Information and Efficiency in Debt Restructuring Offers

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Abstract

The present study analyzes the importance of information in the debt restructuring offer game played between the debtor and her creditors in the presence of the default risk of the debtor. We model it in resemblance with the tender offer game as a two-stage game, with the debtor making an offer in the first round and the creditors choosing to accept or to reject the offer, in the second. A successful (partial) debt restructuring is the efficient outcome of such a game; thereby we examine how private information affects the success of the offer. We first analyze the perfect information case, and then allow creditors to have some private information about the outcome of the game. As well as in the tender offer, ability of the debtor to make her creditors pivotal is crucial to overcome the free-rider problem between the creditors, thus making a successful offer possible. In the perfect information case debt restructuring is always successful and the optimal offer that the debtor can make is to distribute all her assets equally between all creditors. The existence of private information reduces the efficiency of the equilibrium outcome: the Debtor cannot make creditors pivotal to the success of the offer and the probability of a successful debt restructuring is always bounded away from one, because of the free-rider problem. When the loss of default is low, it is optimal for the Debtor to make a minimal (zero) offer and to default. When the loss is high, it may be optimal to ensure a positive probability of successful debt restructuring by offering a non-zero offer.

Keywords: debt restructuring offer, creditor information, efficiency

Özet

Borçlunun temerrüt riski olduğu durumda borcun yeniden yapılandırılması, borçlu ve onun alacaklıları arasında oynanan teklif oyununda bilgilerin önemini analiz eder. Bu oyunu, ihale teklif oyununa benzerlik içerisinde, borçlunun ilk turda bir teklif yaptığı ve alacaklıların kabul etmek veya reddetmek arasındaki seçimi ile iki aşamalı bir oyun olarak modelliyoruz. (Kısmi) borçlarının başarılı bir yeniden yapılandırılması, bu oyunun verimli sonucudur; bu sebeple şahsa ait bilginin teklif başarısını nasıl etkilediğini inceliyoruz. En başta tam bilgi ortamını inceliyoruz. Sonra alacaklıların oyunun sonucu hakkında özel bilgiye sahip olmasına izin veriyoruz. İhale teklifinin yanı sıra, borçlunun kendi alacaklılarını eksene getirme yeteneği, başarılı bir teklifi mümkün kılma ve alacaklılar arasında bedavacılık sorununu aşması için çok önemlidir. Tam bilgi ortamında, borçların yeniden yapılandırılması her zaman başarılı olmakta ve borçlunun yapabileceği optimum teklif, onun tüm varlıklarını tüm alacaklılar arasında eşit dağıtmasıdır. Özel bilgilerin varlığı denge sonucunun verimliliğini azaltır: Borçlu, alacaklıları teklifin başarı eksenine getiremiyor ve alacaklılar arasında bedavacılık sorunu yüzünden borç yeniden yapılandırmasının başarı olasılığı her zaman birden küçük olur. Temerrüt kaybı düşük olduğunda, borçlu için en düşük (sıfır) teklifi yapmak en uygunudur. Kayıp yüksek olduğunda, sıfırdan farklı bir teklif sunmak başarılı borç yeniden yapılandırılmasına pozitif bir olasılık sağlamak için optimal olabilir.

Anahtar Sözcükler: borç yeniden yapılandırma teklifi, alacaklı bilgileri, verimlilik

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Contents

List of Figures

Chapter 1

Research Question

The aim of this paper is to investigate the strategic interaction between a borrower and her creditors in the presence of a default risk of the borrower. We investigate the situation when the debtor is unable to fully meet her obligations to the creditors and is compelled to make a debt restructuring proposal. The debt restructuring may be performed in two ways: by reducing the face-value of the debt or by rescheduling the payment. In both cases, the present value of the debt obligation falls, that is, the proposal may be modeled as the debtor offering to reduce the present value of the initial debt. We analyze the informational problems that could arise in this game, namely the free-rider and "lemons" problems, and the information role in the efficiency of the game's solution, as well as the probability of a successful debt restructuring and the profit of a successful offer that goes to the borrower and to the creditors. We model the problem in resemblance to the relations that develop between a country and its creditors. In last three decades, several countries declared sovereign default (Mexico, 1982; Yugoslavia, 1983; Russia, 1998; Argentina, 2001; Greece, 2012), and some countries currently have difficult financial situations, which is reflected in their low credit risk grades. Since the sovereign default is an outcome which both the debtor and the creditors attempt to avoid, the debt restructuring with reduction of the present value of the debt is one of the methods used in the modern financial relations in the presence of the default risk. As stated in Das, Papaioannou and Trebesch (2012), in the period between 1950 and 2012, 57 restructurings with debt reduction and 129 restructurings with debt rescheduling occurred. Although the debt renegotiation is a complex issue, one major part of which is the legal terms of the debt contract, we simplify the environment and focus on strategic behavior in a given arrangement. In this paper, we investigate the efficiency of a simple debt reduction offer and the associated hazards. By doing this, we endeavor to reveal the incentives of both sides participating in the game and thus understand what may be the motives and the driving forces behind the debt negotiations.

Chapter 2

Literature Review

The problem of debt restructuring is similar to the problem of tender offers, in the sense of underlying processes and motivations. The paper on tender offers by Grossman and Hart (1980) analyses the relation between the shareholders and the raider, when no shareholder can affect the success of the offer (atomistic shareholders assumption). This paper states the free-rider problem between the shareholders that originated from the externality of managers' efforts. The important result of this work is that the shareholder will sell her shares only if she is offered at least the expected value of the share after a successful takeover. Bagnoli and Lipman (1988) consider the finite-stockholder case, where they overcome the problem of free-riding by making every shareholder pivotal. Then, they allow the number of the shareholders to go to infinity. They argue that the offer may succeed with positive probability. Further steps are done by Marquez and Yilmaz (2008), where they introduce private information for shareholders and the notion of private value to the raider in addition to the value created by better management of the firm. They show that when the private value is relatively large, the probability of a successful takeover is high. When the private value generated by the takeover is small, the takeover is successful only when the value created by the better managing of the firm is expected to be low. Detragiache and Garella (1995) provide a thorough analysis of the intrinsic motivations in the debt restructuring offer, focusing on theoretical results of the model and justifying the incentives for the potential preventive and supporting measures. This paper shows that the debt restructuring is successful for high efficiency gains, the analogous result of the Marquez and Yilmaz (2008) model with large private value. This paper also analyses the effect of creditors' heterogeneity on efficiency of the debt restructuring, and measures to overcome the problem of heterogeneous creditors. Unlike in Detragiache and Garella (1995), the model we present considers the exogenous environment and is viewed from the perspective of the similar problem of the tender offer. The difference between our model and tender offers lies in natural differences between the tender and debt restructuring offers. Accordingly, the number of agents that should accept the proposal to make it successful is endogenous in our model and exogenous in takeover models.

Chapter 3

Methodology

The model we analyze represents a game between one debtor and n creditors, each creditor holding one debt obligation. The debtor holds some amount A of assets, which is not enough to cover all her obligations ($A < n$). The game consists of two stages. First, the debtor offers a price p for each obligation that a creditor is willing to sell. Then, all creditors observe this offer and simultaneously decide whether they accept or reject the offer. Thus, the strategy of the debtor is a number p strictly between 0 and 1, and the strategy of the creditor is to accept or to reject the offer, conditional on the price offered. The offer made by the debtor may be of two types.

If the offer is unconditional, then the debtor first pays off the restructured part of the debt to the agents that accepted the offer, and then the payoffs are realized: the debtor's payoff is the amount of money remained after she pays off all her debts in case of a successful debt restructuring process, and the amount left reduced by some positive number L, otherwise. This number represents the loss from the default. It may be the reputation cost - loss from exclusion from financial markets, as in Eaton and Gersovitz (1981); risk of trade sanctions imposed by resident countries of the creditors (Bulow and Rogoff, 1989); political costs - decrease in electoral support, risk of unrest and changes in top economic officials (Panizza and Borensztein, 2010). The reditor's payoff is equal to p in case she accepts the debtor's offer, 1 if she refuses the offer and the debt restructuring is successful, and 0 if she refuses the offer and the debtor defaults.

The offer may be conditional on the number of accepting creditors. Then, the payoff functions are adjusted accordingly: in case of an unsuccessful offer, the debtor gets the amount of her assets A reduced by the loss L only, and both accepting and rejecting creditors get 0 payoff. In case of success, payoffs are the same as in the unconditional offer case.

We presume to focus on the Subgame Perfect Equilibria of this game. We intend to determine the conditions for the existing equilibria to be Pareto Efficient and investigate reasons for potential non-efficiency of the market.

Chapter 4

Perfect Information

4.1 Unconditional Offer

Consider n creditors which hold one debt obligation each, thus the total debt of the Debtor being n. Suppose the Debtor's amount of assets is A with the condition $A < n$. The Debtor proposes to pay a fraction p of each debt obligation, $p < 1$, regardless of the number of creditors accepting the offer and the result of the debt restructuring. Creditors Accept or Reject the offer, and the number of accepting and rejecting creditors is denoted by n_a and $n_r = n - n_a$, respectively. The Debtor first fulfills her obligations to the creditors accepting the offer, paying them in total $n_a \cdot p$. If the remaining amount of assets is enough to cover the full (non-reduced) debt to the creditors that rejected the offer, that is, $A - n_a p \geq n - n_a$, then the remaining obligations of the Debtor are fulfilled and debt restructuring is successful. The payoffs of accepting and rejecting creditors are p and 1, respectively, and the payoff of the Debtor is her profit $A - n_a p - (n - n_a) = A - n + n_a(1 - p)$, where $A - n$ is her initial full debt, and $n_a(1 - p)$ is the debt reduction. Otherwise $A - n_a p < n - n_a$ and the Debtor declares default. The creditor's payoff is p if she accepts the debt restructuring offer and 0 if she refuses, which is a normalization of commonly known expected value of the bond in case of default.

Note that if amount of assets held by the Debtor is strictly less than her debt ($A < n$), then for some high enough $p < 1$ the debt restructuring cannot be performed even if every creditor accepts the offer. Thus, there is an upper bound for the offer p: $A \geq n \cdot p$, or equivalently, $p \leq \frac{A}{n}$.

Later we allow n to go to infinity. In this case, both L and A also increase without bounds but in the same proportion as n:

$$\frac{A}{n} = \alpha < 1 \text{ and } \frac{L}{n} = \lambda < 1$$

Pure Strategy Equilibrium of Creditors' Subgame in the Symmetric Information Model

For any $p \in \left(0, \frac{A}{n}\right]$, the pure strategy subgame equilibrium exists if and only if every accepting creditor is pivotal. The two conditions ensuring that both accepting and rejecting agents do not have profitable deviations, should be satisfied:

$$n - (1-p)n_a \leq A < n - (1-p)n_a + (1-p)$$

or equivalently

$$\frac{n-A}{1-p} \leq n_a < \frac{n-A}{1-p} + 1$$

If the Debtor is compelled to default, that is, $n_a p + (n - n_a) < A$, then any of the rejecting creditors (a rejecting creditor exists for any p between 0 and $\frac{A}{n}$ if the offer is not successful) has a profitable deviation by accepting the offer and getting p instead of 0. If the debt restructuring is successful and not every accepting creditor is pivotal, that is, $(n_a - 1)p + (n - n_a + 1) \geq A$, then each of them has a profitable deviation to reject the proposal so that the number of accepting creditors still ensures the Debtor can pay her debt ($n_a p + (n - n_a) \geq A$) and thus this creditor becomes better off getting 1 instead of p.

This result is similar to the pure strategy equilibrium found by Bagnoli and Lipman (1988). That paper states that the pure strategy equilibrium (in tender offer game) exists when every agent is pivotal. The offer is always successful.

