MOTIVATING THROUGH COMPETITION

by

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Abstract

All-pay auctions have received considerable attention since they provide insight about wide range of competitions where effort is costly regardless of winning or losing. Unlike other types of auction, auctioneer could benefit from these sunk costs depending on his objective. The problem of maximizing total expected effort in all-pay auctions has been studied to manage the sunk costs favoring the designer. However, current literature lacks identifying dependence of prize valuation in ones effort in these games. This dependence is interesting to analyze for two reasons. First, from the designer's perspective, conditioning the prize on the effort could be profitable. Second, the equilibrium characteristics of such a game could be helpful in understanding bidding behavior by identifying the relation between the perception of the prize and their efforts. The result of our analysis show that the equilibrium characteristics significantly differ from the current literature where the prize is predetermined. Moreover, in motivating higher total effort, our model achieves higher designer surplus. Therefore, endogenous prize model could be a valuable tool in experimental studies for understanding players' valuation of their action. Moreover, conditioning prize on the total effort can be beneficial for a designer interested in maximizing the total effort.

Keywords: All-pay, contests, auctions, endogenous prize, effort maximization

Özet

Tam ödemeli ihaleler efor sarf etmenin kazanmadan bağımsız olarak maliyetli olması açısından farklı müsabakaları anlamamıza yardım eder ve bu nedenle son zamanlarda yoğun ilgi görmüştür. Diğer ihale çeşitlerinden farklı olarak, müzayede tasarımcıları hedeflerine bağlı olarak bu maliyetten faydalanabilirler. Müzayede tasarımcısına fayda sağlayacak şekilde toplam efor eniyilemesi tam ödemeli ihaleler çerçevesinde çalışılmıştır. Fakat literatürde tanımlanan ihale ödülleri, oyuncuların efor seviyelerinden bağımsız olarak tayin edilmektedir. Bu çalışmada ödülü oyuncuların efor seviyelerine bağlı olarak belirlemede iki ana neden vardır. İlki, ödülü eforlara bağlı olarak tanımlamak ihale tasarımcısı açısından eforlardan bağımsız duruma göre daha faydalı olabilir. İkincisi, böyle bir oyunun denge nitelikleri oyuncuların ödüle verdiği değer ile sarf ettikleri efor arasındaki ilişkiyi anlama açısından kullanışlı olabilir (olacaktır). Çalışmamızdaki ihale yapısında elde edilen denge nitelikleri, literatürde ödülün önceden belirlendiği ihalelerden önemli ölçüde farklılaşmaktadır. Daha da önemlisi, toplam eforu motive etme yoluyla modelimiz ihale tasarımcısına daha yüksek bir fayda sağlamaktadır. Bu nedenle, içsel ödül modelimiz deneysel çalışmalarda oyuncuların kendi aksiyonlarına verdikleri değeri anlamada önemli bir araç olabilir (olacaktır). Çalışmamız, yarışmada sarf edilecek toplam eforla ilgilenen ihale tasarımcısı için ödülü toplam efora bağlı kılmanın faydalı olacağını göstermektedir.

Anahtar Kelimeler: Tam ödemeli, ihale, içsel ödül, efor eniyilemesi

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1 Introduction

The motivating idea behind this study is to model competitions in which the designer wants to maximize aggregate performance in a contest. First, we examine the characteristics of rivalry as a determinant of aggregate effort. For instance, when two competing parties are not comparable in their chances of winning due to their qualifications, weaker player has incentives to give up, motivating lower effort for the stronger player as well. To overcome this limitation, using heterogeneity between the participants and possible matchings of contestants is considered. As an illustrative example, suppose that a school has 40 students who will be appointed to two groups. There is a scholarship for the student who achieves the highest performance in his/her group throughout the year. Students have different costs for an hour studied which may depend on their intelligence, background, socioeconomic status etc. Assignment of students to groups is important not only for the sake of student learning (by encouraging effort) but also to maintain the overall success of the school. In this scenario, how to make a division in order to achieve highest expected total effort is an exemplar of the question of interest. This analysis serves as a reference point when examining the endogenous prize model in terms of equilibrium behavior and its implications for the aggregate effort.

There are two reasons for studying endogenous prize model. First, from the designer's perspective, if the goal is to maximize the total effort spent in a contest, defining the prize as an increasing function of effort levels can be helpful. As we will show, compared to the case where the prize is exogenously determined, higher effort levels can be encouraged at lower cost using the suggested setting. For example, when a project is assigned to a team of employees, the quality of outcome is determined by the contributions from each member such as the time invested by each employee. Besides their usual salary, employees can be encouraged to exert high effort by using bonuses which depend on the outcome. Employee with the highest effort obtains the bonus whose size is determined by the total time invested in the project.

Second, characteristics of the unique equilibrium in this game could help experimental studies of all-pay auctions. In many competition settings, we observe that players have positive returns (e.g. learning) from their investment (efforts spent in the competition). Moreover, there are peer effects that would make the prize depend on not only ones own effort, but also other players' efforts. Consider a classroom where students compete for a scholarship which is awarded to the student with the highest performance. In this case, each student improves their intellect while trying to obtain the scholarship which they will benefit after the competition. Moreover, competition in the class environment not only motivates further study, but also students learn from each other concurrently. Therefore, the results can be used to understand how players perceive the prize and their strategies depend on the all players' efforts in such a game.

The structure of the study is as follows. Section 2 summarizes the literature. In Section 3 and 4, we study exogenous prize contests and endogenous prize contests, respectively. Section 5 compares the results of the two models and Section 6 concludes.

2 Literature Review

The most comprehensive work on all-pay auctions under complete information is [1]. In his paper, Ron Siegel characterizes payoffs allowing heterogeneity between players, non-ordered cost functions and conditional investments. The model can capture many economic aspects by arranging the parameters accordingly such as risk aversion. Earlier studies on all-pay auctions are highly focused on specific applications such as rent-seeking and lobbying activities ([2], [3], [4], [5]). Some modifications for modeling the phenomena more realistically are considered in [6] by applying financial budget constraints, also in [7] and [8] with prize allocations and structural changes to motivate effort. Scenarios with incomplete information are also studied for the all-pay auctions in [9], [10].

Another subject of interest regarding all-pay contests is the total expected effort. Maximization of the total expected effort has been studied in different contexts. [7] [11] [12] [8] inspect prize and contest structure as determinants of performance in terms of players' efforts. [12] compares the outcomes of a grand contest with a multistage contest analyzing the effects of discriminatory extent of the contest technology. One important limitation is the assumption of identical agents. In a more comprehensive study on all-pay auctions, [8] investigates the outcomes of grand contest and several sub-contests to see the structural effects on maximizing the highest effort and also the total effort. They consider agents whose types are selected from a commonly known distributions with three specification: linear, concave and convex cost functions. Given the linear or concave cost functions, their result does not rely on the underlying population under complete information. When cost functions are convex, however, the prize allocation comes into picture to define optimality as studied in [7]. To introduce heterogeneity, [13] considers four players whose types are either high or low in terms of their valuation of the prize. The paper states that low valuation player has greater incentive to drop-out in a grand contest. Moreover, sequential-elimination results in higher total investment when there is an interior equilibrium. In line with this research, [11] provides an experimental look at the subject by differentiating agents through high and low endowments and analyzes their bids in different contest structures. However, in this work, there are a lot of uncontrolled variables, especially regarding the player characteristics such as risk-aversion etc. Using heterogeneity between players to achieve maximum total expected effort is studied in [14], [15], [16] and [17]. In [14], different than the others, players care about their relative positions within the groups which affects designers objective function. [15], uses heterogeneity in players' valuation for the prize in an all-pay contest under complete information. The paper investigates whether a separating, i.e. allocating similar types to same group or mixing, i.e. allocating high type and low type players to same group is optimal for maximizing the total effort. In order to do that they rely on the payoff results from [1] and calculate total expected score. The main limitation of the model is to consider only two types High type and Low type and the dependence of their analysis in the prize structure. In [16], author analyzes optimal grouping to maximize total effort for geometric or quadratic prize sequences. The results show that depending on the convexity of the prize sequence, mixing is preferred over separating. Although they do not generalize for the general prize sequences, they suggest that different sequences can be approximated to the ones used in the paper. Finally, [17] again consider asymmetric players but unlike the previous works, players' types come from a uniform distribution. Authors show how maximal effort and total expected effort are affected by the homogeneity of contestants within groups.

