Posterior Evidence on US Inflation Volatility Dynamics using a Phillips Curve model with Time Varying Trend Inflation

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Abstract

We construct a New Keynesian Phillips Curve (NKPC) model with time varying trend inflation to analyze the dynamics between the US inflation level and volatility. The model involves a Calvo type pricing of firms where the firms that cannot optimize their prices fix their prices at the previous period's level. The extended model can generate much richer dynamics than the conventional NKPC without any trend inflation. Interestingly, the extended NKPC model indicates that the volatility of US inflation is driven mainly by the level of inflation and shocks to the long-run inflation expectations endogenously. We, further, estimate a simplified version of our NKPC model using US quarterly inflation and labor income share data over the period from 1960 until 2014 using Bayesian inference. The model involves time varying trend inflation (as well as time varying level of labor income share) together with stochastic volatility for inflation. In addition, we use survey based inflation expectations to replace the short-run inflation expectations relaxing the rational expectations assumption. In accordance with theoretical findings, empirical results verify that past volatility does not have a significant impact on the level, whereas the past level of inflation affects the volatility of the inflation.

Özet

A.B.D. enflasyon seviyesi ile oynaklığı arasındaki dinamikleri analiz edebilmek için zamanla değiçen enflasyon seviyesi olan Yeni Keynesyen Phillips eğrisi modeli kurduk. Bu model fiyatları optimumlaştıramayan firmaların geşen dönem fiyatları kullandığı Calvo tipi fiyatlama işermektedir. Bu daha kapsamlı model enflasyon trendi işermeyen geleneksel NKPC modellerine göre daha zengin dinamikler öretmektedir. Şaşırtıcı bir şekilde bu daha kapsamlı model A.B.D. enflasyon oynaklığını büyük ölşöde enflasyon seviyesi ve uzun dünem enflasyon beklentilerinde görülen içsel şoklar tarafından yönlendirildiğini göstermektedir. Ayn zamanda 1960 ve 2014 yılları arasındaki A.B.D. çeyrek enflasyon ve çeyrek emek gelir payı verilerini kullanarak NKPC modelimizin daha basit bir versiyonunu Bayesian ekonometrisi çerşevesinde tahminledik. Bu model zamanla değiçen emek gelir payı seviyesi dıçında zamanla değiçen enflasyon seviyesi ve olasılıksal enflasyon oynaklığı içermektedir. Ayrıca rasyonel enflasyon beklentileri varsayımını zayıflatarak kısa dönem enflasyon beklentileri yerine ankete dayalı enflasyon beklentilerini kullandık. Teorik bulgulara uygun olarak ampirik neticeler geçmiş dönem enflasyon oynaklığının enflasyon seviyesi üzerinde hiç bir etkisi olmadığını ancak geçmiş enflasyon seviyesinin enflasyon oynaklığını etkilediğini göstermektedir.

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Contents

List of Figures

List of Tables

1 Introduction

One of the central issues in macroeconomics is the dynamics of short-run inflation. This issue is crucial for the nature of business cycles and how the monetary policy should be conducted. To address this challenge, recently there have been numerous important advances. This literature depends on early work by Fischer (1997), Taylor (1980), Calvo (1983) and other studies that underlined sticky nominal wage and forward looking price setting. Gali and Gertler (1999) expands this work by putting price setting into an individual utility maximization problem. Aggregating individual behaviours yields a relation between inflation and some measure of overall economic activity, that is, New Keynesian Phillips Curve(NKPC). New Keynesian Phillips Curve can be defined briefly as the dynamic inflation and economic activity relation that emerges in dynamic optimization framework populated by utility maximizing households and profit maximizing firms and modified with some kind of stickiness in price setting. Allowing for both nominal rigidities and market imperfections in these models alters the transmission mechanism for shocks and also provides a more potent role for monetary and fiscal policy Regarding micro-foundations in derivation of NKPC adds new structures on the relation and some important differences in detail.

With further improvements based on previous studies we discuss above, modelling the relation between inflation and economic activity has been one of the building blocks in policy analysis. The NKPC have become enormously important in policy models where the NKPC has become standard approach. NKPC models are theoretically consistent, have explicit microeconomic foundations and provide a rigorous analytical framework for credible welfare and policy analysis. Over the course of time there have been debates about the merits of NKPC and further refinements are incorporated into NKPC models. We will discuss the recent advances occurred in both empirical and theoretical aspects of NKPC.

2 Literature Review

As for the theoretical aspect, Gali and Gertler (1999) extends the baseline NKPC model by allowing for the backward looking firms which set their prices according to past period's inflation. They develop and estimate a hybrid NKPC and showed that both backward looking and forward looking behaviour is statistically significant. They derive the NKPC model by log-linearising the first order conditions of maximization problems of both firms and households around zero trend inflation.However the average inflation rates in recent years for developed countries have been positive and not so close to zero, i.e, from the seventies onwards the average inflation rate has varied from approximately 3% in Germany to almost 10 % in Spain, with the U.S. around 5%. Even from this simple analysis of data, it is obvious that NKPC derived using log-linearisation around zero steady state is ill-suited for describing economies with high rates of inflation. Ascari (2004) shows that when trend inflation is taken into consideration, both the long run and short run nature of NKPC models changes dramatically. The NKPC model is very sensitive to the trend inflation level around which log-linearisation is carried out, therefore it is not surprising that trend inflation is crucial for the dynamics of log-linearised model. As a consequence, the results obtained from a NKPC model with zero steady state inflation might be deceptive.

The other problem in theoretical aspect of NKPC is that to overcome the shortcoming of purely forward-looking versions of the NKPC which generates too little inflation persistence, some authors add ad hoc backward-looking terms as Gali and Gertler (1999) did. Nevertheless the models including backward-looking terms have been criticized because these models lack a reasonable microeconomic foundation. Cogley and Sbordone (2008) showed that to include the variation in the inflation level of the model is important to explain the inflation persistence. Cogley and Sbordone (2008) derives a version of the NKPC as a log-linear approximation around time varying steady state inflation and demonstrates that when the inflation trend is modelled as time varying, a purely forward-looking model fits data very well without requiring a backward-looking term. To derive our model we use the micro-foundations proposed by Ascari(2004) and we treat the inflation trend as time varying following the practice of Cogley and Sbordone(2008).

Various improvements also have taken place in econometric modelling of NKPC. Gali and Gertler (1999) uses labor income share as a indicator of marginal cost instead of ad hoc output gap as theory suggests and their results indicates that labor income share is a statistically significant and quantitatively important determinant of inflation. Following the practice which started with Gali and Gertler (1999), we use labor income share as a proxy of real marginal cost in our empirical model.

The other discrepancy for the econometric aspect of NKPC is that the analysis of NKPC is carried out by using the short run variations in inflation and economic activity. Various studies demean and detrend the data before the analysis to obtain the short run variation, see Gali and Gertler (1999); Smets and Wouters (2003); Mavroedis (2004); DeJong and Dave (2011). However eliminating the low frequency movements may lead to misspecification in the models, see Canova (2012). Numerous works points out the existence of complex low frequency movements, especially in the inflation (McConnell and Perez-Quiros, 2000; Stock and Watson, 2008; Bianchi, 2010). When we examine the non-filtered inflation data, we observe two distinct periods in terms of the inflation trend. The inflation is much higher in the period between the beginning of 1970s and the begining of 1980s compared to the inflation in the latter periods. It is widely believed that this decrease in inflation is caused by the credible monetary policy that made the inflation more stable by committing to a nominal anchor since the early eighties, see McConnell and Perez-Quiros (2000); Stock and Watson (2002); Ahmed et al. (2004); Stock and Watson (2007); Cecchetti et al. (2007). Similarly, the labor income share series follows a time varying trend which is negative and its magnitude increases in the recent periods. The reason for such a trend is most probably technology shocks. The importance of joint analysis of both short and long variation in the data is emphasized by various works (Delle Monache and Harvey, 2010; Canova 2012).

Basturk et al. (2012) shows that prior filtering of data causes misspecification and thus deteriorates posterior inference of NKPC parameters. To address this problem, Basturk et al. (2012) models the low and high frequency movements in the inflation and labor income share series jointly, by expending the NKPC to allow for analysis of non-filtered observed time series.

Following Basturk et al. (2012), we construct a NKPC model with complex time series structures which involves time varying trends in the inflation and marginal cost series. Besides modelling the low frequency movements we also allow high frequency movements to change by adding stochastic volatility structure for the inflation. We use survey based inflation expectations to replace short-run inflation expectations. With further refinements added we investigate the relation that links the inflation trend and volatility.

The structure of this paper is as follows: Section 3 presents different versions of NKPC models derived treating the inflation trend differently as zero first,secondly positive and then time varying. Section 4 provides the empirical models with simplified version of NKPC equation with time varying trend inflation. Section 5 summarizes the likelihood, prior and posterior sampling algorithm. Section 6 presents the estimation results for NKPC parameters and steady state values as well as how the relation between the inflation trend and volatility behaves. Section 6 concludes. Appendix A provides detailed derivation of NKPC equations. In Appendix B details on parametric structures, state space specification of our models and the sampling algorithm are provided.

3 The Theoretical Model

The model we use is a sticky price dynamic general equilibrium model and is based on Calvo(1983) price setting. Moreover, the model is composed of a continuum of infinitely-lived consumers, producers of final and and intermediate goods.

Households

The instantaneous utility function is given by

$$\left\{ \left[bC^{\frac{\eta-1}{\eta}} + (1-b) \left(\frac{M}{P} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta-1}{\eta}} (1-L^e) \right\}^{1-\chi} / (1-\chi) \tag{1}$$

where C is consumption, M is money, P is price of final good, and L is labor.Household's problem is to maximize the expected discounted sum of instantaneous utility subject to a series of budget constraints.

Firms

In the final goods market, a single final good is produced by a perfectly competitive, representative firm. The final good is produced using a continuum of intermediate good, $Y_{j,t}$ indexed by $j \in (0,1)$. Final good producers use the following technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di \right]^{(\epsilon-1)/\epsilon} \tag{2}$$

where $Y_t(i)$ is the output of intermediate goods producer and Y_t is final good output. The demand curve for $Y_t(i)$ is given by

$$Y_t = \left(\frac{P_t(i)}{P_t} \right)^{(-1+\epsilon)/\epsilon} \tag{3}$$

where $P_t(i)$ is the price of intermediate good i and P_t is the aggregate price of the final good. The aggregate price is given by

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\epsilon}} di \right]^{-\epsilon} \tag{4}$$

In the sticky prices model, proposed by Calvo(1983), a fraction $1 - \theta$ of all firms re-optimize their nominal prices while the remaining θ fraction of all firms do not re-optimize their nominal prices. Following Christiano et al. (2005), firms that cannot re-optimize set their price index to lagged inflation are as follows.

$$P_t(i) = \pi_{t-1} P_{t-1}(i) \tag{5}$$

where $\pi_t = P_t/P_{t-1}$. We call this price setting "lagged inflation indexation". The firm i chooses $P_t(i)$ to maximize

$$\begin{aligned} \max \quad & E_t\{\sum_{j=0}^{\infty} \theta^j \gamma^j [\left(\frac{P_t(i)}{P_{t+j}} \right) Y_{t+j}(i) - TC_{i,t+j}(Y_{t+j}(i))]\} \\ \text{s.t.} \quad & Y_{t+j}(i) = \left(\frac{P_t(i)}{P_{t+j}} \right)^{-\epsilon} Y_t \end{aligned} \tag{6}$$

where γ is the discount factor and TC_i is real total costs.

The optimal price fixed by re-setting firms in period t is given by

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{E_t\{\sum_{j=0}^{\infty} \theta^j \gamma^j MC_{i,t+j} (P_{t+j})^{\epsilon} Y_{t+j}\}}{E_t\{\sum_{j=0}^{\infty} \theta^j \gamma^j (P_{t+j})^{\epsilon-1} Y_{t+j}\}} \tag{7}$$

where MC_i is real marginal cost of producer i. This equation represents the core of sticky price models, as thoroughly explained by King and Wolman (1996).

The structure of New Keynesian Phillips Curve depend on how we treat the trend inflation. Analytical investigation shows that dynamic behaviour is highly sensitive to trend inflation. We derive NKPC equations for different types of trend inflation. We start with the well-known case where log-linearisation is taken around the steady state with zero inflation (i.e. $\Pi = 1$). Then we derive the NKPC where log-linearisation is taken around a steady state with trend inflation (i.e $\Pi > 1$). At the last case, we derive a version of NKPC as a log-linear approximation around a time-varying inflation trend. We use upper-case letters and letters with tilde for the steady-state values and log-deviations of variables, respectively.

3.1 NKPC with zero trend inflation

The log-linearised version of (7) is

$$\tilde{p}_t^* - \tilde{p}_t = (1-\theta\gamma)E_t \sum_{j=0}^{\infty} (1-\theta\gamma)\tilde{mc}_{t+j} + \tilde{\pi}_{t,t+j} \tag{8}$$

where $\tilde{\pi}_{t,t+j} = (\tilde{\pi}_{t+1} + \tilde{\pi}_{t+2} + \cdots + \tilde{\pi}_{t+j})$ and $\tilde{\pi}_{t,t} = 0$. The log-linearised version of general price level equation (4) is

$$\tilde{p}_t^* - \tilde{p}_t = \frac{\theta}{1-\theta}\tilde{\pi}_t \tag{9}$$

We combine (8) and (9) in order to get the dynamics of inflation

$$\tilde{\pi}_t = \lambda \tilde{mc}_t + \gamma E_t \tilde{\pi}_{t+1} \tag{10}$$

where $\lambda = \frac{(1-\theta)(1-\theta\gamma)}{\theta}$. As explained by Gali and Gertler (1999), this is the so-called 'New Keynesian Phillips Curve'. We found by iterating (10) forward that the inflation rate today is based on the discounted sum of the future expected marginal costs

$$\tilde{\pi}_t = \lambda \sum_{j=0}^{\infty} \gamma^j E_t mc_{t+j} \tag{11}$$

When we take expected future path of the marginal costs as given, the key parameter for the dynamics of inflation is therefore λ. Gali and Gertler (1999) propose an empirical formulation based on (10) to explain the dynamics of inflation. Gali and Gertler (1999) argue that such a model could explain the behaviour of U.S. inflation in the last thirty years and estimates the structural parameters of the model.

On the contrary Ascari(2004) shows that when the trend inflation is taken into consideration, both the long-run and short-run properties of NKPC models change dramatically, therefore the models obtained by log-linearisation around a zero inflation steady state might be misleading.

3.2 NKPC with trend inflation

The log-linearised version of (7) around a steady state with trend inflation is

$$\begin{aligned} \tilde{p}_t^* - \tilde{p}_t = & E_t \sum_{j=0}^{\infty} (\theta\gamma\Pi^{\epsilon})^j (1-\theta\gamma\Pi^{\epsilon}) \{\tilde{m}c_{t+j} + \epsilon\tilde{\pi}_{t,t+j} + \tilde{y}_{t+j}\} \\ & - E_t \sum_{j=0}^{\infty} (\theta\gamma\Pi^{\epsilon-1})^j (1-\theta\gamma\Pi^{\epsilon-1}) \{(\epsilon-1)\tilde{\pi}_{t,t+j} + \tilde{y}_{t+j}\} \end{aligned} \tag{12}$$

The log-linearised version of general price level around a steady state with trend inflation is

$$\tilde{p}_t^* - \tilde{p}_t = \frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}} \pi_t \tag{13}$$

Combining (12) and (13), we obtain a more generalized version of (10), which can be written as

$$
\begin{aligned}
\tilde{\pi}_t = \quad & \left(\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\right)(1-\theta\gamma\Pi^{\epsilon})\tilde{m}c_t + \gamma E_t\tilde{\pi}_{t+1} \\
& + (1-\Pi)\gamma(1-\theta\Pi^{\epsilon-1})\Bigg[\tilde{y}_t - \left(\epsilon + \frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\right)E_t\tilde{\pi}_{t+1} \\
& - (1-\theta\gamma\Pi^{\epsilon-1})E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j[(\epsilon-1)\tilde{\pi}_{t+1,t+j+1} + \tilde{y}_{t+j+1}]\Bigg] \qquad (14)
\end{aligned}
$$

Setting $\Pi = 1$ results in (10). We can ignore the last additive terms, since the gross inflation rate Π is very close to one. In that case we obtain an expression very close to (10).

$$
\tilde{\pi}_t = \bar{\lambda}(\Pi)\tilde{m}c_t + \gamma E_t\pi_{t+1} \text{ and } \tilde{\pi}_t = \bar{\lambda}(\Pi)\sum_{j=0}^{\infty}\gamma^j E_t mc_{t+j} \qquad (15)
$$

where $\bar{\lambda}(\Pi) = \left(\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\right)(1-\theta\gamma\Pi^{\epsilon})$.

Ascari(2004) shows that the value of λ very much depends on the trend inflation values. Even when we take a small level of inflation trend, i.e 2% annually, the value of λ reduces by % 30 compared to a log linearisation around zero tend inflation. Therefore for any given future expected path of marginal costs, when the trend inflation is not considered, the dynamic response of inflation to marginal costs is overestimated. Moreover, when trend inflation gets higher, the difference between λ and $\bar{\lambda}(\Pi)$ gets bigger.

Ascari(2004) presents some important points from the analysis above. The model shows that the log-linear approximation of dynamics of inflation around zero trend inflation as a function of future expected path of marginal costs gets worse when the trend inflation increases. While the log-linear approximation around zero trend inflation simplifies the process and gives neat results, ignoring the trend inflation may lead to misleading results.

