ANALYSIS OF A BUSINESS MODEL TO OFFER ENERGY SAVING TECHNOLOGIES AS A SERVICE

by

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This is to certify that I have examined this copy of a master's thesis by

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and have found that it is complete and satisfactory in all respects,

and that any and all revisions required by the final

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To my parents

ABSTRACT

Increasing energy efficiency in buildings and industry is a crucial way of improving energy sustainability since buildings account for 40% of the worlds energy use as well as global CO_2 emissions. However, investment costs for applying or replacing existing technologies with energy efficient technologies can prevent the achievement of that goal. In this study, we investigate the feasibility of offering energy saving technologies as a service. In this arrangement, an energy service company offers to make all the necessary energy saving technology investments for a client in exchange for getting a fraction of the savings in energy expenditures for a predetermined time period. In the first part, we analyze the business plan for replacing a single technology with a more efficient one by taking into account uncertainty in energy prices and in the replacement of technologies. In the second part of the study, we present a Mixed Integer Program to minimize CO_2 emissions or maximize profits by selecting the appropriate technologies under budget limitations in multiple periods. As a result, we show that offering energy efficient technologies as a service can be a win-win-win arrangement for the firm, its client, and also for the environment.

ÖZETÇE

Binaların küresel enerji kullanımının ve CO_2 emisyonunun %40'ını oluşturması sebebiyle konutlarda ve sanayide enerji verimliliğinin artırılması enerji sürdürülebilirliğinin iyileştirilmesi için çok önemlidir. Fakat, enerji verimli teknolojilerin uygulanmasının ya da mevcut teknolojilerin değiştirilmesinin yatırım maliyetleri bu amaca ulaşmayı önleyebilir. Bu çalışmada, enerji tasarrufu teknolojilerinin bir hizmet olarak sunulması modelinin fizibilite çalışması yapıldı. Bu bağlamda bir enerji hizmeti şirketi, belirli bir zaman için enerji harcalamarındaki tasarrufun bir kısmı karşılığında enerji tasarrufu yapan teknolojilerle ilgili tüm gerekli yatırımları yapmayı sunuyor. İlk bölümde, fiyatlardaki ve teknolojilerdeki bozulma zamanındaki belirsizliği göz önünde bulundurarak, tek bir teknoloji değişimi için iş planı analizi yapıldı. Çalışmanın ikinci bölümünde, çoklu dönem için bütçe kısıtlamaları altında uygun teknolojileri seçerek CO_2 emisyonlarını en aza indirmek veya karı maksimize etmek için bir Karışık Tamsayılı Programlama (Mixed Integer Program) modeli sunuldu. Çalışmanın sonucu olarak, bir hizmet olarak enerji verimli teknolojileri sunmanın, hem firma, hem müşteri, ve aynı zamanda çevre için kazançlı olduğu gösterildi.

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NOMENCLATURE

CO_2	Carbondioxide
NPV	Net Present Value
MENR	Ministry of Energy and Natural Resources
GHG	Green House Gas
EEM	Energy Efficient Measure
ESCO	Energy Service Company
CFL	Compact Fluorescent Bulb
HDD	Heating Day Degree
VaR	Value at Risk
Kwh	Kilowatt Hour

Chapter 1

INTRODUCTION

1.1 CO_2 Saving and Sustainable Environment

As a consequence of industrialization in the 18th century and the growth of population, energy has been consumed extensively and the demand for energy has increased extensively.World energy consumption has more than doubled since the energy crises of the 1970s, and more than 80 % of this is provided by fossil fuels. Energy demand consumption is forecast to grow by 44% in the next 24 years [1]. People started to consume energy carelessly, which has brought about many environmental problems. Many people have agreed on the concept of sustainability and sustainable development which can be seen as a solution to these environmental problems: global warming, ozone depletion, acid rain, toxic waste, etc.

Today there are over 7.1 billion people living on our planet. By the year 2030, this number will increase to 11 billion [2]. It is estimated that the production of energy needs to increase up to 35 times over today's levels in order to satisfy the basic requirements for all people. Some researchers think that this estimate is an exaggeration since a lot of uncertainty exists concerning the consequences of human acts. Some argue that new technologies and laissez faire capitalism can prevent environmental disruption [3, 4, 5, 6].

Together with limited energy resources,CO_2 emission is another environmental problem related to extensive energy consumption. Emissions from the burning of fossil fuels are the primary cause of the rapid and accelerating growth in atmospheric CO_2 [7]. In the years between 1984 and 2004, primary energy consumption has grown by 49% and CO_2 emissions by 43%, with an average annual increase of 2% and 1.8% respectively [8].

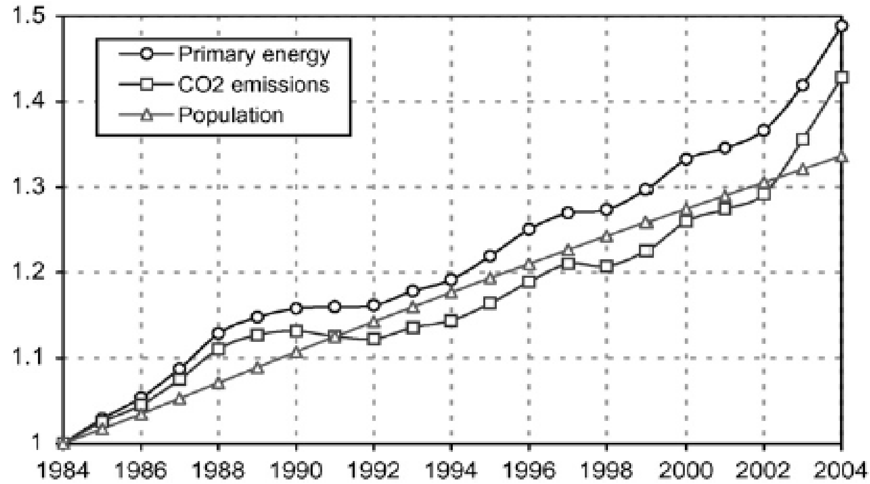

Figure 1.1: Rise in percentages of primary energy consumption, CO_2 emissions and world population.Source: International Energy Agency (IEA)[8].

As a consequence of environmental problems, desire to create eco-economic equilibrium is popularized as "Ecologically Sustainable Development" (ESD) in the Bruntland Commission Report [9]. International commitment to ESD via agreements for dealing with ozone layer depletion and global warming was emphasized in the Earth Summit in 1992 [10]. Yet,the trends in energy supply and demand are not consistent with the aim of sustainable development. The total primary global energy use increased yearly by 2% between 1981 and 2008 [11].

As a consequence, there has been a lot of technology invented for energy saving and CO_2 emission reduction which has become an international target for Ecologically Sustainable Development. Arthur Rosenfeld from the University of California, Berkeley a physicist who has been called the father of energy saving and efficiency stated :" The cheapest energy is what you don't use." [12].

1.2 Energy Saving in Buildings

Architects, construction companies and engineers must create an environmental way to overcome the problem of climate change. Energy usage in buildings is an important source of greenhouse gas emissions which was responsible for 7.85 Gt carbon dioxide (CO_2) emissions in 2002, approximately 33% of the total of energy-related emissions worldwide. In addition, almost 1.5 Gt CO_2 was emitted from fluorinated gases by buildings [13]. In the IPCC Special Report on Emissions Scenarios (SRES) these emissions are projected to increase by 11 Gt (B2 scenario) and 15.6 Gt CO_2

(A1 scenario) by 2030 , with a 34% share of the global total emission with other sectors [14]. In OECD countries, buildings cause about 35-40% of national CO_2 emissions from the consumption of fossil fuels [15]. Natural gas and oil are primarily used for heating and cooling as well as electricity generation in buildings which play important roles in those emissions [16].

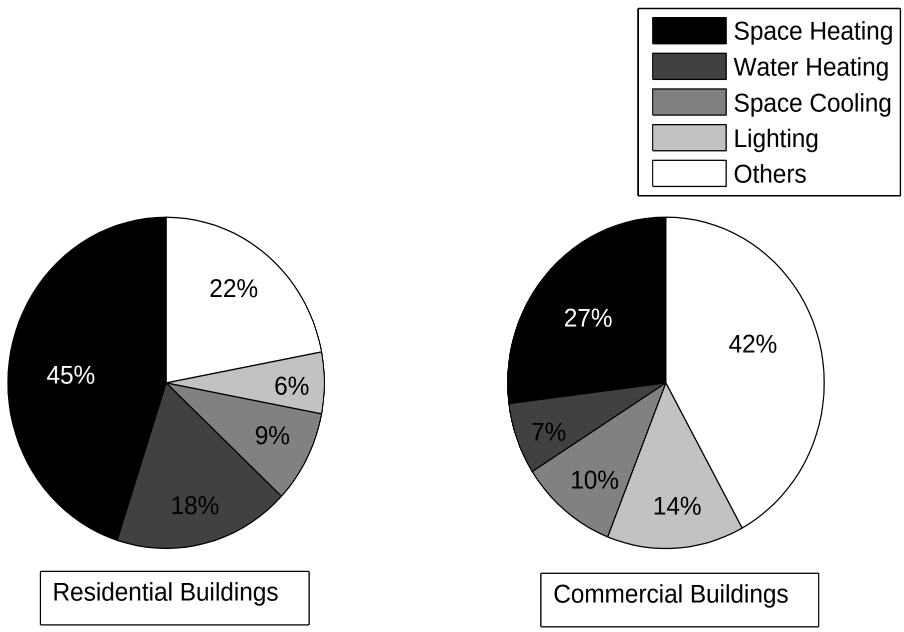

Figure 1.2: Site energy consumption by end use in residential and commercial buildings in U.S. [17].

Figure 1.2 shows the proportion of energy use in residential and commercial buildings in the US. According to the figures above, residential buildings accounts for 78% and commercial buildings accounts for 58% for space heating,water heating,space cooling and lighting in existing buildings.

Environmental facts must be taken into account when constructing new buildings. However, very inefficient buildings exist which will continue to emit large amounts of CO_2 by consumption of energy over time unless they are restored and become suitable for the environment. Cost-effective technologies, electrical fixtures, exterior thermal sheathing, door or window joist, adding insulation in attics or wall cavities, can be implemented to these existing buildings. An approximately 25-30% increase in energy saving for houses built before the 1940s and 12% for houses built in the 1990s can be reached by taking advantage of these technologies [18]. The retrofitting of four houses in the York region of the UK revealed that the air leakage rate can be reduced by 2.5-3.0 times. Moreover, the heating energy requirement is reduced by approximately 35% with improved insulation. Bell and Lowe claim that a reduction of 50% could be achieved by adding additional measures [19].

Energy efficient technologies in buildings provides not only environmental but also cost benefits which can be explained by a representative insulation example. It is assumed that the initial cost of insulation is $1,000, including labor and materials, which saves about 35 million Btus (MBtus) of natural gas per year, natural gas costs $5.61 /MBtu, the house and insulation last 20 years, fuel prices stay the same over time, and the discount rate is 5 percent. Discounting the total savings of $3,900 at 5 percent per year for 20 years yields a net present value of $2,450 which provides $1,450 in profit for implementing insulation [16].

Inefficient buildings prevent our planet from being a livable place. Therefore, a significant decline in GHG emissions for buildings is necessary as fast as possible. Building engineers and architects must be profoundly involved in reducing CO_2 by designing new buildings and retrofitting existing buildings. Approximately 30% - 35% of the total energy supply is consumed in buildings. A lot of technology has been implemented to prevent high CO_2 emissions by using less energy in buildings. Some of these technologies are really simple, which everyone can apply to consume less energy and in due course, reduce CO_2 emissions: changing light bulbs with energy efficient ones, installing exterior thermal sheathing and improving insulation. Developed and developing countries, including Turkey, started to regulate the energy efficiency in buildings. They have passed some laws and regulations to increase energy efficiency of buildings i.e. BEPY, TS825, BEPTR and ASHRAE 90.1 - 2007. In addition to these obligations, some voluntary systems namely, LEED, BREEAM and Energy Star certificate the green buildings. These systems aim to reduce energy consumption and CO_2 emission by encouraging building owners to retrofit energy efficient applications available for both existing and new buildings.

1.2.1 Business Plan Development

Historically, manufacturers' profits have depended on increasing the number of goods produced and sold. As technology has been developing and service has become crucial in business , manufacturing companies have started to find business models to integrate service into their products. With these business models, companies aim to supply exact customer needs , and profit by supplying a product-based service. The Tellus Institute defines this as ”servicizing” which blurs the traditional distinction between manufacturing and traditional service sector enterprises. Instead of selling plain products, the Tellus Institute states that selling functions of products or product-based services is the way to operate for this business model [33].

Operating, leasing and selling functions for the products are the best applicable businesses for these companies. In operating leases, the lessee pays the lessor for the use of a particular piece of equipment over a specified period and retains ownership of the equipment after the lease is over (or the lessee may purchase at fair market value). In selling functions for a product, the purchaser pays the seller for the use of equipment, repair and maintenance, supplies and staff training. The seller guarantees the intended function of the equipment. The seller retains ownership at the end of the contract.

Xerox company's lease model is an example of operating leasing. Xerox only leased its machines and priced them on a per copy basis instead of selling them. Xerox priced $25 + $0.035 per copy for a minimum of 2000 copies per month. Xerox profit was high since variable cost is much less than $0.035 . For instance,a comparison of two companies, the first company making 2000 copies per month and the second one making 10,000 copies per month, yields a difference in price which Xerox can take advantage. The prices amounted to $5700 and $22,500, respectively, until the end of the lifetimes of machines (5 years). Unless Xerox applied operating leasing, it would have only two choices. It could charge at $5700 and sell two machines, and thereby gain $16,800 less, which equals the amount that an intensive user would be paying, or it could charge $22,500 and sell one machine, and thus lose the profit from the user willing to pay $5700 [34].

1.2.2 Business Plan Development Using Energy Saving Returns

The business plan is a new way to construct a business to attain reachable business plans. In our model, the calculations of net present values and energy price forecast yield an interesting question, namely, if we can use these results to develop a business model or not. While comparing the costs of technologies, and the gain from energy saving and CO_2 markets, we came up with a solution, namely that there may exist a threshold value which satisfies the developing business model when it is reachable. This business model is used by Energy Service Companies (ESCOs). The most widely disseminated green business model is the ESCO. The ESCO provides energy saving for companies and public buildings as a service and in return gets paid by part of the savings achieved. The customer does not have to pay for initial investment. ESCOs guarantee energy savings for customers and are paid according to the energy efficiency of technologies that they applied. The customers are compensated if savings are less than guaranteed.

ESCOs are paid according to the size of savings on heat, energy or water. Because of the guarantee, the customer gets a technology with a clear financial profile for the full project period. Customers seem to handle the actual financing, but there will still be a savings guarantee, which means that the ESCOs maintain risk. As a consequence of the financial risk, almost all ESCOs are major companies which have a solid financial structure and capacity. Most of them are part of a corporation that produces key components for the renovation projects. By this way, they are not only creating value for themselves but also increasing sales for their related corporation [35].

Schneider Electric's Energy Saving Performance Contract (ESPC) with the U.S. Coast Guard in Puerto Rico is an example of this model. Schneider Electric has converted from increasingly expensive fossil fuel to renewable energy as key part of this project and it was the first to combine a Renewable Energy Services Agreement in a federal ESPC. The project has saved \$1,862,504 million annually. The total capital of the project is \$49,984,324, and the investment will be returned in 26.8 years. Moreover, this project has lead to a 25% annual reduction in energy consumption and 35% increase in the production of renewable energy [36].

For Turkey's situation ,the EEL (The Energy Efficiency Law) of Turkey legislated in May 2007 and was expected to push 25 - 30% savings in total energy consumption of the country. The law covered administrative structuring, energy auditing, incentives, awareness raising, and the establishment of an ESCO market for energy-efficiency services. Official delivery of ESCO licenses to candidate companies had been stopped in May 2011 due to ongoing changes in the Regulation on Increasing Efficiency in the Use of Energy Resources and Energy issued by the MENR (Ministry of Energy and Natural Resources). It had been declared that ESCO licensing would continue after the approval of modifications in early 2012. However,it was released in July 2012.[37]

1.3 Methodology

Environmental problems and energy saving opportunities in buildings have been discussed by many people. They have started to understand the outcomes of this massive consumption of the nature but there have been a few organized attempts to overcome the situation. Scientists and researchers on the other hand, have focused on the subject and proposed solution to the problems from many views. It has been established in most of the literature that it is useful and gainful to apply EEM investments which have the advantage of saving CO_2 and energy over investment costs. There has been much research conducted on energy efficiency recently (2005-2011). These studies give answers to EEM applications from the view of the environment and finance but do not include a selection process by using optimization [20, 21, 22]. On the other hand, Hens and Verbeeck in 2005 proposed a genetic algorithm for the selection of EEMs by taking into account the finance and environment for designing new buildings [23].

One of the most comprehensive analysis concerning energy efficiency in existing buildings was conducted by Çamlıbel (2011) in his Phd dissertation. This study involves a wide range of issues, including investment cost analysis, energy saving, environmental effects, multiple available EEMs, and optimization techniques using Mixed Integer Programming, for retrofitting low energy buildings at Boğaziçi University Kilyos Campus. The main findings of the research are; (i) Single time decision problem is used for choosing EEMs , (ii) optimization algorithms and heuristics are useful tools for retrofitting buildings, and finally(iii) Investing on these technologies are economically feasible for different budgets. The study however, does not concentrate on the multi time decision problem for a limited budget [24].

In this thesis, ESCO type business plan in single technology replacement and multiple technology selection is examined in two parts. In the first part of the thesis, an analytical approach is proposed to justify the profitability of applying energy efficient projects to the buildings. Cost terms for the replacement and energy consumption are calculated analytically by modeling efficiency improvements, replacement costs, energy usage and energy prices. These cost terms are the key findings of the single technology ESCO type business plan. Risk calculations are presented as the feasibility of the project by using uncertainties in the model i.e. replacement period and energy prices. In the second part, a mathematical program approach is proposed to improve energy efficiency for existing buildings by selecting applicable energy efficient technologies in order to allocate funds properly for the financial and environmental returns. A Mixed integer Programming

(MIP) is developed for both single and multi time separately. Additionally, ESCO type business plan is examined by changing the model to optimize financial savings. The profit rate requirement of the ESCO and the discount rate given to customer are added on the model to make the model applicable in real life.

1.3.1 Energy Demand Calculation

The World Energy Council indicates that the energy demand is forecasted to increase at an annual rate of 1.6% until 2030. Specifically, the main reasons for this trend have been (i) a significant population increase and an even larger increase in the number of buildings; (ii) a decreasing household size related to the changes in the structure of households; (iii) increasing number of household equipment in the buildings and (iv) a long lasting policy of low tariffs. Together with economic and social variables, residential electricity demand is strongly related to climatic factors. These climatic factors can be summarized, for example, by the temperatures registered in different locations. After the 1970s petrol bottleneck, many researchers have analyzed how to use energy efficiently and they have especially investigated forecast of the energy demand with less error in order to use limited sources effectively. Either governments for their energy policies or power plant companies for their sales have to forecast residential energy demand. Since the pioneer work of Houthakker (1951), vast literature on modeling the residential demand for electricity and natural gas examining its determinants has been published [25]. Most of the works have estimated both the short-run and the long-run residential demand by using aggregate data and applying different methodologies [26, 27, 28, 29, 30, 31, 32, 38, 39, 40].

Together with econometrical methodologies determined by price and income elasticity, Heating Day Degrees (HDD) and Cooling Day Degrees (CDD) are also used in many models to forecast the residential natural gas demand [41, 42, 43, 44]. This thesis also employs regression analysis and uses a variable degree-day approach with historical data to forecast natural gas demand [65].

1.3.2 Energy Price Forecast

The price of energy depends on a range of different supply and demand conditions, including the geopolitical situation, import diversification, distribution costs, environmental protection costs, and weather conditions. However, it is common knowledge that when resources are getting insufficient, the energy prices are expected to rise.

Energy producers and consumers regularly attempt to forecast prices of energy over time horizons. Related to resource exploration, reserve development, and production, producers make these forecasts for the general purposes of strategic planning and evaluating investment decisions. Industrial consumers also make these forecasts for the same kinds of reasons. For instance, electric utilities make forecasts for oil and coal prices to decide which energy to use to generate electricity. In our model, since we aim to decide which technologies to select, we calculate energy costs for that technology by forecasting.

We modeled electricity prices in two ways: deterministic and stochastic. The deterministic model, which may not be unrealistic, covers an assumption in which electricity prices are increased with respect to a function. However in the stochastic model, we add a random term to the model for which provides the uncertainty in prices. The standard approach in the literature is to model the logarithmic electricity spot prices through a mean-reverting process, such that in the classical Gaussian setting the spot price dynamics become lognormal [45, 46]. For such models it is notoriously difficult to derive manageable analytical expressions for the corresponding forward and futures contracts [47]. Instead we proposed an autoregressive process with order 1, which is the discrete analogue of the Ornstein Uhlenbeck process [48].

1.3.3 Carbon Markets

CO_2 trading is a market-based approach used to control air pollution by supplying economic incentives for achieving reductions in the emissions of CO_2. As a part of its commitment to the Kyoto Protocol, in January 2005 the European Union presented a project of tradable CO_2 emission permits, whereby restricted allowances were allocated to various industrial emitters of CO_2, specifying the amount of CO_2 they can emit each year. At the end of each year, companies must produce permits to cover their tonnes of CO_2 emitted. Since companies are allowed to trade permits, the project was intended to satisfy not only the reduction of overall CO_2 emissions, but also the abatement cost for firms which again aim to reduce emissions.

1.3.4 Technology Selection Problem

Technology selection is concerned with choosing the best technology from a pool of available technologies. The criteria for a best technology differ depending on the specific requirements of objectives. In order to compete, companies have to continually invest in technologies. However, resource limitations require an analytical skill to strategically allocate resources to a subset of possible projects. Various tools and methods can be used to choose the optimal set of technologies[49]. Many portfolio management tools have been developed to maximize different metrics. Mathematical and scoring models are used where quantitative metrics are available. Graphics and charting evaluate qualitative metrics. Using a combination of qualitative and quantitative tools makes it hard to define an optimum technology portfolio and can lead to information overload [50].

There have been many management techniques, which optimize a portfolio's commercial value within its resource constraints by using a mathematical model [52, 53, 54]. In addition, in recent years, mathematical programming and project selection models have become more practical and realistic [51].

This study includes a combined solution using architectural, engineering, financial and operations research expertise. Through this solution approach, an effective prioritization algorithm, which may enable the most proper placement of the budget is aimed. This way, efficiently prioritized energy efficient technologies may turn into an investment opportunity. The objective of this thesis is to select technologies which satisfy the goal of maximizing net present value of the cash flows comes from the energy saving and reducing CO_2 emissions in the buildings.

1.4 The Objective of the Thesis

The main objective of this thesis is to construct an analytical model to reduce energy usage in buildings, so that a reduction in CO_2 emissions will be achieved. Choosing the best option from available technologies under budget limitations, uncertainty of energy prices, efficiency improvements, reliability and the risk posed by those technologies will be discussed to satisfy these goals. The model provides a straight answer to each problem and gives feasibility conditions to the ESCO type business plans.

Making a decision from the available technologies requires much technical work. The CO_2 saving is calculated for different budgets to see how CO_2 emissions can be achieved with different budget limitations. Efficiency improvements in technologies are also taken into account by assuming exponential growth. Energy prices are forecast to increase with an error bound that is increasing over time. The lifetimes of technologies are modeled to be random which gives an answer to a stochastic problem.

Organization of the remaining part of the thesis is as follows. In Section 2 we give the calculations for the net present value of replacement and energy usage costs for finite time horizon. Both deterministic and stochastic solutions are given. In Section 3, single technology business plan model is developed with expected return and risk calculations by using the derivations from Section 2. In section 4, a Mixed Integer Linear Programming (MILP) is proposed for selecting energy efficient technologies for multiple time by taking into consideration both NPV and CO_2 saving maximization.

Chapter 2

MODEL OF A SINGLE TECHNOLOGY WITH EXPECTED COST CRITERIA

2.1 Model Description

In order to introduce the problem, we first consider the technology replacement problem of a single heating system. We develop an analytical model to decide which of the two technologies should be selected in order to achieve a lower expected net present value by considering uncertainty in product life and energy prices in finite period.

In this chapter, an analytical model is developed by considering replacement and energy costs. A decision making process is used to compare efficiencies and costs of that technologies for two options. Replacement cost of a technology, efficiency of a technology, CO_2 markets, energy demand and energy price are modeled to achieve the goal.

Replacement and energy usage costs are calculated in both deterministic and stochastic cases. In the deterministic cost model, all the variables are assumed to be deterministic that is that is energy price is forecasted with no error, technologies' failure time is periodic, their heating efficiencies are increasing exponentially and the replacement cost of the technologies are decreasing exponentially over time. However, in the stochastic case we calculated the energy price is linearly increasing with error term. Also, the failure (replacement) times of technologies' are considered to be random in which we use exponential distribution. NPV is used as a financial tool for all these calculations.

Energy demand is modeled by linear regression of HDD values in Istanbul. Koç University's monthly consumption of natural gas data is used in the demand calculations. Unit price for the natural gas in Turkey is used to determine parameters in energy price calculations.

The NPV calculations are made for a single technology and then by changing parameters the second technology's NPV of the costs are achieve. In all calculations of cost we compared the two technologies and decided which one to select or find a threshold that satisfies selection criteria of the technologies.

2.1.1 Parameters

Cost of Single Technology

Unit technology replacement price is modeled as an exponentially decreasing function $ce^{-\beta' t}$. Figure 2.1 and Figure 2.2 show real values and exponential function used in this study for Compact Fluorescent Light (CFL) and LED prices. In one of the recent study, Hauri et. al. illustrate that unit cost of CFL bulbs drop in USA[56].

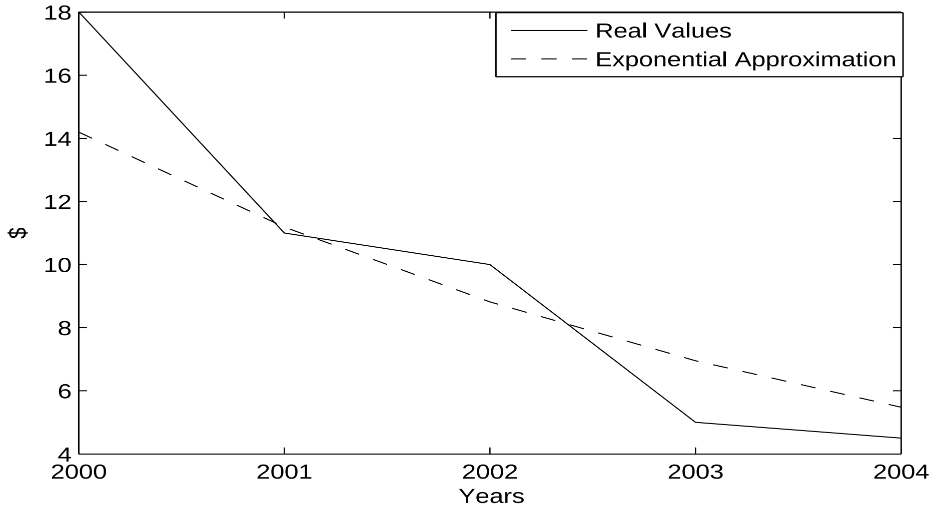

Figure 2.1: CFL Price drop in the US.[56]

According to the LEDinsides survey, the LED light bulb price has also dropped. For instance, the retail prices for LED light bulbs in Japan were about USD 40 in 2010, and now set a record low of USD 18, with a huge drop of 35% in the second quarter of 2010. Figure 2.2 shows our exponential approximation and LED prices in Japan [64].

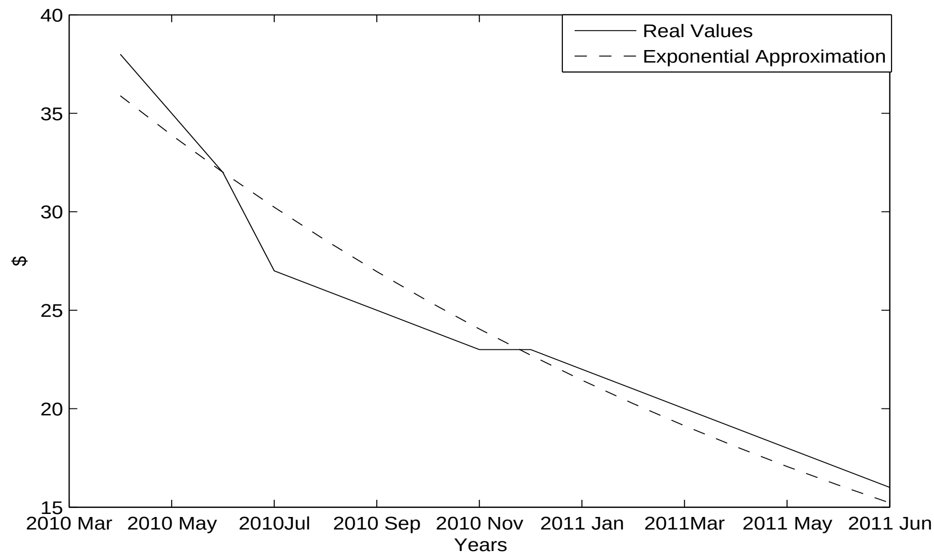

Figure 2.2: LED price decrease over time

Efficiency Improvements

Efficiency, proportion of watt of a bulb to the lumen value, is modeled as an exponentially increasing function $\varepsilon e^{\eta t}$ over time where ε is the initial efficiency and η is the increase rate.Lumen is the unit of enlightenment. Figure 2.3 shows that as time goes on, CFL and LED bulbs are invented. As a result, lumen/watt value that shows the lightning level achieved for the energy spent as an efficiency measure has increased over time.