To the best of our knowledge, there does not exist any work analyzing equilibrium characteristics of asymmetric all-pay auctions under complete information where prize is determined as a linear function of players' bids. Throughout the analysis we refer to this game as "endogenous prize contest" where prize is set as a linear function of bids. Likewise, contests in which the prize is fixed and decided before the outcome is known and hence fixed is referred as "exogenous prize contest". For the case in which prize is a function of player's own bid solely, [18] shows the existence of pure strategy equilibrium under some conditions. However the study is limited with identification of the pure strategy equilibrium and does not specify the conditions for existence explicitly. In this research we look at how endogenous prize structure affects contestants bidding behavior and therefore, total expected effort and compare the results with the exogenous prize contest's.

3 Exogenous Prize Contests

In this chapter, We aim to solve the optimal team formation problem within an all-pay contest framework to maximize total expected effort. During the analysis we assume players face convex cost functions and prize is determined in advance as $V \in \mathbb{R}_+$. This analysis will also stand as a benchmark in assessing the advantages and disadvantages of endogenous prize contest.

3.1 The Model

We analyze the optimal team formation problem in an all-pay auctions setting. There are 2n players and the set of all players is denoted by $\mathcal{I}$. Players are heterogeneous in terms of their abilities which is reflected in their effort cost functions but they have homogeneous valuation of the prizes. Each player $i \in \mathcal{I}$ simultaneously and independently chooses an effort level $e_i \in [0, \infty)$ and bears the cost $c_i(e_i) = c_i e_i^2$ where $c_i \in (0, \infty)$ $\forall i \in \mathcal{I}$. All players have complete information, i.e., cost parameters and the game parameters (c_1, c_2, V) are common knowledge.

In our setting, contestants are grouped such that there are two players in each group. Hence, there are n groups of two players. Players in each group compete with each other. For each two-player contest, the prize structure is given by,

$$\begin{aligned} V_1(e_1, e_2) &= V \\ V_2(e_1, e_2) &= 0 \end{aligned}$$

where V_1 is the prize of the winner and V_2 is the prize of second highest bidder.

Given the profile of efforts $e = (e_i, e_j)$, where $i, j \in \{1, 2\}$ and $i \neq j$, payoff of player i from exerting an effort of e_i is

$$\pi_i(e) = \begin{cases} V - c_i e_i^2, & \text{if } e_i > e_j \\ -c_i e_i^2, & \text{if } e_i < e_j \\ \frac{V}{2} - c_i e_i^2, & \text{if } e_i = e_j \end{cases}$$

In the following, the equilibria of the games will be provided where cumulative and probability distribution function of player i's equilibrium mixed strategy is denoted by $F_i(e_i)$ and $f_i(e_i)$, respectively. More comprehensive study on all-pay auctions are present in [1] and equilibrium characterization can be found in [19].

3.2 Equilibrium Analysis

Consider a two player contest and assume, without loss of generality, that $c_2 \geq c_1$. The equilibrium for all-pay auctions with convex costs are derived in [19].

The unique equilibrium of the game is in mixed strategies where

$$\begin{aligned} F_1^*(e_1) &= \frac{c_2 e_1^2}{V} \\ F_2^*(e_2) &= \frac{c_2 - c_1}{c_2} + \frac{c_1 e_2^2}{V} \end{aligned}$$

for $e \in [0, \sqrt{\frac{V}{c_2}}]$.

The expected efforts of players are:

- $E^1_{fixed} = E[e_1] = \frac{2}{3}\sqrt{\frac{V}{c_2}}$
- $E^2_{fixed} = E[e_2] = \frac{2c_1}{3c_2}\sqrt{\frac{V}{c_2}}$.

Therefore, total expected effort level becomes

$$E^{total}_{fixed} = \frac{2\sqrt{V}}{3}\frac{c_1 + c_2}{c_2\sqrt{c_2}}$$

Winning probability of player 1 is found as $1 - \frac{c_1}{2c_2}$, and winning probability of player 2 is found as $\frac{c_1}{2c_2}$. Three possible options of team formation are:

A. (1-2)(3-4) with total effort level $E_A = \frac{2\sqrt{V}}{3}(\frac{c_1+c_2}{c_2\sqrt{c_2}} + \frac{c_3+c_4}{c_4\sqrt{c_4}})$

B. (1-3)(2-4) with total effort level $E_B = \frac{2\sqrt{V}}{3}(\frac{c_1+c_3}{c_3\sqrt{c_3}} + \frac{c_2+c_4}{c_4\sqrt{c_4}})$.

C. (1-4)(2-3) with total effort level $E_C = \frac{2\sqrt{V}}{3}(\frac{c_1+c_4}{c_4\sqrt{c_4}} + \frac{c_2+c_3}{c_3\sqrt{c_3}})$.

where $c_4 \geq c_3 \geq c_2 \geq c_1$.

We display the matching options in Table 1. Let c^* be defined as $c^* = \frac{c_2^{3/2}c_3^{5/2} - c_2 c_4^{3/2}\left(c_2^{3/2} + \sqrt{c_2}c_3 - c_3^{3/2}\right)}{c_3^{3/2}\left(c_2^{3/2} - c_4^{3/2}\right)}$.

Lemma 1. Given $c_4 \geq c_3 \geq c_2 \geq c_1$,

(i) Option B is always dominated by option C.

Table 1: Two by Two Matchings of Four Players

	Match 1	**Match 2**
Option A	(1-2)	(3-4)
Option B	(1-3)	(2-4)
Option C	(1-4)	(2-3)

(ii) If $c_1 > c^*$, then $E_A > E_C$, that is, option A is optimal. Otherwise, option C is optimal.

Proof

(i) $E_C - E_B = \frac{2\sqrt{V}}{3}(\frac{c_2-c_1}{c_3\sqrt{c_3}} - \frac{c_2-c_1}{c_4\sqrt{c_4}}) \geq 0$. Hence, the designer does not choose option B in any case.

(ii) The two remaining options can be compared by looking at the difference $E_A - E_C = \frac{c_1+c_2}{c_2\sqrt{c_2}} + \frac{c_3-c_1}{c_4\sqrt{c_4}} - \frac{c_2+c_3}{c_3\sqrt{c_3}}$. Whenever this difference is positive, matchings in option A achieve higher expected payoff for the designer. Solving $E_A - E_C > 0$ gives us c^*.

3.3 Discussion

When the optimality of different options are considered, we see that in most of the cases, option A is better than option C for the designer. However, there might be cases in which the loss in the total effort caused by matching the highest and lowest profile players is compensated with the increase that comes from the fierce competition between the middle range players (with costs c_2 and c_3 in our case). The latter case occurs especially when the profiles of the middle range players are very close and option C becomes the optimal one. These ranges are illustrated in Figure 1 and Figure 2. The graphs show the regions where option A and option C is optimal. Difference between the two extreme cost parameters is set as one in the first graph and ten in the second.

This result also suggests that there might be cases in which one option dominates the other one irrespective of the magnitudes of costs as long as costs are distributed in a specific way. For instance, when $c_4 = \delta^4 > c_3 = \delta^3 > c_2 = \delta^2 > c_1 = \delta$

where $\delta > 1$, option A always leads to the maximum total effort regardless of what the actual value of δ is. Likewise, $c_4 = c_1 + 3k > c_3 = c_1 + 2k > c_2 = c_1 + k > c_1$ will result in the optimality of option A $\forall\ c_1$ and $\forall\ k > 0$. A similar exercise can be found in [20] with linear costs functions.