For $p > \frac{A}{n}$, the Debtor is not able to fully pay her obligations, independently of the value of n_a:

$$n_a p + (n - n_a) = n - (1-p)n_a > n - (1-p)n = np > n \cdot \frac{A}{n} = A$$

In this case, the Debtor defaults and is not able to pay off even the restructured part of the debt $n_a p$. We need an additional assumption about what the creditors' payoffs should be in this case. However, we are not interested in this case, since the debt restructuring offer cannot reach its goal - to ease the burden of an unpayable debt.

Choice of the Debtor

The problem of the Debtor is to choose an offer p given the amount of her assets A according to her rational expectations about the creditors' behavior. If the Debtor expects the outcome of the creditors' subgame to be the pure strategy equilibrium, then her maximization problem is

$$\max_p \begin{cases} A - n + (1-p)n_a & : \text{if success} \\ A - n_a p - L & : \text{if failure} \end{cases}$$

Note that for any $p \in \left(0, \frac{A}{n}\right]$ the outcome of the creditors' subgame always leads to a successful partial debt restructuring, whereas for any $p \in \left(\frac{A}{n}, 1\right)$ the default is inevitable and thus $n_a = n$. Since the Debtor has rational beliefs about the outcome of the second stage of the game, her maximization problem becomes

$$\max_p \begin{cases} A - n + (1-p)n_a & : \text{if } p \in \left(0, \frac{A}{n}\right] \\ A - n_a p - L & : \text{if } p \in \left(\frac{A}{n}, 1\right) \end{cases}$$

For $p > \frac{A}{n}$ the Debtor's payoff is

$$\pi = A - n_a p - L = A - np - L < A - n \cdot \frac{A}{n} - L < -L$$

Also, for $p \leq \frac{A}{n}$ the payoff

$$\pi = A - n + (1-p)n_a \in \left[0, (1-p)\right) \text{ since } n_a \in \left[\frac{n-A}{1-p}, \frac{n-A}{1-p} + 1\right)$$

The Debtor is indifferent between choosing any $p \leq \frac{A}{n}$. The above-zero profit of the Debtor originates from the indivisibility of agents and vanishes as $n \to \infty$. Formally, for a finite n the Debtor may earn profit arbitrarily close to $1-p$ by choosing p arbitrarily close from the left to p_0 which makes expression $\frac{n-A}{1-p_0}$ integer. Still, there are many such p_0's spread on the range $\left(0, \frac{A}{n}\right]$, depending on the divisibility of $n - A$.

Thus, the Debtor extracts value just enough to ensure she doesn't default. The partial debt restructuring is successful with the probability one, and the equilibrium outcome is efficient.

The choice of the offer p changes the distribution of the amounts paid to the creditors. For $p = \frac{A}{n}$, all creditors get the same payoff. As p decreases, the heterogeneity of the creditors' payoffs increases. The less is p, the more creditors get payoff of 1 and the less is the gain of accepting creditors.

Symmetric Pure Strategy Equilibrium in Perfect Information Model

Bagnoli and Limpan (1988) investigate pure strategy equilibrium in the case when shareholders may have a different amount of shares (in our model, creditors holding different number of debt obligations). There, the symmetric strategy is defined as "stockholders who are identically situated - i.e., have the same number of shares - behave identically", and the existence of the symmetric pure strategy equilibrium depends on the heterogeneity of the agents. In our model, the creditors are fully homogeneous (every creditor holds a single debt obligation), which in Bagnoli and Lipman (1988) leads to non-existence of symmetric pure strategy equilibrium. However, in debt restructuring model the symmetric equilibrium exists if p is close enough to $\frac{A}{n}$. That is, a symmetric equilibrium always exists when the Debtor offers $p \in \left(\frac{A-1}{n-1}, \frac{A}{n}\right]$ and all creditors accept the offer.

This result is different from the one in Bagnoli and Lipman (1988), because the number of accepting creditors necessary for the offer to be successful, K, depends on the offer itself. Thus, the Debtor may manipulate the necessary number of accepting creditors and can make every creditor pivotal by picking $p = \frac{A}{n}$, whereas in the tender offer model K is fixed.

Symmetric Mixed Strategy Equilibrium in Perfect Information Model

Now suppose that the creditor can choose a mixed strategy; that is, she picks the probability to accept the debt restructuring offer, denote it by γ_i. In the symmetric equilibrium, everybody chooses the same probability γ, so we omit the index. Symmetric mixed strategy equilibrium occurs when γ is strictly between 0 and 1.

We will show that there is no equilibrium in which creditors use symmetric mixed strategies. The Debtor either will offer $p = \frac{A}{n}$ making everyone pivotal and assuring always successful debt restructuring or will certainly default by offering $p = 0$.

Creditor i plays mixed strategy γ when she is indifferent between accepting and rejecting the offer, that is,

$$p = P(success \,|\, i\ rejects) \cdot 1 + P(failure \,|\, i\ rejects) \cdot 0$$

where *success* means the debt restructuring (partial or full) is successful and $failure$ means the Debtor defaults. The left-hand side is the payoff the creditor gets if she accepts the debt restructuring proposal (whether the Debtor defaults or not, creditors that accept the offer receive the payment), and the right-hand side is the expected payoff of rejecting the offer.

For each offer p made by the Debtor, we define the minimum number of acceptances the Debtor has to receive in order not to default as K. If p is taking the largest possible value for a successful offer to occur, $p = \frac{A}{n}$, then the only case when the Debtor doesn't default is if every creditor accepts the offer, that is, $K = n$. For the opposite extreme case, if p is 0, the number n_a is minimized conditional on no-default case when the whole amount A goes to the rejecting creditors, that is, $K = n - A$ or the lowest integer value greater than $n - A$. For any p in between, K is defined by minimizing n_a subject to $A \geq p \cdot n_a + (n - n_a)$, that is, K is the smallest integer number greater than or equal to $\frac{n-A}{1-p}$, an increasing step function of p. As n goes to infinity, $K(p)$ approaches function $\frac{n-A}{1-p}$.

Then we can write the probabilities of *success* and $failure$ as the probabilities that among remaining $n-1$ creditors at least K will accept and reject the offer, respectively.

$$P(success \,|\, i\ rejects) = \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^j (1-\gamma)^{n-j-1}$$

$$P(failure \,|\, i\ rejects) = \sum_{j=0}^{K-1} C_{n-1}^{j} \gamma^j (1-\gamma)^{n-j-1}$$

Then, the indifference condition becomes

$$p = \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^j (1-\gamma)^{n-j-1} \cdot 1 + \sum_{j=0}^{K-1} C_{n-1}^{j} \gamma^j (1-\gamma)^{n-j-1} \cdot 0 \tag{4.1}$$

Graph 1: K as a function of p

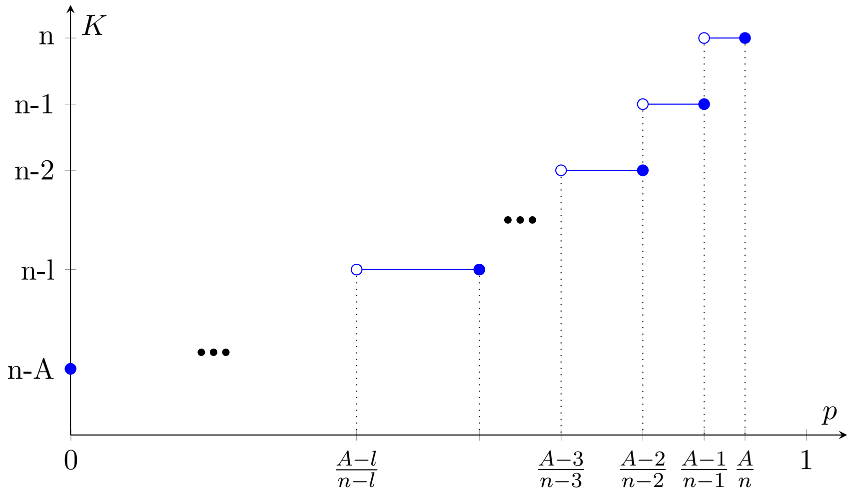


FIGURE 4.1: K as a function of p

for any p satisfying $K(p) < n$, that is, $p \leq \frac{A-1}{n-1}$.

Note that for $p > \frac{A-1}{n-1}$, the payoff of rejecting the offer is 0, because the Debtor will certainly default if at least one creditor rejects the offer. Then, accepting offer p is a strictly dominant strategy, and mixed strategy equilibrium does not exist.

In Appendix A we show that for all $p \in [0, \frac{A-1}{n-1}]$ there is a γ that satisfies the indifference condition (1). Then, we can write such a γ as function $\gamma(p)$.

Choice of the Debtor

The expected payoff of the Debtor is

$$P(success) \cdot (A - E(n_a|success) \cdot p - E(n_r|success)) + P(failure) \cdot (A - E(n_a|failure) \cdot p - L)$$

The expected numbers of accepting and rejecting creditors in case of success and failure are

$$E(n_a|success)P(success) = \sum_{j=K}^{n} \left(C_n^j \gamma^j (1-\gamma)^{n-j} \cdot j \right), \text{ given } j \geq K$$

$$E(n_r|success)P(success) = \sum_{j=K}^{n} \left(C_n^j \gamma^j (1-\gamma)^{n-j} \cdot (n-j) \right), \text{ given } j \geq K$$

$$E(n_a|failure)P(failure) = \sum_{j=0}^{K-1} \left(C_n^j \gamma^j (1-\gamma)^{n-j} \cdot j \right), \text{ given } j < K$$

Then, the expected payoff of the Debtor becomes

$$\pi = A - p \cdot \sum_{j=0}^{n} \left(C_n^j \gamma^j (1-\gamma)^{n-j} \cdot j \right) - \sum_{j=K}^{n} \left(C_n^j \gamma^j (1-\gamma)^{n-j} \cdot (n-j) \right) - L \cdot \sum_{j=0}^{K-1} C_n^j \gamma^j (1-\gamma)^{n-j} =$$

$$= A - \gamma np - n(1-\gamma) \cdot \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^{j} (1-\gamma)^{n-j-1} - L \cdot \sum_{j=0}^{K-1} C_{n}^{j} \gamma^{j} (1-\gamma)^{n-j} =$$

$$= A - \gamma np - n(1-\gamma)p - L \sum_{j=0}^{K-1} C_{n}^{j} \gamma^{j} (1-\gamma)^{n-j} =$$

$$= A - np - L \cdot P(failure) =$$

$$= A - n \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^{j} (1-\gamma)^{n-j-1} - L \sum_{j=0}^{K-1} C_{n}^{j} \gamma^{j} (1-\gamma)^{n-j}$$

which is a function of p.

Expression $\sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^{j} (1-\gamma)^{n-j-1}$ is the probability that among remaining $n-1$ creditors more than K accept the offer. By the Law of Large Numbers, this probability goes to one as n goes to infinity when $\gamma > \frac{K}{n-1}$, and it goes to zero when $\gamma < \frac{K}{n-1}$. Similarly, $\sum_{j=0}^{K-1} C_{n}^{j} \gamma^{j} (1-\gamma)^{n-j}$ is the probability that overall less than K creditors accept the offer, and it goes to zero for $\gamma > \frac{K}{n}$ and to one for $\gamma < \frac{K}{n}$, as n goes to infinity.

For $\gamma < \frac{K}{n-1}$, the creditors' indifference condition implies

$$p = \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^{j} (1-\gamma)^{n-j-1} \rightarrow 0$$

Since all components of the sum are positive, the sum tends to zero only if its every component tends to zero, including γ^{n-1} (when $i = n-1$). Thus, $\gamma \rightarrow 0$. The payoff of the Debtor approaches $A - L$.

For $\gamma > \frac{K}{n-1}$, we get

$$p = \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^{j} (1-\gamma)^{n-j-1} \rightarrow 1$$

which doesn't correspond to an equilibrium, since p is bounded away from 1.

