

**A Discrete Continuous Approach to Wind Power
Investment Decisions**

by

Aysim Gülde Kublay

**A Thesis Submitted to the
Graduate School of Engineering
in Partial Fulfillment of the Requirements for
the Degree of**

**Master of Science
in
Industrial Engineering**

Koc University

September 2014

Koc University

Graduate School of Sciences and Engineering

This is to certify that I have examined this copy of a master's thesis by

Aysim Gülde Kublay

and have found that it is complete and satisfactory in all respects,

and that any and all revisions required by the final

examining committee have been made.

Committee Members:

Metin Türkay, Ph. D. (Advisor)

Sibel Salman, Ph. D.

Belgin Şan Akça, Ph. D.

Date:

ABSTRACT

Harvesting energy from wind is becoming an effective and worthwhile way of meeting the electricity demand. Due to the carbon free structure of the wind energy, some countries started to invest more funds into the construction of wind farms on suitable areas. Before the construction of a wind farm, some variables must be chosen carefully to acquire the optimum results from both the investment cost and efficient design. The location of the wind farm must be convenient in terms of its yearly average wind speed. Other restrictions of wind farm structure include wind turbine design. Rotor length (or blade) is one of the major parameters of a wind turbine in the amount of energy generated. In addition, since wind farms contain multiple wind turbines, their placement also introduces another decision variable. Considering all of the aspects of wind farm, we modeled a multi-period mixed-integer non-linear optimization problem to decide on the investment decisions of a wind farm.

We illustrate the efficiency and accuracy of our model on a real example of Turkey case and implemented the proposed model into GAMS (General Algebraic Modeling System) optimization package by using BARON solver. The result of 40.2 GW of energy that can be generated in the short term with immediate investments is obtained. Due to the uncertainty on wind speed values, we implement two-stage scenario based stochastic approach into the model and compared the results to see the value of perfect information.

Finally, we conclude with the sensitivity analysis in order to see the affects of variations within the variables of the model.

ÖZETÇE

Rüzgârın enerjisinden elde edilen elektrik enerjisi, elektrik talebini karşılamada gittikçe daha önemli ve efektif bir yöntem olmaktadır. Rüzgâr türbinlerinin karbon salınımı yapmamaları ve çevre dostu özellikleri nedeniyle, birçok ülke rüzgar santrallerine yapılan yatırımları arttırmaktadır. Yatırım minimize edilerek, en iyi üretim sonuçlarının alınabilmesi için rüzgâr santralleri kurulmadan önce, birçok değişken göz önüne alınmalıdır. Rüzgâr santrali kurulmasının yapılacağı yer yıllık ortalama rüzgâr hızı bakımından santral kurmaya elverişli olmalıdır. Santrale kurulacak türbin özellikleri de, santralden alınacak verim konusunda oldukça önemlidir. Türbinin rotor uzunluğunun üretilen toplam enerji üzerinde doğrudan etkisi vardır. Diğer yandan, rüzgar santralleri birden fazla türbinden oluştuğundan, türbinlerin yerleşimi de bir diğer karar değişkenidir. Rüzgar santralleri tüm bu açılardan düşünülerek, bu çalışmada çok dönemli, karma tamsayı doğrusal olmayan, rüzgar santralleri yatırım kararlarını en iyileyen problem modellenmiştir.

Geliştirilen matematiksel modelin etkinliği ve doğruluğu Türkiye için elde edilen gerçek veriler uygulanarak, GAMS yazılımında BARON kullanılarak test edilmiştir. Türkiye örneğine göre, Enerji Bakanlığı tarafından belirlenen gelecek 5 yıllık rüzgar enerjisi talebini karşılayacak 38 GW'lık elektrik enerjisi üretim senaryosu belirlenmiştir. Diğer yandan rüzgar hızı belirsizliğini göz önüne alan iki aşamalı rassal model formülasyonu deterministik modele uygulanmıştır.

Son olarak kurulan matematiksel modelin belli varyasyonlar üzerinde nasıl karar verdiğinin duyarlılık analizi yapılmış olup, varyasyonlarla ilgili çeşitli senaryolar temel senaryo ile karşılaştırılmıştır.

ACKNOWLEDGEMENTS

Foremost, I would like to express my sincere gratitude to my advisor Prof. Metin Türkay for the support of my master study and research, for his motivation, enthusiasm, and useful comments, remarks through the learning process of this master thesis.

Furthermore, I would like to thank my committee members, Sibel Salman and Belgin Şan Akca for their times and insightful comments. I also wish to thank all the professors of the Department of Industrial Engineering for sharing their valuable knowledge.

I would like to thank YAPI ve KREDİ BANKASI A.Ş. for providing support through grants for Renewable Energy Scholarship.

A special thanks to my dear friend İrem Dönmez, for her deep friendship and encouragement for every phase of this thesis.

Finally, words cannot express how grateful I am to my fiancé Ertürk, who spent sleepless nights with me, always supporting with patience. My sister Simge, thank you for being my best friend and your unconditional support. And my parents, I owe more than thanks to you for your belief in me and for giving me the chance of pursuing my dreams throughout my life.

Table of Contents

ABSTRACT.....	iii
ÖZETÇE	iv
ACKNOWLEDGEMENTS.....	v
List of Tables	viii
List of Figures	x
NOMENCLATURE	xi
Chapter 1	1
Introduction.....	1
1.1. Wind Energy	2
1.1.1. History of Wind Energy.....	2
1.1.2. Advantages and Disadvantages of Wind Energy.....	3
1.1.3. Wind Power in the World	4
1.1.4. Types of Wind Turbines	5
1.2. Objective of the Study.....	8
1.3. Outline.....	8
Chapter 2.....	9
Literature Review.....	9
2.1. Overview	9
2.2. Optimization Methods for Wind Energy.....	10
Chapter 3.....	13
Problem Description and Modeling	13
3.1. Introduction	13

3.1.1. Mathematical Model Representation	14
Objective Function	15
Constraints.....	16
3.1.2. Model Formulation Using Stochastic Scenario-Based Approach	23
3.2. Computational Results	26
3.2.1. Illustrative Example	26
3.2.2. Application to Turkey Case	29
3.2.3. Two-Stage Stochastic Programming Approach to Turkey Case	38
3.2.4. Real Case Wind Farm Application	44
3.3. Sensitivity Analysis.....	48
3.3.1. Results of Sensitivity Analysis	51
Chapter 4.....	54
Discussion & Conclusions	54
4.1. Main Observations of Wind Energy Investment Model	54
4.2. Future Work	55
BIBLIOGRAPHY	57
VITA	59

List of Tables

- Table 1: Yearly wind speed of illustrative regions
- Table 2: Optimal Energy Generation Plan for Periods
- Table 3: Installed wind turbine numbers through periods
- Table 4: Wind Farm column and row lengths for the locations
- Table 5: Total number of turbines of wind park areas(rows)
- Table 6: Total number of turbines of wind park areas(columns)
- Table 7: Model Results for the 3 period model
- Table 8: Total Wind Potential of Turkey
- Table 9: Yearly Average Wind Speeds of 7 Region of Turkey [20]
- Table 10: Demand Projections (kwh)
- Table 11: Wind Turbine Model Parameters
- Table 12: Demand Projections of Regions (kwh)
- Table 13: Optimal Energy Generation Plan for Periods (MW)
- Table 14: Installed wind turbine numbers through periods
- Table 15: Wind Farm column and row lengths for the locations
- Table 16: Total number of turbines of wind park areas(rows)
- Table 17: Total number of turbines of wind park areas(columns)
- Table 18: Model Results for the Turkey Case
- Table 19: Installed wind turbine numbers through periods
- Table 20: Wind Farm column and row lengths for the locations
- Table 21: Total number of turbines of wind park areas(rows)
- Table 22: Total number of turbines of wind park areas(columns)
- Table 23: Model Results for the Two-Stage Stochastic Approach to Turkey Case
- Table 24: Consolidated Results of All Scenarios
- Table 25: Optimal Objective Function Values
- Table 26: Çanakkale RES Wind Farm
- Table 27: Total number of turbines of wind park areas(rows)

Table 28: Total number of turbines of wind park areas(columns)

Table 29: Wind Farm column and row lengths for the locations

Table 30: Comparison Between Actual Canakkale RES and Optimal Results of Model

Table 31: Compilation results comparison of different periods

Table 32: Effects of adding new locations

Table 33: Affects of adding new wind farm areas

Table 34: Results of extended model

List of Figures

Figure 1. Fuel shares in electricity generation

Figure 2: CO2 emissions by region in 1973 and 2010

Figure 3: Cumulative Top 10 Wind Energy Capacity

Figure 4: Relationship between power capacity and the rotor diameter

Figure 5: Spacing of wind turbines in a wind

Figure 6: Energy resources of Turkey by the year of 2009

Figure 7: Cumulative Distribution of Installed Capacity for Wind Power Plants in Turkey

Figure 8: Wind Map of Turkey at 30m altitude

Figure 9: Çanakkale RES Land Area Lengths

Figure 10: Optimal Placement of Çanakkale RES

Figure 11: Execution Times

Figure 12: Number of Baron Iterations

Figure 13: Total Cost

NOMENCLATURE

Sets

I set of locations

K set of rotor diameter sizes

T set of time periods

M set wind farm areas

Parameters

d_{it} Demand projection of location $i \in I$ in period $t \in T$,

l_k Investment cost of rotor size $k \in K$,

r_k Rotor diameter of rotor size $k \in K$,

$windenergy_{ki}$ Yearly energy output of rotor type $k \in K$ in location $i \in I$,

wf_{mi} Available wind farm area of m in location $i \in I$,

v_i Wind speed of location $i \in I$,

A_k The area that is swept by the rotor type $k \in K$,

Scalars

$Opcost$ Operation cost per kW

$kcol$ Wind turbine installation ratio of y axis of wind area

$krow$ Wind turbine installation ratio of x axis of wind area

r Inflation rate

Decision Variables

P	Total generated energy output ,
$Power_{mit}$	Total generated energy of wind farm $m \in M$ in location $i \in I$ in period $t \in T$,
C_t	Net Present Value (NPV) for yearly operational and management cost of wind turbines in period $t \in T$,
$l_{k,t}$	NPV value of the cost of rotor type $k \in K$ invested in period $t \in T$,
Ly_{mi}	y-axis length of wind farm $m \in M$ in location $i \in I$,
Lx_{mi}	x-axis length of wind farm $m \in M$ in location $i \in I$,
N_{mi}	Total number of wind turbines in wind farm $m \in M$ in location $i \in I$ of all periods,
$N_{col_{mit}}$	Total number of wind turbines in column of wind farm $m \in M$ in location $i \in I$ in period $t \in T$,
$N_{row_{mit}}$	Total number of wind turbines in row of wind farm $m \in M$ in location $i \in I$ in period $t \in T$,
Num_{mikt}	Total number of rotor type $k \in K$ installed in farm $m \in M$ at location $i \in I$ in period $t \in T$,
$N_{total_{mit}}$	Total number of wind turbines in wind farm $m \in M$, in location $i \in I$ in period $t \in T$,
$N_{turbine_{mit}}$	Total number of wind turbine added in period $t \in T$, in wind farm $m \in M$ of location $i \in I$,

$N_added_{m,i,k,t}$	Total number of wind turbine added in period $t \in T$, of type $k \in K$, in wind farm $m \in M$ and location $i \in I$,
Dia_{mit}	Installed rotor diameter of wind turbines in wind farm $m \in M$ in location $i \in I$ in period $t \in T$
y_{mit}	1 if new capacity of wind farm $m \in M$ is added in location $i \in I$ in period $t \in T$, 0 else
x_{mik}	1 if new capacity of rotor type $k \in K$ is added in wind farm $m \in M$ in location $i \in I$, 0 else
z_{mikt}	1 if rotor type $k \in K$ is selected for wind farm $m \in M$ in location $i \in I$ in period $t \in T$, 0 else

Chapter 1

Introduction

The need for resources in a densely populated world requires more and more sources (energy sources such as oil, gas, coal and water). The high density of population is not the only cause for heightened resource consumption; due to the technological developments after the Industrial Revolution, people started to consume more electricity, more gas, more oil and more water in their more civilized society. Whilst in 1973 global electricity generation was 6.115 TWh, in 2010 total amount of electricity generation was reported as 21.431 TWh and only 4% of electricity comes from the renewable resources (see Figure 1).

