**ÇUKUROVA UNIVERSITY**
**INSTITUTE OF NATURAL AND APPLIED SCIENCES**

**MSc THESIS**

**İsmail YILDIZ**

# INVESTIGATION OF BEHAVIOUR OF THE ELECTROMAGNETIC WAVES IN A PARALLEL PLATE WAVEGUIDE WITH GRID TYPE SURFACES

**DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING**

**ADANA, 2015**

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We certify that the thesis titled above was reviewed and approved for the award of degree of the Master of Science by the board of jury on 16/09/2015.

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## ABSTRACT

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Supervisor : Prof. Dr. Hamit SERBEST
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In this study, the behavior of the electromagnetic waves inside and outside of a strip grating parallel plate waveguide is investigated by using ray tracing method. First of all, reflection and transmission at a grid type interface are examined between two simple media. The reflection and transmission coefficients for this interface are obtained by using approximate boundary conditions. With the usage of these coefficients, the effects of waves that are excited outside for TE and TM modes are inspected in both mediums and compared with the results of the field expressions by Floquet mode. Afterwards, the reason of preference of strip grating parallel plate waveguides instead of the conventional parallel plate waveguides is mentioned such that while strip's width increases, dissipated loss also increases. However, smaller selection of the strip's width increases the amount of the expelled wave. Thus, it is shown that the optimization of strip's width is necessary.

**Key Words:** Grid Type Structures, Parallel Plate Waveguides, Approximate Boundary Conditions

# ÖZ

## YÜKSEK LİSANS TEZİ

## IZGARA TİPİ YÜZEYLERLE BİR PARALEL PLAKA DALGA KILAVUZUNDA ELEKTROMANYETİK DALGALARIN DAVRANIŞININ İNCELENMESİ

**İsmail YILDIZ**

**ÇUKUROVA ÜNİVERSİTESİ**
**FEN BİLİMLERİ ENSTİTÜSÜ**
**ELEKTRİK ELEKTRONİK MÜHENDİSLİĞİ ANABİLİM DALI**

Danışman : Prof. Dr. Hamit SERBEST
Yıl: 2015, Sayfa: 75
Jüri : Prof. Dr. Hamit SERBEST
: Prof. Dr. Turgut İKİZ
: Prof. Dr. Ali AKDAĞLI

Bu çalışmada, ızgara tipi duvarlı paralel plaka dalga kılavuzunun içerisindeki ve dışarısındaki elektromanyetik dalgaların davranışı ışın izleme yöntemi ile incelenmiştir. Öncelikle, iki basit ortam arasındaki ızgara tipi bir arayüzde yansıma ve kırılma incelenmiş olup, bu arayüz için yansıma ve kırılma katsayıları yaklaşık sınır koşulları kullanılarak elde edilmiştir. Elde edilen bu katsayılar kullanılarak, dışarıda uyarılan TE ve TM modlarındaki dalganın içeride ve dışarıdaki etkisi incelenmiş ve Floquet modu ile bulunan alan ifadeleri ile karşılaştırılmıştır. Daha sonra, klasik dalga kılavuzu yerine ızgara tipi dalga kılavuzu tercih edilmesine gerekçe olarak, şeritler genişledikçe kaybolan güçte de artış olduğuna değinilmiştir. Ancak şeritlerin çok küçük seçilmesi kılavuzdan dışarı kaçan dalgayı büyüttüğü için, şerit genişliğinin optimize edilmesi gerektiği gösterilmiştir.

**Anahtar Kelimeler:** Izgara Tipi Yapılar, Paralel Plaka Dalga Kılavuzu, Yaklaşık Sınır Koşulları

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## LIST OF SYMBOLS AND ABBREVATIONS

$\mu_1$ : Magnetic Permeability of Medium 1

$\mu_2$ : Magnetic Permeability of Medium 2

$\Pi^*$ : Magnetic Hertzian Potential

$\varepsilon_1$ : Dielectric Permitivity of Medium 1

$\varepsilon_2$ : Dielectric Permitivity of Medium 2

$A_{11}^{TE}$ : Reflection Coefficient for TE mode when incident wave in medium 1

$A_{11}^{TM}$ : Reflection Coefficient for TM mode when incident wave in medium 1

$A_{12}^{TE}$ : Transmission Coefficient for TE mode when incident wave in medium 1

$A_{12}^{TM}$ : Transmission Coefficient for TM mode when incident wave in medium 1

$A_{22}^{TE}$ : Reflection Coefficient for TE mode when incident wave in medium 2

$A_{22}^{TM}$ : Reflection Coefficient for TM mode when incident wave in medium 2

$A_{21}^{TE}$ : Transmission Coefficient for TE mode when incident wave in medium 2

$A_{21}^{TM}$ : Transmission Coefficient for TM mode when incident wave in medium 2

$P_d$ : Dissipated Power

$k_1$ : Wavenumber of Medium 1

$k_2$ : Wavenumber of Medium 2

$\theta_0$ : Incidence Angle

$a$ : Strip Width

$d$ : Strip Period

$E$ : Electric Field Intensity

$f$ : Frequency

| | | |
|---|---|---|
| GHz | : | Gigahertz |
| $H$ | : | Magnetic Field Intensity |
| MHz | : | Megahertz |
| MoM | : | Method of Moments |
| PEC | : | Perfect Electric Conductor |
| SIW | : | Substrate Integrated Waveguide |
| SW | : | Surface Wave |
| TE | : | Transverse Electric |
| THz | : | Terahertz |
| TM | : | Transverse Magnetic |
| URSI | : | International Union of Radio Science |
| UTD | : | Uniform Geometrical Theory of Diffraction |
| $\Pi$ | : | Electric Hertzian Potential |
| $\omega$ | : | Angular Frequency |

# 1. INTRODUCTION

The propagation of an electromagnetic wave and scattering from any interface or edge have been studied since the electromagnetic waves were discovered. In order to transmit a wave, structures called transmission lines have been improved. The parallel plate waveguide which has a cut-off frequency that depends on distance between plates and medium parameters is one of the most well-known and common waveguides where electromagnetic waves oscillate in one direction and propagate in other directions. The cut-off frequency is the critical parameter for a parallel plate waveguide, like other type waveguides; the waves cannot propagate below the cut-off frequency, but occur evanescent due to the fact that propagation constant of the wave become purely imaginary and negative, from the $e^{-jkr}$ with the purely imaginary *k*, it goes to zero when *r* goes to infinity.

In this work, the propagation of plane electromagnetic waves will be considered through a parallel plate waveguide with a grid type structure. In contrast to the conventional waveguide systems, the walls are not full conducting planes. For waveguides with perfect electric conductor, PEC, walls, the electromagnetic wave causes heat loss in the conducting surfaces. By substituting these full surfaces with grating type walls it is expected to be able to reduce the amount of heat loss which may occur during the propagation. This structure is similar to the Substrate Integrated Waveguide (SIW). Also, at microwave frequencies these grating type structures provide the opportunity to work at microelectronic dimensions.

## 1.1. Outline of Dissertation

In Chapter 2, previous studies are mentioned about parallel plate waveguides with strip grating which are studied by different researchers.

In Chapter 3, the problem investigated in this thesis is defined and geometry of this problem is shown in a figure clearly.

In Chapter 4, all coefficients in order to solve this type problem are determined by using approximate boundary conditions.

In Chapter 5, all field expression are calculated by using the coefficients which are determined in chapter 4. For this calculation, Ray Tracing Method is used.

In Chapter 6, the field components are obtained by Floquet modes for same geometry. The electric and magnetic fields which are obtained by using Ray Tracing mode and Floquet mode are compared. The conventional waveguide and strip grating waveguide are compared in terms of dissipated powers and expelled wave.

Finally, related references used in the thesis and biographical information of the author are presented.

# 2. LITERATURE REVIEW

For the last few decades, in waveguide and etc. structures, strip grating are used because of their advantages instead of whole metal plate. There are a lot of studies about periodic structures and strip grating.

Weinstein (1963) studied about the boundary conditions and presented the approximate boundary conditions assuming that the surface at interface is perfectly flat plane instead of the normal boundary conditions with different interface types; for example half space of homogeneous medium with high refractive index, single or multilayered dielectrics, anisotropic dielectric layer, single or multilayer dielectrics coating a metallic substrate, grid structures consisting of flat strips or cylinders etc.

Jacobsen (1970), investigated the electromagnetic waves guided by a periodically strip loaded dielectric slab by using Floquet modes. It's analytical study was presented in such a manner that; at grazing surface, with surface current densities which have the same variation as the field quantities, electric and magnetic Hertzian potentials are expanded in y direction applying Floquet's Theorem. Also, he said that the electric and magnetic field components were obtained in terms of Hertzian potentials by the following formulas $\vec{E} = \vec{\nabla} \times \vec{\nabla} \times \vec{\Pi} + i\omega\mu(\vec{\nabla} \times \vec{\Pi}^{*})$ and $\vec{H} = -i\omega\varepsilon(\vec{\nabla} \times \vec{\Pi}) + \vec{\nabla} \times \vec{\nabla} \times \vec{\Pi}^{*}$ He obtained the determinantal equation with narrow strips and presented numerical and experimental results involving different combinations of the linear and dielectric parameters in terms of *k-α* and *k-β* diagrams and mainly dealt with "backward-radiation" region of the *k-β* diagram. The general behavior and complex solutions around the coupling regions and the boundaries of the *k-α* plane were discussed. Additionally, the numerical results in the backward-radiation region were verified experimentally by measuring the phase and amplitude variation in the near field and by calculation from radiation patterns.

Guglielmi and Oliner, in 1989, published a set of articles, which mentioned periodic structures which separate two different dielectric medium by metal-strip grating. The main aim of these papers was to present a set of multimode equivalent network representation in contradistinction to single mode representation which contains only one reflected wave and only one transmitted wave on both sides of the grating plane. In part I of set of articles, the unit cell approach was applied for two different cases: the aperture phrasing (center of aperture at the origin) and the obstacle phrasing (center of obstacle at the origin). For each case, E-polarized and H-polarized were investigated. Depending on the polarization of incident wave, these two phrasings have different electromagnetic importance. While the obstacle discontinuity is electrically large for E-polarized and electrically small for H-polarized, for the aperture phrasing is vice versa. Fredholm integral equations of the first kind with singular kernels were used in order to represent aperture and obstacle formulations.

In part II of these articles, they presented the approximate solution procedures to solve the integral equation which were obtained in part I with small argument and derived expressions for the elements of the coupling matrices in the four different cases: obstacle phrasing and aperture phrasing for E-polarized and H-polarized. They compared their numerical results with independent numerical solutions.

In part III, Guglielmi and Hochstadt obtained and presented an analytical solution in order to solve Hilbert-type singular integral equations and rigorous multimode equivalent network representations for the main problem mentioned in part I.

In 2000, Burghignoli and his team studied about the radiation from periodic structures at low frequencies (below cut-off) and discovered a new complex solution model which has been found to be an additional solution for n=0 spatial harmonic and has physical meaning in the fast-wave region outside the bound-mode triangle and nonphysical inside. They analyzed the dispersion (normalized phase $\beta/k_0$ and normalized attenuation $\alpha/k_0$ vs. frequency f) when *s/p* ratio, s referred to the width of strips and p was spatial period, from 0, grounded dielectric slab, to 1, the dielectric-filled parallel-plate waveguide, that is, transition from a basically open to a basically closed structures. According to the study of Burghignoli's team, for the large value of *s/p* ratio there was a frequency range where the physical leaky mode had a very low attenuation constant and it could provide an effective narrow-beam leaky wave antenna.

Burghignoli et al. have improved their studies about the dispersion features at low frequencies in 2001. The results of previous paper covered just one dielectric constant and two different values of *s/p* ratio which were 0 and 1, while current work gave results for many dielectric constants and *s/p* ratios systematically. Additionally, in the previous study obtained results have led to many open questions, study in 2001 has answered most of these questions and also proper values of $\beta/k_0$ and $\alpha/k_0$ for leaky-wave antennas were ensured by parameter ranges. The structure analysis has been done by the transverse resonance technique. The behaviors of the dispersion diagrams under the cut-off frequency were different from each other, while the dielectric-filled parallel-plate mode was damping, grounded dielectric slab mode became an improper real mode without physical meaning.

Manara et al. published their study at URSI (2005) which explained the scattering from finite strip grating where large planar semi-transparent strips lied parallel and free-standing, i.e. there was a slop between the grating plane and a coordinate plane like x=a plane or y=b plane when incident wave was an arbitrary polarized plane wave with oblique incidence. Their analysis aimed to prove the efficiency and validity of high frequency asymptotic solutions with approximate homogeneous transition boundary conditions. The expression of the scattered field was obtained by using The Uniform Geometrical Theory of Diffraction (UTD) and the Method of Moments (MoM) was used numerical solution without using any approximate boundary conditions. The excitation and diffraction of the surface wave (SW), appear in both UTD and MoM data, were examined specifically.

Nepa et al. (2005) investigated the electromagnetic scattering at edge of semi-infinite strip grating that strips were parallel to each other and were not parallel to the coordinate plane like x=a plane or y=b plane. Moreover there was an oblique incidence with an arbitrary polarization. The obtained equivalent canonical problem was solved by application of the Sommerfeld-Maliuzhinets method and based along the grating parameters. Additionally, some numerical results were represented predicting a non-vanishing diffracted field for any incident field polarization.

In 2009, Vorobyov numerically solved the nonlinear operator equation for reflection from a plane which consists of periodic semi-infinite grating of thin metal strips when incidence was H-polarized electromagnetic plane wave. Using the successive iteration method, he composed an algorithm to solve a nonlinear operator equation in the Fourier amplitude of the reflected field. Initial approximations and convergence of the solution of the proposed algorithm were numerically verified. The field reflected from semi-infinite grating investigated for different values of the incidence angle, incidence wavelength and strip width.

Shapoval et al. (2013) studied about strip-gratings which were made from not metal but graphene and proposed a novel numerical approach that is based on the surface impedance of graphene. Also, on contrary to commercial software, developed meshless algorithm ensures fast convergence and controlled accuracy of computations. Additionally, they mentioned about hyper-singular integral, obtained by Green functions, and the Nystrom-type discretization. Reflectance, transmittance and absorbance were represented in terms of graphene and grating parameters. Graphene relaxation time is proportional to the resonance number in THz levels, therefore with reduced losses comes higher wave transmittance. They have shown that if graphene strip numbers are low (>10), then Rayleigh anomalies may occur. So that, in most of the applications as tunable absorbers and frequency selective surfaces the graphene is a good alternative material.

Xiong et al. (2014), introduced a new approach in order to analyze large periodic structures with finite length. Since it is impossible to produce infinite length material, truncation of this material is needed to turn into finite length material. This procedure causes discontinuities at the interface and excitation of surface waves that leads to edge effect of finite periodic structures. Finite arrays are analyzed by element-by-element method. While this method is suitable for small finite arrays, it requires complex calculations for large finite arrays. Infinite periodic approximation neglects the edge effect so this results in reduction in calculations but negligence of edge effect is not appropriate for engineering applications. Proposed approach yields proper prediction of the behavior of large finite periodic structures without ignoring the edge effect. This work provides profit of low complexity and memory consumption.

# 3. FORMULATION OF THE PROBLEM

In this study, the propagation of the electromagnetic waves in parallel plate waveguide is analyzed. While one of the plates of waveguide is PEC, the other wall is strip grating. The geometry of the Problem is shown in Figure 3.1.

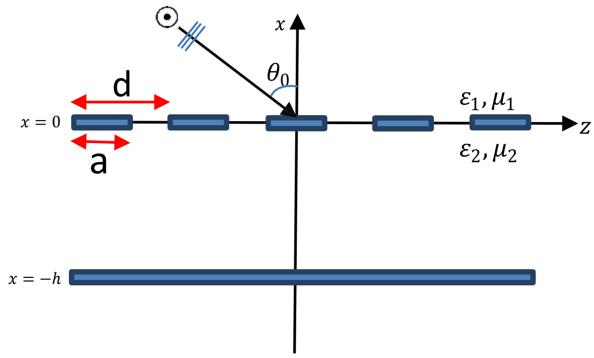


Figure 3.1. Geometry of the Problem

Suppose that an oblique incidence with an incident angle $\theta_0$ has both E-polarized and H-polarized modes. The solution is obtained for these two cases. Here, time dependence is $e^{j\omega t}$. The direction of the propagation is $x-$ and $z-$ direction. There is no propagation in $y-$ direction.

In order to obtain the expressions of electric field and magnetic field in medium 1 and medium 2, the reflection and transmission coefficients should be determined first. The next part covers this study.

# 4. OBTAINING THE COEFFICIENTS BY USING APPROXIMATE BOUNDARY CONDITIONS

When an oblique incidence having an incidence angle travels to an interface as in Figure 4.1, normally, the wave will have a reflection and a transmission according to the reflection coefficient $A_{11}^{TM}$ and transmission coefficient $A_{12}^{TM}$ where the script TM says that this coefficient is obtained for H-polarized. Also, 11 and 12 are reflection and transmission coefficient respectively for incident wave in medium 1. If the incident wave comes from medium 2 to medium 1, subscripts of coefficient become 22 and 21 instead of 11 and 12 respectively.

In order to determine the reflection and transmission coefficients, boundary conditions must be applied on the surface. The normal known boundary conditions cannot be applied in this problem because of the interface is not full PEC. Instead of normal boundary conditions, approximate boundary conditions which were given for the periodic structures by Weinstein in 1963, must be applied because of the strip grating. This part continuous analytically obtaining of the coefficients.

## 4.1. TM Mode When Incidence in Medium 1

For H-polarized, when the incident wave is in medium 1, the reflection and transmission coefficients $A_{22}^{TM}$ and $A_{21}^{TM}$ can be determined by using boundary conditions and the field components are shown in Figure 4.1.

