Assortative Matching Under Information Asymmetry with Search Frictions

by

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Abstract

We analyze a two-sided decentralized search model with transferable utility where heterogeneous sellers and buyers engage in costly search due to time discounting under one-sided information asymmetry. We assume exogenous type distribution which remains unaffected by the consummated matches in the market throughout time. We prove that in such an economy where two types of uninformed sellers and informed buyers are present, assortative matching could be achieved as equilibrium. We show that supermodularity and log-supermodularity of seller-, buyer-specific output functions and their sum along with a higher spread rate for sellers' output function compared to buyers' is necessary and sufficient for assortative matching under any search friction and type distribution. We also find that one-sided information asymmetry does not induce less restrictive asuumptions for assortative matching compared to complete information setting where type heterogeneity is limited to two.
Keywords: assortative matching, information asymmetry, search friction, assignment, log-supermodularity

Özet

Heterojen satıcı ve alıcıların zaman indirimi nedeniyle masraflı arayış içinde olduğu fayda aktarımına izin veren çift-taraflı bir arama modelini tek-taraflı bilgi asimetrisi altında analiz ediyoruz. Zaman boyunca pazarda gerçekleşen eşleşmelerden etkilenmeden kalan dışadayalı tip dağılımı farz ediyoruz. İki tipli, bilgisi olmayan satıcı ve bilgisi olan alıcılardan oluşan böyle bir ekonomide sıralayıcı eşleşmenin denge olarak elde edilebileceğini kanıtlıyoruz. Gösteriyoruz ki satıcı ve alıcıya özel üretim fonksiyonlarının ve bunların toplamının süpermodüler ve log-süpermodülerliğiyle birlikte satıcının üretim fonksiyonun dağılım oranının alıcıya göre daha yüksek olması herhangi bir arama friksiyonu ve tip dağılımı altında sıralayıcı eşleşme için gerekli ve yeterli. Ayrıca tip heterojenliğinin ikiyle sınırlı olduğu durumda tek taraflı bilgi asimetrisinin sıralayıcı eşleşme için tam bilgi ortamına göre daha az sınırlayıcı varsayımlar ortaya çıkarmadığını buluyoruz.
Anahtar Sözcükler: sıralayıcı eşleşme, bilgi asimetrisi, arama friksiyonu, tahsis, log-süpermodülerlik

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Contents

to my aunt Nagihan

1 Introduction

The decision of two impatient agents whether to match is central to many economic situations. The benefit is that when they match, they start to produce and get a high payoff which is not obtainable when unmatched. The flipside is that matching precludes further search and hence finding more suitable partners. The agents then have to weigh the opportunity cost of an immediate match against a delayed but possibly a more profitable match. Examples include the decision of a man and a woman to marry, the decision of a firm and a worker to enter into an employment relationship; or a home owner and a tenant to sign a rental agreement. Therefore, economists have long been interested in markets consisting of heterogeneous agents who differ in various dimensions searching for suitable trading partners, and the matching patterns of those agents. Specifically, what leads to a sorting pattern was a question of maximal importance as it induces an already decentralized efficient allocation without further mechanism design.

Becker (1973) is first to analyze how competition in a marriage market sorts males and females with respect to certain attributes such as wealth and education. Becker asserted that under any core allocation complementarities in production -corresponding to supermodularity of production function- lead to positive assortative matching which is the matching of individuals with similar attributes. However, Becker's model involved no search cost, hence agents were patient enough to find the perfect match. Shimer and Smith (2000) are first to prove the existence of a search equilibrium and to find the sufficient conditions for assortative matching under any search cost and any type distribution. Their finding is that supermodularity of production function does not suffice for assortative matching in a search economy with discounting but its log first- and cross-partial derivatives are also required.

In this paper, we extend Shimer and Smith(2000)'s model by incorporating one-sided information asymmetry. In our model, sellers and buyers search over the infinite horizon for match partners to maximize their total expected payoff. Each period sellers haphazardly match with buyers, then each seller makes an offer to the buyer whom she is matched with. Although the distribution of types are known by both parties, sellers do not observe the type of buyer she is matched with whereas buyers do observe types of the assigned sellers. Agents' utilities are characterized by premuneration values meaning that their value from the match is legally separated, that they are entitled to a certain predetermined output for each contingency. We first show that when sellers are entitled to zero output from any match, log-supermodularity of buyers' premuneration value suffices for assortative matching along with the supermodularity assumption. We then turn to the general case where both parties are entitled to non-zero output and find the sufficient conditions for assortative matching. In the following section, we discuss frictionless matching and find out that Becker's supermodularity condition suffices for assortative matching under one-sided information asymmetry. We finally compare our assumptions with those of complete information setting and observe that log-supermodularity assumption suffices for assortative matching when only two types of sellers and buyers are present.

2 Model

Consider a discrete time, infinite horizon search model where there are uninformed sellers and informed buyers trying to match with potential trading partners and matching is time-consuming. They incur an implicit search cost through a common time discount factor $\delta \in (0, 1)$ at each period. Types are exogeneously given. Assume the economy is at steady state and solely consists of unmatched agents. Let $z = [z_B, z_S]$ represent the steady state probability density functions of buyers and sellers. Each period sellers haphazardly match with buyers, then each seller makes an offer to the buyer whom she is matched with. Although the distribution of types are known by both parties, sellers do not observe the type of buyer she is matched with whereas buyers do observe types of the assigned sellers. A match is formed only if buyer accepts the given offer and matched agents leave the market. However, we assume that the distribution of unmatched agents is not affected by the buyers' acceptance sets, new substitutes for the matched agents arrive in the market at each period and z remains unchanged throughout time.

Preferences: Each agent maximizes her expected present value of payoffs. Agents' utilities are characterized by premuneration values meaning that their value from the match is legally separated, that they are entitled to a certain predetermined output for each contingency. Note that use of premuneration values rather than a common output function as in Shimer and Smith(2000) and Atakan(2006) is a necessary move when asymmetric information is present as otherwise uninformed sellers would not know their exact value for different contingencies.

Let $f : \Theta_S \times \Theta_B \to \mathbb{R}$ and $g : \Theta_S \times \Theta_B \to \mathbb{R}$ where Θ_S and Θ_B are type sets, denote the premuneration values of sellers and buyers, respectively.

We assume that the utility of each party from a consummated match is $u_S = f(s, b) + t(s)$ and $u_B = g(s, b) - t(s)$ where $t(s)$ is the transfer payment demanded by the type s seller.

Strategies: We assume that sellers and buyers use time-invariant strategies. A strategy for seller s is a transfer payment $t_s \in \mathbb{R}^+$ while a strategy for buyer b is a measurable set $A(b) \subseteq \Theta_S$ that represents the type of sellers b is willing to accept, we assume buyers accept the offers at the indifference points. Although what buyers accept is the offer $t(s)$ made by a certain type of seller rather than the types of sellers themselves, the fact that buyers observe the types of sellers and that the buyers are aware of this fact prevent any mimicking by the sellers and for every type of seller there is a distinct corresponding offer that is perfectly representative of her type. Hence we take interest in types of sellers as buyers' strategies for a more convenient analysis of assortative matching. Per-period-payoff of a buyer b using the stationary strategy $A(b)$, when matched with a seller s using the stationary strategy $t(s)$, is

$$\pi_B(s,b,t_s,A(b)) = \begin{cases} g(s,b) - t_s & if \;\; s \in A(b) \\ 0 & if \;\; s \notin A(b) \end{cases}$$

Similarly, per-period-payoff of a seller s using the stationary strategy $t(s)$, when matched with a buyer b using the stationary strategy $A(b)$, is

$$\pi_S(s,b,t_s,A(b)) = \begin{cases} f(s,b) + t_s & if \;\; s \in A(b) \\ 0 & if \;\; s \notin A(b) \end{cases}$$

Given that sellers use the strategies $\{t(s)\}_{s\in\Theta_S}$, each buyer solves the optimization problem

$$v_B(b) = \max_{\hat{A}} \left[E_{z_S} \sum_{i=0}^{\infty} \delta^i \pi_B(s_i, b, t(s_i), \hat{A}) \right] \tag{1}$$

while each seller, given that buyers use the strategies $\{A(b)\}_{b\in\Theta_B}$, solves

$$v_S(s) = \max_{\hat{t_s}} \left[E_{z_B} \sum_{i=0}^{\infty} \delta^i \pi_S(s, b_i, \hat{t_s}, A(b_i)) \right] \tag{2}$$

Note that b_i and s_i are random draws from the steady state type distribution $z = [z_B, z_S]$ if there has been a rejection in the preceding period $i-1$ and $\pi_B = \pi_S = 0$ for all the periods following an acceptance.

