T.C.
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M.Sc. THESIS

COMPARISON OF UNIVARIATE AND HEURISTICS FORECASTING MODELS IN THE EMPLOYMENT/UNEMPLOYMENT SECTOR IN MALI

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FOREWORD

First of all, I would like to thank Istanbul University for giving me the opportunity to accomplish my master program.

I give a special thanks to Yrd. Doc. Dr. Murat Akad for taking me under his supervision, his patience and help during my research which helped me a lot.

Finally, I would like to thank my family as well for their understanding and support without which I would have never been in this position.

February 2018 Hamadou NIANGADOU

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LIST OF SYMBOLS AND ABBREVIATIONS

Symbol	Explanation
α	: Alpha
β	: Beta
ϕ	: Teta

Abbreviation	Explanation
ACF	: Autocorrelation Function
AGO	: Accumulated Generating operations
ARIMA	: Autoregressive Integrated Moving Averages
Eq.	: Equation
Fig.	: Figure
GDP	: Gross Domestic Product
GM	: Grey Model
GPRM	: Grey Prediction with Rolling Mechanism
IAGO	: Inverse Accumulated Generating operations
MA	: Moving Averages
MAE	: Mean Average Error
MSE	: Mean Square error
OEC	: Observatory of Economic Complextity
PACF	: Partial Autocorrelation Function
RMSE	: Root Mean Square error
SES	: Single Exponential Smoothing
StDev	: Standard Deviation
UN	: United Nations
WDA	: World Data Atlas

ÖZET

YÜKSEK LİSANS TEZİ

COMPARISON OF UNIVARIATE AND HEURISTICS FORECASTING MODELS IN THE EMPLOYMENT/UNEMPLOYMENT SECTOR IN MALI

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Mali'de işsizlik her zaman bir problem olmuştur. Emek çok yaygın olmasına rağmen , yetenekli olan kişi yetersizdir. Güvenilir işsizlik verilerini bulmak zordur. Mali'nin hükümeti taraftan 2016'da yapılan bir anket'e göre yüzde 8,1 oranındayken, gerçek rakam muhtemelen yüzde 30'u aşıyor. Yine de, "Dünya Veri Atlası" veri kaynağındaki gerçek verileri kullanarak bu çalışma şunları amaçlamaktadır: hangisinin daha az hata yüzdesine sahip olacağını görmek için, litteratürdeki kabul edilmiş tek de- ğişkenli tahmin modellerinden bazılarını ve sezgisel tahmin modellerini kullanarak bir dizi tahmin operasyonu gerçekleştirin ve böylece gelecekteki sayıların nasıl göründüğü konusunda en iyi tahmini verilmesi.. Ayrıca, Mali'nin ekonomisi üzere bir bakış açısı verilecek, işsizlik oranlarını etkileyen bazı faktörler tartışılacak ve uygulanabilir bazı çözüm önerileri sunulacak.

Şubat 2018, 103. sayfa.

Anahtar kelimeler: Dünya Bankası veri kaynağı, İşsizlik, Tahmin, Mali

SUMMARY

M.Sc. THESIS

COMPARISON OF UNIVARIATE AND HEURISTICS FORECASTING MODELS IN THE EMPLOYMENT/UNEMPLOYMENT SECTOR IN MALI

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Unemployment has always been a problem in Mali. Although labor is widely available, skilled one is in short supply. Reliable unemployment data is difficult to find. While a survey of the Malian government found a rate of 8,1 percent in 2016, the actual figure is likely over 30 percent. Nevertheless, using the actual data from the "World Data Atlas" data source, this study aims to perform a series of forecasting operations using some of the accepted univariate forecasting models in litterature and a set of heuristic ones, so as to see which one will hold less error percentage, and thus give the best estimate on how the future numbers might look like. Also, an insight of the economy of Mali will be given, some of the factors affecting the unemployment rates will be discussed and some feasible solutions will be presented.

February 2018, 103 pages.

Keywords: World Bank data source, Unemployment, Forecasting, Mali

1. INTRODUCTION

Mali has demographic characteristics similar to most sub-Saharan African countries. The population of Mali is very young. The population is estimated to be a little over 18 millions and almost 67% of it is between 0-24 years of age(according to 'index mundi'). This shows how big the labor force is. In Mali, just like in many developing countries, few can afford to be openly unemployed and yet the employment situation has been deteriorating since 1987. Official numbers for the unemployment are around 10% on average for the past few years, but the actual figure is probably bigger than that. The causes of this problem will be explained in this thesis and some solutions will be presented/proposed as well. But before that, future numbers will be estimated through a series of forecasting using some commonly used algorithms in litterature and a few heuristic ones.

Forecasting is an activity or process through which someone predicts or attempts to predict the future, based on previous events or on some information he/she has now. In short it is a guess, but logical and rational, about what is going to happen in the future. It's about capturing the regularities in a data and using them to make predictions. It is used in various fields such as economy, weather, supply chain, planning, manufacturing, quality management, demand, scheduling, etc. Forecasting isn't something which has been created, but rather it has always existed. It is constantly used on regular daily basis. For example a mother of a family which has a monthly limited budget tries to keep the family expenses for food, bills, etc within that budget every month. She spends it over some time, looking at the long term, trying to predict/anticipate any situation that could arise based on past experiences and act accordingly. That is a forecasting process. Predicting inflations or values of certain goods is also a forecasting process. There are two (2) types of forecasting:

Judgement Forecasting: referred to as qualitative forecasting. Here, the data is expressed by means of a natural language description. We don't really use a numerical analysis. This type of forecasting requires only the use of our intuition and experience. It is used best when there is little or no historical information/data. Examples of such forecasting are new products launches, market research, surveys and polls, etc.

Quantitative Forecasting: based on historical/past data or information. The data is a numerical measurement expressed in terms of numbers. That data is analized in order to discern some trends or patterns which repeat more than once and use those to make some predictions. The data is usually spread over a long period of time and is usually continuous, thus it is referred to as “Time Series”. Examples of such forecasting are weather forecasts, demand/sales forecasts, population growth, etc.

There are many available techniques that may be used when working with the second type of forecasting. Most of them are case specific, that is one algorithm may not perform well in every situation. There are a few accepted algorithms/techniques in the litterature. Some of them are: ARIMA models, Artificial Neural Network (ANN), Support Vector Machines (SVM), Moving averages and Exponential smoothing, K-nearest neighbor prediction method (kNN), etc.

However, because of the nature of forecasting itself, that is there will never be a perfect method for every situation, many heuristic techniques have been developed too. Those techniques are mostly case oriented, often the result of different combination of methods (these methods are referred to as ‘hybrid models’). ARIMA+ANN, kNN+SVM, Grey+Evolutionary algorithms are a few examples of such methods. The following lists the different techniques which will be used in this study:

Accepted algorithms in litterature

- Moving averages
- Simple and multiple regression methods
- Exponential smoothing methods
- ARIMA models

Heuristic methods

- Original grey model GM(1,1)
- Grey prediction with Rolling mechanism (GPRM)
- Grey Model with Optimization of Background Value
- Grey_ARIMA model

These heuristic techniques deal with data samples which have a small size (usually less than 40). The reason behind chosing these specifically is that the data which will be used in this study has a size of **27**. Many algorithms usually require a larger amount of data in order to give optimum results. ARIMA is an example of such algorithms. More on the above mentioned algorithms in the following sections.

2. MATERIALS AND METHODS

This section is divided into 4 parts: *section 2-1* discusses the materials which will be used in the study, which comprise the dataset and the software; *section 2-2* discusses a well known and very important notion in forecasting, the principle of parsimony; *section 2-3* describes the multiple forecasting methods which will be used in this study and *section 2-4* shows their application to the dataset.

2-1. Materials

The data which is going to be used in this study is from ***World Data Atlas (WDA)*** [17] which is under "**Knoema**". Knoema is a free resource for statisctical data. It was created through a joint venture by Russian and Indian professionals and it offers an incredibly wide range of data and information about all the countries in the world, collected from highly reliable sources such as the World Health Organisation and United Nations (UN). The data on the Knoema can always be tracked down to check their trustworthiness as every data is linked back to its original source. World Data Atlas not only comes as a stand alone website, but is also available as a Chrome application and as an application for tablets and smartphones [20].

The data taken from WDA consists of the unemployment rates of **Mali** arranged in order from 1990 to 2016, in a yearly basis, that is 27 entries in total as can be seen in *table 2.1*. The values/rates give the number of unemployed person as a percentage of the total labor force. It's very hard to find any information about the employment situation prior to the early 90s. The country, being technologically behind, nothing was really kept digitally until recently. Most data about the country is written down on papers and kept in the archives. And since its independance in 1960, Mali has seen multiple "Coups d'Etat", which lead to the loss of many documents. Therefore, one can find decent data on Mali only through international organisms or institutions such as the "World Bank" or "World Data Atlas" which have done some researches in the past years, and most of the time those do not include any information prior to the 1991 Coup d'Etat (in Mali).

Certain algorithms work best with a minimum entry of 50-55 data. ARIMA is an example of such algorithms, as mentioned before. Since we have got only 27 data, it is therefore normally inconvenient to perform a forecasting exercise with such algorithm. In order to overcome this problem, a popular technique which helps increase sample sizes will be introduced. The technique is called “**Bootstrapping**” [21]. It’s a powerful statistical technique, accepted in litterature, which involves resampling. It generates new data from an initial data sample, which usually has a sample size less than 40. It was first mentioned in 1979 by Bradley Efron [33] and since then different procedures have been developed [34][35][36].

Table 2.1: Yearly Unemployment rates of Mali from 1990-2016.

Year	Unemployment(%)	Year	Unemployment(%)
1990	7	2003	4.5
1991	7.2	2004	8.8
1992	7.1	2005	9.6
1993	12.2	2006	10.4
1994	11.9	2007	11.7
1995	7.4	2008	10.6
1996	8	2009	9.4
1997	3.3	2010	7.3
1998	7.4	2011	6.9
1999	9.3	2012	6.9
2000	7.9	2013	7.3
2001	7.6	2014	8.2
2002	7.3	2015	8.1
		2016	8.1

The method is sometimes referred to as “***Sampling with replacement***”. This basically means that when a value is drawn from a pool/set, instead of putting that value aside, it is possible to draw it a second time, or even more than twice, and because some observations may be resampled more than one time, others might not be sampled at all. Here is how the method works: a first bootstrap sample is generated by drawing random observations from the initial data set, and the average of that sample is calculated. This process is performed **n** times so as to have at the end a “***bootstrap sample of the means***”. This process is vizualized in *Fig. 2.1*.

Bootstrapping is available in many software tools nowadays, however it is also possible to perform the tasks by hand, for small data size. For this study, since we have 27 entries, a new data with a sample size of 27*4 ,which is 108, will be generated.

In order to perform a good forecasting, softwares are needed most of the time. In this study, we will only use 2 of them, namely Microsoft Excel and Minitab. Minitab is a software package for statistical analysis. It’s one of the most popular ones and has ARIMA and linear regression as well as a few other methods already implemented in its library. It is user friendly and easy to use. For more on Minitab, refer to this page [22] or visit the following link: https://libguides.library.kent.edu/statconsulting/minitab

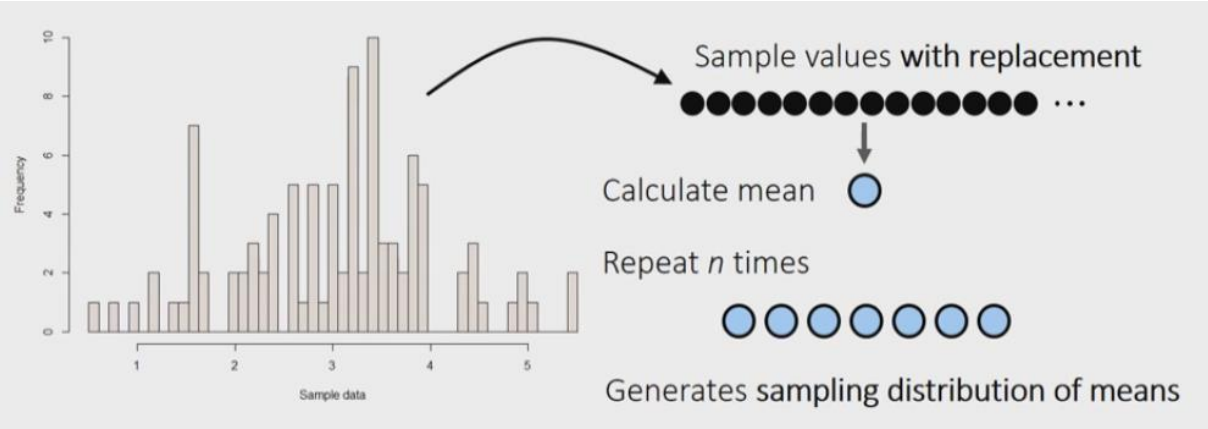

Figure 2.1: Steps to generating a bootstrap data set.

There are many indicative measurement models available for evaluating the accuracy of forecasts. Those models are commonly categorised into two (2) groups: ***scale-dependent*** errors and ***scale-independent*** errors.

A comparison of forecast performance made between different data sets is referred to as *scale-independent*. An example of such model is the Mean absolute percentage error or MAPE.

$$MAPE = mean(absolute\ value(p_i))$$

where $p_i = 100*(e_i / y_i)$ and y_i is the observed value, e_i is the difference between the observed value and the forecast value for a given time i. The disadvantage of this method is that some results may be undefined, when $y_i = 0$, or infinite when y_i is close to zero.

A comparison of forecast methods made on a single data set is referred to as *scale-dependent*. Some popular models are: the Mean squared error (MSE), Mean absolute error (MAE) and Root mean squared error (RMSE).

$$MAE = \frac{\sum|e|}{n}, \quad MSE = \frac{\sum e^2}{n} \quad \text{and} \quad RMSE = \sqrt{MSE}$$

The drawback of using *MSE* is that the square puts a high weight on large deviations/errors, therefore it might return a large forecast error even if the forecast algorithm perfroms well in general. One way to overcome this issue is to use the *RMSE* instead. *RMSE* or *MSE* can be useful when large errors are undesirable. The *MAE* is steady because individual differences have equal weight.

Since only one dataset will be used in this study, it is therefore logical to use scale-dependent measurement techniques to assess the forecast accuracy. There isn't a single best measurement model. However the 3 above mentioned models are widely used in litterature, therefore these 3 will be the reference in this study as well. The lower the values of these are, the more satisfactory the forecasts will be.

It is important to note that the dataset in this study consists of only one variable, which is collected sequentially over equal time measurements, that is from 1990 to 2016. This kind of series is referred to as ***univariate*** time series. On the other hand, a series which has two or more variables is regarded as a ***multivariate*** time series. Different methods are used for each case, when forecasting, but only the models for the univariate time series will be discussed in this study.

2-2. The principle of Parsimony

A highly important principle of reasoning used in science is the principle of parsimony, often referred to as Occam's razor. The principle is named after an English philosopher of the 14th century, William of Occam (1285-1350) [68]. It states that models or explanation should be as simple as possible. His principle is used when choosing among theories, models, equations, explanations, etc. In forecasting, among a number/group of suitable models, the simplest one is always to be chosen. When building a proper time series model, one must consider the principle of parsimony and shouldn't use more parameters than needed. Using many parameters to fit the data at hand is a meaningful approach to building a model. The resulting model is usually a good fit for that particular data, however it will most likely not give good results when used for predicting other datasets.

A parsimonious model is one which has just the right number of predictors needed to describe the model. A model with many parameters is referred to as a *low parsimony* model. One with fewer parameters is referred to as *high parsimony* model. Low parsimony models usually fit better than high ones, but as mentioned earlier, they also tend to be much less effective for predicting other data sets. There are many methods available to help find the right balance between goodness of fit and parsimony. The most popular ones are:

Akaike's Information Criterion (AIC): compares a set of models and rank them from best to worst, the best model being the one which neither under-fits nor over-fits. However, it doesn't tell much about the quality of the models, as it only compares between the given/input models [53].

Bayesian Information Criterion (BIC) often called Schwarz criterion (SBC or SBIC): developed by Gideon E. Schwarz, the method is similar to the AIC but puts more emphasizes on the number of parameters. Models with less parameters are more favored, better [54].

Minimum Description Length (MDL): used in machine learning, says that every data usually has regularities and capturing them can help compress the data. The more we compress the data, the more we learn about it and therefore the model which compresses it the most is best [56].

Bayes Factors [55] is also another method, but is not as popular as the previous ones.

2-3. Overview of the methods

The followings are the methods which will be used in this study. No judgement forecasting model is used here, only ***quantitative*** forecast models.

2-3-1. Accepted forecasting methods in the litterature

All the following methods are linear, that is they do not have a single parameter which is raised to any power greater than one (1).

2-3-1-1. Moving Averages

Moving averages (MA) are about taking the average of the points nearby/around an observation. Observations which are near each other in time are very likely to be close in value. That's the idea behind the technique. That average can be a reasonable estimate for the trend-cycle of that observation. Development of the moving averages goes back to 1901 by R. H. Hooker. It was later on discussed by Yule as '*instantaneous averages*' [37] in 1909, but the name "moving averages" was quickly adopted in 1912 [38]. Later works led to the development of '*exponential moving averages*' or EMAs which is referred to nowadays as Exponential smoothing methods [39]. MAs are very useful when decomposing a time series for advanced forecasting models because they smooth out irregular patterns in the time series data. This helps recognize trends easily. However, seasonality, random events and cyclical patterns may affect the accuracy of the forecasts. It is also important to notice that the more periods we use in the MA, the smoother the time series will be. Therefore, MAs might not be the best forecasting method to use. More on moving averages in this article [23]. There are many kinds of MAs, depending on the number of data points included in the average. MAs can be simple or weighted.

Simple Moving Averages:

SMA is the simplest type of forecasting technique. Here, we are required an odd number of observations to be included in the average. Basically, the last '*n*' period's values are added up and then that sum is divided by '*n*'. The value obtained is referred to as moving average value and is used as the forecast for the next period.

Example: a *3-year MA* → m=3. m = number of observations in the average.

The process is explained in *table 2.3.1*.

Table 2.3.1: Simple moving average process.

Years (t)	Variable (Y)	3-year Moving Totals	3-year Moving Averages
t_1	Y_1	(nothing)	(nothing)
t_2	Y_2	$Y_1 + Y_2 + Y_3$	$\frac{Y1+Y2+Y3}{3} = a_1$
t_3	Y_3	$Y_2 + Y_3 + Y_4$	$\frac{Y1+Y2+Y3}{3} = a_2$
t_4	Y_4	……	……
…..	……	……	……
t_{n-1}	Y_{n-1}	$Y_{n-2} + Y_{n-1} + Y_n$	$\frac{Y(n-2)+Y(n-1)+Y(n)}{3} = a_{n-2}$
t_n	Y_n	(nothing)	(nothing)

The variable *Y* represents the observed values and the variable a_i represents the forecast value for each period.

Centered Moving Averages

This is a MA with an even number of observations to be included in the average. The method is best described through examples. *Table 2.3.2* shows how a 4-year MA is calculated.

The first average a_1 is calculated as follows:

$$a_1 = \frac{1}{4}(Y_1 + Y_2 + Y_3 + Y_4)$$

and the second average a_2 as follows:

$$a_2 = \frac{1}{4}(Y_2 + Y_3 + Y_4 + Y_5)$$

a_1 and a_2 are further averaged to get a new value A_1 whichs is: $A_1 = \frac{1}{2}(a_1 + a_2)$

A_1 is written against t_3 and this is referred to as centering the 4-year moving averages. This process continues until the end of the series.

Table 2.3.2: Centered moving average process deconstructed.

Years (t)	Variable (Y)	4-year Moving Averages	4-year Moving Averages centered
t_1	Y_1	(nothing)	(nothing)
t_2	Y_2	$\frac{Y1+Y2+Y3+Y4}{4} = a_1$	(nothing)
t_3	Y_3	$\frac{Y2+Y3+Y4+Y5}{4} = a_2$	$\frac{a1+a2}{2} = A_1$
t_4	Y_4	$\frac{Y3+Y4+Y5+Y6}{4} = a_3$	$\frac{a2+a3}{2} = A_2$
t_5	Y_5	....	....
......	......	......	....

Double Moving Averages

Any combination of MAs is referred to as a double moving averages or a Moving averages of another Moving averages. The previous example in the Centered MA equivalent to a 2*4MA smoother. a_1, a_2,..., a_n represent the 4MA part, since they are simple averages of the variable Y over 4 periods. A_1, A_2,..., A_n are simply averages of the a_n values over 2 periods. Thus the name 2*4 Moving Averages.

Weighted Moving Averages

Let us look at the previous example. In *Table 2.3.2*, the 2*4-year MA was calculated as follows: The first 4 values were averaged and a_1 was obtained as

$$a_1 = \frac{1}{4}(Y_1 + Y_2 + Y_3 + Y_4) \tag{2.1}$$

then, 4 values were averaged again starting from the second observation Y_2 and a_2 was found to be:

$$a_2 = \frac{1}{4}(Y_2 + Y_3 + Y_4 + Y_5) \tag{2.2}$$

Finally, in order to obtain 2 averages of the 4MAs, successive values of a_n were averaged as:

$$A_1 = \frac{1}{2}(a_1 + a_2)$$

If we replace a_1 and a_2 by their values, the following is obtained:

$$A_1 = \frac{1}{2}(a_1 + a_2) = \frac{1}{2}\left(\frac{Y1+Y2+Y3+Y4}{4} + \frac{Y2+Y3+Y4+Y5}{4}\right) = \frac{1}{8}(Y_1 + 2Y_2 + 2Y_3 + 2Y_4 + Y_5)$$

$$A_1 = \frac{1}{8}(Y_1) + \frac{1}{4}(Y_2) + \frac{1}{4}(Y_3) + \frac{1}{4}(Y_4) + \frac{1}{8}(Y_5) \quad (2.3)$$

Eq. 1 is a weighted Moving Averages of order 5 (because 5 observations are taken into the average) with weights of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{4}$ and $\frac{1}{8}$ for the first, second, third, fourth and fifth terms respectively.

Moving averages have been applied in short-term load forecasting by Ariffin, Karim and Alwi [1].

2-3-1-2. Exponential Smoothing Methods

Smoothing means average (or averaging). With forecasting, the most recent observations provide the best guide as to the future. Exponential smoothing is a weighting algorithm/method that has decreasing weights as observations get older [1]. Unlike Moving Averages, all the values are included in the process. However, recent observations are given relatively more weight values than older ones. Exponential smoothing method is derrived from the moving averages principles. Historically, the method was developed by Holt and Brown. Both scientists worked independently and knew not of each other's works. During world war II, under the US navy, Brown designed a system for tracking submarines. He later on applied that technique to forecast the demand for spare parts and describes his ideas in his book on inventory control problems [39]. Holt worked independently for the Office of Naval Research and developed models for constant processes, processes with linear trends and for seasonal data [40]. 3 years later, in 1960, Peter R. Winters added seasonality to the double exponential smoothing [41]. This model became known as the Holt-Winters method. Exponential Smoothing methods are usually used to remove any randomness in a data. They are best used for short-term forecasting. When the data exhibits no trend nor seasonal pattern, the single exponential smoothing method can be applied to it.

Single Exponential Smoothing

The single exponential smoothing is expressed as follows:

$$F_{t+1} = \alpha * Y_t + (1 - \alpha) * F_t \quad (2.4)$$

where Y_t represents the observation (or observed value) at time t, F_t represents the recent forecast value and α is a weight (the *smoothing constant*). The value of α is always between [0, 1] and is usually chosen arbitrary, according to each case. It is subject to trial and error.

However, it requires only a few trials to figure out which value gives the minimum errors. Also, the first observed value is commonly used as the initial forecast value F_0.

The general exponential smoothing method was applied in Christiaanse [3] for short term hourly MWH(megawatt per hour) load forecasting.

The SES or Simple Exponential Smoothing method does not perform well in long-term forecasting because it is very slow to catch up with sudden level changes in the data. In such cases, it would be best to use a double exponential smoothing method or Holt's (linear) exponential smoothing method.

Holt's Exponential Smoothing

In a SES, the forecast values fall behind when there is an increasing trend and when there is a decreasing trend, the forecast values exceed the observed ones. Holt's method takes care of these problems. To account for the trend component in the series, another *smoothing constant* is added in this method, that is β. β is the *trend* smoothing constant. Now, 3 equations are needed in order to make a forecast:

$$\text{Level:} \quad L_t = \alpha * Y_t + (1 - \alpha) * (L_{t-1} - b_{t-1}) \qquad (2.5)$$

$$\text{Trend component:} \quad b_t = \beta * (L_t - L_{t-1}) + (1 - \beta) * b_{t-1} \qquad (2.6)$$

$$\text{Forecast:} \quad F_{t+1} = L_t + b_t \qquad (2.7)$$

Where α is the smoothing constant for stationary process, β is the the trend-smoothing constant and its value is also between 0 and 1.

L_t is the smoothed constant and b_t is the (smoothed) trend value

As for the single exponential method, starting values for α , β , L_t and b_t must be selected in advance. The following is a way of doing so:

$L_1 = Y_1$ and $b_1 = Y_2 - Y_1$ or $b_1 = (Y_4 - Y_1)/3$.

However, it is important to remember that all initializations are done arbitrary.

Note that when $\alpha = \beta$, Holt's method is referred to as '*Double Exponential Smoothing*' [25].

When a series displays both a trend and a seasonal pattern, '*Holt-Winter*' method is best used in such cases.