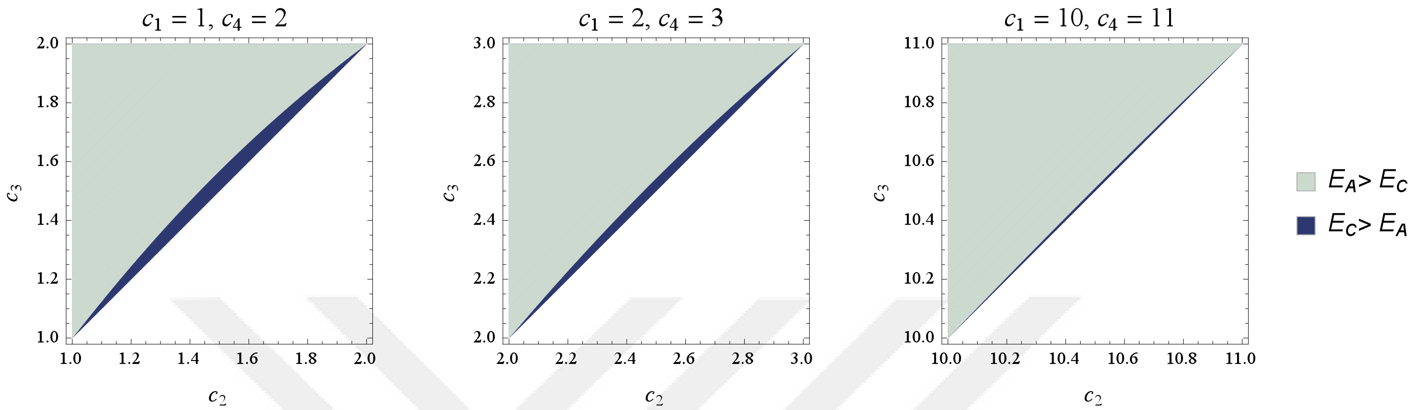

Figure 1: Comparison of Expected Total Effort between Option A and C for $c_4 - c_1 = 1$

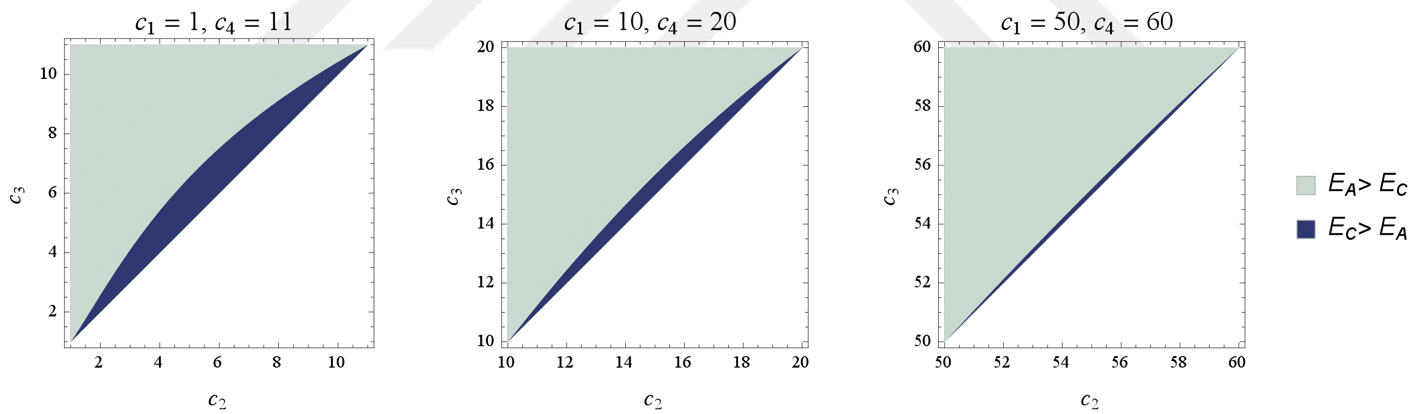

Figure 2: Comparison of Expected Total Effort between Option A and C for $c_4 - c_1 = 10$

4 Endogenous Prize Contests

In this chapter, we aim to characterize the equilibrium of the game where prize is determined endogenously, i.e, as a function of the effort levels of the players who face convex cost functions. We first provide the model of the game where there are two players and the prize is shared between winner and loser according to share parameter denoted by λ. Then, we provide the conditions for existence of pure strategy equilibrium. Finally, the unique mixed strategy equilibrium is characterized and examples for special cases will follow.

4.1 The Model

In our model of complete information endogenous prize contest, there are two players and the set of players is denoted by $\mathcal{I} = \{1, 2\}$. Players are heterogeneous in terms of their abilities which is reflected in their effort cost functions. Each player $i \in \mathcal{I}$ simultaneously and independently chooses an effort level $e_i \in [0, \infty)$ and bears the cost, $c_i(e_i) = c_i e_i^2$ where $c_i \in (0, \infty)$, $\forall i \in \mathcal{I}$. Total prize distributed by the contest designer is set as $\alpha(k_{own}e_1 + k_{competitor}e_2)$ where $\alpha \in \mathbb{R}_+$ is the prize parameter. k_{own} and $k_{competitor}$ are weights of own effort. Therefore, from player 1's perspective the prize is divided between the two players such that:

$$
\begin{aligned}
V_1(e_1, e_2) &= \lambda\alpha(k_{own}e_1 + k_{competitor}e_2) \\
V_2(e_1, e_2) &= (1 - \lambda)\alpha(k_{own}e_1 + k_{competitor}e_2)
\end{aligned}
$$

where $\lambda > \frac{1}{2}$, the share parameter, determines the distribution of total prize among players. Therefore, $V_1(e_1, e_2)$ is the prize for the player with the highest effort level and $V_2(e_1, e_2)$ is for the player with the second highest effort level.

Given the profile of efforts $e = (e_i, e_j)$, where $i, j \in \mathcal{I}$ and $i \neq j$, payoff of player i from exerting effort e_i is

$$
\pi_i(e) = \begin{cases} \lambda V(e) - c_i e_i^2, & \text{if } e_i > e_j \\ (1 - \lambda)V(e) - c_i e_i^2, & \text{if } e_i < e_j \\ \frac{V(e)}{2} - c_i e_i^2, & \text{if } e_i = e_j \end{cases}
$$

Therefore, expected payoff of player i is given by,

$$\pi_i(e_i,\sigma_j) = \lambda\alpha\int_{e_{min}}^{e_i}(k_{own}e_i+k_{competitor}e)f_j(e)de+(1-\lambda)\alpha\int_{e_i}^{e_{max}}(k_{own}e_i+k_{competitor}e)f_j(e)de-c_ie_i^2 \quad (1)$$

where $e_{min} = sup\{e_j : F_j(e_j) = 0\}$, $e_{max} = inf\{e_j : F_j(e_j) = 1\}$ and σ_j is the strategy of player i.

4.2 Characterization of the equilibrium

In the characterization of the equilibrium, we first define the relation between the cost parameters of players and game parameters (α, λ,k_{own} and $k_{competitor}$) to decide whether the equilibrium is in pure or mixed strategies. We find that when the difference between cost parameters are high, there is a pure strategy equilibrium in which the low type player (with the higher cost parameter) gives up. However, when cost parameters are close to each other, strategy of the high type player cannot discourage low type player leading to deviation from the pure strategy equilibrium. Theorem 1 states this condition and identifies the pure strategy equilibrium of the game when the condition is satisfied. Throughout the analysis we will assume, without loss of generality, that $c_2 > c_1$. Let, $\underline{c} = \sqrt{\frac{c_1^2((5\lambda^2-6\lambda+2)(k_{own})^2+2(\lambda^2+\lambda-1)k_{own}k_{competitor}+\lambda^2(k_{competitor})^2)}{\lambda^2(k_{own})^2}} + c_1\left(-\frac{1}{\lambda}+\frac{k_{competitor}}{k_{own}}+2\right)$

Theorem 1. If $c_2 \geq \underline{c}$, then there exists a pure strategy equilibrium where $(e_1^*, e_2^*) = (\frac{k_{own}\lambda\alpha}{2c_1}, \frac{k_{own}(1-\lambda)\alpha}{2c_2})$.

