THE REPUBLIC OF TURKEY
CUKUROVA UNIVERSITY
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DEPARTMENT OF ECONOMETRICS

THE ECONOMETRIC ANALYSIS OF SEASONAL TIME SERIES: APPLICATIONS ON SOME MACROECONOMIC VARIABLES

Sera ŞANLI

MASTER OF ARTS

ADANA / 2015

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ÖZET

MEVSİMSEL ZAMAN SERİLERİNİN EKONOMETRİK ANALİZİ: BAZI MAKROİKTİSADİ DEĞİŞKENLER ÜZERİNE UYGULAMALAR

Sera ŞANLI

Yüksek Lisans Tezi, Ekonometri Ana Bilim Dalı
Danışman: Doç. Dr. Mehmet ÖZMEN
Ağustos 2015, 217 sayfa

Bu çalışmada temel olarak enflasyon, büyüme, işsizlik, tüketim, gayri safi yurt içi hasıla, ihracat gibi bazı makroiktisadi zaman serilerinin mevsimsellik durumunda yapısal özelliklerini değerlendirmek için bu seriler üzerinde analizler yaparak zaman serilerinin mevsimsellik boyutunu tüm yönleriyle ele almak amaçlanmıştır. Çalışmada Mevsimsel Otoregresif Bütünleşik Hareketli Ortalama (SARIMA) modellemesi, Ilmakunnas (1990)'ın çalışmasına dayanan mevsimsel bütünleşme testleri, çeşitli yardımcı regresyon modelleri kullanılarak çeyreklik ve aylık frekanstaki veriler için mevsimsel birim kök testleri, deterministik-stokastik mevsimsellik testleri ve mevsimsel eşbütünleşme analizlerine yer verilmiştir. Mevsimsel birim kök analizleri; OCSB, DHF testlerinin yanısıra temelde Hylleberg, Engle, Granger ve Yoo tarafından geliştirilen en popüler yaklaşım olan HEGY yaklaşımıyla ele alınacaktır. Bu analizler, bize (sıfır frekansın yanı sıra) hangi mevsimsel frekanslarda birim kökün mevcut olup olmadığı bilgisini edinmemize imkan sağlayacaktır. Uygulamalardan elde edilen sonuçlar göstermiştir ki bir seri hangi frekanslarda birim kök içeriyorsa, seriyi durağanlaştırmak için yapılacak dönüşümlerde bu frekanslara karşılık gelen filtreler uygulanmalıdır. Öte yandan, aylık HEGY mevsimsel birim kök uygulamaları, aylık bazlı verilerin mevsimsel frekanslarda hiçbir mevsimsel birim kök içermeyebileceğini ortaya koymuştur. Ayrıca, Türkiye için ele alınan makroiktisadi serilerin yalnızca tek tür mevsimsel davranış sergilediği kesin değildir. Bu makroiktisadi seriler, hem deterministik hem de stokastik bir yapı içerebilir.

<u>Anahtar kelimeler:</u> HEGY yaklaşımı**,** mevsimsel birim kökler, deterministik-stokastik mevsimsellik, mevsimsel bütünleşme, mevsimsel eşbütünleşme.

ABSTRACT

THE ECONOMETRIC ANALYSIS OF SEASONAL TIME SERIES: APPLICATIONS ON SOME MACROECONOMIC VARIABLES

Sera ŞANLI

Master Thesis, Department of Econometrics
Supervisor: Assoc. Prof. Mehmet ÖZMEN

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In this paper, it has been mainly aimed to treat the scope of the seasonality - which is an important component of time series - in all its bearings by making analyses on some macroeconomic time series (such as inflation, growth, unemployment, consumption, gdp, exports etc.) to evaluate the structural properties of these series under seasonality. The conducted analyses include Seasonal Autoregressive Integrated Moving Average (SARIMA) modelling, seasonal integration tests based on the study of Ilmakunnas (1990), seasonal unit root tests for quarterly and monthly data under the various auxiliary regression models, deterministic and stochastic seasonality tests and seasonal cointegration. The analyses of seasonal unit roots have been conducted fundamentally with the most popular approach developed by Hylleberg, Engle, Granger and Yoo called HEGY apart from the OCSB, DHF tests. There are some important implications of the results obtained for these applications: firstly, if a series has unit roots at which frequencies, filters corresponding to those frequencies should be applied to the series in interest in order to make it stationary. On the other hand, monthly HEGY seasonal unit root applications have revealed that even though the data are available on monthly basis, they may not include any seasonal unit roots at seasonal frequencies. In addition, it is not certain to say that all Turkish macroeconomic series display only one type of seasonal behaviour. Thus, they can have both a deterministic and stochastic structure.

Keywords: HEGY procedure, seasonal unit roots, deterministic-stochastic seasonality, seasonal integration, seasonal cointegration.

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LIST OF ABBREVIATIONS

ACF: Autocorrelation Function

ADF: Augmented Dickey-Fuller

ADFSI: Augmented Dickey-Fuller Seasonal Integration

AIC: Akaike Information Criterion

AICc: Corrected Akaike Information Criterion

AO: Additive Outlier

AR: Autoregressive

ARCH: Autoregressive Conditional Heteroscedasticity

ARCH-LM: Autoregressive Conditional Heteroscedasticity-Lagrange Multiplier Test

ARIMA: Autoregressive Integrated Moving Average

ARMA: Autoregressive Moving Average

BIC: Bayesian Information Criterion

BVAR: Bayesian Vector Autoregressive

CBRT: Central Bank of the Republic of Turkey

CH: Canova-Hansen

CLT: Central Limit Theorem

CPI: Consumer Price Index

d.f.: Degrees of Freedom

DF: Dickey-Fuller

DFSI: Dickey-Fuller Seasonal Integration

DGP: Data Generating Process

DHF: Dickey, Hasza and Fuller

DW: Durbin-Watson

FCLT: Functional Central Limit Theorem

GDP: Gross Domestic Product

GLS: Generalized Least Squares

GNP: Gross National Product

HEGY: Hylleberg, Engle, Granger andYoo

HICP: Harmonized Index of Consumer Prices

i.i.d.: Independent and Identically Distributed

IO: Innovative Outlier

KPSS: Kwiatkowski, Phillips, Schmidt and Shin

LBIU: Locally Best Invariant Unbiased

LM: Lagrange Multiplier

LR: Likelihood Ratio

MA: Moving Average

ME: Maximum Eigenvalue

ML: Maximum Likelihood

OCSB: Osborn, Chui, Smith and Birchenhall

OLS: Ordinary Least Squares

PACF: Partial Autocorrelation Function

POI: Point Optimal Invariant

PP: Phillips-Perron

PROC X12: X12 Seasonal Adjustment Procedure

SAR: Seasonal Autoregressive

SARIMA: Seasonal Autoregressive Integrated Moving Average

SIC: Schwarz Information Criterion

SMA: Seasonal Moving Average

TR: Trace

U.K.: United Kingdom

U.S.: United States

VAR: Vector Autoregression

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CHAPTER I

INTRODUCTION

1.1. Statement of the Problem

Economic time series are generally recorded at some fixed interval. When we are dealing with macroeconomic time series, seasonal models are mostly available at monthly or quarterly frequency and for instance, for a variable with quarterly frequency, time series plotting of each quarter as a separate curve gives a useful insight about describing the seasonal behaviour of a series. Such plots are available in Hylleberg (1986). However, if studied with financial time series, our interest is often directed to the seasonal patterns at the daily level. So, seasonality may have many different manifestations and it is a widespread phenomenon observed in many econometric time series (Ghysels & Osborn, 2001, p.3).

The problem of the research is concerned with the concept of seasonality. However, there is no simple answer about what seasonality is. There are some factors underlying the source of seasonal variations like production cycle characteristics, calendar effects (the timing of certain public holidays-such as Christmas and Easter), timing decisions (the timing of school vacations, ending of university sessions etc.) repeating every year in the same month or quarter and differing in magnitude from year to year even the seasonal variations occur regularly (Hansda, 2012, p.1673).

According to Hylleberg (1992), the definition of seasonality in economics is given as:

> *Seasonality is the systematic, although not necessarily regular, intra-year movements caused by the changes of the weather, the calendar and timing of decisions, directly or indirectly through the production and consumption decisions made by the agents of the economy. These decisions are influenced by endowments, the expectations and preferences of the agents, and the production techniques available in the economy* (Hylleberg, 1992, p.4).

All the studies regarding time series methods are useful only in case the series in interest do not display seasonal patterns. That is why it is of great importance to take the time series properties of the series like seasonal patterns or trends into account while dealing with economic time series data and the research on what form of seasonality exists in the data in interest and thus the way of modelling seasonality is also crucial.

Seasonality could be viewed as deterministic or stochastic. The difference between these two types of seasonality can be explained in that way: while in the deterministic seasonal model shocks die out in the long run, in the stochastic seasonal model shocks have a permanent effect. Therefore, in the stochastic seasonal model a positive shock at time t will not only increase the value of a y_t series, but also the value of $y_{t+s}, y_{t+2s},...$ etc. (Özcan, 1994, p.64). Taking seasonal differences can remove the seasonal pattern. However, in the case of deterministic seasonal variation which can be modelled as a deterministic function of time plus stationary noise, this transaction is not required. Since a deterministic seasonal pattern that is subject to differencing results in a noninvertible series; in other words, it contains a unit root in the Moving Average (MA) operator. There are some tests relating to testing the presence of deterministic seasonality which are the Canova-Hansen (CH) Test, the Caner Test and the Tam-Reinsel Test. While Canova and Hansen (1995) adopt a nonparametric approach in handling of autocorrelation problem, Caner (1998) and Tam and Reinsel (1997) adopt a parametric approach and the Monte Carlo study conducted by Caner (1998) has revealed that his proposed test with the parametric approach provides better size and power properties than Canova and Hansen. On the other hand, while Caner (1998) advocates to estimate the deterministic seasonal model in seasonal differences; the others estimate this model in the levels of the series in interest that will be mentioned later. Contrary to the deterministic seasonality, in the case of stochastic seasonality the seasonal differences generate a stationary and invertible process. However, if seasonal differencing is not applied to the series having stochastic seasonality, the series continues to be nonstationary. Therefore, it is of great importance to determine which type of seasonality the series in question displays because nonstationarity and non-invertibility situations create difficulties in parameter estimation and forecasting (Tam & Reinsel, 1997, p.725). In case seasonal time series have unit roots, these roots repeat themselves depending on the seasonal frequencies. As opposed to the conventional unit root tests, in the case of seasonal unit roots taking differences as the number of repeating unit roots in series will remain the series as non-stationary and this application will be able to convert the series into very complex models. In that case, the knowledge of whether unit root in a series is seasonal or not is very crucial (Türe & Akdi, 2005, p.3).

The perception of many econometricians directed at the fact that seasonal variation is often larger and more irregular than being considered and making inference in dynamic models like integration and cointegration tests is disrupted by using seasonal adjustment. All these create a stimulating effect to deal with modelling seasonality. On the other hand, the study of seasonality is tied closely to the study of business cycles. Many analysts tend to work with current data to make inferences about changes in overall economy. The aim is to identify changes in the trend of economic activity from movements in certain indicators, such as data on prices or interest rates, or some other index of economic activity that is reported very frequently (Jaditz, 1994, p.17). In this respect, inferences about the business cycles could be interpreted in a complicate way in the presence of seasonal pattern. This is another justification in order to deal with seasonality.

1.2. The Aim of the Research

In this paper, it has been mainly aimed to treat the scope of the seasonality in all its bearings by making analyses on some macroeconomic time series, to evaluate the structural properties of these series under seasonality and to present the methods that have been suggested and/or employed in the literature including modelling seasonality-what form of seasonality exists in the data worked, deterministic or stochastic-, seasonal integration, seasonal unit root analyses and so on. Since many time series display substantial seasonality, the presence of unit roots corresponding to other frequencies (like seasonal ones) rather than zero is highly possible. The analyses of seasonal unit roots will be conducted with the most popular approach developed by Hylleberg, Engle, Granger and Yoo called HEGY by working with different models that include trends, constants and seasonal dummy variables and with a variety of approaches other than HEGY. These analyses will enable us to understand whether there exist unit roots at seasonal frequencies or not.

It is remarkable to say that the aim in separating of the total variation of a time series into seasonal and other components is to obtain identification of underlying patterns and causal relationships and to lower the possibility of being ill-informed by spurious correlations created by systematic and independent effects (Fromm, 1978, p.26).

1.3. The Importance of the Research

It is very crucial to handle pure analysis of seasonality and determine it exactly in the deterministic time series indicators of the economic system in order to choose a proper policy analysis and carrying it for the economy of the country in question. So, the removal of the knowledge on seasonal factors of an economic variable (i.e. seasonal adjustment procedure) enables the policy maker to differentiate between the seasonal changes and long run changes in a variable and thereby design appropriate policy responses (Hansda, 2012, p.1673).

If necessary to give an example to why seasonality is important to be understood, assume that whether there is an expansion or recession in the economy, there is a significant drop in industrial production in the first quarter of the year. Therefore, it is significant for analysts to make inference about whether a first quarter dip is caused by seasonal factors that will vanish next quarter or whether the decline is an indicator for a change in the business cycle from boom to bust (Jaditz, 1994, p.17).

If we ignore the presence of this seasonality in the series although a series is seasonal in fact, both the knowledge of a description of seasonal fluctuation and a description of the variation in the series with the seasonal fluctuation removed may be disregarded in making useful administrative or policy decisions. Without a prior description, misspecification of the models and incorrect forecasts are highly possible. On the other hand, the knowledge on the amount of seasonal fluctuations may be in great importance for policy makers and administrators for allocating resources in a suitable way. For instance, in case more people are sentenced to prison in the fall, this knowledge will enable the prison administrator to arrange for more beds in the fall months. In addition, in case a nonseasonal series is seasonally adjusted by assuming that this series follows a seasonal pattern, this analysis will also be erroneous and a complex model to be constructed under this assumption will give rise to a misspecified model with an “overadjustment” of seasonality depending on the removal of seasonal fluctuations that are not present in any way (Block, 1983, pp.3-7).

In this research, the results which will be obtained with applications of seasonality analyses on some macroeconomic series will give us an insight about which pattern these series exhibit for a given period at any frequency and whether they are in accordance with the real world expectations or not.

CHAPTER II

LITERATURE REVIEW

2.1. Studies on Seasonal Patterns

There exists a vast literature on seasonality. Nerlove (1964) utilizes the spectral and cross-spectral techniques in order to analyse the effects of seasonal adjustment procedures and mention about a slowly changing and stochastic seasonal pattern to uncover itself in the spectrum of an economic time series through a set of peaks occurring at certain frequencies. Following the work of Box and Jenkins (1970), seasonal ARMA models have been estimated by many time series practitioners**.** In the paper by Kitagawa and Gersch (1984), it is dealt with a smoothness priors –Kalman Filter-Akaike Information Criterion (AIC)- approach to the modelling of time series with trend and seasonality. Kitagawa and Gersch (1984) have supported the usage of a state-space approach with a specific unobservable seasonal component. Through a state-space representation, Thorburn and Tongur (2014) consider the issue of whether seasonal decomposition should be used prior to or after aggregation of time series and have an argument on that the preferable succession order between aggregation and seasonal decomposition must depend on the covariance structure of the series. Hylleberg (1986) provides an extensive discussion of definitions of seasonality. Hasza and Fuller (1982) and Li (1991) discuss the tests for normal and seasonal unit roots on the autoregressive operator. The studies by Otto and Wirjanto (1990), Ghysels, Lee and Siklos (1994b), McDougall (1995) indicate the presence of significant seasonal patterns on many macroeconomic time series. Bell (1987) refers to the discussion that series featuring seasonal unit roots and pure seasonal dummy processes are not regarded as distinct from a practical view of point when $\theta_S = 1$ in equation (4.31). Eiurridge and Wallis (1990) mention about how seasonal patterns in variance should be modelled in the context of Kalman Filter.

Sims (1974) considers the seasonal components of economic time series as “errors in variables”, examines the nature of asymptotic biases in least squares estimates of lag distributions in the case of availability of seasonal noise and analyses procedures for correcting for seasonal bias.

When looked at seasonality from economic viewpoint, Barsky and Miron (1987) outline the estimates of the seasonal patterns in a set of standard macroeconomic variables including consumption, investment, government purchases, employment and money stock concluding that a crucial source of the non-trend variation comes from seasonal fluctuations. Comparing the seasonal cycle to the business cycle, they show the U.S. (United States) economy to display a "seasonal business cycle" and express that it has the significant qualitative features that reflect closely the identical picture of the characteristics of the conventional business cycle. That is, besides the business cycle frequencies, at seasonal frequencies output movements are found to act together across broadly defined sectors and nominal money and real output are found to have a strong correlation.

The paper by Miron (1990) presents some stylized facts about seasonal fluctuations in U.S. and other economies which convey the crucial information about the nature of the business cycle. The results show that the preference shifts have more considerable importance than technology shifts in explaining the important properties of observed seasonal patterns and seasonal cycles and business cycles are closely related. Miron (1990) also discusses the possible welfare implications of seasonal cycles.

In the paper by Beaulieu and Miron (1992a), the cross country variations in seasonal patterns are utilized to describe the basic sources of seasonal cycles. It is shown that a fourth quarter boom in output, a July or August trough in manufacturing production and a first quarter trough in almost every aspect of economic activity constitute the most significant characteristics of seasonal patterns. Even though the model proposed by Beaulieu and Miron (1992a) is in coherence with the stationary stochastic seasonality, only deterministic seasonality is analysed in the study based on the justification that seasonal unit roots and stationary stochastic seasonality are not of quantitative importance whenever dummies have been excluded.

In Jaditz (2000), testing for seasonal components in variance is expressed to be analogous to testing a stationary time series for seasonality in the mean. In the article, all nine common macro time series have been examined to find out if they display seasonality in variance and all series are found to display significant seasonality in variance. Since seasonal variation structures are very crucial to businesses and policy makers with regard to including important signs about the current situation of the economy, from this result it is inferred that the variance of macro time series seems to have a significant seasonal component.

Franses (1992b) presents a general-to-simple test procedure for seasonality which is established upon the tests for parameter restrictions that are associated with seasonal behaviour in a general periodic model. In the study, this procedure is applied to the quarterly U.K. (United Kingdom) stock price index for the period 1963:Q1-1988:Q4 and the U.S. CLI index for the period 1948:Q1-1987:Q4 and it is concluded that cyclical and trend behaviour vary per quarter for CLI index while the U.K. stock price series does not exhibit seasonal patterns.

The study of Canova and Hansen (1995) presents Lagrange Multiplier (LM) tests of the null hypothesis of no unit roots at seasonal frequencies denoting the presence of deterministic seasonality contrary to the tests of Dickey, Hasza and Fuller (DHF) (1984) and Hylleberg, Engle, Granger and Yoo (HEGY) (1990) tests dealing with the null of presence of seasonal unit roots. They generalize the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) (1992) test framework.

With Monte Carlo experiments applied to three data sets which are the quarterly seasonal fluctuations in U.S. macro variables originally used by Barsky and Miron (1989), quarterly European industrial production indexes used by Canova (1993) and stock returns on value weighted indexes for seven industrialized countries, they draw attention to the instable and therefore nonstationary seasonal pattern properties of these variables in most cases. The paper proposed by Canova (1993) presents a methodology for modelling and forecasting the series with common patterns at seasonal and/or other frequencies and attaches the concept of common patterns to a Bayesian Autoregression tradition developed by Litterman (1980), Doan, Litterman and Sims (1984) and Sims (1989) at origin.

In their paper, Raynauld and Simonato (1993) aim to assess a possible alternative based on the adaptation of the Bayesian Vector Autoregressive (BVAR) approach popularized by Litterman (1979, 1984, 1986), Doan, Litterman and Sims (1984) and Sims (1989) to the context of seasonal time series. The forecasting performance of the seasonal BVAR models has been evaluated in the context of a monthly model of the U.S. economy including both seasonal and nonseasonal variables. In the paper proposed by Shaarawy and Ali (2015), it has been aimed basically to develop an approximate Bayesian technique to identify the orders of any seasonal multivariate autoregressive processes and numerical results obtained point out to the sufficiency of using the proposed technique in identifying the orders of seasonal multivariate autoregressive processes for moderate and large sample size in an efficient manner.