Holt-Winter's Exponential Smoothing

To account for the seasonal component of the time series, another smoothing constant 'ϕ' is added, just as in the previous method. This method has 2 variations, depending on the nature of the seasonal component [24]. If the seasonal variations are constant through the series, an *additive* method is preferred whereas a *multiplicative* method is chosen when the seasonal variations change proportionally to the level of the series.

Additive seasonality:

The following equations are used for:

Level:	$L_t = \alpha*(Y_t - S_{t-s}) + (1 - \alpha)*(L_{t-1} + b_{t-1})$	(eq. 1)
Trend component:	$b_t = \beta*(L_t - L_{t-1}) + (1 - \beta)*b_{t-1}$	(eq. 2)
Seasonal component:	$S_t = \phi*(Y_t - L_t) + (1 - \phi)*S_{t-s}$	(eq. 3)
Forecast:	$F_{t+m} = L_t + m*b_t + S_{t-s+m}$	(eq. 4)

Where *s* is the length of the seasonality, that is the number of months or quarters in one season. The series is seasonally adjusted in the level equation (eq. 1) by substracting the seasonal component. The equation for the *trend component* (eq. 2) is the same as in Holt's linear method. Substractions are needed in order to initialize the seasonal indices. They work as follows:

$$S_1 = Y_1 - L_s \; ; \; S_2 = Y_2 - L_s \; ; \; ...... \; ; \; S_s = Y_s - L_s \qquad (2.8)$$

To initialize the level, the average of the *first* season is taken:

$$L_s = \frac{1}{s}(Y_1 + Y_2 + ... + Y_s) \qquad (2.9)$$

It is convenient to use 2 complete seasons when initializing the trend:

$$b_s = \frac{1}{s}\left(\frac{Y(s+1)-Y1}{s} + \frac{Y(s+2)-Y2}{s} + ... + \frac{Y(s+s)-Ys}{s}\right) \qquad (2.10)$$

Each of these elements is an estimate of the trend over one complete season.

Multiplicative seasonality:

The following equations are used:

Level:	$L_t = \alpha*[\frac{Y(t)}{S(t-s)}] + (1 - \alpha)*(L_{t-1} + b_{t-1})$	(2.11)
Trend component:	$b_t = \beta*(L_t - L_{t-1}) + (1 - \beta)*b_{t-1}$	(2.12)
Seasonal component:	$S_t = \phi*[\frac{Y(t)}{L(t)}] + (1 - \phi)*S_{t-s}$	(2.13)

Forecast: $F_{t+m} = (L_t + m*b_t)*S_{t-s+m}$ (2.14)

Note that in a *multiplicative seasonality*, to obtain the level, the series is divided by the seasonal component in order to remove the seasonal effects/patterns, whereas in an *additive model*, the seasonal component is substracted from the series.

The initialization process of the factors b_s and L_s is the same as in the additive model.

The seasonal indices are initialized by taking a ratio of the first data in the first season to the mean of the first year, that is:

$$S_1 = \frac{Y(1)}{L(s)} \; ; \; S_2 = \frac{Y(2)}{L(s)} \; ; \; ...... \; ; \; S_s = \frac{Y(s)}{L(s)} \tag{2.15}$$

The parameter α, β and ϕ are chosen randomly.

Holt and Winter's Exponential Smoothing method was used by Bindiu and Chindriu [4] in a day-ahead load forecasting for a fittings manufacturer. More on this algorithm in the following [42].

2-3-1-3. Simple and Multiple Regressions

Regressions were first studied in depth in the 19th centuries by a scientist named Francis Galton. He was a self-taught statistician, astronomer, anthropologist and naturalist. Regression is a technique used to estimate the relationship between variables. The method is based on the idea that linear relationships are the simplest relationships that can be assumed between two (2) variables. He first presented the regression-line during a lecture in 1877. He later on laid down the principles of multiple regression and the correlation coefficient. However, he wasn't a great mathematician, so he couldn't develop a complete mathematical model which would capture his ideas. His work was later developed into a rigorous mathematical treatment by Karl Pearson under several publications [43]. There exists linear and non-linear regression models. However, since no non-linear model will be used in this study, only the linear ones will be discussed in the following.

Simple linear regression

Any regression of a single variable **Y** (the forecast or dependant variable) on a single variable **X** (the explanatory or independant variable or predictor) is referred to as Simple Regression. Basically, the variable *Y* is forecasted by assuming that it has a linear relationship with the variable *X*. The model is called 'simple' regression because it allows only one predictor variable, that is variable *X*. For example, *Y* could represent the sales of a product and *X* could be the time. The simple linear regression model is expressed as follows:

$$Y = a + b*X + e \qquad (2.16)$$

Where *a* is the intercept, *b* the slope of the line and *e* represent the error factor.
Eq. 1 is the equation of a line, thus the method is often referred to as '*fitting a line through the data*' as the data will be spread out above and below that line.

The ***least squares method*** [26] is used to estimate the parameters *a* and *b*. This method provides an effective way of choosing *a* and *b* by minimizing the sum of the *squared errors*, that is a and *b* are chosen to minimize

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Yi - \hat{Y}i)^2 = \sum_{i=1}^{n} (Yi - a - b * Xi)^2 \qquad (2.17)$$

Using some calculus, the values of *a* and *b* are obtained as follows:

$$b = \frac{\sum_{i=1}^{n} (Xi - \bar{X})(Yi - \bar{Y})}{\sum_{i=1}^{n} (Xi - \bar{X})^2} \quad \text{and} \quad a = \bar{Y} - b*\bar{X} \qquad (2.18)$$

where $\bar{X}$ is the mean or average of the *X* observations and $\bar{Y}$ is the mean of the *Y* observations. *Eq. 1* can therefore be rewritten as follows to forecast values for the next periods:

$$\hat{Y} = a + b*X \qquad (2.19)$$

The following page [27] gives further insights about linear regression models.

2-3-1-3-b. Multiple linear regression

In a multiple linear regression, there is one variable to be predicted (Sales for instance), but there are two or more predictors, assuming that the variable to be predicted has a linear relationship with all the predictors. The general form is as follows:

$$Y = b_0 + b_1*X_1 + b_2*X_2 + ... + b_k*X_k + e \qquad (2.20)$$

Estimating the the coefficients b_k is done with the least squares method again as for the simple regression.

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Yi - \hat{Y}i)^2 =$$
$$\sum_{i=1}^{n} (Yi - b(0) - b(1) * X1 - b(2) * X2 - \cdots - b(k) * X(k))^2 \qquad (2.21)$$

Estimating the values of the coefficients which minimize *eq. 2.21* is a lot harder in a multiple linear regression, thus a computer program would normally be used. More on the model in the following article [28].

2-3-1-4. Non-seasonal ARIMA models

ARIMA stands for Autoregressive (AR) Integrated (I) Moving Average (MA). ARIMA models are the most general class of models for time series forecasting. ARIMA methodologies were first introduced in 1970 by George Box and Gwilym Jenkins in their book [44]. There are seasonal and non-seasonal models[45][46]. The seasonal models are commonly noted *ARIMA(p,d,q)* and the non seasonal models *ARIMA(p,d,q)(P,D,Q)m* where *m* is the number of periods per season. The parameter ***p*** is the order of the non-seasonal autoregressive part, ***d*** is the degree of the non-seasonal first differencing (the number of times successive observations are differenced, needed for stationarity) involved and ***q*** is the order of the non-seasonal moving average part. *P* is the seasonal AR order, *D* the seasonal differencing and *Q* represents the seasonal MA order. Each of these 3 parts is an effort to make the final data stationary, that is the series will have no trend and its statistical properties are all constant over time. Only the non-seasonal models will be discussed in this study.

The term ***AR*** is a simple regression model of the previous values of the forecast variable, in other words time-lagged values of the forecast variable. It is denoted as follows:

$$Y_t = b_0 + b_1 * Y_{t-1} + b_2 * Y_{t-2} + \ldots\ldots + b_p * Y_{t-p} + e_t \qquad (2.22)$$

where $\boldsymbol{e_t}$ is the error term.

The "***I***" term is there to make the series stationary, if needed. If a series is non-stationary in the mean, differencing will usually take care of that irregularity whereas logarithmic and/or power transformations are used when a series is non-stationary in the variance.

The ***MA*** term does not mean a moving average of the observations, but rather one of the series errors.

$$Y_t = c_0 + c_1 * E_{t-1} + c_2 * E_{t-2} + \ldots\ldots + c_p * E_{t-p} + e_t \qquad (2.23)$$

An ARMA model would look like this:

$$Y_t = \mu + b_1 * Y_{t-1} + b_2 * Y_{t-2} + \ldots\ldots + b_p * Y_{t-p} - c_1 * E_{t-1} - c_2 * E_{t-2} - \ldots\ldots - c_p * E_{t-p} \qquad (2.24)$$

By convention, the ***AR*** terms are positive (+) and the ***MA*** terms are negative (-).

In most cases, we don't really deal with values of ***p***, ***d***, ***q*** that are greater than 2, usually ***0***, ***1*** or ***2***. This small range can actually cover a tremendous range of practical forecasting situations. Computer softwares such as *Minitab* or '*IBM SPSS*' are used to facilitate working with this model. These programs either automatically generate values for the parameters *p*, *d* and *q* or let the user manually enter values and compare different results.

However, it is sometimes possible to determine the values of *p* and *q* through the ACF (auto-correlation function) plot and the PACF (partial auto-correlation function) plot. There are many rules to how to do so [29].

The ACF plot shows the relationship between y_t and y_{t-k} for different values of ***k*** for lag 1. If y_t and y_{t-1} are correlated, then y_{t-1} and y_{t-2} must also be correlated and therefore y_t and y_{t-2} should also be correlated through y_{t-1} rather than any new information which could be used in the process of forecasting y_t. The PACF is closely related to the ACF. The PACF plot shows the relationship between y_t and y_{t-k} but for lags 2, 3 and greater, which allows us to retrieve more information from the data. More on ACF and PACF in this article [30].

The following models are some of the special cases of the ARIMA model:

ARIMA(0,0,0)	a white noise
ARIMA(0,1,0) with no constant	a random walk
ARIMA(0,1,0) with a constant	a random walk with drift
ARIMA(*p*,0,0)	an autoregression
ARIMA(0,0,*q*)	a moving average

ARIMA models follow a methodology which is detailed in the work of Box and Jenkins [5].
ARIMA and ARMA models were performed on a household electric consumption time series analysis by Chujai et al.[6]
Abdel-Aal and Al-Garni used an ARIMA $(1, 1, 0)(1, 1, 0)_{12}$ model to forecast monthly electric consumption [7].

Note: For more information about any of the before mentioned algorithms, please refer to this book : "Forecasting methods and applications" [2].

2-3-2. Heuristic models

Forecasting is never perfect, that is there will always be some errors. The goal is to optimize the forecasts by minimizing the errors. In forecasting, there is not a single accepted method which works perfectly in every situation. This characteristic has encouraged many researchers and business practitioners to attempt to develop different forecasting algorithms and models, since the 1970s. Most of the developed techniques are case specific and often the result of the combination of different models, thus the name heuristics. The following described models are used with data sets with small sample sizes (less than 40). They are all non-linear. The data in this study has a size of 27, thus the reason for choosing such models.

2-3-2-1. Grey Model GM(1,1)

Grey system/theory is a non-traditional forecasting technique used in problems where there isn't enough information and the data is discrete. The "*grey*" in Grey theory means a mixture of *black* and *white* where *black* refers to a lack of information and *white* means complete information. The idea was introduced in the early 1980s by Deng [8]. The basic Grey prediction model is the GM(1,1), which is a time series forecasting model in the form of a differential equation. GM(1,1) does not require any prior knowledge to the system. It has the advantage that it can be used with as few as 4 observations [10]. Many variations of the model have been developed throughout the years: a Bayesian GM(1,1) was discussed in [47]; [47][48] discussed genetic algorithms associated with the grey model; the grey prediction with rolling mechanism was used in various studies [49][50]; a Grey-Markov model based on the Markov chains was also used in [51][52]; etc. These efforts are all attempts to improve the original GM(1,1). There are many steps to building a GM(1,1):

Step 1: the original data set, non negative historical sequence, is expressed as follows

$$x^{(0)} = \{x^{(0)}(k)\},\ \mathrm{k} = 1,2,3,\ldots,\mathrm{n} \tag{2.25}$$

Step 2: a new sequence $x^{(1)}$ is created, by a one time accumulated generating operation (AGO) using the initial dataset $x^{(0)}$ in *step 1*. The AGO partially eliminates any fluctuation in the original discrete data

$$x^{(1)}(k) = \sum x^{(0)}(i),\ \mathrm{k} = 1,2,3,\ldots,\mathrm{n} \tag{2.26}$$

Then

$$x^{(1)} = \{\, x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(k) \,\} = \{\, \sum_{i=1}^{1} x^{(0)}(i), \sum_{i=1}^{2} x^{(0)}(i), \ldots, \sum_{i=1}^{n} x^{(0)}(i) \,\}$$

which is a first-order Accumulated Generating Operation series obtained from the initial data set $x^{(0)}$.

Step 3: the grey prediction model GM(1,1) is expressed by the following one-variable first order differential equation

$$\frac{dx^{(1)}}{dt} + a*x^{(1)} = b \tag{2.27}$$

The whitening version of this equation is as follows

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{2.28}$$

where $z^{(1)}(k)$ is referred to as **background value** and is calculated through

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), \quad \text{k = 2,3,..,n} \tag{2.29}$$

$z^{(1)}(k)$ is the mean generation of consecutive neighbors value of accumulating generator sequence.

The parameter *a* is referred to as the development coefficient and *b* as the grey input coefficient.

Step 4: the values of ***a*** and ***b*** are obtained by applying the least-squared method to *eq. 4*

Step 5: Through the use of '***Laplace***' [32] inversion transform, the solution to the differential equation (eq. 2.28) is as follows

$$\hat{x}^{(1)}(k) = [x^{(0)}(1) - \frac{b}{a}] * e^{-a(k-1)} + \frac{b}{a}, \quad \text{k = 1,2,3,....} \tag{2.30}$$

This is called a time response sequence of the basic GM(1,1), it is a forecast result of the one time accumulated generating operation AGO.

Step 6: in order to retrieve the values used in the accumulation process prediction results in *step 5*, the one-time inverse accumulated generating operation (IAGO) is used and the following Grey model is obtained:

$$\hat{x}_0^{(0)}(k) = \hat{x}_0^{(0)}(k) - \hat{x}_0^{(0)}(k-1)$$

Then

$$\boldsymbol{\hat{x}_0^{(0)}(k) = (x^{(0)}(1) - \frac{b}{a})*(1 - e^{a})*e^{-a(k-1)}} \qquad \text{k = 1,2,3,...} \tag{2.31}$$

Where $\hat{x}^{(0)}(1) = x^{(0)}(1)$.

This last equation (eq. 7) is the model which will be used to forecast for future periods.

It is important to note that GM(1,1) accepts only **positive entries**.

For more details, refer to these articles [9] [11] [12].
This method was applied in Turkey by Hamzacebi [69] in 2014.

2-3-2-2. Grey Prediction with Rolling Mechanism (GPRM)

GPRM is a variant of the original GM(1,1). Building a GPRM model is very similar to building a GM(1,1), all the steps are the same through 1 to 6. But, the GPRM adds one (1) extra step to those; recent observations are more likely to give better insight to the future, therefore including them in our model would give better forecast values. That is what GPRM tries to do.
Setting up a GPRM can be summarized in 3 steps:

Step 1: here, we set up our model just like in GM(1,1) and forecast our first value

Step 2: upon obtention of the first predicted value, the oldest data in the original data set $x^{(0)}$ (in step 1 of the GM(1,1)) is removed, that is $x^{(0)}{}_{(1)}$, and the predicted value is inserted at the end of the series. Then, a new GM(1,1) model is set up using the new data set $x^{(0)}$ and we forecast our second value.

Step 3: the processes in step 2 are repeated for every new predicted value until we finish forecasting for a given period of time. This is the reason why the method is called 'rolling mechanism'.

GPRM was applied in Turkey by Akay and Atak for the electricity demand forecasting in 2007 in the following article [13]. This article [14] also contains an application of the method.

It is important to note that this model has a major downside. According to the principle of parsimony, the model is good since it has only 2 parameters. However, the process of repetition can be very exhausting, especially when forecasting for long periods. This makes it highly time consuming. This process could be eased down if there was a software implementation of the algorithm, but unfortunately up to date there are none. Another way around this issue would be to develop a piece of coding which could perform the repetition process for us. But again unfortunately, no codes were found during the course of this study.

2-3-2-3. Grey Model with Optimization of Background Value

It is important to notice that the prediction accuracy of GM(1,1) model is determined by the parameters ***a*** and ***b***, and the values of ***a*** and ***b*** depend on the original data set and the background value, namely the $z^{(1)}(k)$ sequence. So, the prediction precision is directly affected by the equation of background value. At present, most people use the linear value insert method, that is :

$$z^{(1)}(k) = \alpha * x^{(1)}(k-1) + (1 + \alpha) * x^{(1)}(k) \tag{2.32}$$

as the background value equation.

The method used in the above mentioned GM(1,1) model is the original mean value calculating formula:

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1) \tag{2.33}$$

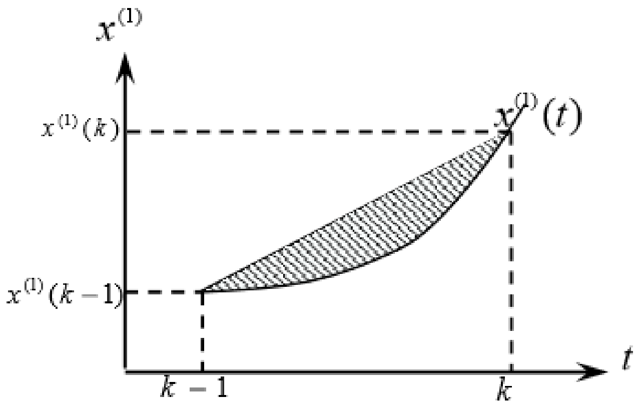

Figure 2.2: Area enclosed by $x^{(1)}(t)$ within ***[k-1,k]*** and the ***t*** axis.

However, it is possible to optimize this equation by calculating the area which is enclosed by $x^{(1)}(t)$ within ***[k-1,k]*** and the ***t*** axis instead of taking an average. This can be seen in *fig. 2.2*. The differential equation of the basic GM(1,1) (which is *eq. 2.27* in step 3 of the GM(1,1) model above) can be rewritten as follows:

$$\frac{dx^{(1)}}{dt} + a * x^{(1)} = b \tag{2.34}$$

Within [k-1, k], that is

$$\int_{k-1}^{k} \frac{dx^{(1)}}{dt} dt \; + \; a * \int_{k-1}^{k} x^{(1)} dt \; = \; b \tag{2.35}$$

The following equation is obtained:

$$x^{(1)}(k) - x^{(1)}(k-1) + a*\int_{k-1}^{k} x^{(1)} dt = b \quad (2.36)$$

The parameters ***a*** and ***b*** estimated by using $\int_{k-1}^{k} x^{(1)} dt$ as background value are more adaptive to whitenization equation.

According to article [19], let's assume that $x^{(0)}(k) = g*e^{a(k-1)}$ and $x^{(1)}(k) = G*e^{a(k-1)} + C$,

$$z^{(1)}(k) = \int_{k-1}^{k} x^{(1)} dt = \int_{k-1}^{k} \left(G * e^{a(t-1)} + C\right) dt = \frac{1}{a}*(G*e^{a(k-1)} - G*e^{a(k-2)})$$
$$\frac{1}{a}*(G*e^{a(k-1)} - G*e^{a(k-2)}) = \frac{1}{a}*[x^{(1)}(k) - x^{(1)}(k-1)] + C$$

$$z^{(1)}(k) = \frac{1}{a}*x^{(0)}(k) + C \quad (2.37)$$

Moreover, $\frac{x^{(0)}(k)}{x^{(0)}(k-1)} = \frac{g*e^{(k-1)}}{g*e^{(k-2)}} = e^{a}$, by applying a logarithm on both sides of the equation,

$$a = \ln x^{(0)}(k) - \ln x^{(0)}(k-1) \quad (2.38)$$

According to article [19] again, $C = -G*e^{-a} = g*(1 - e^{a})^{-1}$;

For $x^{(0)}(k) = g*e^{a(k-1)}$, $g = x^{(0)}(k)*e^{-a(k-1)} = x^{(0)}(k)*e^{a(1-k)}$
We know that:

$$e^{a} = \frac{x^{(0)}(k)}{x^{(0)}(k-1)}; \text{ therefore } g = x^{(0)}(k)*\left[\frac{x^{(0)}(k)}{x^{(0)}(k-1)}\right]^{(1-k)}$$

The value of ***C*** can be computed now:

$$C = g*(1 - e^{a})^{-1} = x^{(0)}(k)*\left[\frac{x^{(0)}(k)}{x^{(0)}(k-1)}\right]^{(1-k)}*\left(1 - \left[\frac{x^{(0)}(k)}{x^{(0)}(k-1)}\right]^{-1}\right) = \frac{[x^{(0)}(k-1)]^{k}}{[x^{(0)}(k)]^{k-2} * [x^{(0)}(k-1) - x^{(0)}(k)]}$$

At last, putting the values of ***a*** and ***C*** in ***eq 2.37***, the new background value formula $z^{(1)}(k)$ is:

$$z^{(1)}(k) = \frac{x(0)(k)}{\ln x^{(0)}(k) - \ln x^{(0)}(k-1)} + \frac{[x^{(0)}(k-1)]^{k}}{[x^{(0)}(k)]^{k-2} * [x^{(0)}(k-1) - x^{(0)}(k)]} \quad (2.39)$$

The parameter ***a*** is estimated by taking the average of the values obtained using ***eq 2.38***. The parameter ***b*** is estimated by using the ***eq 2.28*** in the GM(1,1) model, that is

$$x^{(0)}(k) + az^{(1)}(k) = b \quad \text{so} \quad b = x^{(0)}(k) + az^{(1)}(k)$$

with the new calculated value of ***a*** and the new background value $z^{(1)}(k)$:

$$b(k) = x^{(0)}(k) + a*\left(\frac{x(0)(k)}{\ln x^{(0)}(k) - \ln x^{(0)}(k-1)} + \frac{[x^{(0)}(k-1)]^{k}}{[x^{(0)}(k)]^{k-2} * [x^{(0)}(k-1) - x^{(0)}(k)]}\right) \quad (2.40)$$

for k = 2,3,..,n .The final value of parameter ***b*** is the average of the results obtained using (*2.40*).

The values of a and b are then put in the grey prediction equation and prediction operations can be performed.

$$\hat{x}_0^{(0)}(k) = [x^{(0)}(1) - \frac{b}{a}]*(1 - e^{a})*e^{-a(k-1)} \quad (2.41)$$

2-3-2-4. Grey_ARIMA model

This is a hybrid method based on a simple combination of the two models. Predictions are made using both algorithms separately; the errors for each method is calculated. By assigning weights to both algorithms according to their residuals (errors) and adding their results for respective values of *k* (*k* being the time), a new value is then obtained (forecast value), a value which is normally better. The model is as follows:

$$Hybrid(Grey\_ARIMA) = \alpha*GM(1,1) + \beta*ARIMA(n, p, q) \quad (2.42)$$

Here, the terms *GM(1,1)* and *ARIMA(n,p,q)* represent their respective forecasts at different time values. α and β are parameters assigned to both methods according to the way their residuals corrolate with each other. Their values are between [0, 1]. Note that $\alpha + \beta = 1$. However, it isn't always easy to figure out something just through a study of corrolations. Sometimes, a few trials are needed in order to find the best values of those two parameters. Therefore, it is recommended to try different values of α and β to see which ones give minimum errors. A good example could be as follows:

$$Hybrid = 0.5*GM(1,1) + 0.5*ARIMA(n, p, q)$$

Here, 0.5 means that the residuals of the two models are mutually exclusive, that is when the error of one is positive, the error for the other one is negative, or when the value of one rises, the other one decreases. The predicted value for this model is the average of the results of both GM(1,1) and ARIMA for any given value of *k*.

This particular method has been implemented/used in the following article [15].

The study down here is divided into two parts: in ***Section 2-4-1***, we will work with the *initial* 27 dataset and perform a series of forecasting excersise for 5 periods and get the errors. In ***Section 2-4-2***, the same operations will be performed using the new dataset obtained through *bootstrapping* and the results obatined there will be compared with the ones of *Section 2-4-1* in ***Section 3***.

2-4. Application of the methods

The followings are the applications of the previously mentioned methods. In *section 2-4-1*, the models are applied on the unemployment rates of Mali from (1990-2016), which is referred to as the initial dataset and *section 2-4-2* shows the application of those methods on the data obtained after bootstraping the initial dataset, which is referred to as '*Sample 10*'.

2-4-1. Forecasting with the initial dataset

Here, for every algorithm, the first **22** entries of the data will be used to set up our models, and the last **5** entries to test the model. The time series plot of the data is shown in *figure 2.3*. Looking at the graph, it can be seen that the data shows no trend or seasonal patterns. Therefore we can conclude that Moving avarages and Single exponential smoothing methods are suitable for this case.

The mean of the data is calculated as follows

$$\mu = \left(\sum_{i=1}^{22} Yi \right) / n \qquad (2.43)$$

with *Yi* being the observations at time *i* and *n* equals 22. It is found to be 8,309.

The standard deviation, which is a measure of how numbers are spread out over the mean, is expressed as follows

$$s = \sqrt{\frac{\sum_{i=1}^{n} (Yi - \bar{Y})^2}{n}} \qquad (2.44)$$

with $\bar{Y}$ being the mean of the sample and *n* equals 22 again. It is found to be 2,222. A low standard deviation indicates that the observations are not very distant from the mean whereas a high standard deviation indicates the opposite, that is, observations are quite far off from the mean.

