 - \hat{p}_2)^2 + p_1 k > -a(p_1 - \hat{p}_2)^2 + \frac{k}{2} > -a(p_1 - \hat{p}_2)^2 + (1 - p_1)k$$

Hence, choosing $p_2 = p_1$ is better than choosing a number that is bigger than $p_1 = \hat{p}_2 - \frac{k}{4a}$.

On the other hand, choosing a little bit smaller number than p_1 is better than choosing p_1. However the candidate wants to play the highest number which is smaller than p_1 and such a number does not exist. Thus, there is no best response.

■

Proposition 4 *Under Assumption* 2, *there is a unique Nash equilibrium in which*

$$p_1^* = \hat{p}_1 + \frac{k}{4a} \text{ and } p_2^* = \hat{p}_2 - \frac{k}{4a}.$$

Proof. First, we will show that these strategies constitutes an equilibrium. Fix $p_2^* = \hat{p}_2 - \frac{k}{4a}$. Suppose, for a contradiction, we have $p_1^* < \hat{p}_1 + \frac{k}{4a}$ in equilibrium. However, candidate 1 can get higher payoff by increasing p_1^*. By choosing $p_1 < p_2$ she gets the payoff

$$u_1(p_1, p_2) = -a(p_1 - \hat{p}_1)^2 + \frac{p_1 + p_2}{2}k$$

and the policy that gives the maximum payoff is

$$p_1^* = \hat{p}_1 + \frac{k}{4a}.$$

Thus $p_1^* < \hat{p}_1 + \frac{k}{4a}$ cannot be equilibrium strategy for candidate 1.

Now suppose that $p_1^* > \hat{p}_1 + \frac{k}{4a}$. There are three possibilities:

(i) $\hat{p}_1 + \frac{k}{4a} < p_1^* < p_2$.

Since $p_1 < p_2$, the payoff of candidate 1 is

$$u_1(p_1, p_2) = -a(p_1 - \hat{p}_1)^2 + \frac{p_1 + p_2}{2}k$$

and it is maximized at

$$p_1^* = \hat{p}_1 + \frac{k}{4a}$$

Therefore $p_1^* = \hat{p}_1 + \frac{k}{4a}$ is a profitable deviation for candidate 1 and any policy such that $\hat{p}_1 + \frac{k}{4a} < p_1^* < p_2$ cannot be an equilibrium strategy for candidate 1.

(ii) $p_1^* = p_2$

Candidate 1's payoff is given by

$$u_1(p_2) = -a(p_2 - \hat{p}_1)^2 + \frac{k}{2}$$

Since $p_2 = \hat{p}_2 - \frac{k}{4a} > \frac{1}{2}$ we have the following

$$\begin{aligned} -a(p_2 - \hat{p}_1)^2 + \frac{k}{2} &< -a(\hat{p}_1 + \frac{k}{4a} - \hat{p}_1)^2 + \frac{\hat{p}_1 + \frac{k}{4a} + p_2}{2}k \\ u_1(p_2) &< u_1(\hat{p}_1 + \frac{k}{4a}, p_2) \end{aligned}$$

by equation (4.2). So candidate 1 prefers $p_1^* = \hat{p}_1 + \dfrac{k}{4a}$ to $p_1^* = p_2$. Thus there is no equilibrium with $p_1^* = p_2$.

(iii) $p_1^* > p_2$

Candidate 1's payoff is given by

$$u_1(p_1, p_2) = -a(p_1 - \hat{p}_1)^2 + \left(1 - \frac{p_1 + p_2}{2}\right) k$$

and we know that it is maximized at $p_1^* = \hat{p}_1 - \dfrac{k}{4a} > p_2$. However, by Assumption 2 that is not possible. So we have $\hat{p}_1 - \dfrac{k}{4a} < p_2$ and $p_1^* = \hat{p}_1 - \dfrac{k}{4a}$ cannot be an equilibrium strategy.

Therefore candidate 1's best response to $p_2 = \hat{p}_2 - \dfrac{k}{4a}$ is $p_1^* = \hat{p}_1 + \dfrac{k}{4a}$

Now, fix $p_1^* = \hat{p}_1 + \dfrac{k}{4a}$. Suppose, for a contradiction, we have $p_2^* > \hat{p}_2 - \dfrac{k}{4a}$ in equilibrium. However candidate 2 can get higher payoff by decreasing p_2^*. By choosing $p_2 > p_1$ she obtains the payoff

$$u_2(p_1, p_2) = -a(p_2 - \hat{p}_2)^2 + \left(1 - \frac{p_1 + p_2}{2}\right) k$$

and it is maximized at

$$p_2^* = \hat{p}_2 - \frac{k}{4a}$$

Thus $p_2^* > \hat{p}_2 - \dfrac{k}{4a}$ cannot be an equilibrium strategy for candidate 2.

