QUALITY CONSCIOUS STRATEGIC CUSTOMERS IN QUEUEING TYPE SERVICE SYSTEMS

by

Görkem Sarıyer Ataman

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This is to certify that I have examined this copy of a doctoral dissertation by

Görkem Sarıyer Ataman

and have found that it is complete and satisfactory in all respects,

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examining committee have been made.

Chair of Supervisory Committee:

Reading Committee:

Prof. Fikri Karaesmen (Advisor)

Prof. Zeynep Akşin Karaesmen

Prof. A. Refik Güllü

Assoc. Prof. Lerzan Örmeci

Assist. Prof. Özgür Yılmaz

Date:

To My Mother and Father

ABSTRACT

A service system is a configuration of technology and organizational networks designed to deliver services that satisfy needs, wants and aspirations of customers. Call centers, universities, hospitals, restaurants, shopping centers, hotels, beauty centers are a few examples of the service systems which provide service to customers in different areas. A strategic customer in a service system acts in order to maximize his individual welfare or utility. To maximize his individual utility, a strategic customer compares all of his possible alternatives and chooses the one which brings the highest pay-off to him. The alternative with the highest pay-off is referred as the optimal behavior (action) of this customer. Indeed, each customer's optimal behavior is affected by acts taken by the service provider (system manager) and by the other customers. The result is an aggregate equilibrium pattern of behavior which may not be optimal from the perspective of the society as a whole. So, the optimal decisions of the individual customer, social planner who is responsible from maximizing the social welfare, and the service provider all need to be analyzed in these systems. Many service systems are queueing type systems. So service systems generally include queues, and the individual customer, social planner and the service provider must (directly or indirectly) consider the displeasure of waiting in the queue. Moreover, the service systems that we consider in this thesis have imperfect quality. The service failures which are caused by the service quality problem will require resolutions. Designing the service systems by using different system structures we analyze different service failure resolution alternatives in Strategic Customer Setting. In summary, our goal in this thesis is to integrate service quality issues with strategic behavior and compare different service system structures.

ÖZETÇE

Servis sistemleri, müşterilerin ihtiyaç ve isteklerini karşılamak için oluşturulan tasarım ve teknolojilerdir. Bu sistemlere verilebilecek başlıca örnekler çağrı merkezleri, üniversiteler, hastaneler, restoranlar, alışveriş merkezleri, oteller, güzellik merkezleri olarak sıralanabilir. Servis sistemlerindeki stratejik müşteriler kendi amaç (haz) fonksiyonlarını maksimize etmek için karar verirler. Stratejik bir müşteri karar verirken amaç fonksiyonunu maksimize edebilecek bütün alternatifleri karşılaştırır ve kendisine en çok tatmin verecek (en yüksek hazzı sağlayacak) olanı seçer. Temelde her müşterinin en iyi kararı diğer müşterilerin kararından ve hizmet verenin (sistem yöneticisi) sunduklarından etkilenmektedir. Sonuç olarak, müşterilerin kararı diğer müşterilerinin de kararına bağlı olan bir denge stratejisi olarak düşünülmektedir ve bu strateji sosyal sistem için en iyi strateji olmayabilir. Bu nedenle, bu sistemlerde müşterilerin problemleri, sosyal sistem problemi ve hizmet veren firmanın problemi ayrı ayrı incelenmelidir. Servis sistemleri genelde kuyruk tipi sistemlerdir. Bu nedenle karar verirken müşteriler, sosyal sistemden sorumlu kişiler ve hizmet veren kişiler kuyruktaki beklemenin müşteriler üzerinde yaratacağı memnuniyetsizliği göz önünde bulundurmalıdır. Bunun yanında, bu tezde incelenen sistemler mükemmel kalitede hizmet veren sistemler olarak düşünülmemektedir yani sistemlerdeki hizmet kalitesindeki problemler de göz önünde bulundurulmaktadır. Hizmet kalitesinde problem olduğunda bunun olası çözüm metotları da bu tezde incelenmiştir. Dolayısıyla, bu tezde stratejik müşterilerin olduğu sistemlerde, kalite problemlerini çözme metotları farklı sistem yapıları kullanılarak karşılaştırılmıştır. Kısacası bu tezin amacı kalite konusunu stratejik kararlarla birleştirmek ve farklı sistem dizaynlarını bu bağlamda karşılaştırmaktır.

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I would finally thank my husband, Mustafa Gökalp Ataman, the greatest surprise of my life, for the absolute support he provided and will provide. My life is more meaningful with him.

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NOMENCLATURE

q	:Service quality level
i	:Position of the customer in the queue
$U(i)$	:Utility function of the ith customer in the observable queue
R	:Service reward
C	:Unit waiting cost
$W(i)$	:Waiting time of the ith customer
$E[W(i)]$	:Expected waiting time of the ith customer
μ	:Service rate
λ	:Arrival rate
n^*_{ind}	:Threshold level of the individual's joining strategy in observable queues
ρ_1	:Utilization level of the benchmark model
ρ_2	:Utilization level of the model with resolution
$S_O(n)$	:Social welfare function of observable queues
n^*_{sys}	:Threshold level of the socially optimal joining strategy in observable queues
α	:Joining probability of a customer
$E[W]$	:Expected total waiting time
$U_1(\alpha)$	:Utility function of a customer in the benchmark model
$\alpha_1^{eq}(q)$	:Equilibrium joining probability of a customer as a function of quality level in the benchmark model
$S_1(\alpha)$	:Socially optimal joining function in the benchmark model
$\alpha_1^*(q)$	:Socially optimal joining probability as a function of quality level in the benchmark model
$\Pi_1(\cdot)$	:Profit function of the profit maximizer in the benchmark model
p	:Queue entrance price
a	:Unit cost of quality

$p_{1,\lambda}$:Market capturing price in the benchmark model

$q_{1,\lambda}$:Optimal quality level in the benchmark model for the market capturing pricing

$p_{1,m}(q)$:Monopolistic price as a function of quality level in the benchmark model

$q_{1,m}$:Optimal service quality level in the benchmark model for the monopolistic pricing

$p_{1,m}$:Optimal monopolistic price in the benchmark model

$E[W|i]$:Conditional expected waiting time of a customer who finds i customers in front

$U_2(\alpha)$:Utility function of a customer in the model with resolution

$\alpha_2^{eq}(q)$:Equilibrium joining probability of a customer as a function of quality level in the model with resolution

$S_2(\alpha)$:Socially optimal joining function in the model with resolution

$\alpha_2^*(q)$:Socially optimal joining probability as a function of quality level in the model with resolution

$\Pi_2(\cdot)$:Profit function of the profit maximizer in the model with resolution

$p_{2,\lambda}(q)$:Market capturing price as a function of quality level in the model with resolution

$p_{2,m}(q)$:Monopolistic price as a function of quality level in the model with resolution

$q_{2,m}$:Optimal service quality level in the model with resolution for the monopolistic pricing

$p_{2,m}$:Optimal monopolistic price in the model with resolution

$U_3(\alpha)$:Utility function of a customer in the model with returns

$\alpha_3^{eq}(q)$:Equilibrium joining probability of a customer as a function of quality level in the model with returns

$S_3(\alpha)$:Socially optimal joining function in the model with returns

$\alpha_3^*(q)$:Socially optimal joining probability as a function of quality level in the model with returns

$\Pi_3(\cdot)$	:Profit function of the profit maximizer in the model with returns
ρ_3	:Utilization level of the model with returns
$p_{3,\lambda}(q)$	:Market capturing price as a function of quality level in the model with returns
$q_{3,\lambda}$	:Optimal service quality level in the model with returns for the market capturing pricing
$q_{2,\lambda}$	:Optimal service quality level in the model with resolution for the market capturing pricing
$p_{3,\lambda}$	:Optimal market capturing price in the model with returns
$p_{2,\lambda}$	:Optimal market capturing price in the model with resolution
$\Pi_{3,\lambda}$	:Profit of the profit maximizer in the model with returns when market capturing price is set
$\Pi_{2,\lambda}$	:Profit of the profit maximizer in the model with resolution when market capturing price is set
$\Pi_{1,\lambda}$	:Profit of the profit maximizer in the benchmark model when market capturing price is set
$\Pi_{2,m}$	:Profit of the profit maximizer in the model with resolution when monopolistic price is set
$\Pi_{1,m}$	:Profit of the profit maximizer in the benchmark model when monopolistic price is set
k	:Index for the server in the escalation model
$U_{4,n}(\alpha)$	:Utility function of a customer in the simple escalation model
$\alpha^{eq}_{4,n}(q)$	:Equilibrium joining probability of a customer as a function of quality level in the simple escalation model
$\alpha^{eq}_{4,n-int}(q)$	:Interior equilibrium joining probability of a customer as a function of quality level in the simple escalation model
$U_{4,g}(\alpha)$	:Utility function of a customer in the perfect escalation model

$\alpha^{eq}_{4,g}(q)$:Equilibrium joining probability of a customer as a function of quality level in the perfect escalation model

$\alpha^{eq}_{4,g-int}(q)$:Interior equilibrium joining probability of a customer as a function of quality level in the perfect escalation model

$S_{4,n}(\alpha)$:Socially optimal joining function in the simple escalation model

$\alpha^{*}_{4,n}(q)$:Socially optimal joining probability as a function of quality level in the simple escalation model

$S_{4,g}(\alpha)$:Socially optimal joining function in the perfect escalation model

$\alpha^{*}_{4,g}(q)$:Socially optimal joining probability as a function of quality level in the perfect escalation model

$\Pi_{4,n}(\cdot)$:Profit of the profit maximizer in the simple escalation model

$p_{4,n}(q)$:Queue entrance price as a function of quality level in the simple escalation model

$\Pi_{4,g}(\cdot)$:Profit of the profit maximizer in the perfect escalation model

$p_{4,g}(q)$:Queue entrance price as a function of quality level in the perfect escalation model

$q^{*}_{4,n}$:Optimal service quality level in the simple escalation model

$q^{*}_{4,g}$:Optimal service quality level in the perfect escalation model

$p^{*}_{4,n}$:Optimal queue entrance price in the simple escalation model

$p^{*}_{4,g}$:Optimal queue entrance price in the perfect escalation model

$\rho_{1,\nu}$:Utilization level of the two-parallel-server benchmark model

Π_0 :Steady state probability of having no customer in the system in the parallel server models

Π_{c+} :Steady state probability that a customer has to wait in the system in the parallel server models

$\overline{W}$:Average waiting time in the queue

$\rho_{2,\nu}$:Utilization level of the two-parallel-server model with resolution

$U_{1,\nu}(\alpha)$:Utility function of a customer in the two-parallel-server benchmark model

$\alpha_{1,\nu}^{eq}(q)$	:Equilibrium joining probability of a customer as a function of quality level in the two-parallel-server benchmark model
$U_{2,\nu}(\alpha)$	:Utility function of a customer in the two-parallel-server model with resolution
$\alpha_{2,\nu}^{eq}(q)$	:Equilibrium joining probability of a customer as a function of quality level in the two-parallel-server model with resolution
$S_{1,\nu}(\alpha)$	:Socially optimal joining function in the two-parallel-server benchmark model
$\alpha_{1,\nu}^{*}(q)$	:Socially optimal joining probability as a function of quality level in the two-parallel-server benchmark model
$S_{2,\nu}(\alpha)$	:Socially optimal joining function in the two-parallel-server model with resolution
$\alpha_{2,\nu}^{*}(q)$	:Socially optimal joining probability as a function of quality level in the two-parallel-server model with resolution
$p_{1,\nu}(q)$	:Queue entrance price as a function of quality level in the two-parallel-server benchmark model
$\Pi_{1,\nu}(\cdot)$	:Profit of the profit maximizer in the two-parallel-server benchmark model
$q_{1,\nu}^{*}$	:Optimal service quality level in the two-parallel-server benchmark model
$p_{1,\nu}^{*}$	:Optimal queue entrance price in the two-parallel-server benchmark model
$\Pi_{1,\nu}^{*}$	:Optimal profit in the two-parallel-server benchmark model
$p_{2,\nu}(q)$	:Queue entrance price as a function of quality level in the two-parallel-server model with resolution
$\Pi_{2,\nu}(\cdot)$	:Profit of the profit maximizer in the two-parallel-server model with resolution
$q_{2,\nu}^{*}$	:Optimal service quality level in the two-parallel-server model with resolution
$p_{2,\nu}^{*}$	:Optimal queue entrance price in the two-parallel-server model with resolution
$\Pi_{2,\nu}^{*}$	:Optimal profit in the two-parallel-server model with resolution
$p_{1,d}(q)$	:Queue entrance price as a function of quality level in the single server benchmark model with double rate
$\Pi_{1,d}(\cdot)$	:Profit of the profit maximizer in the single server benchmark model with double rate

$q^*_{1,d}$:Optimal service quality level in the single server benchmark model with double rate

$p^*_{1,d}$:Optimal queue entrance price in the single server benchmark model with double rate

$\Pi^*_{1,d}$:Optimal profit in the single server benchmark model with double rate

$\Pi^*_{4,n}$:Optimal profit in the simple escalation model

$\alpha^{eq}_{1,d}(q)$:Equilibrium joining probability of a customer as a function of quality level in the single server benchmark model with double rate

$\alpha^*_{1,d}(q)$:Socially optimal joining probability as a function of quality level in the single server benchmark model with double rate

$p_{2,d}(q)$:Queue entrance price as a function of quality level in the single server model with resolution with double rate

$\Pi_{2,d}(\cdot)$:Profit of the profit maximizer in the single server model with resolution with double rate

$q^*_{2,d}$:Optimal service quality level in the single server model with resolution with double rate

$p^*_{2,d}$:Optimal queue entrance price in the single server model with resolution with double rate

$\Pi^*_{2,d}$:Optimal profit in the single server model with resolution with double rate

$\Pi^*_{4,g}$:Optimal profit in the perfect escalation model

$U_4(\alpha)$:Utility function of a customer in the improved escalation model

$\mu_1(\alpha)$:Service rate of the first server in the improved escalation model

$p_4(\mu_1, q)$:Queue entrance price as a function of quality level and the service rate in the improved escalation model

$\Pi_4(\cdot)$:Profit of the profit maximizer in the improved escalation model

Π^*_4 :Optimal profit in the improved escalation model

q_1 :Quality level of the first server in the improved escalation model

q_2 :Quality level of the second server in the improved escalation model

$p_4(q_1, q_2)$:Queue entrance price as a function of quality levels in the servers improved escalation model

$q_2^*(q_1)$:Optimal quality level of the second server as a function of the quality level of the first server in the improved escalation model

b :Unit cost of service rate

μ_1^* :Optimal service rate in the benchmark model

$q_{1,\tilde{\mu}}^*$:Optimal service quality level in the benchmark model for the fixed service rate

$\mu_2^*(q)$:Optimal service rate in the model with resolution as a function of quality level

μ_2^* :Optimal service rate in the model with resolution

$q_{2,\tilde{\mu}}^*$:Optimal service quality level in the model with resolution for the fixed service rate

$\mu_3^*(q)$:Optimal service rate in the model with returns as a function of quality level

μ_3^* :Optimal service rate in the model with returns

$q_{3,\tilde{\mu}}^*$:Optimal service quality level in the model with returns for the fixed service rate

$q_{1-\nu,\tilde{\mu}}^*$:Optimal service quality level in the two-parallel-server benchmark model for the fixed service rate

$\mu_{1-\nu}^*$:Optimal service rate in the two-parallel-server benchmark model

$q_{2-\nu,\tilde{\mu}}^*$:Optimal service quality level in the two-parallel-server model with resolution for the fixed service rate

$\mu_{2-\nu}^*$:Optimal service rate in the two-parallel-server model with resolution

$q_{4-n,\tilde{\mu}}^*$:Optimal service quality level in the simple escalation model for the fixed service rate

μ_{4-n}^* :Optimal service rate in the simple escalation model

$q_{4-g,\tilde{\mu}}^*$:Optimal service quality level in the perfect escalation model for the fixed service rate

μ_{4-g}^* :Optimal service rate in the perfect escalation model

Chapter 1

INTRODUCTION

A service system is a configuration of technology and organizational networks designed to deliver services that satisfy needs, wants and aspirations of customers. Call centers, universities, hospitals, restaurants, shopping centers, transportation companies, hotels, beauty centers are a few examples of the service systems which provide service to customers in different areas. Operators, professors, doctors, waiters, salesman, drivers, room-cleaners, cosmeticians are the service providers of these service systems. These workers provide service to customers to satisfy their tangible and in-tangible needs.

A strategic customer in a service system acts independently in order to maximize his individual welfare or utility. To maximize his individual utility, a strategic customer compares all of his possible alternatives and chooses the one which brings the highest pay-off to him. The alternative with the highest pay-off is referred as the optimal behavior (action) of this customer. Indeed, each customer's optimal behavior is affected by acts taken by the service provider (system manager) and by the other customers. The result is an aggregate equilibrium pattern of behavior which may not be optimal from the perspective of the society as a whole. So, the optimal decisions of the individual customer, social planner who is responsible from maximizing the social welfare, and the service provider all need to be analyzed in these systems.

Since the designers and the servers of the service systems are humans who provide service using technological labor and equipments, there is always a probability of the service failure. Some possible reasons for this service failure can be characterized as: The inefficiency, temporary emotional burnout, carelessness, tiredness-illness of the service provider, oldness or the defect of the technological equipment, power blackout or the disconnection of the internet etc. Such kind of failures shows that for a strategic customer assuming a perfect service in all of his visits to a specific service system is unrealistic. So, in real life service systems are imperfect and a strategic customer, the social planner and the service provider must take this imperfection or the service failure probability into account.

Moreover, many service systems are queueing type systems. For example, when a customer makes a call to a call center, if all of the operators are busy, he waits until one of the operator becomes idle. When a patient wants to get examined by a doctor in a policlinic, he generally makes an appointment, since there are other patients who made their appointments earlier. If a customer goes to a restaurant to get his dinner, he waits for the waitress to come after serving the customers who came to the restaurant before. All of these examples illustrate that service systems generally include queues, and the individual customer, social planner and the service provider must (directly or indirectly) consider the displeasure of waiting in the queue.

Based on the above motivation, we analyze how the strategic (rational) customer, the social planner and the service provider behave in imperfect quality queueing systems. The customer is strategic, so he is able to take the best decision for himself among all other alternatives. The possible decisions in these systems typically concern whether or not to join the queue, but can also consider decisions pertaining to the abandonment from the queue. For the social problem, the possible decision strategies of the social planner is to accept some or all of the customers to the system, which maximizes the social welfare. In the problem of the service provider (profit maximizer), the decision is to determine the model parameters which maximize his profit. In this thesis, for the decision parameters of the profit maximizer's problem, we use the queue entrance price, the service quality level and the service rate.

We assume that the satisfaction of receiving a service which satisfies the needs of the customers, is expressed with a monetary value, service reward. So, the customers who receive the satisfactory service take the service reward; on the other hand the ones who are subjected to a service failure cannot take this reward. Since the service systems being considered are queueing type systems, customers choose the decision, in which their service is completed in the shortest time; i.e. they don't want to wait much. To express the displeasure of waiting, we use a monetary value for the unit waiting cost. If a customer decides to join the queue, he pays an entrance fee. To summarize, the utility function of a strategic customer includes three parts: Expected service reward which has a positive sign and increases the satisfaction of the customers, and the expected waiting cost and the queue entrance price which have negative signs and decrease the pleasure of the customers. The social problem includes the first two parts. We consider the entrance fee as a transfer payment in the social problem. So, we express the social welfare function by subtracting total expected waiting cost of the entire system from the total expected reward of the all customers. The revenue of the profit maximizer is formed by the total entrance prices taken from the customers who decide to join the system. However, since providing service with higher quality levels is costly for the profit maximizer, in order to form the problem of the profit maximizer we subtract the total cost of the quality from the total revenue (total entrance price). Although it seems like the expected reward and the waiting cost of the customer are independent from the problem of the profit maximizer, these values affect the profit maximizer's problem indirectly. That is why the service provider must consider these values in setting the entrance price.

Moreover, the service systems that we consider in this thesis have imperfect quality. The service failures which are caused by the service quality problem, will require resolutions. The possible service failure resolution alternatives are listed as:

A customer who is subjected to a service failure

1. can leave this system and go to any other service system to satisfy his needs.

2. returns to the same system,

    immediately, and goes to the end of the queue as a queueing discipline.

    immediately, and keeps the server busy (goes to the beginning of the queue as a queueing discipline).

after a sufficiently long time such that upon next arrival she observes the system in steady state.

3. can be escalated to higher levels of servers in the same system.

In order to analyze the first two service failure resolution alternatives, having a single server in the service system is enough. However, to analyze the third resolution alternative, we must use at least two servers in modeling our systems. Based on this motivation, we first model our service systems using a single server, then we analyze and compare the first two resolution alternatives from the viewpoints of the individual customer, social planner and the service provider. Next, we model our systems using two servers (multi-server models with minimum server number), and analyze all of the three resolution alternatives for the individual customer's, social planner's and the service provider's perspective.

We label the first part as the *Single Stage Models*. This part contains: *The Benchmark Model* which is used to model the first resolution alternative, *The Model With Resolution* and *The Model With Returns* to model the second resolution alternative. We analyze the three problems, the problem of the individual customer, the social problem and the problem of the profit maximizer using these Single Stage Models. We then compare the performances of these models by using the performance criteria of the equilibrium joining probability for the individual problem, socially optimal joining probability for the social problem, and the profit value of the problem of the profit maximizer.

The label of the second part is the Two Server Models. In order to analyze the third service failure resolution alternative we use the *Escalation Models*. As a system design, we model the Escalation Models, as a sequential multi-stage system. That is we assume that there are two sequential servers providing service respectively. To compare the performance of these sequential stage systems based on the performance criteria given before, we also use the other alternative for the Two Server Models. This alternative is the *Two-Parallel-Server Model* which basically is the *M/M/2* model having two identical servers providing service simultaneously. In order to analyze additionally the first two resolution alternatives in these Two Server Models part, we model the Escalation and the Two-Parallel-Server Models in two ways. In order to represent the first resolution alternative, we use the *Simple Escalation* and the *Two-Parallel-Server Benchmark Models* which assume that the unsatisfied customers (for the escalation model, the unsatisfied customers refer the customers who are subjected to a service failure in both of the two servers.) leave the system. We analyze the second resolution alternative by using the *Perfect Escalation Model* and the *Two-Parallel-Server Model With Resolution* which assume that the customers do not leave the system without taking the service reward; i.e. all the customers keep returning until they leave the system satisfied.

In order to combine our analyses given in these two parts, we compare the performances of the Two Server Models with the Single Server Models with a single server who provides service with double server rate.

Up to this point, we assume that the profit maximizer provides service for the fixed service rates. So, he decides on the entrance price and the service quality level. However, since changing the service rate might be easier and less time consuming for the profit maximizer in some settings, we change the structure of the profit maximizer's problem in the last part of this thesis. Here we assume that the profit maximizer decides on the entrance price, service rate, and the service quality level. Since in this case, the entrance price is a function of the other parameters, service rate and service quality level, we consider this problem as an optimization problem with two decisions. In order to analyze how the performance of our models are affected with this change from the profit maximizer's viewpoint, we consider two cases in this part. The first case is the *Fixed Server Rate Case*, which assumes that the profit maximizer provides service with fixed service rate and changes the service quality level. The second case is the *Service Rate Is A Decision Case*, assuming that both the service rate and the service quality level are the decisions.

The outline of this thesis is as follows: The second chapter is the *Literature Review*. We divide this chapter corresponding to the literature survey in two sections. In the first section we briefly mention the related papers published in this area. In the second section, we review and explain the three most relevant references in detail. The third chapter is the *Single Server Models*. We describe the Single Server Models, and then analyze the individual, social and the service provider's problem based on these models. The fourth chapter is the *Two Server Models*. Here after explaining the models corresponding to this chapter, we analyze the three problems based on these models. In the end, by comparing the performance criteria of the previously-mentioned problems based on the different Single Server and Two Server Models, we discuss which models are better in terms of a system design. Moreover, we discuss which resolution alternative is better from the different perspectives, individual, social and service provider. The fifth chapter is the *Service Rate Decision*. In this chapter, we investigate how the performance of the profit maximizer's problem is affected by changing the server rate. The last chapter is the *Conclusions* and we summarize our results and contributions here.

Chapter 2

LITERATURE REVIEW

In recent years, academic research in modeling the strategic customer's behavior in queueing models has shown an increase in volume. In this chapter, we review some of this research in the first section, and then we discuss the most relevant three papers in detail.

2.1 References

The subject of this thesis is related to three main subjects in the literature.
The first subject is "*Strategic Customers in Queueing Systems*". This subject deals with the rational behaviour of the customer, whose goal is to maximize his utility, where he waits in queues of the systems. There are many different topics that are analyzed under this title. The ones which are closely related with the subject of our thesis can be given as: Equilibrium (individual problem of the customer) and the social problem in both of the observable and unobservable queues, the short term (deciding only on the price) and long term (deciding not only on the price but also on the other model parameters) problem of the profit maximizing firm, and the service rate control.
The second subject is "*Queueing Systems Design*". The papers of this subject analyze and compare different queueing system design structures. Queueing systems can be designed in different ways: Single queues can be served by a unique server, by multiple parallel servers providing simultaneous service or by the multiple sequential servers providing service respectively. Similar service design issues can be used for multiple queues. Additionally, in such systems pooling which is the replacement of multi queues by a similar (functionally equivalent) single queue is used in designing

the queueing systems. We give the related references which analyzes parallel systems design, sequential systems design and pooling effect.

The last subject is "Service Quality and Recovery." This subject focuses on the perceived quality of the service, which is generally subjective for the customers, and can be affected by many different measures as the service level, proficiency of servers, outside hearings, loyalty and personal preferences of a customer, waiting in the system etc. The papers of this subject also concern about how the service quality problem can be resolved upon its occurrence. Here we focus on the Service Quality Literature by giving the related references.

For the detailed review of the literature of Strategic Customer, see Hassin and Haviv [20]. In their book, they summarize all the papers that are published on this subject starting from the first known one, namely Naor [30], until the year of 2000's.

Naor [30] observe the aggregate equilibrium pattern of individual where it is not only affected by acts taken by the service providers but also by the acts of the other customers. For a long time, it is known by the economists that this individual equilibrium may not be optimal from the point of social view (society as a whole). But this issue has been considered in the context of queueing theory only after the publication in 1969 of a paper by Naor [30].

The subject of the Naor's paper [30] is the control of *first-come-first-served* (FCFS) *M/M/1* system. In his model, there is a queue manager who announces an entrance price (admission fee) for the customers. Based on this price, customer decides whether or not to join this queue, by comparing his utility value under these two pure strategies. He noticed that in observable queues, where the customer knows their exact position in the queue, the decision of the individual customer is different from the socially optimal one. In order to decrease the mismatch between the individual and social optima Naor suggests imposing an appropriate admission fee. The models given in this thesis are very similar to the one given in Naor [30]. But in his model he assumes that the quality of the service is perfect once the customer joins whereas we are working on the quality (service failure probability) and the recovery problems.

Hassin [19] suggested a way to achieve social optimality without imposing admission fees. He observed that the service discipline regime of LCFS-PR leads to a socially optimal behavior. Here the service discipline regime of LCFS-PR denotes that the newly arrived customer joins the system and is immediately served, possibly preempting the service of another customer. Here preempted customers join a queue where later arrivals get priority over earlier arrivals. When a preempted customer's turn to re-enter service comes, his service is resumed from the point of interruption. In this model, rather then deciding whether to join the queue, individual customer decides when to leave the queue. Olson [32] showed that customers receive priority levels based on their payments under this service discipline regime of LCFS-PR. He shows that the social optimality can be achieved in such a pricing system. These two references deal with the observable queues as we also consider. However, they consider the perfect quality systems and LCFS-PR service discipline, where the service quality is im-perfect and the service discipline is FCFS in our models. Moreover, the decision that an individual faces in our model is whether or not to join to the system rather than when to leave the queue.

For the observable queues, some papers compare the profit maximization decision of the service provider with the socially optima rather than comparing the individual and social decisions. In his model, Naor concludes that the profit maximizing fee is higher compared to socially optimal one. Knudsen [22] generalizes this result to multi-server queues when the waiting cost functions are nonlinear. He shows that the result of Naor holds when the benefit from the service is concave decreasing in total waiting time of the system. Yechiali [39] showed how to compute the profit maximizing fee in *GI/M/1* system. Rusen and Rosenshine [34] investigated the sensitivity of the thresholds and gains in Naor's model to changes in the arrival rates. For the observable queues, Chen and Frank [10] analyzes the state dependent pricing. In their paper, they observed the case where the objective of the profit maximizer matches with the socially optimal one. Thus, they showed that the profit maximizing pricing scheme is to charge the maximum possible fee as long as the queue length is less than a threshold and then charge a higher fee otherwise, when the server is able to adjust the price to the state of the system. Since all of the consumer surplus goes to the server, the optimal profit of the profit maximizer is than socially optimal. Levy and Levy [25] considered an *M/M/1* queue where the server adjusts a price p_i when there are i customers in the system. They proved that under the profit maximizing pricing scheme the expected profit is higher than the related system where customers pay the price before they arrive. When the joining rates are determined through an equilibrium mechanism, Hassin and Haviv [20] suggest an extension of the previous model. In this thesis, we did not consider the problem of the profit maximizing firm when the queue length is observable. But these papers can be references for our future research.

Some of the studies which analyze observable queues with heterogeneous type of customers are listed as: Larsen [23] considered a generalization of Naor's model where the service values of the customers are different. Larsen proves that the profit maximizing fee is greater than or equal to the socially optimal fee. Edelson and Hildebrand [16] showed that the comparison between the profit maximizing and socially optimal price does not necessarily hold when the time values of customers also differ from each other. De Vany [15] considered an observable queue where the service demand is a function of the admission fee. He showed that in Naor's model, the fee charged by a profit maximizing server is too high and thus the rate by which customers join is too small relative to the socially optimal solution. Miller and Buckman [29] considered an *M/M/s/s* model for observable queues.

In this literature, many of the researchers work on the unobservable queue length setting, namely they assume that the customers cannot observe the queue and they decide based on their expected waiting time in the queue. The properties of the basic unobservable single server queue were discovered by Edelson and Hildebrand [16]. They showed that individual optimization leads to queues that are longer than are socially desired, as in the case of observable queues. They also concluded that this gap can be corrected by imposing an admission fee. In this thesis, we also coincide unobservable queueing setting analyzing the equilibrium and social problems of basic *M/M/1* queues, where we similarly conclude that the individually optimal joining rates are higher than the socially optimal ones. By comparing the social problem and the problem of the profit maximizer in unobservable queues, Hassin and Haviv [20] showed in their book that the objectives of the profit maximizer and society coincide. However, we do not have this result in our thesis. Because, we assume that providing service with high quality levels is costly for the profit maximizer and consider this cost in his profit function, whereas since the service is perfect Hassin and Haviv [20] ignore this cost.

Assuming the queue length is unobservable, Chen and Frank [11] considered a short term (pricing problem) and a long term (deciding on the price and the service rate) problems of the profit maximizing firm. In their short term models, assuming all the model parameters are fixed except the queue entrance price, they showed that the comparison between the different pricing schemes (market capturing and monopolistic pricing) depends on the model parameters. In our models, we also consider the short term pricing problem of a profit maximizer and as given in Chen and Frank [11], we show that the comparison between the market capturing and the monopolistic pricing depends also on the model parameters; i.e. when the utilization level of the server is high setting a monopolistic price is more profitable compared to the market capturing pricing. In their long term model, Chen and Frank analyzed the problem of a profit maximizing server in which the cost of maintaining a service rate μ is $b\mu$ per unit of time and the cost of serving the customer is r. They observed that in this model, if a positive profit is possible, then the server will select a processing rate μ and an entrance fee p such that all the potential arrivals will be served. They also observed that the optimal service rate decision does not vary with the cost r and this cost only determines whether a positive profit is possible. They additionally observed that the firm responds to an increase in λ by increasing μ and p. Finally, they observed that the long-run profit maximizing solution is socially optimal as in the short-run. In our thesis, we additionally analyze the long term problem of the profit maximizer. In our long term models, we first assume that the profit maximizer decides on the service quality level for the fixed service rate and then assume that both the service quality level and the service rate are decisions. In our long term analysis, we reach a similar conclusion from a different perspective to the one given in Chen and Frank [11]. That is, in the long term for the market capturing pricing setting, offering a resolution to the unsatisfied in order not to lose them is optimal for the profit maximizer. Thus, when the profit maximizer has chance to play with at least one of his model parameters, it is optimal for him to capture the whole market and not to lose them.

Besides the joining decisions of the individual and social problem and the pricing problem of the profit maximizer, the subject of Service Rate Decisions is explored in the concept of the strategic customers. In these articles the service rate is a decision variable. In most cases, it is the server who determines the service rate, but there are also other models where the service rate decision is made by customers. Mendelson [28] analyzed the long run version of unobservable queueing models of the social planner for the heterogenous service values. In this model, unlike to short run case, the socially optimal solution and profit maximizing solution cannot be compared conclusively. He concluded that the comparison depends on the model parameters. Dewan and Mendelson [14] found that in an *M/M/1* system with linear service rate costs, the optimal admission price is equal to marginal cost of increasing the service rate. Ittig [21] worked with a revenue function depending on the arrival rate and a cost function depending on a number of servers in an *M/M/s* queue or on the service rate in an *M/M/1* queue. For different demand functions, he computed the optimal number of servers and the service rate maximizing the social welfare. In their paper So and Sang [36] characterized the optimal price and capacity for an *M/M/1* system where the service demand function has two parameters: the price and the the α-percentile of the waiting time distribution where α is a decision. Ha [18] considered an unobservable system when customer chooses their own service rate. He showed that the choice of the service rate that the customer chose reflects the amount of service he requests and affects his utility. Cachon and Harker [8] made a similar assumption in their model.

Balachandran and Schaefer [2] described a type of an equilibrium inefficiency: when potential population of customer consists of classes that differ by their cost/reward ratios, a single class dominates in equilibrium and it is not necessarily the socially desired class. They additionally proved that this inefficiency does not exist in the long run model, when the cost of operating the server is linear in the server rate. In their paper, Balachandran and Srinidhi [3] worked with an *M/G/1* model assuming the cost of the service rate μ given an arrival rate λ is proportional to $e^{-(\lambda/\mu)}$. They concluded that the first order optimality conditions of the social problem in the short and long runs coincide. This result holds, since they ignored the dependence between the service rate cost and the arrival rate, in the short run model. Our models related with the service rate decisions are somehow different than the ones given in literature, because previous models all assume that the service quality level is perfect once the customer joins the system. Whereas, since we are analyzing these systems when the service quality is not perfect, in all of our models, we consider two decisions: service quality level and the service rate.

The second subject of the literature which is related with our thesis is the "Queueing Systems Design". This subject analyzes the different design alternatives of the queueing systems. The systems can include a single queue or multiple queues. When a service provider is designing his system, he can use either a single server or multiple servers which provide service to the customers in these queues. The multiple server systems can both be designated in ways that these servers provide simultaneous service (Parallel Servers Systems) and they provide service respectively (Sequential Servers Systems). In his paper, using the service systems which are designated by serial Markovian queues Burnetas [7] analyzed the customer's optimal equilibrium strategies. Namely, the series of *M/M/m* queues with strategic customer behavior is considered in this article. In this model, customers arrive to the first queue and decide whether to join the system, if they join they also decide up to which queue to proceed in this serial system before leaving. Based on the objective of maximizing his own utility value, each customer decides or acts independently. In his model, the decision of a customer is formulated as a game, and using a backward recursive form the unique symmetric Nash equilibrium strategy is identified. By additionally analyzing the social problem and he established the relationship between the equilibrium and social strategies, where these strategies do not coincide. Our models considering the escalations are closely related to the one given in Burnetas [7]. Indeed we restrict this $M/M/m$ systems to an $M/M/2$, and the only decision of the customer is whether or not to join the system. That is the customer decides whether or not to join, and if he joins and is served successfully in the first server, he leaves the system before proceeding to the second server, otherwise he proceeds his service in the second server and leaves the system when his service in this server is completed. Using the priority options, Printezis and Burnetas [33] analyzed $M/M/m$ queues. In their model, the service provider offers priority options, and by deriving the optimal demand of a customer and the policy of a customer as a function of system congestion, remaining options, expiration time, and balking possibility, they identified the optimal option pricing policy. In our models of the escalation systems including $M/M/2$ queues, service provider offers resolution options to unsatisfied customers rather than offering them priorities. The other studies which analyze the multi-server models in the unobservable environment are as follows: In a queueing system, Bell and Stidham [4] analyzed an equilibrium model of routing customers. They analyze both the equilibrium and the social problem, and concluded that, in both of the equilibrium and the socially optimal solution, the active server's input is the service rate minus a portion of the excess of the total rate of service of the active servers over total demand. In social optimization this excess is not distributed uniformly among the active servers whereas in equilibrium solution it is. In the social optimization, the excess is distributed in a way that it is proportional to the square root of their service rates. In a queueing network, Larson [24] analyzes the equilibrium. In this model by choosing their travel speeds, customers move between queues. He thought that customers move at their maximum speeds which is not socially efficient. Mandelbaum and Reiman [26] analyzed the server pooling effects in queueing networks. They concluded that care must be used in pooling. Pooling sometimes helps the service provider by increasing his profit, but it sometimes hurts where its effects can be unbounded. In our analyses, we also showed that if the multi-server models are properly designed they can be profitable for the profit maximizer, otherwise working with a single server is optimal.

The last subject which is related with the subject of this thesis is "Service Quality and Recovery". According to the Gronross [17] the service is a complicated issue. In his paper he discussed the perception of the service quality by the customers. For the perceived quality, Gronross [17] presented a conceptual model. In describing the common characteristics of the perceived quality he used six criteria, namely six criteria of good perceived service quality. They summarize the six criteria as: professionalism and skills, attitudes and behavior, accessibility and flexibility, reliability and trustworthiness, reputation and credibility and recovery. On the other hand, prior research on service failure has focused on four main aspects: types of service failure, recovery strategies, service failure's effect on customer satisfaction, and application of the attribution theory and justice theory. Bell and Zemke [5] focused on the service failure types where they defined the service failures as conditions in which customers are dissatisfied. This dissatisfaction of the customer occurs because the perception of the service that the customer has received is worse than he expected. In our models, we also assume that if a customer receives a perfect quality service he leaves the system satisfied, whereas when the service failure occurs which is not the customer expects, he leaves unsatisfied. By reviewing the prior researches, Smith et al. [35] identified service failure in two types: outcome and process related failure. The outcome related failure refers the what actually occurs during the service. The process related failure contains the manner in which the service is delivered. The service failure type in our models are closer to the former one, which is the outcome related failure, since the failure occurs during the service and as an outcome customer is either satisfied or unsatisfied. According to McColl-Kennedy et al. [27], customers may experience dissatisfaction following a single such failure given that service failures are common in the service industry. When this failure occurs it is important to resolve dissatisfied customers through an appropriate set of actions which is labeled as the customer recovery process. In our models, we also define different resolution actions which deals with the service failure problems. The interaction between service failure and recovery is the one of the common subject in the service management literature. A theory-driven model of customer satisfaction which is related to mental accounting theory is proposed by Chuang, Chang, Yang and Cheng [12]. In this article they considered whether different types of service failure (i.e. outcome- and process-related failure) warrant different types of service recovery (i.e. psychological and tangible recovery). As previously mentioned, the service failure are outcome related in our models, additionally the service recovery, which we refer as the service resolution alternatives can be considered as tangible recoveries, because when the service resolution is proposed to the customer who is subjected to this service failure, he receives the service reward which is defined in monetary values. by investigating the role of service failure severity, Weun, Beatty, Jones [38] developed the previous research on service recovery subject. Results of this paper showed that service failure severity highly affects the satisfaction, trust, commitment and negative word-of-mouth. In our paper, rather than using the severity of the service failure, we use the all or nothing rule, namely the customer either receives the satisfactory service or not. Oh [31] proposed an integrative model of service quality, customer value, and customer satisfaction. This model can be compared with our model, because we also use these three issues in our models. In order to understand the service encounter evaluation Bitner [6] presented a model which synthesizes consumer satisfaction, services marketing, and attribution theories. To assess the effects of physical surroundings and employee responses (explanations and offers to compensate) on attributions and satisfaction in a service failure context, they tested portion of their model experimentally. Cronin and Taylor [13] measured the service quality and the relationships between service quality, consumer satisfaction, and purchase intentions. The purchase intentions of our models can be considered as the joining or not joining decision of the customer.

Namely, in our models we analyze how the service quality level affects the joining decisions of the customer. The results given in Cronin and Taylor [13] provided support that a performance-based measure of service quality can improve the measurements of the service quality construct. They also showed that the service quality is prior to consumer satisfaction. They finally concluded that the consumer satisfaction highly affects the purchase intentions. In our models, we similarly conclude that the joining decisions of the customer's is increasing in the service quality level. All of the summarized researches show that service quality and recovery problem is one of the main subject of the Service Quality literature. However, these papers of the related all ignore the waiting effect of the queueing systems. In this thesis, we try to relate and merge the quality and recovery problems with the waiting effect of the queueing systems.

2.2 Review of the Most Relevant References

In this section, we review three of the main motivating papers, for our study in detail. The purpose is to provide a background for the modeling assumptions in our analysis. We then formulate some basic models with service failure that build on these papers.

- Naor [30]: We review Naor's model which analyzes and compares the individual and social problems when the queue length is observable.
- Edelson and Hildebrand [16]: We review this model comparing the individual and social problems when the queue length is unobservable.
- Chen and Frank [11]: We review the long-run model for the problem of the profit maximizer for the unobservable queue length assumption.
- We start to model basic "Quality Models" and analyze the individual and social problems based on these models. We then compare the performances of the models for the observable queues.