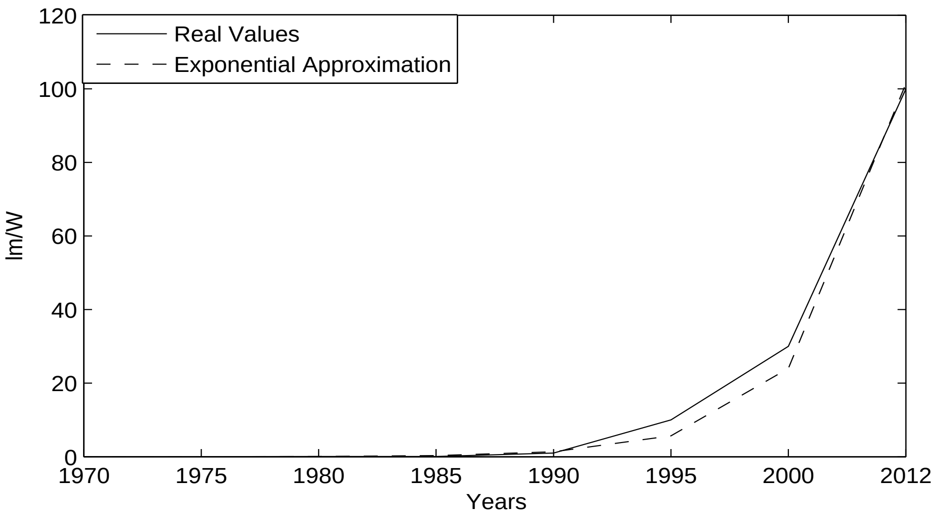

Figure 2.3: Technology improvement in lighting industry

Energy Unit Price

Energy unit price is modeled as an linearly increasing function $f(t) = ct + d$. In order to justify this modeling assumption Figure 2.4 shows the unit price of natural gas in Turkey and a linear approximation.

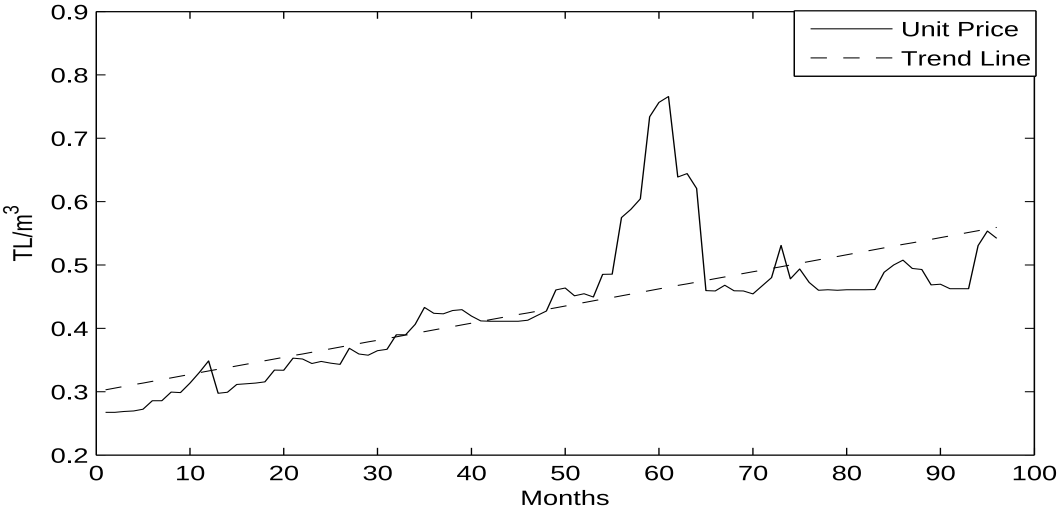

Figure 2.4: Unit price of natural gas over time and trend line

Energy Consumption

In this study, energy consumption is modeled as $aW_t + b$ where W_t is Heating Day Degree (HDD) in month t and a and b are constants determined by using linear regression. HDD is defined as the measurement of the demand of energy to heat a building.

$$W_t = \begin{cases} \sum_{j=1}^{D_t}(T_{it}(j) - T_{ot}(j)), \text{ If } T_i > To, \\ 0, \text{ otherwise,} \end{cases} \tag{2.1}$$

where $T_{it}(j)$ is the inside temperature on the j^{th} day of month t,$T_{ot}(j)$ is the outside temperature on the j^{th} day of month t, and D_t is the number of days in month t.
In our model, we compared W_t values determined by 6-year averages for Koç University Campus with 60-year averages for Istanbul. This comparison showed that using 60-year averages for Istanbul is a better estimator for the energy consumption for Koç University Campus. Table 2.1 shows the 60-year average of HDD values for Istanbul[65].

Table 2.1: Istanbul's 60-year Average of HDDS [65]

Months	60 Year HDD Averages ($°C$ Degree Day)
Jan	363
Feb	341
Mar	315
Apr	171
May	65
Jun	3
Jul	0
Aug	0
Sep	6
Oct	79
Nov	213
Dec	309

Koç University consumption data is used for conducting linear regression analysis. Figure 2.5 shows the natural gas consumption of the Koç University from 2004 to 2011 and the prediction of the consumption.

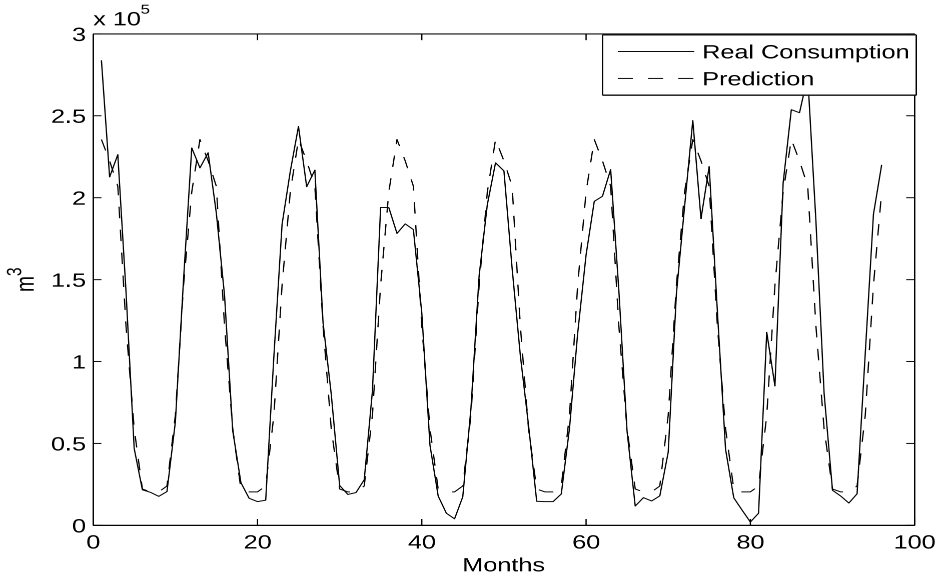

Figure 2.5: Real consumption of natural gas and result of the linear regression of HDD values

Notice that there exists a small decline for the winter months in the real consumption data for Koç University. This occurs because of the break between terms in the academic calendar of the university. Tables 2.2 and 2.3 shows the linear regression results.

Table 2.2: 60 year Average HDD Regression Results-1

Root MSE	19154	**R-Square**	0.9486
Dependent Mean	109544	**Adj R-Sq**	0.9478
Coeff Var	17.48479		

Table 2.3: 60 year Average HDD Regression Results-2

Variable	**DF**	**Parameter Estimate**	**Standard Error**	**t Value**	$Pr > \|t\|$
Intercept	1	20044	3361.55175	5.96	<.0001
60 year Average HDD	1	575.87268	16.02744	35.93	<.0001

CO_2 Markets

According to the Kyoto Protocol, the saved amount of CO_2 can be sold on the CO_2 market. In this arrangement we first calculate the CO_2 amount produced and then as the Kyoto Protocol provides, we will determine the earnings of saved CO_2, which is under the permit in the finite horizon. In the model, L is the permit value which is expected to decrease with $Le^{-\omega t}$ since technology is improving.

Table 2.4 summarizes the parameters used in the model.

Table 2.4: Parameters for Replacement and Energy Usage Cost Calculations

$c(t) = ce^{-\beta' t}$:	Cost of a single technology	dollar
$p(t)$:	Energy unit price	dollar/kwh
$P_1(t) = f(t)$:	Energy price with no error term	dollar
s:	Period of replacement for deterministic case	year
i:	Interest rate	
$\varepsilon(t) = \varepsilon e^{\eta t}$:	Efficiency improvement	
$aW_t + b$:	Natural Gas consumption	m^3
W_t :	Heating Day Degree Value	$^\circ C$ Degree Day
R:	$1xT$ Discount vector	
$\widehat{C}$:	$Tx12$ price matrix	
$\widehat{A}$:	$12x1$ consumption vector	
β':	Cost decrease rate	
$P_2(t) = f(t) + X_t$:	Energy price forecast with error term	dollar
λ:	Failure rate of a technology	
T_n:	Failure time of n^{th} technology	year
L :	Permit value that Kyoto Protocol provides	kg
ω :	CO_2 permit decrease rate	kg
θ:	CO_2 produced of a single technology	kg/KWh
π:	Money earnings from CO_2 saving sale	dollar/kg
$(1+r) = ((1+i)e^{\eta})$:	New parameter of discrete discounting	
α:	New parameter of continuous discounting	

Now, we will explain how cost terms in total cost are constructed. NPV of the total cost is the difference of NPV of the total gain from CO_2 selling from the sum of NPV of replacement cost and NPV of energy usage cost. Without loss of generality, we can show that we can set either cost discount parameter β' or the efficiency improvement parameter η to zero by redefining these parameters. In the remaining part of the thesis, we define $(1+i)e^{\eta} = 1+r$ as the new interest rate and $e^{\beta'-\eta} = e^{\beta}$ and eliminate η.

2.2 Finite Horizon Solution

2.2.1 Deterministic Case

For the deterministic case, lifetimes of the technologies are assumed to be constant, failure times are periodic and the energy unit price is linear without an error term. Consumption is considered to vary according to the Heating Day Degree(HDD) of the place where technologies are implemented. In the model we calculated technology replacement cost and energy usage cost separately and then added them.

NPV of Replacement Cost

In every replacement period for a technology, there needs to be a replacement of old technology with a brand new one. For the technology replacement cost:$T_N = Ns$ is the time of the N^{th} replacement, where s is the constant time period. Hence, we can calculate the total cost for any t such that $sN \leq t < s(N+1)$

$$\begin{aligned} NPV_{rep} &= \sum_t c(1+i)^{-t} e^{-\beta' t} = \sum_t ce^{-(\alpha+\beta)t} = \sum_{i=0}^{N} ce^{-(\alpha+\beta)si} \\ &= c(\sum_{i=0}^{N} (\frac{1}{e^{(\alpha+\beta)s}})^i) = c\frac{1-e^{-(\alpha+\beta)s(N+1)}}{1-e^{-(\alpha+\beta)s}} \end{aligned} \tag{2.2}$$

where $\alpha = ln(1+r)$ is the new continuous discounting parameter.

NPV of Energy Usage Cost

While computing the energy usage cost, the energy price is assumed to change with respect to a function $f(t) = ct + d$ with no error term. Consumption is modeled by linear regression using HDD values. For the analytical solution of the consumption problem the NPV of the energy cost is calculated as follows.

$$
\begin{aligned}
NPV_{energy} =& \sum_t (1+i)^{-t} \frac{aW_t + b}{\varepsilon(t)} p(t) \\
=& \sum_t ((1+i)e^{\eta})^{-t} \frac{aW_t + b}{\varepsilon} p(t) \\
=& \sum_t ((1+r)^{-t} p(t) \frac{aW_t + b}{\varepsilon} \\
=& \sum_{t=1}^{12} (1+r)^t (\frac{aW_t + b}{\varepsilon}) \sum_{t'=1}^{T/12} (1+r)^{12(t'-1)} (c(12(t'-1)+t)+d) \\
=& \sum_{t=1}^{12} (1+r)^t (\frac{aW_t + b}{\varepsilon})(ct+d-12c) \sum_{t'=1}^{T/12} (1+r)^{12(t'-1)} \\
& + \sum_{t=1}^{12} (1+r)^t (\frac{aW_t + b}{\varepsilon}) \sum_{t'=1}^{T/12} (1+r)^{12(t'-1)} 12ct'.
\end{aligned}
\tag{2.3}
$$

NPV of energy cost can be represented in matrix form which provides a more compact representation.

$$
NPV_{energy} = \frac{R\widehat{C}\widehat{A}}{\varepsilon} \tag{2.4}
$$

where $R = \{r_i\}$ is $1xT$ discount vector where $r_i = (1+r')^{-(i-1)}$,
$\widehat{C} = \{\widehat{c}_{i,j}\}$ is the Tx12 matrix where $\widehat{c}_{i,j} = \widehat{c}_{i,j-1} + c$ $\widehat{c}_{i,1} = \widehat{c}_{i-1,12} + c, t \neq 1$ and $\widehat{c}_{1,1} = c + d$
and $\widehat{A} = \{d_j\}$ is 12x1 vector where $\widehat{d_j} = aW_j + b$

NPV of CO_2 Selling

CO_2 trading is also included in the NPV of total cost calculation when calculating total cost. We first calculate the CO_2 amount produced and then, as the Kyoto Protocol provides, we determine the earnings of saved CO_2 which is under the permit in finite horizon. In the model, L is the permit value which is expected to decrease exponentially ($L(t) = Le^{-\omega t}$) since technology is improving.

Total Amount of CO_2 produced The total produced CO_2 if $\eta \neq 0$ is calculated as

$$\theta \sum_{t=1}^{12} \frac{aW_t + b}{\varepsilon} \sum_{0}^{T} e^{-\eta t} = \theta \sum_{t=1}^{12} \frac{aW_t + b}{\varepsilon \eta} \frac{e^{\eta} - e^{-\eta T}}{e^{a} - 1}. \tag{2.5}$$

where θ is CO_2 produced from energy usage of 1 Kwh for a single technology. The calculation of total produced CO_2 if $\eta = 0$:

$$\sum_{t=1}^{12} T \frac{aW_t + b}{\varepsilon} \theta = \frac{T\theta}{\varepsilon} \sum_{t=1}^{12} (aW_t + b) \tag{2.6}$$

Total Revenue from Sale of CO_2 NPV of total revenue from sale of CO_2 is calculated as

$$\begin{aligned} NPV_{CO_2} &= \sum_{t=0}^{T} (1+i)^{-t} \pi L e^{-\omega t} - \sum_{t=1}^{12} \sum_{t'=1}^{T/12} (1+r)^{-12(t'-1)-t} (\frac{aW_t + b}{\varepsilon}) \pi \theta \\ &= \pi L \sum_{t=0}^{T} ((1+i)e^{\omega})^{-t} - \pi \theta \frac{R\widehat{1}\widehat{A}}{\varepsilon} \end{aligned} \tag{2.7}$$

where π is earning from a sale of one kg CO_2 and $\widehat{1}$ is Tx12 matrix and $\widehat{1}_{ij} = 1$ for all i, j.

Deterministic Cost Calculation and Comparison Total Cost in the deterministic case including CO_2 trade is obtained as

NPV of Total cost = NPV of Total cost of replacement + NPV of Total cost of Energy usage

-NPV of Total Revenue from CO_2 sales

$$= c \frac{1 - e^{-(\alpha+\beta) r (N+1)}}{1 - e^{-(\alpha+\beta) r}} + \frac{R\widehat{C}\widehat{A}}{\varepsilon} - \pi L \sum_{t=0}^{T} ((1+i)e^{\omega})^{-t} + \pi \theta \frac{R\widehat{1}\widehat{A}}{\varepsilon} \tag{2.8}$$

.

Theorem 2.2.1. *Let c_1 and c_2 be the unit cost of technologies and ε_1 and ε_2 are the initial efficiencies for technology 1 and technology 2, respectively. The table shows which technology should be selected to achieve a lower expected cost.*

	$\varepsilon_1 > \varepsilon_2$	$\varepsilon_1 = \varepsilon_2$	$\varepsilon_1 < \varepsilon_2$
$c_1 < c_2$	*1*	*1*	$\frac{(c_1-c_2)\varepsilon_1\varepsilon_2}{\varepsilon_1-\varepsilon_2} > \xi \Rightarrow 2$
$c_1 = c_2$	*1*	*1 or 2*	*2*
$c_1 > c_2$	$\frac{(c_1-c_2)\varepsilon_1\varepsilon_2}{\varepsilon_1-\varepsilon_2} > \xi \Rightarrow 2$	*2*	*2*

where $\xi = \frac{(1-e^{-(\alpha+\beta)r})(R\widehat{C}\widehat{A}+\pi\theta R\widehat{1}\widehat{A})}{1-e^{-(\alpha+\beta)r(N+1)}}$.

Proof. The Equation 2.9 below is obtained by subtracting NPV of total cost of technology 2 from NPV of total cost of technology 1 using equation 2.8.

$$\frac{(c_1-c_2)(1-e^{-(\alpha+\beta)r(N+1)})}{1-e^{-(\alpha+\beta)r}} + \frac{(\varepsilon_2-\varepsilon_1)R\widehat{C}\widehat{A}}{\varepsilon_1\varepsilon_2} + \frac{(\varepsilon_2-\varepsilon_1)\pi\theta R\widehat{1}\widehat{A}}{\varepsilon_1\varepsilon_2} \tag{2.9}$$

If Equation 2.9 is greater than zero then choosing technology 2 is more profitable since its total NPV is less than technology 1. In this arrangement, when the unit cost of a technology is less and efficiency of a technology is more than the other one, it is obvious to choose that technology.If both costs and efficiencies are equal then it is indifferent to select. Rewriting the condition

$$\frac{(c_1-c_2)(1-e^{-(\alpha+\beta)r(N+1)})}{1-e^{-(\alpha+\beta)r}} + \frac{(\varepsilon_2-\varepsilon_1)R\widehat{C}\widehat{A}}{\varepsilon_1\varepsilon_2} + \frac{(\varepsilon_2-\varepsilon_1)\pi\theta R\widehat{1}\widehat{A}}{\varepsilon_1\varepsilon_2} > 0$$

yields the following threshold to choose technology 2 over technology 1 where

$$\xi = \frac{(1-e^{-(\alpha+\beta)r})(R\widehat{C}\widehat{A}+\pi\theta R\widehat{1}\widehat{A})}{1-e^{-(\alpha+\beta)r(N+1)}}$$

□

2.2.2 *Stochastic Case*

In this part, we consider the case where the technologies fail randomly with an exponential time to failure distribution and the electricity price is uncertain with a deterministic term and an error term that follows a first order auto-regressive process. We are interested in finding the expected value of NPV of total cost of replacement (NPV_{rep}) and energy usage cost (NPV_{energy}). For the replacement cost, we assume that technologies have an exponentially distributed lifetime. In the energy usage cost we use AR(1) process with a function $f(t)$.

Expected NPV of Replacement Cost

For a realization, we denote the time of the n^{th} failure as T_n, and the present value of the cost of replacement as $ce^{-\alpha T_n}$.Summing this over all n, we obtain the present value of all future replacement costs.

$$NPV_{rep} = \sum_{n=0}^{\infty} ce^{-(\alpha+\beta)T_n 1_{\{T_n \le t\}}} \tag{2.10}$$

Notice that T_n has Erlang distribution.

$$E[NPV_{rep}] = c\sum_{n=0}^{\infty} E[e^{-(\alpha+\beta)T_n} 1_{\{T_n \le t\}}] \tag{2.11}$$

$E[e^{-(\alpha+\beta)T_n}]$ is the Laplace transform of Erlang(n,λ).

Before finding the expectation of replacement cost, we introduced what the replacement cost is.

$$NPV_{rep} = \sum_{n=0}^{\infty} ce^{-(\alpha+\beta)T_n} 1_{\{T_n \le t\}} = c + \sum_{n=1}^{\infty} ce^{-(\alpha+\beta)T_n} 1_{\{T_n \le t\}} = c\sum_{n=1}^{\infty} X_n + c \tag{2.12}$$

where $X_n = e^{-(\alpha+\beta)T_n} 1_{\{T_n \le t\}}$. Then the expected replacement cost is

$$E[NPV_{rep}] = cE[\sum_{n=1}^{\infty} X_n] + c = c\sum_{n=1}^{\infty} E[X_n] + c = c\sum_{n=1}^{\infty} \int_0^t e^{-(\alpha+\beta)s} \frac{\lambda e^{-\lambda s}(\lambda s)^{n-1}}{(n-1)!} ds + c$$

$$= c\int_0^t e^{-(\alpha+\beta)s} \lambda e^{-\lambda s} ds \sum_{n=1}^{\infty} \frac{(\lambda s)^{n-1}}{(n-1)!} + c = c\int_0^t e^{-(\alpha+\beta)s} \lambda ds + c$$

$$E[NPV_{rep}] = \frac{c\lambda}{\alpha+\beta}(1 - e^{-(\alpha+\beta)t}) + c \tag{2.13}$$

Expected NPV of Energy Usage Cost

To model energy prices we use an AR(1) process which is an discrete analogue of the Ornstein Uhlenbeck Process. We assume that $P(t) = f(t) + X_t$ where X_t is AR(1) process.i.e. $X_t = \varphi X_{t-1} + \varepsilon_t$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

$$\begin{aligned}
E[NPV_{energy}] =& E[\sum_{t=1}^{12}\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon})c(12(t'-1)+t)+d+X_{12(t'-1)+t}] \\
=& \sum_{t=1}^{12}(1+r)^{-t}(\frac{aW_t+b}{\varepsilon})\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)}(c(12(t'-1)+t)+d+E[X_{12(t'-1)+t}] \\
=& \sum_{t=1}^{12}(1+r)^{-t}(\frac{aW_t+b}{\varepsilon})(ct+d-12c)\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)} \\
&+ \sum_{t=1}^{12}(1+r)^{-t}(\frac{aW_t+b}{\varepsilon})\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)}12ct'
\end{aligned} \tag{2.14}$$

The NPV of energy cost can be represented in matrix form as follows.

$$NPV_{energy} = \frac{R\widehat{C}\widehat{A}}{\varepsilon} \tag{2.15}$$

where $R = \{r_i\}$ is $1xT$ discount vector where $r_i = (1+r')^{-(i-1)}$,
$\widehat{C} = \{\widehat{c}_{i,j}\}$ is the Tx12 matrix where $\widehat{c}_{i,j} = \widehat{c}_{i,j-1} + c$ $\widehat{c}_{i,1} = \widehat{c}_{i-1,12} + c, t \neq 1$ and $\widehat{c}_{1,1} = c + d$
and $\widehat{A} = \{d_j\}$ is 12x1 vector where $\widehat{d_j} = aW_j + b$
Deterministic and stochastic solutions are the same since $E[X_t] = 0$.

NPV of CO_2 Selling

CO_2 amount produced and the total revenue from the sale of CO_2 are the same as the deterministic solution since the parameters in CO_2 calculation have no stochastic term in our model.

Stochastic Cost Calculation and Comparison Total cost for stochastic case becomes:

Total cost = Expected NPV of total cost of replacement + Expected NPV of total cost of energy usage

− Total revenue from CO_2 sales

$$= \frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)t}) + c + \frac{R\widehat{C}\widehat{A}}{\varepsilon} - \pi L\sum_{t=0}^{T}((1+i)e^{\omega})^{-t} + \pi\theta\frac{R\widehat{1}\widehat{A}}{\varepsilon} \tag{2.16}$$

Theorem 2.2.2. *Let c_1 and c_2 be the unit cost of technologies and ε_1 and ε_2 are the initial efficiencies for technology 1 and technology 2, respectively. The table shows which technology should be selected as a result of given comparisons*

	$\varepsilon_1 > \varepsilon_2$	$\varepsilon_1 = \varepsilon_2$	$\varepsilon_1 < \varepsilon_2$
$c_1\mu_1(t) < c_2\mu_2(t)$	*1*	*1*	$\frac{(c_1\mu_1(t)-c_2\mu_2(t))\varepsilon_1\varepsilon_2}{\varepsilon_1-\varepsilon_2} > \xi \Rightarrow 2$
$c_1\mu_1(t) = c_2\mu_2(t)$	*1*	*1 or 2*	*2*
$c_1\mu_1(t) > c_2\mu_2(t)$	$\frac{(c_1\mu_1(t)-c_2\mu_2(t))\varepsilon_1\varepsilon_2}{\varepsilon_1-\varepsilon_2} > \xi \Rightarrow 2$	*2*	*2*

where $\mu_1(t) = (\lambda_1 P(t) + \alpha + \beta)$, $\mu_2(t) = (\lambda_2 P(t) + \alpha + \beta)$ *and* $\xi = (\alpha + \beta)(R\widehat{C}\widehat{A} - \pi\theta R\widehat{1}\widehat{A})$

Proof. The Equation 2.17 below is obtained by subtracting NPV of total cost of technology 2 from NPV of total cost of technology 1 using Equation 2.16.

$$\frac{(c_1\lambda_1 - c_2\lambda_2)P(t) + (c_1 - c_2)(\alpha+\beta)}{\alpha+\beta} + \frac{(\varepsilon_2 - \varepsilon_1)(R\widehat{C}\widehat{A} - \pi\theta R\widehat{1}\widehat{A})}{\varepsilon_1\varepsilon_2} \tag{2.17}$$

If Equation 2.17 is greater than zero then choosing technology 2 is more profitable since its total NPV is less than technology 1. In this arrangement, when the unit cost of a technology is less and efficiency of a technology is more than the other one, it is obvious to choose that technology.If both costs and efficiencies are equal then it is indifferent to select.
The condition

$$\frac{(c_1\lambda_1 - c_2\lambda_2)P(t) + (c_1 - c_2)(\alpha+\beta)}{\alpha+\beta} + \frac{(\varepsilon_2 - \varepsilon_1)(R\widehat{C}\widehat{A} - \pi\theta R\widehat{1}\widehat{A})}{\varepsilon_1\varepsilon_2} > 0$$

implies a threshold type condition

$$\frac{(c_1\mu 1(t) - c_2\mu_2(t))(\varepsilon_1 - \varepsilon_2)}{\varepsilon_1\varepsilon_2} > (\alpha+\beta)(R\widehat{C}\widehat{A} - \pi\theta R\widehat{1}\widehat{A})$$

where $\mu_1(t) = (\lambda_1 P(t) + \alpha + \beta)$, $\mu_2(t) = (\lambda_2 P(t) + \alpha + \beta)$.

□

Infinite solutions of NPV_{rep}, NPV_{energy} and NPV_{CO_2} is given in the Appendix A.

Chapter 3

BUSINESS PLAN WITH EXPECTED RETURN AND RISK CONSIDERATIONS

In this business plan, there exists a company which provides energy efficient products for the customers and makes a profit by using that technology. Hence, the company makes an agreement with customers based on supplying new energy efficient products and paying the energy usage costs of the customers with applied technology by earning money from the customers energy usage cost with the old technology.In section 3.1, we make an analysis of a business plan based on service agreement by using the model analyzed in Chapter 2. We used the expected NPV of replacement and energy costs calculated in Chapter 2 to calculate the expected total return of the company. After, a feasibility analysis is conducted for ESCOs by using those calculations including giving a discount for the customer.

Together with the expected return, considering risk is also very important for investment decisions. Every investment has risk since, invested amount can be lost. However, taking risk does not mean losing control over the capital invested. In fact, defining the uncertainty and calculating the risk allow managing the investment to advantage.In section 3.2, we calculated the risk of the ESCO-type business plan whose expected return is calculated in the second chapter. Value at Risk metric is used to determine feasibility of the business model. To calculate feasibility, variance of the NPV of the ESCO is also solved analytically by using parameters given in the second chapter.

3.1 Expected Return of the ESCO in Single Technology Business Plan

3.1.1 Single Technology Business Plan NPV Calculation

Table 3.1 summarizes the parameters of the model:

Table 3.1: Parameters for Single Technology Business Plan

ε:	Efficiency of old technology	$^{\circ}C$/watt
ε':	Efficiency of new technology	$^{\circ}C$/watt
$c(t) = ce^{-\beta' t}$:	Cost of a single technology	dollar
$NPV_{energy}(\varepsilon)$:	NPV of energy usage cost with old technology	dollar
$NPV_{energy}(\varepsilon')$:	NPV of energy usage cost with new technology	dollar
C_0:	Initial investment to the new technology	dollar
Δ:	% Discount quantity	
$aW_t + b$:	Natural Gas consumption	m^3
W_t :	Heating Day Degree Value	$^{\circ}C$ Degree Day
R:	$1xT$ Discount vector	
$\widehat{C}$:	$Tx12$ price matrix	
$\widehat{A}$:	$12x1$ consumption vector	
$P_1(t) = f(t)$:	Energy price with no error term	dollar
$P_2(t) = f(t) + X_t$:	Energy price forecast with error term	dollar
π:	Money earnings from CO_2 saving sale	dollar/kg
NPV_{cust}:	NPV of total profit that buyer has	dollar
NPV_{comp}:	NPV of total profit that seller has	dollar
κ:	Operating cost	dollar
λ:	Failure rate of a technology	
L :	Permit value that Kyoto Protocol provides	kg
θ:	CO_2 produced of a single technology	kg/KWh
π:	Money earnings from CO_2 saving sale	dollar/kg

Feasibility

There are some assumptions for our model to be feasible. Firstly, we need to assume that NPV of energy usage cost with old technology must be higher than the NPV of energy usage cost with new technology,that is $NPV_{energy}(\varepsilon) > NPV_{energy}(\varepsilon')$. Also when we consider initial investment to change technology, $NPV_{energy}(\varepsilon) - NPV_{energy}(\varepsilon') - NPV_{rep} > \kappa$ must satisfy to have gain from new technology where κ is the operating cost of the company. This gain can be passed on to the buyer and company. The company can make a discount in the agreement to convince the customer. Δ is the % discount quantity that a company can provide. Profits of customer and company vary according to Δ.Hence Δ is the variable that can be decided to satisfy feasibility in the model.If $\Delta = 0$, then we can say that the company has all the profit. However, if $\Delta = \Delta_{max}$, then the customer makes the highest profit where Δ_{max} is the maximum discount that company can give.Hence it is obvious that for the company, if $\Delta \in (\Delta_{max}, 1]$ then the model is infeasible.