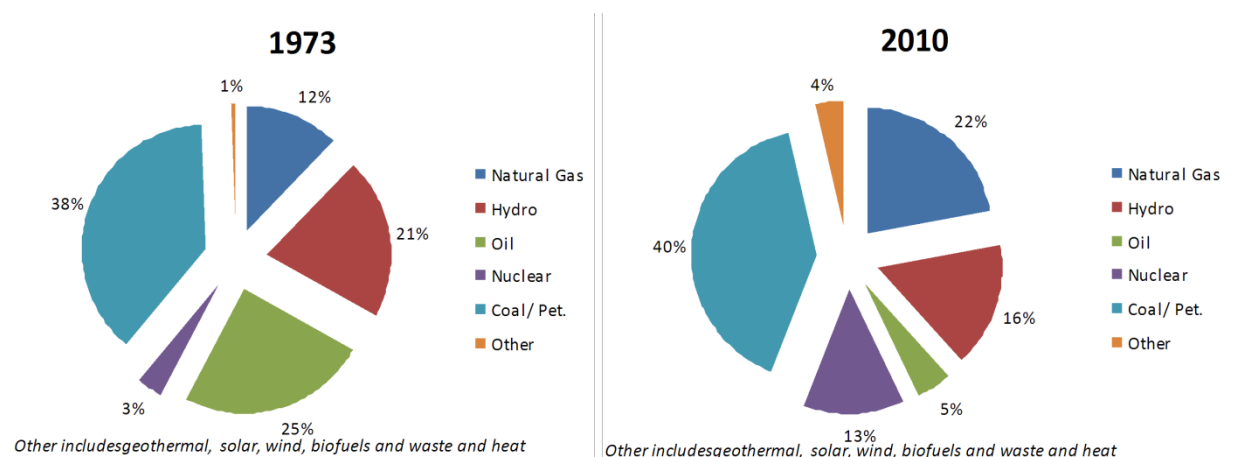


Figure 1. Fuel shares in electricity generation (a) 1973, (b) 2010 [1]

From 1973 to 2010, the major part of world's energy resources consists of carbon heavy resources which results in the high CO₂ emissions to the atmosphere. According to the International Energy Agency's 2012 statistics, CO₂ emissions due to fuel combustion was calculated as 15,637 Mt in 1973 but the total emission doubled up in 2010, with total

amount of 30,326 Mt and there is no doubt that in the future the amount of CO₂ held in the atmosphere will increase if the current energy mix continues to dominate (see Figure 2). Forests absorb CO₂ by doing photosynthesis and also there are some geological structures inside the earth which allow for the storage of CO₂ but these natural remedies cannot deal with the total amount of CO₂ that the humanity outputs in the atmosphere. At this point, the importance of the renewable sources, such as wind, solar, geothermal, ocean and bio energy, comes to the picture because of their carbon free structure.

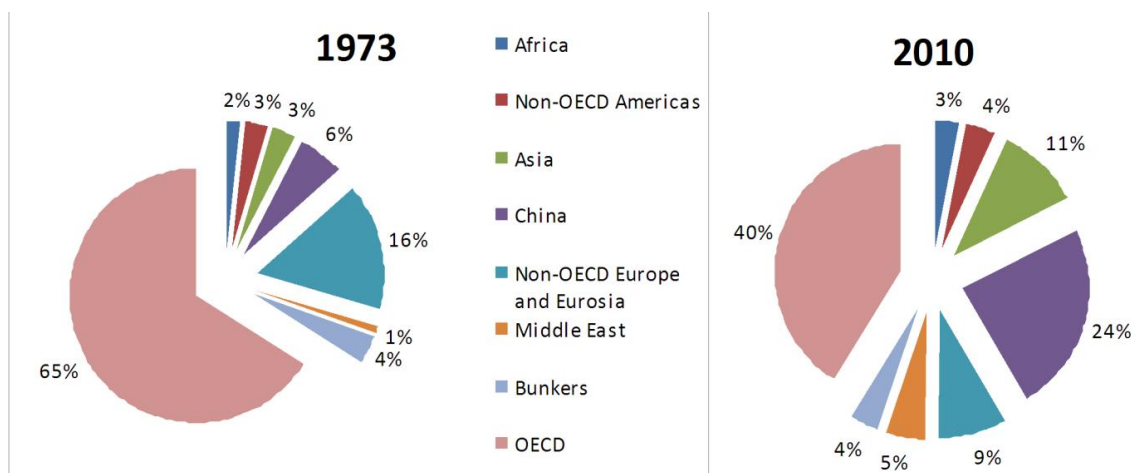


Figure 2: CO₂ emissions by region in 1973 and 2010 (International energy Agency)

1.1. Wind Energy

1.1.1. History of Wind Energy

Since early recorded history, people have been harnessing the energy of the wind. While Egyptians have been using wind energy propelled boats in the Nile, first simple windmill have been constructed in China which were pumping water. At the same time Persians and Middle Easters were using vertical-axis windmills for grinding grain.

Wind energy was spread around Europe by the 11th century by merchants and crusaders who were returning from Middle East, where the windmills are extensively used for food production. Windmills were used for draining lakes and marshes in the Rhine by the Dutch. With the discovery of the new world in 19th century, settlers brought the windmill technology to the new world, where windmills were used for pumping the water to farms and ranches, grinding grain and cutting wood at sawmills. The biggest contribution of windmills to the world energy production was started in the late 1920s, when Americans used small windmills to generate electricity in rural areas, where electricity services were not available. 1930s windmills lost their importance with spread of the power lines in the world. However in 1970s with the oil shortage and increase in the power demand around the world, windmills made a comeback to the energy stage with an important role and seen as an important alternative energy source. At the same times with scientific researches, people realized the effect of the carbon based energy sources to the climate change which increase the importance of the renewable energy sources like wind energy. Since 1970s renewable energy sources are getting more and more popular with technologic developments and the growing concerns about emissions [2].

1.1.2. Advantages and Disadvantages of Wind Energy

Since wind energy becomes an important player in the energy production scene, people started to talk about the advantages and the disadvantages of constructing wind farms and turbines. Although most of the people accepted the importance of the renewable energy sources, some still thinking that further technologic developments are needed to use these types of energy sources [3].

Advantages

- Wind energy is free and easy to capture with the modern technology
- Do not cause emission and effective way to decrease green house effect
- Do not interrupt farming due to small land requirement

- Great resource of energy in remote locations

Disadvantages

- Wind regime cannot be controlled
- Production of the turbines still pollutes the environment
- Large wind farms are needed to supply enough energy for a small town

1.1.3. Wind Power in the World

Worldwide, at the end of 2013 global total of wind power is 318.105 MW with a cumulative market growth more than 12.5% even though the average growth in last decade is about 21 percent.

Although the wind power industry is affected by the global financial crisis, it will be getting more and more important with coming years depending on the forecasts. Wind power market penetration to the electricity grid in the world is expected to be almost 8% of the total by 2018 [4].

Depending on the cumulative capacities in December 2013, PR China and USA are the biggest contributors to the energy sector by the view of wind energy perspective. Moreover PR China is the leading country when new capacity installation is concerned in 2013. The total top ten countries producing 269.773 MW of wind power which is the 84.8% of the total world wind power [5].

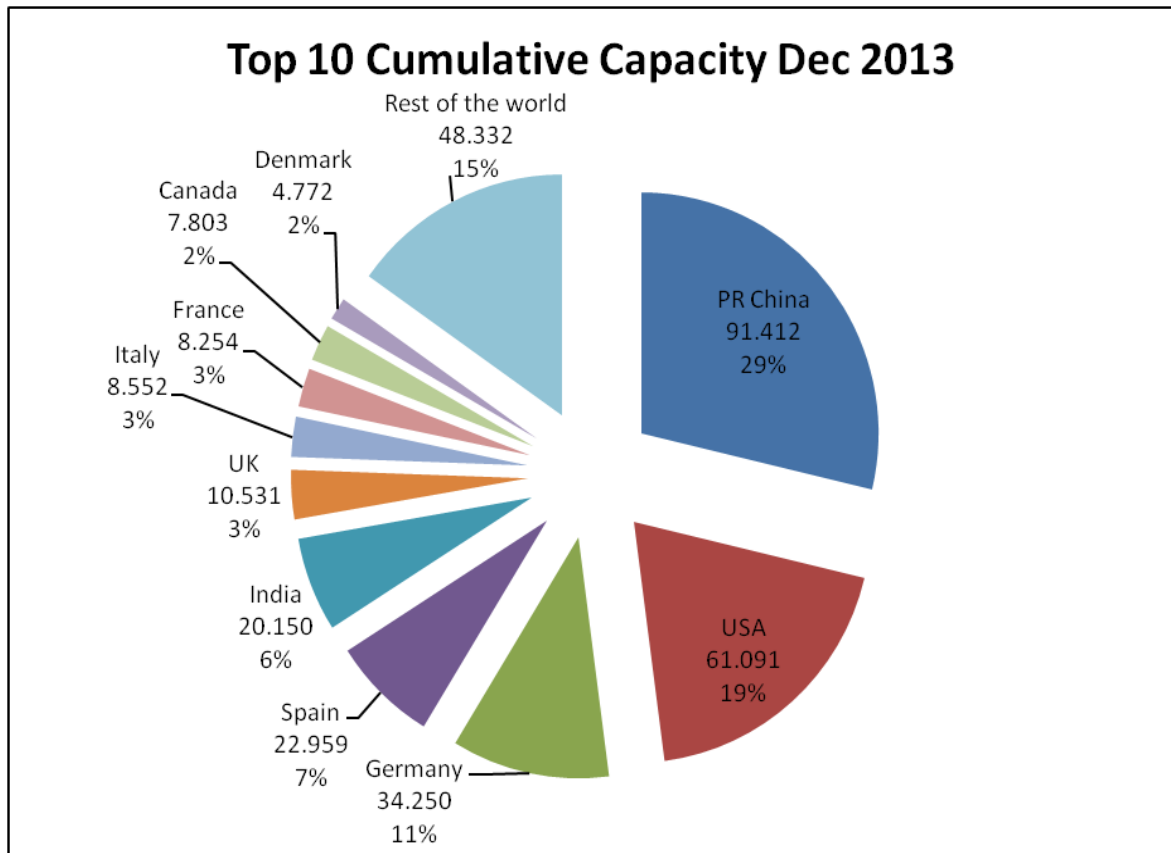


Figure 3: Cumulative Top 10 Wind Energy Capacity [5]

1.1.4. Types of Wind Turbines

Harnessing the energy from wind is considered as an effective way to convert mechanical energy into electricity with the use of wind turbines. Modern wind turbines can be divided into two basic groups; horizontal-axis wind turbines and vertical-axis wind turbines. Most wind parks consist of large turbines that use horizontal-axis wind turbines. Blades, shafts, gears, a generator and a cable are some basic components of a wind turbine. The wind comes through to the blades or rotors and makes them turn and finally energy in the wind is converted through a rotational shaft into the mechanical energy.

There are several types of rotor designs in commercial wind turbines. Longer rotors (or blades) mean more energy would be harvested. The selection of the rotor type highly

depends on the topographical configuration of the land area. In addition, the transportation routes for the turbines and rotors, wind speed data of the area and also grid connection specifications need to be taken into consideration before the production starts. Increasing the length of a rotor as much as possible is not the only consideration in wind farm design. Logistic of those blades can be really costly due to the inconvenience of the roads to the land area of wind farm. To insure the efficiency of the project, the blade type must be chosen carefully to ensure the design of the blade is optimal for the specific geographical topology and wind speed profiles in the particular region. Figure 4 shows the power capacity of different type of rotor diameters.