What Ascari(2004) derived is a purely forward-looking New Keynesian Phillips curve (NKPC): inflation depends on the expected evolution of real marginal costs. However, empirical evidence of significant inflation persistence can not be explained by purely forward-looking models(e.g., see Fuhrer and Moore 1995). Accordingly, a number of studies constructs backward-looking models by introducing some form of price indexation to add lags of inflation to the model in order to enhance the degree of inflation persistence in the model and provide a better fit with the data (e.g., see Lawrence Christiano,Martin Eichenbaum and Charles Evans 2005). However these mechanisms have been criticized because they don't have a reasonable microeconomic foundation.

Cogley and Sbordone(2008) proposes an alternative interpretation of the apparent need for a structural persistence term.They emphasized that to explain the inflation persistence it is important to model the variation in the trend inflation. The trend inflation gives a highly persistent component to the inflation. Cogley and Sbordone(2008) log-linearise the equilibrium conditions of the model around a shifting steady state with a time-varying inflation trend.

3.3 NKPC with time-varying trend inflation

We derive a NKPC model by using the same price indexation as Ascari(2004) did and treating the trend inflation in the same way as Cogley and Sbordone(2008) did. The resulting representation is a log-linear NKPC with time-varying coefficients.

$$\begin{aligned}\tilde{\pi}_t = &\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{\theta\Pi_t^{\epsilon-1}}\right)(1-\theta\gamma\Pi_t^{\epsilon})\tilde{m}c_t + (\gamma - \gamma(1-\Pi_t)(\epsilon(1-\theta\Pi_t^{\epsilon-1}) + \theta\Pi_t^{\epsilon-1})E_t\tilde{\pi}_{t+1} \\ &- (1-\Pi_t)\gamma(1-\theta\Pi_t^{\epsilon-1})E_t\left\{\sum_{j=1}^{\infty}(\theta\gamma\Pi_t^{\epsilon-1})^{j-1}\underbrace{(\tilde{y}_{t+j}-\tilde{y}_{t+j-1})}_{\tilde{g}^y_{t+j}} + \theta\gamma\Pi_t^{\epsilon-1}(\epsilon-1)\tilde{\pi}_{t+1+j})\right\}\end{aligned} \tag{16}$$

where $\tilde{g}^y_{t+j}$ is the log-deviation of gross growth rate from its steady state value which is one and $\bar{\Pi}_t$ is time varying inflation trend. When we take Π_t as constant positive value, (16) boils down to (14) and when we take trend inflation as one, we obtain (10). The last infinite additive term in (16) can be considered as expectational shock, since long-term inflation expectations and growth expectations behave as noise with mean zero. The volatility of the error term is related to the time varying steady state inflation, because the magnitude of this term depends on the trend inflation. Therefore, the theory indicates that the volatility of inflation depends on the inflation trend and shocks to the long-term inflation and growth expectation endogenously.

To further simplify (16), we linearise the coefficients around $\Pi_t = 1$ using first order Taylor approximation and we obtain

$$\tilde{\pi}_t = \left(\frac{1-\theta}{\theta}\right)(1-\theta\gamma)\tilde{mc}_t + \gamma E_t\tilde{\pi}_{t+1} + \left(\frac{1-\epsilon-\theta\gamma(1-\theta\epsilon)}{\theta}(\Pi_t-1)\right)\tilde{mc}_t$$
$$+ (\epsilon(1-\theta)+\theta)(\Pi_t-1)\gamma E_t\tilde{\pi}_{t+1} + (1-\theta)(\Pi_t-1)\gamma E_t\{\sum_{j=1}^{\infty}(\theta\gamma)^{j-1}(\tilde{g}^y_{t+j}+$$
$$\theta\gamma(\epsilon-1))\tilde{\pi}_{t+1+j}\} \tag{17}$$

The two additive terms in (17) gives NKPC expression obtained by log-linearisation around zero steady state. Additionally there are two terms with time varying coefficients for $\tilde{mc}_t$ and $\tilde{\pi}_{t+1}$, respectively. The last infinite additive term can be treated as expectational shock with volatility depending on the trend inflation, since $\tilde{g}^y_{t+j}$ and $\tilde{\pi}^y_{t+1+j}$ are log deviations from steady state values, thus in the infinite horizon they act as noise with mean zero as we discussed above.

Since (17) involves the infinite sum of expectations, a closed form solution only exists when we make certain assumptions such as rational expectations. Rather than following this practice we can model inflation expectations and growth expectations using an unobserved component to be estimated along with other parameters(see Basturk 2012). Specifically, let $\mu_t = E_t\tilde{\pi}_{t+1}$ and $\alpha_t = E_t\tilde{g}^y_{t+1}$. We assume that inflation expectations and growth expectations are anchored around long-term expectations and deviations from this long-term expectations follow an AR(1) process as follows

$$\begin{aligned}\mu_t - \Pi_{t+1} &= \beta(\mu_{t-1} - \Pi_t) + \epsilon_{\pi,t} \\ \alpha_t - G^y_{t+1} &= \beta(\alpha_{t-1} - G^y_t) + \epsilon_{\pi,t}\end{aligned} \tag{18}$$

This formulation states a Bayesian learning rule for the inflation expectations in the sense that each period when the new informations arrives, the states and expectations are updated.

Specifying expectations as in (18), the model becomes

$$\begin{aligned}\tilde{\pi}_t = & \left(\left(\frac{1-\theta}{\theta} \right) (1-\theta\gamma) + \left(\frac{1-\epsilon-\theta\gamma(1-\theta\epsilon)}{\theta} (\Pi_t - 1) \right) \right) \tilde{m}c_t \\ & + \left(\gamma + (\Pi_t - 1)\gamma \left(1 - (\epsilon - 1)(\theta - 1) \left(\frac{\theta\gamma}{1-\theta\gamma\beta} \right) \right) \right) E_t \tilde{\pi}_{t+1} \\ & - \left((\theta - 1)(\Pi_t - 1)\gamma \left(\frac{\theta\gamma}{1-\theta\gamma\beta} \right) \right) \tilde{E}_t g^y_{t+1}\end{aligned} \tag{19}$$

(19) can be estimated, since both inflation expectations and growth expectations data is available from University of Michigan Research Center which provide quarterly one year ahead inflation and growth expectations . This implies that the inflation expectations are anchored around the survey values, see Roberts (1995,1997); Del Negro and Schorfeider (2012) for a similar approach).

Cogley and Sbordone(2008) showed that backward looking component is not needed to model inflation dynamics once variation in trend inflation is taken into account by deriving a version of the NKPC as an approximate equilibrium condition around a time-varying inflation trend and estimating with an unrestricted VAR.

In our empirical analysis we estimate a simplified version of our NKPC model (17) using Bayesian inference. We use a state space framework at which variation in trend inflation is taken into account, also stochastic volatility for inflation. Posterior results are obtained using a simulation based on Bayesian approach.

4 The Empirical Model

We first evaluate the data features for the empirical application. For the empirical analysis, we take into consideration U.S. inflation and real marginal cost series over the period from the first quarter of 1960 until first quarter of 2012. Inflation is computed as the growth rate of the implicit GDP deflator and for the real marginal cost series we use labor share in non-farm business sector. Also we use inflation expectation survey data for the expectation of the next period inflation and we denote it as π_t^S.

Gali and Gerler(1999) showed why labor income share is a good proxy for real marginal cost. Let A_t denote technology, K_t capital, and N_t labor. Then output Y_t is given by

$$Y_t = A_t K_t^{\alpha_k} N_t^{\alpha_n} \tag{20}$$

Real marginal cost series is then given by the ratio of real wage to the marginal product of labor.

$$MC_t = (W_t/P_t)(\partial Y_t/\partial N_t) \tag{21}$$

Hence, given (20) we have

$$MC_t = \frac{Z_t}{\alpha_n} \tag{22}$$

where $Z_t \equiv W_t N_t / P_t Y_t$ is labor income share. Therefore for the log-deviations of labor income share and marginal cost, we have

$$\tilde{mc}_t = \tilde{z}_t \tag{23}$$

For the empirical case, $\tilde{\pi}_t$ and $\tilde{z}_t$ can be interpreted as the transitory components of inflation and marginal cost, in deviation from long-term components. The observed non-filtered inflation and labor income share can be decomposed into permanent and transitory components in a straightforward way as

$$\begin{aligned} \pi_t &= \tilde{\pi}_t + \Pi_t \\ z_t &= \tilde{z}_t + Z_t \end{aligned} \tag{24}$$

where Π_t and Z_t are permanent components of inflation and marginal cost series, i.e, the steady state values of the series.

The structural form representation for the basic NKPC model for filtered data is given as

$$\tilde{\pi}_t = \lambda \tilde{z}_t + \gamma \underbrace{E_t(\tilde{\pi}_{t+1})}_{\pi_t^S - \Pi_t} + \epsilon_{1,t} \tag{25}$$

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \phi_3 \tilde{\pi}_{t-1} + \phi_4 \tilde{\pi}_{t-2} + \phi_5 \tilde{\pi}^S_{t-1} + \phi_6 \tilde{\pi}^S_{t-2} + \epsilon_{2,t} \tag{26}$$

$$\underbrace{E_t(\tilde{\pi}_{t+1})}_{\pi_t^S - \Pi_t} = \psi_1 \tilde{z}_{t-1} + \psi_2 \tilde{z}_{t-2} + \psi_3 \tilde{\pi}_{t-1} + \psi_4 \tilde{\pi}_{t-2} + \psi_5 \tilde{\pi}^S_{t-1} + \psi_6 \tilde{\pi}^S_{t-2} + \epsilon_{3,t} \tag{27}$$

where $(\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t})' \sim NID(0, \Sigma)$

The non-filtered series of U.S inflation, inflations expectations and labor income share is displayed in Figure 2. From Figure 2 we observe two important facts. First there are different periods with different patterns in the inflation series. The period between the beginning of 1970s and the beginning of 1980s has much higher inflation compared to the rest of series. Existing evidence shows that the decline in level and volatility is due to credible monetary policy that stabilized inflationary expectations at a low level via commitment to a nominal anchor since the early eighties, see McConnell and Perez-Quiros(2000); Stock and Watson (2002); Ahmed et al. (2004);

Stock and Watson (2007); Cecchetti et al.(2007).

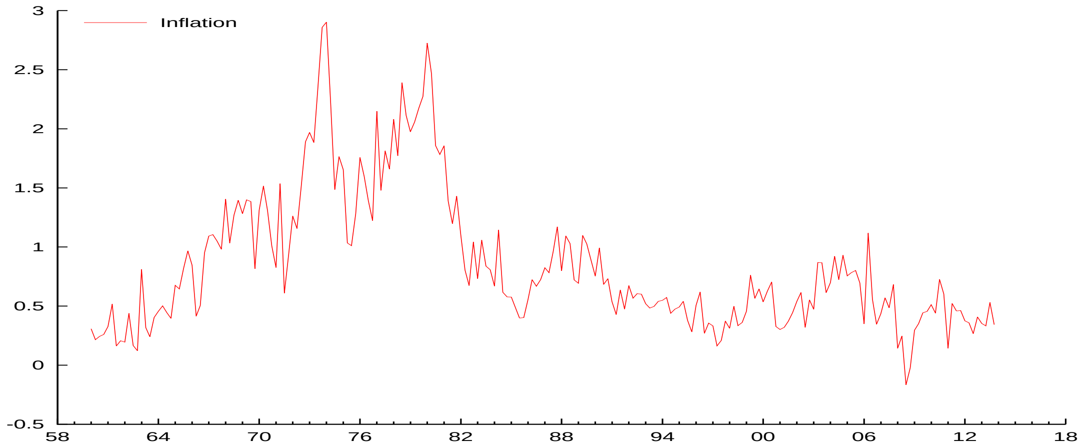
There are two ways to model this changing behaviour of the series. One way is to assign different levels to different periods, i.e, there can be level shifts in the fourth quarter of 1967 and the first quarter of 1983, that is, a higher level is assigned to the period between the fourth quarter of 1967 and the first quarter of 1983 and a lower level is assigned to the remaining periods, see Basturk et. al. (2012). In that way, it is possible to model this changing behaviour of the series to allow for regime changes in parameters to explain the change in the structure of the series, see Cogley and Sargent (2005); Canova and Gambetti(2006); Kim and Nelson (2006); Sims and Zha(2006); Cogley and Sbordone(2008), among others. In our case we assume that the level shifts occur in each time period continuously. Then we can model the time varying inflation level with a random walk process for the level of inflation as follows

$$\Pi_t = \Pi_{t-1} + \eta_{1,t} \tag{28}$$

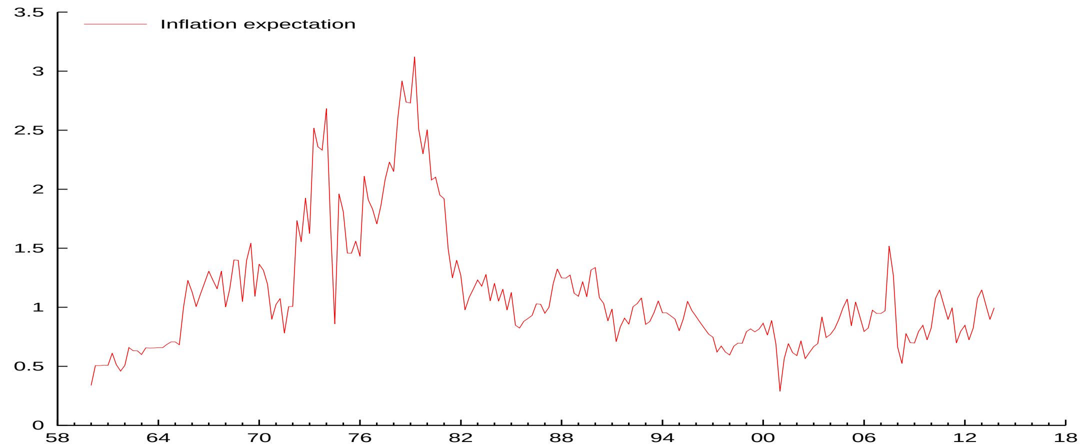
where $\eta_{1,t} \sim NID(0, \sigma^2_{\eta 1})$

The real marginal cost series is analysed in Figure 2. For a visual inspection, we also put a changing trend extracted using the Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1997). Unlike the inflation series we can not spot discrete changes during the sample period for the labor income share series. Instead the labor income share data follows a continuously changing pattern around a negative trend which can be explained with technology shocks. We use local linear trend model to allow for a changing trend because the magnitude of labor income share gets amplified in the second part of sample period

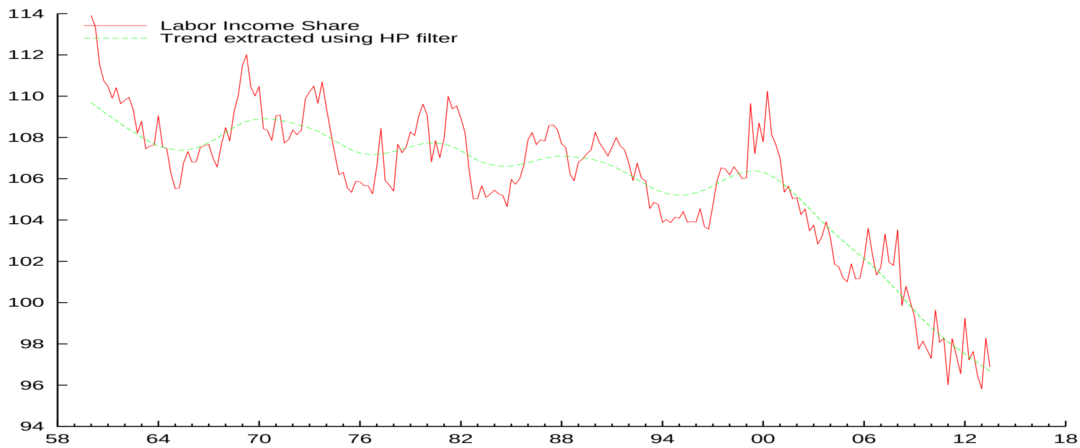
$$\begin{aligned} Z_{t+1} &= \mu_{z,t} + Z_t + \eta_{2,t+1} \\ \mu_{z,t+1} &= \mu_{z,t} + \eta_{2,t+1} \end{aligned} \tag{29}$$



(a) Inflation



(b) Inflation Expectation



(c) Labor Income Share

Figure 1: Inflation, inflation expectations and real marginal cost series over first quarter of 1960 to the first quarter 2014

where see Durbin and Koopman(2001) for details. It is possible to obtain many types of filters by changing the configuration of local linear trend model used for detrending, see see Delle Monache and Harvey (2011), see also Canova (2012) for a similar specification in the more general context of DSGE models. When $\sigma^2_{\eta 3} = 0$, the level of labor income share follows a random walk with drift, $\mu_{z,t}$. When $\sigma^2_{\eta 2} = 0$, a deterministic trend is obtained. On the other hand setting $\sigma^2_{\eta 2} = 0$ and taking $\sigma^2_{\eta 3}$ as positive generates an integrated random walk process which can approximate many different types of nonlinear trends including the Hodrick-Prescott (HP) filter.