For $\gamma = \frac{K}{n-1}$,

$$p = \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^{j} (1-\gamma)^{n-j-1}$$

may take different values between 0 and 1. As we show in Appendix B,

$$\sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^{j} (1-\gamma)^{n-j-1} - \sum_{j=K}^{n} C_{n}^{j} \gamma^{j} (1-\gamma)^{n-j} \rightarrow 0$$

as $n \rightarrow \infty$ for p bounded away from $\frac{A}{n}$. Then, the profit function approaches a linear function in p:

$$\pi \rightarrow A - L - (n - L)p$$

The optimal behaviour of the Debtor depends on the amount of her assets A and the disutility from default decision L. If $A > L$, then the Debtor will prefer to default and get $\pi = A - L > 0$. If $A < L$, then the optimal offer is $p = \frac{A}{n}$ which induces Debtor's profit being $\pi = 0$. If $A = L$, then the Debtor may offer 0 or A/n, both of them yielding zero profit.

Symmetric Equilibria

At the equilibrium of the creditors' subgame, the payoff of the Debtor changes with the offer she makes. For $p = 0$, $\pi = A - L$; for $p = \frac{A}{n}$, $\pi = 0$ and for $p \in (0; \frac{A}{n})$, $\pi \to A - np - L(1-p)$. Since $n > A$, offering $p \in (0; \frac{A}{n})$ in order to compel creditors to mix is never optimal.

Case 1: $A > L$

There is a unique symmetric equilibrium of the game; the Debtor doesn't want to restructure her debt, since restructuring will yield a maximum possible profit slightly over zero (approaching zero as $n \to \infty$), while by defaulting the Debtor may assure a strictly positive profit.

Case 2: $A < L$

There is a unique symmetric equilibrium of the game; the Debtor offers $p = \frac{A}{n}$ and each creditor accepts the offer $\frac{A}{n}$. The Debtor receives zero profit.

Case 3: $A = L$

Both equilibria for previous cases are also equilibria for case $A = L$, yielding zero profit for the Debtor.

Proposition 1: Assume that $\lambda < 1$ and allow n to be arbitrarily large. Then:

a) If $A > L$, then there is a unique equilibrium of the Debt Restructuring Game with $\gamma = 0$, $\pi = A - L$ and the offer being always unsuccessful;

b) If $A < L$, then there is a unique equilibrium of the game with $\gamma = 1$, $\pi = 0$ and the offer being always successful.

Proof in Appendix B

4.2 Conditional Offer

A conditional offer is a pair $(p; M)$, the Debtor offering to pay p for each debt obligation if at least M creditors accept the offer, and declaring default otherwise. The number M may be greater than or equal to $K(p)$. If it is strictly greater, then for $n_a \in [K; M)$ the Debtor is able to pay the (partly) restructured debt but prefers to default. The reason for the Debtor to make a conditional offer may lie in her incentives or legal conditions (as an example, on July 16, 2014 Argentina was obliged not to discriminate between any groups of its bondholders and thus was prohibited to pay her restructured part of the debt prior to the non-restructured part).

The analysis of the conditional offer is identical to the conditional offer in tender games (Bagnoli and Lipman, 1988).

Following Bagnoli and Lipman (1988), one can argue that the Debtor can ensure profit arbitrarily close to maximum by offering infinitesimal amount p conditional on the accepting number of agents being n. The only stable equilibrium of the creditors subgame is to accept the offer. The payoff the Debtor receives is $\pi = A - np \to A$, since the offer is made for a given n.

The offer is always successful and the Debtor extracts all the surplus.

Chapter 5

Private Value Model

Previously, we normalized the creditors' payoffs in case of default to zero. Now, let's investigate the case when every creditor who rejects the offer has a different payoff in case of the offer being unsuccessful, known only by the creditor herself. The distribution of the payoffs $f(a)$, $F(a)$ is a common knowledge; all of the creditors' uncertain payoffs are drawn independently; $F(a)$ - continuous, strictly increasing. Let's order the creditors on the interval $[0, n]$ by their payoff a_j. Then we can define an increasing function

$$a : [0, n] \to [0, \frac{A}{n}],\ a(j) = a_j$$

Let's normalize $a(0) = 0$, so that the offer $p \geq 0$ starts with the smallest payoff a creditor can get if she rejects the offer. None of the creditors knows the true function $a(i)$. However, each one of them has some expectation about how many creditors have their private payoff a_j less than a certain value a (which is the cumulative distribution funciton $F(a)$).

We want to show that the graph of every agent's expectation about the function $a(i)$ coincides with the graph of the inverse of $F(a)$: by the Law of Large Numbers,

$$F(a) = P(a(i) < a) = P(i < a^{-1}(a)) = \frac{a^{-1}(a)}{n}$$

Assume $F(a)$ is strictly increasing. Then, expectation function of $a(i)$ is continuous.

Assumption A1: $n \cdot E[a] < A$. Thus, as n increases, the probability of facing a significant number of "too optimistic" creditors whose outside option is greater than $\frac{A}{n}$ falls and vanishes as $n \to \infty$.

Assumption A2: $F(0) < n - A$. Then, the number of creditors with outside option 0 is not too high.

5.1 Unconditional Offer

5.1.1 Creditors Subgame

Let $P(S \,|\, j\ rejects)$ denote the probability of a successful (partial) debt restructuring when one creditor (j) always rejects the offer and everyone else plays according to the equilibrium strategy.

Graph 2: Inverse of $F(a)$ gives expectation of a

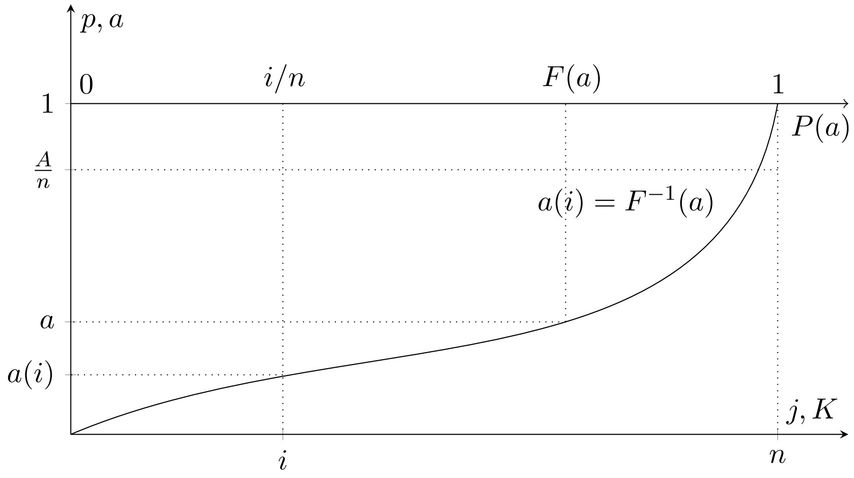


FIGURE 5.1: Inverse of $F(a)$ gives expectation of a

The indifference condition for a creditor j is

$$p = P(S \mid j \ \ rejects) + P(F \mid j \ \ rejects) \cdot a(j)$$

If LHS is strictly greater, then the creditor chooses to accept the offer, if it is strictly less, she rejects.

Let's define sets $\Gamma(p)$ and $\Gamma'(p)$ containing creditors who accept and reject the offer, respectively, as

$$\Gamma(p) = \{j \mid j \text{ accepts offer } p\}$$

$$\Gamma'(p) = \{j \mid j \text{ rejects offer } p\}$$

The offer is successful if and only if

$$A \geq \sum_{j \in \Gamma} p + \sum_{j \in \Gamma'} 1$$

For any creditor of type $a(j) > p$, rejecting the offer is a weakly dominating strategy (strictly dominant if probability of success is greater than zero). It may occur that even if all creditors of type below p accept the offer p, the debt restructuring will be unsuccessful.

Thus, we need to define a minimum number of accepting creditors necessary for the offer to be successful. We are interested in what happens if all creditors below some threshold accept the offer and all creditors above the same threshold reject it. Then, we can define $K(p)$ as index of a creditor with following property: if anyone below such a creditor accepts the offer and everyone above her rejects, then the amount of assets is just enough to pay the restructured debt. More formally,

$$K(p) = \left(i \,|\, A = \sum_{j \leq i} p + \sum_{j > i} 1\right)$$

Claim: $K(p)$ exists and is unique for each p; for $p \neq \frac{A}{n}$, $K(p) \in (0, n)$.

Proof in Appendix C

Graphically, K is such a number j which makes $(1-p)K$ equal to $n - A$, that is, the lower right rectangle cut by lines p_1 and $K(p_1)$ remains constant for all p. As well as in Perfect Information Common Value case, $K(p) = \frac{n-A}{1-p}$.

Graph 3: Visualising $n - A$

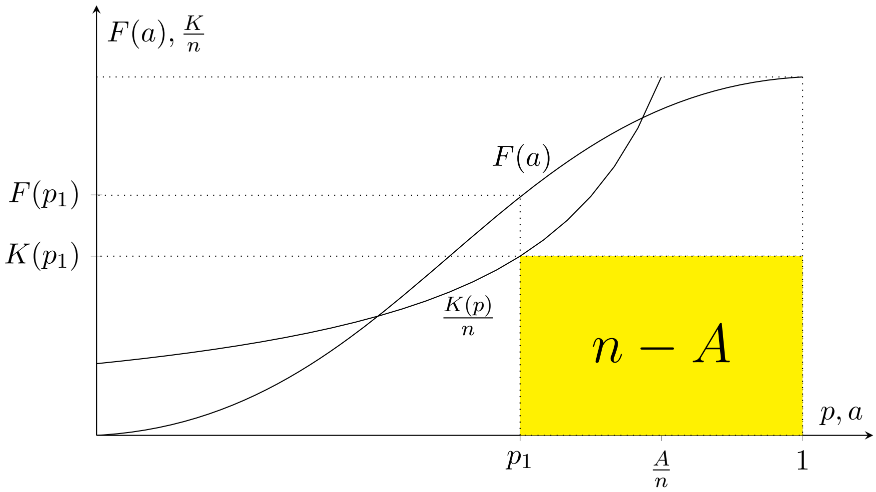


FIGURE 5.2: Visualising $n - A$

First, let's see that for any $p \in \left(0, \frac{A-1}{n-1}\right)$ there always exists a creditor type a^* who is indifferent between accepting and rejecting the offer:

Suppose that all types of creditors are strictly better off accepting the offer. Then, $p \leq (A-1)/(n-1) \implies P(S \mid i \ \ rejects) = 1$ and any creditor of type $a < 1$ would prefer to reject the offer and get 1 instead of $a(i)$, contradiction.

Suppose that all types of creditors are strictly better off rejecting the offer. Then, the probability of success is equal to zero and creditor of type zero would prefer to accept the offer and get $p > 0$ instead of $a(0) = 0$, contradiction.

Suppose that for some $p \in \left(0, \frac{A-1}{n-1}\right)$ there is no such type a^* who is indifferent between accepting and rejecting the offer. Then, according to previous statements, there are some types who strictly prefer to accept and who strictly prefer to reject the offer. Thus, there is $i < n$ and $j < n$ such that

$$P(S \mid i \ \ rejects) + P(F \mid i \ \ rejects) \cdot a(i) < p <$$

$$< P(S \mid j \ \ rejects) + P(F \mid j \ \ rejects) \cdot a(j)$$

Since $P(S \mid i \;\; rejects) = P(S \mid j \;\; rejects)$ for all $i, j < n$ and $a(j)$ is a continuous function in j, there exists $i^* < n$ for whom the indifference condition holds, end of proof.

Claim: If $P(S \mid i \;\; rejects) \neq 1$, then every type $a < a^*$ accepts and every type $a > a^*$ rejects the offer.

Follows from $P(S \mid i \;\; rejects) = P(S \mid j \;\; rejects)$ for all $i, j < n$ and Indifference Condition: For $a = a^*$, the indifference condition holds. For $a < a^*$, LHS of the indifference condition is greater than RHS, thus a-type agent prefers to accept the offer. For $a > a^*$, LHS < RHS and a-type agent rejects the offer.

Claim: Whenever a^* exists, it is unique.

Proof in Appendix D.