Proof of Theorem 1.

i. In equilibrium, player 1 must win with $e_1^* > e_2^*$. Suppose not and let $e_1^* < e_2^*$, that is, player 2 win. If this is an equilibrium, $\pi_2(e_1^*, e_2^*) = \alpha\lambda(k_{competitor}e_1^* + k_{own}e_2^*) - c_2e_2^{*2} \geq 0$ and $\pi_1(e_1^*, e_2^*) = \alpha(1-\lambda)(k_{own}e_1^* + k_{competitor}e_2^*) - c_1e_1^{*2} \geq 0$. $\pi_1(e_2^*+\epsilon, e_2^*) = \alpha\lambda(k_{competitor}e_2^*+k_{own}(e_2^*+\epsilon)) - c_1(e_2*+\epsilon)^2 < \alpha(1-\lambda)(k_{own}e_1^*+ k_{competitor}e_2^*) - c_1e_1^{*2}$ should hold for player 1 not to deviate. This implies that $e_2^* > \frac{k_{own}\lambda\alpha}{2c_1}$. However, then player 2 can always choose can always choose $e_2^* = e_1^* - \epsilon$, $\epsilon > 0$, and wins for sure with a strictly higher payoff. If $e_1^* = e_2^*$, they get the prize with $\frac{1}{2}$ probability. Then, given $\lambda > 1/2$, one player can increase her effort slightly and get a strictly higher payoff. Therefore, $e_1^* > e_2^*$ in equilibrium.

ii. Since player 2 loses with certainty, she chooses the effort level maximizing $\alpha(1-\lambda)(k_{competitor}e_1^* + k_{own}e_2^*) - c_2 e_2^{*2}$. Therefore, player 2's effort level is $e_2^* = \frac{k_{own}(1-\lambda)\alpha}{2c_2}$ in equilibrium. Then, Player 1's best response is $e_1^* = \frac{k_{own}\lambda\alpha}{2c_1}$ maximizing her payoff, $\pi_1(e_1^*, e_2^*) = \alpha\lambda(k_{own}e_1^* + k_{competitor}e_2^*) - c_1 e_1^{*2}$.

iii. Given $(e_1^*, e_2^*) = (\frac{k_{own}\lambda\alpha}{2c_1}, \frac{k_{own}(1-\lambda)\alpha}{2c_2})$ constitutes a pure strategy equilibrium, player 2's should not deviate either. Suppose that $\pi_2(e_1^*, e_1^*+\epsilon) > 0$ for $\epsilon > 0$. In this case player 2 wins for sure because $e_1^*+\epsilon > e_1^*$. Moreover, if $\pi_2(e_1^*, e_1^*+\epsilon) > \pi_2(e_1^*, e_2^*)$, then it is a contradiction that (e_1^*, e_2^*) is an equilibrium. Therefore if $\lim_{\epsilon\to 0} \pi_2(e_1^*, e_1^*+\epsilon) = \alpha\lambda((k_{own}+k_{competitor})e_1^*) - c_2 e_1^{*2} \leq \alpha(1-\lambda)(k_{competitor}e_1^* + k_{own}e_2^*) - c_2 e_2^{*2}$, or equivalently, if $c_2 \geq \underline{c}$, (e_1^*, e_2^*) is a pure strategy equilibrium. For instance, when $k_{competitor} = k_{own} = 1$ and $\lambda = 1$, if $c_2 < 4c_1$, there is equilibrium in pure strategies. Whereas for $\lambda \to \frac{1}{2}$, there is no equilibrium in pure strategies for all $c_2 < c_1$, hence the equilibrium is in pure strategies only.

Corollary 1. If $c_2 < \underline{c}$, then there is no equilibrium in pure strategies.

Theorem 2. When $c_2 < \underline{c}$, there exist a unique equilibrium in mixed strategies. Player 1's equilibrium strategy σ_1^* is as follows:

1. $\alpha_1(k_{own}\alpha(\frac{\lambda p+(1-\lambda)(1-p)}{2c_1})) = q$, i.e, player 1 places an atom at effort level $k_{own}\alpha(\frac{\lambda p+(1-\lambda)(1-p)}{2c_1})$ of size q, which is uniquely determined by the game parameters.

2. Player 1 plays a mixed strategy with its density is given by $F_1(e)$ which is atom-less over the interval $(k_{own}\alpha(\frac{\lambda p+(1-\lambda)(1-p)}{2c_1}), e_h]$ where e_h is determined by the game parameters.

Player 2's strategy σ_2^* is as follows:

1. $\alpha_2(\frac{k_{own}(1-\lambda)\alpha}{2c_2}) = p \geq 0$, i.e, player 2 places an atom at effort level $\frac{k_{own}(1-\lambda)\alpha}{2c_2}$ of size p, which is uniquely determined by the game parameters.

2. Player 2 plays a mixed strategy with its density is given by $F_2(e)$ which is atom-less over the interval $(k_{own}\alpha(\frac{\lambda p+(1-\lambda)(1-p)}{2c_1}), e_h]$ where e_h is determined by game parameters.

Proof of Theorem 2.

Lemma 2. Let $e_{h1} = inf\{e_1 : F_1(e_1) = 1\}$ denote the highest effort in the equilibrium by player 1 and $e_{h2} = inf\{e_2 : F_2(e_2) = 1\}$ by player 2. In equilibrium, $e_{h1} = e_{h2} = e_h$ and $e_h \geq \frac{\lambda\alpha k_{own}}{2c_1}$.

Proof First, let $e_{hi} \neq e_{hj}$ and assume that $e_{hi} > e_{hj}$, $i, j \in \{1, 2\}$, $j \neq i$. Expected payoff of player i becomes $\pi_i(e_{hi}, \sigma_j) = \lambda\alpha(k_{own}e_{hi} + k_{competitor}E_j(e_{hi})) - c_i(e_{hi})$, where $E_j(e) = \int_0^e e_j dF_j(e_j)$. Note that $E_j(e_{hi}) = E_j(e_{hj})$ since $e_{hi} > e_{hj}$. If $e_{hi} > \frac{\lambda\alpha k_{own}}{2c_i}$, player i's payoff is decreasing in effort at e_{hi}. Hence, player i benefits from lowering e_{hi} by transferring mass just below, until setting $e_{hi} = max\{\frac{\lambda\alpha k_{own}}{2c_i}, e_{hj}\}$. If $e_{hi} \leq \frac{\lambda\alpha k_{own}}{2c_i}$, since player i's payoff is weakly increasing in effort at e_{hi} until $\frac{\lambda\alpha k_{own}}{2c_i}$, she is better off by placing an atom of size $F_i(e_{hi}) - F_i(e_{hj})$ at $\frac{\lambda\alpha k_{own}}{2c_i}$ contradicting that $e_{hi} \neq e_{hj}$ in equilibrium. $c_2 \geq c_1$ and $e_{hi} = max\{\frac{\lambda\alpha k_{own}}{2c_i}, e_{hj}\} \forall i \in \mathcal{I}$ implies $e_h \geq \frac{\lambda\alpha k_{own}}{2c_1}$.

Lemma 3. Let S_i denote the support of F_i where $S_i \subseteq [0, e_h]$ $\forall i$ and $e_{li} = inf\{e : e \in S_i\}$. In equilibrium, there is no effort level such that both players place an atom in $S_1 \cap S_2$.

Proof Suppose to the contrary and let both players place an atom at $e^* \neq 0$ of size $\alpha_i(e^*) > 0$ $\forall i$ where $e^* \in S_1 \cap S_2$. Bidding just above e^*, player i can increase her payoff by $\pi_i(e^* + \epsilon, \sigma_j) - \pi_i(e^*, \sigma_j) = \lambda\alpha(k_{own}(e^* + \epsilon)F_j(e^* + \epsilon) + k_{competitor}E_j(e^* + \epsilon)) + (1-\lambda)\alpha(k_{own}(e^* + \epsilon)(1 - F_j(e^* + \epsilon)) + k_{competitor}E_j(e_h) - k_{competitor}E_j(e^* + \epsilon)) - \lambda\alpha(k_{own}e^*(F_j(e^*) - \alpha_j(e^*)) + k_{competitor}(E_j(e^*) - \alpha_i(e^*)e^*) - (1-\lambda)\alpha(k_{own}e^*(1 - F_j(e^*) - \alpha_j(e^*)) + k_{competitor}(E_j(e_h) - E_j(e^*) - \alpha_i(e^*)e^*)) - \frac{\alpha(k_{own}+k_{competitor})\alpha_j(e^*)e^*}{2} - c_i(e^* + \epsilon) + c_i(e^*)$. Notice that the increase in cost of effort $c_i(e^* + \epsilon) - c_i(e^*)$ goes to zero as $\epsilon \to 0$ due to continuity of the quadratic cost function. Moreover, every term that does not include the atom disappears. Therefore, from the first two terms, it is seen that as $\epsilon \to 0$, player i can obtain strictly higher payoff by bidding just above e^* since $\frac{\alpha(k_{own}+k_{competitor})\alpha_j(e^*)e^*}{2}$. Therefore, player i can deviate and get a higher payoff, contradicting that both players places an atom at e^* in equilibrium.