The NKPC model in (6) using (24), (28) and (29) takes the following form

$$\begin{aligned}
\pi_t - \Pi_t &= \lambda_t(mc_t - MC_t) + \gamma_t(\pi^S_t - \Pi_t) + \epsilon_{1,t}, \\
z_t - Z_t &= \phi_1(z_{t-1} - Z_{t-1}) + \phi_2(z_{t-2} - Z_{t-2}) + \phi_3(\pi_{t-1} - \Pi_{t-1}) \\
&\quad + \phi_4(\pi_{t-2} - \Pi_{t-2}) + \phi_5(\pi^S_{t-1} - \Pi_{t-1}) + \phi_6(\pi^S_{t-2} - \Pi_{t-2}) + \epsilon_{2,t}, \\
\pi^S_t - \Pi_t &= \psi_1(z_{t-1} - Z_{t-1}) + \psi_2(z_{t-2} - Z_{t-2}) + \psi_3(\pi_{t-1} - \Pi_{t-1}) \\
&\quad + \psi_4(\pi_{t-2} - \Pi_{t-2}) + \phi_5(\pi^S_{t-1} - \Pi_{t-1}) + \psi_6(\pi^S_{t-2} - \Pi_{t-2}) + \epsilon_{3,t}, \\
\Pi_{t+1} &= \Pi_t + \eta_{1,t+1} \\
Z_{t+1} &= \mu_{z,t} + Z_t + \eta_{2,t+1} \\
\mu_{z,t+1} &= \mu_{z,t} + \eta_{3,t+1}
\end{aligned} \tag{30}$$

where $(\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t})' \sim NID\left(0, \begin{pmatrix} \sigma^2_{\epsilon 1} & \rho_1\sigma_{\epsilon 1}\sigma_{\epsilon 2} & \rho_2\sigma_{\epsilon 1}\sigma_{\epsilon 3} \\ \rho_1\sigma_{\epsilon 1}\sigma_{\epsilon 2} & \sigma^2_{\epsilon 2} & \rho_3\sigma_{\epsilon 2}\sigma_{\epsilon 3} \\ \rho_2\sigma_{\epsilon 1}\sigma_{\epsilon 3} & \rho_3\sigma_{\epsilon 2}\sigma_{\epsilon 3} & \sigma^2_{\epsilon 3} \end{pmatrix}\right)$,
$(\eta_{1,t}, \eta_{2,t}, \eta_{3,t})' \sim NID\left(0, \begin{pmatrix} \sigma^2_{\eta 1} & 0 & 0 \\ 0 & \sigma^2_{\eta 2} & 0 \\ 0 & 0 & \sigma^2_{\eta 3} \end{pmatrix}\right)$ and the residuals $(\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t})'$ and $(\eta_{1,t}, \eta_{2,t}, \eta_{3,t})'$ are independent for all t.

Adding Stochastic Volatility

A further refinement in the NKPC model can be done allowing for time dependency in residual variances. This modification is particularly necessary for the inflation series, since the volatility of this series changes over time considerably, e.g. Stock and Watson(2007) for a reduced form model with a stochastic volatility component. To incorporate stochastic volatility for the inflation level into the NKPC model, we add the following state equation to the system

$$h_{t+1} = h_t + \eta_{4,t+1}, \eta_{4,t+1} \sim NID(0, \sigma^2_{\eta 4}) \tag{31}$$

where the error term of the first equation in (31) has a time-varying variance $\sigma^2_{\epsilon_1,t} = exp(h_t/2)$. We follow the practice of Stock and Watson(2007) by setting the value of $\sigma^2_{\eta 4}$ prior to analysis to facilitate estimation. We assume $\sigma^2_{\eta 4} = 0.5$, which seems to work well for the U.S. inflation series, see Basturk et. al. (2012).

Then we make another modification to the model to investigate the dynamics between US inflation and volatility. We change the state equations for inflation trend and volatility a little bit as follows

$$\begin{aligned} \Pi_{t+1} &= \Pi_t + \tau_h h_t + \eta_{1,t+1} \\ h_{t+1} &= h_t + \tau_\pi \Pi_t + \eta_{4,t+1} \end{aligned} \tag{32}$$

where $(\eta_{1,t}, \eta_{4,t})' \sim NID\left(0, \begin{pmatrix} \sigma^2_{\eta 1} & \rho_4 \sigma_{\eta 1} \sigma_{\eta 4} \\ \rho_4 \sigma_{\eta 1} \sigma_{\eta 4} & \sigma^2_{\eta 4} \end{pmatrix}\right)$. We put additional terms involving h_t and Π_t to state equations for inflation level and stochastic volatility, respectively. With these refinement the model takes following form,

$$
\begin{aligned}
\pi_t - \Pi_t &= \lambda(mc_t - MC_t) + \gamma(\pi_t^S - \Pi_t) + \epsilon_{1,t}, \\
z_t - Z_t &= \phi_1(z_{t-1} - Z_{t-1}) + \phi_2(z_{t-2} - Z_{t-2}) + \phi_3(\pi_{t-1} - \Pi_{t-1}) \\
&\quad + \phi_4(\pi_{t-2} - \Pi_{t-2}) + \phi_5(\pi_{t-1}^S - \Pi_{t-1}) + \phi_6(\pi_{t-2}^S - \Pi_{t-2}) + \epsilon_{2,t}, \\
\pi_t^S - \Pi_t &= \psi_1(z_{t-1} - Z_{t-1}) + \psi_2(z_{t-2} - Z_{t-2}) + \psi_3(\pi_{t-1} - \Pi_{t-1}) \\
&\quad + \psi_4(\pi_{t-2} - \Pi_{t-2}) + \phi_5(\pi_{t-1}^S - \Pi_{t-1}) + \psi_6(\pi_{t-2}^S - \Pi_{t-2}) + \epsilon_{3,t}, \\
\Pi_{t+1} &= \Pi_t + \tau_h h_t + \eta_{1,t+1} \\
Z_{t+1} &= \mu_{z,t} + Z_t + \eta_{2,t+1} \\
\mu_{z,t+1} &= \mu_{z,t} + \eta_{3,t+1} \\
h_{t+1} &= h_t + \tau_\pi \Pi_t + \eta_{4,t+1} \qquad (33)
\end{aligned}
$$

where $(\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t})' \sim NID\left(0, \begin{pmatrix} \sigma_{\epsilon 1}^2 & \rho_1\sigma_{\epsilon 1}\sigma_{\epsilon 2} & \rho_2\sigma_{\epsilon 1}\sigma_{\epsilon 3} \\ \rho_1\sigma_{\epsilon 1}\sigma_{\epsilon 2} & \sigma_{\epsilon 2}^2 & \rho_3\sigma_{\epsilon 2}\sigma_{\epsilon 3} \\ \rho_2\sigma_{\epsilon 1}\sigma_{\epsilon 3} & \rho_3\sigma_{\epsilon 2}\sigma_{\epsilon 3} & \sigma_{\epsilon 3}^2 \end{pmatrix}\right)$,
$(\eta_{1,t}, \eta_{2,t}, \eta_{3,t})' \sim NID\left(0, \begin{pmatrix} \sigma_{\eta 1}^2 & 0 & 0 \\ 0 & \sigma_{\eta 2}^2 & 0 \\ 0 & 0 & \sigma_{\eta 3}^2 \end{pmatrix}\right)$ and $(\eta_{1,t}, \eta_{4,t})' \sim NID\left(0, \begin{pmatrix} \sigma_{\eta 1}^2 & \rho_4\sigma_{\eta 1}\sigma_{\eta 4} \\ \rho_4\sigma_{\eta 1}\sigma_{\eta 4} & \sigma_{\eta 4}^2 \end{pmatrix}\right)$.

The NKPC equation (17) we derived as log-linearisation around zero trend inflation also indicates that the parameters λ and γ also should be time varying, because in the explicit form they involve time varying trend inflation. We assume these parameters as constant to get better identification for the volatility, because for this study volatility is of primary importance.

5 Bayesian Inference

In this section we give summary of Bayesian inference algorithm for the NKPC models which are obtained by the product of the likelihood function and the prior distribution for the parameters. The likelihood functions of the NKPC models are multivariate normal densities, because we assume that errors have normal distributions. We presents details on how the prior distributions are determined and the posterior sampler in this section. More details are presented in Appendix B.

How to determine prior distributions in the NKPC models is crucial because the likelihood of NKPC is often flat(see Kleibergen and Mavroeidis(2011)). One way to deal with this difficulty is to set prior distributions of parameters as informative. However, this may deteriorates posterior inference of parameters. Therefore we use flat prior distributions for parameters and we assign informative prior distributions to the observation variances.

For the parameters of the NKPC model, we use independent flat prior distributions on restricted regions. The range of these regions are based on the underlying economic theory. We determine the intervals for parameters γ and λ as the unit interval. For the states we use a diagonal covariance matrix with an uninformative prior distribution implying that the shocks to the long-run inflation and real marginal cost are independent.

Posterior distributions are obtained as the product of the prior distributions and the likelihood function. Because the number and the location of structural breaks are unknown the likelihood function is hard to tract. Therefore, we construct a MCMC algorithm to sample from the full conditional posterior distributions. Specifically, we use Gibbs sampling. Gibbs sampling steps are based on Kim and Nelson(1999); Gerlach et al.(2000); Cakmakli et al. (2011). Details of MCMC algorithm are given in appendix B.

6 Posterior Evidence

In this section we present posterior evidence on dynamics between the U.S.inflation level and volatility. We estimate two NKPC models, where the first model uses set of equations in (30) with stochastic volatility. The second model is the one at which we included refinements (32) to explain the link between the inflation level and volatility.

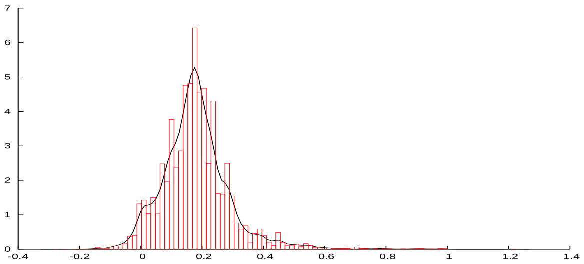
We display the estimation results for the model (30) in Table 1 and the distribution of parameters in Figure 2.

Table 1: Estimation Results of Parameters for standard NKPC model

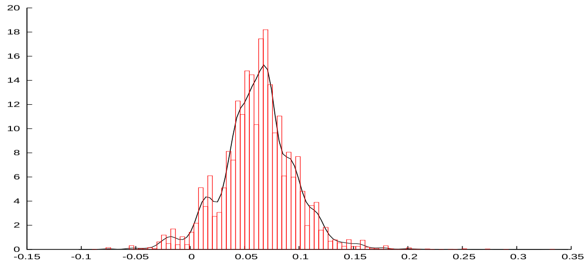
γ	ϕ_1	ϕ_2	ϕ_3	ϕ_4
0.1493 (0.0127)	0.7284 (0.0117)	0.2075 (0.0141)	0.2853 (0.1639)	-0.6180 (0.1364)
λ	ψ_1	ψ_2	ψ_3	ψ_4
0.0621 (0.0012)	0.0094 (0.0015)	-0.0046 (0.0014)	0.0812 (0.0319)	-0.2278 (0.0169)
ϕ_5	ψ_5	ϕ_6	ψ_6	
0.1672 (0.1993)	0.6153(0.0162)	0.0648(0.1064)	0.3332(0.0131)	
ρ_1	ρ_2	ρ_3		
-0.0189 (0.0055)	-0.8889 (0.0067)	0.0057(0.0036)		

Note: The table presents posterior means and standard deviations (in parentheses) of parameters for standard New Keynesian Phillips Curve(NKPC) models estimated for quarterly inflation and real marginal cost over the period from the first quarter of 1960 and the first quarter of 2012. γ is the coefficient of inflation expectations in (30). λ is the slope of the Phillips Curve. $\phi_1, \phi_2, \phi_3, \phi_4, \phi_5$ and ϕ_6 are the parameters for the equation of labor income share in the second line of (30). $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5$ and ψ_6 are the parameters for the equation of inflation expectation survey data in the third line of (30). ρ_1, ρ_2 and ρ_3 are the correlation coefficients between residuals ϵ_1 and ϵ_2, ϵ_1 and ϵ_3, ϵ_2 and ϵ_3, respectively.

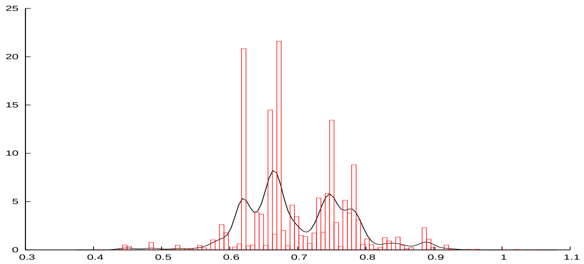
The slope of Phillips curve is estimated approximately as 0.06 that implies an almost flat curve(see e.g. Gali and Gertler (1999); Gali et al. (2005)). The coeffi-