2.2.1 Individual and Social Problem When The Queue Length is Observable

The subject of the paper written by Naor [30], is the control of *first-come-first-served (FCFS) M/M/1* system having strategic customers. This model deals with queueing systems, where an arriving customer observes the length of the queue before making his decision. In this model the queue manager sets an entrance fee, and customers react by setting a pure strategy of whether or not to join this system. He shows that the individual optimization generally defined with a pure threshold strategy. The model assumptions given in Naor [30] is listed as:

- A stationary Poisson stream of customers-with rate λ- arrive to a single server.
- The service times are independent, identically distributed with parameter μ.
- A customer's benefit from completed service is R

  Customers who join the system always take this reward, since Naor assumes that the service is perfect once the customer joins the queue.
- The cost to a customer for staying in the system, either while waiting or while being served, is C per unit of time.
- Customers are risk neutral, that is they maximize the expected value of their net benefit.
- Utility functions of individual customers are identical and additive, from the social point of view.
- $R\mu \geq C^2$, since otherwise not joining is always dominant equilibrium strategy.
- The service discipline is FCFS.
- A decision to join is irrevocable, and reneging is not allowed.

- Upon arrival, a customer inspects the queue length and decides whether to join or not. A customer who leaves, never returns to the system.

The individual's optimization strategy is easy. A customer who joins the queue when i customers are already in the system (including the one who is currently served) expects a benefit $R - \frac{(i+1)C}{\mu}$. The customer then joins if this value is nonnegative. That is if $i + 1 < \frac{R\mu}{C}$. otherwise, the customer leaves. Consequently, the pure threshold strategy, n_e with

$$n_e = \lfloor \frac{R\mu}{C} \rfloor$$

is an equilibrium strategy. Under this strategy an arriving customer joins the queue if he observes $n_e - 1$ or fewer customers and leaves if he observes n_e customers or more. Social optimization is not as trivial. Naor [30] also observes that there exists a pure threshold socially optimal strategy. To show this, he denotes the expected social benefit for unit of time as S_O. Given a maximum queue length of n, the probability of having n customers in the system is:

$$q_n = \frac{\rho^n}{\sum_{i=0}^{n} \rho^i}.$$

Assuming $\rho \neq 1$, the probability that an arriving customer joins is:

$$1 - q_n = \frac{1 - \rho^n}{1 - \rho^{n+1}}.$$

The expected number of customers in the system is:

$$L_n = \frac{\rho}{1-\rho} - \frac{(n+1)\rho^{n+1}}{1-\rho^{n+1}}.$$

Hence,

$$S_O = \lambda R(1-q_n) - CL_n = \lambda R \frac{1-\rho^n}{1-\rho^{n+1}} - C\left[\frac{\rho}{1-\rho} - \frac{(n+1)\rho^{n+1}}{1-\rho^{n+1}}\right] \qquad (2.1)$$

By some lengthy calculations, Naor shows that this social welfare function derived in (2.1) is discretely unimodal in n. In other words, a local maximum is also a global maximum for this function. Thus he seeks a strategy, n_0, which is associated with following inequalities:

$$\lambda R\left[\frac{(1-\rho)\rho^{n_0}}{1-\rho^{n_0+1}} - \frac{(1-\rho)\rho^{n_0+1}}{1-\rho^{n_0+2}}\right] - C\left[\frac{(n_0+1)\rho^{n_0+1}}{1-\rho^{n_0+1}} - \frac{(n_0+2)\rho^{n_0+2}}{1-\rho^{n_0+2}}\right] < 0$$

$$\lambda R\left[\frac{(1-\rho)\rho^{n_0-1}}{1-\rho^{n_0}} - \frac{(1-\rho)\rho^{n_0}}{1-\rho^{n_0+1}}\right] - C\left[\frac{(n_0)\rho^{n_0}}{1-\rho^{n_0}} - \frac{(n_0+1)\rho^{n_0+1}}{1-\rho^{n_0+1}}\right] \geq 0$$

After some algebraic manipulations, he converts the above inequalities to:

$$\frac{n_0(1-\rho) - \rho(1-\rho^{n_0})}{(1-\rho)^2} \leq \frac{R\mu}{C} \leq \frac{(n_0+1)(1-\rho) - \rho(1-\rho^{n_0+1})}{(1-\rho)^2}$$

After deriving threshold inequalities for the individual and the social problem, Naor makes some numerical experiments and concludes that the threshold value of the customer's problem is greater than the social threshold. Namely, let n_e and n^* denote the customer's and social thresholds then $n^* \leq n_e$.

In order to motivate the customers to adopt the threshold n^* rather than n_e, Naor suggested imposing an appropriate admission fee.

2.2.2 Individual and Social Problem When The Queue Length is Unobservable

The properties of the basic unobservable single server queue is discovered by Edelson and Hildebrand [16]. Their model presents the situation where the customers do not observe the queue prior to their actions. They adopt the first eight assumptions of Naor's observable model, and add the following modification: At the time a customer's need for service arises, he irrevocably either joins the queue or balks. It is not possible for him to observe the queue length before making this decision.

As in the observable model, they show that a customer who joins the queue imposes negative externalities on others and therefore individual optimization leads to excessive congestion unless the queue is regulated.

In the paper, Edelson and Hildebrand start by evaluating the customers' behavior in equilibrium when an admission fee of size p is imposed and the potential arrival rate is Λ. There are two pure strategies: to join the queue or not to join. A mixed strategy can be described by a fraction q, $0 \leq q \leq 1.$, which is the probability of joining. Given p, they denote the equilibrium probability of joining by $q_e(p)$, and the effective arrival rate by $\lambda_e(p)$ where $\lambda_e(p) = \Lambda q_e(p) < \mu$. The expected waiting time in the *M/M/1* queue is $W(\lambda) = \frac{1}{\mu-\lambda}$. The net benefit of a customer who joins the queue is $R - CW(\lambda) - p$. They distinguish the possible three cases as:

- $p + CW(0) \geq R$. In this case even if no other customer joins, the net benefit of a customer who joins is non-positive. Therefore, the strategy of not joining is a dominant strategy.

- $p + CW(\Lambda) \leq R$. In this case even if all customers join, they all enjoy a non-negative benefit. So, joining is the dominant strategy.

- $p + CW(0) < R < p + CW(\Lambda)$. In this case if $q_e(p) = 1$, then a customer who joins suffers a negative benefit. So, it cannot be an equilibrium. Likewise if $q_e(p) = 0$, a customer who joins gets a positive benefit, more than by balking. Hence this can also not be an equilibrium. Therefore, there exists a unique equilibrium strategy where $q_e = \frac{\lambda_e(p)}{\Lambda}$ and where $\lambda_e(p)$ solves $CW(\lambda_e(p)) = R - p$. Substituting $W(\lambda) = \frac{1}{\mu-\lambda}$, they obtain the expressions given in the following Table. They then turn their attention to social optimization. They let the socially optimal

Case	$\lambda_e(p)$	$q_e(p)$	$W(\lambda_e(p))$
$\Lambda \leq \mu - \frac{C}{R-p}$	Λ	1	$\frac{1}{\mu-\lambda}$
$0 < \mu - \frac{C}{R-p} \leq \Lambda$	$\mu - \frac{C}{R-p}$	$\frac{\mu - \frac{C}{R-p}}{\Lambda}$	$\frac{R-p}{C}$
$\mu - \frac{C}{R-p} < 0$	0	0	$\frac{1}{\mu}$

Table 2.1: The equilibrium strategy

joining probability be q^*, and the socially optimal joining rate be λ^* where $\lambda^* = q^*\Lambda$. Then,

$$\lambda^* = argmax_{0\leq\lambda\leq\Lambda}\lambda[R - CW(\lambda)]$$

Since $W(\lambda)$ is strictly convex, the function to be maximized is strictly concave and has a unique maximum. Substituting, $W(\lambda) = \frac{1}{\mu-\lambda}$, they get the solution:

$$\mu - \sqrt{\frac{C\mu}{R}} = argmax_{0\leq\lambda\leq\Lambda}\left[\lambda R - \lambda C\frac{1}{\mu-\lambda}\right]$$

is optimal as long as it is in $[0, \Lambda]$. The fact that the solution is nonnegative follows from the assumption that $R\mu \geq C$. Thus if $\Lambda \geq \mu - \sqrt{\frac{C\mu}{R}}$ then $\lambda^* = \mu - \sqrt{\frac{C\mu}{R}}$. Otherwise $\lambda^* = \Lambda$. In the unobservable environment denoting the social welfare under the optimal arrival rate λ^* with, S_U, the socially optimal strategy is summarized as: From the assumption that $R\mu \geq C$, $\lambda_e(0) \geq \lambda^*$. Thus as in the case of observable queues, Edelson and Hildebrand [16] conclude that individual optimization leads to queues that are longer than are socially desired. This gap can be corrected by imposing an appropriate admission fee.

Case	λ^*	q^*	$W(\lambda^*)$	S_U
$\Lambda \geq \mu - \sqrt{\frac{C\mu}{R}}$	$\mu - \sqrt{\frac{C\mu}{R}}$	$\frac{\mu - \sqrt{\frac{C\mu}{R}}}{\Lambda}$	$\sqrt{\frac{R}{C\mu}}$	$(\sqrt{R\mu} - \sqrt{C})^2$
$\Lambda \leq \mu - \sqrt{\frac{C\mu}{R}}$	Λ	1	$\frac{1}{\mu - \Lambda}$	$\Lambda\left(R - \frac{C}{\mu - \Lambda}\right)$

Table 2.2: The socially optimal strategy

2.2.3 Problem of the Profit Maximizer When The Queue Length is Unobservable

Chen and Frank [11] analyzes the pricing problem of a profit maximizing firm who provides service to strategic customers. They classify their analysis in two main parts. In the first part, which is labeled as the short term analysis, assuming all model parameters (R, C, λ and μ) are fixed, they try to maximize the profit of the profit maximizer by comparing different pricing settings. The possible pricing settings are market capturing pricing, which is the minimum price that incites everybody to decide to join, and the monopolistic pricing, in which not all of but some of the customers decide to join with a unique equilibrium joining probability. In the second part, they consider a long-run model of a profit maximizing server in which the cost of maintaining a service rate μ is $b\mu$ per unit of time (other model parameters R, C, and λ are being fixed); namely a linear cost assumption, and the cost of serving a customer is r. They observe in their model that, if a positive profit is possible, then the server will select a processing rate μ, and an admission fee p such that all the potential arrivals will be served. Since the maximum fee that can be charged while maintaining the arrival rate Λ is $p = R - C/(\mu - \Lambda)$, the problem of the service provider becomes

$$max_{r<p<R-\frac{C}{\mu}} \quad (p - r)\Lambda - b\left(\Lambda + \frac{C}{R-p}\right).$$

The solution is:

$$p^* = R - \sqrt{\frac{Cb}{\Lambda}} \qquad \mu^* = \Lambda + \sqrt{\frac{C\Lambda}{b}}.$$

The observations given in Chen and Frank [11] is listed as:

- The solution does not vary with the cost r, of serving a customer. This cost only determines whether a positive profit is possible.

  The condition for a positive profit is $r < R - (Cb/\Lambda)^{1/2}$. The right hand side is the optimal admission fee, p^*.

- The firm responds to an increase in Λ by increasing μ and p.

- As in the short run model, the profit maximizing solution is socially optimal.

2.2.4 Basic Quality Models

In this section, we start to build basic models which consider the possibility of service failure. These models are mainly extensions of the model given in Naor [30]. Naor[30] assumes service is perfect once the customers join the system. Thus, customers decide by comparing their waiting costs with the value of the service. Since service systems do not always provide service with perfect quality, waiting time criteria is not enough for customers while deciding. Customers also take the chance of the service failure probability into account.

Based on this motivation, we add a quality parameter, q where $0 \leq q \leq 1$,to our elementary models and numerically analyze the optimal joining strategies of the individual and social problems in this section.

We keep all the model assumptions given in Naor [30], with only a modification in the third assumption as: *The value of the service is R, but it is taken by only the customers who receive a satisfactory service. Receiving a satisfactory service, depends on the service quality level, q. So, with some probability $1 - q$, service failure occurs,* *and customers leave the system without taking the service reward.*

The first quality model which we analyze in this chapter is the *Benchmark Model*. In this model we assume both the satisfied (customers who are successfully served) and unsatisfied (customers who are subjected to a service failure) customers leave the system without returning.

We secondly analyze a model which takes the returns of the customers into account. We label this model as the *Model With Returns*. In this second model, the satisfied customers who receive a satisfactory service leave the system where the unsatisfied ones return to complete their service. For the Model With Returns, we assume that the returns of the customers are memoryless. Namely, when a customer returns to the system his new decision of whether or not to join the system is assumed to be independent from his past experience. So these returning customers are assumed as new customers for the system.

2.2.4.1 Individual Problem Of The Customers

In this section we assume that the queue length is observable. Thus, the utility of the individual customer is a function of his position, i, in the queue. The reason behind this is: Although the expected service value is the same for all customers (all customers are homogenous), the waiting cost of the i^{th} customer differs from the $(i+1)^{th}$ customer, where the utilities of customers decrease with the increase in the cost of waiting.

For both of the quality models, the Benchmark Model and the Model With Returns, the server provides service with rate μ. Thus for the given model parameters, λ, μ, R, C and q, the utility value of the ith customer of the system is the same in these models. Using the notation $U(i)$, the utility function of the ith customer of the system is represented as:

$$U(i) = Rq - CE[W(i)] = Rq - (i)C\frac{1}{\mu}, \tag{2.2}$$

where $E[W(i)]$ represents the expected waiting time of the i^{th} customer.

Lemma 2.1 $U(i)$ is decreasing in i.

We give the proofs regarding to this section in Appendix A.1.
Since the individual utility function given in (2.2) is decreasing in i as given in Lemma 2.1, to characterize the joining strategies of the customers, a threshold value can be defined. This value represents the queue length up to which customers join. If n^*_{ind} denotes the threshold level of individuals, it can be determined with the following inequality:

$$n^*_{ind} = \lfloor \frac{Rq\mu}{C} \rfloor \tag{2.3}$$

Thus, for the observable queue length conditions the optimal strategy of the customers is characterized as:

$$i = \begin{cases} not\,join, & \frac{C}{\mu} \geq Rq \;\; or \;\; i > n^*_{ind} \\ join, & o.w. \end{cases}$$

Optimal joining strategy of the individuals is interpreted as: If the joining decision is worthwhile, then customers join up to a threshold level. After this level, since the expected waiting cost of customers exceeds the expected service value, they decide not to join.

Corollary 2.1 n^*_{ind} is increasing in R, μ, q decreasing in C, and is independent from λ.

We interpret Corollary 2.1 as: Increases in R and q increase the expected service value. Increasing the service rate decreases the waiting cost. So, these parameters positively affect the joining decision of customers. On the other hand, an increase in the unit waiting cost negatively affects the decision of the customers. Additionally, since when taking their decisions, customers only care about themselves, the individual decisions are independent from the potential arrival rate.

2.2.4.2 Social Problem

In the social problem, the social planner aims to maximize the social welfare (total expected utility function of all customers). For this total expected utility function, the total expected waiting cost of customers must be derived. Thus, we first give the state transition diagram of both of the models. The difference between the two state

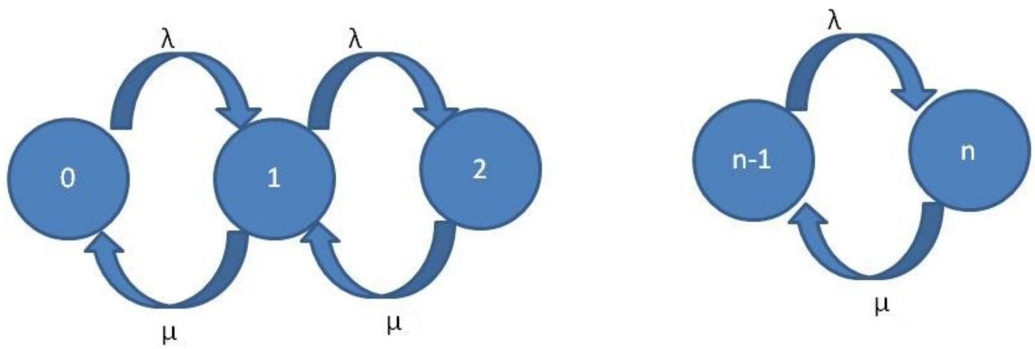

Figure 2.1: State Transition Diagram of Benchmark Model

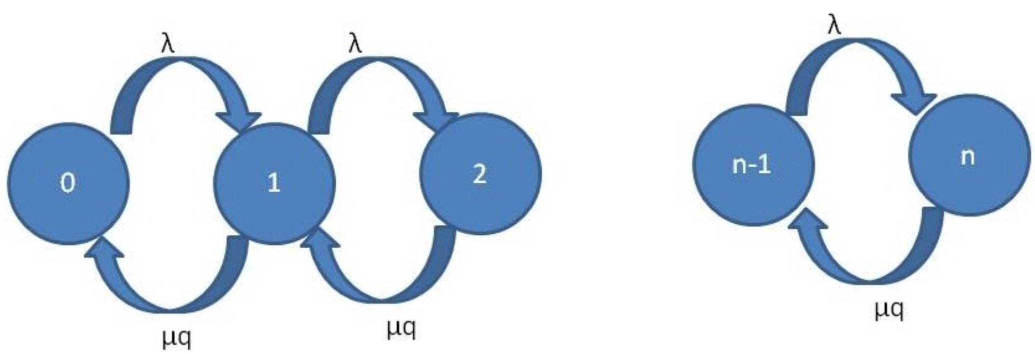

Figure 2.2: State Transition Diagram of The Model With Returns

transition diagrams given in Figures 2.1 and 2.2 is the service rate. In the Benchmark Model, since both of the satisfied and unsatisfied customers leave the system, the service rate is μ. With this service rate and the arrival rate λ, the utilization level of this system is $\rho_1 = \frac{\lambda}{\mu}$. However for the Model With Returns, since unsatisfied customers make memoryless returns, the service rate, which decreases the number of customers in the system is μq. Hence the utilization level of this model is $\rho_2 = \frac{\lambda}{\mu q}$. Since these two models are basic *M/M/1* queueing models, the expressions for steady state probabilities, expected queue length and expected waiting time in the system are all known. Assuming that the social planner accepts n customers to the system, the total expected utility function of the social system, $S_O(n)$, of these two models can be represented as:

$$S_O(n) = Rq\lambda(1-\Pi_n) - CE[W(n)] = Rq\lambda\frac{1-\rho^n}{1-\rho^{n+1}} - C\left[\frac{\rho}{1-\rho} - \frac{(n+1)\rho^{n+1}}{1-\rho^{n+1}}\right], \quad (2.4)$$

where Π_n and $E[W(n)]$ respectively denote the steady state probability of having n customers in the system, and the total expected waiting time.

Lemma 2.2 $S_O(n)$ is discretely unimodular in n.

Since the social welfare function given in (2.4) is unimodular and non-decreasing, as expressed in 2.2, the local maximum is a global maximum. Thus, the pure strategy of the social planner is defined with a threshold value, n^*_{sys}, satisfying the following inequality:

$$\frac{n^*_{sys}(1-\rho) - \rho(1-\rho^{n^*_{sys}})}{(1-\rho)^2} \leq \frac{Rq\mu}{C} < \frac{(n^*_{sys}+1)(1-\rho) - \rho(1-\rho^{(n^*_{sys}+1)})}{(1-\rho)^2} \quad (2.5)$$

The optimal strategy of the social planner for these two models is summarized as:

$$i = \begin{cases} \textit{do not accept}, & \frac{C}{\mu} \geq Rq \ \ or \ \ i > n^*_{sys} \\ \textit{accept}, & o.w. \end{cases}$$

The structure of the social planner's strategy is similar in the two models, except for the difference between the threshold values. Since the utilization values of the two models differ from each other, the optimal strategy of the social planner is characterized with different threshold values in the two models. That is, the utilization value used in (2.5) is ρ_1 for the Benchmark Model, and ρ_2 for the Model With Returns.

Corollary 2.2 n^*_{sys} is increasing in R, μ, q decreasing in C, and λ.

Corollary 2.2 is expressed similarly to Corollary 2.1. The difference is the dependence on the potential arrival rate. Although the individual threshold is not affected by the potential arrival rate, the social threshold is, which shows that the social planner has to take all the customers into consideration in making his decision.

2.2.4.3 Numerical Study

In this section, we numerically explore which represent the optimal strategies of the individual customers and the social planner for the Benchmark Model and the Model With Returns. In the Figures, we use the label Model 1 to denote the Benchmark Model and Model 2 for the Model With Returns.

We list the results of these observations represented in Figure 2.3 as:

- There is a mismatch between the individually and socially optimal thresholds.

  Individual threshold is higher than the socially optimal one.

- The individual threshold is not affected from the potential arrival rate; the social threshold is.

  For the same model parameters, the individual threshold values are the same for $\lambda = 0.5$ and $\lambda = 1.2$, while the threshold of the social system is lower when $\lambda = 1.2$ compared to the case $\lambda = 0.5$.

  Since the congestion of the system is higher for the higher arrival rate, the social planner accepts fewer customers into the system.

- For the lower quality levels, the social planner accepts fewer customers into the system in the Model With Returns, relative to the base model.

The number of the unsatisfied customers is high for low quality levels and these customers return to the system by increasing the load of the server. Thus, the social planner optimizes the social system by using a lower threshold.

For higher quality levels, since the number of returning customers are lower and the utilization of the two models are closer, the socially optimal threshold values of these two models are similar, i.e. models behave similarly.

- Increase in the service reward and the quality level positively affect the individual and social joining strategies.

  Since the expected service value increases, joining decision is optimal.

From the analyses given in this chapter, similar results to Naor [30] are observed. The individually optimal joining strategies of the customers differ from the socially optimal ones. This shows that, joining decisions of the customers negatively affect the social system. Thus, there needs to be a factor which decreases the optimal threshold value of the customers to decrease the mismatch between the individually and socially optimal thresholds.

One factor which decreases this mismatch is setting a queue entrance price, p, for the customers who join the queue. This price decreases the utility of the customers, so fewer customers join the system. On the other hand, since this entrance price is a transfer payment for the social problem, it does not affect the social system. Thus, the mismatch between the optimal values of the individual and social problems decrease.

The other factor is hiding information from the customers. In all the analyses given in this chapter, observable queue length conditions are assumed. Based on these conditions, since customers are informed about the system, they precisely decide if joining is useful for them or not. On the other hand, if the queue length information is kept from them, they only know their expected waiting time. So they can choose not to join based on this expected time, although it is higher than the actual waiting time. Using these two results, from now on, we assume that the queue length is unobservable. In the next chapter, we reanalyze these two problems in the unobservable queue length setting. We additionally analyze the problem of the service provider (profit maximizer). In analyzing this third problem, we add the queue entrance price p to the customers' problem, assuming it is taken by the service provider and generates revenue for him.

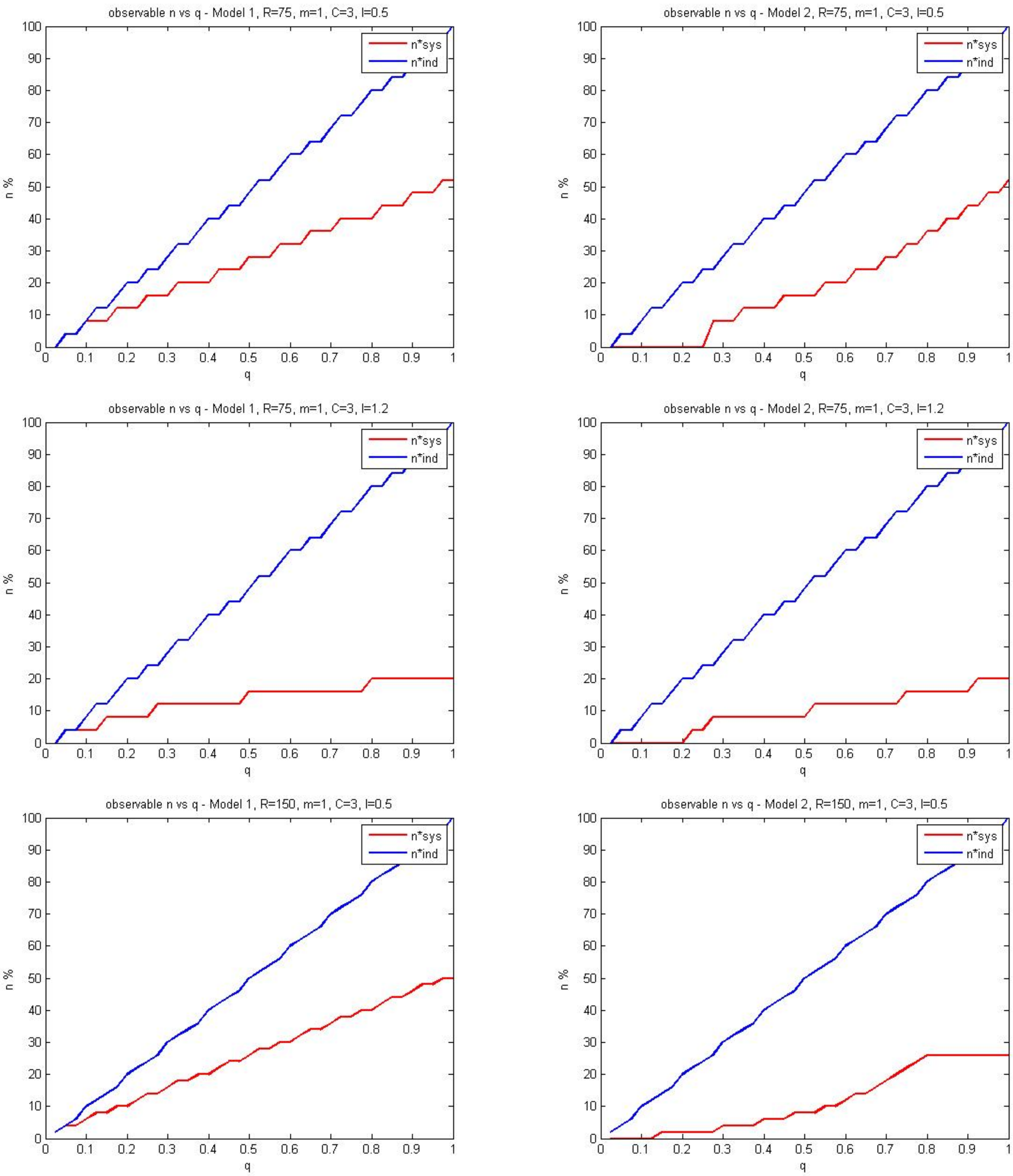

Figure 2.3: Comparison Between Individual and Social Thresholds- Benchmark Model and The Model With Returns

Chapter 3

SINGLE SERVER MODELS

In this chapter the single stage queueing models implementing quality issues that were introduced in Chapter 2 are analyzed in a different setting where the queue length is unobservable and an entrance price is charged to customers who decide to join. Keeping all the model assumptions the same, except the observable queue length assumption, in addition to the two quality models described in the previous chapter, *Benchmark Model and The Model With Returns*, the third model namely *The Model With Resolution* are analyzed in this chapter. Besides the Individual Problem of the Customer and the Social Problem, we also analyze the Problem of the Profit Maximizer. To represent the revenue of the profit maximizer, we add a queue entrance price to the problem of the customers; i.e. the profit maximizer charges an entrance price to customers. This price additionally decreases the mismatch between the individual and social optima as expressed in Naor [30], Edelson and Hildebrand [16] and Chen and Frank [11]. Since providing a service with higher quality levels is more costly for the profit maximizer, we add a cost expression to the problem of the profit maximizer, assuming this cost has a quadratic structure in quality, i.e. assuming the unit cost of the service quality level is a, the total cost of providing service with q units of quality is aq^2.

The organization of this chapter is as follows: In the first three sections of this chapter, we analyze the mentioned problems for the Benchmark Model, The Model With Resolution and The Model With Returns respectively. In the last section, we give the comparison between these models. To compare the models from the individual and the social viewpoint, we use the individually and socially optimal joining probabilities. We give the optimal quality level, optimal price and the optimal profit comparison for the profit maximizer's problem, where the model comparison criteria is the optimal profit value.

We give all the Proofs of the Lemmas, Corollaries and Propositions corresponding this chapter in Appendix A.2. The Observations representing the Numerical Studies of this Chapter can be found in Appendix B.1.

3.1 The Benchmark Model

We introduce the Benchmark Model in Chapter 2 in the observable queue length setting. Since in this chapter the queue length is unobservable for the customer, the representation and the scenario of the model is changed.

We represent the Benchmark Model when the queue length is unobservable, in Figure 3.1.

The scenario of the Benchmark Model is: Customers arrive to the system with the

Figure 3.1: Graphical representation of Benchmark Model

potential arrival rate, λ. Denoting the joining probability with α where $0 \leq \alpha \leq 1$, some of the customers decide not to join the system with rate $\lambda(1-\alpha)$, while others decide to join. The ones who decide to join, enter the queue in front of the server and wait until they are served. Some of them, who join the queue and wait, receive satisfactory service with probability q, while others are subjected to a service failure. Both of these satisfied and unsatisfied customers, with rates $\lambda\alpha q$ and $\lambda\alpha(1-q)$ respectively, leave the system.

We analyze the individual problem of the customers, the social problem and the problem of the profit maximizer of this Benchmark Model in this section.

3.1.1 Individual Problem Of The Customers

The queue length is unobservable and customers are assumed to decide based on the expected waiting time in the system. Since this information, expected waiting time in the system, is the same for all of the customers who are homogenous, they all decide similarly. Thus, the individual problem of customers in an unobservable setting is also defined as the *equilibrium problem* of customers.

The decision variable of the customer's individual problem is the joining probability, α. The equilibrium joining probability of customers is found based on the utility function of the individual customer. The variables of this utility function are: expected service value, total expected waiting cost and the queue entrance price. The customer is assumed to join the queue if his utility function is positive.

However in analyzing the individual problem of the customer, we ignore the entrance price. Because this price is fixed and using it in the individual problem does not change the strategy of the customer, and only shifts the solution (equilibrium joining probability). Based on this reasoning, in analyzing the individual problem of the customers in all of the models, we omit the entrance price and take it into the consideration in analyzing the problem of the profit maximizer.

Using the subscript 1 for the Benchmark Model the utility function of the individual customer is represented as:

$$U_1(\alpha) = Rq - CE[W] \Rightarrow U_1(\alpha) = Rq - \frac{C}{\mu - \lambda\alpha} \tag{3.1}$$

Lemma 3.1 *$U_1(\alpha)$ is concave in α.*

Corollary 3.1 The equilibrium joining probability of the customers for the Benchmark Model is:

$$\alpha_1^{eq} = \begin{cases} 0, & Rq \leq \frac{C}{\mu} \\ \frac{\mu - \frac{C}{Rq}}{\lambda}, & \frac{C}{\mu} < Rq < \frac{C}{\mu-\lambda} \\ 1, & Rq \geq \frac{C}{\mu-\lambda} \end{cases}$$

The interpretation of this result is: In the first case even if no customer joins, the net benefit of a customer who joins is non-positive. Therefore the strategy of not joining is a dominant equilibrium strategy; i.e. no other equilibrium is possible. In the second case, if all potential customers enter, then a customer who joins gets a negative benefit; hence, this cannot be an equilibrium strategy. If no one enters, then a customer who joins gets a positive benefit, more than by balking. Hence this can also not be an equilibrium. Therefore, there exists a unique equilibrium strategy, which solves $U_1(\alpha) = 0$. In the third case, even if all potential customers join, they all enjoy a non-negative benefit. Therefore, joining is a dominant equilibrium strategy.
From the given expression, we conclude that the equilibrium joining probability is increasing in μ, R and q; and decreasing in λ and C. This result is obvious, since the increase in the parameters R and q, increase the expected reward of the customer, so the customers become more desirous to join in this case. On the other hand, increase in λ and C or decrease in μ, increase the total expected waiting cost of a customer, the joining decision of a customer is negatively affected.

3.1.2 Social Problem

For the social problem we assume that there is a social planner who is responsible for maximizing the social welfare (utility function of the whole system).
The queue entrance price is defined as a transfer payment for the social system. Thus, the social problem does not include this price.
To find the socially optimal joining probabilities -rates- of the Benchmark Model, we analyze the total expected utility function of the entire system.

$$S_1(\alpha) = \lambda\alpha\,[Rq - CE[W]] \Rightarrow S_1(\alpha) = \lambda\alpha \left[Rq - \frac{C}{\mu - \lambda\alpha}\right] \qquad (3.2)$$

Lemma 3.2 *$S_1(\alpha)$ is concave in α.*

Since the concavity holds, there is a unique probability value, α_1^*, which maximizes the social function given in (3.2):

$$\frac{\partial S_1(\alpha)}{\partial \alpha} = 0 \Rightarrow \alpha_1^* = \frac{\mu - \sqrt{\frac{C\mu}{Rq}}}{\lambda}$$

Corollary 3.2 The socially optimal joining probability of the Benchmark Model is:

$$\alpha_1^* = \begin{cases} \frac{\mu - \sqrt{\frac{C\mu}{Rq}}}{\lambda}, & \mu - \lambda < \sqrt{\frac{C\mu}{Rq}} \\ 1, & o.w. \end{cases}$$

The socially optimal joining probability is increasing in μ, R and q; and decreasing in λ and C.

Observation 3.1: $\alpha_1^{eq}(q) \geq \alpha_1^*(q)$.

To check the numerical values of this Observation please see Table Tab. B.1. in Appendix. For the different model parameter values, when we compare the second column of this table with the third column, we can observe this result.

As Edelson and Hildebrand [16] represent in their paper, which analyzes and compares the equilibrium and socially optimal joining strategies in perfect quality type queueing systems when the queue length is unobservable, there is a mismatch between these two strategies. Because, a customer only considers himself upon taking his decision and does not take the load (congestion) that his joining decision creates in the whole system into the account.

3.1.3 Problem Of The Profit Maximizer

The aim of the profit maximizer is to maximize his profit by setting the optimal queue entrance price and the service quality level. In optimizing his strategy, he takes the decisions of strategic customers into account. He is not allowed to charge different entrance prices or provide service with different quality levels to different customers.

Customers are assumed to know the entrance price and the expected service quality level. Since the unobservable queue length condition holds, customers know their expected waiting time but do not observe their current state in the queue.
Assuming quadratic cost structure for the service quality level, with the unit cost of a, we analyze the problem of the profit maximizer for the Benchmark Model in this section.
Customers who decide to join pay the queue entrance price before receiving the service. Using the notation $\Pi_1(.)$ for the profit, the problem of the profit maximizer for the Benchmark Model is represented as:

$$\Pi_1(p,q) = p\lambda\alpha_1^{eq} - aq^2 \tag{3.3}$$

So, while setting the optimal entrance price and the quality level, the profit maximizer considers the joining decisions of the customers. Since there are three possible equilibrium joining strategies, the profit maximizer's problem is analyzed in three parts.

1. If $(C/\mu) > Rq - p$, then since the expected utility of the customer is negative even for the empty system, no customer joins.

   To prevent losses, the profit maximizer sets the queue entrance price and the quality level to 0: $p_{1,n} = 0$, $q_{1,n} = 0$, $\Pi_1(p_{1,n}, q_{1,n}) = 0$, where subscript n is used for this no entrance case.

2. If the profit maximizer chooses to serve the whole market, $\alpha_1^{eq} = 1$, he sets the market capturing price, $p_{1,\lambda}$ (notation for market capturing price of the benchmark model).

   This market capturing price, which is a function of the service quality level and is found by equating (3.1) to zero: $p_{1,\lambda}(q) = Rq - \frac{C}{\mu-\lambda}$.

The profit function with this entrance price and equilibrium joining rate is:

$$\Pi_1(p_{1,\lambda}, q) = p_{1,\lambda}\lambda - aq^2 = \left(Rq - \frac{C}{\mu - \lambda}\right)\lambda - aq^2 \tag{3.4}$$

.

Lemma 3.3 $\Pi_1(p_{1,\lambda}, q)$ is concave in q.

Using the concavity given in Lemma 3.3, the unique solution of the optimal quality level for the market capturing price setting which maximizes (3.4) is:

$$\frac{\partial \Pi_1(p_{1,\lambda}, q)}{\partial q} = 0 \Rightarrow \lambda R - 2aq = 0 \Rightarrow q_{1,\lambda} = \frac{\lambda R}{2a} \tag{3.5}$$

.

Plugging the optimal quality level given in (3.5), into the market capturing price and profit functions, the optimal values of the profit maximizer's problem is summarized as:

$$q_{1,\lambda}, p_{1,\lambda}, \Pi_1(p_{1,\lambda}, q_{1,\lambda}) = \begin{cases} \frac{\lambda R}{2a}, \frac{\lambda R^2}{2a} - \frac{C}{\mu-\lambda}, \frac{\lambda^2 R^2}{4a} - \frac{C\lambda}{\mu-\lambda} & if R\lambda < 2a \\ 1, R - \frac{C}{\mu-\lambda}, R\lambda - \frac{C\lambda}{\mu-\lambda} - a & \text{o.w.} \end{cases}$$

Corollary 3.3 $q_{1,\lambda}$ is nondecreasing in λ and R and non increasing in a.

The interpretation of Corollary 3.3 is: For the higher service reward values, more customers decide to enter. To prevent the congestion of the system, the profit maximizer provides service with higher quality levels. This is one of the interesting results which we obtain in this research. Because, if there was no queueing effect in this problem, if the value of the service is high for a customer, then he should choose to join the system even when the service quality level is low. In this case, the service reward and the quality level would be inversely proportional. However, since these systems involve queueing and the increase in the service reward also increases the congestion level of the system (every customer becomes more desirous to join when the service reward increases), the profit maximizer must provide service with higher quality levels to deal with this congestion. This also increases his profit, because a higher entrance price is charged for higher expected service values. With a similar reasoning, an increase in potential arrival rate causes an increase in the quality level. On the other hand, since the value of the quality is inversely proportional with its unit cost, the quality level decreases in unit quality cost.

3. If the profit maximizer decides not to serve all the potential customers, since the waiting cost of the customer who joins is lower, the profit maximizer should set a higher entrance price compared to the market capturing price. We denote this price as the monopolistic price and label it with the subscript m. In this case, customers join with the equilibrium joining probability expressed in Corollary 3.1.

   The profit maximizer's problem with this equilibrium joining probability is:

$$\Pi_1(p,q) = (p\lambda\alpha_1^{eq} - aq^2) = p\lambda\frac{\left(\mu - \frac{C}{Rq-p}\right)}{\lambda} - aq^2. \tag{3.6}$$

Lemma 3.4 $\Pi_1(p,q)$ is concave in p.

Using the concavity given in Lemma 3.4, the value of the monopolistic price of the Benchmark Model depending on the quality level is:

$$\frac{\partial \Pi_1(p,q)}{\partial p} = 0 \Rightarrow \mu - \frac{CRq}{(Rq-p)^2} = 0 \Rightarrow p_{1,m}(q) = Rq - \sqrt{\frac{CRq}{\mu}} \tag{3.7}$$

Plugging this monopolistic price given in (3.7), into the equilibrium joining probability expression, we have: $\alpha_1^{eq}(p_{1,m},q) = \frac{\mu - \sqrt{\frac{C\mu}{Rq}}}{\lambda}$.

Using these optimal joining probability and the monopolistic entrance price the profit maximizer's problem is rewritten as:

$$\Pi_1(p_{1,m},q) = p_{1,m}\lambda\alpha_1^{eq} - aq^2 = \left(Rq - \sqrt{\frac{CRq}{\mu}}\right)\lambda\frac{\mu - \sqrt{\frac{C\mu}{Rq}}}{\lambda} - aq^2 = (\sqrt{Rq\mu} - \sqrt{C})^2 - aq^2 \tag{3.8}$$

Lemma 3.5 *$\Pi_1(p_{1,m},q)$ is convex increasing in q.*

Lemma 3.5 shows that: For the region where the cost of increasing the quality level by one unit is smaller than the unit increase in the profit maximizer's gain, the profit maximizer should increase the quality level as much as possible.

This means, the profit maximizer sets the service quality level to $q_{1,m} = 1$ in the benchmark model when he sets the monopolistic price.

The optimal values of the profit maximizer's problem in the monopolistic case is:

$$q_{1,m} = 1 \quad p_{1,m} = R - \sqrt{\frac{CR}{\mu}} \quad \Pi_1(p_{1,m},q_{1,m}) = \left(\sqrt{R\mu} - \sqrt{C}\right)^2 - a$$

The profit maximizer finds his optimal strategy by comparing his profit values under three possible pricing cases. Assuming the positive profitability condition holds, he sets the market capturing price and serves the whole market if his profit is higher compared to the profit of the monopolistic price case. Otherwise, he sets the monopolistic price and serves not all of the customers but some of them who join based on the equilibrium joining rate. Unfortunately, since the profit functions have different structures in these two pricing strategies, we cannot compare them. But under some conditions we know which strategy is optimal.

Proposition 3.1 If $\mu - \lambda > \sqrt{\frac{C\mu}{R}}$, then market capturing price strategy is optimal.

The interpretation of the result given in Proposition 3.1 is as follows: If the difference between the server rate and the potential arrival rate is greater than the per unit gain of the profit maximizer when he uses first order pricing, then it is better for him to choose the market capturing case.

Proposition 3.2 *If* $\lambda \geq \mu$*,* then the monopolistic price strategy is optimal.

The reasoning behind Proposition 3.2 is obvious. In this case, if all customers join the system, then the queue length and the expected waiting time in the system increase quickly (infinite queue), which negatively affects the utilities of customers.