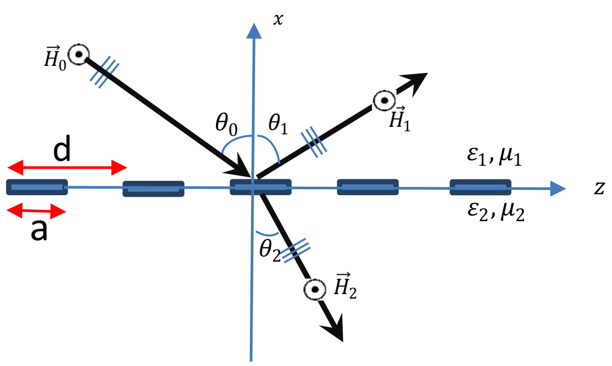


Figure 4.1. Reflection and transmission from grating structure when H-polarized incidence in medium 1

#### 4.1.1. Incident Wave

The wave number of the medium 1 and medium 2 are:

$$k_1 = \omega\sqrt{\varepsilon_1\mu_1}$$

and

$$k_2 = \omega\sqrt{\varepsilon_2\mu_2}$$

where $\omega$ is angular frequency, $\varepsilon_1, \mu_1$ are dielectric permittivity and magnetic permeability of the medium 1 and $\varepsilon_2, \mu_2$ are dielectric permittivity and magnetic permeability of the medium 2, respectively.

The magnetic field of the incident wave is:

$$\vec{H}_{0y} = \hat{e}_y e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$$

By using the Maxwell's equation $\vec{\nabla} \times \vec{H} = j\omega\varepsilon_1\vec{E}$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y e^{-jk_1(-xcos\theta_0+zsin\theta_0)}\right)$$
$$= j\omega\varepsilon_1(E_{0x}\hat{e}_x + E_{0y}\hat{e}_y + E_{0z}\hat{e}_z)$$

$$\hat{e}_z\frac{\partial}{\partial x}\left(e^{-jk_1(-xcos\theta_0+zsin\theta_0)}\right) - \hat{e}_x\frac{\partial}{\partial z}\left(e^{-jk_1(-xcos\theta_0+zsin\theta_0)}\right)$$
$$= j\omega\varepsilon_1(E_{0x}\hat{e}_x + E_{0z}\hat{e}_z)$$

To obtain the x component of the electric field

$$E_{0x} = \frac{-\frac{\partial}{\partial z}\left\{e^{-jk_1(-xcos\theta_0+zsin\theta_0)}\right\}}{j\omega\varepsilon_1} = \frac{jk_1 sin\theta_0 e^{-jk_1(-xcos\theta_0+zsin\theta_0)}}{j\omega\varepsilon_1}$$

$$E_{0x} = \frac{k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$$

To obtain the z component of the electric field

$$E_{0z} = \frac{\frac{\partial}{\partial x}\left\{e^{-jk_1(-xcos\theta_0+zsin\theta_0)}\right\}}{j\omega\varepsilon_1} = \frac{jk_1 cos\theta_0 e^{-jk_1(-xcos\theta_0+zsin\theta_0)}}{j\omega\varepsilon_1}$$

$$E_{0z} = \frac{k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$$

### 4.1.2. Reflected Wave

The H-polarized reflected wave has a magnetic field component as

$$\vec{H}_{1y} = \hat{e}_y A_{11}^{TM} e^{-jk_1(xcos\theta_1+zsin\theta_1)}$$

From the Maxwell's equation $\vec{\nabla} \times \vec{H} = j\omega\varepsilon_1\vec{E}$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y A_{11}^{TM} e^{-jk_1(xcos\theta_1+zsin\theta_1)}\right)$$
$$= j\omega\varepsilon_1(E_{1x}\hat{e}_x + E_{1y}\hat{e}_y + E_{1z}\hat{e}_z)$$

$$\hat{e}_z \frac{\partial}{\partial x}\left\{A_{11}^{TM} e^{-jk_1(xcos\theta_1+zsin\theta_1)}\right\} - \hat{e}_x \frac{\partial}{\partial z}\left\{A_{11}^{TM} e^{-jk_1(xcos\theta_1+zsin\theta_1)}\right\}$$
$$= j\omega\varepsilon_1(E_{1x}\hat{e}_x + E_{1z}\hat{e}_z)$$

To obtain the x component of the electric field

$$E_{1x} = \frac{-\frac{\partial}{\partial z}\left\{A_{11}^{TM} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}\right\}}{j\omega\varepsilon_1} = \frac{jA_{11}^{TM} k_1 sin\theta_1 e^{-jk_1(xcos\theta_1 + zsin\theta_1)}}{j\omega\varepsilon_1}$$

$$E_{1x} = \frac{A_{11}^{TM} k_1 sin\theta_1}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}$$

To obtain the z component of the electric field

$$E_{1z} = \frac{\frac{\partial}{\partial x}\left\{A_{11}^{TM} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}\right\}}{j\omega\varepsilon_1} = \frac{-jA_{11}^{TM} k_1 cos\theta_1 e^{-jk_1(xcos\theta_1 + zsin\theta_1)}}{j\omega\varepsilon_1}$$

$$E_{1z} = \frac{-A_{11}^{TM} k_1 cos\theta_1}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}$$

### 4.1.3. Transmitted Wave

The H-polarized transmitted wave has a magnetic field component as

$$\vec{H}_{2y} = \hat{e}_y A_{12}^{TM} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}$$

The Maxwell's equation for medium 2 becomes $\vec{\nabla} \times \vec{H} = j\omega\varepsilon_2\vec{E}$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y A_{12}^{TM} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}\right)$$
$$= j\omega\varepsilon_2(E_{2x}\hat{e}_x + E_{2y}\hat{e}_y + E_{2z}\hat{e}_z)$$

$$\hat{e}_z \frac{\partial}{\partial x}\left(A_{12}^{TM} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}\right) - \hat{e}_x \frac{\partial}{\partial z}\left(A_{12}^{TM} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}\right)$$
$$= j\omega\varepsilon_2(E_{2x}\hat{e}_x + E_{2y}\hat{e}_y)$$

$$E_{2x} = \frac{-\frac{\partial}{\partial z}\left\{A_{12}^{TM} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}\right\}}{j\omega\varepsilon_2} = \frac{jA_{12}^{TM} k_2 sin\theta_2 e^{-jk_2(-xcos\theta_2+zsin\theta_2)}}{j\omega\varepsilon_2}$$

$$E_{2x} = \frac{A_{12}^{TM} k_2 sin\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}$$

$$E_{2z} = \frac{\frac{\partial}{\partial x}\left\{A_{12}^{TM} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}\right\}}{j\omega\varepsilon_2} = \frac{jA_{12}^{TM} k_2 cos\theta_2 e^{-jk_2(-xcos\theta_2+zsin\theta_2)}}{j\omega\varepsilon_2}$$

$$E_{2z} = \frac{A_{12}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}$$

### 4.1.4. Total Fields For Each Medium

Total fields in medium 1 is equal to the summation of incident and reflected fields.

$$E_x^1 = \frac{k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)} + \frac{A_{11}^{TM} k_1 sin\theta_1}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_1+zsin\theta_1)}$$

$$E_z^1 = \frac{k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)} - \frac{A_{11}^{TM} k_1 cos\theta_1}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_1+zsin\theta_1)}$$

$$H_y^1 = e^{-jk_1(-xcos\theta_0+zsin\theta_0)} + A_{11}^{TM} e^{-jk_1(xcos\theta_1+zsin\theta_1)}$$

The total fields in medium 2 is equal to the transmitted fields.

$$E_x^2 = \frac{A_{12}^{TM} k_2 sin\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}$$

$$E_z^2 = \frac{A_{12}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}$$

$$H_y^2 = A_{12}^{TM} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}$$

### 4.1.5. Approximate Boundary Conditions

The approximate boundary conditions which was obtained by Weinstein are modified for the coordinate system with horizontal z-axis and vertical x-axis.

$$E_y^2 + E_y^1 = (jk_2 l_0 H_z^2 - jk_1 l_0 H_z^1) + l_0 \frac{\partial}{\partial y}(E_x^2 - E_x^1) \quad (4{,}1$$

$$E_y^2 - E_y^1 = (-jk_2 l_2 H_z^2 - jk_1 l_2 H_z^1) - l_2 \frac{\partial}{\partial y}(E_x^2 + E_x^1) \quad (4{,}2)$$

$$H_y^2 - H_y^1 = (-jk_2 l_1 E_z^2 - jk_1 l_1 E_z^1) + l_1 \frac{\partial}{\partial y}(H_x^2 + H_x^1) \quad (4{,}3)$$

$$E_z^2 - E_z^1 = \left(-jk_2 l_3 H_y^2 - jk_1 l_3 H_y^1\right) - l_4 \frac{\partial}{\partial z}(E_x^2 + E_x^1) \quad (4,\ 4)$$

where

$$l_0 = \frac{d}{\pi} \ln\left(\frac{1}{\sin\left(\frac{\pi a}{2d}\right)}\right)$$

$$l_1 = \frac{d}{\pi} \ln\left(\frac{1}{\cos\left(\frac{\pi a}{2d}\right)}\right)$$

$$l_2 = l_3 = l_4 = 0$$

From the equation 4,4

$E_z^2 - E_z^1 = 0$ , at x=0

$$\frac{A_{12}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} - \left(\frac{k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0 + zsin\theta_0)} - \frac{A_{11}^{TM} k_1 cos\theta_1}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}\right) = 0$$

$$\frac{A_{12}^{TM}k_2cos\theta_2}{\omega\varepsilon_2}e^{-jk_2zsin\theta_2}-\left(\frac{k_1cos\theta_0}{\omega\varepsilon_1}e^{-jk_1sin\theta_0}-\frac{A_{11}^{TM}k_1cos\theta_1}{\omega\varepsilon_1}e^{-jk_1zsin\theta_1}\right)=0 \quad ,$$

at x=0

The solution is valid for all z values in this condition,

$$k_1sin\theta_0=k_1sin\theta_1=k_2sin\theta_2$$

It is clearly appeared that

$$\theta_0=\theta_1$$

$$k_1sin\theta_0=k_2sin\theta_2$$

This last equation is known as Snell's Law.

Hence, the equation above becomes

$$A_{12}^{TM}\frac{k_2cos\theta_2}{\omega\varepsilon_2}-(1-A_{11}^{TM})\frac{k_1cos\theta_0}{\omega\varepsilon_1}=0$$

or

$$A_{12}^{TM}\frac{k_2cos\theta_2}{\omega\varepsilon_2}=(1-A_{11}^{TM})\frac{k_1cos\theta_0}{\omega\varepsilon_1}$$

$$A_{12}^{TM}=(1-A_{11}^{TM})\frac{k_1cos\theta_0\varepsilon_2}{k_2cos\theta_2\varepsilon_1} \tag{4,5}$$

From the equation 4,3

$$H_y^2-H_y^1=-jk_2l_1E_z^2-jk_1l_1E_z^1 \quad , \text{ at x=0}$$

$$A_{12}^{TM} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} -$$
$$\left(e^{-jk_1(-xcos\theta_0+zsin\theta_0)} + A_{11}^{TM} e^{-jk_1(xcos\theta_1+zsin\theta_1)}\right) =$$
$$-jk_2 l_1 \left(\frac{A_{12}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}\right) -$$
$$jk_1 l_1 \left(\frac{k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)} - \frac{A_{11}^{TM} k_1 cos\theta_1}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_1+zsin\theta_1)}\right)$$

At x=0;

$$A_{12}^{TM} e^{-jk_2 zsin\theta_2} - \left(e^{-jk_1 zsin\theta_0} + A_{11}^{TM} e^{-jk_1 zsin\theta_1}\right) =$$
$$-jk_2 l_1 \left(\frac{A_{12}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2 zsin\theta_2}\right) -$$
$$jk_1 l_1 \left(\frac{k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1 zsin\theta_0} - \frac{A_{11}^{TM} k_1 cos\theta_1}{\omega\varepsilon_1} e^{-jk_1 zsin\theta_1}\right)$$

By using the Snell's Law, the equation above becomes

$$A_{12}^{TM} - (1 + A_{11}^{TM}) = -jk_2 l_1 \left(\frac{A_{12}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2}\right) - jk_1 l_1 \left(\frac{k_1 cos\theta_0}{\omega\varepsilon_1} - \frac{A_{11}^{TM} k_1 cos\theta_0}{\omega\varepsilon_1}\right)$$

$$A_{12}^{TM} \left[1 + \frac{jk_2^2 l_1 cos\theta_2}{\omega\varepsilon_2}\right] = A_{11}^{TM} \left[1 + \frac{jk_1^2 l_1 cos\theta_0}{\omega\varepsilon_1}\right] + \left[1 - \frac{jk_1^2 l_1 cos\theta_0}{\omega\varepsilon_1}\right] \tag{4,6}$$

We can obtain the expression $A_{11}^{TM}$ and $A_{12}^{TM}$ solving the equations 4,5 and 4,6.

In the equation 4,6; write the equivalent instead of $A_{12}^{TM}$,

$$(1 - A_{11}^{TM}) \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} \left[1 + \frac{jk_2^2 l_1 cos\theta_2}{\omega\varepsilon_2}\right] = A_{11}^{TM} \left[1 + \frac{jk_1^2 l_1 cos\theta_0}{\omega\varepsilon_1}\right] + \left[1 - \frac{jk_1^2 l_1 cos\theta_0}{\omega\varepsilon_1}\right]$$

$$\left[\frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} - A_{11}^{TM} \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1}\right] \left[1 + \frac{jk_2^2 l_1 cos\theta_2}{\omega\varepsilon_2}\right] = A_{11}^{TM} \left[1 + \frac{jk_1^2 l_1 cos\theta_0}{\omega\varepsilon_1}\right] + \left[1 - \frac{jk_2^2 l_1 cos\theta_0}{\omega\varepsilon_1}\right]$$

$$\frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} + \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} \frac{jk_2^2 l_1 cos\theta_2}{\omega \varepsilon_2} - A_{11}^{TM} \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} - A_{11}^{TM} \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} \frac{jk_2^2 l_1 cos\theta_2}{\omega \varepsilon_2} =$$

$$A_{11}^{TM} \left[1 + \frac{jk_1^2 l_1 cos\theta_0}{\omega \varepsilon_1}\right] + \left[1 - \frac{jk_1^2 l_1 cos\theta_0}{\omega \varepsilon_1}\right]$$

$$\frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} + \frac{jk_1 k_2 l_1 cos\theta_0}{\omega \varepsilon_1} - A_{11}^{TM} \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} - A_{11}^{TM} \frac{jk_1 k_2 l_1 cos\theta_0}{\omega \varepsilon_1} = A_{11}^{TM} +$$

$$A_{11}^{TM} \frac{jk_1^2 l_1 cos\theta_0}{\omega \varepsilon_1} + 1 - \frac{jk_1^2 l_1 cos\theta_0}{\omega \varepsilon_1}$$

$$\frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} + \frac{jk_1 k_2 l_1 cos\theta_0}{\omega \varepsilon_1} + \frac{jk_1^2 l_1 cos\theta_0}{\omega \varepsilon_1} - 1 = A_{11}^{TM} \left[1 + \frac{jk_1^2 l_1 cos\theta_0}{\omega \varepsilon_1} + \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} + \right.$$

$$\left. \frac{jk_1 k_2 l_1 cos\theta_0}{\omega \varepsilon_1}\right]$$

We can equalize the denominators at $\omega k_2 cos\theta_2 \varepsilon_1$

$$\frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} + \frac{jk_1 k_2 l_1 cos\theta_0}{\omega \varepsilon_1} + \frac{jk_1^2 l_1 cos\theta_0}{\omega \varepsilon_1} - 1 = A_{11}^{TM} \left[ 1 + \frac{jk_1^2 l_1 cos\theta_0}{\omega \varepsilon_1} + \right.$$

$$\left. \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1} + \frac{jk_1 k_2 l_1 cos\theta_0}{\omega \varepsilon_1}\right]$$

$$\frac{\omega k_1 cos\theta_0 \varepsilon_2 + jk_1 k_2^2 l_1 cos\theta_0 cos\theta_2 + jk_2 k_1^2 l_1 cos\theta_0 cos\theta_2 - \omega k_2 cos\theta_2 \varepsilon_1}{\omega k_2 cos\theta_2 \varepsilon_1} =$$

$$A_{11}^{TM} \left[\frac{\omega k_1 cos\theta_0 \varepsilon_2 + jk_1 k_2^2 l_1 cos\theta_0 cos\theta_2 + jk_2 k_1^2 l_1 cos\theta_0 cos\theta_2 + \omega k_2 cos\theta_2 \varepsilon_1}{\omega k_2 cos\theta_2 \varepsilon_1}\right]$$

The reflection coefficient for H-polarized when incident wave is in medium 1 is found as

$$A_{11}^{TM} = \frac{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1 + k_2) + \omega k_1 cos\theta_0 \varepsilon_2] - [\omega k_2 cos\theta_2 \varepsilon_1]}{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1 + k_2) + \omega k_1 cos\theta_0 \varepsilon_2] + [\omega k_2 cos\theta_2 \varepsilon_1]}$$

To obtain $A_{12}^{TM}$, equation 4,5 can be used.

$$A_{12}^{TM} = (1 - A_{11}^{TM}) \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1}$$

$$A_{12}^{TM} = \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1}\left[1 - \frac{[jk_1k_2l_1 cos\theta_0 cos\theta_2(k_1+k_2)+\omega k_1 cos\theta_0 \varepsilon_2]-[\omega k_2 cos\theta_2 \varepsilon_1]}{[jk_1k_2l_1 cos\theta_0 cos\theta_2(k_1+k_2)+\omega k_1 cos\theta_0 \varepsilon_2]+[\omega k_2 cos\theta_2 \varepsilon_1]}\right]$$

$$A_{12}^{TM} = \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1}\left[\frac{[jk_1k_2l_1 cos\theta_0 cos\theta_2(k_1+k_2)+\omega k_1 cos\theta_0 \varepsilon_2]+[\omega k_2 cos\theta_2 \varepsilon_1]}{[jk_1k_2l_1 cos\theta_0 cos\theta_2(k_1+k_2)+\omega k_1 cos\theta_0 \varepsilon_2]+[\omega k_2 cos\theta_2 \varepsilon_1]} - \right.$$
$$\left.\frac{[jk_1k_2l_1 cos\theta_0 cos\theta_2(k_1+k_2)+\omega k_1 cos\theta_0 \varepsilon_2]-[\omega k_2 cos\theta_2 \varepsilon_1]}{[jk_1k_2l_1 cos\theta_0 cos\theta_2(k_1+k_2)+\omega k_1 cos\theta_0 \varepsilon_2]+[\omega k_2 cos\theta_2 \varepsilon_1]}\right]$$

$$A_{12}^{TM} = \frac{k_1 cos\theta_0 \varepsilon_2}{k_2 cos\theta_2 \varepsilon_1}\left[\frac{2\omega k_2 cos\theta_2 \varepsilon_1}{[jk_1k_2l_1 cos\theta_0 cos\theta_2(k_1+k_2)+\omega k_1 cos\theta_0 \varepsilon_2]+[\omega k_2 cos\theta_2 \varepsilon_1]}\right]$$

Finally, the transmission coefficient for TM mode when incident wave is in medium 1 is found as

$$A_{12}^{TM} = \frac{2\omega k_1 cos\theta_0 \varepsilon_2}{[jk_1k_2l_1 cos\theta_0 cos\theta_2(k_1+k_2)+\omega k_1 cos\theta_0 \varepsilon_2]+[\omega k_2 cos\theta_2 \varepsilon_1]}$$

### 4.2. TM Mode When Incidence in Medium 2

For H-polarized, when the incident wave is in medium 2, the reflection and transmission coefficients $A_{22}^{TM}$ and $A_{21}^{TM}$ can be determined by the same way above and the field components are shown in Figure 4.2.