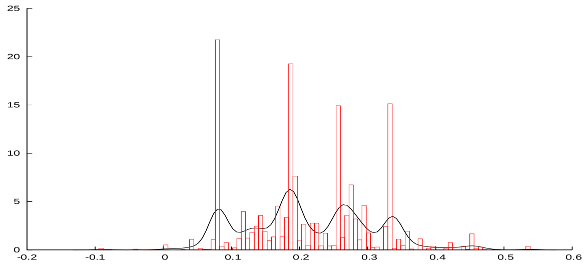
(a) γ



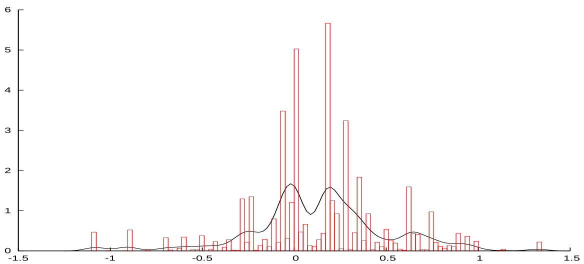
(b) λ



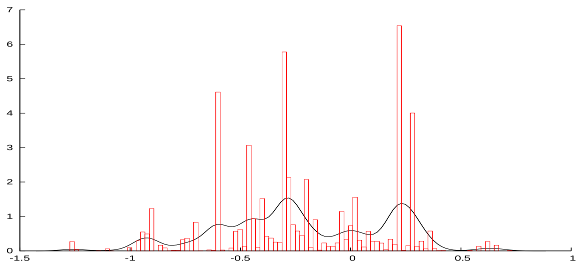
(c) ϕ_1



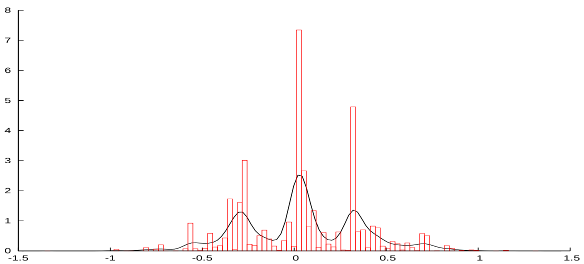
(d) ϕ_2



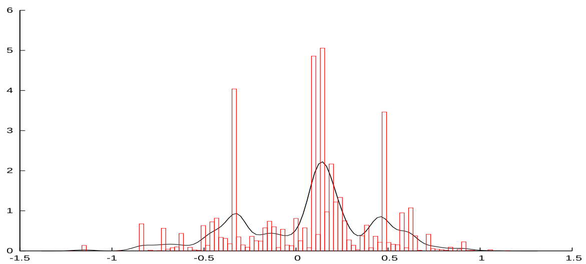
(e) ϕ_3



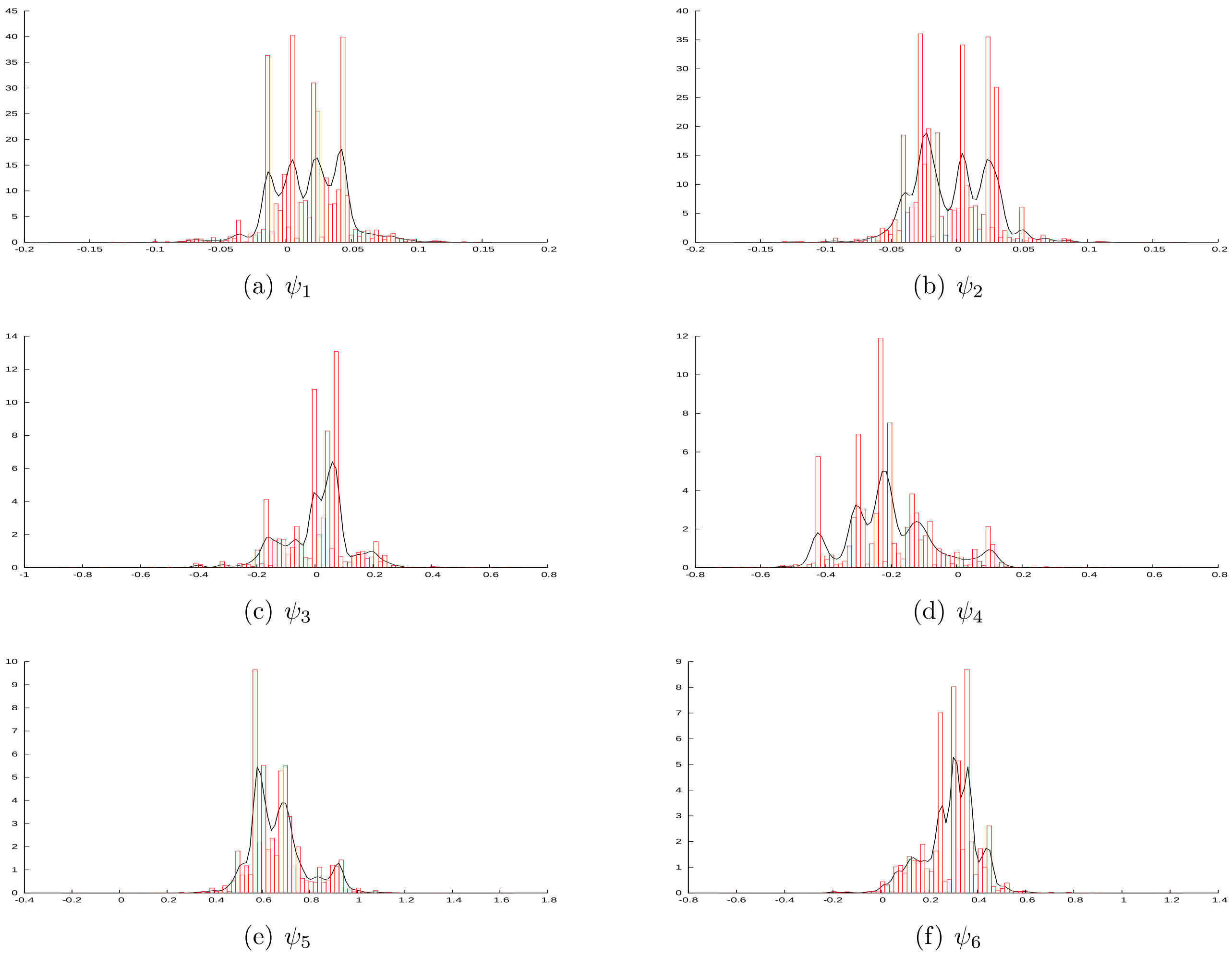
(f) ϕ_4



(g) ϕ_5



(h) ϕ_6



(a) ψ_1 (b) ψ_2

(c) ψ_3 (d) ψ_4

(e) ψ_5 (f) ψ_6

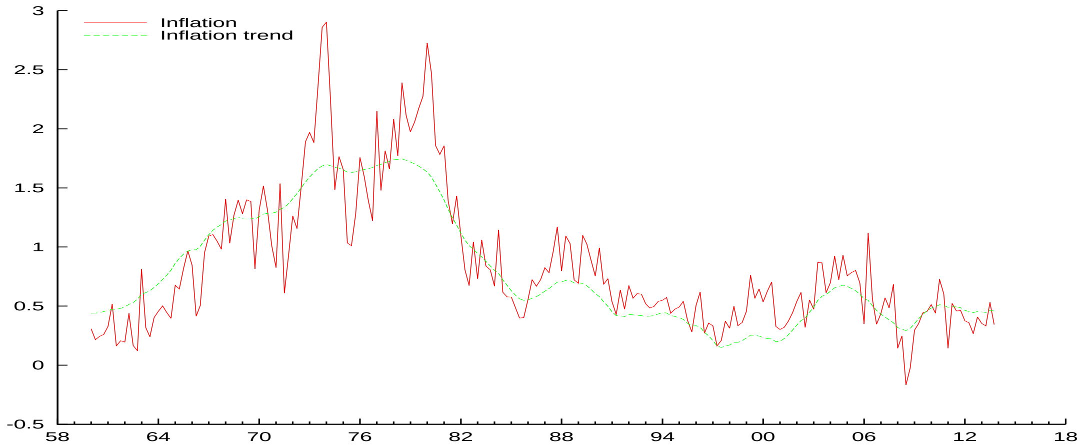
Figure 2: Distribution of parameters for standard NKPC model

cient of the inflation expectation, γ is much lower than the estimates obtained with conventional methods, which is above 0.9 in most of cases. There are two reasons for this finding. First some of its effect is captured by the inflation trend, so the effect of the parameter γ is alleviated. The another reason is that conventional analysis replaces inflation expectation of the next period by the nest period's inflation based on the rational expectational hypothesis, but on the contrary we use survey based inflation expectations to replace the short-run inflation expectations relaxing the rational expectations assumption.

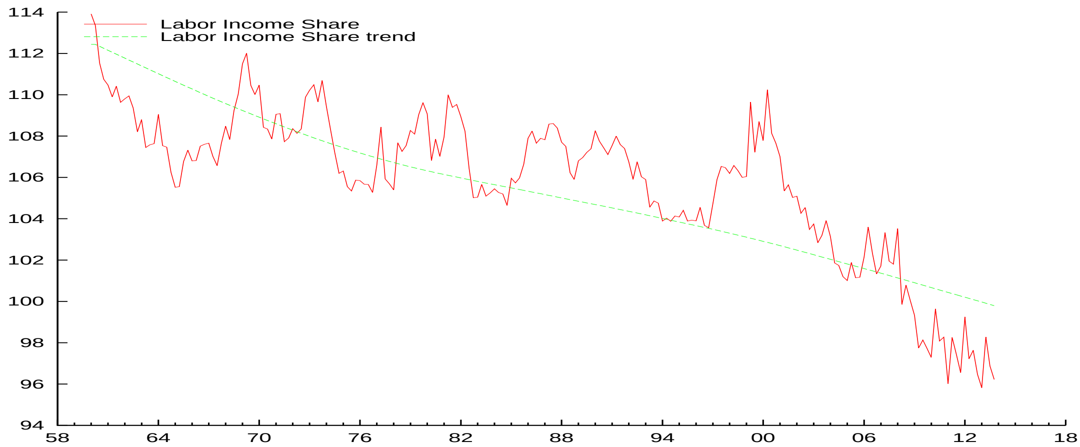
Figure 3 shows the estimated levels for inflation and marginal cost and the volatility of inflation. Estimated inflation level nicely track the observed inflation. One reason for this feature is that the uncertainty of the estimated inflation decreases as part of the uncertainty is reflected in the stochastic volatility process. In other words, a stochastic volatility structure captures part of the inflation uncertainty.

The second panel in Figure 3 presents the estimated results for the real marginal cost. The most prominent feature is the smoothness of estimated result. Marginal cost series follows a slightly nonlinear trend during the sample period.

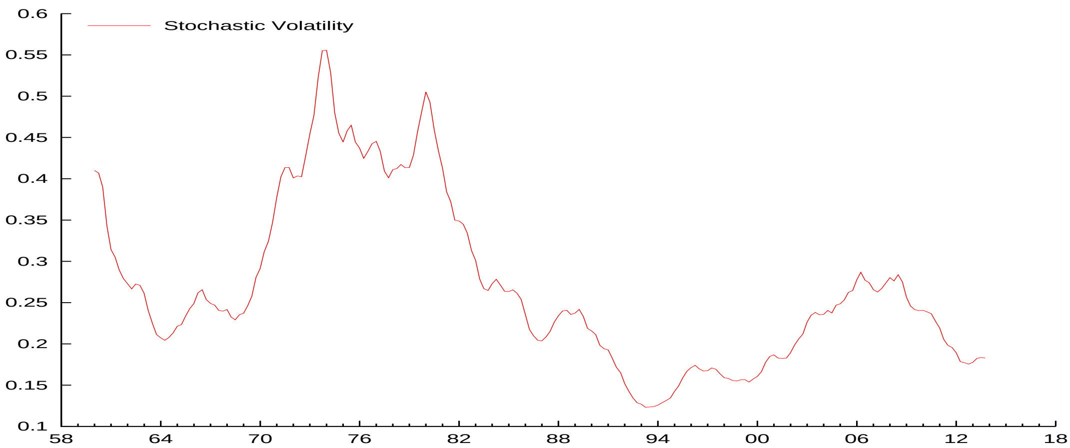
The last panel in Figure 4 presents the estimated inflation volatilities for the NKPC model (30) with stochastic volatility. The stochastic volatility pattern in the figure explains nicely the findings on Great Moderation, which accompanies the decline of the volatility of many U.S. macroeconomic series, see McConell and Perez-Quiros (2000) among others. The period before the beginning of 1980s has high inflation levels with a high volatility, whereas inflation becomes more stable in the second half of the sample period. The decline in inflation volatility after 1980s is related to credible monetary policy that stabilized inflationary expectations at a low level via commitment to a nominal anchor since the early eighties, see Stock and Watson (2002); Ahmed et al. (2004); Stock and Watson (2007). This period of low volatility is followed by a period with high volatility after 2005 and during recent



(g) Inflation and its trend



(h) Labor Income Share and its trend



(i) Inflation volatility

Figure 3: Inflation with its trend, labor income share with its trend and volatility of inflation over first quarter of 1960 to the first quarter 2014

financial crisis.

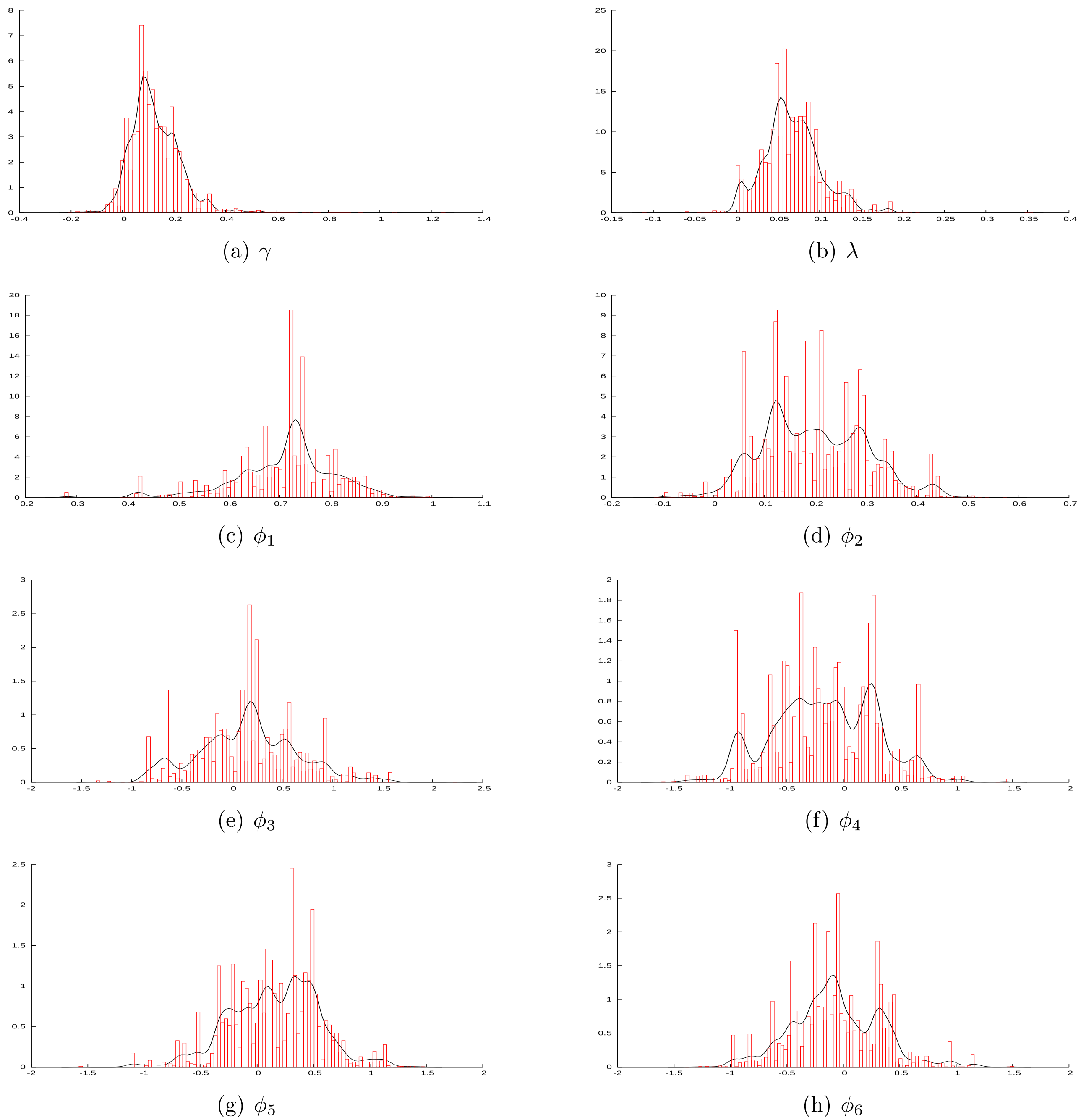
We display estimation results for the NKPC model (33) in Table 2 and distributions of parameters in Figure 4.

Table 2: Estimation Results of Parameters for NKPC model with volatility and level refinements

γ	ϕ_1	ϕ_2	ϕ_3	ϕ_4
0.1272 (0.0097)	0.7126 (0.0093)	0.1990 (0.0102)	0.1562 (0.2400)	-0.1674 (0.2043)
λ	ψ_1	ψ_2	ψ_3	ψ_4
0.0667 (0.0012)	0.0172 (0.0012)	-0.0019 (0.0013)	-0.0547 (0.0254)	-0.1766 (0.0254)
ϕ_5	ψ_5	ϕ_6	ψ_6	ρ_1
0.1540 (0.1511)	0.6761 (0.0231)	-0.0661 (0.1499)	0.2653 (0.0236)	0.020 (0.0055)
ρ_2	ρ_3	τ_h	τ_π	ρ_4
-0.8914 (0.0039)	0.0027(0.001)	-0.0005 (0.0001)	0.0609 (0.0022)	0.0002 (0.0001)

Note: The table presents posterior means and standard deviations (in parentheses) of parameters for the NKPC model which allows for the effect of volatility and level to each other.

The estimates for parameters $\lambda, \gamma, \phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \psi_1, \psi_2, \psi_3, \psi_4, \psi_5$ and ψ_6 are approximately equal to estimates of the first NKPC model. The interesting point here is the estimates of coefficients of inflation level and volatility in the inflation level and volatility state equations, respectively. The coefficient τ_h is estimated very close to zero which indicates that the inflation volatility does not have any effect on the inflation level. On the contrary the coefficient is estimated as 0.0609. This result shows that inflation level has a significant effect on the inflation volatility, i.e. , when the quarterly inflation increases by %2 percent, the volatility increases by 0.12 which is a considerable increase for the volatility. Therefore, empirical results indicates that the past volatility does not have significant impact on the level, whereas the past level of inflation affects the volatility of inflation.