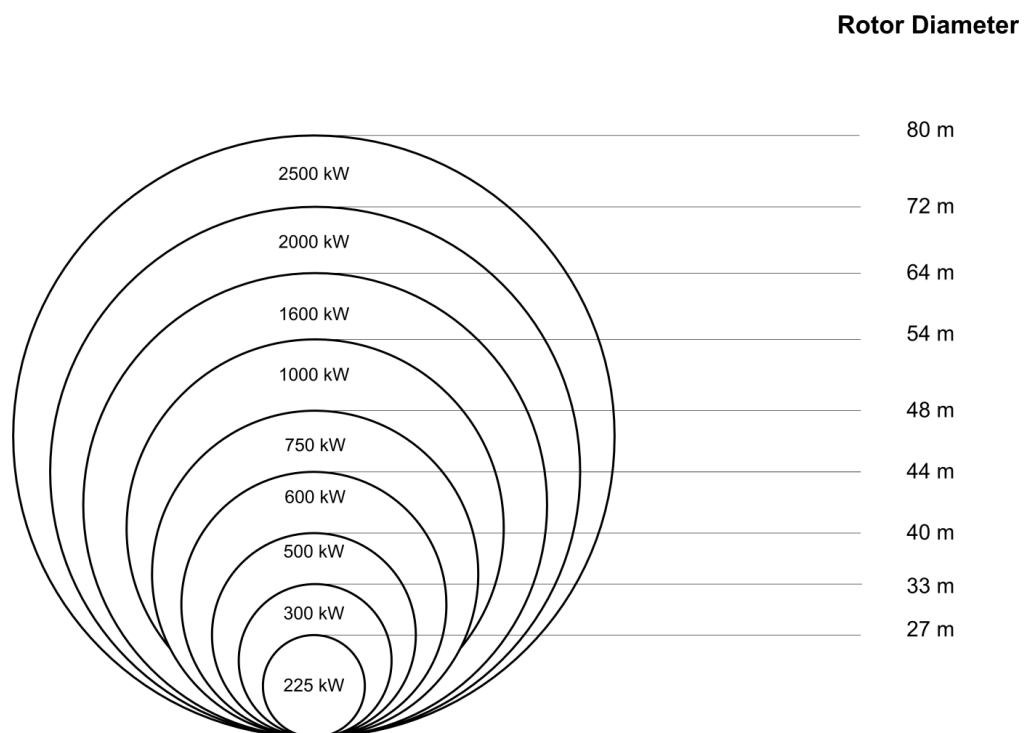


Figure 4: Relationship between power capacity and the rotor diameter (Danish Wind Industry Association, 2009)

In order to generate energy from wind, once wind turbine decision is made, the land area required is approximately 50 acres per MWe [6]. Positioning of turbines must also be taken into account. The required spacing between turbines is given on Figure 5. For example, a 1,000 MWe base load plant would require over 3,400 MWe of widely dispersed wind generation capacity due to its low capacity factor of about 30%.

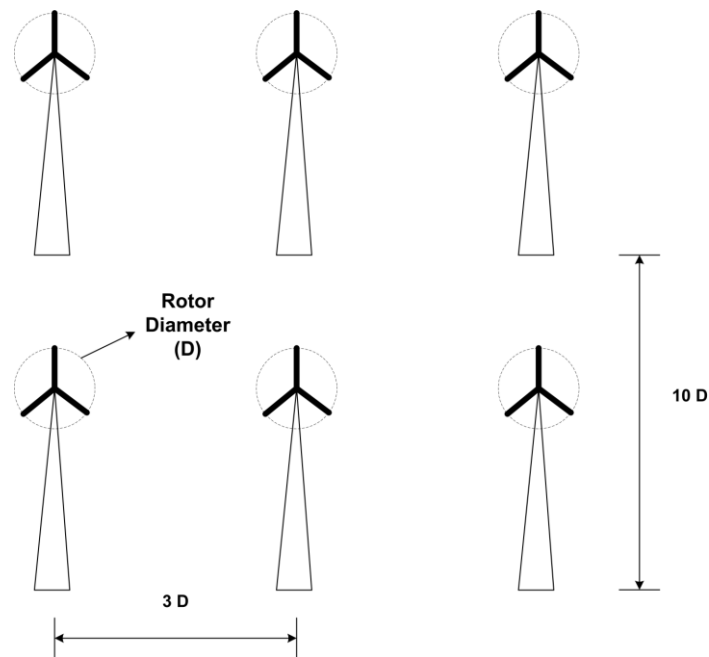


Figure 5: Spacing of wind turbines in a wind farm

Other considerations for optimal planning of wind energy investments are defining the land area that is used and the distance to the main grid connections. The regulation of land area usage for wind energy differs from country to country likewise the costs of specified lands (like forest land, pasture grounds or individual's land) vary with a huge range. The distance of wind farm to the substation also needs an attention during the period of determination average investment costs.

1.2. Objective of the Study

The objective of this work is to formulate a multi-period optimization model to plan the investment decisions on the development of wind farms considering available options and particular characteristics of the region where the wind farm will be installed. Afterwards, illustrations of the features of the proposed model on first a basic hypothetical model and then on Turkey case.

The proposed model in this work considers average regional wind speeds, future wind energy demand, available wind farm area and land area restrictions to be able to determine the optimal type of rotor, area and period of construction of a wind farm and optimal placement of installation new turbines on several areas in 7 regions. This paper presents new multi-period, multi-region and multiple-choice of rotor type optimization model. The model will be formulated as MINLP and be implemented into GAMS solver. BARON solver is used according to the non-convexity of objective and constraints of the model [7].

1.3. Outline

Following the introduction to wind energy technologies and the development of wind energy industry in Chapter 1, Chapter 2 provides a review of the literature for the optimization methods relevant to wind energy investments. Chapter 3 presents problem definition and the mathematical formulation for the deterministic multi-period mixed integer non linear model. Chapter 3 also contains a real case example based on Turkey, a scenario based stochastic programming approach under uncertainty of wind speed and sensitivity analysis study for the different variations of extended Turkey case example.

Chapter 2

Literature Review

2.1. Overview

Wind turbines are the main characters of generating energy from the wind. Several types of wind turbines; with different rotor blade lengths, horizontal or vertical axis type of wind turbines are used to convert the mechanical energy into electrical energy. In the energy converting process, the speed of the wind and the length of the rotor blade play the main roles. Capturing energy from wind highly depends on rotor length of the wind turbine. The amount of energy that wind carries is proportional to the swept area by the rotors. Depending of the magnitude of the wind speed, rotors of the turbine turn to generate the desired energy. In 1920, German physicist Albert Betz showed by his theory that there is no wind turbine that generates more than $16/27$ (59.3%) of the kinetic energy of the wind [8].

$$P = \frac{1}{2} C_p \rho A V^3 \quad (2.1)$$

$$A = \Pi \frac{D^2}{4} \quad (2.2)$$

where P is the wind power output (W), C_p is the power coefficient calculated using appropriate aerodynamic models, ρ is the air density in terms of kg/m^3 , A is the swept area by rotors (m^2), D is the diameter (m) and V is the speed of the wind (m/s).

As can be seen from the formula given above (2.1), wind power increases by a factor of three as the speed of wind increase. According to Betz Theorem, if the length of the rotor blade is doubled, the power obtained from wind energy also increases by factor four.

Another challenge in wind park design is to decide the location and the number of wind turbines.

2.2. Optimization Methods for Wind Energy

Plenty of work has been done that clarifies the potential and opportunities of wind energy in Turkey. Kılıç has presented the potential, current applications and current status of wind energy in Turkey. According to States Electrical Boards report, Turkey's yearly wind potential is estimated around 120 billion kWh which doubles the yearly energy consumption in the country [9].

Kongnam et al. has proposed a mixed integer non-linear mathematical model under the consideration of uncertainty on wind speed. This model determines the optimal number and capacity of wind turbines to be installed at a wind park. Distribution of wind speed data has characterized by Weibull and Rayleigh distribution where Rayleigh distribution was Weibull distribution with shape parameter is taken as 2. Power output was calculated according to wind speed data characterized by these two distributions. Uncertainty of wind speed data made a significant impact on investment decision. Equation 2.3 shows the objective function of the mathematical model where the power output ($P_{i,j}$) was calculated by different approximations on wind speed data [10].

$$\left(\sum_{k=1}^K \frac{r_k}{(1+\delta)^k} \sum_{j=1}^J \sum_{i=1}^I N_i T_j P_{i,j} \right) - \left(\sum_{i=1}^I N_i C_i \right) - \left(\sum_{k=1}^K \frac{(1+\delta)^k - 1}{\delta(1+\delta)^k} \sum_{i=1}^I N_i O_{i,k} \right) \quad (2.3)$$

where k is index of year, i index of turbine, j is index of discretized wind speed level, δ is the discount rate (%/y), r_k is electricity purchasing rate of wind energy (US \$/kWh), N_i is the number of wind turbine i , T_j is the number of hours per year at wind speed level j (h/y), $P_{i,j}$ is the power output (kW), C_i is the capital cost of wind turbine (US \$) and $O_{i,k}$ is operating and the maintenance cost (US \$).

In reference [11] a stochastic programming approach is presented with the goal of maximum profit under the uncertain wind speed data and electricity market prices scenarios. His approach does not meet the global optimum solution to all possible scenarios but provides a robust solution to the individual scenarios. Objective function was constructed as in equation 2.4 where S is the set of scenerios, ρ_s is the probability of occurrence of scenario s , H is the set of hours, λ_{sh} is the forecasted electricity market price, p_{sh} is the power output of the wind farm and $pdev_{sh}$ is the cost of deviation of the wind farm.

$$\sum_{s=1}^S \rho_s \sum_{h=1}^H [\lambda_{sh} p_{sh} - pdev_{sh}] \quad (2.4)$$

Mustarekov presented a combinatorial optimization model for wind turbines type, number choice and design for the wind park. In this work, the model gives the optimal wind turbine numbers within the specified wind park area and specified turbines' spacing recommendations. Turbine spacing is modeled by introducing of variable separation distances coefficients with given lower and upper limits. The approach was tested by numerical experiments by using real data. The rectangular wind park shape is considered as wind park area and wind turbine location is characterized by its two dimensional Cartesian coordinates (x,y) . Total number of wind turbines is calculated as follows:

$$N = N_{row} N_{col} \quad (2.5)$$

$$N_{row} = \frac{L_x}{k_{row} D} + 1 \quad (2.6)$$

$$N_{col} = \frac{L_y}{k_{col} D} + 1 \quad (2.7)$$

where N represents total number of wind turbines, N_{col} and N_{row} are the number of wind turbines in a column and in a row respectively, L_x and L_y are the length of the edges for the

given wind park area and k_{row} and k_{col} are separation distance coefficients for the wind turbine placement in rows and columns respectively [12].

Kusiak has dealt with the optimal placement of wind turbines considering wake loss and developed a multi-objective evolutionary strategy algorithm to optimize the energy output of a wind farm. The optimization model considers wind farm as radius and take into account turbine distance constraints. The model aims to maximize the energy generation by placing wind turbines such a way that the wake loss is minimized [13].

Chapter 3

Problem Description and Modeling

3.1. Introduction

Constructing wind farm requires appropriate decisions of wind turbine types, numbers and area of the farm. Since the wind power equation mentioned in Chapter 2 depends on rotor lengths and wind speed data of the area, these two components plays crucial roles on wind park installations. We propose a multi-period mixed integer non-linear programming (MINLP) model for the wind park investment decisions and implement the model to the Turkey case. The proposed model addresses many realistic features for the investment decisions of a wind park: choosing rotor type to generate desired energy, installation of the turbines considering fitting criterias to the land area, choosing the most suitable location in terms of yearly wind resource records of the at first with non-linear constraints and non-linear objective function.

Using various linearization techniques, some of the constraints and objective function in non-linear forms are got rid of to avoid the computational complexity. By setting optimality criterion to 0, we obtained sensitive results in both linear and non-linear model. The model is implemented into General Algebraic Modeling System (GAMS) using BARON solver for the global optimal solution of mixed integer nonlinear problem. [14]

The organization of the chapter is as follows: Section 3.1.1 explains mathematical formulation and parameter descriptions of the model in detail. Section 3.2 presents computational results. In Section 3.2.1 we performed a computational study for our illustrative example. In Section 3.2.1 we implement our mathematical model in Turkey case.

3.1.1. Mathematical Model Representation

In our multi-period setting, we aim to answer the following questions:

- How much energy to be generated in each period and each wind farm area of the locations?
- Which rotor types should be chosen for the corresponded wind farm areas and periods?
- When, where and how many of wind turbines should be installed?
- For each wind park areas in location, how should be the structure of the turbine installation in terms of number of turbines in x and y axis of the land area?
- How much the optimal net present cost value of our yearly operation and management cost of wind turbines?
- How much the optimal net present investment cost value for the best installment decision of the turbines?

The model is constructed with the following assumptions:

1. All the investment decisions are implemented at the beginning of the time period.
2. All the operation and management cost assumed to be charged at the end of each period.
3. Transmission network cost for the generated energy is not included.
4. Yearly average wind speed data considered for the determination of the wind energy generation.
5. Capacity requirements for the generated wind energy are not considered assuming that there are sufficient battery capacities.
6. Wind park areas within the regions are assumed to be rectangular and of the same size.