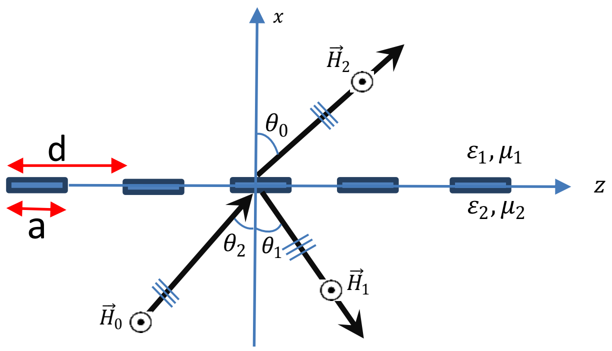


Figure 4.2. Reflection and transmission from grating structure when H-polarized incidence in medium 2

#### 4.2.1. Incident Wave

The magnetic field of the incident wave which is in medium 2 is:

$$\vec{H}_{0y} = \hat{e}_y e^{-jk_2(xcos\theta_2 + zsin\theta_2)}$$

The electric field components of this wave can be found by using the Maxwell's eq.

$$\vec{\nabla} \times \vec{H} = j\omega\varepsilon_2 \vec{E}$$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y e^{-jk_2(xcos\theta_2 + zsin\theta_2)}\right)$$
$$= j\omega\varepsilon_2 (E_{0x}\hat{e}_x + E_{0y}\hat{e}_y + E_{0z}\hat{e}_z)$$

$$\hat{e}_z \frac{\partial}{\partial x}\left(e^{-jk_2(xcos\theta_2 + zsin\theta_2)}\right) - \hat{e}_x \frac{\partial}{\partial z}\left(e^{-jk_2(xcos\theta_2 + zsin\theta_2)}\right)$$
$$= j\omega\varepsilon_2 (E_{0x}\hat{e}_x + E_{0z}\hat{e}_z)$$

For the x-component of electric field

$$E_{0x} = \frac{-\frac{\partial}{\partial z}\left\{e^{-jk_2(xcos\theta_2 + zsin\theta_2)}\right\}}{j\omega\varepsilon_2} = \frac{jk_2 sin\theta_2 e^{-jk_2(xcos\theta_2 + zsin\theta_2)}}{j\omega\varepsilon_2}$$

$$E_{0x} = \frac{k_2 sin\theta_2}{\omega\varepsilon_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)}$$

And, for the z-component of the electric field

$$E_{0z} = \frac{\frac{\partial}{\partial x}\left\{e^{-jk_2(xcos\theta_2 + zsin\theta_2)}\right\}}{j\omega\varepsilon_2} = \frac{-jk_1 cos\theta_2 e^{-jk_2(xcos\theta_2 + zsin\theta_2)}}{j\omega\varepsilon_2}$$

$$E_{0z} = \frac{-k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)}$$

### 4.2.2. Reflected Wave

The reflected wave has the magnetic field component with the reflection coefficient $A_{22}^{TM}$ as below;

$$\vec{H}_{1y} = \hat{e}_y A_{22}^{TM} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}$$

By using the same Maxwell's eq. $\vec{\nabla} \times \vec{H} = j\omega\varepsilon_2 \vec{E}$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y A_{22}^{TM} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}\right)$$
$$= j\omega\varepsilon_2(E_{1x}\hat{e}_x + E_{1y}\hat{e}_y + E_{1z}\hat{e}_z)$$

$$\hat{e}_z \frac{\partial}{\partial x}\left\{A_{22}^{TM} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}\right\} - \hat{e}_x \frac{\partial}{\partial z}\left\{A_{22}^{TM} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}\right\}$$
$$= j\omega\varepsilon_2(E_{1x}\hat{e}_x + E_{1z}\hat{e}_z)$$

The x- and z- components of the reflected field are shown below.

$$E_{1x} = \frac{-\frac{\partial}{\partial z}\left\{A_{22}^{TM} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}\right\}}{j\omega\varepsilon_2} = \frac{jA_{22}^{TM} k_2 sin\theta_1 e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}}{j\omega\varepsilon_2}$$

$$E_{1x} = \frac{A_{22}^{TM} k_2 sin\theta_1}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}$$

$$E_{1z} = \frac{\frac{\partial}{\partial x}\left\{A_{22}^{TM} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}\right\}}{j\omega\varepsilon_2} = \frac{-jA_{22}^{TM} k_2 cos\theta_1 e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}}{j\omega\varepsilon_2}$$

$$E_{1z} = \frac{A_{22}^{TM} k_2 cos\theta_1}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}$$

### 4.2.3. Transmitted Wave

The transmitted wave's magnetic field component is;

$$\vec{H}_{2y} = \hat{e}_y A_{21}^{TM} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

And, electric field components of this wave are found from

$$\vec{\nabla} \times \vec{H} = j\omega\varepsilon_1 \vec{E}$$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y A_{21}^{TM} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}\right)$$
$$= j\omega\varepsilon_1 (E_{2x}\hat{e}_x + E_{2y}\hat{e}_y + E_{2z}\hat{e}_z)$$

$$\hat{e}_z \frac{\partial}{\partial x}\left(A_{21}^{TM} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}\right) - \hat{e}_x \frac{\partial}{\partial z}\left(A_{21}^{TM} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}\right)$$
$$= j\omega\varepsilon_1 (E_{2x}\hat{e}_x + E_{2y}\hat{e}_y)$$

$$E_{2x} = \frac{-\frac{\partial}{\partial z}\left\{A_{21}^{TM} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}\right\}}{j\omega\varepsilon_1} = \frac{jA_{21}^{TM} k_1 sin\theta_0 e^{-jk_1(xcos\theta_0 + zsin\theta_0)}}{j\omega\varepsilon_1}$$

$$E_{2x} = \frac{A_{21}^{TM} k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

$$E_{2z} = \frac{\frac{\partial}{\partial x}\left\{A_{21}^{TM} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}\right\}}{j\omega\varepsilon_2} = \frac{-jA_{21}^{TM} k_1 cos\theta_0 e^{-jk_1(xcos\theta_0 + zsin\theta_0)}}{j\omega\varepsilon_1}$$

$$E_{2z} = \frac{-A_{21}^{TM} k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

### 4.2.4. Total Fields For Each Medium

The fields in medium 2

$$E_x^1 = \frac{k_2 sin\theta_2}{\omega\varepsilon_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} + \frac{A_{22}^{TM} k_2 sin\theta_1}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}$$

$$E_z^1 = \frac{-k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} + \frac{A_{22}^{TM} k_2 cos\theta_1}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}$$

$$H_y^1 = e^{-jk_2(xcos\theta_2 + zsin\theta_2)} + A_{22}^{TM} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}$$

And, in medium 1

$$E_x^2 = \frac{A_{21}^{TM} k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

$$E_z^2 = \frac{-A_{21}^{TM} k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

$$H_y^2 = A_{21}^{TM} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

From the equation 4,4

$$E_z^2 - E_z^1 = 0 \quad \text{, at x=0}$$

$$\frac{-A_{21}^{TM} k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0 + zsin\theta_0)} - \left(\frac{-k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} + \frac{A_{22}^{TM} k_2 cos\theta_1}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}\right) = 0 \quad \text{, at x=0}$$

$$\frac{-A_{21}^{TM}k_1cos\theta_0}{\omega\varepsilon_1}e^{-jk_1zsin\theta_0}-\left(\frac{-k_2cos\theta_2}{\omega\varepsilon_2}e^{-jk_2zsin\theta_2}+\frac{A_{22}^{TM}k_2cos\theta_1}{\omega\varepsilon_2}e^{-jk_2zsin\theta_1}\right)=0$$

From Snell's Law, the equation above become

$$\frac{A_{21}^{TM}k_1cos\theta_0}{\omega\varepsilon_1}-(1-A_{22}^{TM})\frac{k_2cos\theta_2}{\omega\varepsilon_2}=0$$

or

$$A_{21}^{TM}\frac{k_1cos\theta_0}{\omega\varepsilon_1}=(1-A_{22}^{TM})\frac{k_2cos\theta_2}{\omega\varepsilon_2}$$

$$A_{21}^{TM}=(1-A_{22}^{TM})\frac{k_2cos\theta_2\varepsilon_1}{k_1cos\theta_0\varepsilon_2} \tag{4,7}$$

From the equation 4,3

$$H_y^2-H_y^1=-jk_1l_1E_z^2-jk_2l_1E_z^1 \quad , \quad \text{at x=0}$$

$$A_{21}^{TM}e^{-jk_1(xcos\theta_0+zsin\theta_0)}-$$
$$\left(e^{-jk_2(xcos\theta_2+zsin\theta_2)}+A_{22}^{TM}e^{-jk_2(-xcos\theta_1+zsin\theta_1)}\right)=$$
$$-jk_1l_1\left(\frac{-A_{21}^{TM}k_1cos\theta_0}{\omega\varepsilon_1}e^{-jk_1(xcos\theta_0+zsin\theta_0)}\right)-$$
$$jk_2l_1\left(\frac{-k_2cos\theta_2}{\omega\varepsilon_2}e^{-jk_2(xcos\theta_2+zsin\theta_2)}+\frac{A_{22}^{TM}k_2cos\theta_1}{\omega\varepsilon_2}e^{-jk_2(-xcos\theta_1+zsin\theta_1)}\right)$$

At x=0;

$$A_{21}^{TM}e^{-jk_1zsin\theta_0}-\left(e^{-jk_2zsin\theta_2}+A_{22}^{TM}e^{-jk_2zsin\theta_1}\right)=$$
$$-jk_1l_1\left(\frac{-A_{21}^{TM}k_1cos\theta_0}{\omega\varepsilon_1}e^{-jk_1zsin\theta_0}\right)-jk_2l_1\left(\frac{-k_2cos\theta_2}{\omega\varepsilon_2}e^{-jk_2zsin\theta_2}+\right.$$
$$\left.\frac{A_{22}^{TM}k_2cos\theta_1}{\omega\varepsilon_2}e^{-jk_2zsin\theta_1}\right)$$

By using Snell's Law, the equation above become

$$A_{21}^{TM}-(1+A_{22}^{TM})=-jk_1l_1\left(\frac{-A_{21}^{TM}k_1cos\theta_0}{\omega\varepsilon_1}\right)-jk_2l_1\left(\frac{-k_2cos\theta_2}{\omega\varepsilon_2}+\frac{A_{22}^{TM}k_2cos\theta_2}{\omega\varepsilon_2}\right)$$

$$A_{21}^{TM}\left[1-\frac{jk_1^2l_1cos\theta_0}{\omega\varepsilon_2}\right]=A_{22}^{TM}\left[1-\frac{jk_2^2l_1cos\theta_2}{\omega\varepsilon_1}\right]+\left[1+\frac{jk_2^2l_1cos\theta_2}{\omega\varepsilon_1}\right] \qquad (4,8)$$

We can obtain the expression $A_{22}^{TM}$ and $A_{21}^{TM}$ by solving the equations 4,7 and 4,8.

In the equation 4,8; write the equivalent instead of $A_{21}^{TM}$,

$$(1-A_{22}^{TM})\frac{k_2cos\theta_2\varepsilon_1}{k_1cos\theta_0\varepsilon_2}\left[1-\frac{jk_1^2l_1cos\theta_0}{\omega\varepsilon_1}\right]=A_{22}^{TM}\left[1-\frac{jk_2^2l_1cos\theta_2}{\omega\varepsilon_2}\right]+\left[1+\frac{jk_2^2l_1cos\theta_2}{\omega\varepsilon_2}\right]$$

$$\left[\frac{k_2cos\theta_2\varepsilon_1}{k_1cos\theta_0\varepsilon_2}-A_{22}^{TM}\frac{k_2cos\theta_2\varepsilon_1}{k_1cos\theta_0\varepsilon_2}\right]\left[1-\frac{jk_1^2l_1cos\theta_2}{\omega\varepsilon_1}\right]=A_{22}^{TM}\left[1-\frac{jk_2^2l_1cos\theta_2}{\omega\varepsilon_2}\right]+$$
$$\left[1+\frac{jk_2^2l_1cos\theta_2}{\omega\varepsilon_2}\right]$$

$$\frac{k_2cos\theta_2\varepsilon_1}{k_1cos\theta_0\varepsilon_2}-\frac{k_2cos\theta_2\varepsilon_1}{k_1cos\theta_0\varepsilon_2}\frac{jk_1^2l_1cos\theta_0}{\omega\varepsilon_1}-A_{22}^{TM}\frac{k_2cos\theta_2\varepsilon_1}{k_1cos\theta_0\varepsilon_2}+A_{22}^{TM}\frac{k_2cos\theta_2\varepsilon_1}{k_1cos\theta_0\varepsilon_2}\frac{jk_1^2l_1cos\theta_0}{\omega\varepsilon_1}=$$
$$A_{22}^{TM}-A_{22}^{TM}\frac{jk_2^2l_1cos\theta_2}{\omega\varepsilon_2}+1+\frac{jk_2^2l_1cos\theta_2}{\omega\varepsilon_2}$$

$$\frac{k_2cos\theta_2\varepsilon_1}{k_1cos\theta_0\varepsilon_2}-\frac{jk_1k_2l_1cos\theta_2}{\omega\varepsilon_2}-A_{22}^{TM}\frac{k_2cos\theta_2\varepsilon_1}{k_1cos\theta_0\varepsilon_2}+A_{22}^{TM}\frac{jk_1k_2l_1cos\theta_2}{\omega\varepsilon_2}=A_{22}^{TM}-$$
$$A_{22}^{TM}\frac{jk_2^2l_1cos\theta_2}{\omega\varepsilon_2}+1+\frac{jk_2^2l_1cos\theta_2}{\omega\varepsilon_2}$$

We can equalize the denominators at $\omega k_1cos\theta_0\varepsilon_2$

$$\frac{k_2 cos\theta_2 \varepsilon_1}{k_1 cos\theta_0 \varepsilon_2} - \frac{jk_1 k_2 l_1 cos\theta_2}{\omega \varepsilon_2} - A_{22}^{TM} \frac{k_2 cos\theta_2 \varepsilon_1}{k_1 cos\theta_0 \varepsilon_2} + A_{22}^{TM} \frac{jk_1 k_2 l_1 cos\theta_2}{\omega \varepsilon_2} = A_{22}^{TM} - A_{22}^{TM} \frac{jk_2^2 l_1 cos\theta_2}{\omega \varepsilon_2} + 1 + \frac{jk_2^2 l_1 cos\theta_2}{\omega \varepsilon_2}$$

$$\frac{\omega k_2 cos\theta_2 \varepsilon_1}{\omega k_1 cos\theta_0 \varepsilon_2} - \frac{jk_1^2 k_2 l_1 cos\theta_0 cos\theta_2}{\omega k_1 cos\theta_0 \varepsilon_2} - A_{22}^{TM} \frac{\omega k_2 cos\theta_2 \varepsilon_1}{\omega k_1 cos\theta_0 \varepsilon_2} + A_{22}^{TM} \frac{jk_1^2 k_2 l_1 cos\theta_0 cos\theta_2}{\omega k_1 cos\theta_0 \varepsilon_2} = A_{22}^{TM} \frac{\omega k_1 cos\theta_0 \varepsilon_2}{\omega k_1 cos\theta_0 \varepsilon_2} - A_{22}^{TM} \frac{jk_1 k_2^2 l_1 cos\theta_0 cos\theta_2}{\omega k_1 cos\theta_0 \varepsilon_2} + \frac{\omega k_1 cos\theta_0 \varepsilon_2}{\omega k_1 cos\theta_0 \varepsilon_2} + \frac{jk_1 k_2^2 l_1 cos\theta_0 cos\theta_2}{\omega k_1 cos\theta_0 \varepsilon_2}$$

$$A_{22}^{TM} \left[ \frac{-\omega k_2 cos\theta_2 \varepsilon_1 + jk_1^2 k_2 l_1 cos\theta_0 cos\theta_2 + jk_1 k_2^2 l_1 cos\theta_0 cos\theta_2 - \omega k_1 cos\theta_0 \varepsilon_2}{\omega k_1 cos\theta_0 \varepsilon_2} \right] = \frac{-\omega k_2 cos\theta_2 \varepsilon_1 + jk_1^2 k_2 l_1 cos\theta_0 cos\theta_2 + jk_1 k_2^2 l_1 cos\theta_0 cos\theta_2 + \omega k_1 cos\theta_0 \varepsilon_2}{\omega k_1 cos\theta_0 \varepsilon_2}$$

The reflection coefficient for H-polarized, incidence in medium 2, is

$$A_{22}^{TM} = \left[ \frac{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1 + k_2) - \omega k_2 cos\theta_2 \varepsilon_1] + \omega k_1 cos\theta_0 \varepsilon_2}{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1 + k_2) - \omega k_2 cos\theta_2 \varepsilon_1] - \omega k_1 cos\theta_0 \varepsilon_2} \right]$$

To obtain $A_{21}^{TM}$;

$$A_{21}^{TM} = (1 - A_{22}^{TM}) \frac{k_2 cos\theta_2 \varepsilon_1}{k_1 cos\theta_0 \varepsilon_2}$$

$$A_{21}^{TM} = \frac{k_2 cos\theta_2 \varepsilon_1}{k_1 cos\theta_0 \varepsilon_2} \left[ 1 - \frac{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1 + k_2) - \omega k_2 cos\theta_2 \varepsilon_1] + \omega k_1 cos\theta_0 \varepsilon_2}{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1 + k_2) - \omega k_2 cos\theta_2 \varepsilon_1] - \omega k_1 cos\theta_0 \varepsilon_2} \right]$$

$$A_{21}^{TM} = \frac{k_2 cos\theta_2 \varepsilon_1}{k_1 cos\theta_0 \varepsilon_2} \left[ \frac{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1 + k_2) - \omega k_2 cos\theta_2 \varepsilon_1] - \omega k_1 cos\theta_0 \varepsilon_2}{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1 + k_2) - \omega k_2 cos\theta_2 \varepsilon_1] - \omega k_1 cos\theta_0 \varepsilon_2} - \frac{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1 + k_2) - \omega k_2 cos\theta_2 \varepsilon_1] + \omega k_1 cos\theta_0 \varepsilon_2}{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1 + k_2) - \omega k_2 cos\theta_2 \varepsilon_1] - \omega k_1 cos\theta_0 \varepsilon_2} \right]$$

$$A_{21}^{TM} = \frac{k_2 cos\theta_2 \varepsilon_1}{k_1 cos\theta_0 \varepsilon_2} \left[ \frac{-2\omega k_1 cos\theta_0 \varepsilon_2}{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1 + k_2) - \omega k_2 cos\theta_2 \varepsilon_1] - \omega k_1 cos\theta_0 \varepsilon_2} \right]$$

And, the reflection coefficient

$$A_{21}^{TM} = \left[\frac{-2\omega k_2 cos\theta_2 \varepsilon_1}{[jk_1 k_2 l_1 cos\theta_0 cos\theta_2 (k_1+k_2) - \omega k_2 cos\theta_2 \varepsilon_1] - \omega k_1 cos\theta_0 \varepsilon_2}\right]$$

## 4.3. TE Mode When Incidence in Medium 1

For E-polarized, when the incident wave is in medium 1, the reflection and transmission coefficients $A_{11}^{TE}$ and $A_{12}^{TE}$ can be determined by the same way above, the field components is shown in Figure 4.3.