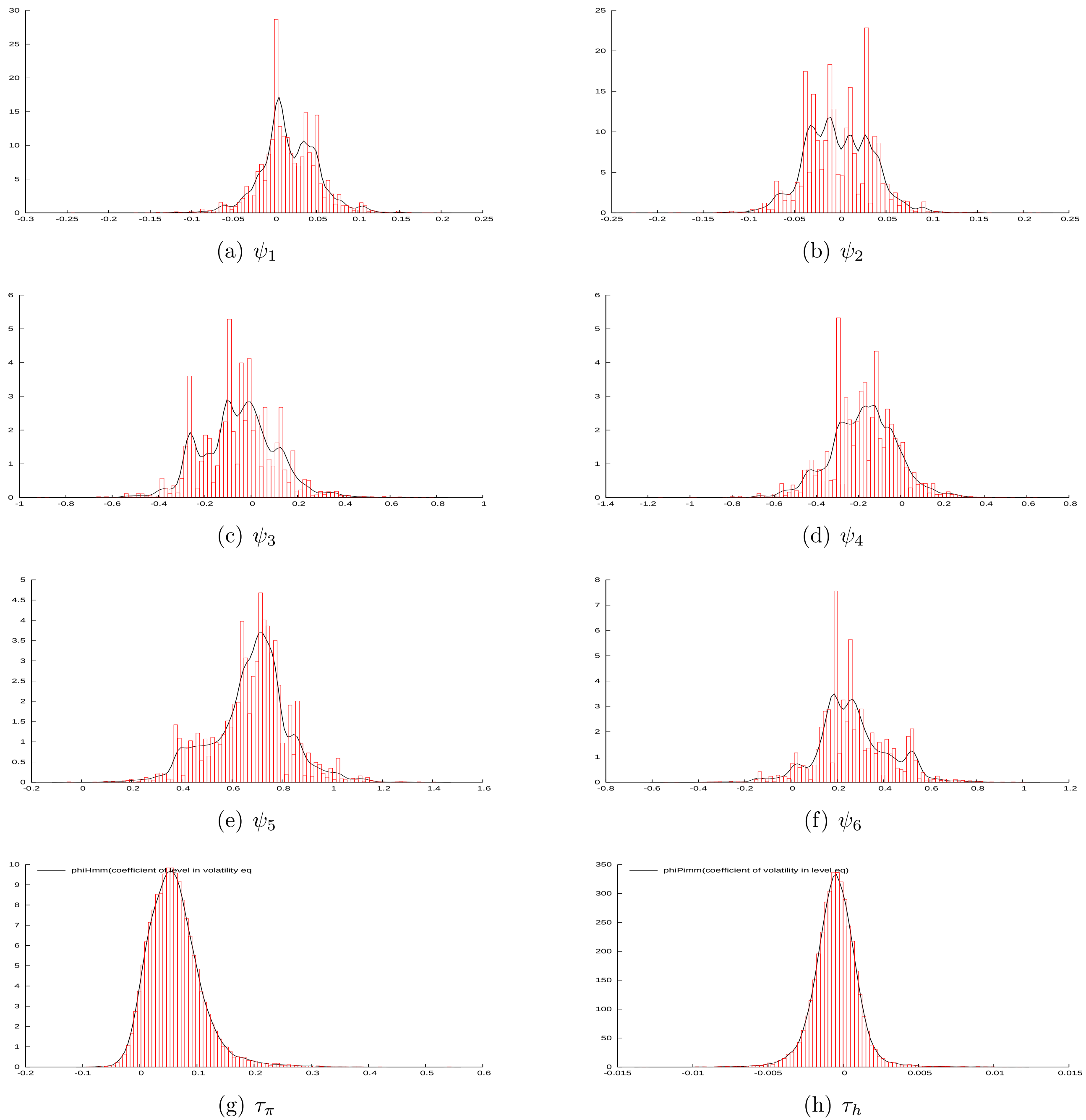
(a) γ (b) λ

(c) ϕ_1 (d) ϕ_2

(e) ϕ_3 (f) ϕ_4

(g) ϕ_5 (h) ϕ_6

Figure 4



(a) ψ_1 (b) ψ_2

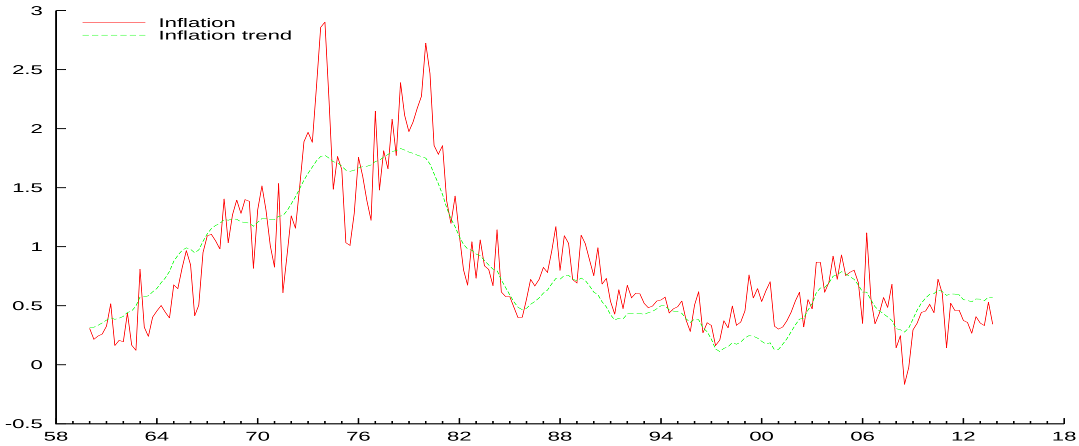
(c) ψ_3 (d) ψ_4

(e) ψ_5 (f) ψ_6

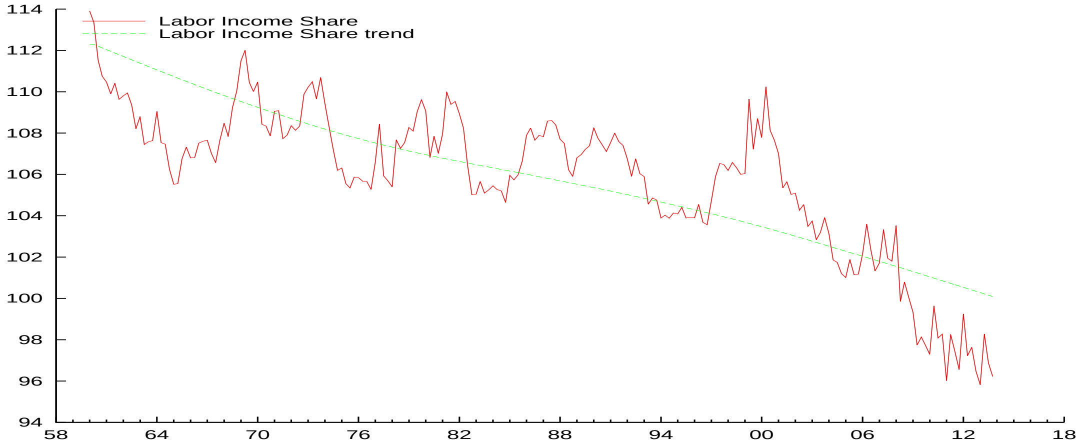
(g) τ_π (h) τ_h

Figure 4: Estimation Results of Parameters for NKPC model with volatility and level refinements

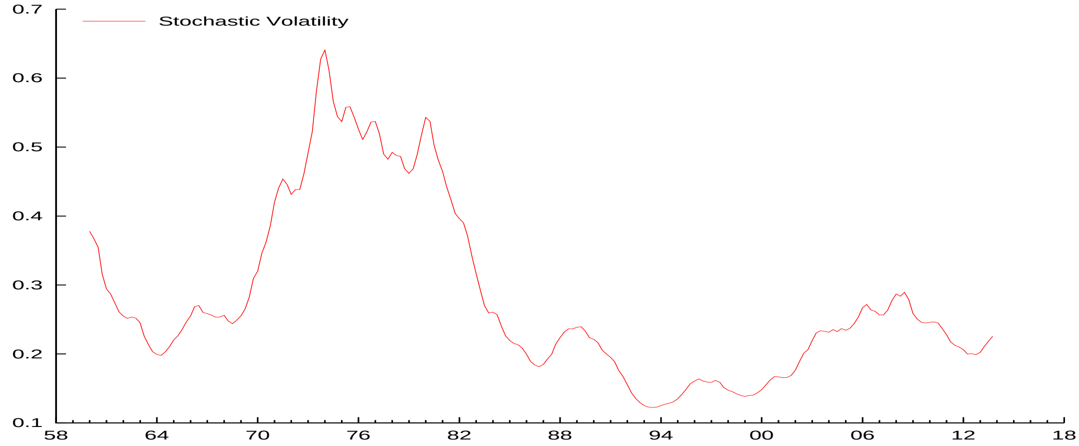
Figure 5 shows the estimated levels for inflation, marginal cost and the inflation volatility.Similar results are obtained compared to the previous model. Estimated inflation level and labor income share level nicely tracks the inflation and labor income share, respectively. The estimated level for labor income share is smooth and approximately linear. The estimated volatility also shows clearly the effect of inflation level on the inflation volatility. In the period from the begining of 1970s to the beginning of 1980s where the inflation level is higher compared to the remaining parts, the volatility is also estimated higher in reference to the rest.



(i) Inflation and its trend



(j) Labor Income Share and its trend



(k) Inflation volatility

Figure 5: Inflation with its trend, labor income share with its trend and volatility of inflation over first quarter of 1960 to the first quarter 2014

7 Conclusion

How we understand monetary policy and its macroeconomic effects has developed substantially owing to the various works done in this literature. The NKPC is an important part of macroeconomic models which is essential for policy analysis. One of the most productive areas in macroeconomic research is New Keynesian Phillips Curves(NKPC). Over time the NKPC models improved and new features are added in order to fit the model to the data better. Following the changes occurred in this area, we derived three different NKPC equations which are NKPC with zero trend inflation, NKPC with trend inflation and NKPC with time varying trend inflation. These models are based on Calvo type pricing where the firms that cannot optimize their prices fix their prices at the previous period's level. We analyzed the dynamics between the U.S. inflation level and volatility using the NKPC model we derived as log-linearisation around time varying steady state inflation. The extendend NKPC shows that the volatility of U.S is guided mainly by inflation volatility and shock to expectations of inflation.

In the empirical part we estimated a simplified version of our NKPC model with time varying inflation trend using US quarterly inflation and labor income share data over the period between the first quarter of 1960 until the first quarter of 2014 using Bayesian inference. The model includes both time varying level of labor income share and stochastic volatility besides time varying inflation trend. Also we used inflation expectation survey data instead expectation of next period inflation relaxing the rational expectation model. By adding some refinements to this model we checked how the link between inflation volatility and level works. As our findings in theoretical part showed, the empirical results indicates that empirical results verify that past volatility does not have a significant impact on the level, whereas the past level of inflation affects the volatility of the inflation.

Appendix

A Derivation of NKPC equations

A.1 Derivation of NKPC with zero trend inflation

Suppose that there is a monopolistic competition with continuum of intermediate goods producers $i \in [0,1]$ and the constant returns to scale production function of each firm is given by $Y_t(i) = A_t K_t(i)^{(1-\sigma)} L_t(i)^{\sigma}$ .All the firms face same demand function $C_t(i) = (\frac{P_t(i)}{P_t})^{-\epsilon} C_t$ and the aggregate price level is given by $P_t = (\int_0^1 P_t(i)^{1-\epsilon} \mathrm{d}i)^{1/(1-\epsilon)}$.

We assume Calvo price setting which means that at each time period t, $(1-\theta)$ fraction of firms can reoptimize their prices while θ fraction of firms cannot change the prices. Let $S(t)$ be the set of firms that are not re-optimizing their prices at time t.Assume that all the firms that re-optimize will choose the same price P_t^* From the definition of the aggregate price level, we can write;

$$P_t = [\int_{S(t)} P_{t-1}(i)^{1-\epsilon} di + (1-\theta)(P_t^*)^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{34}$$

$$= [\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{35}$$

We will log-linearize equation(35). We define log-linearized series $\tilde{x}_t$ around its steady state X as;

$$\tilde{x}_t \equiv \ln X_t - \ln X$$

Rearranging, we can solve this for X_t:

$$X_t = Xe^{\tilde{x}_t} \tag{36}$$

First order Taylor approximation of $e^{\tilde{x}_t}$ around $\tilde{x}_t = 0$ is given below

$$\tilde{x}_t \approx e^0 + e^0(\tilde{x}_t - 0) = 1 + \tilde{x}_t$$

Therefore, equation (36) becomes

$$X_t = X(1 + \tilde{x}_t)$$

To log-linearize equation (35), first write each variable component of equation (35) as;

$$P_t = Pe^{\tilde{p}_t}$$
$$P_{t-1} = P_{-1}e^{\tilde{p}_{t-1}}$$
$$P_t^* = P^*e^{\tilde{p}_t^*}$$

Rearrange equation (35) and substitute the above equalities;

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \tag{37}$$
$$P^{1-\epsilon}e^{(1-\epsilon)\tilde{p}_t} = \theta P_{-1}^{1-\epsilon}e^{(1-\epsilon)\tilde{p}_{t-1}} + (1-\theta)P^{*1-\epsilon}e^{(1-\epsilon)\tilde{p}_t^*}$$
$$P^{1-\epsilon}[1 + (1-\epsilon)\tilde{p}_t] = \theta P_{-1}^{1-\epsilon}[1 + (1-\epsilon)\tilde{p}_{t-1}] + (1-\theta)P^{*1-\epsilon}[1 + (1-\epsilon)\tilde{p}_t^*]$$
$$P^{1-\epsilon} + P^{1-\epsilon}(1-\epsilon)\tilde{p}_t = \theta P_{-1}^{1-\epsilon} + \theta P_{-1}^{1-\epsilon}(1-\epsilon)\tilde{p}_{t-1} + (1-\theta)P^{*1-\epsilon} + (1-\theta)P^{*1-\epsilon}(1-\epsilon)\tilde{p}_t^*$$

Using the steady state expression of equation (37) $P^{1-\epsilon} = \theta P_{-1}^{1-\epsilon} + (1-\theta)(P^*)^{1-\epsilon}$, the equality $P = P_{-1} = P^*$ at the steady state and simplifying;

$$\begin{aligned} P^{1-\epsilon}(1-\epsilon)\tilde{p}_t &= \theta P_{-1}^{1-\epsilon}(1-\epsilon)\tilde{p}_{t-1} + (1-\theta)P^{*1-\epsilon}(1-\epsilon)\tilde{p}_t^* \\ \tilde{p}_t &= \theta\tilde{p}_{t-1} + (1-\theta)\tilde{p}_t^* \end{aligned} \tag{38}$$

The intermediate goods firms that reset prices at time t tries to find the P_t^* that maximizes the expected sum of discounted real profits subject to the demand function

$$\begin{aligned} \max \quad & E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j[\left(\frac{P_t^*}{P_{t+j}}\right)Y_{t+j}(i) - TC_{i,t+j}(Y_{t+j}(i))]\} \\ \text{s.t.} \quad & Y_{t+j}(i) = \left(\frac{P_t^*}{P_{t+j}}\right)^{-\epsilon}Y_t \end{aligned} \tag{39}$$

where γ is stochastic discount factor and TC_i is real total costs.

Inserting the constraint into the objective function, we can write the optimization problem as;

$$\max \qquad E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j[\left(\frac{P_t^*}{P_{t+j}}\right)\left(\frac{P_t^*}{P_{t+j}}\right)^{-\epsilon}Y_t - TC_{i,t+j}(\left(\frac{P_t^*}{P_{t+j}}\right)^{-\epsilon}Y_t)]\}$$

FOC with respect to P_t^* is;

$$E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j(1-\epsilon)\left(\frac{P_t^*}{P_{t+j}}\right)^{-\epsilon}\frac{1}{P_{t+j}}Y_{t+j}\} = E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j MC_{i,t+j}(-\epsilon)(\left(\frac{P_t^*}{P_{t+j}}\right)^{-\epsilon}\frac{1}{P_{t+j}}Y_{t+j})\}$$

Rearranging the terms will give the optimum price;

$$P_t^* = \frac{\epsilon}{\epsilon-1}\frac{E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j MC_{i,t+j}(P_{t+j})^{\epsilon}Y_{t+j}\}}{E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j(P_{t+j})^{\epsilon-1}Y_{t+j}\}} \tag{40}$$

Now, rearrange this by dividing both sides by Pt:

$$
\begin{aligned}
\frac{P_t^*}{P_t} &= \frac{\epsilon}{\epsilon-1}\frac{E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j MC_{i,t+j}\left(\frac{P_{t+j}}{P_t}\right)^{\epsilon}Y_{t+j}\}}{E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j\left(\frac{P_{t+j}}{P_t}\right)^{\epsilon-1}Y_{t+j}\}} \\
\frac{P_t^*}{P_t} &= \frac{\epsilon}{\epsilon-1}\frac{E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j MC_{i,t+j}(\Pi_{t,t+j})^{\epsilon}Y_{t+j}\}}{E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j(\Pi_{t,t+j})^{\epsilon-1}Y_{t+j}\}} \\
\frac{P_t^*}{P_t} &= \frac{\epsilon}{\epsilon-1}\frac{E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j MC_{i,t+j}(\Pi_{t+1}\times\Pi_{t+2}\times\cdots\times\Pi_{t+j})^{\epsilon}Y_{t+j}\}}{E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j(\Pi_{t+1}\times\Pi_{t+2}\times\cdots\times\Pi_{t+j})^{\epsilon-1}Y_{t+j}\}}
\end{aligned} \tag{41}
$$

where $\frac{P_{t+j}}{P_t} = \Pi_{t,t+j} = \Pi_{t+1}\times\Pi_{t+2}\times\cdots\times\Pi_{t+j}$

Log-Linearization of the Variables

Assuming that at steady state, the real variables MC and Y are constant and the nominal variable inflation grow at constant rate Π , the expression for the steady state of equation (41) is given below. We will, for the time being, assume zero steady state inflation, i.e. $Pi = 1$, and log-linearize equation (41) around this steady state.

$$
\begin{aligned}
\frac{P^*}{P} &= \frac{\epsilon}{\epsilon-1}\frac{\sum_{j=0}^{\infty}\theta^j\gamma^j MC_i(\Pi)^{\epsilon}Y}{\sum_{j=0}^{\infty}\theta^j\gamma^j(\Pi)^{\epsilon-1}Y} \\
\frac{P^*}{P} &= \frac{\epsilon}{\epsilon-1}MC_i
\end{aligned} \tag{42}
$$

Then, rewrite the individual components of (41) as follows;

$$P_t^* = P^* e^{\tilde{p}_t^*}$$

$$P_t = P e^{\tilde{p}_t}$$

$$MC_{i,t+j} = MC_i e^{\tilde{m}c_{t+j}}$$

$$Y_{t+j} = Y e^{\tilde{y}_{t+j}}$$

$$\Pi_{t,t+j} = \Pi^j e^{\tilde{\pi}_{t,t+j}}$$

and substitute back in (41) to get;

$$\frac{P^* e^{\tilde{p}_t^*}}{P e^{\tilde{p}_t}} = \frac{\epsilon}{\epsilon - 1} MC_i \frac{E_t\{\sum_{j=0}^{\infty}(\theta\gamma)^j e^{\tilde{m}c_{t+j}} \Pi^{j\epsilon} e^{\epsilon\tilde{\pi}_{t,t+j}} Y e^{\tilde{y}_{t+j}}\}}{E_t\{\sum_{j=0}^{\infty}(\theta\gamma)^j \Pi^{j(\epsilon-1)} e^{(\epsilon-1)\tilde{\pi}_{t,t+j}} Y e^{\tilde{y}_{t+j}}\}} \tag{43}$$

Simplifying this term using the steady state expression in (42) where we impose zero steady state assumption ($\Pi = 1$), we have;

$$\frac{e^{\tilde{p}_t^*}}{e^{\tilde{p}_t}} = \frac{E_t\{\sum_{j=0}^{\infty}(\theta\gamma)^j e^{\tilde{m}c_{t+j}} e^{\epsilon\tilde{\pi}_{t,t+j}} e^{\tilde{y}_{t+j}}\}}{E_t\{\sum_{j=0}^{\infty}(\theta\gamma)^j e^{(\epsilon-1)\tilde{\pi}_{t,t+j}} e^{\tilde{y}_{t+j}}\}} \tag{44}$$