3.2 The Model With Resolution

In Resolution Models, we assume that when the service failure occurs, customers return to the same system for service failure resolution.In this section, we analyze two Resolution Models. So, in these models if a customer decides to join in the beginning, she can not leave the system until she is successfully served. That is, in these models a joining customer takes the service reward, R, for sure.

The thing which differentiates the two Resolution Models is the returning position of the unsatisfied customers. In the first model, we assume that an unsatisfied customer returns to the beginning of the queue, so she occupies the server until she is successfully served. In the second one, an unsatisfied customer is assumed to return to the end of the queue.

We represent the first Resolution Model, in which the unsatisfied customer returns to the beginning of the queue for the resolution in Figure 3.2.

In order to analyze the individual, social and the profit maximizer's problem, we

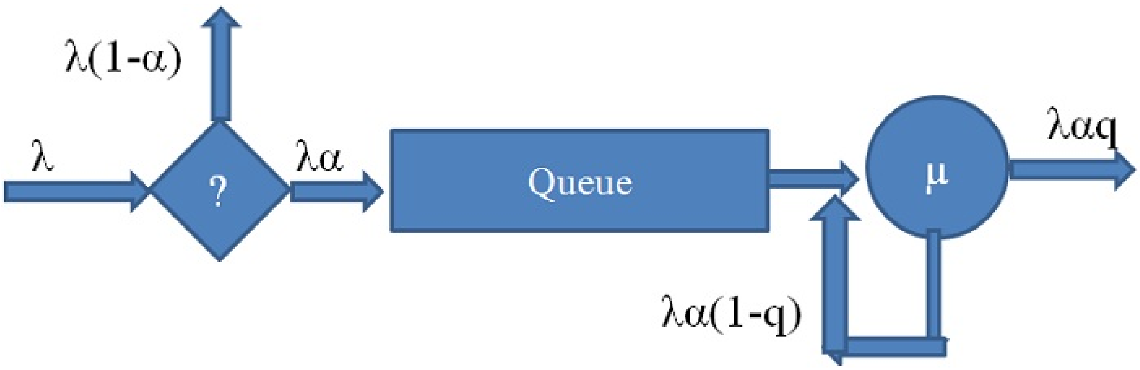

Figure 3.2: Graphical representation of the first Resolution Model

have to derive the total expected waiting time expression for this model. Since only the satisfied customers leave this system, the server utilization is: $\rho_2 = \frac{\lambda}{\mu q}$. The conditional expected waiting time of a customer who finds i customers in-front is:

$$E[W|i] = \left(\frac{i+1}{\mu}\right)\frac{1}{q}$$

Now, we uncondition this waiting time as:

$$E[W] = E[W|i]P(N = i) = \frac{i+1}{\mu}\frac{1}{q}(1-\rho)\rho^i =$$

$$\frac{\mu q - \lambda}{(\mu q)(\mu q)}(i+1)\left(\frac{\lambda}{\mu q}\right)^i = \frac{\mu q - \lambda}{\mu^2 q^2}\frac{\mu^2 q^2}{(\mu q - \lambda)^2} = \frac{1}{\mu q - \lambda}, \tag{3.9}$$

where $E[W_{tot}]$ represents the total waiting time that a customer waits in expectation (considering his probable returns).

In the other Resolution Model, we assume that an unsatisfied customer returns to the end of the queue. Here, we assume that the returns take place after a sufficiently long time and that the returning customer finds the queue in a steady state. We represent this model graphically in Figure 3.3.

The conditional waiting time of a customer who finds i customers in-front in this

Figure 3.3: Graphical representation of the second Resolution Model

model is:

$$E[W|i] = \frac{i+1}{\mu}$$

We give the unconditional waiting time as:

$$E[W] = E[W|i]P(N=i) = \frac{i+1}{\mu}(1-\rho)\rho^i =$$
$$\frac{\mu q - \lambda}{\mu(\mu q)}(i+1)\left(\frac{\lambda}{\mu q}\right)^i = \frac{\mu q - \lambda}{\mu^2 q}\frac{\mu^2 q^2}{(\mu q - \lambda)^2} = \frac{q}{\mu q - \lambda}$$

Since an unsatisfied customer waits this time until she is successfully served; i.e. $1/q$ times in expectation, the total expected waiting time of a customer in this model is:

$$E[W_{tot}] = E[W]\frac{1}{q} = \frac{q}{\mu q - \lambda}\frac{1}{q} = \frac{1}{\mu q - \lambda} \tag{3.10}$$

Proposition 3.3 The total expected waiting time of a customer, in the two Resolution Models are the same.

The interpretation of the result given in Proposition 3.3 is: Since the queue length is assumed as unobservable, the position of a customer does not change anything. The utilization level of the server, which depends on the effective arrival rate and the service rate, is the only thing which affects the total expected waiting time. Since the utilization level of these two models are equal, so are the total expected waiting times.

We showed that the total expected waiting time of the two Resolution Models are the same. Moreover, the expected rewards of a customer are the same in these models. So these two models behave the same. Based on this result, from now on, we will analyze a single Resolution Model.

3.2.1 Individual Problem Of The Customers

We use the subscript 2 to denote the Model With Resolution. The individual utility function of a customer in this model is:

$$U_2(\alpha) = R - CE[W] \Rightarrow U_2(\alpha) = R - \frac{C}{\mu q - \lambda\alpha} \tag{3.11}$$

Lemma 3.6 $U_2(\alpha)$ is concave in α.

Corollary 3.4 The equilibrium joining probability of the customers for the Model With Resolution is:

$$\alpha_2^{eq} = \begin{cases} 0, & Rq \le \frac{C}{\mu} \\ \frac{\mu q - \frac{C}{R}}{\lambda}, & \frac{C}{\mu q} < R < \frac{C}{\mu q - \lambda} \\ 1, & R \ge \frac{C}{\mu q - \lambda} \end{cases}$$

This result is very similar to the one given in Corollary 3.1 which presents the equilibrium strategy of the Benchmark Model.
The equilibrium joining probability of the Model With Resolution is increasing in μ, R and q and decreasing in λ and C, as it is in the Benchmark Model.

3.2.2 Social Problem

The total expected utility function of the entire system in the Model With Resolution is:

$$S_2(\alpha) = \lambda\alpha\left[R - CE[W]\right] \Rightarrow S_2(\alpha) = \lambda\alpha\left[R - \frac{C}{\mu q - \lambda\alpha}\right] \tag{3.12}$$

Lemma 3.7 $S_2(\alpha)$ is concave in α.

Since the concavity holds, there is a unique probability value, α_2^*, which maximizes the social function given in (3.2):

$$\frac{\partial S_2(\alpha)}{\partial \alpha} = 0 \Rightarrow \alpha_2^* = \frac{\mu q - \sqrt{\frac{C\mu q}{R}}}{\lambda}$$

Corollary 3.5 The socially optimal joining probability of the Model With Resolution is:

$$\alpha_2^* = \begin{cases} \frac{\mu q - \sqrt{\frac{C\mu q}{R}}}{\lambda}, & \mu q - \lambda < \sqrt{\frac{C\mu q}{R}} \\ 1, & o.w. \end{cases}$$

The socially optimal joining probability of the Model With Resolution is increasing in μ, R and q; and decreasing in λ and C as in the Benchmark Model.
Observation 3.2: $\alpha_2^{eq}(q) \geq \alpha_2^*(q)$.
We present the numerical values of this Observation in Table Tab. B.2. For the different model parameters since the numbers of the second column is at least equal to or greater than the numbers of the third column, the result of this Observation follows.

This shows, adding a quality parameter in these two problems, and modeling them with a service failure resolution alternative, does not change the result of individually and socially optimal strategies differing from each other.

3.2.3 Problem Of The Profit Maximizer

The problem of the profit maximizer for the Model With Resolution is:

$$\Pi_2(p,q) = p\lambda\alpha_2^{eq} - aq^2 \tag{3.13}$$

We analyze this problem based on the three equilibrium strategies stated in Corollary 3.4.

1. If $(C/\mu) > q(R-p)$, since the expected utility of the customer is negative even for the empty system, no customer joins.

   To prevent losses, the profit maximizer sets the queue entrance price and the quality level to 0: $p_{2,n} = 0$, $q_{2,n} = 0$, $\Pi_2(p_{2,n}, q_{2,n}) = 0$.

2. If the profit maximizer chooses to serve the whole market, $\alpha_2^{eq} = 1$, he then charges the market capturing price $p_{2,\lambda}$.

   The market capturing price of this model which equates (3.10) to zero is :

$$p_{2,\lambda}(q) = R - \frac{C}{\mu q - \lambda} \tag{3.14}$$

   .

   The profit function with this entrance price and the equilibrium joining rate is:

$$\Pi_2(p_{2,\lambda}, q) = p_{2,\lambda}\lambda - aq^2 = \lambda\left(R - \frac{C}{\mu q - \lambda}\right) - aq^2 \tag{3.15}$$

   .

Lemma 3.8 $\Pi_2(p_{2,\lambda}, q)$ is concave in q.

So there is unique solution of this quality level which maximizes (3.15) as:

$$\frac{\partial \Pi_2(p_{2,\lambda}, q)}{\partial q} = 0 \Rightarrow \frac{C\lambda\mu}{(\mu q - \lambda)^2} - 2aq$$

.

Obtaining closed form expressions for the optimal quality level in this market capturing case is complicated, but this problem can be solved numerically.

3. If the profit maximizer decides not to serve all potential customers, the customers join based on the equilibrium joining probability given in Corollary 3.4.

   The profit maximizer's problem with this equilibrium joining probability is:

$$\Pi_2(p, q) = (p\lambda\alpha_2^{eq} - aq^2) = p\lambda\frac{\mu q - \frac{C}{R-p}}{\lambda} - aq^2. \tag{3.16}$$

Lemma 3.9 $\Pi_2(p, q)$ is concave in p.

Corollary 3.6 *The profit maximizer's optimal monopolistic price as a function of the quality level is given by:*

$$p_{2,m}(q) = R - \sqrt{\frac{CR}{\mu q}} \tag{3.17}$$

We rewrite the profit function with this monopolistic price as a function of q as:

$$\Pi_2(p_{2,m}, q) = \mu p_{2,m} q - \frac{C p_{2,m}}{R - p_{2,m}} - aq^2 = \left(\sqrt{R\mu q} - \sqrt{C}\right)^2 - aq^2 \qquad (3.18)$$

Lemma 3.10 $\Pi_2(p_{2,m}, q)$ is convex in q.

As explained in Lemma 3.5, the optimal quality level of the profit maximizer in this monopolistic price case is 1, assuming the positive profitability condition holds; i.e. $q_{2,m} = 1$.

The optimal values of the profit maximizer's problem in the monopolistic case is:

$$q_{2,m} = 1 \quad p_{2,m} = R - \sqrt{\frac{CR}{\mu}} \quad \Pi_2(p_{2,m}, q_{2,m}) = (\sqrt{R\mu} - \sqrt{C})^2 - a$$

3.3 The Model With Returns

In this model, we assume that an unsatisfied customer returns to the same system for resolution. But in her returns, we assume that she decides in a myopic fashion. This means she decides independent from her past experiences of the system, as if she was a new customer for the system. The system also does not distinguish between first time and resolution customers. So, when the service failure occurs, if an unsatisfied customer decides to join the system for service failure resolution, she repays the entrance price.

We represent the Model With Returns in Figure 3.4.

From the Figure 3.4, we observe that all of the unsatisfied customers return to system, however they repetitively decide whether or not to join the system again, and

Figure 3.4: Graphical representation of Model with Returns

not all of the returning customers but some of them takes the resolution alternative and re-joins the system.

3.3.1 Individual Problem Of The Customers

We use the subscript 3 to denote the Model With Returns. The individual utility function of a customer in this model is:

$$U_3(\alpha) = R - CE[W] \Rightarrow U_3(\alpha) = Rq - \frac{C}{\mu q + \mu(1-q)(1-\alpha) - \lambda\alpha} \tag{3.19}$$

Lemma 3.11 $U_3(\alpha)$ is concave in α.

Corollary 3.7 The equilibrium joining probability of the customers for the Model With Resolution is:

$$\alpha_3^{eq} = \begin{cases} 0, & Rq \le \frac{C}{\mu} \\ \frac{\mu - \frac{C}{Rq}}{\mu - \mu q + \lambda}, & \frac{C}{\mu} < Rq < \frac{C}{\mu q - \lambda} \\ 1, & Rq \ge \frac{C}{\mu q - \lambda} \end{cases}$$

We interpret this result as in Corollary 3.1.

The equilibrium joining probability of the Model With Returns is increasing in μ, R and q and decreasing in λ and C, similar to other single stage models.

3.3.2 Social Problem

The total expected utility function of the entire system in the Model With Returns is:

$$S_3(\alpha) = \lambda\alpha\left[Rq - CE[W]\right] \Rightarrow S_3(\alpha) = \lambda\alpha\left[Rq - \frac{C}{\mu q + \mu(1-q)(1-\alpha) - \lambda\alpha}\right] \tag{3.20}$$

Lemma 3.12 $S_3(\alpha)$ is concave in α.

Since concavity holds, there is a unique probability value, α_3^*, which maximizes the social optimization function given in (3.20):

$$\frac{\partial S_3(\alpha)}{\partial \alpha} = 0 \Rightarrow \alpha_3^* = \frac{\mu - \sqrt{\frac{C\mu}{Rq}}}{\mu - \mu q + \lambda}$$

Corollary 3.8 The socially optimal joining probability of the Model With Returns is:

$$\alpha_3^* = \begin{cases} \frac{\mu - \sqrt{\frac{C\mu}{Rq}}}{\mu - \mu q + \lambda}, & \mu q - \lambda < \sqrt{\frac{C\mu}{Rq}} \\ 1, & o.w. \end{cases}$$

The socially optimal joining probability of the Model With Returns is increasing in μ, R and q; and decreasing in λ and C as in the other single stage models.

Observation 3.3: $\alpha_3^{eq}(q) \geq \alpha_3^*(q)$.

The numerical values of this Observation are given in Table Tab. B.3. From the table values, we observe that the comparison follows, since the numbers of the second column is at least equal to or greater than the numbers of the third column.

This shows that retaking the price from the returning customers does not change the result of there is mismatch between the individual and social optima.

3.3.3 Problem Of The Profit Maximizer

The structure of the problem of the profit maximizer is different in the Model With Returns since the returning customers who decide to join the system for the service failure resolution repay the entrance price. Based on this difference, the problem of the profit maximizer is derived in this model as:

$$\Pi_3(p,q) = p\left[\lambda\alpha_3^{eq} + \mu(1-q)\alpha_3^{eq}\rho\right] - aq^2 \tag{3.21}$$

where the server utilization is, $\rho_3 = \frac{\lambda\alpha_3^{eq}}{\mu q + \mu(1-q)(1-\alpha_3^{eq})}$.

Using the equilibrium strategies given in the Corollary 3.7, we analyze the profit maximizer's problem for the Model With Returns.

1. If $(C/\mu) > Rq$, no customer decides to join.

   The optimal model parameters in this case is summarized: $p_{3,n} = 0$, $q_{3,n} = 0$, $\Pi_3(p_{3,n}, q_{3,n}) = 0$.

2. In the market capturing price setting, $\alpha_3^{eq} = 1$.

   This minimum price which equates Equation (3.19) to zero is :

$$p_{3,\lambda}(q) = Rq - \frac{C}{\mu q - \lambda} \tag{3.22}$$

   .

   Using the market capturing price given in (3.22), we rewrite the problem of the profit maximizer as:

$$\Pi_3(p_{3,\lambda}, q) = p_{3,\lambda}\frac{\lambda}{q} - aq^2 = \left(Rq - \frac{C}{\mu q - \lambda}\right)\frac{\lambda}{q} - aq^2 \tag{3.23}$$

   .

Lemma 3.13 $\Pi_3(p_{3,\lambda}, q)$ *is concave in* q*.*

So there is a unique solution of this quality level which maximizes (3.23) as:

$$\frac{\partial \Pi_3(p_{3,\lambda}, q)}{\partial q} = 0 \Rightarrow R - \frac{C\lambda(2\mu q - \lambda)}{(\mu q - \lambda)^2 q^2} - 2aq$$

.

Similar to the market capturing price setting of the Model With Resolution, problem of the profit maximizer can be solved numerically in the Model With Returns.

3. The equilibrium joining probability, when the profit maximizer sets the monopolistic price, which is stated in the Corollary 3.7 is:

$$\alpha_3^{eq} = \frac{\mu - \frac{C}{Rq-p}}{\mu + \lambda - \mu q.}$$

Using this equilibrium joining probability, the utilization of the server is obtained as:

$$\begin{aligned} \rho &= \frac{\lambda \alpha_3^{eq}}{\mu q + \mu(1-q)(1-\alpha_3^{eq})} \\ &= \frac{\lambda(Rq\mu - p\mu - C)}{\lambda R\mu q - \lambda\mu p + C\mu - C\mu q} \end{aligned}$$

So, the problem of the profit maximizer is rewritten as:

$$\Pi_3(p, q) = p\left[\lambda \frac{\mu - \frac{C}{Rq-p}}{\lambda + \mu - \mu q} + \mu(1-q)\frac{\mu - \frac{C}{Rq-p}}{\lambda + \mu - \mu q}\frac{\lambda(Rq\mu - p\mu - C)}{\lambda R\mu q - \lambda\mu p + C\mu - C\mu q}\right] - aq^2 \tag{3.24}$$

Lemma 3.14 $\Pi_3(p,q)$ is concave in p.

Then the optimal price value in this monopolistic price setting strategy is given as:

$$\frac{\partial \Pi_3(p,q)}{\partial p} = 0 \Rightarrow p_{3,m} = \frac{C\mu - C\mu q + \lambda\mu R q - \sqrt{C\mu(\lambda+\mu-\mu q)(C - Cq + \lambda R q)}}{\lambda\mu} \tag{3.25}$$

Plugging this optimal first order price expression in the profit function given in (3.24), and analyzing the obtained function with respect to the quality level, the optimal model parameters can be obtained. However, since the analysis of the function with respect to quality level is analytically intractable, numerical analysis will be used to compare the optimal model parameters of this model with the other models.

3.4 Comparison Between Single Stage Models

In this section, we compare the performances of the three single stage models. The equilibrium and the socially optimal joining probabilities and the optimal profit values are the performance measures of the individual, social, and the profit maximizer's problem respectively.

We divide the comparison in two categories: The first category gives the so called short term comparison. In the short term, we assume that the service quality level is given and fixed. Since changing the quality level requires some time and investment. We give the model comparison from the individual and the social viewpoint in this category, since the service quality level is not a decision of these two problems. We also give the pricing and profit comparison of the problem of the profit maximizer when he uses the market capturing price and the monopolistic price strategies for the given quality level. In the second category we present the long term comparison.

The service quality level is a decision in the long term. So, in this category we give the optimal service quality level, optimal price and the profit values of the problem of the profit maximizer under the market capturing price and the monopolistic price strategies. We compare the long term performances of the models based on the profit values.

3.4.1 Short Term Comparison: Comparison For Given Quality Levels

- Equilibrium Joining Probabilities:

Proposition 3.4 For the given model parameters R, C, λ, μ and q, the equilibrium joining probabilities of the single server model are compared as:

$$\begin{cases} \alpha_3^{eq}(q) \leq \alpha_2^{eq}(q) \leq \alpha_1^{eq}(q), & \lambda \leq \mu q \\ \alpha_2^{eq}(q) \leq \alpha_3^{eq}(q) \leq \alpha_1^{eq}(q), & o.w. \end{cases}$$

For the given model parameters, R, C, λ, μ, and q, the joining decision of a customer depends on the expected model reward and the total expected waiting time. In the Benchmark Model, since the unsatisfied customers leave the system the expected waiting time of this model is the lowest. This explains why a customer is most desirous to join the Benchmark Model although with some probability she leaves the system without taking the service reward. Between the Model With Resolution and the Model With Returns, the comparison follows based on a specific criterion. This criterion is interpreted as: If $\mu q \geq \lambda$, the expected waiting time of the Model With Resolution is lower than the Model With Returns, additionally the expected reward is higher in the first one. This explains why the customers are more willing to join the system in the Model With Resolution. So, for the low congested systems; i.e. lower arrival, higher service rates, the Model With Resolution performs better compared to the Model With Returns from the customer viewpoint.

- Socially Optimal Joining Probabilities:

Proposition 3.5 For the given model parameters R, C, λ, μ and q, the socially optimal joining probabilities of the single server model are compared as:

$$\begin{cases} \alpha_3^*(q) \leq \alpha_2^*(q) \leq \alpha_1^*(q), & \lambda \leq \mu q \\ \alpha_2^*(q) \leq \alpha_3^*(q) \leq \alpha_1^*(q), & o.w. \end{cases}$$

The interpretation of this result is very similar to the one given in Proposition 3.4.

- Problem Of The Profit Maximizer - Market Capturing Price:

Proposition 3.6 For the given model parameters R, C, λ, μ, a and q, the comparison between the market capturing price and profit values as functions of the service quality levels of the single server models is:

$$\begin{cases} p_{3,\lambda}(q) \leq p_{2,\lambda}(q) \leq p_{1,\lambda}(q), & \frac{R}{C\mu} \leq \frac{1}{(\mu q - \lambda)(\mu - \lambda)} \\ p_{3,\lambda}(q) \leq p_{1,\lambda}(q) \leq p_{2,\lambda}(q), & o.w. \end{cases}$$

$$\begin{cases} \Pi_{2,\lambda}(q) \leq \Pi_{1,\lambda}(q), & \frac{R}{C\mu} \leq \frac{1}{(\mu q - \lambda)(\mu - \lambda)} \\ \Pi_{1,\lambda}(q) \leq \Pi_{2,\lambda}(q), & o.w. \end{cases}$$

$$\Pi_{3,\lambda}(q) \leq \Pi_{2,\lambda}(q)$$

In the market capturing price strategy, all the potential customers join the system. Since in the Model With Returns, all unsatisfied customers decide to join again, the expected waiting time of this model is the highest. Additionally since the returning customers repay the entrance price for this congested system, profit maximizer must charge the minimum price to make the customers decide to join. Since the entrance price is the lowest, so is the profit.

The comparison between the other two models, The Benchmark Model and the

Model With Resolution depends on the model parameters. The given condition shows the situations in which per unit gain of a customer is smaller than her additional waiting cost. Namely, if the expected reward of a customer when she returns to the system is smaller than the additional waiting cost that her return creates, it is better for the profit maximizer not to accept this customer in the system.

So, we conclude that in the short term, since the profit maximizer cannot change the service quality level, the waiting cost is the most important criterion to decide on which model to choose. In the Model With Returns, although he recharges the price to returning customers, since these returns increase the waiting costs by creating congestion, the profit maximizer has lower profit in this model. That is, for the conditions stated in Proposition 3.6, it is better for the profit maximizer when the customers choose to leave the system.

- Problem Of The Profit Maximizer - Monopolistic Price:

Proposition 3.7 For the given model parameters R, C, λ, μ, a and $q,$ the comparison between the monopolistic price and profit values as functions of the service quality levels of the single server models is:

$$p_{1,m}(q) \leq p_{2,m}(q)$$

$$\Pi_{1,m}(q) = \Pi_{2,m}(q)$$

From this comparison we conclude that the profit maximizer takes equal profits from the two models, The Benchmark Model and The Model With Returns, by optimizing the equilibrium joining probabilities. The equilibrium joining probability is lower in the Model With Resolution. Since the joining rate is higher in the Benchmark Model compared to Resolution Model, i.e. $\alpha_1^{eq} \geq \alpha_2^{eq}$, the profit maximizer optimizes his profit by decreasing the entrance price in the Benchmark Model.

However, we cannot have the exact comparison between the Model With Returns and the other two single stage models, as we cannot obtain closed form expressions for the Model With Returns in the monopolistic price case.

3.4.2 Long Term Comparison: Comparison When The Quality Level Is A Decision

- Problem Of The Profit Maximizer - Market Capturing Price:

Proposition 3.8 For the given model parameters R, C, λ, μ and a, the comparison between the optimal service quality, market capturing price and the profit values of the single server models is:

$$q_{3,\lambda} \leq q_{2,\lambda} \;\; p_{3,\lambda} \leq p_{2,\lambda} \;\; \Pi_{3,\lambda} \leq \Pi_{2,\lambda}$$

Proposition 3.9 *If* $R\lambda \geq 2a$*, then:*

$$q_{2,\lambda} \leq q_{1,\lambda} \;\; p_{2,\lambda} \leq p_{1,\lambda} \;\; \Pi_{1,\lambda} \leq \Pi_{2,\lambda}$$

Observation 3.4: If $R\lambda < 2a$, then $\Pi_{1,\lambda} \leq \Pi_{2,\lambda}$.
For the numerical values of this Observation please see Table Tab. B.4. From the values of this table, we observe that the profit of the third column is at least equal to or greater than the values of the second column which shows the result of this Observation.

In the long term comparison we conclude that by optimizing the quality level, it is better for the profit maximizer to not to loose the customers; i.e. the profit value of the Benchmark Model is lower than the Model With Resolution. The comparison between the Benchmark Model and the Model With Returns is done in two cases. The first case; $R\lambda \geq 2a$, is given in Proposition 3.4, representing the conditions where it is optimal for the profit maximizer to provide perfect quality service in the Benchmark Model. The comparison between these two models is given numerically in Observation 3.4, where $R\lambda < 2a$ representing the conditions where providing service with interior quality levels is optimal for the profit maximizer.

- Problem Of The Profit Maximizer - Monopolistic Price:

Proposition 3.10 For the given model parameters R, C, λ, μ and a, the comparison between the optimal service quality, monopolistic price and the profit values of the single server models is:

$$q_{1,m} = q_{2,m} \;\; p_{1,m} \leq p_{2,m} \;\; \Pi_{1,m} = \Pi_{2,m}$$

We interpret the result given in the Proposition 3.10 as: When the profit maximizer sets the monopolistic pricing strategy, he can optimize his profit similarly in the Benchmark Model and Model With Resolution, since by changing the price he also changes the equilibrium joining probability.

3.5 Discussion

In this section we summarize our results.

- The relation between the optimal service quality level and the other model parameters is:

  In the Benchmark Model, it is increasing in R and λ, decreasing in a, and independent from μ and C: When the service value increases, more customers decide to join, the optimal action of the profit maximizer in this case is increasing the quality level. Since all of the customers leave the system, the optimal quality level does not affect the waiting cost.

In the Model With Resolution, it is increasing in C, decreasing in a and μ and independent from R: Since only the satisfied customers leave the system, the optimal quality level affects the waiting cost. If the unit waiting cost increase, or the server rate decrease the service provider must provide service with higher quality levels. The optimal quality level is inversely proportional to the unit cost of the quality. All the customers are assumed to receive the service reward, so it does not affect the optimal quality level.

In the Model With Returns, it is increasing in R and C, and decreasing in a and μ: The dependance is very similar to the Model With Resolution except the dependence on the service reward. In the Model With Returns, since only the satisfied customers take the service reward, when this reward increases, customers are more willing to join and in this case the service provider must increase the service quality level to prevent the congestion of the system.

- Short term performance comparison:

  For the social and individual problem the comparison highly depends on the waiting time criteria. Because for this problem, there is no other model parameter that an individual or the social planner can change.

  The Benchmark Model, which assumes that the unsatisfied customers are lost, is better from the individual and social viewpoint.

  For the problem of the profit maximizer, the right of changing the entrance price decreases the waiting time effect.

  Resolution Models can perform better compared to the Benchmark Model depending on the model parameters.

- Long term performance comparison:

  Since the service provider has a power of changing the service quality level, the waiting time effect is decreased compared to given service quality level.

The Resolution Model is always favorable to Benchmark Models; in the long term, it is optimal to provide resolution to the customers for the profit maximizer compared to losing them.

- The Comparison Between The Models Capturing the Returns of the Customers:

  For the market capturing pricing case, since all the returning customers are assumed to join the system, the Resolution Model performs better compared to Model With Returns. Retaking the price is not an optimal action in this case for the profit maximizer: He must charge a low price, since he recharges it for the same waiting cost and lower expected reward.

  For the monopolistic price case, we can not obtain an exact comparison. Since the equilibrium joining probability, entrance price and the service quality level all change in this case, the comparison between the models depends highly on the model parameters.

Before closing this section, by interpreting the model results, we decide to ignore the Model With Returns, in the next chapters. This is because from our analysis given in this section, we conclude that, obtaining closed form expressions and theoretical results are hard in this model. Besides, we also show that the other model, namely the Model With Resolution, which also analyzes the second service failure resolution alternative, performs better not only from the individual and social viewpoint, but also the profit maximizer's viewpoint in the market capturing setting. Besides there is no exact superiority between the two models for the monopolistic pricing case of the profit maximizer's problem.

Chapter 4

TWO SERVER MODELS

Many service systems are multi-stage systems which include many servers within. These systems have some advantages compared to single stage systems. First, since there are many servers in the system, the expected waiting time of a customer is lower compared to the single stage system, assuming the service rates of all of the servers consideration are the same. Also, multi-server systems can offer different service failure resolution alternatives to the customer. If there is a unique server in the system and the service provider aims to offer resolution to the customers in order not to lose them, then the only resolution alternative that he can provide to customers is re-servicing them (accepting the returns of the customers). However, since there is a unique server which provides service with a fixed service rate, these returns of the unsatisfied customers generate congestion in the system. On the other hand, besides accepting the returns of the customers, the escalation alternative can be used as a service failure resolution in the multi-stage systems.

The multi-stage systems can be classified in two main categories. The first one covers the tandem like sequential servers. In this type, there are many servers providing sequential service in the system. As a service discipline, a customer cannot go through the service in higher level of servers before finishing his service in lower server levels, assuming priorities are forbidden, in this type of systems. Such multi-stage systems can be labeled as an Escalation Systems, and if a service failure occurs in lower level of servers, the customers who are subjected a service failure can be escalated to higher server levels as a resolution alternative. The second type is the Parallel-Stage Systems which include many servers, providing simultaneous service. In this second multi-stage system type, similar to single-stage systems, the unsatisfied customers who are subjected to a service failure can be re-serviced as a resolution.

However, analyzing the multi-stage systems can be intractable and complicated. In the queueing literature, analyzing such multi-stage systems, decomposition procedures are generally used. That is, rather than analyzing the system as a whole, we can decompose this system into smaller parts and then combine the results coming from the analysis of the small systems. Additionally, the decomposed parts of the large systems clearly represent the system as a whole, assuming the decomposed parts are identical. So, if we aim to reach general conclusions about the comparisons on the performances of different system types, we can generalize the results obtained from the analysis of the decomposed parts.

Based on this, we work on the smallest multi-stage systems which is the two-stage (two-server) systems in this chapter. We compare the performances of these systems with the single-stage systems which are analyzed in the previous chapter. As before, we analyze the three problems: Individual problem of the customer, social problem and the problem of the profit maximizer.

For the two-stage models, we first analyze the two-sequential-server models which are labeled as the escalation models. In the escalation models, in order to resolve the service failure problem, the customers who leave the first server unsatisfied, are escalated to the second server in the same system. We work on the two different escalation models. The first one is the *Simple Escalation Model* which assumes that the two servers of the system are identical; i.e. unit reward, unit waiting cost, server rate and the service quality level of the two server are the same. The only difference between the two servers is the arrival rate; i.e. since some of the customers leave the system by receiving a satisfactory service from the first server, the arrival rate of the second server is lower. In the Simple Escalation Model, since the second server can be failed; i.e. it provides service with q quality units where $0 \leq q \leq 1$, this system can be compared with the Benchmark Model of the single-stage part (Some of the customers can leave these systems unsatisfied). The second model is the *Perfect Escalation Model*. We assume that in the Perfect Escalation model, the unsatisfied customers coming through the first server, are successfully served for sure in the second server. Thus, the Perfect Escalation Model is similar to the Model With Resolution; i.e. all the customers leave the system satisfied. In the Perfect Escalation Model, not only the arrival rates, but also the quality levels of the two servers of the system are different; i.e. the first servers provides service with q units, but the second one provides perfect quality service.

As another two-server model, we analyze the Two-Parallel Server Models. In these models, there are two identical servers, which simultaneously provide service. The first Two-Parallel Server Model is very similar to the Benchmark Model which we analyze in the previous chapter. Because, we assume that the customers who are subjected to a service failure from any of the two parallel servers leave the system without receiving the service reward as in the Benchmark Model. These unsatisfied customers are assumed to go to another system for the resolution. For this reason, we label this model as the *Two-Parallel-Server-Benchmark Model*. The other model which we analyze in these two-parallel-server models section is the *Two-Parallel-Server Model With Resolution*. As stated in single stage models, the unsatisfied customers who return to the system for the service failure resolution join the system for sure; i.e. they do not take any repetitive decision of whether or not to join and do not repay in their repetitive visits. So, a customer cannot leave the system without taking the service reward in the resolution models. In the previous chapter, in Model With Resolution section, we showed that the strategy of the customer is not affected by the queueing discipline. That is, since the unobservable queue length conditions hold, the expected waiting time of the customer will be the same when he returns to the end of the queue and the beginning of the queue for the resolution. The only parameters that affect the expected waiting time are the effective server rate and the arrival rate, not the position of the returns. So for the resolution alternative, we analyze one model. On the other hand, in the previous chapter, we conclude that the performance of the Model With Returns, which assumes that the unsatisfied customers make memoryless returns to the system and take myopic decisions of whether or not to join, is worse compared to the Model With Resolution, assuming all the customers decide to join the system. This shows, when the service rate is high so that the market capturing pricing strategy is optimal, The Model With Resolution is more profitable compared to the Model With Returns as a system design for the profit maximizer. Based on this, to model the service failure resolution alternative of coming back to the same system, we use the Two-Parallel-Server Model With Resolution, and ignore the Two-Parallel-Server Model With Returns.

In the end, to combine our analyses of the Two-Server-Models with the analyses of the Single-Server-Models presented in the previous chapter, we use the Benchmark Model and the Model With Resolution with a double server rate; i.e. there is a single server who provides service with the rate 2μ.

We compare these models, based on the equilibrium and the socially optimal joining probabilities of the individual customer and the social problem and the profit of the profit maximizer. To make the comparisons fair, we divide the models into two groups: The first group includes the Simple Escalation Model, Two-Parallel-Stage-Benchmark Model, and the Benchmark Model With double-server-rate. The second group covers the perfect escalation model, two-parallel-stage-model with resolution and the resolution model with the double-server-rate.

We give all the Proofs of the Lemmas, Corollaries and Propositions corresponding this chapter in Appendix A.3. For the Observations of the Numerical Studies of this Chapter, you can see Appendix B.2.

4.1 Escalation Models

The motivation behind these models is: The customers who experience a service failure in the first stage of the system can be escalated to the higher level of server for service resolution. Based on this motivation we work on two different escalation (2-sequential stage) models. In the first model, *Simple Escalation Model*, the customers who leave the first stage unsatisfied by experiencing a service failure, go through the queue in front of the second stage and wait to be served. However, in the second stage, there is still a probability of a service failure. So, some of the customers leave this 2-sequential stage system, without taking the service reward. In the second model, *Perfect Escalation Model*, we assume that if a customer experiences a service failure in the first stage, he is escalated to the second server which provides a perfect quality service. So,the customer is successfully served in this system for sure. These two models are represented graphically in Figures 4.1 and 4.2 respectively.

The scenario of the Simple Escalation Model given in Figure 4.1 is summarized

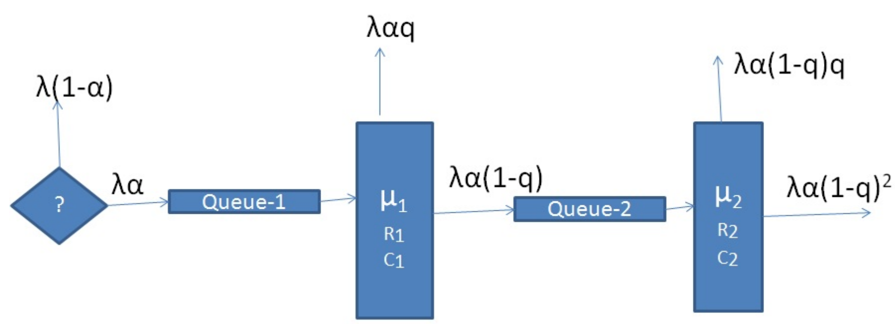

Figure 4.1: Simple Escalation Model

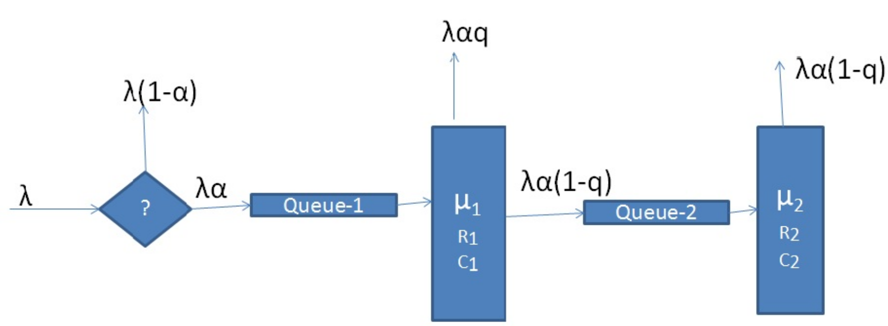

Figure 4.2: Perfect Escalation Model

as: Customers arrive to the system according to a Poisson process with rate λ. By comparing the expected service value with the total expected waiting cost they decide whether or not to join. The joining probability is α, so the joining rate is $\lambda\alpha$. The ones who decide to join, wait in front of the queue of the first server. Some of these customers with probability q, receive satisfactory service in the first server and leave the system, so they do not need any service failure resolution. While the others with probability *1-q*, are subjected to a service failure, and join the queue in front of the second server for the resolution. However since this second server also provides service with q units of quality, there is still a service failure probability, hence the customers with rate $\lambda\alpha(1-q)^2$ leave the system unsatisfied. And the remaining customers with rate $\lambda\alpha(1-q)q$ leave the system satisfied by receiving a good service from the second server.

The scenario is very similar in the perfect escalation model given in Figure 4.2. The difference is the following: The server in the second stage provides perfect quality service, so customers who join the second stage receive a satisfactory service and take the service reward for sure.

We denote these escalation models with the subscript 4, since these are the fourth model that we analyze in this thesis. In order to show the Simple and Perfect Escalation Models, we use subscripts $4, n$ and $4, g$ respectively. We use these notations because the quality of the service in the Simple Escalation Model is not guaranteed whereas in the Perfect Escalation Model it is.

In this section we analyze the individual problem of the customers, social problem and the problem of the profit maximizer for both of the escalation models respectively.

4.1.1 Individual Problem of the Customers

Customers decide based on their utility functions. In this section, to simplify our analysis, we assume that the model parameters R_k, C_k, μ_k where $k = 1, 2$ are equal for the two servers in Figures 4.1 and 4.2.

For the Simple Escalation Model, we assume that the service quality levels of the two servers are equal; i.e. each server provides service with q units. What differentiates the two servers is the congestion levels of the servers; since some of the customers leave the system by receiving a satisfactory service from the first server, the arrival rate of the second server is smaller.

The utility function of the individual of the Simple Escalation Model, $U_{4,n}(\alpha)$ is:

$$U_{4,n}(\alpha) = \sum_{k=1}^{2} R_k(1-q)^{k-1}q - \sum_{k=1}^{2} C_k(1-q)^{k-1}E[W_k] - p$$

$$= Rq + Rq(1-q) - \frac{C}{\mu - \lambda\alpha} - \frac{C(1-q)}{\mu - \lambda\alpha(1-q)} - p \qquad (4.1)$$

Lemma 4.1 $U_{4,n}(\alpha)$ is concave in α.

Using the concavity given in Lemma 4.1, the equilibrium joining probability of the problem of the individual customer for the Simple Escalation Model is derived as:

$$\alpha^{eq}_{4,n}(q) = \begin{cases} 0, & \frac{C(2-q)}{\mu} > Rq(2-q) - p \\ \alpha^{eq}_{4,n-int}(q), & \frac{C(2-q)}{\mu} \leq Rq(2-q) - p \leq \frac{C(2-q)}{\mu-\lambda} \\ 1, & Rq(2-q) - p > \frac{C(2-q)}{\mu-\lambda} \end{cases} \qquad (4.2)$$

where

$$\alpha^{eq}_{4,n-int}(q) = \frac{-2C(1-q) + \mu(2-q)[Rq(2-q)-p] + \sqrt{4C^2(1-q)^2 + \mu^2q^4R^2(2-q)^2 + \mu^2pq^2(p - 4Rq + 2q^2R)}}{2\lambda(1-q)[Rq(2-q)-p]}. \qquad (4.3)$$

The subscript *int* is used for the interior probability levels $0 \leq \alpha_{.-int} \leq 1$.
The utility function of a customer of the Perfect Escalation Model depending on the joining probability, $U_{4,g}(\alpha)$, is written as:

$$U_{4,g}(\alpha) = R - \sum_{k=1}^{2} C_k(1-q)^{k-1}E[W_k] - p = R - \frac{C}{\mu-\lambda\alpha} - \frac{C(1-q)}{\mu - \lambda\alpha(1-q)} - p \qquad (4.4)$$

Lemma 4.2 $U_{4,g}(\alpha)$ is concave in α.