NPV of company and NPV of customer is shown in Equation 3.1. NPV of CO_2 selling is left to customer which can be considered as the lower bound for the company.

$$NPV_{energy}(\varepsilon) - NPV_{energy}(\varepsilon') + NPV_{CO_2}(\varepsilon, \varepsilon') - NPV_{rep} = NPV_{cust} + NPV_{comp} \tag{3.1}$$

If the company makes a discount for the energy usage that the buyer should pay to the seller then,

$$\begin{aligned} NPV_{cust} &= \Delta NPV_{energy}(\varepsilon) + NPV_{CO_2}(\varepsilon, \varepsilon') \\ NPV_{comp} &= (1-\Delta) NPV_{energy}(\varepsilon) - NPV_{energy}(\varepsilon') - NPV_{rep}. \end{aligned} \tag{3.2}$$

where NPV_{CO_2} is

$$\begin{aligned} NPV_{CO_2} &= \pi L \sum_{t=0}^{T} ((1+i)e^{\omega})^{-t} - \pi\theta \frac{\widehat{R1A}}{\varepsilon'} - \pi L \sum_{t=0}^{T} ((1+i)e^{\omega})^{-t} - \pi\theta \frac{\widehat{R1A}}{\varepsilon} \\ &= \frac{L\pi(e^{\omega}(1+i) - (e^{\omega}(1+i))^{-T})}{(e^{\omega}(1+i)-1)} - \pi\theta \frac{\widehat{R1A}}{\varepsilon'} - \frac{L\pi(e^{\omega}(1+i) - (e^{\omega}(1+i))^{-T})}{(e^{\omega}(1+i)-1)} + \pi\theta \frac{\widehat{R1A}}{\varepsilon} \\ &= \pi\theta \widehat{R1A}(\frac{1}{\varepsilon} - \frac{1}{\varepsilon'}). \end{aligned} \tag{3.3}$$

We have calculated the $NPV_{energy}(\varepsilon)$ and $NPV_{energy}(\varepsilon')$ and NPV_{rep} in equations 2.2, 2.3, 2.13 and 2.14 in Section 2. Then the net profits of the company and customer are:

$$\begin{aligned} NPV_{comp} &= (1-\Delta) \sum_{t=1}^{12} \sum_{t'=1}^{T/12} (1+r)^{-12(t'-1)-t} (\frac{aW_t + b}{\varepsilon}) P(t) - NPV_{rep} - \sum_{t=1}^{12} \sum_{t'=1}^{T/12} (1+r)^{-12(t'-1)-t} (\frac{aW_t + b}{\varepsilon'}) P(t) \\ NPV_{cust} &= \Delta \sum_{t=1}^{12} \sum_{t'=1}^{T/12} (1+r)^{-12(t'-1)-t} (\frac{aW_t + b}{\varepsilon}) P(t) + \pi\theta \widehat{R1A}(\frac{1}{\varepsilon} - \frac{1}{\varepsilon'}) \end{aligned} \tag{3.4}$$

where $NPV_{rep} = c\frac{1-e^{-(\alpha+\beta)s(N+1)}}{1-e^{-(\alpha+\beta)s}}$ for the deterministic case and $NPV_{rep} = \frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T}) + c$ for the stochastic case. Figure 3.1 shows the gain of customer and company as Δ increases.

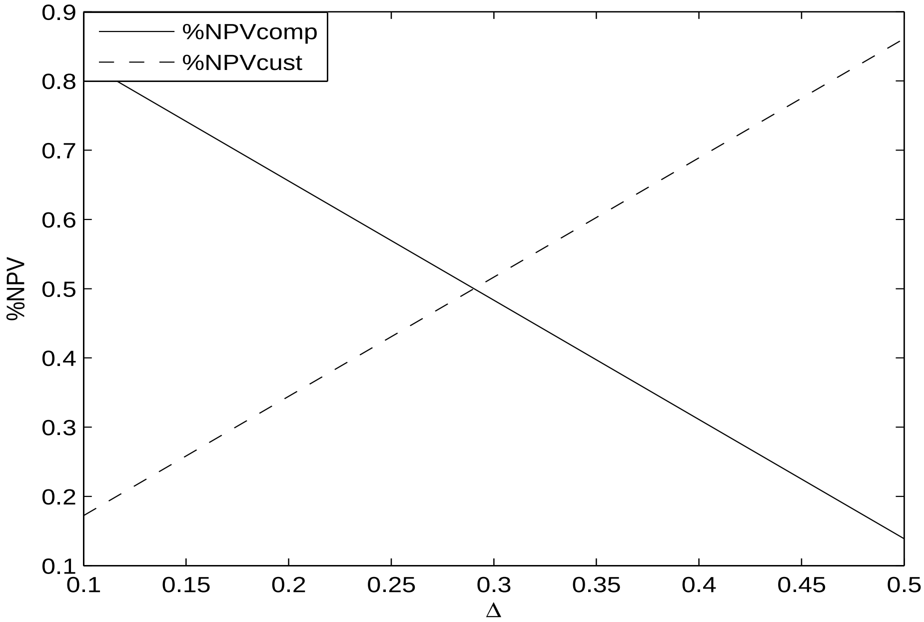

Figure 3.1: Percentage gain that company and customer have as Δ increases

In the following, we show the profits in deterministic and stochastic cases. Then, for feasibility, profit should be positive whenever $\Delta = 0$. Hence the following inequality should be satisfied.

$$NPV_{energy}(\varepsilon) - NPV_{energy}(\varepsilon') - NPV_{rep} > \kappa \tag{3.5}$$

Deterministic Model In the deterministic model we assume that the energy price is increasing with a function $P_1(t) = ct + d$.

$$NPV_{comp} = (1-\Delta)\sum_{t=1}^{12}\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon})P_1(t) - c\frac{1-e^{-(\alpha+\beta)s(N+1)}}{1-e^{-(\alpha+\beta)s}}$$
$$-\sum_{t=1}^{12}\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon'})P_1(t)$$

$$= (1-\Delta)\frac{R\widehat{C}\widehat{A}}{\varepsilon} - c\frac{1-e^{-(\alpha+\beta)s(N+1)}}{1-e^{-(\alpha+\beta)s}} - \frac{R\widehat{C}\widehat{A}}{\varepsilon'}$$

$$= (\frac{1}{\varepsilon} - \frac{\Delta}{\varepsilon} - \frac{1}{\varepsilon'})R\widehat{C}\widehat{A} - c\frac{1-e^{-(\alpha+\beta)s(N+1)}}{1-e^{-(\alpha+\beta)s}} \quad (3.6)$$

where

R is $1xT$ discount vector where	$r_i = (1+r)^{-(i-1)}$
$\widehat{C}$ is the $Tx12$ matrix where	$\widehat{c}_{i,j} = \widehat{c}_{i,j-1} + d,$
	$\widehat{c}_{i,1} = \widehat{c}_{i-1,12} + d, t \neq 1$ and
	$\widehat{c}_{1,1} = c + d$
$\widehat{A}$ is $12x1$ vector where	$\widehat{d}_j aW_j + b$ (3.7)

For feasibility of the model when $\Delta = 0$

$$= (\frac{1}{\varepsilon} - \frac{1}{\varepsilon'})R\widehat{C}\widehat{A} - c\frac{1-e^{-(\alpha+\beta)s(N+1)}}{1-e^{-(\alpha+\beta)s}} > \kappa$$

$$= \frac{1}{\varepsilon} - \frac{1}{\varepsilon'} > \frac{c\frac{1-e^{-(\alpha+\beta)s(N+1)}}{1-e^{-(\alpha+\beta)s}} + \kappa}{R\widehat{C}\widehat{A}}$$

Stochastic Model In the stochastic model we assume that the energy price is increasing with function $P_2(t) = ct + d + X_t$ where X_t is AR(1) process.

$$E[NPV_{comp}] = E[(1-\Delta)\sum_{t=1}^{12}\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon})P_2(t) - \frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T}) - c$$
$$-\sum_{t=1}^{12}\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon'})P_2(t)]$$

$$= (1-\Delta)\sum_{t=1}^{12}\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon})P_1(t) - \frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T}) - c$$
$$-\sum_{t=1}^{12}\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon'})P_1(t)$$

$$= (\frac{1}{\varepsilon} - \frac{\Delta}{\varepsilon} - \frac{1}{\varepsilon'})R\widehat{C}\widehat{A} - \frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T}) - c \tag{3.8}$$

For feasibility of the model when $\Delta = 0$, $E[NPV_{comp}] > 0$ must hold.

$$(\frac{1}{\varepsilon} - \frac{1}{\varepsilon'})R\widehat{C}\widehat{A} - \frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T}) - c > \kappa$$

$$\frac{1}{\varepsilon} - \frac{1}{\varepsilon'} > \frac{\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T}) + c + \kappa}{R\widehat{C}\widehat{A}} \tag{3.9}$$

From now on, we make our calculations by using the stochastic model since the business plan model includes risk.

Δ_{max} **calculation-Contract Selection** In this section, we will find the maximum value of Δ which satisfies the desire of the company to make profit by taking expected value into consideration.

Expected value criteria Firstly, we will make analysis by considering operating expenses that the company has to pay. We will find Δ_{max} which satisfies $E[NPV_{comp}] > \kappa$, where κ is the operating costs of the company. The condition can be rewritten as

$$E[(1-\Delta)\sum_{t=1}^{12}\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon})P_2(t)-\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})-c$$
$$-\sum_{t=1}^{12}\sum_{t'=1}^{T/12}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon'})P_2(t)]\geq\kappa$$

or equivalently

$$(\frac{1}{\varepsilon}-\frac{\Delta}{\varepsilon}-\frac{1}{\varepsilon'})\widehat{R\widehat{CA}}-\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})-c\geq\kappa$$

Then the maximum discount that can be offered to the customer

$$\Delta\leq-\frac{\kappa+\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})+c}{\widehat{R\widehat{CA}}}+1-\frac{\varepsilon}{\varepsilon'}$$
$$\Delta_{max}=-\frac{\kappa+\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})+c}{\widehat{R\widehat{CA}}}+1-\frac{\varepsilon}{\varepsilon'} \quad (3.10)$$

Feasibility The company should determine Δ which satisfies $0<\Delta<\Delta_{max}$ by considering calculations above. There are 2 inequalities to make this model feasible.Weak feasibility can be satisfied with the first analysis when $\kappa=0$, that is:

$$0<\Delta<-\frac{\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})+c}{\widehat{R\widehat{CA}}}+1-\frac{\varepsilon}{\varepsilon'} \quad (3.11)$$

stronger feasibility is satisfied with first analysis with operating costs:

$$0<\Delta<-\frac{\kappa+\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})+c}{\widehat{R\widehat{CA}}}+1-\frac{\varepsilon}{\varepsilon'} \quad (3.12)$$

3.1.2 Initial Investment

Expected value Criteria Initial investment should satisfy the following inequality for the company to make profit with the expected value criteria.

$$(\frac{1}{\varepsilon}-\frac{\Delta}{\varepsilon}-\frac{1}{\varepsilon'})\widehat{R\widehat{CA}}-\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})-c\geq 0$$

that yields

$$c \leq \frac{(\frac{1}{\varepsilon} - \frac{\Delta}{\varepsilon} - \frac{1}{\varepsilon'})R\widehat{C}\widehat{A}}{(\frac{(1-e^{-(\alpha+\beta)T})\lambda}{\alpha+\beta} + 1)} \tag{3.13}$$

Figures 3.2, 3.3 and 3.4 show that as initial investment of a technology increases then discount value that a company can give (Δ) decreases.Figure 3.2 shows that there is no difference in slopes of the decrease for different technologies i.e. $\varepsilon/\varepsilon' = 0.2$ or $\varepsilon/\varepsilon' = 0.5$. Figure 3.3 is the another view of figure 3.2 comparing the proportion of initial investments to the NPV of the invested technology to decrease rate. However, Figure 3.4 illustrates that the slopes of the decrease change for different times since NPV of energy usage is not the same for different times.

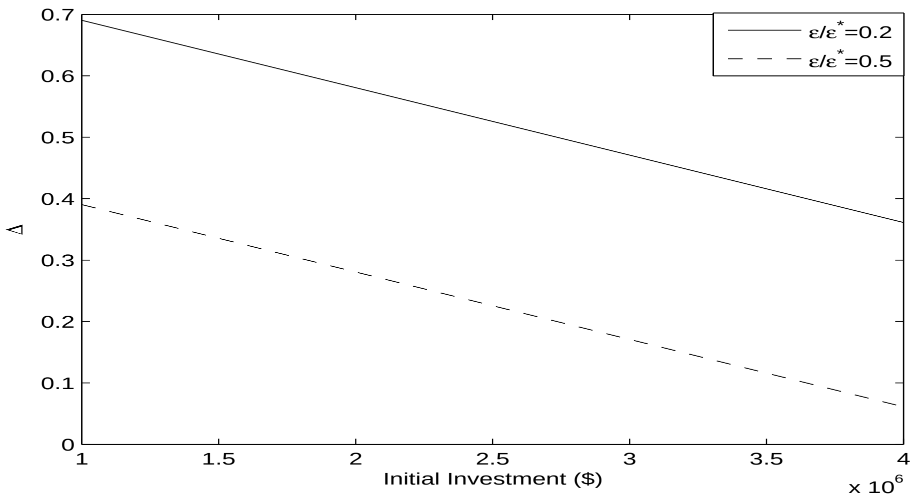

Figure 3.2: Change of discount quantity as initial investment increases in expected cost criteria

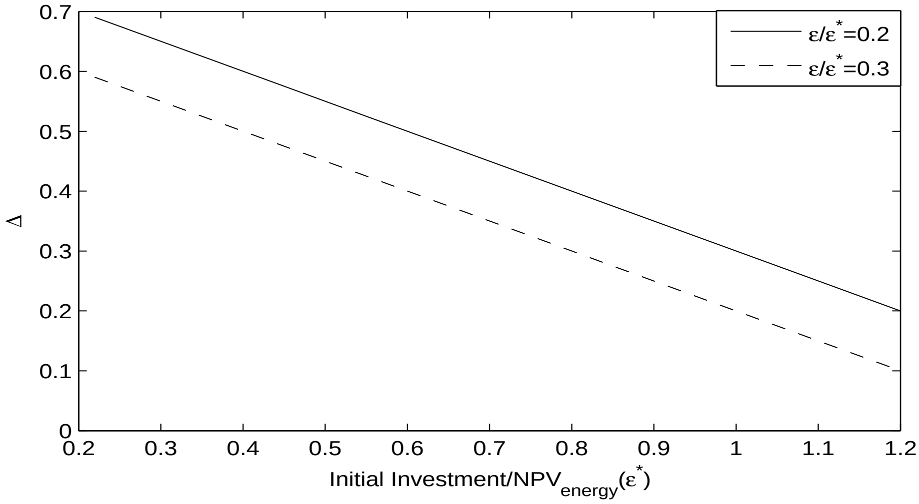

Figure 3.3: Change of discount quantity as proportion of initial investment to energy usage cost increases in expected cost criteria

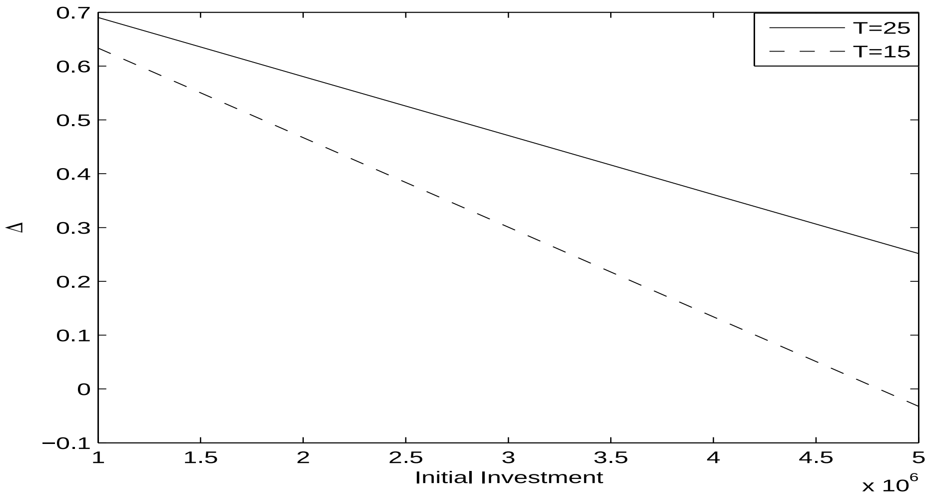

Figure 3.4: Change of discount quantity as initial investment increases in expected cost criteria

Next, we analyze the business plan with additional risk considerations.

3.2 Value at Risk

There have been many measures to define risk one of them which is increasingly used as a first line of defense against financial risks in private sector is called Value at Risk (VaR). VaR summarizes the worst expected loss under over a specific time interval within a given confidence level. In addition to financial reporting, VAR can be used for numerous other purposes such as setting position limits for traders, measuring returns on a risk-adjusted basis and model evaluation [55].

Calculating the variance of the replacement cost and variance of the energy usage cost is not analytically trackable for finite time. Variance of the replacement cost in finite time is approximated by taking infinite horizon solution. In addition, yearly usage is used instead of monthly usage to calculate the variance of energy consumption cost.

3.2.1 Feasibility of Business Plan

Before, we calculated the feasibility of the business plan under expected value criteria. In this part, Value at Risk criteria is included to calculate the feasibility of the model. The feasibility conditions for the VaR calculations are:

1 If there is no discount probability of making at least ψ profit must exceed a confidence level ρ.i.e.$P\{NPV_{comp} > \psi\} > \rho$

2 Discount quantity never exceeds Δ_{max}.i.e.$0 < \Delta < \Delta_{max}$.

The formal definition of Value at risk for our calculation is:$P\{NPV_{comp} > \psi\} > \rho$. Under the normality assumption of NPV(See Section 3.3), the condition $P\{NPV_{comp} > \psi\} > \rho$ can be rewritten as

$$= P\{Z > \frac{\psi - E[NPV_{comp}]}{\sigma_{NPV}}\} > \rho$$

or

$$= E[NPV_{comp}] > \psi - \phi^{-1}(1-\rho)\sigma_{NPV}$$

where ϕ^{-1} is the inverse normal transformation.

Finally,

$$(\frac{1}{\varepsilon}-\frac{\Delta}{\varepsilon}-\frac{1}{\varepsilon'})\widehat{RCA}-\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})-c>\psi-\phi^{-1}(1-\rho)\sigma_{NPV} \tag{3.14}$$

if $\Delta = 0$ the following inequality must hold for the first feasibility condition.

$$(\frac{1}{\varepsilon}-\frac{1}{\varepsilon'})\widehat{RCA}-\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})-c>\psi-\phi^{-1}(1-\rho)\sigma_{NPV} \tag{3.15}$$

Δ_{max} calculation-Contract Selection In this part, Δ_{max} is calculated to satisfy second feasibility condition $0 < \Delta < \Delta_{max}$.

$$(\frac{1}{\varepsilon}-\frac{\Delta}{\varepsilon}-\frac{1}{\varepsilon'})\widehat{RCA}-\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})-c>\psi-\phi^{-1}(1-\rho)\sigma_{NPV}$$

$$(\frac{1}{\varepsilon}-\frac{\Delta}{\varepsilon}-\frac{1}{\varepsilon'})>\frac{\psi-\phi^{-1}(1-\rho)\sigma_{NPV}+\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})+c}{\widehat{RCA}}$$

$$\Delta<(\frac{\psi-\phi^{-1}(1-\rho)\sigma_{NPV}+\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})+c}{\widehat{RCA}}-\frac{1}{\varepsilon}+\frac{1}{\varepsilon'})\varepsilon$$

$$\Delta_{max}=(\frac{\psi-\phi^{-1}(1-\rho)\sigma_{NPV}+\frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T})+c}{\widehat{RCA}}-\frac{1}{\varepsilon}+\frac{1}{\varepsilon'})\varepsilon \tag{3.16}$$

To further the calculations in Equations 3.18 and 3.19 we need to find σ_{NPV}. We assumed that the NPV of replacement cost and the NPV of energy consumption are independent. Hence, the variance of NPV of company is:

$$Var[NPV_{comp}]=Var[(1-\Delta)NPV_{energy}(\varepsilon)-NPV_{energy}(\varepsilon')-NPV_{rep}]$$

$$Var[NPV_{comp}]=Var[(1-\Delta)NPV_{energy}(\varepsilon)]+Var[NPV_{energy}(\varepsilon')]+Var[NPV_{rep}] \tag{3.17}$$

Since $Var[(1-\Delta)NPV_{energy}(\varepsilon)]=0$ then $Var[NPV_{comp}]=Var[NPV_{energy}(\varepsilon')]+Var[NPV_{rep}]$.

Variance of NPV of Replacement Cost We used an infinite variance solution, to approximate finite solution since in finite horizon variance is not analytically traceable.

Infinite Solution Infinite solution of variance is:

$$Var[NPV_{rep}] = Var[c\sum_{n=1}^{\infty} X_n] = E[c^2(\sum_{n=1}^{\infty} X_n)^2] - \frac{c^2\lambda^2}{(\alpha+\beta)^2} \tag{3.18}$$

Firstly, we will calculate $E[NPV_{rep}^2]$. The calculation of $E[NPV_{rep}]$ is given in the Appendix A.

$$E[NPV_{rep}^2] = E[c^2(\sum_{n=1}^{\infty} X_n)^2] = c^2E[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} X_nX_m] = c^2\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} E[X_nX_m] \tag{3.19}$$

We will consider the three situations $n = m, n < m, n > m$

1-)$n = m$;

$$E[NPV_{rep}^2] = \sum_{n=1}^{\infty} c^2E[e^{-2(\alpha+\beta)T_n}] = \sum_{n=1}^{\infty} c^2(\frac{\lambda}{\lambda+2(\alpha+\beta)})^n \tag{3.20}$$

2-)$n < m$;

$$\begin{aligned} E[NPV_{rep}^2] &= \sum_{n=1}^{\infty}\sum_{m=n+1}^{\infty} c^2E[X_nX_m] \\ &= \sum_{n=1}^{\infty}\sum_{m=n+1}^{\infty} c^2E[e^{-(\alpha+\beta)T_n}e^{-(\alpha+\beta)T_m}] \\ &= \sum_{n=1}^{\infty}\sum_{m=n+1}^{\infty} c^2(\frac{\lambda}{\lambda+2(\alpha+\beta)})^n(\frac{\lambda}{\lambda+\alpha+\beta})^{m-n} \end{aligned} \tag{3.21}$$

3-) $n > m$;

$$\begin{aligned} E[NPV_{rep}^2] &= \sum_{m=1}^{\infty}\sum_{n=m+1}^{\infty} c^2E[X_nX_m] \\ &= \sum_{m=1}^{\infty}\sum_{n=m+1}^{\infty} c^2E[e^{-(\alpha+\beta)T_n}e^{-(\alpha+\beta)T_m}] \\ &= \sum_{m=1}^{\infty}\sum_{n=m+1}^{\infty} c^2(\frac{\lambda}{\lambda+2(\alpha+\beta)})^m(\frac{\lambda}{\lambda+\alpha+\beta})^{n-m} \end{aligned} \tag{3.22}$$

Finally, the variance of NPV

$$Var[NPV_{rep}] = \sum_{n=1}^{\infty} c^2 (\frac{\lambda}{\lambda+2(\alpha+\beta)})^n + \sum_{n=1}^{\infty}\sum_{m=n+1}^{\infty} c^2 (\frac{\lambda}{\lambda+2(\alpha+\beta)})^n (\frac{\lambda}{\lambda+\alpha+\beta})^{m-n}$$
$$+ \sum_{m=1}^{\infty}\sum_{n=m+1}^{\infty} c^2 (\frac{\lambda}{\lambda+2(\alpha+\beta)})^m (\frac{\lambda}{\lambda+\alpha+\beta})^{n-m} - \frac{c^2\lambda^2}{(\alpha+\beta)^2}$$

which can be simplified as

$$Var[NPV_{rep}] = \frac{c^2\lambda}{2(\alpha+\beta)} \tag{3.23}$$

In Figure 3.6, we show how variance changes according to time by using a simulation when $\alpha = 0.01$. Variance is increasing until a point, then it is stable after that point. This value is also the variance of replacement cost in the infinite model. In figure 3.7, we tried to learn how parameters affect the variance so we changed $\alpha = 0.07$.The result is as α increases variance reaches its infinite limit faster. Figure 3.8 and Figure 3.9 represent the error of variance when we use a finite t instead of infinite when $\alpha = 0.01$ and $\alpha = 0.07$, respectively. Here we can say that after a certain point we can use the infinite model instead of the finite model. We use this observation to approximate variance for the finite time case.

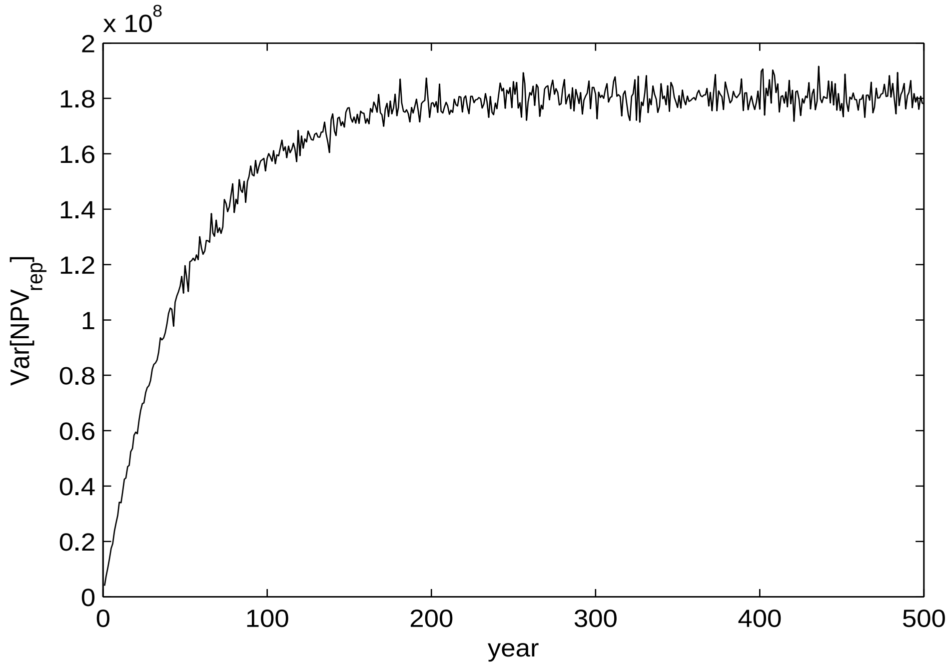

Figure 3.5: Variance of NPV_{rep} when $\alpha = ln(1.01)$

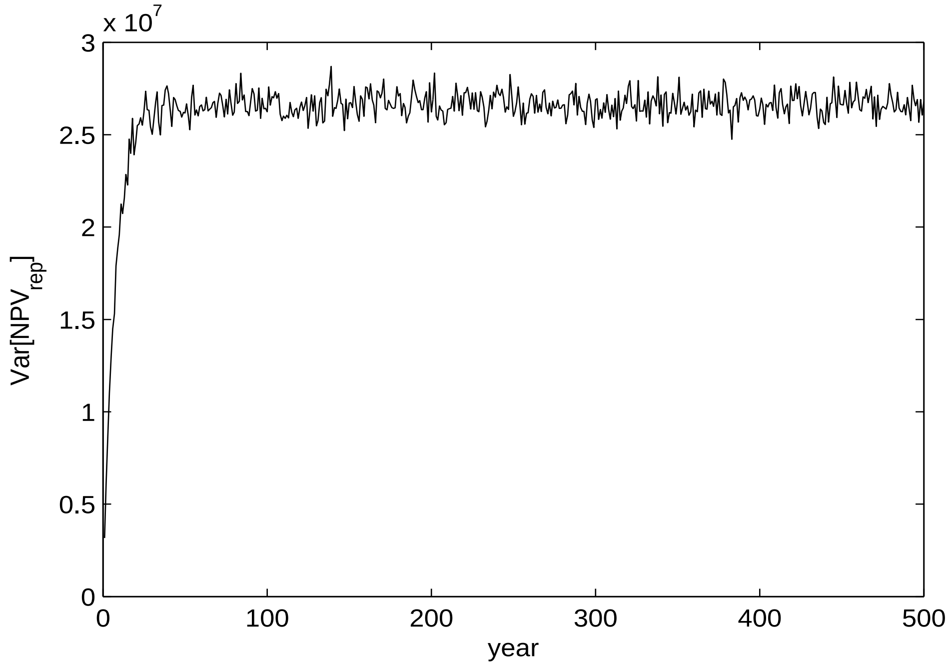

Figure 3.6: Variance of NPV_{rep} when $\alpha = ln(1.07)$

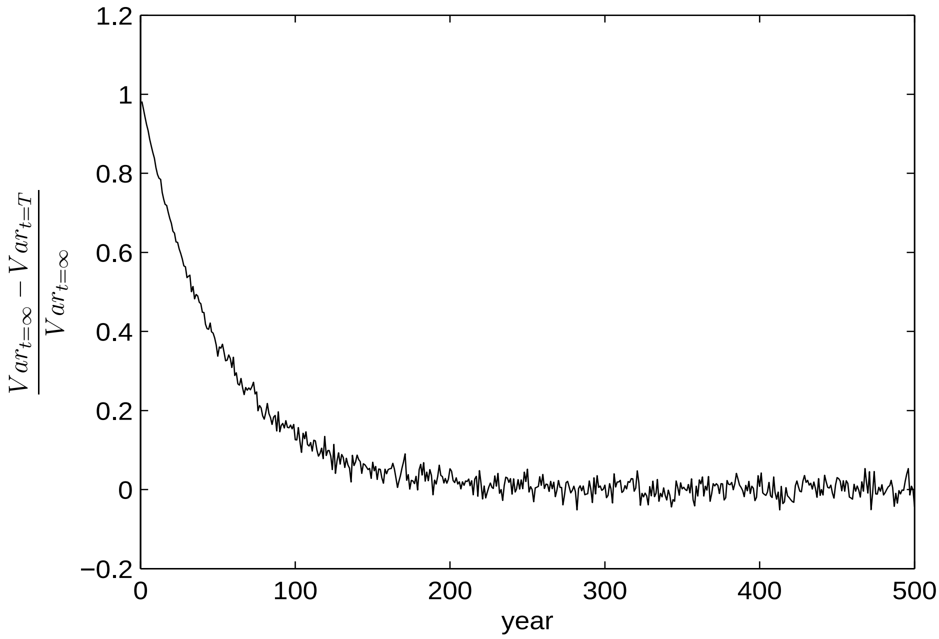

Figure 3.7: Deviation of variance when $\alpha = ln(1.01)$

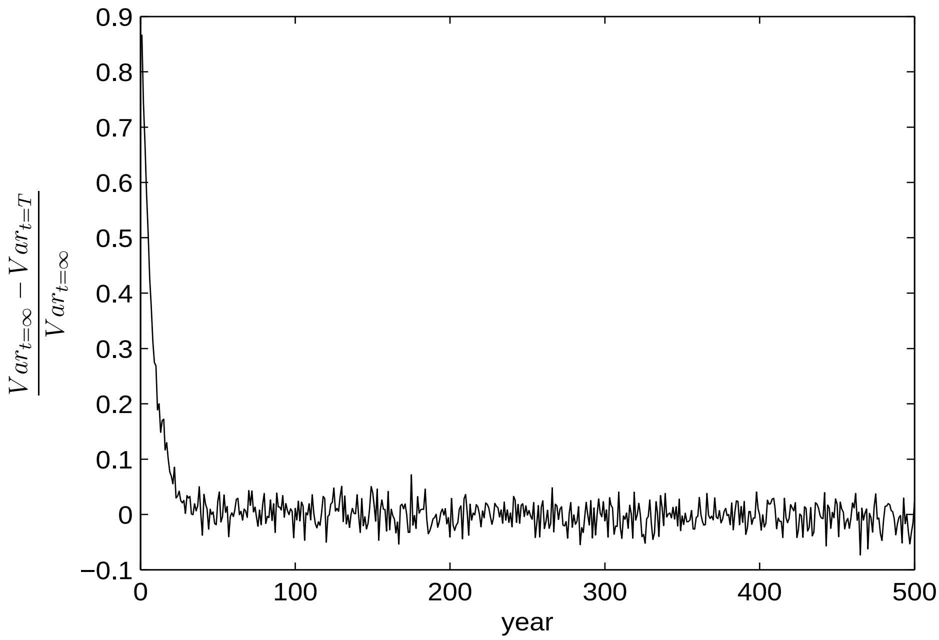

Figure 3.8: Deviation of variance error when $\alpha = ln(1.07)$

By using these observations, we can say that after $t = \frac{2}{\alpha}$ the cost of the finite model is approximately the same as the cost of the infinite model.

