ASSETS, COUNTERPARTY RISK AND BANK NETWORKS

by

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Assets, Counterparty Risk and Bank Networks

Abstract

This thesis consists of two essays discussing shock propagation and counterparty risk in financial networks. The first essay explores the resilience of financial networks to systemic shocks under regulatory solvency constraints. We generalize the contagion under fire sale model of Cifuentes, Ferucci and Shin (2005) by allowing financial institutions to be connected through assets they hold in their portfolios. We simulate the model under different combinations of debt and asset networks and observe how shocks spread across markets. In the second essay, we provide a dynamic model of financial contagion to identify the source of systemic risk when banks can borrow from each other as well as from external creditors to invest in a risky portfolio. Our framework differs from earlier work as it describes how a bank's value function depends on counterparties' risky behavior. We analyze the implications of the model in the case of a ring network of banks where liabilities of a bank are held by a single counterparty. We show that the network effect is positive for the banks whose risky investment is less than the average of the rest. In other words, we show that the counterparties' risky behavior increases the probability of default of a bank. We also show that the uniform Value at Risk (VaR) constraint doesn't reflect the real probability of default when the network effect is considered. Therefore, we propose a policy function which assigns different VaR values for each bank in the network, and obtain that the target level imposed by the social planner is achieved.

Keywords: Financial Networks, Contagion, Systemic Risk, Value at Risk

Varlıklar, Komşu Riskleri ve Banka Ağları

Özet

Bu tez, şokların yayılmasını ve komşu risklerini irdeleyen iki tane bölümden oluşmaktadır. İlk bölüm, devlet düzenlemeleri altında finansal ağların şoklara olan direncini anlamaya çalışmaktadır. Bu amaçla, Cifuentes , Ferruci ve Shin (2005) tarafından ortaya konan model, finansal kurumların portfölyelerinde tuttukları ortak varlıkları da dikkate alarak genelleştirilmiştir. Borç ve varlık ağlarının farklı kombinasyonları üzerinden simulasyonlar yapılmış ve şokların nasıl yayıldığı gözlemlenmiştir. İkinci bölümde ise, bankaların hem birbirlerinden hem de yatırımcılardan topladıkları fonlar ile portfölyö yatırımı yaptıkları bir ortamda, sistemik riskin kaynağını saptamak adına dinamik bir finansal yayılma modeli önerilmiştir. Ortaya konan modelin, önceki modellerden farkı, bankaların değer fonksiyonlarının komşu bankaların riskli davranışlarını içeriyor olmasıdır. Modelin çıkarımlarını halka şeklindeki bir ağda inceledik. Halka ağı, her bankanın borcunun sadece baska bir banka tarafından tutulduğu ağ şeklidir. Ağ efektinin, diğer bankaların ortalamasından daha az oranda riskli yatırım yapan bankalar için pozitif olduğunu gösterdik. Diğer bir deyişle, komşularının aldığı riskin, bankanın batma riskini arttırdığını gösterdik. Bununla beraber, ağ efekti göz önüne alınınca, tek bir Riske Maruz Değer (RMD) ölçütünün gerçek batma olasılığını yansıtmadığını gösterdik. Bu yüzden, ağ içindeki her bir bankaya farklı bir RMD değeri atayan bir fonksiyon önerdik. Bu sayede, sosyal planlayıcının hedef olarak koyduğu RMD değerinin ulaşıldığını ispat ettik.

Anahtar Sözcükler: Finansal Ağlar, Bulaşıcılık, Sistemik Risk, Riske Maruz Değer

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Contents

1 Introduction

Recent economic events have highlighted the importance of networks for capturing the potential effects of shocks to the financial system. Policy actions are reconsidered accordingly and stress-test frameworks are motivated by incorporating the impact of the network structure to the resilience of the financial system to external shocks. The aim of this master thesis is twofold. In the first chapter, it attempts to advance the model proposed by Cifuentes, Ferrucci and Shin (2005) by adding multiple assets through which the financial system is handled as the combination of two seperate categories of networks. On the one hand, banks have mutual debts to each other, as it is the dominant type of connections in the financial network literature. Yet, we additionaly integrate a setup in which banks share common assets, which is the second category of network. In the second chapter, we analyze the role of the counterparty risk in the value function of the banks connected through the interbank debt network. The aim of the proposed setup is not to trace the effect of the shock but to understand the result of the counterparty risky behavior due to the network effect.

In the model proposed by Cifuentes, Ferrucci and Shin (2011), there is a set of financial institutions interlinked through mutual debts to each other. They are required to satifsy a threshold ratio, called the capital asset ratio. It is the ratio of the bank's equity value to the mark to market value of its assets. If any bank violates the ratio, it must sell its assets until the capital asset ratio is equal or above the required ratio. Yet, the prices of the assets endogenously decline, as banks sell their assets. This process causes some other banks to find themselves violating the capital asset ratio as well, depending on the size of the shock and the structure of the connection between the banks.

We modify the CFS model above for two reasons. Firstly, integrating different assets and liability networks and comparing them could give the role of the asset commonality among the financial institutions in times of financial contagion. We ask as to what extent the commonality of assets in banks' balance sheets prepares the ground for an initial shock to propagate. Secondly, we try to identify the asset shocks which result in the decrease of the prices of other assets. This is the recently obversed phenomena in the last financial crises in the U.S.(See, Longstaff (2010)) The shock is represented as the unexpected decline in the cash flow of the asset. We try to provide boundary levels for different type of shocks under different networks. Instead of one asset, we have multiple assets and this feature brings another network structure into analysis. The financial network consists of two sub-networks. Firstly, it is the network determined by banks' debt obligation to each other. Secondly, it is the allocation of assets among financial institutions. It provides a measure indicating how likely for any set of banks are exposed to same shock through the assets in their portfolios. We present an algorithm to obtain the equalibrium when the CFS model is extended with the multiple assets.

In the second chapter, our aim is to investigate how banks' collection of funds from outside increase the probability of default of the banks in the network. In particular, by mixing the two existing models , we examine how counterparty risk arises in an environment where banks have claim on each other, collect deposit and invest in a portfolio. Taking the counterparty risk into account, our model explains why the value at risk value imposed by the goverment fails to achieve the target level. In addition, we illustrate how the heterogeneity among the banks in the network generates the counterparty risk.

We analyze for an infinite horizon economy with n banks. At each date, banks choose how much to invest in the risky asset and to hold the riskless asset. We assume that banks need to borrow from other banks and collect deposits to invest in a portfolio. Therefore, banks are connected through their liability sides. This connectedness over the lending/borrowing relation generates the network structure between the banks. Banks are heterogenous in the sense that they collect different amount of deposits. After shocks on asset returns are realized, banks make payment to each other and to the depositors. The payment equilibrium arises as in the Eisenberg and Noe model (2001). We assume that the depositors has priority over the interbank debts. Hence, if any bank's portfolio return is not sufficient to pay its debts, the payment are done in an order within which the depositors have priority. The bank's income is comprised of the payments from other banks and the portfolio return. If the bank's income is not sufficient to pay its debts, the bank defaults. Therefore, the bank's choice is to minimize probability of default and ,at the same time, to maximize expected cash flow as in Ibragimov, Jaffee and Walden (2011).

One of the implications of our model is shown under the ring network in which all liabilities of a bank are held by a single counterparty. We show that the banks that collects the lower level of deposit than the average of the rest has a positive network effect. It means that for any of the banks in the network, the counterparty risk rises as the bank collects less deposit than the rest. On the contrary, the counterparty risk is zero for the bank that has a higher level of deposit then the average of the rest. It is intiutive in the sense that the default risk is already highest for the banks that collect a higher level of deposit because their portfolio volume is higher. Therefore, the counterparty risk is expected to be zero.

Secondly, in the ring network, we show that the probability of survival is lower than the one that is imposed the VaR constraint due to the network effect and the seniority of deposits. Hence, the VaR level doesn't reflect the real probability of default when banks are put in an interbank lending/borrowing network. Subsequently, our results suggest that as the level of the deposits increases, the probability of survival decreases for all banks in the network. The probability of survival is equal to the level imposed by the VaR constraint only if banks don't collect deposit and invest in portfolio. In other words, funds that are collected from the outside of the network and invested in a portfolio increase the default probability for all banks in the network. Since banks have claims to each other through the interbank borrowing/lending network, an increase in the level of deposit collection and portfolio volume results in a decrease for the probability of survival of all the banks.