Now suppose that $p_2^* < \hat{p}_2 - \dfrac{k}{4a}$. There are three possibilities:

(i) $p_1 < p_2^* < \hat{p}_2 - \dfrac{k}{4a}$.

Since $p_2 > p_1$ the payoff of candidate 2 is

$$u_2(p_1, p_2) = -a(p_2 - \hat{p}_2)^2 + \left(1 - \frac{p_1 + p_2}{2}\right) k$$

and it is maximized at

$$p_2^* = \hat{p}_2 - \frac{k}{4a}.$$

Therefore $p_2^* = \hat{p}_2 - \dfrac{k}{4a}$ is a profitable deviation for candidate 1 and

$$p_1 < p_2^* < \hat{p}_2 - \frac{k}{4a}$$

cannot be an equilibrium strategy for candidate 2.

(ii) $p_2^* = p_1$

Candidate 2's payoff is given by

$$u_2(p_1) = -a(p_1 - \hat{p}_2)^2 + \frac{k}{2}$$

Since $p_1 = \hat{p}_1 + \dfrac{k}{4a} < \dfrac{1}{2}$ we have the following

$$\begin{aligned} -a(p_1 - \hat{p}_2)^2 + \frac{k}{2} &< -a(\hat{p}_2 - \hat{p}_2 + \frac{k}{4a})^2 + \frac{p_1 + \hat{p}_2 - \frac{k}{4a}}{2}k \\ u_2(p_1) &< u_2(p_1, \hat{p}_2 - \frac{k}{4a}) \end{aligned}$$

by the equation (4.6). It means that $p_2^* = \hat{p}_2 - \dfrac{k}{4a}$ is a profitable deviation. Thus there is no equilibrium where $p_2^* = p_1$.

(iii) $p_2^* < p_1$

Candidate 2's payoff is given by

$$u_2(p_1, p_2) = -a(p_2 - \hat{p}_1)^2 + \left(1 - \frac{p_1 + p_2}{2}\right) k$$

and we know that it is maximized at $p_2^* = \hat{p}_2 + \dfrac{k}{4a} < p_1$. However, by Assumption 2, we have $\hat{p}_2 + \dfrac{k}{4a} > p_1$. So $p_2^* = \hat{p}_2 + \dfrac{k}{4a}$ cannot be an equilibrium strategy.

Thus candidate 2's best response to $p_1 = \hat{p}_1 + \frac{k}{4a}$ is $p_2^* = \hat{p}_2 - \frac{k}{4a}$. Therefore in equilibrium we must have

$$p_1^* = \hat{p}_1 + \frac{k}{4a},\ p_2^* = \hat{p}_2 - \frac{k}{4a}.$$

Now we need to prove that $p_1^* = \hat{p}_1 + \frac{k}{4a}$, $p_2^* = \hat{p}_2 - \frac{k}{4a}$ is the unique equilibrium. First we look at if there is an equilibrium in which $p_2(\neq \hat{p}_2 - \frac{k}{4a}) \geq \frac{1}{2}$. By Proposition 1 we know that candidate 1's best response to any $p_2 \geq \frac{1}{2}$ is

$$p_1^* = \hat{p}_1 + \frac{k}{4a}.$$

On the other hand, since $p_1^* = \hat{p}_1 + \frac{k}{4a} < \frac{1}{2}$, again by Proposition 1, candidate 2's best response to $\hat{p}_1 + \frac{k}{4a}$ is $p_2^* = \hat{p}_2 - \frac{k}{4a} > \frac{1}{2}$ and it is unique. So the only possibility for the equilibrium is $p_1^* = \hat{p}_1 + \frac{k}{4a}$, $p_2^* = \hat{p}_2 - \frac{k}{4a}$.

Now we will show that there is no equilibrium where $p_2 < \frac{1}{2}$. We analyze this in three cases:

(1) If $\hat{p}_1 + \frac{k}{4a} < p_2 < \frac{1}{2}$ and $\left(p_2 - \hat{p}_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(2p_2 - 1) \geq 0$ then by Proposition 1 we know that

$$p_1^* = \hat{p}_1 + \frac{k}{4a}.$$

However for this strategy profile to be an equilibrium $\hat{p}_1 + \frac{k}{4a} < p_2 < \frac{1}{2}$ must also be a best response to $p_1^* = \hat{p}_1 + \frac{k}{4a}$.