Lemma 4. If F_i, $i \in \{1, 2\}$, is strictly increasing on some open interval (a, b) where $0 \leq a < b < e_h$, then F_j, $j \neq i$ is strictly increasing on (a, b) as well.

Proof Suppose to the contrary and let $F_i(e)$ strictly increase on (a, b) and there exist a', b', $a < a' < b' < b$ such that $F_j(a') = F_j(b')$, $j \neq i$. Expected payoffs at a' and b' for player i are $\pi_i(a', \sigma_j) = \lambda\alpha(k_{own}a'F_j(a') + k_{competitor}E_j(a')) + (1-\lambda)\alpha(k_{own}a'(1 - F_j(a')) + k_{competitor}(E_j(e_h) - E_j(a'))) - c_i(a')$ and $\pi_i(b') = \lambda\alpha(k_{own}b'F_j(b') + k_{competitor}E_j(b')) + (1-\lambda)\alpha(k_{own}b'(1-F_j(b')) + k_{competitor}(E_j(e_h) - E_j(b'))) - c_i(b')$. Since player j does not play any effort level between a' and b', we have $F_j(a') = F_j(b')$ and $E_j(a') = E_j(b')$. In order that $\pi_i(a') = \pi_i(b')$ holds, we need to have $a' + b' = \frac{k_{own}\lambda\alpha F_j(a') + k_{own}(1-\lambda)\alpha(1-F_j(a'))}{c_i}$. Since $b' > a'$ this implies that $a' < \frac{k_{own}\lambda\alpha F_j(a') + k_{own}(1-\lambda)\alpha(1-F_j(a'))}{2c_i}$. Then it is also true that $a' < \frac{k_{own}\lambda\alpha}{2c_i}$. Therefore, player i is better off by transferring mass from ϵ neighborhood below a' to some δ neighborhood above a' since his payoff is increasing at a'. Player i does not deviate only if $a' = b'$ holds, which leads to a contradiction.

Lemma 5. If F_i, $i \in \{1, 2\}$, is constant on some open interval (a, b), then F_j, $j \neq i$ is constant on the same interval.

Proof The proof directly follows from the previous lemma.

Lemma 6. If F_i, $i \in \{1, 2\}$, is strictly increasing on some open interval (a, b) where $0 \leq a < b < e_h$, then F_i is strictly increasing on $(a, e_h]$ as well.

Proof Suppose that F_i, $i \in \{1, 2\}$, is strictly increasing on some open interval (a, b) and there exist b' and b'' such that F_i is constant on the interval (b', b'') where $b < b' < b'' < e_h$. From the previous lemma, player j, $j \neq i$ does not bid over the same interval as well. Let $b^* = inf\{e : F_i(e) = F_i(b')\}$ and $b^{**} = sup\{e : F_i(e) = F_i(b'')\}$. Expected payoff of player i is $\pi_i(e, \sigma_j) = \lambda\alpha(k_{own}eF_j(e) + k_{competitor}E_j(e)) + (1 - \lambda)\alpha(k_{own}e(1 - F_j(e)) + k_{competitor}(E_j(e_h) - E_j(e))) - c_i(e)$ where $F_j(e) = F_j(b') = F_j(b'')$ and $E_j(e) = E_j(b') = E_j(b'')$ for $e \in (b^*, b^{**})$. Notice that this function has unique maximum given $F_j(b')$, $E_j(b')$. Let $e^* = argmax\{\lambda\alpha(k_{own}eF_j(b') + k_{competitor}E_j(b')) + (1-\lambda)\alpha(k_{own}e(1-F_j(b')) + k_{competitor}(E_j(e_h) - E_j(b'))) - c_i(e)\}$. If $e^* \notin (b^*, b^{**})$ then player i benefits from transferring mass below and above by some $\epsilon > 0$ from b'' or b' if $e^* < b'$ or $e^* > b''$, respectively. On the other hand, if $e^* \in (b^*, b^{**})$, player i can benefit by lowering the upper limit or increasing the lower limit.

Lemma 7. There are no gaps in the interval $(max\{e_{l1}, e_{l2}\}, e_h]$, where $e_{li} = sup\{e_i : F_i(e_i) = 0\}$ $\forall i$.

Proof The proof follows from Lemma 6.

Lemma 8. For all $i \in \{1, 2\}$, F_i contains no atoms in the half open interval $(max\{e_{l1}, e_{l2}\}, e_h]$.

Proof The proof follows from Lemma 6 and 7.

Lemma 9. In equilibrium, $e_{l1} \neq e_{l2}$ where $e_{li} = sup\{e_i : F_i(e_i) = 0\}$.

Proof Suppose to the contrary and let $e_{l1} = e_{l2} = e$.

Case 1: $e = 0$.

$\frac{\partial \pi_1(e,F_j)}{\partial e}\Big|_{e=0} > 0$ at $e = 0$. Therefore, player 1 can increase her lower bound and get a strictly higher payoff.

Case 2: $0 < e \leq \frac{k_{own}(1-\lambda)\alpha}{2c_1}$.

$\frac{\partial \pi_1(e,F_j)}{\partial e}\Big|_{e \leq \frac{k_{own}(1-\lambda)\alpha}{2c_1}} \geq 0$. Therefore, player 1 can increase her lower bound and get a strictly higher payoff.

Case 3:

$0 < e < \frac{k_{own}(1-\lambda)\alpha}{2c_1}$. $\frac{\partial \pi_1(e,\sigma_j)}{\partial e}\Big|_{e > \frac{k_{own}(1-\lambda)\alpha}{2c_1}} < 0$. Therefore, player 1 can decrease her lower bound and get a strictly higher payoff.

Lemma 10. In equilibrium, $F_i(e)$, $\forall i$ is constant on the open interval $(min(e_{l1}, e_{l2}), max(e_{l1}, e_{l2}))$.

Proof Proof follows from Lemma 4.

Lemma 11.

1. In equilibrium, low-type player (Player 2 is the low-type in our problem with $c_2 > c_1$) places an atom of size p at $e_{l2} = \frac{k_{own}(1-\lambda)\alpha}{2c_2}$.
2. Player 1 places an atom of size q at e_{l1} where $e_{l1} = \frac{k_{own}\lambda\alpha p + k_{own}(1-\lambda)\alpha(1-p)}{2c_1}$.

Proof

1. From Lemma 11, there should be an atom at $min(e_{l1}, e_{l2})$ so that an equilibrium can be restored. Suppose $min(e_{l1}, e_{l2}) = e_{l2}$. Therefore, player 2 certainly loses when she plays e_{l2}. Maximizing her payoff given she loses $\pi_2(\sigma_1, e_{l2}) = (1-\lambda)\alpha(k_{own}e_{l2} + k_{competitor}E_1(e_h)) - c_2 e_{l2}^2$, she places an atom at $e_{l2} = \frac{k_{own}(1-\lambda)\alpha}{2c_2}$.
2. Player 1 wins with probability p at e_{l1}. Therefore $\pi_1(e_{l1}, \sigma_2) = \lambda\alpha p(k_{own}e_{l1} + k_{competitor}e_{l2}) + (1-\lambda)\alpha(k_{competitor}(E_2(e_h) - pe_{l1}) + k_{own}(1-p)e_{l1}) - c_1 e_{l1}^2$ is maximized when $e_{l1} = \frac{k_{own}\lambda\alpha p + k_{own}(1-\lambda)\alpha(1-p)}{2c_j}$. Given q, player i does not deviate if $\pi_i(e_{lj} + \epsilon, \sigma_j) = \pi_i(e_{li}, \sigma_j)$ as $\epsilon \to 0$ and $\epsilon > 0$.Solving for q as a function of p,

$$q = \frac{k_{\text{own}}\left(c_1(\lambda-1) + c_2(-\lambda + (2\lambda-1)p + 1)\right)^2}{2c_1c_2(2\lambda-1)(-\lambda+(2\lambda-1)p+1)\left(k_{\text{other}} + k_{\text{own}}\right)} \tag{2}$$

   To complete the proof, we need to show that $min(e_{l1}, e_{l2}) = e_{l2}$ should hold in equilibrium. Suppose otherwise and let $min(e_{l1}, e_{l2}) = e_{l1}$. The same argument will follow for this case as well. Hence, $e_{l1} = \frac{k_{own}(1-\lambda)\alpha}{2c_1}$ and $e_{l1} = \frac{k_{own}\lambda\alpha p + k_{own}(1-\lambda)\alpha(1-p)}{2c_2}$. For $e_{l1} < e_{l2}$ to hold, we need to have $\frac{\text{c2}}{\text{c1+c2}} < \lambda < 1 \wedge \frac{\text{c1}\lambda - \text{c1} - \text{c2}\lambda + \text{c2}}{\text{2c1}\lambda - \text{c1}} < p < 1$. However in this case, it is shown that $q = \frac{\text{k1}(\text{c1}(\lambda - 2\lambda p + p - 1) + \text{c2}(-\lambda) + \text{c2})^2}{\text{2c1c2}(2\lambda-1)(\text{k1+k2})(-\lambda+(2\lambda-1)p+1)} < p$. From Lemma 1 we know that $F_1(e_h) = F_2(e_h) = 1$ should hold which is not possible with $q < p$ since F_1 first order stochastically dominates F_2.