Lastly, in the policy suggestion, we show that the network effect becomes zero, if the government announces different level of VaR contraint for each bank. Although it's practical implication is nearly impossible, it gives a basic idea about the regulation strategy. If the bank collects more deposit and invest in a portfolio, than the regulator should require a lower VaR constraint. In other words, the regulator should demand a lower VaR for the banks who collect deposit more than the rest of the average to reduce the network effect.

1.1 Related Literature

The study of financial networks dates back to the work of Allen and Gale (2000) in which they show that the possibility of contagion depends heavily on the network structure between the banks. They conclude that more equally distributed interbank debts reduce the possibility of contagion. As it is also our starting point, the framework proposed by Eisenberg and Noe (2001) also demonstrates that an initial shock to a bank in a network might cause other banks' default, depending on the structure of the network. Their model forms the basis of many other models within which financial institutions are part of single payment mechanism; see, e.g., Elsinger (2009), Rogers and Veraart (2013), Gourieroux et al. (2013), Acemoglu et al. (2015) There are two important assumptions in their model, which are also followed here. The first one is that remaining assets are distributed pro rata to creditors when any default occurs. The second one is the limited liability which ensures that the payment of a bank can be at most its total income. For the model we propose in chapter 2, our motive is not to demonstrate quantitatively how the initial shock amplifies through the network. (see, for example Glessarman and Young (2014), Gai et al. (2011), Acemoglu et al. (2015)) Yet, we used their model to relate the bank's risky invesment decisions to each other and analyze the bank's value accordingly.

The literature on financial networks is heavily devoted to figure out the trade-off between the risk-sharing and the likelihood of the financial contagion. On the one hand, increasing connections over networks lead to diversification of risky choices. But, they also create channels through which shocks can spread. In their work, Cabrales, Gottardi and Redondo (2016) examine this trade-off with respect to the properties of the distribution of the shocks. They find different optimal choices of networks for different class of distributions. Basically, networks that features uniform level of exposures among the nodes, is proved to be optimal when the distribution of shocks belong to a canonical class. On the contrary, networks having sparser connectivity performs better for the distribution of shocks with mass points. On the same line, Elliot et all (2014) and Acemoglu et al (2015) charactarize the structure of networks and the magnitude of shocks to analyze financial contagion. The former focuses on role of the cross-holdings of assets in the propogation of shocks. The latter inspects the fragility of the financial system over the intensity of interconnections and the magnitude of shocks. The findings of these papers, like the others, are given under various type of conditions. For example, *Elliot et al.* (2014) defines two different phenomena, namely the level of integration and diversification, and examines the financial fragility of the networks over the degree of these terms. Therefore, the question about the trade off between the risk-sharing and the increasing possibility of the contagion doesn't have a clear cut answer. The answers are subjected to change under different conditions. Our model in chapter 2 suggests that the heterogeneity among the banks could be the potential answer in the sense that the network effect is not the same for all banks in the network, it depends on bank-level characteristics.

In the same line of literature, Ibragimov, Jaffee, Walden (2011) discuss risks evaluated by individual intermediaries versus society. Their model provides an explanation how risk diversification, which is the optimal choice for an individual bank, may be suboptimal for society. We contribute to this discussion by integrating networks into analysis. We use their model to show that how counterparties' risky choices affect the banks value function. We formally define the network effect and show the role of the interconnections between the banks in determining banks' values.

The amplification of shocks over different markets attracts a huge attention after the

2008 financial crises. (see, for example, IMF Survey: New Channels Spread U.S. Subprime Crisis to Other Markets, Longstaff (2010)) The related literature on the discussion we have in chapter 1, is mainly on regulatory constraints and fire sales. (see, for example, Brunnermeier et al (2012), Shleifer and Vishny (2011)) Since our aim ,in that chapter, is to generalize the model proposed by Cifuentes, Ferrucci and Shin(2005) with multiple assets, our discussion is directly related with the propogation of shocks over seperate markets. In the CFS model, the setup is a system of interconnected financial institutions and there is only one asset whose price declines as banks sell it to satisfy regulatory constraints. The generalization enables us to compare the performance of different network structures as well as to introduce high-dimensional networks within which banks are connected through interbanks debts and common assets.

2 Liquidity Risk and Asset Holding

Our model is a static one consisting of n financial institutions and m assets that are expected to generate a cash flow. The chain of events begin with a shock to the cash flow of one of the m assets. When the holders of the hitted asset face a difficulty in satisfying the capital asset ratio, they liquidate their balance sheet by selling assets. Since prices are determined endogenously, the prices of the asset declines, as banks choose to sell the asset. After all banks satisfy the capital asset ratio, the interbank debt payments take place. If any bank finds itself in a situation where its income is less than its liabilities, it defaults and the proportional payments are implemented as in the Eisenberg Noe (2000) setup. Subsequently, the new payment equilibrium results in repetition of the above process until all the remaining banks satisfy the required capital asset ratio.

2.1 Model

The liability of bank i to bank j is represented by L_{ij}. By definition, $L_{ij} \geq 0$ and L_{ii} =0 for all i. Thus, the total liability of bank i is the following:

$$L_i = \sum_j L_{ij}$$

The amount of asset j hold by bank i is denoted by A_{ij}. The asset allocation matrix A is:

$$\text{A} = \begin{array}{cc} & \begin{array}{ccccc} Asset_1 & Asset_2 & Asset_3 & ..... & Asset_M \end{array} \\ \begin{array}{c} \text{Bank˙1} \\ Bank_2 \\ Bank_3 \\ . \\ . \\ \text{Bank˙N} \end{array} & \begin{bmatrix} A_{11} & A_{12} & A_{13} & . & A_{1M} \\ A_{21} & . & . & . & . \\ A_{21} & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ A_{N1} & . & . & . & A_{NM} \end{bmatrix} \end{array}$$

We assume that each bank can hold only one type of asset. Assets are identical in terms of their total supply and initial price but they may differ according to their price elasticity. Each asset has a price and they are expected to generate a positive cash flow $\tilde{R}$. But the realized one is $R_j = \tilde{R}_j - \epsilon_j$ where ϵ represents the magnitude of the given shock. Initially, all banks borrow money from each other such that $\alpha_i \geq L_i$ for all i where α denotes income minus deposit payments. In other words, all of the institutions have financially healthy initial positions. The deposit side includes the assets and the cash generated by the assets from the previos period and the debts given to the other institutions. The liability side consists of the issued debts to the rest of the network. Hence, all banks satisfy the following equation initially:

$$\sum_{j\varepsilon M}(P_j + \tilde{R}_j)A_{ij} + \sum_{j\varepsilon N} L_{ij} - D_i \geq L_i \text{ for all } i\varepsilon N$$

Shocks are originated from the changes in cash flows through $\tilde{R}_j$. The amplification of a shock of size ϵ to asset j arises through two different channels:

- The set of banks holding asset j might have liabilities to other banks, we call it debt channel. Suppose bank i holds asset j and borrows from bank k. If bank i is unable to pay its debt after experiencing a reduction in its cash flow due to shock to the asset j, bank k will suffer from the shock as well.

- Yet, there is another channel which is not expressed by the liability matrix. Suppose, bank k is not one of the creditors of bank i ($L_{ik} = 0$). Moreover, we can assume that bank k is not a creditor of creditors of bank i. By assuming so, we weaken the debt channel. But, we assume that bank k shares the same asset or same group of assets with these creditor banks. That is, there exists $m \, \varepsilon M$ and $j \, \varepsilon$ N such that $L_{ij} \neq 0$ and $A_{km}, A_{jm} \geq 0$. The second channel of contagion works over asset commonality. When bank i fails to pay its debts, creditor banks might unexpectedly liquidate their asset m at market price which is determined endogenously by quantity demand and supply. Any reduction in the price of asset m will result in tranmission of shock to bank k.

Therefore, the propogation of a shock depends on its size ϵ, the liability matrix L and the asset Matrix A.