Since the function given in (4.4) is concave, the equilibrium joining probability of customers of the Perfect Escalation Model is derived as:

$$\alpha^{eq}_{4,g}(q) = \begin{cases} 0, & \frac{C(2-q)}{\mu} > R - p \\ \alpha^{eq}_{4,g-int}(q), & \frac{C(2-q)}{\mu} \leq R - p \leq \frac{C}{\mu-\lambda} + \frac{C(1-q)}{\mu-\lambda(1-q)} \\ 1, & R - p > \frac{C}{\mu-\lambda} + \frac{C(1-q)}{\mu-\lambda(1-q)} \end{cases} \tag{4.5}$$

where

$$\alpha^{eq}_{4,g-int}(q) = \frac{2C(1-q) - \mu(2-q)(R-p) + \sqrt{4C^2(1-q)^2 + \mu^2 q^2 (R-p)^2}}{2\lambda(1-q)(R-p)}. \tag{4.6}$$

Proposition 4.1 $\alpha^{eq}_{4,n} \leq \alpha^{eq}_{4,g}$.

The interpretation of the Proposition 4.1 is as follows: Although the total expected service time expressions of the two models are the same, the service value of the Perfect Escalation Model is higher. This explains why the customers are more willing to join the system in the perfect escalation model.

4.1.2 Social Problem

The objective of the social planner is to maximize the total expected utility function of the social system. For the Simple Escalation Model, the total expected utility function of the social system, $S_{4,n}(\alpha)$, is written as:

$$S_{4,n}(\alpha) = \lambda\alpha \left(Rq + Rq(1-q) - \frac{C}{\mu - \lambda\alpha} - \frac{C(1-q)}{\mu - \lambda\alpha(1-q)} \right) \tag{4.7}$$

Lemma 4.3 $S_{4,n}(\alpha)$ *is concave in* α*.*

Using the concavity given in Lemma 4.3 and denoting the interior socially optimal joining probability with $\alpha^*_{4,n-int}(q)$, the socially optimal joining strategy of this Simple Escalation Model is summarized as:

$$\alpha^*_{4,n}(q) = \begin{cases} \alpha^*_{4,n-int}(q), & \text{if} \quad 0 \leq \alpha^*_{4,g-int}(q) < 1 \\ 1, & \text{ow} \end{cases} \tag{4.8}$$

The total expected utility function of the social system in the Perfect Escalation Model, $S_{4,g}(\alpha)$, is written as:

$$S_{4,g}(\alpha) = \lambda\alpha\left(R - \frac{C}{\mu - \lambda\alpha} - \frac{C(1-q)}{\mu - \lambda\alpha(1-q)}\right) \tag{4.9}$$

Lemma 4.4 *$S_{4,g}(\alpha)$ is concave in α.*

Since the concavity follows, there is a unique interior socially optimal joining probability which can be derived from the first order conditions. Assuming this interior joining probability is,$\alpha^*_{4,g-int}(q)$, the socially optimal joining strategy for this Perfect Escalation Model is summarized as:

$$\alpha^*_{4,g}(q) = \begin{cases} \alpha^*_{4,g-int}(q), & \text{if} \quad 0 \leq \alpha^*_{4,g-int}(q) < 1 \\ 1, & \text{ow} \end{cases} \tag{4.10}$$

Proposition 4.2 $\alpha^*_{4,n} \leq \alpha^*_{4,g}$.

The interpretation of Proposition 4.2 is very similar to Proposition 4.1.

Before going through the problem of the profit maximizer, we give the numerical comparison between the equilibrium and the socially optimal joining probabilities of the escalation models.

Observation 4.1. $\alpha^{eq}_{4,g}(q) \geq \alpha^*_{4,g}(q)$, and $\alpha^{eq}_{4,n}(q) \geq \alpha^*_{4,n}(q)$.

We present the numerical analysis of Observation 4.1 in Table Tab. B.5. By comparing the values of the second and the third columns, additionally the fourth and the fifth columns of this table, this result can be observed.

The result of the Observation 4.1. is interpreted as given in the previous chapter. The customers are more desirous to join the system, but in the social system, to prevent the congestion, the social planner optimizes the system; i.e. maximizes the total expected utility function of all of the customers, by accepting lower number of the customers.

4.1.3 Problem of The Profit Maximizer

In this chapter, in all of the analyses considering the problem of the profit maximizer we assume that the profit maximizer charges the minimum entrance price for two reasons. First, in our single stage analysis given in the previous chapter, we conclude that it is optimal for the profit maximizer to set the minimum entrance price and serve the whole market, if the server rate is high enough. Since in this chapter, we work on Two-Server Systems so the service rate of the system is higher, assuming the fixed arrival rate, it is possible to use this result.

Corollary 4.1 *If setting the market capturing price, when the server rate is* μ*, is optimal for the profit maximizer, then it is also optimal when the server rate is* $\mu+\epsilon$*, where* $\epsilon > 0$*.*

The second reason for using this assumption is the theoretical constraints. For the Escalation Models, we fail in showing the concavity of the profit function of the profit maximizer when he charges the monopolistic price. Since the profit function is not concave under monopolistic pricing strategy, we cannot have a unique optimum.
We first analyze the problem of the profit maximizer for the Simple Escalation Model. Assuming the positive profit condition holds, based on the market capturing price setting, the equilibrium joining probability of the customer is: $\alpha_{4,n}^{eq} = 1$. Using this equilibrium probability and equating (4.1) to 0, the market capturing price of the Simple Escalation Model is found as:

$$Rq + Rq(1-q) - \frac{C}{\mu - \lambda\alpha} - \frac{C(1-q)}{\mu - \lambda\alpha(1-q)} - p = 0 \Rightarrow$$

$$p_{4,n}(q) = Rq + Rq(1-q) - \frac{C}{\mu - \lambda} - \frac{C(1-q)}{\mu - \lambda(1-q)} \tag{4.11}$$

Using the market capturing price given in (4.11), the profit function of the profit maximizer is written as:

$$\Pi_{4,n}(q) = \lambda\alpha_{4,n}^{eq} p_{4,n}(q) - 2aq^2 = \lambda\left[Rq + Rq(1-q) - \frac{C}{\mu - \lambda} - \frac{C(1-q)}{\mu - \lambda(1-q)}\right] - 2aq^2. \tag{4.12}$$

Lemma 4.5 $\Pi_{4,n}(q)$ is concave in q.

Based on the concavity given in Lemma 4.5, the fact that the first order condition suffices follows. The profit of the profit maximizer is found by plugging the interior quality level in the equation (4.12).

The market capturing price of the Perfect Escalation Model, which equates (4.4) to 0, is:

$$R - \frac{C}{\mu - \lambda\alpha} - \frac{C(1-q)}{\mu - \lambda\alpha(1-q)} - p = 0 \Rightarrow p_{4,g}(q) = R - \frac{C}{\mu - \lambda} - \frac{C(1-q)}{\mu - \lambda(1-q)} \quad (4.13)$$

Using the price given in (4.13), the profit function of the profit maximizer of the Perfect Escalation Model is derived as:

$$\Pi_{4,g}(q) = \lambda\alpha_{4,g}^{eq} p_{4,g}(q) - aq^2 - a = \lambda\left[R - \frac{C}{\mu - \lambda} - \frac{C(1-q)}{\mu - \lambda(1-q)}\right] - aq^2 - a. \quad (4.14)$$

Lemma 4.6 $\Pi_{4,g}(q)$ is concave in q.

Since the concavity follows, we find the interior quality level using the first derivative function, and calculate the profit of the profit maximizer by plugging this quality level into (4.14).

We close this section, by comparing the performances of the Simple Escalation and Perfect Escalation Models from the profit maximizer's perspective. We first provide the short term comparison, where the profit maximizer decides only on the entrance price for the given service quality level. We then present the long term comparison, where the problem of the profit maximizer includes two decision parameters: price and the service quality level.

Proposition 4.3 *In the short term, for the given model parameters,* R, C, a, q, λ *and* μ,

-Pricing Comparison: $p_{4,g}(q) \geq p_{4,n}(q)$.

-Profit Comparison:

$$\begin{cases} \Pi_{4,g}(q) \geq \Pi_{4,n}(q) & if \ \frac{R\lambda}{a} \geq \frac{1+q}{1-q} \\ \Pi_{4,g}(q) < \Pi_{4,n}(q) & o.w. \end{cases}$$

We interpret Proposition 4.3 as: For the same waiting costs, since the expected reward of the Perfect Escalation Model is at least equal to or greater than the expected reward of the Simple Escalation Model, the profit maximizer charges a higher price in the former one. We conclude that the profit comparison between these models depends on the parameters, i.e. if the excess revenue of the firm exceeds its cost by increasing the quality level then the profit maximizer has higher profit in the Perfect Escalation Model.

Proposition 4.4 In the long term, for the given model parameters, R*,* C*,* a*,* λ and μ*,*
-Quality Level Comparison: $q^*_{4,g} \leq q^*_{4,n}$.
-Pricing Comparison:

$$\begin{cases} p^*_{4,g} \geq p^*_{4,n} & if \ \frac{R}{C\mu} \geq \frac{q_{4,n}-q_{4,g}}{(1-q_{4,n})^2(\mu-\lambda+\lambda q_{4,g})(\mu-\lambda+\lambda q_{4,n})} \\ p^*_{4,g} < p^*_{4,n} & o.w. \end{cases}$$

Proposition 4.4 presents the following: Since the second server provides perfect quality service in the Perfect Escalation Model, the profit maximizer optimizes his profit by decreasing the quality level of the first server; i.e. the cost of the quality of the second server is high. The pricing comparison between the escalation models depends on the model parameters, i.e. if the ratio between the unit reward and the unit waiting cost is higher than the differences between the quality levels of the models, then the profit maximizer sets a higher price in the Perfect Escalation Model. But the profit is not comparable between the two models, since the price and the cost of the quality highly depend on the parameters.

4.2 Parallel Server Models With Two Servers

In this section we analyze two different parallel server models each having two servers. We assume that there are two identical servers in the system each simultaneously provides service with the service quality level, q.

The first parallel server model that we analyze in this section is the Two-Parallel-Server Benchmark Model. This model is represented in Figure 4.3. We denote this model with the subscript $1, \nu$, where 1 denotes that the model is benchmark type and ν is used to denote the parallel servers. The scenario of this model is: A customer who decides to join the system, is served by one of the servers, after she waits for some time in the queue. If she receives a satisfactory service, she leaves the system by receiving the service reward. Otherwise, if she is subjected to a service failure, she leaves the system without taking the service reward, and goes to another system for the resolution.

The other parallel server model is the Two-Parallel-Server Model with Resolution. This model is represented in Figure 4.4. We denote this model with subscript $2, \nu$, since the model is resolution type and ν is to show that it is parallel.

In order to analyze the individual problem of the customers for these parallel server

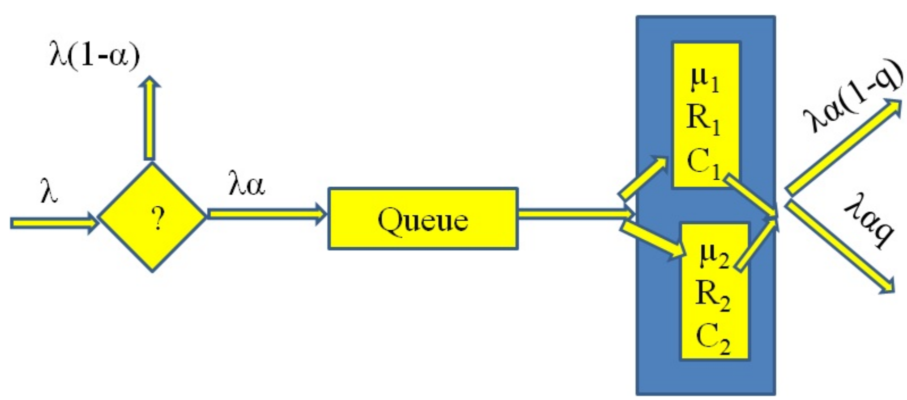

Figure 4.3: Two-Parallel Server Benchmark Model

models, we first give the expected waiting time expression of this parallel server model with two servers; i.e. *M/M/2*.

Since there are two servers in the system each providing service with rate μ, the utilization of the Two-Parallel-Stage Benchmark Model is: $\rho_{1,\nu} = \lambda/c\mu = \lambda/2\mu$. The steady state probability of having no customers in the system, Π_0, is:

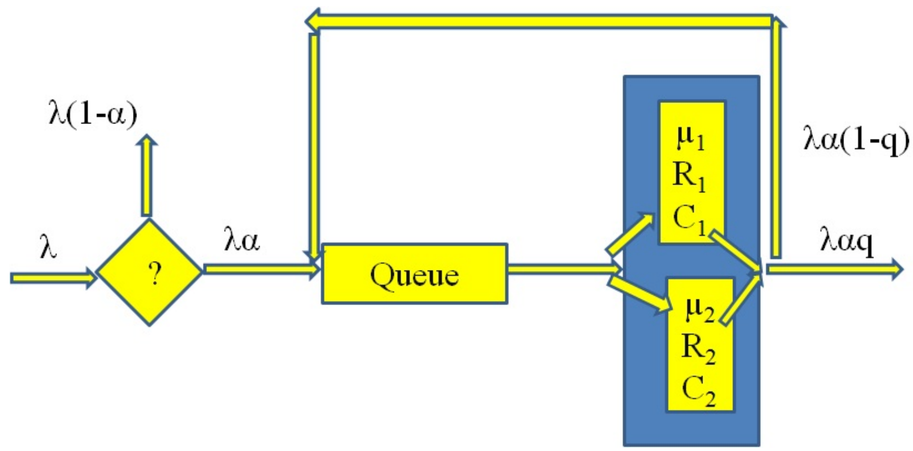

Figure 4.4: Two-Parallel Server Model with Resolution

$$\Pi_0 = \left[\sum_{k=0}^{c-1} \frac{(c\rho)^k}{k!} + \frac{(c\rho)^c}{c!}\frac{1}{1-\rho}\right]^{-1} = \frac{2\mu - \lambda}{2\mu + \lambda} \tag{4.15}$$

The steady state probability that a customer has to wait in the system, Π_{c+} is (since there are at least c customers in the system):

$$\Pi_{c+} = \sum_{k=c}^{\infty} \Pi_k = \frac{(c\rho)^c}{c!(1-\rho)}\Pi_0 = \frac{\lambda^2}{\mu(2\mu+\lambda)} \tag{4.16}$$

The average waiting time in the queue, $E[\overline{W}]$, is:

$$\overline{W} = \frac{\rho}{\lambda(1-\rho)}\Pi_{c+} = \frac{\lambda^2}{\mu(2\mu-\lambda)(2\mu+\lambda)} \tag{4.17}$$

The average waiting time in the system, $E[W]$, is the total of average waiting time in the queue and average service time:

$$E[W] = E[\overline{W}] + \frac{1}{\mu} = \frac{4\mu}{4\mu^2 - \lambda^2\alpha^2} \tag{4.18}$$

The above results are basically the known derivations of basic *M/M/2* queues with $\rho = \lambda/2\mu$. For the Two-Parallel-Server Model With Resolution, since the unsatisfied customers do not leave the system, the utilization of this system is : $\rho_{2,\nu} = \frac{\lambda}{2\mu q}$. Using this utilization level the waiting time expression derived in (4.18) is modified for this resolution model as:

$$E[W] = \frac{4\mu}{4\mu^2 q^2 - \lambda^2 \alpha^2} \tag{4.19}$$

We analyze the individual problem of the customers, the social problem and the problem of the profit maximizer using the total waiting time in the system expressions given in (4.18) and for the benchmark and the resolution type parallel server models respectively.

4.2.1 Individual Problem of the Customers

The utility function of an individual for the Two-Parallel-Server Benchmark Model, $U_{1,\nu}(\alpha)$, is:

$$U_{1,\nu}(\alpha) = Rq - CE[W] - p = Rq - \frac{4C\mu}{4\mu^2 - (\lambda\alpha)^2} - p \tag{4.20}$$

Lemma 4.7 $U_{1,\nu}(\alpha)$ *is concave in* α*.*

Since the concavity follows, the equilibrium joining strategy of this model is derived as before. However, the interior equilibrium joining probability is analytically intractable.

$$\alpha^{eq}_{1,\nu}(q) = \begin{cases} 0, & \frac{C}{\mu} > Rq - p \\ \alpha^{eq}_{1,\nu-int}(q), & \frac{C}{\mu} \leq Rq - p \leq \frac{4C\mu}{4\mu^2-\lambda^2} \\ 1, & Rq - p > \frac{4C\mu}{4\mu^2-\lambda^2} \end{cases} \tag{4.21}$$

For the Two-Parallel-Stage Model With Resolution using the waiting time expression given in (4.2) the utility function of an individual, $U_{2,p}(\alpha)$, is:

$$U_{2,\nu}(\alpha) = R - CE[W] - p = R - \frac{4C\mu}{4\mu^2 q^2 - (\lambda\alpha)^2} - p \tag{4.22}$$

Lemma 4.8 $U_{2,\nu}(\alpha)$ *is concave in* α*.*

Based on the concavity given in Lemma 4.8, the equilibrium joining strategy of this model is obtained as previous models.

$$\alpha_{2,\nu}^{eq}(q) = \begin{cases} 0, & \frac{C}{\mu} > R - p \\ \alpha_{2,\nu-int}^{eq}(q), & \frac{C}{\mu q^2} \leq R - p \leq \frac{4C\mu}{4\mu^2 q^2 - \lambda^2} \\ 1, & R - p > \frac{4C\mu}{4\mu^2 q^2 - \lambda^2} \end{cases} \tag{4.23}$$

The interior equilibrium joining probability expressions of the parallel server models can not be stated in closed form. However using their first order conditions, the comparison can easily be given.

Proposition 4.5 $\alpha_{1,\nu}^{eq} \geq \alpha_{2,\nu}^{eq}$.

Proposition 4.5 can be interpreted as follows: Although, the expected reward of the Two-Parallel-Stage Model With Resolution is higher compared to Two-Parallel-Stage Benchmark Model, since in expectation the customers wait a longer time in the former one, they have a lower probability to join the system. This comparison shows the effect of expected waiting time on individual decision strategy.

4.2.2 Social Problem

The total expected utility function which the social planner optimizes in the Two-Parallel-Server Benchmark Model, $S_{1,\nu}(\alpha)$, is:

$$S_{1,\nu}(\alpha) = \lambda\alpha \left[Rq - \frac{4C\mu}{4\mu^2 - (\lambda\alpha)^2} \right] \tag{4.24}$$

Lemma 4.9 *$S_{1,\nu}(\alpha)$ is concave in α.*

Since the social function of the Two-Parallel-Server Benchmark Model is concave (as in Lemma 4.9), we characterize the socially optimal joining strategy of this model as:

$$\alpha_{1,\nu}^{*}(q) = \begin{cases} \alpha_{1,\nu-int}^{*}(q), & 0 \leq \alpha_{1,p-int}^{*}(q) < 1 \\ 1, & \text{ow} \end{cases} \tag{4.25}$$

For the Two-Parallel-Stage Model With Resolution, the total expected utility function, $S_{2,\nu}(\alpha)$, is:

$$S_{2,\nu}(\alpha) = \lambda\alpha\left[R - \frac{4C\mu}{4\mu^2q^2 - \lambda^2}\right] \tag{4.26}$$

Lemma 4.10 $S_{2,\nu}(\alpha)$ is concave in α.

Since the concavity follows, the characterization of the socially optimal joining strategy of the Two-Parallel-Server Model With Resolution is:

$$\alpha^*_{2,\nu}(q) = \begin{cases} \alpha^*_{2,\nu-int}(q), & 0 \leq \alpha^*_{2,p-int}(q) < 1 \\ 1, & \text{ow} \end{cases} \tag{4.27}$$

Similar to the interior equilibrium joining probabilities, the interior socially optimal joining probabilities can not be presented in closed form, where comparing the first order conditions is straightforward.

Proposition 4.6 $\alpha^*_{1,\nu} \geq \alpha^*_{2,\nu}$.

The interpretation of Proposition 4.6 is very similar to that of Proposition 4.5.
We close this section, by giving the numerical comparison between the equilibrium and socially optimal joining probabilities of the parallel server models.
Observation 4.2. $\alpha^{eq}_{1,\nu}(q) \geq \alpha^*_{1,\nu}(q)$, and $\alpha^{eq}_{2,\nu}(q) \geq \alpha^*_{2,\nu}(q)$.
Please see Table Tab. B.6 for the numerical values of Observation 4.2. Since the values of the second column of this table is at least equal to or greater than the values of the third column and the values of the fourth column is at least equal to or greater than the values of the fifth column, the result stated in Observation 4.2 follows.
The result that the Observation 4.2 presents is the same with our earlier analysis and Naor [30] presents.

4.2.3 Problem of The Profit Maximizer

For the Two-Parallel-Server Benchmark Model, the market capturing price depending on the quality level, $p_{1,\nu}(q)$, which equates the utility function given in (4.20) to zero is:

$$p_{1,\nu}(q) = Rq - \frac{4C\mu}{4\mu^2 - \lambda^2} \tag{4.28}$$

The profit function of the profit maximizer with this entrance price is:

$$\Pi_{1,\nu}(q) = \lambda p_{1,\nu} - 2aq^2 = \lambda \left[Rq - \frac{4C\mu}{4\mu^2 - \lambda^2} \right] - 2aq^2 \tag{4.29}$$

Lemma 4.11 $\Pi_{1,\nu}(q)$ is concave in q.

Corollary 4.2 $q^*_{1,\nu}$ *is:*

$$q^*_{1,\nu} = \frac{\lambda R}{4a} \tag{4.30}$$

Using the optimal service quality level stated in Corollary 4.2, the optimal market capturing price and the profit value of the profit maximizer of the Two-Parallel Server Benchmark model is respectively given as:

$$p^*_{1,\nu} = Rq_{1,\nu} - \frac{4C\mu}{4\mu^2 - \lambda^2} = \frac{\lambda R^2}{4a} - \frac{4C\mu}{4\mu^2 - \lambda^2} \tag{4.31}$$

$$\Pi^*_{1,\nu} = \lambda p_{1,\nu} - 2a(q_{1,\nu})^2 = \frac{\lambda^2 R^2}{8a} - \frac{4\lambda\mu C}{4\mu^2 - \lambda^2} \tag{4.32}$$

Using (4.32), we conclude that the profit value of the profit maximizer in Two-Parallel-Stage Benchmark Model is increasing in R, and decreasing in a and C.

The market capturing price of the Two-Parallel-Stage Model With Resolution, which equates the (4.22) to zero is:

$$p_{2,\nu}(q) = R - \frac{4C\mu}{4\mu^2 q^2 - \lambda^2} \tag{4.33}$$

The profit function of the profit maximizer with this entrance price is:

$$\Pi_{2,\nu}(q) = \lambda p_{2,\nu} - 2aq^2 = \lambda \left[R - \frac{4C\mu}{4\mu^2 q^2 - \lambda^2} \right] - 2aq^2 \tag{4.34}$$

Lemma 4.12 $\Pi_{2,\nu}(q)$ is concave in q.

Corollary 4.3 $q^*_{2,\nu}$ is:

$$q^*_{2,\nu} = \frac{\sqrt{\lambda^2 + \sqrt{\frac{8C\mu^3\lambda}{a}}}}{2\mu} \tag{4.35}$$

As seen in Corollary 4.3, the optimal service quality level of the Two-Parallel-Stage Model With Resolution, is directly proportional with λ and C, and inversely proportional with a and μ. Additionally, the optimal service quality level is not affected by the change in the service reward. The interpretation behind the relation between the model parameters is: When the arrival rate to the system or the cost of waiting in the system, increases the profit maximizer must provide service with higher quality levels. On the other hand, when the service rate increases, since the total expected waiting cost in the system decreases, the profit maximizer can set the quality level to lower values to decrease the cost of the quality. When the unit cost of the quality increases, all the other parameters being the same, the profit maximizer lowers the service quality level to decrease his investment in the quality. Finally, since all the customers receive the service reward in this resolution model, the optimal service quality level is not affected by the change in the service reward.

Using the optimal service quality level stated in the corollary 4.3, the optimal market capturing price and the profit value of the profit maximizer for the two-parallel server model with resolution is:

$$p^*_{2,\nu} = R - \sqrt{\frac{2Ca}{\lambda\mu}} \tag{4.36}$$

$$\Pi^*_{2,\nu} = \lambda p_{2,\nu} - 2a(q_{2,\nu})^2 = R\lambda - \sqrt{\frac{2C\lambda a}{\mu}} - a\frac{\lambda^2 + \sqrt{\frac{8C\lambda\mu^3}{a}}}{2\mu^2} \tag{4.37}$$

From Equation (4.36), we conclude that the optimal market capturing price is increasing in R, λ and μ and decreasing in C and a. The profit function of the profit maximizer expressed in (4.37) shows that the profit value is increasing in R and μ and decreasing in a and C.

Before closing this section, we give the short term and the long term comparison of the problem of the profit maximizer of the Two-Parallel-Stage Models:

Proposition 4.7 In the short term, for the given model parameters, R*,* C*,* a*,* q*,* λ and μ we have the following comparison for the short term price and profit values:

-Pricing Comparison:

$$\begin{cases} p_{1,\nu}(q) \geq p_{2,\nu}(q) & if \ \frac{16C\mu^3(1+q)}{(4\mu^2q^2-\lambda^2)(4\mu^2-\lambda^2)} \geq R \\ p_{1,\nu}(q) < p_{2,\nu}(q) & o.w. \end{cases}$$

-Profit Comparison:

$$\begin{cases} \Pi_{1,\nu}(q) \geq \Pi_{2,\nu}(q) & if \ \frac{16C\mu^3(1+q)}{(4\mu^2q^2-\lambda^2)(4\mu^2-\lambda^2)} \geq R \\ \Pi_{1,\nu}(q) < \Pi_{2,\nu}(q) & o.w. \end{cases}$$

We summarize this result corresponding to the short term comparison as: For given service quality levels, the expected service reward is higher in the Resolution Model compared to the Benchmark model. On the other hand, the expected waiting cost is higher in the Resolution Model. Since the market capturing price is the difference between the expected service reward and the expected waiting cost, the pricing comparison between the Two-Parallel-Server Models depends on the model parameters. Since the arrival rate to the models and the cost of the quality of the models are the same, the profit comparison depends on the pricing comparison.

Proposition 4.8 In the long term, for the given model parameters, R*,* C*,* a*,* λ and μ*,* if $R\lambda \geq 4a$

-Quality Level Comparison: $q^*_{1,\nu} = 1 \geq q^*_{2,\nu}$

-Pricing Comparison: $p^*_{1,\nu} \geq p^*_{2,\nu}$

-Profit Comparison: $\Pi^*_{1,\nu} \leq \Pi^*_{2,\nu}$

The interpretation behind Proposition 4.8 is: Depending on the model parameters; R, λ and a; if the profit maximizer must provide perfect quality service in the Benchmark Model, not to lose many customers, then he charges higher price in this model compared to the Resolution Model, since the optimal service quality level is higher in the former one. However, based on the higher cost of quality, the optimal profit is lower in the Benchmark Model.

For the cases, where the profit maximizer provide service with interior quality levels, in the Benchmark Model, i.e. $R\lambda < 4a$, we give the comparison between the model parameters numerically.

Observation 4.3. *In the long term, for the given model parameters,* R, C, a, λ *and* μ, *if* $R\lambda < 4a$:

-Pricing Comparison: $p^*_{1,\nu} \leq p^*_{2,\nu}$

-Profit Comparison: $\Pi^*_{1,\nu} \leq \Pi^*_{2,\nu}$

Table Tab. B.7 includes the numerical analysis of Observation 4.3. When we compare the numbers of the second and the fourth columns, and the third and the fifth columns the result of this Observation is straightforward.

That is, depending on the model parameters; R, λ and a; if the profit maximizer provides service with interior quality levels (imperfect service), then he charges higher price in the Resolution Model compared to the Benchmark Model. Since the price is higher in the Resolution Model, the profit value is also higher.

By combining the results given in Proposition 4.8 and Observation 4.3, we conclude that in the long term, it is more profitable for the profit maximizer to not loose the customers. So, if the profit maximizer has a chance to change the service quality level, it is optimal for him to offer the resolution option to customers.

4.3 Model Comparison

In this section, we provide a comparison between the Benchmark models; Simple Escalation; Two-Parallel-Stage Benchmark model, Benchmark Model With Double

Server Rate and between the Resolution Models; Perfect Escalation, Two-Parallel-Stage Model With Resolution and Model With Resolution With Double Server Rate. To represent the single stage models with double server rate, we use the subscript d. As a comparison criteria of the problem of the profit maximizer, we use the price and profit for the short term and service quality level, price and profit for the long term. For the individual problem of the customer and the social problem we respectively use the equilibrium joining probability and socially optimal joining probability values. We give the comparisons which we are able to make theoretically as Propositions, and for the ones which are difficult to obtain theoretically we use Observations.

4.3.1 Comparison Between The Benchmark Models

1. Short Term Comparison of the Problem of the Profit Maximizer:

   Proposition 4.9 $p_{1,\nu}(q) \leq p_{1,d}(q)$ and $\Pi_{1,\nu}(q) \leq \Pi_{1,d}(q)$ for all values of q.

2. Long Term Comparison of the Problem of the Profit Maximizer:

   Proposition 4.10 $q^*_{1,\nu} \leq q^*_{1,d}$, $p^*_{1,\nu} \leq p^*_{1,d}$ and $\Pi^*_{1,\nu} \leq \Pi^*_{1,d}$.

   Observation 4.4. *Assuming all the models are profitable*;

   - if $R\lambda < 4a$ then,

     $p^*_{1,d} \geq p^*_{4,n} \geq p^*_{1,\nu}$.

     $\Pi^*_{1,d} \geq \Pi^*_{4,n} \geq \Pi^*_{1,\nu}$.

   - if $R\lambda \geq 4a$ then,

     $p^*_{1,d} \geq p^*_{1,\nu} \geq p^*_{4,n}$.

     $\Pi^*_{1,d} \geq \Pi^*_{4,n}$.

Table Tab. B.8 contains the values of the numerical analysis of Observation 4.4. For the even-valued rows of this table, i.e. $2^{nd}, 4^{th}, ..., 32^{th}$ rows, where $R\lambda \geq 4a$, we observe that the values of the sixth column is at least equal to or greater than the values of the fourth column. And the values of the fourth column is at least equal to or greater than the values of the second column. For the profit comparison of this case we have an exact comparison only for the values of the seventh and the third columns. For the other case we use the odd-valued rows of this table, i.e. $3^{rd}, 5^{th}, ..., 33^{th}$ rows, where $R\lambda \leq 4a$, we observe that the values of the sixth column is at least equal to or greater than the values of the second column, where the values of the latter one is at least equal to or greater than the values of the fourth column. To compare the profit of this case we can compare the values of the third, fifth and seventh columns of the table.
Comparing the results in Proposition 4.10 and Observation 4.4. we conclude that the Single Stage Benchmark Model With Double Service Rate has the highest entrance price and profit values based on the lowest waiting and quality costs. The comparison between the two server models depend on the model parameters. If $R\lambda \geq 4a$, so the profit maximizer provides perfect quality service in the Two-Parallel-Server Benchmark Model, then he charges higher entrance price to customers compared to Simple Escalation Model, since the former one has lower waiting cost. In the cases, when the profit maximizer provides service with interior quality levels, we observe that he sets the price at lower level in the parallel-server model compared to the escalation one.

3. Equilibrium Joining Probability Comparison of The Individual Problem:

Proposition 4.11 $\alpha^{eq}_{1,\nu}(q) \leq \alpha^{eq}_{1,d}(q)$ for all values of q.

4. Socially Optimal Joining Probability Comparison of The Social Problem:

Proposition 4.12 $\alpha^*_{1,\nu}(q) \leq \alpha^*_{1,d}(q)$ for all values of q.

We interpret the results given in Propositions 4.11 and 4.12, which compare these models from the individual and social viewpoint as: Since for the same expected reward, the waiting cost is lower in the Single stage model, joining probabilities of the individual and social problem is higher in this model. By observing the equilibrium and the socially optimal joining probabilities of the Simple Escalation model and the Two-Parallel-Stage Benchmark Model, which are given in Observation 4.1. and Observation 4.2, we also conclude that the joining probability is lowest in the Simple Escalation Model for the similar reasoning.

4.3.2 Comparison Between The Resolution Models

1. Short Term Comparison of the Problem of the Profit Maximizer:

Proposition 4.13 $p_{2,\nu}(q) \leq p_{2,d}(q)$, and $p_{4,g}(q) \leq p_{2,d}(q)$.
$\Pi_{2,\nu}(q) \leq \Pi_{2,d}(q)$, and $\Pi_{4,g}(q) \leq \Pi_{2,d}(q)$. for all values of q.

2. Long Term Comparison of the Problem of the Profit Maximizer:
Observation 4.5.

- *Optimal Quality Level Comparison:* $q^*_{4,g} \leq q^*_{2,\nu}$.
- *Optimal Price Comparison:* $p^*_{4,g} \leq p^*_{2,\nu} \leq p^*_{2,d}$.
- *Optimal Profit Comparison:* $\Pi^*_{4,g} \leq \Pi^*_{2,\nu} \leq \Pi^*_{2,d}$.

The numerical values of Observation 4.5 is presented in Table Tab. B.9.
We interpret the long term comparison between the resolution models, from the profit maximizer's viewpoint as: In Perfect Escalation Model, since the second server provides perfect quality service, the optimization procedure sets low quality level to the first server to decrease total cost of the quality. However, since

the quality level is low, then the arrival rate (the congestion level) of the second server is high. Based on this, not only the waiting cost of the first server, but also the waiting cost of the second server, so the total waiting cost of this Perfect Escalation Model is higher compared to the other Resolution Models. So, in order to decrease the displeasure of the waiting cost effect, the profit maximizer sets lower entrance price in this model. Additionally, since the entrance price is low and the cost of the quality is high, the profit maximizer has the lowest profit in the Perfect Escalation Model compared to the other Resolution Models.

The comparison between the Two-Parallel-Server Model With Resolution and the Single-Stage Model With Resolution With Double Server Rate can be explained as follows: Since for the given quality levels the waiting cost of the Parallel Stage Model is higher, to decrease the latter congestion, profit maximizer must set the service quality level at higher values in this model compared to the Single Stage Model. However, from the observation results, we conclude that setting the higher quality level is not enough for the profit maximizer. That is, the waiting cost of this model is still higher, so the profit maximizer must charge a lower entrance price to customers to induce them to decide to join all. Since the cost of the quality is higher and the entrance price of the model is lower, the profit of the profit maximizer is lower in the Two-Parallel-Stage Model With Resolution compared to the Single Stage Model.

3. Equilibrium Joining Probability Comparison of The Individual Problem:

Proposition 4.14 $\alpha^{eq}_{4,g}(q) \leq \alpha^{eq}_{2,d}(q)$ *and* $\alpha^{eq}_{2,\nu}(q) \leq \alpha^{eq}_{2,d}(q)$ for all values of q.

4. Socially Optimal Joining Probability Comparison of The Social Problem:

Proposition 4.15 $\alpha^*_{4,g}(q) \leq \alpha^*_{2,d}(q)$ *and* $\alpha^*_{2,\nu}(q) \leq \alpha^*_{2,d}(q)$ for all values of q.

The results corresponding the comparison of the individual and social optimal joining probabilities between the resolution models state: Since the service quality level is fixed for the individual and the social problem, and expected reward are exactly the same in the resolution models, the only thing which differentiates the models are the waiting cost. Since the waiting cost is lower in the Single Stage Model, the customers are more willing to join this system, and the social planner can accept more customers to this system.

4.4 Improving The Performance Of The Escalation Models

Although many service systems are designed like escalation models, the comparisons show that their performance in the presence of strategic customers may not be that high. The reasoning of this handicap can be listed as:

- We equally divide the total service rate to the two servers in the model. However, fixing the server rate of the first server to μ is not efficient. Because in this case the expected waiting cost of the first server is high, i.e. all the customers wait to be served in the first server.

- In the Simple Escalation Model, we assume that the two servers are identical; i.e. $R_1 = R_2$, $C_1 = C_2$ and $q_1 = q_2$. In this model, the only thing which creates the difference between the servers is the congestion level. However, if the quality levels of the servers are low, then the congestion level of the second server is also high. So, setting the same quality levels and deciding on only this unique quality level, decreases the performance of this Simple Escalation Model.

- In the Perfect Escalation Model, we assume that the second server provides perfect quality service. Indeed, it seems good in model design at first, when we analyze the model and optimize the service quality level of the first server, we conclude that the profit maximizer sets the quality level to very low values to decrease the total cost of the quality. However, this move of the profit maximizer, increases the total waiting cost in the system, which also decreases the performance of the Perfect Escalation Model.

By identifying the reasons behind the low performance of Escalation Models, we add some studies in this section, to decrease the inefficiency of these models.
To handle the first reason, we decide to change the profit maximizer's single parameter, q, decision problem into the two decision variable problem, μ_1 and q. In this analysis, we fix the total server rate of the system to 2μ, and decide on the server rate of the first server, μ_1, where the remaining $2\mu - \mu_1$ is the rate of the second server.
To deal with the second and third reasons, we again change single parameter, q, problem of the profit maximizer into a two parameter problem, where q_1 and q_2 which respectively denote the service quality levels of the first and the second server.

4.4.1 Escalation Models When the Service Rate and the Service Quality Level Are Decision Variables

In this part we assume that there are two sequential servers providing service with rate 2μ in total, i.e. if the rate of the first server is μ_1, then the rate of the second server is $2\mu - \mu_1$. In addition, the two servers provide service with the service quality level q. So, the problem of the profit maximizer is a two decision parameter problem, where he decides on μ_1 and q.
Based on this scenario, the utility function of the customer is written as:

$$U_4(\alpha) = Rq + Rq(1-q) - \frac{C}{\mu_1 - \lambda\alpha} - \frac{C(1-q)}{(2\mu - \mu_1) - (\lambda\alpha(1-q))} - p$$

For the market capturing price setting, the joining probability of the customer is 1, and the price as a function of μ_1, q is:

$$p_4(\mu_1, q) = Rq + Rq(1-q) - \frac{C}{\mu_1 - \lambda} - \frac{C(1-q)}{(2\mu - \mu_1) - (\lambda(1-q))}$$

The profit of the profit maximizer with this entrance price is:

$$\Pi_4(\mu_1, q) = \lambda p_4(\mu_1, q) - 2aq^2 = \lambda Rq + \lambda Rq(1-q) - \frac{C\lambda}{\mu_1 - \lambda} - \frac{C\lambda(1-q)}{(2\mu - \mu_1) - (\lambda(1-q))} - 2aq^2$$

Lemma 4.13 $\Pi_4(\mu_1, q)$ is jointly concave in μ_1 and q.

Since we have the joint concavity property of the profit function with respect to the decision parameters (as given in Lemma 4.13), we have a unique solution (maximizer) for this problem. We can then give numerical results comparing this Escalation Model with the other models, Two-Parallel-Server-Benchmark-Model and Single Stage Benchmark Model With Double Server Rate.

Observation 4.6. $\Pi_4^*(\mu_1, q) \geq \Pi_{4,n}^*(\mu, q)$.

We give the values of this Observation in table Tab. B.10. From the table values we have some conclusions. We first observe that the optimal service rate of the first server is higher than the service rate of the second server; i.e. values of the second column is higher than $2\mu/2 = \mu$. Secondly, by comparing the values of fourth column of this table with the values of the third column of Tab. B.8, we observe that the profit of the profit maximizer is higher in the former one.

In order to interpret the numbers given in Observation 4.6, we must analyze them together with the numbers given in Observation 4.4. By deciding on the service rate of the first server in the escalation model, the profit maximizer increases his profit; i.e. the profit values of the improved model given in Observation 4.6 are higher than the ones given in Observation 4.4 which represents the Simple Escalation Model. Additionally, by comparing these values with the profit values of the Two-Parallel-Server Benchmark Model (as given in Observation 4.4), we conclude that if they are properly designed, the Escalation Models perform better compared to the parallel-server models. Moreover, the performance of the Escalation Models gets closer to the Single Server Models With Double Rate, i.e. the difference between the profit values is decreased.

The effect of deciding on the service rate on the performance of the Escalation Models is decreasing the waiting cost of the first server, so the total waiting cost. By analyzing the numbers given in Observation 4.6, we see that it is optimal for the profit maximizer to set a larger portion of his total service rate (more than 50%) to the first stage.

4.4.2 Escalation Models When the Servers Provide Service with Different Quality Levels

Assuming that the two sequential servers provide service with q_1 and q_2 quality levels, respectively, the utility function of the individual customer is given as:

$$U_4(\alpha) = Rq_1 + R(1-q_1)q_2 - \frac{C}{\mu - \lambda\alpha} - \frac{C(1-q_1)}{\mu - \lambda\alpha(1-q_1)} - p.$$

Assuming the profit maximizer sets the market capturing price, the equilibrium joining probability of the customer is 1. Based on this equilibrium joining probability, the market capturing price as a function of the quality levels given as:

$$p_4(q_1, q_2) = Rq_1 + R(1-q_1)q_2 - \frac{C}{\mu - \lambda} - \frac{C(1-q_1)}{\mu - \lambda(1-q_1)}.$$

Using the price given in the above equation, the profit function of the profit maximizer is written as:

$$\Pi_4(q_1, q_2) = \lambda \left[Rq_1 + R(1-q_1)q_2 - \frac{C}{\mu - \lambda} - \frac{C(1-q_1)}{\mu - \lambda(1-q_1)} \right] - aq_1^2 - aq_2^2$$

Lemma 4.14 $\Pi_4(q_1, q_2)$ is jointly concave in q_1 and q_2.

Since the concavity follows as stated in Lemma 4.14, we write the optimal service quality level expression of the second server as a function of the service quality level of the first server.

Corollary 4.4 *The optimal quality level of the second server as a function of the quality level of the first server is:*

$$q_2^*(q_1) = \frac{R\lambda(1-q_1)}{2a}$$

The expression given in Corollary 4.4 shows us that, the quality levels of the two servers are inversely proportional. That is if the quality level of the first server is high, then the quality level of the second server is low and vice versa. In other words, if the quality level of the first server is high, the profit maximizer sets the quality level of the second server to lower levels to optimize his profit.