Now, using the approximation $e^x \approx x + 1$ to rewrite once more the equation (44) and rearranging;

$$1 = \frac{E_t\{\sum_{j=0}^{\infty}(\theta\gamma)^j (1 + \tilde{m}c_{t+j})(1 + \epsilon\tilde{\pi}_{t,t+j})(1 + \tilde{y}_{t+j})\}}{E_t\{\sum_{j=0}^{\infty}(\theta\gamma)^j (1 + (\epsilon - 1)\tilde{\pi}_{t,t+j})(1 + \tilde{y}_{t+j})\}} \frac{(1 + \tilde{p}_t^*)}{(1 + \tilde{p}_t)}$$

Rearranging further

$$E_t \sum_{j=0}^{\infty} (\theta\gamma)^j \{1 + (\epsilon - 1)\tilde{\pi}_{t,t+j} + \tilde{y}_{t+j} + \tilde{p}_t^*\} = E_t \sum_{j=0}^{\infty} (\theta\gamma)^j \{1 + \tilde{m}c_{t+j} + \epsilon\tilde{\pi}_{t,t+j} + \tilde{y}_{t+j} + \tilde{p}_t\}$$

$$E_t \sum_{j=0}^{\infty} (\theta\gamma)^j \{\tilde{p}_t^* - \tilde{p}_t\} = E_t \sum_{j=0}^{\infty} (\theta\gamma)^j \{\tilde{m}c_{t+j} + \tilde{\pi}_{t,t+j}\}$$

$$\{\tilde{p}_t^* - \tilde{p}_t\} = (1 - \theta\gamma) E_t \sum_{j=0}^{\infty} (\theta\gamma)^j \{\tilde{m}c_{t+j} + \tilde{\pi}_{t,t+j}\}$$

$$\tilde{p}_t^* = (1 - \theta\gamma) E_t \sum_{j=0}^{\infty} (\theta\gamma)^j \{\tilde{m}c_{t+j} + \tilde{\pi}_{t,t+j}\} + \tilde{p}_t$$

where $\tilde{\pi}_{t,t+j} = (\tilde{\pi}_{t+1} + \tilde{\pi}_{t+2} + \cdots + \tilde{\pi}_{t+j})$ and $\tilde{\pi}_{t,t} = 0$. Now, extend the summation;

$$\begin{aligned}
\tilde{p}_t^* &= (1 - \theta\gamma) E_t \{\tilde{m}c_t + \underbrace{\tilde{\pi}_{t,t}}_{0} + \theta\gamma[\tilde{m}c_{t+1} + \underbrace{\tilde{\pi}_{t,t+1}}_{\tilde{\pi}_{t+1}}] + (\theta\gamma)^2[\tilde{m}c_{t+2} + \underbrace{\tilde{\pi}_{t,t+2}}_{\tilde{\pi}_{t+1}+\tilde{\pi}_{t+2}}] + \ldots\} + \tilde{p}_t \\
&= (1 - \theta\gamma) E_t \{\tilde{m}c_t + \theta\gamma[\tilde{m}c_{t+1} + \underbrace{\tilde{\pi}_{t+1}}_{\frac{\tilde{p}_{t+1}}{\tilde{p}_t}}] + (\theta\gamma)^2[\tilde{m}c_{t+2} + \underbrace{\tilde{\pi}_{t+1}}_{\frac{\tilde{p}_{t+1}}{\tilde{p}_t}} + \underbrace{\tilde{\pi}_{t+2}}_{\frac{\tilde{p}_{t+2}}{\tilde{p}_{t+1}}}] + \ldots\} + \tilde{p}_t \\
&= (1 - \theta\gamma) E_t \{\tilde{m}c_t + \theta\gamma[\ln mc_{t+1} - \ln mc + \ln p_{t+1} - \ln p_t - \ln p + \ln p] \\
&+ (\theta\gamma)^2[\ln mc_{t+2} - \ln mc + \ln p_{t+1} - \ln p_t - \ln p + \ln p + \ln p_{t+2} - \ln p_{t+1} - \ln p + \ln p] + \ldots\} + \tilde{p}_t \\
&= (1 - \theta\gamma) E_t \{\tilde{m}c_t + \theta\gamma[\ln mc_{t+1}^N - \ln mc^N - \ln p_t + \ln p] \\
&+ (\theta\gamma)^2[\ln mc_{t+2}^N - \ln mc^N - \ln p_t + \ln p] + \ldots\} + \tilde{p}_t \\
&= (1 - \theta\gamma) E_t \{\ln mc_t - \ln mc + \theta\gamma[\tilde{m}c_{t+1}^N - \ln p_t + \ln p] + (\theta\gamma)^2[\tilde{m}c_{t+2}^N - \ln p_t + \ln p] + \ldots\} + \tilde{p}_t
\end{aligned}$$

where mc_{t+1}^N denotes nominal marginal cost and we have used the equality $\tilde{x}_t = \ln X_t - \ln X$ at several places. Now, adding to and subtracting from the expression inside the expectations operator the term $(\ln pt - \ln p)$ and rearranging;

$$
\begin{aligned}
\tilde{p}_t^* &= (1-\theta\gamma)E_t\{\ln mc_t - \ln mc + \theta\gamma[\tilde{m}c_{t+1}^N - \ln p_t + \ln p] \\
&+ (\theta\gamma)^2[\tilde{m}c_{t+2}^N - \ln p_t + \ln p] + \ldots + (\ln p_t - \ln p) - (\ln p_t - \ln p)\} + \tilde{p}_t \\
&= (1-\theta\gamma)E_t\{\ln mc_t - \ln mc + \ln p_t - \ln p - (\ln p_t - \ln p)[1 + \theta\gamma + (\theta\gamma)^2 + \ldots] \\
&+ \theta\gamma\tilde{m}c_{t+1}^N + (\theta\gamma)^2\tilde{m}c_{t+2}^N + \ldots\} + \tilde{p}_t \\
&= (1-\theta\gamma)E_t\{-(\ln p_t - \ln p)\frac{1}{1-\theta\gamma} + \tilde{m}c_t^N + \theta\gamma\tilde{m}c_{t+1}^N + (\theta\gamma)^2\tilde{m}c_{t+2}^N + \ldots\} + \tilde{p}_t \\
\tilde{p}_t^* &= (1-\theta\gamma)E_t\sum_{j=0}^{\infty}(\theta\gamma)^j\tilde{m}c_{t+j}^N \qquad (45) \\
&= (1-\theta\gamma)\tilde{m}c_t^N + (\theta\gamma)(1-\theta\gamma)\left(E_t\tilde{m}c_{t+1}^N + E_t\theta\gamma\tilde{m}c_{t+2}^N + \ldots\right) \qquad (46)
\end{aligned}
$$

Now, iterate (45) by one period and take expectations to get;

$$E_t\tilde{p}_{t+1}^* = E_t[(1-\theta\gamma)E_{t+1}\sum_{j=0}^{\infty}(\theta\gamma)^j\tilde{m}c_{t+1+j}^N]$$

Using the law of iterated expectations, this is equal to;

$$
\begin{aligned}
E_t\tilde{p}_{t+1}^* &= (1-\theta\gamma)\sum_{j=0}^{\infty}(\theta\gamma)^j E_t\tilde{m}c_{t+1+j}^N \\
&= (1-\theta\gamma)\left(E_t\tilde{m}c_{t+1}^N + \theta\gamma E_t\tilde{m}c_{t+2}^N + \ldots\right) \qquad (47)
\end{aligned}
$$

Using (47), (46) is equal to;

$$\tilde{p}_t^* = (1-\theta\gamma)\tilde{m}c_t^N + \theta\gamma E_t\tilde{p}_{t+1}^* \qquad (48)$$

From equation (38), we have

$$\tilde{p}_t^* = \frac{1}{1-\theta}(\tilde{p}_t - \theta\tilde{p}_{t-1}) \tag{49}$$

$$E_t\tilde{p}_{t+1}^* = \frac{1}{1-\theta}E_t(\tilde{p}_{t+1} - \theta\tilde{p}_t) \tag{50}$$

Now, using (49) and (50) in (48) and further rearranging;

$$\frac{1}{1-\theta}(\tilde{p}_t - \theta\tilde{p}_{t-1}) = (1-\theta\gamma)\tilde{m}c_t^N + \theta\gamma\frac{1}{1-\theta}E_t(\tilde{p}_{t+1} - \theta\tilde{p}_t)$$

$$(\tilde{p}_t - \theta\tilde{p}_{t-1}) = (1-\theta)(1-\theta\gamma)\tilde{m}c_t^N + \theta\gamma E_t(\tilde{p}_{t+1} - \theta\tilde{p}_t)$$

Now, add $\theta\tilde{p}_t - \tilde{p}_t$ to both sides of the equation;

$$\theta\underbrace{(\tilde{p}_t - \tilde{p}_{t-1})}_{\tilde{\pi}_t} = (1-\theta)(1-\theta\gamma)\tilde{m}c_t^N + \theta\gamma E_t(\tilde{p}_{t+1} - \theta\tilde{p}_t) + \theta\tilde{p}_t - \tilde{p}_t$$

$$= (1-\theta)(1-\theta\gamma)\tilde{m}c_t^N + \theta\gamma E_t\tilde{p}_{t+1} - \gamma\theta^2\tilde{p}_t + \theta\tilde{p}_t - \tilde{p}_t$$

Adding to and subtracting from the right hand side of the equation $\gamma\theta E_t p_t$ in order to get rid of the terms involving $\tilde{p}_t$;

$$\theta\tilde{\pi}_t = (1-\theta)(1-\theta\gamma)\tilde{m}c_t^N + \underbrace{\theta\gamma E_t(\tilde{p}_{t+1} - \theta\tilde{p}_t)}_{E_t\tilde{\pi}_{t+1}} - \underbrace{\gamma\theta^2\tilde{p}_t + \theta\tilde{p}_t - \tilde{p}_t + \gamma\theta\tilde{p}_t}_{-(\gamma\theta^2-\gamma\theta-\theta+1)\tilde{p}_t}$$

$$\theta\tilde{\pi}_t = (1-\theta)(1-\theta\gamma)\tilde{m}c_t^N + \gamma\theta E_t\tilde{\pi}_{t+1} - (1-\theta)(1-\gamma\theta)\tilde{p}_t$$

$$\tilde{\pi}_t = \frac{(1-\theta)(1-\theta\gamma)}{\theta}\tilde{m}c_t + \gamma E_t\tilde{\pi}_{t+1}$$

$$\tilde{\pi}_t = \lambda\tilde{m}c_t + \gamma E_t\tilde{\pi}_{t+1} \tag{51}$$

A.2 Derivation of NKPC with trend inflation

Since we have the same optimization problem given in (39), the optimal price is given by the equation (40). When we divide equation (40) by P_t and rerrange it, we obtain equation (41)

Log-Linearization of the Variables

At this case, we do not assume zero steady state inflation, on the contrary the steady state inflation is greater than zero,that is, $\Pi > 1$. The steady state of equation (41) for positive inflation is given below.

$$
\begin{aligned}
\frac{P^*}{P} &= \frac{\epsilon}{\epsilon-1}\frac{\sum_{j=0}^{\infty}\theta^j\gamma^j MC_i(\Pi)^{j\epsilon}Y}{\sum_{j=0}^{\infty}\theta^j\gamma^j(\Pi)^{j(\epsilon-1)}Y} \\
\frac{P^*}{P} &= \frac{\epsilon}{\epsilon-1}MC_i\frac{1-\theta\gamma\Pi^{\epsilon-1}}{1-\theta\gamma\Pi^{\epsilon}}
\end{aligned}
\tag{52}
$$

For the summations in (52), it must be that $\theta\gamma\Pi^{\epsilon} < 1$

Now we will loglinearize (41) around the steady state equation (52). First we rewrite components of (41) as follows

$$
\begin{aligned}
P_t^* &= P^* e^{\tilde{p}_t^*} \\
P_t &= P e^{\tilde{p}_t} \\
MC_{i,t+j} &= MC_i e^{\tilde{mc}_{t+j}} \\
Y_{t+j} &= Y e^{\tilde{y}_{t+j}} \\
\Pi_{t,t+j} &= \Pi^j e^{\tilde{\pi}_{t,t+j}}
\end{aligned}
$$

and substitute back in (41);

$$\frac{P^* e^{\tilde{p}_t^*}}{P e^{\tilde{p}_t}} = \frac{\epsilon}{\epsilon - 1} MC_i \frac{E_t\{\sum_{j=0}^{\infty}(\theta\gamma)^j e^{\tilde{m}c_{t+j}} \Pi^{j\epsilon} e^{\epsilon\tilde{\pi}_{t,t+j}} Y e^{\tilde{y}_{t+j}}\}}{E_t\{\sum_{j=0}^{\infty}(\theta\gamma)^j \Pi^{j(\epsilon-1)} e^{(\epsilon-1)\tilde{\pi}_{t,t+j}} Y e^{\tilde{y}_{t+j}}\}} \tag{53}$$

When we simplify this term using steady state equation (52), we have;

$$\frac{e^{\tilde{p}_t^*}}{e^{\tilde{p}_t}} \frac{1-\theta\gamma\Pi^{\epsilon-1}}{1-\theta\gamma\Pi^{\epsilon}} = \frac{E_t\{\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^j e^{\tilde{m}c_{t+j}} e^{\epsilon\tilde{\pi}_{t,t+j}} e^{\tilde{y}_{t+j}}\}}{E_t\{\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j e^{(\epsilon-1)\tilde{\pi}_{t,t+j}} e^{\tilde{y}_{t+j}}\}} \tag{54}$$

Now using the approximation $e^x \approx x + 1$ and rearranging;

$$\frac{1-\theta\gamma\Pi^{\epsilon-1}}{1-\theta\gamma\Pi^{\epsilon}} = \frac{E_t\{\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^j(1+\tilde{m}c_{t+j})(1+\epsilon\tilde{\pi}_{t,t+j})(1+\tilde{y}_{t+j})\}}{E_t\{\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j(1+(\epsilon-1)\tilde{\pi}_{t,t+j})(1+\tilde{y}_{t+j})\}} \frac{(1+\tilde{p}_t^*)}{(1+\tilde{p}_t)}$$

Rearranging further;

$$\begin{aligned}(1-\theta\gamma\Pi^{\epsilon-1})E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j\{1+(\epsilon-1)\tilde{\pi}_{t,t+j}+\tilde{y}_{t+j}+\tilde{p}_t^*\} \\ = (1-\theta\gamma\Pi^{\epsilon})E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^j\{1+\tilde{m}c_{t+j}+\epsilon\tilde{\pi}_{t,t+j}+\tilde{y}_{t+j}+\tilde{p}_t\}\end{aligned}$$

$$\begin{aligned}\tilde{p}_t^* - \tilde{p}_t =& E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^j(1-\theta\gamma\Pi^{\epsilon})\{\tilde{m}c_{t+j}+\epsilon\tilde{\pi}_{t,t+j}+\tilde{y}_{t+j}\} \\ &- E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\tilde{\pi}_{t,t+j}+\tilde{y}_{t+j}\}\end{aligned} \tag{55}$$

Calvo Equation:LogLinearization

The aggregate price level is given by the equation (37).The steady state equation of (37) is given below.

$$P^{1-\epsilon} = \theta P_{-1}^{1-\epsilon} + (1-\theta)(P^*)^{1-\epsilon} \tag{56}$$

Individual components of equation (37) can be rewritten as follows

$$P_t = Pe^{\tilde{p}_t}$$

$$P_{t-1} = P_{-1}e^{\tilde{p}_{t-1}}$$

$$P_t^* = P^* e^{\tilde{p}_t^*}$$

We know the relationship between P and P_{-1} which is $P = \Pi P_{-1}$. Using that, we can solve for P^* in the equation (56).

$$P^* = \left(\frac{\Pi^{1-\epsilon} - \theta}{1-\theta} \right)^{1/(1-\epsilon)} P_{-1} \tag{57}$$

Now we will log-linearize the equation (37) around the steady state equation (56).

$$P^{1-\epsilon} e^{(1-\epsilon)\tilde{p}_t} = \theta P_{-1}^{1-\epsilon} e^{(1-\epsilon)\tilde{p}_{t-1}} + (1-\theta) P^{*1-\epsilon} e^{(1-\epsilon)\tilde{p}_t^*}$$

Using approximation $e^x \approx 1 + x$ and the steady state equation (56);

$$P^{1-\epsilon}[1 + (1-\epsilon)\tilde{p}_t] = \theta P_{-1}^{1-\epsilon}[1 + (1-\epsilon)\tilde{p}_{t-1}] + (1-\theta) P^{*1-\epsilon}[1 + (1-\epsilon)\tilde{p}_t^*]$$

$$P^{1-\epsilon} + P^{1-\epsilon}(1-\epsilon)\tilde{p}_t = \theta P_{-1}^{1-\epsilon} + \theta P_{-1}^{1-\epsilon}(1-\epsilon)\tilde{p}_{t-1} + (1-\theta) P^{*1-\epsilon} + (1-\theta) P^{*1-\epsilon}(1-\epsilon)\tilde{p}_t^*$$

$$P^{1-\epsilon}\tilde{p}_t = \theta P_{-1}^{1-\epsilon}\tilde{p}_{t-1} + (1-\theta) P^{*1-\epsilon}\tilde{p}_t^* \tag{58}$$