In their study, Ghysels and Perron (1996) examine the effects of seasonal adjustment filters on the statistical properties of different tests involving structural changes through theoretical discussions and Monte Carlo simulations. The adverse effects of linear filtering in the case of structural change are demonstrated using historical series of economic activity covering the Great Depression. The results indicate that the non-rejection of the unit root null hypothesis with seasonally adjusted series can be grounded on the smoothing properties of many filters requiring a power loss.

Tam and Reinsel (1997) examine the locally best invariant unbiased (LBIU) and point optimal invariant (POI) test procedures for a unit root in the seasonal moving average (SMA) operator for seasonal autoregressive integrated moving average models (SARIMA) and make use of the monthly non-agricultural industry employment series for males age 16-19 modelled by Hillmer, Bell and Tiao (1983). The results for conducted simulations have revealed that for this series, seasonality is stochastic and therefore seasonal differencing is appropriate. They also apply their tests to different types of seasonal time series data and find some of these series to have deterministic seasonality.

In order to distinguish stochastic seasonality from deterministic seasonal pattern, Tam and Reinsel (1998) also examine the LBIU and POI tests for a unit root in SMA model in the presence of a deterministic linear trend and their test is an extension of the framework proposed by Tanaka (1990) and Saikkonen and Luukkonen (1993) nonseasonal MA unit root tests to the seasonal frequencies. The test procedures are applied to the monthly average total ozone data at Boulder, Colorado from 1966 to 1991 and as associated with non-rejection of the null hypothesis, they decide on that modelling seasonality as deterministic is appropriate rather than stochastic.

The article proposed by Caner (1998) suggests a locally best invariant test with the null of seasonal stationarity and the test is derived from the framework of King and Hillier (1985). It is also a generalization of the unit root test proposed by Leybourne and McCabe (1994) from zero frequency case to the seasonal frequency. When compared with the CH test, contrary to it Caner takes the autocorrelation into account in a parametric way and conducted Monte Carlo simulations revealed that his proposed test has better finite sample performance with good power properties than the CH test in an AR type of autocorrelation. Also, in the same manner to Caner (1998), Busetti and Harvey (2003) extend the test procedure proposed by Canova and Hansen (1995) and propose a parametric version of the test. One crucial practical finding from Monte Carlo experiments is that for most economic time series taking first differences can be a good strategy. In addition, they consider a test for breaks in seasonal patterns, also a general test versus any kind of permanent seasonality, deterministic or stochastic.

Lenten and Moosa (1999) aim to model the trend and seasonal behaviour of the alcoholic beverages consumption in the U.K. over the period 1964-1995 by means of the univariate version of Harvey’s (1989) basic structural time series model. Using quarterly seasonally adjusted data, they have found the consumption of beer and wine to display stochastic seasonality and the consumption of spirits to display deterministic seasonality. Also, these three series are expressed to include stochastic trends. According to the goodness of fit measures and diagnostic test statistics, the model with stochastic trend and seasonality has been found to be the most suitable one when compared to other models. Harvey and Scott (1994) examine the implications of explicit modelling of seasonality as an unobserved component which facilitates the dynamic modelling by seperating non-seasonal components from seasonal ones using the consumption model of Davidson, Hendry, Srba and Yeo (1978).

In their paper, Cheung and Coutts (1999) make use of logarithmic daily returns of the Hang Seng Index on the Hong Kong Stock Exchange over the period 1 January 1985 through 30 June 1997 in order to search for a January effect or monthly seasonality. Contrary to the previous studies regarding other stock indices which discover the presence of some type of monthly seasonality in most cases, their study shows strangely that there is no evidence of a January effect or monthly seasonality for the Hang Seng Index over the period in question.

Fang (2000) presents a broad characterization of the presence of significant seasonal patterns in estimated daily and hourly return volatilities using high frequency data for three exchange rates which are mark/dollar, yen/dollar and yen/mark and points out to that disregarding such patterns will result in a biased and insignificant empirical analysis.

In the study by Lim and McAleer (2000), the presence of stochastic seasonality is examined to clarify the nonstationary quarterly international tourist arrivals from Hong Kong and Singapore to Australia from 1975:Q1 to 1996:Q4 using HEGY (1990) procedure. Since the presence of seasonal unit roots gives an insight into a varying seasonal pattern that is against a constant seasonal pattern, the Box Jenkins Seasonal Autoregressive Integrated Moving Average (SARIMA) process is possible to be a more suitable model for tourist arrivals rather than a deterministic seasonal model with seasonal dummy variables.

Seong, Ahn and Jeon (2008) deal with the spurious regression problem in a model including two different types of nonstationary seasonal time series, stochastic and deterministic. With a diverging conventional regression t ratio and an increasing sample size, the conducted Monte Carlo study shows the existence of the phenomenon of spurious regression.

Halim and Bisono (2008) propose a forecasting program for an automatic seasonal nonstationary homogenous forecasting which enables to get the knowledge of the best time series model in the sense of minimum AIC.

In their paper, Chang and Liao (2010) have aimed to forecast the monthly outbound tourism departures of three major destinations from Taiwan to Hong Kong, Japan and U.S.A. respectively using the SARIMA model.

Saz (2011) examines the efficacy of SARIMA models for forecasting Turkish inflation rates from 2003 to 2009 and presents a methodological approach for a combination of a systematic SARIMA forecasting structure and the stepwise selection procedure of the Hyndman-Khandakar (HK) algorithm. This combination is expressed to give rise to choosing a best single SARIMA model which is SARIMA(0,0,0)(1,1,1) model with one degree of seasonal integration, one seasonal AR and one seasonal MA part. According to a structural break analysis, the Turkish inflation rates have been found to display a range of structural breaks with the latest being in mid-2003 and stochastic nature of Turkish inflation has been found to outweigh its deterministic nature.

2.2. Studies on Seasonal Unit Roots, Seasonal Integration and Cointegration

So far, there have been many applications of unit root tests. Univariate unit root tests were first proposed by Fuller (1976) and Dickey and Fuller (1979) whose unit root test is known as the most prominent one. These tests were applied to a number of macroeconomic data by Nelson and Plosser (1982). In their paper, Nelson and Plosser (1982) state that stochastic variation due to real factors lies at the core of macroeconomic fluctuations. They make use of long historical time series for U.S. and application results fail to reject the hypothesis that the series are nonstationary stochastic processes.

As associated with the unit root concept, the relationship between cointegration and error correction models was first suggested by Granger (1981) and then it was also introduced by Granger and Weiss (1983). Engle and Granger (1987) also offer a theorem based on Granger (1983) which associates the moving average (MA), autoregressive (AR) and error correction representations for cointegrated systems and estimation methods. Engle, Granger and Hallman (1989) and Hylleberg, et al. (1990) introduce the concept of seasonal cointegration in their papers. Lee (1992) and Johansen and Schaumburg (1999) examine the seasonal cointegration relationships that are based on maximum likelihood (ML) estimation**.** Joyeux (1992) has dealt with testing for seasonal cointegration using principal components. Kunst (1993) tries to evaluate the effects of modelling seasonal cointegration on predictive accuracy for German and U.K. macroeconomic series. In their study, Ahn and Reinsel (1994) handle the connection between the partially non-stationary vector autoregressive model with seasonal behaviour and seasonal cointegration and the error correction model.

Reimers (1997) analyses the forecasting performance of seasonally cointegrated processes through simulating different data generating processes and using the ML approach proposed by Lee (1992). In this study, there has been made a comparison between forecasts of cointegrated models in fourth differences and first differences including seasonal dummies. The simulation study has shown that the models in first differences with seasonal dummies yield lower forecast errors in the short term than the seasonally cointegrated models for forecast horizons up to four quarters and for larger horizons, the models in fourth differences have been found to outperform the models in first differences.

In their paper, Kunst and Franses (1998) deal with the impact of deleting, restricting or not restricting seasonal constants on forecasting seasonally cointegrated time series for Austria, Germany and the U.K.

Cubadda (2001) introduces the complex error correction model for seasonally cointegrated variables and suggests a reduced rank estimator and a Trace (TR) Test to determine the cointegration rank at frequencies that are different from zero and π.

In the study by Löf and Lyhagen (2002), the comparison of the forecasting performance of the seasonally cointegrated model of Johansen and Schaumburg (1999) and of the specification proposed by Lee (1992) with a parameter restriction included at the annual frequency has been covered. For three data sets from Austria, Germany and U.K., each including six variables: gross domestic product (GDP), private consumption, gross fixed investment, goods exports, real wages and the real interest rate; it is also dealt with how the inclusion of restricted or unrestricted seasonal dummies may have an influence in the seasonal cointegration models. Since the semi-annual frequency for Austria appears to have full rank and the U.K. data set shows a rather weak cointegration evidence at the seasonal frequencies, only the German data are used in the forecasting example. Through Monte Carlo study, Löf and Lyhagen (2002) have found some evidence that for the smaller sample sizes the specification of Johansen and Schaumburg (1999) may result in worse forecasts in the case of the inclusion of more cointegrating relations and for larger sample sizes the study results have been found to favour of this specification.

In their study, Herwartz and Reimers (2003) examine the stochastic nature of the variables in the German money demand equation over the sample period from 1975:1 to 1995:4 by using seasonal unit root tests and prediction tests for structural change are presented for testing the stability of the process subsequent to the German Monetary Union. Depending on the existence of seasonal unit roots, it is concluded that the specification of the German money demand function should be in annual differences. From this point of view, according to a seasonal cointegration analysis the evidence shows the presence of long-run relationships among the included variables for the zero and the seasonal frequencies.

Darné (2004) extends the ML seasonal cointegration procedure proposed by Johansen and Schaumburg (1999) to monthly observed time series.

Cubadda and Omtzigt (2005) introduce iterative reduced rank regression procedures that permit a simultaneous modelling of the cointegration restrictions at the conjugate complex unit root frequencies and examine the small-sample properties using simulations. According to a Monte Carlo study, it is concluded that their new tests for the cointegration rank at the annual frequency perform better than the TR test in Cubadda (2001) for small samples.

Seong, Cho and Ahn (2006) introduce the Maximum Eigenvalue (ME) test for seasonal cointegrating ranks making use of the Cubadda's (2001) approach and make a comparison between the performances of ME test and the TR test in the seasonal case. In the paper by Seong, Cho and Ahn (2007), the inference of seasonal cointegration with common linear restrictions among cointegrating vectors at possibly different frequencies of seasonal unit roots is handled, in order to accommodate linear restrictions in the Gaussian reduced rank (GRR) estimation of Ahn, Cho and Chan Seong (2004) the necessary methods are presented and the related asymptotic distributions are established.

Seong (2009) presents two types of complex error correction models which are the extensions of the complex error correction model of Cubadda (2001) and obtains the limiting distribution of the Likelihood Ratio (LR) test to identify the seasonal cointegrating rank in these two models.

Seong (2013) considers a bootstrap algorithm for identifying seasonal cointegration ranks as an extension of Swensen (2006) who proposes a bootstrap algorithm to test and determine the cointegration rank in a reduced rank VAR (Vector Autoregression) model and Monte Carlo simulations show that the bootsrap algorithm can improve size distortions of the LR test in an efficient manner.

Mert and Demir (2014) have aimed to examine the seasonal patterns to detect if seasonal cointegration relationship exists between export and import series over the 1969:1-2014:1 quarterly periods. Two series have been found to be cointegrated at ¼ and ¾ frequencies with one cointegrating vector and not cointegrated at zero (long-run) frequency. The results have shown that error correction mechanism works at ¼ frequency and the coefficient is negative in accordance with expectations. However, at ¾ frequency, because of the error correction term is positive signed contrast to the expectations, the error correction mechanism has been determined not to operate. In this case, the return to equilibrium of deviations occurring in imports series at ¾ frequency has been expressed not to be fulfilled in the short term.

On the other hand, it is extremely common to come across seasonal economic time series displaying nonstationary stochastic seasonality. This situation has brought about the evolution of several seasonal unit root tests. Dickey, Hasza and Fuller (DHF) (1984) propose a test called DHF which is the extension of the well-known Dickey-Fuller (DF) procedure for the zero frequency unit root case to seasonal time series. They also extend the test to the case of higher-order stationary dynamics. The assumption of DHF test is that the true data generating process (DGP) displays a seasonal autoregressive process of order one or SAR(1) process and thus, seasonal integration is expressed to be tested with the alternative hypothesis of stationary seasonality. One main disadvantage of this test is that it does not allow for unit roots at some but not all of the seasonal frequencies.

The analysis of seasonal unit roots is fundamentally conducted with the most popular HEGY approach developed by Hylleberg et al. (1990) by working with different models that include trends, constants and seasonal dummies and in this paper it is mentioned about that when deterministic components are available in the regression model although not included in the data, the limiting distributions change. HEGY test has originally been derived for quarterly seasonality and extended to data with different frequencies. Contrary to the DHF test proposed by Dickey et al. (1984), HEGY procedure enables to test for unit roots at each seasonal frequency as well as the zero frequency separately and the techniques are applied to quarterly U.K. data for the period 1955:1 to 1984:4 in order to examine the cointegration relationship between consumption and income variables at different frequencies. As a result of application, Hylleberg et al. (1990) find the unit elasticity error correction model to be invalid at any frequency. The asymptotic distributions of the t-statistics from their testable model have been analysed by Chan and Wei (1988). In their paper, Chan and Wei (1988) characterize the limiting distributions of the least square estimates as a functional of stochastic integrals.

Osborn and Smith (1989) examine the performance of periodic autoregressive models in forecasting seasonal (quarterly) U.K. consumption and in the study, the preference between a periodic or non-periodic specification is stated to affect the resulting dynamic properties.

Osborn (1990) examines whether the seasonal component in each variable displays stochastic nonstationarity in quarterly data for 30 important U.K. macroeconomic variables including real GDP and its basic components, employment, price/earning indices, the rate of interest and the exchange rate and she reports that only interest rates and the exchange rate display no significant seasonality and a seasonal unit root exists in only six variables.

Franses (1990) deals with testing for seasonal unit roots in monthly observations. Franses (1994) proposes a seasonal unit root test that grounds on the multivariate representation of univariate seasonal processes. In his paper, the VQ (vectors of quarters) approach which considers an autoregressive model for the vector including annual observations per season is adopted and this approach is expressed to be able to regard as the most appropriate tool for univariate data analysis. The application of Johansen's ML cointegration method shows an extension of HEGY procedure by taking periodically varying coefficients into consideration.

Ghysels, Hall and Lee (1996) suggest their approach as a generalization of the Hylleberg et al. (1990) testing procedure to take the presence of unit roots into account at the zero and seasonal frequencies in periodic AR models as in the approach by

Franses (1994) who considers the seasonal integration hypothesis in a periodic context and a Monte Carlo simulation evidence sheds light upon the advantages of taking periodicity into consideration in testing for unit roots in seasonal time series. In Franses and Vogelsang (1995), the problems of testing for seasonal unit roots (as the extension of HEGY procedure) in the presence of a single break in each season in a specific year are considered. If mean shifts are not included, it is expressed that there is an evidence of seasonal unit root at the bi-annual frequency. However, if seasonal unit roots are tested in the AO (additive outlier) or IO (innovative outlier) model, the evidence for seasonal unit root is said to vanish for quarterly U.S. industrial production data**.** Smith and Otero (1995) mention about how exogenous changes in the level or seasonal pattern of a series have an influence on the HEGY testing procedure for seasonal integration. In this study, it is expressed that the relative position of the “break” can influence the power of the seasonal unit root test statistic substantially and a change in the seasonal pattern has an adverse effect on the seasonal unit roots while a change in the level of the process does not affect them. As the size of the break increases, the ability of a unit root test to discriminate stationarity from nonstationarity is expressed to weaken. In addition, it is specified that for the sample size that is greater than or equal to 100 ($T \geq 100$), the power of the seasonal unit root test to reject the null hypothesis is 100% because the spectrum of the series at seasonal frequency is not influenced from this.

Kawasaki and Franses (1996) propose an alternative approach to determine the number of seasonal unit roots for a large set of quarterly macroeconomic variables by analysing versions of the basic structural model. Through Monte Carlo simulations, they conclude that their method operates very well with regard to having good size and power properties and has a tendency to detect more seasonal unit roots compared to the HEGY method.

Franses and Hobjin (1997) present critical values for a variety of unit root tests in seasonal time series by considering the extensions of Hylleberg et al. (1990) and Osborn, Chui, Smith and Birchenhall (OCSB) (1988) procedures that concern time series with increasing seasonal variation and structural breaks in the seasonal means (in the case of known break point).

Alexander and Jordá (1997) present an empirical research concerning the presence of seasonal unit roots at different frequencies in trade variables for Germany, France, the U.K. and Italy with both quarterly and monthly data by applying to the HEGY test. The findings have shown that the presence of unit roots at most seasonal frequencies is rejected more often in quarterly data than in monthly data; there has been a weak evidence of seasonal unit roots on German trade balance and French industrial production series; no seasonal unit roots have been found in the U.K. series and Italy has been shown as the only country displaying seasonal unit roots in all its three variables. As a conclusion; although the presence of seasonal unit roots appears more apparently in monthly data than in quarterly data, it is expressed that it cannot be referred to the presence of a strong seasonal integration in trade variables of countries in question.

Leong (1997) presents an empirical study focusing on the nature of the seasonality and testing for the presence of seasonal unit roots using HEGY procedure for quarterly observed Australian macroeconomic data (total exports, total imports, expenditure-based GDP, retail trade turnover, total unemployed persons and manufacturers' actual sales for clothing and footwear) and finds that although total exports and total imports include seasonal unit roots, other analysed macroeconomic variables do not have a seasonal unit root and it is seen that the variables exhibit deterministic fluctuations besides stochastic seasonality.

Breitung and Franses (1998) propose a semiparametric "Phillips-Perron (PP) type" extension of the HEGY testing procedure in order to account for serially autocorrelated errors. By following Schmidt and Phillips (1992), Breitung and Franses (1998) have covered score-type tests for integration at seasonal frequencies. As a result of their Monte Carlo simulations it is concluded that since the semiparametric version may suffer from an enormous size bias for some situations, these tests cannot be preferred for general use; from another point of view, in case the parametric tests necessitate a high augmentation lag it is stated that the semiparametric version may be more powerful than the parametric test of HEGY.

Paap, Franses and Hoek (1997) deal with choosing between the deterministic seasonal mean shift model and the seasonal unit root model from a forecasting perspective and the effects of neglecting and allowing for seasonal mean shifts on the forecasting performance through simulation. According to simulation results, it is shown that taking possible deterministic seasonal mean shifts into consideration can create an improved forecasting performance.

In the paper proposed by Rodrigues and Osborn (1999), the empirical performances of Dickey et al. (DHF) (1984), Osborn et al. (OCSB) (1988) and Hylleberg et al. (HEGY) (1990) tests are examined for monthly time series. Although the DHF and

OCSB tests impose restrictions on the autoregressive processes which are not tested explicitly, in case these restrictions are true these tests are said to improve power properties and become preferable with respect to size and power. Balcombe (1999) presents the extensions of the HEGY testing procedures following the sequential approach of Zivot and Andrews (1992) and expresses that the traditional HEGY tests can give rise to low power under the alternative of a zero frequency unit root with structurally unstable deterministic seasonality. An application of the sequential tests to U.S. agricultural price data and macroeconomic data has pointed out to the rejection of seasonal unit roots in all series in question. Rodrigues and Osborn (1999) suggest pretesting the restrictions before applying the seasonal unit root tests. However, since there is no certainty about whether the usual distributions will operate under the seasonal unit root null hypothesis, this pretesting is expressed to entail a further study.

Gil-Alana (1999) considers the different versions of tests of Robinson (1994) in order to test for unit roots and other fractionally integrated hypotheses being settled at the zero and/or at the seasonal frequencies on the interval $[0,\pi]$ with monthly data. A Monte Carlo experiment conducted to control the power of the tests against different fractional alternatives shows that in case there are adequately large numbers of observations, the tests work well in a reasonable manner and an application to the CPI in Spain implies the presence of a single unit root at the zero frequency.

Psaradakis (2000) suggests bootstrap tests for unit roots in a seasonal autoregressive model and the finite sample performance of these tests are examined through simulations.

Shin and Oh (2000) present semiparametric tests that are the extensions of the seasonal unit root tests for the model of Dickey et al. (1984) and based on the feasible generalized least squares estimator instead of the ordinary least squares estimator.