As $\hat{p}_1 + \frac{k}{4a} < \frac{1}{2}$, by the proposition 1 we know that candidate 2's best response to any $p_1 < \frac{1}{2}$ is

$$p_2^* = \hat{p}_2 - \frac{k}{4a}$$

and it is bigger than $\frac{1}{2}$. Hence there is no equilibrium where $\hat{p}_1 + \frac{k}{4a} < p_2 < \frac{1}{2}$.

(2) If $\hat{p}_1 + \frac{k}{4a} < p_2 < \frac{1}{2}$ and $\left(p_2 - \hat{p}_1 - \frac{k}{4a}\right)^2 + \frac{k}{a}(2p_2 - 1) < 0$ or if $\hat{p}_1 - \frac{k}{4a} \leq p_2 \leq \hat{p}_1 + \frac{k}{4a}$ then we know that there is no best response for candidate 1. Thus there cannot be an equilibrium in this case.

(3) If $p_2 < \hat{p}_1 - \frac{k}{4a}$ then the best response of candidate 1 is

$$p_1^* = \hat{p}_1 - \frac{k}{4a}$$

On the other hand, if $p_1 < \frac{1}{2}$ then candidate 2's best response is

$$p_2^* = \hat{p}_2 - \frac{k}{4a}$$

and it is bigger than $\frac{1}{2}$.Thus there is no possible equilibrium with $p_2 \leq x_1 - \frac{k}{4\alpha}$.

Similarly, it can be shown that there is no equilibrium where $p_1 > \frac{1}{2}$. Therefore the unique Nash equilibrium is

$$p_1^* = \hat{p}_1 + \frac{k}{4a},\ p_2^* = \hat{p}_2 - \frac{k}{4a}. \tag{4.8}$$

■

Corollary 5 $y_1^* = x_1 + \frac{kc}{4\alpha},\ \ y_2^* = x_2 - \frac{kc}{4\alpha}$ *is the unique Nash equilibrium.*

Proof. Substituting $p_i = cy_i + (1-c)t_i$ and $\hat{p}_i = cx_i + (1-c)t_i$ in

$$p_1^* = \hat{p}_1 + \frac{k}{4a},\ p_2^* = \hat{p}_2 - \frac{k}{4a}$$

it can be shown that

$$y_1^* = x_1 + \frac{kc}{4\alpha},\ y_2^* = x_2 - \frac{kc}{4\alpha}.$$

is also the unique Nash Equilibrium. ■

As seen in this equilibrium analysis, there is always flip-flopping in the complete information case. It means that although a candidate stated or promised a right (left) policy in the past, she has an incentive to choose a moderate position during the campaign. There are several factors which affect the extent of flip-flopping:

1. α - Cost parameter of choosing a position different from the past announcement. If α is high then the candidates abstain from too much flip-flopping since the lying cost also increases in α. So the campaign promise y_i cannot be far away from the past announcement x_i. Here we can consider α as a factor on the importance of the reputation of a candidate. When α is high, candidates do not move away from their past policy announcements to avoid bad reputation. If α is small then the lying cost will be smaller. Therefore, if a past announcement of a candidate is extreme, she will not hesitate to change her promise into a moderate one and to close to the median because of the low cost of lying.

2. k - Utility of winning the election. It can be easily seen that as k increases the candidates would be more centrist. A higher k makes winning the election more attractive to a candidate and she may neglect the cost of lying to win the election. Even if the lying cost is high the candidate is motivated to choose a more moderate position than her past announcement. Hence, flip-flopping increases in k. On the other hand, if k is too small then the candidates do not demonstrate a willingness to flip-flop because lying to win the election does not provide a significant advantage. Thus they flip-flop a little to keep the lying cost low.

3. c - Commitment level. When c is high, the candidates implement policies close to their campaign promises. Since voters know that the winning candidate (say i) will enact $p_i = cy_i + (1 - c)t_i$ at the office, they vote accordingly knowing that the winning candidate with high c will implement a policy close to her campaign promise. To understand this throughly consider the case where the candidate does not care about commitment, i.e. $c = 0$. The winning candidate implement her ideological policy t_i after the election, even if she announces a very moderate position during the campaign. However, voters do not vote for her because they know that she will implement her ideal policy rather than her campaign promise. Thus when c increases credibility of the candidates arises and depending on this their chance of winning increases. Apart from that, if voters know that a candidate keeps her promise, they do not care much about if she changes her past promise.