The interbank liabilities are of equal seniority and if any bank defaults, the payments will be proportional to the face value of it's liabilities. To this end, the proportion matrix π is defined as follows:

$$\pi_{ij} = \begin{cases} \frac{L_{ij}}{L_i} & \text{if } L_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

That is, the payment by bank i to bank j is given by $C_i \pi_{ij}$ where C_i is the market value of bank i liabilities. Subsequently, the contribution of the payments from other banks to the cash flow of bank i is:

$$\textstyle\sum_j C_j \pi_{ji}$$

The Eisenberg Noe(2000) framework imposes two conditions which are followed here in the same manner. Limited Liability states that $C_i \leq L_i$ for all i and priority of debt states that equity value of bank i is allowed to be greater than zero only if L_i=C_i. In the case where banks are allowed to hold only one type of assets, we obtain the clearing payment vector C=(C_1...C_n) by the following fixed point problem:

$$C_i = max[min[L_i, p_j^* A_{ij} + R_j A_{ij} + \sum_j C_j \pi_{ji} - d_i], 0] \tag{1}$$

where p_j^* is after-shock price of asset j and d_i is the deposit payment of bank i.

With matrix notation, it is

$$C = max[min[L, AP^* + AR + \pi^T C - D_i], 0]$$

where $P^* = (p_1^*, ..., p_m^*)^T$ is the vector of prices and $R = (R_1, ..., R_m)^T$ is the vector of realized cash flow for each asset.

2.2 Capital Ratio and Equilibrium Pricing of Assets

Following the CFS framework, we impose for each bank to satisfy the capital asset ratio $\bar{r}$ over the equation :

$$r_i = \frac{\sum_{M_1}(p_{im}A_{im}) + \sum_{M_1} R_{im}A_{im} + \sum_j C_j\pi_{ji} - C_i - D_i}{\sum_{M_1} p_{im}(A_{im} - s_{im}) + \sum_j C_j\pi_{ji} + \sum_{M_1} R_{im}A_{im}} \geq \bar{r}$$

Again, one asset case reduces the equation of capital asset ratio to the following equation:

$$r_i = \frac{p_j A_{ij} + R_j A_{ij} + \sum_j C_j \pi_{ji} - C_i - D_i}{p_j(A_{ij} - S_{ij}) + \sum_j C_j \pi_{ji} + R_j A_{ij}} \tag{2}$$

Then, the optimal amount of S_i, which is the minimum amount of the illiquid assets banks need to sell in order to safisy $\bar{r}$:

$$S_i(p_j) = \begin{cases} \frac{(\bar{r}-1)(p_j A_{ij} + \sum_j C_j \pi_{ji} + R_j A_{ij}) + C_i + D_i}{p_j \bar{r}} & S_i \leq A_i \\ A_i & S_i > A_i \\ 0 & S_i \leq 0 \end{cases}$$

where p_j is the initial price of the asset j.

When banks holding asset j simultaneously decide how much asset j to be sold to satisfy $\bar{r}$, the pricing function is as follows:

$$p_j^* = e^{-\alpha_j \sum_i S_i(p_j^*)} \tag{3}$$

2.3 Algorithm for Obtaining Equilibrium

Suppose, we have n banks, 3 assets and each bank holds only one asset. Any shock to the cash flow of asset 1, that is ϵ_1 is given to the system.

Step 1 : Fix everything in (2), find s_{i1} as a function of p_j which is the minimum amount of asset 1 sold to satisfy $\bar{r}$ for all banks holding asset 1. After obtaining s_{i1} for all i, we put them into equation (3), which gives the new price for asset 1, p_1^1.

Remark 1 : If for any R and $\sum(c_j \pi_{ji})$ value , if any bank is unable to meet the given ratio, then the bank is required to sell all of its assets.

Step 2: Calculating the equation (1) for each bank with the new price for asset 1, p_1^1, yields the new clearing vector, we call it C^1. Given C^1, if there is no bank such that $\alpha_i < L_i$, then the shock causes only the reduction of the asset price itself. We can terminate the algorithm.

Definition 1: If an asset shock causes only the reduction of the asset price itself, it is called *idiosyncratic* and *isolated shock*.

Definition 2: If an asset shock causes only any subset of the banks holding that asset to default, it is called *isolated default shock*.

Step 3: If the given shock causes $\alpha_i < L_i$ for some bank i, we turn back to equation (2) and calculate S_i for the remaining banks. This is the first step for tracing the effects of the initial shock to the other parts of the network. Calculating s_{i2} and s_{i3} will give the required amount of assets of type 2 and 3 to be sold, for banks holding them to satisfy $\bar{r}$.

The new prices for asset 2 and 3 are p_2^1 and p_3^1, respectively. At this step, we capture the effect of the shock to asset 1 on the prices of asset 2 and 3.

Definition 3: If an asset shock causes the price reduction for other assets, it is called *idiosyncratic and knock − onshock*.

Step 4: Given p_1^1, p_2^1 and p_3^1, we look for the clearing vector in $Eq.(2)$ again and call it C^2.

At the end of step 4, if for any bank, holding asset 2 or asset 3, $\alpha_i < L_i$, then we conclude that the initial shock has lead some banks to default.

Definition 4: If an asset shock causes financial contagion as described in the 2^{nd} result, it is called *contagious default shock*.

Step 5: We have p_1^1, p_2^1 , p_3^1 and C^2. We turn back to $Eq.(2)$ and look whether the capital asset ratio is satisfied for all banks.

If the ratio is satisfied for all banks, the shock is eliminated and it causes only first-round effects.

Definition 5: An asset shock is called $first - round\ effect\ shock$, if it is eliminated after step 5.

After step 5, if there are banks that are unable to satisfy the capital asset ratio, then these banks are forced to sell their remaining assets. This process will yield second-round effects.

Step 6: We first determine the prices then put them in equation (1) and finally obtain the clearing vector, C^3. Among the remaining banks, we look whether $\alpha_i < L_i$. Any bank whose cash flow is less than its liabilities becomes insolvent. Banks that are insolvent just after C^3, are called second-round insolvent banks.

This step is identical with step 5, only differing in terms of the prices and clearing vector. We repeat the process until all banks satifsy the given ratio.

Definition 6: Banks which default after C^n, are called $(n-1)^{th}$-round defaulting banks.

Definition 7: Having obtained p_1^{n-1}, p_2^{n-1} , p_3^{n-1} and C^n, if any bank is unable to satisfy the capital asset ratio, then we define the initial shock as $n^{th} - round\ effect$ shock.

We terminate the algorithm when the remaining banks satisfy the capital asset ratio $\bar{r}$.

Definition 8 (*Equilibrium*): Given the initial shock to the cash flow of asset j, the liability network π_{ij}, and the asset network A_{ij}, if the interbank payments C_{ij}, the prices of the assets $(p_1, ..., p_m)$ and all banks satisfy the equations (1), (3) and (2), then $(\mathbf{C},\mathbf{p})$ is the equilibrium.

2.4 Simulations

Here, we point out the effect of the asset commonality under different network structures. Banks are considered as identical institutions in order to keep the network structure in the center of attraction. For the same reason , we set the elasticity of all assets equal. Given that the total quantity is 15 for all assets, their elasticity is equal to a value such that it ensures the price will decline by 50% if all assets are sold. Initially, all assets have the same price and the same expected cash flow, both of them are 1. Yet, the realized cash flow is 1-ϵ. We characterize the size of the given shock as the portion of the balance sheet that is wiped out. For instance, if the realized shock to the asset i is ϵ_i, then the portion that is wiped out from the asset side of the balance sheet of the bank j is $A_{ji}\epsilon_i$ divided by the value of the total assets.

