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SOVEREIGN DEFAULT RISK AND AMBIGUITY AVERSION

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Sovereign Default Risk and Ambiguity Aversion

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Abstract

We build a real business cycle model that contains interactions between fiscal policy instruments and sovereign default risk. Government is assumed to collect distortionary tax on labor income to finance its expenditure and transfers to households and service its debt. Calibrating Greek economy between 1971-2010 we follow Bi (2012) and construct fiscal limit through which we measure default probability of sovereign debt. Our benchmark results are in line with earlier literature that steadily rising transfers deteriorate fiscal balance and cause debt overhang. As a result sovereign default probability and risk premium rise sharply. Following Ju and Miao (2012) we extend our model preferences to allow for risk aversion and ambiguity aversion characterized separately. The nonlinear simulations show that under the assumption that investors are ambiguity averse a much higher sovereign risk premium is demanded and thus government cannot service its ever-increasing debt at even earlier stages. While stabilizing transfers as a fiscal reform can ameliorate default risk in baseline case, under ambiguity we find that government should put a higher default rate into practice and repudiate a further portion of its debt to land its economy safely.

Keywords: Fiscal policy, sovereign default risk, fiscal limit, sovereign risk premium, risk aversion, ambiguity aversion

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Özet

Çalışma mali politika enstrümanları ve temerrüt riski arasındaki dinamikleri içeren bir reel iş döngüsü modelini merkeze almaktadır. Devlet harcamalarının, yapılan sosyal transferlerin ve geçmiş dönemden gelen borçların finanse edilebilmesi için devletin emek gelirleri üzerinden vergi topladığı varsayılmaktadır. Yunanistan ekonomisinin 1971-2010 yıllarına ait verilerinden hareketle Bi (2012) çalışması dikkate alınarak devlet borcu için mali eşik hesaplanmaktadır. Borcun mali eşiğe yaklaştığı ölçüde artan temerrüt riski modellenmektedir. İlk sonuçlar önceki çalışmalara uygun biçimde sürekli artan sosyal transferlerin onyıllar içinde mali dengeyi bozarak aşırı borçlanmaya yol açtığını yansıtmaktadır. Aşırı borç yükü temerrüt olasılığını artırırken yatırımcıların devlet tahvili için istediği risk primini yükseltmektedir. Ju ve Miao (2012) çalışması esas alınarak yatırımcıların tercihleri riskten kaçınma ve belirsizlikten kaçınma davranışları ile daha sofistike hale getirildiğinde simülasyon sonuçları belirsizlikten kaçınan yatırımcıların daha yüksek risk primleri talep ettiği ve devletin sürekli artan borçlarını ilk modele göre daha önce ödeyememe durumuna geldiği saptanmaktadır. Mali reform çerçevesinde sosyal transferlerin stabilize edilmesi temerrüt riskinin düşürülmesini sağlayabilirken belirsizlik altında riskin azaltılması ve ekonominin normale dönmesi ancak devletin borcunun bir bölümünü ödemeyi reddetmesiyle mümkün olmaktadır.

Anahtar Sözcükler: Mali politika, temerrüt riski, mali eşik, risk primi, riskten kaçınma, belirsizlikten kaçınma

1 Introduction

Until last decade sovereign default has been identified with developing countries. However latest economic crises in Euro Area raised concerns about sustainability of sovereign debt in developed countries as well. The most recent case with Greece reminded us the consequences of badly run public finances. Greece's credit rating was A in early 2000s from where it was downgraded to CC in last years of decade.[1] In 2011 Greece's 10-Year bond yield reached unprecedented levels where the spread between Greek and German long-term bond rates rose to almost 15%. One quick explanation could be that the Greek debt to GDP ratio increased from 100% to around 170% as of 2012 where for Germany the same ratio lingered between 70% and 80%. However many developed countries evidently have better credit ratings with much higher debt to GDP ratios. For instance as of 2012 Japan was rated as AA- by Standard & Poor's where its debt to GDP ratio was around 210%. Therefore government's ability to pay its debt needs to be investigated rather than the level of debt.

In the case of Greece, we focus on its historical fiscal policy to explain recently emerged risk of sovereign default. Figure 1 depicts that since 1970s Greek government steadily increased its transfers to households. The share of transfers in GDP rose from 5% to 16% until 2000s. Rising transfers led to an extended period of fiscal deficit which was financed through accumulating debt and taxation. Without considerable economic growth or austerity measures the riskiness of Greek debt raised concerns. Rising probability of default was priced by creditors in sovereign risk premium which rose sharply as well.

To simulate Greek economy we build a real business cycle model with a simple production setting that contains interactions between fiscal policy instruments and sovereign default risk. Government is assumed to collect distortionary tax on labor income to finance its expenditure and transfers to households and service its debt. Calibrating Greek economy

[1] See Figure 1.

between 1971-2010 we follow Bi (2012) and construct fiscal limit through which we measure default probability of sovereign debt. We put Laffer curve into use where by definition of Laffer curve to an extent raising tax rate on labor income increases tax revenue however after reaching its maximum revenue starts to decrease each time government raises taxes. Our benchmark results are compatible with earlier literature that steadily rising transfers deteriorate fiscal balance and cause debt overhang. Such prolonged periods of fiscal deficit can harm investors' expectations about government's ability to pay its debt. As a result sovereign default probability and risk premium rise sharply.

Following Ju and Miao (2012) we extend our model preferences to allow for risk aversion and ambiguity aversion which are characterized separately. For this model we assume that regime-switching transfers follow an unobservable state. Therefore households have to develop beliefs about current state which the economy is in. These beliefs are actually distorted state probabilities in such a way that ambiguity averse households attach more weight to low continuation value. For comparison we also build recursive utility model which allows only risk aversion over continuation value.[2] The nonlinear simulations show that under the assumption that investors are ambiguity averse a much higher sovereign risk premium is demanded. Government cannot service its ever-increasing debt where increasing tax only decreases consumption and labor supply to critical levels. While stabilizing transfers as a fiscal reform can ameliorate default risk in baseline case, under ambiguity we find that government should put a higher default rate into practice and repudiate a further portion of its debt to land its economy safely.

[2] See Epstein and Zin (1989)

2 Literature Review

This study is related to a growing body of literature that studies the link between fiscal dynamics and sovereign default. Sovereign default has been commonly identified as an optimal strategy of government. Aguiar and Gopinath (2006) builds an open economy model for emerging markets where government decides on the endogenous default. Arellano (2008) finds that in a small open economy model economic recessions increase default incentives for governments. Our approach instead follows Bi (2012) and Bi and Traum (2014) where sovereign default is modeled as an exogenous case where a fiscal limit is calculated based on fiscal data. Bi (2012) develops a closed production economy where government collects tax to finance its debt and regime-switching transfers. Current fiscal position of government and all possible future fiscal scenarios together determine whether sovereign default occurs or not. When the current debt level is closer to fiscal limit probability of default rises. In a similar setting Bi and Traum (2014) uses Bayesian methods to estimate the distribuition of fiscal limit for Greek economy. We adopt their model as a baseline case for our study.

Departing from this early literature, we extend our model preferences to allow for more sophisticated attitudes toward risk and uncertainty. Instead of leisure-valued log-utility model used in Bi (2012) we make use of recursive utility model from Epstein and Zin (1989) and generalized recursive smooth ambiguity model proposed by Ju and Miao (2012). Inspired by Kreps and Porteus (1978), Epstein and Zin (1989) and Weil (1989) use a recursive utility model to obtain solution for a consumption-based asset pricing model assuming only two-way separation between intertemporal subtitution and risk aversion. As a genuine innovation, Ju and Miao (2012), following also Klibanoff, Marinacci, and Mukerji (2005) proposes generalized recursive smooth ambiguity (GRSA) model which permits a three-way separation among risk aversion, ambiguity aversion, and intertemporal substitution assuming a consumption-growth process with hidden states. Using GRSA model, Ju and Miao (2012) can match the mean equity premium, the mean risk-free rate, and the volatility of the equity premium and explain other key financial phenomena. In another study to explain the same issues Jahan-Parvar and Liu (2012) sets up a production economy where technological productivity follows an unobservable state.

As a contribution to both literatures modifying GRSA model to only create a separation between risk aversion and ambiguity aversion we apply generalized recursive smooth ambiguity model to explain the sovereign debt dynamics and movement of real interest rate of government bond. We contribute to sovereign default risk literature by enriching model preferences. For smooth ambiguity model literature we explore a new area which is long-term sovereign bond market. We assume that transfers have an ambiguous regime-swtiching process leaving other shock processes to move stochastically. With hidden states and sensitivity to ambiguity and risk we aim to explain sovereign default risk phenomenon in the case of Greece.

Section 3 introduces the settings of baseline model. Section 4 discusses the sovereign default and fiscal limit. Section 5, 6 and 7 contain the solution procedures for our models. Section 8 explains the techniques used in numerical procedures. Section 9 depicts our calibration of data. Section 10 presents the results of our nonlinear simulation and Section 11 concludes.

3 Model Settings

We build a simple RBC model following Bi and Traum (2014) where production takes place with technological productivity A_t and labor supply n_t, or simply $(1 - L_t)$ where L_t is leisure choice of households. Resource constraint is set as the sum of households' consumption c_t and government expenditure g_t:

$$y_t = A_t n_t = A_t(1 - L_t) \tag{1}$$

$$c_t + g_t = y_t \tag{2}$$

We assume that the deviation of technological productivity from its steady state value $\bar{A}$ follows an AR(1) process:

$$\ln\left(\frac{A_t}{\bar{A}}\right) = \rho_A \ln\left(\frac{A_{t-1}}{\bar{A}}\right) + \varepsilon_t^A, \qquad \varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2) \tag{3}$$

We also build government budget constraint where government imposes distortionary taxation τ_t on labor income and issues one-period bond b_t with unenforceable contract in order to finance government expenditure g_t, transfers to household z_t and pay its debt from previous period b_{t-1} which is a post-default value. Post-default means that a partial default may occur with some probability. We denote this ratio of partial default as Δ_t[3] and thus, paid amount of government debt from previous period can be written as

$$b_t^d = (1 - \Delta_t) b_{t-1} \tag{4}$$

Similar to A_t, we assume that government expenditure g_t is an exogenous variable that follows an AR(1) process:

$$\ln\left(\frac{g_t}{\bar{g}}\right) = \rho_g \ln\left(\frac{g_{t-1}}{\bar{g}}\right) + \varepsilon_t^g, \qquad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2) \tag{5}$$

Our modelling of tax rate τ_t again follows Bi (2012). Government is assumed to collect taxes on labor income in order to adjust its finances to be able to pay its debt. So, tax rate can

[3] We explain Δ_t in detail in Section 4.

be seen as an endogenous response of government to changes in its debt. Thus, we write the deviation of tax rate from its steady state value as

$$\tau_t = \bar{\tau} + \gamma_\tau (b_t^d - \bar{b}) \tag{6}$$

where $\gamma^\tau > 0$ implying that government tends to increase tax rate when its debt rises. Lump-sum transfers z_t to households made by government is assumed to follow a regime-switching process with two states under a Markov chain

$$\begin{pmatrix} \lambda_{11} & 1-\lambda_{11} \\ 1-\lambda_{22} & \lambda_{22} \end{pmatrix}$$

Following Hamilton (1985) we use Markov Switching Regression to identify different regimes in Greek government's transfers data between 1971 and 2010. We estimate that transfers switch between two states: stationary regime ($s_t = 1$) and explosive regime ($s_t = 2$). We first assume that the government gives out transfers in order to stabilize the effects of business cycles on household income. Then, we expect that transfers move countercyclically implying that the government tries to revert the effects of changes in technological productivity A_t. Hence in a stationary regime we can write transfers as

$$\ln\left(\frac{z_t}{\bar{z}}\right) = \gamma_z \cdot \ln\left(\frac{A_t}{\bar{A}}\right), \qquad \gamma_z < 0, \qquad (s_t = 1) \tag{7}$$

given out by government can be also identified with an explosive pattern.[4] Therefore, transfers in explosive regime can be written as

$$\ln\left(\frac{z_t}{\bar{z}}\right) = \rho_z \cdot \ln\left(\frac{z_{t-1}}{\bar{z}}\right) + \gamma_z \cdot \ln\left(\frac{A_t}{\bar{A}}\right), \qquad \gamma_z < 0, \qquad (s_t = 2) \tag{8}$$

[4]See Figure 1

where $\rho_z > 1$ which assures that transfers will rise. Assuming that q_t is the price of government bond b_t we can now write down the government budget constraint as

$$b_t q_t + \tau_t A_t n_t \quad = \quad g_t + z_t + b_t^d \tag{9}$$

4 Sovereign Default and Fiscal Limit

It is assumed that after sovereign default a ratio of the government debt, Δ_t is not paid to investors. If $\Delta_t = 0$ then it means that there is no default and government services its debt fully. If $\Delta_t = \delta \in (0, 1]$ there will be a *haircut* on the debt when government is not able to pay its debt back to investors. Since sovereign default is modeled as an exogenous case rather than a strategic decision of government, in each state a stochastic fiscal limit is constructed for government debt beyond which default will occur. Fiscal limit is characterized as the government's ability to service its debt by emitting fiscal surplus. The amount of this surplus depends on the current fiscal position of government and to achieve this end we adopt Laffer curve approach following Bi (2012).