The variance, similar and closely related to the variance, is expressed simply as the square of the standard deviation, as follows

$$v = s^2 = \frac{\sum_{i=1}^{n} (Yi - \bar{Y})^2}{n} \qquad (2.45)$$

It is found to be 4,937.

The *Pearson correlation coefficient* is used to determine the type of relationship between two (2) variables. Its value can range between -1 and +1. A negative value indicates a negative correlation, that is if the value of one variable increases, the value of the second variable decreases, or vice versa. A positive value indicates a positive correlation, that is both variables decrease or increase together. The formula of the pearson correlation is expressed as follows :

$$r = \frac{n*\sum(x*y) - (\sum x)*(\sum y)}{\sqrt{[n*\sum x^2 - (\sum x)^2]*[n*\sum y^2 - (\sum y)^2]}} \tag{2.46}$$

where *x* represents the first variable which is the time in our case, and *y* represents the second variable which is the unemployment rates. The Pearson correlation coefficient of the *Year* and *Unemployment* is found to be 0.023, which is very low. This indicates that there is very little or no correlation between the time and the change of values for the unemployment. Therefore, a linear regression is not suitable for this data. Nonetheless, we will still use it, for the sake of comparison with other algorithms. Also no ARIMA model will be used in this section, that is because the size of the dataset is only 27 and as mentioned before, a minimum of 50 data are needed in order to be able to set up an ARIMA model.

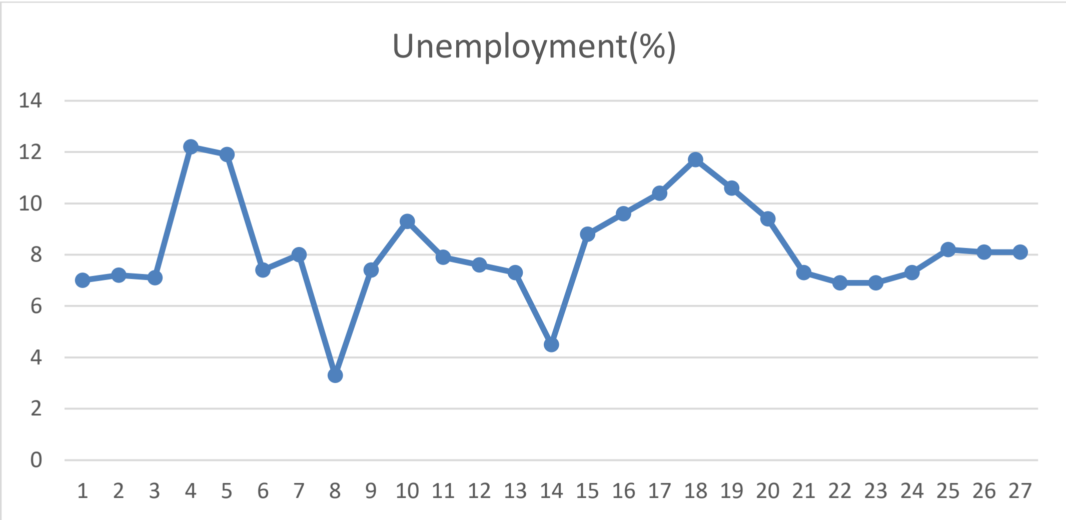

Figure 2.3: Unemployment rates of Mali from World Data Atlas.

2-4-1-1. Simple Linear regression

The data consists of only two (2) variables, the time which is in year and the unemployment rates. Therefore a simple linear regression model is best suitable here.The first 22 entries of the data are inserted into *Minitab* and the method of simple linear regression is applied to them.

Again the methodology/algorithm is already implemented in Minitab. All the parameters are automatically estimated by the program. Similarly to *eq 2.16* in *Section 2-2-1-3-a*, the following linear regression equation is obtained:

$$\text{Unemployment}(\%) = -79 + (0{,}0437 * \text{year})$$

Using this equation, the next 5 years values are predicted as follows:
Year 2012: Unemployment(%) = -79 + (0,0437 * 2012) = 8,9244;
Year 2013: Unemployment(%) = -79 + (0,0437 * 2013) = 8,9681;
Year 2014: Unemployment(%) = -79 + (0,0437 * 2014) = 9,0118;
Year 2015: Unemployment(%) = -79 + (0,0437 * 2015) = 9,0555;
Year 2016: Unemployment(%) = -79 + (0,0437 * 2016) = 9,0992;

These results are now compared with the observed ones and the MAE, MSE and RMSE are computed.

Table 2.3.3: Error estimation for the simple linear regression.

Year	2012	2013	2014	2015	2016
Observed value	6.9	7.3	8.2	8.1	8.1
Predicted value	8.9244	8.9681	9.0118	9.055	9.0992
Error (absolute)	2.0244	1.6681	0.8118	0.9555	0.9992
Error (square)	4.098195	2.782558	0.659019	0.91298	0.998401

The MAE is equal to 1.2918, the MSE is 1.89 and the RMSE is 1.37

2-4-1-2. Simple moving averages

As explained before, the more periods we use in a moving average, the worst our forecats will be. Therefore it is convenient to use a 3-period simple moving average here, that is, 3 observations will be included in each average. The following is the equation for that:

$$T_t = \frac{1}{3}(Y_{t-1} + Y_t + Y_{t+1}) , \quad \text{where } t = 1, 2, 3, 4, ..., n-1$$

where Y is the observed value at time t.

The whole data set will be used here, that is all 27 entries will be needed.

For t = 1, the following is calculated:

$$T_1 = \frac{1}{3}(Y_0 + Y_1 + Y_2) = \frac{1}{3}(7 + 7{,}2 + 7{,}1) = \frac{1}{3}(Y_{1990} + Y_{1991} + Y_{1992}) = 7{,}1.$$

This is the forecast value for the year 1991, not 1990.

For t = 2 (year 1992 now),

$$T_2 = \frac{1}{3}(Y_1 + Y_2 + Y_3) = \frac{1}{3}(Y_{1991} + Y_{1992} + Y_{1993}) = 8{,}83.$$

This operation is repeated over and over until the end of the data. The results obtained are shown in *table 2.3.4*.

Comparing those results with the observed ones, the errors are computed and the MAE, the MSE and the RMSE are found to be 0.8611, 1.4619 and 1.2 respectively.

Table 2.3.4: Simple Moving average results.

Year 1991	T_{1991}	7.1		Year 2004	T_{2004}	7.63
Year 1992	T_{1992}	8.83		Year 2005	T_{2005}	9.6
Year 1993	T_{1993}	10.4		Year 2006	T_{2006}	10.57
Year 1994	T_{1994}	10.5		Year 2007	T_{2007}	10.9
Year 1995	T_{1995}	9.1		Year 2008	T_{2008}	10.57
Year 1996	T_{1996}	6.233		Year 2009	T_{2009}	9.1
Year 1997	T_{1997}	6.233		Year 2010	T_{2010}	7.87
Year 1998	T_{1998}	6.67		Year 2011	T_{2011}	7.03
Year 1999	T_{1999}	8.2		Year 2012	T_{2012}	7.03
Year 2000	T_{2000}	8.266		Year 2013	T_{2013}	7.47
Year 2001	T_{2001}	7.6		Year 2014	T_{2014}	7.87
Year 2002	T_{2002}	6.466		Year 2015	T_{2015}	8.13
Year 2003	T_{2003}	6.87				

2-4-1-3. Single exponential smoothing

The time series plot of the data (displayed in *fig. 2.3*) shows no sign of any trend nor seasonality. Therefore, a single exponential smoothing model is suitable for this data. The general equation for a single exponential smoothing, as seen in *section 2-3-1-2-a*, is as follows:

$$F_{t+1} = F_t + \alpha*(Y_t - F_t)$$

where Y_t was the observed value at time t and F_{t+1} the predicted value. It is commonly assumed that the initial value F_0 is the first oberved value (first entry in the table). For this case, the

Table: 2.3.5: Results of the single exponential smoothing method.

α values	0.2		0.5		0.7		0.9
	F_{t+1}		F_{t+1}		F_{t+1}		F_{t+1}
t=0	7	F_1	7		7		7
t=1	7.04	F_2	7.1		7.14		7.18
t=2	7.072	F_3	7.1		7.112		7.108
t=3	8.0976	F_4	9.65		10.6736		11.6908
t=4	8.85808	F_5	10.775		11.53208		11.87908
t=5	8.566464	F_6	9.0875		8.639624		7.847908
t=6	8.453171	F_7	8.54375		8.191887		7.984791
t=7	7.422537	F_8	5.921875		4.767566		3.768479
t=8	7.41803	F_9	6.6609375		6.61027		7.036848
t=9	7.794424	F_{10}	7.9804688		8.493081		9.073685
t=10	7.815539	F_{11}	7.9402344		8.077924		8.017368
t=11	7.772431	F_{12}	7.7701172		7.743377		7.641737
t=12	7.677945	F_{13}	7.5350586		7.433013		7.334174
t=13	7.042356	F_{14}	6.0175293		5.379904		4.783417
t=14	7.393885	F_{15}	7.4087646		7.773971		8.398342
t=15	7.835108	F_{16}	8.5043823		9.052191		9.479834
t=16	8.348086	F_{17}	9.4521912		9.995657		10.30798
t=17	9.018469	F_{18}	10.576096		11.1887		11.5608
t=18	9.334775	F_{19}	10.588048		10.77661		10.69608
t=19	9.34782	F_{20}	9.9940239		9.812983		9.529608
t=20	8.938256	F_{21}	8.6470119		8.053895		7.522961
t=21	8.530605	F_{22}	7.773506		7.246168		6.962296
t=22	8.204484	F_{23}	7.336753		7.003851		6.90623
t=23	8.023587	F_{24}	7.3183765		7.211155		7.260623
t=24	8.05887	F_{25}	7.7591882		7.903347		8.106062
t=25	8.067096	F_{26}	7.9295941		8.041004		8.100606
t=26	8.073677	F_{27}	8.0147971		8.082301		8.100061

values 0.2, 0.5, 0.7 and 0.9 have been used for the parameter α. *Table 2.3.5* shows the results for every case (the calculations have been done in Microsoft Excel).

The error for each case is as follows:

For α = 0.2, the MAE = 1.203 and MSE = 2.935

For α = 0.5, the MAE = 0.784 and MSE = 1.152

For α = 0.7, the MAE = 0.464 and MSE = 0.410

For α = 0.9, the MAE = 0.146 and MSE = 0.046

The value of **α = 0.9** yields the minimum MAE and MSE, therefore that value of α is the most appropriate for this case. Its RMSE value is 0.214

2-4-1-4. Original GM(1,1)

As explained in *section 2-3-2-1*, there are 6 steps to building a grey differential equation or model. The first 22 entries of the data set will be used ,through step 1 to 6, to set up the model.

Step 1: the initial series $X^{(0)}$ is equal to the first 22 entries, that is $X^{(0)}(k)$ = {7, 7.2 , 7.1 , 12.2 , ..., 9.4 , 7.3, 6.9}.

Step 2: the new series $X^{(1)}$ is generated using $X^{(0)}$ in *eq. 2.26* of *section 2-2-2-1*. $X^{(1)}(k)$ = {7, 14.2 , 21.3 , 33.5 , 45.4 , 52.8 , 60.8 , 64.1 , 71.5 , 80.8 , 88.7 , 96.3 , 103.6 , 108.1 , 116.9 , 126.5 , 136.9 , 148.6 , 159.2, 168.6 , 175.9, 182.8}

Step 3-4: the following equation (*eq. 2.28*) represents the basic GM(1,1)

$$X^{(0)}(k) + a*Z^{(1)}(k) = b \quad \Rightarrow \quad X^{(0)}(k) = b - a*Z^{(1)}(k).$$

This equation must be solved in oder to estimate the best values for *a* and *b*, for the values of *k* starting from *2* to *n*. We can do so by applying the OLS technique and and with the help of matrice calculations. However, it is possible to make this task a bit easier. Instead of using matrices, some transformations will be introduced here.

Let's name 3 variables *X*, *Y* and *A* such that

$$X = Z^{(1)}(k) , \quad Y = X^{(0)}(k) \quad \text{and} \quad A = -a$$

By substituting these variables into the previous equation, we obtain the fitted equation:

$$Y = b + A*X$$

For each observed response Y_i, with a corresponding predictor X_i, we obtain a fitted value

$$\hat{Y}_i = b + A*X_i .$$

We would like to minimize the sum of squares error, that is minimize the squared distances between each observed value to its fitted/predicted value.

$$SSE = \sum(Yi - \hat{Y}i)^2 = \sum(Yi - (b + A*Xi))^2 \quad \text{for } i = 1,....,n$$

A little bit of calculus is introduced in order to solve for this .

- $SS_{xx} = \sum(x_i - \bar{x})^2 = \sum(x_i)^2 - [(\sum x_i)^2]/n$
- $SS_{xy} = \sum(x_i - \bar{x})(y_i - \bar{y}) = \sum(x_i y_i) - [(\sum x_i)*(\sum y_i)]/n$
- $SS_{yy} = \sum(y_i - \bar{y})^2 = \sum(y_i)^2 - [(\sum y_i)^2]/n$
- $A = SS_{xy} / SS_{xx}$
- $b = [(\sum y_i)/n] - A*[(\sum x_i)/n]$

$\bar{x}$ and $\bar{y}$ represent the averages for the values of $\mathbf{x_i}$ and $\mathbf{y_i}$ that are included in the calculations.

$$\bar{x} = [\sum x_i]/n \quad \text{and} \quad \bar{y} = [\sum y_i]/n$$

All the calculations are performed in Excel. Table 2.3.6 summarizes them.

The following results are obtained:

$\sum(x_i) = 1968.6$ $\qquad \sum(x_i)^2 = 237174.7$

$\sum(y_i) = 175.8$ $\qquad \sum(y_i)^2 = 1573.58$

$\sum(x_i*y_i) = 16683.42$

The values of SS_{xx} , SS_{xy}, *A* and *b* can now be easily computed:

$SS_{xx} = 237174.7 - [(1968.6)^2]/21 = 52632.5$

$SS_{xy} = 16683.42 - [1968.6*175.8]/21 = 203.42$

$A = SS_{xy} / SS_{xx} = 203.42 / 52632.5 = 0.00386$

$b = [175.8 / 21] - 0.00386*[1968.6 / 21] = 8.009$

We said earlier that $A = -a$, which means that $a = -A$. Therefore $a = -0.00386$.

The estimate values of ***a*** and ***b*** are -0.00386 and 8.009 respectively.

Step 6: the values of *a* and *b* are put into the Grey model and the following equation is obtained

$$\hat{x}_0^{(0)}(k) = [x^{(0)}(1) - \frac{b}{a}]*(1 - e^{a})*e^{-a(k-1)} = [x^{(0)}(1) + \frac{8.009}{0.00386}]*(1 - e^{-0.00386})*e^{0.00386(k-1)}$$

with the initial value of $x^{(0)}{}_{(1)} = 7$.

The values for the next 5 years can now be predicted.

for k = 23, $\hat{x}_0^{(0)}(23) = [7 + 2074.87]*(1 – e^{-0.00386})*e^{0.00386(23-1)} =$ 8.72

for k = 24, $\hat{x}_0^{(0)}(24) = [7 + 2074.87]*(1 – e^{-0.00386})*e^{0.00386(24-1)} =$ 8.75

for k = 25, $\hat{x}_0^{(0)}(25) = [7 + 2074.87]*(1 – e^{-0.00386})*e^{0.00386(25-1)} =$ 8.79

for k = 26, $\hat{x}_0^{(0)}(26) = [7 + 2074.87]*(1 – e^{-0.00386})*e^{0.00386(26-1)} =$ 8.82

for k = 27, $\hat{x}_0^{(0)}(27) = [7 + 2074.87]*(1 – e^{-0.00386})*e^{0.00386(27-1)} =$ 8.86

The MAE, MSE and RMSE are calculated and their values are 1.068, 1.3718 and 1.17 respectively.

Table 2.3.6: Summary of the calculations performed in Excel.

k	$x^{(0)}$	$x^{(1)}(k)$	$x^{(1)}(k-1)$	$Z^{(1)}(k)$	Y	X	Xsquare	X*Y	Ysquare
1	7	7			7				
2	7.2	14.2	7	10.6	7.2	10.6	112.36	76.32	51.84
3	7.1	21.3	14.2	17.75	7.1	17.75	315.0625	126.025	50.41
4	12.2	33.5	21.3	27.4	12.2	27.4	750.76	334.28	148.84
5	11.9	45.4	33.5	39.45	11.9	39.45	1556.303	469.455	141.61
6	7.4	52.8	45.4	49.1	7.4	49.1	2410.81	363.34	54.76
7	8	60.8	52.8	56.8	8	56.8	3226.24	454.4	64
8	3.3	64.1	60.8	62.45	3.3	62.45	3900.003	206.085	10.89
9	7.4	71.5	64.1	67.8	7.4	67.8	4596.84	501.72	54.76
10	9.3	80.8	71.5	76.15	9.3	76.15	5798.823	708.195	86.49
11	7.9	88.7	80.8	84.75	7.9	84.75	7182.563	669.525	62.41
12	7.6	96.3	88.7	92.5	7.6	92.5	8556.25	703	57.76
13	7.3	103.6	96.3	99.95	7.3	99.95	9990.003	729.635	53.29
14	4.5	108.1	103.6	105.85	4.5	105.85	11204.22	476.325	20.25
15	8.8	116.9	108.1	112.5	8.8	112.5	12656.25	990	77.44
16	9.6	126.5	116.9	121.7	9.6	121.7	14810.89	1168.32	92.16
17	10.4	136.9	126.5	131.7	10.4	131.7	17344.89	1369.68	108.16
18	11.7	148.6	136.9	142.75	11.7	142.75	20377.56	1670.175	136.89
19	10.6	159.2	148.6	153.9	10.6	153.9	23685.21	1631.34	112.36
20	9.4	168.6	159.2	163.9	9.4	163.9	26863.21	1540.66	88.36
21	7.3	175.9	168.6	172.25	7.3	172.25	29670.06	1257.425	53.29
22	6.9	182.8	175.9	179.35	6.9	179.35	32166.42	1237.515	47.61

2-4-1-5. Grey prediction with rolling mechanism

Since predictions for five (5) periods need to be made, five (5) different GM(1,1) models will be needed. The process of setting up one Grey model is already a bit tiring, setting up five (5) comes with much more difficulties. However, for simplicity, only the results will be shown down here, that is the final equations and the predicted values. The first step of the method is the combination of all the steps/work in a basic GM(1,1). Therefore, the final equation obtained in the above section (*section 2-4-1-d*) will be used in the first step.

Step 1: the first model is as follows

$$\hat{x}_0^{(0)}(k) = [x^{(0)}(1) + \frac{8.009}{0.00386}]*(1 - e^{-0.00386})*e^{0.00386(k-1)}$$

with $x^{(0)}_{(1)} = 7$.

For $k = 23$, the predicted value is $\hat{x}_0^{(0)}(23) = 8{,}72$.

The first data $x^{(0)}_{(1)} = 7$ is removed and 8,72 is the new entry added to our data.

Step 2: the new value of $x^{(0)}_{(1)}$ is the second element of the initial series $X^{(0)}$, that is

$$x^{(0)}_{(1)new} = x^{(0)}_{(2)} = 7{,}2.$$

Also as mentioned before, 8.72 is added to the end of the data. The new model is generated using the new data.

Step 3: the new model is

$$\hat{x}_0^{(0)}(k) = [x^{(0)}(1) + \frac{8.21}{0.00237}]*(1 - e^{-0.00237})*e^{0.00237(k-1)} \qquad \text{with } x^{(0)}_{(1)} = 7{,}2.$$

For k = 24, the following value is obtained: $x^{(0)}_{(24)} = 8{,}67$.

This new value is added to the series/data and $x^{(0)}_{(1)} = 7{,}2$ is removed. Now

$$x^{(0)}_{(1)new} = x^{(0)}_{(3)} = 7.1$$

of the initial data series used in step 1. Another model is generated again.

Step 4: the new model is as follows

$$\hat{x}_0^{(0)}(k) = [x^{(0)}(1) + \frac{8.47}{0.00057}]*(1 - e^{-0.00057})*e^{0.00057(k-1)} \qquad \text{with } x^{(0)}_{(1)} = 7{,}1.$$

For k = 25, the following is obtained :$x^{(0)}_{(25)}$ = 8,59.

Again this value is added, $x^{(0)}_{(1)}$ = 7,1 is removed from the data and

$$x^{(0)}_{(1)new} = x^{(0)}_{(4)} = 12,2$$

from the original data and another model is generated.

Step 5: the new model is as follows

$$\hat{x}_0^{(0)}(k) = [x^{(0)}(1) + \frac{7.68}{0.0068}]*(1 - e^{-0.068})*e^{0.068(k-1)} \qquad \text{with } x^{(0)}_{(1)} = 12,2.$$

For k = 26, the predicted value is: $x^{(0)}_{(26)}$ = 9.17.

Now, the last model is generated with $x^{(0)}_{(1)new} = x^{(0)}_{(5)}$ = 11,9.

Step 6: the last model is as follows

$$\hat{x}_0^{(0)}(k) = [x^{(0)}(1) + \frac{6.63}{0.0186}]*(1 - e^{-0.0186})*e^{0.0186(k-1)} \qquad \text{and } x^{(0)}_{(1)} = 11,9.$$

For k = 27, the prediction for the last period is: $x^{(0)}_{(27)}$ = 11,01.

In summary, the forecast values for period 23 to 27 are:

k = 23, $\hat{x}_0^{(0)}(23)$ = 8,72

k = 24, $\hat{x}_0^{(0)}(24)$ = 8,67

k = 25, $\hat{x}_0^{(0)}(25)$ = 8,59

k = 26, $\hat{x}_0^{(0)}(26)$ = 9,17

k = 27, $\hat{x}_0^{(0)}(27)$ = 11,01

The errors for the last five periods are 1.512, 2.99 and 1.729 for the MAE, the MSE and the RMSE respectively.

2-4-1-6. Grey model with Optimization of Background Value

The first thing to do is to estimate the value of parameter *a* using *eq. 2.38* of *section 2-3-2-3*

$$a(k) = \ln x^{(0)}(k) - \ln x^{(0)}(k-1)$$

For every value of k = 2,3,...,22 a new value of $\boldsymbol{a}$ is obtained and at the end, those values are averaged to obtain the final value of $\mathbf{a} = \frac{\sum a(k)}{21} = \mathbf{-0{,}0003}$. *Table 2.3.7* displays the numerical calculations for the parameter *a*.

Table 2.3.7: Microsoft Excel results for parameter *a*.

K value	Ln $x^{(0)}(k)$	Ln $x^{(0)}(k-1)$	a(k)
1		0.84509804	
2	0.857332496	0.857332496	0.0122345
3	0.851258349	0.851258349	-0.006074
4	1.086359831	1.086359831	0.2351015
5	1.075546961	1.075546961	-0.010813
6	0.86923172	0.86923172	-0.206315
7	0.903089987	0.903089987	0.0338583
8	0.51851394	0.51851394	-0.384576
9	0.86923172	0.86923172	0.3507178
10	0.968482949	0.968482949	0.0992512
11	0.897627091	0.897627091	-0.070856
12	0.880813592	0.880813592	-0.016813
13	0.86332286	0.86332286	-0.017491
14	0.653212514	0.653212514	-0.21011
15	0.944482672	0.944482672	0.2912702
16	0.982271233	0.982271233	0.0377886
17	1.017033339	1.017033339	0.0347621
18	1.068185862	1.068185862	0.0511525
19	1.025305865	1.025305865	-0.04288
20	0.973127854	0.973127854	-0.052178
21	0.86332286	0.86332286	-0.109805
22	0.838849091		-0.024474

Next, the background value needs to be estimated for every value of *k* between [2, 22] using *eq. 2.39* of the method. The right-hand side of the equation is divided in 2 parts to make the calculations in Excel easier, thus the reason for introducing the parameters J and F.

$$z^{(1)}(k) = \frac{x(0)(k)}{ln\, x^{(0)}(k) - ln\, x^{(0)}(k-1)} + \frac{[x^{(0)}(k-1)]^{k}}{[x^{(0)}(k)]^{k-2} * [x^{(0)}(k-1) - x^{(0)}(k)]} = J + F$$

Table 2.3.8 shows the numerical results of the background value $\boldsymbol{z^{(1)}(k)}$ for k = 2,3,....,22
After that, the parameter *b* needs to be estimated in its turn, for every single value of $z^{(1)}(k)$, using *eq. 40* of the method (*section 2-2-2-3*).

$$b(k) = x^{(0)}(k) + a * z^{(1)}(k) \qquad \text{for } a = -0,0003$$

which is the the value calculated above and k = 2,3,...,22. The final estimate of *b* is the average of all the *b(k)* which is found to be $\mathbf{b} = \frac{\sum b(k)}{21}$ = **8,1489**.