7. Yearly average wind speeds of the regions are taken for the calculations so that wind turbine height and wind speeds at different heights are not considered.

Objective Function

The objective function that minimizes the total net present value of the operation and investment cost of overall periods of the wind park requirements is given as following:

$$\text{Min } z = \sum_t C_t + \sum_m \sum_i \sum_k \sum_t Ntotal_{mit} x_{mik} l_{kt} \quad (3.1)$$

Because of the existence of nonlinear term in the objective function, new variable which is Num_{mikt} is introduced to the model instead of the cross product of $Ntotal_{mit}$ and x_{mik} hold the linearity on objective function.

$$Num_{mikt} = Ntotal_{mit} * x_{mik} \quad \forall m, i, k \quad (3.2)$$

To make sure that the putting the new variable will yield the same result as it has in nonlinear form we add some additional constraints for Num_{mikt} that is presented in (3.3), (3.4) and (3.5):

$$Num_{mikt} \leq M * x_{mik} \quad \forall m, i, k, t \quad (3.3)$$

$$Num_{mikt} \geq Ntotal_{mit} - (1 - x_{mik}) * M \quad \forall m, i, k, t \quad (3.4)$$

$$Num_{mikt} \leq Ntotal_{mit} + (1 - x_{mik}) * M \quad \forall m, i, k, t \quad (3.5)$$

Where M is a sufficiently large parameter of big-M constraints within the mixed integer non-linear programming framework.

l_{kt} represents the total net present value for the investment decision of rotor type k in period t . The mathematical formula for l_{kt} presented in equation (3.6)

$$l_{kt} = \frac{l_k}{(1+r)^{t-1}} \quad \forall k, t \quad (3.6)$$

Our yearly operation and management cost represented as C_t is formulated as follows:

$$C_t = \sum_m \sum_i \frac{Power_{mit} Op_{cost}}{(1+r)^t} \quad \forall t \quad (3.7)$$

Since our investment plan based on a yearly time horizon, the cost of wind turbines depends on the added wind turbine values in each year. The calculation of the yearly new wind turbine installations is as follows:

$$Nadded_{mikt} = Num_{mikt} - Num_{mik,t-1} \quad \forall m, i, k, t \quad (3.8)$$

Thus, our final linear objective function can be represented as below:

$$\text{Min } z = \sum_t C_t + \sum_m \sum_i \sum_k \sum_t Nadded_{mikt} l_{kt} \quad (3.9)$$

Where r is discount rate.

Constraints

The constraints that need to be considered are given as the following:

Annual energy demand:

Annual generated energy from wind farms must satisfy the energy demand of location i in period t . Here we must clarify that since once you install a wind farm you will continue to generate electricity from there for the following periods, we must consider cumulative sum of total generated energy until the period t in order to satisfy the total demand of period t .

$$\sum_{m,i} Power_{mit} \geq d_t \quad \forall i, t \quad (3.10)$$

Total generated energy for each period, location and period is calculated as the following equation (3.10):

$$Power_{mit} = \sum_k N_{total_{mit}} x_{mik} windenergy_{ki} \quad \forall m, i, t \quad (3.11)$$

Wind energy capacity:

Based on power output calculations, annual energy from the wind turbine was expressed as in above form [6]:

$$windenergy_{ki} = \frac{1}{2} * Rho * v_i^3 * A_k * C_p * \frac{8760}{1000} \quad \forall k, i \quad (3.12)$$

The values of parameter $windenergy_{ki}$ in equation (3.12) are calculated by means of equation (3.11) for each type of rotor and location. As a result, total generated power of wind farm m in location i , in period t can be calculated as in equation (3.12):

$$Power_{mit} = \sum_k windenergy_{ki} N_{total_{mit}} x_{mik} \quad \forall m, i, t \quad (3.13)$$

Since non-linearity exists also in equation (3.12) because of the cross product of binary and integer variable, we can linearize it by using the same technique likewise in the equations (3.3), (3.4) and (3.5).

We introduce a new variable that is;

$$Num_{mikt} = N_{total_{mit}} x_{mik} \quad \forall m, i, k, t \quad (3.14)$$

$$Num_{mikt} \leq M * x_{mik} \quad \forall m, i, k, t \quad (3.15)$$

$$Num_{mikt} \geq N_{total_{mit}} - (1 - x_{mik}) * M \quad \forall m, i, k, t \quad (3.16)$$

$$Num_{mikt} \leq N_{total_{mit}} + (1 - x_{mik}) * M \quad \forall m, i, k, t \quad (3.17)$$

Now, our equation (3.12) turns into the linear form presented below:

$$Power_{mit} = \sum_k windenergy_{ki} * Num_{mikt} \quad \forall m, i, t \quad (3.18)$$

Choosing rotor type of a wind farm:

We can choose one type of diameter for the rotor of wind turbine in a specific wind farm area. Equation (3.18) stands for this constraint:

$$\sum_k x_{mik} = 1 \quad \forall m, i \quad (3.19)$$

We use the equation (3.20) to determine the rotor diameter:

$$Dia_{mit} = \sum_k x_{mik} * r_k * y_{mit} \quad \forall m, i, t \quad (3.20)$$

Since equation (3.20) not linear because of the cross product of binary variables x_{mik} and y_{mit} , here we introduce a new binary variable, which is z_{mikt} :

$$z_{mikt} = x_{mik} * y_{mit} \quad \forall m, i, k, t \quad (3.21)$$

To obtain the same results as in nonlinear form some additional constraints need to be added (3.22), (3.23), (3.24):

$$z_{mikt} - x_{mik} \leq 0 \quad \forall m, i, k, t \quad (3.22)$$

$$z_{mikt} - y_{mit} \leq 0 \quad \forall m, i, k, t \quad (3.23)$$

$$x_{mik} + y_{mit} - z_{mikt} \leq 1 \quad \forall m, i, k, t \quad (3.24)$$

Finally, we can write the linear form of the constraint (3.20) above;

$$Dia_{mit} = \sum_k z_{mikt} * r_k \quad \forall m, i, t \quad (3.25)$$

On the other hand, to ensure that we cannot assign a positive number to diameter choice variable Dia_{mit} if the decision of not installing a new wind farm is given, we need to add one more constraint (3.26) on this variable:

$$Dia_{mit} \leq y_{mit} * M \quad \forall m, i, t \quad (3.26)$$

Number of wind turbines:

To determine the number of wind turbines that will be installed in a row we need to divide our available area by wind turbine interval length which can be realized according

to the rotor type. Due to multi-period decision character of our model, we need to subtract the previous installations of wind turbine in that specified area and time:

$$Nrow_{miT} \leq \frac{Lx_{mi} - \sum_t k_{row} Dia_{mit} Nrow_{mit}}{k_{row} Dia_{mit}} + 1 \quad \forall m, i, t < T \quad (3.27)$$

Similarly, number of wind turbines through the column of a wind farm area can be determined as:

$$Ncol_{miT} \leq \frac{Ly_{mi} - \sum_t k_{col} Dia_{mit} Ncol_{mit}}{k_{col} Dia_{mit}} + 1 \quad \forall m, i, t < T \quad (3.28)$$

Some of the values of the variable Dia_{mit} can be assigned to zero that cause division by zero in evaluating the derivative of the functions during optimization. In order to address this numerical evaluation error, we can express our number of wind turbine and diameter constraints by the following manner to avoid calculation errors.

$$(1 - y_{miT})(Lx_{mi} - \sum_{t=0}^{t=T-1} k_{row} Dia_{mit} Nrow_{mit}) + k_{row} Dia_{miT}(Nrow_{miT} - 1) = Lx_{mi} - \sum_{t=0}^{t=T-1} k_{row} Dia_{mit} Nrow_{mit} \quad (3.29)$$

$$(1 - y_{miT})(Ly_{mi} - \sum_{t=0}^{t=T-1} k_{col} Dia_{mit} Ncol_{mit}) + k_{col} Dia_{miT}(Ncol_{miT} - 1) = Ly_{mi} - \sum_{t=0}^{t=T-1} k_{col} Dia_{mit} Ncol_{mit} \quad (3.30)$$

Total number of wind turbines in a farm:

Since the geometry of the wind farm area was considered as rectangle or square, we can calculate the total number of wind turbines in a specific wind farm area as following:

$$N_{mi} = \sum_t Ncol_{mit} * \sum_t Nrow_{mit} \quad \forall m, i \quad (3.31)$$

Due to the installation decision of a wind farm area, number of wind turbines in a row or column of the wind farm area must be restricted as:

$$Ncol_{mit} \leq y_{mit} * M \quad \forall m, i, t \quad (3.32)$$

$$Nrow_{mit} \leq y_{mit} * M \quad \forall m, i, t \quad (3.33)$$

Therefore, the number of wind turbines in a column or row must be calculated by themselves. If we do not install any wind turbine on a column of a specific area in any location and in any period then the corresponding number of wind turbines on the row must be zero.

$$Nrow_{mit} \leq Ncol_{mit} * M \quad \forall m, i, t \quad (3.34)$$

$$Ncol_{mit} \leq Nrow_{mit} * M \quad \forall m, i, t \quad (3.35)$$

Total number of wind turbines in a farm until the end of the period t :

$$N_total_{miT} = \sum_t^T Ncol_{mit} \sum_t^T Nrow_{mit} \quad (3.36)$$

Land area restriction:

We are restricted by the land area that is available for the wind farm in each location so equation 25 needs to be added in order to hold the feasibility in the wind farm area:

$$wf_{mi} \geq Ly_{mi} * Lx_{mi} \quad \forall m, i \quad (3.37)$$

Non-Negativity and integrality constraints

Decision variables for determining rotor types, investment periods and locations for the installation are assigned as binary variables. Decision variables for the number of wind turbines and the length of rotor types are set as integer variables. Other decision variables are set as non-negative variables.

$$P, Power_{mit}, Ly_{mi}, Lx_{mi} \geq 0 \quad (3.38)$$

$$N_{mit}, Ncol_{mit}, Nrow_{mit}, Num_{mikt}, Dia_{mit} \text{ is integer} \quad (3.39)$$

$$x_{mik}, y_{mit}, z_{mikt} \in \{0,1\} \quad (3.40)$$

3.1.2. Model Formulation Using Stochastic Scenario-Based Approach

We extend our mathematical model formulation in a way that considers uncertainty of wind speed. For this purpose we applied two-stage stochastic programming approach to our deterministic model [7]. Two-stage stochastic programming approach is frequently used when any one of the model parameters is uncertain. In this approach, the set of the constraints and decision variables is divided into two groups (two stages). First-stage variables the ones that need to be determined when environmental information is barely available. Given the first-stage decisions, the second subset of decisions can be determined based on the realization of a number of random events which are called second-stage decisions. The objective is to minimize the sum of first stage costs that consist of deterministic and the expectation of second stage costs.

The classical two-stage stochastic program with fixed recourse is as follows [8]

$$\begin{aligned} & \text{Min}_{x \in \mathbb{R}^n} c^T x + E[Q(x, \varepsilon)] \\ & \text{s. t. } Ax = b, x \geq 0 \end{aligned} \quad (3.41)$$

where x is the first-stage decision variable with an associated cost vector c . $Q(x, \varepsilon)$ is the optimal value of second stage problem and $\varepsilon = (q, h, T, W)$ are the parameters of second stage problem (3.42)

$$\begin{aligned} & Q(x, \varepsilon) = \min_{y \in \mathbb{R}^m} q^T y \\ & \text{s. t. } Tx + Wy = h, y \geq 0 \end{aligned} \quad (3.42)$$

In our approach, we will consider the discrete distributions only, which means that vector ε has a finite number of realizations (scenarios) with respect to probabilities p_k where $k=1,2,\dots,K$. Thus, the expectation of the optimal value of second stage problem can be written as below:

$$E[Q(x, \varepsilon)] = \sum_{k=1}^K p_k Q(x, \varepsilon) \quad (3.43)$$

Hence, for a taken first-stage decision x , the linear programming for the expectation $E[Q(x, \varepsilon)]$ given in equation (3.44)

$$\begin{aligned} & \text{Min}_{y_1, y_2, \dots, y_k} \sum_{k=1}^K p_k q_k^T y_k \\ & \text{s. t. } T_k x + W_k y = h_k \\ & \quad y_k \geq 0, \forall k \end{aligned} \tag{3.44}$$

Using the equations (3.43) and (3.44) we can formulate a linear program that forms the deterministic equivalent of the stochastic problem:

$$\begin{aligned} & \text{Min}_{x, y_1, \dots, y_k} c^T x + \sum_{k=1}^K p_k q_k^T y_k \\ & \text{s. t. } T_k x + W_k y = h_k \quad \forall k \\ & \quad Ax = b \\ & \quad x \geq 0, y_k \geq 0, \forall k \end{aligned} \tag{3.45}$$

For the proposed deterministic multi-period mixed integer non-linear mathematical model, we classified the decision variables as first and second-stage decision variables in order to fit into two-stage stochastic programming approach. The number of wind turbine decisions is turned to be the first-stage decision variables and the power generation decision variables are categorized as second-stage decision variables. The uncertainty in our model is the average wind speed of the location that effects on the power generation in these locations. We considered 3 discrete scenarios one of which is the base case (Scenario 1). In Scenario 2, we suppose that the average wind speed is 10% less comparing with base case scenario in each location based on the observation on meteorological results. Scenario 2 can also be considered as “worst case scenario” because of the lowest wind speed datas. In Scenario 3 or in best case scenario, we considered average wind speed is 10% high,

In our stochastic approach, we add one more set, set of scenarios to our deterministic model. This new index is also added to other parameters and variables presented below.