We present the numerical study comparing the performance of this Escalation Model with the other models. Since the quality level of the second server can take any value; $0 \leq q_2 \leq 1$, some of the customers can leave this system unsatisfied with probability $(1-q_1)(1-q_2)$. So to make the fair comparisons, as the other models we use the Two-Parallel-Stage Benchmark Model and the Single-Stage Benchmark Model with Double Server Rate.

Observation 4.7. $\Pi_4^*(q_1, q_2) \geq \Pi_{4,n}^*(\mu, q)$.

Table Tab. B.11 includes the numerical values of this Observation. By comparing the second and third columns of this table, we observe that if the values of the second column is high, then the values of the third column is low, so the optimal quality values of the two servers are inversely proportional. Additionally by comparing the fourth column of this table with the fourth column of Tab. B.8, it is straightforward that the values are higher in the former one.

By combining the numbers given in Observation 4.4 and Observation 4.7, we come up with similar conclusions as we present in Observation 4.6. That is, by re-designing our Escalation Models with different service quality levels in each of the server, the profit maximizer's profit from these models increased where they are higher compared to parallel-server models and closer to the Single Server Model With Double Server Rate.

Moreover, from the numbers given in Observation 4.7, we reach such a conclusion that assuming unit cost of the service quality are the same in each state, it is optimal for the profit maximizer to set high quality level to the first server. The reason behind this is to decrease the total waiting cost in the system. Since the quality levels of the servers are inversely proportional, we see that the service quality level of the second server is smaller compared to the first one.

4.5 Discussion

In the last section of this chapter, we discuss and compare the models.

- As a system design issue, we observe that the Escalation Models are not very efficient. These models have lower joining probabilities for the individual and the social problem, and lower profits for the problem of the profit maximizer. We conclude that the reason behind this result is the high waiting cost of these models. The first server provides service with the rate μ, and all of the customers who join, wait in the queue in front of this server. Moreover, in the perfect escalation model, since the second server provides perfect quality service, to decrease the cost of the quality, the profit maximizer sets the optimal quality level of the first server to low levels, which increase the congestion and the waiting cost of the second server. Thus the inefficiency of the Escalation Models is caused by the parameter choices in system design. After a redesign which entails optimization of model parameters, the performance of this model is increased; i.e. the profit of the profit maximizer gets closer to the Single Stage Models With Double Server Rate.

- The Single Server Model which provides service with a double rate performs best between all of the models that we analyze in this chapter. The interpretation behind this result as: It is optimal to provide service with a single but a more qualified server; i.e. the service rate of the server is high and the optimal quality of the server is high since the quality cost is low; i.e. profit maximizer invests the quality cost for only a unique server. This result may not always be applicable as a doubly fast server may not be feasible.

- The Two-Parallel-Stage Models performs well, but not as much as the Single Stage Model. From the individual and social viewpoint, the performance of the Two-Parallel-Stage Model is lower compared to the Single Stage Model, because the waiting cost of the system is higher in the former one. From the profit maximizer's viewpoint the similar comparison holds based not only on the higher waiting cost but also the higher cost of the quality. So, as a system design issue the M/M/1 Model with a double server rate is better than the *M/M/2* Model.

- For the individual problem, we conclude that it is optimal for the customer to leave the system rather than taking the resolution of the service failure; i.e. Benchmark Models have higher joining probabilities compared to Resolution Models. Additionally, for the social problem, it is optimal for the profit maximizer to not the offer resolution for the quality problem of the system. That is the Benchmark Models have also higher joining probabilities compared the Resolution Models. These result shows the importance of the waiting costs for the individual and the social problem. Since the service quality level is fixed and there is no other parameters for this problem which can decrease the displeasure of the waiting cost effect, both the individual and the social planner chooses the models which have the lowest waiting costs.

- When we compare the different resolution alternatives from the profit maximizer's viewpoint in the short term, where the quality level is fixed, we conclude that the performances of the models depend on the model parameters. For example, the Benchmark Models, in which the profit maximizer loses some of his customers, can be preferable if the service quality level or the service rate is low, or the arrival rate is high. For the inverse situations; the service rate or the service quality level is high or the arrival rate is low, it is optimal for the profit maximizer to provide resolution to unsatisfied customers and not to lose them. When we compare the short term results of the profit maximizer's problem and the individual and social problem, we also conclude that, since the profit maximizer has one decision parameter, queue entrance price, he has the chance to mitigate the displeasure of the waiting cost by decreasing the entrance price. So, in the short term results of the profit maximizer's problem, we conclude that depending on the model parameters the Resolution Models can be more profitable for the profit maximizer.

- When we analyze the problem of the profit maximizer in the long term, we observe that the Resolution Models are always better and more profitable compared to the Benchmark Models. So, since in the long term, the profit maximizer has the chance to change both the queue entrance price and the service quality level, it is optimal for him to offer resolution to customers and not to lose them.

- From the service quality level expressions of all of the models that we analyze, we conclude that the optimal service quality level is affected by the change of the service reward in Benchmark Models, since not all of the customers receive this reward. However, for the Resolution Models, since all of the customers receive the service reward, we conclude that the optimal service quality level is independent from the service reward.

Chapter 5

SERVICE RATE DECISIONS

In previous chapters, we assume that in the long term, to maximize his profit, the profit maximizer decides on the service quality level. However, changing the service quality level may require investment and time. For this reason, in some service systems, rather than changing the service quality level, the profit maximizer may choose to change the service rate.

In this chapter, we analyze the profit maximizer's problem in our different quality models, when the service rate is also a decision for the profit maximizer. So in this chapter, we investigate the optimal price, service rate and the service quality level selection problem of the profit maximizer in the quality models which are discussed in the previous chapters. By working on such a service decision problem, we will be able to conclude how the strategy of the profit maximizer is affected from the changes in the service rate and the service quality level.

In the analysis of this chapter, we build on the results given in Chen and Frank [11]. In their paper, they analyze the short and long term problem of the profit maximizer in different pricing settings assuming that the service quality level of the system is perfect. In the long term analysis, Chen and Frank assume that the service rate of the system is a decision for the profit maximizer, so in this problem, the profit maximizer optimizes his profit by deciding on the entrance price and the service rate. In their problem, they assume the linear cost structure holds for the service rate and the unit cost of adding server to their model is q. By also considering the fixed cost, F, of the profit maximizer, the total cost function of the long term problem given in Chen and Frank [11] is represented as $q\mu + F$. In the long term, by comparing the profit values of the profit maximizer under different pricing settings; monopolistic and market capturing, they showed that it is optimal for the profit maximizer to set the market capturing price and serve the whole market.

Since the service rate is also a decision in our analysis, based on the long term analysis result given in Chen and Frank [11], we focus on the market capturing pricing setting in this chapter. We also use the linear cost function for the service rate decision as given in Chen and Frank [11]. But for the unit cost parameter of the service rate we use the notation b, since their notation is used for the service quality level in our thesis. Moreover, we omit the fixed costs in all of our analysis because in this thesis we assume that the positive profitability condition holds in the profit maximizer's problem (the fixed costs are canceled in optimization). Since our main problem in this thesis is the service quality or the failure problem, in this chapter we still analyze this problem. So the decisions of the long term problem of the profit maximizer are the price, service rate and the service quality level. But since in the market capturing pricing setting, the optimal price exactly depends on the optimal service rate and the quality level, the decision of the price can be omitted of our long term problem. That is, our long term problem in this chapter is a maximization problem with two decisions, service rate and the service quality level, where the optimal entrance price is denoted by the optimal values of these parameters. As before, we use a quadratic cost structure for changing the service quality level since the linear cost structure is not suitable (in service systems, the cost of changing the service level from 80% to 85% is different from changing it from 85% to 90%).

In the first section of this chapter, we will analyze how the analysis changes when we decide additionally on the service rate for the earlier single stage models. In the second section, we repeat a similar analysis for the two-stage models. In order to compare this analysis with the earlier ones and see how the change in the service rate affects our analysis, we classify our analysis for all of the models in two cases. In the first case, we analyze the profit maximizer's problem when the service rate and the service quality level are the decisions, and label this case as the "Service Rate Is A Decision Case". In the second case, we use our earlier results where only the decision

is the service quality level, and label this as the "Fixed Service Rate Case".
For the detailed Proofs of the Lemmas, Corollaries and Propositions of this chapter, you can see Appendix A.4 and for the Observations you can see Appendix B.3.

5.1 *The Single Stage Models*

5.1.1 *The Benchmark Model*

Using the individual utility function given in (3.1), and setting the joining probability value 1; $\alpha_1^{eq} = 1$, the market capturing price of the benchmark model as a function of the service rate, μ, and the service quality level, q, is represented as:

$$p_1(\mu, q) = Rq - \frac{C}{\mu - \lambda} \tag{5.1}$$

Using the entrance price (5.1) and the linear cost function of the service rate and the quadratic cost function of the service quality level, the profit function of the profit maximizer of the Benchmark Model, under the market capturing setting is shown as:

$$\Pi_1(\mu, q) = \lambda p_1(\mu, q) - b\mu - aq^2 = \lambda \left[Rq - \frac{C}{\mu - \lambda} \right] - b\mu - aq^2 \tag{5.2}$$

Lemma 5.1 $\Pi_1(\mu, q)$ *is concave in* μ*.*

Corollary 5.1 The optimal service rate as a function of the model parameters is:

$$\mu_1^*(q) = \lambda + \sqrt{\frac{C\lambda}{b}}$$

As concluded in Chen and Frank [11], the firm responds to an increase in λ by increasing μ. When the service rate, μ, increases, the entrance price, p, also increases. The optimal service rate decision does not vary with the unit reward R. Compared to Chen and Frank [11], our problems additionally include the quality parameter. In our analysis we conclude that the optimal service rate parameter is independent of this service quality parameter in the Benchmark Model. The reasoning of this result

can be explained as: In the Benchmark Model all of the customers leave the system independent of the service quality that they receive (the dissatisfaction of the customers does not create any congestion in the system, since they do not return).
Using the optimal service rate obtained in Corollary 5.1, the profit function of the profit maximizer is rewritten as:

$$\Pi_1(\mu_1^*, q) = R\lambda q - 2\sqrt{C\lambda b} - b\lambda - aq^2 \tag{5.3}$$

Lemma 5.2 $\Pi_1(\mu_1^*, q)$ is concave in q.

Corollary 5.2 The optimal service quality level is given by:

$$q_1^* = \begin{cases} \frac{R\lambda}{2a}, & \textit{if } (R\lambda - a)q > 2\sqrt{C\lambda b} + b\lambda \textit{ for all } 0 < q \leq 1 \\ 0, & o.w. \end{cases}$$

In order to observe how the analysis change when we additionally decide on the service rate, we compare this optimal quality level, with the one we give earlier. In Chapter 3, we analyze the problem of the profit maximizer under market capturing pricing setting, when we decide only on the service quality level. In this chapter, we denote this optimal quality level with, $q_{1,\tilde{\mu}}^*$, since this is obtained when the service rate is not a decision (fixed-service-rate).

Proposition 5.1 If positive profit is possible for the Service Rate is a Decision and the Fixed Service Rate Case, then:

$$q_1^* = q_{1,\tilde{\mu}}^*$$

The result given in Proposition 5.1 seems interesting. Because it shows that if the profit maximizer is in the positive profit conditions, his choice of service quality level is not affected from his service rate decision. Because, the terms representing the service quality level and the service rate are not inter-related.

5.1.2 The Model With Resolution

For the individual utility function of the Model With Resolution, we use (3.14). Since the market capturing pricing strategy is optimal when the service rate is a decision for the profit maximizer, all customers are assumed to join, so α_2^{eq}=1. Then, the market capturing price of the Model With Resolution is:

$$p_2(\mu, q) = R - \frac{C}{\mu q - \lambda} \tag{5.4}$$

Using the entrance price given in (5.4), the profit function of the profit maximizer of this model is:

$$\Pi_2(\mu, q) = \lambda p_2(\mu, q) - b\mu - aq^2 = \lambda \left[R - \frac{C}{\mu q - \lambda} \right] - b\mu - aq^2 \tag{5.5}$$

Lemma 5.3 $\Pi_2(\mu, q)$ is concave in μ.

Corollary 5.3 *The optimal service rate as a function of the model parameters and the service quality level is:*

$$\mu_2^*(q) = \frac{\lambda + \sqrt{\frac{C\lambda q}{b}}}{q}$$

The interpretation of the result given in the Corollary 5.3 is: The optimal service rate is not affected by the change in R. The service rate increases when the arrival rate increase. Compared to the Benchmark Model, we conclude that the optimal service rate decision is affected by the change in the service quality level in the Resolution Model. The reasoning is: The returns of the unsatisfied customers generate the congestion of the system so when the service quality level is low so return rate to the system is high and the profit maximizer must increase his service rate. That is the optimal service rate is inversely proportional to the service quality level.

We rewrite the profit function of the profit maximizer, by plugging the optimal service rate expression, as a function of the service rate, which is derived in Corollary 5.3 in place.

$$\Pi_2(\mu_2^*, q) = R\lambda - 2\sqrt{\frac{C\lambda b}{q}} - \frac{b\lambda}{q} - aq^2 \tag{5.6}$$

Lemma 5.4 $\Pi_2(\mu_2^*, q)$ is concave in q.

Since concavity follows, the optimal service quality level is found numerically from the first order condition.

We compare the optimal quality level of this service rate is a decision case, q_2^*, with the one we give earlier in the third chapter, fixed service rate case, $q_{2,\tilde{\mu}}^*$. The first order conditions representing the service quality level of these two cases, Case 1 where the service rate is a decision and the Case 2 with a fixed service rate, cannot be compared analytically. Because, for the first case when the service rate is a decision, the first order condition representing the service quality level depends on the unit service rate cost, b, and is independent from the service rate, μ. However in the second case, for the fixed service rate, the first order condition describing the service quality level depends on the service rate, and is independent from the unit cost of the service rate. So, the optimal quality levels and the optimal profits of these two cases can only be compared numerically.

Observation 5.1:

- *If* $\mu \leq \mu_2^*$ *then* $q_2^* \leq q_{2,\tilde{\mu}}^*$.
- *If* $\mu > \mu_2^*$ *then* $q_2^* \geq q_{2,\tilde{\mu}}^*$.
- $\Pi\left(\mu_2^*, q_2^*\right) \geq \Pi\left(\mu, q_{2,\tilde{\mu}}^*\right)$

The numerical values of this Observation is presented in Table Tab. B.12. To interpret these numbers we first compare the second and fifth columns of the table and observe that if the value of the second column is higher than the value of the fifth column, then we also observe that the value of the third column is smaller than the value of the sixth column. Additionally the numbers of the seventh column is always higher than the values of the fourth column.

The numerical result given in the Observation 5.1. shows the following: The service rate and the service quality level are substitutes of each other. So, if the profit maximizer sets his service rate to a value that is less than its optimal, then he maximizes his profit by increasing his service quality level. Similarly, if he holds an excess service capacity, then he compromises from his cost regarding the service quality level, by decreasing it. On the other hand, the profit maximizer increases his profit by additionally deciding on the service rate.

5.1.3 The Model With Returns

The individual utility function of the model with returns is represented in (3.23). All customers are assumed to join under the market capturing pricing strategy where α_3^{eq}=1. The market capturing entrance price of the Model with Returns is:

$$p_3(\mu, q) = Rq - \frac{C}{\mu q - \lambda} \tag{5.7}$$

As derived in the third chapter, the profit function of the profit maximizer under the market capturing price setting of the Model With Returns is:

$$\Pi_3(\mu, q) = \left(Rq - \frac{C}{\mu q - \lambda}\right)\left(\frac{\lambda}{q}\right) - b\mu - aq^2 \tag{5.8}$$

Lemma 5.5 $\Pi_3(\mu, q)$ is concave in μ.

Corollary 5.4 The optimal service rate as a function of the service quality level is:

$$\mu_3^*(q) = \frac{\lambda + \sqrt{\frac{C\lambda}{b}}}{q}$$

The optimal service rate expression of the Model With Returns also depend on the service quality level. As in the Resolution Model, the optimal service rate is inversely proportional with the service quality level.

Proposition 5.2

$$\mu_3^*(q) \geq \mu_2^*(q) \geq \mu_1^*(q)$$

The result given in Proposition 5.2 can be interpreted as follows: Since the profit maximizer offers resolutions in the second and third models which increase the congestion of the system, he has to keep the service rate higher in these models compared to the Benchmark Model. Moreover, since he does not offer a full resolution in the Model With Returns; i.e. the returning customers have still chance of being failed in the service, he has to keep the maximum service rate in this model between the single-stage-models.

Plugging the optimal service rate expression derived in Corollary 5.4 in place of (5.8), the profit function of the profit maximizer is obtained as:

$$\Pi_3(\mu_3^*, q) = R\lambda - 2\sqrt{\frac{C\lambda b}{q^2}} - \frac{b\lambda}{q} - aq^2 \tag{5.9}$$

Lemma 5.6 $\Pi_3(\mu_3^*, q)$ is concave in q.

Since concavity follows, the optimal service quality level is found numerically from the first order condition.

Since we showed the concavities of the profit functions of the Model With Resolution and Model With Returns, with respect to the service quality level, as given in Lemmas 5.4 and 5.6, we can compare the optimal model parameters and the optimal profit by comparing the first order conditions.

Proposition 5.3 *$q_3^* \geq q_2^*$ and $\mu_3^* \geq \mu_2^*$.*

Based on Proposition 5.3, we conclude that: Since the market capturing price is set, all customers are assumed to join the system in the models. Based on this, the waiting time expressions are the same in the Model With Resolution and the Model

With Returns. Since for the same waiting time expressions, the profit maximizer offers full resolution in the Model With Resolution; i.e. customers cannot leave the system without taking the reward, he must set his optimal model parameters to higher levels in the Model With Returns, since some customers still have a chance of being unsatisfactorily served. That is, to decrease the waiting cost of the customers,the profit maximizer sets the service rate and the service quality level at higher levels in the Model With Returns.

For the Model With Returns, we compare the model parameters of the Service Rate is a Decision Case with the ones given in Chapter 3, as a Fixed Service Rate Case. The numerical comparison for these two cases is as follows:

Observation 5.2:

- *If* $\mu \leq \mu_3^*$ *then* $q_3^* \leq q_{3,\tilde{\mu}}^*$.
- *If* $\mu > \mu_3^*$ *then* $q_3^* \geq q_{3,\tilde{\mu}}^*$.
- $\Pi\left(\mu_3^*, q_3^*\right) \geq \Pi\left(\mu, q_{3,\tilde{\mu}}^*\right)$.

Table B.13 contains the numerical values of Observation 5.2. The interpretation of this table is very similar to the interpretation of Table 5.1.

We interpret our findings given in the Observation 5.2 as given after Observation 5.1.

Before closing this section we compare the profit values of the profit maximizer in both of the cases in the Model With Resolution and the Model With Returns.

Observation 5.3:

- $\Pi\left(\mu, q_{2,\tilde{\mu}}^*\right) \geq \Pi\left(\mu, q_{3,\tilde{\mu}}^*\right)$.
- $\Pi\left(\mu_2^*, q_2^*\right) \geq \Pi\left(\mu_3^*, q_3^*\right)$.

For Observation, we use the values of Tables Tab B.12 and Tab B.13. When we compare the fourth and seventh columns of these two tables, we see that the values are higher in Table B.12.

Using the result of Observation 5.3, we conclude that the Resolution Model, which offers full resolution and does not retake the entrance price from the returning customers, is more profitable compared to the Model With Returns, which retakes the entrance price.

5.2 The Two-Server-Models

In our Two-Server-Models, we use a server in each stage. So, in our analysis during this section, we assume that the profit maximizer invests for both of the stages; i.e. double costs for the service rate and the service quality level.

5.2.1 Two Parallel Stage Models

In this section, we will analyze the concavity of the Two-Parallel-Stage Benchmark and the Resolution Models respectively.
For the Two-Parallel-Stage-Benchmark Model, the market capturing price which pushes all the potential customers to decide to join is as given in (4.28). With this entrance price and proper cost functions of this model, the profit function of the profit maximizer is written as:

$$\Pi_{1-\nu}(\mu, q) = \lambda p_{1-\nu} - 2\mu b - 2aq^2 = \lambda \left(Rq - \frac{4C\mu}{4\mu^2 - \lambda^2} \right) - 2\mu b - 2aq^2 \qquad (5.10)$$

Lemma 5.7 $\Pi_{1-\nu}(\mu, q)$ *is jointly concave in* μ *and* q*.*

Since the profit function is jointly concave as given in Lemma 5.7, optimal solution of the problem of the profit maximizer is unique. Since the derivative function with respect to the service rate, μ, is analytically hard to analyze, we analyze this model and the model parameters numerically.
We compare the result of this analysis, with the one given in the fourth chapter, which works on the same problem with the only decision parameter, service quality level; Fixed Service Rate Case. This analysis is the same with the service rate is a decision case.

Proposition 5.4 If the positive profitability is achieved in both of the cases: Service Rate is a Decision and the Fixed Service Rate,

$$q^*_{1-\nu} = q^*_{1-\nu,\tilde{\mu}} = \frac{\lambda R}{4a}$$

The interpretation of Proposition 5.4 is: Since in the profit function, the service rate and the service quality level are not inter-related, the optimal service quality level of the Two-Parallel-Stage-Benchmark Model are the same in the two cases into consideration: Service Rate is a Decision and the Fixed Service Rate.

The numerical analysis, representing the optimal model parameters for the two cases; i.e. $q^*_{1-\nu,\tilde{\mu}}, \Pi\left(\mu, q^*_{1-\nu,\tilde{\mu}}\right)$ for the fixed service rate case and $\mu^*_{1-\nu}, q^*_{1-\nu}, \Pi\left(\mu^*_{1-\nu}, q^*_{1-\nu}\right)$ for the service rate is a decision case; is represented in the Observation 5.4.

Observation 5.4: *For the Parallel Server Model, optimal quality level values and the profits of the fixed service rate and service rate is a decision case is compared as:*

- $$\begin{cases} q^*_{1-\nu,\tilde{\mu}} = q^*_{1-\nu} = \frac{R\lambda}{4a}, & (R\lambda - 4aq)q > \frac{4C\lambda\mu}{4\mu^2-\lambda^2} + b\mu \\ q^*_{1-\nu,\tilde{\mu}} = 0 < q^*_{1-\nu}, & \text{o.w.} \end{cases}$$

- $\Pi\left(\mu, q^*_{1-\nu,\tilde{\mu}}\right) \leq \Pi\left(\mu^*_{1-\nu}, q^*_{1-\nu}\right)$

Please see Table Tab. B.14 for the numerical values of Observation 5.4. To interpret this table we first use second, third, fifth and sixth columns of the table and observe that if the values of the second column is lower than the values of the fifth column, then the values of the third column is higher than the values of the sixth column, and vice versa. Additionally when we compare the fourth and the seventh columns we observe that the values of the former one is always smaller than the values of the seventh one.

From the numbers representing the optimal model parameters of the Two-Parallel-Stage-Benchmark Model for the two cases we have the following observations: The profit maximizer achieves higher profit when he additionally decides on the service rate. Since the service rate and the service quality level parameters are not interrelated the optimal service quality level are the same in both of the cases assuming the positive profitability conditions hold in both of the cases. For the congested systems, arrival rate is high and the server rate is low, the positive profitability of the system is hard to achieve in the fixed server rate case.

We now analyze the Two-Parallel-Stage Model With Resolution for these two cases. The market capturing price of this model is given in (4.33). Using this entrance price, the profit function of the profit maximizer is:

$$\Pi_{2-\nu}(\mu, q) = \lambda p_{2-p} - 2\mu b - 2aq^2 = \lambda \left(R - \frac{4C\mu}{4\mu^2 q^2 - \lambda^2} \right) - 2\mu b - 2aq^2 \qquad (5.11)$$

Lemma 5.8 $\Pi_{2-\nu}(\mu, q)$ is jointly concave in μ *and* q.

Since the profit function is jointly concave with respect to the model parameters as given in Lemma 5.8, we have the unique solution of this maximization problem. However, since the optimal model parameters are untractable, we will use Observations to perform our comparisons.

In order to compare the results above, with our earlier analysis given in Chapter 4, representing the Fixed Service Rate Case, we present Observations based on numerical results.

The numerical analysis, representing the optimal model parameters for the two cases; i.e. $q^*_{2-\nu,\tilde{\mu}}, \Pi\left(\mu, q^*_{2-\nu,\tilde{\mu}}\right)$ for the fixed service rate case and $\mu^*_{2-\nu}, q^*_{2-\nu}, \Pi\left(\mu^*_{2-\nu}, q^*_{2-\nu}\right)$ for the service rate is a decision case; is summarized in the Observation 5.5.

Observation 5.5:

- If $\mu \leq \mu^*_{2-\nu}$ then $q^*_{2-\nu,\tilde{\mu}} \geq q^*_{2-\nu}$.
- If $\mu > \mu^*_{2-\nu}$ then $q^*_{2-\nu} > q^*_{2-\nu,\tilde{\mu}}$.
- $\Pi\left(\mu^*_{2-\nu}, q^*_{2-\nu}\right) \geq \Pi\left(\mu, q^*_{2-\nu,\tilde{\mu}}\right)$.

We present the numerical results of Observation 5.5 in Table Tab. B.15. We interpret this Table as Table Tab. B.14, which shows the numerical values of Observation 5.5.

We interpret this Observation as follows: The profit maximizer increases his profit when the service rate is also a decision parameter as in the Two-Parallel-Stage-Benchmark Model. By comparing the optimal quality levels of the two cases, we observe that the optimal quality level of the fixed server rate case is higher than the optimal quality level of the server rate is a decision case when the fixed server rate is smaller than it is optimal.

Now we give the comparison between the service rate expressions depending on the service quality level, of the Two-Parallel-Server Models.

Proposition 5.5 $\mu^*_{1-\nu}(q) \leq \mu^*_{2-\nu}(q)$ if $\mu q > \lambda$ for all values of μ and $0 < q \leq 1$.

The interpretation of Proposition 5.5 is: When the quality level is fixed, the profit maximizer must increase the service rate if he offers resolution, since the congestion of the system increases based on this resolution.

We close this section by comparing the profit values of the profit maximizer in the different Two-Parallel-Server Models for the Fixed Service Rate and the Service Rate is a Decision cases, by using the numbers given in the Observations 5.4 and 5.5.

Observation 5.6:

- $\Pi\left(\mu, q^*_{1-\nu,\tilde{\mu}}\right) \leq \Pi\left(\mu, q^*_{2-\nu,\tilde{\mu}}\right)$.
- $\Pi\left(\mu^*_{1-\nu}, q^*_{1-\nu}\right) \leq \Pi\left(\mu^*_{2-\nu}, q^*_{2-\nu}\right)$.

We reuse the numerical values of Tables Tab. B.14 and Tab. B.15 to reach Observation 5.6. From the tables values we observe that the fourth and seventh columns of Table Tab. B.14 is always lower than the same column values of Table Tab. B.15.

From the model comparison given in Observation 5.6, we have the following result: In the long term, when the profit maximizer has chance to change at least one of the model parameters, only the quality level or the service rate and the quality level, it is more profitable for him to give resolution to the customers and not to loose them.

5.2.2 The Escalation Models

In this section, we analyze the two escalation models. Since the profit functions of these models are analytically hard to analyze with respect to the model parameters μ and q, and deriving explicit closed form expressions of these model parameters are intractable, we give the numerical analysis representing the optimal model parameters for the two cases: fixed service rate and the service rate is a decision.
We first give the results of the numerical analysis of the Simple Escalation Model.
Observation 5.7:

- $q^*_{4-n,\tilde{\mu}} \cong q^*_{4-n}$.
- $\Pi\left(\mu, q^*_{4-n,\tilde{\mu}}\right) \leq \Pi\left(\mu^*_{4-n}, q^*_{4-n}\right)$.

We present the numerical values of Observation 5.7. in Tab. B.16. The table is interpreted similarly as we give in Observations 5.4 and 5.5.
We interpret Observation 5.7 as follows: Since in the profit function the terms having μ and q are independent, the optimal quality levels are nearly the same in both of the cases: fixed service rate and the service rate is a decision. Additionally, as in all of the single stage models and the two parallel stage models, in this model we observe that the profit maximizer increases his profit by deciding on the service rate.
For the Perfect Escalation Model, we summarize our observations in Observation 5.7.
Observation 5.8:

- If $\mu \leq \mu^*_{4-g}$ then $q^*_{4-g,\tilde{\mu}} \geq q^*_{4-g}$.
- If $\mu > \mu^*_{4-g}$ then $q^*_{4-g} \geq q^*_{4-g,\tilde{\mu}}$.
- $\Pi\left(\mu^*_{4-g}, q^*_{4-g}\right) \geq \Pi\left(\mu, q^*_{4-g,\tilde{\mu}}\right)$.

Table Tab. B.17 contains the numerical values of Observation 5.8. The interpretation of this table is similar to previous one.
The results which we summarized in Observation 5.8 are interpreted as: The two

decision variables, the service rate and the service quality level are the substitutes of each other. This means, if the fixed service rate is smaller than it is optimal, then to maximize his profit, the profit maximizer should increase the service quality level. Moreover, the profit of the profit maximizer is higher when he decides not only on the service quality level but also on the service rate.

Finally, we give the long term comparison between the two models in the Observation 5.8.

Observation 5.9:

- $q^*_{4-g,\tilde{\mu}} \leq q^*_{4-n\tilde{\mu}}$ and $q^*_{4-g} \leq q^*_{4-n}$.
- $\mu^*_{4-g} \geq \mu^*_{4-n}$
- $\Pi\left(\mu, q^*_{4-n,\tilde{\mu}}\right) \leq \Pi\left(\mu, q^*_{4-g,\tilde{\mu}}\right)$.
- $\Pi\left(\mu^*_{4-n}, q^*_{4-n}\right) \leq \Pi\left(\mu^*_{4-g}, q^*_{4-g}\right)$.

To see the numbers of Observation 5.9 we use Tables Tab. B.16 and B.17.

The interpretation of Observation 5.9. is as follows: In the Perfect Escalation Model, since the second server provides perfect quality service, the quality level of the first server is smaller compared to the Simple Escalation Model. On the other hand, since the service quality level of the first server is small, to decrease the waiting costs in the perfect escalation model, the profit maximizer should keep the server rate higher in the Perfect Escalation Model compared to the Simple Escalation Model. Moreover, as before, we again conclude that in the long term, when the profit maximizer has chance to decide on at least one of the model parameters, then to increase his profit it is optimal for him to offer resolution to the customers rather than losing them.

5.3 Discussion

In this chapter, we analyze how the problem of the profit maximizer is affected when he decides not only on the service quality level but also on the service rate in all of the models represented in this thesis.

We summarize our results related to this chapter as:

- Deciding on more parameters positively affects the profit of the profit maximizer.

  The profit values are higher when Service Rate is a Decision compared to the Fixed Service Rate Case in all of the models.

- The service quality level and the service rate are the substitutes of each other.

  If the fixed service rate is smaller than it is optimal, the profit maximizer must set the service quality level at least equal or greater than it is optimal.

  If the fixed service rate is higher than it is optimal, than the profit maximizer sets the service quality level at most equal or smaller levels than it is optimal.

- If the profit maximizer offers resolution to customers, then he must increase at least one of his model parameters, service rate and the service quality level, to prevent the system from the congestion of the returning customers create.

- In the long term, when the profit maximizer has chance to change at least one of the model parameters, it is optimal for him to offer resolution to customers rather than losing them.

  The profit value of the Resolution type models are higher than the Benchmark type models.

- Especially for the congested systems; i.e. $\lambda = 9.5$, $\mu = 6$ or $\mu = 10$, deciding on μ rather than fixing it highly affects the performance of the system.

  The difference between the profits of the Service Rate is a Decision and Fixed Service Rate Cases is large in all of the models.

Chapter 6

CONCLUSIONS

In this thesis, we analyze the behavior of strategic customers in service systems by considering the service quality. We mainly focus on three problems: Problem of the individual customer, social problem and the problem of the profit maximizer.

By adding a service quality perspective in these problems, first we are able to analyze how the behaviour of the strategic customer and the social planner is affected when service failure occurs with given probability levels. In our analysis, in both of the observable and unobservable queue length settings, we see that the joining rates of the individual and the social problems are increasing in the service quality level. Thus, the higher the service quality level is, so are the individually and socially optimal joining rates. We also see that the individually and socially optimal joining rates differ from each other, where the former one is higher compared to later one. This result is the same with the ones given in Naor [30] and Edelson and Hildebrand [16] which analyze these two problems in the observable and unobservable queues respectively. One of the results, that we obtain differently compared to these papers in literature, in our thesis is the following: The difference between the individual and social problem is increasing in the service quality level. So, for higher quality levels the mismatch between these two problems is higher, where it reaches the maximum for the perfect quality of service. This shows us that providing service with lower quality levels can be used to decrease the difference between the individual and social optima.

We also analyze the profit maximization of the profit maximizer when he provides service to strategic customers under service failures. In their book, Hassin and Havviv [20], by comparing the social and profit maximizer's problems assuming the service quality level is perfect, conclude that the objectives of the profit maximizer and society coincide. However, we do not obtain this result in our thesis, since we include the cost of the quality to the profit maximizer's problem.

On the other hand, since we are dealing with the quality problem, we also need to analyze the resolution of this quality problem. The possible resolution alternatives are listed in this thesis as: Customers can leave the system and go another system, they can return to the same system or they can be escalated to the higher level of servers in the same system. To compare these resolution alternatives from the individual, social and the profit maximizer's perspective, we develop different quality models.

In the comparisons, which compare the individually and socially optimal joining rates of these different models, we conclude that the Benchmark Type Models, which assume that the customers who are subjected to a service failure in their first trials and go to another system for the resolution, are favorable compared to the Resolution Type Models, in which unsatisfied customers return to the same server or escalated to the higher levels of servers in the same system. This result shows the waiting effect on these problems. Because in Resolution Type Models, when the unsatisfied customers retry the same system for the service failure resolution, the system becomes more congested. Thus, in Resolution Type Models, to prevent themselves from the later congestion (higher waiting time) that the unsatisfied customers create, the customer decides to join with lower rates. Similarly, the social planner accepts the customers in the Resolution Type Systems with lower rates compared to Benchmark Type Models.

For the profit maximizer's problem, we divide our conclusions in two parts. The first part is labeled as the Short Term Comparison and compares the optimal model parameters of this problem when the quality level is fixed (profit maximizer has no enough time or budget to change the quality of the service) and only the decision is the entrance price. The second part is labeled as the Long Term Comparison where the service quality level is also decision for the profit maximizer.

In the short term comparisons, we conclude that the Benchmark Type Models can be preferable compared to Resolution Type Models; i.e. if the additional unit waiting time that the returning customer creates is higher than the ratio between the unit reward and the waiting cost. Thus in the short term, losing some of the customers (because in Benchmark Type Models, unsatisfied customers leave the system) can be preferable for the profit maximizer. This result resembles the one given in Chen and Frank [11] who conclude that based on the model parameters, when the system load is high, the monopolistic price setting in which not all of but some of the customers join the system is more profitable.

In the long term, when the service quality level is a decision, we show that Resolution Type Models are always the winner. So, in the long term, by choosing not only the entrance price but also the quality level, the profit maximizer optimizes his profit by offering resolutions to unsatisfied customers in order to not to lose them. The possible resolution alternatives in which the unsatisfied customers are not lost for the profit maximizer are: Customers can return to the same server and they are escalated to the higher levels of servers in the same system. When the profit maximizer offers the former one to resolve the service failure problem of the unsatisfied customer, we conclude that it is optimal for the profit maximizer to not to retake the entrance price from the returning customers. In the latter one, when the customers are escalated to the higher levels of servers in the same system, we observe that the system (escalation models) needs to be properly designed; i.e. it is optimal for the profit maximizer to decide on the additional parameters (service rate of the first server, μ_1 or the quality levels of the servers q_1 and q_2.). Comparison between these two resolution alternatives of designing the system with a rapid but a unique server, Single Stage Models With Double Rate, or with two sequential servers, Escalation Models, shows that the former design is favorable although it cannot always be possible. This result corresponding to the long term comparison also resembles the one given in Chen and Frank [11]. By comparing the market capturing and monopolistic pricing setting in the long term, when additional to entrance price the profit maximizer decides on the service rate, Chen and Frank [11] shows that the market capturing pricing setting in which every customer joins the system is optimal.

Changing the service quality level is costly and time consuming and the profit maximizer may not be able to do this due to the budget and time constraints. Thus in the end we change the definition of the profit maximizer's problem to see what else can be done under this scenario; i.e. the profit maximizer has time and budget constraints. In this part (Chapter 5), for all of the models that we analyze earlier, we change our long term problem of deciding on the service quality level into that of deciding on both of the service quality level and the service rate. Comparing the long term problem defined in the end (decision parameters are the service quality level and the service rate) with the one previously defined (only the decision parameter is the service quality level), we first conclude that the service rate and the service quality level are substitutes of each other. Thus, if the profit maximizer has some constraints regarding time or budget, he can change his service rate rather than the service quality level. However, for the proper conditions (no time or budget constraints), it is optimal for him to decide on both of the model parameters, service rate and the service quality level.

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Appendix A

PROOFS

A.1 Proofs Of Chapter-II

Lemma 2.1: $U(i)$ *is decreasing in* i.

Proof:

$$\frac{\partial U(i)}{\partial i} = -\frac{C}{\mu} < 0 \qquad \frac{\partial^2 U(i)}{\partial i^2} = 0$$

Corollary 2.1: n^*_{ind} *is increasing in* R, μ, q *decreasing in* C, *and is independent from* λ.

Proof:

$$\frac{\partial n^*_{ind}}{\partial R} = \frac{\mu q}{C} > 0, \ \frac{\partial^2 n^*_{ind}}{\partial R^2} = 0$$
$$\frac{\partial n^*_{ind}}{\partial q} = \frac{\mu R}{C} > 0, \ \frac{\partial^2 n^*_{ind}}{\partial q^2} = 0$$
$$\frac{\partial n^*_{ind}}{\partial \mu} = \frac{Rq}{C} > 0, \ \frac{\partial^2 n^*_{ind}}{\partial \mu^2} = 0$$
$$\frac{\partial n^*_{ind}}{\partial C} = -\frac{\mu Rq}{C^2} < 0, \ \frac{\partial^2 n^*_{ind}}{\partial C^2} = \frac{2\mu Rq}{C^3} > 0$$
$$\frac{\partial n^*_{ind}}{\partial \lambda} = 0, \ \frac{\partial^2 n^*_{ind}}{\partial \lambda^2} = 0$$

Lemma 2.2: $S_O(n)$ *is discretely unimodular in* n.

Proof:

For detailed proof one can see Naor [30].

Corollary 2.2: n^*_{sys} *is increasing in* R, μ, q *decreasing in* C, *and* λ.

Proof:

For detailed proof one can see Naor [30].

A.2 Proofs Of Chapter-III

Lemma 3.1: $U_1(\alpha)$ *is concave in* α.

Proof:

$$\frac{\partial U_1(\alpha)}{\partial \alpha} = -\frac{C\lambda}{(\mu - \lambda\alpha)^2}$$

$$\frac{\partial^2 U_1(\alpha)}{\partial \alpha^2} = -\frac{2C\lambda^2}{(\mu - \lambda\alpha)^3} < 0$$

Corollary 3.1: *The equilibrium joining probability of the customers for the Benchmark Model is:*

$$\alpha_1^{eq} = \begin{cases} 0, & Rq \leq \frac{C}{\mu} \\ \frac{\mu - \frac{C}{Rq}}{\lambda}, & \frac{C}{\mu} < Rq < \frac{C}{\mu-\lambda} \\ 1, & Rq \geq \frac{C}{\mu-\lambda} \end{cases}$$

Proof:

Assume $Rq - \frac{C}{\mu} < 0$, and customers decide to join with a joining probability α. Then,

$$U_1(\alpha) = Rq - CE[W] \Rightarrow U_1(\alpha) = Rq - \frac{C}{\mu - \lambda\alpha} < 0,$$

since $\frac{C}{\mu} < \frac{C}{\mu - \lambda\alpha}$. This contradicts with the utility maximization of customers, because they have negative utility, while not joining has non-negative. Hence not joining is the dominant equilibrium strategy.

Now, assume $Rq - \frac{C}{\mu - \lambda} \geq 0$, and customers decide to join with probability α, where $0 \leq \alpha \leq 1$. Then,

$$U_1(\alpha) = Rq - CE[W] \Rightarrow U_1(\alpha) = Rq - \frac{C}{\mu - \lambda\alpha} > 0,$$

since $\frac{C}{\mu - \lambda} > \frac{C}{\mu - \lambda\alpha}$. However this can not be an equilibrium strategy, since the ones with probability $1 - \alpha$, who decide not to join have a zero utility, while others have positive utility. Hence joining is the dominant equilibrium strategy.

For the region where $\frac{C}{\mu} < Rq < \frac{C}{\mu - \lambda}$, utility value of a tagged customer is positive for the low joining probabilities, and it decreases with an increase in α. So, there is a joining probability value in which, utility of the tagged customer turns to negative. This equilibrium joining probability is the one which makes customers indifferent between joining or not. This unique value is found as:

$$\begin{aligned} U_1(\alpha) &= 0 \\ \Rightarrow 0 &= Rq - \frac{C}{\mu - \lambda\alpha} \\ \Rightarrow C &= (Rq)(\mu - \lambda\alpha) \\ \Rightarrow (\mu - \lambda\alpha) &= \frac{C}{Rq} \\ \Rightarrow \alpha &= \frac{\mu - \frac{C}{Rq}}{\lambda} \end{aligned}$$

Lemma 3.2: $S_1(\alpha)$ *is concave in* α.