Variance of NPV of Energy Usage Cost For risk calculation we calculate variance of the NPV for a single technology business plan. In order to do that, we assume that consumer is getting paid yearly to simplify calculation of the variance. Hence, we assume the energy price is increasing yearly with function $P(t) = mt + n + X_t$. Also, since the consumption is the same for every year in period we take $\sum_{t=1}^{12} \frac{aW_t+b}{\varepsilon} = G$ as a constant variable. Hence NPV of the electricity cost becomes:

$$\sum_{t=0}^{T}(1+r)^{-t}G(mt+n+X_t) \tag{3.24}$$

and

$$E[NPV_{energy}] = \sum_{t=0}^{T}(1+r)^{-t}G(mt+n) \tag{3.25}$$

The variance of NPV of energy cost for finite T is:

$$Var[NPV_{energy}] = E[NPV_{energy}^2] - E[NPV_{energy}]^2$$

$$= E[\sum_{t=0}^{T}\sum_{t'=0}^{T}(1+r)^{-t}G^2(mt+n+X_t)(1+r)^{-t'}(mt'+n+X_t')] - E[NPV_{energy}]^2$$

which can be evaluated as

$$= \frac{G^2\sigma_\varepsilon^2 B(T)}{(1-\varphi^2)} + \frac{2G^2\sigma_\varepsilon^2}{(1-\varphi^2)}\sum_{t=0}^{T-1}(1+r)^{-2t}\frac{\varphi-\varphi(\frac{\varphi}{(1+r)})^{T-t}}{(1+r)-\varphi}$$

$$= \frac{G^2\sigma_\varepsilon^2 B(T)}{(1-\varphi^2)} + \frac{2G^2\sigma_\varepsilon^2\varphi}{(1-\varphi^2)((1+r)-\varphi)}(\frac{(1+r)^{2-2T}((1+r)^{2T}-1)}{(1+r)^2-1}$$
$$-(\frac{\varphi}{(1+r)})^T\frac{((1+r)\varphi)^{-T+1}(((1+r)\varphi)^T-1)}{((1+r)\varphi)-1}) \tag{3.26}$$

where $B(T) = \frac{(1+r)^{-2T}((1+r)^{2(T+1)}-1)}{(1+r)^2-1}$.

Since $Var[NPV_{comp}] = Var[NPV_{energy}(\varepsilon')] + Var[NPV_{rep}]$

$$Var[NPV_{comp}] = \frac{G^2\sigma_\varepsilon^2 B(T)}{(1-\varphi^2)} + \frac{2G^2\sigma_\varepsilon^2\varphi}{(1-\varphi^2)((1+r)-\varphi)}\Big(\frac{(1+r)^{2-2T}((1+r)^{2T}-1)}{(1+r)^2-1} - \Big(\frac{\varphi}{(1+r)}\Big)^T\frac{((1+r)\varphi)^{-T+1}(((1+r)\varphi)^T-1)}{((1+r)\varphi)-1}\Big) + \frac{c^2\lambda}{2(\alpha+\beta)} \tag{3.27}$$

Now, Equations 3.15 and 3.16 can be clarified by using variance calculation in Equation 3.27.

Feasibility when $\Delta = 0$ Feasibility condition 3.2 with the σ_{NPV} is calculated as follows:

$$\Big(\frac{1}{\varepsilon} - \frac{1}{\varepsilon'}\Big)R\widehat{C}\widehat{A} - \frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T}) - c > \psi - \phi^{-1}(1-\rho)\sigma_{NPV} \tag{3.28}$$

where $\sigma_{NPV} = \sqrt{Var[NPV_{comp}]}$ given in Equation 3.27.

Δ_{max} calculation-Contract Selection After calculating σ_{NPV} with its value equation 3.3 becomes:

$$\Delta_{max} = \Big(\frac{\psi - \phi^{-1}(1-\rho)\sigma_{NPV} + \frac{c\lambda}{\alpha+\beta}(1-e^{-(\alpha+\beta)T}) + c}{R\widehat{C}\widehat{A}} - \frac{1}{\varepsilon} + \frac{1}{\varepsilon'}\Big)\varepsilon \tag{3.29}$$

where $\sigma_{NPV} = \sqrt{Var[NPV_{comp}]}$ given in Equation 3.27. Feasibility condition $0 < \Delta < \Delta_{max}$ must hold for the calculated Δ_{max} for company to make profit of the investment.

3.3 Distribution of NPV and Probability of Bankruptcy

In this section we aim to find the distribution of NPV. Monte Carlo simulation is used to find the distribution. Figure 3.9 shows the simulation results. As a result of the Central Limit Theorem, NPV is expected to have a Normal Distribution for large T. In Figure 3.9 the simulation is run for 10,000 sample and supports the normality assumption. As a result, we approximate NPV with a Normal Distribution with calculated mean and variance in Section 3.1.1 and Section 3.2.1 respectively.

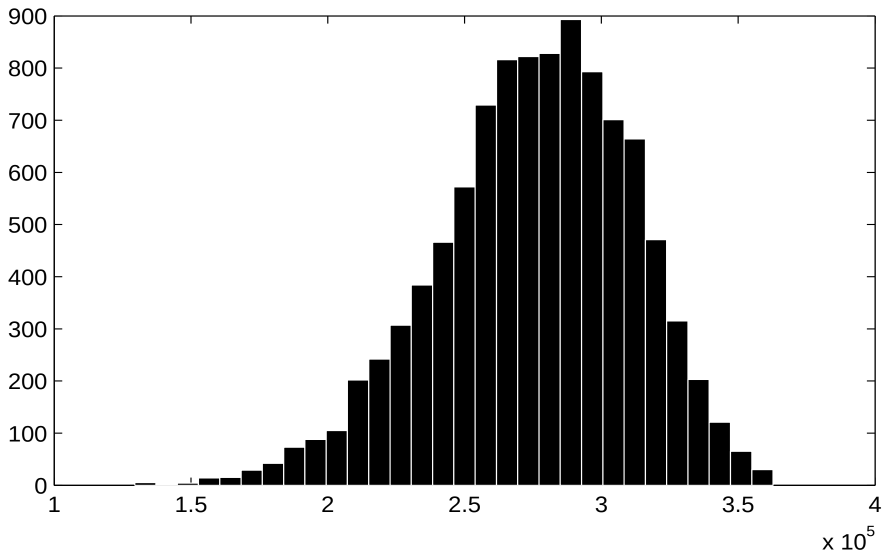

Figure 3.9: Distribution of NPV_{comp}

$$NPV_{comp} \sim N(\mu^*, \sigma^*)$$

where μ^* is given in Equation 3.8 and σ^* is given in Equation 3.27.

Probability of bankruptcy can be calculated by using this distribution.

$$\begin{aligned}
&P[NPV_{comp} < 0] \\
&= P\{\frac{NPV_{comp} - E[NPV_{comp}]}{\sigma_{NPV}} > \frac{-E[NPV_{comp}]}{\sigma_{NPV}}\} \\
&= P\{Z > \frac{-E[NPV_{comp}]}{\sigma_{NPV}}\} \\
&= \phi(\frac{-E[NPV_{comp}]}{\sigma_{NPV}})
\end{aligned}$$

where ϕ is the cdf of standard normal distribution.

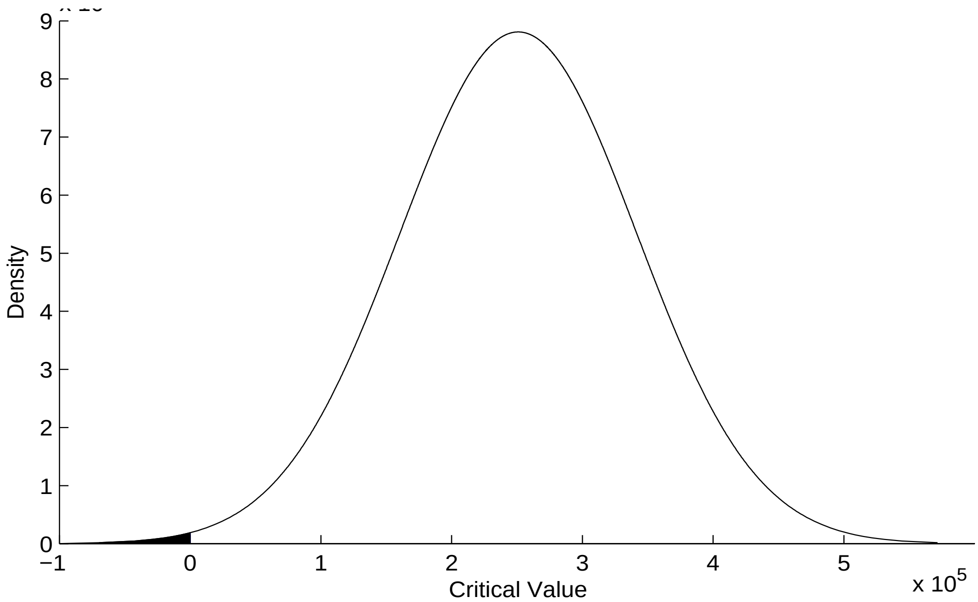

Figure 3.10: Probability of bankruptcy

In Figure 3.10 shaded area $(0, -\infty)$ shows the probability of bankruptcy.

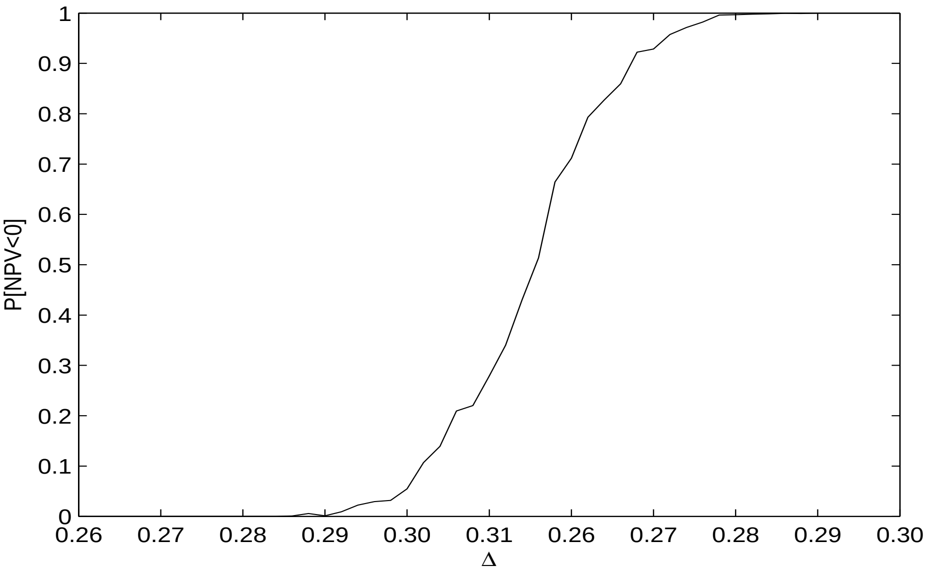

Figure 3.11: Change in probability of bankruptcy as Δ increases

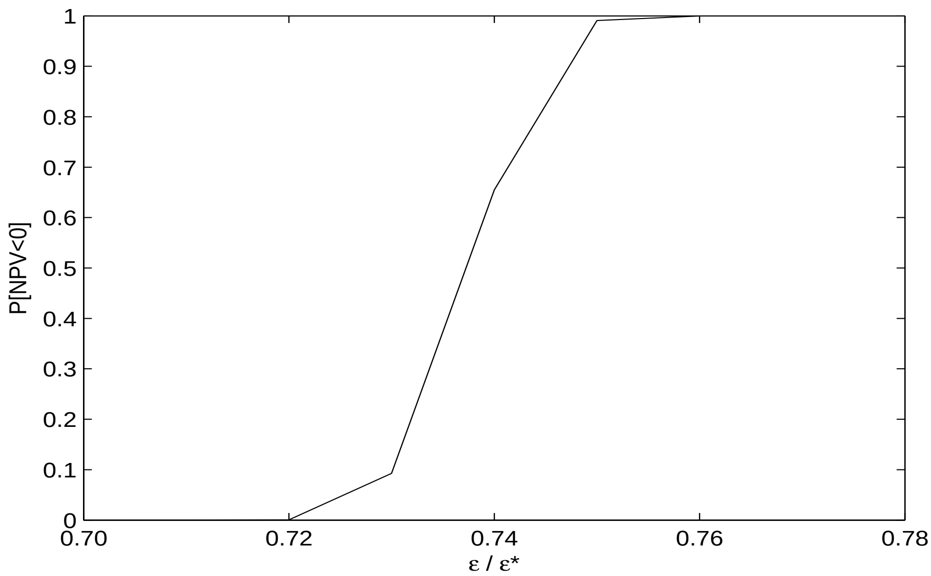

Figure 3.12: Change in probability of bankruptcy as ratio of the efficiency of old technology to the new technology increases

Figure 3.11 and 3.12 show the changes in the probability of bankruptcy as Δ and ratio of the efficiency of old technology to the new technology increases, respectively. Figure 3.11 indicates that giving higher discounts can cause bankruptcy. Figure 3.12 indicates that investing in almost similar technology increase the probability of bankruptcy. More efficient technologies (less ratios) are more appropriate to avoid bankruptcy.

3.4 Comparing Uncertainty in the Model

In this section, we aimed to find which uncertainty of model affects the variability most. As we know, there exists two uncertain variables in the model, which are replacement periods and energy price variation. To do this we plotted how variance changes as parameters increase, are held constant and are held at zero.

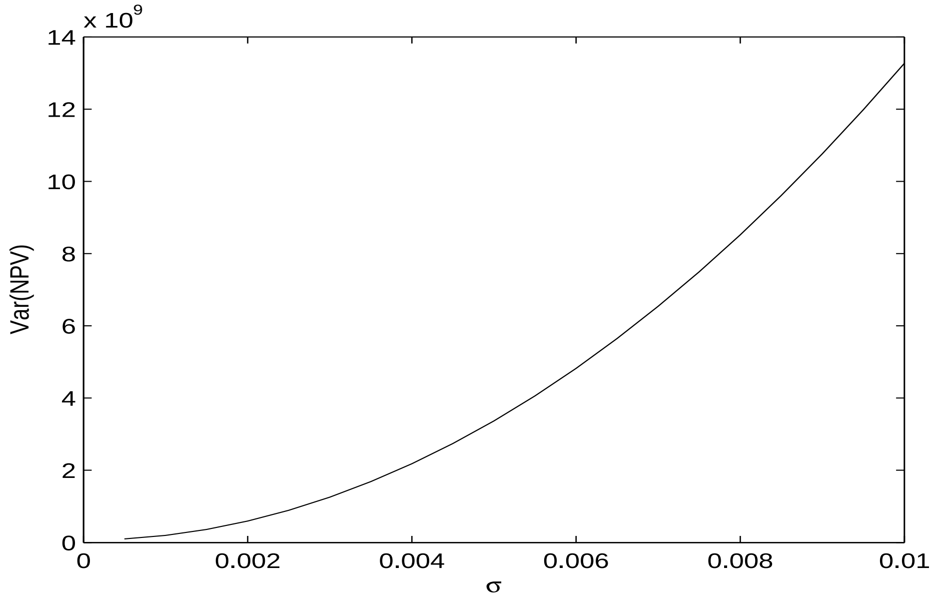

Figure 3.13: Variance of NPV_{comp}as σ increases

In Figure 3.13 we take λ to be a constant variable and we increased σ to learn how the variance of the NPV changes.

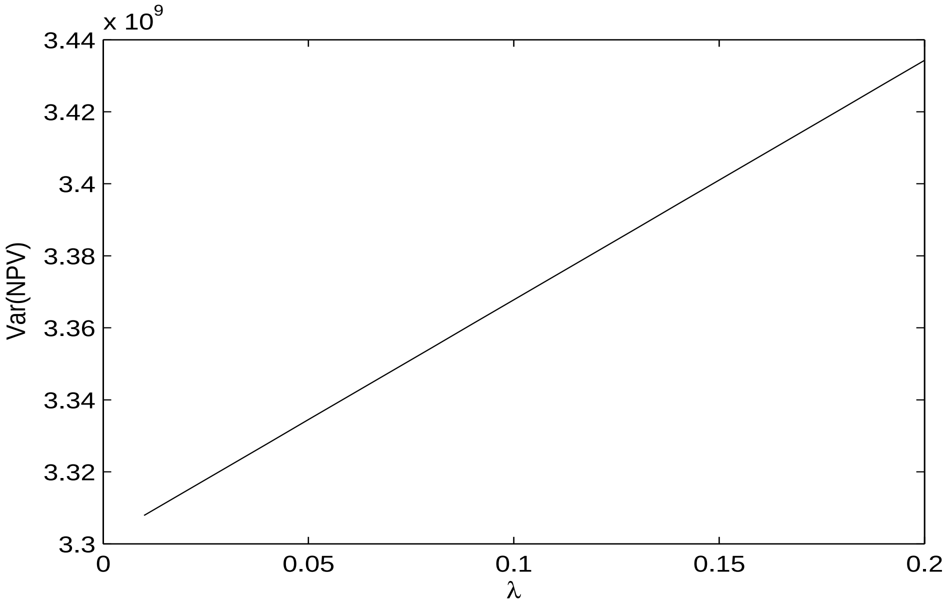

Figure 3.14: Variance of NPV_{comp} as λ increases

In Figure 3.14 we take σ to be a constant variable and we increased λ to learn how the variance of the NPV changes.

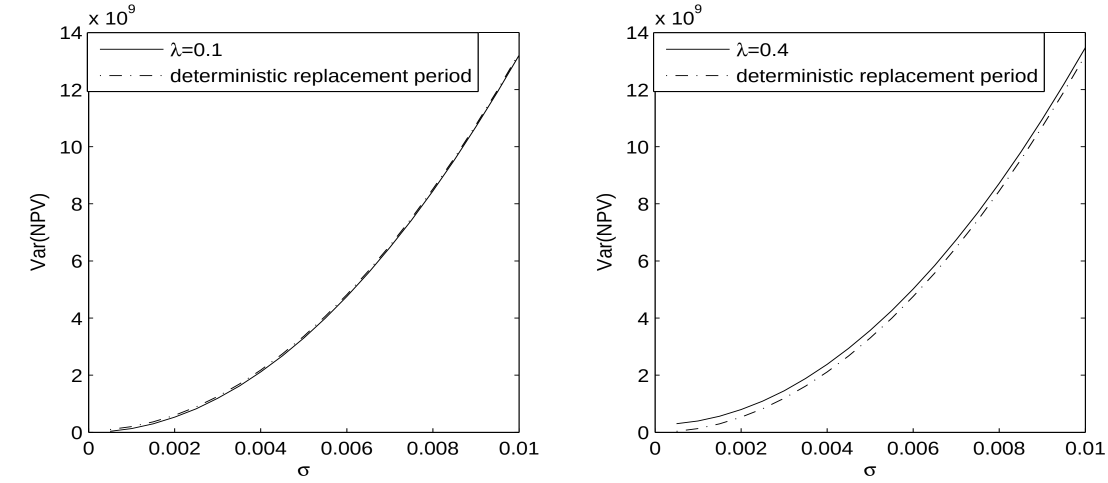

Figure 3.15: Comparison of variances in the models when λ is constant and replacement is deterministic

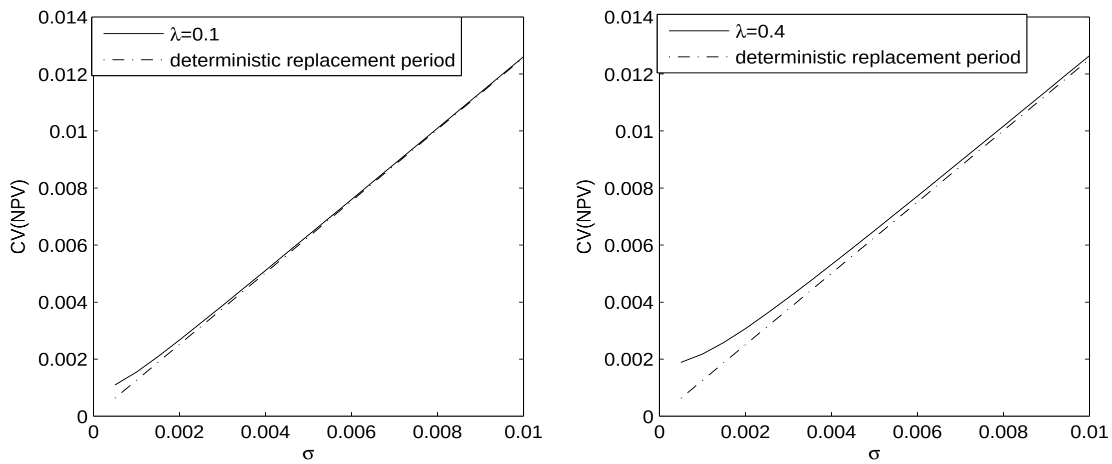

Figure 3.16: Comparison of CVs in the models when λ is constant and replacement is deterministic

In the Figures 3.15 and 3.16, the graphs represent the variance and coefficient of variation of the NPV when λ is constant and $\lambda = 0$. We can see the difference of λ in the variation of NPV. Figures show that uncertainty of λ does not affect the variation of model very much, which can be negligible.

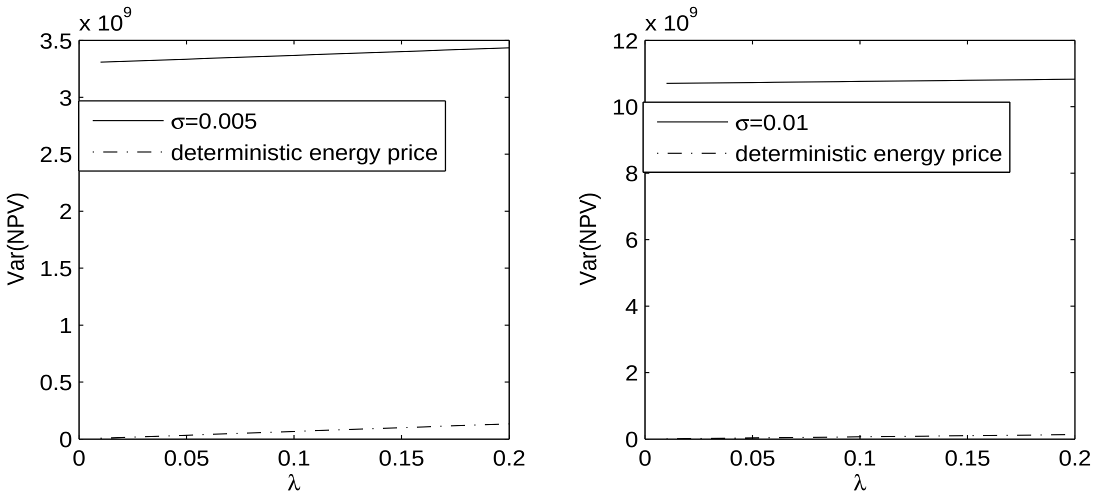

Figure 3.17: Comparison of variances in the models when σ is constant and the energy price is deterministic

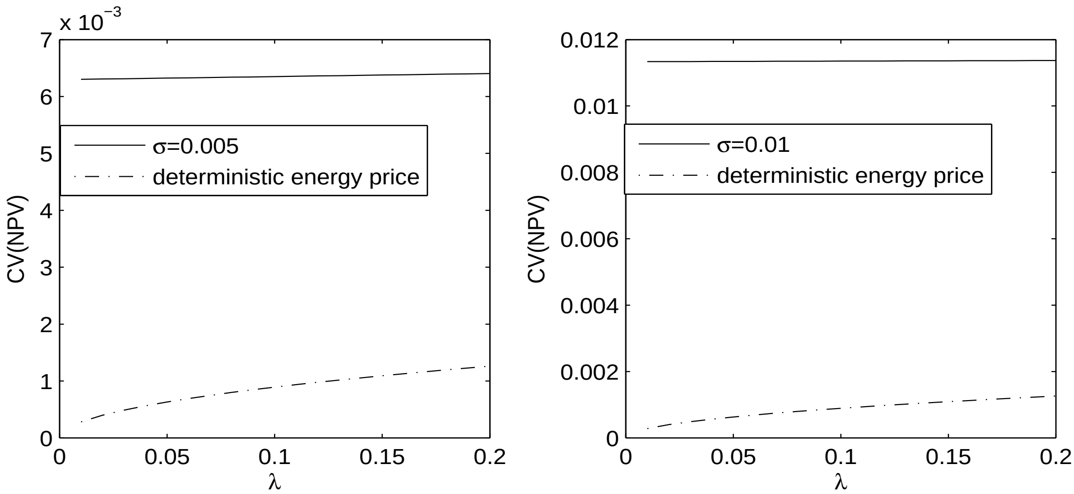

Figure 3.18: Comparison of CVs in the models when σ is constant and replacement is deterministic

In the Figure 3.17 and Figure 3.18, the graph represents the variance and coefficient of variation of NPV when σ is constant and $\sigma = 0$. Figures show that uncertainty of σ affects the variation of model very much, which is more than the effect of λ.

Chapter 4

MULTI-PRODUCT TECHNOLOGY SELECTION PROBLEM AND ESCO BUSINESS PLAN ANALYSIS

After discussing the single technology case, we focus on multiple technology selection now. As an important case study, we summarize a PhD dissertation by Camlibel on building energy efficient technologies that can be implemented in the Boğaziçi University Kilyos Campus [24].In that study, after carrying out technical and mechanical measurements in buildings, there is data collected in Kilyos Campus which give information about how much CO_2 can be saved and how much these Energy Efficient Measures cost. This data is used to minimize CO_2 emissions and maximize dollar savings under budget constraints. Afterwards, a heuristic solution is proposed to this problem, which can be applied easily and has little error when compared to optimized value.

We extend this study in two ways. First, we focus on variability of objective function values as a sensitivity analysis. Second, multi-time optimization is used to select technologies for CO_2 saving maximization. We compared the results of single-time and multi-time optimizations in both CO_2 and NPV savings. In addition, we add a profit rate constraint to the problem which satisfies making at least a level of profit.

In the last part of this chapter, we aim to develop a business plan like the energy service companies. We changed the objective function as maximizing the dollar saving for a predetermined time period. We added discount rate constraint to show the profitability of the company.Finally, a linearly increasing price is used instead of constant price to calculate financial savings which affect both CO_2 and NPV savings maximization in our model.

4.1 Technology Selection as Deterministic Knapsack Problem in Single Period

In all technologies implemented, there exists an investment cost and a $C0_2$ saving amount of the investment. Hence we can think of this problem as the Binary Knapsack Problem. However,the objective function can be changed whether we want to maximize CO_2 saving, $ saving or kWh saving. A usual Binary Knapsack Problem is:

c_i:	Cost of technology i	dollar
e_i:	Saved amount of CO_2 from technology i	kg
B:	Budget	dollar
x_i:	Decision variable of investing or not	0,1

$$x_i = \begin{cases} 1, & \text{if technology i is implemented,} \\ 0, & \text{if technology i is not implemented} \end{cases} \tag{4.1}$$

$$\begin{aligned} \text{maximize} \quad & \sum_i e_i x_i \\ \text{subject to} \quad & \sum_i c_i x_i \leq B, \\ & x_i \in \{0,1\} \end{aligned} \tag{4.2}$$

4.1.1 Case Study of Boğaziçi University Kilyos Campus

The technologies that are implemented in the Boğaziçi Kilyos Campus are:

- Envelope insulation retrofit 6cm
- Envelope insulation retrofit 5cm
- Envelope insulation retrofit 4cm
- Tromble Wall and sunroof application
- Boiler retrofit
- Heating Loop Piping Insulation Retrofit
- Thermostatic Valve Installation in all Radiators
- Domestic Hot Water Heating Setpoint
- Heating Water Circulation Pumps Retrofit
- Lighting Ballast Retrofit

Table 4.1 shows the investment cost and saving amounts of energy, money and CO_2 of the technologies that are implemented in the Boğaziçi Kilyos Campus. Detailed tables are given in the Appendix B.