It naturally follows that a match is consummated if and only if it generates non-negative surplus for the buyer. Therefore, matching set $M(b)$ for a type b buyer is exactly her acceptance set $A(b)$ as any match with a seller she accepts is already such a match, hence

$$M(b) = A(b) = \{s : g(s,b) - t(s) \geq v_B(b)\} \;\; \forall b \in \Theta_B$$

Symmetrically, matching set $M(s)$ for a type s seller consists of buyers that accept seller s,

$$M(s) = \{b : s \in A(b)\} = \{b : g(s,b) - t(s) \geq v_B(b)\}$$

Buyer's value function is her expected gain from future matches

$$v_B(b) = \delta\Big[E_{z_S} max\{g(s,b) - t(s), v_B(b)\}\Big] \tag{3}$$

As we are in search of a stationary strategy in an environment with exogenous probabilities waiting for another period is never optimal for the seller. Hence seller's value function is just the discounted value of her optimized present payoff [seller solves the monopolist's problem]

$$v_S(s) = \delta \max_{t_s} \Big[\int_{M(s)} \big(f(s,b) + t(s) - v_S(s)\big) dZ_B(b) + v_S(s)\Big] \tag{4}$$

Search Equilibrium: A search equilibrium $\mathbb{E} = [v_B, v_S, A(\cdot), t(\cdot)]$ composed of functions $v_B : \Theta_B \to \mathbb{R}$ and $v_S : \Theta_S \to \mathbb{R}$, the maximized values for each buyer b and seller s of participating in the economy; and strategies $A(b)$, $t(s)$ for each b and s where $A(b)$ is an optimizer for $v_B(b)$ and $t(s)$ for $v_S(s)$ for each $b \in \Theta_B$, $s \in \Theta_S$.

Assortative Matching

Definition: Let Θ_B, Θ_S be ordered sets. There is assortative matching if the matching sets form a lattice in $\Theta_B \times \Theta_S$ i.e. for any $(b,s), (b',s') \in \bigcup_{b \in \Theta_B} \{b\} \times M(b) \subseteq \Theta_B \times \Theta_S$, we have

$$\big(min\{b,b'\}, min\{s,s'\}\big), \big(max\{b,b'\}, max\{s,s'\}\big) \in \bigcup_{b \in \Theta_B} \{b\} \times M(b)$$

Our definition is simply an adaptation of Shimer and Smith's lattice definition for assortative matching which is a generalization of Becker's definition. In a frictional environment, individuals are willing to accept sets of agents; mismatch is acceptable unlike Becker's definition. However, the mismatch should describe a preference for matching with similar types. In our definition, if a high type seller matches with a low type buyer and a low type seller matches with a high type buyer in equilibrium, then the sellers should also be willing to match with their own types. The cases that this does not hold do not fully represent a preference for the same or higher types and that is the main motivation for this definition.

The following are the assumptions that we use to prove assortative matching.

A1: *The premuneration values f and g are non-negative and increasing in s, b.*

A2: *The premuneration values are strictly supermodular i.e. if $s_1 > s_2$ and $b_1 > b_2$ where $s \in \Theta_S$ and $b \in \Theta_B$, then $f(s_1, b_1) + f(s_2, b_2) > f(s_1, b_2) + f(s_2, b_1)$; and $g(b_1, s_1) + g(b_2, s_2) > g(b_1, s_2) + g(b_2, s_1)$.*

A3: *The premuneration values are strictly log-supermodular i.e. if $s_1 > s_2$ and $b_1 > b_2$ where $s \in \Theta_S$ and $b \in \Theta_B$, then*

$$log f(s_1, b_1) + log f(s_2, b_2) > log f(s_1, b_2) + log f(s_2, b_1)$$

or alternatively

$$f(s_1, b_1) f(s_2, b_2) > f(s_1, b_2) f(s_2, b_1)$$

Corollary. If we assume f, g to be twice differentiable along with A1, then log-supermodularity condition implies supermodularity condition.

Proof. Let f be log-supermodular.

$$\frac{\partial^2 log f}{\partial s \partial b} = \frac{1}{f^2} \left(\frac{\partial^2 f}{\partial s \partial b} f - \frac{\partial f}{\partial s} \frac{\partial f}{\partial b} \right) \geq 0$$

As f and its first partial derivatives are non-negative, $\frac{\partial^2 f}{\partial s \partial b}$ is non-negative. □

3 Analysis for Two Types

3.1 Zero Value for Sellers

We proceed by investigating the simplest case where there are only two types of sellers and buyers, namely high and low for each, and the premuneration values of the sellers are taken to be zero, that is

$$\Theta_S = \{H, L\} \quad and \quad \Theta_B = \{h, l\}, \quad f(b,s) = 0 \ \forall (b,s) \in \Theta_B \times \Theta_S \tag{5}$$

Let z be a density function such that

$$z_B(H) = p \quad z_S(h) = q \quad where \quad p, q \in (0,1)$$

Theorem 1: *If g satisfies A1 to A3, then matching is assortative for any z and δ.*

Proof. We prove the theorem case by case

Case 1: $A(h) = A(l) = \{t_H, t_L\}$

Continuation values of buyers:

$$v(h) = \delta[p(g_{Hh} - t_H) + (1-p)(g_{Lh} - t_L)]$$
$$v(l) = \delta[p(g_{Hl} - t_H) + (1-p)(g_{Ll} - t_L)]$$

As we are in search for a competitive equilibrium in which both types of buyers accept both types of sellers, the following must hold:

$$g_{Hh} - t_H \geq v(h), \ \ g_{Lh} - t_L \geq v(h)$$

$$g_{Hl} - t_H \geq v(l), \ \ g_{Ll} - t_L \geq v(l)$$

Assume that sellers make their offers in order to be accepted by both types of buyers. Observe that under supermodularity assumption any utility maximizing offer made by high type seller t_H that is accepted by low type buyer is also accepted by high type buyer as there is no better match for high type buyer other than high type seller.

Claim: For any $t_H \leq g_{Hl}$ and $t_L \leq g_{Ll}$, $g_{Hl} - v(l) \leq g_{Hh} - v(h)$.