Proof:

$$\frac{\partial S_1(\alpha)}{\partial \alpha} = \lambda R q - \frac{C\lambda\mu}{(\mu - \lambda\alpha)^2}$$

$$\frac{\partial^2 S_1(\alpha)}{\partial \alpha^2} = -\frac{2C\lambda^2\mu}{(\mu - \lambda\alpha)^3} < 0$$

Corollary 3.2: *The socially optimal joining probability of the Benchmark Model is:*

$$\alpha_1^* = \begin{cases} \frac{\mu - \sqrt{\frac{C\mu}{Rq}}}{\lambda}, & \mu - \lambda < \sqrt{\frac{C\mu}{Rq}} \\ 1, & \text{o.w.} \end{cases}$$

Proof:

Similar proof follows as given in corollary 3.1.

Lemma 3.3: $\Pi_1(p_{1,\lambda}, q)$ *is concave in* q.

Proof:

$$\frac{\partial \Pi_1(p_{1,\lambda}, q)}{\partial q} = \lambda R - 2aq$$

$$\frac{\partial^2 \Pi_1(p_{1,\lambda}, q)}{\partial q^2} = -2a < 0$$

Corollary 3.3: $q_{1,\lambda}$ *is nondecreasing in* λ *and* R *and non increasing in* a.

Proof:

$$\frac{\partial q_{1,\lambda}}{\partial\ \lambda} = \frac{R}{2a} \qquad \frac{\partial^2 q_{1,\lambda}}{\partial \lambda^2} = 0$$

$$\frac{\partial q_{1,\lambda}}{\partial R} = \frac{\lambda}{2a} \qquad \frac{\partial^2 q_{1,\lambda}}{\partial R^2} = 0$$

$$\frac{\partial q_{1,\lambda}}{\partial a} = -\frac{\lambda R}{2a^2} \qquad \frac{\partial^2 q_{1,\lambda}}{\partial a^2} = \frac{\lambda R}{a^3}$$

Lemma 3.4: $\Pi_1(p,q)$ *is concave in* p.

Proof:

$$\frac{\partial \Pi_1(p,q)}{\partial p} = \mu - \frac{CRq}{(Rq-p)^2}$$

$$\frac{\partial^2 \Pi_1(p,q)}{\partial p^2} = -\frac{2CRq}{(Rq-p)^3} < 0$$

Lemma 3.5: $\Pi_1(p_{1,m},q)$ *is convex increasing in* q.

Proof:

$$\frac{\partial \Pi_1(p_{1,m},q)}{\partial q} = \mu - \sqrt{\frac{CR\mu}{q}} - 2aq$$

The first order condition shows that:

- for $q = 0$, $\frac{\partial \Pi_1(p_{1,m},q)}{\partial q} = -Infinity$.

- for $q = 1$, $\frac{\partial \Pi_1(p_{1,m},q)}{\partial q} > 0$ if $\mu > \sqrt{\frac{CR}{\mu}} + 2a$

Proposition 3.1: *If* $\mu - \lambda > \sqrt{\frac{C\mu}{R}}$, *then market capturing price strategy is optimal.*

Proof:

Assume for the market capturing price case $\lambda R \geq 2a$, so $q_{1,\lambda} = 1$. For the first order pricing case, $q_{1,m} = 1$. We want to prove that $\Pi_1(p_{1,\lambda}, q_{1,\lambda}) \geq \Pi_1(p_{1,m}, q_{1,m})$ if $\mu - \lambda > \sqrt{\frac{C\mu}{R}}$. That is:

$$\Pi_1(p_{1,\lambda}, 1) \geq \Pi(p_{1,m}, 1) \tag{A.1a}$$

$$\lambda p_{1,\lambda} - aq^2 \geq \lambda \alpha_1^{eq} p_{1,m} - aq^2 \tag{A.1b}$$

$$\lambda(R - \frac{C}{\mu - \lambda}) - a \geq (\mu - \sqrt{\frac{C\mu}{R}})(R - \sqrt{\frac{CR}{\mu}}) - a \tag{A.1c}$$

$$\lambda(R - \frac{C}{\sqrt{\frac{C\mu}{R}}}) \geq \lambda(R - \sqrt{\frac{CR}{\mu}}) \tag{A.1d}$$

$$(R - \sqrt{\frac{CR}{\mu}}) \geq (R - \sqrt{\frac{CR}{\mu}}) \tag{A.1e}$$

In obtaining the inequality (A.1d), we use the given condition as follows: We put the maximum value of $\mu - \lambda$ in left hand side of the inequality (A1.c), and for the expression $(\mu - \sqrt{\frac{C\mu}{R}})$ given in the right hand side of (B.1c) we plug its maximum value which is λ. Since the left hand side of the equation is still greater than or equal to the right side (although the minimum value is used in left hand side and the maximum value is used in right hand side), we conclude that if the given condition is satisfied then the profit maximizer chooses market capturing price case, i.e. it is more profitable.

Now assume $\lambda R < 2a$, so $q_{1,\lambda} = \frac{\lambda R}{2a}$. To prove that the market capturing price case is better for the same condition, we use similar strategy, with using this $q_{1,\lambda}$ in the comparison.

$$\Pi_1(p_{1,\lambda}, q_{1,\lambda}) \geq \Pi(p_{1,m}, 1) \tag{A.2a}$$

$$\lambda p_{1,\lambda} - aq^2 \geq \lambda \alpha_1^{eq} p_{1,m} - aq^2 \tag{A.2b}$$

$$\lambda(R\frac{\lambda R}{2a} - \frac{C}{\mu - \lambda}) - a\frac{\lambda^2 R^2}{4a^2} \geq (\mu - \sqrt{\frac{C\mu}{R}})(R - \sqrt{\frac{CR}{\mu}}) - a \tag{A.2c}$$

$$-\lambda\frac{C}{\sqrt{\frac{C\mu}{R}}} + \frac{\lambda^2 R^2}{4a^2} \geq \lambda(R - \sqrt{\frac{CR}{\mu}}) - a \tag{A.2d}$$

$$-\lambda(R + \sqrt{\frac{CR}{\mu}} - \sqrt{\frac{CR}{\mu}}) \geq -a - \frac{\lambda^2 R^2}{4a^2} \tag{A.2e}$$

$$-\lambda R \geq -(\frac{\lambda R}{2} + \frac{\lambda R}{2}) \tag{A.2f}$$

In the above equations (A2.), we use maximum value of λ and minimum value of a (since it has negative sign), in the right side of the inequality. Under these changes since the inequality is still satisfied, we again conclude that the market capturing price is optimal for the profit maximizer if $\mu - \lambda > \sqrt{\frac{C\mu}{R}}$.

Proposition 3.2: *If $\lambda \geq \mu$, then monopolistic price strategy is optimal.*

Proof:

If $\lambda \geq \mu$, assume the profit maximizer sets market capturing price, and all customers decide to join. The equilibrium joining probability is 1 in this case. Thus the expected waiting time is:

$$E\{W(1)\} = \frac{1}{\mu - \lambda} < 0$$

Since expected waiting time can not take negative values, this equilibrium joining probability can not be the solution of this problem.

Proposition 3.3: *The total expected waiting time of a customer, in the two Reso-*

lution Models are the same.

Proof:

Comparing the total expected waiting time expressions given in (3.9) and (3.10), the result follows.

Lemma 3.6: $U_2(\alpha)$ *is concave in* α.

Proof:

$$\frac{\partial U_2(\alpha)}{\partial \alpha} = -\frac{C\lambda}{(\mu q - \lambda\alpha)^2}$$

$$\frac{\partial^2 U_1(\alpha)}{\partial \alpha^2} = -\frac{2C\lambda^2}{(\mu q - \lambda\alpha)^3} < 0$$

Corollary 3.4: *The equilibrium joining probability of the customers for the Model With Resolution is:*

$$\alpha_2^{eq} = \begin{cases} 0, & Rq \leq \frac{C}{\mu} \\ \frac{\mu q - \frac{C}{R}}{\lambda}, & \frac{C}{\mu q} < R < \frac{C}{\mu q - \lambda} \\ 1, & R \geq \frac{C}{\mu q - \lambda} \end{cases}$$

Proof:

The similar proof follows as given in Corollary 3.1.

Lemma 3.7: $S_2(\alpha)$ *is concave in* α.

Proof:

$$\frac{\partial S_2(\alpha)}{\partial \alpha} = \lambda R - \frac{C\lambda\mu q}{(\mu q - \lambda\alpha)^2}$$

$$\frac{\partial^2 S_2(\alpha)}{\partial \alpha^2} = -\frac{2C\lambda^2\mu}{(\mu q - \lambda\alpha)^3} < 0$$

Corollary 3.5: *The socially optimal joining probability of the Model With Resolution is:*

$$\alpha_2^* = \begin{cases} \frac{\mu q - \sqrt{\frac{C\mu q}{R}}}{\lambda}, & \mu q - \lambda < \sqrt{\frac{C\mu q}{R}} \\ 1, & \text{o.w.} \end{cases}$$

Proof:

Similar proof follows as given in Corollary 3.2.

Lemma 3.8: $\Pi_2(p_{2,\lambda}, q)$ *is concave in* q.

Proof:

$$\frac{\partial \Pi_2(p_{2,\lambda}, q)}{\partial q} = \frac{C\lambda\mu}{(\mu q - \lambda)^2} - 2aq$$

$$\frac{\partial^2 \Pi_2(p_{2,\lambda}, q)}{\partial q^2} = -\frac{2C\lambda\mu^2}{(\mu q - \lambda)^3} - 2a < 0$$

Lemma 3.9: $\Pi_2(p, q)$ *is concave in* p.

Proof:

$$\frac{\partial \Pi_2(p, q)}{\partial p} = \mu q - \frac{CR}{(R-p)^2}$$

$$\frac{\partial^2 \Pi_2(p, q)}{\partial p^2} = -\frac{2CRq}{(R-p)^3} < 0$$

Corollary 3.6: *The profit maximizer's optimal monopolistic price as a function of the quality level is given by:*

$$p_{2,m}(q) = R - \sqrt{\frac{CR}{\mu q}}$$

Proof:

Since the concavity follows as given in Lemma 3.9, the optimal monopolistic price as a function of q is found by equating the first derivative function to 0 as:

$$\frac{\partial \Pi_2(p,q)}{\partial p} = 0 \Rightarrow \mu q - \frac{CR}{(R-p)^2} \Rightarrow p_{2,m}(q) = R - \sqrt{\frac{CR}{\mu q}}$$

Lemma 3.10: $\Pi_2(p_{2,m}, q)$ *is convex in* q.

Proof:

Proof is same as given in Lemma 3.5.

Lemma 3.11: $U_3(\alpha)$ *is concave in* α.

Proof:

$$\frac{\partial U_3(\alpha)}{\partial \alpha} = -\frac{C(\mu - \mu q + \lambda)}{(\mu - \alpha(\mu - \mu q + \lambda))^2}$$

$$\frac{\partial^2 U_1(\alpha)}{\partial \alpha^2} = -\frac{2C(\mu - \mu q + \lambda)^2}{(\mu - \alpha(\mu - \mu q + \lambda))^3} < 0$$

Corollary 3.7: *The equilibrium joining probability of the customers for the Model With*

Resolution is:

$$\alpha_3^{eq} = \begin{cases} 0, & Rq \leq \frac{C}{\mu} \\ \frac{\mu - \frac{C}{Rq}}{\mu - \mu q + \lambda}, & \frac{C}{\mu} < Rq < \frac{C}{\mu q - \lambda} \\ 1, & Rq \geq \frac{C}{\mu q - \lambda} \end{cases}$$

Proof:

Proof follows as given in Corollary 3.1.

Lemma 3.12: *$S_3(\alpha)$ is concave in α.*

Proof:

$$\frac{\partial S_3(\alpha)}{\partial \alpha} = \lambda R - \frac{C\lambda\mu}{(\mu - \alpha(\mu - \mu q + \lambda))^2}$$

$$\frac{\partial^2 S_3(\alpha)}{\partial \alpha^2} = -\frac{2C\lambda\mu(\mu - \mu q + \lambda)}{(\mu - \alpha(\mu - \mu q + \lambda))^3} < 0$$

Corollary 3.8: *The socially optimal joining probability of the Model With Returns is:*

$$\alpha_3^* = \begin{cases} \frac{\mu - \sqrt{\frac{C\mu}{Rq}}}{\mu - \mu q + \lambda}, & \mu q - \lambda < \sqrt{\frac{C\mu}{Rq}} \\ 1, & \text{o.w.} \end{cases}$$

Proof:

Similar proof follows as given in other models.

Lemma 3.13: $\Pi_3(p_{3,\lambda}, q)$ *is concave in q.*

Proof:

$$\frac{\partial \Pi_3(p,q)}{\partial q} = \frac{C\lambda(2\mu q - \lambda)}{(\mu q - \lambda)^2 q^2}$$

$$\frac{\partial^2 \Pi_2(p,q)}{\partial p^2} = -\frac{C\lambda(2\mu q - \lambda)(4\mu^2 q^3 - 6\mu q^2 \lambda + 2\lambda^2 q)}{(\mu q - \lambda)^4 q^4} - 2a < 0$$

Lemma 3.14: $\Pi_3(p,q)$ *is concave in* p.

Proof:

$$\frac{\partial \Pi_3(p,q)}{\partial p} = \frac{\lambda(C^2(-1+q) + \lambda\mu(Rq-p)^2 + C(\lambda Rq + \mu(1-q)(Rq-2p)))}{(\lambda(Rq-p) + C(-1+q)^2}$$

$$\frac{\partial^2 \Pi_3(p,q)}{\partial p^2} = -\frac{2C\lambda(\lambda + \mu - \mu q)(\lambda Rq - C - Cq)}{(\lambda(Rq-p) - C(1-q))^3} < 0$$

Proposition 3.4: *For the given model parameters* $R, C, \lambda, \mu,$ *and* q *, the equilibrium joining probabilities of the single server model are compared as:*

$$\begin{cases} \alpha_3^{eq} \leq \alpha_2^{eq} \leq \alpha_1^{eq}, & \lambda \leq \mu q \\ \alpha_2^{eq} \leq \alpha_3^{eq} \leq \alpha_1^{eq}, & \text{o.w.} \end{cases}$$

Proof:

To prove this result we first compare the equilibrium joining probabilities of the Benchmark Model and the Model With Resolution. We use the obtained probabilities given in Corollaries 3.1 and 3.4.

$$\begin{cases} 0,0 & Rq \leq \frac{C}{\mu} \\ \frac{\mu - \frac{C}{Rq}}{\lambda}, \frac{\mu q - \frac{C}{R}}{\lambda} & \frac{C}{\mu} < Rq < \frac{C}{\mu - \lambda} \\ 1, \frac{\mu q - \frac{C}{R}}{\lambda} & Rq \geq \frac{C}{\mu - \lambda} \text{ and } Rq \leq \frac{C}{\mu q - \lambda} \\ 1,1, & \text{o.w.} \end{cases}$$

Comparing the equilibrium joining probabilities in the second region, since $\frac{R\mu q - C}{R\lambda q} \geq \frac{R\mu q - C}{R\lambda}$, the comparison follows.

We now compare the equilibrium probabilities of the Benchmark Model and the Model With Returns by using the probabilities given in corollaries 3.1 and 3.7.

$$\begin{cases} 0,0 & Rq \leq \frac{C}{\mu} \\ \frac{\mu - \frac{C}{Rq}}{\lambda}, \frac{\mu - \frac{C}{Rq}}{\mu - \mu q + \lambda} & \frac{C}{\mu} < Rq < \frac{C}{\mu - \lambda} \\ 1, \frac{\mu - \frac{C}{Rq}}{\mu - \mu q + \lambda} & Rq \geq \frac{C}{\mu - \lambda} \text{ and } Rq \leq \frac{C}{\mu q - \lambda} \\ 1,1, & \text{o.w.} \end{cases}$$

In the second region, since $\mu(1-q) \geq 0$, the comparison between these models follows. The equilibrium joining probability comparison between the Model With Resolution and Model With Returns depend on the model parameters. To show this, assume $\alpha_2^{eq} \geq \alpha_3^{eq}$, then:

$$\frac{R\mu q - C}{R\lambda} \geq \frac{R\mu q - C}{Rq(\mu - \mu q + \lambda)} \Rightarrow q(\mu - \mu q + \lambda) \geq \lambda \Rightarrow \mu q(1-q) \geq \lambda(1-q) \Rightarrow \mu q \geq \lambda$$

So the comparison follows.

Proposition 3.5: *For the given model parameters R, C, λ, μ, and q , the socially optimal joining probabilities of the single server model are compared as:*

$$\begin{cases} \alpha_3^* \leq \alpha_2^* \leq \alpha_1^*, & \lambda \leq \mu q \\ \alpha_2^* \leq \alpha_3^* \leq \alpha_1^*, & \text{o.w.} \end{cases}$$

Proof:

To compare the socially optimal joining probabilities of the Benchmark Model and The Model With Resolution, we use the probability expressions given in Corollaries 3.2 and 3.5. Assume $\alpha_2^{eq} \leq \alpha_1^{eq}$, then:

$$\mu - \sqrt{\frac{C\mu}{Rq}} \geq \mu q - \sqrt{\frac{C\mu q}{R}} \Rightarrow \mu(1-q) \geq \sqrt{\frac{C\mu}{R}} \frac{1-q}{\sqrt{q}} \Rightarrow Rq \geq \frac{C}{\mu}$$

Since if the above condition does not holds, the customer does not decide to join, the comparison follows.

Using the expressions given in Corollaries 3.2 and 3.8, we compare the socially optimal joining probabilities of the Benchmark Model and the Model With Returns. Since $\mu - \mu q + \lambda \geq \lambda$, the comparison follows.

Proposition 3.6: *For the given model parameters $R, C, \lambda, \mu, a,$ and q , the comparison between the price and profit values as functions of the service quality levels of the the single server models is:*

$$\begin{cases} p_{3,\lambda}(q) \leq p_{2,\lambda}(q) \leq p_{1,\lambda}(q), & \frac{R}{C\mu} \leq \frac{1}{(\mu q-\lambda)(\mu-\lambda)} \\ p_{3,\lambda}(q) \leq p_{1,\lambda}(q) \leq p_{2,\lambda}(q), & \text{o.w.} \end{cases}$$

$$\begin{cases} \Pi_{2,\lambda}(q) \leq \Pi_{1,\lambda}(q), & \frac{R}{C\mu} \leq \frac{1}{(\mu q-\lambda)(\mu-\lambda)} \\ \Pi_{1,\lambda}(q) \leq \Pi_{2,\lambda}(q), & \text{o.w.} \end{cases}$$

$$\Pi_{3,\lambda}(q) \leq \Pi_{2,\lambda}(q)$$

Proof:

First we compare the market capturing price values as a function of the quality levels for the models. We first compare the market capturing price of the Model With Returns with the Model With Resolution and the Benchmark Model.

$$p_{1,\lambda}(q) = Rq - \frac{C}{\mu - \lambda} \quad p_{3,\lambda}(q) = Rq - \frac{C}{\mu q - \lambda}$$

Since for the same expected reward, the expected waiting cost is smaller in the Benchmark Model; i.e. $\frac{C}{\mu-\lambda} \leq \frac{C}{\mu q-\lambda}$, the comparison follows.

For the Model With Resolution and the Model With Returns, we have:

$$p_{2,\lambda}(q) = R - \frac{C}{\mu q - \lambda} \quad p_{3,\lambda}(q) = Rq - \frac{C}{\mu q - \lambda}$$

Since for the same expected waiting cost, the expected reward is greater in the first one, the comparison follows.

Now we compare the market capturing price values of the Benchmark Model and the Model With Resolution, assuming the market capturing price is greater in the Benchmark Model for the given service quality level. Then:

$$Rq - \frac{C}{\mu - \lambda} \geq R - \frac{C}{\mu q - \lambda} \Rightarrow \frac{C}{\mu q - \lambda} - \frac{C}{\mu - \lambda} \geq R(1-q) \Rightarrow \frac{R}{C\mu} \leq \frac{1}{(\mu q - \lambda)(\mu - \lambda)}$$

So the pricing comparison follows.

We now compare the profit values of the Model With Resolution and the Model With Returns as:

$$\Pi_{2,\lambda}(q) = R\lambda - \frac{C\lambda}{\mu q - \lambda} - aq^2 \quad \Pi_{3,\lambda}(q) = R\lambda - \frac{C\lambda}{(\mu q - \lambda)q} - aq^2$$

Since for the same expected reward the expected waiting cost is smaller in the Model With Resolution, the comparison follows.

The profit comparison between the Benchmark Model and the Model With Resolution is same as the pricing comparison, since the profit function structures are the same for these models.

Proposition 3.7: *For the given model parameters R, C, λ, μ, and a , the socially optimal joining probabilities of the single server model are compared as:*

$$p_{1,m}(q) \leq p_{2,m}(q)$$

$$\Pi_{1,m}(q) = \Pi_{2,m}(q)$$

Proof:

First we compare the monopolistic price values of the two models for the given service quality levels, assuming the price is smaller in the Benchmark Model.

$$Rq - \sqrt{\frac{CRq}{\mu}} \leq R - \sqrt{\frac{CR}{\mu q}} \Rightarrow \sqrt{\frac{CR}{\mu}}\left(\frac{1}{\sqrt{q}} - \sqrt{q}\right) \leq R(1-q) \Rightarrow C \leq \mu q$$

Since the given condition always holds, monopolistic price comparison holds.

To compare the profit values, we look at the (3.9) and (3.22). Since the profit functions are the same, the monopolistic profits of the models are the same.

Proposition 3.8: *For the given model parameters R, C, λ, μ and a , the comparison between the service quality level, market capturing price and the profit values of the single server models is:*

$$q_{3,\lambda} \leq q_{2,\lambda} \;\; p_{3,\lambda} \leq p_{2,\lambda} \;\; \Pi_{3,\lambda} \leq \Pi_{2,\lambda}$$

Proof:

To compare the optimal quality levels of the Model With Returns and Model With

Resolution, in the market capturing price strategy, we compare the first derivative functions. Since we showed the concavity of the profit functions in Lemmas 3.8 and 3.13. If the first derivative function of the Model With Returns takes smaller value than the Model With Resolution for all service quality levels, $0 \leq q \leq 1$, then we conclude that the optimal quality level of the Model With Returns is smaller than the Model With Resolution. To show this;

$$\frac{C\lambda(2\mu q-\lambda)}{(\mu q-\lambda)^2(q)^2} - 2aq \leq \frac{C\lambda\mu}{(\mu q-\lambda)^2} - 2aq \Rightarrow (2-q) \leq \frac{\lambda}{\mu q}$$

The condition holds, so we conclude that the optimal quality level of the Model With Returns is smaller than the Model With Resolution.

In short term comparison we see that the market capturing price and the profit value of the third model, is smaller than the second model for the given quality levels. Since in this result we showed that the optimal quality level is smaller in the second model, optimal price and profit comparison follows.

Proposition 3.9: *If* $R\lambda \geq 2a$, then:

$$q_{2,\lambda} \leq q_{1,\lambda} \;\; p_{2,\lambda} \leq p_{1,\lambda} \;\; \Pi_{1,\lambda} \leq \Pi_{2,\lambda}$$

Proof:

In the Benchmark Model, we give that the optimal quality level in the market capturing price strategy is $\frac{R\lambda}{2a}$. However, since the maximum value of a quality level is 1, we shift it to 1 when $R\lambda$ exceeds $2a$. For the Model With Resolution, we showed in the lemma 3.8 that the profit function is concave with respect to the quality level under the market capturing price strategy. So there is an interior quality level, which maximizes the profit value. So we conclude that the optimal profit is higher in the Model With Resolution. Since the market capturing price expression of the Model With Resolution; i.e. $R - \frac{C}{\mu q-\lambda}$, is increasing in the service quality level, we conclude that it is lower for the lower quality levels.

Proposition 3.10: *For the given model parameters R, C, λ, μ and a , the comparison between the service quality level, monopolistic price and the profit values of the single server models is:*

$$q_{1,m} = q_{2,m} \;\; p_{1,m} \leq p_{2,m} \;\; \Pi_{1,m} = \Pi_{2,m}$$

Proof:
Since the profit functions under the monopolistic price strategy of the first and the second model are the same, (3.9) and (3.22), the optimal quality levels and the profit values are the same. For the same quality level, the optimal pricing comparison is same as the short term comparison which is given in the Proposition 3.5. So the result follows.

A.3 Proofs Of Chapter-IV

Lemma 4.1: $U_{4,n}(\alpha)$ *is concave in* α.
Proof:

$$\frac{\partial U_{4,n}(\alpha)}{\partial \alpha} = -\frac{C\lambda}{(\mu - \lambda\alpha)^2} - \frac{C\lambda(1-q)^2}{(\mu - \lambda\alpha(1-q))^2}$$

$$\frac{\partial^2 U_{4,n}(\alpha)}{\partial \alpha^2} = -\frac{2C\lambda^2}{(\mu - \lambda\alpha)^3} - \frac{2C\lambda^2(1-q)^3}{(\mu - \lambda\alpha(1-q))^3} < 0$$

Lemma 4.2: $U_{4,g}(\alpha)$ *is concave in* α.
Proof:

$$\frac{\partial U_{4,g}(\alpha)}{\partial \alpha} = -\frac{C\lambda}{(\mu - \lambda\alpha)^2} - \frac{C\lambda(1-q)^2}{(\mu - \lambda\alpha(1-q))^2}$$

$$\frac{\partial^2 U_{4,g}(\alpha)}{\partial \alpha^2} = -\frac{2C\lambda^2}{(\mu - \lambda\alpha)^3} - \frac{2C\lambda^2(1-q)^3}{(\mu - \lambda\alpha(1-q))^3} < 0$$

Proposition 4.1: $\alpha^{eq}_{4,n}(q) \leq \alpha^{eq}_{4,g}(q)$.

Proof:

Since the expected waiting cost expressions of the two escalation models are the same, and the expected reward is higher in the Perfect Escalation Model; i.e. $R \geq Rq + Rq(1-q)$, customers are more desirous to join the system in the Perfect Escalation Model. So the result follows.

Lemma 4.3: $S_{4,n}(\alpha)$ *is concave in* α.

Proof:

$$\frac{\partial S_{4,n}(\alpha)}{\partial \alpha} = R\lambda q(2-q) - \frac{C\lambda\mu}{(\mu - \lambda\alpha)^2} - \frac{C\lambda\mu(1-q)}{(\mu - \lambda\alpha(1-q))^2}$$

$$\frac{\partial^2 S_{4,n}(\alpha)}{\partial \alpha^2} = -\frac{2C\lambda^2\mu}{(\mu - \lambda\alpha)^3} - \frac{2C\lambda^2\mu(1-q)^2}{(\mu - \lambda\alpha(1-q))^3} < 0$$

Lemma 4.4: $S_{4,g}(\alpha)$ *is concave in* α.

Proof:

$$\frac{\partial S_{4,g}(\alpha)}{\partial \alpha} = R\lambda - \frac{C\lambda\mu}{(\mu - \lambda\alpha)^2} - \frac{C\lambda\mu(1-q)}{(\mu - \lambda\alpha(1-q))^2}$$

$$\frac{\partial^2 S_{4,g}(\alpha)}{\partial \alpha^2} = -\frac{2C\lambda^2\mu}{(\mu - \lambda\alpha)^3} - \frac{2C\lambda^2\mu(1-q)^2}{(\mu - \lambda\alpha(1-q))^3} < 0$$

Proposition 4.2: $\alpha^{*}_{4,n}(q) \leq \alpha^{*}_{4,g}(q)$.

Proof:

Since the concavity given in Lemmas 4.3 and 4.4 follows, and the first derivative function of the social utility function of the perfect escalation model is always greater than the first derivative function of the social utility function of the simple escalation model for the same α values, the result follows.

Lemma 4.5: $\Pi_{4,n}(q)$ *is concave in* q.

Proof:

$$\frac{\partial \Pi_{4,n}(q)}{\partial q} = 2R\lambda(1-q) + \frac{C\lambda\mu}{(\mu-\lambda+\lambda q)^2} - 4aq$$

$$\frac{\partial^2 \Pi_{4,n}(q)}{\partial q^2} = -2R\lambda - \frac{2C\lambda^2\mu}{(\mu-\lambda+\lambda q)^3} - 4a < 0$$

Lemma 4.6: $\Pi_{4,g}(q)$ *is concave in* q.

Proof:

$$\frac{\partial \Pi_{4,g}(q)}{\partial q} = \frac{C\lambda\mu}{(\mu-\lambda+\lambda q)^2} - 2aq$$

$$\frac{\partial^2 \Pi_{4,g}(q)}{\partial q^2} = -\frac{2C\lambda^2\mu}{(\mu-\lambda+\lambda q)^3} - 2a < 0$$

Proposition 4.3: *In the short term for the given model parameters,* R, C, a, q, λ *and* μ,

-Pricing Comparison: $p_{4,g}(q) \geq p_{4,n}(q)$.

-Profit Comparison:

$$\Pi_{4,g}(q), \Pi_{4,n}(q) = \begin{cases} \Pi_{4,g}(q) \geq \Pi_{4,n}(q) & \text{if } \frac{R\lambda}{a} \geq \frac{1+q}{1-q} \\ \Pi_{\lambda,g}(q) < \Pi_{\lambda,n}(q) & o.w. \end{cases}$$

Proof:

Using the functions (4.11) and (4.13), we compare the market capturing prices of the two escalation models. Assume $p_{4,g}(q) \geq p_{4,n}(q)$, then;

$$R > Rq + Rq(1-q) \Rightarrow R > Rq(2-q) \Rightarrow 1 > q(2-q)$$

since the above comparison condition always holds, the result follows.

If the expressions of the cost of the quality were the same in these two escalation models, then the pricing and profit comparison would also be the same for the given quality level. However since the cost of the quality are not the same in these models, the profit comparison depends on the model parameters. Using the profit functions given in (4.11) and (4.14), we obtain a comparison for the profit expressions which depend on the model parameters. If we assume that the profit value of the profit maximizer in the Perfect Escalation Model is at least greater than his profit in the Simple Escalation Model, then;

$$R\lambda - aq^2 - a > R\lambda q + R\lambda q(1-q) - 2aq^2 \Rightarrow R\lambda(1-q)^2 > a(1-q)(1+q) \Rightarrow \frac{R\lambda}{a} > \frac{1+q}{1-q}.$$

From the above result, we obtain a comparison for the profit values as given in this Proposition.

Proposition 4.4: *In the long term for the given model parameters,* R, C, a, λ *and* μ,

-Quality Level Comparison: $q^*_{4,g} \leq q^*_{4,n}$.

-Pricing Comparison:

$$p^*_{4,g}, p^*_{4,n} = \begin{cases} p^*_{4,g} \geq p^*_{4,n} & \text{if } \frac{R}{C\mu} \geq \frac{q^*_{4,n} - q^*_{4,g}}{(1-q^*_{4,n})^2(\mu-\lambda+\lambda q^*_{4,g})(\mu-\lambda+\lambda q^*_{4,n})} \\ p^*_{4,g} < p^*_{4,n} & o.w. \end{cases}$$

Proof:

Since the profit functions are concave as given in Lemmas 4.5 and 4.6, to compare the optimal service quality level of the escalation models, we use the first derivative functions. If the first derivative function of the Perfect Escalation Model given in Lemma 4.5 is smaller than the first derivative function of the Simple Escalation Model for all quality levels, we conclude that the optimal quality level of the perfect escalation model is smaller compared to the simple escalation model. Assuming this holds, we have the following comparison;

$$2aq > 4aq - 2R\lambda(1-q) \Rightarrow aq < R\lambda(1-q)$$

The above comparison holds, since otherwise unit cost of quality exceeds the unit revenue of the firm from the second server, which ends causes negative profit.
To compare the optimal price values of the escalation models, we plug the optimal quality values in the price expressions given in (4.11) and (4.13). Assume $p_{4,g} \geq p_{4,n}$, then:

$$\begin{aligned} R - \frac{C}{\mu-\lambda} - \frac{C(1-q_{4,g})}{\mu-\lambda+\lambda q_{4,g}} &\geq Rq_{4,n} + Rq_{4,n}(1-q_{4,n}) - \frac{C}{\mu-\lambda} - \frac{C(1-q_{4,n})}{\mu-\lambda+\lambda q_{4,n}} \\ R - Rq_{4,n} - Rq_{4,n}(1-q_{4,n}) &\geq \frac{C(1-q_{4,g})}{\mu-\lambda+\lambda q_{4,g}} - \frac{C(1-q_{4,n})}{\mu-\lambda+\lambda q_{4,n}} \\ R(1-q_{4,n})^2 &\geq \frac{C\mu(q_{4,n}-q_{4,g})}{(\mu-\lambda+\lambda q_{4,n})(\mu-\lambda+\lambda q_{4,g})} \\ \frac{R}{C\mu} &\geq \frac{q_{4,n}-q_{4,g}}{(1-q_{4,n})^2(\mu-\lambda+\lambda q_{4,n})(\mu-\lambda+\lambda q_{4,g})} \end{aligned}$$

The above equations show that the optimal price comparison between the escalation models depend on the model parameters.

Lemma 4.7: $U_{1,\nu}(\alpha)$ *is concave in* α.
Proof:

$$\begin{aligned} \frac{\partial U_{1,\nu}(\alpha)}{\partial \alpha} &= -\frac{8C\alpha\lambda^2\mu}{(4\mu^2-\lambda^2\alpha^2)^2} \\ \frac{\partial^2 U_{1,\nu}(\alpha)}{\partial \alpha^2} &= -\frac{8\mu(3\lambda^4\alpha^2+4\lambda^2\mu^2)}{(4\mu^2-\lambda^2\alpha^2)^3} < 0 \end{aligned}$$

Lemma 4.8: $U_{2,\nu}(\alpha)$ *is concave in* α.
Proof:

$$\frac{\partial U_{2,\nu}(\alpha)}{\partial \alpha} = -\frac{8C\alpha\lambda^2\mu}{(4\mu^2q^2 - \lambda^2\alpha^2)^2}$$
$$\frac{\partial^2 U_{2,\nu}(\alpha)}{\partial \alpha^2} = -\frac{8\mu(3\lambda^4\alpha^2 + 4\lambda^2\mu^2q^2)}{(4\mu^2q^2 - \lambda^2\alpha^2)^3} < 0$$

Proposition 4.5: $\alpha^{eq}_{1,p}(q) \geq \alpha^{eq}_{2,p}(q)$.

Proof:

Since the individual utility functions are concave as given in Lemmas 4.7 and 4.8, we can compare the first derivative functions of the two models. Since the first derivative equations are negative and the $0 \leq q \leq 1$, the first derivative function of the Two-Parallel-Stage Model With Resolution is smaller compared to the Two-Parallel-Stage Benchmark Model for all values of α, where $0 \leq \alpha \leq 1$. So, the result follows.

Lemma 4.9: $S_{1,\nu}(\alpha)$ *is concave in* α.

Proof:

$$\frac{\partial S_{1,\nu}(\alpha)}{\partial \alpha} = R\lambda q - \frac{8\lambda^3\alpha^2\mu}{(4\mu^2 - \lambda^2\alpha^2)^2} - \frac{4\lambda\mu}{(4\mu^2 - \lambda^2\alpha^2)}$$
$$\frac{\partial^2 S_{1,\nu}(\alpha)}{\partial \alpha^2} = -\frac{8\mu(\lambda^5\alpha^3 + 12\alpha\lambda^3\mu^2)}{(4\mu^2 - \lambda^2\alpha^2)^3} < 0$$

Lemma 4.10: $S_{2,\nu}(\alpha)$ *is concave in* α.

Proof:

$$\frac{\partial S_{2,\nu}(\alpha)}{\partial \alpha} = R\lambda - \frac{8\lambda^3\alpha^2\mu}{(4\mu^2q^2 - \lambda^2\alpha^2)^2} - \frac{4\lambda\mu}{(4\mu^2q^2 - \lambda^2\alpha^2)}$$
$$\frac{\partial^2 S_{2,\nu}(\alpha)}{\partial \alpha^2} = -\frac{8\mu(\lambda^5\alpha^3 + 12\alpha\lambda^3\mu^2q^2)}{(4\mu^2q^2 - \lambda^2\alpha^2)^3} < 0$$

Proposition 4.6: $\alpha^{*}_{1,\nu}(q) \geq \alpha^{*}_{2,p}(q)$.

Proof:

Using the concavity conditions given in Lemmas 4.9 and 4.10, and comparing the first derivative functions for all values of α, the result follows.

Lemma 4.11: $\Pi_{1,\nu}(q)$ *is concave in* q.

Proof:

$$\frac{\partial \Pi_{1,\nu}(q)}{\partial q} = R\lambda - 4aq$$

$$\frac{\partial^2 \Pi_{1,\nu}(q)}{\partial q^2} = -4a < 0$$

Corollary 4.2: $q^*_{1,\nu}$ *is:*

$$q^*_{1,\nu} = \frac{\lambda R}{4a} \tag{A.3}$$

Proof:

Using the concavity given in Lemma 4.11, and equating the first derivative function to 0 with respect to the quality level the result follows.

Lemma 4.12: $\Pi_{2,p}(\nu)$ *is concave in* q.

Proof:

$$\frac{\partial \Pi_{2,\nu}(q)}{\partial q} = \frac{32C\lambda\mu^3 q}{(4\mu^2q^2 - \lambda^2)^2} - 4aq$$

$$\frac{\partial^2 \Pi_{2,\nu}(q)}{\partial q^2} = -\frac{4\left[a(4\mu^2q^2 - \lambda^2)^3 + 8C\mu^3(\lambda^2 + 12\mu^2q^2)\right]}{(4\mu^2q^2 - \lambda^2)^3} < 0$$

Corollary 4.3: $q^*_{2,\nu}$ *is:*

$$q^*_{2,\nu} = \frac{\sqrt{\lambda^2 + \sqrt{\frac{8C\mu^3\lambda}{a}}}}{2\mu} \tag{A.4}$$

Proof:

Using the concavity given in Lemma 4.12, the optimal quality level is found by equating the first derivative function to 0 with respect to the quality level; i.e.

$$\frac{\partial \Pi_{2,\nu}(q)}{\partial q} = 0 \Rightarrow q_{2,\nu} = \frac{\sqrt{\lambda^2 + \sqrt{\frac{8C\mu^3\lambda}{a}}}}{2\mu}$$

Proposition 4.7: *In the short term for the given model parameters, R, C, a, q, λ* and *μ, we have the following comparison for the price and profit values.*

-Pricing Comparison:

$$\begin{cases} p_{1,\nu}(q) \geq p_{2,\nu}(q) & \text{if } \frac{16C\mu^3(1+q)}{(4\mu^2q^2-\lambda^2)(4\mu^2-\lambda^2)} \geq R \\ p_{1,\nu}(q) < p_{2,\nu}(q) & o.w. \end{cases}$$

-Profit Comparison:

$$\begin{cases} \Pi_{1,\nu}(q) \geq \Pi_{2,\nu}(q) & \text{if } \frac{16C\mu^3(1+q)}{(4\mu^2q^2-\lambda^2)(4\mu^2-\lambda^2)} \geq R \\ \Pi_{1,\nu}(q) < \Pi_{2,\nu}(q) & o.w. \end{cases}$$

Proof:

For the given service quality levels, in order to compare the market capturing prices of the two models, we compare the functions expressed in (4.28) and (4.33). Assume the market capturing price as a function of the service quality level of the benchmark model is at least equal to or greater than the price of the resolution model. Then;

$$Rq - \frac{4C\mu}{4\mu^2 - \lambda^2} \geq R - \frac{4C\mu}{4\mu^2q^2 - \lambda^2} \Rightarrow$$

$$\frac{4C\mu}{4\mu^2q^2-\lambda^2}-\frac{4C\mu}{4\mu^2-\lambda^2}\geq R(1-q)\Rightarrow$$

$$\frac{4C\mu(4\mu^2-4\mu^2q^2)}{(4\mu^2q^2-\lambda^2)(4\mu^2-\lambda^2)}\geq R(1-q)\Rightarrow$$

$$\frac{16C\mu^3(1+q)}{(4\mu^2q^2-\lambda^2)(4\mu^2-\lambda^2)}\geq R$$

To compare the profit values we use the expressions given in (4.29) and (4.34). Since in these functions, the arrival rate and the cost of the quality are the same, the profit comparison depends only on the pricing comparison. So, the result follows.

Proposition 4.8: *In the long term for the given model parameters,* R*,* C*,* a*,* λ *and* μ, if $R\lambda \geq 4a$

-*Quality Level Comparison:* $q^*_{1,\nu} = 1 \geq q^*_{2,\nu}$

-*Pricing Comparison:* $p^*_{1,\nu} \geq p^*_{2,\nu}$

-*Profit Comparison:* $\Pi^*_{1,\nu} \leq \Pi^*_{2,\nu}$

Proof:

If $R\lambda \geq 4a$, the optimal service quality level expression given in Corollary 4.2 takes higher values than 1, in which case we set it to 1. This means, the profit maximizer provides perfect quality service in the Benchmark Model. On the other hand, since the profit function of the Resolution Model is concave with respect to the quality level as given in Lemma 4.12, it can be more profitable for the profit maximizer to set the interior quality level. So, the service quality level comparison follows.

The price functions given in (4.28) and (4.33) are increasing in the quality level. They receive the same value in the perfect quality level. This shows that the optimal price of the Resolution Model is lower compared to the Benchmark Model, since the optimal quality level is lower.