The new index:

$s = \{1, 2, \dots, S\}$ scenarios

Parameters:

We add one more parameter to our model that indicates the probability of scenarios.

P_s probability of scenarios

The new “s” indice is added to the parameters $windenergy_{ki}$ and d_t .

$windenergy_{kis}$ Yearly energy output of rotor type $k \in K$ in location $i \in I$, for scenario $s \in S$,

d_{ts} Demand projection of location $i \in I$ in period $t \in T$, for scenario $s \in S$

Variables:

The new dimension of scenarios is added to second-stage variables.

$Power_{mits}$ Total generated energy of wind farm $m \in M$ in location $i \in I$ in period $t \in T$ and for scenario $s \in S$

C_{ts} NPV for yearly operational and management cost of wind turbines in period $t \in T$

Objective Function

$$\text{Min } z = \sum_t P_s C_{ts} + \sum_m \sum_i \sum_k \sum_t N_{total_{mit}} x_{mik} l_{kt} \quad (3.46)$$

$$C_{ts} = \sum_m \sum_i \frac{Power_{mits} Op_{cost}}{(1+r)^t} \quad \forall t, s \quad (3.47)$$

Constraints

Annual Energy Demand

$$\sum_{m,i} \text{Power}_{mits} \geq d_{ts} \quad \forall i, t, s \quad (3.48)$$

$$\text{Power}_{mits} = \sum_k N_{total_{mit}} x_{mik} \text{windenergy}_{kis} \quad \forall m, i, t, s \quad (3.49)$$

Wind Energy Capacity

$$\text{windenergy}_{kis} = \frac{1}{2} * \text{Rho} * v_{is}^3 * A_k * C_p * \frac{8760}{1000} \quad \forall k, i, s \quad (3.50)$$

$$\text{Power}_{mits} = \sum_k \text{windenergy}_{kis} \text{Num}_{mikt} \quad \forall m, i, t, s \quad (3.51)$$

3.2. Computational Results

In this section, we provide results for a small scale problem for illustration purposes and application of our both deterministic and stochastic approaches to a real case study in Turkey. In the examples presented in next sections, periods are considered as years and interest rate for the net present value calculations is taken as 10%.

3.2.1. Illustrative Example

The proposed model first is applied to a basic model where hypothetical parameters are used. 2 regions are considered as A and B with the following yearly wind speed data:

Table 1: Yearly wind speed of illustrative regions

Region	Wind Speed
A	2
B	2,5

For the wind turbine rotor type, small-scale wind turbines are used with 10, 15 and 20 m rotor length and with 30,000 €, 95,000 € and 150,000 € of investment costs respectively. Yearly wind energy outputs of three wind turbines are calculated according to the equation (3.11). This hypothetic wind farm that will be installed in 2 different regions is planned to satisfy the energy demand of whole regions for the next 3 periods. Total energy demand of regions for the next 3 periods is given as 500 MW, 600 MW and 700 MW. Land areas of wind farms are restricted by 3000 square meter for the region A and 2000 square meter for the region B.

General Algebraic Modeling System (GAMS) [14] software is used for the compilation of model with the BARON [7] solver to the 0 optimality tolerance criteria.

Table 2: Optimal Energy Generation Plan for Periods (MW)

$(m,i) \setminus t$	Period 1	Period 2	Period 3
(1,1)			3,6
(1,2)	462	462	462
(2,2)	234.5	234.5	234.5

Model decides to establish two wind farms in the first period and a small one for the last period to fulfill the energy demand over periods. First and second region is chosen for the wind farm installation due to their more advantageous wind availability.

Table 3: Installed wind turbine numbers through periods

$(m,i) \setminus t$	Period 1	Period 2	Period 3	Total
(1,1)			2	2
(1,2)	33	33	33	33
(2,2)	67	67	67	67

Total of 102 wind turbines will be installed into 3 different wind farm areas. For the first period, a farm of a number of 33 wind turbines with 20 m rotor length and a farm of number of 67 wind turbines with 10 m rotor length will be placed in region B on different wind farm areas. For the last period, additionally one small farm will be added to region A of 2 wind turbines with 10 m rotor length. (Table 3)

Proposed model also tends to decide the lengths of edges for the wind farm. Table 4 shows the length decisions for the rows and columns of the wind farm for the optimum placement.

Table 4: Wind Farm column and row lengths for the locations

(m,i)	L_x (m)	L_y (m)
(1,1)	100	-
(1,2)	-	1920
(2,2)	-	1980

Table 5: Total number of turbines of wind park areas(rows)

$(m,i) \setminus t$	Period 1	Period 2	Period 3
(1,1)			2
(1,2)	1		
(2,2)	1		

Table 6: Total number of turbines of wind park areas(columns)

$(m,i) \setminus t$	Period 1	Period 2	Period 3
(1,1)			1
(1,2)	33		
(2,2)	67		

Model results (see Table 7) show us the optimal results of illustrative example application of our model.

Table 7: Model Results for the 3 period model

Execution time(sec)		554.08
Total Generated Power (MW)		2093.1
Total Cost (€)		7,131,022.577
Rotor Type Choice (k)		1&3
# of Constraints		449
# of Variables	# of Binary Variables	60
	# of Continuous Variables	33
	# of Integer Variables	100
	Total # of Variables	193
# of Iterations with Baron		22464

3.2.2. Application to Turkey Case

Turkey's energy need over the years has been steadily increasing as a result of being one of the fastest developing countries in terms of installed industrial factories, total population etc. Energy Market Regulatory Authority of Turkey (EPDK) has reported that total amount of energy consumption by the year 2011 was 228 TWh while installed capacity of energy production was calculated as only 55.4 GW which means that energy market of Turkey depends on foreign sources with the rate of 77%. At this point, when we question the total potential of Turkey's renewable energy sources, theoretically we end up really modest results in terms of hidden potentials of wind, solar, hydro, and geothermal and bio energy. Yearly average solar radiation in Turkey is 1311 kWh/m² per year and 3.6 kWh/m² per day. Total yearly insolation period is approximately 2,460 hours per year [16]. The overall geothermal heat generation potential of Turkey corresponds to 31,500 MW of

energy which puts it among the first 10 countries in the world in terms of the usage of geothermal energy [17]. Among all renewable energy sources, hydropower dominates the others in terms of energy generation. Total gross hydropower potential of Turkey is 433 GWh/year but only 125 GW/h of it can be used economically [18]. Turkey's total biomass capacity is expected to be 32 Mtoe and 17 Mtoe of this amount can be presented as usable biomass potential [19]. Turkey's energy mix share among all the resources is given in Figure 6.

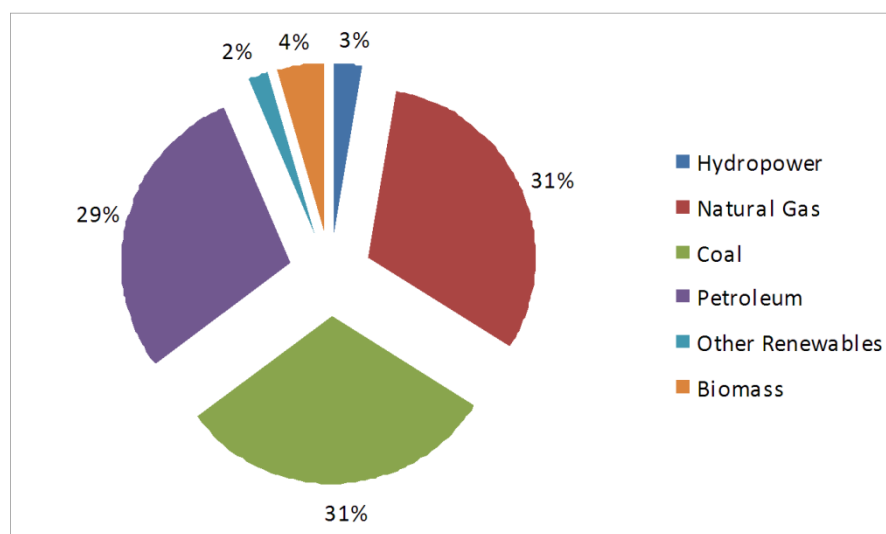


Figure 6: Energy resources of Turkey by the year of 2009 (Ministry of Environment and Urbanism of Turkey)

Due to being situated at the meeting point of three continents (Asia, Europe and Africa), Turkey has the great potential of usage of wind power as an energy resource. Table 7 mentions total potential of different wind power classes at 50m height. Turkey entered to wind power energy market in 1998 with 8.7 MW capacity. After 10 years, investments on wind power energy increased noticeably and so did the number of wind power plants and total installed capacity of wind power. For the past 4 years, growth rate of wind power capacity of Turkey reached its highest value and by the end of year 2011 total capacity was reported as 1805.85 MW (see Figure 7).

Table 8: Total Wind Potential of Turkey (Ministry of Energy and Natural Resources of Turkey)

Total Wind Potential (50 m height)			
Wind Power Class	Wind Power Density (W/m ²)	Wind Speed (m/s)	Total Potential (MW)
3	300-400	6.5-7	83906.96
4	400-500	7.0-7.5	29259.36
5	500-600	7.5-8	12994.320
6	600-800	8.0-9.0	5399.92
7	>800	>9.0	195.84
Total			131756.4

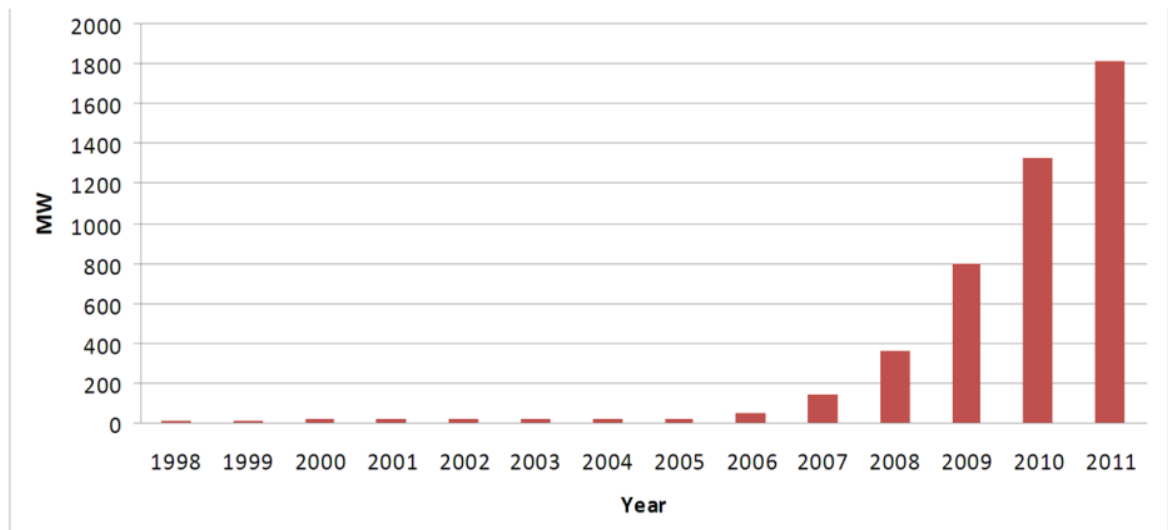


Figure 7: Cumulative Distribution of Installed Capacity for Wind Power Plants in Turkey (Turkish Wind Energy Association)

Currently, Turkey is using only 1.37% of its wind power potential with 49 wind parks in operation and with 13 new wind park projects which corresponds to a total of 517.55 MW of power [20]. However there is still a great wind power potential that can be used to the fullest, in order to decrease the energy dependence on the outside sources for Turkey and to increase the usage of environmentally friendly sources like carbon free energy sources, a future work can be done to satisfy some part of Turkey's energy need with wind energy.