Then,

$$P(Success) = \sum_{j=K(p)}^{n} C_n^j \left[F(a^*)\right]^j \left[1 - F(a^*)\right]^{n-j}$$

$$P(S \mid i \;\; rejects) = \sum_{j=K(p)}^{n-1} C_{n-1}^j \left[F(a^*)\right]^j \left[1 - F(a^*)\right]^{n-j-1}$$

There are three different cases:

1) If $a^* > F^{-1}(\frac{K}{n})$, then by the Law of Large Numbers $P(Success) \to 1$ and indifference condition implies $p \to 1$, condtradiction (p is bounded away from 1, $p \leq \frac{A}{n}$).

2) If $a^* < F^{-1}(\frac{K}{n})$, then by the LLN, $P(Success) \to 0$ and indifference condition implies $a^* \to p$ as $n \to \infty$.

3) If $a^* = F^{-1}(\frac{K}{n})$, then $P(Success) = \sum_{j \geq K} C_n^j \left(\frac{K}{n}\right)^j \left(\frac{n-K}{n}\right)^{n-j}$ may approaching zero or bounded away from zero.

5.1.2 Debtor's Choice with Continuum of Creditors

The profit function of the Debtor is

$$\pi(p) = A - P(S) \cdot \Big(p \cdot E[n_a \,|\, n_a \geq K] + E[n_r \,|\, n_a \geq K]\Big) - (1 - P(S)) \cdot$$

$$\cdot \Big(p \cdot E[n_a \,|\, n_a < K] + E[n_r \,|\, n_a < K] \cdot E[a(j) \,|\, n_a < K;\, a > a^*] + L\Big)$$

Let's write down conditional expectations:

$$E[n_a \,|\, n_a \geq K] = \frac{\sum_{i=K}^{n} i \cdot P(n_a = i)}{\sum_{i=K}^{n} P(n_a = i)} = \frac{\sum_{i=K}^{n} i \cdot C_n^i \left[F(a^*)\right]^i \left[1 - F(a^*)\right]^{n-i}}{\sum_{i=K}^{n} C_n^i \left[F(a^*)\right]^i \left[1 - F(a^*)\right]^{n-i}}$$

$$E[n_r \,|\, n_a \geq K] = \frac{\sum_{i=K}^{n} (n-i) \cdot C_n^i \left[F(a^*)\right]^i \left[1 - F(a^*)\right]^{n-i}}{\sum_{i=K}^{n} C_n^i \left[F(a^*)\right]^i \left[1 - F(a^*)\right]^{n-i}}$$

$$E[n_a \,|\, n_a < K] = \frac{\sum_{i=0}^{K-1} i \cdot C_n^i \left[F(a^*)\right]^i \left[1 - F(a^*)\right]^{n-i}}{\sum_{i=0}^{K-1} C_n^i \left[F(a^*)\right]^i \left[1 - F(a^*)\right]^{n-i}}$$

$$E[n_r \mid n_a < K] = \frac{\sum_{i=0}^{K-1}(n-i)\cdot C_n^i\left[F(a^*)\right]^i\left[1-F(a^*)\right]^{n-i}}{\sum_{i=0}^{K-1} C_n^i\left[F(a^*)\right]^i\left[1-F(a^*)\right]^{n-i}}$$

Then, the profit becomes

$$\pi = A - p\cdot n\cdot F(a^*) - n\cdot(1-F(a^*)) + (1-E[a \mid a > a^*])\sum_{i=0}^{K-1}(n-i)C_n^i\left[F(a^*)\right]^i\left[1-F(a^*)\right]^{n-i}$$

$$-L\cdot\sum_{i=0}^{K-1} C_n^i\left[F(a^*)\right]^i\left[1-F(a^*)\right]^{n-i} = A - p\cdot n\cdot F(a^*) - n\left[1-F(a^*)\right] + (1-E[a \mid a > a^*])\cdot$$

$$\cdot\left[1-F(a^*)\right]\sum_{i=0}^{K-1} nC_{n-1}^i\left[F(a^*)\right]^i\left[1-F(a^*)\right]^{n-1-i} - L\cdot\sum_{i=0}^{K-1} C_n^i\left[F(a^*)\right]^i\left[1-F(a^*)\right]^{n-i} =$$

$$= A - n + n(1-p)F(a^*) + n(1-E[a \mid a > a^*])\left[1-F(a^*)\right]P(F \mid a \ rejects) - L\cdot P(F)$$

We analyze the profit function for different cases, depending on the probability of success:

1) $p = 0$: assumption A2 ($F(0) < A - n$) implies $P(Success) \to 0$. Then all types $a > 0$ surely reject the offer, and

$$\pi(p=0) \to A - n\left[1-F(0)\right]E[a \mid a > 0] - L = A - nE[a] - L$$

2) p that induces $a^* < F^{-1}(\frac{K}{n})$:
From the creditors' subgame, $a^* \to p$ and $P(Success) \to 0$. Then

$$\pi \to A - npF(p) - n\left[1-F(p)\right]E[a \mid a > p] - L$$

3) p that induces $a^* = F^{-1}(\frac{K}{n})$:
a) If $a^* \to p$, then $P(Success \mid a \ rejects) \to 0$ and

$$\pi \to A - Kp - (n-K)E[a \mid a > p] - P(F)\cdot L$$

b) If $a^* < p$ and bounded away from it, then $P(Success \mid a \ rejects) > 0$ and

$$\pi = A - pK - (n-K)\frac{p-a^*}{1-a^*} - (n-K)\frac{1-p}{1-a^*}E[a \mid a > a^*] - P(F)\cdot L,$$

since the Indifference Condition implies

$$P(Success \mid a^* \ rejects) = \frac{p-a^*}{1-a^*}$$

4) No offer p induces a^* such that $F(a^*) > \frac{K}{n}$, since in this case $P(Success) \to 1$ and $p \to 1$, contradiction.

Proposition 2: Among all offers p that induce $P(Success) \to 0$, the optimal offer is $p = 0$.

Proof:

Take any $p' > 0$ which induces $P(Success) \to 0$ (we will show later that such an offer p' exists). Then for large n, the profit approaches

$$\pi(p') \to A - np'F(p') - n[1 - F(p')]E[a \mid a > p'] - L <$$

$$< A - nF(p')E[a \mid a < p'] - n[1 - F(p')]E[a \mid a > p'] - L =$$

$$= A - nE[a] - L = \pi(p = 0)$$

Assumption A2 and continuity of the $F(a)$ imply that difference $np'F(p') - nF(p)E[a \mid a < p]$ remains constant as $n \to \infty$. Thus, $\pi(p') < \pi(0)$ for n large enough, end of proof.

As well as in Perfect Information case, for $p < \frac{A}{n}$, $P(S \mid a^* \ rejects) \to P(Success)$ as $n \to \infty$ (Appendix B), that is, no offer $p < \frac{A}{n}$ can make creditors pivotal with significant probability.

However, if $F(\frac{A}{n}) = 1$, then offer $p = \frac{A}{n}$ induces $P(S \mid i \ rejects) = 0$, but $P(Success) = 1$, since every creditor will prefer $p = \frac{A}{n}$ to the outside option $a(j) < \frac{A}{n}$. Then, every creditor becomes pivotal with probability one. The problem becomes exactly the same as Perfect Information Symmetric Strategies case, with $\pi(p = \frac{A}{n}) = 0$, which is an optimal offer for the Debtor when $L > (n - A)[1 - E[a]]$.

We want to find values of L for which the Debtor will be willing to make an offer which yields $P(Success) > 0$, that is, when the gain of a decrease in probability of default will be greater than the difference in payment to the creditors that assures a positive probability of successful debt restructuring:

$$\pi(p_1) - \pi(p = 0) = -pK - (n - K)\frac{p - a^*}{1 - a^*} - (n - K)\frac{1 - p}{1 - a^*}\frac{\int_{a^*}^1 a\,dF(a)}{1 - \frac{K}{n}}$$

$$-P(F) \cdot L + n\int_0^1 a\,dF(a) + L > 0$$

$$P(Success) \cdot L > pK - n\int_0^{a^*} a\,dF(a) + \frac{p - a^*}{1 - a^*}((n - K) - n\int_{a^*}^1 a\,dF(a))$$

Apart from the case when $F(\frac{A}{n}) = 1$, $P(Success) - P(S \mid i \ rejects) \to 0$ and offering $p > 0$ is optimal when

$$\frac{p - F^{-1}(\frac{1-\alpha}{1-p})}{1 - F^{-1}(\frac{1-\alpha}{1-p})}L > n\Bigg[p\frac{1 - \alpha}{1 - p} + \Big(1 - \frac{1 - \alpha}{1 - p}\Big)\frac{p - F^{-1}(\frac{1-\alpha}{1-p})}{1 - F^{-1}(\frac{1-\alpha}{1-p})}$$

$$-\int_0^{F^{-1}\left(\frac{1-\alpha}{1-p}\right)} a\,dF(a) - \frac{p - F^{-1}(\frac{1-\alpha}{1-p})}{1 - F^{-1}(\frac{1-\alpha}{1-p})}\int_{F^{-1}\left(\frac{1-\alpha}{1-p}\right)}^1 a\,dF(a)\Bigg]$$

Denote $D_1(p) = \big(pK - n\int_0^{a^*} a\,dF(a)\big)/n$ and $D_2(p) = \big((n - K) - n\int_{a^*}^1 a\,dF(a)\big)/n$. Then $nD_1 + nD_2 = n - nE[a] - (n - A) = n \cdot const$ and offering $p > 0$ is optimal when

$$P(Failure)\Big[(n-K)-n\int_{a^*}^{1} a\, dF(a^*)-L\Big] = P(F)[nD_2-L] > A-nE[a]-L$$

or

$$P(Failure)[D_2-\lambda] > \alpha - E[a] - \lambda$$

In appendix D we show an example when this condition holds and offering a $p>0$ is indeed optimal.

Graph 4: Visualising different cases

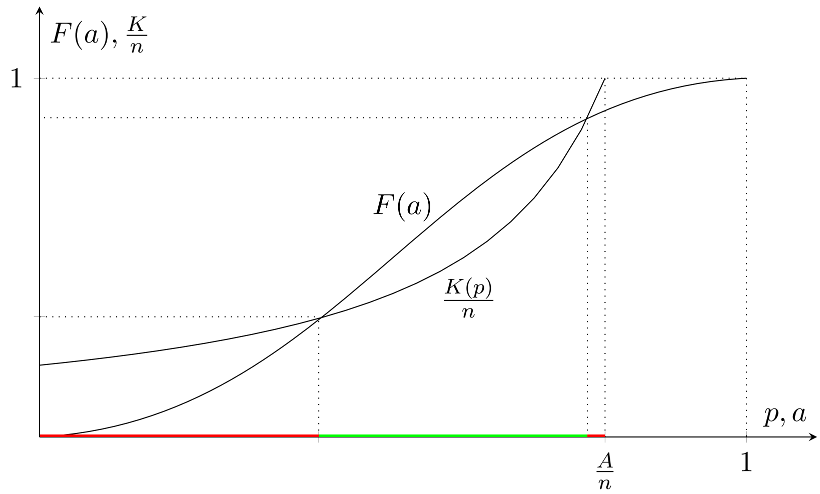


Figure 5.3: Visualising different cases

Are these three cases even possible? That is, are there such offers p that induce a^* such that $F(a^*) < \frac{K(p)}{n}$; $F(a^*) = \frac{K(p)}{n}$ and $a^* \to p$; $F(a^*) = \frac{K(p)}{n}$ and $a^* < p$?

Assumption A2 ($F(0) < n - A$) and continuity of functions $F(a)$ and $\frac{K(p)}{n}$ imply that there exist $p > 0$ such that $F(p) < \frac{K(p)}{n}$ (red region on the graph). Since $a^* \leq p$, we have $F(a^*(p)) < \frac{K(p)}{n}$ for p satisfying $F(p) < \frac{K(p)}{n}$.