Finally, the values of ***a*** and ***b*** are put into *eq. 2.41* of *section 2-2-2-3* and the following Grey prediction model is obtained:

$$\hat{x}_0^{(0)}(k) = [x^{(0)}(1) - \frac{b}{a}]*(1 - e^{a})*e^{-a(k-1)} = [7 + \frac{8,1489}{0,0003}]*(1 - e^{-0,0003})*e^{0,0003(k-1)}$$

Now, the values of year 2012 to 2016 can be predicted through *k* values of 23, 24, 25, 26 and 27 respectively. They are:

Year 2012: k = 23 → $\hat{x}^0(23) = 8{,}203$

Year 2013: k = 24 → $\hat{x}^0(24) = 8{,}206$

Year 2014: k = 25 → $\hat{x}^0(25) = 8{,}208$

Year 2015: k = 26 → $\hat{x}^0(26) = 8{,}211$

Year 2016: k = 27 → $\hat{x}^0(27) = 8{,}213$

Finally, the MAE, the MSE and the RMSE are computed and their values are 0.4882, 0.5087 and 0.71 respectively.

Table 2.3.8: Microsoft Excel results for the background value $z^{(1)}(k)$.

J	$[x^{(0)}(k-1)]^k$	$[x^{(0)}(k)]^{k-2}$	$x^{(0)}(k-1) - x^{(0)}(k)$	F	$Z^{(1)}(k)$
588.5018	49	1	-0.2	-245	343.5018
-1168.89	373.248	7.1	0.1	525.7014	-643.187
51.89248	2541.1681	148.84	-5.1	-3.34768	48.54481
-1100.54	270270.8163	1685.159	0.3	534.6099	-565.931
-35.8674	2839760.855	2998.6576	4.5	210.4468	174.5794
236.2791	1215128.027	32768	-0.6	-61.8046	174.4745
-8.58088	16777216	1291.46797	4.7	2764.002	2755.421
21.09959	46411.4844	1215128.03	-4.1	-0.00932	21.09027
93.70161	492399039.7	55958181	-1.9	-4.63127	89.07034
-111.494	45010354568	119851596	1.4	268.2505	156.7566
-452.018	59091511032	642888893	0.3	306.3853	-145.632
-417.364	2.82213E+11	3137266856	0.3	299.85	-117.514
-21.4173	1.22045E+12	68952523.6	2.8	6321.379	6299.962
30.2125	6283298709	1.8979E+12	-4.3	-0.00077	30.21173
254.0451	1.29337E+15	5.6467E+13	-0.8	-28.6309	225.4142
299.1763	4.99587E+16	1.8009E+15	-0.8	-34.6754	264.501
228.7277	2.02582E+18	1.233E+17	-1.3	-12.6381	216.0896
-247.202	1.97484E+20	2.6928E+17	1.1	666.7131	419.5116
-180.153	3.20714E+20	3.2832E+17	1.2	814.0193	633.8668
-66.4815	2.727E+20	2.5301E+16	2.1	5132.528	5066.046
-281.935	9.84244E+18	5.9839E+16	0.4	411.2076	129.2731

2-4-2) Forecasting with the bootstrap dataset

As explained before, a set with the sample size of 108 will be generated from the initial data set. ***Minitab*** is used for this purpose. In order to find a bootstrap data set which ressembles the most to the initial data set, ***10*** bootstrap data sets (Sample 1 through 10) were created and the one with the closest *mean* and *standard deviation* to the original/initial data set's was chosen. The following table displays the statistics for each bootstrap data set:

Table 2.3.9: Descriptive statistics of the 10 bootstrap data sets.

Variables	Mean	StDev
Unemployment	8,200	2,024
Sample 1	8,330	2,043
Sample 2	7,959	1,601
Sample 3	8,106	2,213
Sample 4	8,327	1,985
Sample 5	8,228	2,188
Sample 6	8,358	1,918
Sample 7	8,424	1,946
Sample 8	8,441	2,078
Sample 9	7,905	1,877
Sample 10	8,192	1,941

Unemployment represents the original set. As it can be seen, it is a bit difficult to pick a *sample* according to both statistics. However, "***Sample 10***" has a mean which is the closest to the one of the unemployment set and its standard deviation is very close to it too. Therefore, it is chosen as the data set which will be used in this part of the thesis. Note that it has a sample size of 108. "*Sample 10*" is divided into multiple ***quartiles***, that is, every year has 4 quartiles, therefore for example the first 4 entries represent the values of quartile 1, quartile 2, quartile 3 and quartile 4 of year 1990. The next 4 entries are the values of the first, second, third and fourth quartile of 1991 (an so on and so forth). The last 4 entries are the values of quartile 1 through 4 of year 2016. *Fig. 2.3.2* shows the time series plot of the data from time $t = 1$, which represents the first quartile of year 1990, to time $t = 108$, which is quartile 4 of year 2016. The table of *Sample 10* is attached in the appendices. For every algorithm used next, the first ***88*** entries of '***Sample 10***' will be used to set up each model, and the remaining ***20*** entries will be used to test each one of them and get the errors. Since the results here will be compared with the one obtained from the initial data set (of 27 entries), it is therefore logical to predict for *5* periods with the next algorithms as well. And also since we've assumed that every year in '***Sample 10***' has 4 quartiles, 4*5 = 20 quartiles for the last 5 years. The last 20 entries therefore represent the values of the years 2012, 2013, 2014, 2015 and 2016 respectively.

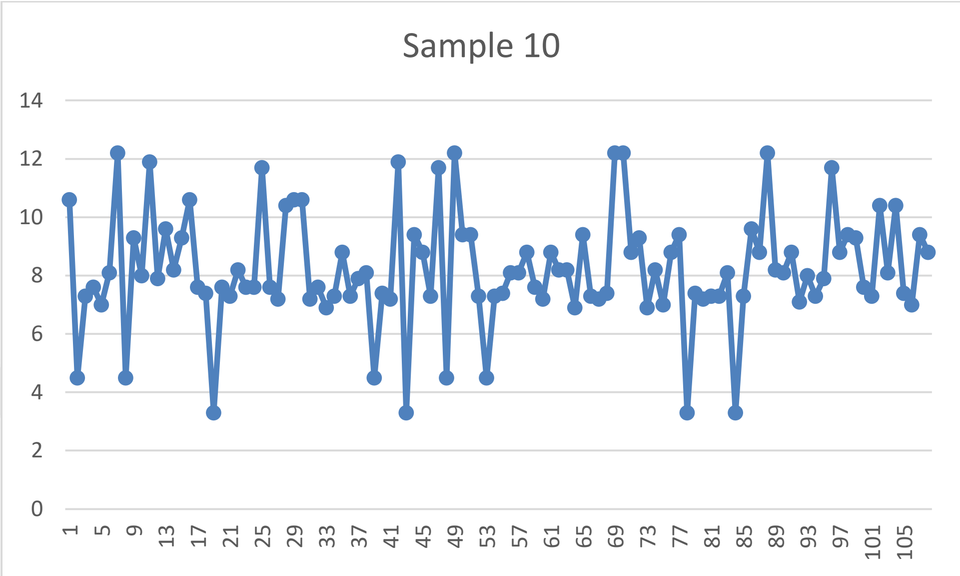

Figure 2.4: Time series plot of 'Sample 10'.

2-4-2-1. Simple Linear regression

The data of '*Sample 10*' consists of only two (2) variables again, the time which is in year and the unemployment rates. Therefore a simple linear regression model is chosen to be suitable.The first 88 entries of the data are inserted into *Minitab* and the method of simple linear regression is applied to them. All the parameters are automatically estimated by the program. Similarly to *eq. 2.16* in *Section 2-2-1-3-a*, the following linear regression equation is obtained:

unemployment = 8.21 - (0,00224 * t)

Using this equation, the next 20 periods will be forecasted. Again, every period represent a quartile of a year, and therefore to get the yearly rate, we would simply average 4 periods at a time. For

t = 89: Unemployment = 8.21 - (0,00224 * 89) = 8.010

t = 90: Unemployment = 8.21 - (0,00224 * 90) = 8.008

t = 91: Unemployment = 8.21 - (0,00224 * 91) = 8.006

t = 92: Unemployment = 8.21 - (0,00224 * 92) = 8.003

These 4 values represent quartile 1, quartile 2, quartile 3 and quartile 4 values of year 2012. This process is carried on until the last 4 elements, being the different quartiles of year 2016 are forecasted.

t = 105: Unemployment = 8.21 - (0,00224 * 105) = 7.974

t = 106: Unemployment = 8.21 - (0,00224 * 106) = 7.972

t = 107: Unemployment = 8.21 - (0,00224 * 107) = 7.970

t = 108: Unemployment = 8.21 - (0,00224 * 108) = 7.968

These results are now compared with the observed ones and the MAE, MSE and RMSE are calculated just as in *section 2-4-1-a*. They are found to be 0.99, 1.82 and 1.35 respectively.

2-4-2-2. Simple moving averages

As explained before in *section 2-3-1-1*, the more periods we use in a moving average, the worst our forecats will be. Therefore it is convenient to use a 3-period simple moving average here as well. The following is the equation for that:

$$T_t = \frac{1}{3}(Y_{t-1} + Y_t + Y_{t+1}) , \qquad \text{where } t = 1, 2, 3, 4, ..., n-1$$

where *Y* is the observed value at time *t* and $n = 108$.

The whole data set is used here, that is all 108 entries will be needed. The numerical calculations are the same as in the previous example in *section 2-4-1-b* and are performed in Excel. The file is attached in the appendices as *Appendix 1*. It also contains '*Sample 10*'.

The results obtained are compared with the observed values, the errors are computed and the MAE, the MSE and RMSE are found to be equal to 1.29, 2.83 and 1.68 respectively.

2-4-2-3. Single exponential smoothing

Fig. 2.4 shows no sign of any trend nor seasonality. Therefore, a single exponential smoothing model is suitable for this data. The general equation for a single exponential smoothing, as seen in *section 2-3-1-2-a*, is as follows:

$$F_{t+1} = F_t + \alpha*(Y_t - F_t)$$

where Y_t was the observed value at time *t* (*t* goes from 0 to 107) and F_{t+1} the predicted value. It is commonly assumed that the initial value F_0 is the first oberved value (first entry in the table). The values 0.2, 0.5, 0.7 and 0.9 have been used again for the parameter α. The calculations (predictions) have been performed using Microsoft Excel and the resulting file is attached in the appendices as *Appendix 2*. The error for each value of α is as follows:

For α = 0.2, the MAE = 1.195 and MSE = 2.839

For α = 0.5, the MAE = 0.831 and MSE = 1.296

For α = 0.7, the MAE = 0.539 and MSE = 0.540

For α = 0.9, the MAE = 0.194 and MSE = 0.072

The value of **α = 0.9** yields the minimum MAE and MSE, therefore that value of α is the most appropriate for this case. The resulting RMSE is equal to 0.268

2-4-2-4. ARIMA model

Looking at *Fig. 2.4*, we can see that the series seems to be constant in the mean. Therefore it can be assumed that it is in a state of stationarity. No differencing is needed here. Also, the graphs of the *ACF*(auto correlation function) and *PACF*(partial auto correlation function) support that deduction (see *Fig. 2.5* and *Fig. 2.6*).

Moreover, it is known that an ACF that dies out gradually and a PACF that cuts off sharply after a few lags show the presence of an AR term in a series. *Fig. 2.5* shows the example of an AR(1). On the other hand, an ACF that cuts off (usually negative at lag 1) sharply after a few lags and a PACF that dies out gradually show the presence of a MA term in a series. Fig. 2.6 shows the case for a MA(1).

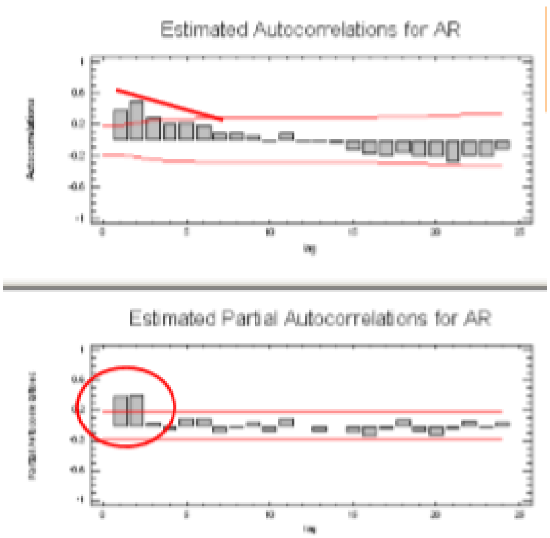

Figure 2.5: an AR(1) signature.

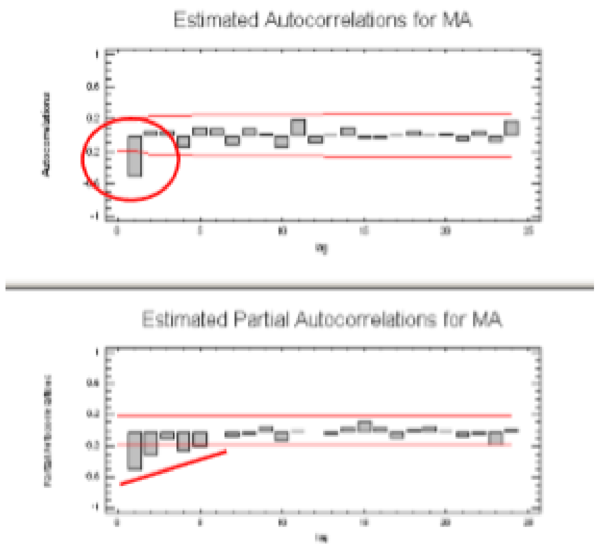

Figure 2.6: a MA(1) signature.

Fig. 2.7 and *Fig. 2.8* don't display any sign of such characteristics, therefore it can be concluded that there are no AR and MA terms in *Sample 10*. In addition, the observations on those repesctive graphs seem to be under the control limits. This is the characteristic of a '***White noise***' series, referred to as *ARIMA(0,0,0)*. By definition, if a series is white noise, it cannot be forecasted, at least not with the ARIMA methodology, because its values at different times are statistically independent. It is therefore meaningless to attempt to forecast this data. The following equation is the representation of a white noise model

$$Y_t = C + e_t \qquad \text{for } t = 1, 2, \ldots, n$$

The variable *c* is a constant; it represents the level of the series, in other terms, its mean. e_t is the error term, from $t = 1$ to $t = n$, and is uncorrelated from period to period.

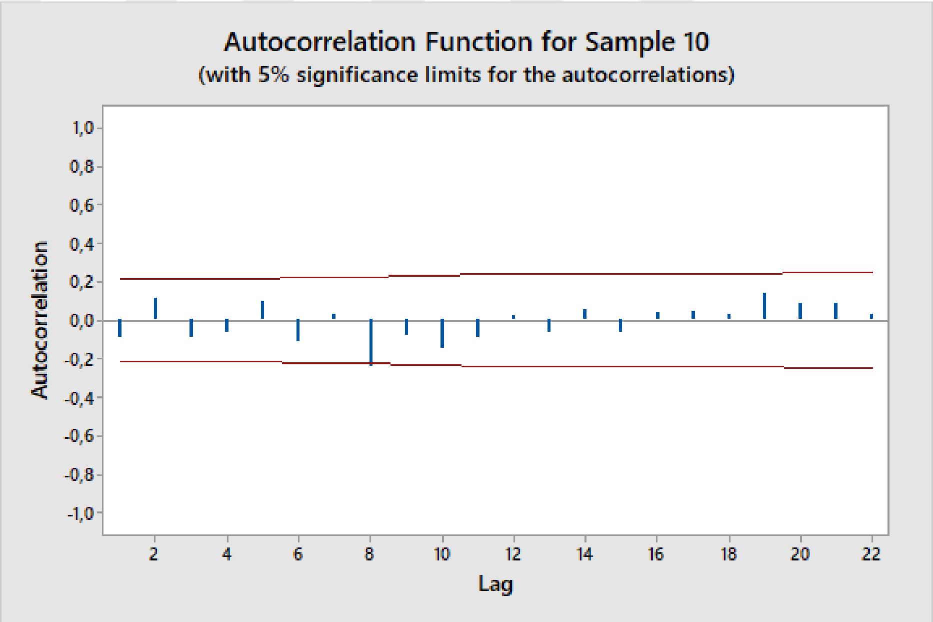

Figure 2.7: ACF graph of '*Sample 10*'.

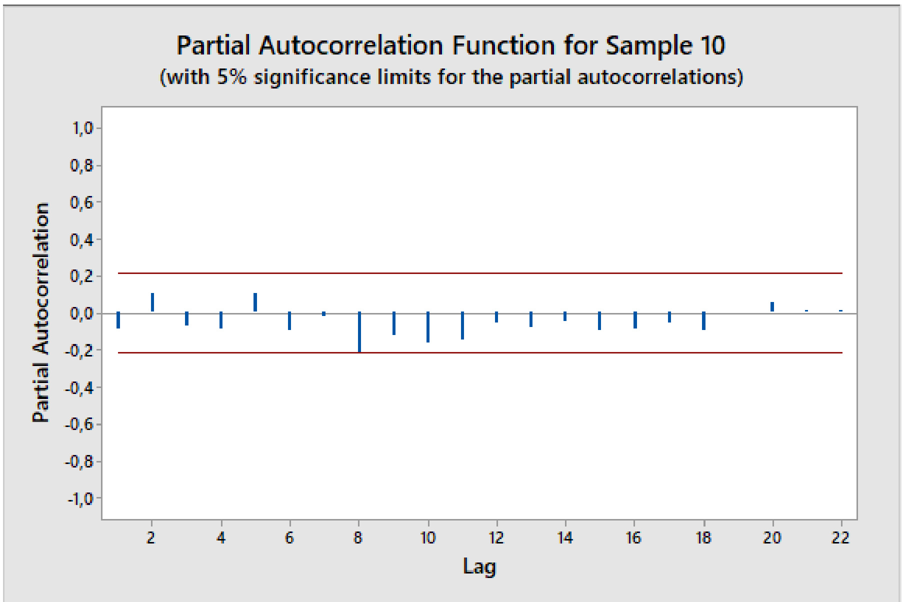

Figure 2.8: PACF graph of '*Sample 10*'.

2-4-2-5. Original GM(1,1)

The process here will be the same as in the example in '*section 2-4-1-d*', but with the data of '*Sample 10*'. The first 88 entries of the data set will be used to set up the model.

Step 1: the initial series $X^{(0)}$ is equal to the first 88 entries, that is $X^{(0)}(k)$ = {10.6 , 4.5 , 7.3 , 7.6 , ... , 8.8 , 12.2}.

Step 2: the new series $X^{(1)}$ is generated using the series $X^{(0)}$ in *eq. 2.26* of *section 2-2-2-1*. $X^{(1)}(k)$ = {10.6, 15.1, 22.4, 30, 37, ..., 701.5, 713.7 }

Step 3-4: as given before, the following equation represents the basic GM(1,1)

$$X^{(0)}(k) + a*Z^{(1)}(k) = b \quad \rightarrow \quad X^{(0)}(k) = b - a*Z^{(1)}(k)$$

The parameters *a* and *b* need to be estimated using this equation, for the values of *k* between *2* and *n* (*n* being equal to 88 – 1 = 87, because the first obrservation will not be included in the final calculations). OLS technique with some transformations is applied again in order to do so. We name variable *X*, variable *Y* and variable *A* again such that

$$X = Z^{(1)}(k)\,, \quad Y = X^{(0)}(k) \quad \text{and} \quad A = -a$$

By substituting these variables into the basic GM(1,1) equation, the following equation is obtained

$Y = b + A*X$

For each observed response Y_i, with a corresponding predictor X_i, we obtain a fitted value

$\hat{Y}_i = b + A*X_i$.

We would like to minimize the sum of squares error, that is minimize the squared distances between each observed value to its fitted/predicted value.

$SSE = \sum(Yi - \hat{Y}i)^2 = \sum(Yi - (b + A*Xi))^2$ for i = 1,....,n

A little bit of calculus was introduced before in order to solve for this.

- $SS_{xx} = \sum(x_i - \bar{x})^2 = \sum(x_i)^2 - [(\sum x_i)^2]/n$
- $SS_{xy} = \sum(x_i - \bar{x})(y_i - \bar{y}) = \sum(x_i y_i) - [(\sum x_i)*(\sum y_i)]/n$
- $SS_{yy} = \sum(y_i - \bar{y})^2 = \sum(y_i)^2 - [(\sum y_i)^2]/n$
- $A = SS_{xy} / SS_{xx}$
- $b = [(\sum y_i)/n] - A*[(\sum x_i)/n]$

$\bar{x}$ and $\bar{y}$ represent the averages for the values of $\boldsymbol{x_i}$ and $\boldsymbol{y_i}$ that are included in the calculations.

$\bar{x} = [\sum x_i]/n$ and $\bar{y} = [\sum y_i]/n$; $n=87(88-1)$

All the calculations are done in Microsoft Excel and the file is attached as *Appendix 3*.
The following results are obtained:

$\sum(x_i) = 31524.75$ $\sum(x_i)^2 = 15030203$
$\sum(y_i) = 703.1$ $\sum(y_i)^2 = 6045.93$
$\sum(x_i*y_i) = 254627.7$

The values of SS_{xx} , SS_{xy}, *A*, *a* and *b* can now be easily calculated:
$SS_{xx} = 15030203 - [(31524.75)^2]/87 = 3607101.131$
$SS_{xy} = 254627.7 - [31524.75*703.1]/87 = -143.009$

$A = SS_{xy} / SS_{xx}$ = -143.009 / 3607101.131 = -0.0000396

b = [703.1 / 87] + 0.0000396*[31524.75 / 87] = ***8.095***

We said earlier that $A = -a$, which means that $a = -A$. Therefore ***a = 0.0000396***.

The estimate values of ***a*** and ***b*** are 0.0000396 and 8.095 respectively.

Step 6: the values of *a* and *b* are put into the Grey model (*eq. 2.31* of *section 2-2-2-1*) and the following equation is obtained

$$\hat{x}_0^{(0)}(k) = [10.6 - \frac{8.095}{0.0000396}]*(1 - e^{0.0000396})*e^{-0.0000396(k-1)}$$

This equation is used to forecast the next 20 data which account for the last five (5) years, for k = 89, 90, ..., 108. The results obtained are summarized in *table 2.4.1*.

Comparing the forecast results with the observed values in '*Sample 10*', the MAE, MSE and RMSE are computed. Their values are 0.977, 1.740 and 1.319 respectively.

Table 2.4.1: Forecast results of the GM(1,1).

Observed Value	k values	Forecasts	Errors absolute
8.2	89	8.06658106	0.133419
8.1	90	8.06626163	0.033738
8.8	91	8.06594221	0.734058
7.1	92	8.0656228	0.965623
8	93	8.06530341	0.065303
7.3	94	8.06498403	0.764984
7.9	95	8.06466466	0.164665
11.7	96	8.06434531	3.635655
8.8	97	8.06402597	0.735974
9.4	98	8.06370664	1.336293
9.3	99	8.06338732	1.236613
7.6	100	8.06306802	0.463068
7.3	101	8.06274873	0.762749
10.4	102	8.06242945	2.337571
8.1	103	8.06211018	0.03789
10.4	104	8.06179093	2.338209
7.4	105	8.06147169	0.661472
7	106	8.06115246	1.061152
9.4	107	8.06083325	1.339167
8.8	108	8.06051404	0.739486

2-4-2-6. Grey method with Optimization of Background Value

The first thing to do is to estimate the value of parameter *a* using *eq. 2.38* in *section 2-3-2-3*.

$$a(k) = \ln x^{(0)}(k) - \ln x^{(0)}(k-1)$$

For every value of k = 2,3,...,88 a new value of *a* is obtained and at the end, those values are averaged to obtain the final value of $\mathbf{a} = \frac{\sum a(k)}{87} = \mathbf{0{,}0016}$.

The excel table file summarizing the numerical calculations for parameter *a* as well as all the calculations for this method is attached in the appendices as *Appendix 4*.

Next, the background value needs to be estimated for every value of k between [2, 88] using *eq 2.39* of the method in *section 2-3-2-3*. The right-hand side of the equation is divided in 2 parts to make the calculations in Excel easier, thus the reason for introducing the parameters J and F.

$$z^{(1)}(k) = \frac{x(0)(k)}{\ln x^{(0)}(k) - \ln x^{(0)}(k-1)} + \frac{[x^{(0)}(k-1)]^k}{[x^{(0)}(k)]^{k-2} * [x^{(0)}(k-1) - x^{(0)}(k)]} = J + F$$

The excel table file summarizing the numerical calculations for the parameter *Z(k)* is found in *Appendix 4*.

After that, the parameter *b* needs to be estimated in its turn, for every single value of $z^{(1)}(k)$, using *eq. 2.40* of the method as follows:

$$b(k) = x^{(0)}(k) + a * z^{(1)}(k) \qquad \text{for } a = 0{,}0016$$

which is the the value calculated above and *k* = 2,3,...87.

The excel table file summarizing the numerical calculations for the parameter *b(k)* is also attached in *Appendix 4*.

You would notice that there are some ***undefined*** numbers for the values of the parameters ***Z(k)*** and ***b(k)***, that is because some successive entries of '*Sample 10*' have the same values and the difference $\boldsymbol{x^{(0)}(k-1) - x^{(0)}(k)}$ will have some results equal to ***zero***, and any division by the number zero will give an undefined result. A few assumptions are made below:

1. Any undefined result will be ignored while averaging the values of *b(k)* (7 in total).
2. High values (values which are hundred and thousand times bigger than the maximum observation in the $x^{(0)}(k)$ series) of *b(k)* will also be ignored during the averaging process (21 in total).