4.1 Fiscal Limit and Laffer Curve

We build Laffer curve where government collect distortionary tax on labor income to finance its debt. To an extent government is able to raise its tax rate to pay its debt and avoid hitting the bounds of fiscal limit. However, by the definition of Laffer curve when tax rate exceeds a certain threshold, τ_{max} which maximizes tax revenue, tax revenue will start to decrease and thus government will not be able to use this tool to further finance its debt. We build "dynamic" Laffer curves which implies that the peak of Laffer curve may change with the state of economy.[5] For instance, higher technological productivity A_t or lower government

[5]See Trabandt and Uhlig (2009) for a similar analysis of Laffer curves.

expenditure g_t can give government more fiscal space to raise its taxes. Conversely, with low productivity levels government is assumed to have no ample room for fiscal policy without hurting its credibility.

We construct fiscal limit under the assumption that government will keep its tax rate in such a way that it will always have its tax revenue at maximum in all future periods. Basically we calculate the amount of debt that can be sustained if starting from today government will be in its best behavior with maximum fiscal surplus in future. Therefore we write fiscal limit which relies on current state of the economy as the sum of discounted maximum fiscal surplus of government as

$$\mathcal{B}(A_t, g_t, s_t) = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{U_c^{max}(A_{t+j}, g_{t+j})}{U_c^{max}(A_t, g_t)} \left(\mathcal{T}_{max}(A_{t+j}, g_{t+j}) - g_{t+j} - z(A_{t+j}, s_{t+j})\right) \tag{10}$$

where $\mathcal{T}_t^{max} = \tau_t^{max}(c_t^{max} + g_t)$. We use Markov Chain Monte Carlo simulation and derive conditional distribuiton of fiscal limit $\mathcal{B}(.)$ on the current state.[6] Effective fiscal limit b_t^* drawn from the distribution determines whether default will occur or not as

$$\Delta_t = \begin{cases} 0, & b_{t-1} \leq b_t^* \\ \delta, & b_{t-1} > b_t^* \end{cases}$$

where for our analysis we adopt default rate as $\delta = 0.0947$ which was calibrated using haircuts and sovereign bond default rates data between 1983 and 2010 for emerging countries[7]

[6] Procedure is explained in detail in Section 4.2

[7] See Bi (2012), Panizza (2008), and Sturzenegger and Zettelmeyer (2008) for greater details.

4.2 Markov Chain Monte Carlo Simulation Procedure

For this procedure, we adopt utiliy function form, $U_t = c_t(1-n_t)^{\phi}$. Households' optimization problem characterizes the optimal consumption level as

$$c_t = \frac{(A_t - g_t)(1-\tau_t)}{1+\phi-\tau_t}$$

Given state variables A_t and g_t, government chooses tax rate τ_t that maximizes tax revenue $\mathcal{T}_t$ as

$$\begin{aligned}\mathcal{T}_t &= \tau_t(c_t + g_t)\\ &= \tau_t\frac{A_t(1-\tau_t)+\phi g_t}{1+\phi-\tau_t}\end{aligned}$$

First-order condition of the maximization problem for tax revenue is:

$$\frac{\partial \mathcal{T}_t}{\partial \tau_t} = -A_t + \frac{(1+\phi)(A_t-g_t)\phi}{(1+\phi-\tau)^2} = 0$$

which gives revenue maximizing tax rate, τ_{max} as

$$\tau_{max} = 1+\phi-\sqrt{\frac{(1+\phi)(A_t-g_t)\phi}{A_t}}$$

We already defined the state-dependent fiscal limit as the sum of discounted fiscal surplus for all future periods as

$$b^*_{i,t} = \mathbb{E}_t\sum_{j=0}^{\infty}\beta^j\frac{U_{max}(A_{t+j},g_{t+j})}{U_{max}(A_t,g_t)}\frac{c_{max}(A_t,g_t)}{c_{max}(A_{t+j},g_{t+j})}\left(\mathcal{T}_{max}(A_{t+j},g_{t+j})-g_{t+j}-z(A_{t+j},s_{t+j})\right)$$

In Monte Carlo simulation procedure, for all possible states (A_t, g_t, s_t) we define the future horizon as 100 periods. We randomly draw shock processes for A_{t+j} and g_{t+j}, and starting from existing state s_t we use Markov chain to update for s_{t+j}. Repeating this procedure, we obtain $b^*_{i,t}$ for $i = 1, ..., 10,000$ and construct conditional distribution of fiscal limit $\mathcal{B}^*(A_t, g_t, s_t)$ using Kernel estimation.

4.3 Fiscal Limit Exercises

Fiscal limit is characterized on the existing state which implies that change in the current levels of state variables affects the conditional distribution of fiscal limit. To this end, we calculate fiscal limits for different levels of technological productivity and government expenditure and for two different transfers regimes. Figure 2 shows that while in the stationary regime difference in productivity levels has no significant effect on cumulative probability of current debt-to-GDP ratio's reaching the fiscal limit, or we can simply say default probability of debt, in the explosive regime lower productivity levels are matched with higher default probabilities of debt. A striking result in Figure 2 indicates that where government debt reached 2 times the output level probability of sovereign default is almost 0.8 if transfers follow an explosive pattern and productivity level is low $(A = 0.8763)$. The same debt-to-GDP ratio accounts for a much lower default probability, almost 0.3 if transfers are in the stationary regime.

Additionally, under the assumption that transfers are in the explosive regime where government debt is 1.5 times the output GDP probability of default is 0.4 for low productivity level comparing to 0.2 probability of default for high productivity $(A = 1.1411)$. Furthermore, when debt-to-GDP ratio rises to 2 default probability also rises to approximately 0.7 for low productivity level where high productivity level prevents default probability from rising above nearly 0.5. Although different levels of productivity evidently change the default probability, Figure 3 indicates that higher or lower government expenditure levels have no significant effect on the probability of default in both regimes. Our results for government expenditure differs from the results in Bi (2012) since we adopt households' utility as $U_t = c_t(1 - n_t)^{\phi}$ instead of leisure-valued log-utility used in Bi (2012) and Bi and Traum (2014). This characterization leads to a substantial reduction in the subjective discount factor and effects of lowering or raising government expenditure on fiscal surplus fade out due to subjective discounting.

Lastly, assuming that technological productivity and government expenditure are in their steady-state values Figure 4 shows that when transfers follow stationary regime instead of being in explosive regime default probability is drastically lower comparing to transfers in explosive regime. The difference in default probability between two regimes raises to as high as 0.3 when government debt is 2 times the output. However, when debt-to-GDP ratio reaches 2.5 default occurs in both regimes.

5 Baseline Model

In this section we build the baseline version of the model following Bi (2012) and Bi and Traum (2014) in order to make comparisons with our later models. We characterize households' preferences as risk neutrality and ambiguity neutrality which are crucial concepts to be explained in later sections. Therefore, for now we only say that households have no specific preferences over continuation value of their optimization problem which is

$$V_t = U_t + \beta \cdot \mathbb{E}_t \left[V_{t+1} \right] \tag{11}$$

Clearly current continuation value of utility maximization problem is equal to the sum of current utility and time-discounted expected value of future continuation value. V_t is defined to depend on the exogenous variables such as A_t and g_t with the state variable s_t dictating which regime transfers will follow. Then, V_{t+1} will depend on again A_{t+1} and g_{t+1} which are determined by shock processes and s_{t+1} switches between regimes following Markov Chain. Thus, households are assumed to take expectation over future continuation value which cannot be known at time t.

For the utility part, we adopt a leisure-valued function to show the effects of distortionary taxation on labor decision. Also, we assume that consumption c_t and leisure $(1-n_t)$ have different shares in utility function where consumption has a higher share, and thus, we define utility as $U_t = c_t(1-n_t)^{\phi}$ where $\phi \in (0,1)$

5.1 Model Solution

Households solve optimization problem below by choosing consumption c_t, labor n_t, and government bond b_t:

$$\begin{aligned} V_t &= \max_{\{c_t, n_t, b_t\}} \{U_t + \beta \cdot \mathbb{E}_t[V_{t+1}]\} && (12) \\ s.t. \quad & c_t + b_t q_t = (1-\Delta_t)b_{t-1} + z_t + (1-\tau_t)A_t n_t \end{aligned}$$

where we denote $U(c_t, 1-n_t) = c_t(1-n_t)^{\phi}$. Then households' first-order conditions are:

$$\begin{aligned} c_t &: \lambda_t = (1-n_t)^{\phi} \\ n_t &: \lambda_t A_t(1-\tau_t) = \phi c_t (1-n_t)^{\phi-1} \\ b_{t-1} &: \frac{\partial V_t}{\partial b_{t-1}} = \lambda_t(1-\Delta_t) \\ b_t &: \lambda_t q_t = \mathbb{E}_t\left\{\beta \cdot \frac{\partial V_{t+1}}{\partial b_t}\right\} \end{aligned}$$

which yield following consumption-labor decision and Euler Equation

$$\begin{aligned} \phi c_t &= A_t(1-\tau_t)(1-n_t) && (13) \\ q_t &= \beta \mathbb{E}_t\left\{(1-\Delta_{t+1})\left(\frac{U_{t+1}}{U_t}\right)\left(\frac{c_t}{c_{t+1}}\right)\right\} && (14) \\ q_t &= \frac{g_t + z_t + b_t^d - A_t n_t \tau_t}{b_t} \end{aligned}$$

where this optimal solution also satisfies transversality condition

$$\lim_{j\to\infty} \mathbb{E}_t \left\{ \beta^{j+1}(1-\Delta_{j+t+1}) \left(\frac{U_{j+t+1}}{U_t}\right) \left(\frac{c_t}{c_{j+t+1}}\right) b_{t+j} \right\} = 0$$

Additionally, resource constraint (2) and consumption-labor decision (13) together implies

$$c_t = \frac{(A_t - g_t)(1-\tau_t)}{1+\phi-\tau_t} \tag{15}$$

To show how sovereign default risk enters Euler Equation and build a gateway for numerical integration we should write equation (14) in a more detailed fashion as

$$\begin{aligned} q_t &= \underbrace{[p_{t,1} \cdot \lambda_{s_t,1} + p_{t,2} \cdot \lambda_{s_t,2}]} \cdot \beta \int\limits_{\varepsilon^A_{t+1}} \int\limits_{\varepsilon^g_{t+1}} (1-\delta) \cdot \left(\frac{U_{t+1}|d}{U_t}\right) \left(\frac{c_t}{c_{t+1}|d}\right) \\ &+ \underbrace{[(1-p_{t,1}) \cdot \lambda_{s_t,1} + (1-p_{t,2}) \cdot \lambda_{s_t,2}]} \cdot \beta \int\limits_{\varepsilon^A_{t+1}} \int\limits_{\varepsilon^g_{t+1}} \left(\frac{U_{t+1}|nd}{U_t}\right) \left(\frac{c_t}{c_{t+1}|nd}\right) \end{aligned} \tag{16}$$

where $(.|d)$ and $(.|nd)$ denote next period variables where government debt will default or not. Government debt (b_t) will hit the fiscal limit (b_t^*) with the probability $p_{t,1} = p(b_t, A_{t+1}, g_{t+1}, 1)$ if next period transfers follow stationary regime ($s_{t+1} = 1$) plus the probability $p_{t,2} = p(b_t, A_{t+1}, g_{t+1}, 2)$ if next period transfers follow explosive regime ($s_{t+1} = 2$).

5.2 Nonlinear Solution and Policy Function Iteration

Current period debt level b_t is adjusted considering government budget constraint under the assumption that households price the sovereign bond using bond-pricing Euler equation. However when bond price q_t is determined next period default probability $p_{t,\cdot}$ must be taken in the account as well. As we model next period default probability to be calculated with current period debt level b_t, it can be seen that b_t enters both equations in a nonlinear fashion. Therefore, whole set of equations turns into a system of nonlinear equations. In order to solve this system we construct a discretized grid space using our exogenous state variables where we take b_t as a policy variable to be determined.