Substituting (57) into the equation (58) and using $P = \Pi P_{-1}$, we have;

$$
\begin{aligned}
& P_{-1}^{1-\epsilon}\Pi^{1-\epsilon}\tilde{p}_t = \theta P_{-1}^{1-\epsilon}\tilde{p}_{t-1} + (1-\theta)\left(\left(\frac{\Pi^{1-\epsilon}-\theta}{1-\theta}\right)^{1/(1-\epsilon)} P_{-1}\right)^{1-\epsilon}\tilde{p}_t^* \\
& \Pi^{1-\epsilon}\tilde{p}_t = \theta\tilde{p}_{t-1} + (\Pi^{1-\epsilon}-\theta)\tilde{p}_t^* \\
& (\Pi^{1-\epsilon}-\theta)\tilde{p}_t + \theta\tilde{p}_t = \theta\tilde{p}_{t-1} + (\Pi^{1-\epsilon}-\theta)\tilde{p}_t^* \\
& (\Pi^{1-\epsilon}-\theta)(\tilde{p}_t^* - \tilde{p}_t) = \theta(\underbrace{\tilde{p}_t - \tilde{p}_{t-1}}_{\pi_t}) \\
& \tilde{p}_t^* - \tilde{p}_t = \theta(\tilde{p}_t - \tilde{p}_{t-1})/(1/\Pi^{\epsilon-1} - \theta) \\
& \tilde{p}_t^* - \tilde{p}_t = \frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\pi_t
\end{aligned} \tag{59}
$$

Derivation of NKPC

Combining the equation (55) with the equation (59);

$$
\begin{aligned}
\frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\pi_t =& E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^j(1-\theta\gamma\Pi^{\epsilon})\{1+\tilde{m}c_{t+j}+\epsilon\tilde{\pi}_{t,t+j}+\tilde{y}_{t+j}+\tilde{p}_t\} \\
& - E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\tilde{\pi}_{t,t+j}+\tilde{y}_{t+j}\}
\end{aligned}
$$

$$\pi_t = \frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\{E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^j(1-\theta\gamma\Pi^{\epsilon})\{\tilde{m}c_{t+j}+\epsilon\tilde{\pi}_{t,t+j}+\tilde{y}_{t+j}\}$$

$$-E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\tilde{\pi}_{t,t+j}+\tilde{y}_{t+j}\}\} \quad (60)$$

$$\pi_t = \left(\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\right)(1-\theta\gamma\Pi^{\epsilon})\tilde{m}c_t+\left(\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\right)(1-\theta\gamma\Pi^{\epsilon}-(1-\theta\gamma\Pi^{\epsilon-1}))\tilde{y}_t$$

$$+\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\{E_t\sum_{j=1}^{\infty}(\theta\gamma\Pi^{\epsilon})^j(1-\theta\gamma\Pi^{\epsilon})\{\tilde{m}c_{t+j}+\epsilon\tilde{\pi}_{t,t+j}+\tilde{y}_{t+j}\}$$

$$-E_t\sum_{j=1}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\tilde{\pi}_{t,t+j}+\tilde{y}_{t+j}\}\}$$

$$\pi_t = \left(\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\right)(1-\theta\gamma\Pi^{\epsilon})\tilde{m}c_t+(1-\Pi)\gamma(1-\theta\Pi^{\epsilon-1})\tilde{y}_t$$

$$+\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\{E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^{j+1}(1-\theta\gamma\Pi^{\epsilon})\{\tilde{m}c_{t+j+1}+\epsilon\tilde{\pi}_{t,t+j+1}+\tilde{y}_{t+j+1}\}$$

$$-E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^{j+1}(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\tilde{\pi}_{t,t+j+1}+\tilde{y}_{t+j+1}\}\}$$

$$\pi_t = \left(\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\right)(1-\theta\gamma\Pi^{\epsilon})\tilde{m}c_t+(1-\Pi)\gamma(1-\theta\Pi^{\epsilon-1})\tilde{y}_t$$

$$+\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\{E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^{j+1}(1-\theta\gamma\Pi^{\epsilon})\{\tilde{m}c_{t+j+1}+\epsilon\tilde{\pi}_{t,t+j+1}-\epsilon\tilde{\pi}_{t+1}+\epsilon\tilde{\pi}_{t+1}+\tilde{y}_{t+j+1}\}$$

$$-E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^{j+1}(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\tilde{\pi}_{t,t+j+1}-(\epsilon-1)\tilde{\pi}_{t+1}+(\epsilon-1)\tilde{\pi}_{t+1}+\tilde{y}_{t+j+1}\}\}$$

$$\pi_t = \left(\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\right)(1-\theta\gamma\Pi^{\epsilon})\tilde{m}c_t+\gamma E_t\tilde{\pi}_{t+1}+(1-\Pi)\gamma(1-\theta\Pi^{\epsilon-1})[\tilde{y}_t-\epsilon E_t\tilde{\pi}_{t+1}]-\gamma\theta\Pi^{\epsilon-1}E_t\tilde{\pi}_{t+1}$$

$$+\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\{E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^j(\theta\gamma\Pi^{\epsilon})(1-\theta\gamma\Pi^{\epsilon})\{\tilde{m}c_{t+j+1}+\epsilon\underbrace{(\tilde{\pi}_{t,t+j+1}-\tilde{\pi}_{t+1})}_{\tilde{\pi}_{t+1,t+j+1}}+\tilde{y}_{t+j+1}\}$$

$$-E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j(\theta\gamma\Pi^{\epsilon-1})(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\underbrace{(\tilde{\pi}_{t,t+j+1}-\tilde{\pi}_{t+1})}_{\tilde{\pi}_{t+1,t+j+1}}+\tilde{y}_{t+j+1}\}\}$$

$$\pi_t = \left(\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\right)(1-\theta\gamma\Pi^{\epsilon})\tilde{m}c_t+\gamma E_t\tilde{\pi}_{t+1}+(1-\Pi)\gamma(1-\theta\Pi^{\epsilon-1})[\tilde{y}_t-\epsilon E_t\tilde{\pi}_{t+1}]-\gamma\theta\Pi^{\epsilon-1}E_t\tilde{\pi}_{t+1}$$

$$+\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}(\theta\gamma\Pi^{\epsilon})\{E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^j(1-\theta\gamma\Pi^{\epsilon})\{\tilde{m}c_{t+j+1}+\epsilon\tilde{\pi}_{t+1,t+j+1}+\tilde{y}_{t+j+1}\}$$

$$-E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j\Pi^{-1}(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\tilde{\pi}_{t+1,t+j+1}+\tilde{y}_{t+j+1}\}\}$$

(61)

$$\begin{aligned}
\pi_t = & \left(\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\right)(1-\theta\gamma\Pi^{\epsilon})\tilde{m}c_t + \gamma E_t\tilde{\pi}_{t+1} + (1-\Pi)\gamma(1-\theta\Pi^{\epsilon-1})[\tilde{y}_t - \epsilon E_t\tilde{\pi}_{t+1}] - \gamma\theta\Pi^{\epsilon-1}E_t\tilde{\pi}_{t+1} \\
& + \frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}(\theta\gamma\Pi^{\epsilon})\{E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^j(1-\theta\gamma\Pi^{\epsilon})\{\tilde{m}c_{t+j+1} + \epsilon\tilde{\pi}_{t+1,t+j+1} + \tilde{y}_{t+j+1}\} \\
& - E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\tilde{\pi}_{t+1,t+j+1} + \tilde{y}_{t+j+1}\}\} \\
& + \frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}(\theta\gamma\Pi^{\epsilon})(1-\Pi^{-1})E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\tilde{\pi}_{t+1,t+j+1} + \tilde{y}_{t+j+1}\}
\end{aligned} \tag{62}$$

When we iterate (60) by one period and take expectation, we have

$$\begin{aligned}
E_t\pi_{t+1} = & \frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\{E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon})^j(1-\theta\gamma\Pi^{\epsilon})\{\tilde{m}c_{t+1+j} + \epsilon\tilde{\pi}_{t+1,t+1+j} + \tilde{y}_{t+1+j}\} \\
& - E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\tilde{\pi}_{t+1,t+1+j} + \tilde{y}_{t+1+j}\}
\end{aligned} \tag{63}$$

Using (63) in the equation (62);

$$\begin{aligned}
\pi_t = & \left(\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\right)(1-\theta\gamma\Pi^{\epsilon})\tilde{m}c_t + \gamma E_t\tilde{\pi}_{t+1} + (1-\Pi)\gamma(1-\theta\Pi^{\epsilon-1})[\tilde{y}_t - \epsilon E_t\tilde{\pi}_{t+1}] - \gamma\theta\Pi^{\epsilon-1}E_t\tilde{\pi}_{t+1} \\
& + (\theta\gamma\Pi^{\epsilon})E_t\tilde{\pi}_{t+1} \\
& + \frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}(\theta\gamma\Pi^{\epsilon})(1-\Pi^{-1})E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j(1-\theta\gamma\Pi^{\epsilon-1})\{(\epsilon-1)\tilde{\pi}_{t+1,t+j+1} + \tilde{y}_{t+j+1}\} \\
\pi_t = & \left(\frac{1-\theta\Pi^{\epsilon-1}}{\theta\Pi^{\epsilon-1}}\right)(1-\theta\gamma\Pi^{\epsilon})\tilde{m}c_t + \gamma E_t\tilde{\pi}_{t+1} \\
& + (1-\Pi)\gamma(1-\theta\Pi^{\epsilon-1})\left[\tilde{y}_t - \left(\epsilon + \frac{\theta\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\right)E_t\tilde{\pi}_{t+1}\right. \\
& \left. - (1-\theta\gamma\Pi^{\epsilon-1})E_t\sum_{j=0}^{\infty}(\theta\gamma\Pi^{\epsilon-1})^j[(\epsilon-1)\tilde{\pi}_{t+1,t+j+1} + \tilde{y}_{t+j+1}]\right]
\end{aligned} \tag{64}$$

A.3 Derivation of NKPC with time varying trend inflation

Log-linear approximation of aggregate prices

To derive NKPC with time varying trend inflation, we use the model in Ascari(2004). We begin with the definition of aggregate price level;

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \tag{65}$$

We first divide (65) by P_t to have

$$1 = \theta(\Pi_t^{-1})^{1-\epsilon} + (1-\theta)x_t^{1-\epsilon} \tag{66}$$

where $x_t \equiv P_t^*/P_t$ is the optimizing firms' relative price. We also define the stationary variables $\tilde{\Pi}_t = \Pi_t/\bar{\Pi}_t$ and $\tilde{x}_t = x_t/\bar{x}_t$ where a bar over a variable indicates its steady state value.

Then we transform (66) to express it in terms of the stationary variables defined previously.

$$1 = \theta\tilde{\Pi}_t^{\epsilon-1}\bar{\Pi}_t^{\epsilon-1} + (1-\theta)\tilde{x}_t^{1-\epsilon}\bar{x}_t^{1-\epsilon} \tag{67}$$

In steady state $\tilde{\Pi}_t = 1$ and $\tilde{x}_t = 1$ and (66) defines $\bar{x}_t$.

$$\bar{x}_t = \left[\frac{1-\theta\bar{\Pi}_t^{\epsilon-1}}{1-\theta}\right]^{\frac{1}{1-\epsilon}} \tag{68}$$

Defining hat variables $\hat{x}_t \equiv \ln\tilde{x}_t$ and $\hat{\pi}_t \equiv \ln\tilde{\Pi}_t \equiv \ln(\Pi_t/\bar{\Pi}_t) \equiv \pi_t - \bar{\pi}_t$, the log-linear approximation of (67) around its steady state is:

$$0 = \theta\bar{\Pi}_t^{\epsilon-1}\hat{\pi}_t - (1-\theta)\bar{x}_t^{1-\epsilon}\hat{x}_t \tag{69}$$

After substituting $\bar{x}_t$ from (38), (39) becomes

$$0 = \theta\bar{\Pi}_t^{\epsilon-1}\hat{\pi}_t - (1-\theta\bar{\Pi}_t^{\epsilon-1})\hat{x}_t \tag{70}$$

This equation gives a solution for $\hat{x}_t$:

$$\hat{x}_t = \frac{1}{\varphi_{0t}}\hat{\pi}_t \tag{71}$$

where we set $\varphi_{0t} = \frac{1-\theta\bar{\Pi}_t^{\epsilon-1}}{\theta\bar{\Pi}_t^{\epsilon-1}}$

Log-linear approximation of firm's FOC

We start with definition of optimum price obtained from firm's problem.

$$P_t^* = \frac{\epsilon}{\epsilon-1}\frac{E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j MC_{i,t+j}\left(\frac{P_{t+j}}{\bar{\pi}_{t,t+j}}\right)^{\epsilon}Y_{t+j}\}}{E_t\{\sum_{j=0}^{\infty}\theta^j\gamma^j\left(\frac{P_{t+j}}{\bar{\pi}_{t,t+j}}\right)^{\epsilon-1}Y_{t+j}\}} \equiv \frac{C_t}{D_t} \tag{72}$$

We can express C and D in recursive form

$$C_t = \frac{\epsilon}{\epsilon-1}MC_tP_t^{\epsilon}Y_t + \theta\gamma E_t(C_{t+1}); \tag{73}$$

and

$$D_t = P_t^{\epsilon-1} Y_t + \theta\gamma E_t(D_{t+1}); \tag{74}$$

Deflating appropriately (73) and (74) we obtain

$$\tilde{C}_t \equiv \frac{C_t}{Y_t P_t^{\epsilon}} = \frac{\epsilon}{\epsilon - 1} MC_t + \theta\gamma E_t(\tilde{C}_{t+1} g_{t+1}^{y} \Pi_{t+1}^{\epsilon}); \tag{75}$$

$$\tilde{D}_t \equiv \frac{D_t}{Y_t P_t^{\epsilon-1}} = 1 + \theta\gamma E_t(\tilde{D}_{t+1} g_{t+1}^{y} \Pi_{t+1}^{\epsilon-1}); \tag{76}$$

where $g_{t+1}^{y} = Y_{t+1}/Y_t$.

$$\frac{\tilde{C}_t}{\tilde{D}_t} = \frac{P_t^*}{P_t} = x_t \tag{77}$$

From (75) and (76) evaluated at steady state we can solve for

$$\bar{C}_t = \frac{\frac{\epsilon}{\epsilon-1} \bar{M}C_t}{1 - \theta\gamma \bar{g}^{y} \bar{\Pi}_t^{\epsilon}} \tag{78}$$

$$\bar{D}_t = \frac{1}{1 - \theta\gamma \bar{g}^{y} \bar{\Pi}_t^{\epsilon-1}} \tag{79}$$

To derive a log-linear approximation of (45), we first define $\hat{C}_t = \ln \frac{\tilde{C}_t}{\bar{C}_t}$, $\hat{D}_t = \ln \frac{\tilde{D}_t}{\bar{D}_t}$ and $\hat{m}c_t = \ln \frac{MC_t}{\bar{M}C_t}$ and then derive

$$\hat{C}_t = (1 - \varphi_{2t})\hat{m}c_t + \varphi_{2t} E_t[\hat{g}_{t+1}^{y} + \epsilon\hat{\pi}_{t+1}] + \varphi_{2t} E_t \tilde{C}_{t+1} \tag{80}$$

and

$$\hat{D}_t = \varphi_{1t} E_t[\hat{g}^y_{t+1} + (\epsilon - 1)\hat{\pi}_{t+1}] + \varphi_{1t} E_t \tilde{D}_{t+1} \tag{81}$$

where

$$\varphi_{1t} = \theta\gamma\bar{g}^y \bar{\Pi}_t^{\epsilon-1}$$
$$\varphi_{2t} = \theta\gamma\bar{g}^y \bar{\Pi}_t^{\epsilon}$$

The log-linearization of (45) is then:

$$\hat{x}_t = \hat{C}_t - \hat{D}_t \tag{82}$$

From this equality we can solve for $\hat{\pi}_t$ using (71):

$$\hat{\pi}_t = \varphi_{0t}\hat{x}_t \tag{83}$$

By some manipulations on expressions (83), (80) and (81) we obtain following two equations:

$$\hat{\pi}_t = \varphi_{0t}(1 - \varphi_{2t})\hat{m}c_t + \varphi_{0t}\varphi_{2t}E_t(\hat{\pi}_{t+1}) + \varphi_{2t}E_t\hat{\pi}_{t+1} + \varphi_{0t}\left(\frac{\varphi_{2t} - \varphi_{1t}}{\varphi_{1t}}\right)\hat{D}_t \tag{84}$$

$$\hat{D}_t = \varphi_{1t}E_t(\hat{g}^y_{t+1}) + \varphi_{1t}E_t((\epsilon - 1)\hat{\pi}_{t+1}) + \varphi_{1t}E_t\tilde{D}_{t+1} \tag{85}$$

Finally we expand forward second equation, substitute it into the first and we obtain:

$$\hat{\pi}_t = \varphi_{0t}(1 - \varphi_{2t})\hat{m}c_t + (1 + \varphi_{0t})(\varphi_{2t})E_t(\hat{\pi}_{t+1}) + \varphi_{0t}(\varphi_{2t} - \varphi_{1t})(\epsilon - 1)E_t\hat{\pi}_{t+1} + \tag{86}$$
$$[E_t\sum_{j=0}^{\infty}\varphi_{0t}(\varphi_{2t} - \varphi_{1t})\varphi_{1t}^j\hat{g}^y_{t+1+j}] + [E_t\sum_{j=1}^{\infty}\varphi_{0t}(\varphi_{2t} - \varphi_{1t})(\epsilon - 1)\varphi_{1t}^j\hat{\pi}_{t+1+j}]$$