Hamori and Tokihisa (2000) have aimed to analyse whether there exists seasonal integration in Japanese macro data or not for the targeted variables of GDP and its components from 1955:Q2 to 1996:Q1. It is concluded that if the seasonal integration test is applied without allowing for a structural break, the existence of seasonal integration is more likely in terms of the real variables and the evidence is in the direction that no noteworthy seasonality exists in the deflators when compared with the real variables. In their paper, Hamori and Tokihisa (2001) analyse the stability of Japanese money demand function using seasonal integration and seasonal cointegration and they find that there exist unit roots in money balances, interest rates and real GDP series in different cycles. Because of the rejection of seasonal cointegration in every case, it is also expressed that there is no stable relationship between money supply and the real economy for the period under consideration.

Gil-Alana and Robinson (2001) discuss the seasonal behaviour of quarterly U.K. and Japanese consumption and income series from an autoregressive unit root viewpoint using the tests of Robinson (1994). They conclude that resorting to seasonal fractional integration is a reasonable alternative for modelling these series.

The article proposed by Taylor and Smith (2001) deals with the problem of testing for a nonstochastic seasonal unit root in a seasonally observed time series process against a randomized seasonal unit root hypothesis (in other saying, a seasonal heteroscedastic integrated alternative).

Harvey, Leybourne and Newbold (2001b) have tried to analyse the behaviour of AO and IO tests for seasonal unit roots in the presence of seasonal mean shifts under the null hypothesis for quarterly data using Monte Carlo simulation. Simulation studies are expressed to show that the use of innovational outlier test with a break date selection that is based on the significance of shift dummy variables may result in an erroneously estimated break point, leading to spurious rejection of the null. Also, Ghysels (1994) presents a study on the effect of seasonal mean shifts on seasonal unit root testing.

Harvey, Leybourne and Newbold (2001a) have analysed the performance of unit root tests that allow for an endogenously determined break in level.

da Silva Lopes (2001) presents a comparison of the power properties of the tests proposed by Dickey et al. (1984), Osborn et al. (1988) and Hylleberg et al. (1990) for the seasonal differencing filter in the presence of seasonal mean shifts.

Rubia (2001) presents the extension of HEGY testing procedure to analyse the weekly seasonality of the daily electricity demand series quoted in several deregulated electricity markets and the evidence shows that the Spanish, Argentine and Australian electricity markets exhibit different seasonal patterns.

Kunst and Reutter (2002) present a combination of seasonal unit root tests in which some of them have absence of unit roots while other tests employ the presence of unit roots as their null hypothesis and they evaluate the outstanding qualities of such seasonal unit root test combinations founded upon a pseudo-Bayesian structure which strayed from the cited study of Hatanaka (1996) or Hylleberg (1995).

Osborn and Rodrigues (2002) present a general approach for derivation of the asymptotic distributions of various seasonal unit root tests, including those of Dickey et al. (1984), Osborn et al. (1988), Hylleberg et al. (1990), Franses (1994), Ghysels et al. (1996) and Kunst (1997). Their unifying approach reveals that the asymptotic distributions of all these test procedures depend on the same vector of Wiener processes (i.e., the elements of the vector Brownian motions which are composed of the s distinct processes implied by the null hypothesis that the s observations within each year follow independent integrated processes in an asymptotical manner). This dependence leads to the conclusion that in order to generate all critical values, linear transformations of a single set of replications of the underlying process can be utilized instead of applying separate Monte Carlo simulations for each test. In addition, in this paper the OCSB and DHF test regressions are referred to be restricted forms of the Kunst and HEGY regressions which require nonstandard distributions and F-tests from the latter ones are shown to be exactly equivalent.

In her paper, Çağlayan (2003) investigates the presence of seasonal unit root for the monthly series of personal consumption expenditures made to non-durable and semi-durable goods and services, per capita disposable income and stock market returns that are concerned with the life-long permanent income hypothesis over the period 1988:01-2000:04 and examines if cointegration exists among given variables by using HEGY procedure. In her study, the presence of seasonal unit root has been found in consumption expenditures and disposable income series for both 0 and ¼ frequencies and in stock market returns series for ¼ frequency. Also, it is concluded that consumption expenditures and disposable income variables are cointegrated at zero frequency.

Kadılar and Erdemir (2003) focus on the problem of determining the lag number of multivariate seasonal models and express that in the case of seasonal patterns, using AIC gives rise to a poor performance in order to select the order of the seasonal vector autoregressive (SVAR) models. To overcome this problem, they develop a seasonally modified AIC which performs better than the usual AIC.

Gil-Alana (2003) has tried to construct confidence intervals for the seasonal fractional differencing parameter for several measures of the U.S. monetary aggregate by means of fractionally integrated techniques following the tests of Robinson (1994). In the study, it is utilized from quarterly and seasonally unadjusted time series data for the period of 1960Q1:1998Q4 and it is expressed that the conclusion is in the direction of the rejection of seasonal unit roots in favour of smaller integration degrees.

Gil-Alana (2004) introduces a version of the tests of Robinson (1994) that enable to test different orders of integration at zero and each seasonal frequencies applying to the Italian consumption and income series and in the study results it has been given emphasis to the importance of the long run (zero) frequency for both consumption and income series but the seasonal frequency π in case of the differences.

da Silva Lopes and Montanes (2005) analyse the behaviour of HEGY seasonal unit root tests for quarterly time series in the presence of seasonal mean shifts.

Lucey and Whelan (2004) investigate the monthly and half-yearly seasonality of the Irish equity market in the long term and show that the Irish equity market displays a month-of-the-year effect with a January peak, in addition April effect and semi-annual seasonality.

Rodrigues and Franses (2005) introduce a sequential seasonal unit root testing procedure for high frequency data focusing on quarterly and monthly data. According to simulation results, it is shown that their new sequential approach is more powerful than the traditional HEGY procedure, particularly in small samples.

Ayvaz (2006) investigates the seasonal behaviours of Gross National Product (GNP), consumption, export and import series in Turkish Economy using HEGY procedure and tries to detect the presence of stochastic or deterministic seasonality for these quarterly data for the period 1989:Q1-2004:Q4. The evidence has shown that consumption series displays stochastic seasonality, GNP and export series include seasonal unit roots at semi-annual and annual frequencies. In addition, imports series is expressed to have a non-seasonal unit root (at zero frequency).

Coşar (2006) has tried to examine the seasonal properties of the Turkish consumer price index (CPI) through Beaulieu and Miron’s (1993) extension of the classical HEGY test developed by Hylleberg et al. (1990) and the LM-type CH seasonal unit root test procedures with the aim to specify the seasonality accurately in econometric models. In the Coşar’s (2006) study, there has been an evidence of both deterministic and nonstationary stochastic seasonality in the CPI series of Turkey.

Gagea (2007) studies the identification methods of the nature of the seasonal component of Romania’s quarterly exports between 1990-2006 by using HEGY seasonal unit root testing procedure. Conducted test shows that the seasonal component may be deterministic, stochastic or mixed and since the deterministic seasonal component situation seems to be rather weak; the appropriate filter to eliminate seasonal variations is expressed to be the seasonal difference operator $(1-L^4)$.

Otero, Smith and Giulietti (2007) deal with testing for seasonal unit roots in heterogeneous panels in the presence of cross section dependence.

In their study, Caporale and Gil-Alana (2008) introduce a version of the tests of Robinson's (1994) procedure that is appropriate for testing the integration order of the trend and seasonal components of a time series at the same time. The tests enable to test for both unit and fractional degrees of integration. An application of the tests to monthly non-seasonally adjusted data on four U.S. monetary aggregates results in the presence of a unit root at the zero frequency together with possibly fractional values for the monthly component for all series.

Tasseven (2008) presents the extension of HEGY procedure based on an IO model for testing seasonal unit roots by considering seasonal mean shifts in more than one year with exogenous break points. Following the study of Franses and Vogelsang (1995), Tasseven (2008) applies to double break points considering the 1994 and 2001 major financial crises. Apart from seasonal unit roots, the study allows for the effects of shocks to the system such as policy interventions or other crises which can affect the domestic macroeconomic developments. Based on the empirical money demand model for Turkish economy for the 1986:1 – 2003:1 period; the GDP deflator, real M2 and the expected inflation variables are found to contain seasonal unit roots and in case the possible structural changes are taken into consideration, seasonal unit roots are seen to disappear for the real M1 balances. Proietti (2002) introduces a class of seasonal specific structural time series models in the context of unobserved components framework focusing on the time domain representation rather than the frequency domain.

Jiménez-Martin and Flores de Frutos (2009) propose a new equilibrium model of the exchange rates which takes seasonal shocks in preferences into account for five industrialized countries using seasonally unadjusted data and which makes a generalization of standard dynamic equilibrium models of exchange rates. The proposed model explains how agents smooth seasonal movements in fundamental variables for their investment decisions although the fundamental variables explaining exchange rates exhibit seasonal fluctuations.

Shin and Oh (2009) deal with testing for seasonal unit roots for each seasonal frequency in panel models of cross-sectionally correlated time series in the basis of the instrumental variable estimation.

Khedhiri and El Montasser (2010) try to build up the asymptotic theory of the test of Lyhagen (2006) -who presents an extension of the KPSS framework to the seasonal case- in the time domain in the presence of AO. According to Monte Carlo studies conducted to examine the finite-sample performance of the seasonal KPSS test, it is concluded that the seasonal KPSS test has good power properties.

Harvey (2011) deals with modelling the inflation-output gap relationship by using unobserved components.

Kunst and Franses (2011) deal with the problem of testing for seasonal unit roots in monthly panel data through the generalization of the quarterly cross-sectionally augmented HEGY test to the monthly case.

In her study, Ayvaz Kızılgöl (2011) has examined if GDP, export, consumption and investment series have seasonal unit roots and display a seasonal cointegration relationship by using quarterly series for the period 1987Q1-2007Q3. For this aim, Ayvaz Kızılgöl (2011) has utilized from HEGY (1990) and Engle, Granger, Hylleberg and Lee (1993) tests. In the study, it is concluded that there is no seasonal cointegration relationship between series at zero and biannual frequencies. However, a seasonal cointegration relationship has been detected between gross domestic product and consumption series at ¼ (and ¾ frequency) for the model with intercept and seasonal dummy variables.

In the paper suggested by Chirico (2012), Italian daily electricity price data in the years 2008-2011 are analysed in order to detect the type of seasonality for the application of ARIMA (Autoregressive Integrated Moving Average) modelling. When HEGY test is performed on the sub-periods 2008-2009 and 2010-2011, it is concluded that 2008-2009 prices are seen to display a random walk movement contrary to 2010-2011 daily prices that do not include such a movement. In addition, the seasonality features non-stochasticity in both sub-periods pointing out to the absence of seasonal unit roots and thus the presence of deterministic seasonality in the short run.

In their study, Gürel and Tiryakioğlu (2012) have analysed the seasonal patterns of the seasonally unadjusted quarterly Turkish Industrial Production Index estimated by the Turkish Statistical Institute (TURKSTAT) and the sub-sectors of the mining industry, the manufacturing industry and electricity, gas and water sectors at constant

1997 prices over the period 1977:1–2008:4 by using the HEGY approach. The main findings have shown that all these four series contain seasonal unit roots at long-run (zero) frequency indicating to the presence of non-seasonal unit roots and the electricity and total industry production series are not stationary at each seasonal frequency. According to the evidence, the presence of both deterministic and non-stationary stochastic seasonality has been detected in the Turkish manufacturing industry series.

Tıraşoğlu (2012) has carried out HEGY procedure for the series that are composed of CPI and its expenditure groups. Important results of the study are those: all series have unit roots at zero frequency and at semi-annual frequency unit root exists for CPI and its some expenditure groups.

The aim of the paper proposed by Meng and He (2012) is to propose a HEGY-type test based on the study by Hylleberg, Engle, Granger and Yoo (1990) in order to test seasonal unit roots in data with other frequencies not studied until that time such as hourly and daily data. In their study, Meng and He (2012) present the asymptotic distributions of the HEGY-type test statistics by following the work of Beaulieu and Miron (1992b), Chan and Wei (1988) and Hamilton (1994) and critical values for hourly and daily data at different frequencies. The study reveals that the HEGY-type test for hourly data suffers from the size distortion problem depending on the presence of the negative strong seasonal MA component in the series. Meng and He (2012) have tried to detect the presence of seasonal unit roots in hourly wind power production data in Sweden in warm season and cold season separately for 2008-2009 years and compare the performance of their test when deterministic components are included or not. For these separate two series, they conclude that are no seasonal unit roots in both series; however, zero frequency unit root exists in both. Regarding the size and power properties of the HEGY test, they also show that the smallest size distortion is satisfied when lag augmentations in auxiliary regression are included without lag elimination and tests with seasonal dummies included in auxiliary regression have more power than the tests without seasonal dummies.

Meng (2013) proposes corrected test statistics in order to test seasonal unit roots in the case of serially correlated residuals of the HEGY test equation following the commonly used PP unit root test technique. As a result of the simulation studies, Meng (2013) compares the corrected statistics and commonly used HEGY test statistics and expresses that the former for monthly data has more power when compared to the latter.

In Rodrigues, Rubia and Valle e Azevedo (2013), the finite sample properties of the frequency-domain test suggested by Robinson (1994) and its time-domain equivalent suggested by Hassler, Rodrigues and Rubia (2009) are compared in order to test for seasonal integration in fractional context. Montasser (2011) focuses on the performance of the overall Fisher statistics of the Kunst and HEGY tests for seasonal integration by utilizing from the procedure introduced by Osborn and Rodrigues (2002) when the DGP exhibits a non-stationary alternative treated by del Barrio Castro (2006). According to simulation results, Kunst F-type statistic has been found to keep up high power in case all unit roots implied by the filter in question are not present. In the study, it is also concluded that the augmentation of the regression model of the test with lagged dependent variables maintains these high power properties. For the frequency-domain test, Rodrigues et al. (2013) make an extension of the analysis in Gil-Alana (2000).

In their study, Hindrayanto, Aston, Koopman and Ooms (2013) have tried to examine the dynamic properties of the frequency-specific basic structural models for seasonal time series in which the time-varying trigonometric terms associated with distinct seasonal frequencies have different variances for their disturbances.

Cellini and Cuccia (2014) investigate the seasonal processes that the Euro-U.S. dollar exchange rate exhibit over the period January 1999 (starting from birth of Euro) to December 2012. This study indicates to the statistical significance of specific month effects in the first-difference form of exchange rate and heterogeneity in their variance across months to a noteworthy extent. The evidence shows the presence of significant seasonality in the form of both day effect and month effect. In the study, U.S. dollar has been found to be inclined to appreciate with respect to the Euro in January while the Euro-Dollar exchange rate displays higher daily variability in December and lower variability in January, other things being equal. On the other hand, there has been no evidence of structural instability in the exchange rate level dynamics.

Alves (2014) considers the scope to which seasonal variations are present in the performances of 5349 Equity Europe or Equity Eurozone investment funds. Considering worldwide, results of the study indicate to the greater performance in the intermediate and final months of each quarter when compared to the first month.

del Barrio Castro, Osborn and Taylor (2014) have tried to discover the small sample performance of diverse methods in order to detect the lag augmentation polynomial in a HEGY seasonal unit root test regression using Monte Carlo methods and whether the results are improved by using seasonal generalized least squares detrending developed by Rodrigues and Taylor (2007) or not. They have made an extension of the modified information criteria of Ng and Perron (2001) to the seasonal unit root testing context applying for lag specification with both ordinary least squares (OLS) and generalized least squares (GLS) detrending. In the study, it is concluded that in the proper use MAIC (modified AIC) and AIC present more reliable size compared to the lag selection methods on the basis of hypothesis testing or BIC (Bayesian Information Criterion) and the presence of seasonal unit roots at the semi-annual and annual frequencies is rejected very often by the results with OLS detrending than the ones with GLS detrending. On the other hand, Kunst (2014) has presented a paper considering a combined nonparametric test for seasonal unit roots.

Ben Zaied and Binet (2015) deal with modelling seasonal effects of residential water demand for quarterly data from 1980 to 2007 utilizing from seasonal integration and cointegration. As a result, they touch on the important role of seasonality in modelling residential water demand.

CHAPTER III

METHODOLOGY

3.1. Introduction to the Concept of Time Series

Before referring to the concept of seasonality which is a component of time series, the concept of time series and its components will be introduced.

A time series is a set of random variables indexed in time, $\{X_1, X_2, \ldots, X_T\}$. From this point of view, an observed time series is denoted by $\{x_1, x_2, \ldots, x_T\}$, where the sub-index represents the time to which the observation x_t is relevant. For instance, the first observed value x_1 gives the realization of the random variable X_1 and in the same manner, x_2 is the realization of X_2 and so on. The characterization of a T-dimensional vector of random variable is possible by different probability distribution (Cholette & Dagum, 2006, p. 15). Time series are measured at regular intervals of time, generally monthly or quarterly, over relatively long periods (they are also mentioned as raw data, non-adjusted or original series). This enables us to reveal and analyse the behaviour of patterns and establish the current estimates in a more meaningful and historical perspective (Central Bureau of Statistics, 2011, chap. 2). Examples are available in various fields: the annual crop yield of sugar beets and their price per ton for instance are recorded in agriculture; daily stock prices, weekly interest rates or monthly unemployment rates are reported on newspapers' business sections; social sciences make researches on annual death and birth rates, the number of accidents in the home and various forms of criminal activities; meteorology reports hourly wind speeds, daily maximum and minimum temperatures and so on. Time series analyses contain methods that attempt to have a better understanding of the data generating mechanism or to predict future events based on known past events (Falk et al., 2011, p. 1).

Generally, methods for time series analyses are separated into three classes as descriptive methods, time domain methods and frequency domain methods. Descriptive methods include the decomposition of an observed time series into trend, seasonal, cyclical and irregular components (Iwueze, Nwogu, Johnson & Ajaraogu, 2011, p. 633). Although time domain and frequency domain approaches operate in a distinct way, they are complementary procedures which are related mathematically. In time domain methods, data series are tried to be characterized in the same terms in which they are observed and reported and characterization of relationships between data values is feasible through the autocorrelation function (ACF). Also, different time domain procedures are available for discrete and continuous data (Wilks, 2006, p. 339). Frequency domain methods centre on spectral analysis which is a procedure to estimate the spectral density function. This type of function is helpful in describing how the variation in a time series may be accounted for by cyclic components at different frequencies (Chatfield, 2004, p. 8). So, in frequency-domain analysis the overall time series is considered to emerge from the combined effects of a collection of sine and cosine waves oscillating at different rates. The original data is generated by the sum of these waves. However, the primary interest is on the relative strengths of the individual component waves. Briefly, frequency-domain analyses occur in the mathematical space described by this aggregation of sine and cosine waves (Wilks, 2006, p. 339). For the relation between frequency-domain and time-domain analyses also see Warner (1998).

A time series has a characteristic feature that the data are *not generated independently*. Data are frequently under the control of a trend effect and contain cyclic components. Hence, statistical methods assuming *independent and identically distributed* data are removed from the time series analysis. For this reason, proper methods are needed under the heading of *time series analysis* (Falk et al., 2011, p. 1).

3.1.1. Components of Time Series

The components of a time series consist of various elements that can be separated from the observed data. These components can be classified in a wide manner as shown in Figure 1. Briefly, a time series includes four components:

1) Secular Trend (T): Trend or secular trend shows the long term-tendency of the time series to move in an upward or downward direction and reflects how its behaviour is over the entire period being analysed. It does not contain any short term variations such as seasonality, irregularity etc. (Jain & Jhunjhunwala, 2007, pp. 8.2-8.3). In case a time series does not include any trend component, then the data is said to be stationary (Gaynor & Kirkpatrick, 1994, p. 80).