Lemma 12. There exist a unique p value such that $F_1(e_h) = F_2(e_h) = 1$ holds.

Proof Let e^* be defined as $e^* = \{e : F_1(e) = 1\}$ when $p = 1$. If there exist a $p \in [0, 1]$ such that $F_2(e^*) = F_1(e^*) = 1$, then there exist an equilibrium where $e_h = e^*$. To show the existence of this p value, define $d(p') = (F_2(e^*) - F_1(e^*))\Big|_{p=p'}$, which is strictly increasing in p and decreasing in e. S

Case 1: $p' = 0$ From Lemma . we know that $q \geq 0$. Moreover, in equilibrium, $f_1(e) > f_2(e)$ since $c_2 > c_1$. Therefore we have $F_1(e^*) > F_2(e^*)$ which implies $d(0) < 0$.

Case 2: $p = 1$ Again from Lemma . we know that $0 < q < 1$. Hence, $d(1) > 0$. Hence, there exists a $p^* = p^*(c_1, c_2, \lambda)$ such that $d(p^*) = 0$ and $e_h = e^*_{p^*}$ in equilibrium.

In the equilibrium, player i decides on the strategy by maximizing the expected payoff.

$$\begin{aligned}\frac{\partial \pi_1(e_i, \sigma_j)}{\partial e_i} = & \quad \lambda\alpha(k_{own}F_j(e_i) + (k_{own} + k_{competitor})e_i f_j(e_i)) \\ & +(1-\lambda)\alpha(k_{own}(1 - F_j(e_i)) - (k_{own} + k_{competitor})e_i f_j(e_i)) - 2c_i e_i\end{aligned}$$

Solving $\frac{\partial \pi_1(e_i,\sigma_j)}{\partial e_i} = 0$, we have,

$$F_1(e) = k_1(ek_{own} + ek_{competitor})^{-\frac{k_{own}}{k_{own}+k_{competitor}}} + \frac{(e(k_{own}+k_{competitor}))^{\frac{k_{own}}{k_{own}+k_{competitor}}}(ek_{own}+ek_{competitor})^{-\frac{k_{own}}{k_{own}+k_{competitor}}}(2c_2e+\alpha(\lambda-1)(2k_{own}+k_{competitor}))}{\alpha(2\lambda-1)(2k_{own}+k_{competitor})}$$

$$F_2(e) = k_2(ek_{own} + ek_{competitor})^{-\frac{k_{own}}{k_{own}+k_{competitor}}} + \frac{(e(k_{own}+k_{competitor}))^{\frac{k_{own}}{k_{own}+k_{competitor}}}(ek_{own}+ek_{competitor})^{-\frac{k_{own}}{k_{own}+k_{competitor}}}(2c_1e+\alpha(\lambda-1)(2k_{own}+k_{competitor}))}{\alpha(2\lambda-1)(2k_{own}+k_{competitor})}$$

where k_i's are constants from the integration to be determined later as a function of players' cost parameters, c_1 and c_2, prize parameter α and division parameter, λ by imposing initial conditions found from equilibrium characterization.

4.3 Closed form solution for $\lambda = 1$ and $k_{own} = k_{competitor} = 1$

In the contest where the prize is allocated to the player with the highest effort level. Thus, we will take $\lambda = 1$ and analyze the game in which there is positive prize for the winner only. For $k_{own} = k_{competitor} = 1$, there exist a closed form solution which is represented in this section. Maximizing of the expected payoffs given in Eqn. 1, we obtain the effort distribution for each player.

$$F_1(e) = \frac{2c_2 e}{3\alpha} + \frac{k_1}{\sqrt{e}}$$

$$F_2(e) = \frac{2c_1 e}{3\alpha} + \frac{k_2}{\sqrt{e}}$$

for $e \in (e_l, e_h)$. Solving $F_1(\frac{\alpha p}{2c_1}) = q$ for k_1 gives $k_1 = (q - \frac{c_2 p}{3c_1})\sqrt{\frac{\alpha p}{2c_1}} = (-\frac{c_2 p}{12c_1})\sqrt{\frac{\alpha p}{2c_1}}$. Similarly, $F_2(\frac{\alpha p}{2c_1}) = p$ implies that $k_2 = \frac{2p}{3}\sqrt{\frac{\alpha p}{2c_1}}$.

$$F_1(e) = \frac{2c_2 e}{3\alpha} - \frac{c_2 p}{12c_1}\sqrt{\frac{\alpha p}{2c_1 e}}$$

$$F_2(e) = \frac{2c_1 e}{3\alpha} + \frac{2p}{3}\sqrt{\frac{\alpha p}{2c_1 e}}$$

Therefore, in a 2-player complete information all-pay auction in which only the winner obtains a positive prize, player 1's strategy σ_1^* is as follows:

1. $\alpha(\frac{\alpha p}{2c_1}) = q(c_1, c_2) > 0$, i.e, player 1 places an atom at effort level $\frac{\alpha p}{2c_1}$ of size q, which is uniquely determined by c_1 and c_2.

2. Player 1 plays a mixed strategy which is atom-less over the interval $(\frac{\alpha p}{2c_1}, e_h]$ where $e_h = \frac{a(8c_1+c_2)}{6c_1c_2}$ and its density is given by $F_1(e)$.

Player 2's strategy σ_2^* is as follows:

1. $\alpha_2(0) > 0$, i.e, player 2 places an atom at effort level 0 of size $\alpha_2(0)$, which is uniquely determined by c_1 and c_2.

2. Player 2 plays a mixed strategy which is atom-less over the interval $(\frac{\alpha p}{2c_1}, e_h]$ where $e_h = \frac{a(8c_1+c_2)}{6c_1c_2}$ and its density is given by $F_2(e)$.

Since $F_1(e_h) = F_2(e_h) = 1$, solving for e_h gives $e_h = \frac{ap(8c_1+c_2)^{2/3}}{c_1\sqrt[3]{128c_1^2 - 256c_1c_2 + 128c_2^2}}$. At e_h, both functions must be equal to one. Therefore, from $F_1(e_h) = F_2(e_h) = 1$, p is uniquely found as $p = \frac{2\sqrt[3]{2}\sqrt[3]{(c_2-c_1)^2(8c_1+c_2)}}{3c_2}$. Substituting p into the equation for e_h, it is found that $e_h = \frac{a(8c_1+c_2)}{6c_1c_2}$.

The expected efforts of players are:

- $E^1_{endo} = \frac{\alpha\left(16c_1^2+10c_2c_1+c_2^2\right)}{36c_1^2c_2}$
- $E^2_{endo} = \frac{\alpha\left(64c_1^2-32c_2c_1-5c_2^2+4\ 2^{2/3}(c_2-c_1)^{4/3}(8c_1+c_2)^{2/3}\right)}{36c_1c_2^2}$

leading the total expected effort of

$$E^{total}_{endo} = \frac{\alpha\left(64c_1^3 - 16c_2c_1^2 + 5c_2^2c_1 + 4\ 2^{2/3}\left(c_2-c_1\right)^{4/3}\left(8c_1+c_2\right)^{2/3}c_1 + c_2^3\right)}{36c_1^2c_2^2}$$

For $k_{own} \neq k_{competitor}$, the equilibrium characteristics are derived and are delineated in Figure 3 under comparison section for specific parameters.