For the profit comparison, the profit functions stated in (4.29) and (4.34) receive same value for the quality level 1. Since the profit function of the Resolution Model is concave as given in Lemma 4.12, we conclude that the profit maximizer maximizes his profit by setting the lower service quality level. This shows the profit comparison.

Proposition 4.9: $p_{1,\nu}(q) \leq p_{1,d}(q)$ *and* $\Pi_{1,\nu}(q) \leq \Pi_{1,d}(q)$ *for all values of* q.

Proof:

To compare the price values between the two-parallel-stage and the single stage benchmark values, we use their price expressions:

$$p_{1,\nu}(q) = Rq - \frac{4C\mu}{4\mu^2 - \lambda^2}$$
$$p_{1,d}(q) = Rq - \frac{C}{2\mu - \lambda}$$

If $p_{1,\nu}(q) \leq p_{1,d}(q)$, then;

$$\frac{4C\mu}{4\mu^2 - \lambda^2} \geq \frac{C}{2\mu - \lambda} \Rightarrow 2\mu \geq \lambda$$

Since the above condition holds, the pricing comparison follows.

Since the price is higher, and the cost of the quality is lower in the single server model, the profit comparison also follows.

Proposition 4.10: $q^*_{1,\nu} \leq q^*_{1,d}$, $p^*_{1,\nu} \leq p^*_{1,d}$ *and* $\Pi^*_{1,\nu} \leq \Pi^*_{1,d}$.

Proof:

The optimal service quality levels of the single server and two-parallel-server benchmark models are:

$$q^*_{1,\nu} = \frac{R\lambda}{4a}$$
$$q^*_{1,d} = \frac{R\lambda}{2a}$$

So the optimal quality level of the two-parallel-server model is lower.

For the optimal price comparison we have:

$$p^*_{1,\nu} = Rq_{1,p} - \frac{4C\mu}{4\mu^2 - \lambda^2}$$
$$q^*_{1,d} = Rq_{1,d} - \frac{C}{2\mu - \lambda}$$

Since the first term is higher and the second term is lower in the Two-Parallel-Server Benchmark Model compared to the single server, the comparison holds.

To compare the optimal profits, we use the following profit functions:

$$\Pi^*_{1,\nu} = \lambda\left[R\frac{R\lambda}{4a} - \frac{4C\mu}{4\mu^2 - \lambda^2}\right] - 2a\frac{\lambda^2 R^2}{16a^2} = \frac{\lambda^2 R^2}{8a} - \frac{4C\lambda\mu}{4\mu^2 - \lambda^2}$$
$$\Pi^*_{1,d} = \lambda\left[R\frac{R\lambda}{2a} - \frac{C}{2\mu - \lambda}\right] - 2a\frac{\lambda^2 R^2}{4a^2} = \frac{\lambda^2 R^2}{4a} - \frac{C\lambda\mu}{2\mu - \lambda}$$

Since the first term is higher, and the second term is lower in the Single-Stage Model, the profit comparison follows.

Proposition 4.11: $\alpha^{eq}_{1,\nu}(q) \leq \alpha^{eq}_{1,d}(q)$ for all values of q.

Proof:

Using the equilibrium joining probabilities of the 2-parallel-stage benchmark model and the benchmark model with double rate, we have the following comparison:

$$\begin{cases} 0, 0 & Rq \leq \frac{C}{2\mu} \\ 0, \frac{2\mu - \frac{C}{Rq}}{\lambda} & \frac{C}{2\mu} < Rq \leq \frac{C}{\mu} \\ \frac{\sqrt{4\mu^2 - \frac{4C\mu}{Rq}}}{\lambda}, \frac{2\mu - \frac{C}{Rq}}{\lambda} & \frac{C}{\mu} < Rq \leq \frac{C}{2\mu - \lambda} \\ \frac{\sqrt{4\mu^2 - \frac{4C\mu}{Rq}}}{\lambda}, 1 & \frac{C}{2\mu - \lambda} < Rq \leq \frac{4C\mu}{4\mu^2 - \lambda^2} \\ 1, 1 & \frac{4C\mu}{4\mu^2 - \lambda^2} < Rq \end{cases} \tag{A.5}$$

and in the third region , to compare the equilibrium joining probabilities, we take the squares of the expressions as:

$$\left(\alpha_{1,\nu}^{eq}(q)\right)^2 = \frac{4\mu^2 - \frac{4C\mu}{Rq}}{\lambda^2}$$
$$\left(\alpha_{1,d}^{eq}(q)\right)^2 = \frac{4\mu^2 - \frac{4C\mu}{Rq} + \frac{C^2}{R^2q^2}}{\lambda^2}$$

since the second one is greater, the result follows.

Proposition 4.12: $\alpha_{1,\nu}^*(q) \leq \alpha_{1,d}^*(q)$ for all values of q.

Proof:

To compare the socially optimal joining probabilities, we use the first derivative expressions of the social functions as:

$$\frac{\partial S_{1,\nu}(\alpha)}{\alpha} = R\lambda q - \frac{4\lambda C\mu(4\mu^2 + \lambda^2\alpha^2)}{(2\mu + \lambda\alpha)^2(2\mu - \lambda\alpha)^2}$$
$$\frac{\partial S_{1,d}(\alpha)}{\alpha} = R\lambda q - \frac{2\lambda C\mu}{(2\mu - \lambda\alpha)^2}$$

If $\alpha_{1,\nu}^*(q) \leq \alpha_{1,d}^*(q)$, then:

$$\frac{4C\mu(4\mu^2 + \lambda^2\alpha^2)}{(2\mu + \lambda\alpha)^2} \geq 2C\lambda\mu \Rightarrow$$
$$8\mu^2 + 2\lambda^2\alpha^2 \geq 4\mu^2 + 4\lambda\mu\alpha + \lambda^2\alpha^2 \Rightarrow$$
$$4\mu^2 + \lambda^2\alpha^2 - 4\lambda\mu\alpha \geq 0 \Rightarrow$$
$$(2\mu - \lambda\alpha)^2 \geq 0$$

Since the above condition always holds, the result follows.

Proposition 4.13: $p_{2,p}(\nu) \leq p_{2,d}(q)$, and $p_{4,g}(q) \leq p_{2,d}(q)$.

$\Pi_{2,\nu}(q) \leq \Pi_{2,d}(q)$, and $\Pi_{4,g}(q) \leq \Pi_{2,d}(q)$.

for all values of q.

Proof:

We first write the price expressions of these three resolution models as a function of q as:

$$p_{2,\nu}(q) = R - \frac{4C\mu}{4\mu^2q^2 - \lambda^2}$$
$$p_{2,d}(q) = R - \frac{C}{2\mu q - \lambda}$$

Assuming that $p_{2,\nu}(q) \leq p_{2,d}(q)$, then:

$$\frac{4C\mu}{4\mu^2q^2 - \lambda^2} \geq \frac{C}{2\mu q - \lambda} \Rightarrow$$

$$\frac{4C\mu}{4\mu^2q^2 - \lambda^2} \geq \frac{C}{2\mu q - \lambda} \Rightarrow$$

$$2\mu q + \lambda \leq 4\mu$$

In the above condition, the left hand side reaches its maximum value for $q = 1$. Since the left hand side is at most equal to the right hand side for this maximum value, then we conclude that the condition already holds for the other values of $0 \leq q < 1$. So, the result follows.

Now, we write the market capturing price functions of the Perfect Escalation Model and The Model with Resolution With Double Server Rate as a function of the quality level.

$$p_{2,d}(q) = R - \frac{C}{2\mu q - \lambda}$$
$$p_{4,g}(q) = R - \frac{C}{\mu - \lambda} - \frac{C(1-q)}{\mu - \lambda(1-q)}$$

Since for the same expected reward, the expected waiting cost of the Perfect Escalation Model in only the first server is greater than, the total waiting cost of the Single Stage Double Server Rate Model With Resolution, the comparison holds.

Proposition 4.14: $\alpha^{eq}_{4,g}(q) \leq \alpha^{eq}_{2,d}(q)$ *and* $\alpha^{eq}_{2,\nu}(q) \leq \alpha^{eq}_{2,d}(q)$ *for all values of* q.

Proof:

The expected reward expressions of the three resolution models are the same so we compare the expected waiting costs. The total expected waiting cost of the perfect escalation model is :

$$E[W_{4,g}] = \frac{C}{\mu - \lambda} + \frac{C(1-q)}{\mu - \lambda(1-q)}$$

For the Single Stage Double Server Rate Model With Resolution it is:

$$E[W_{2,d}] = \frac{C}{2\mu q - \lambda}$$

Since the waiting cost in the first server of the perfect escalation model is higher than the cost in the single stage model, we conclude that the customers are more willing to join (the equilibrium probability) is higher in the Single Stage Model. This proves the first part.

Now let us compare the waiting costs of the Two-Parallel Stage Model and the Single Stage Model.

$$E[W_{2,\nu}] = \frac{4C\mu}{4\mu^2 q^2 - \lambda^2}$$

$$E[W_{2,d}] = \frac{C}{2\mu q - \lambda}$$

Assume $E[W_{2,\nu}] \geq E[W_{2,d}]$, then

$$\frac{4C\mu}{4\mu^2 q^2 - \lambda^2} \geq \frac{C}{2\mu q - \lambda} \Rightarrow 4\mu \geq 2\mu q + \lambda$$

As given in Proposition 4.12, the condition always holds. So, since the expected waiting time of the Two-Parallel-Stage Model is higher than, the equilibrium joining probabilities are lower.

Proposition 4.15: $\alpha^*_{4,g}(q) \leq \alpha^*_{2,d}(q)$ and $\alpha^*_{2,\nu}(q) \leq \alpha^*_{2,d}(q)$ *for all values of* q.
Proof:
The similar result follows based on the reasoning given in Proposition 4.14.

Lemma 4.13: $\Pi_4(\mu_1, q)$ *is jointly concave in* μ_1 and q.
Proof:

$$\frac{\partial \Pi_4(\mu_1, q)}{\partial \mu_1} = \frac{C\lambda}{(\mu_1 - \lambda)^2} - \frac{C\lambda(1-q)}{(2\mu - \mu_1 - \lambda + \lambda q)^2}$$

$$\frac{\partial^2 \Pi_4(\mu_1, q)}{\partial \mu_1^2} = -\frac{2C\lambda}{(\mu_1 - \lambda)^3} - \frac{2C\lambda(1-q)}{(2\mu - \mu_1 - \lambda + \lambda q)^3} < 0$$

$$\frac{\partial \Pi_4(\mu_1, q)}{\partial q} = 2R\lambda - 2R\lambda q - \frac{C\lambda(-2\mu + \mu_1)}{(2\mu - \mu_1 - \lambda + \lambda q)^2} - 4aq$$

$$\frac{\partial^2 \Pi_4(\mu_1, q)}{\partial q^2} = -2R\lambda - \frac{2C\lambda^2(2\mu - \mu_1)}{(2\mu - \mu_1 - \lambda + \lambda q)^3} - 4a < 0$$

$$\frac{\partial^2 \Pi_4(\mu_1, q)}{\partial q, \partial \mu_1} = \frac{C\lambda(-2\mu + \mu_1 + \lambda(-1+q))}{(\lambda - 2\mu + \mu_1 - \lambda q)^3}$$

$$\frac{\partial^2 \Pi_4(\mu_1, q)}{\partial \mu_1, \partial q} = \frac{C\lambda(-2\mu + \mu_1 + \lambda(-1+q))}{(\lambda - 2\mu + \mu_1 - \lambda q)^3}$$

Since in the above two derivative functions, the cross terms of the Heissian functions are the same, and the second derivative functions of the model parameters are negative, the result follows.

Lemma 4.14: $\Pi_4(q_1, q_2)$ *is jointly concave in* q_1 and q_2.
Proof:

$$\frac{\partial \Pi_4(q_1, q_2)}{\partial q_1} = R\lambda - R\lambda q_2 + \frac{C\lambda\mu}{(\mu - \lambda + \lambda q_1)^2} - 2aq_1$$

$$\frac{\partial^2 \Pi_4(q_1, q_2)}{\partial q_1^2} = -\frac{2C\lambda^2\mu}{(\mu - \lambda + \lambda q_1)^3} - 2a < 0$$

$$\frac{\partial \Pi_4(q_1, q_2)}{\partial q_2} = R\lambda(1 - q_1) - 2aq_2$$

$$\frac{\partial^2 \Pi_4(q_1, q_2)}{\partial q_2^2} = -2a < 0$$

$$\frac{\partial^2 \Pi_4(q_1, q_2)}{\partial q_1, \partial q_2} = -R\lambda < 0$$

$$\frac{\partial^2 \Pi_4(q_1, q_2)}{\partial q_2, \partial q_1} = -R\lambda < 0$$

As in Lemma 4.13, the result follows.

Corollary 4.4: *The optimal quality level of the second server as a function of the quality level of the first server is:*

$$q_2^*(q_1) = \frac{R\lambda(1-q_1)}{2a}$$

Proof:

By equating the function $\partial\Pi_4(q_1, q_2)/\partial q_2$ to 0 in Lemma 4.14, the result follows.

A.4 Proofs Of Chapter-V

Lemma 5.1: $\Pi_1(\mu, q)$ *is concave in* μ.

Proof:

$$\frac{\partial\Pi_1(\mu, q)}{\partial\mu} = \frac{C\lambda}{(\mu-\lambda)^2} - b$$

$$\frac{\partial^2\Pi_1(\mu, q)}{\partial\mu^2} = -\frac{2C\lambda}{(\mu-\lambda)^3} < 0$$

Corollary 5.1: *Optimal service rate as a function of the model parameters is:*

$$\mu_1^*(q) = \lambda + \sqrt{\frac{C\lambda}{b}}$$

Proof:

Since the function is concave as given in Lemma 5.1, the optimal service rate is found by equating the first derivative function to 0.

$$\frac{\partial \Pi_1(\mu, q)}{\partial \mu} = 0 \Rightarrow \frac{C\lambda}{(\mu-\lambda)^2} - b = 0 \Rightarrow \mu_1^*(q) = \lambda + \sqrt{\frac{C\lambda}{b}}$$

So,the result follows.

Lemma 5.2: $\Pi_1(\mu_1^*, q)$ *is concave in* q.

Proof:

$$\frac{\partial \Pi_1(\mu_1^*, q)}{\partial q} = R\lambda - 2aq$$

$$\frac{\partial^2 \Pi_1(\mu_1^*, q)}{\partial q^2} = -2a < 0$$

Corollary 5.2: *Optimal service quality level is:*

$$q_1^* = \begin{cases} \frac{R\lambda}{2a}, & \text{if } (R\lambda - aq)q > 2\sqrt{C\lambda b} + b\lambda \text{ for all } 0 < q \leq 1 \\ 0, & \text{o.w.} \end{cases}$$

Proof:

Using the concavity given in Lemma 5.2, and setting the first derivative of the profit function with respect to the service quality level, the result follows.

Proposition 5.1: *If positive profit is possible for the Service Rate is a Decision and the Fixed Service Rate Case, then:*

$$q_1^* = q_{1,\tilde{\mu}}^*$$

Proof:

Using the concavity properties of the profit functions as given in Lemmas 5.1 and 5.2, and the optimal service quality level expressions which are derived by equating the first derivative of the profit functions with respect to the service quality level to 0, we obtain the result.

Lemma 5.3: $\Pi_2(\mu, q)$ *is concave in* μ.

Proof:

$$\frac{\partial \Pi_2(\mu, q)}{\partial \mu} = \frac{C\lambda q}{(\mu q - \lambda)^2} - b$$

$$\frac{\partial^2 \Pi_2(\mu, q)}{\partial \mu^2} = -\frac{2C\lambda q^2}{(\mu q - \lambda)^3} < 0$$

Corollary 5.3: *Optimal service rate as a function of the model parameters and the service quality level is:*

$$\mu_2^*(q) = \frac{\lambda + \sqrt{\frac{C\lambda q}{b}}}{q}$$

Proof:

Based on the concavity given in Lemma 5.3, the optimal service rate as a function of the service quality level as obtained from the first order conditions as:

$$\frac{\partial \Pi_2(\mu, q)}{\partial \mu} = 0 \Rightarrow \frac{C\lambda q}{(\mu q - \lambda)^2} - b = 0 \Rightarrow \mu_2^*(q) = \frac{\lambda + \sqrt{\frac{C\lambda q}{b}}}{q}$$

So,the result follows.

Lemma 5.4: $\Pi_2(\mu_2^*, q)$ *is concave in* q.

Proof:

$$\frac{\partial \Pi_2(\mu_2^*, q)}{\partial q} = \sqrt{\frac{C\lambda b}{q^3}} + \frac{b\lambda}{q^2} - a$$

$$\frac{\partial^2 \Pi_2(\mu_2^*, q)}{\partial q^2} = -\frac{3}{2}\sqrt{\frac{C\lambda b}{q^5}} - \frac{2b\lambda}{q^3} < 0$$

Lemma 5.5: $\Pi_3(\mu, q)$ *is concave in* μ.

Proof:

$$\frac{\partial \Pi_3(\mu, q)}{\partial \mu} = \frac{C\lambda}{(\mu q - \lambda)^2} - b$$

$$\frac{\partial^2 \Pi_3(\mu, q)}{\partial \mu^2} = -\frac{2C\lambda q}{(\mu q - \lambda)^3} < 0$$

Corollary 5.4: *Optimal service rate as a function of the model parameters and the service quality level is:*

$$\mu_3^*(q) = \frac{\lambda + \sqrt{\frac{C\lambda}{b}}}{q}$$

Proof:

Using the concavity given in Lemma 5.5, and equating the first derivative function to

0, the result follows, as in Corollary 5.3.

Proposition 5.2:

$$\mu_2^*(q) \geq \mu_2^*(q) \geq \mu_1^*(q)$$

Proof:

Comparing the optimal service rate expressions as a function of the model parameters given in Corollaries 5.1, 5.3 and 5.4 the result follows.

Lemma 5.6: $\Pi_3(\mu_3^*, q)$ *is concave in* q.

Proof:

$$\frac{\partial \Pi_3(\mu_3^*, q)}{\partial q} = \sqrt{\frac{4C\lambda b}{q^4}} + \frac{b\lambda}{q^2} - 2aq$$

$$\frac{\partial^2 \Pi_3(\mu_3^*, q)}{\partial q^2} = -\frac{2\left(b\lambda + 2\sqrt{bC\lambda}\right)}{q^3} - 2a < 0$$

Proposition 5.3: $q_3^* \geq q_2^*$ and $\mu_3^* \geq \mu_2^*$.

Proof:

To prove this result, we start with comparing the optimal quality levels. Since the profit functions are concave as given in Lemmas 5.4 and 5.6, to compare the optimal service quality levels, we compare the first order conditions as:

$$\frac{\partial \Pi_2(\mu_2^*, q)}{\partial q} = \sqrt{\frac{C\lambda b}{q^3}} + \frac{b\lambda}{q^2} - 2aq$$

$$\frac{\partial \Pi_3(\mu_3^*, q)}{\partial q} = \sqrt{\frac{4C\lambda b}{q^4}} + \frac{b\lambda}{q^2} - 2aq$$

For all the values of the service quality level, $0 \leq q \leq 1$, the second and third expressions of the two first derivative functions given above are the same. Since the first expression of the derivative function of the Model With Returns is higher than the first expression of the derivative function of the Model With Resolution, the optimal quality level expression comparison follows.

For the same service quality level, the service rate of the Model With Returns is higher than the Model With Resolution. Since the optimal quality level is higher in the Model With Returns, so the optimal service rate comparison also follows.

Lemma 5.7: $\Pi_{1-\nu}(\mu, q)$ *is jointly concave in* μ *and* q.

Proof:

$$\frac{\partial \Pi_{1-\nu}^2(\mu, q)}{\partial \mu^2} = -\frac{32\lambda C(3\lambda^2\mu + 4\mu^3)}{(4\mu^2 - \lambda^2)^3} < 0$$

$$\frac{\partial^2 \Pi_{1-\nu}(\mu, q)}{\partial q^2} = -4a < 0$$

$$\frac{\partial^2 \Pi_{1-\nu}(\mu, q)}{\partial q \partial \mu} = 0$$

$$\frac{\partial^2 \Pi_{1-\nu}(\mu, q)}{\partial \mu \partial q} = 0$$

Since the eigen-values of the Heissian matrix, $\frac{\partial \Pi^2_{1-\nu}(\mu,q)}{\partial \mu^2}$ and $\frac{\partial^2 \Pi_{1-\nu}(\mu,q)}{\partial q^2}$, are negative and
$\frac{\partial \Pi^2_{1-\nu}(\mu,q)}{\partial \mu^2} \frac{\partial^2 \Pi_{1-\nu}(\mu,q)}{\partial q^2} - \frac{\partial^2 \Pi_{1-\nu}(\mu,q)}{\partial q \partial \mu} \frac{\partial^2 \Pi_{1-\nu}(\mu,q)}{\partial \mu \partial q}$ is positive, the result follows.

Proposition 5.4: *If the positive profitability is achieved in both of the cases: Service Rate is a Decision and the Fixed Service Rate,*

$$q^*_{1-\nu} = q^*_{1-\nu,\tilde{\mu}} = \frac{\lambda R}{4a}$$

Proof:
Since the profit function is concave with respect to the service quality level, and the service rate and the service quality level terms are not inter-related, equating the first derivative functions in both of the cases, the result follows.

Lemma 5.8: $\Pi_{2-\nu}(\mu, q)$ *is jointly concave in* μ *and* q.
Proof:

$$\frac{\partial^2 \Pi_{2-\nu}(\mu, q)}{\partial \mu^2} = -\frac{32C\lambda(3\lambda^2 \mu q^2 + 4\mu^3 q^4)}{(4\mu^2 q^2 - \lambda^2)^3} < 0$$

$$\frac{\partial^2 \Pi_{2-\nu}(\mu, q)}{\partial q^2} = -\frac{32C\lambda\mu^3 \left(\lambda^2 + 12\mu^2 q^2\right)}{(4\mu^2 q^2 - \lambda^2)^3} < 0$$

$$\frac{\partial^2 \Pi_{2-\nu}(\mu, q)}{\partial q \partial \mu} = -\frac{32C\lambda\mu^2 q(3\lambda^2 + 4\mu^2 q^2)}{(4\mu^2 q^2 - \lambda^2)^3} < 0$$

$$\frac{\partial^2 \Pi_{2-\nu}(\mu, q)}{\partial \mu \partial q} = -\frac{32C\lambda\mu^2 q(3\lambda^2 + 4\mu^2 q^2)}{(4\mu^2 q^2 - \lambda^2)^3} < 0$$

Since the eigen-values of the Heissian matrix, $\frac{\partial \Pi^2_{1-\nu}(\mu,q)}{\partial \mu^2}$ and $\frac{\partial^2 \Pi_{1-\nu}(\mu,q)}{\partial q^2}$, are negative and

$\frac{\partial^2 \Pi_{2-\nu}(\mu,q)}{\partial \mu^2}\frac{\partial^2 \Pi_{2-\nu}(\mu,q)}{\partial q^2} - \frac{\partial^2 \Pi_{2-\nu}(\mu,q)}{\partial q \partial \mu}\frac{\partial^2 \Pi_{2-\nu}(\mu,q)}{\partial \mu \partial q}$ is positive, the result follows.

Proposition 5.5: $\mu^*_{1-\nu}(q) \leq \mu^*_{2-\nu}(q)$ if $\mu q > \lambda$ *for all values of* μ *and* $0 < q \leq 1$.

Proof:

To compare the service rate expressions as functions of the quality level, we use the first derivative functions derived in Lemmas 5.7 and 5.8. When we look at the first derivative functions, we observe that the numerator and the denominator of the first derivative function of the two parallel stage model with resolution which is derived in Lemma 5.8 is smaller. However since the denominator reduces more; i.e. it is a squared expression, we conclude that the first derivative function of the two parallel stage model with resolution takes higher values compared to the two parallel stage benchmark model. This shows us that optimal service rate of the model with resolution is higher than the optimal service rate of the benchmark model in the two parallel stage case for the given quality levels. However to end up with this result, we have to reach the steady state in both of the models, which forms a condition of $\mu q > \lambda$ for all values of μ and $0 < q \leq 1$. Otherwise, positive profitability conditions can not be reached in model with resolution and the comparison seems unfair.

Appendix B

NUMERICAL RESULTS

B.1 Numerical Results of Chapter-III

Observation 3.1:$\alpha_1^{eq} \geq \alpha_1^*$.

Observation 3.2: $\alpha_2^{eq} \geq \alpha_2^{*}$.

Observation 3.3:$\alpha_3^{eq} \geq \alpha_3^*$.

Observation 3.4: If $R\lambda < 2a$, then $\Pi_{1,\lambda} \leq \Pi_{2,\lambda}$.

B.2 Numerical Results of Chapter-IV

Observation 4.1. $\alpha_{4,g}^{eq}(q) \geq \alpha_{4,g}^{*}(q)$, and $\alpha_{4,n}^{eq}(q) \geq \alpha_{4,n}^{*}(q)$.

Observation 4.2. $\alpha_{2,\nu}^{eq}(q) \leq \alpha_{1,\nu}^{*}(q)$, and $\alpha_{2,\nu}^{eq}(q) \leq \alpha_{1,\nu}^{*}(q)$.

Observation 4.3. *In the long term, for the given model parameters,* R, C, a, λ *and* μ, *if* $R\lambda < 4a$:

-Pricing Comparison: $p^*_{1,\nu} \leq p^*_{2,\nu}$

-Profit Comparison: $\Pi^*_{1,\nu} \leq \Pi^*_{2,\nu}$

Observation 4.4. Assuming all the models are profitable;

- if $R\lambda < 4a$ then,

  $p^*_{1,d} \geq p^*_{4,n} \geq p^*_{1,\nu}$.

  $\Pi^*_{1,d} \geq \Pi^*_{4,n} \geq \Pi^*_{1,\nu}$.

- if $R\lambda \geq 4a$ then,

  $p^*_{1,d} \geq p^*_{1,\nu} \geq p^*_{4,n}$.

  $\Pi^*_{1,d} \geq \Pi^*_{4,n}$.

Observation 4.5.

- *Optimal Quality Level Comparison:* $q^*_{4,g} \leq q^*_{2,\nu}$.
- *Optimal Price Comparison:* $p^*_{4,g} \leq p^*_{2,\nu} \leq p^*_{2,d}$.
- *Optimal Profit Comparison:* $\Pi^*_{4,g} \leq \Pi^*_{2,\nu} \leq \Pi^*_{2,d}$.

Observation 4.6. $\Pi_4^*(\mu_1, q) \geq \Pi_{4,n}^*(\mu, q)$.

Observation 4.7.$\Pi_4^*(q_1, q_2) \geq \Pi_{4,n}^*(\mu, q)$.

B.3 Numerical Results of Chapter-V

Observation 5.1:

- If $\mu \leq \mu_2^*$ then $q_2^* \leq q_{2,\tilde{\mu}}^*$.

- If $\mu > \mu_2^*$ then $q_2^* \geq q_{2,\tilde{\mu}}^*$.

- $\Pi\left(\mu_2^*, q_2^*\right) \geq \Pi\left(\mu, q_{2,\tilde{\mu}}^*\right)$

Observation 5.2:

- If $\mu \leq \mu_3^*$ then $q_3^* \leq q_{3,\tilde{\mu}}^*$.
- If $\mu > \mu_3^*$ then $q_3^* \geq q_{3,\tilde{\mu}}^*$.
- $\Pi\left(\mu_3^*, q_3^*\right) \geq \Pi\left(\mu, q_{3,\tilde{\mu}}^*\right)$.

Observation 5.3:

- $\Pi\left(\mu, q^*_{3,\tilde{\mu}}\right) \leq \Pi\left(\mu, q^*_{2,\tilde{\mu}}\right)$.

- $\Pi\left(\mu^3_*, q^*_3\right) \leq \Pi\left(\mu^2_*, q^*_2\right)$.

See Observations 5.1 and 5.2.

Observation 5.4: For the Parallel Server Model, optimal quality level values and the profits of the fixed service rate and service rate is a decision case is compared as:

- $$\begin{cases} q^*_{1-\nu,\tilde{\mu}} = q^*_{1-\nu} = \frac{R\lambda}{4a}, & (R\lambda - 4aq)q > \frac{4C\lambda\mu}{4\mu^2-\lambda^2} + b\mu \\ q^*_{1-\nu,\tilde{\mu}} = 0 < q^*_{1-\nu}, & \text{o.w.} \end{cases}$$
- $\Pi\left(\mu, q^*_{1-\nu,\tilde{\mu}}\right) \leq \Pi\left(\mu^*_{1-\nu}, q^*_{1-\nu}\right)$

Observation 5.5:

- If $\mu \leq \mu^*_{2-\nu}$ then $q^*_{2-\nu,\tilde{\mu}} \geq q^*_{2-\nu}$.

- If $\mu > \mu^*_{2-\nu}$ then $q^*_{2-\nu} > q^*_{2-\nu,\tilde{\mu}}$.

- $\Pi\left(\mu^*_{2-\nu}, q^*_{2-\nu}\right) \geq \Pi\left(\mu, q^*_{2-\nu,\tilde{\mu}}\right)$.

Observation 5.6:

- $\Pi\left(\mu, q^*_{1-\nu,\tilde{\mu}}\right) \leq \Pi\left(\mu, q^*_{2-\nu,\tilde{\mu}}\right)$.

- $\Pi\left(\mu^*_{1-\nu}, q^*_{1-\nu}\right) \leq \Pi\left(\mu^*_{2-\nu}, q^*_{2-\nu}\right)$.

See Observations 5.4 and 5.5.

Observation 5.7:

- $q^*_{4-n,\tilde{\mu}} \cong q^*_{4-n}$.
- $\Pi\left(\mu, q^*_{4-n,\tilde{\mu}}\right) \leq \Pi\left(\mu^*_{4-n}, q^*_{4-n}\right)$.

Observation 5.8:

- If $\mu \leq \mu^*_{4-g}$ then $q^*_{4-g,\tilde{\mu}} \geq q^*_{4-g}$.

- If $\mu > \mu^*_{4-g}$ then $q^*_{4-g} \geq q^*_{4-g,\tilde{\mu}}$.

- $\Pi\left(\mu^*_{4-g}, q^*_{4-g}\right) \geq \Pi\left(\mu, q^*_{4-g,\tilde{\mu}}\right)$.

Observation 5.9:

- $q^*_{4-g,\tilde{\mu}} \leq q^*_{4-n\tilde{\mu}}$ and $q^*_{4-g} \leq q^*_{4-n}$.
- $\mu^*_{4-g} \geq \mu^*_{4-n}$
- $\Pi\left(\mu, q^*_{4-n,\tilde{\mu}}\right) \leq \Pi\left(\mu, q^*_{4-g,\tilde{\mu}}\right)$.
- $\Pi\left(\mu^*_{4-n}, q^*_{4-n}\right) \leq \Pi\left(\mu^*_{4-g}, q^*_{4-g}\right)$.

See Observations 5.7 and 5.8.

parameters	α_1^{eq}	α_1^*
$R = 1, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.4$	0.91	0.57
$R = 1, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.8$	1	0.81
$R = 1, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.4$	1	0.94
$R = 1, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.8$	1	1
$R = 1, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.4$	0.72	0.46
$R = 1, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.8$	0.91	0.65
$R = 1, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.4$	1	0.75
$R = 1, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.8$	1	0.97
$R = 1, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.4$	0.44	0.24
$R = 1, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.8$	0.9	0.57
$R = 1, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.4$	0.94	0.55
$R = 1, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.8$	1	0.94
$R = 1, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.4$	0.35	0.19
$R = 1, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.8$	0.72	0.46
$R = 1, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.4$	0.75	0.44
$R = 1, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.8$	1	0.75
$R = 1.5, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.4$	1	0.72
$R = 1.5, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.8$	1	0.91
$R = 1.5, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.4$	1	1
$R = 1.5, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.8$	1	1
$R = 1.5, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.4$	0.85	0.58
$R = 1.5, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.8$	0.97	0.73
$R = 1.5, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.4$	1	0.89
$R = 1.5, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.8$	1	1
$R = 1.5, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.4$	0.75	0.45
$R = 1.5, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.8$	1	0.72
$R = 1.5, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.4$	1	0.79
$R = 1.5, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.8$	1	1
$R = 1.5, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.4$	0.6	0.36
$R = 1.5, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.8$	0.85	0.58
$R = 1.5, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.4$	0.99	0.63
$R = 1.5, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.8$	1	0.89

Table B.1: Comparison Between The Equilibrium and Socially Optimal Joining Probabilities of the Benchmark Model

parameters	α_2^{eq}	α_2^*
$R = 1, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.4$	0.36	0.23
$R = 1, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.8$	0.91	0.65
$R = 1, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.4$	0.56	0.37
$R = 1, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.8$	1	0.97
$R = 1, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.4$	0.28	0.18
$R = 1, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.8$	0.73	0.52
$R = 1, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.4$	0.45	0.3
$R = 1, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.8$	1	0.78
$R = 1, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.4$	0.17	0.1
$R = 1, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.8$	0.72	0.46
$R = 1, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.4$	0.37	0.22
$R = 1, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.8$	1	0.75
$R = 1, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.4$	0.14	0.08
$R = 1, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.8$	0.58	0.37
$R = 1, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.4$	0.3	0.18
$R = 1, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.8$	0.9	0.6
$R = 1.5, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.4$	0.42	0.29
$R = 1.5, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.8$	0.97	0.73
$R = 1.5, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.4$	0.62	0.44
$R = 1.5, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.8$	1	1
$R = 1.5, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.4$	0.34	0.23
$R = 1.5, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.8$	0.78	0.58
$R = 1.5, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.4$	0.5	0.36
$R = 1.5, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.8$	1	0.85
$R = 1.5, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.4$	0.3	0.18
$R = 1.5, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.8$	0.85	0.58
$R = 1.5, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.4$	0.5	0.32
$R = 1.5, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.8$	1	0.89
$R = 1.5, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.4$	0.24	0.14
$R = 1.5, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.8$	0.68	0.46
$R = 1.5, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.4$	0.4	0.25
$R = 1.5, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.8$	1	0.71

Table B.2: Comparison Between The Equilibrium and Socially Optimal Joining Probabilities of the Resolution Model

parameters	α_3^{eq}	α_3^*
$R = 1, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.4$	0.49	0.31
$R = 1, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.8$	0.89	0.63
$R = 1, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.4$	0.66	0.44
$R = 1, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.8$	1	0.88
$R = 1, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.4$	0.43	0.28
$R = 1, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.8$	0.74	0.53
$R = 1, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.4$	0.58	0.39
$R = 1, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.8$	1	0.75
$R = 1, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.4$	0.24	0.13
$R = 1, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.8$	0.71	0.45
$R = 1, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.4$	0.44	0.26
$R = 1, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.8$	1	0.68
$R = 1, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.4$	0.21	0.12
$R = 1, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.8$	0.59	0.38
$R = 1, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.4$	0.39	0.23
$R = 1, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.8$	0.86	0.58
$R = 1.5, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.4$	0.54	0.39
$R = 1.5, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.8$	0.94	0.71
$R = 1.5, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.4$	0.73	0.52
$R = 1.5, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.8$	1	0.96
$R = 1.5, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.4$	0.43	0.35
$R = 1.5, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.8$	0.79	0.60
$R = 1.5, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.4$	0.59	0.46
$R = 1.5, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.8$	1	0.81
$R = 1.5, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.4$	0.41	0.24
$R = 1.5, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.8$	0.82	0.55
$R = 1.5, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.4$	0.59	0.37
$R = 1.5, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.8$	1	0.81
$R = 1.5, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.4$	0.35	0.22
$R = 1.5, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.8$	0.69	0.47
$R = 1.5, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.4$	0.52	0.33
$R = 1.5, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.8$	0.96	0.68

Table B.3: Comparison Between The Equilibrium and Socially Optimal Joining Probabilities of the Model With Returns

parameters	$\Pi^*_{1,\lambda}$	$\Pi^*_{2,\lambda}$
$R=5, C=0.25, \lambda=1, \mu=5.5, a=5.5$	1.08	4.12
$R=5, C=0.25, \lambda=1, \mu=5.5, a=7.5$	0.78	3.95
$R=5, C=0.25, \lambda=1, \mu=3.5, a=5.5$	1.04	3.50
$R=5, C=0.25, \lambda=1, \mu=3.5, a=7.5$	0.73	3.18
$R=5, C=0.25, \lambda=2, \mu=5.5, a=5.5$	4.40	7.96
$R=5, C=0.25, \lambda=2, \mu=5.5, a=7.5$	3.19	7.49
$R=5, C=0.25, \lambda=2, \mu=3.5, a=5.5$	4.21	6.20
$R=5, C=0.25, \lambda=2, \mu=3.5, a=7.5$	2.99	5.22
$R=5, C=1, \lambda=1, \mu=5.5, a=5.5$	0.91	3.29
$R=5, C=1, \lambda=1, \mu=5.5, a=7.5$	0.61	3.01
$R=5, C=1, \lambda=1, \mu=3.5, a=5.5$	0.74	2.30
$R=5, C=1, \lambda=1, \mu=3.5, a=7.5$	0.43	1.80
$R=5, C=1, \lambda=2, \mu=5.5, a=5.5$	3.97	6.48
$R=5, C=1, \lambda=2, \mu=5.5, a=7.5$	2.76	5.80
$R=5, C=1, \lambda=2, \mu=3.5, a=5.5$	3.21	4.00
$R=5, C=1, \lambda=2, \mu=3.5, a=7.5$	1.99	2.70
$R=3, C=0.25, \lambda=1, \mu=5.5, a=5.5$	0.35	2.12
$R=3, C=0.25, \lambda=1, \mu=5.5, a=7.5$	0.24	1.95
$R=3, C=0.25, \lambda=1, \mu=3.5, a=5.5$	0.31	1.50
$R=3, C=0.25, \lambda=1, \mu=3.5, a=7.5$	0.2	1.18
$R=3, C=0.25, \lambda=2, \mu=5.5, a=5.5$	1.49	3.96
$R=3, C=0.25, \lambda=2, \mu=5.5, a=7.5$	1.06	3.49
$R=3, C=0.25, \lambda=2, \mu=3.5, a=5.5$	1.30	2.20
$R=3, C=0.25, \lambda=2, \mu=3.5, a=7.5$	0.87	1.22
$R=3, C=1, \lambda=1, \mu=5.5, a=5.5$	0.19	1.29
$R=3, C=1, \lambda=1, \mu=5.5, a=7.5$	0.08	1.21
$R=3, C=1, \lambda=1, \mu=3.5, a=5.5$	0.01	0.30
$R=3, C=1, \lambda=1, \mu=3.5, a=7.5$	0	0
$R=3, C=1, \lambda=2, \mu=5.5, a=5.5$	1.06	2.48
$R=3, C=1, \lambda=2, \mu=5.5, a=7.5$	0.63	1.80
$R=3, C=1, \lambda=2, \mu=3.5, a=5.5$	0.30	0.32
$R=3, C=1, \lambda=2, \mu=3.5, a=7.5$	0	0

Table B.4: Comparison Between The Profits of the Benchmark Model and Model With Resolution When $R\lambda < 2a$

parameters	$\alpha_{4,g}^{eq}$	$\alpha_{4,g}^{*}$	$\alpha_{4,n}^{eq}$	$\alpha_{4,n}^{*}$
$R = 1, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.4$	0.92	0.64	0.79	0.51
$R = 1, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.8$	0.94	0.68	0.93	0.67
$R = 1, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.4$	1	0.98	1	0.83
$R = 1, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.4$	1	0.8	0.98	0.64
$R = 1, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.8$	1	0.85	1	0.83
$R = 1, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.4$	1	1	1	1
$R = 1, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.4$	0.69	0.42	0.84	0.22
$R = 1, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.8$	0.78	0.5	0.76	0.49
$R = 1, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.4$	1	0.73	0.85	0.5
$R = 1, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.4$	0.86	0.53	0.51	0.27
$R = 1, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.8$	1	0.81	1	0.79
$R = 1, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.8$	1	1	1	0.98
$R = 1, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.8$	0.97	0.63	0.95	0.61
$R = 1, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.4$	1	0.91	1	0.62
$R = 1.5, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.4$	0.98	0.73	0.91	0.63
$R = 1.5, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.8$	0.99	0.76	0.98	0.75
$R = 1.5, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.4$	1	1	1	0.96
$R = 1.5, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.4$	1	0.92	1	0.79
$R = 1.5, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.8$	0.99	0.76	0.98	0.75
$R = 1.5, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1.5, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.8$	1	0.95	1	0.94
$R = 1.5, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.4$	1	1	1	1
$R = 1.5, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.4$	0.84	0.56	0.67	0.4
$R = 1.5, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.8$	0.89	0.61	0.88	0.6
$R = 1.5, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.4$	1	0.82	1	0.7
$R = 1.5, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.4$	1	0.7	0.84	0.51
$R = 1.5, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.8$	1	0.94	1	0.93
$R = 1.5, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1.5, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.8$	1	0.77	1	0.76
$R = 1.5, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.4$	1	1	1	0.89