2) Seasonal Variations (S): These variations are the part of the variations in a time series which indicate the regular periodic changes occurring within a period of less than a year and which may be observed as daily, weekly, monthly or quarterly (Jain & Jhunjhunwala, 2007, p. 8.3). Therefore, the fluctuations recurring every year with more

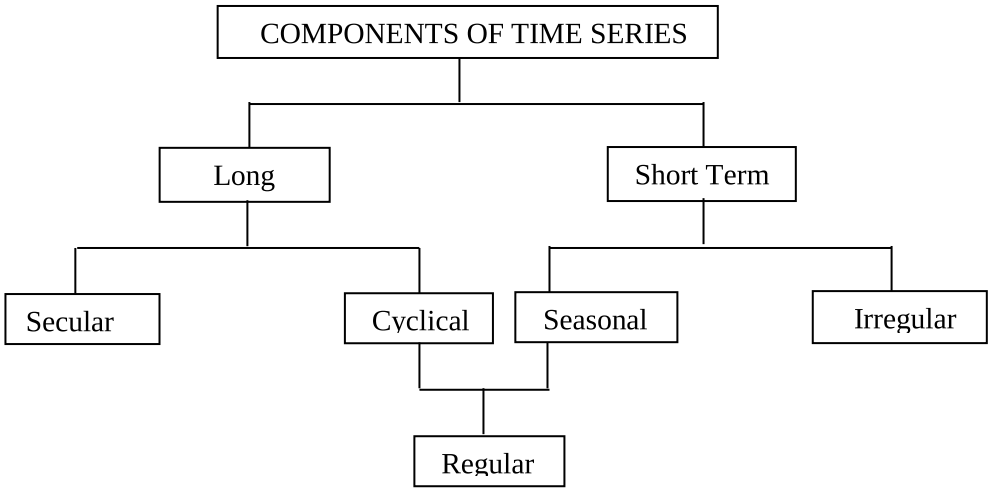

Figure 1. Components of time series (Source: Jain & Jhunjhunwala, 2007, p. 8.2)

or less the same timing and intensity generate a seasonal pattern. To give an example, the demand for cold beverages becomes low during the winter, starts to increase during the spring, peaks during the summer months and then starts to decline in the autumn or pizza delivery peaks on the weekends (Collier & Evans, 2010, p. 196). Systematic calendar related effects as one type of component of a time series include both seasonal and calendar effects. Seasonal fluctuations can be stemmed from climatic variations like summer, winter or rainfalls; administrative or legal measures like starting and ending dates of school year or fiscal year; social/cultural/traditional/religious and calendar-related effects (e.g., the timing of certain public holidays such as Christmas, Valentine's Day; variation in the length of months and quarters depending on the nature of the calendar). Other calendar effects are associated with the factors not necessarily happening in the same month (or quarter) each year (Central Bureau of Statistics, 2011, chap. 2). These effects are given as trading day effects and moving holidays:

*** Trading Day Effects:** It is well known that from year to year the number of weekdays of months differs so that there are always four of each weekday and a few additional days in each month. Therefore, some days recur more often than others and the availability of such an effect (how often each of the seven days of the week occurs in each month) has an influence on the volume of economic activity, particularly in retail sales and stock market activities. For instance, the retail grocery sales volume in U.S. is smaller on Mondays, Tuesdays and Wednesdays than on days later in the week. So, in a year in which March, say, has an excess of early weekdays and higher when March has five Thursdays, Fridays and Saturdays; there will be proportionately low sales volume sales in this month in a year. Besides this, since the length of February is not the same in each year, this leap year effect which is not incorporated into the seasonal component comprises trading day effects along with recurring day-of-week effects and when these effects are disregarded in case they have significant size, modelling and forecasting are obstructed to an extreme degree. For this reason, it is of great importance to detect, estimate and remove such effects during the seasonal adjustment process (see Bleikh & Young, 2014, p. 45; Soukup & Findley, 2000)

*** Moving Holidays:** These are holidays like New Year celebrations and religious holidays whose exact timing shift over the year by affecting economic activity and should be taken into consideration in time series analysis and forecasting. One example to this is Easter which can happen in either March or April (Bleikh & Young, 2014, p. 45). It can be said that calendar-related systematic effects related to the dates of moving holidays are not considered as seasonal, since they happen in different calendar months depending on the dates of the holidays (Central Bureau of Statistics, 2011, chap. 2).

2.1) Stable and Moving Seasonality: During the period being analysed, in case seasonal pattern is unchanged or it remains virtually the same in time, in magnitude and shape; the series in question is said to display stable seasonality. If the seasonal pattern changes gradually over time in amplitude or shape, or both, the series is said to have moving seasonality. The causes for this type of seasonality may be a gradually evolving seasonal pattern as economic behaviour, economic structures, technological advances, and institutional and social arrangements change. To give an example, the magnitude of the seasonal component for agriculture series may show a gradual decrease which is stemmed from the technological changes that reduce the effect of weather on growth and sales of fruits and vegetables (Central Bureau of Statistics, 2011, chap. 2 - p. 5). In case there is too much moving seasonality, this could lead to inaccurate estimation of the seasonal component of the series (Branch & Mason, 2006, p. 14).

3) Cyclical Variations (C): These variations have a longer duration than a year and extend over long periods of two to fifteen years. However, they may not exhibit a regular periodicity. Generally, they are referred to as business cycles, which are the periodic movements in the time series around the trend line. These cycles do not occur at a uniform frequency. A cyclical variation consists of four phases: (i) prosperity (ii) recession (iii) depression (iv) recovery. Starting from prosperity phase, all economic activities are inclined to be at their peak (output, employment prices, investment, profit, etc. increase). After prosperity phase, these activities begin to fall and recession starts. And then this fall comes to the lowest level, namely depression level. In this level, the production of goods and services, employment, income, prices etc. show a significant drop. Then, the recovery phase starts for economic activities. Therefore, the cycle becomes completed from period of one boom to another boom. It should be noted that business cycles do not have a regular period as the period of various seasons. For this reason, they are called cyclical fluctuations rather than periodic fluctuations. The study of business cycles enables us to frame suitable policies for stabilizing the level of business activity, examine the characteristics of fluctuations of a business and forecast and estimate the future behaviour.

Table 1
Distinctions between Seasonal Variations and Cyclical Variations

	Basis of Distinction	Seasonal Variations	Cyclical Variations
1.	Causes	The causes of seasonal variations are seasons, festivals, weather conditions, customs, traditions.	The causes of cyclical variations are disparity between demand and supply, working of the economic system.
2.	Duration	These variations occur during less than a year.	These variations occur during different periods of two to fifteen years.
3.	Area of occurrence	These variations happen in any economy.	These variations generally occur in capitalists economy.
4.	Periodicity	These variations are less powerful.	These variations are more powerful.
5.	Degree of Accuracy	These variations can be estimated with a high degree of accuracy.	These variations cannot be accurately estimated because of lack of their regularity.
6.	Regularity	There is regular periodicity in these variations.	There is no regularity in the periodicity of these variations.
7.	Activities of Preceding Variations	These variations do not depend on the activities of preceding period.	These variations depend upon the activities of preceding period.

Source: Jain & Jhunjhunwala, 2007, p. 8.4.; Jain & Sandhu, 2006-07, pp. 8.5-8.6.

4) Irregular or Random Variations (I) (sometimes called white noise): These are erratic fluctuations in a time series which do not have a definable pattern. They are mostly stemmed from the effect of “outside” events on the data and occur at once or unexpectedly in a time series. Thus, the main causes can be considered as strikes, floods, wars, earthquakes etc. Since there is no regularity in their periodicity, they are called random or chance fluctuations and so they are unpredictable by their nature. To a usual extent, they indicate short term variations and because of their irregular structure, isolating these variations from the time series completely becomes complicated. (Gaynor & Kirkpatrick, 1994, p. 80; Jain & Sandhu, 2006-07, p. 8.6).

3.1.2. Time Series Models

The relationship among the different time series components can be expressed through either the *additive* or *multiplicative* models:

1. Additive Model: According to the additive model, all the components of time series are not affected by one another and they are expressed in absolute values. Let Y be represented as observed value in a given time series (original data). In this case, Y can be expressed as a sum of four components in the following way:

$$Y = T + C + S + I \tag{3.1}$$

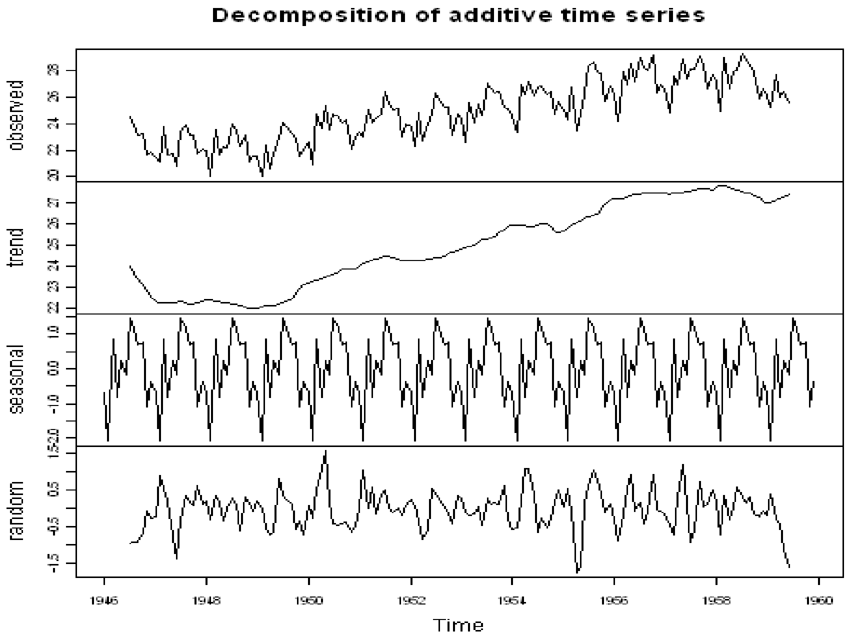

Figure 2. Decomposition of additive time series (Source: Using R for Time Series Analysis)

2. Multiplicative Model: A multiplicative model assumes that the trend, seasonal, cyclical and irregular components are represented by a multiplicative function. This model is widely used in practice. Its assumption is that all the four components mentioned above are not necessarily independent and they can be influenced by one another. In this model, only the trend (T) component is expressed in absolute value while other components are expressed as rate or percentage. Therefore, Y is modelled by:

$$Y = T \times C \times S \times I \tag{3.2}$$

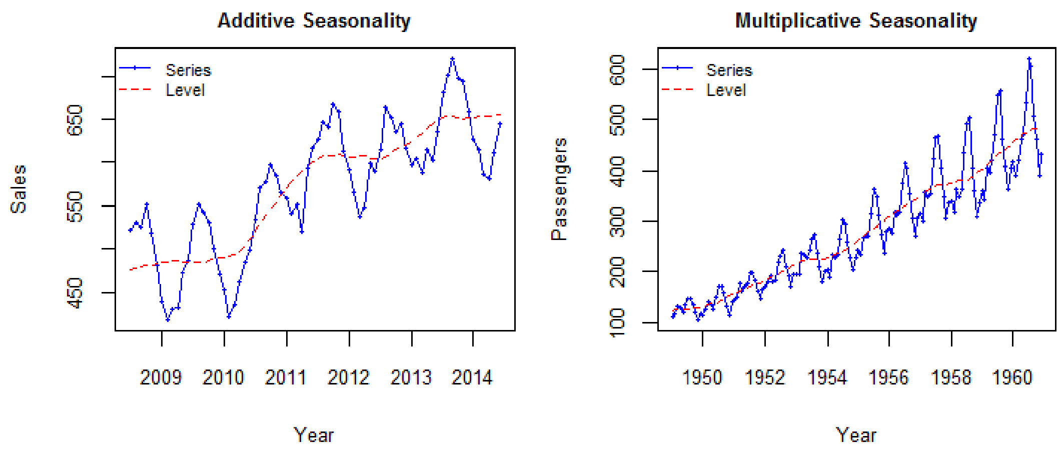

Figure 3. Graphical representations of additive and multiplicative seasonality (Source: Forecasting Society, 2014).

To summarize, diverging aspects of two types of models from one another is given as:

Table 2

Distinctions between Additive and Multiplicative Seasonality Models

Basis of distinction	Additive Model	Multiplicative Model
1. Basic Assumptions	It assumes that all the four components of time series are independent of each other.	It assumes that all the four components of time series are due to different causes but they are not necessarily independent and they can affect one another.
2. Expression	$Y = T + C + S + I$	$Y = T * C * S * I$
3. Absolute values/rates	All components of a time series are expressed as absolute values.	Only trend (T) is expressed as an absolute value while other components (S,C,I) are expressed as rate or percentages.

Source: Jain & Jhunjhunwala, 2007, p. 8.5.

Note: Unfortunately, many real-life time series are not classified as additive or multiplicative models. It is usual to come across a time series corresponding to a multiplicative model. However, it should not be multiplied with some type of irregular component. In this case, a more realistic model is given as a mixed multiplicative model with an addition of the irregular component to other components, namely a *pseudo additive model* in equation (3.3):

$$Y = (T \times C \times S) + I \tag{3.3}$$

Apart from the pseudo additive model given above, a *pseudo multiplicative model* is also likely to fit some real-life data most closely given as:

$$Y = T \times (C + S + I) \tag{3.4}$$

(Davis & Pecar, 2013, p. 189).

3.2. The Definition of Seasonality

Seasonality is a widespread phenomenon observed in many economic time series. Everyone knows what it is. However, it is not common to think about an applicable definition of it. It is certain for the concept of seasonality that there should be something like 'a systematic intra-year movement' in its definition. At this point, it is of great importance to consider the causes of such a systematic movement with regard to finding out how systematic it is. In Hylleberg (1992), the concept of seasonality is defined as: "Seasonality is the systematic, although not necessarily regular, intra-year movement caused by the changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by the agents of the economy. These decisions are influenced by endowments, the expectations and preferences of the agents, and the production techniques available in the economy." (Hylleberg, 1992, pp. 3-4).

3.2.1. What is Seasonal Adjustment?

The most well-known statistics like Balance of Payments (BOP) and Gross Domestic Product (GDP) are regular time series. By analysing those series, it is tried to get an idea about the general pattern of the data, the long term movements, and whether any unusual occurrences have had major effects on the series. However, this type of analysis is not free from ambiguity when studied with original series. Since, some short-term effects associated with the time of the year which obscure other movements are inevitable to occur. For instance, retail sales rise each December depending on Christmas. The main aim in applying to seasonal adjustment is to remove common seasonal fluctuations and typical calendar effects in the time series mentioned. Or in other saying, we can say that the nonseasonal fluctuations are filtered from the raw data with seasonal adjustment. The seasonally adjusted series are frequently made available to the public and mentioned in financial press. Getting a seasonally adjusted series requires subtracting from (or divide) the raw series (by) an estimate of the seasonal component. When we adjust the original data for these effects, this makes the comparisons between consecutive time periods more effectively. (Ghysels & Osborn, 2001, pp. 1-2; Office for National Statistics, 2007, chap. 2). The concept of seasonal adjustment brings with it an important question of whether unit root tests should be applied to seasonally adjusted or seasonally unadjusted data. In their study, Ghysels and Perron (1993) draw attention to that although correlation in the data at seasonal frequencies is removed by seasonal adjustment, seasonal adjustment introduces a bias in the ACF at lags less than the seasonal period which does not disappear even asymptotically. Therefore, in case seasonally adjusted (or filtered) data are used, there will be a bias in ADF and PP statistics toward non-rejection of the unit root. Briefly, in an asymptotical manner if worked with unadjusted data rather than adjusted data, the unit root tests are expected to be more powerful (Maddala & Kim, 1998, pp. 364-365).

3.2.2. Tests for Seasonality

In their paper, Cellini and Cuccia (2011) deal with the seasonality of monthly time series of bilateral nominal exchange rates. In order to detect the presence of seasonality, for each of the considered monthly time series, they examine the following tests:

1) The D8 F-test for evaluating the presence of stable seasonality, F_S ; F_S is a one-way analysis-of-variance test which measures the degree of stability of the seasonal component of a time series. Basically, this test is based on the quotient of two variances: the between-month variance and the residual variance and it checks for the equality of monthly means with the hypotheses given as,

$$H_0 : m_1 = m_2 = ... = m_{12}$$

$$H_1 : m_p \neq m_q \text{ for at least one pair (p,q)} \tag{3.5}$$

where $m_1,......., m_{12}$ are the monthly means of the seasonal irregular (SI) component (the detrended series) taken place in table D8. The assumption of this test is that SI values are independently normally distributed with means denoted by m_i and common standard deviation given by σ. This assumption could be true to a certain extent for the underlying true SI ratios in a conceptual manner. However, it does not hold for the estimates of the SI ratios which are in fact dependent and heteroscedastic and thus has an influence on the behaviour of the resulting F-statistic. A traditional solution to this issue is using a cut-off value of 7 as critical value rather than using a critical value from the F-distribution (Lytras, Feldpausch & Bell, 2007, p. 848; US Census Bureau, 2010, p. 2). In case F_S is greater than 7.0, the null hypothesis of no stable seasonality is rejected and the series is regarded as seasonal (presence of stable seasonality). As a consequence, the series will be seasonally adjusted. By contrast, in case F_S is less than 7.0 this results in the acceptance of the null hypothesis (for instance, for quarterly time series this indicates that all four quarterly seasonal means are equal) implying that there is no seasonal variability in the data. In this case, the series will not be seasonally adjusted.

2) The Kruskall-Wallis statistic, K, which examines the equality of median values across different months (the implication of a value of this statistic falling into the rejection region is that median values are not constant across months). The Kruskal-Wallis test statistic, K is given as:

, $$K = \left[\frac{12}{N(N+1)} \sum_{i=1}^{n} \frac{R_i^{\,2}}{N_i} \right] - 3(N+1) \qquad (3.6)$$

where the *k* seasons are first ordered and assigned ranks (R_i) and

R_i is the rank total for the i^th^ season;

N_i is the number of observations in the i^th^ season;

N is the total number of observations; and

k is the number of seasons.

This test statistic has an approximate Chi-square distribution on *k* – 1 degrees of freedom (Paquette, 2009, pp. 17-18.)

3) The F-test for evaluating the presence of moving seasonality, F_M ; this test is applied to the sum of the seasonal and irregular components of the time series (that is, the series without trend and cyclical components) and is based on the quotient of two variances, the variance between years and the residual variance. With the null hypothesis of no moving seasonality, a test value falling into the rejection region indicates that the seasonal irregular component of the series is not stable across years; that is, the seasonal component is moving over years. Since moving seasonality can lead to distortions, this situation complicates the process of disentangling seasonality. On the other hand, the acception of the null means that *identifiable stable seasonality* is present.

All tests mentioned above are computed by the X-12- ARIMA program, which is a widely used program in applied economic analyses and provided by the U.S. Census Bureau for seasonal adjustment and this program is an enhanced version of the X-11 Variant of the Census Method II.

Getting a statistically efficient estimate of the seasonal component of a series may not be possible even in case a series is found to be seasonal according to the results of F test for evaluating stable seasonality. In other saying, this seasonal component may not be *identifiable* in a statistical manner. For deciding about the identifiability situation and evaluating the goodness of the de-seasonal procedure (or in other saying for deciding whether to seasonally adjust a series), apart from the F statistic for stable seasonality F_S , analysts use some quality control measures as guidelines like M7 statistic and Q statistic. M7 is one of the quality assessment statistics developed by Statistics Canada in the 1970s (see Lothian & Morry, 1978) and commonly preferred in applied economic researches. This statistic is calculated by using F_S and F_M statistics which represent the D8 F-statistics for stable seasonality and moving seasonality respectively (see Ladiray & Quenneville, 2001) and given as

$$M7 = \sqrt{\frac{1}{2}\left(\frac{7}{F_S} + \frac{3F_M}{F_S}\right)} \tag{3.7}$$

(Lytras et al., 2007, pp. 848-849). It varies over the interval [0,3] and the values that are smaller than 1 are an indicator for an accurate de-seasonal procedure (that is, the series has identifiable seasonality). Another quality measure is Q statistic. The Q statistic is a weighted average of 11 M statistics (that is, 11 quality measures from M1 to M11 describing the extent to which the seasonal decomposition is successful) that test for different kinds of problems concerning the overall quality of the seasonal adjustment like large variances, and the nonexistence of randomness in the irregular component. Evaluating estimates of the irregular and seasonal components is the main aim with the use of M-statistics. Both calculation and interpretation of all M values are given in detail in Lothian and Morry (1978). An irregular component of the series which is statistically random and neither too large nor too small relative to the remaining components and the series entirely and a stable seasonal component are preferred to be able to measure the seasonal component of a series precisely. Six of the M-statistics which are M1, M2, M3, M4, M5 and M6 measure the size of the irregular component compared to the other components. More obviously, M3 and M5 measure the size of the irregular component compared to the trend component. M4 measures the autocorrelation in the irregular component. M7 measures the extent to which the seasonal effect is identifiable. M8 to M11 quality measures deal with the extent to which the seasonal pattern changes. The smaller the value of a quality measure, the better the seasonal decomposition scores. The M statistics are normalized to 1.0. The implication of the values of M7 and Q that are less than 1.0 together is that the seasonality of a series being analysed is identifiable. A value of M7 that is greater than 1.0 is a sign of substantial moving seasonality. A value of Q that is greater than 1.0 indicates some kind of problems such as large variances, the absence of randomness in the irregular component or too much change in the seasonal component. In either case, we could not measure the seasonality of the series. Therefore, it can be said that in order to conclude that the deseasonal series is acceptable, it is necessary for Q statistic to be lower than 1 and M values that are smaller than 1.0 indicate that the seasonal adjustment may be considered as successful (Branch & Mason, 2006, pp. 14,18; Cellini & Cuccia, 2011, pp. 44-45; Van Velzen, Wekker & Ouwehand, 2011, pp. 16-18).