The number of banks is 9 and they are divided into 3 groups. Each group holds one type of assets. So, the number of different assets is 3. The balance sheet of a typical bank is as follows:

Balance Sheet	
Assets	Liabilities
5 Illiquid assets	7 Deposits
5 Liquid assets(Cash Flow)	15 Interbank liabilities
15 Interbank assets	3 Equity
25 Total Assets	25 Net worth and liabilities

2.5 The Ring Liability Network and Totally Seperated Asset Network

The ring financial network defines an interbank debt relation in which bank i is the single creditor of bank j. Additionally, totally seperated asset network characterizes an asset network in which bank i represents the set of banks holding only asset i. As a result, banks holding only asset i become the single creditor of banks holding asset j. The figure 1 below shows the structure of the ring liability network and totally seperated asset network:

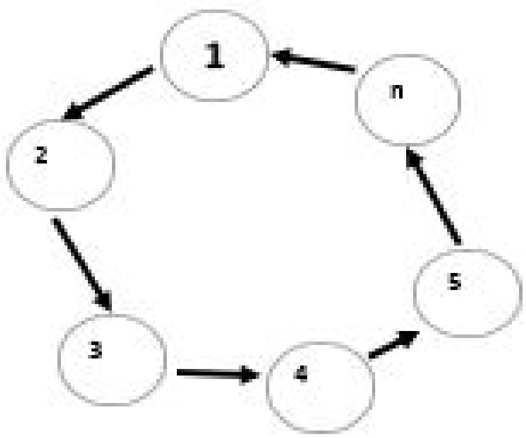


Figure 1: The network on the left represents the ring debt network. Node i includes all banks holding asset i. The matrix below imposes that banks in node i hold only asset i.

$$\begin{array}{cc} & \begin{array}{ccccc} Asset_1 & Asset_2 & Asset_3 & ..... & Asset_N \end{array} \\ \begin{array}{c} \text{Node˙1} \\ Node_2 \\ Node_3 \\ . \\ . \\ \text{Node˙N} \end{array} & \begin{bmatrix} A_{11} & 0 & 0 & . & 0 \\ 0 & A_{22} & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & 0 & 0 & A_{NN} \end{bmatrix} \end{array}$$

Result 1: Let α denote the portion of the balance sheet that is wiped out after the given shock. For the financial networks composed of the ring liability network and the totally seperated asset network, for any idiosyncratic shock, there exists α_l^R, α_h^R and α_*^R such that if

- $\alpha_h^R > \alpha > \alpha_l^R$, then α_1 is isolated default shock.
- $\alpha_*^R > \alpha \geq \alpha_h^R$, then α_1 is knock-on shock.
- $\alpha \geq \alpha_*^R$, then α is contagious default shock.

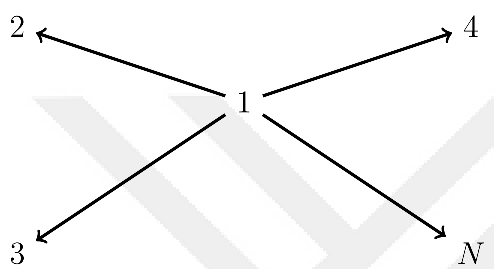
Given the numerical values as described above, the result for the ring liability and totally seperated asset network is given in table 1. For example, if the shock to the asset 1 wipes out the portion of the balance sheet amounting to the any rate between 8% and 11.1%, then the shock will be an isolated default shock. If it amounts to a ratio between 11.1% and 11.5%, the shock will not be a knock-on shock and reduce the price of the asset held by the creditor banks. Finally, we find that, if the part of the balance sheet that is wiped out after the shock, amounts to more than 11.4%, then the shock will cause the default of creditor banks as well.

2.6 The Star Liability Network and Totally Seperated Asset Network

The star financial network constitutes a debt network such that only bank i has debt obligations to other banks. Again, totally seperated asset network characterizes an asset network in which bank i represents the set of banks holding only asset i. Here, banks holding asset 1 are debtor to the all other banks holding different types of assets. The figure below shows the structure of the star liability network and totally seperated asset network:

Table 1: Shock to Balance Sheet Ratio

Totally Seperated Asset Network(n=3)		
Shock	Ring Liability	Star Liability
Isolated Default:	$11.1 > \alpha_1 > 8$	$17 > \alpha_1 > 8.6$
Knock-on:	$11.5 > \alpha_1 \geq 11.1$	-
Contagious Default:	$\alpha_1 \geq 11.5$	$\alpha_1 \geq 17$



$$
\begin{array}{c c}
 & \begin{array}{ccccc} Asset_1 & Asset_2 & Asset_3 & ..... & Asset_N \end{array} \\
\begin{array}{c} \text{Node˙1} \\ Node_2 \\ Node_3 \\ . \\ . \\ \text{Node˙N} \end{array} &
\begin{bmatrix}
A_{11} & 0 & 0 & . & 0 \\
0 & A_{22} & 0 & 0 & 0 \\
0 & 0 & A_{33} & 0 & 0 \\
. & . & . & . & . \\
. & . & . & . & . \\
0 & 0 & 0 & 0 & A_{NN}
\end{bmatrix}
\end{array}
$$

Result 2: Let α denote the portion of the balance sheet that is wiped out after the given shock. For the financial networks composed of the star liability network and the totally seperated asset network, for any idiosyncratic shock, there exists α_l^S, α_h^S and α_*^S such that if

- $\alpha_h^S \geq \alpha > \alpha_l^S$, then α is isolated default shock.
- $\alpha > \alpha_*^S$, then α is contagious default shock.

Comparing the results under the ring and star liability network yields that the star network performs better in absorbing the shock. For the same level of shock, the star network is able to keep the shock isolated by transmitting the shock to several neighbors in a smaller amount. But, the shock is spread over the ring network without being divided, therefore the effect to the neighbors of the banks whose asset is hit, is heavier. In other words, it represents the difference between the situations where the bank borrows from a creditor or from several creditors to invest in an asset. If the volume of the credit is equal, then the amount of the debt on which the bank default per creditor is greater when it has only one creditor. For the reason, the lower bound for the contagious default shock is higher. That is, the size of the balance sheet wiped out after the given shock is greater for the star liability network for the shock to be the contagious default shock.

The other interesting feature of the star network here is that it doesn't allow for a shock to be a knock-on shock. Hence, the shock causes other banks to default whenever it spreads to other assets and reduce their price. As long as the shock to the asset is between the knock-on values given in the table 1, the ring liability network causes the price of other assets to decline, but the holders don't default.

For the time being, there are two important limitations for advancing the model to capture more general cases. The first problem arises when banks choose to have more than one asset. The optimal choice of which asset to sell is dependent on several factors which are hard to be incorporated in the model. As it may depend on many other factors, we think that price elasticity of the assets and the share of the asset held by the bank are decisive factors. Since the level of the price reduction is mostly determined by these factors, the optimal decision should take them into consideration. The second problem is the lack of empirically grounded inverse demand function. Although there are some studies (Thiery), the observed values are not captured by a functional form. Overcoming these problems will help us to analyze high-dimensional networks within which interconnections among the banks are over the mutual debts and commonly held assets. Moreover, it will prepare the ground for explaining the spread of shocks not only from one institutions to the other, but also from one market to the other.

3 Networks and Counterparty Risk

We analyze for an infinite horizon economy with n banks. At each date, banks choose how much to invest in the risky asset and to hold the riskless asset. We assume that banks need to borrow from other banks and collect deposits to invest in a portfolio. Therefore, banks are connected through their liability sides. This connectedness over the lending/borrowing relation generates the network structure between the banks. Banks are heterogenous in the sense that they collect different amount of deposits. After shocks on asset returns are realized, banks make payment to each other and to the depositors. The payment equilibrium arises as in the Eisenberg and Noe model (2001). We assume that the depositors has priority over the interbank debts. Hence, if any bank's portfolio return is not sufficient to pay its debts, the payment are done in an order within which the depositors have priority. The bank's income is comprised of the payments from other banks and the portfolio return. If the bank's income is not sufficient to pay its debts, the bank defaults. Therefore, the bank's choice is to minimize probability of default and ,at the same time, to maximize expected cash flow as in Ibragimov, Jaffee and Walden (2011).