3. In some cases, *b(k)* might have negative values. Those values will also be neglected.

The final estimate of *b* is the average of all the positive values of *b(k)* which is found to be

$$\mathbf{b} = \frac{\sum \boldsymbol{positive}(\boldsymbol{b}(\boldsymbol{k}))}{\mathbf{87-(7+21)}} = \mathbf{9.4}.$$

Finally, the values of ***a***, ***b*** and $\boldsymbol{x^{(0)}}$ are put into *eq. 2.41* of the method and the following Grey prediction model is obtained:

$$\hat{x}_0^{(0)}(k) = [x^{(0)}(1) - \frac{b}{a}]*(1 - e^{a})*e^{-a(k-1)} = [10.6 - \frac{9.4}{0{,}0016}]*(1 - e^{0{,}0016})*e^{-0.0016(k-1)}$$

Using this equation, the values for the next 20 periods, corresponding to the 4 quartiles of every successive year from 2012 through 2016, can be predicted for ***k*** values of 89, 90, 91, ..., 107, 108 respectively. *Table 2.4.2* summarizes the calculations.

Table 2.4.2: Forecast results of the optimized Grey Model.

k values	Observed value	Forecast value	Error absolute
89	8.2	8.157223967	0.042776033
90	8.1	8.144182845	0.044182845
91	8.8	8.131162571	0.668837429
92	7.1	8.118163113	1.018163113
93	8	8.105184438	0.105184438
94	7.3	8.092226512	0.792226512
95	7.9	8.079289302	0.179289302
96	11.7	8.066372775	3.633627225
97	8.8	8.053476898	0.746523102
98	9.4	8.040601638	1.359398362
99	9.3	8.027746962	1.272253038
100	7.6	8.014912837	0.414912837
101	7.3	8.00209923	0.70209923
102	10.4	7.989306108	2.410693892
103	8.1	7.976533439	0.123466561
104	10.4	7.96378119	2.43621881
105	7.4	7.951049329	0.551049329
106	7	7.938337822	0.938337822
107	9.4	7.925646637	1.474353363
108	8.8	7.912975742	0.887024258

Using the error values, the MAE, MSE and RMSE can easily be calculated. Their results are found to be 0.99, 1.79 and 1.33 respectively.

2-4-2-7. Grey_ARIMA model

This particular method cannot be used here because no ARIMA model couldn't be derrived from the data ('*Sample 10*'). '*Sample 10*' being a white noise, it is therefore impossible to do any prediction through the ARIMA methodology. So instead of '*Sample 10*', a new data set will be used here in order to show how the method works. The GDP of Mali from 1967 to 2016 will be used here. The GDP, meaning gross domestic product, is the total value of everything that the people and companies in one country have produced during a year. Whether the citizens are foreigners or the companies are foreign-owned doesn't matter. As long as they are located withing the country's boundaries, their production is added to the GDP. The reason for choosing this data is that it is one of the few available data about Mali which date from 1960s. Also, the data has 50 entries, which is an acceptable sample size for using ARIMA.

All the data set will be used here to set up both GM(1,1) and ARIMA models. Later on, with the models obtained, the data will be forecasted from t = 1 to t = n (*n* is the sample size, which is 50 here) and the errors will be calculated. Those errors are called '*residuals*'. For both models, a graph of the residuals will be plotted and an analysis of both graphs will determine the values of the parameters α and β of *eq. 2.42* of the method in *section 2-3-2-4*. *Fig. 2.9* shows the time series plot of the data set.

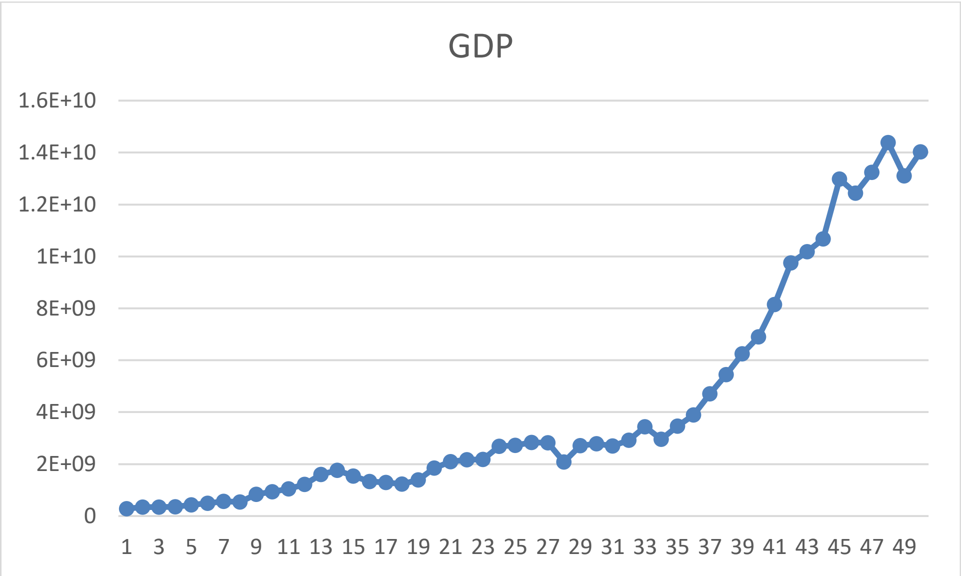

Figure 2.9: Time series plot of the GDP of Mali from 1967 to 2016.

Setting up the ARIMA model:

It is clear that the data in Fig. 2.3.2.e has an up-going trend, therefore it can be concluded that it is not stationary. The data needs to be made stationary. There are 2 ways of doing so

1. through the method of differencing, which is in the form

$$Y'_t = Y_t - Y_{t-1}$$

This method usually removes any non-stationarity in the mean.

2. through logarithmic or power transformations

$$Y'_t = \ln Y_t$$

This usually takes care of any non-stationarity in the variance.

Fig 3.1 and *fig. 3.2* display the new series obtained after the first and second differencing operations made on the GDP data. A first differencing operation was performed, but the resulted series didn't seem to be stationary, so a second differencing operation was performed on the resulted series, which is often referred to as the differences of the first-differences. The obtained series still doesn't seem to be stationary. We would logically try to do a third differencing operation, hoping that the non-stationarity will be completely removed, but differencing a series too much can result in inaccurate forecasts. Thus, it is not recommended to difference more than 2 times.

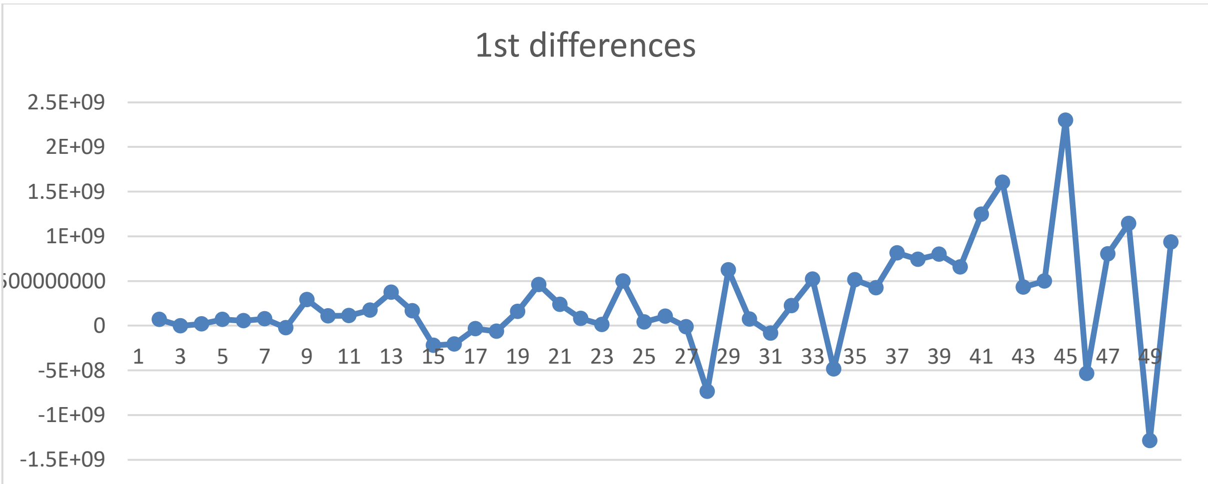

Figure 3.1: 1st differencing operation.

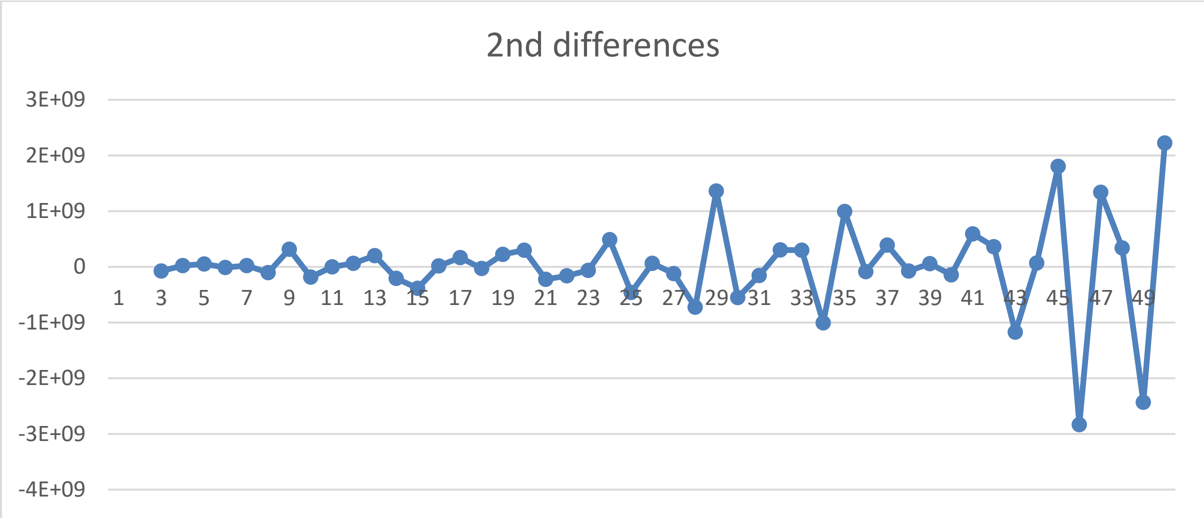

Figure 3.2: 2nd differencing operation.

On the other hand, *fig. 3.3* displays the result obtained after performing a natural logarithm operation on the GDP data. The resulted series seems to be stationary. Therefore, this differencing method is assumed to be best in this case.

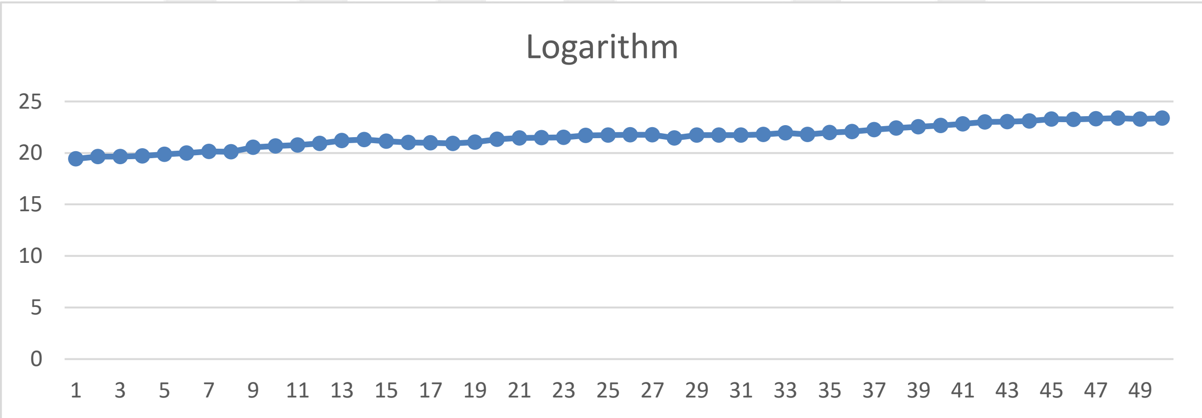

Figure 3.3: Result of the logarithmic operation on the GDP.

Furthermore, the ACF graph of the GDP shows a slowly decreasing pattern and the PACF graph shows a spike at lag 1. These are the characteristics of a AR term. Since there is only one spike, the parameter *n* will be assigned the value 1 (AR(1)).

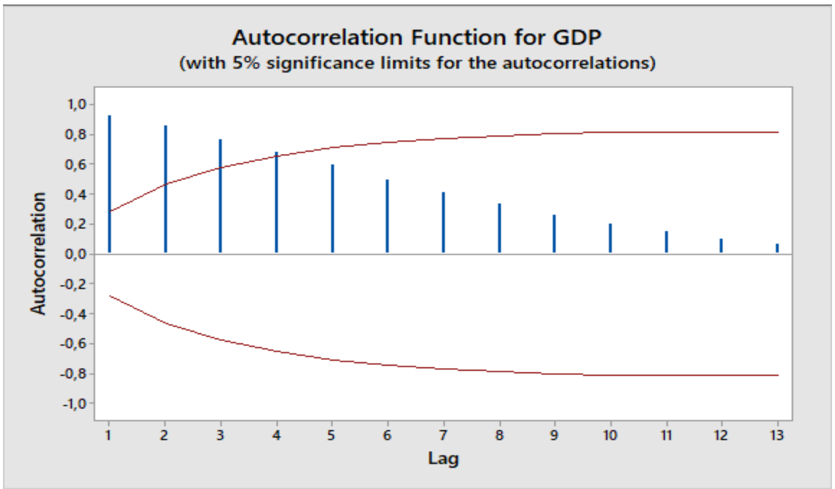

Figure 3.4: ACF graph of the GDP.

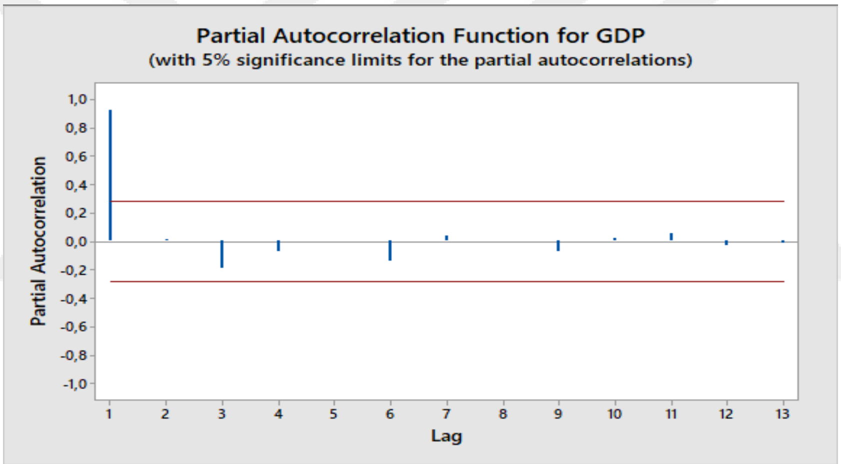

Figure 3.5: PACF graph of the GDP.

There are no signs of a MA term in the ACF and PAFC graphs, therefore the final ARIMA model is ARIMA(1,1,0) with the differencing operation being the natural logarithm.

The logarithmic difference isn't supported in *Minitab*, that is the program does not perform a logartihm difference on the series if the value 1 is given to the *Integrated* (I) term. Therefore it is better to apply an ARIMA(1,0,0) to the ***differenced series*** and retrieve the final forecasts by taking the exponential of each of the results of the ARIMA(1,0,0). The model for an ARIMA(1,0,0) or AR(1) is as follows:

$$Y_t = C + \phi_1 * Y_{t-1} + e_t \qquad (2.47)$$

Where C is a constant, ϕ_1 is a parameter and e_t is the error term.

After performing AR(1) in Minitab, the following values are found:

$$\phi_1 = 1.0256 \quad \text{and} \quad c = -0.4981$$

The final equation is therefore:

$$Y_t = -0.4981 + 1.0256 * Y_{t-1} \tag{2.48}$$

And $Y_1 = Y_0$.

The values of Y which will be used here are the ones of the differenced series (the logarithm values). For each value of Y_t calculated using *eq. 2*, the actual forecast value is retrieved through

$$F_t = e^{Y_t} \tag{2.49}$$

where F_t is the real forecast value at time t.

The steps in *eq. 2.48* and *eq. 2.49* account for the ARIMA(1,1,0).

The calculations are performed in Microsoft Excel and the results are saved there. The file is attached in the appendices as *Appendix 5*. Here is a small screenshot of the file:

Table 2.4.3: Screenshot of the ARIMA results in Microsoft Excel.

Year	t values	GDP	Logarithm	AR(1) results	Ft values	Errors
1967	1	2.75E+08	19.4340783	19.4340783	275494520.1	0
1968	2	3.44E+08	19.6554891	19.4334907	275332688.3	-68439276.35
1969	3	3.4E+08	19.64420271	19.66056962	345522949.5	5609116.448
1970	4	3.6E+08	19.70098207	19.6489943	341546469	-18225894.31
1971	5	4.3E+08	19.87952071	19.70722721	362026222.9	-68070515.51
1972	6	4.87E+08	20.00298861	19.89033644	434773796	-51843536.4
1973	7	5.64E+08	20.15000377	20.01696512	493466294.4	-70217365.92
1974	8	5.39E+08	20.10475713	20.16774386	573772688.4	35025420.03
1975	9	8.31E+08	20.53779206	20.12133891	547755135.2	-282955480
1976	10	9.39E+08	20.66056881	20.56545953	854015184.4	-85212809.25
1977	11	1.05E+09	20.77190217	20.69137937	968616549.8	-81221942.8
1978	12	1.22E+09	20.92432929	20.80556287	1085778272	-136924084
1979	13	1.6E+09	21.19040492	20.96189212	1269504015	-325919270.4
1980	14	1.76E+09	21.28840396	21.23477929	1667813438	-91877373.78
1981	15	1.54E+09	21.1543806	21.3352871	1844155151	305182992.4

Setting up the grey model GM(1,1)

Similarly to the examples in *section 2.4.1.d* and *section 2.4.2.b*, the model is set up using the GDP data. All the calculations are performed in Microsoft Excel and the file is attached in the appendices. The resulting grey equation is as follows:

$$\hat{x}_0^{(0)}(k) = (275494520.1 + \frac{240880408.2}{0.078})*(1 - e^{-0.078})*e^{0.078(k-1)} \tag{2.50}$$

where the parameter a = -0.078 and parameter b = 240880408.2

Using *eq. 2.50*, forecasts are made for the values of k from 1 to 50 and the errors are calculated. The errors for both algorithms (ARIMA and Grey model) are called residuals. Some of those errors have positive values and the others have negative values. The residuals for both algorithms are plotted in *fig. 3.6*

The first remark that can be made is that the ARIMA(1,1,0) model performs better than the GM(1,1) model, that is the residuals of the ARIMA(1,1,0) are much closer to the zero axis.

The second remark that is made from *fig. 3.6* is that the graph of the residuals of the GM(1,1) is, for the most time, below the one of the ARIMA(1,1,0).
Recall that the eqaution of a Grey_ARIMA model is as follows:

$$\text{Hybrid(Grey\_ARIMA)} = \alpha*\text{GM}(1,1) + \beta*\text{ARIMA}(n,p,q)$$

According to the second remark, it would be logical to take the average of the forecats of the two algorithms at different periods, and this would give a better result. That is, α and β would both be equal to 0,5. But the first remark states that the ARIMA model greatly outperforms the Grey one. Therefore it would be reasonable to assign a bigger weight to the ARIMA term, resulting in the grey term having a lower weight. The values of 0.2, 0.1 and 0.05 for the parameter α are chosen here while the parameter β has values 0.8, 0.9 and 0.95, for trial.

Recall that α + β = 1.
The MAE for the ARIMA(1,1,0) was equal to 360931525.1
The MAE for the GM(1,1) was equal to 1128039365
Both of these errors are very huge because the GDP is expressed in terms of ‘billions’, therefore the errors should be in the ‘millions’ or in the ‘thousands’ (the best case would be in the

hundreds). Neither of these two (2) algorithms performs particularly well with the GDP data. Nonetheless, after computing the hybrid forecasting with the different values of α and β above,

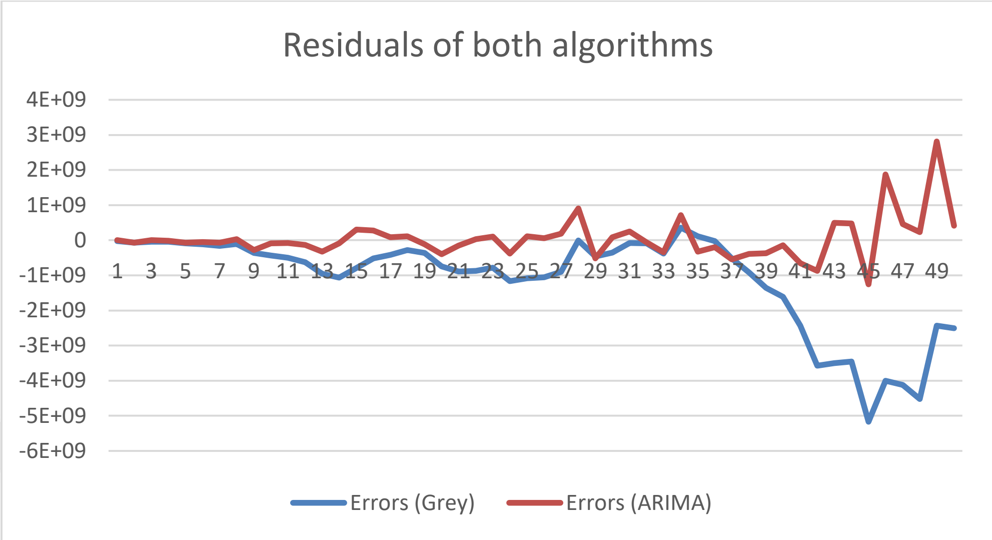

Figure 3.6: Residual plots of the ARIMA(1,1,0) and GM(1,1).

the lowest value of the MAE was 324379042.3 for the corresponding values of α = 0.1 and β = 0,9. Again this error is very big, but it is less than the MAEs of both the ARIMA and the grey models. All the calculations were done in Microsoft Excel. The file is attached in the appendices as *Appendix 6*. However, below (*table 2.4.4*) is a small screenshot of it.

Table 2.4.4: Screenshot of the Hybrid model calculations in Microsoft Excel.

t values	Forecasts (Grey)	Forecasts (Arima)		Hybrid	errors
1	252397524.8	275494520.1		270875121	4619399.077
2	272872683	275332688.3	alpha 0.2	274840687	68931277.41
3	295008840.5	345522949.5	beta 0.8	335420128	4493705.354
4	318940742	341546469		337025324	22747039.7
5	344814063	362026222.9		358583791	71512947.49
6	372786296.5	434773796		422376296	64241036.3
7	403027712	493466294.4		475378578	88305082.41
8	435722391.5	573772688.4		546162629	7415360.657
9	471069350.3	547755135.2		532417978	298292636.9
10	509283748.5	854015184.4		785068897	154159096.4
11	550598200.1	968616549.8		885012880	164825612.7
12	595264190	1085778272		987675456	235026900.4
13	643553603.7	1269504015		1144313933	451109352.7
14	695760383	1667813438		1473402827	286287984.7
15	752202315.1	1844155151		1625764583	86792425.3
16	813222966.8	1607317929		1448498936	114744902.3
17	879193775.9	1387891780		1286152179	11613269.01
18	950516312.5	1349497079		1269700926	36768917.58
19	1027624723	1280398297		1229843582	162352351.6

3. RESULTS

This section is divided into two (2) parts: *section 3.1* contains the results and errors from the forecasting operations performed on the initial data (which is the unemployment rates of Mali from 1990 to 2016) and *section 3.2* shows the results and errors of the ones performed on the bootstrap data (referred to as '*Sample 10*').

3.1 Results from the unemployment dataset

Table 2.4.3 summarizes the different methods which have been used with the unemployment dataset, which was referred to as the initial dataset. A total of six algorithms were used. The dataset had a sample size of 27. Recall that the first twenty-two (22) entries were used to set up the models for each of the six algorithms, and the last five (5) entries were used to test the models and compute the errors. The estimators used for the error were the MAE, the MSE and the MRSE.

Table 2.4.5: Errors from the unemployment dataset.

Methods	MAE	MSE	RMSE
Simple linear regression	1.2918	1.89	1.37
Simple moving averages	0.8611	1,4619	1,20
Single exponential smoothing	0,146	0,046	0,214
ARIMA	-	-	-
GM(1,1)	1,068	1,3718	1,17
Grey prediction with rolling mechanism	1,512	2,99	1,729
Grey model with optimization of background value	0,4882	0,5087	0,71

The row of the ARIMA method has blank entries because no ARIMA model has been performed on this data since it has a size of only 27 and in order to set up an ARIMA model, at least 50-54 data are needed. The Grey_ARIMA model couldn’t be applied as well because of the same reasons. Surprisingly, the single exponential smoothing method has the lowest errors, this is because SES methods work very well with series which do not exhibits any trend or seasonal pattern. The data used in the experiment is a good example of such series. However, the Grey model with optimization of the background value has the second lowest errors. It outperforms both the original grey model GM(1,1) and the grey model with rolling mechanism.

3.2 Results from the bootstrap dataset

A total of eight (8) methods was supposed to be used on the bootstrap data or ‘*Sample 10*’, but due to some unexpected issues, only five (5) have been applied successfully. Recall that the dataset had a sample size of 108: the first 88 entries were used to set up each model and the last 20 entries to evaluate them and compute the errors. The method of Grey prediction with rolling mechanism is very exhaustive with long-term forecasting. It is best used in short-term forecasting. In order to use it for long periods predictions, it is best to have it implemented in a software or develop a piece of coding and let a computer do the calculations if possible. Unfortunately, as explained in the previous sections, there is no code available for this method yet, therefore it hasn’t been used with the bootstrap dataset. In addition, ‘*Sample 10*’ turned out to be a white noise series and such series cannot be predicted through ARIMA models. Since it wasn’t possible to get any ARIMA model, it was therefore impossible to perform a Grey_ARIMA model on this data. However this particular method was applied on the Malian GDP data, to show how the method works, but its results are not going to be taken into consideration during the comparison process of all the methods. *Table 2.4.4* summarizes the errors obtained after the prediction operations of each of the five (5) models.