Table B.5: Comparison Between The Equilibrium and Socially Optimal Joining Probabilities of the Escalation Models

parameters	$\alpha_{1,\nu}^{eq}$	$\alpha_{1,\nu}^{*}$	$\alpha_{2,\nu}^{eq}$	$\alpha_{2,\nu}^{*}$
$R = 1, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.4$	1	1	0	0
$R = 1, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.8$	1	1	1	1
$R = 1, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.4$	1	1	0	0
$R = 1, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.4$	1	1	0.85	0
$R = 1, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.8$	1	1	1	1
$R = 1, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.4$	1	1	1	0.74
$R = 1, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.4$	1	1	0	0
$R = 1, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.8$	1	1	1	1
$R = 1, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.4$	1	1	0	0
$R = 1, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.4$	1	1	0	0
$R = 1, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.8$	1	1	1	1
$R = 1, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.4$	1	1	0	0
$R = 1.5, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.4$	1	1	0	0
$R = 1.5, C = 0.75, \lambda = 5, \mu = 5.5, q = 0.8$	1	1	1	1
$R = 1.5, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.4$	1	1	0	0
$R = 1.5, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.4$	1	1	1	0.66
$R = 1.5, C = 0.75, \lambda = 5, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1.5, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1.5, C = 0.75, \lambda = 4, \mu = 5.5, q = 0.8$	1	1	1	1
$R = 1.5, C = 0.75, \lambda = 4, \mu = 7.5, q = 0.4$	1	1	1	1
$R = 1.5, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.4$	1	1	0	0
$R = 1.5, C = 1.5, \lambda = 5, \mu = 5.5, q = 0.8$	1	1	1	1
$R = 1.5, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.4$	1	1	0	0
$R = 1.5, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.4$	1	1	1	1
$R = 1.5, C = 1.5, \lambda = 5, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1.5, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.8$	1	1	1	1
$R = 1.5, C = 1.5, \lambda = 4, \mu = 5.5, q = 0.8$	1	1	1	1
$R = 1.5, C = 1.5, \lambda = 4, \mu = 7.5, q = 0.4$	1	1	1	1

Table B.6: Comparison Between The Equilibrium and Socially Optimal Joining Probabilities of the Parallel Server Models

parameters	$p^*_{1,\nu}$	$\Pi^*_{1,\nu}$	$p^*_{2\nu}$	$\Pi^*_{2,\nu}$
$R=5, C=0.25, \lambda=1, \mu=5, a=3$	2.03	0.99	4.45	3.84
$R=5, C=0.25, \lambda=1, \mu=5, a=6$	0.99	0.47	4.23	3.33
$R=5, C=0.25, \lambda=1, \mu=7.5, a=3$	2.05	1.01	4.55	4.08
$R=5, C=0.25, \lambda=1, \mu=7.5, a=6$	1.01	0.49	4.37	3.68
$R=5, C=0.25, \lambda=2, \mu=5, a=3$	4.11	4.06	4.61	8.21
$R=5, C=0.25, \lambda=2, \mu=5, a=6$	2.03	1.98	4.45	7.33
$R=5, C=0.25, \lambda=2, \mu=7.5, a=3$	4.13	4.10	4.68	8.63
$R=5, C=0.25, \lambda=2, \mu=7.5, a=6$	2.05	2.02	4.55	8.00
$R=5, C=1, \lambda=1, \mu=5, a=3$	1.88	0.84	3.90	2.75
$R=5, C=1, \lambda=1, \mu=5, a=6$	0.84	0.32	3.45	1.78
$R=5, C=1, \lambda=1, \mu=7.5, a=3$	1.95	0.91	4.11	3.18
$R=5, C=1, \lambda=1, \mu=7.5, a=6$	0.91	0.39	3.74	2.42
$R=5, C=1, \lambda=2, \mu=5, a=3$	3.96	3.75	4.23	6.66
$R=5, C=1, \lambda=2, \mu=5, a=6$	1.88	1.67	3.90	5.14
$R=5, C=1, \lambda=2, \mu=7.5, a=3$	4.03	3.90	4.37	7.36
$R=5, C=1, \lambda=2, \mu=7.5, a=6$	1.95	1.81	4.11	6.21
$R=3, C=0.25, \lambda=1, \mu=5, a=3$	0.7	0.32	2.45	1.84
$R=3, C=0.25, \lambda=1, \mu=5, a=6$	0.32	0.14	2.23	1.33
$R=3, C=0.25, \lambda=1, \mu=7.5, a=3$	0.72	0.34	2.55	2.08
$R=3, C=0.25, \lambda=1, \mu=7.5, a=6$	0.34	0.15	2.37	1.68
$R=3, C=0.25, \lambda=2, \mu=5, a=3$	1.45	1.4	2.61	4.21
$R=3, C=0.25, \lambda=2, \mu=5, a=6$	0.7	0.65	2.45	3.33
$R=3, C=0.25, \lambda=2, \mu=7.5, a=3$	1.47	1.43	2.68	4.63
$R=3, C=0.25, \lambda=2, \mu=7.5, a=6$	0.72	0.68	2.55	4.00
$R=3, C=1, \lambda=1, \mu=5, a=3$	0.55	0.17	1.90	0.75
$R=3, C=1, \lambda=1, \mu=5, a=6$	0	0	0	0
$R=3, C=1, \lambda=1, \mu=7.5, a=3$	0.62	0.24	2.11	1.18
$R=3, C=1, \lambda=1, \mu=7.5, a=6$	0.24	0.05	1.74	0.42
$R=3, C=1, \lambda=2, \mu=5, a=3$	1.29	1.08	2.23	2.66
$R=3, C=1, \lambda=2, \mu=5, a=6$	0.54	0.33	1.90	1.14
$R=3, C=1, \lambda=2, \mu=7.5, a=3$	1.36	1.23	2.37	3.36
$R=3, C=1, \lambda=2, \mu=7.5, a=6$	0.61	0.48	2.11	2.21

Table B.7: Optimal Price and Profit Comparison of the Parallel Server Models

parameters	$p^*_{4,n}$	$\Pi^*_{4,n}$	$p^*_{1,\nu}$	$\Pi^*_{1,\nu}$	$p^*_{1,d}$	$\Pi^*_{1,d}$
$R = 5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 1$	3.99	4.37	4.83	5.24	4.88	6.31
$R = 5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 3$	3.02	2.52	2.92	2.08	4.88	1.31
$R = 5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 1$	4.53	5.11	4.89	5.34	4.93	6.40
$R = 5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 3$	3.77	3.70	2.99	2.18	4.93	4.40
$R = 5, C = 0.25, \lambda = 1, \mu = 1.75, a = 1$	4.26	3.19	4.84	2.84	4.9	3.9
$R = 5, C = 0.25, \lambda = 1, \mu = 1.75, a = 3$	3.15	1.83	1.94	0.89	4.05	1.98
$R = 5, C = 0.25, \lambda = 1, \mu = 2.5, a = 1$	4.41	3.37	4.89	2.90	4.94	3.94
$R = 5, C = 0.25, \lambda = 1, \mu = 2.5, a = 3$	3.31	2.01	1.99	0.94	4.09	2.22
$R = 5, C = 1, \lambda = 1.5, \mu = 1.75, a = 1$	0	0	4.3	4.45	4.5	5.80
$R = 5, C = 1, \lambda = 1.5, \mu = 1.75, a = 3$	0	0	2.4	1.29	4.5	3.75
$R = 5, C = 1, \lambda = 1.5, \mu = 2.5, a = 1$	3.78	4.29	4.56	4.84	4.71	6.07
$R = 5, C = 1, \lambda = 1.5, \mu = 2.5, a = 3$	2.94	2.32	2.66	1.68	4.71	4.07
$R = 5, C = 1, \lambda = 1, \mu = 1.75, a = 1$	3.25	2.07	4.38	2.38	4.6	3.6
$R = 5, C = 1, \lambda = 1, \mu = 1.75, a = 3$	2.07	0.52	1.48	0.42	3.75	1.68
$R = 5, C = 1, \lambda = 1, \mu = 2.5, a = 1$	3.91	2.79	4.58	2.58	4.75	3.75
$R = 5, C = 1, \lambda = 1, \mu = 2.5, a = 3$	2.72	1.34	1.68	0.62	3.9	1.83
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 1$	6.33	8.02	7.33	8.99	7.38	10.06
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 3$	5.56	5.73	6.88	5.01	7.38	8.06
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 1$	7.08	9.15	7.39	9.09	7.43	10.14
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 3$	6.34	6.90	6.94	5.11	7.43	8.14
$R = 7.5, C = 0.25, \lambda = 1, \mu = 1.75, a = 1$	6.83	5.55	7.34	5.34	7.4	6.4
$R = 7.5, C = 0.25, \lambda = 1, \mu = 1.75, a = 3$	5.63	3.75	4.49	2.19	7.4	4.4
$R = 7.5, C = 0.25, \lambda = 1, \mu = 2.5, a = 1$	7.01	5.73	7.40	5.39	7.44	6.44
$R = 7.5, C = 0.25, \lambda = 1, \mu = 2.5, a = 3$	5.83	3.95	4.55	2.24	7.44	4.44
$R = 7.5, C = 1, \lambda = 1.5, \mu = 1.75, a = 1$	3.34	3.43	6.8	8.2	7	9.5
$R = 7.5, C = 1, \lambda = 1.5, \mu = 1.75, a = 3$	2.59	0.95	6.35	4.12	7	7.5
$R = 7.5, C = 1, \lambda = 1.5, \mu = 2.5, a = 1$	6.34	7.96	7.06	8.60	7.21	9.82
$R = 7.5, C = 1, \lambda = 1.5, \mu = 2.5, a = 3$	5.58	5.59	6.61	4.61	7.21	7.82
$R = 7.5, C = 1, \lambda = 1, \mu = 1.75, a = 1$	5.84	4.47	6.88	4.88	7.1	6.1
$R = 7.5, C = 1, \lambda = 1, \mu = 1.75, a = 3$	4.59	2.51	4.03	1.72	7.1	4.1
$R = 7.5, C = 1, \lambda = 1, \mu = 2.5, a = 1$	6.48	5.07	7.08	5.18	7.25	6.25
$R = 7.5, C = 1, \lambda = 1, \mu = 2.5, a = 3$	5.31	3.29	4.23	1.93	7.25	4.25

Table B.8: Optimal Price and Profit Comparison of the Benchmark Type Models

parameters	$q^*_{4,g}$	$p^*_{4,g}$	$\Pi^*_{4,g}$	$q^*_{2,\nu}$	$p^*_{2,\nu}$	$\Pi^*_{2,\nu}$	$q^*_{2,d}$	$p^*_{2,d}$	$\Pi^*_{2,d}$
$R = 5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 1$	0.42	3.84	4.58	0.71	4.55	5.82	0.43	4.88	7.13
$R = 5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 3$	0.26	3.71	2.36	0.61	4.24	4.13	0.43	4.88	6.76
$R = 5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 1$	0.25	4.61	5.86	0.6	4.63	6.22	0.31	4.53	7.3
$R = 5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 3$	0.11	4.56	3.80	0.5	4.38	5.06	0.31	4.93	7.1
$R = 5, C = 0.25, \lambda = 1, \mu = 1.75, a = 1$	0.23	4.47	3.42	0.59	4.66	3.77	0.29	4.9	4.82
$R = 5, C = 0.25, \lambda = 1, \mu = 1.75, a = 3$	0.1	4.40	1.37	0.49	4.12	2.66	0.3	4.9	4.65
$R = 5, C = 0.25, \lambda = 1, \mu = 2.5, a = 1$	0.12	4.70	3.68	0.51	4.54	4.02	0.21	4.94	4.90
$R = 5, C = 0.25, \lambda = 1, \mu = 2.5, a = 3$	0.04	4.68	1.67	0.41	4.22	3.21	0.21	4.94	4.81
$R = 5, C = 1, \lambda = 1.5, \mu = 1.75, a = 1$	0	0	0	0.92	4.14	4.51	0.43	4.5	6.57
$R = 5, C = 1, \lambda = 1.5, \mu = 1.75, a = 3$	0	0	0	0.75	3.49	1.86	0.43	4.5	6.20
$R = 5, C = 1, \lambda = 1.5, \mu = 2.5, a = 1$	0.56	3.76	4.33	0.8	4.27	5.13	0.31	4.71	6.96
$R = 5, C = 1, \lambda = 1.5, \mu = 2.5, a = 3$	0.3	3.52	2.01	0.64	3.75	3.16	0.31	4.71	6.78
$R = 5, C = 1, \lambda = 1, \mu = 1.75, a = 1$	0.53	3.30	2.02	0.78	3.92	2.70	0.29	4.6	4.52
$R = 5, C = 1, \lambda = 1, \mu = 1.75, a = 3$	0	0	0	0.62	3.11	0.81	0.29	4.6	4.35
$R = 5, C = 1, \lambda = 1, \mu = 2.5, a = 1$	0.36	3.99	2.86	0.7	4.11	3.13	0.21	4.75	4.71
$R = 5, C = 1, \lambda = 1, \mu = 2.5, a = 3$	0.15	3.82	0.75	0.55	3.48	1.66	0.21	4.75	4.62
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 1$	0.42	6.34	8.32	0.71	7.05	9.57	0.43	7.38	10.88
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 3$	0.26	6.21	6.11	0.61	6.74	7.8	0.43	7.38	10.51
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 1$	0.25	7.11	9.61	0.6	7.12	9.98	0.31	7.43	11.05
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 3$	0.11	7.06	7.55	0.5	6.8	8.81	0.31	7.43	10.85
$R = 7.5, C = 0.25, \lambda = 1, \mu = 1.75, a = 1$	0.23	6.97	5.92	0.59	6.96	6.27	0.29	7.4	7.3
$R = 7.5, C = 0.25, \lambda = 1, \mu = 1.75, a = 3$	0.1	6.90	3.87	0.49	6.60	5.15	0.29	7.4	7.15
$R = 7.5, C = 0.25, \lambda = 1, \mu = 2.5, a = 1$	0.12	7.20	6.18	0.51	7.04	6.53	0.21	7.43	7.40
$R = 7.5, C = 0.25, \lambda = 1, \mu = 2.5, a = 3$	0.04	7.18	4.17	0.41	6.72	5.71	0.21	7.44	7.31
$R = 7.5, C = 1, \lambda = 1.5, \mu = 1.75, a = 1$	0.73	3.30	3.42	0.92	6.64	8.26	0.43	7.00	10.32
$R = 7.5, C = 1, \lambda = 1.5, \mu = 1.75, a = 3$	0.47	2.94	0.76	0.75	5.99	5.61	0.43	7.00	9.95
$R = 7.5, C = 1, \lambda = 1.5, \mu = 2.5, a = 1$	0.56	6.26	8.07	0.8	6.77	8.88	0.31	7.21	10.72
$R = 7.5, C = 1, \lambda = 1.5, \mu = 2.5, a = 3$	0.3	6.02	5.76	0.64	6.25	6.92	0.31	7.21	10.53
$R = 7.5, C = 1, \lambda = 1, \mu = 1.75, a = 1$	0.53	5.80	4.52	0.78	6.42	5.20	0.29	7.1	7.02
$R = 7.5, C = 1, \lambda = 1, \mu = 1.75, a = 3$	0.28	5.47	2.23	0.62	5.61	3.31	0.29	7.1	6.85
$R = 7.5, C = 1, \lambda = 1, \mu = 2.5, a = 1$	0.36	6.49	5.36	0.7	6.61	5.63	0.21	7.25	7.21
$R = 7.5, C = 1, \lambda = 1, \mu = 2.5, a = 3$	0.15	6.32	3.25	0.55	5.97	4.16	0.21	7.25	7.12

Table B.9: Optimal Price and Profit Comparison of the Resolution Type Models

parameters	μ_1^*	$q*$	$\Pi_4^*(\mu_1, q)$
$R = 5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 1$	2.76	0.85	5.48
$R = 5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 3$	2.38	0.61	3.43
$R = 5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 1$	3.75	0.82	5.68
$R = 5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 3$	3.25	0.58	3.80
$R = 5, C = 0.25, \lambda = 1, \mu = 1.75, a = 1$	2.49	0.75	3.31
$R = 5, C = 0.25, \lambda = 1, \mu = 1.75, a = 3$	2.14	0.48	1.89
$R = 5, C = 0.25, \lambda = 1, \mu = 2.5, a = 1$	3.45	0.73	3.42
$R = 5, C = 0.25, \lambda = 1, \mu = 2.5, a = 3$	3.00	0.47	2.06
$R = 5, C = 1, \lambda = 1.5, \mu = 1.75, a = 1$	3.47	1	4.74
$R = 5, C = 1, \lambda = 1.5, \mu = 1.75, a = 3$	2.55	0.73	1.58
$R = 5, C = 1, \lambda = 1.5, \mu = 2.5, a = 1$	4.95	1	5.07
$R = 5, C = 1, \lambda = 1.5, \mu = 2.5, a = 3$	3.35	0.63	2.77
$R = 5, C = 1, \lambda = 1, \mu = 1.75, a = 1$	2.70	0.85	2.62
$R = 5, C = 1, \lambda = 1, \mu = 1.75, a = 3$	2.21	0.54	0.81
$R = 5, C = 1, \lambda = 1, \mu = 2.5, a = 1$	3.6	0.79	2.97
$R = 5, C = 1, \lambda = 1, \mu = 2.5, a = 3$	3.05	0.5	1.42
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 1$	2.87	0.89	9.17
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 3$	2.49	0.69	6.72
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 1$	3.95	0.87	9.34
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 3$	3.4	0.67	7.02
$R = 7.5, C = 0.25, \lambda = 1, \mu = 1.75, a = 1$	2.63	0.81	5.69
$R = 7.5, C = 0.25, \lambda = 1, \mu = 1.75, a = 3$	2.24	0.57	3.83
$R = 7.5, C = 0.25, \lambda = 1, \mu = 2.5, a = 1$	3.65	0.8	5.78
$R = 7.5, C = 0.25, \lambda = 1, \mu = 2.5, a = 3$	3.15	0.56	3.97
$R = 7.5, C = 1, \lambda = 1.5, \mu = 1.75, a = 1$	3.47	1	8.49
$R = 7.5, C = 1, \lambda = 1.5, \mu = 1.75, a = 3$	2.63	0.78	5.12
$R = 7.5, C = 1, \lambda = 1.5, \mu = 2.5, a = 1$	4.95	1	8.82
$R = 7.5, C = 1, \lambda = 1.5, \mu = 2.5, a = 3$	3.5	0.71	6.12
$R = 7.5, C = 1, \lambda = 1, \mu = 1.75, a = 1$	3.47	1	5.09
$R = 7.5, C = 1, \lambda = 1, \mu = 1.75, a = 3$	2.31	0.62	2.88
$R = 7.5, C = 1, \lambda = 1, \mu = 2.5, a = 1$	3.75	0.84	5.39
$R = 7.5, C = 1, \lambda = 1, \mu = 2.5, a = 3$	3.2	0.59	3.40

Table B.10: Optimal Model Parameters of the Escalation Models When The Service Rate of the First Server is a Decision

parameters	q_1^*	q_2^*	$\Pi_4^*(q_1^*, q_2^*)$
$R = 5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 1$	0.99	0.04	4.94
$R = 5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 3$	0.99	0.01	2.98
$R = 5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 1$	0.99	0.04	6.07
$R = 5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 3$	0.99	0.01	4.11
$R = 5, C = 0.25, \lambda = 1, \mu = 1.75, a = 1$	0.99	0.02	3.64
$R = 5, C = 0.25, \lambda = 1, \mu = 1.75, a = 3$	0.59	0.34	1.86
$R = 5, C = 0.25, \lambda = 1, \mu = 2.5, a = 1$	0.99	0.02	3.80
$R = 5, C = 0.25, \lambda = 1, \mu = 2.5, a = 3$	0.54	0.38	2.04
$R = 5, C = 1, \lambda = 1.5, \mu = 1.75, a = 1$	0.99	0.04	0.44
$R = 5, C = 1, \lambda = 1.5, \mu = 1.75, a = 3$	0	0	0
$R = 5, C = 1, \lambda = 1.5, \mu = 2.5, a = 1$	0.99	0.04	4.94
$R = 5, C = 1, \lambda = 1.5, \mu = 2.5, a = 3$	0.99	0.01	2.98
$R = 5, C = 1, \lambda = 1, \mu = 1.75, a = 1$	0.99	0.02	2.63
$R = 5, C = 1, \lambda = 1, \mu = 1.75, a = 3$	0.83	0.14	0.7
$R = 5, C = 1, \lambda = 1, \mu = 2.5, a = 1$	0.99	0.02	3.30
$R = 5, C = 1, \lambda = 1, \mu = 2.5, a = 3$	0.73	0.23	1.42
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 1$	0.99	0.06	8.65
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 1.75, a = 3$	0.99	0.02	6.69
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 1$	0.99	0.06	9.78
$R = 7.5, C = 0.25, \lambda = 1.5, \mu = 2.5, a = 3$	0.99	0.02	7.82
$R = 7.5, C = 0.25, \lambda = 1, \mu = 1.75, a = 1$	0.99	0.04	6.11
$R = 7.5, C = 0.25, \lambda = 1, \mu = 1.75, a = 3$	0.99	0.01	4.15
$R = 7.5, C = 0.25, \lambda = 1, \mu = 2.5, a = 1$	0.99	0.04	6.28
$R = 7.5, C = 0.25, \lambda = 1, \mu = 2.5, a = 3$	0.99	0.01	4.32
$R = 7.5, C = 1, \lambda = 1.5, \mu = 1.75, a = 1$	0.99	0.26	4.15
$R = 7.5, C = 1, \lambda = 1.5, \mu = 1.75, a = 3$	0.99	0.02	2.19
$R = 7.5, C = 1, \lambda = 1.5, \mu = 2.5, a = 1$	0.99	0.06	8.66
$R = 7.5, C = 1, \lambda = 1.5, \mu = 2.5, a = 3$	0.99	0.02	6.69
$R = 7.5, C = 1, \lambda = 1, \mu = 1.75, a = 1$	0.99	0.04	5.11
$R = 7.5, C = 1, \lambda = 1, \mu = 1.75, a = 3$	0.99	0.01	3.15
$R = 7.5, C = 1, \lambda = 1, \mu = 2.5, a = 1$	0.99	0.04	5.78
$R = 7.5, C = 1, \lambda = 1, \mu = 2.5, a = 3$	0.99	0.01	3.81

Table B.11: Optimal Model Parameters of the Escalation Models When The Quality Levels of the Servers are Different

Model Parameters	μ	$q^*_{2,\tilde{\mu}}$	$\Pi\left(\mu, q^*_{2,\tilde{\mu}}\right)$	μ^*_2	q^*_2	$\Pi\left(\mu^*_2, q^*_2\right)$
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 0.5$	10	0.72	88.84	12	0.63	89.07
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 2$	10	0.72	73.84	6.75	0.97	76.21
$R = 20, C = 1, \lambda = 5, a = 15, b = 0.5$	10	0.66	85.34	14.75	0.49	86.78
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 0.5$	10	1	158.5	17.58	0.76	174.42
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 0.5$	10	0.92	82.7	15.25	0.72	84.31
$R = 20, C = 1, \lambda = 5, a = 15, b = 2$	10	0.66	70.34	8.5	0.75	70.93
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 2$	10	1	136	12.83	0.91	147.55
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 2$	10	1	60	14.73	0.98	136.51
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 2$	10	1	143.5	11.88	1	154.75
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 0.5$	10	1	151	21.38	0.6	171.06
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 2$	10	0.92	67.7	8.5	1	68.36
$R = 20, C = 5, \lambda = 5, a = 15, b = 0.5$	10	0.82	77.10	18.5	0.56	81.38
$R = 20, C = 5, \lambda = 5, a = 15, b = 2$	10	0.82	62.10	10	0.82	62.10
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 0.5$	10	1	75	26.6	0.66	164.27
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 0.5$	10	1	82.5	21.85	0.85	168.42
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 2$	10	1	67.5	14.25	1	144
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 0.5$	15	0.54	88.70	12	0.63	89.07
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 2$	15	0.54	66.20	6.75	0.97	76.21
$R = 20, C = 1, \lambda = 5, a = 15, b = 0.5$	15	0.48	86.77	14.75	0.49	86.78
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 0.5$	15	0.86	174.16	17.58	0.76	174.42
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 0.5$	15	0.72	84.30	15.25	0.72	84.31
$R = 20, C = 1, \lambda = 5, a = 15, b = 2$	15	0.48	64.27	8.5	0.75	70.93
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 2$	15	0.8	146.6	12.83	0.91	147.55
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 2$	15	0.96	136.48	14.73	0.98	136.51
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 2$	15	0.86	151.66	11.88	1	154.75
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 0.5$	15	0.8	169.1	21.38	0.6	171.06
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 2$	15	0.72	61.8	8.5	1	68.36
$R = 20, C = 5, \lambda = 5, a = 15, b = 0.5$	15	0.63	80.93	18.5	0.56	81.38
$R = 20, C = 5, \lambda = 5, a = 15, b = 2$	15	0.63	58.43	10	0.82	62.10
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 0.5$	15	0.96	158.98	26.6	0.66	164.27
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 0.5$	15	1	166.36	21.85	0.85	168.42
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 2$	15	1	143.86	14.25	1	144

Table B.12: Model With Resolution- Optimal Model Parameters: Fixed Service Rate, Service Rate Is A Decision

Model Parameters	μ	$q^*_{3,\tilde{\mu}}$	$\Pi\left(\mu, q^*_{3,\tilde{\mu}}\right)$	μ^*_3	q^*_3	$\Pi\left(\mu^*_3, q^*_3\right)$
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 0.5$	10	0.77	88.15	11.25	0.72	88.25
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 2$	10	0.77	73.15	6.5	1	76.17
$R = 20, C = 1, \lambda = 5, a = 15, b = 0.5$	10	0.71	84.08	14.25	0.57	85.19
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 0.5$	10	1	158.5	16.63	0.84	173.86
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 0.5$	10	1	82.5	14	0.86	83.32
$R = 20, C = 1, \lambda = 5, a = 15, b = 2$	10	0.71	69.09	8	0.82	70.01
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 2$	10	1	136	11.88	0.98	147.31
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 2$	10	1	60	14.25	1	136.51
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 2$	10	1	143.5	11.88	1	154.75
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 0.5$	10	1	151	20.43	0.68	169.67
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 2$	10	1	67.5	8.5	1	68.36
$R = 20, C = 5, \lambda = 5, a = 15, b = 0.5$	10	0.89	75.92	17.75	0.68	78.99
$R = 20, C = 5, \lambda = 5, a = 15, b = 2$	10	0.89	60.92	9.25	0.93	61.06
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 0.5$	10	1	75	24.7	0.78	162.29
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 0.5$	10	1	82.5	19.48	0.99	168
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 2$	10	1	67.5	14.25	1	144
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 0.5$	15	0.62	87.74	11.25	0.72	88.25
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 2$	15	0.62	65.24	6.5	1	76.17
$R = 20, C = 1, \lambda = 5, a = 15, b = 0.5$	15	0.56	85.17	14.25	0.57	85.19
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 0.5$	15	0.89	173.79	16.63	0.84	173.86
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 0.5$	15	0.84	83.29	14	0.86	83.32
$R = 20, C = 1, \lambda = 5, a = 15, b = 2$	15	0.56	62.67	8	0.82	70.01
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 2$	15	0.83	145.79	11.88	0.98	147.31
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 2$	15	1	136.36	14.25	1	136.51
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 2$	15	0.89	151.29	11.88	1	154.75
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 0.5$	15	0.83	168.29	20.43	0.68	169.67
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 2$	15	0.84	60.79	8.5	1	68.36
$R = 20, C = 5, \lambda = 5, a = 15, b = 0.5$	15	0.73	78.75	17.75	0.68	78.99
$R = 20, C = 5, \lambda = 5, a = 15, b = 2$	15	0.73	56.25	9.25	0.93	61.06
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 0.5$	15	1	158.86	24.7	0.78	162.29
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 0.5$	15	1	166.36	19.48	0.99	168
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 2$	15	1	143.86	14.25	1	144

Table B.13: Model With Returns- Optimal Model Parameters: Fixed Service Rate, Service Rate Is A Decision

Model Parameters	μ	$q^*_{1-\nu,\tilde{\mu}}$	$\Pi\left(\mu, q^*_{1-\nu,\tilde{\mu}}\right)$	$\mu^*_{1-\nu}$	$q^*_{1-\nu}$	$\Pi\left(\mu^*_{1-\nu}, q^*_{1-\nu}\right)$
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 0.5$	6	1	77.98	5.06	1	78.63
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 2$	6	1	59.99	5.01	1	63.66
$R = 20, C = 1, \lambda = 5, a = 15, b = 0.5$	6	1	62.99	5.06	1	63.63
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 0.5$	6	0	0	9.7	1	164.01
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 0.5$	6	1	73.96	6.28	1	73.99
$R = 20, C = 1, \lambda = 5, a = 15, b = 2$	6	1	44.99	5.00	1	48.66
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 2$	6	0	0	9.58	1	120.38
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 2$	6	0	0	9.77	1	114.54
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 2$	6	0	0	9.58	1	135.38
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 0.5$	6	0	0	9.7	1	149.01
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 2$	6	1	55.96	5.12	1	58.11
$R = 20, C = 5, \lambda = 5, a = 15, b = 0.5$	6	1	58.96	6.28	1	58.99
$R = 20, C = 5, \lambda = 5, a = 15, b = 2$	6	1	40.96	5.12	1	43.11
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 0.5$	6	0	0	10.01	1	143.87
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 0.5$	6	0	0	10.01	1	158.87
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 2$	6	0	0	9.77	1	129.54
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 0.5$	10	1	72.33	5.06	1	78.63
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 2$	10	1	42.33	5.01	1	63.66
$R = 20, C = 1, \lambda = 5, a = 15, b = 0.5$	10	1	57.33	5.06	1	63.63
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 0.5$	10	0	0	9.7	1	164.01
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 0.5$	10	1	72.33	6.28	1	73.99
$R = 20, C = 1, \lambda = 5, a = 15, b = 2$	10	1	29.47	5.00	1	48.66
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 2$	10	0	0	9.58	1	120.38
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 2$	10	0	0	9.77	1	114.54
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 2$	10	0	0	9.58	1	135.38
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 0.5$	10	0	0	9.7	1	149.01
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 2$	10	1	42.33	5.12	1	58.11
$R = 20, C = 5, \lambda = 5, a = 15, b = 0.5$	10	1	57.33	6.28	1	58.99
$R = 20, C = 5, \lambda = 5, a = 15, b = 2$	10	1	27.33	5.12	1	43.11
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 0.5$	10	0	0	10.01	1	143.87
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 0.5$	10	0	0	10.01	1	158.87
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 2$	10	0	0	9.77	1	129.54

Table B.14: Two Parallel Stage Benchmark Model- Optimal Model Parameters: Fixed Service Rate, Service Rate Is A Decision

Model Parameters	μ	$q^*_{2-\nu,\tilde{\mu}}$	$\Pi\left(\mu, q^*_{2-\nu,\tilde{\mu}}\right)$	$\mu^*_{2-\nu}$	$q^*_{2-\nu}$	$\Pi\left(\mu^*_{2-\nu}, q^*_{2-\nu}\right)$
$R=20, C=1, \lambda=5, a=7.5, b=0.5$	6	0.84	81.85	8.16	0.62	83.96
$R=20, C=1, \lambda=5, a=7.5, b=2$	6	0.84	62.85	5.08	0.92	63.78
$R=20, C=1, \lambda=5, a=15, b=0.5$	6	0.84	71.26	8.16	0.62	78.19
$R=20, C=1, \lambda=9.5, a=7.5, b=0.5$	6	0	0	13.59	0.7	167.17
$R=20, C=5, \lambda=5, a=7.5, b=0.5$	6	0.84	75.58	8.93	0.72	76.93
$R=20, C=1, \lambda=5, a=15, b=2$	6	0.84	53.26	6.58	0.76	54.59
$R=20, C=1, \lambda=9.5, a=15, b=2$	6	0	0	11.68	0.82	121.51
$R=20, C=5, \lambda=9.5, a=15, b=2$	6	0	0	10.35	0.92	115.99
$R=20, C=1, \lambda=9.5, a=7.5, b=2$	6	0	0	9.58	1	135.38
$R=20, C=1, \lambda=9.5, a=15, b=0.5$	6	0	0	13.58	0.7	159.82
$R=20, C=5, \lambda=5, a=7.5, b=2$	6	0.84	57.58	5.12	0.99	58.35
$R=20, C=5, \lambda=5, a=15, b=0.5$	6	0.84	64.99	11.35	0.56	70.94
$R=20, C=5, \lambda=5, a=15, b=2$	6	0.84	46.99	6.65	0.76	47.44
$R=20, C=5, \lambda=9.5, a=15, b=0.5$	6	0	0	15.39	0.64	152.5
$R=20, C=5, \lambda=9.5, a=7.5, b=0.5$	6	0	0	11.93	0.82	160.24
$R=20, C=5, \lambda=9.5, a=7.5, b=2$	6	0	0	9.77	0.98	129.78
$R=20, C=1, \lambda=5, a=7.5, b=0.5$	10	0.52	83.54	8.16	0.62	83.96
$R=20, C=1, \lambda=5, a=7.5, b=2$	10	0.52	53.54	5.08	0.92	63.78
$R=20, C=1, \lambda=5, a=15, b=0.5$	10	0.52	77.48	8.16	0.62	78.19
$R=20, C=1, \lambda=9.5, a=7.5, b=0.5$	10	0	0	13.59	0.7	167.17
$R=20, C=5, \lambda=5, a=7.5, b=0.5$	10	0.68	76.81	8.93	0.72	76.93
$R=20, C=1, \lambda=5, a=15, b=2$	10	0.52	49.48	6.58	0.76	54.59
$R=20, C=1, \lambda=9.5, a=15, b=2$	10	0	0	11.68	0.82	121.51
$R=20, C=5, \lambda=9.5, a=15, b=2$	10	0	0	10.35	0.92	115.99
$R=20, C=1, \lambda=9.5, a=7.5, b=2$	10	0	0	9.58	1	135.38
$R=20, C=1, \lambda=9.5, a=15, b=0.5$	10	0	0	13.58	0.7	159.82
$R=20, C=5, \lambda=5, a=7.5, b=2$	10	0.52	53.54	5.12	0.99	58.35
$R=20, C=5, \lambda=5, a=15, b=0.5$	10	0.52	69.48	11.35	0.56	70.94
$R=20, C=5, \lambda=5, a=15, b=2$	10	0.52	46.48	6.65	0.76	47.44
$R=20, C=5, \lambda=9.5, a=15, b=0.5$	10	0	0	15.39	0.64	152.5
$R=20, C=5, \lambda=9.5, a=7.5, b=0.5$	10	0	0	11.93	0.82	160.24
$R=20, C=5, \lambda=9.5, a=7.5, b=2$	10	0	0	9.77	0.98	129.78

Table B.15: Two Parallel Stage Model With Resolution- Optimal Model Parameters: Fixed Service Rate, Service Rate Is A Decision

Model Parameters	μ	$q^*_{4-n,\tilde{\mu}}$	$\Pi\left(\mu, q^*_{4-n,\tilde{\mu}}\right)$	μ^*_{4-n}	q^*_{4-n}	$\Pi\left(\mu^*_{4-n}, q^*_{4-n}\right)$
$R=20, C=1, \lambda=5, a=7.5, b=0.5$	6	0.87	75.84	7.25	0.87	77.39
$R=20, C=1, \lambda=5, a=7.5, b=2$	6	0.87	57.84	6.25	0.87	57.84
$R=20, C=1, \lambda=5, a=15, b=0.5$	6	0.77	65.69	7.25	0.77	67.26
$R=20, C=1, \lambda=9.5, a=7.5, b=0.5$	6	0	0	12.83	0.93	160.36
$R=20, C=5, \lambda=5, a=7.5, b=0.5$	6	0.89	55.40	10	0.88	71.62
$R=20, C=1, \lambda=5, a=15, b=2$	6	0.77	47.69	6.25	0.77	47.70
$R=20, C=1, \lambda=9.5, a=15, b=2$	6	0	0	10.93	0.87	113.59
$R=20, C=5, \lambda=9.5, a=15, b=2$	6	0	0	12.83	0.87	97.96
$R=20, C=1, \lambda=9.5, a=7.5, b=2$	6	0	0	10.93	0.93	125.66
$R=20, C=1, \lambda=9.5, a=15, b=0.5$	6	0	0	12.83	0.87	148.29
$R=20, C=5, \lambda=5, a=7.5, b=2$	6	0.89	37.40	7.5	0.89	46.51
$R=20, C=5, \lambda=5, a=15, b=0.5$	6	0.79	44.81	10.25	0.78	61.30
$R=20, C=5, \lambda=5, a=15, b=2$	6	0.79	26.81	7.5	0.79	36.05
$R=20, C=5, \lambda=9.5, a=15, b=0.5$	6	0	0	16.63	0.87	140.39
$R=20, C=5, \lambda=9.5, a=7.5, b=0.5$	6	0	0	16.63	0.93	152.6
$R=20, C=1, \lambda=5, a=7.5, b=0.5$	6	0	0	12.83	0.94	110.24
$R=20, C=1, \lambda=5, a=7.5, b=0.5$	10	0.87	75.89	7.25	0.87	77.39
$R=20, C=1, \lambda=5, a=7.5, b=2$	10	0.87	45.89	6.25	0.87	57.84
$R=20, C=1, \lambda=5, a=15, b=0.5$	10	0.77	65.80	7.25	0.77	67.26
$R=20, C=1, \lambda=9.5, a=7.5, b=0.5$	10	0	0	12.83	0.93	160.36
$R=20, C=5, \lambda=5, a=7.5, b=0.5$	10	0.88	71.62	10	0.88	71.62
$R=20, C=1, \lambda=5, a=15, b=2$	10	0.77	35.79	6.25	0.77	47.70
$R=20, C=1, \lambda=9.5, a=15, b=2$	10	0	0	10.93	0.87	113.59
$R=20, C=5, \lambda=9.5, a=15, b=2$	10	0	0	12.83	0.87	97.96
$R=20, C=1, \lambda=9.5, a=7.5, b=2$	10	0	0	10.93	0.93	125.66
$R=20, C=1, \lambda=9.5, a=15, b=0.5$	10	0	0	12.83	0.87	148.29
$R=20, C=5, \lambda=5, a=7.5, b=2$	10	0.88	41.62	7.5	0.89	46.51
$R=20, C=5, \lambda=5, a=15, b=0.5$	10	0.78	61.29	10.25	0.78	61.30
$R=20, C=5, \lambda=5, a=15, b=2$	10	0.78	31.29	7.5	0.79	36.05
$R=20, C=5, \lambda=9.5, a=15, b=0.5$	10	0	0	16.63	0.87	140.39
$R=20, C=5, \lambda=9.5, a=7.5, b=0.5$	10	0	0	16.63	0.93	152.6
$R=20, C=1, \lambda=5, a=7.5, b=0.5$	10	0	0	12.83	0.94	110.24

Table B.16: Simple Escalation Model- Optimal Model Parameters: Fixed Service Rate, Service Rate Is A Decision

Model Parameters	μ	$q^*_{4-g,\tilde{\mu}}$	$\Pi\left(\mu, q^*_{4-g,\tilde{\mu}}\right)$	μ^*_{4-g}	q^*_{4-g}	$\Pi\left(\mu^*_{4-g}, q^*_{4-g}\right)$
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 0.5$	6	0.31	79.43	7.75	0.19	81.57
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 2$	6	0.31	61.43	6.25	0.29	61.55
$R = 20, C = 1, \lambda = 5, a = 15, b = 0.5$	6	0.22	71.42	7.75	0.12	73.90
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 0.5$	6	0	0	12.83	0.25	165.10
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 0.5$	6	0.61	56.30	11	0.32	74.33
$R = 20, C = 1, \lambda = 5, a = 15, b = 2$	6	0.22	53.42	6.25	0.2	53.62
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 2$	6	0	0	10.93	0.25	121.82
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 2$	6	0	0	13.3	0.38	103.16
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 2$	6	0	0	10.93	0.33	129.62
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 0.5$	6	0	0	13.3	0.15	157.32
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 2$	6	0.61	38.30	7.75	0.48	48.16
$R = 20, C = 5, \lambda = 5, a = 15, b = 0.5$	6	0.46	46.74	11.5	0.18	66.40
$R = 20, C = 5, \lambda = 5, a = 15, b = 2$	6	0.46	28.74	7.75	0.33	39.47
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 0.5$	6	0	0	18.05	0.24	147.20
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 0.5$	6	0	0	17.1	0.41	155.45
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 2$	6	0	0	13.3	0.53	112.17
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 0.5$	10	0.11	80.61	7.75	0.19	81.57
$R = 20, C = 1, \lambda = 5, a = 7.5, b = 2$	10	0.11	50.61	6.25	0.29	61.55
$R = 20, C = 1, \lambda = 5, a = 15, b = 0.5$	10	0.06	73.06	7.75	0.12	73.90
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 0.5$	10	0	0	12.83	0.25	165.10
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 0.5$	10	0.36	74.18	11	0.32	74.33
$R = 20, C = 1, \lambda = 5, a = 15, b = 2$	10	0.06	43.06	6.25	0.2	53.62
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 2$	10	0	0	10.93	0.25	121.82
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 2$	10	0	0	13.3	0.38	103.16
$R = 20, C = 1, \lambda = 9.5, a = 7.5, b = 2$	10	0	0	10.93	0.33	129.62
$R = 20, C = 1, \lambda = 9.5, a = 15, b = 0.5$	10	0	0	13.3	0.15	157.32
$R = 20, C = 5, \lambda = 5, a = 7.5, b = 2$	10	0.36	44.18	7.75	0.48	48.16
$R = 20, C = 5, \lambda = 5, a = 15, b = 0.5$	10	0.22	66.08	11.5	0.18	66.40
$R = 20, C = 5, \lambda = 5, a = 15, b = 2$	10	0.22	36.08	7.75	0.33	39.47
$R = 20, C = 5, \lambda = 9.5, a = 15, b = 0.5$	10	0	0	18.05	0.24	147.20
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 0.5$	10	0	0	17.1	0.41	155.45
$R = 20, C = 5, \lambda = 9.5, a = 7.5, b = 2$	10	0	0	13.3	0.53	112.17

Table B.17: Perfect Escalation Model- Optimal Model Parameters: Fixed Service Rate, Service Rate Is A Decision