Proof. Suppose not true. Then

$$v(h) - v(l) = \delta[p(g_{Hh} - g_{Hl}) + (1-p)(g_{Lh} - g_{Ll})] > g_{Hh} - g_{Hl}$$

Hence

$$1 > \frac{\delta(1-p)}{1-\delta p} > \frac{g_{Hh} - g_{Hl}}{g_{Lh} - g_{Ll}}$$

which contradicts supermodularity of g. □

However, low type seller's any utility maximizing offer t_L that is accepted by low type buyer is not necessarily accepted by high type buyer as high type buyer h could always wait another period to possibly match with a high type seller H instead of accepting low type seller's offer t_L. Hence, we require a lower bound on high type buyer's relative gain from a match with low type seller compared to the gain from matching with a high type seller that makes an instant match more desirable i.e.

$$\frac{g_{Lh} - t_L}{g_{Hh} - t_H} \geq \frac{\delta p}{1 - \delta(1-p)}$$

Indeed, this lower bound is an increasing function of the probability of meeting a high type seller p and the level of time friction δ.

Now we solve the utility maximization problem of sellers under these assumptions. At an optimum we have $t_H = g_{Hl} - v(l)$ and $t_L = g_{Ll} - v(l)$ because $u_H = t_H \leq g_{Hl} - v(l)$ and $u_L = t_L \leq g_{Ll} - v(l)$. Substituting the offers t_H, t_L in continuation values, we have $v(l) = 0$ and $v(h) > 0$. Hence $t_H^* = g_{Hl}$ and $t_L^* = g_{Ll}$ is an equilibrium candidate.

We further need to ensure that the sellers do not deviate by increasing their offers at the expense of not matching with low type buyers. Suppose low type buyer deviates from $t_L^* = g_{Ll}$ by offering $\widetilde{t_L} = g_{Lh} - v(h)$ as this is the best deviation from the candidate equilibrium offer while high type seller offers $t_H^* = g_{Hl}$. If we revise the continuation values, we have

$$\widetilde{t_L} = g_{Lh} - \frac{\delta p}{1 - \delta(1-p)}(g_{Hh} - g_{Hl})$$

Similarly, if H deviates while $t_L^* = g_{Ll}$, her best deviation is

$$\widetilde{t_H} = g_{Hh} - \frac{\delta(1-p)}{1-\delta p}(g_{Lh} - g_{Ll})$$

Observe that no-deviation-constraint for low type seller

$$u_L(t_L^*) \geq u_L(\widetilde{t_L}) \implies g_{Ll} \geq q\widetilde{t_L} + (1-q)\delta g_{Ll} \implies q \leq \frac{(1-\delta)g_{Ll}}{\widetilde{t_L} - \delta g_{Ll}}$$

Similarly, no-deviation-constraint for high type seller

$$u_H(t_H^*) \geq u_H(\widetilde{t_H}) \implies g_{Hl} \geq q\widetilde{t_H} + (1-q)\delta g_{Hl} \implies q \leq \frac{(1-\delta)g_{Hl}}{\widetilde{t_H} - \delta g_{Hl}}$$

Set

$$\bar{q} = min\left\{\frac{(1-\delta)g_{Ll}}{\widetilde{t_L} - \delta g_{Ll}}, \frac{(1-\delta)g_{Hl}}{\widetilde{t_H} - \delta g_{Hl}}\right\}$$

Hence for any $q \leq \bar{q}$ and strictly supermodular g s.t.

$$\frac{g_{Lh} - g_{Ll}}{g_{Hh} - g_{Hl}} \geq \frac{\delta p}{1-\delta(1-p)}$$

we have an equilibrium that is trivially assortative.

Case 2: $A(h) = \{t_H\}, A(l) = \{t_H, t_L\}$

Consider a matching structure where high type seller H is matched with both type of buyers whereas low type seller L is only matched with low type of buyer.

Buyers' continuation values are

$$v(h) = \delta[p(g_{Hh} - t_H) + (1-p)v(h)] \tag{6}$$

$$v(l) = \delta[p(g_{Hl} - t_H) + (1-p)(g_{Ll} - t_L)] \tag{7}$$

Note that

$$g_{Hh} - t_H \geq v(h), \;\; g_{Lh} - t_L < v(h)$$

$$g_{Hl} - t_H \geq v(l), \;\; g_{Ll} - t_L \geq v(l)$$

Sellers' utility maximization problems yield $t_L^* = g_{Ll}$ and $t_H^* = g_{Hl}$. Observe that t_H and t_L respect each type of buyer's constraints under our construction if we assume

$$\frac{g_{Lh} - g_{Ll}}{g_{Hh} - g_{Hl}} < \frac{\delta p}{1 - \delta(1-p)}$$

Note that H might increase his offer to $\widetilde{t_H} = g_{Hh} - v(h)$ and get accepted by only high type buyer while $t_L^* = g_{Ll}$. Then no-deviation-constraint for H becomes $q \leq \frac{(1-\delta)g_{Hl}}{g_{Hh} - \delta g_{Hl}}$. Low type seller L cannot increase his utility by matching with high type buyer. Suppose $\widetilde{t_L} = g_{Lh} - v(h)$ while $t_H = t_H^*$. Then

$$\widetilde{t_L} = g_{Lh} - \frac{\delta p}{1 - \delta(1-p)}(g_{Hh} - g_{Hl})$$

. This amount is always less than t_L^* due to $\frac{g_{Lh} - g_{Ll}}{g_{Hh} - g_{Hl}} < \frac{\delta p}{1 - \delta(1-p)}$.

Hence for any $q \leq \frac{(1-\delta)g_{Hl}}{g_{Hh} - \delta g_{Hl}}$ and strictly supermodular g s.t.

$$\frac{g_{Lh} - g_{Ll}}{g_{Hh} - g_{Hl}} < \frac{\delta p}{1 - \delta(1-p)}$$

we have an assortative equilibrium in which $A(h) = \{t_H\}, A(l) = \{t_H, t_L\}$ with $t_L^* = g_{Ll}, t_H^* = g_{Hl}$.

Case 3: $A(h) = \{t_H, t_L\}, A(l) = \{t_L\}$

Consider a matching structure where high type seller H is only matched with high type buyer h whereas low type seller L matches with both types of buyers.

Buyers' continuation values are

$$v(h) = \delta[p(g_{Hh} - t_H) + (1-p)(g_{Lh} - t_L)] \quad (8)$$

$$v(l) = \delta[pv(l) + (1-p)(g_{Ll} - t_L)] \quad (9)$$

Note that

$$g_{Hh} - t_H \geq v(h), \;\; g_{Lh} - t_L \geq v(h)$$

$$g_{Hl} - t_H < v(l), \;\; g_{Ll} - t_L \geq v(l)$$

Low type seller L's utility maximization problem yields $t_L^* = g_{Ll}$ and $v(l) = 0$. Moreover, H only matches with high type buyer so H offers $t_H = g_{Hh} - v(h)$ to maximize his own utility. Substituting t_H and t_L in explicit form of $v(h)$, we have

$$t_H^* = g_{Hh} - \frac{\delta(1-p)}{1-\delta p}(g_{Lh} - g_{Ll})$$

Observe that t_H and t_L respect each type of buyer's constraints under our construction. Note that H might reduce his offer to $\widetilde{t_H} = g_{Hl} - v(l)$ and get accepted by both type of buyers as in previous case while $t_L^* = g_{Ll}$. Then no-deviation-constraint for H is

$$qt_H^* + (1-q)v(H) \geq g_{Hl} \;\; where \;\; v(H) = \frac{\delta q}{1-\delta(1-q)} t_H^* \implies q \geq \frac{(1-\delta)g_{Hl}}{t_H^* - \delta g_{Hl}}$$

Low type seller L could also increase his utility by only matching with high type buyer. Suppose $\widetilde{t_L} = g_{Lh} - v(h)$ while $t_H = t_H^*$. Then

$$\widetilde{t_L} = g_{Lh} - \frac{\delta p}{1-\delta(1-p)} \frac{\delta(1-p)}{1-\delta p}(g_{Lh} - g_{Ll})$$

No-deviation-constraint for L is

$$g_{Ll} \geq q\widetilde{t_L} + (1-q)v(L) \;\; where \;\; v(L) = \delta g_{Ll} \implies q \leq \frac{(1-\delta)g_{Ll}}{\widetilde{t_L} - \delta g_{Ll}}$$

Note that such q exists iff

$$g_{Hh}g_{Ll} - g_{Hl}g_{Lh} \geq \frac{\delta(1-p)}{1-\delta p}(g_{Lh} - g_{Ll})(g_{Ll} - \frac{\delta p}{1-\delta(1-p)} g_{Hl})$$

Hence for any such q and strictly supermodular g, we have an assortative equilibrium in which $A(h) = \{t_H, t_L\}, A(l) = \{t_L\}$ with $t_L^* = g_{Ll}, t_H^* = g_{Hh} - \frac{\delta(1-p)(g_{Lh} - g_{Ll})}{1-\delta p}$.