Here again the SES performed better than all the other algorithms. It is so because the bootstrap series also doesn’t exhibit any trend nor seasonal pattern. The GM(1,1) and the Grey model with optimization of the background value have the second and third lowest errors. This time around, the GM(1,1) performed slightly better than the Grey model with optimization of the background value, with a difference of only 0.02 in the MAE and 0.01 in the RMSE.

Table 2.4.6: Errors from the bootstrap dataset.

Methods	MAE	MSE	RMSE
Simple linear regression	0.99	1.82	1.35
Simple moving averages	1.29	2.83	1.68
Single exponential smoothing	0.194	0.072	0.268
ARIMA	-	-	-
GM(1,1)	0.977	1.74	1.32
Grey prediction with rolling mechanism	-	-	-
Grey model with optimization of the background value	0.99	1.79	1.33
Grey_ARIMA	-	-	-

4. DISCUSSION

The original grey method and the optimized model have proven to be good forecasting models for both datasets. Apart from the SES method, they performed better than every other algorithms. However, it is important to notice that since the grey equation has exponential factors, the values of the powers of each exponential term have a high impact in the prediction operations. Negative values of the parameter *a* will cause the predicted values to follow an upward trend, meaning that their values will increase over time, whereas positive values of the same parameter will cause the opposite effect. For long-term predictions, this situation can result in significant deviations of the predicted values from the observed ones. Therefore it can be concluded that the grey models are more suitable for short-term predictions. The grey prediction with rolling mechanism GPRM is also best used in short-term predictions since its methodology hasn't been implemented in any software program yet. Furthermore, when working with the optimized grey model (the grey model with optimization of the background value) on the bootstrap data, another interesting thing was discovered. It was observed that successive entries of '*Sample 10*' had sometimes the same value, for example quartile 1 and 2 of year 1997 both have an unemployment rate of 10,6. One aspect of the optimized model is that it has some division operations. The dividends are often the result of the substraction of two (2) successive entries, and since some of those entries have equal values, that result can be equal to zero (0) sometimes. Therefore, any division by zero will give an undefined result. Such results were observed a few times during the application of the method. They have been discarded, meaning that they haven't been considered in the final averaging operation. Despite this issue, the method still gave great results.

On another hand, as explained above, the series in both data sets used in this study turned out to be white noise. Due to this unexpected situation, it wasn't possible to set up any ARIMA model, and therefore no Grey_ARIMA model could be set up either dataset. Although the later was applied on a different dataset (Malian GDP), it is not really possible to make a good comparison with the other models because of that same reason. Furthermore, there is no correlation between the unemployment rates and the GDP. Looking at the graph of the unemployment rates, it could be seen that their values were not really changing that much over time, while the GDP kept increasing. It can be deducted that even though the GDP was better every year, this didn't affect the unemployment rates. The Pearson correlation of the GDP (values from 1990 to 2016) and the unemployment rates (the original/initial set) is equal to 0.024 which is very low and thus confirms the previously made conclusion. There are many theories/reasons as to why this is the case.

The GDP was pretty much steady until 2001, showing no large or sudden increase nor decrease. In 2002, Mali hosted the African Nations Cup football tournament. This event boosted the economy, infrastructures improved and there was a lot of encouragemnt for the private sector. Furthermore, in the same year was held a presidential and parliament elections. Since its independance in 1960, every change of power has happened through violent Coups d'Etat. 2002 was the first time that the power was peacefully transferred from one governemnt to the other one. Furthermore, prior to the year 2000, Mali relied heavily on agricultural export, mainly on cotton production. It still does nowadays. The country was the second largest producer in Africa in 2016, according to Bloomberg Markets [57], and 12th in the world in 2017, according to 'index mundi' [58]. However, after the opening of the Sadiola gold mine in 1997, Gold became the second biggest export product of the country and the country rapidly became the thrid largest producer in Africa. Also since 2001, there was a speedy discovery of many new gold mines and their exploitation was just as fast. *Fig. 3.7* shows the gold production of the country in the thousands of kilogrammes, according to CEIC [59] (a Euromoney Institutional Investor company that provides data used for busineses decisions, economic analysis, etc), from 1990 to 2014.

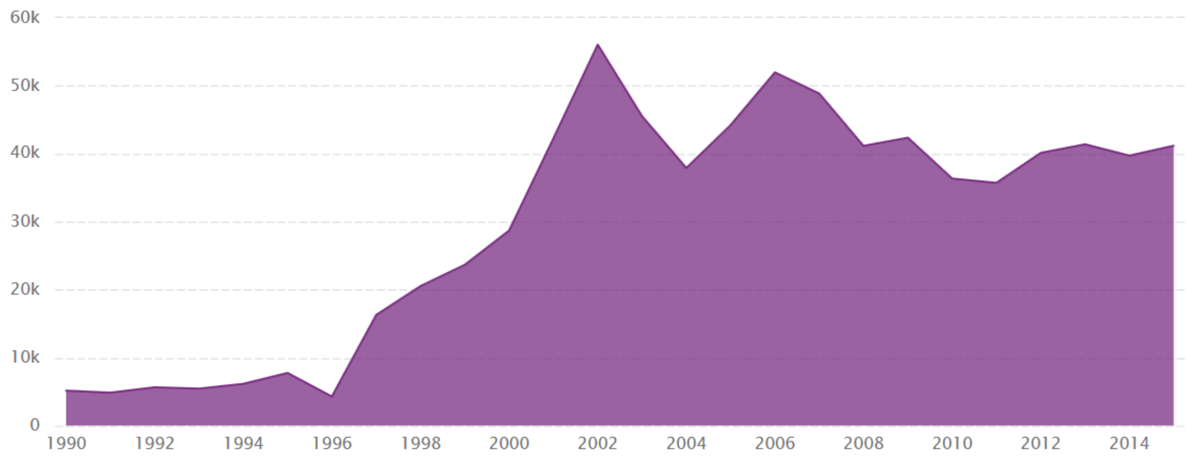

Figure 3.7: Mali Gold production in kg from 1990-2014.

Notice that, on one hand, the GDP started rising exponentially from 2000 (see *fig. 2.9*) and on the other one, the gold production increased since 1996. Thus it can be deducted that there is a strong positive correlation between the GDP and the production of gold. As of today, gold represents **72%** of the total exports of the country, according to OEC [63] and Trading Economics [60]. Furthermore, *fig. 3.8* shows the price of the kilo of gold in US dollars since 1998, from the 'Gold Price' [61] where the gold price history for the past 1 day up to the past 43 years can be found. Their content is updated on daily basis.

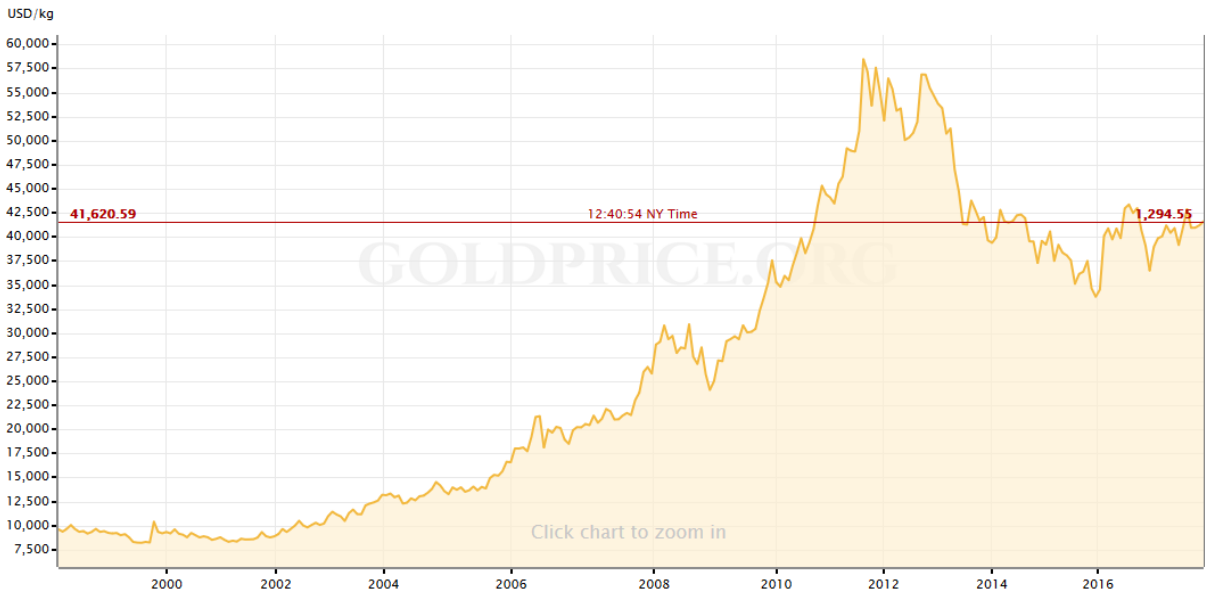

Figure 3.8: Price of the kilo of gold in US dollars since 1998.

It can be seen that from 2002, the price of the gold just ketp increasing exponentially, until 2012, when it showed a small decrease. A similar decrease was observed in the GDP as well (see *fig. 2.9*). From 2013 to 2016, the price went down again but started increasing shortly afterwards. A similar pattern can be observed in *fig. 2.9* (of the GDP). These observations shows what triggered the change in the GDP. Although it was rising, the discovery and exploitation of many gold mines hasn't impacted much the unemployment rates, because they didn't create many new jobs. Only a very little proportion of the population works in the mines. According to 'index mundi [58]', the total labor force of the country was over 4 million in 2008 and well over 5 million in 2013, whereas a study from the 'World Bank' [62], of Mali and Tanzania, shows that only about 3000 to 3900 people were working in the mining sector, from 2008 to

2013 in Mali. *Fig. 4.1* shows the number of employees of the study in Tanzania and Mali, from 2005 to 2013.

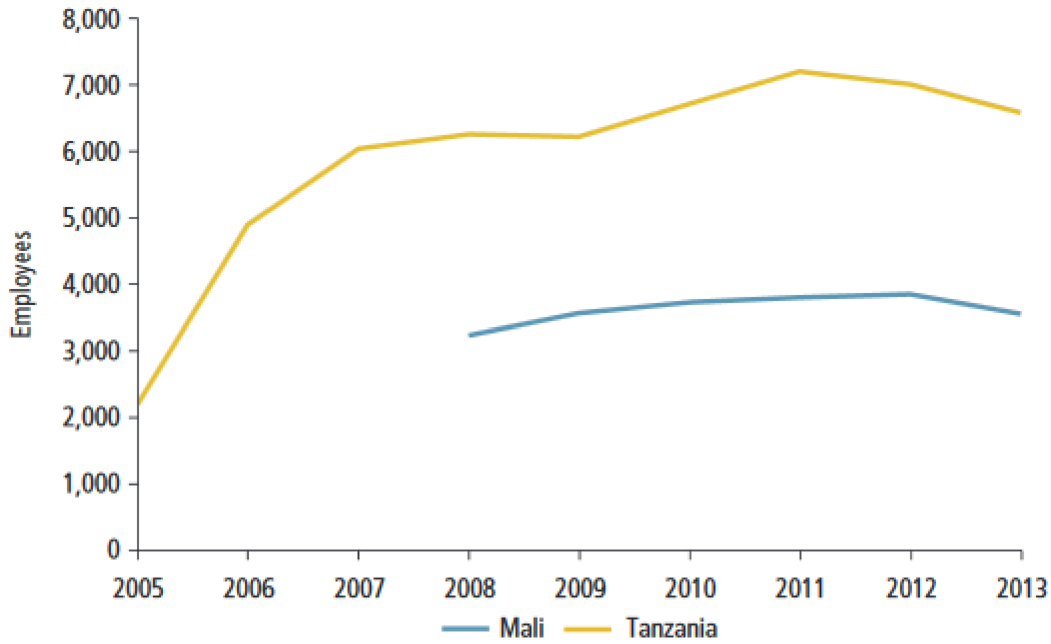

Figure 3.9: Employment in Mining in Mali and Tanzania.

These are extremely small numbers, considering that gold has always represented more than half of all the exports of the country (72% as of 2016).

So if jobs don't come from the mines, then where do they come from? The answer is very simple for someone who grew up in the country and has been exposed to the realities. The vast majority of the population is self-employed, meaning that most people work for themselves. Mali being a third world country, there aren't many public and privates institutions in the country, therefore not many official jobs available. Most people are entrepreneurs. They all have their own small businesses which goes from selling food and clothes on the streets, to being a self-owned taxi driver. Many people's lives revolve around fishing and raising cattles. The fish and part of the livestock are used to feed the people, the milk extracted from the cattle is usually consumed in the country, by its population. Some of the cattle is sold abroad and used as meat. All these small businesses are not registered officially in the government and they are not regulated by any laws, to some extent. In 2016, sheep, goats and bovine accounted for 7.9% of total exports of the country, according to OEC [63]. In addition, 63% of the workforce worked in the farming sector in 2010, according to the 'African Department of the International

Monetary Fund' [64], but only about 6% worked in the modern formal sector (which has only a few private companies and public administration). Most farmers work for themselves, and their harvest is either sold to the government, or to private companies within or outside the country. Only the cotton and the rice farming are really monitored by the government. In 2016, cotton accounted for 9.2% of the total exports of the country, according to OEC [63].

5. CONCLUSION AND RECOMMENDATIONS

The new methods discussed in this study have given good, if not great results. However, they are not very easy to implement. The principle of parsimony [68] states that amongst a set of models, the one which is the simplest (simple in the number of parameters as well as in the application) should be chosen to work with. Although the new ones performed well, it is best to choose methods such as the Simple Exponential Smoothing or the ARIMA or the Regression models which are easier to implement, unless being advised to. These heuristics can be used if time and complexity are no issues for the forecaster. Further work needs to be done on the GPRM model because it has the potential to give great forecasts, if its algorithm is implemented in a software. The same should be done with all the other heuristic methods discussed in this study as this will greatly help facilitate their use and thus raised them to a desired level of parsimony. It is also important to recall that they are mostly used with small size datasets (because of their complexity) and for short-term predictions. It would be interesting to use them with big datasets and see how they behave. This could reveal more about their potential and perhaps help increase their performances.

As for the unemployment in Mali, this study has shown that even though the government isn't creating many new jobs, the rates are very low every year compared with other countries in the world. According to the Central Intelligence Agency (CIA) bureau [65], Mali had the 106th lowest unemployment rates in the world in 2016, showing better numbers than countries such as France, Turkey and Saudi Arabia which are considered to be 'developed' while Mali is still regarded as a 'third world country'. Furthermore, the poverty rates in those countries, according to the CIA World Factbook [66], were 7.9% and 16.9% in 2014 for France and Turkey respectively, whereas it was 36.1% in Mali during that same year. These numbers are quite contradictory. Some sources, such as index mundi [67], suggest that the actual unemployment rates could be above 30% in Mali, which is most likely to be true. This is characteristic of third world countries, that is numbers don't add up most of the time. So as for the malian government, they need to redefine what they mean by or accept as work, and also make a better effort in assessing their future statistics. This could perhaps help solve some of the problems they're faced with.

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APPENDICES

APPENDIX 1: Simple Moving Averages results with '*Sample 10*'

Year	Time t	Sample 10	3-period MA	errors	errors square
1990	1	10.6			
	2	4.5	7.466666667	2.966667	8.801111111
	3	7.3	6.466666667	0.833333	0.694444444
	4	7.6	7.3	0.3	0.09
1991	5	7	7.566666667	0.566667	0.321111111
	6	8.1	9.1	1	1
	7	12.2	8.266666667	3.933333	15.47111111
	8	4.5	8.666666667	4.166667	17.36111111
1992	9	9.3	7.266666667	2.033333	4.134444444
	10	8	9.733333333	1.733333	3.004444444
	11	11.9	9.266666667	2.633333	6.934444444
	12	7.9	9.8	1.9	3.61
1993	13	9.6	8.566666667	1.033333	1.067777778
	14	8.2	9.033333333	0.833333	0.694444444
	15	9.3	9.366666667	0.066667	0.004444444
	16	10.6	9.166666667	1.433333	2.054444444
1994	17	7.6	8.533333333	0.933333	0.871111111
	18	7.4	6.1	1.3	1.69
	19	3.3	6.1	2.8	7.84
	20	7.6	6.066666667	1.533333	2.351111111
1995	21	7.3	7.7	0.4	0.16
	22	8.2	7.7	0.5	0.25
	23	7.6	7.8	0.2	0.04
	24	7.6	8.966666667	1.366667	1.867777778
1996	25	11.7	8.966666667	2.733333	7.471111111
	26	7.6	8.833333333	1.233333	1.521111111
	27	7.2	8.4	1.2	1.44
	28	10.4	9.4	1	1
1997	29	10.6	10.53333333	0.066667	0.004444444
	30	10.6	9.466666667	1.133333	1.284444444
	31	7.2	8.466666667	1.266667	1.604444444
	32	7.6	7.233333333	0.366667	0.134444444
1998	33	6.9	7.266666667	0.366667	0.134444444
	34	7.3	7.666666667	0.366667	0.134444444
	35	8.8	7.8	1	1
	36	7.3	8	0.7	0.49
1999	37	7.9	7.766666667	0.133333	0.017777778

APPENDIX 1 (cont.): Simple Moving Averages results with ‘Sample 10’

	38	8.1	6.833333333	1.266667	1.604444444
	39	4.5	6.666666667	2.166667	4.694444444
	40	7.4	6.366666667	1.033333	1.067777778
2000	41	7.2	8.833333333	1.633333	2.667777778
	42	11.9	7.466666667	4.433333	19.65444444
	43	3.3	8.2	4.9	24.01
	44	9.4	7.166666667	2.233333	4.987777778
2001	45	8.8	8.5	0.3	0.09
	46	7.3	9.266666667	1.966667	3.867777778
	47	11.7	7.833333333	3.866667	14.95111111
	48	4.5	9.466666667	4.966667	24.66777778
2002	49	12.2	8.7	3.5	12.25
	50	9.4	10.33333333	0.933333	0.871111111
	51	9.4	8.7	0.7	0.49
	52	7.3	7.066666667	0.233333	0.054444444
2003	53	4.5	6.366666667	1.866667	3.484444444
	54	7.3	6.4	0.9	0.81
	55	7.4	7.6	0.2	0.04
	56	8.1	7.866666667	0.233333	0.054444444
2004	57	8.1	8.333333333	0.233333	0.054444444
	58	8.8	8.166666667	0.633333	0.401111111
	59	7.6	7.866666667	0.266667	0.071111111
	60	7.2	7.866666667	0.666667	0.444444444
2005	61	8.8	8.066666667	0.733333	0.537777778
	62	8.2	8.4	0.2	0.04
	63	8.2	7.766666667	0.433333	0.187777778
	64	6.9	8.166666667	1.266667	1.604444444
2006	65	9.4	7.866666667	1.533333	2.351111111
	66	7.3	7.966666667	0.666667	0.444444444
	67	7.2	7.3	0.1	0.01
	68	7.4	8.933333333	1.533333	2.351111111
2007	69	12.2	10.6	1.6	2.56
	70	12.2	11.06666667	1.133333	1.284444444
	71	8.8	10.1	1.3	1.69
	72	9.3	8.333333333	0.966667	0.934444444
2008	73	6.9	8.133333333	1.233333	1.521111111
	74	8.2	7.366666667	0.833333	0.694444444
	75	7	8	1	1
	76	8.8	8.4	0.4	0.16
2009	77	9.4	7.166666667	2.233333	4.987777778
	78	3.3	6.7	3.4	11.56
	79	7.4	5.966666667	1.433333	2.054444444
	80	7.2	7.3	0.1	0.01

APPENDIX 1 (cont.): Simple Moving Averages results with ‘Sample 10’

2010	81	7.3	7.266666667	0.033333	0.001111111
	82	7.3	7.566666667	0.266667	0.071111111
	83	8.1	6.233333333	1.866667	3.484444444
	84	3.3	6.233333333	2.933333	8.604444444
2011	85	7.3	6.733333333	0.566667	0.321111111
	86	9.6	8.566666667	1.033333	1.067777778
	87	8.8	10.2	1.4	1.96
	88	12.2	9.733333333	2.466667	6.084444444
2012	89	8.2	9.5	1.3	1.69
	90	8.1	8.366666667	0.266667	0.071111111
	91	8.8	8	0.8	0.64
	92	7.1	7.966666667	0.866667	0.751111111
2013	93	8	7.466666667	0.533333	0.284444444
	94	7.3	7.733333333	0.433333	0.187777778
	95	7.9	8.966666667	1.066667	1.137777778
	96	11.7	9.466666667	2.233333	4.987777778
2014	97	8.8	9.966666667	1.166667	1.361111111
	98	9.4	9.166666667	0.233333	0.054444444
	99	9.3	8.766666667	0.533333	0.284444444
	100	7.6	8.066666667	0.466667	0.217777778
2015	101	7.3	8.433333333	1.133333	1.284444444
	102	10.4	8.6	1.8	3.24
	103	8.1	9.633333333	1.533333	2.351111111
	104	10.4	8.633333333	1.766667	3.121111111
2016	105	7.4	8.266666667	0.866667	0.751111111
	106	7	7.933333333	0.933333	0.871111111
	107	9.4	8.4	1	1
	108	8.8			

APPENDIX 2: Single Exponential Smoothing results with *'Sample 10'*

t	**Sample 10**		**alpha**	0.2				0.5			0.7			0.9		
				F(t+1)		**errors**	**errors sq.**		**errors**	**errors sq.**		**errors**	**errors sq.**		**errors**	**errors sq.**
1	10.6		t = 0	10.6	F1	0	0	10.6	0	0	10.6	0	0	10.6	0	0
2	4.5		t = 1	9.38	F2	4.88	23.8144	7.55	3.05	9.3025	6.33	1.83	3.3489	5.11	0.61	0.3721
3	7.3		t = 2	8.964	F3	1.664	2.768896	7.425	0.125	0.015625	7.009	0.291	0.084681	7.081	0.219	0.047961
4	7.6		t = 3	8.6912	F4	1.0912	1.190717	7.5125	0.0875	0.007656	7.4227	0.1773	0.031435	7.5481	0.0519	0.002694
5	7		t = 4	8.35296	F5	1.35296	1.830501	7.25625	0.25625	0.065664	7.12681	0.12681	0.016081	7.05481	0.05481	0.003004
6	8.1		t = 5	8.302368	F6	0.202368	0.040953	7.678125	0.421875	0.177979	7.808043	0.291957	0.085239	7.995481	0.104519	0.010924
7	12.2		t = 6	9.081894	F7	3.118106	9.722583	9.939063	2.260938	5.111838	10.88241	1.317587	1.736036	11.77955	0.420452	0.17678
8	4.5		t = 7	8.165516	F8	3.665516	13.436	7.219531	2.719531	7.39585	6.414724	1.914724	3.666167	5.227955	0.727955	0.529918
9	9.3		t = 8	8.392412	F9	0.907588	0.823715	8.259766	1.040234	1.082088	8.434417	0.865583	0.749234	8.892795	0.407205	0.165816
10	8		t = 9	8.31393	F10	0.31393	0.098552	8.129883	0.129883	0.01687	8.130325	0.130325	0.016985	8.08928	0.08928	0.007971
11	11.9		t = 10	9.031144	F11	2.868856	8.230335	10.01494	1.885059	3.553446	10.7691	1.130902	1.27894	11.51893	0.381072	0.145216
12	7.9		t = 11	8.804915	F12	0.904915	0.818871	8.957471	1.057471	1.118244	8.760729	0.860729	0.740855	8.261893	0.361893	0.130966
13	9.6		t = 12	8.963932	F13	0.636068	0.404582	9.278735	0.321265	0.103211	9.348219	0.251781	0.063394	9.466189	0.133811	0.017905
14	8.2		t = 13	8.811146	F14	0.611146	0.373499	8.739368	0.539368	0.290917	8.544466	0.344466	0.118657	8.326619	0.126619	0.016032

APPENDIX 2 (cont.): Single Exponential Smoothing results with ‘Sample 10’