5 Comparison

In this chapter, we will provide comparison of equilibria in exogenous and endogenous prize contests when $\lambda = 1$, $k_{own} = k_{competitor} = 1$. In order to do that we first focus on equilibrium characteristics which might help experimental studies of all-pay contests and then discuss the equilibrium implications for the designer.

5.1 Equilibrium Characteristics

First aspect we are going to analyze is the participation behavior of the players. For the exogenous prize contests, both players participate by mixing continuously on the interval $[0, \sqrt{\frac{V}{c_2}}]$. Whereas in endogenous prize contests, participation behavior of low type player (player 2 in our case since $c_2 > c_1$) is determined by the difference in abilities of two players. To be more precise, low type players stays out of the competition when her cost parameter c_2 is greater than four times of high type's cost parameter, that is, $c_2 \geq 4c_1$. If $c_2 < 4c_1$, the equilibrium is in mixed strategies when prize is set endogenously, whereas the equilibrium is always in mixed strategies with fixed prize. In that regard, endogenous prize model is a better representative of the all-pay nature of these contests. In other words, since effort is costly, one needs to make sure that she wins with high enough probability to cover the costs. Hence, there is more weight on the higher effort levels. When

$c_2 \geq 4c_1$, costs are always higher than potential benefits of choosing a positive effort level encouraging low type player to give up. However, this is never the case with the exogenous prize.

Secondly, equilibrium strategies of players between two types of contests significantly differ. As stated earlier, there is a pure strategy equilibrium given $c_2 \geq 4c_1$. Moreover, when $c_2 < 4c_1$ high type player places an atom at $\frac{\alpha p}{2c_1}$, low type places an atom at zero and both types mixes continuously from this point on until e_h whose size is determined by cost parameters of players and game parameters. Hence, unlike equilibrium strategies in fixed prize contest, both players have atoms in their strategies and also there is an interval such that effort levels in that interval is never played. These differences in strategies may shed light on experimental studies trying the understand how players perceive the prize.

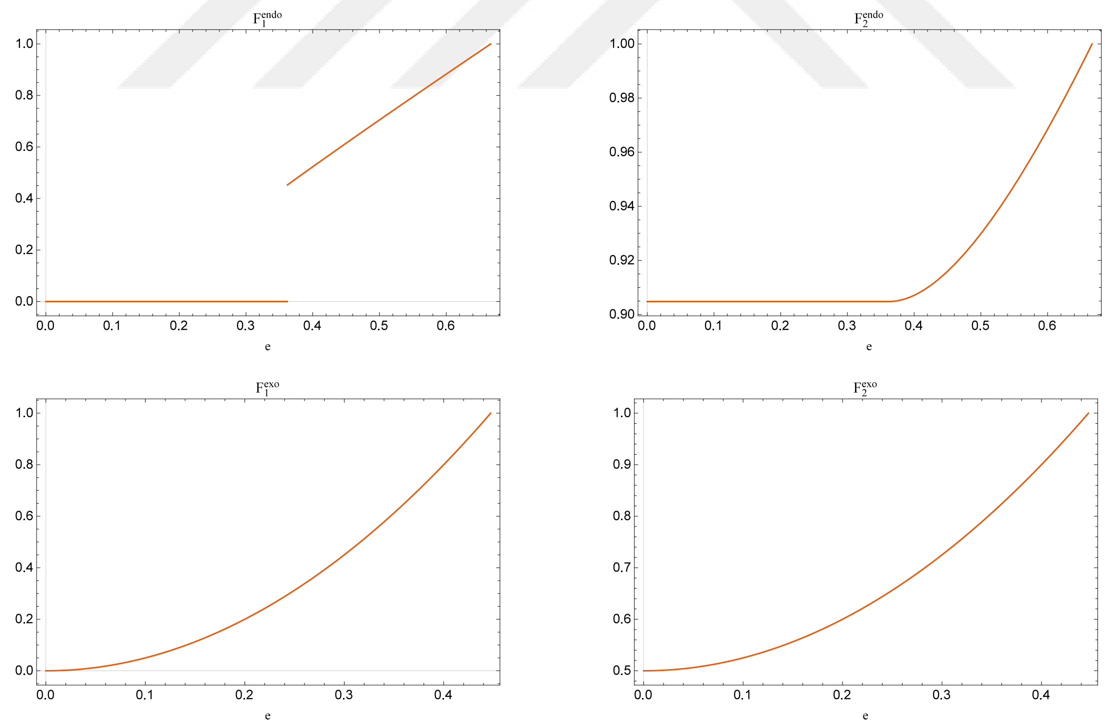

Figure 3: Distribution of Efforts for Equilibrium Strategies for $c_1 = 1$, $c_2 = 2$, $\alpha = 0.8$, $k_{own} = k_{competitor} = 1$ and $V = 0.4$

In Figure 3, we choose prize parameter α in endogenous prize contest and prize V in exogenous prize contest so that total expected prize distributed in both contests are the same given the cost parameters of players. First thing to notice is that, when auctioneer offers the same prize on expectation in both types of contests, endogenous prize contests achieve higher maximum effort. Secondly, expected total effort of high type player is higher in endogenous prize contests.

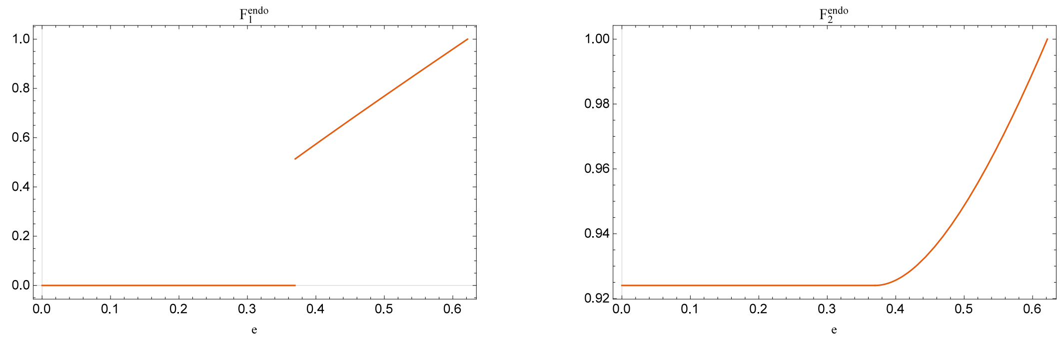

Figure 4: Distribution of Efforts for Equilibrium Strategies for $c_1 = 1$, $c_2 = 2$, $\alpha = 0.8$, $k_{own} = 1$ and $k_{competitor} = 0.8$

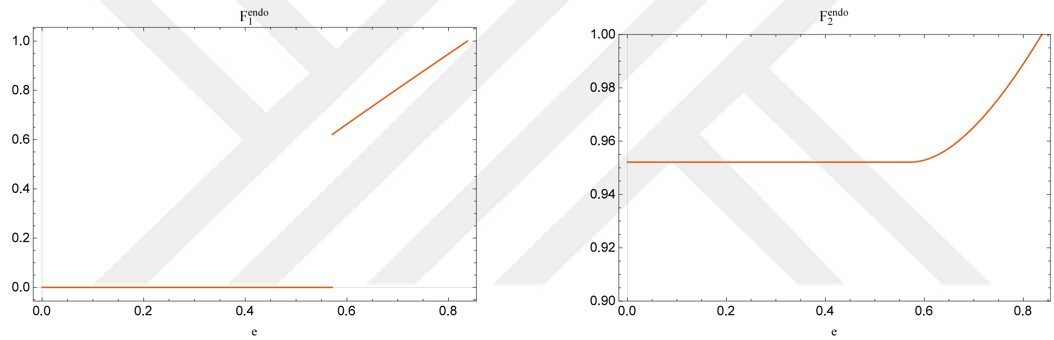

Figure 5: Distribution of Efforts for Equilibrium Strategies for $c_1 = 1$, $c_2 = 2$, $\alpha = 0.8$, $k_{own} = 1.5$ and $k_{competitor} = 0.8$

To illustrate the effect of weights in determining the prize in the endogenous prize contest, in Figure 4, we keep k_{own} constant and decrease $k_{competitor}$ to 0.8 and in Figure 5 we increase k_{own} to 1.5 and decrease $k_{competitor}$ to 0.8. We observe that decreasing $k_{competitor}$ lower the maximum effort level and increases the probability of low type giving up by bidding zero. The former observation can be explained by the decrease in the expected prize since the support from the competitor is now less than before. The same reasoning applies to latter observation since such a change affects low type player more than the high type. When we increase k_{own}, we observe an increase in the maximum effort level and also in the expected total effort level of high type player. On the other hand, expected total effort level of low type player decreases since increasing weight in own effort is more beneficial for the high type.