Our setup is a mixture of the two models, mentioned above. It leads us to understand the results of bank's behavior of bringing money through deposits into the system within which banks have interbank claims on each other. Since banks increase their portfolio volume as they collect more deposit, this situation makes the counterparty risk rise depending on the network type. Our setup takes the collection of deposits of the banks and the network structure as exogenously given. Subsequently, the date 0 value of each bank and the payment equilibrium arises endogenously as a consequence of the portfolio choice of the banks and the network structure. We first solve the model under the Value at Risk (VaR) constraint without taking the network effect into consideration. We obtain the portfolio choices of banks whose probability of default don't exceed the maximum level that is allowed by the government. Secondly, we solve the model with the network effect. That is, we allow the probability of survival of the banks in the network to be exposed to the counterparty risk. When the counterparty risk is accounted, we show that, the portfolio choice of the banks under the VaR constraint violates the level imposed by the government. We characterize the equilibrium under the network effect and point out the effects of the heterogenous deposit collection on the counterparty risk for a particular network type.

In our model, the structure in which banks are part of a single clearing mechanism, is taken by Eisenberg and Noe (2001). Since banks are connected through interbank borrowing/lending network, defaults of a single bank has negative effect on the rest of the banks. This feature enables us to relate the probability of default of a bank with the behaviours of its neighbors. In our setup, defaults may occur due to risky portfolio choice. Therefore, the amount of the fund that a bank invests in a portfolio increases the probability of default for all banks in the network. To formalize this idea, we also use the model proposed by Ibragimov, Jaffee and Walden (2011). The main point of interest in their model for us is that it incorporates the probability of default and the expected income jointly in the value function of the banks. In other words, banks aim to maximize their expected cash flow, but also they gain disutility as the risk of default increases. Hence, the portfolio choice is subjected to a trade-off between the probability of default and the expected invesment return. By integrating these two models, we came up with a model which helps us to understand how one of the banks decision over this trade-off affects its neighbors and the whole network.

3.1 Model

Consider an infinite horizon economy in which there are n heterogenous banks denoted by $(1, 2, ...n)$ and two assets, whose share in banks' portolios are denoted by $\alpha_1^n \quad and \quad \alpha_2^n$, respectively. At the initial date, banks borrow from each other, collect deposit and invest in portfolio of two assets which yield returns at the next date. As in the "coconut" model of Diamond (1982) and in Acemoglu et all (2013), we assume that banks can't use their own funds to invest in a portfolio. Asset 1 is a risk-free bond and asset 2 is a risky asset. Therefore, asset 1 pays off one dollar at $t+1$ and costs $\delta < 1$ at t. The liability of bank i to bank j is represented by L_{ij}. The amount of deposit collected by bank i is denoted by x_i. By definition, $L_{ij} \geq 0$ and L_{ii} =0 for all i. The total liability of bank is the sum

$$L_i + x_i \quad where \quad L_i = \sum_j L_{ij} \tag{4}$$

As in the Eisenberg-Noe framework, interbank liabilities are of equal seniority and if any bank defaults, payments will be proportional to the face value of its liabilities. The network matrix π is defined as follows:

$$\pi_{ij} = \begin{cases} \frac{L_{ij}}{L_i} & \text{if } L_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

That is, the payment by bank i to bank j is given by $P_i \pi_{ij}$ where P_i is the market value of bank i liabilities and $P_i \leq L_i$. Subsequently, the contribution of the payments from other banks to the cash flow of bank i is:

$$\textstyle\sum_j P_j \pi_{ji}$$

For each bank, the amount of available fund to invest in portfolio is equal to its liabilities.

$$c_i = [\alpha_2^i(1+r) + \alpha_1^i)](L_i + x_i) \quad \text{and} \quad (1+r) = \tilde{R} \quad where \quad r \quad is \quad R.V \tag{5}$$

Interbank lending and portfolio investment constitute the incomes of the banks. The fixed point characterization is as follows:

$$p_i = max[0, min[l_i, \sum_j p_j \pi_{ji} + c(\alpha_1^i, \alpha_2^i, \tilde{R}) - x_i]] \tag{6}$$

The first term in the minimum expression shows the amount bank i borrows. The second term is the total income of bank i minus the deposit payment. Since, deposit payment has priority over interbank debts, banks' payment to other banks in the network is the minimum of its total liabilities and its net income after the deposit payment. Hence, the clearing vector P^* is a vector in which banks pay each other the minimum of what they borrow and what they have after the realization of return shocks and the deposit payment. By matrix notation, it is $P^* = (p_1, p_2.....p_n)$:

$$P^* = max[0, min[\mathbf{L}, \pi^t P^* + \mathbf{C}(\alpha_1^i, \alpha_2^i, \tilde{R}) - \mathbf{X}]] \tag{7}$$

Definition 1: Given the deposits $(x_1, x_2, ...x_n)$, the debt network Π and the realization of the asset return $(\tilde{R})$, P^* is the *clearing payment vector*, if it solves $Eq.(4)$.

After the realization of the asset return $\tilde{R}$ and the clearing payment vector P^*, the value of the financial institution i at $t = 1$ is the following:

$$\begin{aligned} & max[\sum_j p_j^* \pi_{ji} + c(\alpha_1^i, \alpha_2^i, R) - L_i - x_i, 0] \\ & = Q + \sum_j p_j^* \pi_{ji} + c(\alpha_1^i, \alpha_2^i, R) - L_i - x_i \end{aligned} \tag{8}$$

where $Q = max[\sum_j p_j^* \pi_{ji} + c(\alpha_1^i, \alpha_2^i, R) - L_i - x_i, 0] + L_i + x_i - \sum_j p_j^* \pi_{ji} - c(\alpha_1^i, \alpha_2^i, R)$ is the realized value of the option to default. (Ibramigox,Jaffee, Walden, 2011). Since the limited liability, the value of the banks can't be lower than zero. The price paid at $t = 0$ is the discounted expected value of the option to default and expected return of the risky asset minus premium d, per unit risk :

$$\delta E(\alpha_2^i \tilde{R}(L_i + x_i) + Q(\tilde{R})) - d \tag{9}$$

The premium d increases as the share of the risky asset in the portfolio α_2 rises. The ex-ante value of the bank i between $t = 0$ and $t = 1$ is then:

$$
\begin{aligned}
&\underbrace{d^i - \delta(\alpha_1^i)(L_i + x_i) - \delta E(\alpha_2^i \tilde{R}(L_i + x_i) + Q(\tilde{R}))}_{\text{t=0}} \\
&\qquad + \underbrace{\delta E(\alpha_2^i \tilde{R}(L_i + x_i) + Q(\tilde{R}) + (\alpha_1^i)(L_i + x_i) - x_i)}_{\text{t=1}} \\
&\qquad = d^i - \delta x_i + E[L_i - \sum_j p_j^* \pi_{ji} | I_i] \\
&\qquad = d^i - \delta x_i
\end{aligned}
$$

The result follows because $E[\sum_j p_j^* \pi_{ji} - L_i | I_i] = 0$, where I_i is the information set of bank i.

Definition 2: The probability of survival q is the probability that the banks' portfolio invesment is greater than zero.

Definition 3: The VaR value is the maximum level for the probability of default imposed by the government. It is denoted by β.

The probability of survival depends on the information through which the probability is calculated. We assume that banks in the network know to whom they owe and from whom they borrow, but they do not know neighbor's deposits. The information set of bank i is denoted by I_i. Therefore, for an individual bank i the probability of survival is simply:

$$Pr([\alpha_2^i \tilde{R} + (1 - \alpha_2^i)](L_i + x_i) > 0) = q_i \tag{11}$$

It is because of the fact that bank i's expectation about the difference between its network income and liability is equal to zero. It is again because of the fact that we have:

$$E[L_i - \sum_j p_j^* \pi_{ji} | I_i] = 0 \tag{12}$$

On the other, in the reality, there is a counterpary risk, therefore the probability of survival for node i depends also on the deposits of the banks on the network.