In order to ensure that there exist offers p that induce two remaining cases, we need additional assumptions:

Assumption A3: $\exists\, p$ s.t. $F(p) = \frac{K(p)}{n}$. The indifference type a^* cannot be bounded away from p, because in this case $F(a^*) < F(p) = \frac{K(p)}{n}$, $P(Success \,|\, a^* \textit{ rejects}) \to 0$ and Indifference Condition implies $a^* \to p$, contradiction. Thus, $a^* \to p$ and $P(Success \,|\, a^* \textit{ rejects}) \to 0$.

Assumption A4: $\exists\, p$ s.t. $F(p) > \frac{K(p)}{n}$, green region on the graph (A4, A2 and continuity of $F(a)$ imply A3). If $a^* \to p$, then $P(Success) \to 1$, contradiction. Then, $F(a^*)$ cannot be bounded away from $\frac{K(p)}{n}$ because $F(a^*) < \frac{K(p)}{n}$ implies $P(Success) \to 0$ and $a^* \to p$, contradiction.

In Appendix E we give some simulations on some particular cases, showing the profit function for different offers p.

Chapter 6

Conclusion

When the outside options of every creditor is same, the outcome of the game is always efficient. Creditors allow the Debtor to extract profit just enough to guarantee she doesn't default. The Debtor is forced to make a maximal offer, paying to all creditors equally.

This result extends to the model with private outside option, in case that outside option of each creditor is not bigger than the maximal offer that the Debtor can make (that is, $F(\frac{A}{n}) = 1$). The exact value of the outside option is not crucial for the outcome of the game, if none of the creditors expects a "too optimistic" gain in case of default.

However, if there is a positive probability of some agents being "too optimistic" with expectation about outside option higher than the Debtor's maximal offer, then the probability of successful debt restructuring becomes

$$P(Success) = \frac{p - F^{-1}(\frac{n-A}{n(1-p)})}{1 - F^{-1}(\frac{n-A}{n(1-p)})}$$

bounded away from one, and the profit

$$\pi = n\frac{1-p}{1-a^*}\Big(\frac{\alpha - p}{1-p} - \int_{a^*}^{1} a\, dF(a) - \lambda\Big)$$

In order to overcome the free-rider problem between the creditors, the Debtor has to make them pivotal. This allows to keep the probability of successful debt restructuring at one. The uncertainty about the existence of "too optimistic" creditors and volatility of their amount doesn't allow the Debtor to make other creditors pivotal. The probability of a successful restructuring may be positive only when the Debtor offers a high offer p to avoid loss from default (not too high, though, as she has to reserve some amount to pay noon-restructured part of the debt to the "optimistic" creditors).

The simulations we provide in Appendix E corroborate our findings. For offers inducing imperceptible probability of success, the profit is less than $\pi(p = 0)$. For the loss L high enough, there is an internal solution to the profit maximization problem with probability of a successful restructuring bounded away from zero.

Appendix A

Existence of $\gamma(p)$

For a fixed p (thus, K also fixed), $p = \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^j (1-\gamma)^{n-j-1}$ has a unique solution γ:

$$f(\gamma) = \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^j (1-\gamma)^{n-j-1} \text{ is a continuous function of } \gamma.$$

$$\frac{d}{d\gamma} f(\gamma) = \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^{j-1} (1-\gamma)^{n-j-2} [j(1-\gamma) - (n-j-1)\gamma] =$$

$$= \sum_{j=K}^{n-1} (n-1) C_{n-2}^{j-1} \gamma^{j-1} (1-\gamma)^{n-j-1} - \sum_{j=K}^{n-1} (n-1) C_{n-2}^{j} \gamma^{j} (1-\gamma)^{n-j-2} =$$

$$= \sum_{j=K-1}^{n-2} (n-1) C_{n-2}^{j} \gamma^{j} (1-\gamma)^{n-j-2} - \sum_{j=K}^{n-1} (n-1) C_{n-2}^{j} \gamma^{j} (1-\gamma)^{n-j-2} =$$

$$= (n-1) C_{n-2}^{K-1} \gamma^{K-1} (1-\gamma)^{n-K-1} > 0$$

which means that $f(\gamma)$ is strictly increasing; the extreme values of p are attained by the function $f(\gamma)$:

For $p = 0$, $K = n - A$ and the equation is satisfied by $\gamma = 0$.

For $p = \frac{A-1}{n-1}$, $K = n - 1$ and the equation is satisfied by $\gamma = \sqrt[n-1]{p} \in (0, 1)$.

Thus, γ exists for any $p \in \left[0, \frac{n-A}{1-p}\right]$ and is unique. Therefore, we can view γ which satisfies the indifference condition (1) as a function of p, $\gamma(p)$.

Furthermore, $\gamma(p)$ is discontinuous at the points where K changes ($p = \frac{A-l}{n-l}$, $l \leq A$) and continuous on all remaining open intervals. For any p on these intervals we may take derivative of the $\gamma(p)$:

$$\frac{d}{dp} p = \frac{d}{dp} \left[\sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^j (1-\gamma)^{n-j-1} \right] =$$

$$= \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^{j-1} (1-\gamma)^{n-j-2} [j(1-\gamma) - (n-j-1)\gamma] \frac{d\gamma}{dp} =$$

$$= (n-1) \frac{d\gamma}{dp} C_{n-2}^{K-1} \gamma^{K-1} (1-\gamma)^{n-K-1} = 1$$

that is, $\frac{d\gamma}{dp} > 0$ for $n \geq 2$ and $K < n$.

The second derivative:

$$(n-1)\Big(\frac{d\gamma}{dp}\Big)^2 C_{n-2}^{K-1}(K-1)\gamma^{K-2}(1-\gamma)^{n-K-1} - (n-1)\Big(\frac{d\gamma}{dp}\Big)^2 C_{n-2}^{K-1}(n-K-1)\gamma^{K-1}\cdot$$

$$\cdot(1-\gamma)^{n-K-2} + (n-1)\frac{d^2\gamma}{dp^2}C_{n-2}^{K-1}\gamma^{K-1}(1-\gamma)^{n-K-1} = 0$$

$$\gamma(1-\gamma)\frac{d^2\gamma}{dp^2} = \Big(\frac{d\gamma}{dp}\Big)^2\big[\gamma(n-2)-(K-1)\big]$$

Appendix B

Equilibrium in Perfect Information

Suppose each creditor accepts the offer with probability γ (which in imperfect information case will be equal to $F(a^*)$). Let's write probabilities of success, if every creditor sticks to the strategy γ, except for one, who is either surely rejecting or accepting the offer (denote these probabilities by $P(Success \,|\, i\ rejects)$ and $P(Success \,|\, i\ accepts)$):

$$P(Success \,|\, i\ rejects) = \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^j (1-\gamma)^{n-j-1}$$

$$P(Success \,|\, i\ accepts) = \sum_{j=K-1}^{n-1} C_{n-1}^{j} \gamma^j (1-\gamma)^{n-j-1}$$

Then,

$$P(Success \,|\, i\ accepts) - P(Success \,|\, i\ rejects) = C_{n-1}^{K-1} \gamma^{K-1} (1-\gamma)^{n-K}$$

Using Stirling approximation, we can rewrite

$$P(Success \,|\, i\ accepts) - P(Success \,|\, i\ rejects) =$$

$$= \frac{1}{\sqrt{2\pi}} \sqrt{\frac{n-1}{(K-1)(n-K)}} \left(\frac{K}{K-1}\right)^{K-1} \left(\frac{n-K-1}{n-K}\right)^{n-K} \to$$

$$const \cdot \sqrt{\frac{(1-p)^2}{n(1-\alpha)(\alpha-p)}}$$

For $p < \alpha = \frac{A}{n}$, last expression tends to zero as $n \to \infty$.

Since $P(Success \,|\, i\ rejects) \leq P(Success) \leq P(Success \,|\, i\ accepts)$, we have $P(Success) - P(Success \,|\, i\ rejects) \to 0$ as $n \to \infty$ for $p < \alpha$.

Proof of Proposition 1:

a) Suppose that $A > L$ and $n > L$. The profit function is equal to

$$\pi = A - L - n \cdot p + L \cdot P(success)$$

Claim: If $0 < \gamma < \frac{K}{n}$, then $n \cdot p > L \cdot P(success)$ for n big enough. In order to prove this, we show that $\frac{p}{P(success)} > \lambda = \frac{L}{n}$ for all $\lambda \in (0, 1)$.

If $K = n$, then $p = \frac{A}{n}$ and $\gamma = 1$. For $K < n$,

$$P(success) = \sum_{j=K}^{n} C_n^j \gamma^j (1-\gamma)^{n-j} = C_n^K \gamma^K (1-\gamma)^{n-K} + \sum_{j=K+1}^{n} C_n^j \gamma^j (1-\gamma)^{n-j} <$$

$$< C_n^K \gamma^K (1-\gamma)^{n-K} + \sum_{j=K}^{n-1} C_{n-1}^j \gamma^j (1-\gamma)^{n-j-1} = C_n^K \gamma^K (1-\gamma)^{n-K} + p$$

since $\frac{j+1}{n} > Kn > \gamma$ implies $C_{n-1}^j \gamma^j (1-\gamma)^{n-j-1} > C_n^{j+1} \gamma^{j+1} (1-\gamma)^{n-j-1}$.

Then, $\forall \epsilon > 0 \; \exists N$ such that

$$\frac{p}{P(success)} > 1 - \frac{C_n^K \gamma^K (1-\gamma)^{n-K}}{\sum_{j=K}^{n} C_n^j \gamma^j (1-\gamma)^{n-j}} > 1 - \epsilon \text{ for all } n > N$$

For $1 > \gamma \geq \frac{K}{n}$,

$$\pi(p) = A - L + \gamma^n - \sum_{j=K}^{n-1} C_n^j \gamma^j (1-\gamma)^{n-j-1} \left[(n-j) - L(1-\gamma)\right]$$

Since for all $j = K...n-1$, $n - j \geq n - K = n(1 - \frac{K}{n}) \geq n(1-\gamma) > L(1-\gamma)$ and $P(success) \geq 1/2$, we can write

$$\pi(p) < A - L + \gamma^n - P(success) \cdot (n - L) \leq A - L - \frac{n-L}{2} + \gamma^n < A - L$$

for all $n > N_4$, N_4 big enough.

Thus, for all $p \in (0, \frac{A}{n})$ such that $1 > \gamma(p) \geq \frac{K}{n}$, there exists N_4 big enough, such that $\forall n \geq N_4$,

$$\pi(p) < A - L \tag{B.1}$$

From equation (1) and Claim it follows that for all $p \in (0, \frac{A}{n})$, there exists an N such that $\forall n \geq N$,

$$\pi(p) < A - L$$

Since Debtor can always assure profit of $\pi = A - L$ by offering $p = 0$, for which optimal creditors' symmetric strategy is always reject the offer ($\gamma = 0$), unique equilibrium of the game is as follows:

Debtor offers $p = 0$;

Creditors accept the offer with probability γ, reject with probability $1 - \gamma$, where γ solves

$$p = \sum_{j=K}^{n-1} C_{n-1}^j \gamma^j (1-\gamma)^{n-j-1}$$

If LHS is strictly greater for all $\gamma \in (0, 1)$, then creditors always accept the offer. If RHS is strictly greater for all $\gamma \in (0, 1)$, then creditors always reject the offer.

For the completeness of the proof of part a), we should consider the only remaining symmetric strategy of the creditors: $\gamma = 1$, which may be induced only by the offer $p = \frac{A}{n}$ (otherwise, any creditor would strictly prefer to reject the offer). In this case, all creditors accept the offer and Debtor receives zero profit, which is less than $A - L$ and thus cannot be an equilibrium payoff.

b) Suppose that $A < L$ and $n > L$. As we have shown in part a), for every p in the open interval $(0, 1)$, the Debtor's profit is less than $A - L$ (for a high enough number of creditors), which in this case is negative. If Debtor offers $p = 0$, then all creditors accept and she receives exactly $A - L$. If Debtor offers $p = \frac{A}{n}$, then the optimal symmetric strategy of creditors is to accept the offer with probability one, and the payoff of the Debtor becomes equal to zero, which is the highest payoff she can attain in this case.