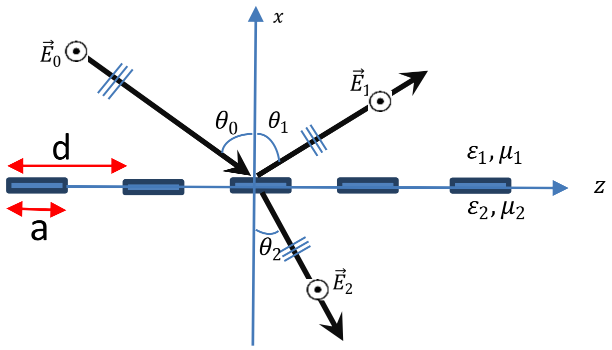


Figure 4.3. Reflection and transmission from grating structure when E-polarized incidence in medium 1

### 4.3.1. Incident Wave

For E-polarized, while the wave is propagating in the x- and z- directions, electric field has y-component.

$$\vec{E}_{0y} = \hat{e}_y e^{-jk_1(-xcos\theta_0 + zsin\theta_0)}$$

Magnetic field is obtained by Maxwell's equation

$$\vec{\nabla} \times \vec{E} = -j\omega\mu_1 \vec{H}$$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y e^{-jk_1(-xcos\theta_0 + zsin\theta_0)}\right)$$
$$= -j\omega\mu_1 (H_{0x}\hat{e}_x + H_{0y}\hat{e}_y + H_{0z}\hat{e}_z)$$

$$\hat{e}_z \frac{\partial}{\partial x}\left(e^{-jk_1(-xcos\theta_0+zsin\theta_0)}\right) - \hat{e}_x \frac{\partial}{\partial z}\left(e^{-jk_1(-xcos\theta_0+zsin\theta_0)}\right)$$
$$= -j\omega\mu_1(H_{0x}\hat{e}_x + H_{0z}\hat{e}_z)$$

Magnetic field has x-component given below;

$$H_{0x} = \frac{\frac{\partial}{\partial z}\left\{e^{-jk_1(-xcos\theta_0+zsin\theta_0)}\right\}}{j\omega\mu_1} = \frac{-jk_1 sin\theta_0 e^{-jk_1(-xcos\theta_0+zsin\theta_0)}}{j\omega\mu_1}$$

$$H_{0x} = \frac{-k_1 sin\theta_0}{\omega\mu_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$$

and, z component as

$$H_{0z} = \frac{-\frac{\partial}{\partial x}\left\{e^{-jk_1(-xcos\theta_0+zsin\theta_0)}\right\}}{j\omega\mu_1} = \frac{-jk_1 cos\theta_0 e^{-jk_1(-xcos\theta_0+zsin\theta_0)}}{j\omega\mu_1}$$

$$H_{0z} = \frac{-k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$$

### 4.3.2. Reflected Wave

Reflection wave’s electric field is

$$\vec{E}_{1y} = \hat{e}_y A_{11}^{TE} e^{-jk_1(xcos\theta_1+zsin\theta_1)}$$

From the same Maxwell’s equation $\vec{\nabla} \times \vec{E} = -j\omega\mu_1\vec{H}$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y A_{11}^{TE} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}\right)$$
$$= -j\omega\mu_1(H_{1x}\hat{e}_x + H_{1y}\hat{e}_y + H_{1z}\hat{e}_z)$$

$$\hat{e}_z \frac{\partial}{\partial x}\left(A_{11}^{TE} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}\right) - \hat{e}_x \frac{\partial}{\partial z}\left(A_{11}^{TE} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}\right)$$
$$= -j\omega\mu_1(H_{1x}\hat{e}_x + H_{1z}\hat{e}_z)$$

Magnetic field's components are

$$H_{1x} = \frac{\frac{\partial}{\partial z}\left\{A_{11}^{TE} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}\right\}}{j\omega\mu_1} = \frac{-jA_{11}^{TE} k_1 sin\theta_1 e^{-jk_1(xcos\theta_1 + zsin\theta_1)}}{j\omega\mu_1}$$

$$H_{1x} = \frac{-A_{11}^{TE} k_1 sin\theta_1}{\omega\mu_1} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}$$

and

$$H_{1z} = \frac{-\frac{\partial}{\partial x}\left\{A_{11}^{TE} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}\right\}}{j\omega\mu_1} = \frac{jA_{11}^{TE} k_1 cos\theta_1 e^{-jk_1(xcos\theta_1 + zsin\theta_1)}}{j\omega\mu_1}$$

$$H_{1z} = \frac{A_{11}^{TE} k_1 cos\theta_1}{\omega\mu_1} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}$$

### 4.3.3. Transmitted Wave

The transmitted wave has the electric field

$$\vec{E}_{2y} = \hat{e}_y A_{12}^{TE} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}$$

Magnetic field's components are found from Maxwell's equation

$$\vec{\nabla} \times \vec{E} = -j\omega\mu_2\vec{H}$$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y A_{12}^{TE} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}\right)$$
$$= -j\omega\mu_2(H_{2x}\hat{e}_x + H_{2y}\hat{e}_y + H_{2z}\hat{e}_z)$$

$$\hat{e}_z \frac{\partial}{\partial x}\left(A_{12}^{TE} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}\right) - \hat{e}_x \frac{\partial}{\partial z}\left(A_{12}^{TE} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}\right)$$
$$= -j\omega\mu_2(H_{2x}\hat{e}_x + H_{2z}\hat{e}_z)$$

$$H_{2x} = \frac{\frac{\partial}{\partial z}\left\{A_{12}^{TE} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}\right\}}{j\omega\mu_2} = \frac{-jA_{12}^{TE} k_2 sin\theta_2 e^{-jk_2(-xcos\theta_2+zsin\theta_2)}}{j\omega\mu_2}$$

x- component of the transmitted wave is

$$H_{2x} = \frac{-A_{12}^{TE} k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}$$

$$H_{2z} = \frac{-\frac{\partial}{\partial x}\left\{A_{12}^{TE} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}\right\}}{j\omega\mu_2}$$
$$= \frac{-jA_{12}^{TE} k_2 cos\theta_2 e^{-jk_2(-xcos\theta_2+zsin\theta_2)}}{j\omega\mu_2}$$

z- component of the transmitted wave is

$$H_{2z} = \frac{-A_{12}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}$$

### 4.3.4. Total Fields For Each Medium

Total fields in medium 1 are obtained by summation of components the incident and reflected waves

$$H_x^1 = \frac{-k_1 sin\theta_0}{\omega\mu_1} e^{-jk_1(-xcos\theta_0 + zsin\theta_0)} - \frac{A_{11}^{TE} k_1 sin\theta_1}{\omega\mu_1} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}$$

$$H_z^1 = \frac{-k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(-xcos\theta_0 + zsin\theta_0)} + \frac{A_{11}^{TE} k_1 cos\theta_1}{\omega\mu_1} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}$$

$$E_y^1 = e^{-jk_1(-xcos\theta_0 + zsin\theta_0)} + A_{11}^{TE} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}$$

Total fields in medium 2 are equal to the components of the transmitted wave.

$$H_x^2 = \frac{-A_{12}^{TE} k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}$$

$$H_z^2 = \frac{-A_{12}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}$$

$$E_y^2 = A_{12}^{TE} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}$$

From the equation 4,2

$$E_y^2 - E_y^1 = 0 \quad \text{at x=0}$$

$$A_{12}^{TE} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} - \left(e^{-jk_1(-xcos\theta_0 + zsin\theta_0)} + A_{11}^{TE} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}\right) = 0 \quad , \text{ at x=0}$$

$$A_{12}^{TE} e^{-jk_2 zsin\theta_2} - \left(e^{-jk_1 zsin\theta_0} + A_{11}^{TE} e^{-jk_1 zsin\theta_1}\right) = 0$$

With the Snell's Law, the equation above becomes

$$A_{12}^{TE} - (1 + A_{11}^{TE}) = 0$$

or

$$A_{12}^{TE} = 1 + A_{11}^{TE} \tag{4, 9}$$

From the equation 4,1

$$E_y^2 + E_y^1 = (jk_2 l_0 H_z^2 - jk_1 l_0 H_z^1) \text{ , at x=0}$$

$$A_{12}^{TE} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} +$$
$$\left(e^{-jk_1(-xcos\theta_0 + zsin\theta_0)} + A_{11}^{TE} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}\right) =$$
$$jk_2 l_0 \left(\frac{-A_{12}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}\right) -$$
$$jk_1 l_0 \left(\frac{-k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(-xcos\theta_0 + zsin\theta_0)} + \frac{A_{11}^{TE} k_1 cos\theta_1}{\omega\mu_1} e^{-jk_1(xcos\theta_1 + zsin\theta_1)}\right)$$

At x=0;

$$A_{12}^{TE} e^{-jk_2 zsin\theta_2} + \left(e^{-jk_1 zsin\theta_0} + A_{11}^{TE} e^{-jk_1 zsin\theta_1}\right) =$$
$$jk_2 l_0 \left(\frac{-A_{12}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2 zsin\theta_2}\right) -$$
$$jk_1 l_0 \left(\frac{-k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1 zsin\theta_0} + \frac{A_{11}^{TE} k_1 cos\theta_1}{\omega\mu_1} e^{-jk_1 zsin\theta_1}\right)$$

When the Snell's Law is applied to the above

$$A_{12}^{TE} + (1 + A_{11}^{TE}) = jk_2 l_0 \left(\frac{-A_{12}^{TE} k_2 cos\theta_2}{\omega\mu_2}\right) - jk_1 l_0 \left(\frac{-k_1 cos\theta_0}{\omega\mu_1} + \frac{A_{11}^{TE} k_1 cos\theta_0}{\omega\mu_1}\right)$$

$$A_{12}^{TE}\left[1+\frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}\right]=A_{11}^{TE}\left[-1-\frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}\right]+\left[-1+\frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}\right] \qquad (4,10)$$

We can obtain the expression $A_{11}^{TE}$ and $A_{12}^{TE}$ by solving the equations 4,9 and 4, 10.

In the equation 4,10; write the equivalent instead of $A_{12}^{TE}$,

$$(1+A_{11}^{TE})\left[1+\frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}\right]=A_{11}^{TE}\left[-1-\frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}\right]+\left[-1+\frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}\right]$$

$$1+\frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}+A_{11}^{TE}+A_{11}^{TE}\frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}=A_{11}^{TE}\left[-1-\frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}\right]+$$
$$\left[-1+\frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}\right]$$

$$A_{11}^{TE}\left[2+\frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}+\frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}\right]=-2-\frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}+\frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}$$

We can equalize the denominators at $\omega\mu_2\mu_1$

$$A_{11}^{TE}\left[2+\frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}+\frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}\right]=-2-\frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}+\frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}$$

$$A_{11}^{TE}\left[2\frac{\omega\mu_2\mu_1}{\omega\mu_2\mu_1}+\frac{jk_2^2 l_0 cos\theta_2\mu_1}{\omega\mu_2\mu_1}+\frac{jk_1^2 l_0 cos\theta_0\mu_2}{\omega\mu_2\mu_1}\right]=-2\frac{\omega\mu_2\mu_1}{\omega\mu_2\mu_1}-\frac{jk_2^2 l_0 cos\theta_2\mu_1}{\omega\mu_2\mu_1}+$$
$$\frac{jk_1^2 l_0 cos\theta_0\mu_2}{\omega\mu_2\mu_1}$$

$$A_{11}^{TE}\left[\frac{2\omega\mu_2\mu_1+jk_2^2 l_0 cos\theta_2\mu_1+jk_1^2 l_0 cos\theta_0\mu_2}{\omega\mu_2\mu_1}\right]=\left[\frac{-2\omega\mu_2\mu_1-jk_2^2 l_0 cos\theta_2\mu_1+jk_1^2 l_0 cos\theta_0\mu_2}{\omega\mu_2\mu_1}\right]$$

$$A_{11}^{TE}=\frac{jk_1^2 l_0 cos\theta_0\mu_2-[2\omega\mu_2\mu_1+jk_2^2 l_0 cos\theta_2\mu_1]}{jk_1^2 l_0 cos\theta_0\mu_2+[2\omega\mu_2\mu_1+jk_2^2 l_0 cos\theta_2\mu_1]}$$

To obtain $A_{12}^{TE}$ ;

$$A_{12}^{TE} = (1 + A_{11}^{TE})$$

$$A_{12}^{TE} = \left(1 + \frac{jk_1^2 l_0 cos\theta_0 \mu_2 - [2\omega\mu_2\mu_1 + jk_2^2 l_0 cos\theta_2 \mu_1]}{jk_1^2 l_0 cos\theta_0 \mu_2 + [2\omega\mu_2\mu_1 + jk_2^2 l_0 cos\theta_2 \mu_1]}\right)$$

$$A_{12}^{TE} = \left(\frac{jk_1^2 l_0 cos\theta_0 \mu_2 + [2\omega\mu_2\mu_1 + jk_2^2 l_0 cos\theta_2 \mu_1]}{jk_1^2 l_0 cos\theta_0 \mu_2 + [2\omega\mu_2\mu_1 + jk_2^2 l_0 cos\theta_2 \mu_1]} + \frac{jk_1^2 l_0 cos\theta_0 \mu_2 - [2\omega\mu_2\mu_1 + jk_2^2 l_0 cos\theta_2 \mu_1]}{jk_1^2 l_0 cos\theta_0 \mu_2 + [2\omega\mu_2\mu_1 + jk_2^2 l_0 cos\theta_2 \mu_1]}\right)$$

Finally, the transmission coefficient for E-polarized with incidence in medium 1 is

$$A_{12}^{TE} = \left(\frac{2jk_1^2 l_0 cos\theta_0 \mu_2}{jk_1^2 l_0 cos\theta_0 \mu_2 + [2\omega\mu_2\mu_1 + jk_2^2 l_0 cos\theta_2 \mu_1]}\right)$$

### 4.4. TE Mode When Incidence in Medium 2

For E-polarized, when the incident wave is in medium 2, the reflection and transmission coefficients $A_{22}^{TE}$ and $A_{21}^{TE}$ can be determined by the same way above and the field components are shown in Figure 4.4.