3.2 Bank's Problem under VaR Constraint

The net present value of operating the bank between $t = 0$ and $t = 1$ is equal to the ex-ante value between $t = 0$ and $t = 1$, which is d_i as calculated above. If the bank survives with the probability q_i, the portfolio invesment and interbank lending process is repeating. The important part is the fact that VaR constraint doesn't take counterparty risk into consideration and it only focus on bank's invesment position. The value of the bank in recursive form, as in IJW model, is:

$$\begin{aligned} V^i = d^i - \delta x_i + \delta q_i V^i \\ = \frac{d^i - \delta x_i}{1 - \delta q_i} \end{aligned} \tag{13}$$

Therefore, banks' individual problem is to choose over one risky and one risk-free asset. Assuming that VaR constraint is imposed by the government and short-sale is not allowed, for all the banks in the network, the problem is the following:

$$V_i = \max_{\alpha_1^i, \alpha_2^i} \frac{d^i - \delta x_i}{1 - \delta q_i} \tag{14}$$

$$\alpha_1^i + \alpha_2^i = 1 \tag{15}$$

$$\alpha_1^i, \alpha_2^i \in [0, 1] \tag{16}$$

$$q_i = Pr([\alpha_2^i \tilde{R} + (1 - \alpha_2^i)](L + x_i) > 0) \geq 1 - \beta \tag{17}$$

For the time being, the risk premium d is only affected by the amount of risky invesment, in general it is also a function of counterparties'risky behavior. The trade-off that banks face is through the risky asset choice. It causes numerator to increase by rising the risk premium, yet denumerator also rises as the probability of survival declines after each additional risky asset choice.

Theorem 1: Following the result in IJW model, bank's problem under VaR constraint yields that maximum amount of risky asset is chosen, that is $Eq.(13)$ is binding, and the rest is invested to risk-free asset:

$$\alpha_2^i = \frac{1}{1 - F_{\tilde{R}}^{-1}(\beta))} \tag{18}$$

$$\alpha_1^i = 1 - \frac{1}{1 - F_{\tilde{R}}^{-1}(\beta))} \tag{19}$$

where $F_{\tilde{R}}^{-1}$ is the inverse function of the cumulative distribution function of the random variable $\tilde{R}$.

Condition 1 (No Short-Sale) : Since short-sale is not allowed, the results require the condition that:

$$0 \geq F^{-1}(\beta) \tag{20}$$

$Eq.(19)$ describes the set of $\beta's$ that are implementable for the model to be solved under the short-sale constraint.

3.3 Redefining Bank's Problem under the Network Effect

Now, we formalize the same situation with the network effect. When any of the bank isn't capable of paying its deposits, other banks in the network are exposed to the counterparty risk. The expected difference between the total payments to the bank and the liabilities of the bank to the network is no longer zero. In fact, it is a function of the deposits of the banks in the network. Hence, for some function h, we have:

$$E[\sum_j p_j^* \pi_{ji} - l_i] = h(x_{-i}) \tag{21}$$

Therefore, the probability of survival when the network effect is considered for bank i is:

$$q_i^{**} = Pr([\sum_j p_j^* \pi_{ji} - l_i + \alpha_2^i \tilde{R} + (1 - \alpha_2^i)]L - x_i > 0) \tag{22}$$

Subsequently, bank i's problem is reformulated as follows:

$$V_i = \underset{\alpha_1^i, \alpha_2^i}{max} \frac{d^i - \delta x_i}{1 - \delta q_i^{**}} \tag{23}$$

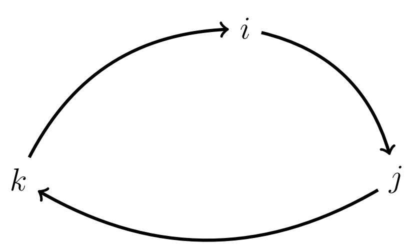
$$\alpha_1, \alpha_2 \in [0, 1] \tag{24}$$

$$q_i^{**} = Pr([\sum_j p_j^* \pi_{ji} - l_i + \alpha_2^i \tilde{R} + (1 - \alpha_2^i)](L_i + x_i) - x_i > 0) \geq 1 - \beta \tag{25}$$

Hence, as long as $\sum_j p_j^* \pi_{ji} - l_i \neq 0$ and $x_i \neq 0$, we have $q_i^{**} \neq q_i$.

3.4 Implication on the Ring Network

In this part, we construct a ring financial network with three heterogenous banks (i, j, k) such that $x_i < x_j < x_k$. The Ring network between (i, j, k) is:



The direction of the arrows represents interbank lending. The direction from i to j describes the liability of bank j to bank i. We denote net portfolio income of bank i, which is portfolio income minus deposits by $\epsilon_i (= c_i - x_i - L_i)$.

Proposition 1: In a ring financial network with 3 banks, the failure of bank i happens, if one of the three events below occurs:

1. $(\epsilon_i < 0)$
2. $(\epsilon_i + \epsilon_k < 0) \cap (\epsilon_i > 0)$
3. $(\epsilon_i + \epsilon_j + \epsilon_k < 0) \cap (\epsilon_i + \epsilon_k > 0)$

Note that these events are not necessarily disjoint sets, whether their intersection is empty set or not depends on the node's relative level of deposit collection to the rest of the network. See the Appendix.

Bank i's problem without counterparty risk considers only the first event. $((\epsilon_i < 0))$. We denote the solution as α_2^i which is the share of risky asset in bank i's portfolio after the optimal choice under only individual risk. Therefore, for a given β satisfying $Eq.(19)$,

$$\alpha_2^i = \frac{1}{(1 - F_{\tilde{R}}^{-1}(\beta))} \tag{26}$$

Now, we solve the same problem under network knowledge which deals also with counterpary default risk when investing in portfolio.

Theorem 2: For a given β under $Eq.(19)$, if $x_i < x_j < x_k$, then in the ring financial network represented as above, the probability of survival for node i, j and k are :

$$q_i^{**} = 1 - F\left(\frac{\sum_n x_i + \sum_n (\alpha_2^i - 1)(L + x_i)}{\sum_n (L + x_i)(\alpha_2^i)}\right) \tag{27}$$

$$q_j^{**} = \begin{cases} 1 - F((1 - \frac{L}{\alpha_2^j(L+x_j)}) & \frac{\alpha_2^i(L+X_i)+\alpha_2^k(L+X_k)}{2} < \alpha_2^j(L+X_j) \\ 1 - F(\frac{\sum_n x_i + \sum_n(\alpha_2^i - 1)(L+x_i)}{\sum_n (L+x_i)(\alpha_2^i)}) & otherwise \end{cases}$$

$$q_k^{**} = 1 - F(1 - \frac{L}{\alpha_2^k(L+x_k)}) \tag{28}$$

Corollary 1: For a given β under $Eq.$(19), if the VaR constraint(β) is identical for all nodes such that $x_i < x_j < x_k$ then in the ring financial network represented as above, the probability of survival for node i, j and k are :

$$q_i^{**} = 1 - F(1 - \frac{3L}{(\alpha_2)\sum_n(L+x_i)}) \tag{29}$$

$$q_j^{**} = \begin{cases} 1 - F(1 - \frac{L}{\alpha_2(L+x_j)}) & \frac{(x_i+x_k)}{2} < x_j \\ 1 - F(1 - \frac{3L}{(\alpha_2)\sum_n(L+x_j)}) & otherwise \end{cases}$$

$$q_k^{**} = 1 - F(1 - \frac{L}{(\alpha_2)(L+x_k)}) \tag{30}$$

The corollary above displays that the probability of survival for the bank that has the lowest amount of deposit is affected not only by its own deposit but also the deposits of the other banks. On the other hand, the bank that has the highest amount of deposit, doesn't influenced by the holdings of the others. It's probability of survival only depends on its own deposit.

Corollary 2: For a given β under $Eq.$(19), if the VaR constraint(β) is identical for all nodes such that $x_i < x_j < x_k$, then the probability of survival for node i is a decreasing function of x_i, x_j and x_k. The probability of survival for node k is a decreasing function of only x_k.

The probability of survival for all nodes decreases as their own deposits increases. Yet, Corollary 1 and Corollary 2 indicates that in the ring financial network, the probability of survival for the bank that has lowest amount of deposit decreases as the other banks rises their deposits.

Corollary 3: For a given β under $Eq.$(19), if the VaR constraint(β) is imposed for all nodes such that $x_i < x_j < x_k$,, then in the ring financial network represented as above, the following equalities hold:

$$q(\beta) > q_i^{**} = q_j^{**} > q_k^{**} \quad if \quad \frac{x_i + x_k}{2} \geq x_j \tag{31}$$

$$q(\beta) > q_i^{**} > q_j^{**} > q_k^{**} \quad if \quad \frac{x_i + x_k}{2} < x_j \tag{32}$$

The corollary 3 shows that the probability of survival decreases as the bank collects more deposit and invests in a portfolio. The probability of survival imposed by the VaR constraint is higher than the one realized when network effect is included. That is, the probability of default for all nodes is higher due to the network effect and seniority of deposit payments.