Thus, there is a unique equilibrium of the game:

Debtor offers $p = \frac{A}{n}$;

Creditors accept the offer with probability γ, reject with probability $1 - \gamma$, where γ solves

$$p = \sum_{j=K}^{n-1} C_{n-1}^{j} \gamma^{j} (1 - \gamma)^{n-j-1}$$

If LHS is strictly greater for all $\gamma \in (0, 1)$, then creditors always accept the offer. If RHS is strictly greater for all $\gamma \in (0, 1)$, then creditors always reject the offer.

Appendix C

Existence of $K(p)$

Existence: Define $f : [0, n] \to \mathbb{R}$

$$f(i) = A - \sum_{j \leq i} p - \sum_{j > i} 1 = A - p \cdot i - (n - i) = A - n + (1 - p)i$$

for a fixed $p \in [0, \frac{A}{n}]$. Note that $f(i)$ is continuous on $[0, n]$ for all values of p.

$$f(0) = A - n < 0$$

$$f(n) = A - pn \leq A - n \cdot \frac{A}{n} = 0$$

$$f(n) = A - pn < A - n \cdot \frac{A}{n} = 0 \text{ for } p \neq \frac{A}{n}$$

By the Intermediate Value Theorem, $\exists i \in [0, n]$ such that $f(i) = 0$, thus $K(p)$ exists for all p. For $p \neq \frac{A}{n}$, $K(p) \neq n$ and $K(p) \in (0, n)$.

Uniqueness: Suppose that $K < K'$ both satisfy $f(K) = f(K') = 0$ for a fixed $p \in [0, \frac{A}{n}]$. Then

$$f(K') - f(K) = (1 - p)(K' - K) \neq 0, \text{ contradiction.}$$

Appendix D

Private Value with Uniform Distribution

The indifference condition:

$$p = P(S \mid a^* \;\; rejects) + P(F \mid a^* \;\; rejects) \cdot a^* =$$
$$= a^* + (1-a^*) \sum_{j=K(p)}^{n-1} C_{n-1}^{j} \big[F(a^*)\big]^{j} \big[1 - F(a^*)\big]^{n-j-1}$$

Derivative of the RHS with respect to a^* is (see Appendix A):

$$(n-1)\frac{dF(a^*)}{da^*} C_{n-2}^{K-1} \gamma^{K-1} (1-\gamma)^{n-K-1} > 0$$

Thus, the RHS if the indifference condition is strictly monotone in a^* and a^* is unique for each p. Therefore, a^* is a function of p, $a^*(p)$, end of proof.

Suppose that a is drawn from the uniform distribution, $F(a) = a$. If $\alpha < \frac{3}{4}$, then lines $F(p) = p$ and $\frac{K}{n} = \frac{n-A}{1-p}$ do not intersect and there is no offer p inducing $P(Success)$ bounded away from zero, as $n \to \infty$. Suppose that $\alpha \in \big(\frac{3}{4}, 1\big)$. Then for

$$p \in \Big(\frac{1-\sqrt{4\alpha-3}}{2}, \frac{1+\sqrt{4\alpha-3}}{2}\Big)$$

we have $F(p) > \frac{K(p)}{n}$ and

$$a^* = \frac{K}{n} = \frac{1-\alpha}{1-p}$$

$$P(Success) = \frac{p-a^*}{1-a^*} = \frac{p(1-p)-(1-\alpha)}{\alpha-p}$$

$$(1-a^*)E[a \,|\, a > a^*] = \int_{a^*}^{1} a\, da = \frac{1}{2}\big[1-(a^*)^2\big] = \frac{1}{2}\frac{(1-p)^2-(1-\alpha)^2}{(1-p)^2}$$

$$\frac{\pi}{n} = \alpha - \frac{p(1-\alpha)}{1-p} - \frac{\alpha-p}{1-p}\frac{p(1-p)-(1-\alpha)}{\alpha-p} - \frac{1}{2}\frac{(1-p)^2}{\alpha-p}\frac{(1-p)^2-(1-\alpha)^2}{(1-p)^2}$$
$$-\frac{(1-p)^2}{\alpha-p}\lambda = 1 - p - \frac{1}{2}\frac{(1-p)^2}{\alpha-p} + \frac{1}{2}\frac{(1-\alpha)^2}{\alpha-p} - \frac{(1-p)^2}{\alpha-p}\lambda =$$

$$= \frac{1}{2}(\alpha - p) - \frac{(1-p)^2}{\alpha - p}\lambda$$

The derivative of the profit function is

$$\frac{d(\pi(p))}{dp} = -\frac{1}{2} + \frac{2(1-p)(\alpha-p)}{(\alpha-p)^2}\lambda - \frac{(1-p)^2}{(\alpha-p)^2}\lambda =$$

$$= \frac{1}{2(\alpha-p)^2}\left[2(1-p)(\alpha-p)\lambda - 2(1-p)(1-\alpha)\lambda - (\alpha-p)^2\right]$$

$$= \frac{1}{2(\alpha-p)^2}\left[(2\lambda-1)(p-a)^2 - 2\lambda(a-1)^2\right]$$

If $\lambda \leq \frac{1}{2}$, then the profit function is decreasing on interval $p \in \left(\frac{1-\sqrt{4\alpha-3}}{2}, \frac{1+\sqrt{4\alpha-3}}{2}\right)$. Since any offer p outside this interval induces $P(Success) \to 0$ with $\pi(p) < pi(0)$, in this case offering $p > 0$ is never optimal. The optimal offer will always be $p = 0$.

If $\lambda > \frac{1}{2}$, then the profit function increases on

$$p \in \left(\frac{1-\sqrt{4\alpha-3}}{2},\ a - (1-a)\sqrt{\frac{2\lambda}{2\lambda-1}}\right)$$

and decreases on

$$p \in \left(a - (1-a)\sqrt{\frac{2\lambda}{2\lambda-1}},\ \frac{1+\sqrt{4\alpha-3}}{2}\right)$$

for any $a \in (\frac{3}{4}, 1)$.

Then, the profit of the Debtor at $p = a - (1-a)\sqrt{\frac{2\lambda}{2\lambda-1}}$ is

$$\pi = \frac{1}{2}(\alpha-p) - \frac{(1-p)^2}{\alpha-p}\lambda = \frac{1}{2}(1-\alpha)\sqrt{\frac{2\lambda}{2\lambda-1}} - (1-\alpha)\lambda\frac{\left(1+\sqrt{\frac{2\lambda}{2\lambda-1}}\right)^2}{\sqrt{\frac{2\lambda}{2\lambda-1}}} =$$

$$= \frac{1}{2}(1-\alpha)\sqrt{\frac{2\lambda}{2\lambda-1}} - \frac{(1-\alpha)\lambda}{\sqrt{\frac{2\lambda}{2\lambda-1}}} - 2\lambda(1-\alpha) - (1-\alpha)\lambda\sqrt{\frac{2\lambda}{2\lambda-1}} =$$

$$= (1-\alpha)(1-2\lambda)\sqrt{\frac{2\lambda}{2\lambda-1}} - 2\lambda(1-\alpha) > \alpha - \frac{1}{2} - \lambda = \pi(0)$$

when $\frac{\alpha-1/2}{1-\alpha} > \sqrt{\frac{2\lambda}{2\lambda-1}}$.

This condition requires a higher λ for α approaching $\frac{3}{4}$. This condition shows how big λ should be to offset the small probability of a successful debt restructuring:

For α close to $\frac{3}{4}$, there is small room for the difference between p and $a^*(p)$, which influences the $P(Success) \to \frac{p-a^*}{1-a^*}$, putting an upper bound on the maximum possible probability of success.

To sum up, if the type of the creditors is distributed uniformly on $[0, 1]$, then:

If $\alpha \leq \frac{3}{4}$, then $P(Success) \to 0$ for all $p \in [0, \alpha]$

If $\alpha \leq \frac{3}{4}$ and $\lambda \leq \frac{1}{2}$, then the optimal offer for the Debtor is $p = 0$. The Debtor could induce $P(Success)$ bounded away from zero, but the loss of offering $p > 0$ is greater than gain from the increase in probability of successful restructuring.

If $\alpha \leq \frac{3}{4}$, $\lambda \leq \frac{1}{2}$ and $\frac{\alpha-1/2}{1-\alpha} < \sqrt{\frac{2\lambda}{2\lambda-1}}$, then the optimal offer is still $p = 0$.

If $\alpha \leq \frac{3}{4}$, $\lambda \leq \frac{1}{2}$ and $\frac{\alpha-1/2}{1-\alpha} > \sqrt{\frac{2\lambda}{2\lambda-1}}$, then the optimal offer is

$$p = a - (1-a)\sqrt{\frac{2\lambda}{2\lambda - 1}}$$

Appendix E

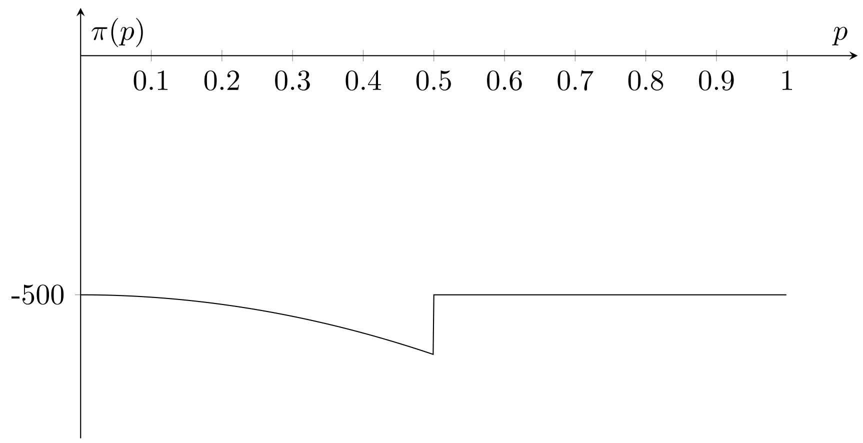
Simulations with Normal and Uniform Distribution Functions

Here we give some examples of different profit functions, depending on the values of n, A and L.

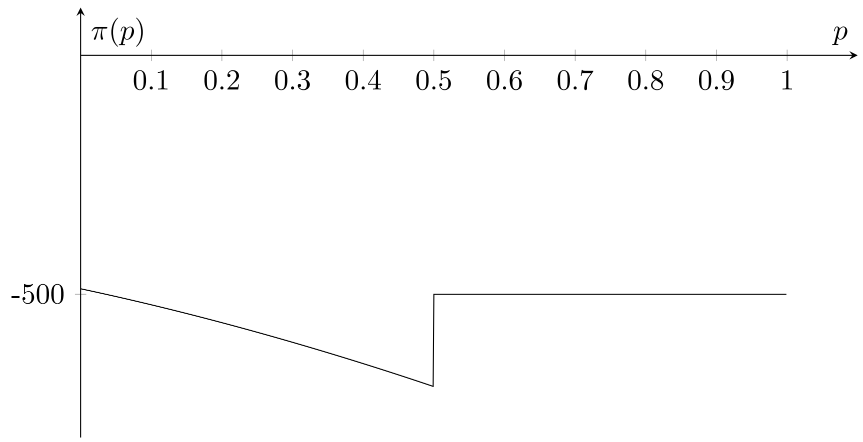
The uniform distribution is taken as $F(a) = a$, thus $F(0) = 0$.

The normal distribution is taken with $\mu = \frac{1}{2}$ and $\sigma = \frac{1}{4}$, so that $F(0) = 0.023$.