Table 4.1: Technologies investment cost and saving amounts wrt kWh,$ and CO_2 [24]

Code	Investment USD	kWh Saving/year	$ Saving/year	kg CO_2 saving/year
D1	1.250	76.827	3.129	17.977
H1	500	3.295	134	771
I1	1.250	55.557	2.263	13.000
A2	1.071	23.443	955	5.486
B2	1.071	23.443	955	5.486
C2	964	21.099	859	4.937
E2	2.330	50.989	2.077	11.931
F2	6.750	229.189	9.335	53.630
I2	1.969	43.077	1.754	10.080
D3	41.250	250.005	10.182	58.501
A4	5.850	61.050	2.486	14.286
B4	5.850	60.764	2.475	14.219
C4	1.300	17.402	709	4.072
I4	9.100	63.969	2.605	14.969
D5	10.938	24.599	3.296	15.178
G5	12.500	37.426	5.015	23.092
H5	1.563	1.972	264	1.216
I5	6.250	17.200	2.305	10.613
A6	34.402	95.692	3.514	22.392
B6	34.402	95.611	3.511	22.373
C6	17.084	27.329	1.004	6.395
E6	33.448	156.501	5.747	36.621
F6	22.182	69.278	2.544	16.211
I6	38.998	92.882	3.411	21.734
A7	30.850	81.789	3.003	19.139
B7	30.850	81.210	2.982	19.003
C7	15.320	23.286	855	5.449
E7	29.995	152.049	5.583	35.579
F7	19.892	59.272	2.177	13.870
I7	34.972	79.314	2.912	18.559
A8	27.055	61.993	2.276	14.506
B8	27.055	61.948	2.275	14.496
C8	13.436	17.706	650	4.143
E8	26.305	145.914	5.358	34.144
F8	17.445	45.575	1.674	10.664
I8	30.670	60.237	2.212	14.095
D9	3.750	4.455	597	2.749
G9	6.250	11.880	1.592	7.330
I9	3.750	5.940	796	3.665
A10	65.655	31.912	1.172	7.467
B10	65.655	31.890	1.171	7.462
E10	119.824	65.837	2.418	15.406

After observing data, there should be a new constraint to limit investing in Envelope insulation retrofit in 4 cm, 5 cm and 6 cm in the same building. Our problem is now to maximize CO_2, kwh and money saving according to different budgets in a single period by using Kilyos Campus data. Hence our binary programme is :

$$\begin{aligned}
\text{maximize} \quad & \sum_{i=1}^{42} e_i x_i \\
\text{subject to} \quad & \sum_{i=1}^{42} c_i x_i \leq B, \\
& x_{19} + x_{25} + x_{31} \leq 1 \\
& x_{20} + x_{26} + x_{32} \leq 1, \\
& x_{21} + x_{27} + x_{33} \leq 1, \\
& x_{22} + x_{28} + x_{34} \leq 1, \\
& x_{23} + x_{29} + x_{35} \leq 1,, \\
& x_{24} + x_{30} + x_{36} \leq 1,
\end{aligned} \tag{4.3}$$

CPLEX 12.1 with MATLAB interface is used to solve this Binary Knapsack Problem. Figure 4.1 illustrates the maximum values of CO_2 saved according to different budgets. Simply, by changing the objective function to maximize kwh saving or money saving then Figure 4.2 and Figure 4.3 occur for different budgets. The saving values are increasing as it is expected.

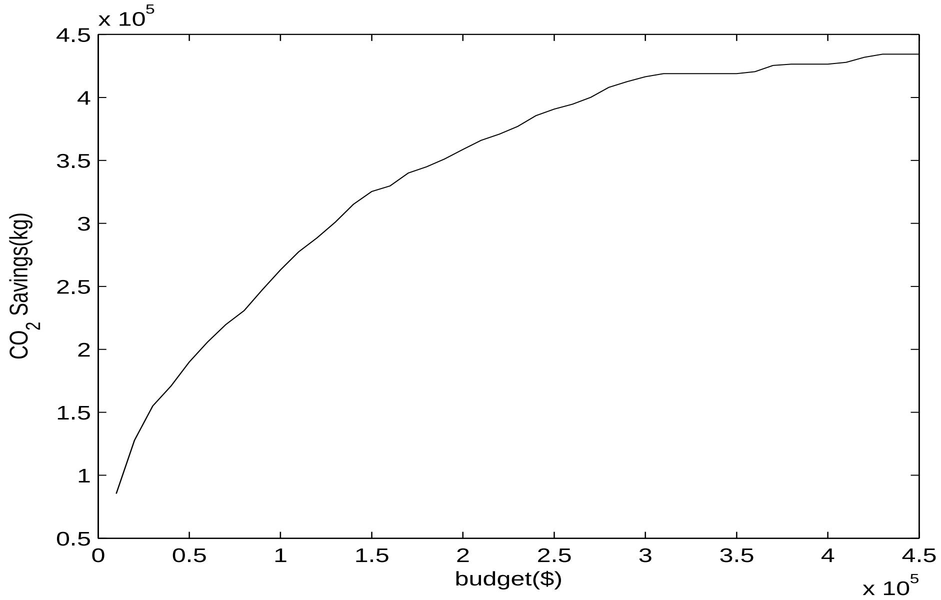

Figure 4.1: CO_2 saving amount with respect to CO_2 saving maximization according to different budgets

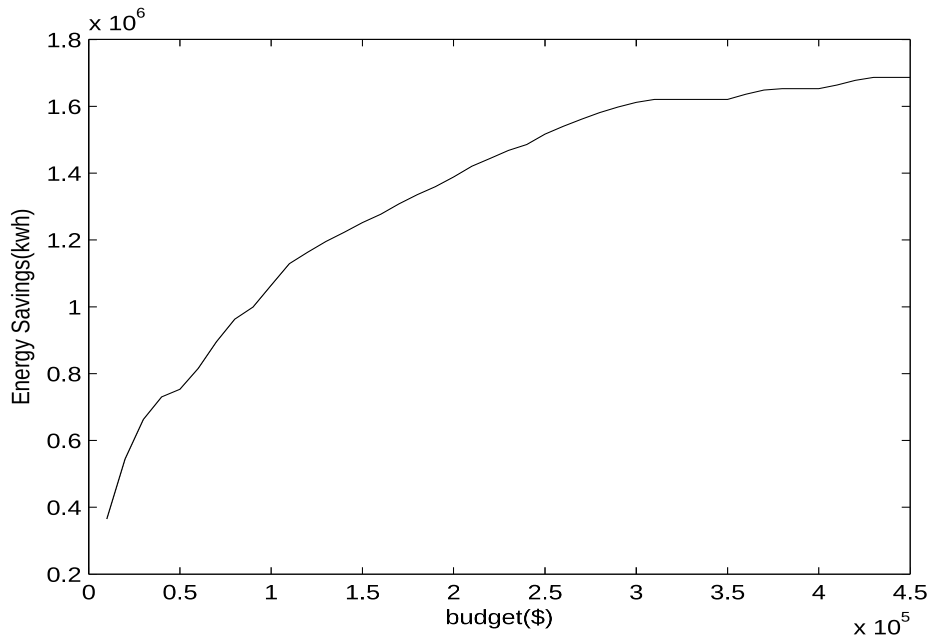

Figure 4.2: Kwh saving amount with respect to kwh saving maximization according to different budgets

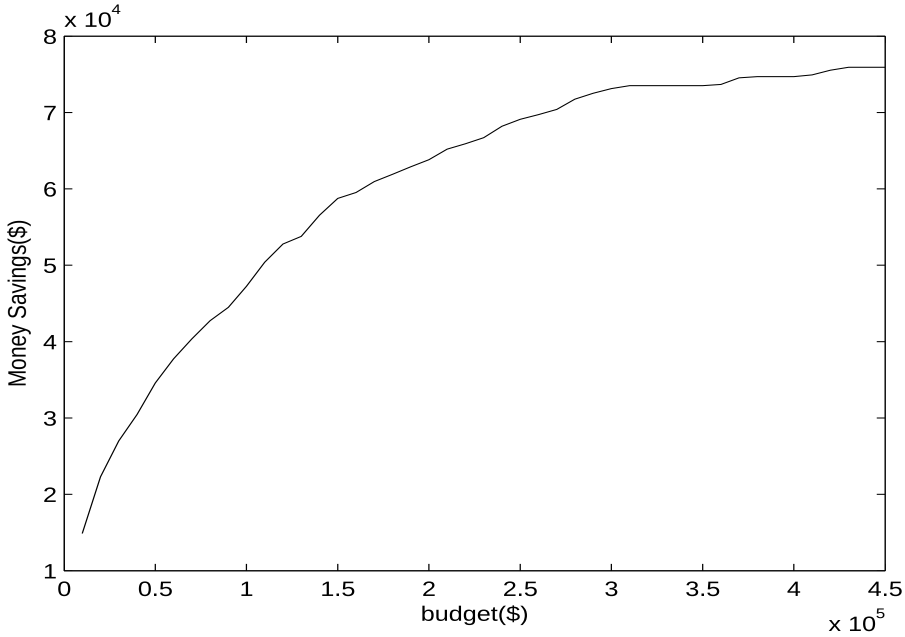

Figure 4.3: Money saving amount with respect to money saving maximization according to different budgets

In Figure 4.4, we compare what will change in the amount of CO_2 saved if the objective functions are changed as maximizing kwh saving and money saving. It has almost the same trend but there occurs some big amounts when kwh saving is set to be the objective function. When comparing kwh maximization and CO_2 maximization the biggest difference, 33,036 kg, occurs in budget 200,000. On the other hand, a comparison of money maximization and CO_2 maximization give almost the same results. Amounts when CO_2 saving maximization is used are larger than the amounts when money saving maximization is used in only 8 different budgets. Additionally, the biggest difference is 7,800 kg in budget 130,000.

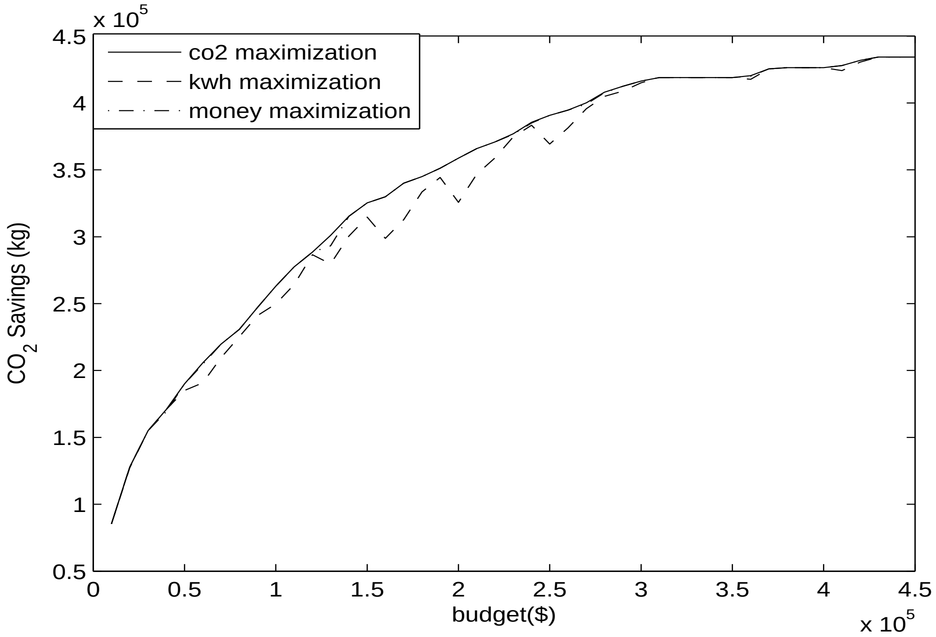

Figure 4.4: CO_2 saving amount with respect to CO_2,kwh and money saving maximization according to different budgets

4.2 Value of Information in Single Period Technology Selection

In this section, we compared the value of information for CO_2 saving uncertainty. We take CO_2 saving amounts in the Boğaziçi Kilyos Case as mean savings. Then for the uncertainty we use

$$e_i = \bar{e}_i \pm \varphi_i \sim N(0, \sigma_i) \tag{4.4}$$

as real CO_2 saving for 42 technologies where where e_i is real CO_2 saving amounts $\bar{e}_i$ is mean CO_2 saving amounts and $\sigma_i = CV\bar{e}_i$. Notice that $e_i \sim N(\bar{e}_i, CV\bar{e}_i)$. We generate real CO_2 saving amounts by Monte Carlo simulation for all technologies for CV=0.1, 0.2 and 0.3. After, we solve a single time CO_2 maximization problem with generated values for given budgets given in Equation 4.3. Table 4.2 shows the comparison of the optimization results of savings for CV=0.1, 0.2, 0.3 and mean CO_2 saving amounts.

Budget	CO_2 saving/year where $e_i = \bar{e}_i$	CO_2 saving/year where $e_i \sim N(\bar{e}_i, 0.1\bar{e}_i)$	CO_2 saving/year where $e_i \sim N(\bar{e}_i, 0.2\bar{e}_i)$	CO_2 saving/year where $e_i \sim N(\bar{e}_i, 0.3\bar{e}_i)$
10.000	85.378	85.454	86.019	87.767
20.000	127.815	127.828	128.214	129.709
30.000	155.104	155.126	155.462	156.879
40.000	170.844	171.564	173.623	176.564
50.000	190.025	190.476	192.042	194.883
60.000	205.765	206.223	208.113	212.216
70.000	219.727	220.071	223.390	229.077
80.000	230.722	232.966	238.465	245.229
90.000	247.310	249.249	254.393	260.967
100.000	263.050	264.226	268.759	275.088
110.000	277.457	277.946	281.910	288.142
120.000	288.452	289.573	293.849	300.634
130.000	300.988	302.713	307.021	313.586
140.000	315.266	316.484	319.881	325.407
150.000	325.345	327.243	330.543	335.385
160.000	329.809	332.969	337.354	343.782
170.000	340.023	342.226	346.803	353.340
180.000	344.978	348.743	354.278	361.311
190.000	351.159	355.180	361.415	369.220
200.000	358.717	362.650	369.455	377.632
210.000	365.935	370.387	377.128	385.071
220.000	370.881	374.986	382.537	391.363
230.000	376.949	381.740	389.729	398.981
240.000	385.559	389.879	397.559	406.325
250.000	390.785	394.560	402.427	411.544
260.000	394.703	399.269	407.787	417.389
270.000	400.094	405.925	414.688	424.040
280.000	408.055	412.915	420.768	429.534
290.000	412.519	416.597	424.798	433.830
300.000	416.437	420.780	428.261	436.607

Table 4.2: Value of Information for CO_2 Saving Amounts with respect to Different Budgets and CV

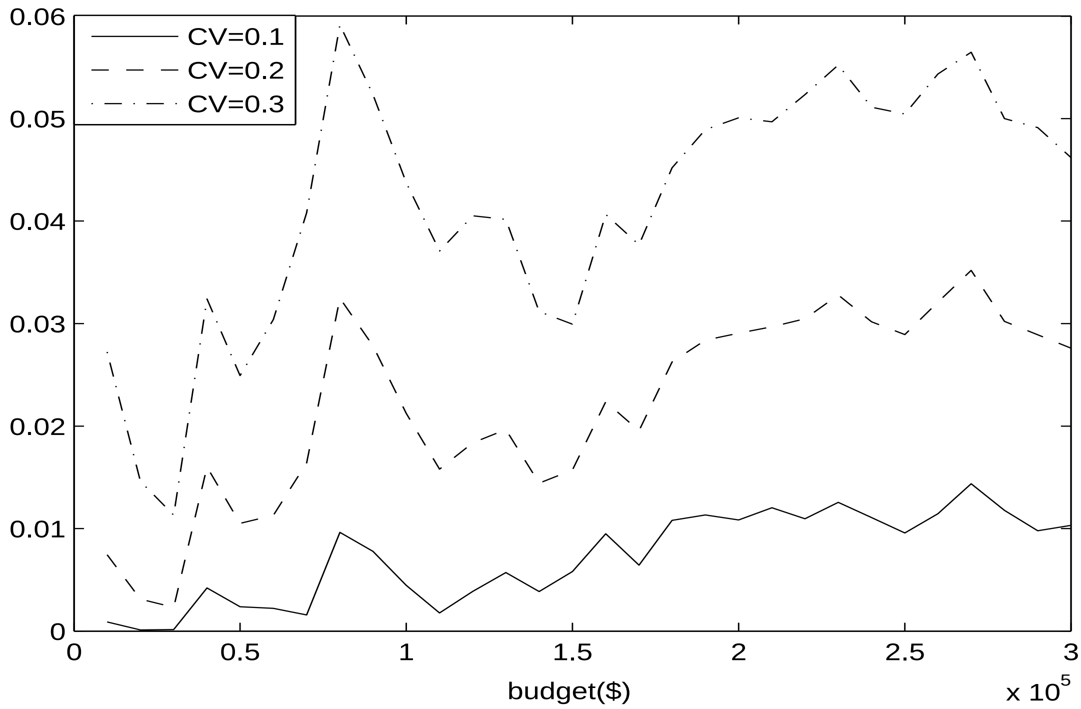

Figure 4.5: % error of CO_2 saving amounts for CV= 0.1,0.2 and 0.3

Figure 4.5 shows that as coefficient of variation gets higher the deviation from mean becomes higher.

4.3 Technology Selection Problem in Multiple Periods

In the previous section, we explained how the CO_2 amount can be maximized in single decision time with a given budget. We assumed that money savings can not be used to invest another technology.

On the other hand, in this section we analyze multiperiod decisions to invest in ecologically friendly technologies. At the beginning of each year we invest in technologies and at the end of that year we obtain CO_2 and \$ saving from those technologies. Hence we can use those savings to invest again at the beginning of the next year. We also assumed that at the beginning of each year excess money will be deposited in a bank account to gain interest for one year, which again can be used for determining next year's optimal decisions.

In other words, optimization model tries to maximize CO_2 saving for a certain time and given initial budget. The model allows using yearly \$ saving amounts as budget for the next year's decision. Also, for better results, the model allows depositing money into bank to gain interest to invest in more expensive technology which has higher CO_2 saving. For instance, model solves CO_2 saving maximization problem for T years with initial budget B. The solution that model gives is the CO_2 saving amount after T years and is reached by the investment decisions made in the beginning of each year. Model may suggest to invest technology i,j and k for the first year and deposit the remaining money into the bank. Then it uses the money savings form technology i,j and k and interest to decide second year's investments.

We used CPLEX Class API MATLAB interface to optimize this multiperiod technology selection problem.

4.3.1 Modeling

$I_{i,t}$:	Investment required for project i at the beginning of year t	dollar
$E_{i,t}$:	Carbondioxide saving received from project i at the begining of year t	kg
$S_{i,t}$:	Financial saving received from project i at the ending of year t	dollar
C_t	Cash available at the begining of year t	dollar
r:	Annual interest rate	
B:	Initial Budget	dollar

$$Y_{i,t} = \begin{cases} 1, & \text{If you invest in project i at the beginning of year t,} \\ 0, & \text{otherwise,} \end{cases} \tag{4.5}$$

$$
\begin{aligned}
\text{maximize} \quad & \sum_i \sum_t E_{i,t} Z_{i,t} \\
\text{subject to} \quad & \sum_i Y_{i,t} I_{i,t} \leq C_t, \ \forall t, \\
& C_t = (C_{t-1} - \sum_i Y_{i,t-1} I_{i,t-1})(1+r) + \sum_i S_{i,t-1} Z_{i,t-1}, \ t = 1, \ldots, T, \\
& C_1 = B, \\
& \sum_t Y_{i,} \leq 1, \ \forall i, \\
& Z_{i,t} = \sum_{t'}^{t} Y_{i,t'}, \ \forall i, \\
& C_t \geq 0, \ \forall t, \\
& Y_{i,t} \in \{0,1\}, \\
& Z_{i,t} \in \{0,1\}.
\end{aligned}
\tag{4.6}
$$

4.3.2 CO_2 Saved Amounts Comparison Between Single Time and Multi-Period Optimization Problems

In this section we aim to compare saved CO_2 amounts of the multiperiod optimization(Equation 4.6) values with single time values. To compare the results, for the single time values, we take the single time yearly saved values and multiply them with the year compared with multiperiod optimization.Figure 4.6, 4.7, 4.8 and 4.9 show how CO_2 saved amounts are changed in multiple time optimization and single time optimization used in multiple time for the budgets 10,000, 50,000, 100,000 and 250,000 respectively.

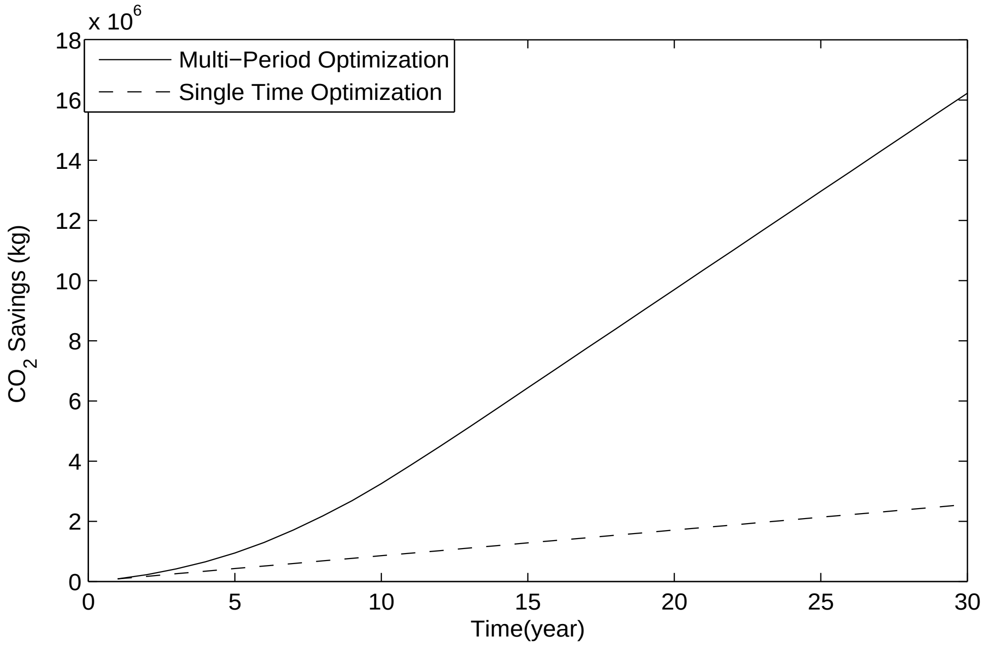

Figure 4.6: Difference of CO_2 saving amount in single and multiple time optimization problems when initial budget =10,000$

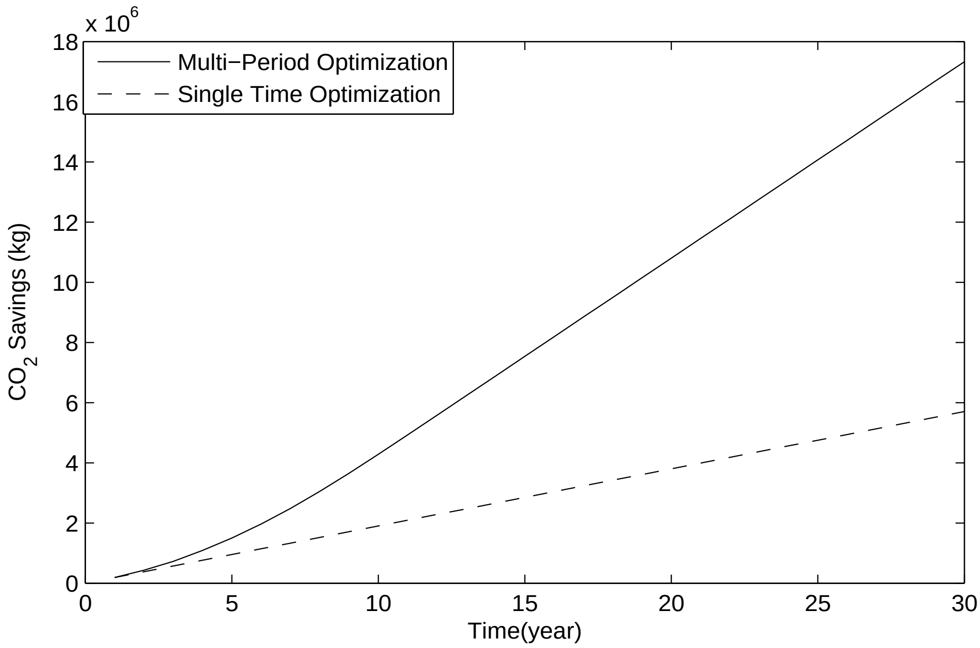

Figure 4.7: Difference of CO_2 saving amount in single and multiple time optimization problems when initial budget=50,000$

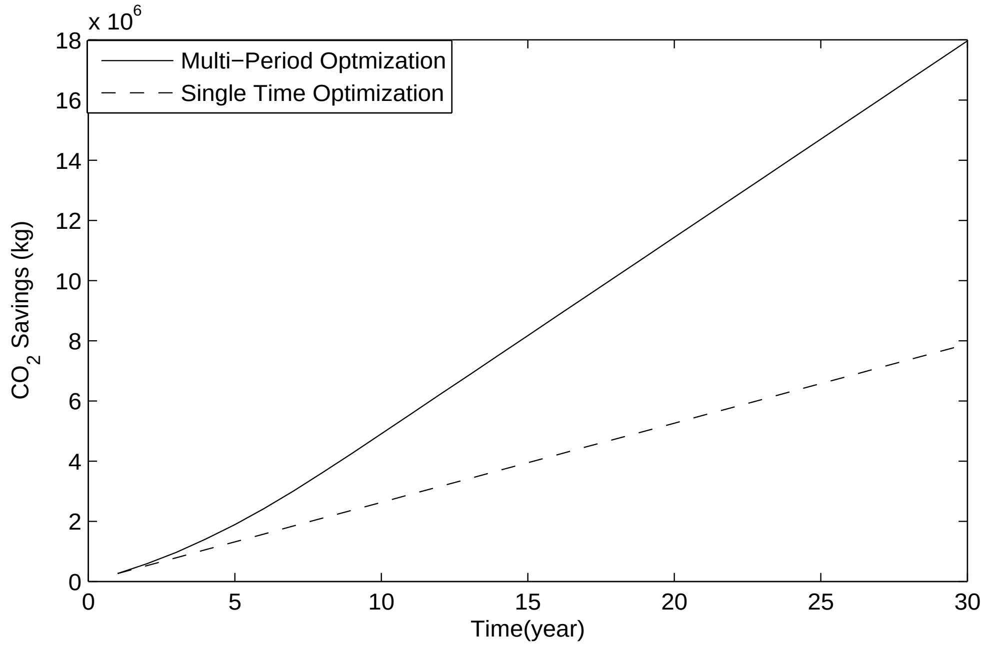

Figure 4.8: Difference of CO_2 saving amount in single and multiple time optimization problems when initial budget=100,000$

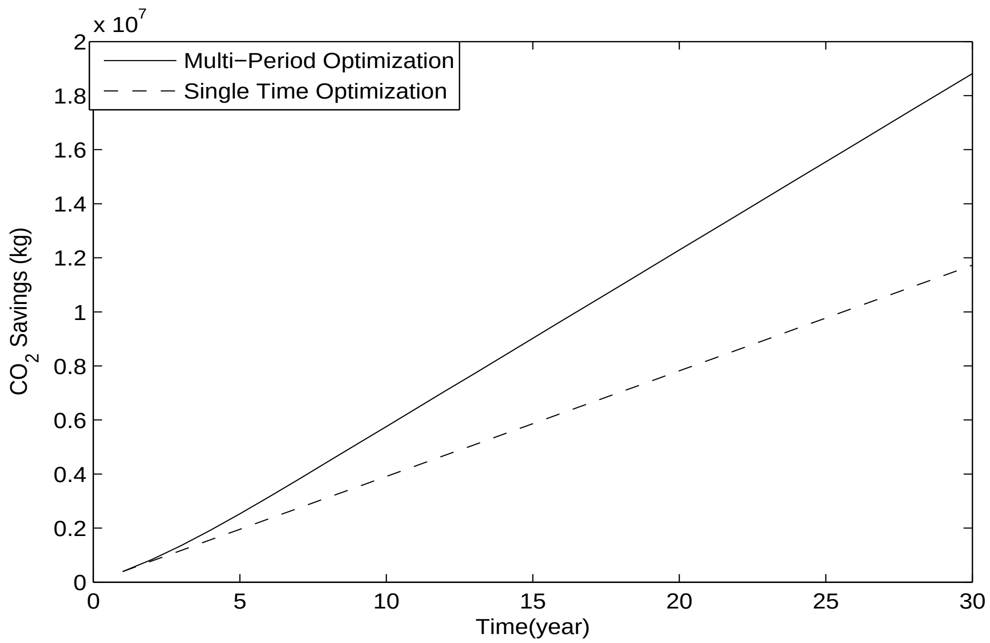

Figure 4.9: Difference of CO_2 saving amount in single and multiple time optimization problems when initial budget=250,000$

It is easily observed from the figures that as the initial budget increases, the gap between time optimized value and single time optimized value decreases since higher budgets yields more technologies to be invested in.