5.2 Equilibrium Implications

From the designers perspective, the main motivation of this research is to maximize the total expected effort produced in the equilibrium. To compare two types contest in terms of designer surplus when both contests achieve the same total expected effort level the prize value V for the exogenous and the prize parameter α in the endogenous prize contest is decided accordingly. In the analysis we set $c_2 = kc_1$ where $1 < k < 4$, ensuring both contests have mixed strategy equilibrium. Solving for the prize parameter α that yields same total expected effort given V and $c_1 = 1$ is found as,

$$\alpha = \frac{24\sqrt{k}(k+1)}{4\ 2^{2/3}(k+8)^{2/3}(k-1)^{4/3} + (k+8)((k-3)k+8)}$$

Total expected effort levels and expected surplus for both type of contest are plotted against prize parameter α for three different cost scenarios, where the surplus of the designer is calculated as total expected effort - total prize. We observe that when the total expected effort levels are the same, designer surplus is always higher endogenous prize contest.

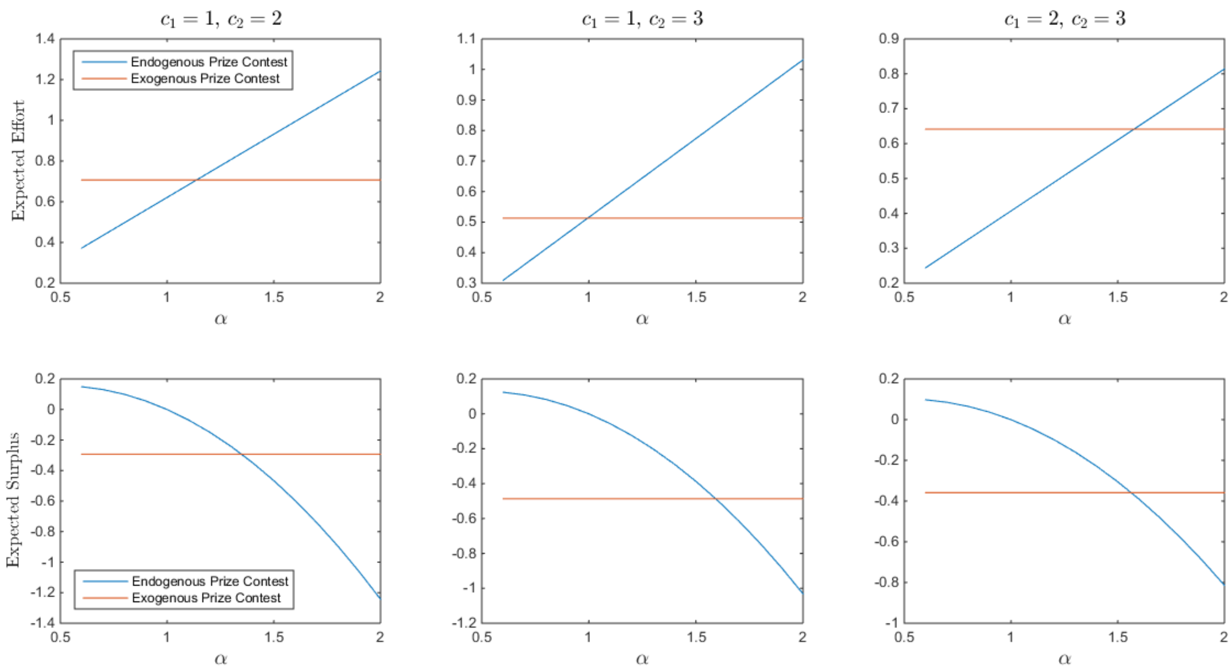

Figure 6: Total Expected Effort and Expected Surplus vs Prize Parameter

6 Conclusion

This study analyzes all-pay contests where the prize is a linear function of players' bids. To the best of our knowledge, there is no work investigating the equilibrium under such prize formation. We first show that there exist a unique equilibrium in this game and then characterize this equilibrium analyzing its effect on motivating effort. Unlike the findings in contests where prize is predetermined, we show that there is a unique equilibrium in pure strategies depending on the cost parameters of players. This shows that there is a threshold above which the difference between the types of players encourage low type to drop out. Furthermore, the characteristics of mixed strategy equilibrium also differs from the ones in the exogenous prize contests in which high type plays an atom-less strategy. These aspects could be interesting to look at from an experimental perspective. We, then, compare the total expected effort in exogenous and endogenous prize contests. Conditioning on total effort is shown to motivate higher effort at lower cost to the auctioneer. An interesting direction to follow could be considering effect of setting a minimum prize other than zero as in our case, that is, what if there is a combination of a fixed prize and endogenous prize for the winner. Future research may also include generalization of the model to n players which may differ in participation behavior from the current literature. Moreover, different functional forms for prize and cost functions can be considered. Last, informational asymmetries through prize structure or heterogeneity between players might be an interesting question.

References

[1] R. Siegel, "All-pay contests," *Econometrica*, vol. 77, no. 1, pp. 71–92, 2009.

[2] A. L. Hillman and D. Samet, "Characterizing equilibrium rent-seeking behavior: A reply to tullock," *Public Choice*, vol. 54, no. 1, pp. 85–87, 1987.

[3] A. L. Hillman and J. G. Riley, "Politically contestable rents and transfers," *Economics & Politics*, vol. 1, no. 1, pp. 17–39, 1989.

[4] M. R. Baye, D. Kovenock, and C. G. De Vries, "Rigging the lobbying process: an application of the all-pay auction," *The american economic review*, vol. 83, no. 1, pp. 289–294, 1993.

[5] Y. Barut and D. Kovenock, "The symmetric multiple prize all-pay auction with complete information," *European Journal of Political Economy*, vol. 14, no. 4, pp. 627–644, 1998.

[6] Y.-K. Che and I. Gale, "Expected revenue of all-pay auctions and first-price sealed-bid auctions with budget constraints," *Economics Letters*, vol. 50, no. 3, pp. 373–379, 1996.

[7] B. Moldovanu and A. Sela, "The optimal allocation of prizes in contests," *American Economic Review*, pp. 542–558, 2001.

[8] B. Moldovanu and A. Sela, "Contest architecture," *Journal of Economic Theory*, vol. 126, no. 1, pp. 70–96, 2006.

[9] E. Amann and W. Leininger, "Asymmetric all-pay auctions with incomplete information: the two-player case," *Games and Economic Behavior*, vol. 14, no. 1, pp. 1–18, 1996.

[10] Y. Barut, D. Kovenock, and C. N. Noussair, "A comparison of multiple-unit all-pay and winner-pay auctions under incomplete information," *International Economic Review*, vol. 43, no. 3, pp. 675–708, 2002.

[11] R. M. Sheremeta, "Experimental comparison of multi-stage and one-stage contests," *Games and Economic Behavior*, vol. 68, no. 2, pp. 731–747, 2010.

[12] M. Gradstein and K. A. Konrad, "Orchestrating rent seeking contests," *The Economic Journal*, vol. 109, no. 458, pp. 536–545, 1999.

[13] R. Stracke, "Contest design and heterogeneity," *Economics Letters*, vol. 121, no. 1, pp. 4–7, 2013.

[14] B. Moldovanu, A. Sela, and X. Shi, "Contests for status," *Journal of Political Economy*, vol. 115, no. 2, pp. 338–363, 2007.

[15] J. Xiao, "Grouping players across all-pay contests," 2013.

[16] J. Xiao, "Asymmetric all-pay contests with heterogeneous prizes," *Journal of Economic Theory*, vol. 163, pp. 178–221, 2016.

[17] S. O. Parreiras and A. Rubinchik, "Group composition in contests," *Haifa University*, 2015.

[18] J. A. Amegashie, "An all-pay auction with a pure-strategy equilibrium," *Economics Letters*, vol. 70, no. 1, pp. 79–82, 2001.

[19] H. Vartiainen, "Comparing complete information all pay auctions," 2007.

[20] J. Xiao, "Ability grouping in all-pay contests," *Browser Download This Paper*, 2015.