Our solution method is based on Coleman (1991), Davig (2004) and Richter et al. (2013) where we lay out candidate policy functions that reduce the system of nonlinear equations to a set of expectation first-order difference equations. We start with an initial guess $b(\psi_0)$.[8] Given the current state $\psi_t = (b_{t-1}, \Delta_t, A_t, g_t, z_t, s_t, b_t^*)$ we obtain updated policy function $b_t = b(\psi_t)$ which solves following nonlinear equation using Sims (2002) algorithm:

$$\frac{b_t^d + g_t + z_t - \tau(\psi_t)A_t n(\psi_t)}{b(\psi_t)} = \beta \mathbb{E}_t \left\{ (1 - \Delta(\psi_{t+1})) \left(\frac{U(\psi_{t+1})}{U(\psi_t)} \right) \left(\frac{c(\psi_t)}{c(\psi_{t+1})} \right) \right\} \tag{17}$$

where we construct $\psi_{t+1} = (b(\psi_t), \Delta_{t+1}, A_{t+1}, g_{t+1}, z_{t+1}, s_{t+1}, b_{t+1}^*)$ and take expectation $\mathbb{E}_t$ using linear interpolation and numerical integration technniques (namely Trapezoid Rule)[9]

We calculate distance between our initial guess $b(\psi_0)$ and updated policy function $b_t = b(\psi_t)$. If the distance is smaller than the desired tolerance $(1e-6)$ then policy function convergence is satisfied. Otherwise, we update our guess as $b(\psi_t)$ and repeat earlier steps until a convergence is met.

6 Recursive Utility Model

We now build a model with Epstein-Zin[10] preferences which allows for risk aversion. This model will constitute a middle step for us to build our final model.[11] In a setting with risk-neutral households, we model attitudes toward risk over continuation value of households' optimization problem with a linear transformation $u(x) = x$. Then certainty equivalent

[8] Initial guess is obtained by log-linearized solution. See Appendix

[9] Section 8 provide more detailed procedure

[10] See Epstein and Zin (1989)

[11] Generalized Recursive Smooth Ambiguity Utility Model was built in Ju and Miao (2012)

becomes $u^{-1}(x) = x$ as well. Therefore, we can write certainty equivalent as

$$u^{-1}\left(\mathbb{E}_t\left[u(V_{t+1})\right]\right) = \mathbb{E}_t\left[V_{t+1}\right] \tag{18}$$

We can define V_t as the sum of current utility and time-discounted expectation of continuation value

$$V_t = U_t + \beta\mathbb{E}_t\left[V_{t+1}\right] \tag{19}$$

Epstein and Zin (1989) dictate preferences over timing of resolution of uncertainty. Therefore EZ preferences allow for intertemporal substitution and risk aversion. However we only focus on risk aversion part where we model our households as they are indifferent to intertemporal substitution. In order to parametrize risk aversion over continuation value with risk aversion parameter γ, we now use a concave transformation $u(x) = x^{1-\gamma}/(1-\gamma)$ where indirect value function is $u^{-1}(x) = [(1-\gamma)x]^{\frac{1}{1-\gamma}}$. This way of modelling implies that households prefer early resolution of uncertainty to late.[12] Therefore certainty equivalent can be written as

$$u^{-1}\left(\mathbb{E}_t\left[u(V_{t+1})\right]\right) = \left((1-\gamma)\mathbb{E}_t\left[\frac{V_{t+1}^{1-\gamma}}{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} = \left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \tag{20}$$

Then V_t becomes

$$V_t = U_t + \beta\left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \tag{21}$$

[12]For a greater discussion see Kreps and Porteus (1978)

In risk-neutrality condition, risk aversion parameter is set to become $\gamma = 0$ and continuation value V_t turns into its baseline version as

$$V_t = U_t + \beta \mathbb{E}_t [V_{t+1}]$$

6.1 Model Solution

Following Epstein and Zin (1989) and Weil (1989) households solve optimization problem below by choosing consumption c_t, labor hours n_t, and government bond b_t:

$$V(\psi_t) = \max_{\{c_t, n_t, b_t\}} \left\{ U_t + \beta \Big(\mathbb{E}_t [V(\psi_{t+1})]^{1-\gamma} \Big)^{\frac{1}{1-\gamma}} \right\} \tag{22}$$

$$s.t. \qquad c_t + b_t q_t = (1-\Delta_t) b_{t-1} + z_t + (1-\tau_t) A_t n_t \tag{23}$$

where we denote $U(c_t, 1-n_t) = c_t(1-n_t)^{\phi}$ and state variables are given as $\psi_t = (b_{t-1}, \Delta_t, A_t, g_t, z_{t-1}, s_t)$. Then households' first-order conditions are:

$$c_t \; : \; \lambda_t = (1-n_t)^{\phi} \tag{24}$$

$$n_t \; : \; \lambda_t A_t (1-\tau_t) = \phi c_t (1-n_t)^{\phi-1} \tag{25}$$

$$b_{t-1} \; : \; \frac{\partial V_t}{\partial b_{t-1}} = \lambda_t (1-\Delta_t) \tag{26}$$

$$b_t \; : \; \lambda_t q_t = \beta \mathbb{E}_t \left\{ \frac{1}{1-\gamma} \Big(\mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right] \Big)^{\frac{\gamma}{1-\gamma}} \cdot (1-\gamma) V_{t+1}^{-\gamma} \cdot \frac{\partial V_{t+1}}{\partial b_t} \right\} \tag{27}$$

which yield following consumption-labor decision and Euler Equation

$$\phi c_t = A_t (1-\tau_t)(1-n_t) \tag{28}$$

$$q_t = \frac{g_t + z_t + b_t^d - A_t n_t \tau_t}{b_t} \tag{29}$$

$$q_t = \beta \mathbb{E}_t \left\{ (1-\Delta_{t+1}) \left(\frac{U_{t+1}}{U_t} \right) \left(\frac{c_t}{c_{t+1}} \right) V_{t+1}^{-\gamma} \Big(\mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right] \Big)^{\frac{\gamma}{1-\gamma}} \right\} \tag{30}$$

where this optimal solution also satisfies transversality condition

$$\lim_{j\to\infty} \mathbb{E}_t \left\{ M_{t,t+j+1} \cdot b_{t+j} \right\} = 0 \tag{31}$$

where $M_{t,t+1}$ is the term inside expectation on the right-hand side of equation (30). Since future continuation value depends on whether sovereign debt will default in next period or not certainty equivalent should be written as

$$\begin{aligned} \left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} &= \Bigg(\int_{\varepsilon^A_{t+1}}\int_{\varepsilon^g_{t+1}} \Big[p_{t,1}\cdot\lambda_{s_t,1}\cdot\left(V_{1,t+1}^{1-\gamma}|d\right) + p_{t,2}\cdot\lambda_{s_t,2}\cdot\left(V_{2,t+1}^{1-\gamma}|d\right) \\ &+ \quad (1-p_{t,1})\cdot\lambda_{s_t,1}\cdot\left(V_{1,t+1}^{1-\gamma}|nd\right) + (1-p_{t,2})\cdot\lambda_{s_t,2}\cdot\left(V_{2,t+1}^{1-\gamma}|nd\right)\Big]\Bigg)^{\frac{1}{1-\gamma}} \end{aligned}$$

where for instance $(V_{1,t+1}|nd)$ is the future continuation value if transfers are in stationary regime ($s_{t+1} = 1$) and default does not occur in next period ($t+1$). Then Euler equation becomes

$$\begin{aligned} q_t = \beta \quad \cdot \quad & \Bigg(\int_{\varepsilon^A_{t+1}}\int_{\varepsilon^g_{t+1}} p_{t,1}\cdot\lambda_{s_t,1}\cdot(1-\delta)\left(\frac{U_{t+1}|d}{U_t}\right)\left(\frac{c_t}{c_{t+1}|d}\right)\left(V_{1,t+1}^{-\gamma}|d\right)\left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{\gamma}{1-\gamma}} \\ + \quad & p_{t,2}\cdot\lambda_{s_t,2}\cdot(1-\delta)\left(\frac{U_{t+1}|d}{U_t}\right)\left(\frac{c_t}{c_{t+1}|d}\right)\left(V_{2,t+1}^{-\gamma}|d\right)\left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{\gamma}{1-\gamma}} \\ + \quad & (1-p_{t,1})\cdot\lambda_{s_t,1}\cdot\left(\frac{U_{t+1}|nd}{U_t}\right)\left(\frac{c_t}{c_{t+1}|nd}\right)\left(V_{1,t+1}^{-\gamma}|nd\right)\left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{\gamma}{1-\gamma}} \\ + \quad & (1-p_{t,2})\cdot\lambda_{s_t,2}\cdot\left(\frac{U_{t+1}|nd}{U_t}\right)\left(\frac{c_t}{c_{t+1}|nd}\right)\left(V_{2,t+1}^{-\gamma}|nd\right)\left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{\gamma}{1-\gamma}}\Bigg) \end{aligned}$$

6.2 Nonlinear Solution and Policy Function Iteration

In baseline model we only consider government debt b_t as a policy variable. However, current continuation value also enters bond-pricing Euler equation so we choose V_t as a policy variable as well. Similar to earlier procedure we use Monotone Mapping method in discretized state space around state variables and obtain iterated policy functions $b_t = b(\psi_t)$ and $V_t = V(\psi_t)$

using initial guesses $b(\psi_0)$ and $V(\psi_0)$. Given the current state $\psi_t = (b_{t-1}, \Delta_t, A_t, g_t, z_{t-1}, s_t, b_t^*)$ our updated policy functions $b(\psi_t)$ and $V(\psi_t)$ solve following system of nonlinear equations:

$$
\begin{aligned}
\frac{b_t^d + g_t + z_t - \tau(\psi_t) A_t n(\psi_t)}{b(\psi_t)} &= \beta \mathbb{E}_t \left\{ (1-\Delta_{t+1}) \left(\frac{U(\psi_{t+1})}{U(\psi_t)} \frac{c(\psi_t)}{c(\psi_{t+1})} \right) \left(\frac{V(\psi_{t+1})}{\left[\mathbb{E}_t \left(V(\psi_{t+1})^{1-\gamma} \right)\right]^{1/(1-\gamma)}} \right)^{-\gamma} \right\} \\
V(\psi_t) &= U(\psi_t) + \beta \cdot \left[\mathbb{E}_t \left(V(\psi_{t+1})^{1-\gamma} \right)\right]^{1/(1-\gamma)}
\end{aligned}
$$

where we construct $\psi_{t+1} = (b(\psi_t), \Delta_{t+1}, A_{t+1}, g_{t+1}, z_t, s_{t+1}, b_{t+1}^*)$ and take expectation $\mathbb{E}_t$ using linear interpolation and numerical integration technniques

We calculate distance between our initial guesses $b(\psi_0)$, $V(\psi_0)$ and updated policy functions $b_t = b(\psi_t)$, $V_t = V(\psi_t)$. If the maximum distance is smaller than the desired tolerance $(1e-6)$ then policy function convergence is satisfied. Otherwise, we update our guesses as $b(\psi_t)$ and $V(\psi_t)$ and repeat earlier steps until a convergence is met.

7 Generalized Recursive Smooth Ambiguity Utility Model

As we have seen in recursive utility model, V_{t+1} depends on which state transfers will be in next period. In earlier two models, transfers were assumed to move between states via Markov transition matrix. Therefore, households think that V_{t+1} will take two different values for two possible states $(s_{t+1} = 1)$ and $(s_{t+1} = 2)$. However, in this section we assume that transfers follow a hidden state implying that households cannot observe s_{t+1} directly via predetermined transition probabilities $\lambda_{s_t, s_{t+1}}$ but they can only have beliefs μ_t about underlying state. These beliefs are basically distorted state probabilities which create an additional sensitivity behavior over future continuation value.

Ju and Miao (2012) proposed Generalized Recursive Smooth Ambiguity Utility Model which creates a further separation between risk aversion and ambiguity aversion. Similar to recursive utility model, risk sensitivity over continuation value is again characterized by concave transformation $u(x) = x^{1-\gamma}/(1-\gamma)$. However unlike earlier model since now households have ambiguous beliefs over states of economy we cannot compute certainty equivalence right away. As it is mentioned, households have different beliefs over different certainty equivalence computed under different states. Thus, following Ju and Miao (2012) we compute an additional concave transformation $v(x) = x^{1-\eta}/(1-\eta)$ over certainty equivalent of risk aversion as

$$v^{-1}\left\{\mathbb{E}_{\mu_t}\left[v \circ u^{-1}\left(\mathbb{E}_{s_{t+1}}\left[u(V_{t+1})\right]\right)\right]\right\} \tag{32}$$

Notice that if $(v \circ u^{-1})(.)$ is a linear transformation, or simply $v = u$ notation above will reduce to certainty equivalence in recursive utility model. Additionally, in this model households cannot possibly know that which state transfers follow therefore Markov transition matrix cannot be used for inference in a traditional way. Instead households construct beliefs about unobservable state. For instance, in our model we set state belief $\mu_t = Pr(s_{t+1} = 2)$ as the probability that transfers will be in explosive state in next period. Given prior belief μ_0 households update their beliefs using Bayes' Rule:

$$\mu_{t+1} = \frac{\lambda_{22} \cdot f(\ln(z_{t+1}/\bar{z}), 2) \cdot \mu_t + (1-\lambda_{11}) \cdot f(\ln(z_{t+1}/\bar{z}), 1) \cdot (1-\mu_t)}{f(\ln(z_{t+1}/\bar{z}), 2) \cdot \mu_t + f(\ln(z_{t+1}/\bar{z}), 1) \cdot (1-\mu_t)} \tag{33}$$

where $f(\ln\left(\frac{z_{t+1}}{\bar{z}}\right), s) = \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{1}{2} \cdot \left(\frac{\ln\left(\frac{z_{t+1}}{\bar{z}}\right) - \kappa_s}{\sigma_s}\right)^2\right)$ is the density function of the normal distribution with mean κ_s and variance σ_s^2 dependent on hidden state s as

$$\begin{aligned}
\kappa_1 &= \kappa_2 = 0 \\
\sigma_1^2 &= \frac{\eta_z^2 \sigma_A^2}{1-\rho_A^2} \\
\sigma_2^2 &= \frac{\eta_z^2 \sigma_A^2}{(1-\rho_A^2)(1-\rho_z)^2}
\end{aligned}$$

7.1 Model solution

Following Ju and Miao (2012), household solves optimization problem below by choosing consumption c_t, labor hours n_t, and government bond b_t:

$$
\begin{aligned}
V(\mu_t,\psi_t) &= \max_{\{c_t,n_t,b_t\}} \{U(c_t,1-n_t) + \beta\cdot\mathcal{R}_t\left(V(\mu_{t+1},\psi_{t+1})\right)\} \\
where &\quad \mathcal{R}_t\left(V(\mu_{t+1},\psi_{t+1})\right) = \left\{\mathbb{E}_{\mu_t}\left(\mathbb{E}_{s_{t+1}}[V(\mu_{t+1},\psi_{t+1})]^{1-\gamma}\right)^{\frac{1-\eta}{1-\gamma}}\right\}^{\frac{1}{1-\eta}} \\
s.t. &\quad c_t + b_t q_t = (1-\Delta_t)b_{t-1} + z_t + (1-\tau_t)A_t n_t
\end{aligned}
$$

where state variables are given as $\psi_t = (b_{t-1},\Delta_t,A_t,g_t,z_t)$.