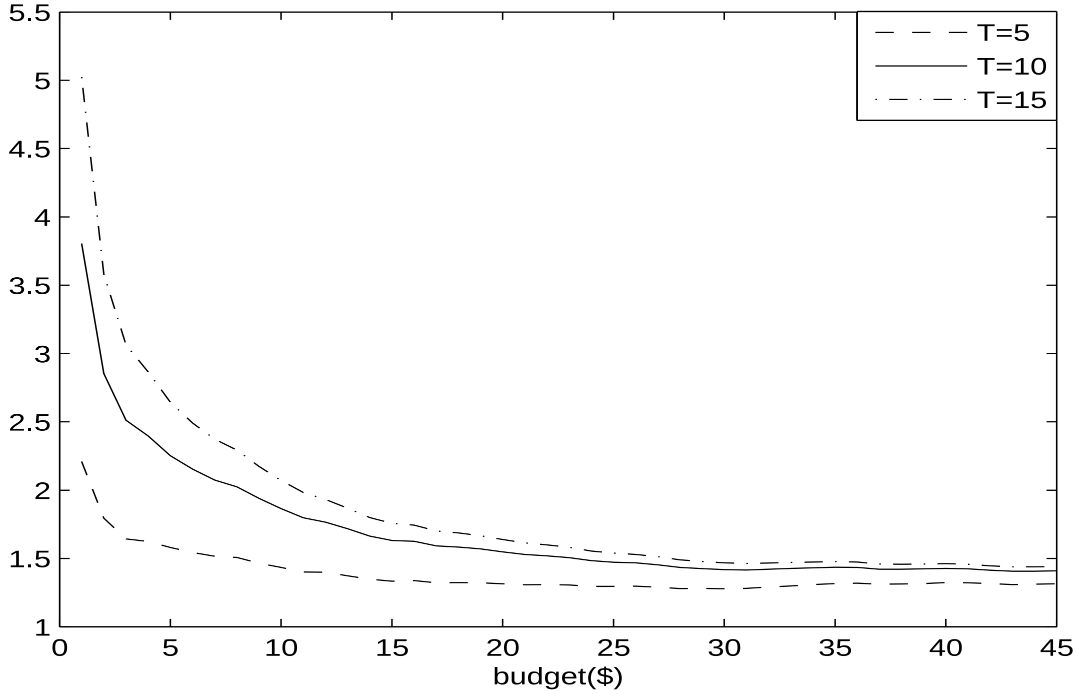

Figure 4.10: Multiplier of CO_2 amounts which is calculated by time optimization value/single time optimization in t=5,t=10 and t=15

In Figure 4.10 we analyzed how multi-period optimization is better than single period optimization with respect to CO_2 saving for different budgets. Hence we divided amounts which are calculated by multi time optimization to the single time optimization values to find the multiplier. As seen in the figure, in smaller budgets, multi time optimization is much better than single time optimization values independent of time. When time increases the multiplier increases especially in small budgets. In addition, Figure 4.10 shows that the ratio remain constant at 1.5 instead of 1 for greater budgets. The reason of this result is, envelope insulation retrofit of the buildings. For multiperiod optimization, model allows investing in envelope insulation retrofit options (4cm,5cm and 6 cm) in different decision times. In other words, for greater budgets, in multiperiod optimization model invests more technology when compared to single time optimization. In single time optimization, we can only invest one envelope insulation option.

4.4 ESCO-Business Plan

ESCOs guarantee the energy savings and the same level of energy service at a lower cost by implementing an energy efficiency technology. A performance guarantee can revolve around the actual flow of energy savings from a technology, can stipulate that the same level of energy service will be provided for less money. ESCOs typically finance the installation of an energy efficient technology they implement by providing a savings guarantee. Also, ESCOs retain an on-going operational role in measuring and verifying the savings over the financing term.

NPV is used as constraints and objective function to determine several models.We present 4 different sections. In the first part given in section 4.1.1 we compared NPV in CO_2 saving maximization which is modeled in the previous section. A comparison is made between the NPVs in single time and multi-time CO_2 saving maximizations. In section 4.2.2 a new MIP is modeled by adding NPV in the constraint as satisfying a certain level of NPV is guaranteed for ESCO. In the section 4.3.3 NPV is maximized and the results are compared in CO_2 saving maximization and NPV maximization. Finally, in section 4.4.4 discount rate is added in the constraint for both CO_2 saving maximization and NPV maximization.

4.4.1 NPV Changes in CO_2 Maximization for Single and Multiple Time

In this part NPV is compared in CO_2 saving maximization for both single and multiple time periods. Figure 4.11, 4.12, 4.13 and 4.14 show how NPV changes in CO_2 saving maximization for single and multiple time optimization for budgets 10,000, 50,000, 100,000, 250,000, respectively.

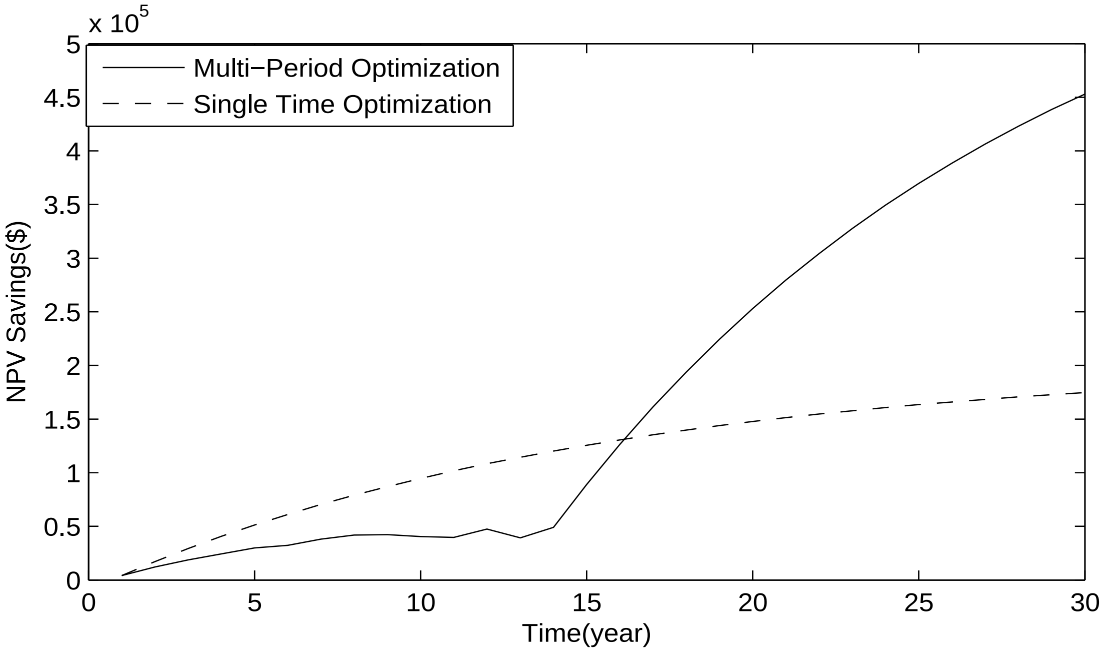

Figure 4.11: Difference of NPV saving amount in single and multiple time optimization problems when initial budget =10,000$

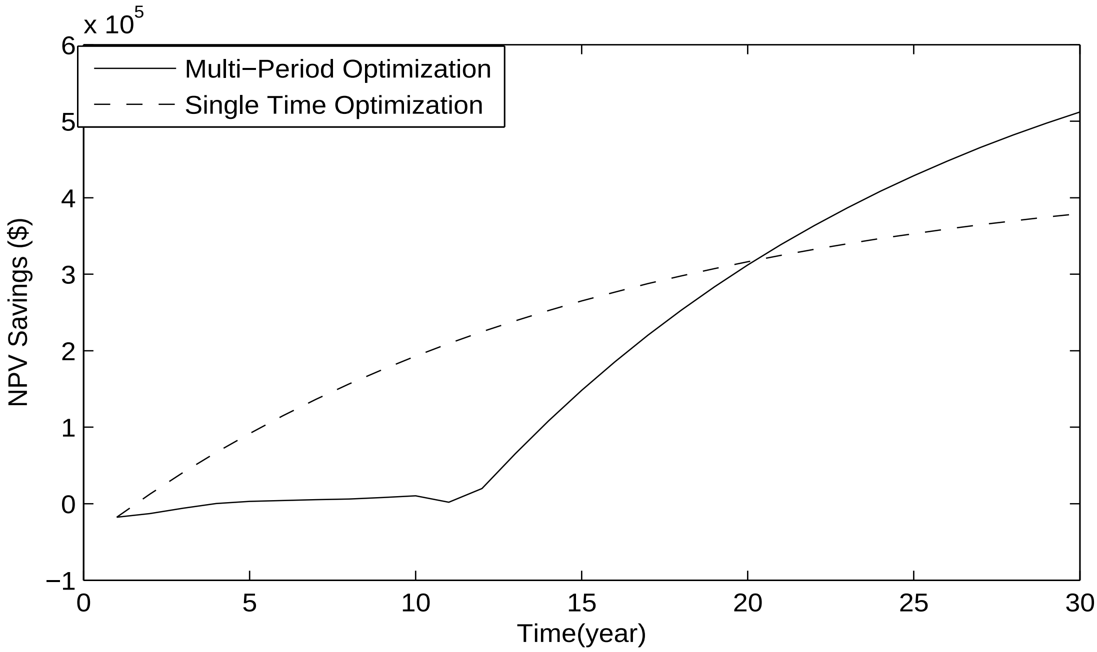

Figure 4.12: Difference of NPV saving amount in single and multiple time optimization problems when initial budget=50,000$

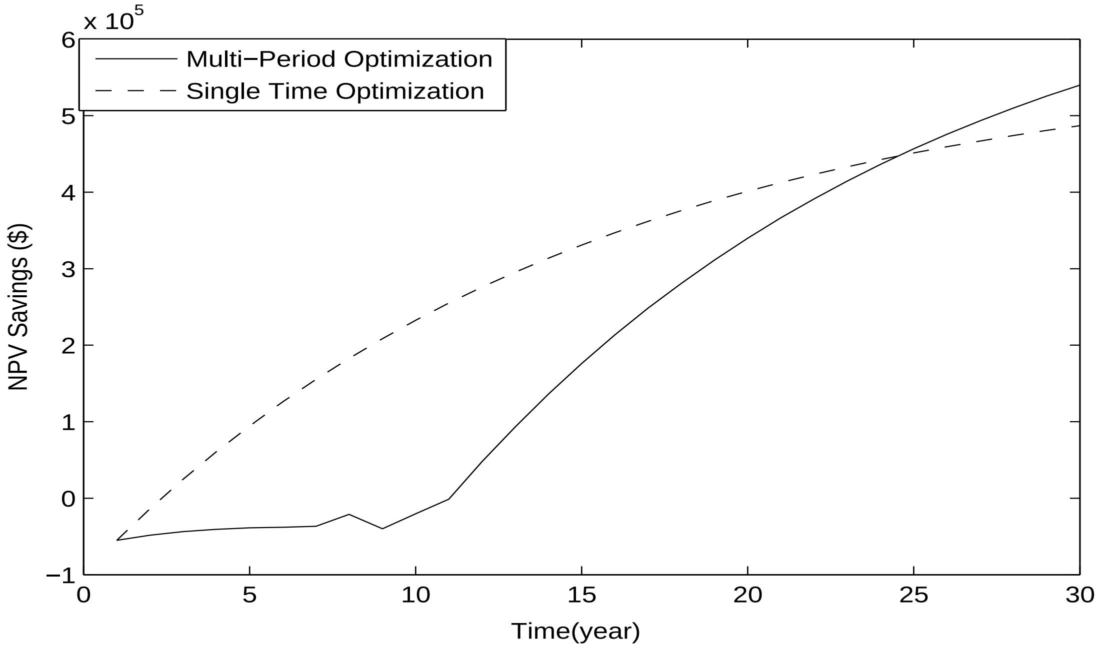

Figure 4.13: Difference of NPV saving amount in single and multiple time optimization problems when initial budget=100,000$

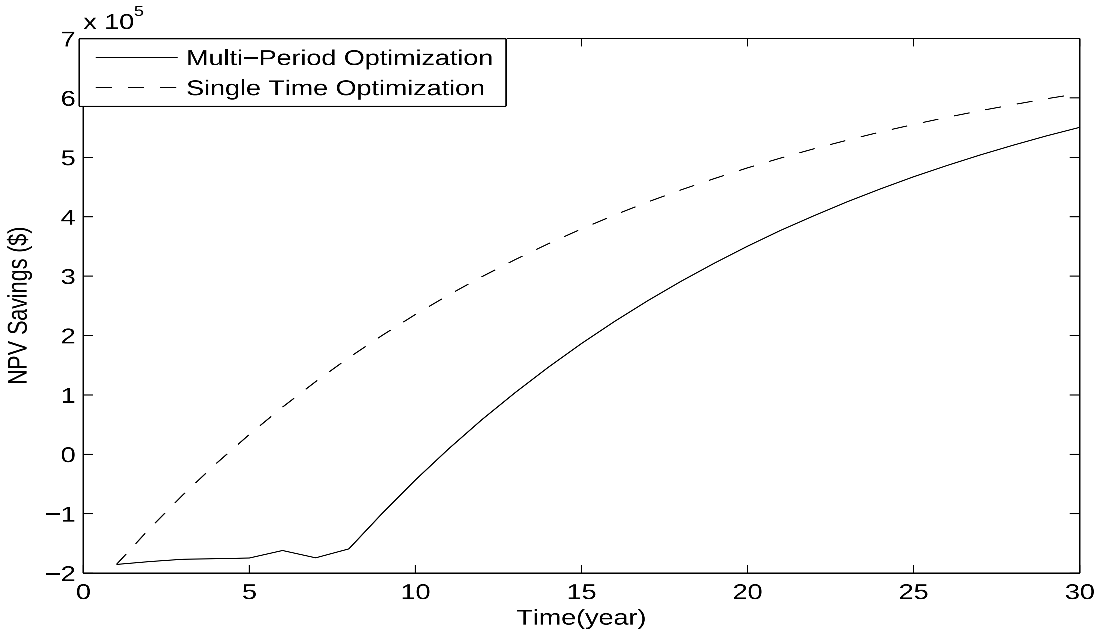

Figure 4.14: Difference of NPV saving amount in single and multiple time optimization problems when initial budget=250,000$

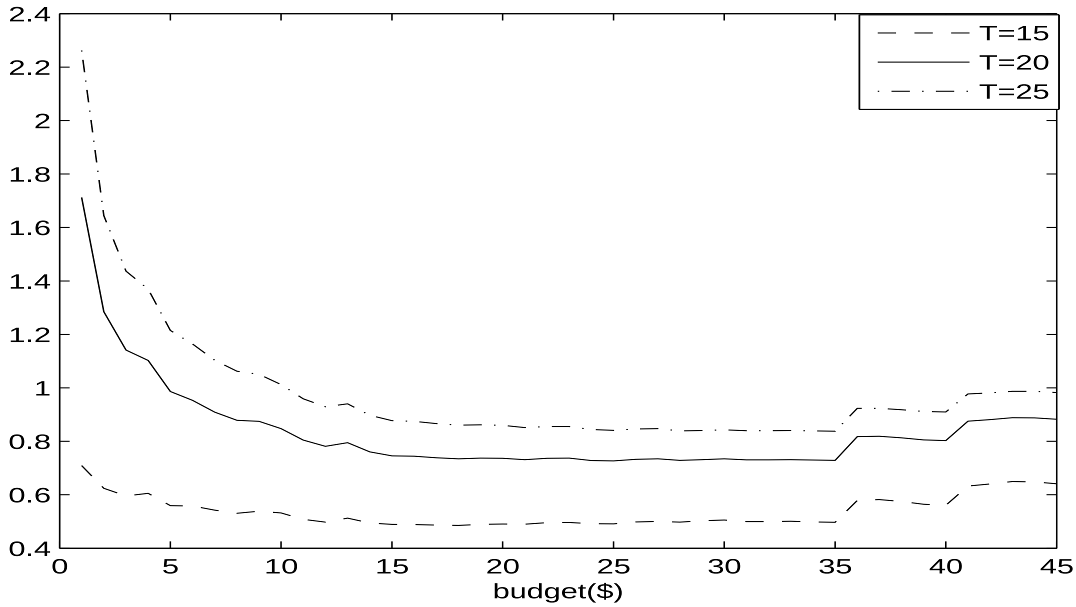

Figure 4.15: Multiplier of NPV amounts which is calculated by time optimization value/single time optimization in t=15,t=20 and t=25

After analyzing figures 4.11, 4.12, 4.13 and 4.14 we can say that in time optimization methodology, the system pushes as many investments as it can. Hence, for smaller times NPV is negative. After applying more technologies than single time optimization has, the growth rate of NPV in time optimization is larger than single time optimization has.

Figure 4.15 presents the change of rate between time optimization and single time optimization according to budgets. In this analysis CO_2 is maximized and NPV is analyzed according to CO_2 maximization. We can see that for smaller budgets time optimization is also better than single time optimization in terms of NPV.

4.4.2 Profit Rate Constraint and Results

In this section we aimed to add a new constraint to the multi period model. Financial savings are also important for the investor. This constraint provides a certain value of NPV is guaranteed. We define a profit rate and we want that for specific time and initial budget, NPV must be bigger than the multiplication of profit rate and initial budget. So our model becomes:

$$
\begin{aligned}
\text{maximize} \quad & \sum_i \sum_t E_{i,\,t} Z_{i,\,t} \\
\text{subject to} \quad & \sum_i Y_{i,\,t} I_{i,\,t} \leq C_t,\ \forall t, \\
& C_t = (C_{t-1} - \sum_i Y_{i,\,t-1} I_{i,\,t-1})(1+r) + \sum_i S_{i,\,t-1} Z_{i,\,t-1},\ t = 1, \ldots, T, \\
& C_1 = B, \\
& NPV \geq \rho B, \\
& \sum_t Y_{i,} \leq 1,\ \forall i, \\
& Z_{i,t} = \sum_{t'}^{t} Y_{i,\,t'},\ \forall i, \\
& C_t \geq 0,\ \forall t, \\
& Y_{i,\,t} \in \{0,1\}, \\
& Z_{i,\,t} \in \{0,1\}.
\end{aligned}
\tag{4.7}
$$

The mathematical constraint of NPV is:

$$
\sum_i \sum_t^{T-1} (1+r)^{-T+1} \frac{S_{i,t}}{(1+r)} Y_{i,t} + \sum_i (1+r)^{-T+1} \left(\frac{S_{i,T}}{(1+r)} - I_{i,T} \right) Y_{i,t} - C_1 + (1+r)^{-T+1} C_T \geq \rho B \tag{4.8}
$$

Figure 4.16, 4.17 and 4.18 show that when profit rate increases saved CO_2 decreases. Notice that in every figure there are different maximum profit rates. These maximum profit rates are calculated by maximizing NPV.

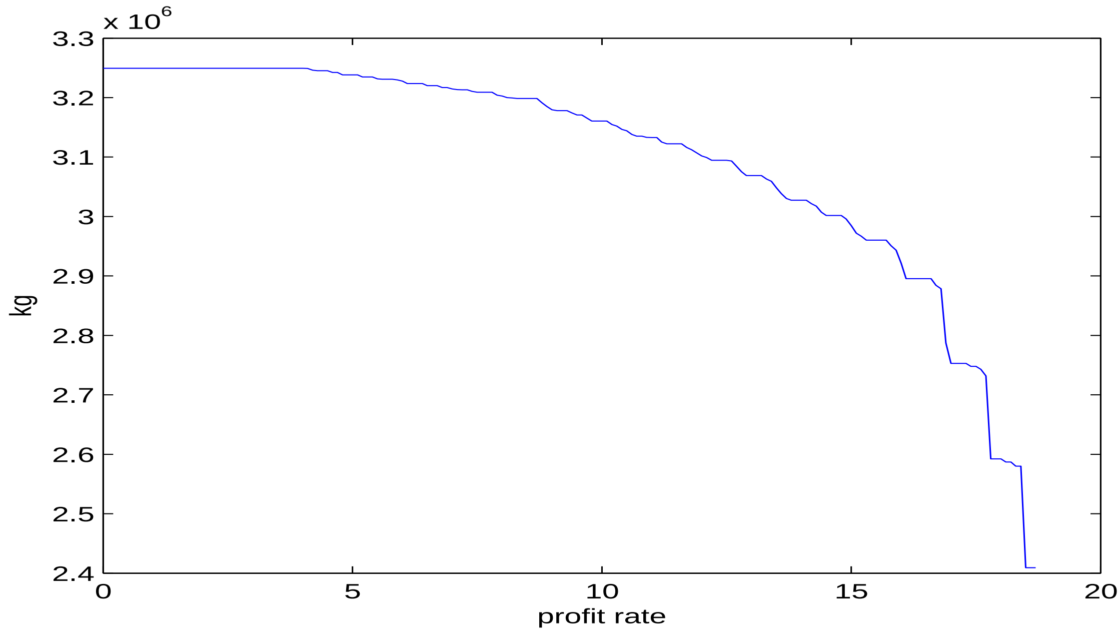

Figure 4.16: CO_2 saved amounts when adding profit rate constraint for different profit rates in budget 10.000 and t=10

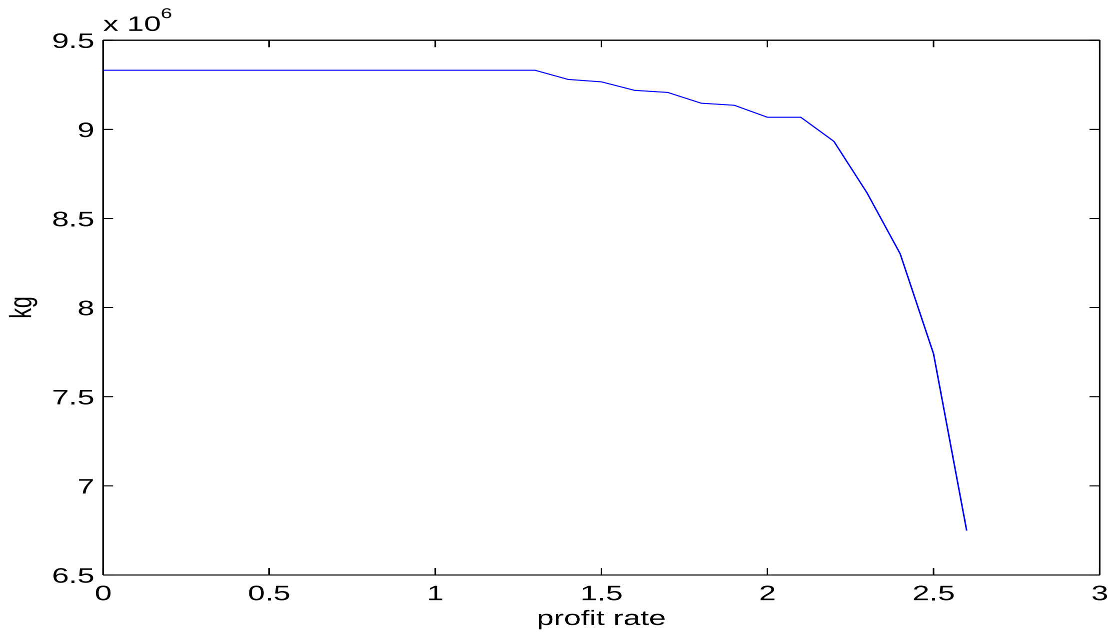

Figure 4.17: CO_2 saved amounts when adding profit rate constraint for different profit rates in budget 120.000 and t=17

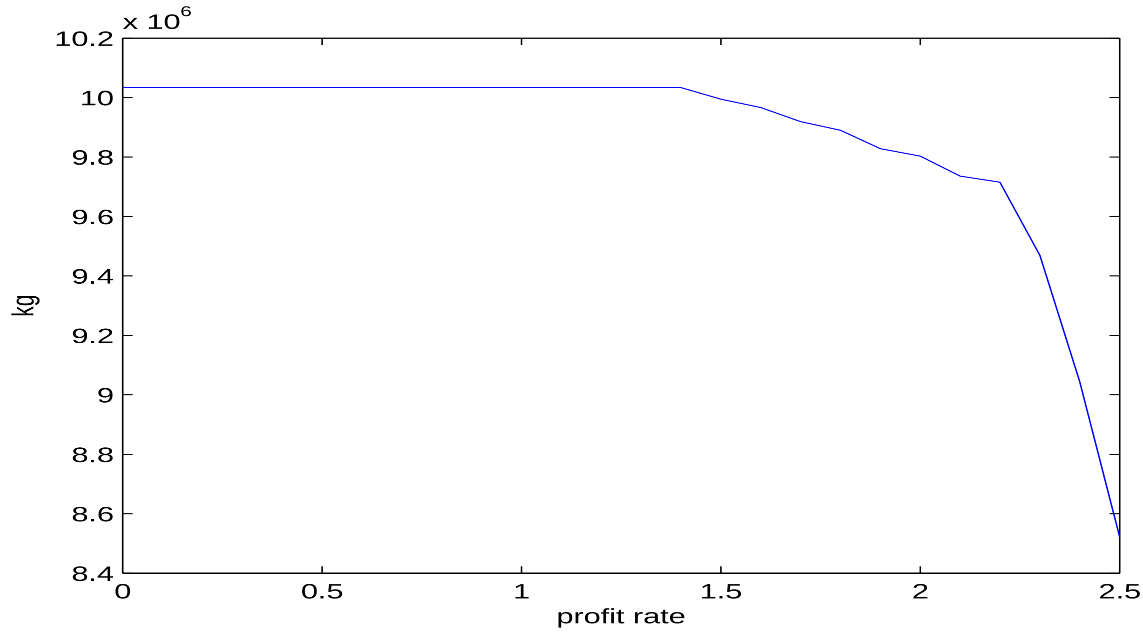

Figure 4.18: CO_2 saved amounts when adding profit rate constraint for different profit rates in budget 180.000 and t=17

4.4.3 NPV Maximization

In this section, we analyze the NPV of the investments in single time optimization and multiple time optimization. We assume that in single time optimization we just invest in the beginning of year 1 and we have money savings at the end of each year. However in the multiple time optimization, we started with the initial investment and at the beginning of each year we invest again (in a bank or technology). In this way, we calculated the NPV as if we invest in the first time and get all the savings at the end of the period that NPV is going to be calculated since all intermediaries are zero.

We thought that with this point of view ESCOs are going to maximize their NPV to make profit. We maximize the NPV and compare it with the NPV when CO_2 saving is maximized.

$$Y_{i,t} = \begin{cases} 1, & \text{If you invest in project i at the beginning of year t,} \\ 0, & \text{otherwise,} \end{cases} \tag{4.9}$$

$$\begin{aligned}
\text{maximize} \quad & NPV \\
\text{subject to} \quad & \sum_i Y_{i,\,t} I_{i,\,t} \leq C_t,\ \forall t, \\
& C_t = (C_{t-1} - \sum_i Y_{i,\,t-1} I_{i,\,t-1})(1+r) + \sum_i S_{i,\,t-1} Z_{i,\,t-1},\ t = 1, \ldots, T, \\
& C_1 = B, \\
& \sum_t Y_{i,} \leq 1,\ \forall i, \\
& Z_{i,t} = \sum_{t'}^{t} Y_{i,\,t'},\ \forall i, \\
& C_t \geq 0,\ \forall t, \\
& Y_{i,\,t} \in \{0,1\}, \\
& Z_{i,\,t} \in \{0,1\}.
\end{aligned} \tag{4.10}$$

where $NPV = \sum_i \sum_t^{T-1} (1+r)^{-T+1} \frac{S_{i,t}}{(1+r)} Y_{i,t} + \sum_i (1+r)^{-T+1} (\frac{S_{i,T}}{(1+r)} - I_{i,T}) Y_{i,t} - C_1 + (1+r)^{-T+1} C_T$
Figures 4.19, 4.20, 4.21 and 4.22 show the comparison of the NPVs when CO_2 is maximized and NPV is maximized for the budgets, 10,000, 50,000, 100,000, and 250,000 respectively. Even for the budget 250000 NPV maximization supplies more saving since model tries to invest in all technologies for CO_2 maximization in the earlier times. Increase in the budget provides that all technologies are invested earlier for CO_2 maximization.