Consequently, since there is a positive utility of winning the election ($k > 0$), there is always flip-flopping in the complete information case. However, the degree of flip-flopping goes to zero as α goes to infinity. So if α is low, a candidate chooses a policy which is closer to the center rather than her past announcement gets elected. On the other hand, she does not choose median ($\frac{1}{2}$) as in Downsian model because she has to consider the cost of lying. The intuition behind this result is that in real life, a left-wing party chooses a policy close to the left and a right-wing party picks a position close to the right instead of adopting ($\frac{1}{2}$) to ensure the office. Therefore, the equilibrium platforms are divergent in our model different from the standard election model.

4.2 The Election Model with Incomplete Information

In this section, we analyze that what happens if we add imperfect information into our model. Differently from the game with complete information there are types of candidates and these types are private information. We have two types of candidates, $\theta = \{c, n\}$. With probability q a candidate is *commitment type, c*, and with probability $(1-q)$ a candidate is *normal type, n*. The commitment type makes a promise in the past, she announces this promise during the campaign and after the election she commits to her position. However, the normal type of candidate is free to misrepresent her policy intention and she may flip-flop during the campaign or after the election.

We can also explain this notion as follows: Both types of candidates announced a policy x_i in the past. The commitment type candidate promises $y_i^c = x_i$, during the campaign. Later if she gets elected he implements $p_i^c = x_i$. The normal type candidate *may* promise $y_i^n \neq x_i$ at the campaign stage and if she wins the election she implements $p_i = cy_i + (1-c)t_i$ where $i = 1, 2$. We can formulate this situation as a Bayesian game $G_B = (N, (S_i), (\Theta_i), (q), (u_i))$:

- The set of players: $N = \{1, 2\}$,
- The set of strategies: $S_i = P$,
- The set of types: : $\Theta_i = \{c, n\}$,
- The probabilities: $prob(\Theta_i = c) = q \in (0, 1)$ and $prob(\Theta_i = n) = 1 - q$,
- The payoff function: $u_i : P \times P \to \mathbb{R}$ for player i of normal type

$$u_i(p_1, p_2) = -\frac{\alpha}{c^2}(p_i - (cx_i + (1-c)t_i))^2 + \pi_i(p_1, p_2)k$$

Therefore there are two possible types of equilibria:

(1) Separating equilibria; where each type announces a different position. If the types separate, electorate can deduce the type of candidate from the announcements.

(2) Pooling equilibria, where each type chooses the same policy. When the types pool the voters cannot infer the exact type of the candidate.

In this model, a normal type of candidate has a chance to manipulate voters by pretending to be commitment type during the campaign stage. Since she announces her past policy statement during the election, voters cannot decide her true intention based on this policy choice, but they believe that after the election she will implement x_i with probability q, and $cx_i+(1-c)t_i$ with probability $1-q$. If pooling is more profitable for the candidate than separating, the normal type can obtain more votes by imitating the commitment type and not revealing her true intention.

This notion is interesting but it is too complicated to fully characterize and interpret the equilibrium behavior of this model.[1] Thus we describe a more convenient model in the next chapter.

[1]We can characterize the separating equilibrium under certain conditions but we cannot specify under what conditions, a pooling equilibrium exists.

CHAPTER 5

AN EXTENSION TO INCOMPLETE INFORMATION

Different from the main model here after the election, the winning candidate has to implement the campaign promise y_i, where $i = 1, 2$. In other words, we could also think of this as the cost of implementing a policy different from the promise as being too high, i.e., $c = 1$. We have

$$p_i = y_i$$

The set of players $N = \{1, 2\}$, the set of strategies $S_i = P$ for each player i and the payoff function $u_i : P \times P \to \mathbb{R}$.

There is also a cost which comes from announcing different from x_i (announcing $y_i \neq x_i$) during the campaign. Thus expected payoff of the candidate i is

$$u_i(y_1, y_2) = -\alpha(x_i - y_i)^2 + \pi_i(y_1, y_2)k$$

where $k > 0$ is constant and

$$\pi_i(y_1, y_2) = \begin{cases} \dfrac{y_1 + y_2}{2} & \text{if } y_i < y_j \\ \dfrac{1}{2} & \text{if } y_i = y_j \\ 1 - \dfrac{y_1 + y_2}{2} & \text{if } y_i > y_j \end{cases} \qquad (i \neq j)$$

is the probability of winning for the candidate i.

The strategic form of the game $G = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ is as follows:

- The set of players: $N = \{1, 2\}$,

- The set of strategies: $S_i = P$ for each player i
- The payoff function: $u_i : P \times P \to \mathbb{R}$ for player i is

$$u_i(y_1, y_2) = -\alpha(x_i - y_i)^2 + \pi_i(y_1, y_2)k$$

The intuition behind the condition $c = 1$ is can be considered as follows: If a candidate announces a policy different from her past statement during the campaign and enacts her past policy statement at the office then she suffers from a very high cost of lying. Therefore, we choose $c = 1$ to avoid this behavior.