Departing from our earlier two models we now write certainty equivalence $\mathcal{R}_t\left(V_{t+1}\right)$ by taking expectation with respect to state belief, μ_t as

$$
\mathcal{R}_t(V_{t+1}) = \left\{(1-\mu_t)\left(\mathbb{E}_{s_{t+1}=1}[V_{t+1}^{1-\gamma}]\right)^{\frac{1-\eta}{1-\gamma}} + \mu_t\left(\mathbb{E}_{s_{t+1}=2}[V_{t+1}^{1-\gamma}]\right)^{\frac{1-\eta}{1-\gamma}}\right\}^{\frac{1}{1-\eta}} \qquad (34)
$$

where

$$
\begin{aligned}
\mathbb{E}_{s_{t+1}=1}\left[V_{t+1}^{1-\gamma}\right] &= \int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g} p_{t,1}\left(V_{1,t+1}^{1-\gamma}|d\right) + (1-p_{t,1})\left(V_{1,t+1}^{1-\gamma}|nd\right) \\
\mathbb{E}_{s_{t+1}=2}\left[V_{t+1}^{1-\gamma}\right] &= \int_{\varepsilon_{t+1}^A}\int_{\varepsilon_{t+1}^g} p_{t,2}\left(V_{2,t+1}^{1-\gamma}|d\right) + (1-p_{t,2})\left(V_{2,t+1}^{1-\gamma}|nd\right)
\end{aligned}
$$

What makes this model different from the model with Epstein-Zin preferences is that we now introduce an additional parameter η and we sum different certainty equivalents of risk aversion across distorted state beliefs . We denote η as ambiguity aversion parameter since it creates a new separation between risk aversion and ambiguity aversion. This separation can be observed in our certainty equivalent equation (34). We model ambiguity neutrality as taking $\eta=\gamma$ because if there is no ambiguity then $\mathbb{E}_t = \mathbb{E}_{s_{t+1}=1} = \mathbb{E}_{s_{t+1}=2}$ and certainty equivalent reduces to

$$\mathcal{R}_t(V_{t+1}) = \left\{(1-\mu_t)\Big(\mathbb{E}_t[V_{t+1}^{1-\gamma}]\Big)^{\frac{1-\eta}{1-\gamma}} + \mu_t\Big(\mathbb{E}_t[V_{t+1}^{1-\gamma}]\Big)^{\frac{1-\eta}{1-\gamma}}\right\}^{\frac{1}{1-\eta}} = \Big(\mathbb{E}_t[V_{t+1}^{1-\gamma}]\Big)^{\frac{1}{1-\gamma}} \tag{35}$$

Above equation tells us that our model with Epstein-Zin preferences is only a specific case where households are ambiguity neutral. Therefore, we say that households are ambiguity averse if and only if $\eta > \gamma$. Then the households' first-order conditions are,

$$\begin{aligned}
c_t &: \lambda_t = \Big[(1-n_t)^{\phi}\Big] \\
n_t &: \lambda_t A_t(1-\tau_t) = \Big[\phi c_t(1-n_t)^{\phi-1}\Big] \\
b_{t-1} &: \frac{\partial V_t}{\partial b_{t-1}} = \lambda_t(1-\Delta_t) \\
b_t &: \lambda_t q_t = \beta\mathbb{E}_t\left\{\frac{\partial V_{t+1}}{\partial b_t}\cdot V_{t+1}^{-\gamma}\cdot\Big(\mathbb{E}_{s_{t+1}}\Big[V_{t+1}^{1-\gamma}\Big]\Big)^{\frac{\gamma-\eta}{1-\gamma}}\cdot\left[\mathcal{R}_t\left(V_{t+1}\right)\right]^{\eta}\right\}
\end{aligned}$$

which yield following consumption-labor decision and Euler Equation

$$\frac{\phi c_t}{1-n_t} = A_t(1-\tau_t)$$

$$q_t = \beta\mathbb{E}_t\left\{(1-\Delta_{t+1})\left(\frac{U_{t+1}}{U_t}\right)\left(\frac{c_t}{c_{t+1}}\right)\cdot\left(\frac{V_{t+1}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{-\gamma}\left(\frac{\Big(\mathbb{E}_{s_{t+1}}\Big[V_{t+1}^{1-\gamma}\Big]\Big)^{\frac{1}{1-\gamma}}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{\gamma-\eta}\right\} \tag{36}$$

Notice that as we mentioned about ambiguity neutral case if $\eta = \gamma$ expression on the right-hand side of Euler equation reduces to its recursive utility version. Therefore, ambiguity aversion clearly has an effect on the sovereign risk premium.

7.2 Nonlinear Solution and Policy Function Iteration

Nearly identical to earlier procedure we use Monotone Mapping method in discretized state space around state variables and obtain iterated policy functions $b_t = b(\mu_t, \psi_t)$ and $V_t = V(\mu_t, \psi_t)$. However we now set recursive utility model, or we can say that ambiguity-neutral case as our baseline case instead of log-linerized model. Therefore, we obtain policy functions $b(\mu_0, \psi_0)$ and $V(\mu_0, \psi_0)$ from recursive utility model as initial guesses. Given the current state $\psi_t = (b_{t-1}, \Delta_t, A_t, g_t, z_t, b_t^*)$ our updated policy functions $b(\mu_t, \psi_t)$ and $V(\mu_t, \psi_t)$ solve following system of nonlinear equations:

$$\begin{aligned}
\frac{b_t^d + g_t + z_t - \tau(\mu_t, \psi_t) A_t n(\mu_t, \psi_t)}{b(\mu_t, \psi_t)} &= \beta \mathbb{E}_t \Bigg[(1 - \Delta(\mu_{t+1}, \psi_{t+1})) \left(\frac{U(\mu_{t+1}, \psi_{t+1})}{U(\mu_t, \psi_t)} \frac{c(\mu_t, \psi_t)}{c(\mu_{t+1}, \psi_{t+1})} \right) \\
&\times \left(\frac{V(\mu_{t+1}, \psi_{t+1})}{\mathcal{R}_t(V(\mu_{t+1}, \psi_{t+1}))} \right)^{-\gamma} \left(\frac{\left[\mathbb{E}_t \left(V(\mu_{t+1}, \psi_{t+1})^{1-\gamma} \right) \right]^{1/(1-\gamma)}}{\mathcal{R}_t(V(\mu_{t+1}, \psi_{t+1}))} \right)^{\gamma - \eta} \Bigg] \\
V(\mu_t, \psi_t) &= U(\mu_t, \psi_t) + \beta \cdot \left\{ \mathbb{E}_{\mu_t} \left(\mathbb{E}_{s_{t+1}} [V(\mu_{t+1}, \psi_{t+1})]^{1-\gamma} \right)^{\frac{1-\eta}{1-\gamma}} \right\}^{\frac{1}{1-\eta}}
\end{aligned}$$

where we construct $\psi_{t+1} = (b(\psi_t), \Delta_{t+1}, A_{t+1}, g_{t+1}, z_{t+1}, b_{t+1}^*)$ and take expectation $\mathbb{E}_t$ using linear interpolation and numerical integration technniques

We calculate distance between our initial guesses $b(\mu_0, \psi_0)$, $V(\mu_0, \psi_0)$ and updated policy functions $b_t = b(\mu_t, \psi_t)$, $V_t = V(\mu_t, \psi_t)$. If the maximum distance is smaller than the desired tolerance $(1e - 6)$ then policy function convergence is satisfied. Otherwise, we update our guesses as $b(\mu_t, \psi_t)$ and $V(\mu_t, \psi_t)$ and repeat earlier steps until a convergence is met.

8 Numerical Techniques

8.1 Linear Interpolation

Future Continuation Value

We use linear interpolation to find future continuation value V_{t+1} in our solution procedure. We already can find current continuation value V_t by the iteration of approximating function $V(\mu, \psi)$ for all possible combinations of nodes in predetermined discrete grid space. However when we update to obtain state belief and variables in next period $(\mu^{'}, \psi^{'})$ we may not have the exact combination of nodes in discrete grid space that gives us $V(\mu^{'}, \psi^{'})$. Thus, we apply linear interpolation by using nearest nodes to proxy future continuation value.

The nearest perimeter around $V(b'_d, A', g', z', \mu')$ is constructed starting from $V(b^d_{i_1}, A_{i_2}, g_{i_3}, z_{i_4}, \mu_{i_5})$ and using following values at $2^5 = 32$ points

$$\mathcal{V} = \left\{ V(b^d_{i_1+a}, A_{i_2+b}, g_{i_3+c}, z_{i_4+d}, \mu_{i_5+e}) | a, b, c, d, e \in \{0, 1\} \right\} \tag{37}$$

First we locate the grid point to left of b'_d as

$$loc_{b'_d} = \min(n_b - 1, max(1, floor(dist/step) + 1)) \tag{38}$$

where $step = b^d_2 - b^d_1$ and $dist = b'_d - b^d_1$. After similarly locating each grid we start to interpolate in the b^d direction holding others fixed

$$\begin{aligned} V(b'_d, \cdot) &= V(b^d_{i_1}, \cdot) + \left(b'_d - b^d_{i_1}\right) \frac{V(b^d_{i_1+1}, \cdot) - V(b^d_{i_1}, \cdot)}{b^d_{i_1+1} - b^d_{i_1}} \\ &= \left(\frac{b^d_{i_1+1} - b'_d}{b^d_{i_1+1} - b^d_{i_1}}\right) V(b^d_{i_1}, \cdot) + \left(\frac{b'_d - b^d_{i_1}}{b^d_{i_1+1} - b^d_{i_1}}\right) V(b^d_{i_1+1}, \cdot) \\ &= w_{b^d_{i_1}} V(b^d_{i_1}, \cdot) + w_{b^d_{i_1+1}} V(b^d_{i_1+1}, \cdot) \end{aligned}$$

Second we interpolate in the A direction holding others fixed

$$\begin{aligned}
V(b'_d, A', \cdot) &= V(b'_d, A_{i_2}, \cdot) + \left(A' - A_{i_2}\right) \frac{V(b'_d, A_{i_2+1}, \cdot) - V(b'_d, A_{i_2}, \cdot)}{A_{i_2+1} - A_{i_2}} \\
&= \left(\frac{A_{i_2+1} - A'}{A_{i_2+1} - A_{i_2}}\right) V(b'_d, A_{i_2}, \cdot) + \left(\frac{A' - A_{i_2}}{A_{i_2+1} - A_{i_2}}\right) V(b'_d, A_{i_2+1}, \cdot) \\
&= w_{A_{i_2}} V(b'_d, A_{i_2}, \cdot) + w_{A_{i_2+1}} V(b'_d, A_{i_2+1}, \cdot) \\
&= w_{b^d_{i_1}} w_{A_{i_2}} V(b^d_{i_1}, A_{i_2}, \cdot) + w_{b^d_{i_1+1}} w_{A_{i_2}} V(b^d_{i_1+1}, A_{i_2}, \cdot) \\
&+ w_{b^d_{i_1}} w_{A_{i_2+1}} V(b^d_{i_1}, A_{i_2+1}, \cdot) + w_{b^d_{i_1+1}} w_{A_{i_2+1}} V(b^d_{i_1+1}, A_{i_2+1}, \cdot)
\end{aligned}$$