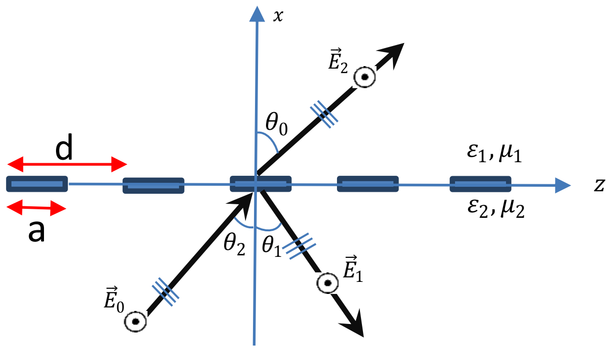


Figure 4.4. Reflection and transmission from grating structure when E-polarized incidence in medium 2

#### 4.4.1. Incident Wave

The electric field of the incident wave for E-polarized with incidence in medium 2 is

$$\vec{E}_{0y} = \hat{e}_y e^{-jk_2(xcos\theta_2 + zsin\theta_2)}$$

Magnetic field can be obtained by using that,

$$\vec{\nabla} \times \vec{E} = -j\omega\mu_2\vec{H}$$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y e^{-jk_2(xcos\theta_2 + zsin\theta_2)}\right)$$
$$= -j\omega\mu_2(H_{2x}\hat{e}_x + H_{2y}\hat{e}_y + H_{2z}\hat{e}_z)$$

$$\hat{e}_z \frac{\partial}{\partial x}\left(e^{-jk_2(xcos\theta_2 + zsin\theta_2)}\right) - \hat{e}_x \frac{\partial}{\partial z}\left(e^{-jk_2(xcos\theta_2 + zsin\theta_2)}\right)$$
$$= -j\omega\mu_2(H_{2x}\hat{e}_x + H_{2z}\hat{e}_z)$$

x- and z- components of the incident wave are equal to;

$$H_{0x} = \frac{\frac{\partial}{\partial z}\left\{e^{-jk_2(xcos\theta_2 + zsin\theta_2)}\right\}}{j\omega\mu_2} = \frac{-jk_2sin\theta_2 e^{-jk_2(xcos\theta_2 + zsin\theta_2)}}{j\omega\mu_2}$$

$$H_{0x} = \frac{-k_2sin\theta_2}{\omega\mu_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)}$$

And,

$$H_{0z} = \frac{-\frac{\partial}{\partial x}\left\{e^{-jk_2(xcos\theta_2 + zsin\theta_2)}\right\}}{j\omega\mu_2} = \frac{jk_1cos\theta_2 e^{-jk_2(xcos\theta_2 + zsin\theta_2)}}{j\omega\mu_2}$$

$$H_{0z} = \frac{k_2cos\theta_2}{\omega\mu_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)}$$

### 4.4.2. Reflected Wave

The components of the reflection wave are found below. Electric field of this wave

$$\vec{E}_{1y} = \hat{e}_y A_{22}^{TE} e^{-jk_2(-xcos\theta_1+zsin\theta_1)}$$

From the equation $\vec{\nabla} \times \vec{E} = -j\omega\mu_2\vec{H}$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y A_{22}^{TE} e^{-jk_2(-xcos\theta_1+zsin\theta_1)}\right) = -j\omega\mu_2(H_{1x}\hat{e}_x + H_{1y}\hat{e}_y + H_{1z}\hat{e}_z)$$

$$\hat{e}_z \frac{\partial}{\partial x}\left\{A_{22}^{TE} e^{-jk_2(-xcos\theta_1+zsin\theta_1)}\right\} - \hat{e}_x \frac{\partial}{\partial z}\left\{A_{22}^{TE} e^{-jk_2(-xcos\theta_1+zsin\theta_1)}\right\} = -j\omega\mu_2(H_{1x}\hat{e}_x + H_{1z}\hat{e}_z)$$

The x- component of magnetic field is

$$H_{1x} = \frac{\frac{\partial}{\partial z}\left\{A_{22}^{TE} e^{-jk_2(-xcos\theta_1+zsin\theta_1)}\right\}}{j\omega\mu_2} = \frac{-jA_{22}^{TE} k_2 sin\theta_1 e^{-jk_2(-xcos\theta_1+zsin\theta_1)}}{j\omega\mu_2}$$

$$H_{1x} = \frac{-A_{22}^{TE} k_2 sin\theta_1}{\omega\mu_2} e^{-jk_2(-xcos\theta_1+zsin\theta_1)}$$

The z- component of magnetic field is

$$H_{1z} = \frac{-\frac{\partial}{\partial x}\left\{A_{22}^{TE} e^{-jk_2(-xcos\theta_1+zsin\theta_1)}\right\}}{j\omega\mu_2} = \frac{-jA_{22}^{TE} k_2 cos\theta_1 e^{-jk_2(-xcos\theta_1+zsin\theta_1)}}{j\omega\mu_2}$$

$$H_{1z} = \frac{-A_{22}^{TE} k_2 cos\theta_1}{\omega\mu_2} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}$$

### 4.4.3. Transmitted Wave

Last wave has an electric field component as;

$$\vec{E}_{2y} = \hat{e}_y A_{21}^{TE} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

Magnetic field components are found by using $\vec{\nabla} \times \vec{E} = -j\omega\mu_1 \vec{H}$

$$\left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y A_{21}^{TE} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}\right)$$
$$= -j\omega\mu_1(H_{2x}\hat{e}_x + H_{2y}\hat{e}_y + H_{2z}\hat{e}_z)$$

$$\hat{e}_z \frac{\partial}{\partial x}\left(A_{21}^{TE} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}\right) - \hat{e}_x \frac{\partial}{\partial z}\left(A_{21}^{TE} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}\right)$$
$$= -j\omega\mu_1(H_{2x}\hat{e}_x + H_{2y}\hat{e}_y)$$

$$H_{2x} = \frac{\frac{\partial}{\partial z}\left\{A_{21}^{TE} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}\right\}}{j\omega\mu_1} = \frac{-jA_{21}^{TE} k_1 sin\theta_0 e^{-jk_1(xcos\theta_0 + zsin\theta_0)}}{j\omega\mu_1}$$

x- component of the magnetic field is

$$H_{2x} = \frac{-A_{21}^{TE} k_1 sin\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

$$H_{2z} = \frac{-\frac{\partial}{\partial x}\left\{A_{21}^{TE} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}\right\}}{j\omega\mu_1} = \frac{jA_{21}^{TE} k_1 cos\theta_0 e^{-jk_1(xcos\theta_0 + zsin\theta_0)}}{j\omega\mu_1}$$

And, the z- component is

$$H_{2z} = \frac{A_{21}^{TE} k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

### 4.4.4. Total Fields For Each Medium

Total fields in medium 2 are the summation of the components of the incident and reflected waves

$$H_x^1 = -\frac{k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} - \frac{A_{22}^{TE} k_2 sin\theta_1}{\omega\mu_2} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}$$

$$H_z^1 = \frac{k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} - \frac{A_{22}^{TE} k_2 cos\theta_1}{\omega\mu_2} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}$$

$$E_y^1 = e^{-jk_2(xcos\theta_2 + zsin\theta_2)} + A_{22}^{TE} e^{-jk_2(-xcos\theta_1 + zsin\theta_1)}$$

And, in medium 1,

$$H_x^2 = \frac{-A_{21}^{TE} k_1 sin\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

$$H_z^2 = \frac{A_{21}^{TE} k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

$$E_y^2 = A_{21}^{TE} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

From Eq. 2

$$E_y^2 - E_y^1 = 0 \quad , \text{ at x=0}$$

$$A_{21}^{TE} e^{-jk_1(x cos\theta_0 + z sin\theta_0)} - \left(e^{-jk_2(x cos\theta_2 + z sin\theta_2)} + A_{22}^{TE} e^{-jk_2(-x cos\theta_1 + z sin\theta_1)}\right) = 0 \text{ , at x=0}$$

$$A_{21}^{TE} e^{-jk_1 z sin\theta_0} - \left(e^{-jk_2 z sin\theta_2} + A_{22}^{TE} e^{-jk_2 z sin\theta_1}\right) = 0$$

With the Snell's Law, the equation above becomes

$$A_{21}^{TE} - (1 + A_{22}^{TE}) = 0$$

or

$$A_{21}^{TE} = 1 + A_{22}^{TE} \tag{4,11}$$

From the equation 4,1

$$E_y^2 + E_y^1 = (jk_1 l_0 H_z^2 - jk_2 l_0 H_z^1) \text{ , at x=0}$$

$$A_{21}^{TE} e^{-jk_1(x cos\theta_0 + z sin\theta_0)} + \left(e^{-jk_2(x cos\theta_2 + z sin\theta_2)} + A_{22}^{TE} e^{-jk_2(-x cos\theta_1 + z sin\theta_1)}\right) = jk_1 l_0 \left(\frac{A_{21}^{TE} k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(x cos\theta_0 + z sin\theta_0)}\right) - jk_2 l_0 \left(\frac{k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(x cos\theta_2 + z sin\theta_2)} - \frac{A_{22}^{TE} k_2 cos\theta_1}{\omega\mu_2} e^{-jk_2(-x cos\theta_1 + z sin\theta_1)}\right)$$

At x=0;

$$A_{21}^{TE} e^{-jk_1 z sin\theta_0} + \left(e^{-jk_2 z sin\theta_2} + A_{22}^{TE} e^{-jk_2 z sin\theta_1}\right) = jk_1 l_0 \left(\frac{A_{21}^{TE} k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1 z sin\theta_0}\right) - jk_2 l_0 \left(\frac{k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2 z sin\theta_2} - \frac{A_{22}^{TE} k_2 cos\theta_1}{\omega\mu_2} e^{-jk_2 z sin\theta_1}\right)$$

Then,

$$A_{21}^{TE} + (1 + A_{22}^{TE}) = jk_1 l_0 \left(\frac{A_{21}^{TE} k_1 cos\theta_0}{\omega\mu_1}\right) - jk_2 l_0 \left(\frac{k_2 cos\theta_2}{\omega\mu_2} - \frac{A_{22}^{TE} k_2 cos\theta_1}{\omega\mu_2}\right)$$

$$A_{21}^{TE}\left[1 - \frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}\right] = A_{22}^{TE}\left[-1 + \frac{jk_2^2 l_0 cos\theta_0}{\omega\mu_2}\right] + \left[-1 - \frac{jk_2^2 l_0 cos\theta_0}{\omega\mu_2}\right]$$

(4,12)

We can obtain the expressions $A_{22}^{TE}$ and $A_{21}^{TE}$ by solving the equations 4,11 and 4,12.

In the equation 4,12; write the equivalent instead of $A_{21}^{TE}$,

$$(1 + A_{22}^{TE})\left[1 - \frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}\right] = A_{22}^{TE}\left[-1 + \frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}\right] + \left[-1 - \frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}\right]$$

$$1 - \frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1} + A_{22}^{TE} - A_{22}^{TE}\frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1} = A_{22}^{TE}\left[-1 + \frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}\right] + \left[-1 - \frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2}\right]$$

$$A_{22}^{TE}\left[2 - \frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2} - \frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}\right] = -2 - \frac{jk_2^2 l_0 cos\theta_2}{\omega\mu_2} + \frac{jk_1^2 l_0 cos\theta_0}{\omega\mu_1}$$

We can equalize the denominators at $\omega\mu_2\mu_1$

$$A_{22}^{TE}\left[2\frac{\omega\mu_2\mu_1}{\omega\mu_2\mu_1} - \frac{jk_2^2 l_0 cos\theta_2\mu_1}{\omega\mu_2\mu_1} - \frac{jk_1^2 l_0 cos\theta_0\mu_2}{\omega\mu_2\mu_1}\right] = -2\frac{\omega\mu_2\mu_1}{\omega\mu_2\mu_1} - \frac{jk_2^2 l_0 cos\theta_2\mu_1}{\omega\mu_2\mu_1} + \frac{jk_1^2 l_0 cos\theta_0\mu_2}{\omega\mu_2\mu_1}$$

$$A_{22}^{TE}\left[\frac{-2\omega\mu_2\mu_1 + jk_2^2 l_0 cos\theta_2\mu_1 + jk_1^2 l_0 cos\theta_0\mu_2}{\omega\mu_2\mu_1}\right] = \frac{2\omega\mu_2\mu_1 + jk_2^2 l_0 cos\theta_2\mu_1 - jk_1^2 l_0 cos\theta_0\mu_2}{\omega\mu_2\mu_1}$$

The reflection coefficient for E-polarized when incident wave is in medium 2 can be written as;

$$A_{22}^{TE} = \frac{jk_2^2 l_0 cos\theta_2 \mu_1 - [jk_1^2 l_0 cos\theta_0 \mu_2 - 2\omega\mu_2\mu_1]}{jk_2^2 l_0 cos\theta_2 \mu_1 + [jk_1^2 l_0 cos\theta_0 \mu_2 - 2\omega\mu_2\mu_1]}$$

To obtain $A_{21}^{TE}$ ;

$$A_{21}^{TE} = (1 + A_{22}^{TE})$$

$$A_{21}^{TE} = \left(1 + \frac{jk_2^2 l_0 cos\theta_2 \mu_1 - [jk_1^2 l_0 cos\theta_0 \mu_2 - 2\omega\mu_2\mu_1]}{jk_2^2 l_0 cos\theta_2 \mu_1 + [jk_1^2 l_0 cos\theta_0 \mu_2 - 2\omega\mu_2\mu_1]}\right)$$

$$A_{21}^{TE} = \left(\frac{jk_2^2 l_0 cos\theta_2 \mu_1 + [jk_1^2 l_0 cos\theta_0 \mu_2 - 2\omega\mu_2\mu_1]}{jk_2^2 l_0 cos\theta_2 \mu_1 + [jk_1^2 l_0 cos\theta_0 \mu_2 - 2\omega\mu_2\mu_1]} + \frac{jk_2^2 l_0 cos\theta_2 \mu_1 - [jk_1^2 l_0 cos\theta_0 \mu_2 - 2\omega\mu_2\mu_1]}{jk_2^2 l_0 cos\theta_2 \mu_1 + [jk_1^2 l_0 cos\theta_0 \mu_2 - 2\omega\mu_2\mu_1]}\right)$$

And, the transmission coefficient for E-polarized when incident wave is in medium 2 is

$$A_{21}^{TE} = \left(\frac{2jk_2^2 l_0 cos\theta_2 \mu_1}{jk_2^2 l_0 cos\theta_2 \mu_1 + [jk_1^2 l_0 cos\theta_0 \mu_2 - 2\omega\mu_2\mu_1]}\right)$$

# 5. SOLUTION WITH GEOMETRICAL OPTICS

The electric field and magnetic field expressions for each medium can be obtained by summation of the all fields in each medium. All fields are shown in Figure 5.1. Fields after the second scattering are neglected because their amplitude is too small as compared to the amplitude of the components of incident wave.

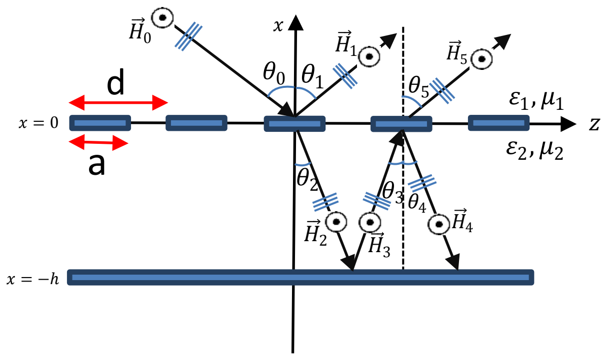


Figure 5.1. All waves in the problem for H-polarized

When magnetic field component of the incident wave is $\vec{H}_{0y}^{TM} = \hat{e}_y e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$ (for H-polarized), electric field components can be found by using Maxwell's Equations. The electric field of this wave will contain x and z components, it will not contain y component. Samely, (for E-polarized) the magnetic field of the incident wave will contain x and z components without y component if the electric field component of the incident wave is $\vec{E}_{0y}^{TE} = \hat{e}_y e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$. Thus, both electric and magnetic field expressions will have x, y and z components since E-polarized and H-polarized exist. Reflected and transmitted waves have the same form with the incident wave by multiplying with the reflection and transmission coefficient, respectively.

Also, $\theta_0 = \theta_1 = \theta_5$ and $\theta_2 = \theta_3 = \theta_4$ from Snell's Law.

All field components in medium 1 and medium 2 are listed below.

$$H_{0y}^{TM} = e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$$

$$E_{0x}^{TM} = \frac{k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$$

$$E_{0z}^{TM} = \frac{k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$$

$$H_{1y}^{TM} = A_{11}^{TM} e^{-jk_1(xcos\theta_0+zsin\theta_0)}$$

$$E_{1x}^{TM} = \frac{A_{11}^{TM} k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)}$$

$$E_{1z}^{TM} = \frac{-A_{11}^{TM} k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)}$$

$$H_{2y}^{TM} = A_{12}^{TM} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}$$

$$E_{2x}^{TM} = \frac{A_{12}^{TM} k_2 sin\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}$$

$$E_{2z}^{TM} = \frac{A_{12}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)}$$

$$H_{3y}^{TM} = -A_{12}^{TM} e^{-jk_2(xcos\theta_2+zsin\theta_2)} e^{-jhk_2\cos\theta_2}$$

$$E_{3x}^{TM} = \frac{-A_{12}^{TM} k_2 sin\theta_2}{\omega\varepsilon_2} e^{-jk_2(xcos\theta_2+zsin\theta_2)} e^{-jhk_2\cos\theta_2}$$

$$E_{3z}^{TM} = \frac{A_{12}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(xcos\theta_2+zsin\theta_2)} e^{-jhk_2 \cos\theta_2}$$

$$H_{4y}^{TM} = -A_{12}^{TM} A_{22}^{TM} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} e^{-j2hk_2 \cos\theta_2}$$

$$E_{4x}^{TM} = \frac{-A_{12}^{TM} A_{22}^{TM} k_2 sin\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} e^{-j2hk_2 \cos\theta_2}$$

$$E_{4z}^{TM} = \frac{-A_{12}^{TM} A_{22}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} e^{-j2hk_2 \cos\theta_2}$$

$$H_{5y}^{TM} = -A_{12}^{TM} A_{21}^{TM} e^{-jk_1(xcos\theta_0+zsin\theta_0)} e^{-j2hk_2 \cos\theta_2}$$

$$E_{5x}^{TM} = \frac{-A_{12}^{TM} A_{21}^{TM} k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} e^{-j2hk_2 \cos\theta_2}$$

$$E_{5z}^{TM} = \frac{A_{12}^{TM} A_{21}^{TM} k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} e^{-j2hk_2 \cos\theta_2}$$

$$E_{0y}^{TE} = e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$$

$$H_{0x}^{TE} = \frac{-k_1 sin\theta_0}{\omega\mu_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$$

$$H_{0z}^{TE} = \frac{-k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)}$$

$$E_{1y}^{TE} = A_{11}^{TE} e^{-jk_1(xcos\theta_0+zsin\theta_0)}$$

$$H_{1x}^{TE} = \frac{-A_{11}^{TE} k_1 sin\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)}$$

$$H_{1z}^{TE} = \frac{A_{11}^{TE} k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0 + zsin\theta_0)}$$

$$E_{2y}^{TE} = A_{12}^{TE} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}$$

$$H_{2x}^{TE} = \frac{-A_{12}^{TE} k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}$$

$$H_{2z}^{TE} = \frac{-A_{12}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)}$$

$$E_{3y}^{TE} = -A_{12}^{TE} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} e^{-jhk_2 \cos\theta_2}$$

$$H_{3x}^{TE} = \frac{A_{12}^{TE} k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} e^{-jhk_2 \cos\theta_2}$$

$$H_{3z}^{TE} = \frac{-A_{12}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} e^{-jhk_2 \cos\theta_2}$$

$$E_{4y}^{TE} = -A_{12}^{TE} A_{22}^{TE} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} e^{-j2hk_2 \cos\theta_2}$$

$$H_{4x}^{TE} = \frac{A_{12}^{TE} A_{22}^{TE} k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} e^{-j2hk_2 \cos\theta_2}$$

$$H_{4z}^{TE} = \frac{A_{12}^{TE} A_{22}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} e^{-j2hk_2 \cos\theta_2}$$

$$E_{5y}^{TE} = -A_{12}^{TE} A_{21}^{TE} e^{-jk_1(xcos\theta_0 + zsin\theta_0)} e^{-j2hk_2 \cos\theta_2}$$

$$H_{5x}^{TE} = \frac{A_{12}^{TE} A_{21}^{TE} k_1 sin\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0 + zsin\theta_0)} e^{-j2hk_2 \cos\theta_2}$$

$$H_{5z}^{TE} = \frac{-A_{12}^{TE}A_{21}^{TE}k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)}e^{-j2hk_2\cos\theta_2}$$