Case 4: $A(h) = \{t_H, t_L\}, A(l) = \{t_H\}$

Consider a matching structure where low type seller L is only matched with high type buyer h whereas high type seller H matches with both types of buyers.

Buyers' continuation values are

$$v(h) = \delta[p(g_{Hh} - t_H) + (1-p)(g_{Lh} - t_L)] \tag{10}$$

$$v(l) = \delta[p(g_{Hl} - t_H) + (1-p)v(l)] \tag{11}$$

Note that

$$g_{Hh} - t_H \geq v(h), \;\; g_{Lh} - t_L \geq v(h)$$

$$g_{Hl} - t_H \geq v(l), \;\; g_{Ll} - t_L < v(l)$$

High type seller H's utility maximization problem yields $t_H^* = g_{Hl}$ and $v(l) = 0$. Moreover, L only matches with high type buyer so L offers $t_L = g_{Lh} - v(h)$ to maximize his own utility. Substituting t_H and t_L in explicit form of $v(h)$, we have

$$t_L^* = g_{Lh} - \frac{\delta p}{1-\delta(1-p)}(g_{Hh} - g_{Hl})$$

Observe that t_H and t_L respect each type of buyer's constraints under our construction. Note that L might reduce his offer to $\widetilde{t_L} = g_{Ll} - v(l)$ and get accepted by both type of buyers as in previous case while $t_H^* = g_{Hl}$. Then no-deviation-constraint for L becomes

$$qt_L^* + (1-q)v(L) \geq g_{Ll} \;\; where \;\; v(L) = \frac{\delta q}{1-\delta(1-q)} t_L^* \implies q \geq \frac{(1-\delta)g_{Ll}}{t_L^* - \delta g_{Ll}}$$

High type seller H could increase his utility by only matching with high type buyer. Suppose $\widetilde{t_H} = g_{Hh} - v(h)$ while $t_L = t_L^*$. Then

$$\widetilde{t_H} = g_{Hh} - \frac{\delta p}{1-\delta(1-p)} \frac{\delta(1-p)}{1-\delta p}(g_{Hh} - g_{Hl})$$

No-deviation-constraint for H is

$$g_{Hl} \geq b\widetilde{t_H} + (1-b)v(H) \;\; where \;\; v(H) = \delta g_{Hl} \implies q \leq \frac{(1-\delta)g_{Hl}}{\widetilde{t_H} - \delta g_{Hl}}$$

Note that such q does not exist due to log-supermodularity assumption since

$$-g_{Hh}g_{Ll} + g_{Hl}g_{Lh} < \frac{\delta p}{1-\delta(1-p)}(g_{Hh} - g_{Hl})(g_{Hl} - \frac{\delta(1-p)}{1-\delta p} g_{Ll})$$

Case 5: $A(h) = \{t_H\}, A(l) = \{t_L\}$

Consider a matching structure where high type seller H is only matched with high type buyer h and low type seller L is matched with low type of buyer l.

Buyers' continuation values are

$$v(h) = \delta[p(g_{Hh} - t_H) + (1-p)v(h)] \tag{12}$$

$$v(l) = \delta[pv(l) + (1-p)(g_{Ll} - t_L)] \tag{13}$$

Note that

$$g_{Hh} - t_H \geq v(h), \;\; g_{Lh} - t_L < v(h)$$

$$g_{Hl} - t_H < v(l), \;\; g_{Ll} - t_L \geq v(l)$$

Sellers' utility maximization problems yield $t_L^* = g_{Ll} - v(l)$ and $t_H^* = g_{Hh} - v(l)$, hence $v(l) = v(h) = 0$. But then A1 is contradicted

$$g_{Lh} - t_L < v(h) \implies g_{Lh} - g_{Ll} < 0$$

Therefore such an equilibrium does not exist.

Case 6 : $A(h) = \{t_L\}, A(l) = \{t_H, t_L\}$

Consider a matching structure where high type seller H is matched with only low type buyers whereas low type seller L matches with both types of buyer.

Buyers' continuation values are

$$v(h) = \delta[pv(h) + (1-p)(g_{Lh} - t_L)] \tag{14}$$

$$v(l) = \delta[p(g_{Hl} - t_H) + (1-p)(g_{Ll} - t_L)] \tag{15}$$

Note that

$$g_{Hh} - t_H < v(h), \;\; g_{Lh} - t_L \geq v(h)$$

$$g_{Hl} - t_H \geq v(l), \;\; g_{Ll} - t_L \geq v(l)$$

Sellers' utility maximization problems yields $t_L^* = g_{Ll} - v(l)$ and $t_H^* = g_{Hl} - v(l)$, hence $v(l) = 0$ and $v(h) = \frac{\delta a}{1-\delta(1-a)}(g_{Lh} - g_{Ll})$. But then the supermodularity of g is contradicted

$$g_{Hh} - t_H < v(h) \implies \frac{g_{Hh} - g_{Hl}}{g_{Lh} - g_{Ll}} < \frac{\delta(1-p)}{1-\delta p} < 1$$

Therefore such an equilibrium does not exist.

Case 7 : $A(h) = \{t_L\}, A(l) = \{t_H\}$

Consider a matching structure where high type seller H is matched with only low type buyer whereas low type seller L matches with only hight type buyer.

Buyers' continuation values are

$$v(h) = \delta[pv(h) + (1-p)(g_{Lh} - t_L)] \tag{16}$$

$$v(l) = \delta[p(g_{Hl} - t_H) + (1-p)v(l)] \tag{17}$$

Note that

$$g_{Hh} - t_H < v(h), \;\; g_{Lh} - t_L \geq v(h)$$

$$g_{Hl} - t_H \geq v(l), \;\; g_{Ll} - t_L < v(l)$$

Sellers' utility maximization problems yield $t_L^* = g_{Lh} - v(l)$ and $t_H^* = g_{Hl} - v(l)$, hence $v(l) = v(h) = 0$. But then A1 is contradicted

$$g_{Hh} - t_H < v(h) \implies g_{Hh} - g_{Hl} < 0$$

Therefore such an equilibrium does not exist.

Claim: $t_L, t_H \in A(b)$ for some b in any equilibrium.