We begin with the characterization of the equilibria of the game under complete information.

5.1 The Election Model with Complete Information

The equilibrium analysis is similar to the main model. The only difference in the new model is that y_i is substituted for p_i.

Assumption 6 $x_1 + \frac{k}{4\alpha} < \frac{1}{2} < x_2 - \frac{k}{4\alpha}$

The following proposition gives the best response correspondences:

Proposition 7 *Under Assumption 6, the best response correspondence of candidate* 1 *is given by*

$$BR_1(y_2) = \begin{cases} x_1 + \frac{k}{4\alpha}, & y_2 \geq \frac{1}{2} \\ x_1 + \frac{k}{4\alpha}, & x_1 + \frac{k}{4\alpha} < y_2 < \frac{1}{2} \;\&\; \left(y_2 - x_1 - \frac{k}{4\alpha}\right)^2 + \frac{k}{\alpha}(2y_2 - 1) \geq 0 \\ \emptyset, & x_1 + \frac{k}{4\alpha} < y_2 < \frac{1}{2} \;\&\; \left(y_2 - x_1 - \frac{k}{4\alpha}\right)^2 + \frac{k}{\alpha}(2y_2 - 1) < 0 \\ \emptyset, & x_1 - \frac{k}{4\alpha} \leq y_2 \leq x_1 + \frac{k}{4\alpha} \\ x_1 - \frac{k}{4\alpha}, & y_2 < x_1 - \frac{k}{4\alpha} \end{cases}$$

and the best response correspondence of candidate 2 is given by

$$BR_2(y_1) = \begin{cases} x_1 - \dfrac{k}{4\alpha}, & y_1 \le \dfrac{1}{2} \\ x_2 - \dfrac{k}{4\alpha}, & \dfrac{1}{2} < y_1 < x_2 - \dfrac{k}{4\alpha} \;\; \& \;\; \left(y_1 - x_2 - \dfrac{k}{4\alpha}\right)^2 + \dfrac{k}{\alpha}(1-2y_1) \ge 0 \\ \emptyset, & \dfrac{1}{2} < y_1 < x_2 - \dfrac{k}{4\alpha} \;\; \& \;\; \left(y_1 - x_2 - \dfrac{k}{4\alpha}\right)^2 + \dfrac{k}{\alpha}(1-2y_1) < 0 \\ \emptyset, & x_2 - \dfrac{k}{4\alpha} \le y_1 \le x_2 + \dfrac{k}{4\alpha} \\ x_2 + \dfrac{k}{4\alpha}, & y_1 > x_2 + \dfrac{k}{4\alpha} \end{cases}$$

Proof. The proof is similar to the proof of Proposition 3. ■

Proposition 8 $y_1^* = x_1 + \dfrac{k}{4\alpha}$, $y_2^* = x_2 - \dfrac{k}{4\alpha}$ *is the unique Nash equilibrium.*

Proof. The proof is similar to the proof of Proposition 4. ■

As in the model in Chapter 1, Proposition 4 shows that politicians always flip-flop under certainty. The only difference in the new model is that candidates do not change their election promises once they are elected, i.e. $c = 1$. However, there is still flip-flopping and α and k determine its degree in the same way as in the previous model.

5.2 The Election Model with Incomplete Information

In this model, there are two types of candidates, $\theta = \{c, n\}$, and these types are private information. With probability $q \in (0,1)$ a candidate is commitment type, c, and with probability $(1-q)$ a candidate is normal type, n. The commitment type makes a commitment in the past, she sticks to her past commitment during the campaign and if she wins she keeps her promise. However, the normal type is free to change her past commitment, that is, she may flip-flop during the campaign but if she gets elected she still has to implement her election promise.

We can also explain this notion as follows: Both types of candidates announced a policy x_i in the past. The commitment type candidate promises $y_i^c = x_i$, during the campaign. Later if she gets elected she implements $p_i = x_i$. On the other hand, the normal type candidate promises which may be different from x_i during the campaign but if she wins the election she implements $p_i = y_i^n$, where $i = 1, 2$. We can formulate this situation as a Bayesian game $G_B = (N, (S_i), (\Theta_i), (q), (u_i))$:

- The set of players: $N = \{1, 2\}$,
- The set of strategies: $S_i = P$ for each player i
- The set of types: $\Theta_i = \{c, n\}$
- The probabilities: $prob(\Theta_i = c) = q \in (0, 1)$ and $prob(\Theta_i = n) = 1 - q$

The payoff function: $u_i : P \times P \to \mathbb{R}$ for player i of normal type

$$u_i(y_1, y_2) = -\alpha(x_i - y_i)^2 + \pi_i(y_1, y_2)k$$

Proposition 9 *Under Assumption 6, the unique equilibrium is a separating equilibrium in which*

$$(y_1^c, y_1^n) = (x_1, x_1 + \frac{k}{4\alpha}), \quad (y_2^c, y_2^n) = (x_2, x_2 - \frac{k}{4\alpha})$$

Proof. The equilibrium can be investigated in four possibilities: (a) Both candidates separates $(b), (c)$ Only one candidate separates (d) Both candidates pool.