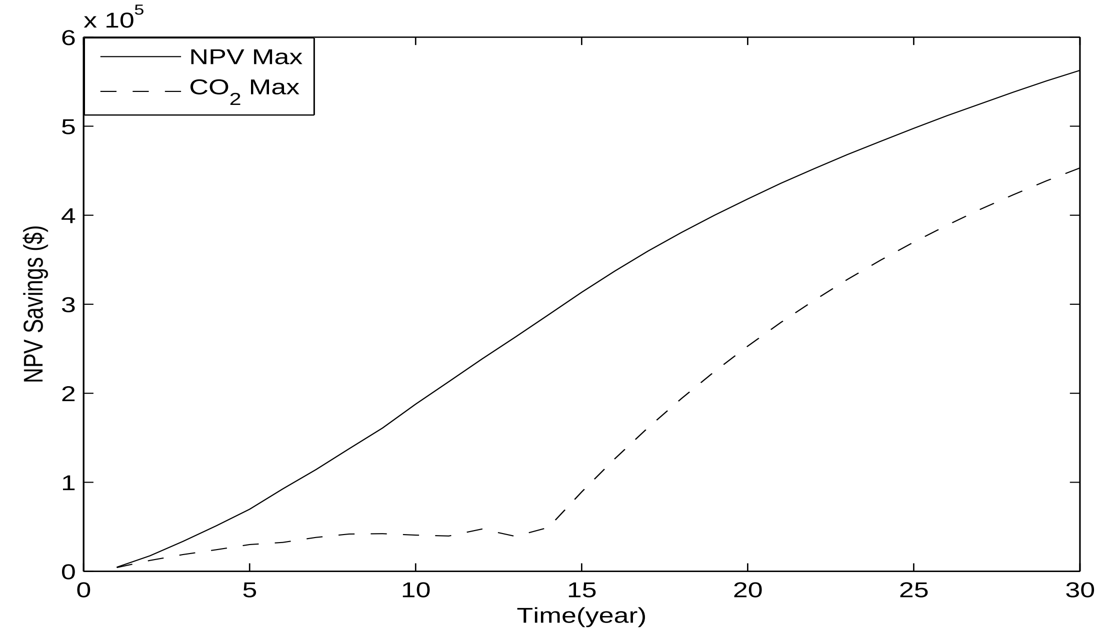

Figure 4.19: Comparison of NPV when NPV is maximized and CO_2 is maximized in budget 10,000$

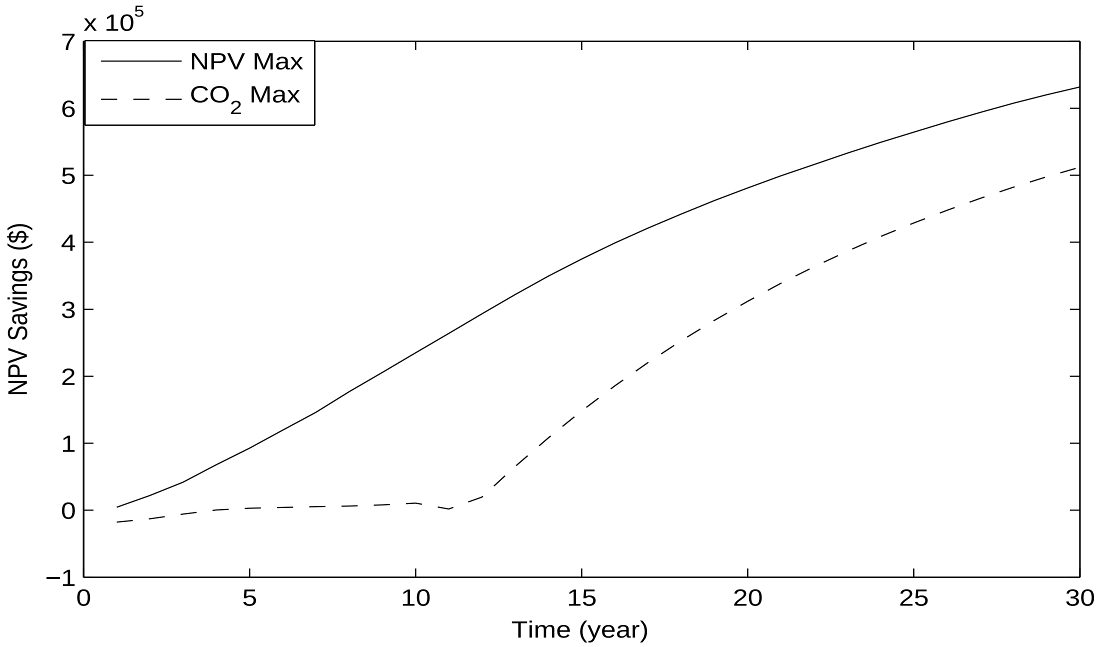

Figure 4.20: Comparison of NPV when NPV is maximized and CO_2 is maximized in budget 50,000$

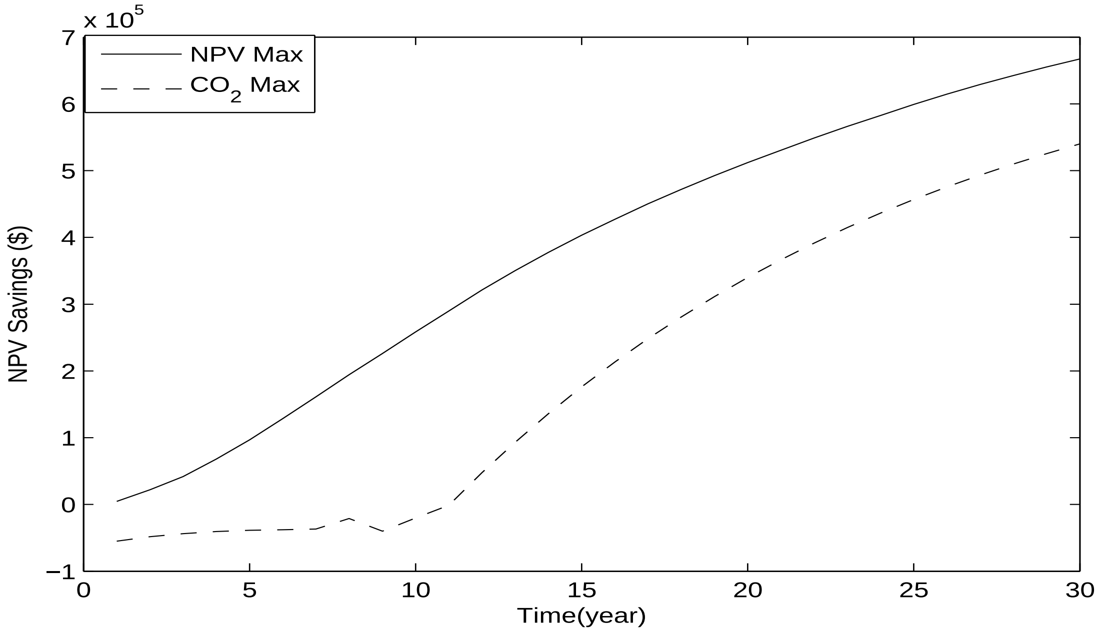

Figure 4.21: Comparison of NPV when NPV is maximized and CO_2 is maximized in budget 100,000$

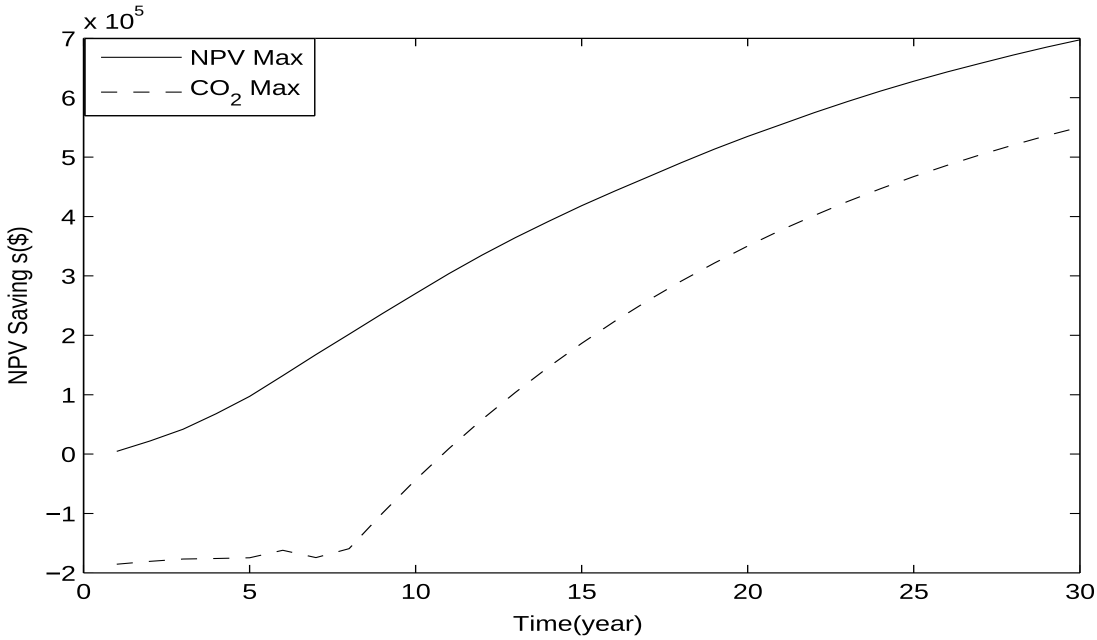

Figure 4.22: Comparison of NPV when NPV is maximized and CO_2 is maximized in budget 250,000$

4.4.4 Discount Rate

In the model which includes discount, we aim to find maximized saved CO_2 and NPV under the condition of lower financial savings. The company gives a discount to the customers and so it has less money to spend on other investments which affect CO_2 and NPV of the model.

$$Y_{i,t} = \begin{cases} 1, & \text{If you invest in project i at the beginning of year t,} \\ 0, & \text{otherwise,} \end{cases}$$

CO_2 Saving Maximization

$$\begin{aligned}
\text{maximize} \quad & \sum_i \sum_t E_{i,\,t} Z_{i,\,t} \\
\text{subject to} \quad & \sum_i Y_{i,\,t} I_{i,\,t} \le C_t,\ \forall t, \\
& C_t = (C_{t-1} - \sum_i Y_{i,\,t-1} I_{i,\,t-1})(1+r) + \sum_i (1-\Delta) S_{i,\,t-1} Z_{i,\,t-1},\ t = 1, \ldots, T, \\
& C_1 = B, \\
& \sum_t Y_{i,} \le 1,\ \forall i, \\
& Z_{i,t} = \sum_{t'}^{t} Y_{i,\,t'},\ \forall i, \\
& C_t \ge 0,\ \forall t, \\
& Y_{i,\,t} \in \{0,1\}, \\
& Z_{i,\,t} \in \{0,1\}.
\end{aligned} \tag{4.11}$$

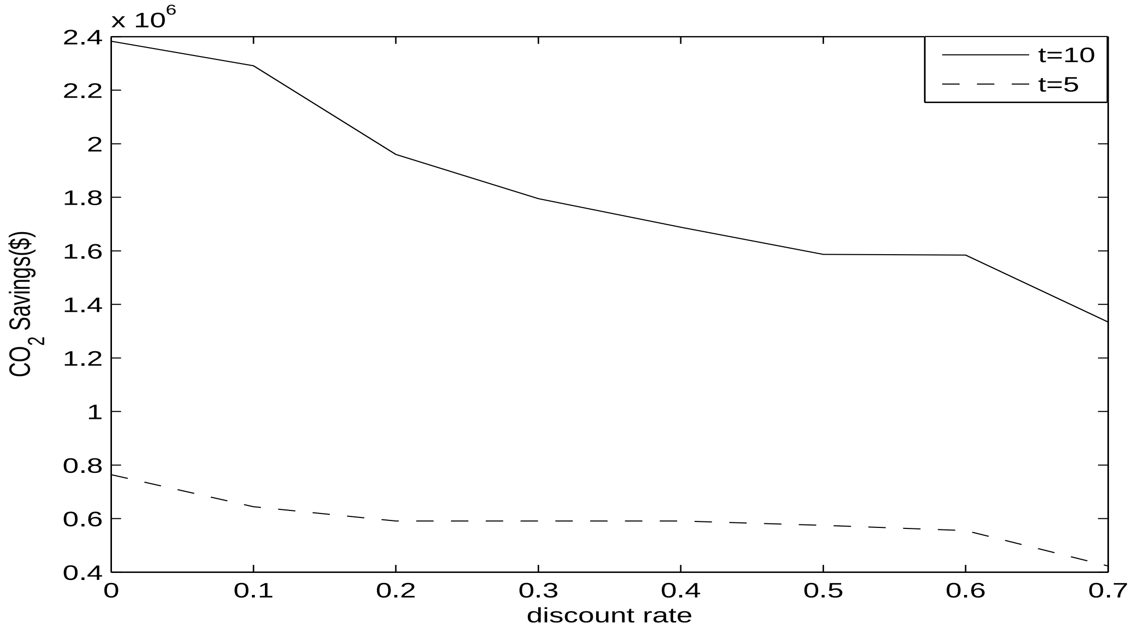

Figure 4.23: Saved CO_2 amounts with respect to different discount rates in budget 10.000 for t=5 and t=10

Figure 4.23 shows the saved CO_2 amounts for times 5 and 10 when the discount rate that the company gives to the customer increases. As discount rate increases, number of technologies that are invested in decreases as well as the saved CO_2 amount.

NPV Maximization

$$\begin{aligned}
\text{maximize} \quad & NPV_2 \\
\text{subject to} \quad & \sum_i Y_{i,\,t} I_{i,\,t} \le C_t,\ \forall t, \\
& C_t = (C_{t-1} - \sum_i Y_{i,\,t-1} I_{i,\,t-1})(1+r) + \sum_i (1-\Delta) S_{i,\,t-1} Z_{i,\,t-1},\ t = 1,\dots,T, \\
& C_1 = B, \\
& \sum_t Y_{i,} \le 1,\ \forall i, \\
& Z_{i,t} = \sum_{t'}^{t} Y_{i,\,t'},\ \forall i, \\
& C_t \ge 0,\ \forall t, \\
& Y_{i,\,t} \in \{0,1\}, \\
& Z_{i,\,t} \in \{0,1\}.
\end{aligned} \tag{4.12}$$

where $NPV_2 = \sum_i \sum_t^{T-1} (1+r)^{-T+1} \frac{(1-\Delta)S_{i,t}}{(1+r)} Y_{i,t} + \sum_i (1+r)^{-T+1} (\frac{(1-\Delta)S_{i,T}}{(1+r)} - I_{i,T}) Y_{i,t} - C_1 + (1+r)^{-T+1} C_T$

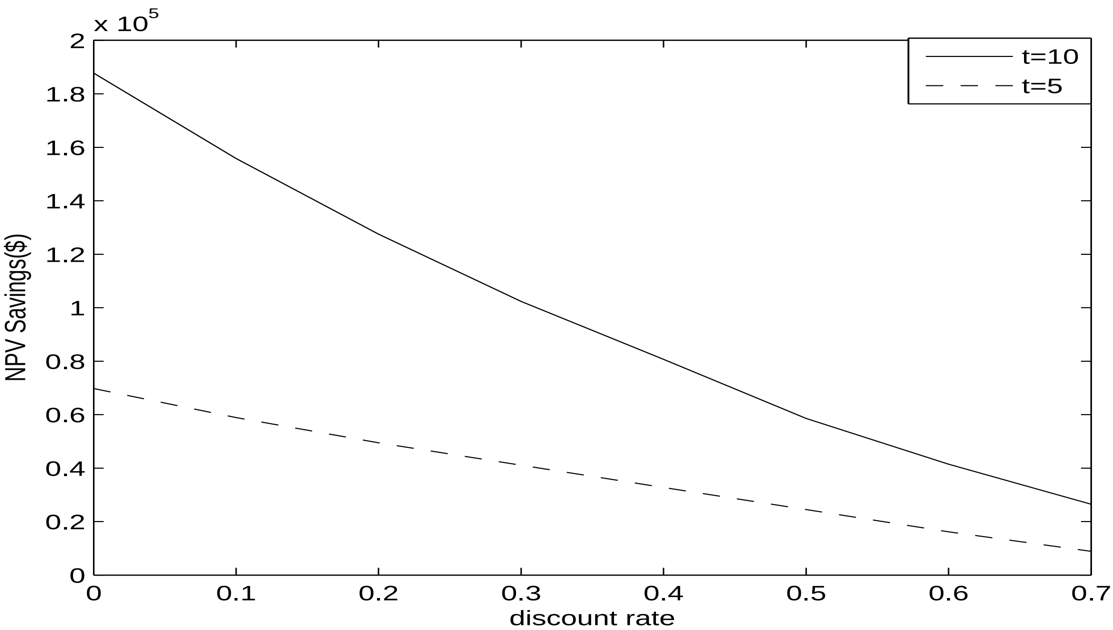

Figure 4.24: Saved NPV amounts with respect to different discount rates in budget 10.000 for t=5 and t=10

Figure 4.24 shows the saved NPV amounts for times 5 and 10 when the discount rate that the company gives to the customer increases. As discount rate increases, number of technologies that are invested decreases as well as the saved NPV amount.

4.5 Linearly Increasing Price Condition

In this section, we consider that the price is linearly increasing instead of constant all over the period. That means we use the Boğaziçi University data as the initial saving and forecasted the unit price of the following years with a function $f(t) = at + b$ for electricity and natural gas separately.

CO_2 Saving Maximization

$$
\begin{aligned}
\text{maximize} \quad & \sum_i \sum_t E_{i,t} Z_{i,t} \\
\text{subject to} \quad & \sum_i Y_{i,t} I_{i,t} \le C_t, \ \forall t, \\
& C_t = (C_{t-1} - \sum_i Y_{i,t-1} I_{i,t-1})(1+r) + \sum_i F_{i,t-1} Z_{i,t-1}, \ t = 1, \ldots, T, \\
& C_1 = B, \\
& \sum_t Y_{i,} \le 1, \ \forall i, \\
& Z_{i,t} = \sum_{t'}^{t} Y_{i,t'}, \ \forall i, \\
& C_t \ge 0, \ \forall t, \\
& Y_{i,t} \in \{0,1\}, \\
& Z_{i,t} \in \{0,1\}.
\end{aligned}
\tag{4.13}
$$

where $F_{i,t}$ is the financial saving of technologies which is linearly increasing and calculated by the constant energy saving and linearly increasing unit price for electricity and natural gas technologies separately. Figures 4.25, 4.26 and 4.27 illustrates the comparison of saved CO_2 amounts when energy price is constant and linearly increasing for the budgets 10,000, 100,000 and 250,000 respectively. Figures indicate that saved CO_2 amount is almost the same when price is increasing or constant.

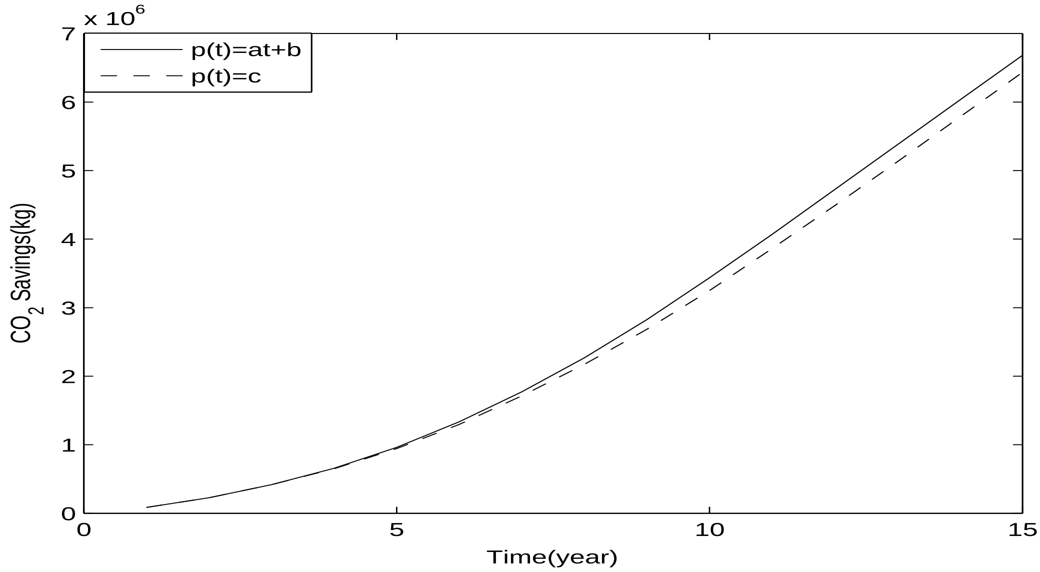

Figure 4.25: Comparison of CO_2 saving amount when price is linearly increasing and constant with budget=10,000

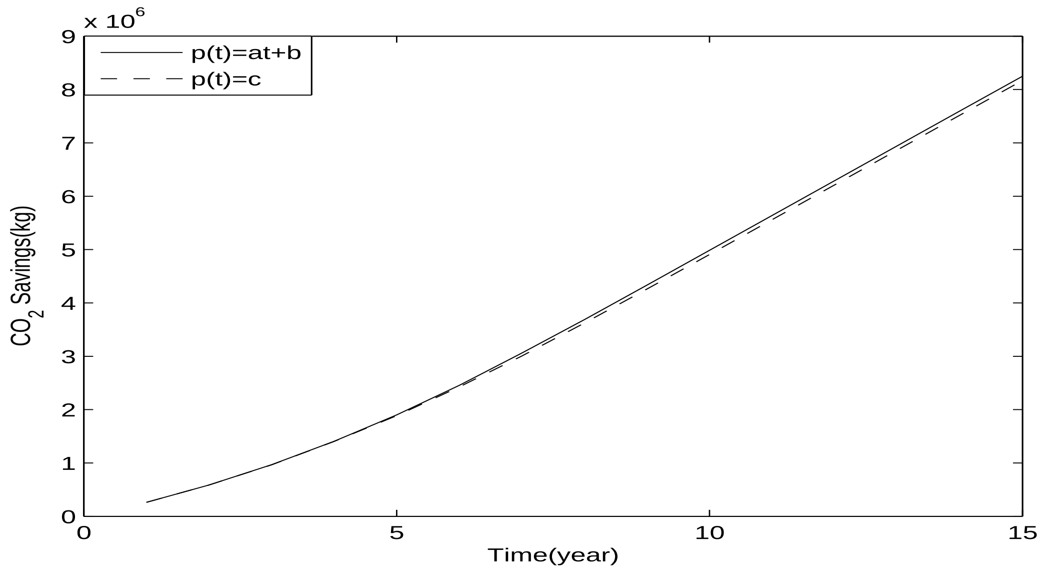

Figure 4.26: Comparison of CO_2 saving amount when price is linearly increasing and constant with budget=100,000

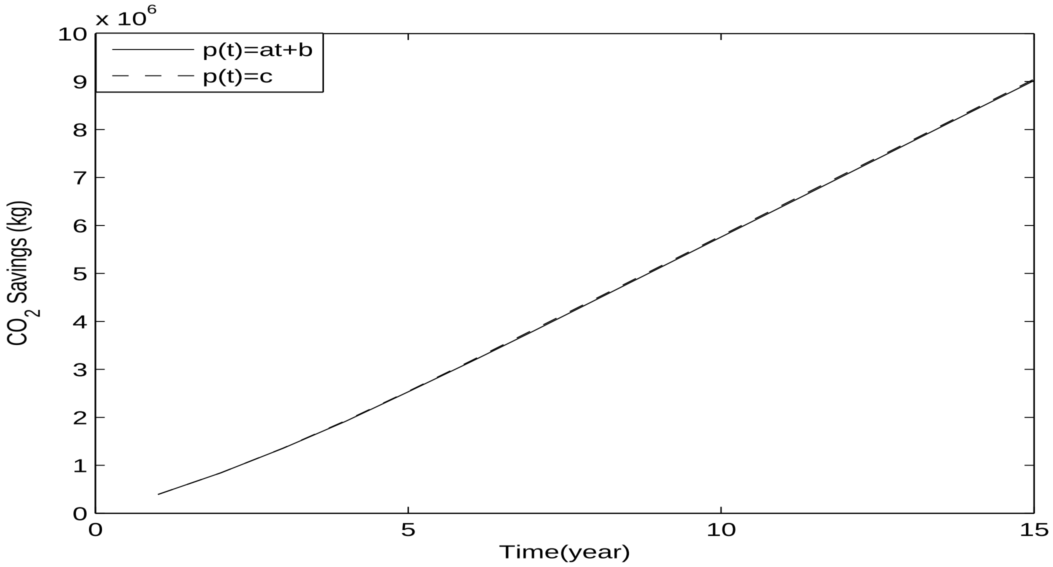

Figure 4.27: Comparison of CO_2 saving amount when price is linearly increasing and constant with budget=250,000

NPV Saving Maximization

$$
\begin{aligned}
\text{maximize} \quad & NPV_3 \\
\text{subject to} \quad & \sum_i Y_{i,t} I_{i,t} \leq C_t,\ \forall t, \\
& C_t = (C_{t-1} - \sum_i Y_{i,t-1} I_{i,t-1})(1+r) + \sum_i F_{i,t-1} Z_{i,t-1},\ t = 1, \ldots, T, \\
& C_1 = B, \\
& \sum_t Y_{i,} \leq 1,\ \forall i, \\
& Z_{i,t} = \sum_{t'}^{t} Y_{i,t'},\ \forall i, \\
& C_t \geq 0,\ \forall t, \\
& Y_{i,t} \in \{0,1\}, \\
& Z_{i,t} \in \{0,1\}.
\end{aligned}
\tag{4.14}
$$

where $NPV_3 = \sum_i \sum_t^{T-1} (1+r)^{-T+1} \frac{F_{i,t}}{(1+r)} Y_{i,t} + \sum_i (1+r)^{-T+1} (\frac{F_{i,T}}{(1+r)} - I_{i,T}) Y_{i,t} - C_1 + (1+r)^{-T+1} C_T$ and $F_{i,t}$ is the financial saving of technologies which is linearly increasing and calculated by the constant energy saving and linearly increasing unit price for electricity and natural gas technologies separately. Figures 4.28, 4.29 and 4.30 illustrate the comparison of saved NPV amounts when energy price is constant and linearly increasing for the budgets 10,000, 100,000 and 250,000 respectively. Figures indicate that saved NPV amount may differ as the unit energy price increases over time.

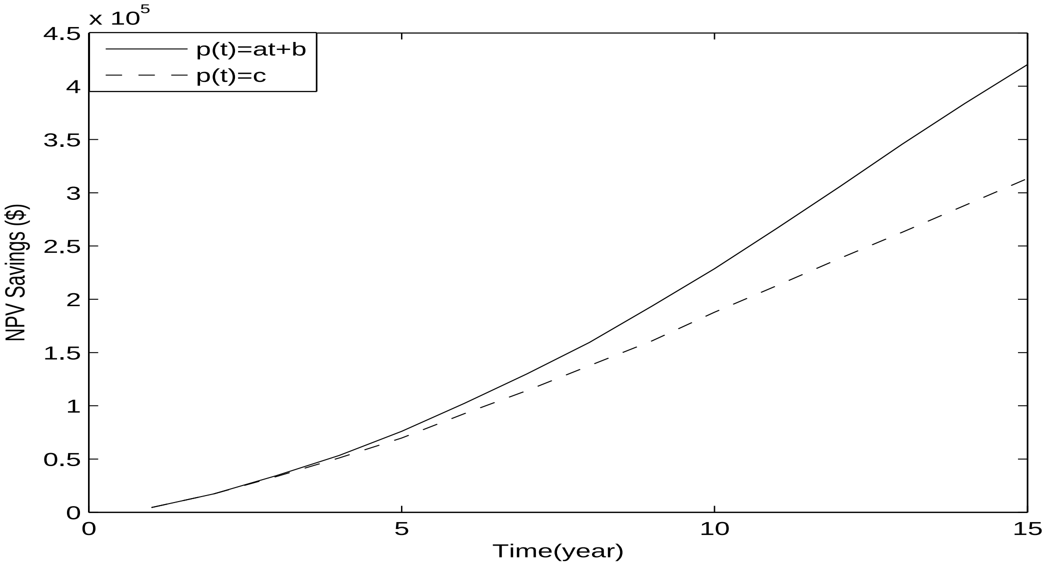

Figure 4.28: Comparison of NPV amount when price is linearly increasing and constant with budget=10,000

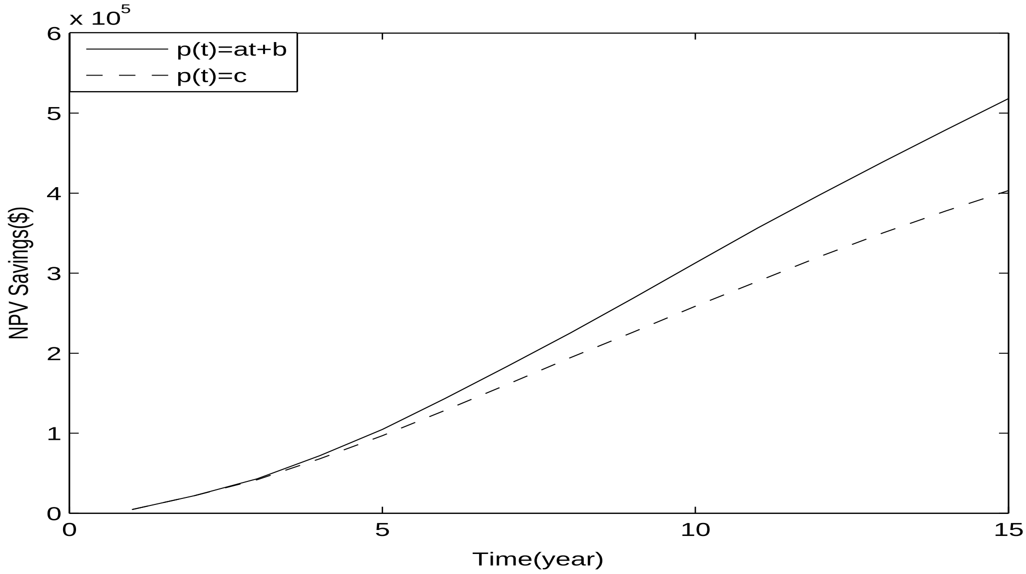

Figure 4.29: Comparison of NPV amount when price is linearly increasing and constant with budget=100,000

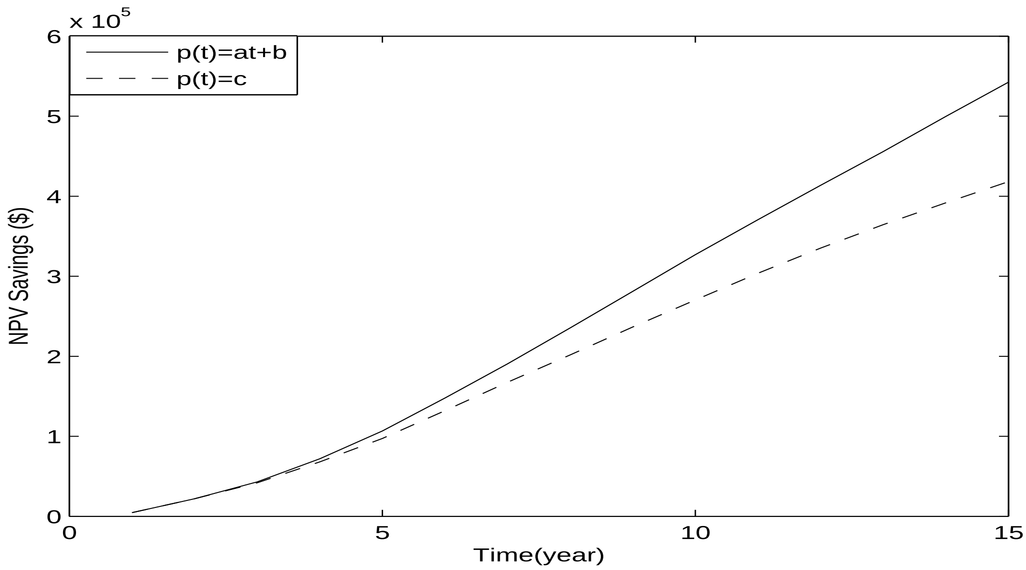

Figure 4.30: Comparison of NPV amount when price is linearly increasing and constant with budget=250,000

Chapter 5

CONCLUSIONS

Energy use in buildings is a key contributor to climate change: buildings are responsible for 33% of total global CO_2 emissions and 35%-40% of total global energy. One of the ways of reducing these is retrofitting the existing buildings by investing in energy efficient technologies.