Proof. Suppose, wlog, $t_L \notin A(b)$ $\forall b$ in some equilibrium. Then $t_L > g_{Lb}$ $\forall b$ and $u_L = 0$. But then, there exists a more profitable deviation $t_L = g_{L\bar{b}}$ where $\bar{b} = max\ b$ making $u_L > 0$. □

By the claim, the only case left is
Case 8: $A(h) = \{t_H, t_L\}, A(l) = \emptyset$

Buyers' continuation values are

$$v(h) = \delta[p(g_{Hh} - t_H) + (1-p)(g_{Lh} - t_L)] \tag{18}$$

$$v(l) = 0 \tag{19}$$

Note that

$$g_{Hh} - t_H \geq v(h),\ \ g_{Lh} - t_L \geq v(h)$$

$$g_{Hl} - t_H < v(l),\ \ g_{Ll} - t_L < v(l)$$

Hence $t^*_H = g_{Hh}$ and $t^*_L = g_{Lh}$ are candidate equilibrium transfers. No-deviation-constraints are

$$q \geq \frac{(1-\delta)g_{Hl}}{g_{Hh} - \delta g_{Hl}}$$

and

$$q \geq \frac{(1-\delta)g_{Ll}}{g_{Hl} - \delta g_{Ll}}$$

Set

$$\bar{q} = max\left\{\frac{(1-\delta)g_{Hl}}{g_{Hh} - \delta g_{Hl}}, \frac{(1-\delta)g_{Ll}}{g_{Hl} - \delta g_{Ll}}\right\}$$

Hence for any $q \geq \bar{q}$ and strictly supermodular g, such equilibrium exists.

□

The strategy profile of sellers can be written as

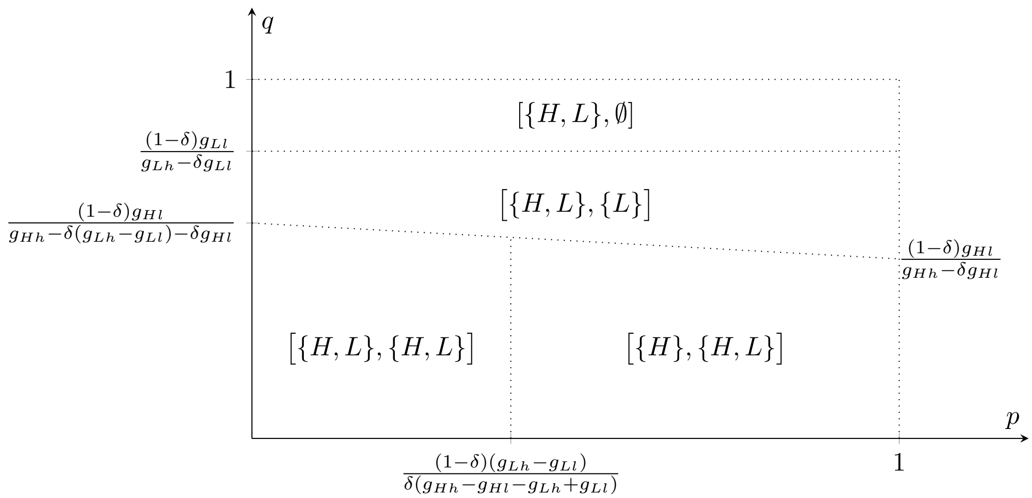
$$(t^*_H, t^*_L) = \begin{cases} (g_{Hh}, g_{Lh}) & if\ \ q \geq \frac{(1-\delta)g_{Ll}}{g_{Lh} - \delta g_{Ll}} \\ (g_{Hh} - \frac{\delta(1-p)}{1-\delta p}(g_{Lh} - g_{Ll}), g_{Ll}) \quad if\ \ \frac{(1-\delta)g_{Ll}}{g_{Lh}-\delta g_{Ll}} > q \geq & \frac{(1-\delta)g_{Hl}}{g_{Hh} - \frac{\delta(1-p)}{1-\delta p}(g_{Lh} - g_{Ll}) - \delta g_{Hl}} \\ (g_{Hl}, g_{Ll}) & otherwise \end{cases}$$

When probability of meeting a high type buyer q is sufficiently high, sellers propose as to match with only high type buyers. As $\frac{\partial}{\partial\delta}\frac{(1-\delta)g_{Ll}}{g_{Lh}-\delta g_{Ll}} < 0$, this lower bound increases along with level of friction and that requires a much higher probability q for proposing as to match with only high types. When q is below $\frac{(1-\delta)g_{Ll}}{g_{Lh}-\delta g_{Ll}}$, low type seller cannot afford to be proposing as to match with only high type buyers, so she deviates and makes an offer as to match with low type and hope to be accepted by high type buyers if possible. Low type seller deviating reduces high type seller's expected payoff through continuation values. Although high type seller could still afford to match with only high type buyers, high type buyer's continuation value has increased as matching with a low type seller is now more valuable for her. However, high type seller also deviates when $q < \frac{(1-\delta)g_{Hl}}{g_{Hh}-\frac{\delta(1-p)}{1-\delta p}(g_{Lh}-g_{Ll})-\delta g_{Hl}}$, they both propose as to match with both types of buyers. Although seller's strategy remains the same below this level of q, high type buyer's strategy depends on whether the instant gain from a match with a low type seller exceeds the value of waiting. Hence if $\frac{g_{Lh}-g_{Ll}}{g_{Hh}-g_{Hl}} \geq \frac{\delta p}{1-\delta(1-p)}$ does not hold, high type buyer deviates and only match with high type seller.

Therefore the following equilibrium matching structure arises

$$\big[A(h), A(l)\big] = \begin{cases} [\{H,L\},\emptyset] & if \;\; q \geq \frac{(1-\delta)g_{Ll}}{g_{Lh}-\delta g_{Ll}} \\ \big[\{H,L\},\{L\}\big] & if \;\; \frac{(1-\delta)g_{Ll}}{g_{Lh}-\delta g_{Ll}} > q \geq \frac{(1-\delta)g_{Hl}}{g_{Hh}-\frac{\delta(1-p)}{1-\delta p}(g_{Lh}-g_{Ll})-\delta g_{Hl}} \\ \big[\{H\},\{H,L\}\big] & if \;\; \left(\frac{(1-\delta)g_{Hl}}{g_{Hh}-\frac{\delta(1-p)}{1-\delta p}(g_{Lh}-g_{Ll})-\delta g_{Hl}} > q\right) \wedge \left(p \geq \frac{(1-\delta)(g_{Lh}-g_{Ll})}{\delta(g_{Hh}-g_{Hl}-g_{Lh}+g_{Ll})}\right) \\ \big[\{H,L\},\{H,L\}\big] & if \;\; \left(\frac{(1-\delta)g_{Hl}}{g_{Hh}-\frac{\delta(1-p)}{1-\delta p}(g_{Lh}-g_{Ll})-\delta g_{Hl}} > q\right) \wedge \left(\frac{(1-\delta)(g_{Lh}-g_{Ll})}{\delta(g_{Hh}-g_{Hl}-g_{Lh}+g_{Ll})} > p\right) \end{cases}$$

Graph 1: Equilibrium Matching Structure when $f = 0$



3.2 Non-Zero Value for Both Parties

Now we turn to the general case where $f(s,b) \neq 0 \ \forall (b,s) \in \Theta_B \times \Theta_S$.

Theorem 2: *Assume f, g satisfy A1 and A2.*
If

$$\frac{g_{Hh} + f_{Hh}}{g_{Lh} + f_{Lh}} \geq \frac{g_{Hl}}{g_{Ll}}$$

$$\frac{g_{Hh} + f_{Hh}}{g_{Lh} + f_{Lh}} \geq \frac{f_{Hl}}{f_{Ll}}$$

and

$$\frac{g_{Hh}}{g_{Lh}} \geq \frac{f_{Hh}}{f_{Lh}}$$

then matching is assortative for any z and δ.