The electric field in medium 1 is;

$$\vec{E}^1 = \hat{e}_x E_x + \hat{e}_y E_y + \hat{e}_z E_z$$

where

$$E_x = E_{0x}^{TM} + E_{1x}^{TM} + E_{5x}^{TM}$$

$$E_y = E_{0y}^{TE} + E_{1y}^{TE} + E_{5y}^{TE}$$

$$E_z = E_{0z}^{TM} + E_{1z}^{TM} + E_{5z}^{TM}$$

x- component of the electric field in medium 1 is

$$E_x = \frac{k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)} + \frac{A_{11}^{TM}k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} - \frac{A_{12}^{TM}A_{21}^{TM}k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)}e^{-j2hk_2\cos\theta_2}$$

y- component of the electric field in medium 1 is

$$E_y = e^{-jk_1(-xcos\theta_0+zsin\theta_0)} + A_{11}^{TE} e^{-jk_1(xcos\theta_0+zsin\theta_0)} - A_{12}^{TE}A_{21}^{TE} e^{-jk_1(xcos\theta_0+zsin\theta_0)}e^{-j2hk_2\cos\theta_2}$$

z- component of the electric field in medium 1 is

$$E_z = \frac{k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)} - \frac{A_{11}^{TM} k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} +$$
$$\frac{A_{12}^{TM} A_{21}^{TM} k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} e^{-j2hk_2 \cos\theta_2}$$

The magnetic field in medium 1 is;

$$\vec{H}^1 = \hat{e}_x H_x + \hat{e}_y H_y + \hat{e}_z H_z$$

where

$$H_x = H_{0x}^{TE} + H_{1x}^{TE} + H_{5x}^{TE}$$

$$H_y = H_{0y}^{TM} + H_{1y}^{TM} + H_{5y}^{TM}$$

$$H_z = H_{0z}^{TE} + H_{1z}^{TE} + H_{5z}^{TE}$$

x- component of the magnetic field in medium 1 is

$$H_x = \frac{-k_1 sin\theta_0}{\omega\mu_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)} - \frac{A_{11}^{TE} k_1 sin\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} +$$
$$\frac{A_{12}^{TE} A_{21}^{TE} k_1 sin\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} e^{-j2hk_2 \cos\theta_2}$$

y- component of the magnetic field in medium 1 is

$$H_y =$$
$$e^{-jk_1(-xcos\theta_0+zsin\theta_0)} + A_{11}^{TM} e^{-jk_1(xcos\theta_0+zsin\theta_0)} -$$
$$A_{12}^{TM} A_{21}^{TM} e^{-jk_1(xcos\theta_0+zsin\theta_0)} e^{-j2hk_2 \cos\theta_2}$$

z- component of the magnetic field in medium 1 is

$$H_z = \frac{-k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)} + \frac{A_{11}^{TE} k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} -$$
$$\frac{A_{12}^{TE} A_{21}^{TE} k_1 cos\theta_0}{\omega\mu_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} e^{-j2hk_2 \cos\theta_2}$$

The electric field in medium 2 is;

$$\vec{E}^2 = \hat{e}_x E_x + \hat{e}_y E_y + \hat{e}_z E_z$$

where

$$E_x = E_{2x}^{TM} + E_{3x}^{TM} + E_{4x}^{TM}$$

$$E_y = E_{2y}^{TE} + E_{3y}^{TE} + E_{4y}^{TE}$$

$$E_z = E_{2z}^{TM} + E_{3z}^{TM} + E_{4z}^{TM}$$

x- component of the electric field in medium 2 is

$$E_x = \frac{A_{12}^{TM} k_2 sin\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} - \frac{A_{12}^{TM} k_2 sin\theta_2}{\omega\varepsilon_2} e^{-jk_2(xcos\theta_2+zsin\theta_2)} e^{-jhk_2 \cos\theta_2} - \frac{A_{12}^{TM} A_{22}^{TM} k_2 sin\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} e^{-j2hk_2 \cos\theta_2}$$

y- component of the electric field in medium 2 is

$$E_y = A_{12}^{TE} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} - A_{12}^{TE} e^{-jk_2(xcos\theta_2+zsin\theta_2)} e^{-jhk_2 \cos\theta_2}$$
$$-A_{12}^{TE} A_{22}^{TE} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} e^{-j2hk_2 \cos\theta_2}$$

z- component of the electric field in medium 2 is

$$E_z = \frac{A_{12}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} + \frac{A_{12}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} e^{-jhk_2 \cos\theta_2} - \frac{A_{12}^{TM} A_{22}^{TM} k_2 cos\theta_2}{\omega\varepsilon_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} e^{-j2hk_2 \cos\theta_2}$$

The magnetic field in medium 2 is;

$$\vec{H}^2 = \hat{e}_x H_x + \hat{e}_y H_y + \hat{e}_z H_z$$

where

$$H_x = H_{2x}^{TE} + H_{3x}^{TE} + H_{4x}^{TE}$$

$$H_y = H_{2y}^{TM} + H_{3y}^{TM} + H_{4y}^{TM}$$

$$H_z = H_{2z}^{TE} + H_{3z}^{TE} + H_{4z}^{TE}$$

x- component of the magnetic field in medium 2 is

$$H_x = \frac{-A_{12}^{TE} k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} + \frac{A_{12}^{TE} k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} e^{-jhk_2 \cos\theta_2} + \frac{A_{12}^{TE} A_{22}^{TE} k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} e^{-j2hk_2 \cos\theta_2}$$

y- component of the magnetic field in medium 2 is

$$H_y = A_{12}^{TM} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} - A_{12}^{TM} e^{-jk_2(xcos\theta_2+zsin\theta_2)} e^{-jhk_2\cos\theta_2} - A_{12}^{TM} A_{22}^{TM} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} e^{-j2hk_2\cos\theta_2}$$

z- component of the magnetic field in medium 2 is

$$H_z = \frac{-A_{12}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} - \frac{A_{12}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(xcos\theta_2+zsin\theta_2)} e^{-jhk_2\cos\theta_2} + \frac{A_{12}^{TE} A_{22}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2+zsin\theta_2)} e^{-j2hk_2\cos\theta_2}$$

Thus, the electric and magnetic field's expressions in medium 1 and medium 2 can be written as,

- **Electric field in medium 1**

$$\vec{E}^1 = \hat{e}_x \left\{ \frac{k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)} + \frac{A_{11}^{TM} k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} - \frac{A_{12}^{TM} A_{21}^{TM} k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} e^{-j2hk_2\cos\theta_2} \right\} + \hat{e}_y \left\{ e^{-jk_1(-xcos\theta_0+zsin\theta_0)} + A_{11}^{TE} e^{-jk_1(xcos\theta_0+zsin\theta_0)} - A_{12}^{TE} A_{21}^{TE} e^{-jk_1(xcos\theta_0+zsin\theta_0)} e^{-j2hk_2\cos\theta_2} \right\} + \hat{e}_z \left\{ \frac{k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(-xcos\theta_0+zsin\theta_0)} - \frac{A_{11}^{TM} k_1 cos\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} + \frac{A_{12}^{TM} A_{21}^{TM} k_1 sin\theta_0}{\omega\varepsilon_1} e^{-jk_1(xcos\theta_0+zsin\theta_0)} e^{-j2hk_2\cos\theta_2} \right\}$$

- **Magnetic field in medium 1**

$$\vec{H}^1 = \hat{e}_x\left\{\frac{-k_1 sin\theta_0}{\omega\mu_1}e^{-jk_1(-xcos\theta_0+zsin\theta_0)} - \frac{A_{11}^{TE}k_1 sin\theta_0}{\omega\mu_1}e^{-jk_1(xcos\theta_0+zsin\theta_0)} + \frac{A_{12}^{TE}A_{21}^{TE}k_1 sin\theta_0}{\omega\mu_1}e^{-jk_1(xcos\theta_0+zsin\theta_0)}e^{-j2hk_2\cos\theta_2}\right\} + \hat{e}_y\left\{e^{-jk_1(-xcos\theta_0+zsin\theta_0)} + A_{11}^{TM}e^{-jk_1(xcos\theta_0+zsin\theta_0)} - A_{12}^{TM}A_{21}^{TM}e^{-jk_1(xcos\theta_0+zsin\theta_0)}e^{-j2hk_2\cos\theta_2}\right\} + \hat{e}_z\left\{\frac{-k_1 cos\theta_0}{\omega\mu_1}e^{-jk_1(-xcos\theta_0+zsin\theta_0)} + \frac{A_{11}^{TE}k_1 cos\theta_0}{\omega\mu_1}e^{-jk_1(xcos\theta_0+zsin\theta_0)} - \frac{A_{12}^{TE}A_{21}^{TE}k_1 cos\theta_0}{\omega\mu_1}e^{-jk_1(xcos\theta_0+zsin\theta_0)}e^{-j2hk_2\cos\theta_2}\right\}$$

- **Electric field in medium 2**

$$\vec{E}^2 = \hat{e}_x\left\{\frac{A_{12}^{TM}k_2 sin\theta_2}{\omega\varepsilon_2}e^{-jk_2(-xcos\theta_2+zsin\theta_2)} - \frac{A_{12}^{TM}k_2 sin\theta_2}{\omega\varepsilon_2}e^{-jk_2(xcos\theta_2+zsin\theta_2)}e^{-jhk_2\cos\theta_2} - \frac{A_{12}^{TM}A_{22}^{TM}k_2 sin\theta_2}{\omega\varepsilon_2}e^{-jk_2(-xcos\theta_2+zsin\theta_2)}e^{-j2hk_2\cos\theta_2}\right\} + \hat{e}_y\left\{\begin{matrix}A_{12}^{TE}e^{-jk_2(-xcos\theta_2+zsin\theta_2)} - A_{12}^{TE}e^{-jk_2(xcos\theta_2+zsin\theta_2)}e^{-jhk_2\cos\theta_2} \\ -A_{12}^{TE}A_{21}^{TE}e^{-jk_2(-xcos\theta_2+zsin\theta_2)}e^{-j2hk_2\cos\theta_2}\end{matrix}\right\} + \hat{e}_z\left\{\frac{A_{12}^{TM}k_2 cos\theta_2}{\omega\varepsilon_2}e^{-jk_2(-xcos\theta_2+zsin\theta_2)} + \frac{A_{12}^{TM}k_2 cos\theta_2}{\omega\varepsilon_2}e^{-jk_2(xcos\theta_2+zsin\theta_2)}e^{-jhk_2\cos\theta_2} - \frac{A_{2TM}A_{3TM}k_2 cos\theta_2}{\omega\varepsilon_2}e^{-jk_2(-xcos\theta_2+zsin\theta_2)}e^{-j2hk_2\cos\theta_2}\right\}$$

- **Magnetic field in medium 2**

$$\vec{H}^2 =$$

$$\hat{e}_x \left\{ \frac{-A_{12}^{TE} k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} + \right.$$

$$\frac{A_{12}^{TE} k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} e^{-jhk_2 \cos\theta_2} +$$

$$\left. \frac{A_{12}^{TE} A_{21}^{TE} k_2 sin\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} e^{-j2hk_2 \cos\theta_2} \right\} +$$

$$\hat{e}_y \left\{ A_{12}^{TM} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} - A_{12}^{TM} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} e^{-jhk_2 \cos\theta_2} - \right.$$

$$\left. A_{12}^{TM} A_{21}^{TM} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} e^{-j2hk_2 \cos\theta_2} \right\} +$$

$$\hat{e}_z \left\{ \frac{-A_{12}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} - \right.$$

$$\frac{A_{12}^{TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(xcos\theta_2 + zsin\theta_2)} e^{-jhk_2 \cos\theta_2} +$$

$$\left. \frac{A_{2TE} A_{3TE} k_2 cos\theta_2}{\omega\mu_2} e^{-jk_2(-xcos\theta_2 + zsin\theta_2)} e^{-j2hk_2 \cos\theta_2} \right\}$$

# 6. COMPARISON WITH THE OTHER KNOWN RESULTS

Jacobsen (1970), studied about the electromagnetic waves guided by a periodically strip loaded dielectric slab by using Floquet modes. He worked numerically and experimentally, but analytically he just gave the electric and magnetic field formulation, didn't show obtaining the expressions of electric and magnetic fields. In this part of the work, these expressions are also obtained from,

$$\vec{E} = \vec{\nabla} \times \vec{\nabla} \times \vec{\Pi} + i\omega\mu(\vec{\nabla} \times \vec{\Pi}^*)$$

$$\vec{H} = -i\omega\varepsilon(\vec{\nabla} \times \vec{\Pi}) + \vec{\nabla} \times \vec{\nabla} \times \vec{\Pi}^*$$

The geometry of Jabobsen's problem is shown in Figure 6.1. He defined the medium 1 as free space and medium 2 as slab.

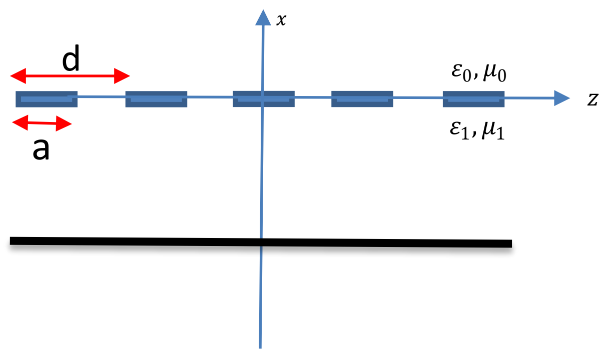


Figure 6.1. Geometry of the problem for Jacobsen's work

$$\vec{E} = \vec{\nabla} \times \vec{\nabla} \times \vec{\Pi} + i\omega\mu(\vec{\nabla} \times \vec{\Pi}^*)$$

$$\vec{H} = -i\omega\varepsilon(\vec{\nabla} \times \vec{\Pi}) + \vec{\nabla} \times \vec{\nabla} \times \vec{\Pi}^*$$

where

$$\vec{\Pi} = \hat{e}_y\Pi_y \quad \text{and} \quad \vec{\Pi}^* = \hat{e}_y\Pi_y^*$$

where $\Pi$ and $\Pi^*$ are electric and magnetic Hertzian potentials, respectively.

$$\Pi_y^{(0)} = e^{i\zeta_0 z} \sum A_n^{(0)} \, e^{\left[in\left(\frac{2\pi}{d}\right)z\right]} e^{i\kappa_n^{(0)}x} \quad , x > 0 \text{ (in medium 0)}$$

$$\Pi_y^{(1)} = e^{i\zeta_0 z} \sum \left\{a_n^{(1)} e^{i\kappa_n^{(1)}x} + A_n^{(1)} e^{-i\kappa_n^{(1)}x}\right\} e^{\left[in\left(\frac{2\pi}{d}\right)z\right]} \quad , \quad -h < x < 0 \quad \text{(in medium 1)}$$

$$\Pi_y^{*(0)} = e^{i\zeta_0 z} \sum B_n^{(0)} \, e^{\left[in\left(\frac{2\pi}{d}\right)z\right]} e^{i\kappa_n^{(0)}x} \quad , x > 0 \text{ (in medium 0)}$$

$$\Pi_y^{*(1)} = e^{i\zeta_0 z} \sum \left\{b_n^{(1)} e^{i\kappa_n^{(1)}x} + B_n^{(1)} e^{-i\kappa_n^{(1)}x}\right\} e^{\left[in\left(\frac{2\pi}{d}\right)z\right]} \quad , \quad -h < x < 0 \quad \text{(in medium 1)}$$

$${\kappa_n^{(0)}}^2 = k_0^2 - \zeta_n^2$$

$${\kappa_n^{(1)}}^2 = k_1^2 - \zeta_n^2$$

To determine the field expressions, we should find the results below:

- $\vec{\nabla} \times \vec{\nabla} \times \vec{\Pi}$
- $\vec{\nabla} \times \vec{\nabla} \times \vec{\Pi}^*$
- $\vec{\nabla} \times \vec{\Pi}$
- $\vec{\nabla} \times \vec{\Pi}^*$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{\Pi} = \left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y \Pi_y\right)$$

$$= \left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\left(\hat{e}_x \times \hat{e}_y\right)\frac{\partial}{\partial x}\Pi_y + \left(\hat{e}_y \times \hat{e}_y\right)\frac{\partial}{\partial y}\Pi_y + \right.$$

$$\left.\left(\hat{e}_z \times \hat{e}_y\right)\frac{\partial}{\partial z}\Pi_y\right)$$

$$= \left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_z\frac{\partial}{\partial x}\Pi_y - \hat{e}_x\frac{\partial}{\partial z}\Pi_y\right)$$

$$= (\hat{e}_x \times \hat{e}_z)\frac{\partial^2}{\partial x^2}\Pi_y - (\hat{e}_x \times \hat{e}_x)\frac{\partial^2}{\partial x \partial z}\Pi_y + \left(\hat{e}_y \times \hat{e}_z\right)\frac{\partial^2}{\partial x \partial y}\Pi_y$$
$$- \left(\hat{e}_y \times \hat{e}_x\right)\frac{\partial^2}{\partial y \partial z}\Pi_y + (\hat{e}_z \times \hat{e}_z)\frac{\partial^2}{\partial x \partial z}\Pi_y$$
$$- (\hat{e}_z \times \hat{e}_x)\frac{\partial^2}{\partial z^2}\Pi_y$$

Since,

$$\frac{\partial}{\partial y}\Pi_y = 0$$

$$\frac{\partial}{\partial y}\Pi_y^* = 0$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{\Pi} = -\hat{e}_y\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\Pi_y$$

$$\vec{\nabla} \times \vec{\Pi} = \left(\frac{\partial}{\partial x}\hat{e}_x + \frac{\partial}{\partial y}\hat{e}_y + \frac{\partial}{\partial z}\hat{e}_z\right) \times \left(\hat{e}_y\Pi_y\right)$$

$$\vec{\nabla} \times \vec{\Pi} = \left(\left(\hat{e}_x \times \hat{e}_y\right)\frac{\partial}{\partial x}\Pi_y + \left(\hat{e}_y \times \hat{e}_y\right)\frac{\partial}{\partial y}\Pi_y + \left(\hat{e}_z \times \hat{e}_y\right)\frac{\partial}{\partial z}\Pi_y\right)$$

$$\vec{\nabla}\times\vec{\Pi} = -\hat{e}_x \frac{\partial}{\partial z}\Pi_y + \hat{e}_z \frac{\partial}{\partial x}\Pi_y$$

$$\vec{\nabla}\times\vec{\Pi}^* = -\hat{e}_x \frac{\partial}{\partial z}\Pi_y^* + \hat{e}_z \frac{\partial}{\partial x}\Pi_y^*$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{\Pi} = -\hat{e}_y \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\Pi_y$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{\Pi}^* = -\hat{e}_y \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\Pi_y^*$$