Turkey is divided in 7 different geographical regions as Marmara, Aegean, Mediterranean, Central Anatolia, Black Sea, South Eastern Anatolia and Eastern Anatolia. Each region has specific characteristics in terms of yearly wind speed, geological structure of land area etc. Marmara and Aegean regions have better wind energy potential compared to the other regions as shown by the yearly average wind speeds in Table 2.

Table 9: Yearly Average Wind Speeds of 7 Region of Turkey [20]

Region	Average Wind Speed (m/s)
Marmara	3.29
Aegean	2.65
Mediterranean	2.45
Central Anatolia	2.46
Black Sea	2.38
South Eastern Anatolia	2.69
Eastern Anatolia	2.12

In Figure 8, wind speed scattering data in 30m altitude is demonstrated. As can be seen from this figure, western parts of Turkey, especially the Aegean and Mediterranean coasts

and northwestern parts are promising areas in terms of wind energy potential. Taking into account the industrial development and intense human population of Marmara and Aegean regions, meeting some portion of the electricity demand with wind energy can be advantageous.

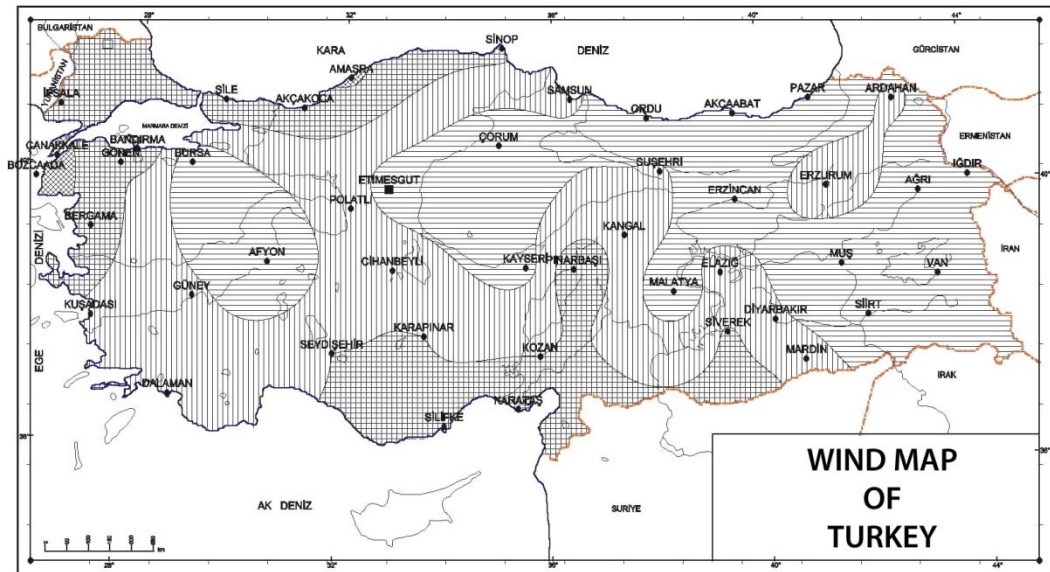


Figure 8: Wind Map of Turkey at 30m altitude

Table 10: Demand Projections (kwh) (EIE- Electrical Power Resources Survey And Development Administration)

Periods (years)	Total wind energy demand projection (kWh)
1	6636000
2	6985000
3	7334000
4	7684000
5	8033000
6	8382000

The proposed mathematical model for optimal number, installation and choice of wind turbines type are entered into GAMS as a multi period mixed-integer nonlinear problem using BARON solver. Different types of wind turbine models that are our wind turbine set are given in Table 11. For the annual wind power calculations, equation (3.11) is used where ρ corresponds to the air density (1.23 kg/m^3) and c_p corresponds to efficiency factor of a wind turbine (0.4).

Table 11: Wind Turbine Model Parameters

No (k)	Wind Turbine Model	Rated Power (kW)	Rotor Diameter
1	Hypothetic Model	230	28
2	ENERCON'S E33	330	33
3	ENERCON'S E48	800	48

Seven different regions of Turkey were listed but since the most promising areas are located in Marmara and Aegean these regions are considered as potential wind park areas throughout the model. Yearly wind power output is calculated according to the average wind speed information of the corresponded region. The data for investment and operation costs of the wind turbine are taken from Electrical Power Resources Survey and Development Administration (EIE) of Turkey [17]

The aim of the model is to cover the energy need of the regions by minimizing the total cost at "0" level of optimality criteria using wind energy demand forecasts for the next 3 periods. In Turkey case, periods are considered to be years. Total energy demand that is going to be generated for regions between years 2015-2020 is listed in Table 12.

Table 12: Demand Projections of Regions (kwh)[22]

Periods \ Regions	Marmara	Aegean	Mediterranean	South Eastern Anatolia
2015	2586946	1053116	899529.6	427336.2
2016	2722999	1108502	946837.6	449810.6
2017	2859051	1163887	994145.6	472285
2018	2722999	1108502	946837.6	449810.6
2019	3131546	1274817	1088897	517298.3
2020	3267598	1330202	1136205	539772.7

According to the optimal results of the GAMS compilation, new wind farms are installed in the first period in available areas of Marmara Region. Both wind farm areas are used in Marmara Region due to its high wind capacity. 79 and 38 wind turbines of type 1 (28 m rotor blade) are decided to installed on the first and second wind farm areas of the location, respectively. Energy generation amounts of the installed wind turbines during the periods are presented in Table 13.

Table 13: Optimal Energy Generation Plan for Periods (MW)

$(m,i) \setminus t$	Period 1	Period 2	Period 3
(1,1)	4964.913	4964.913	4964.913
(2,1)	2388.186	2388.186	2388.186

Total number of wind turbines that are planned to be installed for the energy generation are presented in Table 14. Depending on the wind power potential of the regions and energy need for the next 3 periods, 2 different wind farms are installed. Due to the prosperous wind resource in Marmara Region, all of the turbines are located within the region.

Table 14: Installed wind turbine numbers through periods

$(m,i) \setminus t$	Period 1	Period 2	Period 3	Total
(1,1)	95	95	95	95
(2,1)	22	22	22	22

Because of high investment cost requirements for the production of wind turbines with large blades our model tends to choose relatively smaller wind turbine that corresponds to 28m rotor blade that was mentioned in Table 4.

We assume the wind farm area to be a rectangular surface for the optimum wind turbine placement. Model is aimed to count the number of columns and rows of the wind farm area to clarify the placement of wind turbines and their total number in each column and row. Table 15 indicates the optimum column and row length for the wind farm areas and Table 16 and Table 17 present number of rows and columns of corresponded wind farm areas.

Table 15: Wind Farm column and row lengths for the locations

(m,i)	L_x (m)	L_y (m)
(1,1)	26320	
(2,1)		5880

Table 16: Total number of turbines of wind park areas(rows)

$(m,i) \setminus t$	Period 1	Period 2	Period 3
(1,1)	95	95	95
(2,1)	1	1	1

Table 17: Total number of turbines of wind park areas(columns)

$(m,i) \setminus t$	Period 1	Period 2	Period 3
(1,1)	1	1	1
(2,1)	22	22	22

The results obtained shows that by using the wind energy in potential areas, 22.05 GW of energy can be generated by the end of the third period from a carbon free resource. Net present value of the installation and operation cost of several wind parks on different sides of Turkey is calculated approximately 29.5 Million Euros. (Total cost excludes logistic costs of wind turbines, grid connection costs and land area specific costs driven by the governmental regulations.)

Table 18: Model Results for the Turkey Case

Execution time (sec)		477.960
Total Generated Power (MW)		22,059.297
Total Cost (€)		29,535,520
Rotor Type Choice		1
# of Constraints		449
# of Variables	# of Binary Variables	90
	# of Continuous Variables	49
	# of Integer Variables	150
	Total # of Variables	289
# of Iterations with Baron		16867

3.2.3. Two-Stage Stochastic Programming Approach to Turkey Case

In two-stage stochastic approach we considered the wind speed of each location is uncertain. We constructed 3 scenarios (worst, base and best) which allow us to investigate the results with different wind availabilities. We assume that Turkey case model presented in 3.2.1 is our base model and occurs with 50% probability and other scenarios occur with 25% probabilities.

Table 19 shows the total number of turbines for corresponded locations. Compared to Turkey case results, model tends to invest more wind turbines and as it seen from Table 19, optimal placement of wind turbines change due to the uncertainty of wind speed. On the other hand, total cost (36,418,950 €) also increases because of the higher number of wind turbine investments.

Table 20 states the wind farm area's optimal dimensions where to put the wind turbines mentioned in Table 21 and Table 22.

Table 23 gives the detail information about the GAMS compilation results. Adding one more index to the model increased total number of constraints and number of variables likewise the total number of Baron Iterations.

Table 19: Installed wind turbine numbers through periods

$(m,i) \setminus t$	Period 1	Period 2	Period 3	Total
(1,1)	86	86	86	86
(1,2)	-	-	2	2
(2,1)	57	57	57	57

Table 20: Wind Farm column and row lengths for the locations

(m,i)	L_x (m)	L_y (m)
(1,1)	23800	
(1,2)	280	
(2,1)		4704

Table 21: Total number of turbines of wind park areas(rows)

$(m,i) \setminus t$	Period 1	Period 2	Period 3
(1,1)	86		
(1,2)			2
(2,1)	1		

Table 22: Total number of turbines of wind park areas(columns)

$(m,i) \setminus t$	Period 1	Period 2	Period 3
(1,1)	1		
(1,2)			1
(2,1)	57		

Table 23: Model Results for the Two-Stage Stochastic Approach to Turkey Case

Execution time (sec)	500.220	
Total Generated Power (MW)	82,704.977	
Total Cost (€)	36,418,950 €	
Rotor Type Choice	1	
# of Constraints	485	
# of Variables	# of Binary Variables	90
	# of Continuous Variables	79
	# of Integer Variables	150
	Total # of Variables	319
# of Iterations with Baron	21573	

In order to compare the results and see the effects of the change in the wind speed availability, we computed scenarios individually. In each scenario, optimal placement and number of wind turbines changed but the chosen diameter remained the same because of the high investment costs of other type of wind turbines. In scenario 2, as a result of decrease in the average wind speed of the locations, annual operational cost is decreased and so the objective value is. In scenario 3, which is the best case in terms of wind availability, we obtained the minimum cost compared to other scenarios. Since we have the highest wind speed values in Scenario 3, total number of wind turbine that needs to be installed is reduced and optimal placement of these wind turbines are also changed. Other important observation about the computational results of Scenario 3 is that the execution time of the model decreased almost 6 times compared to our base case Turkey example or two-stage stochastic approach model.

Two-stage stochastic approach gives us the optimal results by taking into account the different wind speed situations that results with more investment and operational cost compared to our base case scenario (Turkey case example presented in Chapter 3).

Table 24 shows the consolidated results of all scenarios including the basic Turkey case model. In all scenarios, chosen rotor type does not change from 28m rotor blade because of the lower investment cost. Moreover, in each scenario Marmara Region is considered for potential wind farm areas. In every case, most of the wind turbines are installed in this region.

Uncertainty in the average wind speed (Two-Stage Stochastic Approach) or lower wind speed availability (Scenario 2) result with more wind turbine investment in order to satisfy the energy demand in yearly basis.