15	9.3		t = 14	8.908917	F15	0.391083	0.152946	9.019684	0.280316	0.078577	9.07334	0.22666	0.051375	9.202662	0.097338	0.009475
16	10.6		t = 15	9.247133	F16	1.352867	1.830248	9.809842	0.790158	0.62435	10.142	0.457998	0.209762	10.46027	0.139734	0.019526
17	7.6		t = 16	8.917707	F17	1.317707	1.736351	8.704921	1.104921	1.22085	8.362601	0.762601	0.58156	7.886027	0.286027	0.081811
18	7.4		t = 17	8.614165	F18	1.214165	1.474197	8.05246	0.65246	0.425705	7.68878	0.28878	0.083394	7.448603	0.048603	0.002362
19	3.3		t = 18	7.551332	F19	4.251332	18.07383	5.67623	2.37623	5.64647	4.616634	1.316634	1.733525	3.71486	0.41486	0.172109
20	7.6		t = 19	7.561066	F20	0.038934	0.001516	6.638115	0.961885	0.925223	6.70499	0.89501	0.801043	7.211486	0.388514	0.150943
21	7.3		t = 20	7.508853	F21	0.208853	0.043619	6.969058	0.330942	0.109523	7.121497	0.178503	0.031863	7.291149	0.008851	7.83E-05
22	8.2		t = 21	7.647082	F22	0.552918	0.305718	7.584529	0.615471	0.378805	7.876449	0.323551	0.104685	8.109115	0.090885	0.00826
23	7.6		t = 22	7.637666	F23	0.037666	0.001419	7.592264	0.007736	5.98E-05	7.682935	0.082935	0.006878	7.650911	0.050911	0.002592
24	7.6		t = 23	7.630133	F24	0.030133	0.000908	7.596132	0.003868	1.5E-05	7.62488	0.02488	0.000619	7.605091	0.005091	2.59E-05
25	11.7		t = 24	8.444106	F25	3.255894	10.60085	9.648066	2.051934	4.210433	10.47746	1.222536	1.494594	11.29051	0.409491	0.167683
26	7.6		t = 25	8.275285	F26	0.675285	0.45601	8.624033	1.024033	1.048644	8.463239	0.863239	0.745182	7.969051	0.369051	0.136199
27	7.2		t = 26	8.060228	F27	0.860228	0.739992	7.912017	0.712017	0.506968	7.578972	0.378972	0.14362	7.276905	0.076905	0.005914
28	10.4		t = 27	8.528182	F28	1.871818	3.503702	9.156008	1.243992	1.547515	9.553692	0.846308	0.716238	10.08769	0.312309	0.097537
29	10.6		t = 28	8.942546	F29	1.657454	2.747154	9.878004	0.721996	0.521278	10.28611	0.313893	0.098529	10.54877	0.051231	0.002625
30	10.6		t = 29	9.274037	F30	1.325963	1.758179	10.239	0.360998	0.13032	10.50583	0.094168	0.008868	10.59488	0.005123	2.62E-05

APPENDIX 2 (cont.): Single Exponential Smoothing results with 'Sample 10'

31	7.2		t = 30	8.859229	F31	1.659229	2.753042	8.719501	1.519501	2.308883	8.19175	0.99175	0.983567	7.539488	0.339488	0.115252
32	7.6		t = 31	8.607383	F32	1.007383	1.014821	8.159751	0.559751	0.313321	7.777525	0.177525	0.031515	7.593949	0.006051	3.66E-05
33	6.9		t = 32	8.265907	F33	1.365907	1.865701	7.529875	0.629875	0.396743	7.163257	0.263257	0.069304	6.969395	0.069395	0.004816
34	7.3		t = 33	8.072725	F34	0.772725	0.597105	7.414938	0.114938	0.013211	7.258977	0.041023	0.001683	7.266939	0.033061	0.001093
35	8.8		t = 34	8.21818	F35	0.58182	0.338514	8.107469	0.692531	0.479599	8.337693	0.462307	0.213728	8.646694	0.153306	0.023503
36	7.3		t = 35	8.034544	F36	0.734544	0.539555	7.703734	0.403734	0.163001	7.611308	0.311308	0.096913	7.434669	0.134669	0.018136
37	7.9		t = 36	8.007635	F37	0.107635	0.011585	7.801867	0.098133	0.00963	7.813392	0.086608	0.007501	7.853467	0.046533	0.002165
38	8.1		t = 37	8.026108	F38	0.073892	0.00546	7.950934	0.149066	0.022221	8.014018	0.085982	0.007393	8.075347	0.024653	0.000608
39	4.5		t = 38	7.320887	F39	2.820887	7.957402	6.225467	1.725467	2.977236	5.554205	1.054205	1.111349	4.857535	0.357535	0.127831
40	7.4		t = 39	7.336709	F40	0.063291	0.004006	6.812733	0.587267	0.344882	6.846262	0.553738	0.306626	7.145753	0.254247	0.064641
41	7.2		t = 40	7.309367	F41	0.109367	0.011961	7.006367	0.193633	0.037494	7.093878	0.106122	0.011262	7.194575	0.005425	2.94E-05
42	11.9		t = 41	8.227494	F42	3.672506	13.4873	9.453183	2.446817	5.986912	10.45816	1.441836	2.078892	11.42946	0.470542	0.22141
43	3.3		t = 42	7.241995	F43	3.941995	15.53933	6.376592	3.076592	9.465416	5.447449	2.147449	4.611537	4.112946	0.812946	0.660881
44	9.4		t = 43	7.673596	F44	1.726404	2.98047	7.888296	1.511704	2.285249	8.214235	1.185765	1.406039	8.871295	0.528705	0.279529
45	8.8		t = 44	7.898877	F45	0.901123	0.812023	8.344148	0.455852	0.207801	8.62427	0.17573	0.030881	8.807129	0.007129	5.08E-05
46	7.3		t = 45	7.779102	F46	0.479102	0.229538	7.822074	0.522074	0.272561	7.697281	0.397281	0.157832	7.450713	0.150713	0.022714

APPENDIX 2 (cont.): Single Exponential Smoothing results with 'Sample 10'

47	11.7		t = 46	8.563281	F47	3.136719	9.839005	9.761037	1.938963	3.759578	10.49918	1.200816	1.441958	11.27507	0.424929	0.180564
48	4.5		t = 47	7.750625	F48	3.250625	10.56656	7.130518	2.630518	6.919628	6.299755	1.799755	3.239119	5.177507	0.677507	0.459016
49	12.2		t = 48	8.6405	F49	3.5595	12.67004	9.665259	2.534741	6.424911	10.42993	1.770073	3.13316	11.49775	0.702249	0.493154
50	9.4		t = 49	8.7924	F50	0.6076	0.369178	9.53263	0.13263	0.017591	9.708978	0.308978	0.095467	9.609775	0.209775	0.044006
51	9.4		t = 50	8.91392	F51	0.48608	0.236274	9.466315	0.066315	0.004398	9.492693	0.092693	0.008592	9.420978	0.020978	0.00044
52	7.3		t = 51	8.591136	F52	1.291136	1.667032	8.383157	1.083157	1.17323	7.957808	0.657808	0.432711	7.512098	0.212098	0.044985
53	4.5		t = 52	7.772909	F53	3.272909	10.71193	6.441579	1.941579	3.769728	5.537342	1.037342	1.076079	4.80121	0.30121	0.090727
54	7.3		t = 53	7.678327	F54	0.378327	0.143131	6.870789	0.429211	0.184222	6.771203	0.528797	0.279627	7.050121	0.249879	0.06244
55	7.4		t = 54	7.622662	F55	0.222662	0.049578	7.135395	0.264605	0.070016	7.211361	0.188639	0.035585	7.365012	0.034988	0.001224
56	8.1		t = 55	7.718129	F56	0.381871	0.145825	7.617697	0.482303	0.232616	7.833408	0.266592	0.071071	8.026501	0.073499	0.005402
57	8.1		t = 56	7.794503	F57	0.305497	0.093328	7.858849	0.241151	0.058154	8.020022	0.079978	0.006396	8.09265	0.00735	5.4E-05
58	8.8		t = 57	7.995603	F58	0.804397	0.647055	8.329424	0.470576	0.221441	8.566007	0.233993	0.054753	8.729265	0.070735	0.005003
59	7.6		t = 58	7.916482	F59	0.316482	0.100161	7.964712	0.364712	0.133015	7.889802	0.289802	0.083985	7.712927	0.112927	0.012752
60	7.2		t = 59	7.773186	F60	0.573186	0.328542	7.582356	0.382356	0.146196	7.406941	0.206941	0.042824	7.251293	0.051293	0.002631
61	8.8		t = 60	7.978549	F61	0.821451	0.674782	8.191178	0.608822	0.370664	8.382082	0.417918	0.174655	8.645129	0.154871	0.023985
62	8.2		t = 61	8.022839	F62	0.177161	0.031386	8.195589	0.004411	1.95E-05	8.254625	0.054625	0.002984	8.244513	0.044513	0.001981

APPENDIX 2 (cont.): Single Exponential Smoothing results with ‘Sample 10’

63	8.2		t = 62	8.058271	F63	0.141729	0.020087	8.197795	0.002205	4.86E-06	8.216387	0.016387	0.000269	8.204451	0.004451	1.98E-05
64	6.9		t = 63	7.826617	F64	0.926617	0.858619	7.548897	0.648897	0.421068	7.294916	0.394916	0.155959	7.030445	0.130445	0.017016
65	9.4		t = 64	8.141294	F65	1.258706	1.584342	8.474449	0.925551	0.856645	8.768475	0.631525	0.398824	9.163045	0.236955	0.056148
66	7.3		t = 65	7.973035	F66	0.673035	0.452976	7.887224	0.587224	0.344832	7.740542	0.440542	0.194078	7.486304	0.186304	0.034709
67	7.2		t = 66	7.818428	F67	0.618428	0.382453	7.543612	0.343612	0.118069	7.362163	0.162163	0.026297	7.22863	0.02863	0.00082
68	7.4		t = 67	7.734742	F68	0.334742	0.112052	7.471806	0.071806	0.005156	7.388649	0.011351	0.000129	7.382863	0.017137	0.000294
69	12.2		t = 68	8.627794	F69	3.572206	12.76066	9.835903	2.364097	5.588954	10.75659	1.443405	2.083419	11.71829	0.481714	0.232048
70	12.2		t = 69	9.342235	F70	2.857765	8.16682	11.01795	1.182048	1.397239	11.76698	0.433022	0.187508	12.15183	0.048171	0.00232
71	8.8		t = 70	9.233788	F71	0.433788	0.188172	9.908976	1.108976	1.229827	9.690094	0.890094	0.792266	9.135183	0.335183	0.112348
72	9.3		t = 71	9.24703	F72	0.05297	0.002806	9.604488	0.304488	0.092713	9.417028	0.117028	0.013696	9.283518	0.016482	0.000272
73	6.9		t = 72	8.777624	F73	1.877624	3.525473	8.252244	1.352244	1.828564	7.655108	0.755108	0.570189	7.138352	0.238352	0.056812
74	8.2		t = 73	8.662099	F74	0.462099	0.213536	8.226122	0.026122	0.000682	8.036533	0.163467	0.026722	8.093835	0.106165	0.011271
75	7		t = 74	8.32968	F75	1.32968	1.768048	7.613061	0.613061	0.375844	7.31096	0.31096	0.096696	7.109384	0.109384	0.011965
76	8.8		t = 75	8.423744	F76	0.376256	0.141569	8.20653	0.59347	0.352206	8.353288	0.446712	0.199552	8.630938	0.169062	0.028582
77	9.4		t = 76	8.618995	F77	0.781005	0.609969	8.803265	0.596735	0.356092	9.085986	0.314014	0.098605	9.323094	0.076906	0.005915
78	3.3		t = 77	7.555196	F78	4.255196	18.10669	6.051633	2.751633	7.571482	5.035796	1.735796	3.012987	3.902309	0.602309	0.362777

APPENDIX 2 (cont.): Single Exponential Smoothing results with 'Sample 10'

79	7.4		t = 78	7.524157	F79	0.124157	0.015415	6.725816	0.674184	0.454524	6.690739	0.709261	0.503051	7.050231	0.349769	0.122338
80	7.2		t = 79	7.459325	F80	0.259325	0.06725	6.962908	0.237092	0.056213	7.047222	0.152778	0.023341	7.185023	0.014977	0.000224
81	7.3		t = 80	7.42746	F81	0.12746	0.016246	7.131454	0.168546	0.028408	7.224166	0.075834	0.005751	7.288502	0.011498	0.000132
82	7.3		t = 81	7.401968	F82	0.101968	0.010398	7.215727	0.084273	0.007102	7.27725	0.02275	0.000518	7.29885	0.00115	1.32E-06
83	8.1		t = 82	7.541575	F83	0.558425	0.311839	7.657864	0.442136	0.195485	7.853175	0.246825	0.060923	8.019885	0.080115	0.006418
84	3.3		t = 83	6.69326	F84	3.39326	11.51421	5.478932	2.178932	4.747744	4.665952	1.365952	1.865826	3.771989	0.471989	0.222773
85	7.3		t = 84	6.814608	F85	0.485392	0.235606	6.389466	0.910534	0.829072	6.509786	0.790214	0.624439	6.947199	0.352801	0.124469
86	9.6		t = 85	7.371686	F86	2.228314	4.965382	7.994733	1.605267	2.576882	8.672936	0.927064	0.859448	9.33472	0.26528	0.070374
87	8.8		t = 86	7.657349	F87	1.142651	1.305651	8.397366	0.402634	0.162114	8.761881	0.038119	0.001453	8.853472	0.053472	0.002859
88	12.2		t = 87	8.565879	F88	3.634121	13.20683	10.29868	1.901317	3.615005	11.16856	1.031436	1.06386	11.86535	0.334653	0.111992
89	8.2		t = 88	8.492703	F89	0.292703	0.085675	9.249342	1.049342	1.101118	9.090569	0.890569	0.793114	8.566535	0.366535	0.134348
90	8.1		t = 89	8.414163	F90	0.314163	0.098698	8.674671	0.574671	0.330247	8.397171	0.297171	0.08831	8.146653	0.046653	0.002177
91	8.8		t = 90	8.49133	F91	0.30867	0.095277	8.737335	0.062665	0.003927	8.679151	0.120849	0.014604	8.734665	0.065335	0.004269
92	7.1		t = 91	8.213064	F92	1.113064	1.238912	7.918668	0.818668	0.670217	7.573745	0.473745	0.224435	7.263467	0.163467	0.026721
93	8		t = 92	8.170451	F93	0.170451	0.029054	7.959334	0.040666	0.001654	7.872124	0.127876	0.016352	7.926347	0.073653	0.005425
94	7.3		t = 93	7.996361	F94	0.696361	0.484919	7.629667	0.329667	0.10868	7.471637	0.171637	0.029459	7.362635	0.062635	0.003923

APPENDIX 2 (cont.): Single Exponential Smoothing results with ‘Sample 10’

95	7.9		t = 94	7.977089	F95	0.077089	0.005943	7.764833	0.135167	0.01827	7.771491	0.128509	0.016515	7.846263	0.053737	0.002888
96	11.7		t = 95	8.721671	F96	2.978329	8.870443	9.732417	1.967583	3.871384	10.52145	1.178553	1.388986	11.31463	0.385374	0.148513
97	8.8		t = 96	8.737337	F97	0.062663	0.003927	9.266208	0.466208	0.21735	9.316434	0.516434	0.266704	9.051463	0.251463	0.063233
98	9.4		t = 97	8.869869	F98	0.530131	0.281038	9.333104	0.066896	0.004475	9.37493	0.02507	0.000628	9.365146	0.034854	0.001215
99	9.3		t = 98	8.955896	F99	0.344104	0.118408	9.316552	0.016552	0.000274	9.322479	0.022479	0.000505	9.306515	0.006515	4.24E-05
100	7.6		t = 99	8.684716	F100	1.084716	1.17661	8.458276	0.858276	0.736638	8.116744	0.516744	0.267024	7.770651	0.170651	0.029122
101	7.3		t = 100	8.407773	F101	1.107773	1.227161	7.879138	0.579138	0.335401	7.545023	0.245023	0.060036	7.347065	0.047065	0.002215
102	10.4		t = 101	8.806219	F102	1.593781	2.540139	9.139569	1.260431	1.588686	9.543507	0.856493	0.73358	10.09471	0.305293	0.093204
103	8.1		t = 102	8.664975	F103	0.564975	0.319197	8.619785	0.519785	0.270176	8.533052	0.433052	0.187534	8.299471	0.199471	0.039789
104	10.4		t = 103	9.01198	F104	1.38802	1.9266	9.509892	0.890108	0.792292	9.839916	0.560084	0.313695	10.18995	0.210053	0.044122
105	7.4		t = 104	8.689584	F105	1.289584	1.663027	8.454946	1.054946	1.112911	8.131975	0.731975	0.535787	7.678995	0.278995	0.077838
106	7		t = 105	8.351667	F106	1.351667	1.827004	7.727473	0.727473	0.529217	7.339592	0.339592	0.115323	7.067899	0.067899	0.00461
107	9.4		t = 106	8.561334	F107	0.838666	0.703361	8.563737	0.836263	0.699337	8.781878	0.618122	0.382075	9.16679	0.23321	0.054387
108	8.8		t = 107	8.609067	F108	0.190933	0.036455	8.681868	0.118132	0.013955	8.794563	0.005437	2.96E-05	8.836679	0.036679	0.001345

APPENDIX 3: Excel results of the Grey model applied to '*Sample 10*'

t	Sample 10	x(0)	x^1(k)	x^1(k-1)	Z^1(k)	Y	X	Xsquare	Ysquare	X*Y
1	10.6	10.6	10.6			10.6				
2	4.5	4.5	15.1	10.6	12.85	4.5	12.85	165.1225	20.25	57.825
3	7.3	7.3	22.4	15.1	18.75	7.3	18.75	351.5625	53.29	136.875
4	7.6	7.6	30	22.4	26.2	7.6	26.2	686.44	57.76	199.12
5	7	7	37	30	33.5	7	33.5	1122.25	49	234.5
6	8.1	8.1	45.1	37	41.05	8.1	41.05	1685.103	65.61	332.505
7	12.2	12.2	57.3	45.1	51.2	12.2	51.2	2621.44	148.84	624.64
8	4.5	4.5	61.8	57.3	59.55	4.5	59.55	3546.203	20.25	267.975
9	9.3	9.3	71.1	61.8	66.45	9.3	66.45	4415.603	86.49	617.985
10	8	8	79.1	71.1	75.1	8	75.1	5640.01	64	600.8
11	11.9	11.9	91	79.1	85.05	11.9	85.05	7233.503	141.61	1012.095
12	7.9	7.9	98.9	91	94.95	7.9	94.95	9015.503	62.41	750.105
13	9.6	9.6	108.5	98.9	103.7	9.6	103.7	10753.69	92.16	995.52
14	8.2	8.2	116.7	108.5	112.6	8.2	112.6	12678.76	67.24	923.32
15	9.3	9.3	126	116.7	121.35	9.3	121.35	14725.82	86.49	1128.555
16	10.6	10.6	136.6	126	131.3	10.6	131.3	17239.69	112.36	1391.78
17	7.6	7.6	144.2	136.6	140.4	7.6	140.4	19712.16	57.76	1067.04
18	7.4	7.4	151.6	144.2	147.9	7.4	147.9	21874.41	54.76	1094.46
19	3.3	3.3	154.9	151.6	153.25	3.3	153.25	23485.56	10.89	505.725
20	7.6	7.6	162.5	154.9	158.7	7.6	158.7	25185.69	57.76	1206.12
21	7.3	7.3	169.8	162.5	166.15	7.3	166.15	27605.82	53.29	1212.895
22	8.2	8.2	178	169.8	173.9	8.2	173.9	30241.21	67.24	1425.98
23	7.6	7.6	185.6	178	181.8	7.6	181.8	33051.24	57.76	1381.68
24	7.6	7.6	193.2	185.6	189.4	7.6	189.4	35872.36	57.76	1439.44
25	11.7	11.7	204.9	193.2	199.05	11.7	199.05	39620.9	136.89	2328.885
26	7.6	7.6	212.5	204.9	208.7	7.6	208.7	43555.69	57.76	1586.12
27	7.2	7.2	219.7	212.5	216.1	7.2	216.1	46699.21	51.84	1555.92
28	10.4	10.4	230.1	219.7	224.9	10.4	224.9	50580.01	108.16	2338.96
29	10.6	10.6	240.7	230.1	235.4	10.6	235.4	55413.16	112.36	2495.24
30	10.6	10.6	251.3	240.7	246	10.6	246	60516	112.36	2607.6
31	7.2	7.2	258.5	251.3	254.9	7.2	254.9	64974.01	51.84	1835.28
32	7.6	7.6	266.1	258.5	262.3	7.6	262.3	68801.29	57.76	1993.48
33	6.9	6.9	273	266.1	269.55	6.9	269.55	72657.2	47.61	1859.895
34	7.3	7.3	280.3	273	276.65	7.3	276.65	76535.22	53.29	2019.545
35	8.8	8.8	289.1	280.3	284.7	8.8	284.7	81054.09	77.44	2505.36
36	7.3	7.3	296.4	289.1	292.75	7.3	292.75	85702.56	53.29	2137.075
37	7.9	7.9	304.3	296.4	300.35	7.9	300.35	90210.12	62.41	2372.765
38	8.1	8.1	312.4	304.3	308.35	8.1	308.35	95079.72	65.61	2497.635
39	4.5	4.5	316.9	312.4	314.65	4.5	314.65	99004.62	20.25	1415.925
40	7.4	7.4	324.3	316.9	320.6	7.4	320.6	102784.4	54.76	2372.44

APPENDIX 3 (cont.): Excel results of the Grey model applied to 'Sample 10'

41	7.2	7.2	331.5	324.3	327.9	7.2	327.9	107518.4	51.84	2360.88
42	11.9	11.9	343.4	331.5	337.45	11.9	337.45	113872.5	141.61	4015.655
43	3.3	3.3	346.7	343.4	345.05	3.3	345.05	119059.5	10.89	1138.665
44	9.4	9.4	356.1	346.7	351.4	9.4	351.4	123482	88.36	3303.16
45	8.8	8.8	364.9	356.1	360.5	8.8	360.5	129960.3	77.44	3172.4
46	7.3	7.3	372.2	364.9	368.55	7.3	368.55	135829.1	53.29	2690.415
47	11.7	11.7	383.9	372.2	378.05	11.7	378.05	142921.8	136.89	4423.185
48	4.5	4.5	388.4	383.9	386.15	4.5	386.15	149111.8	20.25	1737.675
49	12.2	12.2	400.6	388.4	394.5	12.2	394.5	155630.3	148.84	4812.9
50	9.4	9.4	410	400.6	405.3	9.4	405.3	164268.1	88.36	3809.82
51	9.4	9.4	419.4	410	414.7	9.4	414.7	171976.1	88.36	3898.18
52	7.3	7.3	426.7	419.4	423.05	7.3	423.05	178971.3	53.29	3088.265
53	4.5	4.5	431.2	426.7	428.95	4.5	428.95	183998.1	20.25	1930.275
54	7.3	7.3	438.5	431.2	434.85	7.3	434.85	189094.5	53.29	3174.405
55	7.4	7.4	445.9	438.5	442.2	7.4	442.2	195540.8	54.76	3272.28
56	8.1	8.1	454	445.9	449.95	8.1	449.95	202455	65.61	3644.595
57	8.1	8.1	462.1	454	458.05	8.1	458.05	209809.8	65.61	3710.205
58	8.8	8.8	470.9	462.1	466.5	8.8	466.5	217622.3	77.44	4105.2
59	7.6	7.6	478.5	470.9	474.7	7.6	474.7	225340.1	57.76	3607.72
60	7.2	7.2	485.7	478.5	482.1	7.2	482.1	232420.4	51.84	3471.12
61	8.8	8.8	494.5	485.7	490.1	8.8	490.1	240198	77.44	4312.88
62	8.2	8.2	502.7	494.5	498.6	8.2	498.6	248602	67.24	4088.52
63	8.2	8.2	510.9	502.7	506.8	8.2	506.8	256846.2	67.24	4155.76
64	6.9	6.9	517.8	510.9	514.35	6.9	514.35	264555.9	47.61	3549.015
65	9.4	9.4	527.2	517.8	522.5	9.4	522.5	273006.3	88.36	4911.5
66	7.3	7.3	534.5	527.2	530.85	7.3	530.85	281801.7	53.29	3875.205
67	7.2	7.2	541.7	534.5	538.1	7.2	538.1	289551.6	51.84	3874.32
68	7.4	7.4	549.1	541.7	545.4	7.4	545.4	297461.2	54.76	4035.96
69	12.2	12.2	561.3	549.1	555.2	12.2	555.2	308247	148.84	6773.44
70	12.2	12.2	573.5	561.3	567.4	12.2	567.4	321942.8	148.84	6922.28
71	8.8	8.8	582.3	573.5	577.9	8.8	577.9	333968.4	77.44	5085.52
72	9.3	9.3	591.6	582.3	586.95	9.3	586.95	344510.3	86.49	5458.635
73	6.9	6.9	598.5	591.6	595.05	6.9	595.05	354084.5	47.61	4105.845
74	8.2	8.2	606.7	598.5	602.6	8.2	602.6	363126.8	67.24	4941.32
75	7	7	613.7	606.7	610.2	7	610.2	372344	49	4271.4
76	8.8	8.8	622.5	613.7	618.1	8.8	618.1	382047.6	77.44	5439.28
77	9.4	9.4	631.9	622.5	627.2	9.4	627.2	393379.8	88.36	5895.68
78	3.3	3.3	635.2	631.9	633.55	3.3	633.55	401385.6	10.89	2090.715
79	7.4	7.4	642.6	635.2	638.9	7.4	638.9	408193.2	54.76	4727.86
80	7.2	7.2	649.8	642.6	646.2	7.2	646.2	417574.4	51.84	4652.64
81	7.3	7.3	657.1	649.8	653.45	7.3	653.45	426996.9	53.29	4770.185
82	7.3	7.3	664.4	657.1	660.75	7.3	660.75	436590.6	53.29	4823.475
83	8.1	8.1	672.5	664.4	668.45	8.1	668.45	446825.4	65.61	5414.445

APPENDIX 3 (cont.): Excel results of the Grey model applied to 'Sample 10'

84	3.3	3.3	675.8	672.5	674.15	3.3	674.15	454478.2	10.89	2224.695
85	7.3	7.3	683.1	675.8	679.45	7.3	679.45	461652.3	53.29	4959.985
86	9.6	9.6	692.7	683.1	687.9	9.6	687.9	473206.4	92.16	6603.84
87	8.8	8.8	701.5	692.7	697.1	8.8	697.1	485948.4	77.44	6134.48
88	12.2	12.2	713.7	701.5	707.6	12.2	707.6	500697.8	148.84	8632.72