The first condition is that high type buyer's marginal contribution to total value by matching with high type seller (instead of low type seller) is higher than low type buyer's marginal contribution to her own value by matching with high type seller (instead of low type seller). The second condition is high type buyer's marginal contribution to total value by matching with high type seller (instead of low type seller) is higher than high type seller's relative gain by matching with a low type buyer compared to a low type seller. The third condition is high type buyer's marginal contribution to her own value by matching with a high type seller is higher than high type seller's relative gain by matching with a high type buyer compared to a low type seller.

Proof. Note that the proof is only different in terms of the utility of sellers which is now $u_S = f(s,b) + t(s)$.

Case 1: $A(h) = A(l) = \{t_H, t_L\}$

Observe that no-deviation-constraint for low type seller

$$u_L(t_L^*) \geq u_L(\widetilde{t_L}) \implies g_{Ll} + pf_{Lh} + (1-p)f_{Ll} \geq q(\widetilde{t_L} + f_{Lh}) + (1-q)\delta(g_{Ll} + pf_{Lh} + (1-p)f_{Ll})$$

$$\implies q \leq \frac{(1-\delta)(g_{Ll} + pf_{Lh} + (1-p)f_{Ll})}{\widetilde{t_L} + f_{Lh} - \delta(g_{Ll} + pf_{Lh} + (1-p)f_{Ll})}$$

Similarly, no-deviation-constraint for high type seller

$$u_H(t_H^*) \geq u_H(\widetilde{t_H}) \implies g_{Hl} + pf_{Hh} + (1-p)f_{Hl} \geq q(\widetilde{t_H} + f_{Hh}) + (1-q)\delta(g_{Hl} + pf_{Hh} + (1-p)f_{Hl})$$

$$\implies q \leq \frac{(1-\delta)(g_{Hl} + pf_{Hh} + (1-p)f_{Hl})}{\widetilde{t_H} + f_{Hh} - \delta(g_{Hl} + pf_{Hh} + (1-p)f_{Hl})}$$

Set

$$\bar{q} = min\left\{ \frac{(1-\delta)(g_{Ll} + pf_{Lh} + (1-p)f_{Ll})}{\widetilde{t_L} + f_{Lh} - \delta(g_{Ll} + pf_{Lh} + (1-p)f_{Ll})}, \frac{(1-\delta)(g_{Hl} + pf_{Hh} + (1-p)f_{Hl})}{\widetilde{t_H} + f_{Hh} - \delta(g_{Hl} + pf_{Hh} + (1-p)f_{Hl})} \right\}$$

Hence for any $q \leq \bar{q}$ and strictly supermodular g s.t.

$$\frac{g_{Lh} - g_{Ll}}{g_{Hh} - g_{Hl}} \geq \frac{\delta p}{1 - \delta(1-p)}$$

we have an equilibrium that is trivially assortative.

Case 2: $A(h) = \{t_H\}, A(l) = \{t_H, t_L\}$

Sellers' utility maximization problems yield $t_L^* = g_{Ll}$ and $t_H^* = g_{Hl}$. Observe that t_H and t_L respect each type of buyer's constraints under our construction if we assume

$$\frac{g_{Lh} - g_{Ll}}{g_{Hh} - g_{Hl}} < \frac{\delta p}{1 - \delta(1 - p)}$$

Note that H might increase his offer to $\widetilde{t_H} = g_{Hh} - v(h)$ and get accepted by only high type buyer while $t_L^* = g_{Ll}$. Then no-deviation-constraint for H becomes

$$q \leq \frac{(1 - \delta)(g_{Hl} + pf_{Hh} + (1 - p)f_{Hl}}{g_{Hh} + f_{Hh} - \delta(g_{Hl} + pf_{Hh} + (1 - p)f_{Hl})}$$

Low type seller L cannot increase his utility by matching with high type buyer. Suppose $\widetilde{t_L} = g_{Lh} - v(h)$ while $t_H = t_H^*$. Then

$$\widetilde{t_L} = g_{Lh} - \frac{\delta p}{1 - \delta(1 - p)}(g_{Hh} - g_{Hl})$$

This amount is always less than t_L^* due to $\frac{g_{Lh} - g_{Ll}}{g_{Hh} - g_{Hl}} < \frac{\delta p}{1 - \delta(1 - p)}$.

Hence for any

$$q \leq \frac{(1 - \delta)(g_{Hl} + pf_{Hh} + (1 - p)f_{Hl})}{g_{Hh} + f_{Hh} - \delta(g_{Hl} + pf_{Hh} + (1 - p)f_{Hl})}$$

and strictly supermodular g s.t.

$$\frac{g_{Lh} - g_{Ll}}{g_{Hh} - g_{Hl}} < \frac{\delta p}{1 - \delta(1 - p)}$$

we have an assortative equilibrium in which $A(h) = \{t_H\}, A(l) = \{t_H, t_L\}$ with $t_L^* = g_{Ll}, t_H^* = g_{Hl}$.

Case 3: $A(h) = \{t_H, t_L\}, A(l) = \{t_L\}$

Low type seller L's utility maximization problem yields $t_L^* = g_{Ll}$ and $v(l) = 0$. Moreover, H only matches with high type buyer so H offers $t_H = g_{Hh} - v(h)$ to maximize his own utility. Substituting t_H and t_L in explicit form of $v(h)$, we have

$$t_H^* = g_{Hh} - \frac{\delta(1-p)}{1-\delta p}(g_{Lh} - g_{Ll})$$

Observe that t_H and t_L respect each type of buyer's constraints under our construction. Note that H might reduce his offer to $\widetilde{t_H} = g_{Hl} - v(l)$ and get accepted by both type of buyers as in previous case while $t_L^* = g_{Ll}$. Then no-deviation-constraint for H is

$$q(t_H^*+f_{Hh})+(1-q)v(H) \geq g_{Hl}+pf_{Hh}+(1-p)f_{Hl} \;\; where \;\; v(H) = \frac{\delta q}{1-\delta(1-q)}(t_H^*+f_{Hh})$$

$$\implies q \geq \frac{(1-\delta)(g_{Hl}+pf_{Hh}+(1-p)f_{Hl})}{t_H^*+f_{Hh}-\delta(g_{Hl}+pf_{Hh}+(1-p)f_{Hl})}$$

Low type seller L could also increase his utility by only matching with high type buyer. Suppose $\widetilde{t_L} = g_{Lh} - v(h)$ while $t_H = t_H^*$. Then

$$\widetilde{t_L} = g_{Lh} - \frac{\delta p}{1-\delta(1-p)}\frac{\delta(1-p)}{1-\delta p}(g_{Lh} - g_{Ll})$$

No-deviation-constraint for L is

$$g_{Ll}+pf_{Lh}+(1-p)f_{Ll} \geq q(\widetilde{t_L}+f_{Lh})+(1-q)v(L) \;\; where \;\; v(L) = \delta(g_{Ll}+pf_{Lh}+(1-p)f_{Ll})$$

$$\implies q \leq \frac{(1-\delta)(g_{Ll}+pf_{Lh}+(1-p)f_{Ll}}{\widetilde{t_L}+f_{Lh}-\delta(g_{Ll}+pf_{Lh}+(1-p)f_{Ll})}$$

Note that such q exists iff

$$(g_{Hh}+f_{Hh})g_{Ll}-g_{Hl}(g_{Lh}+f_{Lh})+p(f_{Lh}g_{Hh}-f_{Hh}g_{Lh})+(1-p)[(f_{Hh}+g_{Hh})f_{Ll}-f_{Hl}(f_{Lh}+g_{Lh})]$$

$$\geq \frac{\delta(1-p)}{1-\delta p}(g_{Lh}-g_{Ll})\left(g_{Ll}+pf_{Lh}+(1-p)f_{Ll}-\frac{\delta p}{1-\delta(1-p)}(g_{Hl}+pf_{Hh}+(1-p)f_{Hl}\right)$$

Hence for any such q and strictly supermodular g, we have an assortative equilibrium in which $A(h) = \{t_H, t_L\}, A(l) = \{t_L\}$ with $t_L^* = g_{Ll}, t_H^* = g_{Hh} - \frac{\delta(1-p)(g_{Lh}-g_{Ll})}{1-\delta p}$.