Table 24: Consolidated Results of All Scenarios

	Scenario 1			Scenario 2			Scenario 3			Two-Stage Stochastic Model		
	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3
$(m,i) \setminus t$	95	95	95	100	100	100	88	88	88	86	86	86
N_total	22	22	22	43	43	43	8	8	8	57	57	57
N_row	95	95	95	1	1	1	88	88	88	86	86	86
N_col	22	22	22	43	43	43	1	1	1	1	1	1
Execution Time (sec)	477,960			235,060			81,480			500,220		
Total Cost (€)	29,535,520			36,076,810			24,565,590			36,418,950		
Rotor Type Choice	1			1			1			1		
# of Iterations with Baron	16867			12009			3077			21573		

The Value of Perfect Information (EVPI)

We computed all scenarios individually in the previous section and we acquired different optimum values for each scenario. Multiplying with these values by their corresponded occurring probabilities we can find the expected value of all possible scenarios. The expected value of this solution is called as the ‘wait-and-see’ solution (WS) in literature [15].

Let OV_s be the optimum value of the scenario s , WS can be computed as follows:

$$WS = \sum_s P_s OV_s \quad (3.52)$$

The optimum value of the deterministic equivalent (DE) problem (two-stage stochastic approach) represents the expected value without perfect information. Since the expected value of perfect information (EVPI) is the difference between expected outcome with the perfect information and the expected outcome without perfect information, the value of information can be calculated by the following equation:

$$EVPI = DE - WS \quad (3.53)$$

Table 25: Optimal Objective Function Values

	Probability	Optimal Value (€)
Scenario 1	0.5	29,535,520
Scenario 2	0.25	36,076,810
Scenario 3	0.25	24,565,590
DE of Two-Stage Stochastic Program		36,418,950

$$EVPI = 36,418,950 - [(0.5 \times 29,535,520) + (0.25 \times 36,076,810) + (0.25 \times 24,565,590)]$$

$$EVPI = 6,490,590 \text{ €}$$

This EVPI value represents the value of perfect information of the wind condition for the locations.

3.2.4. Real Case Wind Farm Application

We applied our mathematical modeling approach to an existing wind farm in Çanakkale province, in Marmara Region, in Turkey. Actually, there are 15 wind turbines with 71 meter rotor length within this farm. Table 26 represents each wind turbines location in terms of Universal Transverse Mercator (UTM) coordinates.

By using UTM coordinate data, one can easily calculate the lengths of the wind farm area. The difference between maximum and minimum value of the “East” values presented in Table 26, gives the x length (L_x) of the area and the difference between maximum and minimum value of the “North” values, gives the y length (L_y) of the area in meter. Thus, the wind farm’s total land area is calculated as shown in Figure 9.

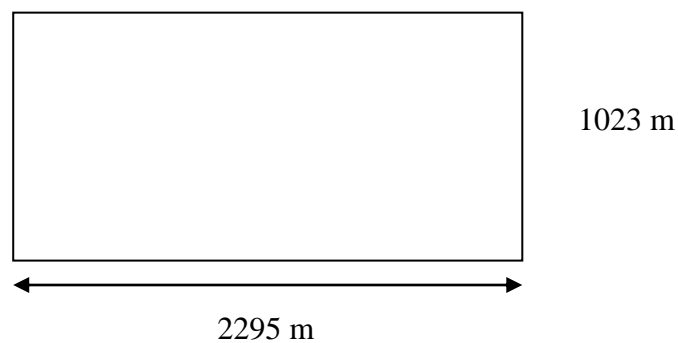


Figure 9: Çanakkale RES Land Area Lengths

Three different types of wind turbines are considered in our mathematical model. One of them is the wind turbine with 71 m rotor length that is already used in the wind farm. The

other wind turbines are chosen according to the usage rate in wind industry with 48 and 52 m rotor length. Since Çanakkale province is located in Marmara Region, this region's annual average wind speed data is entered to the model for the yearly generated energy calculations of the corresponded wind turbines.

In this example, we deal with one wind farm area with known dimensions. After this point, upper bounds for L_x and L_y variables are defined into the model that mentioned in the following equations:

$$L_x \leq 2295 \quad (3.54)$$

$$L_y \leq 1023 \quad (3.55)$$

Table 26: Çanakkale RES Wind Farm–(Ministry of Energy and Natural Resources) [23]

Turbine Number	UTM Coordinate (6 Degree)		Turbine Power (MW)	Rotor Length (m)
	East	North		
T1	433232	4413900	2	71
T2	434542	4413240	2	71
T3	433772	4413180	2	71
T4	434292	4413200	2	71
T5	432582	4414110	2	71
T6	432902	4414070	2	71
T7	434142	4413200	2	71
T8	433402	4413900	2	71
T9	432422	4414120	2	71
T10	433952	4413190	2	71
T11	432752	4414100	2	71
T12	433564	4414151	2	71
T13	434717	4413314	2	71
T14	433610	4413128	2	71
T15	433042	4414040	2	71

After the compilation our new example in GAMS, we obtained the solutions presented in the following Tables:

Table 27: Total number of turbines of wind park areas(rows)

$(m,i) \setminus t$	Period 1	Period 2	Period 3
(1,1)	5		

Table 28: Total number of turbines of wind park areas(columns)

$(m,i) \setminus t$	Period 1	Period 2	Period 3
(1,1)	7		

Table 29: Wind Farm column and row lengths for the locations

(m,i)	L_x (m)	L_y (m)
(1,1)	2080	936

According to the optimal results, within the available land area, we can install 35 wind turbines with 52 m rotor length (Figure 10). Due to the regulatory constraints, existing wind farm's turbine placement differs from our approach. Our approach tends to show the wind investment decisions that provide maximum efficiency in terms of energy generation by ignoring regulatory affairs.

Real case wind farm application shows that our mathematical approach is also applicable to single farm optimal investment and placement decisions. Among given rotor types choices and land area dimensions, our model is able to give the best wind turbine settlement within the area, optimal number of wind turbines and the chosen rotor type.

Table 30 gives the comparative results between actual Çanakkale RES wind farm and optimal results of our approach applied the same farm. In terms of both investment cost and generated energy by the turbines, our approach gives the better results.

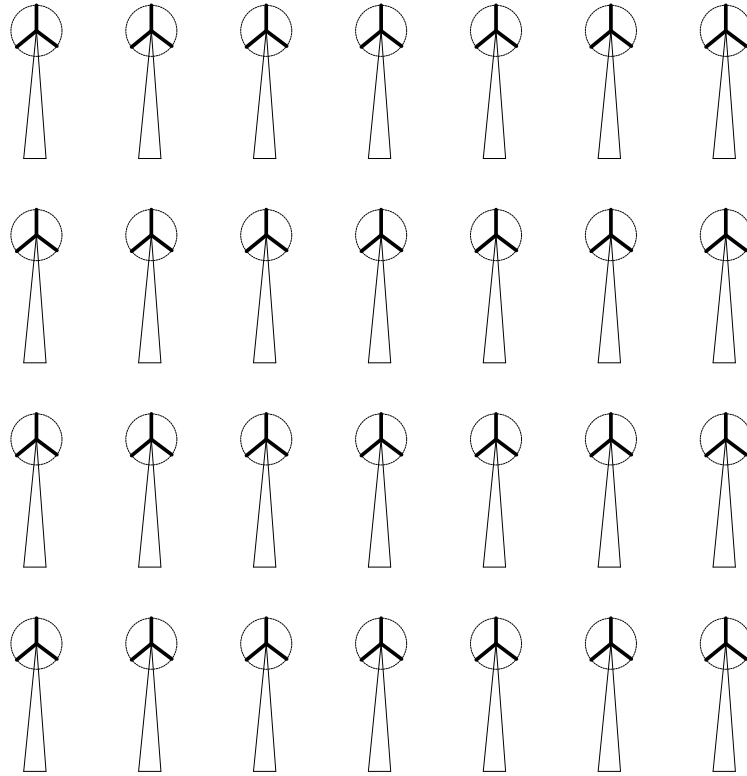


Figure 10: Optimal Placement of Çanakkale RES

Table 30: Comparison Between Actual Çanakkale RES and Optimal Results of Model

Investment Cost for the Actual Wind Turbines	36,225,000 €
Investment Cost for Optimal Result	31,237,500 €
Annual Generated Energy of Actual Wind Turbines	6,061.47 MW
Annual Generated Energy of Optimal Result	7,586.55 MW

3.3. Sensitivity Analysis

In order to determine the limitations and impacts of the parameters of the model, we performed a sensitivity analysis. Tests are conducted by enlarging the three sets of the model; location, time period and wind farm areas in locations.

In this chapter basic model is considered as the numerical example we conduct in Section 3.2.2. We compare different variations of periods, locations and wind farm areas. Finally we enlarge our basic model to see the applicability of the model to the large real case problems.

Variation 1: Increasing Periods

In our basic model adapted to Turkey case in Section 3.2.2, we have 3 periods of energy demand to satisfy. In order to see the changes in model solve statements we added one new period and performed the compilation.

Table 31 shows that execution time and number of iterations are proportional with the number of periods. Moreover with increasing periods total power and cost are also increasing due to higher energy demand. In addition to given results, the table below indicates that the model is also capable of solving real case scenarios with even higher number of periods.

Table 31: Compilation results comparison of different periods

	Basic Model	4periods
Execution Time (sec)	293.3	2510.230
Baron Iterations	15369	73133
Total Power (MW)	22,059.297	30,702.178
Total Cost (€)	29,535,520	31,347,320

Variation 2: Adding New Locations

Since yearly average wind speed in Marmara and Aegean Regions is higher, we chose these regions for our basic model calculations. In order to see the affect of adding more locations (regions) to the compilation results, we add Mediterranean and South Eastern Anatolia Regions to the numerical example respectively. Table 31 represents the results with new locations.

Total power and total cost values of the Table 32 are the same since newly added locations wind availability is lower than the locations used in basic model. This shows that the model does not tend to choose the regions with poor wind resource.

Table 32: Effects of adding new locations

	Basic Model	3 Locations	4 Locations
Execution Time (sec)	293.3	918.070	677.930
Baron Iterations	15369	30678	16079
Total Power (MW)	22,059.297	22,059.297	22,059.297
Total Cost (€)	29,535,520	29,535,520	29,535,520

Variation 3: Adding New Wind Farm Areas

In our basic model, we present 2 wind farm areas for each location assuming that the wind farm areas within the same region have the same shape and size. In order to see the affect of increasing available wind park areas, we add one more wind farm areas in each location.

Increasing wind farm areas enables to install relatively smaller and cheaper wind turbines rather than setting bigger ones due to not having land restrictions which decreases total cost of installation.

Table 33: Affects of adding new wind farm areas

	Basic Model	3 Wind Farm Areas
Execution Time (sec)	293.3	34247.540
Baron Iterations	15,369	925,873
Total Power (MW)	22,059.297	21,367.980
Total Cost (€)	29,535,520	29,250,030

Variation 4: Enlarging the model

In the previous variations, we change the number of variable in each set one by one to see the effects on the results individually. However, in this case, each set is enlarged simultaneously in order to experience the cumulative effect of the variations on total cost. Moreover, this approach proves that the proposed model is adaptable to more complex real case scenarios.

While we consider 2 locations, 2 wind farm areas and 3 periods in our basic model, we extend the model by adding 1 location, 1 wind farm area and 2 more periods to the real case example of Turkey. The more model sets and parameters get complex, the more time and iterations involves the model but still the model is able to calculate the global optimal solution.

Table 34: Results of extended model

	Basic Model	Extended Model
Execution Time (sec)	293.3	80,880
Baron Iterations	15,369	734,357
Total Power (MW)	22,059.297	40.222
Total Cost (€)	29,535,520	33,046,630

3.3.1. Results of Sensitivity Analysis

Our multi-period mixed integer mathematical approach was executed under different scenarios in order to compare the results and to prove the model's adaptiveness to various circumstances. Increasing periods, locations and/or available wind farm areas make compilation even harder in terms of execution time or number of baron iteration but not impossible to solve.

Sensitivity analysis is performed to observe the effects on execution times, number of baron iterations and objective function. Turkey case example is taken for the basic model and some variations made on this example. Total periods are increased by 1, 2 more locations are added one by one and one more wind farm area is included to the basic example separately. Figure 11, 12 and 13 show the effect of variations on each parameter.

Adding one more wind farm area to each location has the extreme effect on execution time, it increases compilation time by 117 times (Figure 11). The same effect goes also for number of baron iterations. The model executes 60 times more iterations when wind farm area variable increased by 1 as shown in Figure 12. Apart from deciding number of wind turbines, where to invest and when to invest, one can easily say that "how to invest" (placement of the turbines) is the most challenging decision for our model.