The followings are the forecasts

Observed Value	k values	forecasts	errors	errors sq.
8.2	89	8.06658106	0.133419	0.017801
8.1	90	8.06626163	0.033738	0.001138
8.8	91	8.06594221	0.734058	0.538841
7.1	92	8.0656228	0.965623	0.932427
8	93	8.06530341	0.065303	0.004265
7.3	94	8.06498403	0.764984	0.585201
7.9	95	8.06466466	0.164665	0.027114
11.7	96	8.06434531	3.635655	13.21799
8.8	97	8.06402597	0.735974	0.541658
9.4	98	8.06370664	1.336293	1.78568
9.3	99	8.06338732	1.236613	1.529211
7.6	100	8.06306802	0.463068	0.214432
7.3	101	8.06274873	0.762749	0.581786
10.4	102	8.06242945	2.337571	5.464236
8.1	103	8.06211018	0.03789	0.001436
10.4	104	8.06179093	2.338209	5.467222
7.4	105	8.06147169	0.661472	0.437545
7	106	8.06115246	1.061152	1.126045
9.4	107	8.06083325	1.339167	1.793368
8.8	108	8.06051404	0.739486	0.546839

APPENDIX 4: Numerical operations of the Grey model with Optimization of Background Value applied to '*Sample 10*'

t	Sample 10	k	Ln X_k	Ln x_{k-1}	a_k	a	J	$(x_{k-1})^k$	$(x_k)^{k-2}$	$X_{k-1} - X_k$	F	Z=J+F	b_k
1	10.6	1				0.001616							
2	4.5	2	1.504077	2.360854	-0.856776604		-5.25224	112.36	1	6.1	18.41967	13.16743	4.521068
3	7.3	3	1.987874	1.5040774	0.483796951		15.08897	91.125	7.3	-2.8	-4.45817	10.6308	7.317009
4	7.6	4	2.028148	1.9878743	0.040273899		188.7078	2839.824	57.76	-0.3	-163.886	24.82139	7.639714
5	7	5	1.94591	2.0281482	-0.082238098		-85.1187	25355.25	343	0.6	123.2034	38.08467	7.060935
6	8.1	6	2.091864	1.9459101	0.145953913		55.49697	117649	4304.672	-1.1	-24.8459	30.65103	8.149042
7	12.2	7	2.501436	2.0918641	0.40957189		29.7872	2287679	270270.8	-4.1	-2.06449	27.72271	12.24436
8	4.5	8	1.504077	2.501436	-0.997358555		-4.51192	4.91E+08	8303.766	7.7	7675.609	7671.097	16.77376
9	9.3	9	2.230014	1.5040774	0.725937003		12.81103	756680.6	6017009	-4.8	-0.0262	12.78483	9.320456
10	8	10	2.079442	2.2300144	-0.150572858		-53.1304	4.84E+09	16777216	1.3	221.9046	168.7741	8.270039
11	11.9	11	2.476538	2.0794415	0.397096858		29.9675	8.59E+09	4.79E+09	-3.9	-0.46026	29.50724	11.94721
12	7.9	12	2.066863	2.4765384	-0.409675641		-19.2835	8.06E+12	9.47E+08	4	2129.279	2109.996	11.27599
13	9.6	13	2.261763	2.0668628	0.194900339		49.25594	4.67E+11	6.38E+10	-1.7	-4.30249	44.95345	9.671926
14	8.2	14	2.104134	2.2617631	-0.157628944		-52.0209	5.65E+13	9.24E+10	1.4	436.4183	384.3974	8.815036

APPENDIX 4 (cont.): Numerical operations of the Grey model with Optimization of Background Value applied to 'Sample 10'

15	9.3	15	2.230014	2.1041342	0.125880246		73.87974	5.1E+13	3.89E+12	-1.1	-11.8997	61.98002	9.399168
16	10.6	16	2.360854	2.2300144	0.130839601		81.01523	3.13E+15	2.26E+14	-1.3	-10.6537	70.3615	10.71258
17	7.6	17	2.028148	2.360854	-0.332705754		-22.843	2.69E+17	1.63E+13	3	5506.486	5483.643	16.37383
18	7.4	18	2.00148	2.0281482	-0.026668247		-277.484	7.16E+15	8.09E+13	0.2	442.4938	165.0102	7.664016
19	3.3	19	1.193922	2.00148	-0.807557532		-4.0864	3.28E+16	6.53E+08	4.1	12242831	12242827	19591.82
20	7.6	20	2.028148	1.1939225	0.834225779		9.110244	2.35E+10	7.16E+15	-4.3	-7.6E-07	9.110243	7.614576
21	7.3	21	1.987874	2.0281482	-0.040273899		-181.259	3.14E+18	2.53E+16	0.3	413.8375	232.5787	7.672126
22	8.2	22	2.104134	1.9878743	0.116259806		70.53168	9.84E+18	1.89E+18	-0.9	-5.78873	64.74295	8.303589
23	7.6	23	2.028148	2.1041342	-0.075985907		-100.019	1.04E+21	3.14E+18	0.6	552.6904	452.6718	8.324275
24	7.6	24	2.028148	2.0281482	0		#DIV/0!	1.38E+21	2.39E+19	0	#DIV/0!	#DIV/0!	#DIV/0!
25	11.7	25	2.459589	2.0281482	0.431440595		27.11845	1.05E+22	3.7E+24	-4.1	-0.00069	27.11776	11.74339
26	7.6	26	2.028148	2.4595888	-0.431440595		-17.6154	5.93E+27	1.38E+21	4.1	1048389	1048371	1684.994
27	7.2	27	1.974081	2.0281482	-0.054067221		-133.168	6.05E+23	2.71E+21	0.4	557.9491	424.7815	7.87965
28	10.4	28	2.341806	1.974081	0.36772478		28.28202	1.01E+24	2.77E+26	-3.2	-0.00114	28.28088	10.44525
29	10.6	29	2.360854	2.3418058	0.019048195		556.4832	3.12E+29	4.82E+27	-0.2	-323.354	233.129	10.97301
30	10.6	30	2.360854	2.360854	0		#DIV/0!	5.74E+30	5.11E+28	0	#DIV/0!	#DIV/0!	#DIV/0!
31	7.2	31	1.974081	2.360854	-0.386772975		-18.6156	6.09E+31	7.29E+24	3.4	2456747	2456729	3937.966

APPENDIX 4 (cont.): Numerical operations of the Grey model with Optimization of Background Value applied to ‘Sample 10’

32	7.6	32	2.028148	1.974081	0.054067221		140.5658	2.72E+27	2.66E+26	-0.4	-25.596	114.9698	7.783952
33	6.9	33	1.931521	2.0281482	-0.096626836		-71.4087	1.17E+29	1.01E+26	0.7	1649.79	1578.381	9.42541
34	7.3	34	1.987874	1.9315214	0.056352937		129.5407	3.32E+28	4.23E+27	-0.4	-19.61	109.9307	7.475889
35	8.8	35	2.174752	1.9878743	0.186877373		47.0897	1.65E+30	1.47E+31	-1.5	-0.07452	47.01518	8.875224
36	7.3	36	1.987874	2.1747517	-0.186877373		-39.063	1E+34	2.25E+29	1.5	29669.33	29630.26	54.70842
37	7.9	37	2.066863	1.9878743	0.078988411		100.0147	8.77E+31	2.61E+31	-0.6	-5.5956	94.41908	8.051071
38	8.1	38	2.091864	2.0668628	0.025001302		323.9831	1.29E+34	5.08E+32	-0.2	-126.864	197.119	8.41539
39	4.5	39	1.504077	2.0918641	-0.587786665		-7.65584	2.7E+35	1.48E+24	3.6	5.08E+10	5.08E+10	81258768
40	7.4	40	2.00148	1.5040774	0.497402603		14.87728	1.34E+26	1.07E+33	-2.9	-4.3E-08	14.87728	7.423804
41	7.2	41	1.974081	2.00148	-0.027398974		-262.784	4.35E+35	2.73E+33	0.2	797.0823	534.2987	8.054878
42	11.9	42	2.476538	1.974081	0.502457374		23.6836	1.02E+36	1.05E+43	-4.7	-2.1E-08	23.6836	11.93789
43	3.3	43	1.193922	2.4765384	-1.282615932		-2.57287	1.77E+46	1.82E+21	8.6	1.13E+24	1.13E+24	1.82E+21
44	9.4	44	2.24071	1.1939225	1.046787221		8.979857	6.53E+22	7.44E+40	-6.1	-1.4E-19	8.979857	9.414368
45	8.8	45	2.174752	2.2407097	-0.065957968		-133.418	6.18E+43	4.1E+40	0.6	2511.003	2377.585	12.60414
46	7.3	46	1.987874	2.1747517	-0.186877373		-39.063	2.79E+43	9.69E+37	1.5	192267.5	192228.4	314.8655
47	11.7	47	2.459589	1.9878743	0.471714494		24.80314	3.77E+40	1.17E+48	-4.4	-7.3E-09	24.80314	11.73969
48	4.5	48	1.504077	2.4595888	-0.955511445		-4.70952	1.87E+51	1.12E+30	7.2	2.33E+20	2.33E+20	3.73E+17
49	12.2	49	2.501436	1.5040774	0.997358555		12.23231	1.02E+32	1.15E+51	-7.7	-1.2E-20	12.23231	12.21957

APPENDIX 4 (cont.): Numerical operations of the Grey model with Optimization of Background Value applied to 'Sample 10'

50	9.4	50	2.24071	2.501436	-0.260726262		-36.0531	2.08E+54	5.13E+46	2.8	14477599	14477562	23173.5
51	9.4	51	2.24071	2.2407097	0		#DIV/0!	4.26E+49	4.82E+47	0	#DIV/0!	#DIV/0!	#DIV/0!
52	7.3	52	1.987874	2.2407097	-0.252835341		-28.8725	4.01E+50	1.47E+43	2.1	13010263	13010234	20823.68
53	4.5	53	1.504077	1.9878743	-0.483796951		-9.30142	5.7E+45	2.06E+33	2.8	9.89E+11	9.89E+11	1.58E+09
54	7.3	54	1.987874	1.5040774	0.483796951		15.08897	1.88E+35	7.81E+44	-2.8	-8.6E-11	15.08897	7.324142
55	7.4	55	2.00148	1.9878743	0.013605652		543.8916	3.04E+47	1.17E+46	-0.1	-259.105	284.7864	7.855658
56	8.1	56	2.091864	2.00148	0.090384061		89.61757	4.75E+48	1.14E+49	-0.7	-0.59386	89.02371	8.242438
57	8.1	57	2.091864	2.0918641	0		#DIV/0!	6.08E+51	9.26E+49	0	#DIV/0!	#DIV/0!	#DIV/0!
58	8.8	58	2.174752	2.0918641	0.08288766		106.1678	4.92E+52	7.78E+52	-0.7	-0.90366	105.2641	8.968423
59	7.6	59	2.028148	2.1747517	-0.146603474		-51.8405	5.3E+55	1.61E+50	1.2	274739.8	274688	447.1008
60	7.2	60	1.974081	2.0281482	-0.054067221		-133.168	7.06E+52	5.31E+49	0.4	3322.544	3189.377	12.303
61	8.8	61	2.174752	1.974081	0.200670695		43.85294	1.98E+52	5.3E+55	-1.6	-0.00023	43.85271	8.870164
62	8.2	62	2.104134	2.1747517	-0.070617567		-116.118	3.61E+58	6.74E+54	0.6	8931.888	8815.77	22.30523
63	8.2	63	2.104134	2.1041342	0		#DIV/0!	3.72E+57	5.53E+55	0	#DIV/0!	#DIV/0!	#DIV/0!
64	6.9	64	1.931521	2.1041342	-0.172612743		-39.9739	3.05E+58	1.02E+52	1.3	2298791	2298751	3684.902
65	9.4	65	2.24071	1.9315214	0.309188278		30.40219	3.35E+54	2.03E+61	-2.5	-6.6E-08	30.40219	9.448643
66	7.3	66	1.987874	2.2407097	-0.252835341		-28.8725	1.68E+64	1.79E+55	2.1	4.48E+08	4.48E+08	717266.2
67	7.2	67	1.974081	1.9878743	-0.013793322		-521.992	6.96E+57	5.33E+55	0.1	1306.229	784.2374	8.45478

APPENDIX 4 (cont.): Numerical operations of the Grey model with Optimization of Background Value applied to 'Sample 10'

68	7.4	68	2.00148	1.974081	0.027398974		270.0831	1.99E+58	2.34E+57	-0.2	-42.49	227.5932	7.764149
69	12.2	69	2.501436	2.00148	0.499955952		24.40215	9.48E+59	6.11E+72	-4.8	-3.2E-14	24.40215	12.23904
70	12.2	70	2.501436	2.501436	0		#DIV/0!	1.11E+76	7.46E+73	0	#DIV/0!	#DIV/0!	#DIV/0!
71	8.8	71	2.174752	2.501436	-0.32668423		-26.9373	1.35E+77	1.48E+65	3.4	2.7E+11	2.7E+11	4.31E+08
72	9.3	72	2.230014	2.1747517	0.055262679		168.2872	1.01E+68	6.22E+67	-0.5	-3.23576	165.0514	9.564082
73	6.9	73	1.931521	2.2300144	-0.298492989		-23.1161	5E+70	3.62E+59	2.4	5.76E+10	5.76E+10	92231352
74	8.2	74	2.104134	1.9315214	0.172612743		47.50518	1.19E+62	6.23E+65	-1.3	-0.00015	47.50504	8.276008
75	7	75	1.94591	2.1041342	-0.158224005		-44.2411	3.44E+68	4.92E+61	1.2	5817023	5816979	9314.166
76	8.8	76	2.174752	1.9459101	0.228841572		38.45455	1.69E+64	7.79E+69	-1.8	-1.2E-06	38.45455	8.861527
77	9.4	77	2.24071	2.1747517	0.065957968		142.515	5.31E+72	9.65E+72	-0.6	-0.91712	141.5979	9.626557
78	3.3	78	1.193922	2.2407097	-1.046787221		-3.1525	8.02E+75	2.55E+39	6.1	5.15E+35	5.15E+35	8.24E+32
79	7.4	79	2.00148	1.1939225	0.807557532		9.163434	9.17E+40	8.53E+66	-4.1	-2.6E-27	9.163434	7.414661
80	7.2	80	1.974081	2.00148	-0.027398974		-262.784	3.46E+69	7.45E+66	0.2	2320.453	2057.67	10.49227
81	7.3	81	1.987874	1.974081	0.013793322		529.2416	2.78E+69	1.59E+68	-0.1	-174.352	354.8899	7.867824
82	7.3	82	1.987874	1.9878743	0		#DIV/0!	6.2E+70	1.16E+69	0	#DIV/0!	#DIV/0!	#DIV/0!
83	8.1	83	2.091864	1.9878743	0.103989714		77.89232	4.53E+71	3.87E+73	-0.8	-0.01464	77.87768	8.224604
84	3.3	84	1.193922	2.0918641	-0.897941593		-3.67507	2.05E+76	3.3E+42	4.8	1.3E+33	1.3E+33	2.08E+30
85	7.3	85	1.987874	1.1939225	0.79395188		9.194512	1.18E+44	4.53E+71	-4	-6.5E-29	9.194512	7.314711
86	9.6	86	2.261763	1.9878743	0.27388875		35.05073	1.76E+74	3.24E+82	-2.3	-2.4E-09	35.05073	9.656081

APPENDIX 4 (cont.): Numerical operations of the Grey model with Optimization of Background Value applied to 'Sample 10'

87	8.8	87	2.174752	2.2617631	-0.087011377		-101.136	2.87E+85	1.91E+80	0.8	187706.9	187605.7	308.9692
88	12.2	88	2.501436	2.1747517	0.32668423		37.34493	1.3E+83	2.67E+93	-3.4	-1.4E-11	37.34493	12.25975

The followings are the forecasts

k values	Observed value	Forecast value	Error absolute	Error sq.
89	8.2	8.157223967	0.042776033	0.00183
90	8.1	8.144182845	0.044182845	0.001952
91	8.8	8.131162571	0.668837429	0.447344
92	7.1	8.118163113	1.018163113	1.036656
93	8	8.105184438	0.105184438	0.011064
94	7.3	8.092226512	0.792226512	0.627623
95	7.9	8.079289302	0.179289302	0.032145
96	11.7	8.066372775	3.633627225	13.20325
97	8.8	8.053476898	0.746523102	0.557297
98	9.4	8.040601638	1.359398362	1.847964
99	9.3	8.027746962	1.272253038	1.618628
100	7.6	8.014912837	0.414912837	0.172153
101	7.3	8.00209923	0.70209923	0.492943
102	10.4	7.989306108	2.410693892	5.811445
103	8.1	7.976533439	0.123466561	0.015244
104	10.4	7.96378119	2.43621881	5.935162
105	7.4	7.951049329	0.551049329	0.303655
106	7	7.938337822	0.938337822	0.880478
107	9.4	7.925646637	1.474353363	2.173718
108	8.8	7.912975742	0.887024258	0.786812

APPENDIX 5: Numerical operations of the Grey_Arima model applied on the Malian GDP

GDP	Errors (Grey)	Errors (ARIMA)	t	Forecasts (Grey)	Forecasts (ARIMA)		Hybrid	errors		Hybrid	errors		Hybrid	errors
275494520	-23096995.38	0	1	252397524.8	275494520.1		270875121	4619399.077		273184820.6	2309699.538		274339670.4	1154849.769
343771965	-70899281.65	-68439276.35	2	272872683	275332688.3	alpha 0.2	274840687	68931277.41	alpha 0.1	275086687.8	68685276.88	alpha 0.05	275209688.1	68562276.61
339913833	-44904992.56	5609116.448	3	295008840.5	345522949.5	beta 0.8	335420128	4493705.354	beta 0.9	340471538.6	557705.5467	beta 0.95	342997244.1	3083410.997
359772363	-40831621.25	-18225894.31	4	318940742	341546469		337025324	22747039.7		339285896.3	20486467		340416182.6	19356180.65
430096738	-85282675.41	-68070515.51	5	344814063	362026222.9		358583791	71512947.49		360305006.9	69791731.5		361165614.9	68931123.5
486617332	-113831035.9	-51843536.4	6	372786296.5	434773796		422376296	64241036.3		428575046	58042286.35		431674421	54942911.37
563683660	-160655948.3	-70217365.92	7	403027712	493466294.4		475378578	88305082.41		484422436.1	79261224.16		488944365.3	74739295.04
538747268	-103024876.8	35025420.03	8	435722391.5	573772688.4		546162629	7415360.657		559967658.7	21220390.34		566870173.5	28122905.19
830710615	-359641264.8	-282955480	9	471069350.3	547755135.2		532417978	298292636.9		540086556.7	290624058.5		543920846	286789769.2
939227994	-429944245.2	-85212809.25	10	509283748.5	854015184.4		785068897	154159096.4		819542040.8	119685952.8		836778612.6	102449381

APPENDIX 5 (cont.): Numerical operations of the Grey_Arima model applied on the Malian GDP

1049838493	-499240292.4	-81221942.8	11	550598200.1	968616549.8		885012880	164825612.7		926814714.8	123023777.8		947715632.3	102122860.3
1222702356	-627438166.1	-136924084	12	595264190	1085778272		987675456	235026900.4		1036726864	185975492.2		1061252568	161449788.1
1595423286	-951869681.9	-325919270.4	13	643553603.7	1269504015		1144313933	451109352.7		1206908974	388514311.6		1238206495	357216791
1759690812	-1063930429	-91877373.78	14	695760383	1667813438		1473402827	286287984.7		1570608132	189082679.3		1619210785	140480026.5
1538972158	-786769843.1	305182992.4	15	752202315.1	1844155151		1625764583	86792425.3		1734959867	195987708.8		1789557509	250585350.6
1333754034	-520531067.5	273563894.7	16	813222966.8	1607317929		1448498936	114744902.3		1527908433	194154398.5		1567613181	233859146.6
1297765449	-418571672.6	90126331.88	17	879193775.9	1387891780		1286152179	11613269.01		1337021980	39256531.43		1362456880	64691431.66
1232932008	-282415695.6	116565070.9	18	950516312.5	1349497079		1269700926	36768917.58		1309599002	76666994.23		1329548041	96616032.56
1392195933	-364571210.5	-111797636.8	19	1027624723	1280398297		1229843582	162352351.6		1255120939	137074994.2		1267759618	124436315.5
1852163475	-741175102	-401866274.1	20	1110988373	1450297200		1382435435	469728039.7		1416366318	435797156.9		1433331759	418831715.5
2090629723	-889515019.6	-147016591.5	21	1201114703	1943613131		1795113446	295516277.1		1869363288	221266434.3		1906488210	184141512.9
2169040742	-870488420.4	31625256.61	22	1298552321	2200665998		2020243263	148797478.8		2110454630	58586111.09		2155560314	13480427.24

APPENDIX 5 (cont.): Numerical operations of the Grey_Arima model applied on the Malian GDP

2181821902	-777927564.6	103535254.3	23	1403894338	2285357157		2109064593	72757309.48		2197210875	15388972.41		2241284016	59462113.36
2681912030	-1164130051	-382742530	24	1517781979	2299169500		2142891996	539020034.3		2221030748	460881282.1		2260100124	421811906.1
2724131545	-1083223057	116995321.1	25	1640908488	2841126866		2601083191	123048354.4		2721105029	3026516.661		2781115947	56984402.23
2830673389	-1056650041	56333582.74	26	1774023348	2887006972		2664410247	166263142		2775708609	54964779.65		2831357790	684401.542
2818280876	-900344037.9	184585856.4	27	1917936838	3002866732		2785880754	32400122.47		2894373743	76092866.96		2948620238	130339361.7
2081846483	-8321509.126	907538094.9	28	2073524974	2989384578		2806212657	724366174.1		2897798617	815952134.5		2943591597	861745114.7
2706425298	-464690466.1	-515241004	29	2241734832	2191184294		2201294402	505130896.4		2196239348	510185950.2		2193711821	512713477.1
2780422212	-356831890.3	87341081.67	30	2423590322	2867763294		2778928700	1493512.731		2823345997	42923784.47		2845554645	65132433.07
2697105694	-76907281.71	251100831.3	31	2620198412	2948206525		2882604903	185499208.7		2915405714	218300020		2931806120	234700425.6
2920358587	-87602712.92	-62722823.73	32	2832755874	2857635763		2852659785	67698801.57		2855147774	65210812.65		2856391769	63966818.19
3439463140	-376906578.7	-338980772.2	33	3062556562	3100482368		3092897207	346565933.5		3096689787	342773352.9		3098586078	340877062.6
2954129566	356869726.5	712801443.2	34	3310999292	3666931009		3595744666	641615099.8		3631337837	677208271.5		3649134423	695004857.4

APPENDIX 5 (cont.): Numerical operations of the Grey_Arima model applied on the Malian GDP

3465305993	114290364.3	-328046414.7	35	3579596358	3137259579		3225726935	239579058.9		3181493257	283812736.8		3159376418	305929575.7
3889758024	-19775292.5	-194567010.9	36	3869982731	3695191013		3730149356	159608667.3		3712670185	177087839.1		3703930599	185827425
4703504467	-519578447.1	-543416422.9	37	4183926019	4160088044		4164855639	538648827.8		4162471841	541032625.4		4161279942	542224524.2
5444474268	-921137046.7	-389563760	38	4523337222	5054910508		4948595851	495878417.3		5001753180	442721088.7		5028331844	416142424.3
6245031690	-1354749327	-371837771	39	4890282363	5873193919		5676611608	568420082.1		5774902764	470128926.5		5824048341	420983348.7
6899799786	-1612804716	-139308582.4	40	5286995070	6760491203		6465791977	434007809.1		6613141590	286658195.7		6686816397	212983389.1
8145694632	-2429804465	-657301754.7	41	5715890167	7488392877		7133892335	1011802297		7311142606	834552025.8		7399767742	745926890.3
9750822511	-3571244138	-872604532.5	42	6179578374	8878217979		8338490058	1412332454		8608354018	1142468493		8743285999	1007536513
1.0181E+10	-3500139570	495717221.3	43	6680882201	10676738992		9877567634	303454136.9		10277153313	96131542.21		10476946152	295924381.8
1.0679E+10	-3455896340	481367429.4	44	7222853127	11160116897		1.0373E+10	306085324.6		10766390520	87641052.4		10963253708	284504240.9
1.2978E+10	-5169317385	-1258085249	45	7808790176	11720022312		1.0938E+10	2040331676		11328899098	1649208462		11524460705	1453646856
1.2443E+10	-4000487901	1872125085	46	8442259996	14314872982		1.314E+10	697602487.7		13727611684	1284863786		14021242333	1578494436

APPENDIX 5 (cont.): Numerical operations of the Grey_Arima model applied on the Malian GDP

1.3246E+10	-4119293459	463165686.2	47	9127118572	13709577718		1.2793E+10	453326142.9		13251331803	4919771.665		13480454760	234042728.9
1.4388E+10	-4520825367	230109023.7	48	9867534697	14618469088		1.3668E+10	720077854.5		14143375649	244984415.4		14380922368	7437695.849
1.31E+10	-2432042754	2812291255	49	10668015346	15912349355		1.4863E+10	1763424453		15387915954	2287857854		15650132655	2550074555
1.4035E+10	-2501547220	417864324.7	50	11533433114	14452844658		1.3869E+10	166017984.3		14160903504	125923170.2		14306874081	271893747.5

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