Case 4: $A(h) = \{t_H, t_L\}, A(l) = \{t_H\}$

High type seller H's utility maximization problem yields $t_H^* = g_{Hl}$ and $v(l) = 0$. Moreover, L only matches with high type buyer so L offers $t_L = g_{Lh} - v(h)$ to maximize his own utility. Substituting t_H and t_L in explicit form of $v(h)$, we have

$$t_L^* = g_{Lh} - \frac{\delta p}{1-\delta(1-p)}(g_{Hh} - g_{Hl})$$

Observe that t_H and t_L respect each type of buyer's constraints under our construction. Note that L might reduce his offer to $\widetilde{t_L} = g_{Ll} - v(l)$ and get accepted by both type of buyers as in previous case while $t_H^* = g_{Hl}$. Then no-deviation-constraint for L becomes

$$q(t_L^*+f_{Lh})+(1-q)v(L) \geq g_{Ll}+pf_{Lh}+(1-p)f_{Ll} \;\; where \;\; v(L) = \frac{\delta q}{1-\delta(1-q)}(t_L^*+f_{Lh})$$

$$\implies q \geq \frac{(1-\delta)(g_{Ll} + pf_{Lh} + (1-p)f_{Ll})}{t_L^* + f_{Lh} - \delta(g_{Ll} + pf_{Lh} + (1-p)f_{Ll})}$$

High type seller H could increase his utility by only matching with high type buyer. Suppose $\widetilde{t_H} = g_{Hh} - v(h)$ while $t_L = t_L^*$. Then

$$\widetilde{t_H} = g_{Hh} - \frac{\delta p}{1-\delta(1-p)}\frac{\delta(1-p)}{1-\delta p}(g_{Hh} - g_{Hl})$$

No-deviation-constraint for H is

$$g_{Hl}+pf_{Hh}+(1-p)f_{Hl} \geq q(\widetilde{t_H}+f_{Hh})+(1-q)v(H) \;\; where \;\; v(H) = \delta(g_{Hl}+pf_{Hh}+(1-p)f_{Hl})$$

$$\implies q \leq \frac{(1-\delta)(g_{Hl} + pf_{Hh} + (1-p)f_{Hl})}{\widetilde{t_H} + f_{Hh} - \delta(g_{Hl} + pf_{Lh} + (1-p)f_{Ll})}$$

Note that such q does not exist due to the additional assumptions we make

$$-(g_{Hh}+f_{Hh})g_{Ll}+g_{Hl}(g_{Lh}+f_{Lh})-p(f_{Lh}g_{Hh}-f_{Hh}g_{Lh})-(1-p)[(f_{Hh}+g_{Hh})f_{Ll}-f_{Hl}(f_{Lh}+g_{Lh})]$$

$$< \frac{\delta p}{1-\delta(1-p)}(g_{Hh}-g_{Hl})\left(g_{Hl}+pf_{Hh}+(1-p)f_{Hl}-\frac{\delta(1-p)}{1-\delta p}(g_{Ll}+pf_{Lh}+(1-p)f_{Ll}\right)$$

Note that Cases 5-7 do not represent an equilibrium irrespective of the value of f.

Case 8: $A(h) = \{t_H, t_L\}, A(l) = \emptyset$

Note that $t_H^* = g_{Hh}$ and $t_L^* = g_{Lh}$ are candidate equilibrium transfers. No-deviation-constraints are

$$q \geq \frac{(1-\delta)(g_{Hl} + pf_{Hh} + (1-p)f_{Hl})}{g_{Hh} + f_{Hh} - \delta(g_{Hl} + pf_{Hh} + (1-p)f_{Hl})}$$

and

$$q \geq \frac{(1-\delta)(g_{Ll} + pf_{Lh} + (1-p)f_{Ll})}{g_{Hl} + f_{Hl} - \delta(g_{Ll} + pf_{Lh} + (1-p)f_{Ll})}$$

Set

$$\bar{q} = max\left\{\frac{(1-\delta)(g_{Hl} + pf_{Hh} + (1-p)f_{Hl})}{g_{Hh} + f_{Hh} - \delta(g_{Hl} + pf_{Hh} + (1-p)f_{Hl})}, \frac{(1-\delta)(g_{Ll} + pf_{Lh} + (1-p)f_{Ll})}{g_{Hl} + f_{Hl} - \delta(g_{Ll} + pf_{Lh} + (1-p)f_{Ll})}\right\}$$

Hence for any $q \geq \bar{q}$ and strictly supermodular g, such equilibrium exists.

□

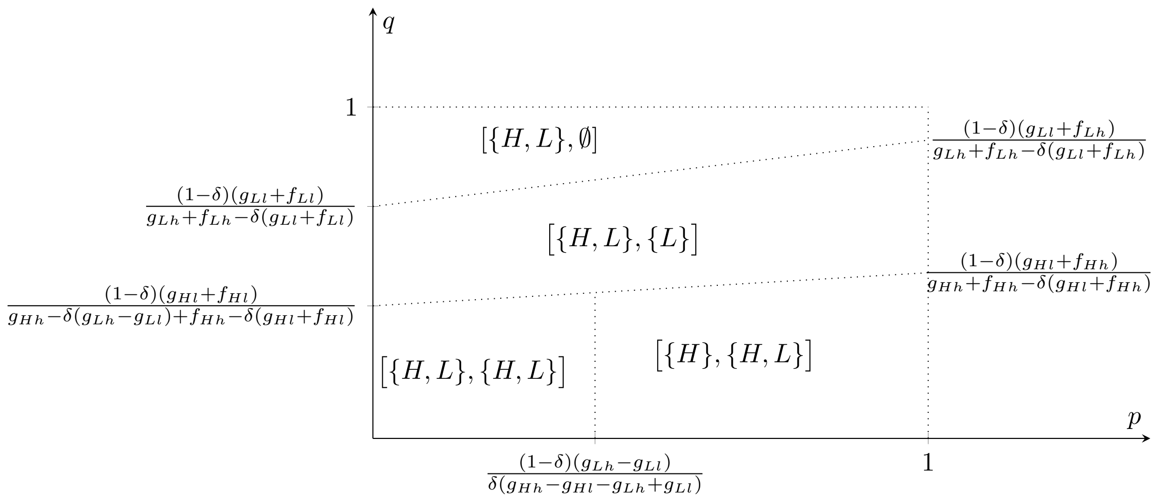
The strategy profile of sellers can be written as

$$(t_H^*, t_L^*) = \begin{cases} (g_{Hh}, g_{Lh}) & if \;\; q \geq \frac{(1-\delta)(g_{Ll}+pf_{Lh}+(1-p)f_{Ll})}{g_{Lh}+f_{Lh}-\delta(g_{Ll}+pf_{Lh}+(1-p)f_{Ll})} \\ (g_{Hh} - \frac{\delta(1-p)}{1-\delta p}(g_{Lh} - g_{Ll}), g_{Ll}) \quad if \;\; \frac{(1-\delta)(g_{Ll}+pf_{Lh}+(1-p)f_{Ll})}{g_{Lh}+f_{Lh}-\delta(g_{Ll}+pf_{Lh}+(1-p)f_{Ll})} > q \geq \frac{(1-\delta)(g_{Hl}+pf_{Hh}+(1-p)f_{Hl})}{t_H^*+f_{Hh}-\delta(g_{Hl}+pf_{Hh}+(1-p)f_{Hl})} & \\ (g_{Hl}, g_{Ll}) & otherwise \end{cases}$$