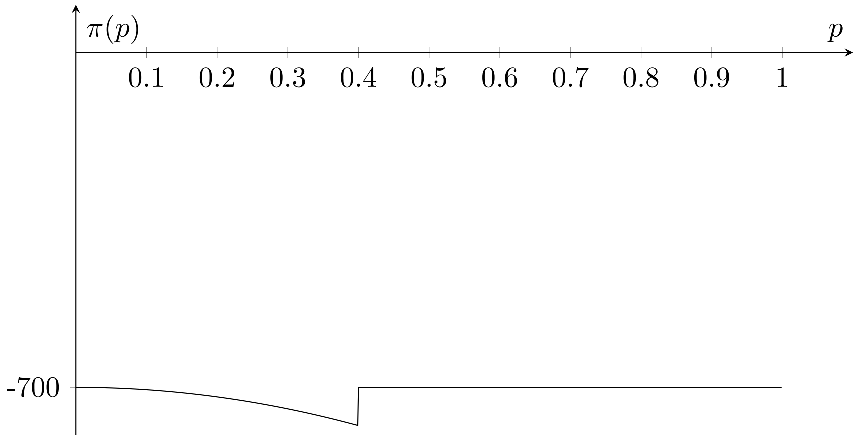
Graph 4: Simulation with unifrom dist., n=1000, L=500, A=500



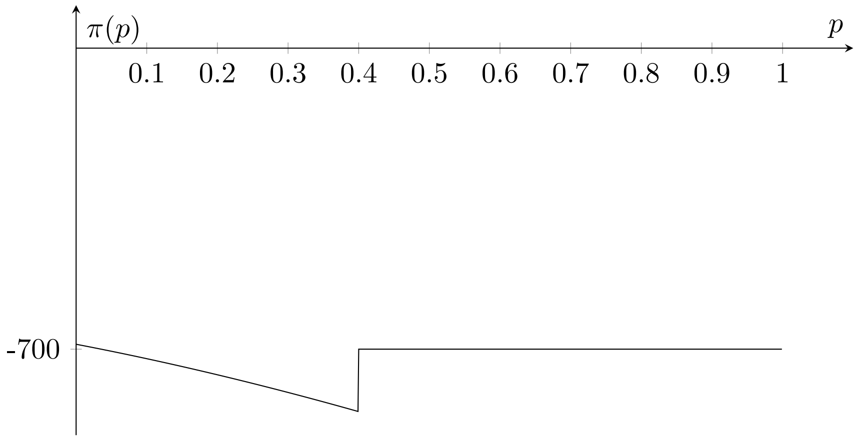
Graph 5: Simulation with normal dist., n=1000, L=500, A=500



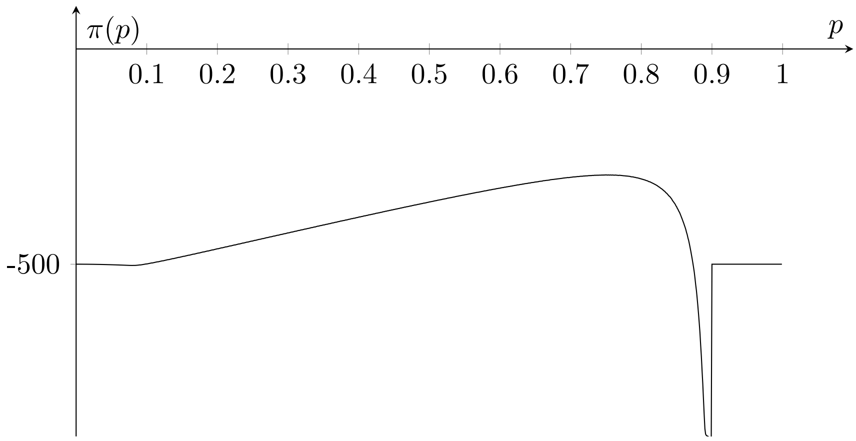
Graph 6: Simulation with unifrom dist., n=1000, L=600, A=400



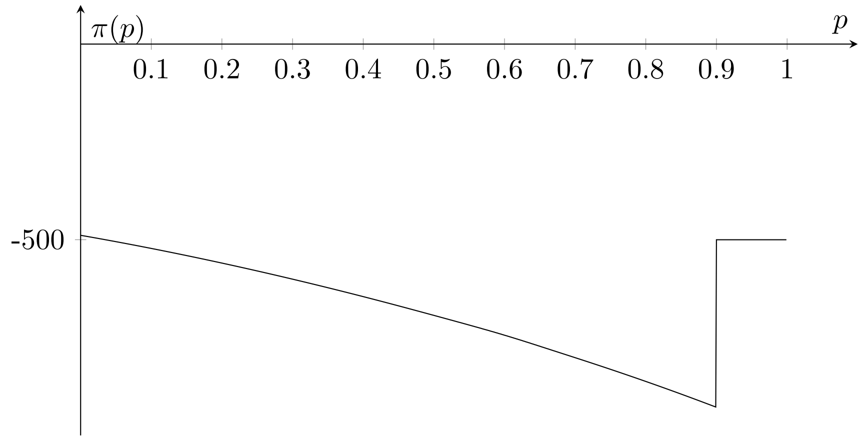
Graph 7: Simulation with normal dist., n=1000, L=600, A=400



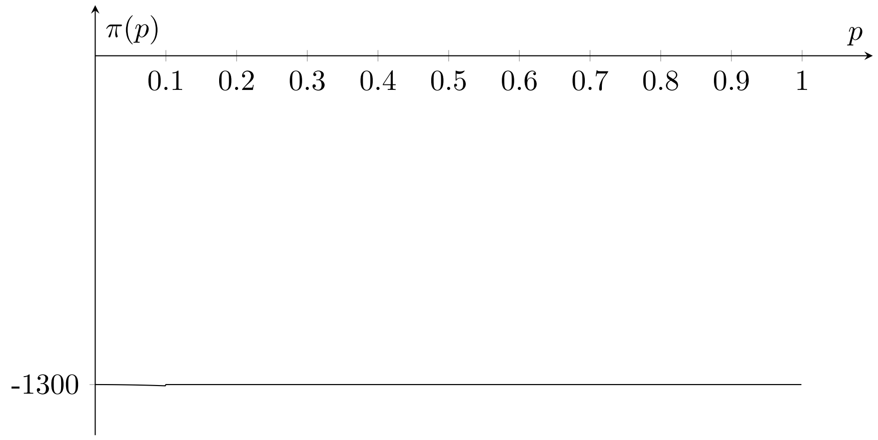
Graph 8: Simulation with unifrom dist., n=1000, L=900, A=900



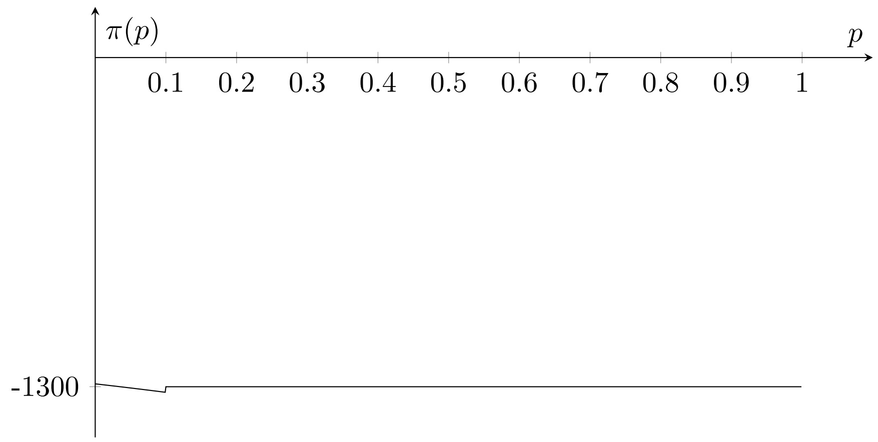
Graph 9: Simulation with normal dist., n=1000, L=900, A=900



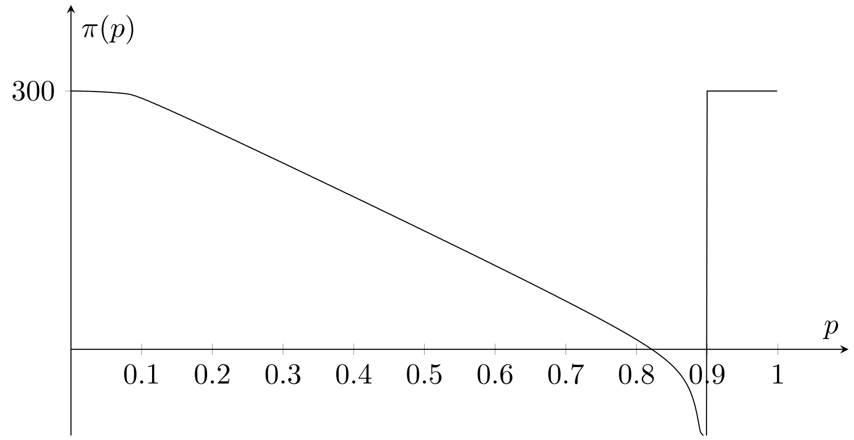
Graph 10: Simulation with unifrom dist., n=1000, L=900, A=100



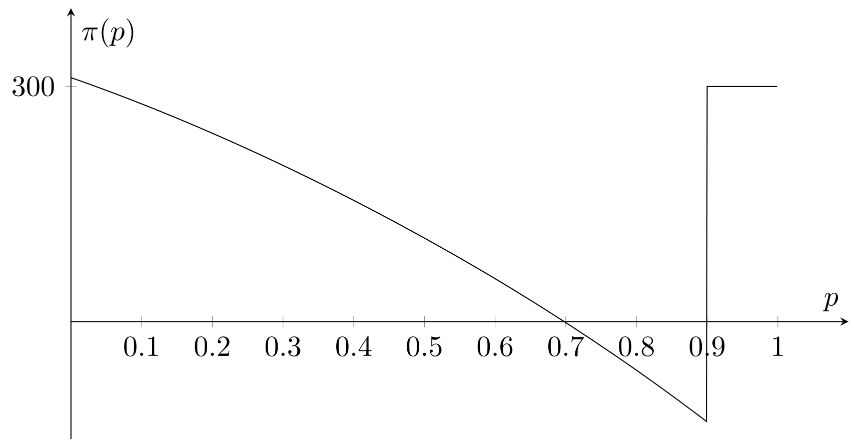
Graph 11: Simulation with normal dist., n=1000, L=900, A=100



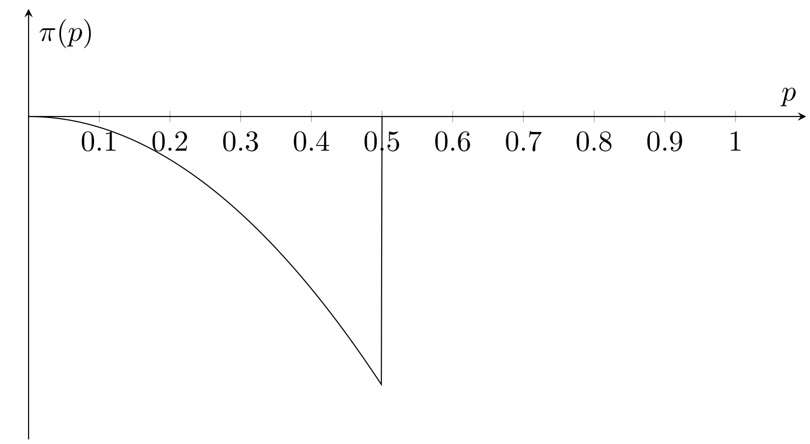
Graph 12: Simulation with unifrom dist., n=1000, L=100, A=900