Rearranging the terms;

$$\hat{\pi}_t = \left(\frac{1-\theta\bar{\Pi}_t^{\epsilon-1}}{\theta\bar{\Pi}_t^{\epsilon-1}}\right)(1-\theta\gamma\bar{g}^y\bar{\Pi}_t^{\epsilon})\hat{m}c_t + \gamma\bar{g}^y E_t\hat{\pi}_{t+1} - \epsilon(1-\bar{\Pi}_t)(1-\theta\bar{\Pi}_t^{\epsilon-1})\gamma\bar{g}^y E_t\hat{\pi}_{t+1}$$

$$- (1-\bar{\Pi}_t)\theta\bar{\Pi}_t^{\epsilon-1}\gamma\bar{g}^y E_t\hat{\pi}_{t+1} - (1-\bar{\Pi}_t)\gamma\bar{g}^y(1-\theta\bar{\Pi}_t^{\epsilon-1})E_t\Bigg\{\sum_{j=1}^{\infty}(\theta\gamma\bar{g}^y\bar{\Pi}_t^{\epsilon-1})^{j-1}(\hat{g}^y_{t+j}$$

$$+ \theta\gamma\bar{g}^y\bar{\Pi}_t^{\epsilon-1}(\epsilon-1)\hat{\pi}_{t+1+j})\Bigg\} \tag{87}$$

We take $\bar{g}^y$ as one, so NKPC becomes as follows

$$\tilde{\pi}_t = \left(\frac{1-\theta\Pi_t^{\epsilon-1}}{\theta\Pi_t^{\epsilon-1}}\right)(1-\theta\gamma\Pi_t^{\epsilon})\tilde{m}c_t + (\gamma - \gamma(1-\Pi_t)(\epsilon(1-\theta\Pi_t^{\epsilon-1}) + \theta\Pi_t^{\epsilon-1})E_t\tilde{\pi}_{t+1}$$
$$- (1-\Pi_t)\gamma(1-\theta\Pi_t^{\epsilon-1})E_t\Bigg\{\sum_{j=1}^{\infty}(\theta\gamma\Pi_t^{\epsilon-1})^{j-1}\underbrace{(\tilde{y}_{t+j}-\tilde{y}_{t+j-1})}_{\tilde{g}^y_{t+j}} + \theta\gamma\Pi_t^{\epsilon-1}(\epsilon-1)\tilde{\pi}_{t+1+j})\Bigg\} \tag{88}$$

Linearising the coefficients around $\bar{\Pi}_t = 1$ using first order Taylor approximation, we obtain;

$$\tilde{\pi}_t = \left(\frac{1-\theta}{\theta}\right)(1-\theta\gamma)\tilde{m}c_t + \gamma E_t\tilde{\pi}_{t+1} + \left(\frac{1-\epsilon-\theta\gamma(1-\theta\epsilon)}{\theta}(\Pi_t-1)\right)\tilde{m}c_t$$
$$+ (\epsilon(1-\theta)+\theta)(\Pi_t-1)\gamma E_t\tilde{\pi}_{t+1} + (1-\theta)(\Pi_t-1)\gamma E_t\{\sum_{j=1}^{\infty}(\theta\gamma)^{j-1}(\tilde{g}^y_{t+j}+$$
$$\theta\gamma(\epsilon-1))\tilde{\pi}_{t+1+j}\} \tag{89}$$

We assume $\mu_t = E_t\pi_{t+1}$, $\alpha_t = E_t g_{t+1}$ and the following partial adjustment mechanisms

$$
\begin{aligned}
\mu_t - \Pi_{t+1} &= \beta(\mu_{t-1} - \Pi_t) + \epsilon_{\pi,t} \\
\alpha_t - G^y_{t+1} &= \beta(\alpha_{t-1} - G^y_t) + \epsilon_{\pi,t}
\end{aligned} \tag{90}
$$

Specifying expectations as in (90) the model becomes,

$$
\begin{aligned}
\tilde{\pi}_t = & \left(\left(\frac{1-\theta}{\theta} \right) (1-\theta\gamma) + \left(\frac{1-\epsilon-\theta\gamma(1-\theta\epsilon)}{\theta} (\Pi_t - 1) \right) \right) \tilde{m}c_t \\
& + \left(\gamma + (\Pi_t - 1)\gamma \left(1 - (\epsilon - 1)(\theta - 1) \left(\frac{\theta\gamma}{1-\theta\gamma\beta} \right) \right) \right) E_t \tilde{\pi}_{t+1} \\
& - \left((\theta - 1)(\Pi_t - 1)\gamma \left(\frac{\theta\gamma}{1-\theta\gamma\beta} \right) \right) \tilde{E}_t g^y_{t+1}
\end{aligned} \tag{91}
$$

B Bayesian inference of the NKPC models

$$Y_t = HX_t + BU_t + \epsilon_t, \epsilon_t \sim N(0, Q_t)$$
$$X_t = FX_{t-1} + R_t\eta_t, \eta_t \sim N(0, I) \tag{92}$$

where

$$Y_t = \begin{bmatrix} \pi_t \\ mc_t \\ \pi_t^S \end{bmatrix}, X_t = (\Pi_t, MC_t, \mu_{mc,t}, MC_{t-1}, MC_{t-2}, \Pi_{t-1}, \Pi_{t-2})', U_t = \begin{bmatrix} \pi_t^S \\ mc_t \\ mc_{t-1} \\ mc_{t-2} \\ \pi_{t-1} \\ \pi_{t-2} \end{bmatrix}$$

$$H = \begin{bmatrix} (1-\gamma) & -\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\phi_1 & -\phi_2 & -(\psi_3+\psi_5) & -(\phi_4+\phi_6) \\ 1 & 0 & 0 & -\psi_1 & -\psi_2 & -(\psi_3+\psi_5) & -(\psi_4+\psi_6) \end{bmatrix},$$

$$B = \begin{bmatrix} \gamma & \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 & \phi_3 & \phi_5 & \phi_5 & \phi_6 \\ 0 & 0 & \psi_1 & \psi_2 & \psi_3 & \psi_4 & \psi_5 & \psi_6 \end{bmatrix}, \eta_t = \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \end{bmatrix}$$

$$Q_t = \begin{bmatrix} \sigma^2_{\epsilon 1,t} & \rho_1\sigma_{\epsilon 1,t}\sigma_{\epsilon 2} & \rho_2\sigma_{\epsilon 1,t}\sigma_{\epsilon 3} \\ \rho_1\sigma_{\epsilon 1,t}\sigma_{\epsilon 2} & \sigma^2_{\epsilon 2} & \rho_3\sigma_{\epsilon 2}\sigma_{\epsilon 3} \\ \rho_2\sigma_{\epsilon 1,t}\sigma_{\epsilon 3} & \rho_3\sigma_{\epsilon 2}\sigma_{\epsilon 3} & \sigma^2_{\epsilon 3} \end{bmatrix}, F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, R_t = \begin{bmatrix} \sigma_{\eta 1} & 0 & 0 \\ 0 & \sigma_{\eta 2} & 0 \\ 0 & 0 & \sigma_{\eta 3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This section provides the MCMC algorithm for the posterior inference of the NKPC model. Specifically, we use a Gibbs sampler (see Geman and Geman, 1984; Tanner and Wong, 1987). The NKPC model in (30) can be transformed into the state-space form as above. Once the state space form of the model is determined as in (22) standard Bayesian techniques in state-space models can be applied. Let $Y_{1:T} = (Y_1, Y_2, ..., Y_T)'$, $X_{1:T} = (X_1, X_2, ..., X_T)'$, $U_{1:T} = (U_1, U_2, ..., U_T)'$,$\sigma^2_{\epsilon_1,1:T} = (\sigma^2_{\epsilon_1,1}, \sigma^2_{\epsilon_1,2}, ..., \sigma^2_{\epsilon_1,T})'$ and $\theta = (\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \gamma, \lambda)$ For the NKPC model (30), the simulation scheme is as follows

1. Initialize the parameters by drawing θ,h_t, Q_t and R_t using prior distributions. Initialize $m = 1$.

2. Sample X_t^m from $p(X_t|Y_{1:T}, h_{1:T}, U_{1:T}, R_{1:T}, Q_{1:T})$ for $t = 1, 2...T$

3. Sample θ^m from $p(\theta|Y_{1:T}, X^m_{1:T}, U_{1:T}, R_{1:T}, Q_{1:T})$

4. Sample h_t^m from $p(h_t|Y_{1:T}, X^m_{1:T}, \theta^m, U_{1:T}, R_{1:T}, Q_{1:T}, \sigma^{2,m-1}_{\eta 4})$ for $t = 1, 2...T$

5. Sample $\sigma^{2,m}_{\eta i}$ from $p(\sigma^2_{\eta i}|X^m_{1:T}, h^m_{1:T})$ for $i = 1, 2, 3, 4$

6. Sample ρ_i^m from $p(\rho_i|X^m_{1:T}, h_t^m, Y_{1:T}, U_{1:T}, \theta^m)$ for $i = 1, 2, 3$

7. Sample $\sigma^{2,m}_{\epsilon 2}$ and $\sigma^{2,m}_{\epsilon 3}$ from $p(\sigma^2_{\epsilon 2}, \sigma^2_{\epsilon 3}|X^m_{1:T}, h_t^m, Y_{1:T}, U_{1:T}, \theta^m)$

8. Set $m = m + 1$, repeat $(2) - (8)$ until $m = M$

Steps (3)-(5) are common to many models in the Bayesian state-space framework, see for example Kim and Nelson (1999); Gerlach et al. (2000).

Sampling of θ

Conditional on the states Π_t, Z_t and h_t for $t = 1, 2, ...T$, redefining the variables such that $\tilde{\pi}_t = \pi_t - \Pi_t$,$\tilde{z}_t = z_t - Z_t$ and $\varepsilon_t = \epsilon_t / exp(h_t/2)$, the measurement equation can be written as

$$\tilde{\pi}_t = \lambda \tilde{z}_t + \gamma(\pi_t^S - \Pi_t) + \epsilon_{1,t} \tag{93}$$

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \phi_3 \tilde{\pi}_{t-1} + \phi_4 \tilde{\pi}_{t-2} + \phi_5 \tilde{\pi}_{t-1}^S + \phi_6 \tilde{\pi}_{t-2}^S + \epsilon_{2,t} \tag{94}$$

$$(\pi_t^S - \Pi_t) = \psi_1 \tilde{z}_{t-1} + \psi_2 \tilde{z}_{t-2} + \psi_3 \tilde{\pi}_{t-1} + \psi_4 \tilde{\pi}_{t-2} + \psi_5 \tilde{\pi}_{t-1}^S + \psi_6 \tilde{\pi}_{t-2}^S + \epsilon_{3,t} \tag{95}$$

Posterior distributions of the parameters under flat priors are non-standard since $\tilde{z}_t$ term is also on the right hand side of (23). We therefore use two Metropolis Hastings steps to sample these parameters(Metropolis et al., 1953; Hastings, 1970). First step is to sample γ and λ jointly and the other step is to sample $\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \psi_1, \psi_2, \psi_3, \psi_4, \psi_5$ and ψ_6 jointly. For both Metropolis Hastings we use multivariate studentd-t density as candidate density and actual posterior distributions are derived using flat priors for all parameters.

Sampling of states, X_t

Conditional on the remaining model parameters, $X_{0:T}$ can be drawn using standard Bayesian inference. This includes implementation of the Kalman first and a simulation smoother using the filtered values for drawing smoothed states as in Carter and Kohn (1994) and Frhwirth-Schnatter (1994). We start the recursion for $t = 1, ...T$

$$
\begin{aligned}
&\textit{Prediction}\\
&X_{t|t-1} = FX_{t-1|t-1}\\
&P_{t|t-1} = FP_{t-1|t-1}F' + R_t'R_t\\
&\eta_{t|t-1} = y_t - HXt|t-1 - BU_t\\
&f_{t|t-1} = HP_{t|t-1}H' + Q_t \qquad (96)\\
&\textit{Updating}\\
&X_{t|t} = X_{t|t-1} + K_t\eta_{t|t-1}\\
&P_{t|t} = P_{t|t-1} - K_tH'f_{t|t-1}'\\
&\text{where } K_t = P_{t|t-1}H'f_{t|t-1}' \text{ is the Kalman Gain}
\end{aligned}
$$

and store $X_{t|t}$ and $P_{t|t}$. The last filtered state $X_{T|T}$ and its covariance matrix $P_{T|T}$ correspond to the smoothed estimates of the mean and the covariance matrix of states for period T. After having saved all the filtered values, simulation smoother does the following recursions for $t = T-1, ...1$

$$
\begin{aligned}
&\textit{Smoothing}\\
&X_{t|t,X_{t+1}^*} = X_{t|t} + P_{t|t}F^{*'}(F^*P_{t|t}F^{*'} + R_{t+1}^{*'}R_{t+1}^*)^{-1}(X_{t+1}^* - F^*X_{t|t})\\
&P_{t|t,X_{t+1}^*} = P_{t|t} - P_{t|t}F^{*'}(F^*P_{t|t}F^{*'} + R_{t+1}^{*'}R_{t+1}^*)^{-1}F^*P_{t|t}
\end{aligned}
$$

where X_{t+1}^*, F^*, R^* are the parts of $X_{t+1|t+1}, F, R$ which correspond to positive definite part of R. Intuitively, the simulation smoother updates the state values using the same fundamentals as in Kalman filter, where at each step filtered values are updated using the smoothed values obtained from backward recursion. In order to update the initial states, using the state equation $X_{0|t,X_1^*} = F^{-1}(X_1)$ and

$P_{0|t,X_1^*} = F^{-1}(P_1 + R_1'R_1)F'^{-1}$ can be written for the first observation. Given the mean $X_{t|t,X_{t+1}^*}$ and the covariance matrix $P_{t|t,X_{t+1}^*}$, samples of states can be drawn from $X_t \sim N(X_{t|t,X_{t+1}^*}, P_{t|t,X_{t+1}^*})$ for $t = 0, ..., T$

Sampling of inflation volatilities, h_t

Conditional on the remaining model parameters, drawing $h_{0:T}$ can be implemented using standard Bayesian inference as in the case of X_t. One important difference, however, is the logarithmic transformation of the variance in (31). As the transformation affects the error structure, the square of which follows a χ^2 distribution, the system is not Gaussian but has a log-χ^2 distribution. regarding the properties of log-χ^2 distribution, Kim et al. (1998) and Omori et al. (2007) approximate this distribution using mixture of Gaussian distributions. Hence, conditional on these mixture components the system is kept as Gaussian allowing for standard inference outlined above. For details, see Omori et al. (2007).

Sampling of state error variances, σ_η^2

Using standard results form a linear regression model with a conjugate prior for the variances in (30), it follows that the conditional posterior distribution of $\sigma_{\eta i}^2$, with $i = 1, 2, 3, 4$ is an inverted χ^2 distribution with scale parameter $\Phi_{\eta i} + \sum_{t=1}^{T} \eta_{i,t}^2$ and with $T + \upsilon_{\eta i}$ degrees of freedom for $i = 1, 2, 3, 4$ where $\Phi_{\eta i}$ and $\upsilon_{\eta i}$ are the scale and degrees of freedom parameters of the prior density.

Sampling of marginal cost variance, inflation expectation survey variance and correlation coefficients

First, we decompose the Q_t in the following form

$$Q_t = \begin{bmatrix} \sigma_{\epsilon 1,t} & 0 & 0 \\ 0 & \sigma_{\epsilon 2} & 0 \\ 0 & 0 & \sigma_{\epsilon 3} \end{bmatrix} \begin{bmatrix} 1 & \rho_1 & \rho_2 \\ \rho_1 & 1 & \rho_3 \\ \rho_2 & \rho_3 & 1 \end{bmatrix} \begin{bmatrix} \sigma_{\epsilon 1,t} & 0 & 0 \\ 0 & \sigma_{\epsilon 2} & 0 \\ 0 & 0 & \sigma_{\epsilon 3} \end{bmatrix} \tag{97}$$

For all covariance matrices, the transformation above is possible. Then, all parameters in Q_t, $\sigma_{\epsilon 1,t}, \sigma_{\epsilon 2}, \sigma_{\epsilon 3}, \rho_1, \rho_2$ and ρ_3 are estimated separately. $\sigma_{\epsilon 1,t}$ is estimated using estimated values of h_t. Similar to the estimation of state error variances, $\sigma_{\epsilon 2}$ and $\sigma_{\epsilon 3}$ are estimated using inverse-Wishart posterior distribution under conjugate priors. To estimate ρ_1, ρ_2 and ρ_3 we use griddy Gibbs sampler by setting up a grid in the interval $\rho \in (-1, 1)$ based on the precision we desire about value of ρ_1, ρ_2 and ρ_3.

For the NKPC model (33), the bayesian inference structure is almost same as the inference structure of the model (30). One difference is that for the state equations of h and Π_t the terms,$\tau_h h_t$ and τ_π are added as constants. τ_h and τ_π are estimated jointly using multivariate student-t distribution with flat priors. Inflation level error variance, volatility error variance and the correlation coefficient betwwen inflation level and volatility are also estimated jointly using inverse-Wishart distribution.

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