3.2.3. Combined Test for the Presence of Identifiable Seasonality

The test for identifiable seasonality is also feasible to be performed by combining the *F* tests for stable and moving seasonality along with a Kruskal-Wallis test for stable seasonality. The following description is based on Lothian and Morry (1978). Recall that F_S and F_M stand for the F value for the stable and moving seasonality tests respectively. This combined test operates in the following way:

1. If the null hypothesis in the stable seasonality test cannot be rejected at the 0.10% significance level (,001), then since the series is said to be not seasonal, PROC X12 (X12 Seasonal Adjustment Procedure) displays "Identifiable Seasonality Not Present."

2. In case the null hypothesis is rejected in the first step, the quantities given below are calculated:

$$T_1 = \frac{7}{F_M} \quad , \quad T_2 = \frac{3F_M}{F_S} \tag{3.8}$$

Let T represent the simple average of T_1 and T_2:

$$T = \frac{(T_1 + T_2)}{2} \tag{3.9}$$

If the null hypothesis of no moving seasonality is rejected at the 5% significance level (,05) and in the case of $T \geq 1.0$, this means that we fail to reject the null hypothesis of identifiable seasonality *not* present and PROC X12 displays "Identifiable Seasonality Not Present."

3. If the null hypothesis of identifiable seasonality *not* present is not accepted, however $T_1 \geq 1.0$, $T_2 \geq 1.0$, or the Kruskal-Wallis chi-squared test cannot reject at the 0.10% significance level (,001), then PROC X12 displays “Identifiable Seasonality Probably Not Present”.

4. In case the null hypotheses of no stable seasonality that are related to F_S and Kruskal-Wallis chi-squared tests are rejected and if none of the combined measures expressed in steps 2 and 3 fail, then we reject the null hypothesis of identifiable seasonality *not* present, and PROC X12 displays “Identifiable Seasonality Present**”** (Statistical Analysis Software).

3.3. Seasonality in the Mean

In this section, we will have a look at a class of time series processes in which seasonal mean behaviour can be modelled. This class consists of deterministic seasonal mean shifts, stochastic stationary and nonstationary processes and unobserved components ARIMA models. Each class will be mentioned in details in Chapter 4. Here, there will be a brief introduction only to stochastic processes which appear more difficult to be understood than other processes for creating a basic understanding.

3.3.1. Linear Stationary Seasonal Processes (Stochastic Stationary Processes)

Stochastic seasonality can be mostly depicted by the autoregressive – moving average (ARMA) processes. A famous example for this class of processes is represented by the first order seasonal autoregressive (SAR) process:

$$y_t = \phi_s y_{t-s} + \varepsilon_t \tag{3.10}$$

where ε_t ~ i.i.d. $(0, \sigma_\varepsilon{}^2)$ with $|\phi_s| < 1$ (i.i.d.: Independent and Identically Distributed). With a use of lag operator $(L^k y_t = y_{t-k})$, (3.10) can be written as

$$(1 - \phi_s L^s) y_t = \varepsilon_t \tag{3.11}$$

Since there is no intercept in this process, the unconditional mean of the process regardless of the season becomes equal to zero. However in the deterministic seasonal process, this property differs as will be seen later. On the other hand, the mean conditional on past y_t is not equal to zero and it displays seasonal patterns:

$$E(y_t | y_{t-1},.....) = \phi_s y_{t-s} \tag{3.12}$$

and ACF for the process is represented by

$$\rho(kS) = \phi_s^{\ k}, \quad k = 1,2,........, \tag{3.13}$$

and $\rho(k) = 0$ for all other k values. As implied by (3.13), we can mention about the presence of autocorrelation only for seasonal lags and the magnitude of this autocorrelation diminishes over time. In conclusion, the only pattern in y_t is connected to seasonality. But, since the series is mean reverting toward its expected value of zero regardless of the season, this seasonality in y_t has a transitory characteristic. With seasonal MA polynomials, the case of first order SAR process can be extended to higher order processes, namely

$$\phi_s(L^s) y_t = \theta_s(L^s)\varepsilon_t \tag{3.14}$$

where $\phi_s(L^s) \equiv 1 - \sum_{i=1}^{p} \phi_{is} L^{is}$ and $\theta_s(L^s) \equiv 1 - \sum_{i=1}^{Q} \theta_{is} L^{is}$, where the roots of both polynomials lie outside the unit circle and both polynomials have no common roots. Such processes are namely seasonal ARMA processes. For the ACFs of such processes, see Box, Jenkins and Reinsel (2008) (Ghysels & Osborn, 2001, pp. 8-9).

3.3.2. Nonstationary Unit Root Processes

A typical characteristic of most economic time series is that they display unit root nonstationarity. This characteristic is also possible to be observed in the seasonal. Seasonal random walk process is the simplest one showing those features:

$$\Delta_S y_t = \varepsilon_t \tag{3.15}$$

where $\Delta_S \equiv (1 - L^S)$ stands for the seasonal differencing operator. Because this process can be obtained from (3.10) with $\phi_s = 1$, it is also namely a seasonal unit root process. Again, taking the stationary case as basis with the assumption of $y_{-s+1} = ... = y_0 = 0$, there will be no seasonality in the mean. However, the seasonality becomes persistent in the case of $\phi_s = 1$ in (3.12) in the sense that

$$E(y_t | y_{t-1},.....) = E(y_{t+ks} | y_{t-1},.....) = y_{t-s}, \quad \text{k=1,2,……} \tag{3.16}$$

Therefore we can say that mean reversion does not apply to this process (Ghysels & Osborn, 2001, pp. 11-12).

3.4. Seasonal Moving Average (SMA) Model

SMA model can be specified by:

$$Y_t = (1-\theta_0 L^s)\varepsilon_t = \varepsilon_t - \theta_0 \varepsilon_{t-s} \tag{3.17}$$

where θ_0 is the parameter of interest which is on or near the unit circle, L is the lag operator and ε_t ~ i.i.d. $(0,\sigma^2)$. Here, the null hypothesis $H_0 : \theta = 1$ implies the presence of seasonal MA unit root against the alternative of $H_1 : \theta < 1$. Tam and Reinsel (1997) have also mentioned in their paper about testing for deterministic seasonality to be identical to testing for seasonal MA unit root in the ARIMA model and expressed that seasonal differencing for a deterministic seasonal series results in a seasonal unit root in the MA operator (Tam & Reinsel, 1997, p. 727). By using the parameterization $\theta_0 = (1-\gamma / T)$, where T is the sample size, Davis, Chen and Dunsmuir (1996) have tried to build up the convergence in distribution of the maximum likelihood (ML) estimator of θ_0 (Davis et al., 1996, pp. 160-161). For more information, see Davis et al. (1996) or Tam and Reinsel (1998).

3.5. Seasonal ARIMA Models (SARIMA)

The characterization of seasonal series occurs by a strong serial correlation at the seasonal lag. As known, the classical decomposition of the time series consists of a trend component, a seasonal component and a random noise component. But, in practice it may not be logical to assume that the seasonality component repeats itself exactly in the same way cycle after cycle. SARIMA models allow for randomness in the seasonal pattern from one cycle to the next (Brockwell & Davis, 2006, p. 320).

Box and Jenkins (1970) presents an extension of the ARIMA model in order to take seasonal effects into consideration. At the core of idea for adding this seasonal component, trying to adjust a cyclical effect takes place. For example, in the case of monthly data, the observation y_t may depend in part on y_{t-12} accounting for an annual effect. In the same manner, for daily data, the dependence may be realized through y_{t-7} representing a weekly effect. Coping with these dependencies in order to remove the seasonal effect in question may be possible via differencing the data. However, one can also specify AR or MA relationships at the seasonal interval in question. For this case, Box and Jenkins (1970) define a general multiplicative SARIMA model shown as

ARIMA $(p,d,q)\,\mathrm{x}\,(P,D,Q)_s$, where the lower-case letters p,d,q indicate the nonseasonal orders and the upper-case letters P,D,Q indicate the seasonal orders of the process with period *s* (that is, *s* is the number of observations per year). The parentheses mean that the seasonal and nonseasonal elements are multiplied (Hamaker & Dolan, 2009, pp. 198-199, Pankratz, 1983, p. 281).

Before giving a clear definition for SARIMA, assume that X_t $(t = 0,\pm1,\pm2,......)$ is an ARMA (p,q) process if { X_t } is stationary and if for every *t*,

$$X_t - \phi_1 X_{t-1} - .... - \phi_p X_{t-p} = Z_t + \theta_1 Z_{t-1} + ... + \theta_q Z_{t-q} \tag{3.18}$$

where $\{Z_t\} \sim WN(0,\sigma^2)$. (3.18) can be written symbolically as

$$\phi(L)X_t = \theta(L)Z_t\,, \quad (t = 0,\pm1,\pm2,......) \tag{3.19}$$

where ϕ and θ are the p^{th} and q^{th} degree polynomials

$$\phi(z) = 1 - \phi_1 z - .... - \phi_p z^p \tag{3.20}$$

and

$$\theta(z) = 1 + \theta_1 z + ... + \theta_q z^q \tag{3.21}$$

and *L* is the backward shift operator defined by $L^j X_t = X_{t-j}$, $j = 0,\pm1,\pm2,......$. These ϕ and θ polynomials are mentioned as the autoregressive (AR) and moving average (MA) polynomials respectively of the difference equations (3.19) (Brockwell & Davis, 2006, p. 78). If we fit an ARMA(*p,q*) model $\phi(L)Y_t = \theta(L)Z_t$ to the differenced series $Y_t = (1 - L^s)X_t$, then the model for the original series becomes $\phi(L)(1 - L^s)X_t = \theta(L)Z_t$. This is a special case of the general SARIMA model which will be defined as follows:

Definition: If d and D are nonnegative integers, then { X_t } is a *seasonal ARIMA* $(p,d,q)\,\mathrm{x}\,(P,D,Q)_s$ process with period *s* if the differenced series $Y_t = (1-L)^d (1-L^S)^D X_t$ is a causal ARMA process defined by

$$\phi(L)\Phi(L^s)Y_t = \theta(L)\Theta(L^s)Z_t\,,\ \{Z_t\} \sim WN(0,\sigma^2) \tag{3.22}$$

where $\Phi(z) = 1 - \Phi_1 z - .... - \Phi_P z^P$ (seasonal AR(P) characteristic polynomial), $\Theta(z) = 1 + \Theta_1 z + ... + \Theta_Q z^Q$ (seasonal MA(Q) characteristic polynomial) with $\phi(z)$ and $\theta(z)$ expressed in (3.20) and (3.21) respectively (Brockwell & Davis, 2002, p. 203).

On the other hand, a more general multiplicative SARIMA model can be expressed by adding a constant term δ to take the case of a deterministic trend into consideration as follows:

$$\phi(L)\Phi(L^s)Y_t = \delta + \theta(L)\Theta(L^s)Z_t \tag{3.23}$$

and substituting $Y_t = (1-L)^d (1-L^s)^D X_t = \Delta_S{}^D \Delta^d X_t$ into (3.23), it becomes

$$\phi(L)\Phi(L^s)\Delta_S{}^D \Delta^d X_t = \delta + \theta(L)\Theta(L^s)Z_t \tag{3.24}$$

(Shumway & Stoffer, 2011, p. 157).

As seen in the definition given above, derivation of $\{Y_t\}$ comes from the original series $\{X_t\}$ using both simple differencing (in order to remove trend) and seasonal differencing $\Delta_s = 1 - L^s$ to remove seasonality. For instance, when $d = D = 1$ and $s = 12$, then Y_t becomes

$$\begin{aligned} Y_t &= \Delta\Delta_{12} X_t = \Delta_{12} X_t - \Delta_{12} X_{t-1} \\ &= (X_t - X_{t-12}) - (X_{t-1} - X_{t-13}) \end{aligned} \tag{3.25}$$

Now take a SARIMA model of order $(1,0,0)\,\mathrm{x}\,(0,1,1)_{12}$. Then this model can be written in the following equation:

$$(1-\phi L)Y_t = (1+\theta L^{12})Z_t \tag{3.26}$$

where $Y_t = \Delta_{12} X_t$. Then we find

$$X_t = X_{t-12} + \phi(X_{t-1} - X_{t-13}) + Z_t + \theta Z_{t-12} \tag{3.27}$$

so that X_t depends on X_{t-1}, X_{t-12} and X_{t-13} as well as the innovation at time $(t-12)$ (Chatfield, 1996, p. 60).

Now, let us take an ARIMA $(0,0,1)\,\mathrm{x}\,(0,1,1)_4$ process with a periodicity of four (since, s=4) as an example. Here, D = 1 implies that Y_t is differenced once by length four. With d = 0, it can be inferred that there is no seasonal differencing. There is one seasonal MA term at lag 4 (Q = 1) and one nonseasonal MA term at lag 1 (q = 1). Moreover, the two MA operators are multiplied by each other. Using lag operator, this model can be written as

$$(1-L^4)X_t = (1+\theta_1 L)(1+\Theta_1 L^4)Z_t \tag{3.28}$$

As another example, take the case of ARIMA $(1,0,0)\,\mathrm{x}\,(1,0,1)_{12}$ process with a periodicity of length 12 (since, s=12). In this example, it is obvious that X_t does not require seasonal and nonseasonal differencing at all since $d=0$ and $D=0$. On the other hand, the seasonal part of the process is composed of one AR $(P=1)$ and one MA $(Q=1)$ component at lag 12. In addition, there is a nonseasonal AR term at lag 1 $(p=1)$. The multiplication of the two AR operators in the lag operator form can be expressed as

$$(1-\phi_1 L)(1-\Phi_1 L^{12})X_t = (1+\Theta_1 L^{12})Z_t \tag{3.29}$$

(Pankratz, 1983, p. 281).

In identifying SARIMA model, the first task is to find values d and D which reduce the series to stationarity and remove most of the seasonality. Then, we need to assess the values of p, P, q and Q by examining the sample ACF and partial autocorrelation function (PACF) of the differenced series at lags which are multiples of s and choosing a SARIMA model in which ACF and PACF have a similar shape. Ultimately, the model parameters may be estimated through an appropriate iterative procedure. For details, see Box and Jenkins (1970, chap. 9) (Chatfield, 1996, pp. 60-61) (all AR and MA polynomial representations have been taken from Brockwell & Davis, 2006, p. 78).

3.5.1. Stationarity and Invertibility Conditions

Representing a model in a multiplicative form is a big convenience in terms of expressing the seasonal and nonseasonal components separately and controlling the stationarity and invertibility conditions. For instance, take an ARIMA $(2,0,1)\,\mathrm{x}\,(1,0,2)_s$ model and express it in a lag operator form as follows:

$$(1-\phi_1 L-\phi_2 L^2)(1-\Phi_1 L^s)X_t = (1+\Theta_1 L^s+\Theta_2 L^{2s})(1+\theta_1 L)Z_t \tag{3.30}$$

The stationary requirement applies only to the AR coefficients. The nonseasonal part of (3.30) has the same stationarity conditions as for an $AR(2)$: $|\phi_2|<1,\quad \phi_2-\phi_1<1,$ and $\phi_2+\phi_1<1$. Now we need to apply a seperate stationary condition for the AR seasonal part. It is the same as for a nonseasonal AR(1) model, except in this case we have a (seasonal) $AR(1)_s$ component; so the condition becomes $|\Phi_1|<1$.

As in the case of stationarity, we need to consider invertibility condition which applies only to the MA part of (3.30) for nonseasonal and seasonal components separately. For the nonseasonal part, the condition is $|\theta_1| < 1$. The conditions on the seasonal part are the same as for a nonseasonal MA(2) model, except in this case there exists an $MA(2)_s$ component. Therefore the joint conditions are given as $|\Theta_2| < 1$, $\Theta_2 - \Theta_1 < 1$ and $\Theta_2 + \Theta_1 < 1$ (Pankratz, 1983, p. 285). (AR and MA polynomial representations have been taken from Brockwell & Davis, 2006, p. 78).

3.5.2. The Expanded Model

It should be noted that all multiplicative SARIMA models can be telescoped out into an ordinary ARMA (p,q) model in the variable

$$Y_t \overset{def}{=} \Delta_S{}^D \Delta^d X_t \tag{3.31}$$

For instance, consider that the series $\{x_t\}_{t=1}^{T}$ follows a SARIMA $(0,1,1)\,\text{x}\,(12,0,1,1)$ or ARIMA $(0,1,1)\,\text{x}\,(0,1,1)_{12}$ model. For this process, we have

$$(1-L^{12})(1-L)X_t = (1+\theta_1 L)(1+\Theta_1 L^{12})Z_t \tag{3.32}$$

and it becomes

$$Y_t = (1+\theta_1 L + \Theta_1 L^{12} + \theta_1 \Theta_1 L^{13})Z_t \tag{3.33}$$

where $Y_t = (1-L^{12})(1-L)X_t$. Hence, it can be said that this multiplicative SARIMA model has an ARMA(0,13) representation where only the coefficients θ_1, $\theta_{12} \overset{def}{=} \Theta_1$ and $\theta_{13} \overset{def}{=} \theta_1 \Theta_1$ are not zero and all other coefficients of the MA polynomial are equal to zero. So, if the model in question is SARIMA $(0,1,1)\,\text{x}\,(12,0,1,1)$ given in (3.32), only the two coefficients which are θ_1 and Θ_1 have to be estimated. However, for the ARMA(0,13), instead we have to estimate the three coefficients which are θ_1, θ_{12} and θ_{13}. Therefore, it is apparent that SARIMA models take a parsimonious model structure into account and a model specification such as (3.33) is called an *expanded model*. In addition, we can say that only an expanded multiplicative model can be estimated directly (Chen, Schulz & Stephan, 2003, pp. 233-234).

3.5.3. Theoretical ACFs and PACFs for Seasonal Processes

In SARIMA models, estimated acfs and pacfs display the same expected behaviours as in the structure of nonseasonal models. For seasonal time series data, observations s time periods apart $(z_t, z_{t-s}, z_{t+s}, z_{t-2s}, z_{t+2s}, ...)$ have characteristics in common. So, observations s periods apart are expected to be correlated and in this manner, acfs and pacfs for seasonal series should have nonzero coefficients at one or more multiples of lag s $(s, 2s, 3s, ...)$. If we observe nonseasonal and purely seasonal acfs and pacfs, it is seen that the coefficients appearing at lags 1,2,…. in the former appear at lags $s, 2s, 3s, ...$ in the latter. For instance, theoretical acf of a nonseasonal AR(1) process having $\phi_1 = 0,7$ tails off exponentially in this manner (where k is the lag length and ρ_k represents the autocorrelation coefficient):

Table 3

Behaviour of the Theoretical ACF of a Non-seasonal Process

k	ρ_k
1	$\rho_1 = 0.7$
2	$\rho_2 = 0.49$
3	$\rho_3 = 0.34$
⋮	⋮

For instance, for quarterly data (that is, s=4), a stationary seasonal process including one seasonal AR coefficient also has a theoretical acf decaying in an exponential manner, however at the seasonal lags $(4, 8, 12, ...)$ which are multiples of 4, the representation becomes as shown in Table 4.

Table 4

Behaviour of the Theoretical ACF of a Seasonal Process

k	ρ_k
1	$\rho_1 = 0$
2	$\rho_2 = 0$
3	$\rho_3 = 0$
4	$\rho_4 = 0.7$
5	$\rho_5 = 0$
6	$\rho_6 = 0$
7	$\rho_7 = 0$
8	$\rho_8 = 0.49$
9	$\rho_9 = 0$
10	$\rho_{10} = 0$
11	$\rho_{11} = 0$
12	$\rho_{12} = 0.34$
$\vdots$	$\vdots$

This similarity between nonseasonal and seasonal acfs and pacfs makes the seasonal analysis simpler. So, having knowledge of nonseasonal acfs and pacfs helps give a description of identical patterns occurring at multiples of lag *s* (Pankratz, 1983, pp. 270-271). For more details, see Box and Jenkins (1976, chap. 9).