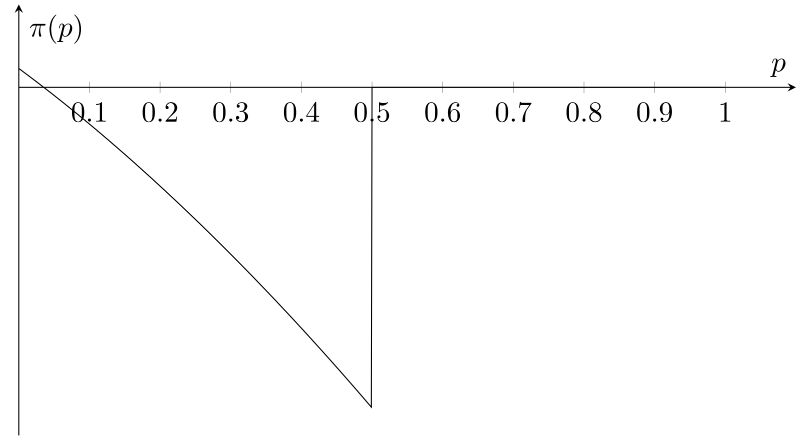
Graph 13: Simulation with normal dist., n=1000, L=100, A=900



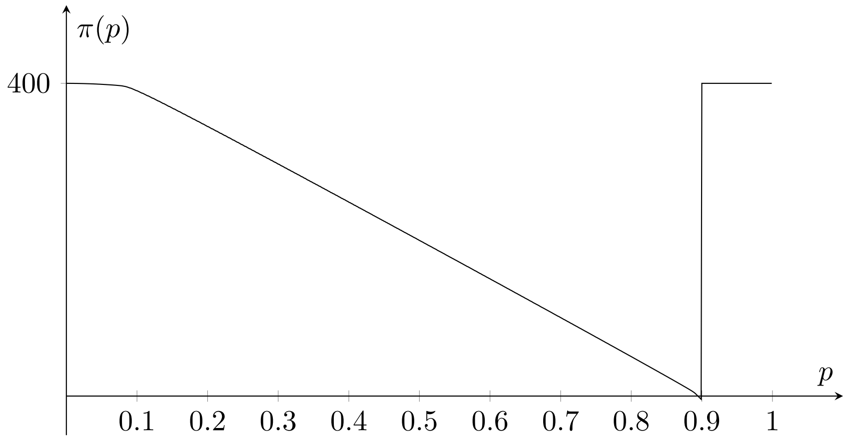
Graph 14: Simulation with unifrom dist., n=1000, L=0, A=500



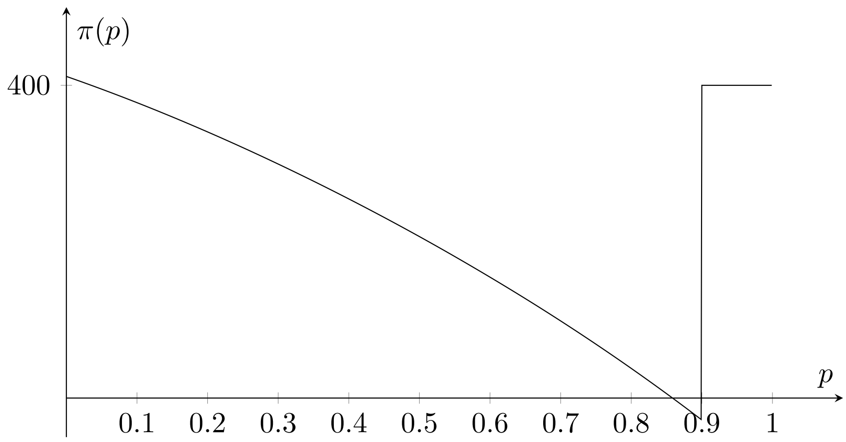
Graph 15: Simulation with normal dist., n=1000, L=0, A=500



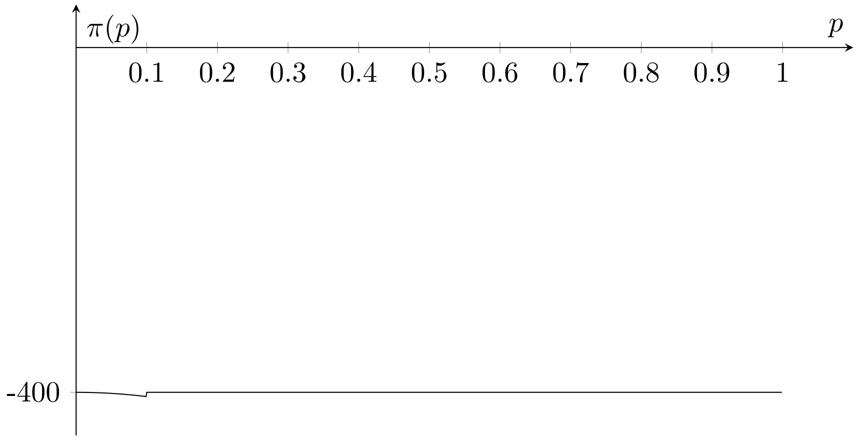
Graph 16: Simulation with unifrom dist., n=1000, L=0, A=900



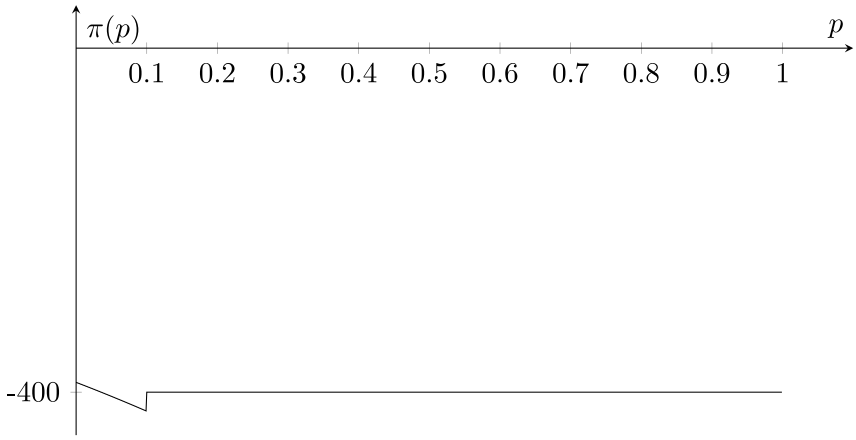
Graph 17: Simulation with normal dist., n=1000, L=0, A=900



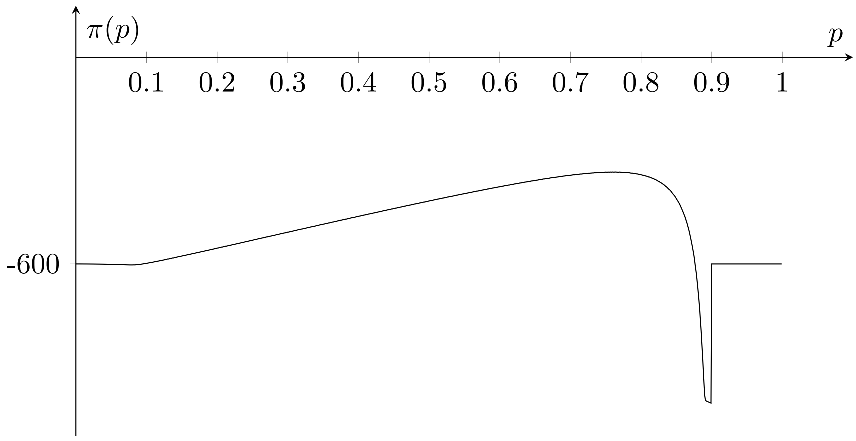
Graph 18: Simulation with unifrom dist., n=1000, L=0, A=100



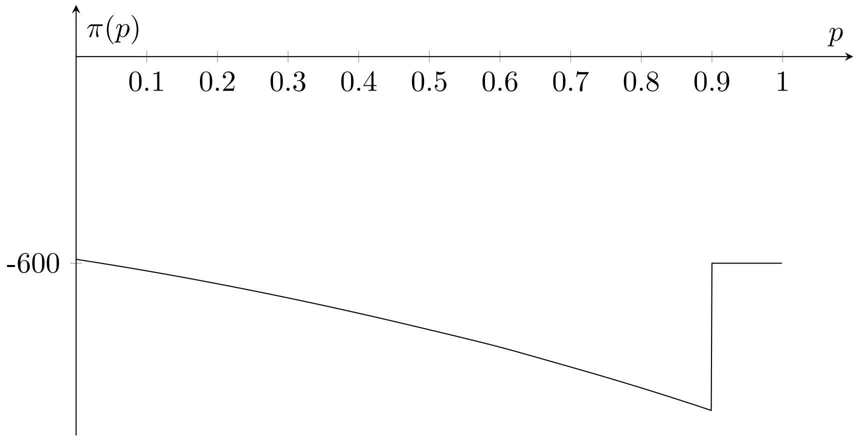
Graph 19: Simulation with normal dist., n=1000, L=0, A=100



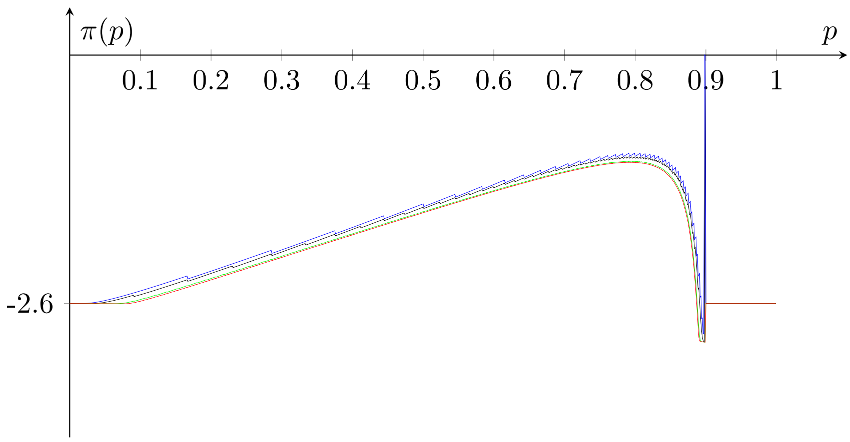
Graph 20: Simulation with unifrom dist., n=1000, L=1000, A=900



Graph 21: Simulation with normal dist., n=1000, L=1000, A=900

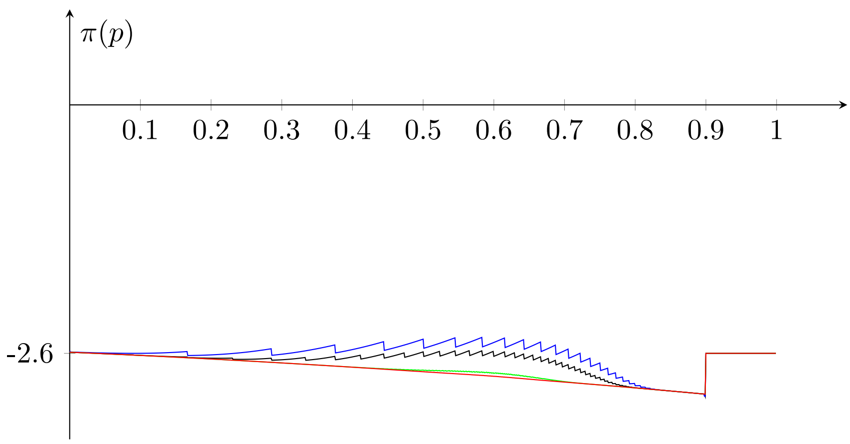


Graph 22: Profit function as $n \to \infty$, $\lambda = 3$, $\alpha = 0.9$, unif. dist.



On Graph 22, blue line corresponds to $n = 50$, grey - $n = 100$, green - $n = 500$, red - $n = 1000$. According to the result in Appendix D, the profit function increases starting with p around 0.11 and reaches its maximum at $0.9 - 0.1\sqrt{\frac{6}{5}} \approx 0.79$.

Graph 23: Profit function as $n \to \infty$, $\lambda = 3$, $\alpha = 0.9$, norm. dist.



On Graph 23, blue line corresponds to $n = 50$, grey - $n = 100$, green - $n = 500$, red - $n = 1000$.

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