Repeating the procedure for all directions we obtain

$$\begin{aligned}
V(b'_d, A', g', z', \mu') = \sum_{a=0}^{1}\sum_{b=0}^{1}\sum_{c=0}^{1}\sum_{d=0}^{1}\sum_{e=0}^{1} & w_{b^d_{i_1+a}} w_{A_{i_2+b}} w_{g_{i_3+c}} w_{z_{i_4+d}} w_{\mu_{i_5+e}} \cdot \\
& V(b^d_{i_1+a}, A_{i_2+b}, g_{i_3+c}, z_{i_4+d}, \mu_{i_5+e})
\end{aligned} \tag{39}$$

Default Probability

In Section 2, we already characterized conditional distribution of fiscal limit for all possible nodes of A_t, g_t and s_t as defined in discretized grid space. Therefore we can easily locate $p(b, A, g, s)$ given $b_{t-1} = b$, $A_t = A$, $g_t = g$ and $s_t = s$. However, throughout nonlinear solution we obtain updated $b_t = b'$, $A_{t+1} = A'$, $g_{t+1} = g'$ and $s_{t+1} = s'$ which may not correspond to any exact node in the prespecified grid. In this situation, as we did for the future continuation value, to find the closest perimeter around $p(b', A', g', 1)$ and $p(b', A', g', 2)$ we locate starting point $p(b_{j_1}, A_{j_2}, g_{j_3}, 1)$ and $p(b_{j_1}, A_{j_2}, g_{j_3}, 2)$ and use following two sets of $2^3 = 8$ points

$$\mathcal{P}_1 = \{p(b_{j_1+a}, A_{j_2+b}, g_{j_3+c}, 1) | a, b, c \in \{0, 1\}\}$$

$$\mathcal{P}_2 = \{p(b_{j_1+a}, A_{j_2+b}, g_{j_3+c}, 2) | a, b, c \in \{0, 1\}\}$$

First we locate the grid point to left of b' as

$$loc_{b'} = \min(n_b - 1, max(1, floor(dist/step) + 1)) \tag{40}$$

where $step = b_2 - b_1$ and $dist = b' - b_1$. After similarly locating each grid we start to interpolate in the b direction holding others fixed

$$\begin{aligned}
p(b', A_{j_2}, g_{j_3}, .) &= p(b_{j_1}, A_{j_2}, g_{j_3}, .) + \left(b' - b_{j_1}\right) \frac{p(b_{j_1+1}, A_{j_2}, g_{j_3}, .) - p(b_{j_1}, A_{j_2}, g_{j_3}, .)}{b_{j_1+1} - b_{j_1}} \\
&= \left(\frac{b_{j_1+1} - b'}{b_{j_1+1} - b_{j_1}}\right) p(b_{j_1}, A_{j_2}, g_{j_3}, .) + \left(\frac{b' - b_{j_1}}{b_{j_1+1} - b_{j_1}}\right) p(b_{j_1+1}, A_{j_2}, g_{j_3}, .) \\
&= w_{b_{j_1}} \cdot p(b_{j_1}, A_{j_2}, g_{j_3}, .) + w_{b_{j_1+1}} \cdot p(b_{j_1+1}, A_{j_2}, g_{j_3}, .)
\end{aligned}$$

Second we interpolate in the A direction holding others fixed

$$\begin{aligned}
p(b', A', g_{j_3}, .) &= p(b', A_{j_2}, g_{j_3}, .) + \left(A' - A_{j_2}\right) \frac{p(b', A_{j_2+1}, g_{j_3}, .) - p(b', A_{j_2}, g_{j_3}, .)}{A_{j_2+1} - A_{j_2}} \\
&= \left(\frac{A_{j_2+1} - A'}{A_{j_2+1} - A_{j_2}}\right) p(b', A_{j_2}, g_{j_3}, .) + \left(\frac{A' - A_{j_2}}{A_{j_2+1} - A_{j_2}}\right) p(b', A_{j_2+1}, g_{j_3}, .) \\
&= w_{A_{j_2}} \cdot p(b', A_{j_2}, g_{j_3}, .) + w_{A_{j_2+1}} \cdot p(b', A_{j_2+1}, g_{j_3}, .) \\
&= w_{b_{j_1}} w_{A_{j_2}} \cdot p(b_{j_1}, A_{j_2}, g_{j_3}, .) + w_{b_{j_1+1}} w_{A_{j_2}} \cdot p(b_{j_1+1}, A_{j_2}, g_{j_3}, .) \\
&+ w_{b_{j_1}} w_{A_{j_2+1}} \cdot p(b_{j_1}, A_{j_2+1}, g_{j_3}, .) + w_{b_{j_1+1}} w_{A_{j_2+1}} \cdot p(b_{j_1+1}, A_{j_2+1}, g_{j_3}, .)
\end{aligned}$$

Third we interpolate in the g direction holding others fixed

$$\begin{aligned}
p(b', A', g', .) &= p(b', A', g_{j_3}, .) + \left(g' - g_{j_3}\right) \frac{p(b', A', g_{j_3+1}, .) - p(b', A', g_{j_3}, .)}{g_{j_3+1} - g_{j_3}} \\
&= \left(\frac{g_{j_3+1} - g'}{g_{j_3+1} - g_{j_3}}\right) p(b', A', g_{j_3}, .) + \left(\frac{g' - g_{j_3}}{g_{j_3+1} - g_{j_3}}\right) p(b', A', g_{j_3+1}, .) \\
&= w_{g_{j_3}} \cdot p(b', A', g_{j_3}, .) + w_{g_{j_3+1}} \cdot p(b', A', g_{j_3+1}, .)
\end{aligned}$$

Writing first and second steps into third we obtain default probability in next state

$$p(b', A', g', 1) = \sum_{a=0}^{1}\sum_{b=0}^{1}\sum_{c=0}^{1} w_{b_{j_1+a}} w_{A_{j_2+b}} w_{g_{j_3+c}} \cdot p(b_{j_1+a}, A_{j_2+b}, g_{j_3+c}, 1) \tag{41}$$

$$p(b', A', g', 2) = \sum_{a=0}^{1}\sum_{b=0}^{1}\sum_{c=0}^{1} w_{b_{j_1+a}} w_{A_{j_2+b}} w_{g_{j_3+c}} \cdot p(b_{j_1+a}, A_{j_2+b}, g_{j_3+c}, 2) \tag{42}$$

8.2 Trapezoid Rule

We use Trapezoid Rule for pricing kernel's numerical integration over ε^A_{t+1} and ε^g_{t+1} where each has $n = 10$ stochastic realizations. When we construct the grids for both stochastic variables we truncate their distributions at four standard deviations. Assuming that the pricing kernel M_{t+1} is a continuous over variables we first write the expectation over ε^g_{t+1} holding $\varepsilon^A_{t+1} = \varepsilon^A_1$ as constant

$$\begin{aligned}
\mathbb{E}_{j=1}[M(.)] \quad &\approx \quad \left(\frac{Pr(\varepsilon^A_1)Pr(\varepsilon^g_1)\cdot M(\varepsilon^A_1,\varepsilon^g_1) + Pr(\varepsilon^A_1)Pr(\varepsilon^g_2)\cdot M(\varepsilon^A_1,\varepsilon^g_2)}{2}\right)\Delta\varepsilon^g \\
&+ \quad \left(\frac{Pr(\varepsilon^A_1)Pr(\varepsilon^g_2)\cdot M(\varepsilon^A_1,\varepsilon^g_2) + Pr(\varepsilon^A_1)Pr(\varepsilon^g_3)\cdot M(\varepsilon^A_1,\varepsilon^g_3)}{2}\right)\Delta\varepsilon^g \\
&+ \quad ... \\
&+ \quad \left(\frac{Pr(\varepsilon^A_1)Pr(\varepsilon^g_9)\cdot M(\varepsilon^A_1,\varepsilon^g_9) + Pr(\varepsilon^A_1)Pr(\varepsilon^g_{10})\cdot M(\varepsilon^A_1,\varepsilon^g_{10})}{2}\right)\Delta\varepsilon^g \\
&= \quad \left(\frac{\Delta\varepsilon^g}{2}\right)Pr(\varepsilon^A_1)\left[\left(2\sum_{i=1}^{10}Pr(\varepsilon^g_i)M(\varepsilon^A_1,\varepsilon^g_i)\right) - Pr(\varepsilon^g_1)M(\varepsilon^A_1,\varepsilon^g_1) - Pr(\varepsilon^g_{10})M(\varepsilon^A_1,\varepsilon^g_{10})\right]
\end{aligned}$$

Doing the same summation for all j we write expectation over ε^A_{t+1}

$$\mathbb{E}[M(.)] \approx \left(\frac{\Delta\varepsilon^A}{2}\right)\left[\left(2\sum_{j=1}^{10}\mathbb{E}_j[M(.)]\right) - \mathbb{E}_{j=1}[M(.)] - \mathbb{E}_{j=10}[M(.)]\right] \tag{43}$$

9 Calibration to the Greek Data

Using data between 1971 and 2010 we calibrate parameters to the Greek economy. We collected our data for fiscal variables from OECD Economic Outlook No. 86. For government debt we refer to European Commission (2009). Following Camerer (1999) and Ju and Miao (2012) we adopt risk aversion and ambiguity aversion parameters as 2 and 8. We also set ambiguity aversion paramter as 5 to draw comparisons between two values. We calibrate at the steady-state tax revenue as 33.12% percent of GDP, government expenditure as 16.74% percent of GDP, and transfers as 14.04% percent of GDP. We define productivity as real GDP per worker which is extracted from Penn World Table Version 8.1. HP filtered productivity is estimated to have 0.45 persistence and a standard deviation of 0.033. Similarly using HP filtering we estimate that detrended government expenditure has 0.426 persistence of and 0.03 standard deviation. We regress tax rate on government debt and find mean response of tax to debt as 0.37. This supports our assumption that government raises tax rate when its debt rises. For government transfers we run Markov switching regression and estimate that non-switching response of transfers to productivity as -0.45 which is negative as we assume that transfers move countercyclically. Transfers are found to grow at a rate of 1.015 in the explosive regime. Both regimes are estimated to be highly persistent where probability of staying at current regime in next period is 0.975 for both which leaves 0.025 probability to switch to other regime.

Table 1: Parameter Values

Preferences	
Discount factor, β	0.95
Risk aversion, γ	2
Ambiguity aversion, η	5-8
Steady-state values	
Expenditure-GDP ratio, $\bar{g}/\bar{y}$	0.1674
Transfers-GDP ratio, $\bar{z}/\bar{y}$	0.1404
Tax rate, $\bar{\tau}$	0.3312
Labor ratio, $\bar{n}$	0.25
Other parameters	
Productivity persistence, ρ_A	0.45
Expenditure persistence, ρ_g	0.426
Standard deviation of productivity, σ_A	0.033
Standard deviation of expenditure, σ_g	0.03
Response of tax to debt, γ_τ	0.37
Response of transfers to productivity, γ_z	-0.45
Transfers growth, ρ_z	1.015

10 Nonlinear Simulations

In this section we run nonlinear simulations to replicate downturn of Greek economy because of ever-rising transfers and inertia in fiscal policy. For baseline model we observe that after an extended period of increasing transfers even a mild economic recession can put sustainability of government debt payroll in danger implying a rise in sovereign default risk and real interest rate. In this scenario, we find that while fiscal measures such as expenditure cut cannot be a remedy to palliate default risk, a structural fiscal reform namely changing the regime that transfers follow from explosive to stationary can ameliorate the sovereign default risk and thus decrease sovereign risk premium. However as Figure 5 supplies the comparisons between cases with different preferences for models with recursive and ambiguous preferences, government cannot handle its fiscal problems so easily and the case of rising transfers and fiscal apathy might result with serious economic crisis and total breakdown in economic variables. In these cases, we find that there may be an only way out that government has to put a higher default rate into practice and further repudiate a part of its debt.

10.1 Baseline Model

Figure 6 indicates that the economy stays at the steady-state with stationary transfers until the period $t = 5$ and then transfers switch to grow in the explosive regime for the next 35 years. With constantly increasing government tranfers and in the absence of positive productivity shock or surplus generating fiscal action, government starts to accumulate debt as a result and tries to finance it through further taxation as tax rate raises from 0.3 to 0.4. However raising tax rate, by definition, has a distortionary effect on consumption and labor supply and thus both decrease consequently. Despite these negative economic indicators, real interest rate sustain its level, 2.5% for long periods since decrease in consumption raises the marginal utility consumption therefore price of government bond increases.

Nevertheless, this economic perspective of households changes when debt reaches excessive levels and transfers continue to rise and seem to steadily follow explosive regime. When debt-to-GDP ratio goes up from 0.5 to 1.5 households start to price the sovereign default risk since the probability of debt hitting the fiscal limit rises from 0.1 to 0.3. Consequently, real interest rate nearly doubles its value.