CHAPTER IV

DETERMINISTIC SEASONALITY

4.1. Introduction to Deterministic Seasonality

Deterministic seasonality gives a description of varying unconditional mean behaviour with the season of the year. It is the known part of the seasonal cycle when "the process is started" and is limited to time constant seasonal means or time constant growth rates that show differences across quarters/months (Kunst, 2012). In this section, deterministic seasonality will be handled in detail with its dummy variable and trigonometric representations. It would be unfair to consider recent developments associated with deterministic seasonality apart from the issues surrounding stochastic, and especially nonstationary stochastic seasonality. Therefore a further analysis will be realized on various types of seasonal processes and finally some tests will be proposed in order to distinguish between deterministic and nonstationary stochastic seasonality by testing the null hypothesis that seasonality is of the deterministic type. When we consider topics pertaining to seasonality, it is mostly convenient to realize the season and the year to which a specific observation *t* relates in an explicit manner. For this realization, it is preferable to use two subscripts for a variable with the first one referring to the season and the second to the year. From the knowledge of the season in which the initial observation falls, we can infer about the season for all subsequent values of *t*. By making a simple assumption that $t=1$ corresponds to the first season of a year (that is, the first quarter for quarterly data or January for monthly data as $s=1$) and s denotes the season in which observation *t* falls, the series of observations $y_1, y_2, ..., y_s, y_{s+1}, ...$ could be written in the double subscript notation as $y_{11}, y_{21}, ..., y_{s1}, y_{12}, ..., y_{s2}, y_{13}, ...$. Generally, y_t could be written as identical to $y_{s\tau}$, where $s=1+[(t-1)\bmod S]$ (that is, s_t is one plus the integer remainder obtained when $t-1$ is divided by S that denotes the number of observations per year) and $\tau = 1+\text{int}[(t-1)/S]$ which is a notation for the year in which a specific observation t falls with "int" denoting the integer part. In the case of that y_t includes T observations, we will assume that there are exactly T_τ years of data, so that $T_\tau = T/S$ (Ghysels & Osborn, 2001, pp. 6, 19).

4.2. Representations of Deterministic Seasonality

There are two representations of deterministic seasonality which will be mentioned in following sections:

1. The Dummy Variable Representation
2. The Trigonometric Representation

4.2.1. The Dummy Variable Representation

The most frequently used dummy variable representation of seasonality can be expressed as follows:

$$y_t = \sum_{s=1}^{S} \gamma_s \delta_{st} + z_t \ , \quad t = 1, \ldots\ldots\ldots, T_\tau \tag{4.1}$$

where y_t is a univariate process, δ_{st} is a seasonal dummy variable that takes the value one in season s (that is, δ_{st} = 1 if $s_t = s$ for s = 1,…..,S) and is zero otherwise and finally the process z_t is a weakly stationary stochastic process with mean zero. Thus, for season s of year τ ,

$$E(y_{s\tau}) = \gamma_s \ , \ s = 1, \ldots\ldots\ldots, S \tag{4.2}$$

This property is of primary interest with respect to implying that the process has a seasonally shifting mean. This time varying mean gives information about nonstationarity of process. Since it is very easy to remove this nonstationarity position of the process so that the deviations $y_t - E(y_t) = z_t$ are weakly stationary, this nonstationarity is often ignored. The disadvantage of this dummy representation in (4.2) is that it cannot distinguish seasonality from the overall mean when the latter becomes nonzero. The overall mean of y_t is given as:

$$E(y_t) = \mu = \frac{1}{S} \sum_{s=1}^{S} \gamma_s \tag{4.3}$$

The deterministic seasonal effect for season s denoted by m_s is found by subtracting this overall mean. That is, $m_s = \gamma_s - \mu$. It is very clear from this equation that when observations are summed over a year, there will be no deterministic seasonality. Since this equation comes with a restriction of $\sum_{s=1}^{S} m_s = 0$. If the level of the series (here denoted as μ) is isolated from the seasonal component, it will take the form of

$$y_t = \mu + \sum_{s=1}^{S} m_s \delta_{st} + z_t \tag{4.4}$$

This equation can be reformulated in a way to include a trend component that is unchanged over the seasons by putting $\mu_0 + \mu_1 t$ instead of μ. A further reformulation is realized by writing separate trends for each season:

$$y_t = \mu_0 + \mu_1 t + \sum_{s=1}^{S} (m_{0s} + m_{1s} t)\delta_{st} + z_t \tag{4.5}$$

Here, we encounter again with same restrictions that are $\sum_{s=1}^{S} m_{0s} = \sum_{s=1}^{S} m_{1s} = 0$ mentioned above. However, this type of trending deterministic seasonality has such an implication that observations for seasons of the year diverge over time and that is why it may seem unrealistic for many applications. For both (4.4) and (4.5) processes, each observation deviates from its respective seasonal mean with a constant variance over both s and τ as implied by stationarity for $z_t = y_t - E(y_t) = y_s - E(y_{s\tau})$. This result points out to that when we have a deterministic seasonal process, the observations cannot wander too far from their underlying mean (Ghysels & Osborn, 2001, pp. 20-21).

4.2.2. The Trigonometric Representation

A deterministic seasonal process with period S can also be equivalently written in terms of sines and cosines corresponding to (4.4) as follows:

$$y_t = \mu + \sum_{k=1}^{S/2} \left[\alpha_k \cos\left(\frac{2\pi kt}{S}\right) + \beta_k \sin\left(\frac{2\pi kt}{S}\right) \right] + z_t \tag{4.6}$$

where

$$\sum_{s=1}^{S} \delta_{st} m_s = \sum_{k=1}^{S/2} \left[\alpha_k \cos\left(\frac{2\pi kt}{S}\right) + \beta_k \sin\left(\frac{2\pi kt}{S}\right) \right] \quad \text{for } t = 1, \ldots\ldots\ldots, T \tag{4.7}$$

with

$$\alpha_k = \frac{2}{S} \sum_{s=1}^{S} m_s \cos\left(\frac{2\pi kj}{s}\right), \quad k = 1, 2, \ldots\ldots\ldots, \frac{S}{2} \tag{4.8}$$

$$\alpha_{S/2} = \frac{1}{S} \sum_{s=1}^{S} m_s \cos(\pi j) \tag{4.9}$$

$$\beta_k = \frac{2}{S} \sum_{s=1}^{S} m_s \sin\left(\frac{2\pi kj}{s}\right), \quad k = 1, 2, \ldots\ldots\ldots, \frac{S}{2} - 1 \tag{4.10}$$

Thus, both dummy variable representation and trigonometric representation will be the same. However, the trigonometric representation is seen to be more useful in separating seasonality from the overall mean μ than the dummy variable representation. In equation (4.7), α_k is considered only for $k = 1,2,.......,\frac{S}{2}$ and β_k only for $k = 1,2,.......,\frac{S}{2} - 1$. Because, $\beta_{S/2}$ multiplies a sine term that is always zero. This representation provides a basis to spectral analysis of seasonality and seasonal adjustment (see Hannan, Terrel & Tuckwell, 1970).

If we took the case of quarterly data, as an implication of equation (4.6) the seasonal dummy variable coefficients of equation (4.1) are connected with the deterministic components of the trigonometric representation in the following way:

$$\begin{aligned} \gamma_1 &= \mu + \beta_1 - \alpha_2 \\ \gamma_2 &= \mu - \alpha_1 + \alpha_2 \\ \gamma_3 &= \mu - \beta_1 - \alpha_2 \\ \gamma_4 &= \mu + \alpha_1 + \alpha_2 \end{aligned} \qquad (4.11)$$

For quarterly data (that is, S = 4), the trigonometric components can easily be expressed in a clear way:

For k=1, $\cos\left(\frac{2\pi t}{4}\right) = \cos\left(\frac{\pi t}{2}\right) = 0,-1,0,1,....$ So, $\alpha_1 = \frac{1}{2}\sum_{s=1}^{4} m_s \cos\left(\frac{s\pi}{2}\right) = \frac{1}{2}(-m_2 + m_4)$

k=2, $\cos\left(\frac{4\pi t}{4}\right) = \cos(\pi t) = -1,+1,-1,+1,....$ $\alpha_2 = \frac{1}{4}\sum_{s=1}^{4} m_s \cos(s\pi) = \frac{1}{4}(-m_1 + m_2 - m_3 + m_4)$

k=1, $\sin\left(\frac{2\pi t}{4}\right) = \sin\left(\frac{\pi t}{2}\right) = 1,0,-1,0,....$ $\beta_1 = \frac{1}{2}\sum_{s=1}^{4} m_s \sin\left(\frac{s\pi}{2}\right) = \frac{1}{2}(m_1 - m_3)$

k=2, $\sin\left(\frac{4\pi t}{4}\right) = 0$

with α_1 and β_1 denoting the annual wave and α_2 denoting the half-year component (Kunst, 2012).

As seen above, the coefficients α_1 and β_1 are related with the spectral frequency $\frac{\pi}{2}$, because they multiply $\cos\left(\frac{\pi t}{2}\right)$ and $\sin\left(\frac{\pi t}{2}\right)$ respectively for $t = 1,2,........$ (through the values 1, 0, -1, 0; it can be inferred that α_1 and β_1 have a half-cycle every two periods and a full cycle every four periods even though α_1 is associated with the second and fourth quarters while β_1 is associated with the first and third quarters). By the same logic, it is obvious that α_2 is related with the spectral frequency π, since it multiplies $\cos(\pi t)$ for $t = 1,2,........$ in (4.6). Also because the terms of $\cos(\pi t)$ alternate between -1 and 1, α_2 displays a full cycle every two periods. In the case of quarterly data, these spectral frequencies also mean the seasonal frequencies; since any deterministic seasonal pattern over the four quarters of the year can be specified as a linear function of terms at these $\frac{\pi}{2}$ and π frequencies, such that $\alpha_1 \cos(t\pi/2) + \beta_1 \sin(t\pi/2) + \alpha_2 \cos(t\pi)$. By construction of these functions, in an essential manner the seasonal pattern sums to zero over any four sequential values of t.

(4.11) can also be represented in a different notation as:

$$\Gamma = R.B, \tag{4.12}$$

where $\Gamma = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)'$, $B = (\mu, \alpha_1, \beta_1, \alpha_2)'$ and

$$R = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 1 & -1 & 0 & 1 \\ 1 & 0 & -1 & -1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \tag{4.13}$$

This 4x4 non-singular matrix handles the one-to-one relationship between the dummy variable representation expressed in (4.1) and the trigonometric representation (4.6) for the quarterly case. Equation (4.12) can also be applied for data at sampled other frequencies. For instance, if we take monthly data with S=12, then the seasonal frequencies become $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}$ and π. For monthly data, it is also possible to express any deterministic seasonal pattern by using the trigonometric cosine and sine functions at these spectral frequencies. However, recall one more time that the representation holds the overall mean μ separate from the deterministic component with the latter necessarily summing to zero over any twelve successive values of t. Now return to the general case of S seasons. There are some good properties concerned with matrix R in (4.13). When μ is included in the vector B, the matrix R becomes a square matrix and must be non-singular because there is a one-to-one relationship between the seasonal dummy and trigonometric representations. The columns of the matrix R are orthogonal to each other meaning that when the vector R_i represents the *i*th column, so that $R = (R_1, ..., R_S)$, then $R_i' R_j = 0, i \neq j$. This quality of R assures that $R'R = D$ is a diagonal matrix. So, if the *i*th diagonal element of D is shown as d_i, then

$$R^{-1} = \begin{bmatrix} \frac{1}{d_1} R_1{}' \\ \frac{1}{d_2} R_2{}' \\ \vdots \\ \frac{1}{d_s} R_S{}' \end{bmatrix} \tag{4.14}$$

so that the inverse of R becomes the transpose of itself. For instance, let's verify this in the quarterly case:

$$R^{-1} = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0 & -0.5 & 0 & 0.5 \\ 0.5 & 0 & -0.5 & 0 \\ -0.25 & 0.25 & -0.25 & 0.25 \end{bmatrix} \tag{4.15}$$

Thus, it should be noted that the first column of R includes a vector of ones. In this case, $R_1{}' R_1 = d_1 = S$ and consequently, each element of the first row of R^{-1} corresponds to 1/S and this inverse provides us a definitional relationship $\mu = (1/S)\sum_{s=1}^{S} \gamma_s$. It is also remarkable to say that equations (4.8) and (4.10) - which describe the coefficients α_k and β_k as cosine and sine functions - efficiently reveal the elements of R^{-1}. Sometimes, it is very practical to identify the overall mean with the zero spectral frequency. So, μ can be expressed with respect to trigonometric functions as $\alpha_0 \cos(2\pi kt/S)$ with $k = 0$ and (4.6) becomes equivalent to

$$y_t = \sum_{k=0}^{S/2} \left[\alpha_k \cos\left(\frac{2\pi kt}{S}\right) + \beta_k \sin\left(\frac{2\pi kt}{S}\right) \right] + z_t \tag{4.16}$$

since sin(0)=0. It is realized that the overall mean μ has a spectral interpretation, so it is convenient to write it as μ and therefore use the representation (4.6) in preference to (4.11).

Even if not explicitly stated here, it is obvious that the trend coefficients in the seasonally varying trend model can also be expressed by using a trigonometric representation and then with suitable definitions of the elements of Γ, R and B in (4.12), the relationships between the trend coefficients in the dummy variable and trigonometric representations can easily be observed (Ghysels & Osborn, 2001, pp. 21-24).

4.3. Stochastic and Deterministic Seasonality

4.3.1. Stochastic Seasonal Processes

The discussion on previous section dealing with deterministic seasonality which has an unconditional mean, $E(y_{s\tau})$. In this section, we will cover stochastic seasonality.

Let's take the case of first order seasonal autoregressive AR(1) process for z_τ:

$$z_{s\tau} = \phi_S z_{s,\tau-1} + \varepsilon_{s\tau}, \quad s = 1,........,S, \;\; \tau = 1,2,........,T_\tau \qquad (4.17)$$

where $\varepsilon_t = \varepsilon_{S\tau}$ is i.i.d. $(0,\sigma^2)$. This seasonal AR(1) process puts emphasis on that the autoregressive relationship for $z_{s\tau}$ in season s is associated with the same season in the preceding year. The process can be written in a more general form using starting value as follows:

$$z_{s\tau} = \phi_s^{\ \tau} z_{s0} + \sum_{j=0}^{\tau-1} \phi_s^{\ j} \varepsilon_{s,\tau-j} \qquad (4.18)$$

In the model defined here, there are two sources of seasonality: the first one is the unobserved starting value for season s, z_{s0}, which influences the subsequent observations for that season through $\phi_s^{\ \tau} z_{s0}$. The second one is that $z_{s\tau}$ is affected by disturbances for the specific season s in previous years ($\varepsilon_{s,\tau-j}$ for $j > 0$), so that patterns that occur by chance through the disturbances are inclined to be repeated. On the other hand, in both cases ϕ_s should be greater than zero for these repeating patterns. If the process is also stationary $(0 < \phi_s < 1)$, the effects of z_{s0} and of any specific $\varepsilon_{s,\tau-j}$ diminish over time.

The variance of (4.17) is expressed as follows:

$$Var(z_{s\tau}) = \phi_s^{\ 2\tau} Var(z_{s0}) + \sigma^2 \sum_{j=0}^{\tau-1} \phi_s^{\ 2j} \qquad (4.19)$$

If $-1 < \phi_s < +1$ or in other words if the process is stationary and $Var(z_{s\tau}) = Var(z_{s0})$, the variance becomes:

$$Var(z_{s\tau}) = \frac{\sigma^2}{(1-\phi_s^{\ 2})} \qquad (4.20)$$

which is constant over both seasons $s = 1,.........,S$ and years $\tau = 1,2,........,T_\tau$.

For the case of quarterly data, the seasonal autoregression process in (4.17) can be expressed as:

$$z_{s\tau} = \phi_4 z_{s,\tau-1} + \varepsilon_{s\tau}\,, \quad s = 1,....,4 \quad \tau = 1,2,........,T_\tau \tag{4.21}$$

So, the model can be written as $(1-\phi_4 L^4) z_t = \varepsilon_t$ and decomposition of the polynomial operator for this seasonal AR(1) process is given as:

$$(1-\phi_4 L^4) = (1-\sqrt[4]{\phi_4}L)(1+\sqrt[4]{\phi_4}L)(1+\sqrt[2]{\phi_4}L^2) \tag{4.22}$$

Here the first component $(1-\sqrt[4]{\phi_4}L)$ represents a nonseasonal factor and other two components namely $(1+\sqrt[4]{\phi_4}L)$ and $(1+\sqrt[2]{\phi_4}L^2)$ contribute to seasonality. We can say that AR processes $(1+\sqrt[4]{\phi_4}L)y_t = \varepsilon_t$ and $(1+\sqrt[2]{\phi_4}L^2)y_t = \varepsilon_t$ have peaks at frequencies π and $\frac{\pi}{2}$, respectively in their spectral densities. The term $(1+\sqrt[2]{\phi_4}L^2)$ can also be decomposed as $(1+i\sqrt[4]{\phi_4}L).(1-i\sqrt[4]{\phi_4}L)$ where $i=\sqrt{-1}$. However, we cannot separate this complex pair of factors with $\frac{\pi}{2}$ frequency, because they should be together to have a real-valued process. On the other hand in the case of monthly data, the factorization becomes $(1-\phi_{12}L^{12})$ with $\phi_{12} > 0$. As analogous to the quarterly case, this polynomial operator can be factorized to a nonseasonal factor and eleven seasonal factors and the seasonal factors consist of five complex pairs that are related to the seasonal frequencies $\frac{\pi}{6},\frac{\pi}{3},\frac{\pi}{2},\frac{2\pi}{3}$ and $\frac{5\pi}{6}$ (these can be obtained by using $\frac{2\pi j}{s}$, where s=12 and $j = 1,2,......,s/2$ or $j = 1,2,......,6$) together with a real factor associated with the frequency π (Ghysels & Osborn, 2001, pp. 24-26).

4.3.2. The Seasonal Random Walk

In the case of $\phi_s = 1$ in (4.17), we refer to the seasonal random walk process and the observed process $y_{s\tau}$ will be $y_{s\tau} = y_{s,\tau-1} + \varepsilon_{s\tau}$, or by expressing it with a starting value, we get

$$y_{s\tau} = y_{s0} + \sum_{j=1}^{\tau} \varepsilon_{sj} \tag{4.23}$$

This seasonal random walk process will include S random walks since $s = 1,........,S$. Because the disturbances are independent over the seasons, these S random walks (for S seasons of the year) are also independent of each other. Therefore, any linear combination of these processes can itself be exhibited as a random walk. The accumulation of disturbances enables the differences to wander far from the mean over time, leading to the phenomenon that "summer may become winter" (Baltagi, 2001, p. 657).

Contrary to the stationary stochastic seasonal process, here the effects of the starting value for season s which is y_{s0}, and of any specific disturbance for season s (ε_{sj}) do not diminish over time as τ increases. Hence, in the case of $E(y_{s0}) = \gamma_s \neq 0$ for all τ, $E(y_{s\tau})$ becomes equal to γ_s. More clearly, any deterministic seasonal component in the starting value for the seasonal random walk in season s is carried over to all following observations associated with that season[1]. It should be noted that a significant distinction between the deterministic seasonal process mentioned previously and seasonal random walk process exists so that in the first case, $Var(y_{s\tau})$ is constant over both s and τ. However, in equation (4.23), $Var(y_{s\tau}) = Var(y_{s0}) + \tau\sigma^2$ and as seen, it linearly increases with τ. So, one more time we can state that with this increasing variance, $y_{s\tau}$ can wander far from its unconditional mean γ_s over time (Ghysels & Osborn, 2001, pp. 26-27).