(a) In order to organize our characterization, suppose first that candidate 1 separates in equilibrium, i.e. $(y_1^c = x_1,\ y_1^n \neq x_1)$. Candidate 2 might be separating or pooling. Suppose first that candidate 2 pools, i.e. $y_2^c = y_2^n = x_2$. We will show that $y_1^n = x_1 + \frac{k}{4\alpha}$.

By choosing $x_1 \neq y_1 < x_2$ the best she can get is found by maximizing

$$u_1(y_1, x_2) = -\alpha(x_1 - y_1)^2 + \frac{y_1 + x_2}{2}k$$

This is solved at $y_1^* = x_1 + \frac{k}{4\alpha}$ which gives the payoff of

$$\frac{k^2}{16\alpha} + \frac{x_1 + x_2}{2}k$$

By choosing $y_1 = x_2$ candidate 2 gets the payoff of

$$u_1(x_2) = -\alpha(x_1 - x_2)^2 + \frac{k}{2}$$

We have

$$\lim_{y_1 \to x_2^-} -\alpha(x_1 - y_1)^2 + \frac{y_1 + x_2}{2}k = -\alpha(x_1 - x_2)^2 + x_2 k$$

As $x_2 > \frac{1}{2}$, the following is true

$$-\alpha(x_1 - x_2)^2 + x_2 k > -\alpha(x_1 - x_2)^2 + \frac{k}{2}$$

and when $y_1 < x_2$ the maximum payoff is $\frac{k^2}{16\alpha} + \frac{x_1 + x_2}{2}k$, so we have

$$\frac{k^2}{16\alpha} + \frac{x_1 + x_2}{2}k > -\alpha(x_1 - x_2)^2 + x_2 k > -\alpha(x_1 - x_2)^2 + \frac{k}{2}$$

Thus, $y_1 = x_2$ is not a profitable deviation.

By choosing $y_1 > x_2$, her payoff is

$$u_1(y_1, x_2) = -\alpha(x_1 - y_1)^2 + \left(1 - \frac{y_1 + x_2}{2}\right)k$$

We have

$$\lim_{y_1 \to x_2^+} -\alpha(x_1 - y_1)^2 + \left(1 - \frac{y_1 + x_2}{2}\right)k = -\alpha(x_1 - x_2)^2 + (1 - x_2)k$$

Again as $x_2 > \frac{1}{2}$,

$$\frac{k^2}{16\alpha} + \frac{x_1 + x_2}{2}k > -\alpha(x_1 - x_2)^2 + \frac{k}{2} > -\alpha(x_1 - x_2)^2 + (1 - x_2)k$$

Hence, $y_1 > x_2$ is not a profitable deviation either. Now we will show that it is not optimal for candidate 1 to pool, i.e choosing $y_1 = x_1$ is not a profitable deviation. Pooling means that both commitment and normal types announces x_1 during the campaign, that is, $y_1^c = y_1^n = x_1$. If there is a pooling equilibrium, the normal type candidate does not have a profitable deviation. We already know the policy which gives the maximum payoff is $y_1^* = x_1 + \frac{k}{4\alpha}$. So to $y_1 = x_1$ be optimal, the payoff of choosing x_1 must be greater than the payoff of choosing $y_1^n = x_1 + \frac{k}{4\alpha}$. However, we have

$$\begin{aligned} u_1(x_1, x_2) &< u(y_1^n, y_2) \\ \frac{x_1 + x_2}{2}k &< \frac{k^2}{16\alpha} + \frac{x_1 + x_2}{2}k \end{aligned}$$

so $y_1 = x_1$ is not a profitable deviation. We showed that the only candidate for the equilibrium in which candidate 1 separates and candidate 2 pools is $y_1^n = x_1 + \frac{k}{4\alpha}$.