Corollary 4: For a given β under $Eq.$(19), if the VaR constraint(β) is imposed and $x_i = x$ for some $x \geq 0$ for all i, then

$$q = q_i^{**} = q_j^{**} = q_k^{**}, \quad if \quad x = 0 \tag{33}$$

$$q > q_i^{**} = q_j^{**} = q_k^{**}, \quad if \quad x > 0 \tag{34}$$

Corollary 4 says that the probability of survival for the bank i is less than VaR constraint when the network effect is considered. As the theorem 2 shows, the survival probability for the bank i is q_i^{**}. Since the probability of survival considered under the network effect and deposit payments, is lower than the VaR constraint for all banks, any potential policy designed over VaR doesn't hit the target level. In other words, the probability of default exceeds the maximum level that is allowed.

Definition 4: The probability of survival neutralized from the seniority of deposit assumption is denoted by q^*. For bank i,

$$q_i^* = P([(\alpha_2^i)\tilde{R} + (1-\alpha_2^i)](L+x_i) - x_i > 0) \tag{35}$$

Corollary 5: For a given β under $Eq.(19)$, in the ring financial network, the VaR constraint with $x_i < x_j < x_k$ causes positive network effect for all i whose deposit is less than the average of the rest of the network. The network effect is silent for the banks whose deposit is higher than the average deposit of the rest.That is:

$$q_i^* - q_i^{**} > 0$$

$$q_k^{**} = q_k^*$$

$$q_j^* - q_j^{**} > 0 \text{ if } \frac{x_i + x_k}{2} \geq x_j$$

$$q_j^* = q_j^{**} \text{ if } \frac{x_i + x_k}{2} < x_j$$

The above corollary establishes that *the network effect* is positive for the banks whose deposit, therefore the portfolio volume, is less than the average of the rest. In other words, for those banks, the financial network affects the survival probability in a negative way because of the fact that they are exposed to counterparty risk. On the contrary, there is no counterparty risk for the banks whose deposit is higher than the average of the rest. Thus, although each bank chooses to have a portfolio regulated by the VaR constraint, the banks that collect less deposit are exposed to counterparty risk. For those banks, the ring financial network causes the probability of survival to decrease.

3.5 Policy Suggestion

In this part, we deal with the problem which appears when the network effect is ignored. If the banks are subjected to the same VaR constraint, although their amount of collected deposit is different, the network effect is not silent. As the previous section demonstrates, it leads to a decrease in the probability of survival for some nodes under the ring financial network. Therefore, we suggest that the network effect disappears if the VaR constraint is not identical for all banks. We show that is possible to regulate invesment choices of the banks depending on their leverage level so that the network effect is zero.

Suppose that the target level for the VaR constraint, imposed by the social planner is β for all banks in the network. If the social planner announces different β_i^p for all banks, then it is possible to achieve a portfolio choice such that the network effect is silent.

Theorem 3: If the publicly announced β_i^p is a function of β such that:

$$\beta_i^p = F_{\tilde{R}}(1 - \frac{1 - F_{\tilde{R}}^{-1}(\beta)(L + x_i)}{L}) \tag{36}$$

then the network effect is zero, that is $q_i = q_i^{**}$ for all i. We denote the relation above by the function g. That is, $\beta_i^p = g(\beta, x_i)$.

The theorem above reflects the fact the counterparty risk through the interbank borrowing/lending network is prevalent whenever the identical VaR constraint is imposed to the banks that are heterogenous in terms of their deposits, therefore, of portfolio volume. The counterparty risk problem can be overcome by imposing different VaR constraint depending on the collected deposit size. If the government targets the VaR level of the magnitude β, then it should announce for each different bank $\beta_i^p = g(\beta, x_i)$. Thus, the optimally chosen invesment decisions would not generate counterparty risk.

4 Conclusion

In the first chapter, by extending the CFS model with multiple assets, we incorporate a new channel over which the propagation of shocks occur. The connections between the banks are not only determined by the debt claims, but also by the assets held in common. In financial contagion literature, the role of the sudden decrease in asset prices are studied and the consequences of the fire sale are presented in theoretical models. Yet, the networks of high-dimensions are still an open question. In this chapter, we present a model of a financial network which is more than one dimension in the sense that mutual debts and asset commonality are two seperate networks. The main finding is that the shock can spread from one bank to other one, although their connection over the liability network is weak. Since banks are connected through the asset network as well, the asset shock propagates over the commonly held assets. Therefore, combining networks of different categories is of signifance to understand the amplification mechanism of shocks. Here, we propose an algorithm to obtain the equilibrium of such high-dimensional network examples. We limit the simulations to the ring and star type of liability networks and to the totally seperated asset network.

For the time being, there are two important limitations for advancing the model to capture more general cases. The first problem arises when banks choose to have more than one asset. The optimal choice of which asset to sell is dependent on several factors which are hard to be incorporated in the model. As it may depend on many other factors, we think that price elasticity of the assets and the share of the asset held by the bank are decisive factors. Since the level of the price reduction is mostly determined by these factors, the optimal decision should take them into consideration. The second problem is the lack of empirically grounded inverse demand function. Although there are some studies (Greenwoord et.al.(2012)), the observed values are not captured by a functional form.

In the second chapter, by mixing two existing models, we provide a dynamic model of financial contagion. Our framework figures out how banks' value are affected by the decision of the other banks in the network. We show that the network effect increases as the banks collect deposit from outside the system and invest in a portfolio. Since the deposits collected by the banks have priority over the credits taken by the interbank network, the likelihood of defaults increases as the banks collect more fund and invest in a portfolio.

We also show that the network effect prevents the VaR value to hit its target level. As a policy instrument, the VaR measure is used to set a limit to the potential loss in value of a portfolio. Yet, our framework explains how banks' loss might exceed the VaR constraint when the network effect is considered. We find that the bank that has the highest amount of deposit doesn't influenced by the choices of the other banks. Therefore, the VaR constraint reflects the true probability of default only for the bank that collect the highest amount of deposit. On the other hand, the VaR value doesn't mirror the true default probability for the banks whose deposits are less than the average of the rest. In other words, banks that involve in a portfolio invesment less in a volume than the others are exposed to the counterparty risk. Hence, we show that the systemic risk arises from the banks that have more indebted to the outside of the system than the other banks and invest in a portfolio under the same VaR constraint.

5 Appendix

In all the proofs below, the function F denotes the cumulative distribution fuction of the asset return.

Proof of Theorem 2

$$P(\epsilon_i < 0) = P((\alpha_2^i)\tilde{R}(L+x_i)+(1-\alpha_2^i)(L+x_i)-x_i < 0) = P(\tilde{R} < \frac{x_i+(\alpha_2^i-1)(L+x_i)}{\alpha_2^i(L+x_i)})$$
$$= F(1-\frac{L}{\alpha_2^i(L+x_i)})$$

$$P(\epsilon_i+\epsilon_k < 0) = P(\tilde{R}(\alpha_2^i(L+x_i)+\alpha_2^k(L+x_k))+(1-\alpha_2^i)(L+X_i)+\alpha_2^k(L+x_k)-x_i-xk < 0) =$$
$$P(\tilde{R} < \frac{x_i+xk+(\alpha_2^i-1)(L+x_i)+(\alpha_2^k-1)(L+x_k)}{\alpha_2^i(L+x_i)+\alpha_2^k(L+x_k)})$$

$$P((\epsilon_i+\epsilon_k < 0)\cap(\epsilon_i > 0)) = P(\frac{x_i+(\alpha_2^i-1)(L+x_i)}{\alpha_2^i(L+x_i)} < \tilde{R} <$$
$$\frac{x_i+xk+(\alpha_2^i-1)(L+x_i)+(\alpha_2^k-1)(L+x_k)}{\alpha_2^i(L+x_i)+\alpha_2^k(L+x_k)}) =$$
$$F(\frac{x_i+x_k+(\alpha_2^i-1)(L+x_i)+(\alpha_2^k-1)(L+x_k)}{\alpha_2^i(L+x_i)+\alpha_2^k(L+x_k)}) - F(\frac{x_i+(\alpha_2^i-1)(L+x_i)}{\alpha_2^i(L+x_i)})$$

$$P((\epsilon_i+\epsilon_j+\epsilon_k < 0)\cap(\epsilon_i+\epsilon_j > 0)) =$$
$$F(\frac{\sum_n x_i+\sum_n(\alpha_2^i-1)(L+x_i)}{\sum_n \alpha_2^i(L+x_i)}) - F(1-\frac{2L}{\alpha_2^k(L+x_k)+\alpha_2^i(L+x_i)})$$