- **In Free Space (medium 0)**

In medium 0, the necessary terms in order to obtain the electric and magnetic fields expressions are acquired below one by one.

$$\frac{\partial^2}{\partial x^2}\Pi_y = \frac{\partial^2}{\partial x^2}\left\{e^{i\zeta_0 z}\sum A_n^{(0)}\, e^{\left[in\left(\frac{2\pi}{d}\right)z\right]} e^{i\kappa_n^{(0)}x}\right\}$$
$$= -\kappa_n^{(0)^2}\sum A_n^{(0)}\, e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]} e^{i\kappa_n^{(0)}x}$$

$$\frac{\partial^2}{\partial z^2}\Pi_y = \frac{\partial^2}{\partial z^2}\left\{e^{i\zeta_0 z}\sum A_n^{(0)}\, e^{\left[in\left(\frac{2\pi}{d}\right)z\right]} e^{i\kappa_n^{(0)}x}\right\}$$
$$= -\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)^2 \sum A_n^{(0)}\, e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]} e^{i\kappa_n^{(0)}x}$$

$$\frac{\partial^2}{\partial x^2}\Pi_y^* = \frac{\partial^2}{\partial x^2}\left\{e^{i\zeta_0 z}\sum B_n^{(0)}\, e^{\left[in\left(\frac{2\pi}{d}\right)z\right]} e^{i\kappa_n^{(0)}x}\right\}$$
$$= -\kappa_n^{(0)^2}\sum B_n^{(0)}\, e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]} e^{i\kappa_n^{(0)}x}$$

$$\frac{\partial^2}{\partial z^2}\Pi_y^* = \frac{\partial^2}{\partial z^2}\left\{e^{i\zeta_0 z}\sum B_n^{(0)}\, e^{\left[in\left(\frac{2\pi}{d}\right)z\right]}e^{i\kappa_n^{(0)}x}\right\}$$

$$= -\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)^2 \sum B_n^{(0)}\, e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}e^{i\kappa_n^{(0)}x}$$

$$\frac{\partial}{\partial x}\Pi_y = \frac{\partial}{\partial x}\left\{e^{i\zeta_0 z}\sum A_n^{(0)}\, e^{\left[in\left(\frac{2\pi}{d}\right)z\right]}e^{i\kappa_n^{(0)}x}\right\}$$

$$= i\kappa_n^{(0)}\sum A_n^{(0)}\, e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}e^{i\kappa_n^{(0)}x}$$

$$\frac{\partial}{\partial z}\Pi_y = \frac{\partial}{\partial z}\left\{e^{i\zeta_0 z}\sum A_n^{(0)}\, e^{\left[in\left(\frac{2\pi}{d}\right)z\right]}e^{i\kappa_n^{(0)}x}\right\}$$

$$= i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)\sum A_n^{(0)}\, e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}e^{i\kappa_n^{(0)}x}$$

$$\frac{\partial}{\partial x}\Pi_y^* = \frac{\partial}{\partial x}\left\{e^{i\zeta_0 z}\sum B_n^{(0)}\, e^{\left[in\left(\frac{2\pi}{d}\right)z\right]}e^{i\kappa_n^{(0)}x}\right\}$$

$$= i\kappa_n^{(0)}\sum B_n^{(0)}\, e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}e^{i\kappa_n^{(0)}x}$$

$$\frac{\partial}{\partial z}\Pi_y^* = \frac{\partial}{\partial z}\left\{e^{i\zeta_0 z}\sum B_n^{(0)}\, e^{\left[in\left(\frac{2\pi}{d}\right)z\right]}e^{i\kappa_n^{(0)}x}\right\}$$

$$= i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)\sum B_n^{(0)}\, e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}e^{i\kappa_n^{(0)}x}$$

For the electric field in free-space

$$\vec{E}_{fs} = \vec{\nabla}\times\vec{\nabla}\times\vec{\Pi} + i\omega\mu\left(\vec{\nabla}\times\vec{\Pi}^*\right)$$

$$= -\hat{e}_y\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\Pi_y + i\omega\mu\left(-\hat{e}_x\frac{\partial}{\partial z}\Pi_y^* + \hat{e}_z\frac{\partial}{\partial x}\Pi_y^*\right)$$

$$\vec{E}_{fs} = \hat{e}_x \left[ -i\omega\mu i \left( \zeta_0 + n\left(\frac{2\pi}{d}\right)\right) \sum B_n^{(0)}\, e^{\left[i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)z\right]} e^{i\kappa_n^{(0)}x} \right]$$

$$+ \hat{e}_y \left[ \kappa_n^{(0)^2} \sum A_n^{(0)}\, e^{\left[i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)z\right]} e^{i\kappa_n^{(0)}x} \right.$$

$$\left. + \left( \zeta_0 + n\left(\frac{2\pi}{d}\right)\right)^2 \sum A_n^{(0)}\, e^{\left[i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)z\right]} e^{i\kappa_n^{(0)}x} \right]$$

$$+ \hat{e}_z \left[ i\omega\mu i \kappa_n^{(0)} \sum B_n^{(0)}\, e^{\left[i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)z\right]} e^{i\kappa_n^{(0)}x} \right]$$

Hence, electric field in free space can be written as

$$\vec{E}_{fs} = \sum A_n^{(0)}\, e^{\left[i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)z\right]} e^{i\kappa_n^{(0)}x} \left\{ \kappa_n^{(0)^2} + \left( \zeta_0 + n\left(\frac{2\pi}{d}\right)\right)^2 \right\} \hat{e}_y$$

$$+ \sum B_n^{(0)}\, e^{\left[i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)z\right]} e^{i\kappa_n^{(0)}x} \left\{ \left[ \omega\mu \left( \zeta_0 + n\left(\frac{2\pi}{d}\right)\right)\right] \hat{e}_x \right.$$

$$\left. - \omega\mu\kappa_n^{(0)} \hat{e}_z \right\}$$

For the magnetic field in free-space

$$\vec{H}_{fs} = -i\omega\varepsilon\left(\vec{\nabla} \times \vec{\Pi}\right) + \vec{\nabla} \times \vec{\nabla} \times \vec{\Pi}^* =$$

$$= -i\omega\varepsilon \left( -\hat{e}_x \frac{\partial}{\partial z} \Pi_y + \hat{e}_z \frac{\partial}{\partial x} \Pi_y \right) + -\hat{e}_y \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \Pi_y^*$$

$$\vec{H}_{fs} = \hat{e}_x \left[ i\omega\varepsilon i \left( \zeta_0 + n\left(\frac{2\pi}{d}\right)\right) \sum A_n^{(0)} \, e^{\left[ i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right) z \right]} e^{i\kappa_n^{(0)} x} \right]$$

$$+ \hat{e}_y \left[ {\kappa_n^{(0)}}^2 \sum B_n^{(0)} \, e^{\left[ i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right) z \right]} e^{i\kappa_n^{(0)} x} \right.$$

$$\left. + \left( \zeta_0 + n\left(\frac{2\pi}{d}\right)\right)^2 \sum B_n^{(0)} \, e^{\left[ i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right) z \right]} e^{i\kappa_n^{(0)} x} \right]$$

$$+ \hat{e}_z \left[ -i\omega\varepsilon i\kappa_n^{(0)} \sum A_n^{(0)} \, e^{\left[ i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right) z \right]} e^{i\kappa_n^{(0)} x} \right]$$

Hence, magnetic field in free space can be written as

$$\vec{H}_{fs} = \sum B_n^{(0)} \, e^{\left[ i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right) z \right]} e^{i\kappa_n^{(0)} x} \left\{ {\kappa_n^{(0)}}^2 + \left( \zeta_0 + n\left(\frac{2\pi}{d}\right)\right)^2 \right\} \hat{e}_y$$

$$+ \sum A_n^{(0)} \, e^{\left[ i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right) z \right]} e^{i\kappa_n^{(0)} x} \left\{ \left[ -\omega\varepsilon \left( \zeta_0 + n\left(\frac{2\pi}{d}\right)\right)\right] \hat{e}_x \right.$$

$$\left. + \omega\varepsilon\kappa_n^{(0)} \hat{e}_z \right\}$$

- **In Slab (medium 1)**

In medium 1, the necessary terms in order to obtain the electric and magnetic fields expressions are acquired below one by one.

$$\frac{\partial^2}{\partial x^2}\Pi_y = \frac{\partial^2}{\partial x^2}\left\{ \sum \left\{ \left[ a_n^{(1)} e^{i\kappa_n^{(1)} x} + A_n^{(1)} e^{-i\kappa_n^{(1)} x} \right] e^{\left[ i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right) z \right]} \right\} \right\}$$

$$= -{\kappa_n^{(1)}}^2 \sum \left\{ \left[ a_n^{(1)} e^{i\kappa_n^{(1)} x} + A_n^{(1)} e^{-i\kappa_n^{(1)} x} \right] e^{\left[ i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right) z \right]} \right\}$$

$$\frac{\partial^2}{\partial z^2}\Pi_y = \frac{\partial^2}{\partial z^2}\left\{\Sigma\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x} + A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\} =$$

$$-\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)^2 \Sigma\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x} + A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}$$

$$\frac{\partial^2}{\partial x^2}\Pi_y^* = \frac{\partial^2}{\partial x^2}\left\{\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x} + B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}$$

$$= -\kappa_n^{(1)^2}\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x} + B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}$$

$$\frac{\partial^2}{\partial z^2}\Pi_y^* = \frac{\partial^2}{\partial z^2}\left\{\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x} + B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}$$

$$= -\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)^2\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x} + B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}$$

$$\frac{\partial}{\partial x}\Pi_y = \frac{\partial}{\partial x}\left\{\sum\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x} + A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}$$

$$= i\kappa_n^{(1)}\sum\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x} - A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}$$

$$\frac{\partial}{\partial z}\Pi_y = \frac{\partial}{\partial z}\left\{\sum\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x} + A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}$$

$$= i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)\sum\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x} + A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}$$

$$\frac{\partial}{\partial x}\Pi_y^* = \frac{\partial}{\partial x}\left\{\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x} + B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}$$

$$= i\kappa_n^{(1)}\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x} - B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}$$

$$\frac{\partial}{\partial z}\Pi_y^* = \frac{\partial}{\partial z}\left\{\sum\left\{b_n^{(1)}e^{i\kappa_n^{(1)}x} + B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right\}e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}$$

$$= i\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x} + B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}$$

For the electric field in slab

$$\vec{E}_s = \vec{\nabla}\times\vec{\nabla}\times\vec{\Pi} + i\omega\mu\left(\vec{\nabla}\times\vec{\Pi}^*\right)$$

$$= -\hat{e}_y\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\Pi_y + i\omega\mu\left(-\hat{e}_x\frac{\partial}{\partial z}\Pi_y^* + \hat{e}_z\frac{\partial}{\partial x}\Pi_y^*\right)$$

$$\vec{E}_s = -\hat{e}_y\left\{\left(-\kappa_n^{(1)^2}\sum\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x} + A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\} - \right.\right.$$
$$\left.\left.\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)^2\sum\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x} + A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right)\right\} -$$
$$\hat{e}_x\left\{i\omega\mu i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x} + B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\} +$$
$$\hat{e}_z\left\{i\omega\mu i\kappa_n^{(1)}\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x} - B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}$$

Hence, electric field in slab can be written as

$$\vec{E}_s =$$
$$\hat{e}_x\left\{\omega\mu\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x}+B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}+$$
$$\hat{e}_y\left\{\sum\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x}+A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\left[{\kappa_n^{(1)}}^2+\left(\zeta_0+\right.\right.\right.$$
$$\left.\left.\left.n\left(\frac{2\pi}{d}\right)\right)^2\right]\right\}-\hat{e}_z\left\{\omega\mu\kappa_n^{(1)}\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x}-B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}$$

For the magnetic field in slab

$$\vec{H}_s=-i\omega\varepsilon\left(\vec{\nabla}\times\vec{\Pi}\right)+\vec{\nabla}\times\vec{\nabla}\times\vec{\Pi}^*=$$
$$=-i\omega\varepsilon\left(-\hat{e}_x\frac{\partial}{\partial z}\Pi_y+\hat{e}_z\frac{\partial}{\partial x}\Pi_y\right)+-\hat{e}_y\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\Pi_y^*$$

$$\vec{H}_s=\hat{e}_x\left\{i\omega\varepsilon i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)\sum\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x}+A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}$$
$$-\hat{e}_y\left\{-{\kappa_n^{(1)}}^2\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x}+B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right.$$
$$\left.-\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)^2\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x}+B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}$$
$$-\hat{e}_z\left\{i\omega\varepsilon i\kappa_n^{(1)}\sum\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x}-A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}$$

Hence, magnetic field in slab can be written as

$$\vec{H}_s =$$
$$\hat{e}_x\left\{-\omega\varepsilon\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)\sum\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x}+A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}+$$
$$\hat{e}_y\left\{\left[{\kappa_n^{(1)}}^2+\right.\right.$$

$$\left(\zeta_0 + n\left(\frac{2\pi}{d}\right)\right)^2\Bigg]\sum\left\{\left[b_n^{(1)}e^{i\kappa_n^{(1)}x} + B_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\Bigg\} +$$

$$\hat{e}_z\left\{\omega\varepsilon\kappa_n^{(1)}\sum\left\{\left[a_n^{(1)}e^{i\kappa_n^{(1)}x} - A_n^{(1)}e^{-i\kappa_n^{(1)}x}\right]e^{\left[i\left(\zeta_0+n\left(\frac{2\pi}{d}\right)\right)z\right]}\right\}\right\}$$

To obtain a graph of electric field and magnetic field expressions vs frequency, the first 3 modes are used because the amplitudes of the fields for third and bigger modes are too small as compared to the first 3 modes i.e. n=0, n=1 and n=2.

In Figure 6.2, it is shown that the electric fields in free space for obtaining with Floquet modes with the constants A=1 and B=0,001 and medium 1 obtained by ray tracing solution with the 90 incidence angle are matched. In order to realize the oscillation on electric field when frequency increases, the expanded version of the graph is shown in Figure 6.3.

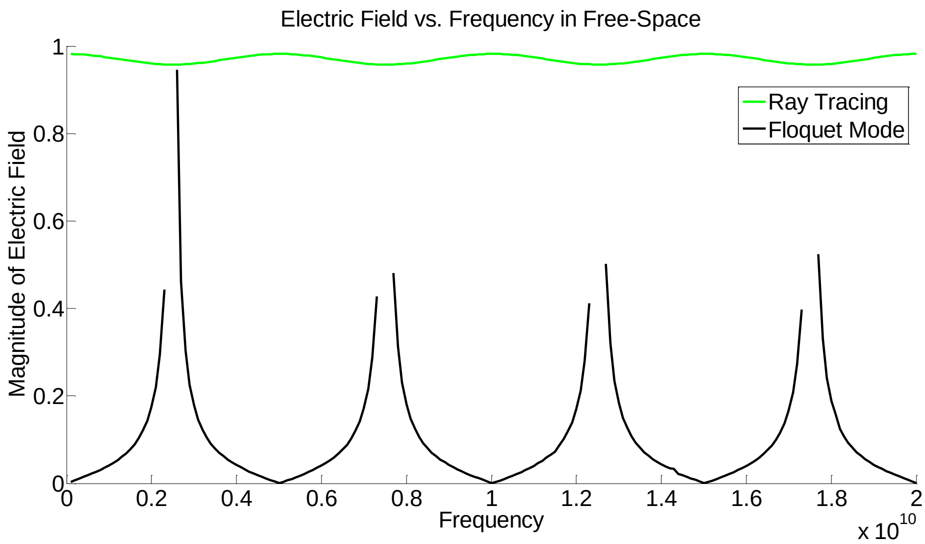


Figure 6.2. Electric field in free space (medium 1)

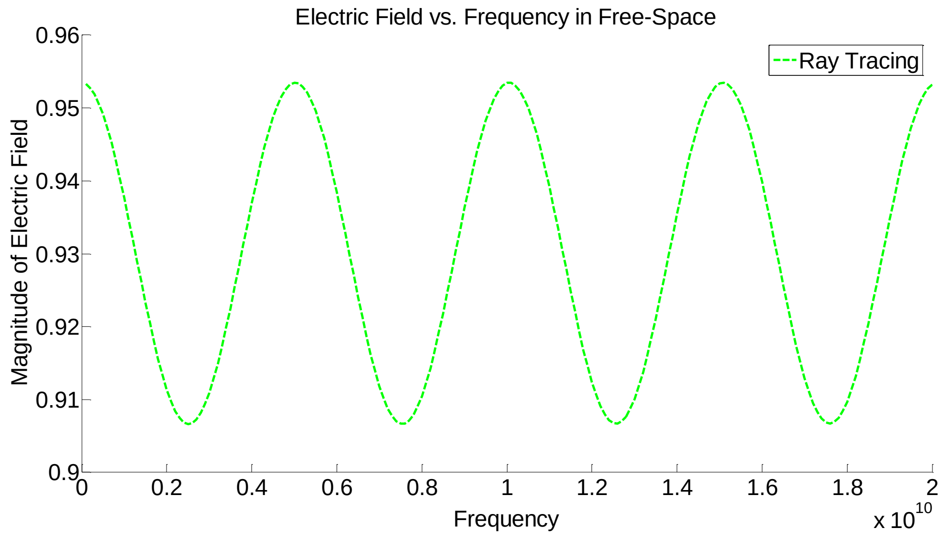


Figure 6.3. The expanded version of the electric field in medium 1 for ray tracing solution

The same work is done for magnetic fields and shown in Figure 6.4 and Figure 6.5.