Suppose now candidate 2 separates at $y_2^n \neq x_2$ but $y_2^n \geq \frac{1}{2}$. Since candidate 1 thinks that candidate 2 is commitment type with probability q, and normal type with probability $(1 - q)$ it follows that $y_2 = qx_2 + (1 - q)y_2^n$. As $y_2^n > \frac{1}{2}$ and $x_2 > \frac{1}{2}$ it can be easily seen that

$$y_2 = qx_2 + (1 - q)y_2^n > \frac{1}{2}$$

Thus by choosing $y_1 < y_2 = qx_2 + (1 - q)y_2^n$ she gets the payoff

$$u_1(y_1, y_2) = -\alpha(x_1 - y_1)^2 + \frac{y_1 + qx_2 + (1 - q)y_2^n}{2}k$$

and $y_1^* = x_1 + \dfrac{k}{4\alpha}$ maximizes this payoff. Since $qx_2 + (1-q)y_2^n > \dfrac{1}{2}$, it can be shown that $y_1 = qx_2 + (1-q)y_2^n$ or $y_1 > qx_2 + (1-q)y_2^n$ or $y_1 = x_1$ is not a profitable deviation in a similar way as above. Now we will check that if there is an equilibrium with $y_2^n < \dfrac{1}{2}$. Suppose that candidate 2 chooses a policy with $y_2^n < \dfrac{1}{2}$. Candidate 1 thinks that her opponent is commitment type with probability q, and normal type with probability $(1-q)$. So there are two possibilities: $y_2 = qx_2 + (1-q)y_2^n < \dfrac{1}{2}$ and $y_2 = qx_2 + (1-q)y_2^n \geq \dfrac{1}{2}$. In the first case, we can easily show that there is no possible equilibrium with $y_2 < \dfrac{1}{2}$ by applying the steps similar to (1)-(3) in Proposition 4. Let's look at the second case. If $y_2 = qx_2 + (1-q)y_2^n \geq \dfrac{1}{2}$ then we know that best response of candidate 1 is

$$y_1^* = x_1 + \frac{k}{4\alpha}.$$

It is obvious that candidate 1 separates, too (as $y_1^* \neq x_1$). Now this to be an equilibrium, the strategy $(y_2^c = x_2,\ y_2^n < \dfrac{1}{2})$ must be a best response to $(y_1^c = x_1, y_1^n = x_1 + \dfrac{k}{4\alpha})$. By choosing $y_2 > y_1 (= qx_1 + (1-q)y_1^n)$, candidate 2 gets the payoff

$$u_2(y_1, y_2) = -\alpha(x_2 - y_2)^2 + \left(1 - \frac{qx_1 + (1-q)y_1^n + y_2}{2}\right) k$$

and it is maximized at

$$y_2^* = x_2 - \frac{k}{4\alpha}.$$

However by Assumption 7, $x_2 - \dfrac{k}{4\alpha} > \dfrac{1}{2}$. Additionally, it can be shown that neither $y_2 = qx_1 + (1-q)y_1^n$ nor $y_2 < qx_1 + (1-q)y_1^n$ is a profitable deviation.Therefore, we can conclude that there is no equilibrium in which $y_2^n < \dfrac{1}{2}$. So far we showed that in any separating equilibrium, we have $y_1^n = x_1 + \dfrac{k}{4\alpha}$ and if candidate 2 is also separates, we have $y_2^n \geq \dfrac{1}{2}$. Similar calculations show that if candidate 2 separates then she must be playing $y_2^n = x_2 - \frac{k}{4\alpha}$. This implies that the unique equilibrium in which both candidates separate is

$$y_1^n = x_1 + \frac{k}{4\alpha},\ y_2^n = x_2 - \frac{k}{4\alpha}.$$

$(b)\&(c)$ Now we will show that there is no equilibrium in which only one candidate separates. Suppose that candidate 1 separates and candidate 2 pools. Since candidate 2 pools her strategy is $y_2^c = y_2^n = x_2$. As $x_2 > \frac{1}{2}$, candidate 1's optimal strategy is $y_1^c = x_1 < \frac{1}{2}$, $y_1^n = x_1 + \frac{k}{4\alpha} < \frac{1}{2}$. Now we check that if it is optimal to pool for candidate 2. Since candidate 2 thinks that candidate 1 is commitment type with probability q, and normal type with probability $(1-q)$, candidate 2's expected payoff to $y_2 = x_2 (= qx_2 + (1-q)x_2)$ is

$$\left[q\frac{x_1 + x_2}{2} + (1-q)\frac{y_1^n + x_2}{2}\right] k = \frac{qx_1 + (1-q)y_1^n + x_2}{2} k.$$

By choosing $y_2^n \neq x_2$, the worst that can happen is that voters believe she is the normal type. We have shown that in this case her best choice is $y_2^n = x_2 - \frac{k}{4\alpha}$ and whenever $y_1 \leq \frac{1}{2}$ this is the unique best response, i.e., strictly better than x_2. Since $qx_1 + (1-q)y_1^n < \frac{1}{2}$, this shows that $y_2 = x_2$ is not a best response. Hence, there cannot be an equilibrium in which candidate 1 separates and candidate 2 pools. Similarly, it can be shown that there is no equilibrium in which candidate 1 pools and candidate 2 separates.