In addition, since $x_i < x_j < x_k$, we have $P((\epsilon_i+\epsilon_j+\epsilon_k < 0)\cap(\epsilon_i+\epsilon_k > 0)\cap(\epsilon_i < 0)) = \emptyset$

Hence, we get:

$q_i^{**} = 1 - F(\frac{x_i+x_k+x_j+(\alpha_2^i-1)(L+x_i)}{\alpha_2^i(L+x_i)+\alpha_2^j(L+x_j)+\alpha_2^k(L+x_k)})$

For k, we have:

$$P(\epsilon_k < 0) = P((\alpha_2^k)\tilde{R}(L+x_k)+(1-\alpha_2^k)(L+x_k)-x_k < 0) = P(\tilde{R} < \frac{x_k + (\alpha_2^k - 1)(L + x_k)}{\alpha_2^k(L + x_k)}) \\ = F(1 - \frac{L}{\alpha_2^i(L + x_k)})$$

$P((\epsilon_k + \epsilon_j < 0) \cap (\epsilon_k > 0)) = \emptyset$, because $x_j < x_k$

$P((\epsilon_i + \epsilon_j + \epsilon_k < 0) \cap (\epsilon_k + \epsilon_j > 0)) = \emptyset$, again because $x_i < x_j < x_k$
Therefore, we have:

$q_k^{**} = 1 - F(1 - \frac{L}{\alpha_2^i(L+x_k)})$

$$P(\epsilon_j < 0) = P((\alpha_2^j)\tilde{R}(L+x_j)+(1-\alpha_2^j)(L+x_j)-x_j < 0) = P(\tilde{R} < \frac{x_j + (\alpha_2^j - 1)(L + x_j)}{\alpha_2^j(L + x_j)}) \\ = F(1 - \frac{L}{\alpha_2^j(L + x_j)})$$

$P((\epsilon_i + \epsilon_j < 0) \cap (\epsilon_j > 0)) = \emptyset$, because $x_i < x_j$

$$P((\epsilon_i + \epsilon_j + \epsilon_k < 0) \cap (\epsilon_i + \epsilon_j > 0)) = F(\frac{\sum_n x_i + \sum_n (\alpha_2^i - 1)(L + x_i)}{\sum_n \alpha_2^i (L + x_i)}) - F(1 - \frac{2L}{\alpha_2^j (L + x_j) + \alpha_2^i (L + x_i)})$$

If we assume that:

$$\frac{\alpha_2^i (L + X_i) + \alpha_2^k (L + X_k)}{2} < \alpha_2^j (L + X_j) \tag{37}$$

Then, the probability of the intersection is:

$$P((\epsilon_i + \epsilon_j + \epsilon_k < 0) \cap (\epsilon_i + \epsilon_j > 0) \cap \epsilon_j < 0) = F(\frac{\sum_n x_i + \sum_n (\alpha_2^i - 1)(L + x_i)}{\sum_n \alpha_2^i (L + x_i)}) - F(1 - \frac{2L}{\alpha_2^j (L + x_j) + \alpha_2^i (L + x_i)})$$

So, we get:

$q_j^{**} = 1 - F(1 - \frac{L}{\alpha_2^j (L + x_j)})$

Otherwise, given the condition that: $\alpha_2^j (L + x_j) > \alpha_2^i (L + x_i)$

If we assume that:

$$\frac{\alpha_2^i (L + X_i) + \alpha_2^k (L + X_k)}{2} > \alpha_2^j (L + X_j) \tag{38}$$

Then, the probability of the intersection is:

$F(1-\frac{L}{\alpha_2^j(L+x_j)})-F(1-\frac{2L}{\alpha_2^j(L+x_j)+\alpha_2^i(L+x_i)})$

So, we get:

$q_j^{**}=1-F(\frac{\sum_n x_i+\sum_n(\alpha_2^i-1)(L+x_i)}{\sum_n \alpha_2^i(L+x_i)}))$

<u>Proof of Corollary 1</u>

If we put $\alpha_2^i=\alpha_2^k=\alpha_2^j=\alpha_2$ for $q_i^{**},q_j^{**},q_k^{**}$, then we get the result.

<u>Proof of Corollary 2</u>

Since the function f denotes the probability distribution of the asset return and it is always greater than zero, it is straightforward to show that:

$\frac{dq_i^*}{dx_i}=-L\alpha_2^i(L+x_i)^{-2}f(1-L\alpha_2^i(L+x_i)^{-1})<0$ for all i.

<u>Proof of Corollary 3</u>

For the proof, we assume that $\frac{x_i+x_k}{2}<x_j$. The same proof can be applied when $\frac{x_i+x_k}{2}>x_j$.

By $Eq.(14)$, we have $F(\frac{\alpha_2-1}{\alpha_2})=\beta$, therefore we get:

$q=1-F(\frac{\alpha_2-1}{\alpha_2})$.

Since F is strictly increasing function, we have $q>q_i^{**}$, if $\frac{1}{\alpha_2}>\frac{3L}{\alpha_2\sum_n(L+x_i)}$. The equality holds for all x_i such that $\sum_n x_i>0$

By the same reason, we have $q_i^{**}>q_j^{**}$, if $\frac{3L}{\alpha_2\sum_n(L+x_i)}>\frac{L}{\alpha_2(L+x_j)}$ and $q_j^{**}>q_k^{**}$, if $\frac{L}{\alpha_2(L+x_j)}>\frac{L}{\alpha_2(L+x_k)}$. These equalities hold trivially when $x_k>x_j>x_i$.

Proof of Corollary 4

Simply put $x = x_i$ for all i in $Eq.(25) - 26$ and also in q_j^{**} .

Proof of Corollary 5

$q_i^* = P([(\alpha_2^i)\tilde{R} + (1-\alpha_2^i)](L+x_i) - x_i > 0) \geq 1-\beta =$

$P(R > 1 - \frac{L}{\alpha_2(L+x_i)}) \geq 1-\beta$

Therefore, we have:

$q_i^* = 1 - F(1 - \frac{L}{\alpha_2(L+x_i)})$

Since, $\frac{3L}{\alpha_2 \sum_n (L+x_i)} < \frac{L}{\alpha_2(L+x_i)}$, we have

$q_i^* - q_i^{**} > 0$

When the same argument is applied for the other nodes, the corollary 5 can be proved to be true.

Proof of Theorem 3

Since the network effect is non-zero for the node i and also for the node j, depending on x_j, we prove it only for the node i. The same method can be applied to node j, whenever it has non-zero network effect.

We denote the solution under q^* for node i as α_2^{i*}:

$$\alpha_2^{i*} = \frac{L}{1 - F^{-1}(\beta)(L+x_i)} \tag{39}$$

We claim that $q_i^* = q_i^{**}$ for all i, if $\alpha_2^i = \alpha_2^{i*}$ for all i. This can be proved simply by putting $Eq.(34)$ into $Eq.(25)$.

Therefore, the condition $\alpha_2^i = \alpha_2^{i*}$ yields that

$$\frac{1}{1 - F^{-1}(\beta_i^p)} = \frac{L}{1 - F^{-1}(\beta)(L + x_i)} \tag{40}$$

where β_i^p is publicly annouced VaR value for bank i and β is the target VaR value for the government. By rearranging $Eq.(35)$, we obtain:

$$\beta_i^p = F_{\tilde{R}}(1 - \frac{1 - F_{\tilde{R}}^{-1}(\beta)(L + x_i)}{L})$$

6 References

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