4.3.2.1. Asymptotic Properties of a Seasonal Random Walk

If we are to mention about asymptotic properties of a seasonal random walk process, firstly we assume that the initial values of the data generating process (DGP) are equal to zero ($y_{s0} = 0$ for $s = 1,........,S$). In that case, S independent partial sum processes can be obtained as:

$$y_{s\tau} = \sum_{j=1}^{\tau} \varepsilon_{sj} \quad (\text{for } s = 1,........,S) \tag{4.24}$$

As $T_\tau \to \infty$, then the behaviour of each scaled partial sum converges to a Brownian motion. That is,

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[1] Sometimes, it may be misleading to think that the seasonal random walk has no deterministic component. Because for simplicity, there is an assumption often made such as $y_{s0} = 0$ or at least $E(y_{s\tau}) = 0$. Therefore even though the seasonal random walk process does not explicitly include deterministic seasonal effects, these are implicitly included when $E(y_{s\tau})$ is nonzero (see Baltagi, 2001, p. 657).

$$T_\tau^{-1/2}\sum_{j=1}^{\tau}\varepsilon_{sj} = \frac{1}{\sqrt{T_\tau}} y_{s\tau} \Rightarrow \sigma W_s(r) \quad (4.25)$$

where $\Rightarrow$ indicates convergence in distribution and $W_s(r)$ for $s = 1,\dots\dots\dots,S$ are independent standard Brownian motions, derived from an i.i.d. (0,1) disturbance (see also Baltagi, 2001, p. 659). Then while the DGP is the seasonal random walk with initial values equal to zero, as $T_\tau \to \propto$

1) $T_\tau^{-3/2}\sum_{\tau=1}^{T_\tau} y_{s\tau} \Rightarrow \sigma\int_0^1 W_s(r)dr, \quad s = 1,\dots\dots\dots,S$ (4.26)

2) $T_\tau^{-1}\sum_{\tau=1}^{T_\tau}\varepsilon_{s\tau} y_{s,\tau-1} \Rightarrow \sigma^2\int_0^1 W_s(r)dW_s(r), \quad s = 1,\dots\dots\dots,S$
(4.27)

3) $T_\tau^{-2}\sum_{\tau=1}^{T_\tau} y_{s\tau}^{\ 2} \Rightarrow \sigma^2\int_0^1 [W_s(r)]^2 dr, \quad s = 1,\dots\dots\dots,S$ (4.28)

(see Banerjee, Lumsdaine, & Stock, 1992).

Note that the disturbance terms $\varepsilon_{s\tau}$ underlying Brownian motions are independent over seasons and for this reason $W_s(r)$ are independently distributed over $s = 1,\dots\dots\dots,S$ as well.

An extension of seasonal random walk process of (4.23) to a more general seasonal unit root process in which $\Delta_s y_t = z_t$ is a stationary and invertible ARMA process (Ghysels & Osborn, 2001, pp. 27-28).

4.3.3. Deterministic Seasonality versus Seasonal Unit Roots

Recent discussions on seasonality so far have focused on whether an observed series should be modelled as a deterministic seasonal process or a seasonal unit root process. For both cases, it is possible to have a stationary stochastic seasonal component. In later sections, we will discuss the tests of the deterministic seasonality null hypothesis. Before handling these tests, it is remarkable to mention about the relationship between the competing hypotheses. According to Bell (1987), the two competing processes are the simple deterministic seasonality model which is

$$y_{s\tau} = \gamma_s + \varepsilon_{s\tau}, \quad s = 1,\dots\dots\dots,S, \quad \tau = 1,\dots\dots\dots,T_\tau \quad (4.29)$$

and the process of

$$\Delta_S y_{s\tau} = (1-\theta_S L^S)\varepsilon_{s\tau}, \quad s = 1,\dots\dots\dots,S, \quad \tau = 1,\dots\dots\dots,T_\tau \quad (4.30)$$

These two specifications are equivalent in the special case of $\theta_S = 1$. It is very straightforward to get this result: if $\varepsilon_{s\tau}$ is replaced by $(1-\theta_S L^S)\varepsilon_{s\tau}$ in (4.23), then the implication of the annual differenced process (4.30) implies

$$y_{s\tau} = y_{s,\tau-1} + (1-\theta_S L^S)\varepsilon_{s\tau},$$
$$= y_{s0} + \sum_{j=1}^{\tau}(1-\theta_S L^S)\varepsilon_{sj}, \quad (4.31)$$

For $\theta_S = 1$, (4.31) becomes

$$y_{s\tau} = y_{s0} + \sum_{j=1}^{\tau}(\varepsilon_{sj} - \varepsilon_{s,j-1}),$$
$$= y_{s0} - \varepsilon_{s0} + \varepsilon_{s\tau}$$
$$= \gamma_s + \varepsilon_{s\tau},$$

when $y_{s0} = \gamma_s + \varepsilon_{s0}$. As well known, this last line is the deterministic seasonality equation. So, using a simplified assumption about the starting values which is $y_{s0} = \gamma_s + \varepsilon_{s0}$, the two processes of (4.29) and (4.30) with a special case of $\theta_S = 1$ are equivalent for any $y_{s\tau}$.

Sometimes, to discriminate between deterministic seasonality and a seasonal unit root process may be hard. There is a prevalent view about the cancellation of the seasonal differencing operator Δ_S and the noninvertible MA operator $1-L^S$ in (4.30) with $\theta_S = 1$. The extension of this logic also occurs when there is "near cancellation" situation with θ_S close to but less than unity in (4.30). In this case, in an empirical manner the properties of the seasonal unit root process for finite T values become similar to the properties of the deterministic seasonal process given in (4.29). Therefore, generally it may be a hard task to distinguish a deterministic seasonal process from a seasonal unit root process (Ghysels & Osborn, 2001, pp. 28-29).

4.3.4. Unobserved Components Approach

Observed time series are assumed to be a function of several components which are trends, cycles, seasonality and irregularity. Traditionally, these components are accepted to be separately generated and this idea takes place at the core of the seasonal adjustment. A linear unobserved component model is expressed in that way:

$$y_t = y_t^{tr} + y_t^{c} + y_t^{s} + y_t^{i}, \quad (4.32)$$

where the superscripts respectively denote the trend, business cycle, seasonal and irregular components which are mutually independent and there is no general consensus about the nature of these components. However, for some specific cases, some simple decompositions are available. For example, for a seasonal random walk process an unobserved component model can be written as:

$$(1-L)y_t^{ns} = \varepsilon_t^{ns},$$
$$(1+L+.....+L^{S-1})y_t^{s} = \varepsilon_t^{s}, \tag{4.33}$$

where y_t^{ns} denotes the nonseasonal component of y_t which is composed of y_t^{tr}, y_t^{c} and y_t^{i}. The first line of (4.33) displays a nonseasonal random walk and therefore includes the zero frequency component of the seasonal random walk. It is very common to decompose the operator Δ_S into (1 – L) and $(1+L+.....+L^{S-1})$ in testing of seasonal unit roots that will be discussed later. However, there is no unique decomposition of y_t into separate orthogonal components (see, Bell & Hilmer, 1984) and seasonal adjustment methods for the time series require to be established on a specific decomposition (Ghysels & Osborn, 2001, p. 12).

Harvey (1989) specifies the seasonal component summed over a year as random with a zero mean rather than summing to zero over the year in a deterministic way given in (4.1). Thus, the second line of (4.33) which is $(1+L+.....+L^{S-1})y_t^{s} = \varepsilon_t^{s}$ is taken as basis with ε_t^{s} being i.i.d. $(0, w^2)$ and independent of the disturbances driving the other components. With $w^2 = 0$, the deterministic seasonal model becomes a special case of the second line of (4.33). If disturbance term has a nonzero variance, in that case the unobserved components approach enables seasonality to evolve over time. However, in that case the addition of a disturbance term with nonzero variance has a drawback in terms of specifying a seasonal component transformed into a nonstationary process. Indeed, for autoregressive process in $(1+L+.....+L^{S-1})y_t^{s} = \varepsilon_t^{s}$ there will be S – 1 unit roots occurring at seasonal frequencies.

A different type of unobserved components approach for a nonstationary stochastic seasonal component is available in the case of allowing α and β coefficients which take place in the trigonometric representation in subsection 4.2.2. to evolve as random walks so that

$$\alpha_{kt} = \alpha_{k,t-1} + \eta_{kt}, \quad k = 1,......,S/2$$
$$\beta_{kt} = \beta_{k,t-1} + \eta_{kt}^{*}, \quad k = 1,......,(S/2)-1 \tag{4.34}$$

where η_{kt} and η_{kt}^{*} are i.i.d. $(0, w^2)$ processes. This generalization also underlies the Canova and Hansen (1995) framework. So, at each seasonal frequency $\frac{2\pi k}{s}$, ($k = 1, ..., S/2$), seasonality evolves over time and available nonstationarity observed in (4.34) is directly associated with the corresponding seasonal frequencies for any given k. The implication of the unobserved components approach which makes the generalization of deterministic seasonality and allows it to evolve over time is that with this evolvement, seasonality displays a nonstationary stochastic process with unit roots available at all seasonal frequencies. However, what we are trying to mention here is not that an unobserved components model can never be expressed with stationary seasonality (such models are covered to some extent by Nerlove, Grether, & Carvalho 1995), rather in recent studies unobserved components models of interest are based on nonstationary seasonality. Whether the unobserved components approach is in the form of the second line of (4.33) or (4.34), it also shows a typical nonstationary stochastic process for the nonseasonal component and this process is simply specified as the random walk by which the nonseasonal component is $y_t^{ns} = \mu_t$ with

$$\mu_t = \mu_{t-1} + \varepsilon_t^{ns} \tag{4.35}$$

(Ghysels & Osborn, 2001, pp. 29-30).

4.3.5. A Summary of Seasonality Models

As mentioned above, there exist two basic types of seasonality:

1) *Deterministic Seasonality* features time-constant seasonal means: These are nonstationary, however when looked at sub-series for seasons, they are stationary. The nature of this seasonality can be expressed with “summer remains summer”. Since stochastic models generally include deterministic parts, deterministic can be seen as a special case of stochastic and so it does not mean that deterministic seasonality is non-stochastic. However, in current usage, stochastic seasonality is described with stationary patterns and deterministic seasonality is described with complete dummy patterns.

2) *Stochastic Seasonality* is separated into two parts:

a. Stationary Stochastic Seasonality features time-constant means (not so good models)

b. Unit Root Seasonality implies nonstationarity. This type of seasonality has evolving seasonal means over time.

In stochastic seasonality, we can say that "summer may become winter" (Kunst, 2012).

4.4. Testing Deterministic Seasonality

It is very crucial to test about whether seasonality of a series is of this type or not. In this respect, Harvey (1989) handles the subject of unobserved components model in the context of $(1+L+....+L^{S-1})y_t^{\,s}=\varepsilon_t^{\,s}$ with $\varepsilon_t^{\,s}$ being i.i.d. $(0,w^2)$ and suggests testing the null hypothesis of deterministic seasonality by means of a test of $w^2=0$ against $w^2>0$. Although this approach is attractive in its simplicity, it is also restrictive in terms of depending on the assumption that the specified unobserved components model sufficiently represents the DGP for y_t. Especially, there is no allowance for stationary stochastic seasonality. In this section, under this framework mentioned above, the discussion will be on more general types of processes (Ghysels & Osborn, 2001, p. 30).

4.4.1. Canova-Hansen (CH) Test

The study of Canova and Hansen (1995) presents Lagrange Multiplier (LM) tests of the null hypothesis of no unit roots at seasonal frequencies against the alternative of a unit root at either a specific seasonal frequency or a set of selected seasonal frequencies. So the test statistics of CH are derived from the LM principle that necessitates only the estimation of the model under the null using least square techniques and they are fairly simple functions of the residuals. These tests are also a framework for testing seasonal stability. CH tests complement the tests of Dickey, Hasza and Fuller (DHF) (1984) and Hylleberg, Engle, Granger and Yoo (HEGY) (1990) that examine the null of seasonal unit roots at one or more seasonal frequencies. So, it is clear that contrary to these seasonal unit root tests, the null hypothesis of CH test is that the process is stationary (that is, stationary seasonality rather than nonstationary seasonality). Here the rejection of the null hypothesis would imply the nonstationarity of the data. Although the null of CH test is stationary seasonality, for simplicity they refer to their tests as seasonal unit root tests. On the other hand, Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) (1992) suggest an LM statistic for the null of stationarity against the alternative of a unit root at the zero frequency. Tanaka (1990) and Saikkonen and Luukkonen (1993) have developed tests with the null hypothesis of a moving average unit root which have an analogue structure with the KPSS test. As HEGY have generalized the Dickey-Fuller (DF) framework from zero frequency to the seasonal frequencies, by the same token Canova and Hansen have generalized the KPSS framework from the zero frequency to the seasonal frequencies. Since seasonal intercepts stand for the deterministic components of seasonality and they are assumed to be constant over the sample, under the null hypothesis of stationarity the tests by Canova and Hansen can also be introduced as the tests for constancy of seasonal intercepts over time. In this context, Canova and Hansen adopt the methodology of Nyblom (1989) and Hansen (1990) who designed LM tests for parameter instability. What is interesting is that the LM test for joint instability of the seasonal intercepts numerically shows equivalence to the LM tests for unit roots at all seasonal frequencies. Therefore CH tests can also be considered as a test for seasonal unit roots or a test for instability in the seasonal pattern. Since the asymptotic distribution is not unaffected by any trending regressors such as a unit root process or a deterministic trend, Canova and Hansen exclude such variables from the regression and they also require that the dependent variable be used as free of trends and thus assume an appropriately transformed data in order to eliminate unit roots at the zero frequency. In their study, Canova and Hansen (1995) deal with Monte Carlo experiments, derive an asymptotic distribution theory for their tests and examine the power of them. The large sample distributions of their test statistics are not standard but they are free from nuisance parameters and affiliated with only one “degrees-of-freedom” parameter. Canova and Hansen compare the power and size properties of their tests with a test for the presence of stochastic (stationary) seasonality and the HEGY tests for seasonal unit roots. As a result, they point out to their tests with reasonable size and power properties. They examine three data sets for their tests: the first one is the data set originally used by Barsky and Miron (1989) asserting the hypothesis that quarterly seasonal fluctuations in U.S. macro variables can be well characterized by deterministic patterns. So, Canova and Hansen have been interested in detecting if this hypothesis is appropriate or not. The second data set is the set of quarterly European industrial production indexes used by Canova (1993) and third one is the set on stock returns on value weighted indexes for seven industrialized countries. As a result of their test applications to these three seasonal variables, Canova and Hansen show us that the seasonal patterns of these variables in most cases display important instabilities and therefore nonstationarity (Canova & Hansen, 1995, pp. 237-238).

The approach of Canova and Hansen (1995) is fundamentally based on the trigonometric representation of deterministic seasonality. Here we show this representation in a different notation as:

$$y_t = \sum_{s=1}^{S} F_S' B_t \delta_{st} + z_t \tag{4.36}$$

where the 1 x S vector F_S' is the *s*th row of the matrix R expressed in (4.13) for quarterly case, $B_t = (\mu_t, \alpha_{1t}, \beta_{1t}, ..., \alpha_{S/2,t})'$ and z_t are disturbances. The assumption is that z_t are normally distributed and stationary, but not necessarily uncorrelated over time. If B_t becomes identical to B for all *t* values, then this equation will be equivalent to (4.6). Canova and Hansen base their theories for this test on all elements of B evolving according to a (vector) random walk:

$$\begin{aligned} B_t &= B_{t-1} + V_t \\ &= B_0 + \sum_{i=1}^{t} V_i \end{aligned} \tag{4.37}$$

where the disturbance term V_t is i.i.d. with $E(V_t V_t') = w^2 H$ where H is a known positive definite matrix and V_t is independent of z_t.

Under the null hypothesis of $w^2 = 0$, B is unchanged over time and therefore there exists deterministic seasonality. The alternative one which is $w^2 > 0$ says that there are unit roots in each element of B_t that are related with the zero and seasonal frequencies. So this alternative hypothesis is also an implication that is equivalent to the nonstationarity of y_t at both zero and seasonal frequencies. In other words, we can say that under the alternative hypothesis, the process is seasonally integrated.

For this test proposed by Canova and Hansen, under the null hypothesis of deterministic seasonality, DGP of (4.36) and (4.37) can be written in vector notation:

$$Y_\tau = \Gamma + Z_\tau = R.B + Z_\tau \tag{4.38}$$

where $Y_\tau = (y_{1\tau}, ..., y_{S\tau})'$ is the vector of observations for year τ and the disturbance process Z_τ is stationary with zero mean and its covariance matrix denoted as $\Omega_Z = E(Z_t Z_t')$ and R matrix is the same as the one discussed in subsection 4.2.2. The columns of this matrix are mutually orthogonal. That is, when the vector R_i represents the *i*th column of R, then $R_i' R_j = 0$ for $i \neq j$. This information makes us guarantee that $R'R = D$ is a diagonal matrix. By multiplying equation (4.38) by R', we get

$$R'Y_\tau = D.B + R'Z_\tau \qquad (4.39)$$

with $E(R'Z_\tau Z_\tau' R) = \Omega_{RZ} = R'\Omega_Z R$. Because D is known to be diagonal, $D.B$ becomes a scaled version of B. For instance, for quarterly case $D.B = (4\mu, 2\alpha_1, 2\beta_1, 4\alpha_2)'$ or in a more explicit way:

$$R'R = D = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \text{ and } D.B = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}_{4x4} \begin{bmatrix} \mu \\ \alpha_1 \\ \beta_1 \\ \alpha_2 \end{bmatrix}_{4x1} = (4\mu, 2\alpha_1, 2\beta_1, 4\alpha_2)'$$

In order to get a test statistic free of nuisance parameters, the covariance matrix H of V_t in (4.37) and used in the CH test is assumed to be associated with the covariance matrix of $R'Z_\tau$ such that $H = \Omega_{RZ}^{-1}$.

As discussed before, the dummy variable and trigonometric representations are equivalent to each other. So, in this case we have identical OLS (ordinary least squares) residuals for both (4.1) and (4.36) models. In order to build the test statistics, OLS residuals under the null of $w^2 = 0$ are utilized. From the OLS residuals, $\hat{z}_t (t = 1,.........., T)$ form the S x 1 vectors of partial sums $\hat{Z}_t^{\,a}$, where $\hat{Z}_t^{\,a'} = (\sum_{j=1}^{t} \hat{z}_j \delta_{1j}, ..., \sum_{j=1}^{t} \hat{z}_j \delta_{Sj})$ for $t = 1,.........., T$ aggregating over time periods to t the residuals for each of the season s. So, sth element of $\hat{Z}_t^{\,a}$ shows the aggregated residuals for season s.

Canova and Hansen propose an LM test statistic as follows:

$$L = \frac{S}{T^2} \sum_{t=1}^{T} (R'\hat{Z}_t^{\,a})' \hat{\Omega}_{RZ}^{-1} (R'\hat{Z}_t^{\,a}) = \frac{S}{T^2} \sum_{t=1}^{T} \hat{Z}_t^{\,a'} \hat{\Omega}_Z^{-1} \hat{Z}_t^{\,a} \qquad (4.40)$$

In order to find out its asymptotic distribution, it should be noted that for the annual disturbance vector Z_τ, the elements of $(\Omega_Z)^{-1/2} Z_\tau$ are mutually independent with the assumption of normality. If we sum up over years 1 to τ, we will get $X_\tau = (\Omega_Z)^{-1/2} \sum_{j=1}^{\tau} Z_j$ as a vector of independent I(1) processes, each having a disturbance with a variance of unity and zero starting value. Then in a similar fashion to (4.25),

$$\frac{1}{\sqrt{T_\tau}} X_\tau \Rightarrow W^x(r) \tag{4.41}$$

where $W^x(r)$ is an S x 1 vector of independent standard Brownian motions (see Appendix A1-A2). Continuing with the same logic of (4.28),

$$\frac{1}{{T_\tau}^2} \sum_{j=1}^{T_\tau} {X_\tau}' X_\tau \Rightarrow \int_0^1 W^x(r)' W^x(r) dr \tag{4.42}$$

Here there are T observations for $T/S = T_\tau$ years and it should be noted that the summation over t implies ST_τ.

Under the null hypothesis, (4.40) will converge to a distribution very closely associated with (4.42):

$$L \Rightarrow \int_0^1 [W^x(r) - rW^x(1)]'[W^x(r) - rW^x(1)] dr \tag{4.43}$$

In this result, a subtraction of $rW^x(1)$ term can be considered as a correction of the vector Brownian motion $W^x(r)$ to estimate B in (4.39); see Nyblom (1989). The distribution of (4.43) is nonstandard. The description of the limit distribution occurs by an integral over a Brownian bridge (see Appendix A3) starting at zero for $r = 0$ and coming down to zero again for $r = 1$(Kunst, 2012). This S x 1 vector $W^x(r)$ is sometimes called the Von-Mises distribution with S degrees of freedom or VM(S)**.** This asymptotic distribution is tabulated by Nyblom (1989) and Canova and Hansen (1995).