(d) Now we will look at the case where both candidates pools. Suppose there is an equilibrium in this case. Candidate 1 and 2 pools and choose $y_1^c = y_1^n = x_1$, $y_2^c = y_2^n = x_2$ respectively. We have already shown that in (a), when candidate 2 pools it is not profitable for candidate to pool. Thus, there is no equilibrium in which both candidates pool.

Therefore, the only candidate for an equilibrium is that both candidates separate at $y_1^n = x_1 + \frac{k}{4\alpha}, \;\; y_2^n = x_2 - \frac{k}{4\alpha}$.

Finally, it remains to show that the strategies and the beliefs' specification in which

$$\begin{aligned} (y_1^c &= x_1, \; y_1^n = x_1 + \frac{k}{4\alpha}), \; (y_2^c = x_2, \; y_2^n = x_2 - \frac{k}{4\alpha}), \\ \mu &= \begin{cases} \mu(c|x_i) = 1 \;\text{ for } \quad\; y_i = x_i \\ \mu(n|y_i) = 1 \text{ for all } y_i \neq x_i \end{cases} \end{aligned} \tag{5.1}$$

is a perfect Bayesian equilibrium, where $i = 1, 2$.

First we will show that given $(y_2^c = x_2, \, y_2^n = x_2 - \frac{k}{4\alpha})$ candidate 1's best choice is $(y_1^c = x_1, \, y_1^n = x_1 + \frac{k}{4\alpha})$. Since candidate 1 separates, the voters' beliefs are $\mu(c|x_1) = 1$ and $\mu(n|y_1) = 1$. Commitment type always chooses $y_1^c = x_1$ during the election so we need to find the normal type's best response. We have shown that $y_1^n = x_1 + \frac{k}{4\alpha}$ is the unique best response to any $y_2 \geq \frac{1}{2}$. Since $y_2 = qy_2^c + (1-q)y_2^n > \frac{1}{2}$ we can conclude that $(y_1^c = x_1,$ $y_1^n = x_1 + \frac{k}{4\alpha})$ is the unique best response to $(y_2^c = x_2, \, y_2^n = x_2 - \frac{k}{4\alpha})$.

Similarly we can show that given $(y_1^c = x_1, \, y_1^n = x_1 + \frac{k}{4\alpha})$ candidate 2's unique best response is $(y_2^c = x_2, \, y_2^n = x_2 - \frac{k}{4\alpha})$.

Therefore, (5.1) is the unique separating perfect Bayesian equilibrium.

■

We find out that under uncertainty the candidates diverge from the median as in the complete information case and the degree of flip-flopping is determined by cost of lying and the utility of winning the election. Since candidates commit to their election promises, the normal types of candidates cannot confuse voters by imitating the commitment types. Therefore, pooling equilibria do not exist.

CHAPTER 6

CONCLUSION

We develop an alternative theory of divergent platforms by analyzing two models of electoral competition under complete and incomplete information. First, we present an election model that allows candidates to switch their past proposals and campaign promises under complete information, but this misrepresentation of positions has costs. We investigate how these costs affect the equilibrium policies and flip-flopping behavior. One of our results shows that if the cost of changing past statements is too high then candidates abstain from flip-flopping and select policies close to their past proposals. On the other hand, if this cost is low, they see no harm to flip-flop to increase their chance of winning. The other result shows that if credibility of a candidate is high, she has an incentive to flip-flop and make herself closer to the median voter's platform. The last result illustrates that candidates always flip-flop, since the utility of winning the election is always positive.

Second, we extend the former model to a model with incomplete information in which candidates have to commit their campaign promises after the election, i.e. the cost of defecting is too high. However they still incur the cost of changing past political statements during the election. We find that the lying cost determines the extent of flip-flopping in the same way as in the previous model.

As seen in these two equilibrium analyzes, candidates generally do not adopt the median voter's policy in contrast to the Downsian model. Additionally, we find out that flip-flopping is not necessarily bad as opposed to the general view.

As we mention in Chapter 4, the analysis of our first model with uncertainty is attractive because candidate have their freedom to change campaign promises. There might be a pooling equilibrium in which a normal type can pretend to be commitment type and once gets elected she alters her policy. However, the full characterization of the equilibria requires more detailed analysis and it will be the subject of future work.

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