As an addition to growing fiscal deficit and critical position of debt, we assume that now economy enters a recession when productivity level decreases starting from the period $t = 35$. The recession generating values are replicated from Bi (2012) as for instance -3.25% implies productivity level is 3.25 percent lower than its steady-state level. Figure 7 illustrates that

	t=35	t=36	t=37	t=38	t=39	t=40
Productivity	-3.25%	-4.67%	-5.31%	-5.6%	-2.56%	-1.16%

by the definition of fiscal limit lower productivity levels influence the government's ability to service its debt adversely where government debt nearly reaches twice the value of output and this further raises the probability of sovereign default to about 0.4. This upward shift in default probability is reflected on risk premium and real interest rate reaches 6%.

Fiscal Responses

As we mentioned in the short-run an expenditure cut can supply enough fiscal space via generating fiscal surplus for government to repair its credibility. However after prolonged period of upward-trending transfers and debt overhang Figure 8 shows that lowering government expenditure as simulated below cannot alleviate the situation as it comes at last minute and households do not change their worries over sustainability of debt. Aside from short-run

	t=35	t=36	t=37	t=38	t=39	t=40
Government Expenditure	-11.3%	-15.7%	-17.5%	-18.3%	-8.25%	-3.6%

austerity measures we now assume that as a structural reform government switches transfers to stationary regime at the period t=35. Figure 9 illustrates that fiscal reform can cause a substantial decrease in default probability and real interest rate as two almost return its steady-state levels. Contrary to a temporary cut in government expenditure regime switching transfers repair the government's ability to pay its debt through exhibiting potential fiscal surplus in the future, shift the fiscal limit up and restore the expectations of households. Thus, economy can safely be landed via fiscal reform.

10.2 Recursive and Smooth Ambiguity Models

We already showed that with recursive utility households demand higher risk premium since they are risk averse over continuation value. In the case of ambiguity aversion risk premium gets even higher values.[13] Expectations of forward-looking households deteriorates earlier than they do in the baseline case since their beliefs are distorted in such a way that they attach more weight to low continuation value where transfers follow explosive regime. Therefore with lower bond price government has to accumulate even more debt to be able to service its debt from earlier periods. This leads to a vicious cycle where consumption, labor supply and tax

[13] See Figure 5

revenue plummet while tax rate reaches almost 1. Figure 10 shows that at period $t = 35$ debt hits the fiscal limit. Default occurs even earlier in the smooth ambiguity model where probability of sovereign default reaches 1 at period $t = 25$ in Figure 14. In the baseline case we simulate an economic recession for last five periods where sovereign debt dynamics get worse. However in recursive preferences model since default already occurs at period $t = 35$ economy already stays in the crisis path at last periods.[14] Therefore we do not report figures indicating additional recession data for our smooth ambiguity model where economy enters fiscal crisis at around period $t = 25$.

Fiscal Responses

Similar to baseline case, decreasing government expenditure leads no improvement on the expectations of households about debt dynamics in the recursive utility model. Although government spends significantly lesser at last five periods this cannot change neither the future fiscal surplus nor the fiscal limit. Therefore government bond still is priced very low since households view this action as a short-run fiscal policy. However when transfers switch to be stabilizer in the economy government starts to give fiscal surplus. This policy lowers the amount of debt that government needs to borrow where Figure 13 indicates that debt-GDP ratio is 2.5 at $t = 35$ comparing to the value of 4 for the same period in Figure 12. Recovery in the fiscal balance delays the sovereign default but cannot avert it completely. Figure 13 shows that although at $t = 30$ default probability reaches as low as 0.2 and real interest rate is around 10% at last period due to risk sensitive behavior of households risk premium bursts again and probability of default rises sharply and takes the value of 1. Comparing to baseline case in the recursive utility model fiscal reform at late stage cannot completely undone the effects of fiscal deficit.

For the smooth ambiguity model, fiscal reform has no effect on the bond pricing behavior

[14] See Figure 11

because although transfers are switched to follow stationary regime households are ambiguous about latent state that transfers follow therefore still demand high risk premium as they put more weight to the case of explosive transfers. A quick comparison between Figure 14 and Figure 15 states that even if final period debt-GDP ratio falls from 15 to 10 bond-pricing dynamcics still does not change and sovereign default occurs similar to the case of fiscal apathy.

We propose a fiscal reform other than switching transfers' regime that if sovereign debt hits the fiscal limit government declares a partial default which is a rate $\delta' = 0.3$ higher than the prespecified default rate $\delta = 0.1$.Figure 15 shows that this repudiation of debt prevent consumption and labor supply from crashing and decrease level of government debt to be twice the value of output. This recovery of fiscal dynamics change the views of households where default probability falls sharply and real interest rate decreases by more than 5% percent.

11 Conclusion

We develop three models to explore sovereign debt dynamics of Greek economy. First model is the baseline case based on Bi (2012) where households are indifferent to risk and ambiguity. We find that steadily rising transfers deteriorate fiscal balance and cause debt overhang. As a result sovereign default probability and risk premium rise sharply. As a rescue plan stabilizing transfers can alleviate default risk. Second model contains recursive utility based on Epstein and Zin (1989) where households are risk averse over future continuation value that adds to sovereign risk premium. Following Ju and Miao (2012) we build our final model where preferences allow for risk aversion and ambiguity aversion characterized separately. Under the assumption that households are ambiguity averse sovereign risk premium gets even higher. As households price the sovereign bond this low government starts to collect even more debt to roll over its initial debt. Without a forward leap in productivity government uses its taxation tool and increases tax rate. Because of the shape of Laffer curve after some point tax revenue starts to decrease and government enters a cycle where debt is financed with more debt. Consequently government cannot service its ever-increasing debt and default occurs at earlier stages than it does in other two models. Since ambiguity averse investors cannot directly observe the state that transfers follow, stabilizing transfers cannot be a remedy to default risk. Therefore we find that instead government should renounce a higher portion of its debt for an economic recovery.

Figure 1: Greece Data between 1971 and 2010 (Standard & Poor's, OECD Economic Outlook No. 84 (2009) and European Commission (2009))

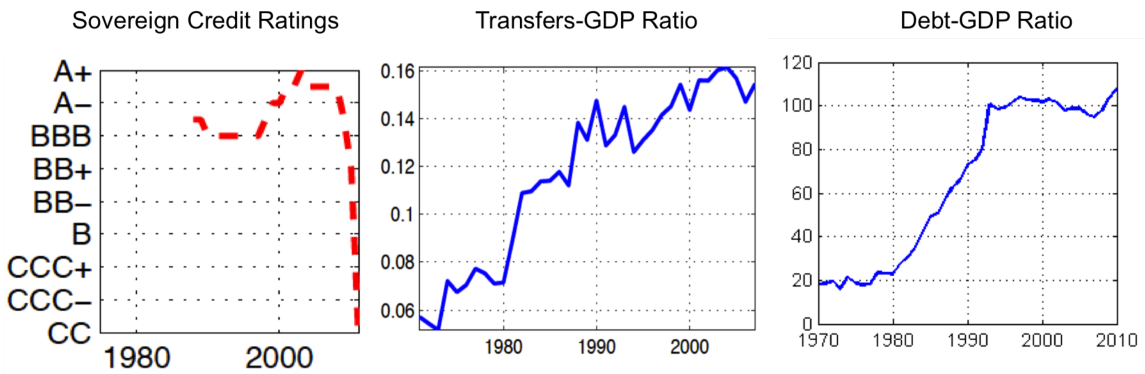


Figure 2: Distributions of Fiscal Limits with different productivity levels

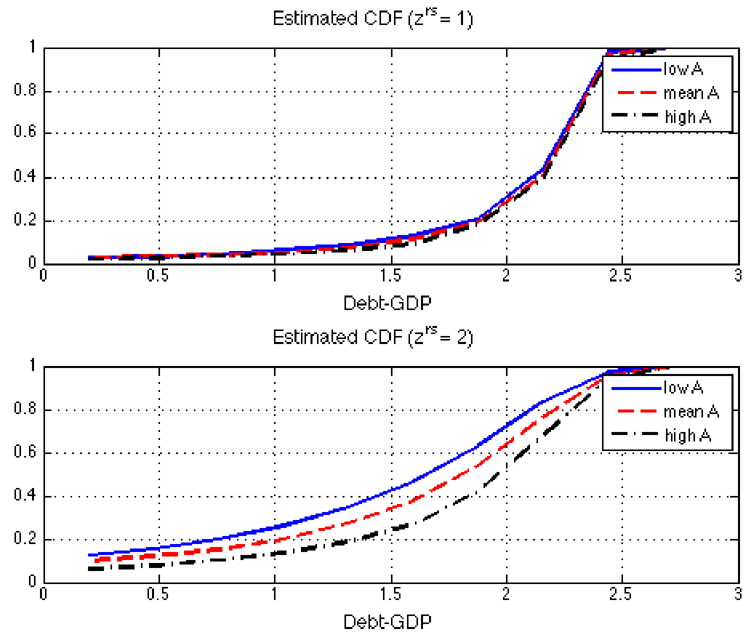


Figure 3: Distributions of Fiscal Limits with different government expenditure levels

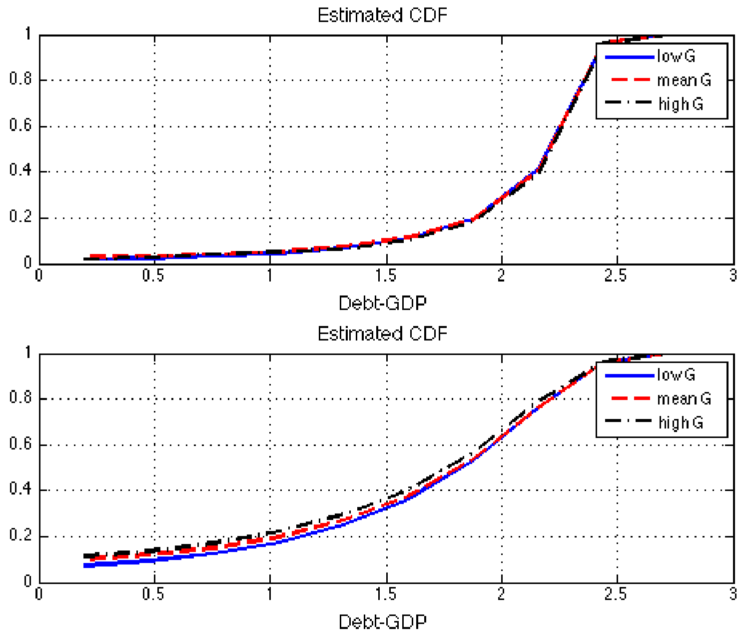


Figure 4: Distributions of Fiscal Limits under different transfers regimes

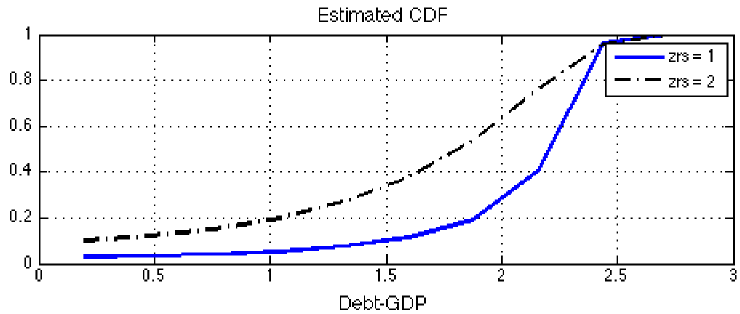


Figure 5: Nonlinear Simulation with Comparison of Different Preferences: Growing transfers for 40 years

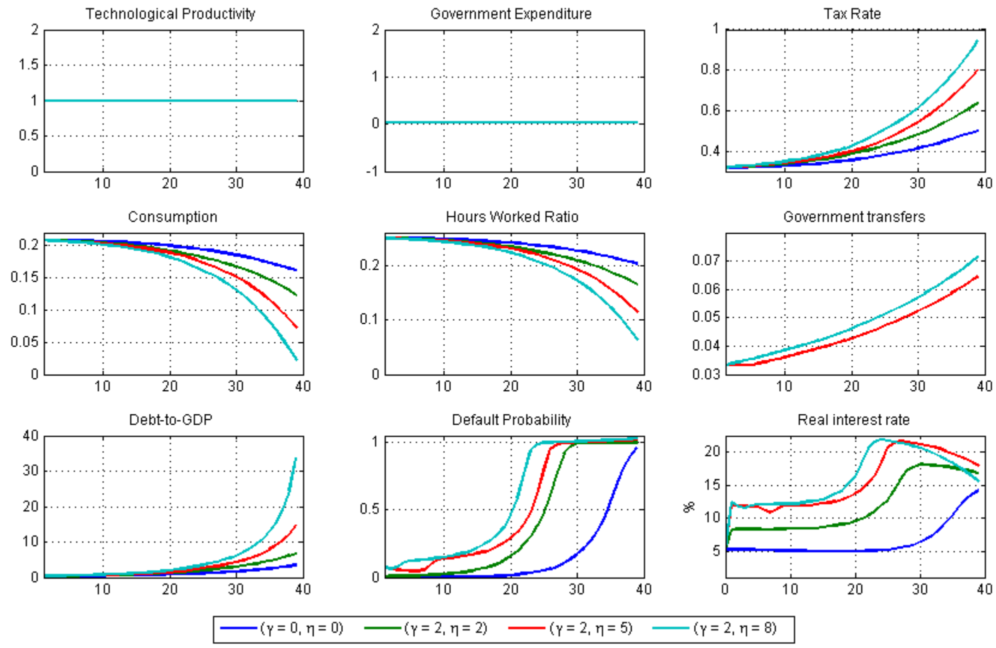


Figure 6: Nonlinear Simulation with Baseline Preferences ($\gamma = 0, \eta = 0$): Growing transfers starting from $t = 5$