And the equilibrium matching structure is as the following

$$\big[A(h), A(l)\big] = \begin{cases} [\{H, L\}, \emptyset] & if \;\; q \geq \frac{(1-\delta)(g_{Ll}+pf_{Lh}+(1-p)f_{Ll})}{g_{Lh}+f_{Lh}-\delta(g_{Ll}+pf_{Lh}+(1-p)f_{Ll})} \\ [\{H, L\}, \{L\}] & if \;\; \frac{(1-\delta)(g_{Ll}+pf_{Lh}+(1-p)f_{Ll})}{g_{Lh}+f_{Lh}-\delta(g_{Ll}+pf_{Lh}+(1-p)f_{Ll})} > q \geq \frac{(1-\delta)(g_{Hl}+pf_{Hh}+(1-p)f_{Hl})}{t_H^*+f_{Hh}-\delta(g_{Hl}+pf_{Hh}+(1-p)f_{Hl})} \\ [\{H\}, \{H, L\}] & if \;\; \left(\frac{(1-\delta)(g_{Hl}+pf_{Hh}+(1-p)f_{Hl})}{t_H^*+f_{Hh}-\delta(g_{Hl}+pf_{Hh}+(1-p)f_{Hl})} > q\right) \wedge \left(p \geq \frac{(1-\delta)(g_{Lh}-g_{Ll})}{\delta(g_{Hh}-g_{Hl}-g_{Lh}+g_{Ll})}\right) \\ [\{H, L\}, \{H, L\}] & if \;\; \left(\frac{(1-\delta)(g_{Hl}+pf_{Hh}+(1-p)f_{Hl})}{t_H^*+f_{Hh}-\delta(g_{Hl}+pf_{Hh}+(1-p)f_{Hl})} > q\right) \wedge \left(\frac{(1-\delta)(g_{Lh}-g_{Ll})}{\delta(g_{Hh}-g_{Hl}-g_{Lh}+g_{Ll})} > p\right) \end{cases}$$

Graph 2: Equilibrium Matching Structure when $f \neq 0$



3.3 Frictionless Matching under One-Sided Information Asymmetry

If there is no search friction due to time discounting, then sellers are patient enough to search for the highest profit generating partner. Since the probabilities are exogenous and constant, supermodularity assumptions on f and g suffice for Case 8 to be the unique assortative equilibrium as in Becker's.

3.4 Comparison to Complete Information

If we remove one-sided information asymmetry and take a common output function, then our model is reduced to Shimer and Smith(2000). Our initial guess was that one-sided information asymmetry could induce less restrictive assumptions on assortative matching compared to complete information setting as uninformed sellers are bound to make less scrutinized offers due to lack of information. For that purpose, we analyze a two-type-case under complete information but we find out that log-supermodularity assumption still suffices for assortative matching.

Theorem 3: *Assume a common output function g under complete information. If g is supermodular and log-supermodular, then any equilibrium is assortative.*

Proof. Observe that the only non-assortative equilibrium that could occur under supermodularity assumption are the following

Case 1: $A(h) = \{h, l\}, A(l) = \{h\}$

Let us write the Bellman equations for each type

$$v(h) = \delta\big(\frac{p}{2}(g_{hh} - v(h) + v(h)) + \frac{1-p}{2}(g_{hl} - v(l) + v(h)\big)$$

$$v(l) = \delta\big(\frac{p}{2}(g_{hl} - v(h) + v(l)) + (1-p)v(l)\big)$$

where

$$g_{hh} \geq 2v(h), \;\; g_{hl} \geq v(h) + v(l) \;\text{ and }\; 2v(l) > g_{ll}$$

Then

$$v(l) = \frac{2\delta p(1-\delta+\delta p)g_{hl} - \delta^2 p^2 g_{hh}}{4 - 6\delta + 2\delta^2 + 2\delta p(2 - 2\delta + \delta p)}$$

Hence by log-supermodularity

$$\frac{2\delta p(1-\delta+\delta p)g_{hl} - \delta^2 p^2 g_{hh}}{4 - 6\delta + 2\delta^2 + 2\delta p(2 - 2\delta + \delta p)} > \frac{g_{ll}}{2} > \frac{g_{hl}^2}{2g_{hh}}$$

It follows that

$$0 > \frac{1}{2}\big(\frac{g_{hl}}{g_{hh}}\big)^2 - \frac{2\delta p(1-\delta+\delta p)}{4 - 6\delta + 2\delta^2 + 2\delta p(2 - 2\delta + \delta p)}\frac{g_{hl}}{g_{hh}} + \frac{\delta^2 p^2}{4 - 6\delta + 2\delta^2 + 2\delta p(2 - 2\delta + \delta p)}$$

As $\Delta < 0 \;\; \forall \delta, p$, we have a contradiction.

Case 2: $A(h) = \{l\}, A(l) = \{h, l\}$

The Bellman equations for each type

$$v(h) = \delta\big(pv(h) + \frac{1-p}{2}(g_{hl} - v(l) + v(h)\big)$$

$$v(l) = \delta\big(\frac{p}{2}(g_{hl} - v(h) + v(l)) + \frac{1-p}{2}(g_{ll} - v(l) + v(l))\big)$$

where

$$g_{hh} < 2v(h), \;\; g_{hl} \geq v(h) + v(l) \;\text{ and }\; 2v(l) \leq g_{ll}$$

Then

$$v(h) = \frac{2\delta(1-p)(1-\delta p)g_{hl} - \delta^2(1-p)^2 g_{ll}}{4 - 2\delta - 4\delta p + 2\delta^2 p^2}$$

Hence by log-supermodularity

$$\frac{2\delta(1-p)(1-\delta p)g_{hl} - \delta^2(1-p)^2 g_{ll}}{4 - 2\delta - 4\delta p + 2\delta^2 p^2} > \frac{g_{hh}}{2} > \frac{g_{hl}^2}{2g_{ll}}$$

It follows that

$$0 > \frac{1}{2}\big(\frac{g_{hl}}{g_{ll}}\big)^2 - \frac{2\delta(1-p)(1-\delta p)}{4 - 2\delta - 4\delta p + 2\delta^2 p^2}\frac{g_{hl}}{g_{ll}} + \frac{\delta^2(1-p)^2}{4 - 2\delta - 4\delta p + 2\delta^2 p^2}$$

As $\Delta < 0 \;\; \forall \delta, p$, we have a contradiction.

$\square$

4 Conclusion

We analyzed a frictional dencentralized search model under one-sided information asymmetry. We found that assortative matching is achievable under log-supermodularity assumption when sellers value from the match is zero. If sellers are entitled to a non-zero value from the match, then more restrictive assumptions are needed in order to restore assortative matching. We compare our model with Shimer and Smith(2000)'s model in order to see how binding our assumptions are and we find out that log-supermodularity also suffices for assortative matching under complete information meaning that incorporating one-sided information asymmetry does not induce less restrictive assumptions when sellers value are taken to be zero. However, our model is limited in terms of type heterogeneity and distribution endogeneity. Hence a full and healthy characterization is only possible through extending our model to a continuum of types and allowing the distribution of types to be affected by the acceptance sets of agents.

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