For the objective function, increasing period has eventually additive effect on total cost but on the other hand 1 more wind farm availability in each location caused reverse impact on objective function. Having more wind farm area means that we are not constricted by land area so relatively small and more wind turbines can be chosen with less investment cost (Figure 13).

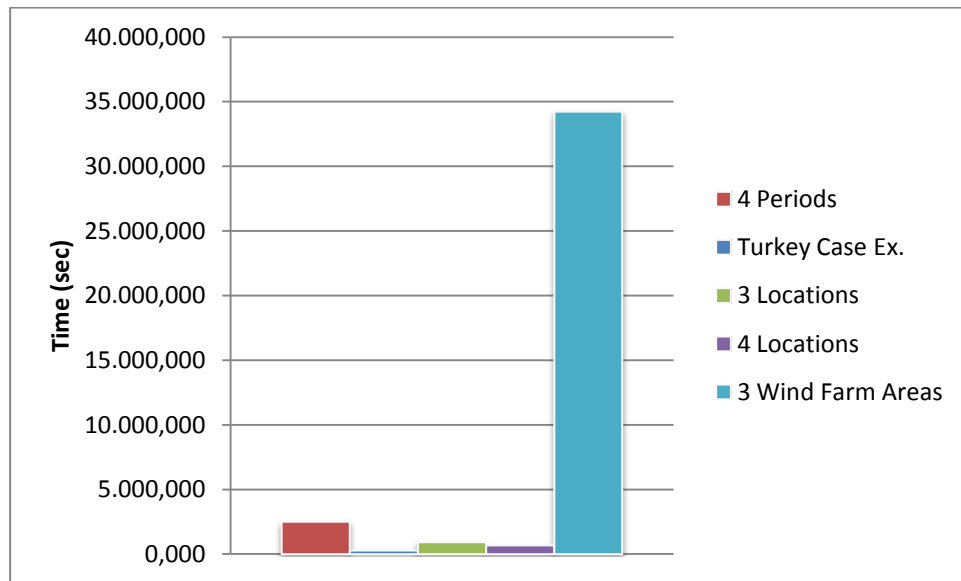


Figure 11: Execution Times

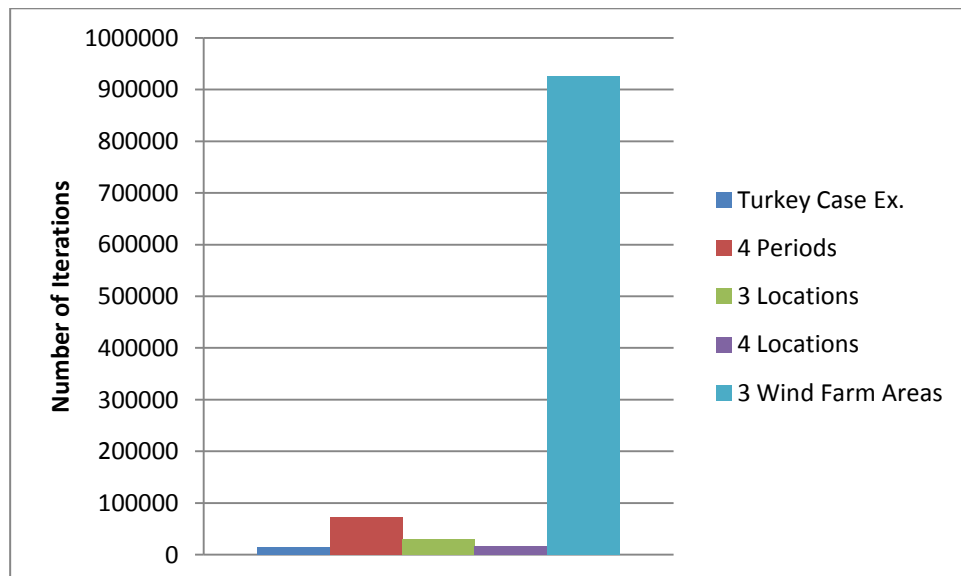


Figure 12: Number of Baron Iterations

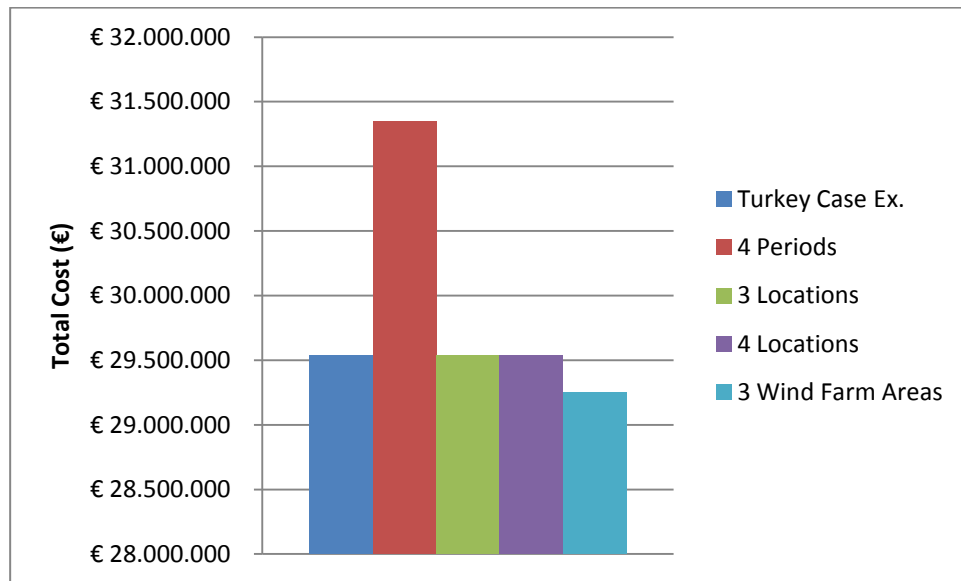


Figure 13: Total Cost

Chapter 4

Discussion & Conclusions

4.1. Main Observations of Wind Energy Investment Model

We present a multi-period mixed integer non-linear programming model to design wind farms with optimum number of wind turbines, selection of optimal rotor type, and the optimal placement of turbines within the available land area. The decision variables included in the optimization process are the total generated energy of corresponded area, total number of wind turbines, chosen rotor type for the wind turbine, and x -and y -axis lengths of the wind farm area (given that the land area is rectangular). We also reformulate some of the nonlinearities in the optimization model that can be done through big-M constraints. The proposed model is tested on an illustrative example and the real case of Turkey by BARON solver [7] using GAMS software [14]. Taking into account the uncertainty on wind speed, a scenario based two-stage stochastic approach is applied into model in order to see the effect of this uncertainty. Under different wind circumstances, model gave the optimal results. With this stochastic approach, we are able to observe the effect of the uncertainty on objective value. On the other hand, real data taken from an existing wind farm entered into deterministic model and compatibility of our deterministic model for single farm problems is experienced.

All computational work has been executed on a personal computer (32-bit operating system, 2.50 GHz CPU, and 4.00 GB). The illustrative example is solved in a run time of approximately 2.24 minutes while extended real Turkey case example presented in Chapter 3.3, is solved in 3 hours and 34 minutes.

The specific goals that are accomplished by the scope of this research are as follows:

- A deterministic MINLP mathematical model is developed and implemented.
- The data used in computational examples is described in detail.
- The deterministic model is first applied to an illustrative example, then is extended for the real Turkey case.
- Cost-oriented objective function is constructed and the optimal net present value for the both investment and operating cost are calculated.
- Sensitivity analysis is conducted to see the affects of setting up different variations.
- Turkey case example is enlarged to prove that the model is usable for more complex real case scenarios.

For the Turkey case, we have foreseen that it would be possible to generate more than 40,2 GW of energy within a 5 years period to satisfy the energy demand by using the existing wind potential. Considering the energy dependence to outside sources of countries and increased greenhouse gas emissions to the atmosphere, wind can be a prosperous option of harvesting energy for the future.

4.2. Future Work

While this study applied to real case data of Turkey, the program can be extended to apply other locations in Turkey or other countries. Stochastic programming can be a great extension to this study since wind speed does not follow a deterministic way and varies a lot during the year in real life. Moreover, more sensitivity analysis such as wind speed fluctuation and energy demand forecast can be applied to gather better results.

Another concern for the future work can be transmission network for the generated energy. In wind related problems and wind investment decisions grid connection of the wind specified area plays an important role for the investment plans.

This work also can be expanded as multi-objective model with applying evolutionary algorithms. Along with the minimizing cost objective, minimizing wake loss can be considered in order to increase the quality of wind investment decisions.

BIBLIOGRAPHY

- [1] IEA (International Energy Agency), Key World Energy Statistics, 2012, <http://www.iea.org>
- [2] EIA (Energy Information Administration), History of Wind Power, 2013, <http://www.eia.gov>
- [3] Baroudi, Jamal A., Venkata Dinavahi, and Andrew M. Knight. "A Review of Power Converter Topologies for Wind Generators." *Renewable Energy* 32.14 (2007): 2369-385.
- [4] Rick Tidball and others, "Cost and Performance Assumptions for Modeling Electricity Generation Technologies", US National Renewable Energy Laboratory, Nov. 2010, p.63.)
- [5] GWEC (Global Wind Energy Council), Global Wind Report – Annual Market Update, 2013, <http://www.gwec.net/>
- [6] Energy Resources & Systems, Tushar K. Gosh, Mark A. Prelas, 2010
- [7] Branch And Reduce Optimization Navigator, Nikolaos Sahinidis and Mohit Tawarmalani, The Optimization Firm, LLC.
- [8] Betz, Albert. Introduction to the Theory of Flow Machines. Oxford: Pergamon, 1966.
- [9] Kılıç, Cumali. "Wind Energy and Assessment of Wind Energy Potential in Turkey." *Renewable and Sustainable Energy Reviews* 16 (2012): 1165-173
- [10] Kongnam, C., S. Nuchprayoon, S. Premrudeepreechacharn, and S. Uatrungjit. "Decision Analysis on Generation Capacity of a Wind Park." *Renewable and Sustainable Energy Reviews* 13.8 (2009): 2126-133
- [11] Pousinho, H.M.I, V.M.F Mendes, and J.P.S Catalao. "A Stochastic Programming Approach for the Development of Offering Strategies for a Wind Power Producer." *Electric Power Systems Research* 89 (2012): 45-53
- [12] Ivan Mustarekov, D. B. (2010). Wind turbines type and number choice using combinatorial optimization. *Renewable energy* , 1887-1894xx
- [13] Kusiak, Andrew, and Zhe Song. "Design of Wind Farm Layout for Maximum Wind Energy Capture." *Renewable Energy* (2009)].

- [14] Rosenthal, R. E., 2012. GAMS – A User’s Guide. GAMS Development Group, Washington, DC
- [15] Birge, J. R., & Louveaux, F. V. (1997). *Introduction to stochastic programming*. Springer.
- [16] Furkan D. (2010) Overview of the photovoltaic technology status and perspective in Turkey, *Renewable and Sustainable Energy Reviews*, 15:3768-3779
- [17] EIE (Electrical Power Resources Survey And Development Administration). Geothermal Energy in Turkey; 2011, <http://www.eie.gov.tr> [accessed 10.11.2011]].
- [18] I. Yuksel, (2010), As a renewable energy hydropower for sustainable development in Turkey, *Renewable and Sustainable Energy Reviews*, 14:3213-3219
- [19] N. Saracoglu (2010), The Biomass Potential of Turkey for Energy Production: Part I, Energy Sources, Part B: Economics, Planning, and Policy, 5:3, 272-278
- [20] Turkey’s Wind Energy Association (TUREB), Turkish Wind Energy Statistics Report, 2012
- [21] Kılıç, Cumali. "Wind Energy and Assessment of Wind Energy Potential in Turkey." *Renewable and Sustainable Energy Reviews* 16 (2012): 1165-173
- [22] "VIII." Blue Book. Ankara: Ministry of Energy and Natural Sources, 2011. 19-22
- [23] EIE (Electrical Power Resources Survey And Development Administration). Wind Energy, <http://www.eie.gov.tr/yenilenebilir/ruzgar.aspx> [accessed 10.09.2014]].

VITA

Aysim Gülde Kublay was born in İzmir, Turkey, on 23 August, 1988. She is graduated from Nilüfer Milli Piyango Anatolian High School in 2006. She received her B.S. degree in Industrial Engineering from Galatasaray University, Istanbul, in July 2011. In 2011, she joined the M.S. program in Industrial Engineering at Koç University.