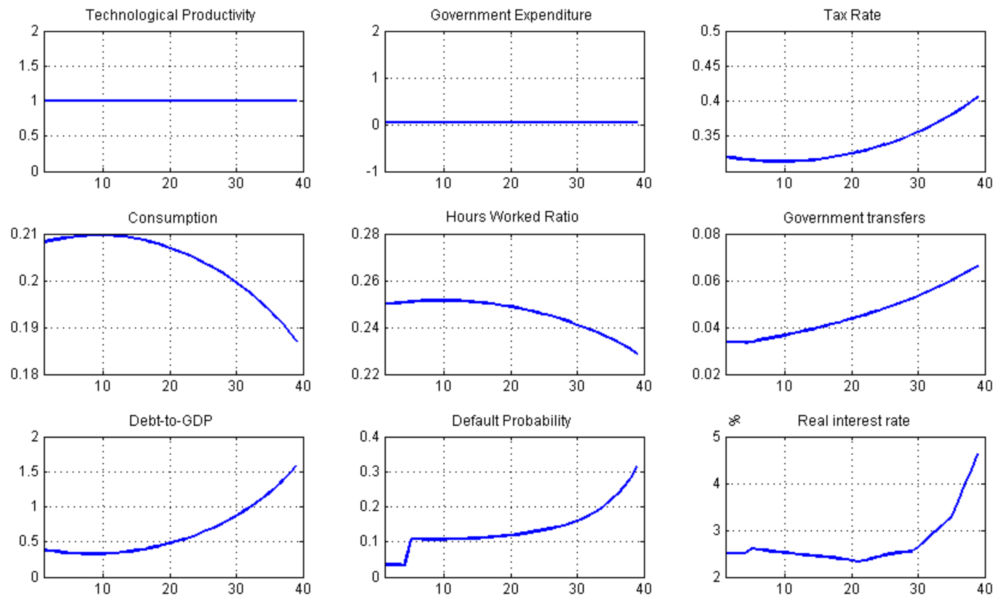


Figure 7: Nonlinear Simulation with Baseline Preferences ($\gamma = 0, \eta = 0$): Economic recession with lower productivity levels

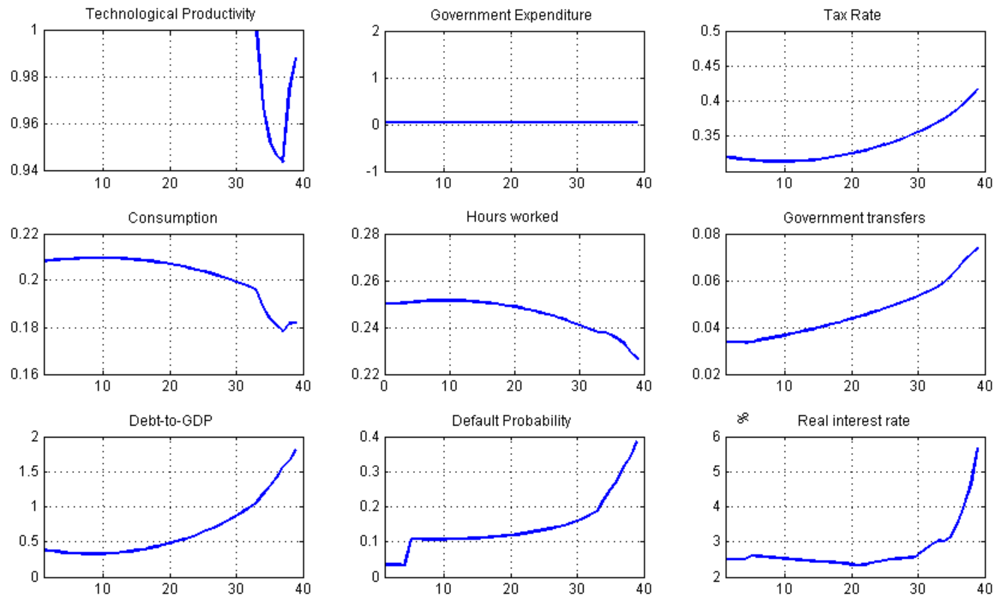


Figure 8: Nonlinear Simulation with Baseline Preferences ($\gamma = 0, \eta = 0$): Austerity measure as an expenditure cut

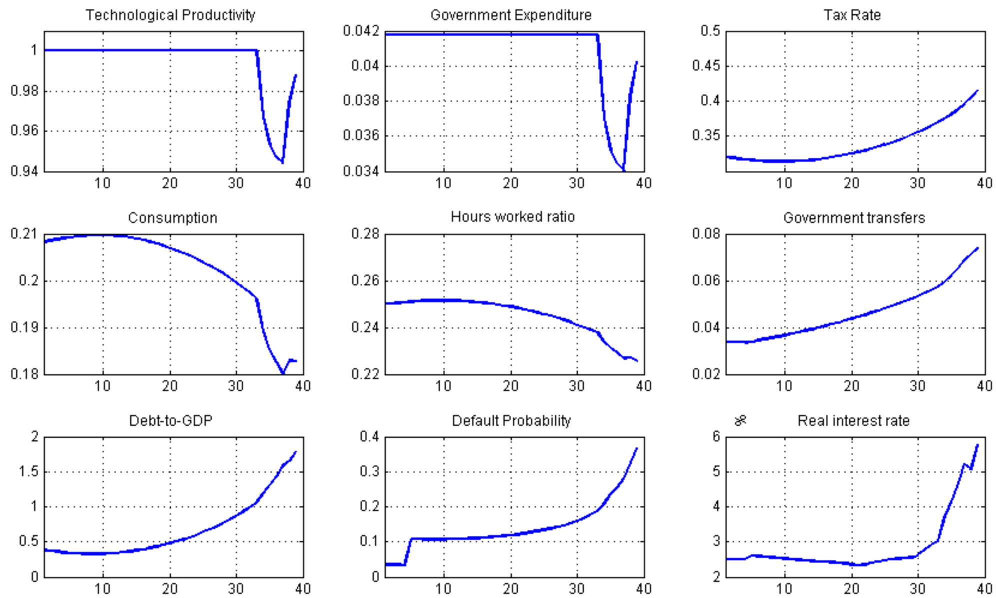


Figure 9: Nonlinear Simulation with Baseline Preferences ($\gamma = 0, \eta = 0$): Fiscal reform where transfers return to stationary regime

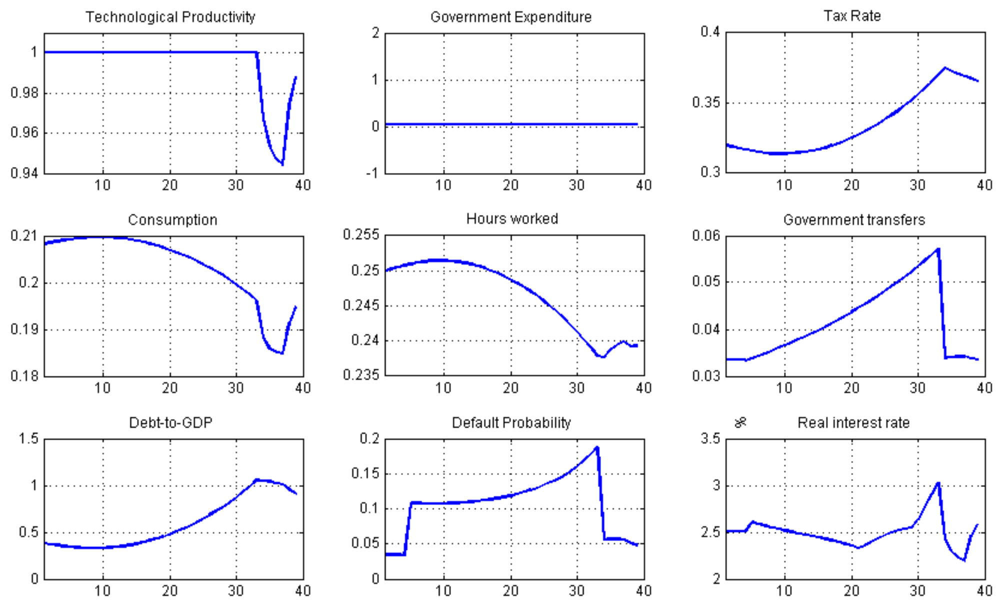


Figure 10: Nonlinear Simulation with Recursive Preferences ($\gamma = 2, \eta = 2$): Growing transfers starting from $t = 5$

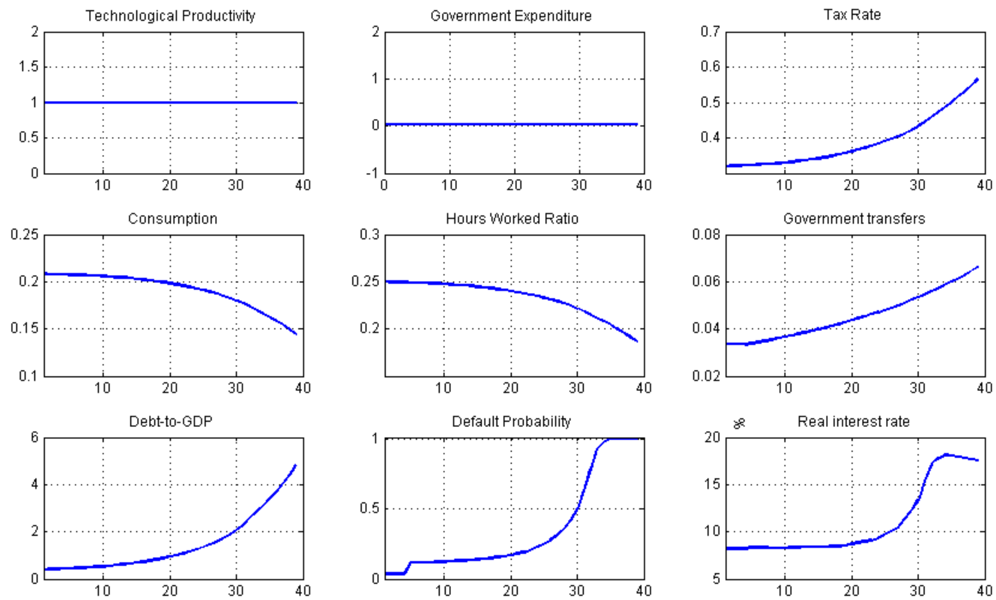


Figure 11: Nonlinear Simulation with Recursive Preferences ($\gamma = 2, \eta = 2$): Economic recession with lower productivity levels

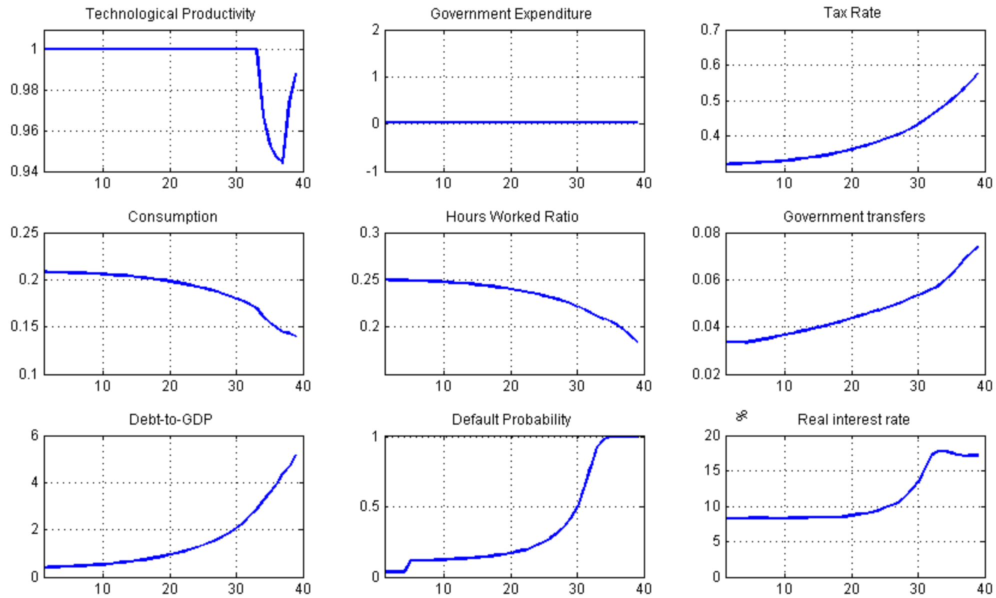


Figure 12: Nonlinear Simulation with Recursive Preferences ($\gamma = 2, \eta = 2$): Austerity measure as an expenditure cut