The study includes two parts. First, a model for the single technology is constructed and an ESCO type business model is analyzed for the replacement of the technologies and energy consumption. In the single technology replacement, we modeled replacement and energy consumption costs separately and use the outcomes in the business plan. We also added efficiency improvements and life cycle costs of the technologies to the model. After solving the model analytically, we calculated the expected return and risk for the energy service company. We came to the conclusion that, even with single technology, an energy service company and clients can make profit from the efficient technologies with the right parameters.

The second part of the thesis includes a decision making problem to select optimal technologies from the available technologies which are installed to Boğaziçi University Kilyos Campus under budgetary conditions. A Mixed Integer Programming algorithm is used to achieve that goal in the multiple time period. In addition, sensitivity analysis is performed for the single time period case. Finally, we approached the problem as a business plan and added profit constraints to make the model feasible for energy service companies.

Two main conclusions are achieved through the two different parts in this thesis.

For the first part, the solutions can be used for a ESCO business plan. In the literature, there is no detailed model of ESCO type business plan. This model allows the entrepreneurs to calculate the financial returns and risks of the business. They can use the model to give decision whether to invest in the business or not. ESCO type business plan can be feasible by setting parameters right. Companies can also control their risks of losing money by using the calculations in the Chapter 3. In addition, the distribution of the NPV and probability of bankruptcy is given in the thesis. For our example, for a single technology, we construct the model as changing a technology about heating system. Results show that, company can get expected profit of almost 275,000 $ within 15 years by increasing efficiency 1.4 times with an acceptable risk.

For the second part, we develop this model as an MILP optimization problem which considers multiple technologies. There exist multiple decision times for these technologies.In the literature a similar study has been done as a Phd. Thesis. In this thesis, model was developed to include single time decision making for a given budget. We extend this study as it includes multiple decision times. We compared the results for single time and multiple time.It concludes that using multiperiod optimization gives almost 5 times better solutions for smaller budgets when CO_2 saving is maximized. In addition, we used these data for the business plan, we extend the study to develop 4 different models which are profit rate constraint, net present value maximization, discount rate and linearly increasing price.

Firstly, while maximizing CO_2 saving we add a profit constraint which guarantees making at least a certain value of NPV. Saved amounts of CO_2 values decreased crucially for higher NPVs. The result indicates that for higher NPV return, company can reduce CO_2 savings. Although model allows saving 3,1 million kg CO_2 under the constraint getting 10 times initial budget NPV, for the profit rate 18, model saves only 2,5 million kg for 10 years and initial budget is 10,000$.

NPV maximization is another model whose results show that even with 10,000$ initial budget, ESCO can get almost 200,000$ NPV in 10 years. Another remarkable solution show that for greater budget, 250,000$, company can get 100,000$ NPV for 5 years.

For the ESCO type business plan similar to the model we developed in second chapter, we also include discount rate in the third model which allows, giving a proportion of the earnings to the customer. We both maximize CO_2 savings and NPV savings while considering discount rate to see how saving amounts change as proportion increases. Saved amount of CO_2 for discount rate 0.7 decreased almost half of the saving amount when there is no discount. However, NPV savings change more than CO_2 savings as discount rate increases. Saved amount of NPV for discount rate 0.7 decreases 10 times when compared to the the saving amount if there is no discount. On the other hand, smaller discounts can be given to customer to convince them for the business. Results show that saving amounts don't decrease crucially for smaller discount rates.

Last model includes linearly increasing price as energy price instead of constant price. We both maximize NPV and CO_2 savings with linearly increasing price consideration. Since saving amounts of money increase, saved CO_2 amounts and saved NPV amounts increase when compared to the constant energy price. For the CO_2 saving maximization, the results show increase in the CO_2 amounts are less than the increase in the NPV savings. For the budget,10,000 $ and for T=15, linearly increasing energy price model gives 35 % higher NPV when compared to the constant price. Also, for initial budget 250,000 $ and T=13 linearly increasing price model gives 100,000$ more NPV when compared to constant energy price. It is shown that making multi year arrangement with customers can be a win-win-win arrangement for the company, its customer, and also for the environment with the right agreement.

This thesis can be extended to the future studies which involve risk considerations for the ESCO business plan for multiple years. Third party financing can be considered for the sharing risk between customer and the company. In addition, it provides the insight for the building owners to invest in environment friendly technologies since it is more profitable. Also this thesis can be a benchmark for such studies which have different data for the technologies implemented.

Developing and using a new business model to expedite the process of increasing energy efficiency can be an effective way to achieve energy sustainability. The benefit of such systems will be more pronounced in the mega cities of developing countries like Turkey.

Appendix A

INFINITE HORIZON SOLUTION

Infinite Horizon Solution

Deterministic Case

For the deterministic case, lifetimes of the technologies are assumed to be constant, failure times are periodic and the energy price is forecasted linearly, which has no error term. In the model, we calculated NPV of technology replacement cost and NPV of energy usage cost separately and added them.

NPV of Replacement Cost

In a deterministic replacement calculation, after every s period we replaced the technology with a brand new one.Hence, the technology replacement cost is calculated as:

$$NPV_{rep} = \sum_{n=0}^{\infty} ce^{-(\alpha+\beta)sn} = c\sum_{n=0}^{\infty}(\frac{1}{e^{(\alpha+\beta)s}})^n = \frac{c}{1-e^{-(\alpha+\beta)s}}. \quad (A.1)$$

NPV of Energy Usage Cost

NPV of energy usage cost is calculated by modeling the unit price of used energy and the consumption of that energy given in the section 2.1.1.
The energy usage cost is calculated as

$$\begin{aligned} NPV_{energy} &= \sum_{t=1}^{12}\sum_{t'=1}^{\infty}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon})(c(12(t'-1)+t)+d) \\ &= \sum_{t=1}^{12}(1+r)^{-t}(\frac{aW_t+b}{\varepsilon})R(r)((ct+d-12c)+12cR(r)) \quad (A.2) \end{aligned}$$

where R(r)=$\frac{(r+1)^{12}}{r^{12}+12r^{11}+66r^{10}+220r^9+495r^8+792r^7+924r^6+792r^5+495r^4+220r^3+66r^2+12r}$.

CO_2 markets

CO_2 Amount Produced in a Single Technology The total CO_2 produced depends on the amount of energy while using a technology. θ is the amount of CO_2 produced for energy usage. The calculation of total produced CO_2:

$$\begin{aligned}&\sum_{t=1}^{12}\frac{aW_t+b}{\varepsilon}\sum_{t'=0}^{\infty}e^{-\eta t}\theta\\&=\frac{\theta}{\varepsilon}\sum_{t=1}^{12}(aW_t+b)\frac{e^{\eta}}{e^{\eta}-1}\end{aligned} \tag{A.3}$$

where $\eta \neq 0$. If $\eta = 0$ then the result is ∞

Total Revenue from Sale of CO_2 NPV of total revenue from sale of CO_2 is calculated as

$$\begin{aligned}&\sum_{t=0}^{T}(1+i)^{-t}\pi L e^{-\omega t}-\sum_{t=1}^{12}\frac{aW_t+b}{\varepsilon}\sum_{t'=1}^{\infty}(1+r)^{-(12(t-1)-t')}\pi\theta\\&=\pi L\sum_{t=0}^{T}((1+i)e^{\omega})^{-t}-\frac{\pi\theta}{\varepsilon}\sum_{t=1}^{12}(1+r)^{t}(aW_t+b)R_2(r)\end{aligned} \tag{A.4}$$

where $R_2(r)$=$\frac{12(r+1)^{13}}{r^{12}+12r^{11}+66r^{10}+220r^9+495r^8+792r^7+924r^6+792r^5+495r^4+220r^3+66r^2+12r}$ and π is the earning from a sale of one kg CO_2.

Deterministic Cost Calculation and Comparison NPV of total cost for the deterministic case is calculated as

$$
\begin{aligned}
\text{NPV of Total Cost} = {} & \text{NPV of Total Cost of Replacement + NPV of Total Cost of Energy Usage} \\
& - \text{NPV of Total Revenue from } CO_2 \text{ Sales} \\
& = \frac{c}{1-e^{-(\alpha+\beta)s}} + \sum_{t=1}^{12}(1+r)^{-t}\left(\frac{aW_t+b}{\varepsilon}\right)R(r)((ct+d-12c)+12cR(r)) \\
& - \pi L \sum_{t=0}^{T}((1+i)e^{\omega})^{-t} + \frac{\pi\theta}{\varepsilon}\sum_{t=1}^{12}(1+r)^{t}(aW_t+b)R_2(r) \\
& = \frac{c}{1-e^{-(\alpha+\beta)s}} + \sum_{t=1}^{12}(1+r)^{-t}\left(\frac{aW_t+b}{\varepsilon}\right)(R(r)((ct+d-12c)+12cR(r))+\pi\theta R_2(r)) \\
& - \pi L \sum_{t=0}^{T}((1+i)e^{\omega})^{-t}
\end{aligned} \tag{A.5}
$$

.

Theorem A.1.1. *Let c_1 and c_2 be the unit cost of technologies and ε_1 and ε_2 are the initial efficiencies for technology 1 and technology 2, respectively. The table shows which technology should be selected as a result of given comparisons*

	$\varepsilon_1 > \varepsilon_2$	$\varepsilon_1 = \varepsilon_2$	$\varepsilon_1 < \varepsilon_2$
$c_1 < c_2$	*1*	*1*	$\frac{(c_1-c_2)\varepsilon_1\varepsilon_2}{\varepsilon_1-\varepsilon_2} < \xi \Rightarrow 2$
$c_1 = c_2$	*1*	*1 or 2*	2
$c_1\ c_2$	$\frac{(c_1-c_2)\varepsilon_1\varepsilon_2}{\varepsilon_1-\varepsilon_2} > \xi \Rightarrow 2$	2	2

where $\xi = (\sum_{t=1}^{12}(1+r)^{-t}(aW_t+b)(R(r)((ct+d-12c)+12cR(r))+\pi\theta R_2(r)))(1-e^{-(\alpha+\beta)s})$.

Proof. The equation A.6 below is obtained by subtracting NPV of total cost of technology 2 from NPV of total cost of technology 1 using equation A.5.

$$
\frac{c_1-c_2}{1-e^{-(\alpha+\beta)s}} + \frac{(\varepsilon_2-\varepsilon_1)(\sum_{t=1}^{12}(1+r)^{-t}(aW_t+b)(R(r)((ct+d-12c)+12cR(r))+\pi\theta R_2(r)))}{\varepsilon_2\varepsilon_1} \tag{A.6}
$$

If Equation A.6 is greater than zero then choosing technology 2 is more profitable since its total NPV is less than technology 1. In this arrangement, when the unit cost of a technology is less and efficiency of a technology is more than the other one, it is obvious to choose that technology.If both costs and efficiencies are equal then it is indifferent to select.
The condition

$$\frac{c_1 - c_2}{1 - e^{-(\alpha+\beta)s}} + \frac{(\varepsilon_2 - \varepsilon_1)(\sum_{t=1}^{12}(1+r)^{-t}(aW_t + b)(R(r)((ct + d - 12c) + 12cR(r)) + \pi\theta R_2(r)))}{\varepsilon_2 \varepsilon_1} > 0$$

gives

$$\frac{(c_1 - c_2)\varepsilon_2\varepsilon_1}{(\varepsilon_1 - \varepsilon_2)} > (\sum_{t=1}^{12}(1+r)^{-t}(aW_t + b)(R(r)((ct + d - 12c) + 12cR(r)) + \pi\theta R_2(r)))(1 - e^{-(\alpha+\beta)s})$$

where

$$\xi = (\sum_{t=1}^{12}(1+r)^{-t}(aW_t + b)(R(r)((ct + d - 12c) + 12cR(r)) + \pi\theta R_2(r)))(1 - e^{-(\alpha+\beta)s})$$

□

Stochastic Case

In this section, technologies are considered to have a random lifetime whose distributions are exponential with parameter λ. They are continuously replaced with a brand new one when failure happens. An energy price which is forecasted with an AR(1) process with error $X_t = \varphi X_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and efficiency improvements are considered in the energy usage cost. We are interested in finding the expected value of total cost of replacement (NPV_{rep}) and Energy usage cost(NPV_{energy}).As it is calculated for the deterministic case, we also calculate replacement cost and energy usage cost separately for the stochastic case.

Expected NPV of Replacement Cost

Technologies are assumed to have a random lifetime whose distribution is exponential with parameter λ. They are continuously replaced with a brand new one when failure happens.

$$NPV_{rep} = \sum_{n=0}^{\infty} ce^{-(\alpha+\beta)T_n} \tag{A.7}$$

Summing up this over all n, we obtain the present value of all future replacement costs where T_n has Erlang distribution. Expected value of the NPV of replacement cost is

$$E[NPV_{rep}] = c\sum_{n=0}^{\infty} E[e^{-(\alpha+\beta)T_n}] \tag{A.8}$$

.

$E[e^{-(\alpha+\beta)T_n}]$ is the Laplace transform of Erlang(n,λ).

$$E[NPV_{rep}] = c\sum_{n=0}^{\infty} E[e^{-(\alpha+\beta)T_n}] = c\sum_{n=0}^{\infty}(\frac{\lambda}{\lambda+\alpha+\beta})^n = c(\frac{1}{1-\frac{\lambda}{\lambda+\alpha+\beta}}) = \frac{c\lambda}{\alpha+\beta}+c = \frac{c}{s'(\alpha+\beta)}+c \tag{A.9}$$

where $s' = \frac{1}{\lambda}$ is the expected replacement period.
We observed that the first two terms in the Taylor expansion of the deterministic replacement costs give the stochastic replacement cost when $s = s'$.

Expected NPV of Energy Usage Cost

The expectation of NPV of energy usage cost is used to determine the technology selection. To model energy prices we use an AR(1) process which is an discrete analogy of the Ornstein Uhlenbeck Process so that we assume that $P(t) = f(t) + X_t$ where X_t is AR(1) process.i.e. $X_t = \varphi X_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

$$NPV_{energy} = \sum_{t=1}^{12}\sum_{t'=1}^{\infty}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon})(c(12(t'-1)+t)+d+X_{12(t'-1)+t}) \tag{A.10}$$

The expected value of NPV of energy consumption cost is calculated by using $f(t) = ct + d + X_t$ as

$$E[NPV_{energy}] = E[\sum_{t=1}^{12}\sum_{t'=1}^{\infty}(1+r)^{-12(t'-1)-t}(\frac{aW_t+b}{\varepsilon})(c(12(t'-1)+t)+d+X_{12(t'-1)+t})]$$

$$= \sum_{t=1}^{12}(1+r)^{-t}(\frac{aW_t+b}{\varepsilon})R(r)((ct+d-12c)+12cR(r)) \tag{A.11}$$

where R(r)=$\frac{(r+1)^{12}}{r^{12}+12r^{11}+66r^{10}+220r^9+495r^8+792r^7+924r^6+792r^5+495r^4+220r^3+66r^2+12r}$. Since $E[X_t] = 0$ deterministic and the stochastic NPV of costs are the same.

CO_2 Markets

The CO_2 amount produced and the total revenue from sale of CO_2 is the same as the deterministic solution since the parameters in the CO_2 calculation have no stochastic term in our model.

Stochastic Cost Calculation and Comparison NPV of Total cost for the stochastic case is calculated as

$$\begin{aligned}
\text{NPV of Total Cost} &= \text{NPV of Total Cost of Replacement + NPV of Total Cost of Energy Usage} \\
&- \text{NPV of Total Revenue from } CO_2 \text{ Sales} \\
&= \frac{c\lambda}{\alpha+\beta} + c + \sum_{t=1}^{12}(1+r)^{-t}(\frac{aW_t+b}{\varepsilon})R(r)((ct+d-12c)+12cR(r)) - \pi L\sum_{t=0}^{T}((1+i)e^{\omega})^{-t} \\
&+ \frac{\pi\theta}{\varepsilon}\sum_{t=1}^{12}(1+r)^t(aW_t+b)R_2(r) \\
&= \frac{c\lambda}{\alpha+\beta} + c + \sum_{t=1}^{12}(1+r)^{-t}(\frac{aW_t+b}{\varepsilon})(R(r)((ct+d-12c)+12cR(r))+\pi\theta R_2(r)) \\
&- \pi L\sum_{t=0}^{T}((1+i)e^{\omega})^{-t}
\end{aligned} \tag{A.12}$$

Theorem A.1.3. *Let c_1 and c_2 be the unit cost of technologies and ε_1 and ε_2 are the initial efficiencies for technology 1 and technology 2, respectively. The table shows which technology should be selected as a result of given comparisons*

	$\varepsilon_1 > \varepsilon_2$	$\varepsilon_1 = \varepsilon_2$	$\varepsilon_1 < \varepsilon_2$
$c_1\mu_1 < c_2\mu_2$	*1*	*1*	$\frac{(c_1\mu_1-c_2\mu_2)\varepsilon_1\varepsilon_2}{\varepsilon_1-\varepsilon_2} < \xi \Rightarrow 2$
$c_1\mu_1 = c_2\mu_2$	*1*	*1 or 2*	*2*
$c_1\mu_1 > c_2\mu_2$	$\frac{(c_1\mu_1-c_2\mu_2)\varepsilon_1\varepsilon_2}{\varepsilon_1-\varepsilon_2} > \xi \Rightarrow 2$	*2*	*2*

where $\mu_1 = \lambda_1+\alpha+\beta$, $\mu_2 = \lambda_2+\alpha+\beta$ *and* $\xi = \sum_{t=1}^{12}(1+r)^{-t}(aW_t+b)(R(r)((ct+d-12c)+12cR(r))+\pi\theta R_2(r))(\alpha+\beta)$

Proof. The Equation A.13 below is obtained by subtracting NPV of total cost of technology 2 from NPV of total cost of technology 1 using Equation A.12.

$$\frac{c_1\lambda_1 - c_2\lambda_2}{\alpha+\beta} + c_1 - c_2 + \frac{(\varepsilon_2-\varepsilon_1)(\sum_{t=1}^{12}(1+r)^{-t}(aW_t+b)(R(r)((ct+d-12c)+12cR(r))+\pi\theta R_2(r)))}{\varepsilon_2\varepsilon_1} \tag{A.13}$$

If Equation A.13 is greater than zero then choosing technology 2 is more profitable since its total NPV is less than technology 1. In this arrangement, when the unit cost of a technology is less and efficiency of a technology is more than the other one, it is obvious to choose that technology.If both costs and efficiencies are equal then it is indifferent to select.
The condition

$$\frac{c_1\lambda_1 - c_2\lambda_2}{\alpha+\beta} + c_1 - c_2 + \frac{(\varepsilon_2-\varepsilon_1)(\sum_{t=1}^{12}(1+r)^{-t}(aW_t+b)(R(r)((ct+d-12c)+12cR(r))+\pi\theta R_2(r)))}{\varepsilon_2\varepsilon_1} > 0$$

yields

$$\frac{(c_1\mu_1 - c_2\mu_2)\varepsilon_2\varepsilon_1}{(\varepsilon_1-\varepsilon_2)} > (\sum_{t=1}^{12}(1+r)^{-t}(aW_t+b)(R(r)((ct+d-12c)+12cR(r))+\pi\theta R_2(r)))(\alpha+\beta)$$

where

$$\xi = \sum_{t=1}^{12}(1+r)^{-t}(aW_t+b)(R(r)((ct+d-12c)+12cR(r))+\pi\theta R_2(r))(\alpha+\beta)$$

□

Appendix B

TECHNOLOGIES IMPLEMENTED TO BOĞAZIÇI UNIVERSITY KILYOS CAMPUS

Below, figures show various technologies that are implemented in buildings at the Boğaziçi University Kilyos Campus. In the figures, cost, kWh saving/year, money saving/year and CO_2 saving/year is also given which are calculated after technical and mechanical work [24].

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
A6	1st Dorm - North Block	Envelope Insulation Retrofit-6cm XPS	3cm XPS application-poor quality in laboring and material	6cm XPS application-as per TS 825 requirements with no thermal bridge	34.402	95.692	3.514	22.392
B6	1st Dorm - South Block	Envelope Insulation Retrofit-6cm XPS	3cm XPS application-poor quality in laboring and material	6cm XPS application-as per TS 825 requirements with no thermal bridge	34.402	95.611	3.511	22.373
C6	1st Dorm - Apartments	Envelope Insulation Retrofit-6cm XPS	3cm XPS application-poor quality in laboring and material	6cm XPS application-as per TS 825 requirements with no thermal bridge	17.084	27.329	1.004	6.395
E6	Prep School-Block A	Envelope Insulation Retrofit-6cm XPS	No insulation	6cm XPS application-as per TS 825 requirements with no thermal bridge	33.448	156.501	5.747	36.621
F6	Prep School-Block B	Envelope Insulation Retrofit-6cm XPS	3cm XPS application-poor quality in laboring and material	6cm XPS application-as per TS 825 requirements with no thermal bridge	22.182	69.278	2.544	16.211
I6	2nd Dorm	Envelope Insulation Retrofit-6cm XPS	3cm XPS application-poor quality in laboring and material	6cm XPS application-as per TS 825 requirements with no thermal bridge	38.998	92.882	3.411	21.374

Table B.1: Envelope insulation retrofit-6cm XPS [24]

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
A7	1st Dorm - North Block	Envelope Insulation Retrofit-5cm XPS	3cm XPS application-poor quality in laboring and material	5cm XPS application-as per TS 825 requirements with no thermal bridge	30.850	81.789	3.003	19.139
B7	1st Dorm - South Block	Envelope Insulation Retrofit-5cm XPS	3cm XPS application-poor quality in laboring and material	5cm XPS application-as per TS 825 requirements with no thermal bridge	30.850	81.210	2.982	19.003
C7	1st Dorm - Apartments	Envelope Insulation Retrofit-5cm XPS	3cm XPS application-poor quality in laboring and material	5cm XPS application-as per TS 825 requirements with no thermal bridge	15.320	23.286	855	5.449
E7	Prep School-Block A	Envelope Insulation Retrofit-5cm XPS	No insulation	5cm XPS application-as per TS 825 requirements with no thermal bridge	29.995	152.049	5.583	35.579
F7	Prep School-Block B	Envelope Insulation Retrofit-5cm XPS	3cm XPS application-poor quality in laboring and material	5cm XPS application-as per TS 825 requirements with no thermal bridge	19.892	59.272	2.177	13.870
I7	2nd Dorm	Envelope Insulation Retrofit-5cm XPS	3cm XPS application-poor quality in laboring and material	5cm XPS application-as per TS 825 requirements with no thermal bridge	34.972	79.314	2.912	18.559

Table B.2: Envelope insulation retrofit-5cm XPS [24]

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
A8	1st Dorm - North Block	Envelope Insulation Retrofit-4cm XPS	3cm XPS application-poor quality in laboring and material	4cm XPS application- as per TS 825 requirements with no thermal bridge	27.055	61.993	2.276	14.506
B8	1st Dorm - South Block	Envelope Insulation Retrofit-4cm XPS	3cm XPS application-poor quality in laboring and material	4cm XPS application- as per TS 825 requirements with no thermal bridge	27.055	61.948	2.275	14.496
C8	1st Dorm - Apartments	Envelope Insulation Retrofit-4cm XPS	3cm XPS application-poor quality in laboring and material	4cm XPS application- as per TS 825 requirements with no thermal bridge	13.436	17.706	650	4.143
E8	Prep School- Block A	Envelope Insulation Retrofit-4cm XPS	No insulation	4cm XPS application- as per TS 825 requirements with no thermal bridge	26.305	145.914	5.358	34.144
F8	Prep School- Block B	Envelope Insulation Retrofit-4cm XPS	3cm XPS application-poor quality in laboring and material	4cm XPS application- as per TS 825 requirements with no thermal bridge	17.445	45.575	1.674	10.664
I8	2nd Dorm	Envelope Insulation Retrofit-4cm XPS	3cm XPS application-poor quality in laboring and material	4cm XPS application- as per TS 825 requirements with no thermal bridge	30.670	60.237	2.212	14.095

Table B.3: Envelope insulation retrofit-4cm XPS [24]

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
A10	1st Dorm -North Block	Passive Measure-Trombe Wall Application on South Façade	No trombe wall, just standard exterior wall	Trombe wall applied, over TS 825 standard exterior wall	65.655	31.912	1.172	7.467
B10	1st Dorm - South Block	Passive Measure-Trombe Wall Application on South Façade	No trombe wall, just standard exterior wall	Trombe wall applied, over TS 825 standard exterior wall	65.655	31.890	1.171	7.462
E10	Prep School- Block A	Passive Measure- Sunroom Application on the Roof where there's skylight currently	Poor performance polycarbonate skylight	Sunroof Application using low-E double pane glass	119.824	65.837	2.418	15.406

Table B.4: Trombe wall application and sunroom on the roof [24]

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
D3	1st Dorm -ALL	Boiler retrofit by capacity utilization with high efficiency in partial loads	Old, cast iron, large size boilers	New technology,steel, condensing, multi small size boilers operating in an integrated fashion	41.250	250.005	10.182	58.501

Table B.5: Boiler retrofit [24]

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
A2	1st Dorm - North Block	Heating Loop Piping Insulation Retrofit	No insulation	Properly insulated as per local "Building Energy Performance" Directive	1.071	23.443	955	5.486
B2	1st Dorm - South Block	Heating Loop Piping Insulation Retrofit	No insulation	Properly insulated as per local "Building Energy Performance" Directive	1.071	23.443	955	5.486
C2	1st Dorm - Apartments	Heating Loop Piping Insulation Retrofit	No insulation	Properly insulated as per local "Building Energy Performance" Directive	964	21.099	859	4.937
E2	Prep School- Block A	Heating Loop Piping Insulation Retrofit	No insulation	Properly insulated as per local "Building Energy Performance" Directive	2.330	50.989	2.077	11.931
F2	Prep School- Block B	Heating Loop Piping Insulation Retrofit	No insulation	Properly insulated as per local "Building Energy Performance" Directive	6.750	229.189	9.335	53.630
I2	2nd Dorm	Heating Loop Piping Insulation Retrofit	No insulation	Properly insulated as per local "Building Energy Performance" Directive	1.969	43.077	1.754	10.080

Table B.6: Heating loop piping insulation retrofit [24]

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
A4	1st Dorm - North Block	Thermostatic valve installation in all radiators	No thermostatic valves	Thermostatic valve installed	5.850	61.050	2.486	14.286
B4	1st Dorm - South Block	Thermostatic valve installation in all radiators	No thermostatic valves	Thermostatic valve installed	5.850	60.764	2.475	14.219
C4	1st Dorm - Apartments	Thermostatic valve installation in all radiators	No thermostatic valves	Thermostatic valve installed	1.300	17.402	709	4.072
I4	2nd Dorm	Thermostatic valve installation in all radiators	No thermostatic valves	Thermostatic valve installed	9.100	63.969	2.605	14.969

Table B.7: Thermostatic valve installation in all radiators [24]

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
A1	1st Dorm - North Block	DHW Heating Setpoint Optimization	Heating Setpoint 60 Deg C	Heating setpoint 45 Deg C	Applied together with all other blocks of 1st Dorm	33.334	1.358	7.800
B1	1st Dorm - South Block	DHW Heating Setpoint Optimization	Heating Setpoint 60 Deg C	Heating setpoint 45 Deg C		3.334	1.358	7.800
C1	1st Dorm - Apartments	DHW Heating Setpoint Optimization	Heating Setpoint 60 Deg C	Heating setpoint 45 Deg C		10.158	414	2.377
D1	1st Dorm ALL	DHW Heating Setpoint Optimization	Heating Setpoint 60 Deg C	Heating setpoint 45 Deg C	1.250	76.827	3.129	17.977

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
H1	Hotel	DHW Heating Setpoint Optimization	Heating Setpoint 60 Deg C	Heating setpoint 45 Deg C	500	3.295	134	771

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
I1	2nd Dorm	DHW Heating Setpoint Optimization	Heating Setpoint 60 Deg C	Heating setpoint 45 Deg C	1.250	55.557	2.263	13.000

Table B.8: Domestic hot water heating setpoint [24]

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
D9	1st Dorm -ALL	Heating Water Circulation Pumps Retrofit by Variable Speed Drive Pumps	Old, constant speed, demand controlled circulation pumps	New variable speed, demand controlled circulation pumps	3.750	4.455	597	2.749
G9	Prep School-ALL	Heating Water Circulation Pumps Retrofit by Variable Speed Drive Pumps	Old, constant speed, demand controlled circulation pumps	New variable speed, demand controlled circulation pumps	6.250	11.880	1.592	7.330
I9	2nd Dorm	Heating Water Circulation Pumps Retrofit by Variable Speed Drive Pumps	Old, constant speed, demand controlled circulation pumps	New variable speed, demand controlled circulation pumps	3.750	5.940	796	3.665

Table B.9: Heating water circulation pumps retrofit [24]

EEM	Building	Energy Efficiency Measure	Current Status	Status After EEM	Cost	Annual Performance(Saving/Reduction) Results		
Code	Name/Block	Description	Description	Description	USD	kWh Saving/year	$ saving/year	kg CO2/year
D5	1st Dorm-ALL	Lighting Ballast Retrofit	Mechanical Ballast Lighting	Electronic Ballast Lighting	10.938	24.599	3.296	15.178
G5	Prep School-ALL	Lighting Ballast Retrofit	Mechanical Ballast Lighting	Electronic Ballast Lighting	12.500	37.426	5.015	23.092
H5	Hotel	Lighting Ballast Retrofit	Mechanical Ballast Lighting	Electronic Ballast Lighting	1.563	1.972	264	1.216
I5	2nd Dorm	Lighting Ballast Retrofit	Mechanical Ballast Lighting	Electronic Ballast Lighting	6.250	17.200	2.305	10.613

Table B.10: Lighting ballast retrofit [24]

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