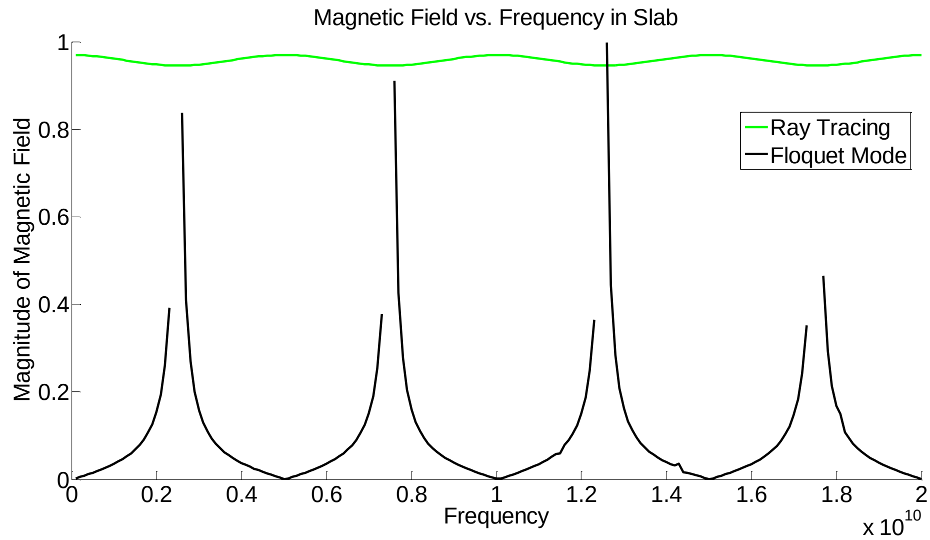


Figure 6.4. Magnetic field in free space (medium 1)

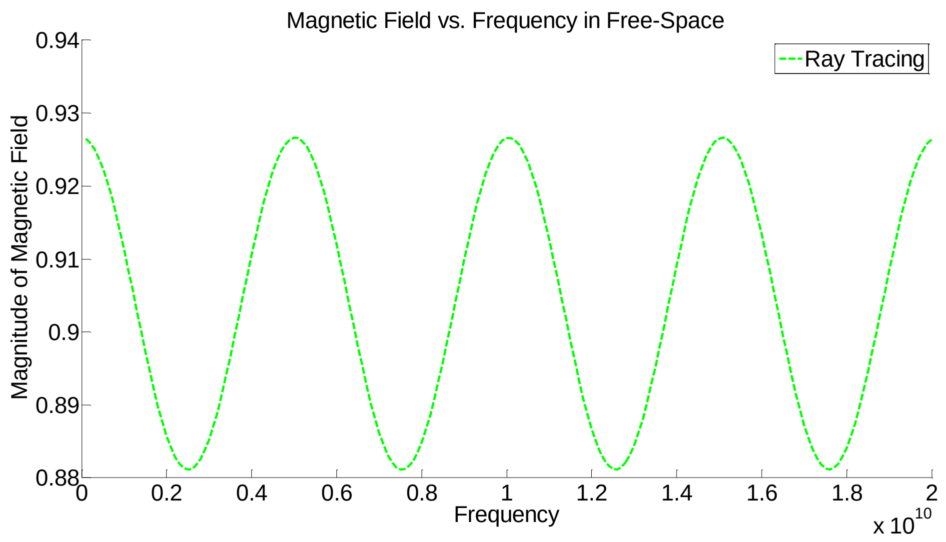


Figure 6.5. The expanded version of the magnetic field in medium 1 for ray tracing solution

In slab, like in medium 1, electric and magnetic fields with ray tracing solution are compared with the fields from the other solution technique and also shown in the following figures below.

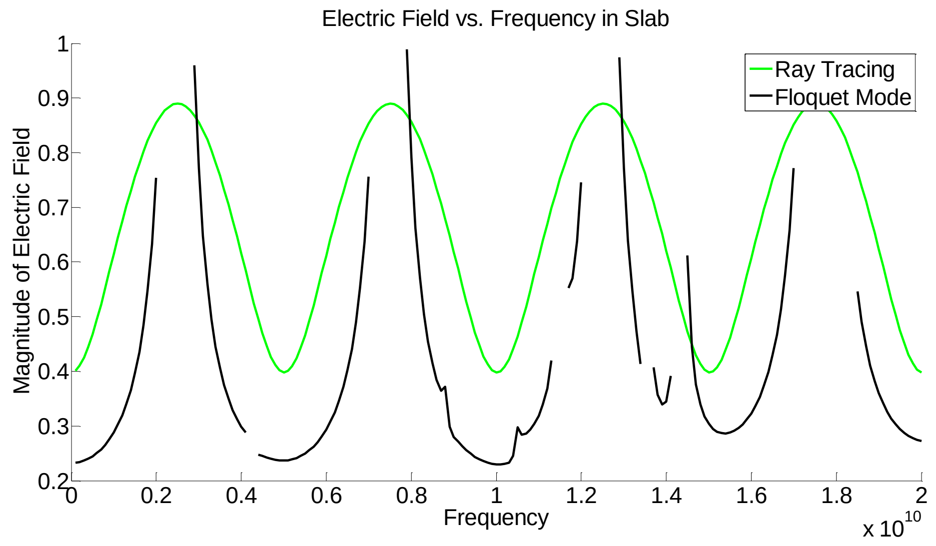


Figure 6.6. Electric field in slab (medium 2)

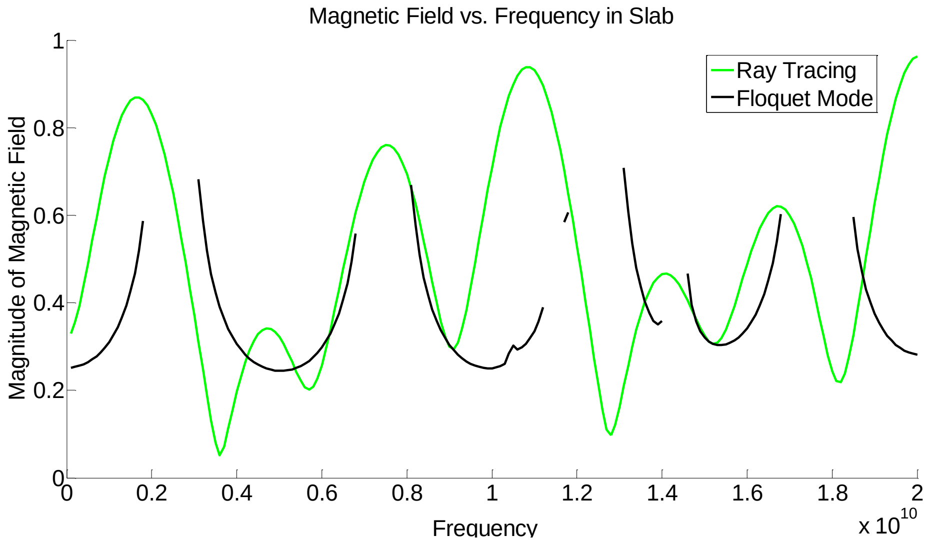


Figure 6.7. Magnetic field in slab (medium 2)

According to the graphs in free space and in slab, it can be said that the solution by using the Ray Tracing Method can be used as an alternative solution for this problem instead of solution with Floquet modes, because the graphs show us that the curves of electric and magnetic fields have the same waveform. There are some differences at curves since they are not same problem.

On the other hand, in order to answer the question that why the strip grating prefers to the parallel plate waveguide with full metallic surface, the advantage of this structure should announce such as low dissipated loss.

The parallel plate waveguides with full metal and with strip grating wall are shown in Figure 6.8 and Figure 6.9. The top views of these structures are shown in Figure 6.10 and 6.11. To determine the dissipated loss, the waveguide has to get a wall which has resistance value instead of PEC, so that the conductivity of the material of wall has to get a finite value. Also, the electric and magnetic fields are able to propagate in the material least along one skin depth distance.

The dissipated power in terms of the electric field $\vec{E}$ and the conductivity $\sigma_s$ is defined as:

$$\vec{P}_d = \frac{1}{2}\iiint \vec{J}\cdot\vec{E}^* dV = \frac{1}{2}\iiint \sigma_s\vec{E}\cdot\vec{E}^* dV = \frac{1}{2}\iiint \sigma_s \left|\vec{E}\right|^2 dV$$

To evaluate the volume integral, the wall of the waveguide must be nonzero but it can be very small.

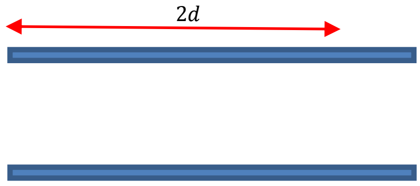


Figure 6.8. Parallel Plate Waveguide with Full Metallic Surface

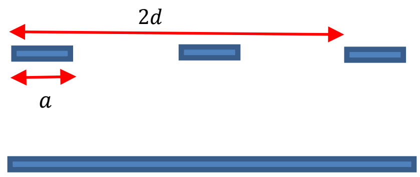


Figure 6.9. Parallel Plate Waveguide with Strip Grating

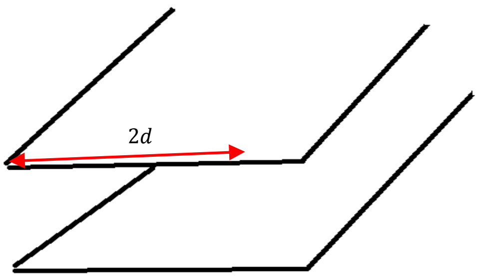


Figure 6.10. Parallel Plate Waveguide with Full Metallic Surface view from top

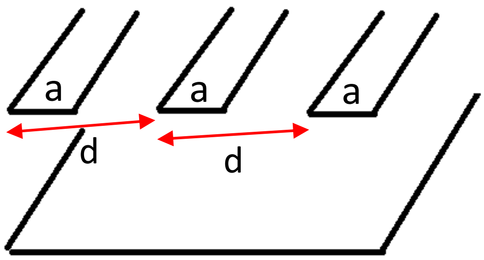


Figure 6.11. Parallel Plate Waveguide with Strip Grating view from top

Supposing that the material used for the walls of the waveguides is the same and the same electric field propagates in the slabs. Let the length of the wall and strips is equal to $d$ and thickness is $w$. The dissipated losses for each case:

- The case when the wall is full;

$$\vec{P}_{dfw} = \frac{1}{2}\iiint \sigma_s \left|\vec{E}\right|^2 dV = \frac{1}{2}\sigma_s\left|\vec{E}\right|^2 wl2d$$

- The case when the wall has strip grating;

$$\vec{P}_{dsg} = \frac{1}{2}\iiint \sigma_s \left|\vec{E}\right|^2 dV = \frac{1}{2}\sigma_s\left|\vec{E}\right|^2 wl2a$$

Thus, the ratio of the dissipated losses between grating strip and full wall is proportional to the width of the strips $a$.

$$\frac{\vec{P}_{dsg}}{\vec{P}_{dfw}} = \frac{\frac{1}{2}\sigma_s\left|\vec{E}\right|^2 wl2a}{\frac{1}{2}\sigma_s\left|\vec{E}\right|^2 wl2d} = a/d$$

The meaning of that the ratio of the $a$ / $d$ , that is, $a$ determines the dissipated loss of the parallel plate waveguide with strip grating wall. In order to decrease the dissipated loss, the width of strips $a$ must be decreased.

On the other hand, $a$ should not go to zero. Because the electromagnetic wave in the waveguide must not escape to out of waveguide. It is clearly explained as below.

Let an electromagnetic wave exists in the parallel plate waveguide with the strip grating which has width $a$ and periodicity $d$. Reflected and transmitted waves shown in Figure 6.12 are proportional to the reflection and transmission coefficients, respectively.

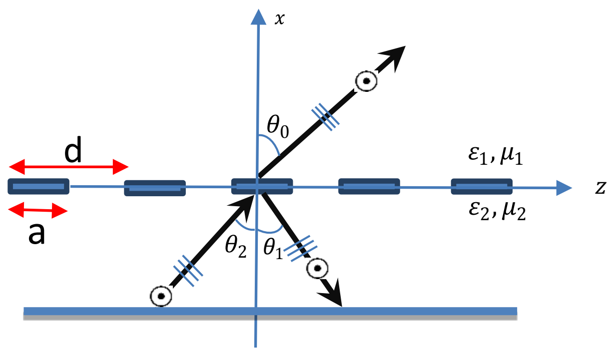


Figure 6.12. Incident, reflected and transmitted waves at an interface with strips

The transmission coefficient determines the amount of the wave going to other side of the interface, that is, the transmitted wave. The good performance of the waveguide, transmitted wave on the other side of the waveguide must be minimum. Let investigate the transmitted wave with respect to the ratio of $a/d$.

The reflection and transmission coefficients for E-polarized and H-polarized are obtained separately when incident wave in medium1 and in medium2 above.

The variation of the coefficients with respect to the ratio $a/d$ is shown in Figure 6.13 and Figure 6.14 at 300 MHz and 3 GHz

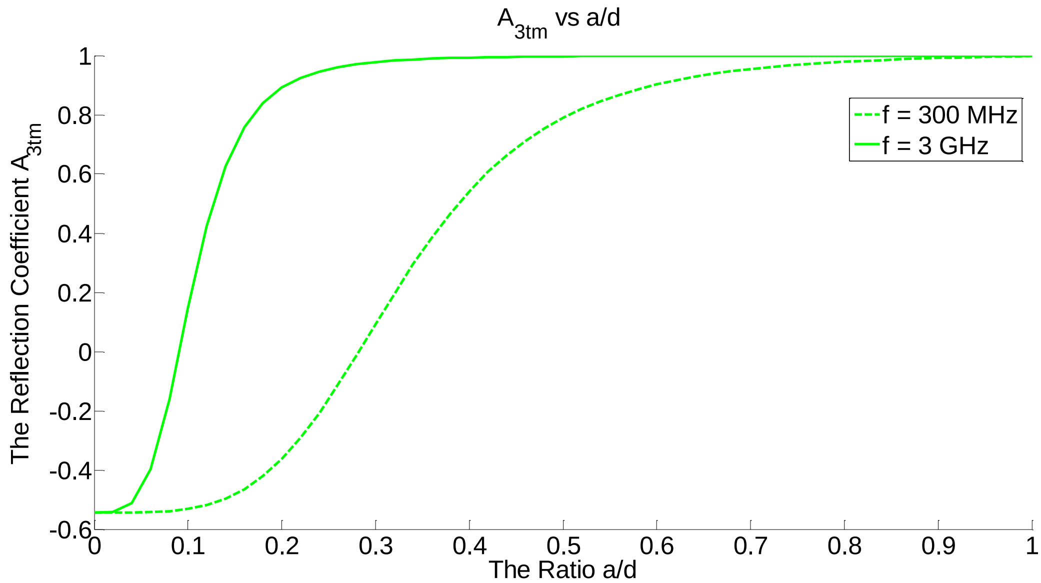


Figure 6.13. Reflection Coefficient vs. the Ratio $a/d$

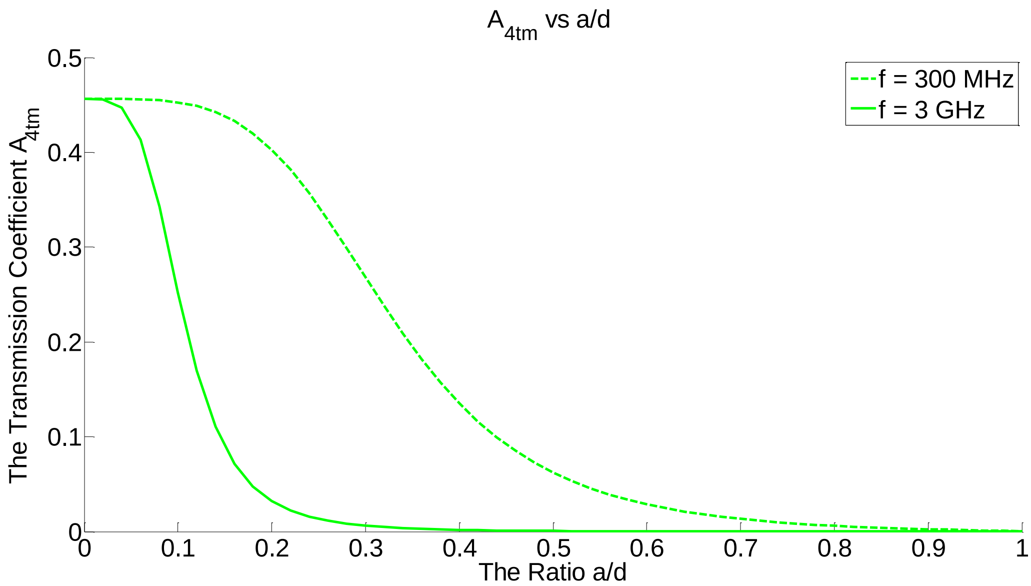

Figure 6.14. Transmission Coefficient vs. the Ratio $a/d$

According to the graphs, while the width of strips is increasing, the reflection coefficient goes to 1 and transmission coefficient goes to 0. This means that if the width is increased, then the amount of the wave which can escape to out of waveguide will decrease. So, the width of the strips must be increased.

Now, the strips width should be decreased in order to decrease the dissipated loss and should be increased in order to prevent escaping of wave. So that, the strips have to get an optimum width to access both of cases

# 7. CONCLUSION

In literature, there are lots of studies about transmission of electromagnetic waves. Waveguides are one of them among these studies. In this thesis, parallel plate waveguides are investigated.

Parallel plate waveguide is a structure where two metal parallel layers strict wave in one dimension while wave can propagate in other two dimensions. According to the nature of the walls of the structure, during the propagation, expelled wave and dissipated power should be taken into account.

In this thesis, instead of conventional parallel plate waveguide, parallel plate waveguide with strip grating is studied. The advantages of the strip grating are investigated when it’s compared with the waveguide with full metallic walls. First of all, the strip grating metal is placed at an interface between two simple different dielectric mediums and the reflection and transmission coefficients at interface are obtained for both E-polarized and H-polarized by using the Weinstein’s approximate boundary conditions. Then, with using these coefficients, waves which are assumed to be excited in medium 1 are analyzed for E-polarized and H-polarized by using ray tracing method. According to the incident wave, the components of the other waves in medium 1 and medium 2 are expressed by using Maxwell’s Equations. Jacobsen had investigated the same problem with Floquet modes. The electric and magnetic field expressions that are not expressed in his study are obtained. With the comparison of the electric and magnetic field expressions acquired by using both ray tracing mode and Floquet mode and it is observed that same results are obtained. Thus, it can be easily said that ray tracing mode is an alternative solution method for this problem.

In this thesis, dissipated power is investigated in order to show/prove that waveguides with strip gratings are preferred instead of conventional waveguides. There are advantages in the topic of dissipated power with the ratio of strip width (a) over period (d). While strip width is decreasing, the dissipated power is also decreasing which is a desired situation. On the other hand, while strip width is decreasing, the amount of the wave which escapes out of the waveguide is increasing. So that, strip width should not be very small. Additionally, in the consideration of dissipated power and expelled wave, strip widths should be optimized in terms of different values of strip period, medium parameters, incident angle and frequency.

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## BIOGRAPHY

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