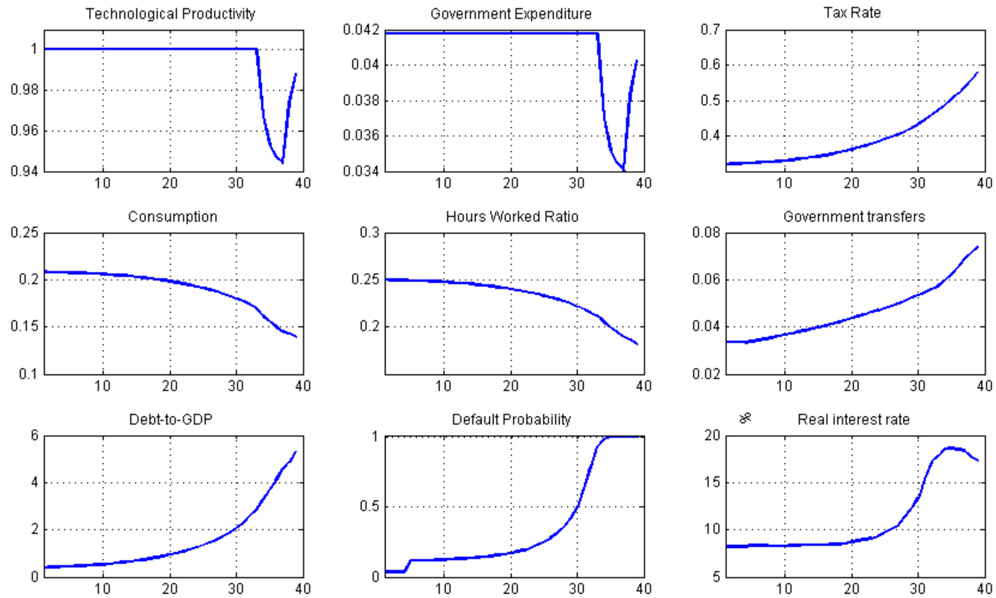


Figure 13: Nonlinear Simulation with Recursive Preferences ($\gamma = 2, \eta = 2$): Fiscal reform where transfers return to stationary regime

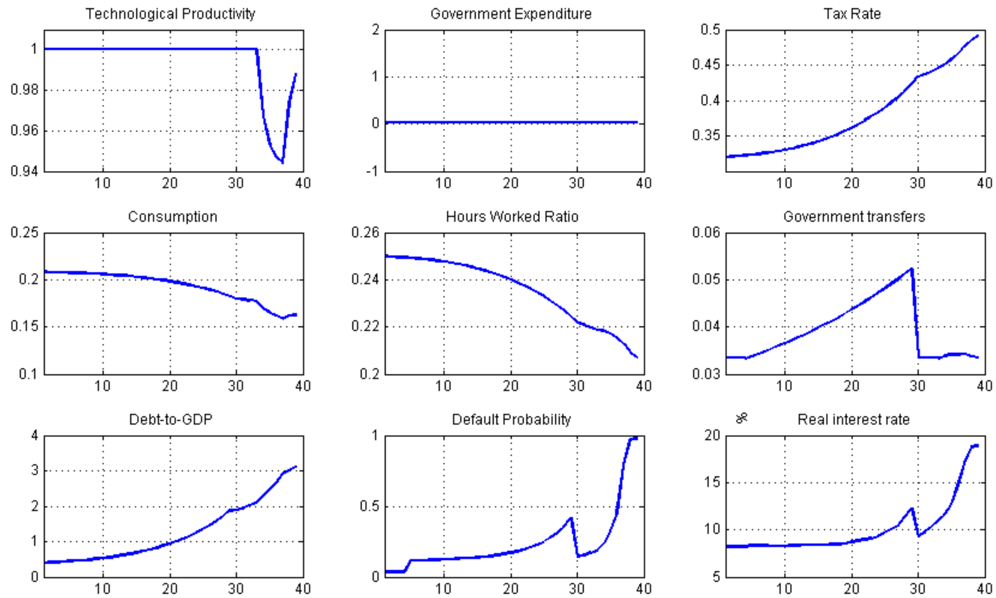


Figure 14: Nonlinear Simulation with Ambiguous Preferences ($\gamma = 2, \eta = 5$): Growing transfers starting from $t = 5$

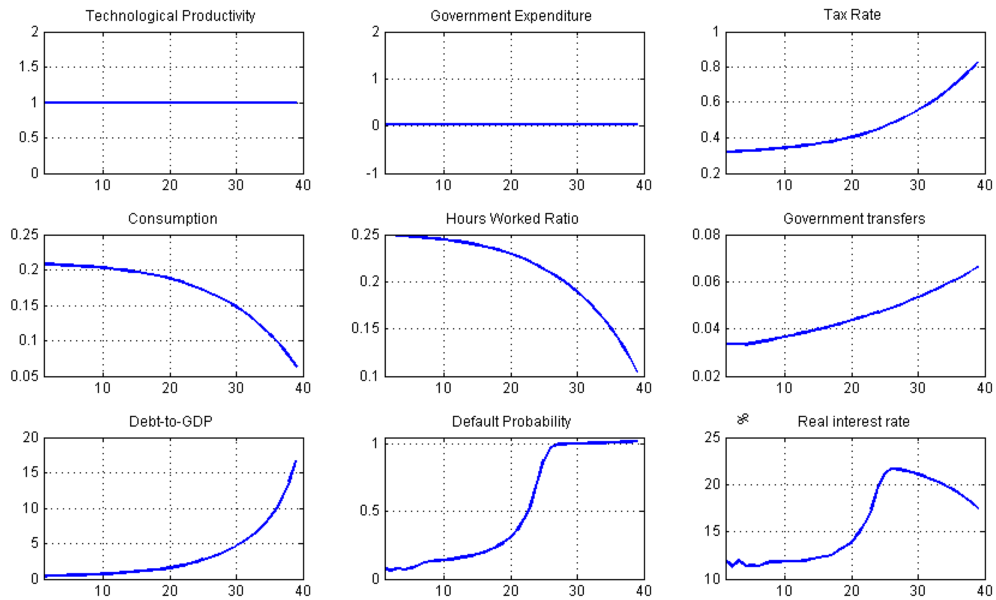


Figure 15: Nonlinear Simulation with Ambiguous Preferences ($\gamma = 2, \eta = 5$): Fiscal reform where transfers return to stationary regime at $t = 30$

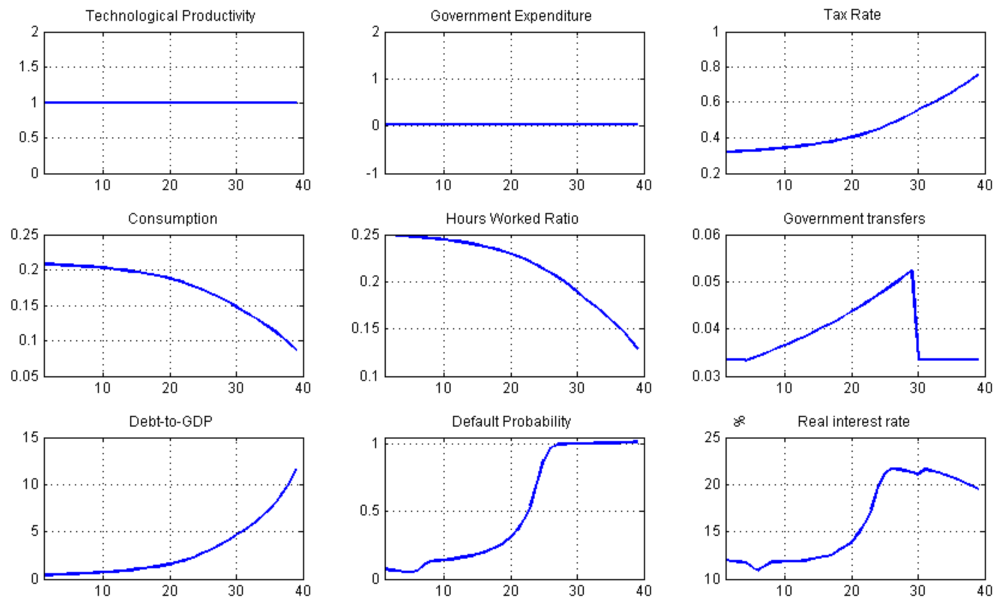
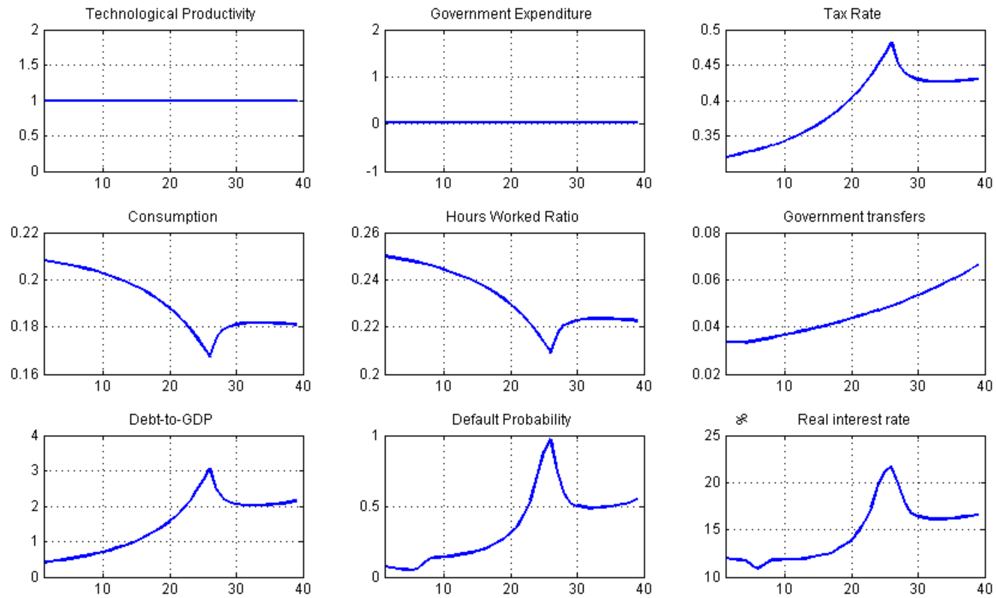


Figure 16: Nonlinear Simulation with Ambiguous Preferences ($\gamma = 2, \eta = 5$): Application of higher default rate starting from $t = 25$ when default probability reaches 1



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Appendix

Log-Linearized Equilibrium

$$
\begin{aligned}
\hat{A}_t &= \rho_A \cdot \hat{A}_{t-1} + \varepsilon_t^A && (44)\\
\hat{g}_t &= \rho_g \cdot \hat{g}_{t-1} + \varepsilon_t^g && (45)\\
\hat{\tau}_t - \left(\gamma^\tau \frac{\bar{b}}{\bar{\tau}}\right) \cdot \hat{b}_t &= 0 && (46)\\
\frac{\bar{b}\bar{R}}{\bar{A}\bar{n}} \cdot (\hat{R}_{t-1} + \hat{b}_{t-1}) &= \frac{\bar{b} \cdot \hat{b}_t - \bar{g} \cdot \hat{g}_t - \bar{z} \cdot \hat{z}_t}{\bar{A}\bar{n}} + \bar{\tau} \cdot \hat{\tau}_t + \bar{A} \cdot \hat{A}_t + \bar{n} \cdot \hat{n}_t && (47)\\
\frac{\bar{c}}{\bar{A}\bar{n}} \cdot \hat{c}_t + \frac{\bar{g}}{\bar{A}\bar{n}} \cdot \hat{g}_t - \hat{A}_t - \hat{n}_t &= 0 && (48)\\
\mathbb{E}_t \hat{c}_{t+1} - \mathbb{E}_t \hat{U}_{t+1} &= \hat{c}_t - \hat{U}_t + \hat{R}_t && (49)\\
\hat{c}_t - \hat{A}_t + \frac{\bar{\tau}}{1-\bar{\tau}} \cdot \hat{\tau}_t + \frac{\bar{n}}{1-\bar{n}} \cdot \hat{n}_t &= 0 && (50)\\
\hat{U}_t - \hat{c}_t + \frac{\phi\bar{n}}{1-\bar{n}} \cdot \hat{n}_t &= 0 && (51)\\
\beta \cdot \mathbb{E}_t \hat{V}_{t+1} &= \hat{V}_t + (-1+\beta) \cdot \hat{U}_t && (52)\\
\hat{z}_t - \gamma_z \cdot \hat{A}_t &= 0 && (53)\\
& \mathit{or} && (54)\\
\hat{z}_t - \gamma_z \cdot \hat{A}_t &= \rho_z \cdot \hat{z}_{t-1} && (55)
\end{aligned}
$$