For this test of Canova and Hansen, the rejection of the null hypothesis is possible for large values of L. Under the alternative hypothesis, the residuals from (4.1) that construct the test statistic show random walk behavior in B_t. Then, the partial sums expressed in (4.40) emphasize this behavior.

It is obvious here that the number of possible unit roots under test results is the degrees of freedom for the Von-Mises distribution. Therefore it can be realized that if the overall mean is excluded while testing deterministic seasonality, then under the alternative hypothesis, S – 1 seasonal unit roots will be taken into consideration and the asymptotic distribution of L becomes VM(S – 1) (Ghysels & Osborn, 2001, pp. 31-34).

There are some theorems regarding LM test statistic. Let $\underset{d}{\rightarrow}$ denote convergence in distribution:

<u>Theorem 1</u>*:* Under H_0, $L \underset{d}{\rightarrow} VM(a)$.

This theorem says that the large sample distribution of the L statistic does not depend on any nuisance parameters other than a (the rank of A that will be given in equation (4.46)) which explains the number of elements that are being tested for constancy.

Let L_f denote the test statistic for joint test for unit roots at all seasonal frequencies:

Theorem 2: Under H_0, $L_f \underset{d}{\rightarrow} VM(S-1)$.

According to this theorem, the large sample distribution of the test for unit roots at all seasonal frequencies is given by Von-Mises distribution with S – 1 degrees of freedom (d.f.). Hence for quarterly data, in the table for Von-Mises critical values given in Canova and Hansen (1995) the appropriate critical values are found in this table by using the row corresponding to $p = S - 1 = 3$; if worked with monthly data, $p = S - 1 = 11$.

Now let $L_{\pi j/q}$ be the test statistic to test for a seasonal unit root at frequency $(j/q)\pi$ (for $q = s/2$ and $j = 1,....,q$) and L_{π} be the one to test for a seasonal unit root at frequency π:

Theorem 3: Under H_0, 1) for $j < q$, $L_{\pi j/q} \rightarrow_d VM(2)$

2) $L_{\pi} \rightarrow_d VM(1)$.

As well known, for quarterly data there are two seasonal frequencies: $\frac{\pi}{2}$ (annual) and π (biannual). This theorem says that for frequencies which are different from π, the large sample distributions of the tests for seasonal unit roots are given by the generalized Von-Mises distribution with 2 d.f. and for frequency π, the large sample distribution is given with 1 d.f.

For testing the stability of the a^{th} seasonal intercept (where $1 \le a \le s$), choose A to be the unit vector with a 1 in the a^{th} element and zeros elsewhere. In that case, let L_a be the test statistic for testing for instability in an individual season:

Theorem 4: Under H_0, $L_a \rightarrow_d VM(1)$ for each $a = 1,........,s$.

This theorem states that for this test statistic, critical values are given in the first row of the table for Von-Mises critical values. Since the a^{th} dummy variable is zero for all but one in out of every s observations, the test statistic L_a can be calculated using only the residuals from the a^{th} season (Canova & Hansen, 1995, pp. 241-242).

To obtain $\hat{\Omega}_Z$, Canova and Hansen suggest a nonparametric kernel estimator proposed by Newey and West (1987). Their form of kernel estimated enables explanatory variables. However, while their opinion is based on that these explanatory variables should contain a lagged dependent variable y_{t-1}, according to Hylleberg (1995) they should not include y_{t-1}. Since y_{t-1} may display at least some of the effect of the seasonal unit root of -1. If the test statistic includes zero frequency, then y_{t-1} may similarly show the effect of zero frequency unit root of +1. Also, longer lagged dependent variables should not take place. Because these may capture one or more seasonal unit roots and therefore the tests may have no power. To sum up, the approach proposed by Canova and Hansen is most suitable one amongst the others. Since all serial correlation is overcome in a nonparametric way. So, there is no need to include any lagged dependent variables in the test regression (Ghysels & Osborn, 2001, p.34).

In order to describe the data generating processes of Monte Carlo experiments by Canova and Hansen (1995), we can express their regression models in a somewhat different way:

$$y_i = \mu + x_i'\beta + f_i'\gamma_i + e_i \tag{4.44}$$

with

$$\gamma_i = \gamma_{i-1} + u_i \tag{4.45}$$

where x_i is a k x 1 vector of explanatory variables which are not collinear with $f_i'\gamma_i$, f_i is an s–1 vector with $f_{ji}' = (\cos((j/q)\pi i), \sin((j/q)\pi i))$ for $q = s/2$ and $j < q$ and $f_{ji} = \cos(\pi i) = (-1)^i$ for $j = q$ where this latter expression holds since $\sin(\pi i)$ is identically zero for all integer i. The components of f_i represent the cyclical processes at the seasonal frequencies: $(j/q)\pi$, $j = 1,....,q$ and the coefficients γ_i stand for the contribution of each cycle to the seasonal process S_i. The formulation (4.44) is useful in terms of allowing seasonality to be interpreted as cyclical. Here the specification for the deterministic seasonal component is written as $S_i = \sum_{j=1}^{q} f_{ji}'\gamma_j$. Note also that γ_i is an s–1 vector. To allow for unit roots potentially at only a subset of the seasonal frequencies, equation (4.45) is modified as:

$$A'\gamma_i = A'\gamma_{i-1} + u_i \tag{4.46}$$

where A is a (s – 1) x (s – 1) matrix with rank equal to a and a is the number of roots for testing stationarity. Most of the elements in A matrix are zero excluding the elements on the diagonal which correspond to the elements in γ_i for the stationarity test. For instance, on quarterly data, in order to test for the presence of all seasonal unit roots set $A = I_{s-1} = I_3$(here, DGP has no unit roots if $\omega = 0$ but has unit roots at both seasonal frequencies when $\omega \neq 0$), set $A = A_1$ to test for the presence of roots ±i (here the test $L_{\pi/2}$ is designed: so that in the case of $\omega = 0$, there exist no unit roots; but if $\omega \neq 0$, this implies a pair of complex conjugate roots at frequency $\pi/2$) and set $A = A_2$ to test for the presence of root -1 (here L_π test is designed: when $\omega = 0$, there are no unit roots, but when $\omega \neq 0$, there exists a unit root at frequency π) where $A = I_{s-1} = I_3$, A_1 and A_2 are defined as

$$A = I_{s-1} = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In short, under the null hypothesis of $H_0 : w^2 = 0$, S_i is purely deterministic and stationary; thus the series is stationary and u_i is a vector of zero ($E(u_i u_i') = 0$). But if y_i has all seasonal unit roots, $E(u_i u_i') > 0$ (Canova & Hansen, 1995, pp. 239-240, 243-245).

Example: The Simplest Quarterly Case

Let's take the case of quarterly data for which R is given as (4.13). For the sake of simplicity, assume that errors are not autocorrelated ($z_t = \varepsilon_t$) and not heteroscedastic (that is, $E(Z_\tau Z_\tau') = \sigma^2 I_S$). As a result $\Omega_{RZ} = E(R' Z_\tau Z_\tau' R)$ becomes equal to

$$\Omega_{RZ} = \sigma^2 R'R = \sigma^2 D$$
$$= \sigma^2 \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \quad (4.47)$$

If the disturbance $z_t = \varepsilon_t$ satisfies the standard properties expressed here under the null hypothesis, then the optimal choice is to estimate the deterministic seasonal component by OLS. For this, assume that we apply to equation (4.1) and OLS residuals are

$\hat{\varepsilon}_t = \hat{\varepsilon}_{S\tau}$, where $\hat{\varepsilon}_{S\tau}$ is a notation for season s of year τ. In order to keep the analysis as simple as possible, there is a simplifying assumption that period t is the fourth quarter of year τ, then

$$R'\hat{Z}_t^{\,a} = R'\hat{E}_t^{\,a} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix} . \begin{bmatrix} \sum_{j=1}^{\tau} \hat{\varepsilon}_{1j} \\ \sum_{j=1}^{\tau} \hat{\varepsilon}_{2j} \\ \sum_{j=1}^{\tau} \hat{\varepsilon}_{3j} \\ \sum_{j=1}^{\tau} \hat{\varepsilon}_{4j} \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{j=1}^{\tau} \hat{\varepsilon}_{1j} + \sum_{j=1}^{\tau} \hat{\varepsilon}_{2j} + \sum_{j=1}^{\tau} \hat{\varepsilon}_{3j} + \sum_{j=1}^{\tau} \hat{\varepsilon}_{4j} \\ \sum_{j=1}^{\tau} \hat{\varepsilon}_{4j} - \sum_{j=1}^{\tau} \hat{\varepsilon}_{2j} \\ \sum_{j=1}^{\tau} \hat{\varepsilon}_{1j} - \sum_{j=1}^{\tau} \hat{\varepsilon}_{3j} \\ \sum_{j=1}^{\tau} \hat{\varepsilon}_{4j} - \sum_{j=1}^{\tau} \hat{\varepsilon}_{3j} + \sum_{j=1}^{\tau} \hat{\varepsilon}_{2j} - \sum_{j=1}^{\tau} \hat{\varepsilon}_{1j} \end{bmatrix} \tag{4.48}$$

More generally, for a t which corresponds to season $s < 4$ of year τ, the summations will be up to year $\tau - 1$ for quarters $s+1,.....,4$ since these are following *t*.

With all assumptions given above, by using equations (4.47) and (4.48), CH test statistic as given in (4.40) can be written as:

$$L = \frac{1}{T^2\tilde{\sigma}^2} \sum_{t=1}^{T} \begin{bmatrix} \left(\sum_{j=1}^{\tau} \hat{\varepsilon}_{1j} + \sum_{j=1}^{\tau} \hat{\varepsilon}_{2j} + \sum_{j=1}^{\tau} \hat{\varepsilon}_{3j} + \sum_{j=1}^{\tau} \hat{\varepsilon}_{4j} \right)^2 + \\ 2\left(\sum_{j=1}^{\tau} \hat{\varepsilon}_{4j} - \sum_{j=1}^{\tau} \hat{\varepsilon}_{2j} \right)^2 + 2\left(\sum_{j=1}^{\tau} \hat{\varepsilon}_{1j} - \sum_{j=1}^{\tau} \hat{\varepsilon}_{3j} \right)^2 \\ + \left(\sum_{j=1}^{\tau} \hat{\varepsilon}_{4j} - \sum_{j=1}^{\tau} \hat{\varepsilon}_{3j} + \sum_{j=1}^{\tau} \hat{\varepsilon}_{2j} - \sum_{j=1}^{\tau} \hat{\varepsilon}_{1j} \right)^2 \end{bmatrix} \tag{4.49}$$

or

$$L = L_0 + L_{\pi/2} + L_{\pi} \tag{4.50}$$

Here $\hat{\Omega}_{RZ} = \tilde{\sigma}^2 D$ and $\tilde{\sigma}^2$ stands for the usual OLS estimator of σ^2. L_0, $L_{\pi/2}$ and L_{π} test statistics which are relevant to the 0, $\pi/2$ and π frequencies are given in the first, second and third lines of equation (4.49) respectively (in each case aggregated over $t = 1,........,T$ and scaled by division by $T^2\tilde{\sigma}^2$).

With some straightforward algebra, the CH test statistic can be written in accordance with the form given in the second line in (4.40) as

$$L = \frac{4}{T^2 \tilde{\sigma}^2} \sum_{t=1}^{T} \left[\left(\sum_{j=1}^{\tau} \hat{\varepsilon}_{1j} \right)^2 + \left(\sum_{j=1}^{\tau} \hat{\varepsilon}_{2j} \right)^2 + \left(\sum_{j=1}^{\tau} \hat{\varepsilon}_{3j} \right)^2 + \left(\sum_{j=1}^{\tau} \hat{\varepsilon}_{4j} \right)^2 \right] \tag{4.51}$$

which refers to the statistic in terms of separate squared partial sums for each of the four quarters.

Since the number of possible unit roots is four under the overall null hypothesis, L ~ $VM(4)$. When the constant μ is excluded, the test statistic would be distributed as $VM(3)$. If we apply for separate tests at the spectral frequencies of $\pi/2$ and π, under the null hypothesis of deterministic seasonality they would be distributed as $VM(2)$ and $VM(1)$ respectively.

For the general quarterly case if the disturbances of (4.36) are not i.i.d. $(0, \sigma^2)$, then the summation of the separate test statistics of (4.50) cannot be performed. In this case, the test statistics at different frequencies are not mutually independent anymore (Ghysels & Osborn, 2001, pp. 34-36).

4.4.2. The Caner Test

Caner (1998) adopts the CH framework. However, instead of the nonparametric Newey-West correction to autocorrelation adopted by Canova and Hansen (1995) he proposes a parametric autoregressive augmentation. Hence, in the Caner test the disturbances are assumed to be i.i.d. Although his test excludes the overall constant μ from consideration, the null hypothesis model of deterministic seasonality can be generalized by including this overall μ as follows:

$$\phi(L) y_t = \sum_{s=1}^{S} F_s{}' B_t \delta_{st} + \varepsilon_t \tag{4.52}$$

where B_t is constant over t.

By making suitable assumptions about the starting values, this process given in (4.52) is also equivalent to

$$\phi(L) \Delta_S y_t = \theta(L) \varepsilon_t \tag{4.53}$$

with $\theta(L) = 1 - L^S$. Because of its better properties near the invertibility boundary, generally for MA processes the exact ML procedure is preferred (Ansley & Newbold, 1980). That is why Caner mostly proposes the use of ML estimation for $\theta(L)$ instead of least squares. The alternative hypothesis is that $\theta(L)\varepsilon_t$ in (4.53) represents a general

MA(S) process [not $(1-L^S)\varepsilon_t$]. So, this alternative implies that B_t includes one or more unit roots (therefore y_t is nonstationary). In the case of including exactly S unit roots, it becomes $\theta(L)\varepsilon_t = \varepsilon_t$ (so, Δ_S will remove the nonstationarity).

Caner test statistic is given as:

$$L = \frac{S}{\tilde{\sigma}^2 T^2} \sum_{t=1}^{T} (R'\hat{E}_t{}^a)' D^{-1} (R'\hat{E}_t{}^a), \tag{4.54}$$

where $\tilde{\sigma}^2$ is a consistent estimator of $\sigma^2 = Var(\varepsilon_t)$ and $\hat{E}_t{}^a$ is obtained using the residuals of (4.52). Under the null hypothesis, $L \Rightarrow VM(S)$. As in (4.50), the decomposition of this statistic is likely to be expressed as the sum of statistics that test the null hypothesis for the zero frequency and for each of the seasonal frequencies and therefore if the overall mean is excluded, deterministic seasonality is tested with the test statistic given by $L_{\pi/2} + L_{\pi} \Rightarrow VM(3)$ (Ghysels & Osborn, 2001, pp. 36-37).

4.4.3. Tam-Reinsel Test

Tam and Reinsel (1997) also consider the validity for the null hypothesis of deterministic seasonality. Taking annual differences in the dummy variable representation in (4.1), their test is based on the null of $\theta_S = 1$ in

$$\Delta_S y_t = z_t - \theta_S z_{t-S}, \qquad t = 1,\ldots\ldots,T \tag{4.55}$$

Here the initial assumption is that $z_t = \varepsilon_t \sim$ i.i.d. $(0,\sigma^2)$. When (4.55) is compared with the Caner's representation (4.53), the approach is seen to be similar to that of Caner (1998).

Tam and Reinsel consider their test as LBIU and present two equivalent forms for their test statistic. The first approach is realized through (4.55). Under the null hypothesis, the equation with i.i.d. disturbance assumption and $\theta_S = 1$ can be written as

$$\Delta_S y_{s\tau} = \varepsilon_{s\tau} - \varepsilon_{s,\tau-1}, \tag{4.56}$$

In (4.56) it is obvious that only observations and disturbances for season s are considered. The vector of the differenced values relating to year τ is given as $\Delta_S Y_\tau = (\Delta_S y_{1\tau},\ldots\ldots,\Delta_S y_{S\tau})'$ and for this vector, the covariance properties when the null hypothesis is true are:

$$E(\Delta_S Y_{\tau 1} \Delta_S Y'_{\tau 2}) = \begin{cases} 2\sigma^2 I_S, & \tau_2 = \tau_1 \\ -\sigma^2 I_S, & \tau_2 = \tau_1 \pm 1 \\ 0, & otherwise \end{cases} \tag{4.57}$$

If the consideration is directed to the complete sample period vector (that is, $\Delta_s Y = (\Delta_S Y'_1, ..., \Delta_S Y'_{T_\tau})'$), in that case, under the null hypothesis T x T covariance matrix is known. If Ω_Y denotes $E(\Delta_S Y \Delta_S Y')$ for $\theta_S = 1$, the test statistic for the null hypothesis of $\theta_S = 1$ against the invertible seasonal moving average alternative hypothesis $\theta_S < 1$ is written as:

$$L_{MA} = \frac{1}{T_\tau} \frac{\Delta_S Y' [\Omega_Y]^{-2} \Delta_S Y}{\Delta_S Y' [\Omega_Y]^{-1} \Delta_S Y} \tag{4.58}$$

and of course, the rejection of the null hypothesis is possible for large values of L_{MA}.

When it comes to the second approach for test statistic, it makes use of equivalence between deterministic seasonality and the seasonally differenced process with $\theta_S = 1$ as expressed in Bell (1987) and discussed in subsection (4.3.3.). So, instead of (4.56) with seasonal MA representation, corresponding representation becomes

$$y_t = \sum_{s=1}^{S} \gamma_s \delta_{st} + \varepsilon_t, \quad t = -S+1, ..., 1, ..., T \tag{4.59}$$

In this second approach, for the equivalence of Bell it is crucially needed to contain the starting values. So, the period starts at year $\tau = 0$. When OLS procedure is applied to all T+S observations of (4.59), $\hat{\varepsilon}_t$ becomes equal to $\hat{\varepsilon}_{s\tau}$ and the test statistic becomes

$$L_{MA} = \frac{1}{ST_\tau (T_\tau + 1)\tilde{\sigma}^2} \sum_{s=1}^{S} \sum_{\tau=0}^{T_\tau} \left[\sum_{j=0}^{\tau} \hat{\varepsilon}_{sj} \right]^2,$$
$$= \frac{1}{ST_\tau (T_\tau + 1)\tilde{\sigma}^2} \sum_{\tau=0}^{T_\tau} (\hat{E}_\tau{}^a)' \hat{E}_\tau{}^a, \tag{4.60}$$

where $\tilde{\sigma}^2$ is the OLS estimator of σ^2 and $\hat{E}_\tau{}^a$ represents the vector of accumulated season-specific residuals at the end of year τ $(E_\tau{}^a = (\sum_{j=1}^{\tau} \varepsilon_{1j}, ..., \sum_{j=1}^{\tau} \varepsilon_{sj})')$. However, since only end of year values are considered in the first approach for test statistic, this approach applies to $t = S\tau$ over $\tau = 0, 1, ..., T_\tau$.

When we compare test statistics, Tam Reinsel approach essentially constructs the overall test statistic by examining each of the seasons while the test statistic proposed by Caner has a decomposition allowing for each of the seasonal frequencies to be examined separately. Therefore, the Tam Reinsel form has a practical use in order to examine about whether deviation from the null hypothesis process of constant deterministic components is associated with, say, only one or two specific seasons. So it can be said that the form of the test statistic to be used should depend on the issues of interest in a specific case. One difference between CH and Tam-Reinsel approaches in terms of test statistic is that while the Tam-Reinsel statistic sums over years, the former sums over all observations. The precise relationship between these statistics is $L_{MA} = \frac{T_\tau}{S(T_\tau - 1)}.L$. This relation refers to the overall test statistic for the null hypothesis of constancy of the parameters $\gamma_1, \gamma_2, ..., \gamma_s$ of the deterministic component of (4.1) over time. On the other hand, when the scaling in test statistics is taken into account, the tabulated asymptotic critical values in Canova and Hansen (1995) and in Tam and Reinsel (1997) are seen to be very similar (Ghysels & Osborn, 2001, pp.39-40).

Tam and Reinsel (1997) also consider their analysis with the disturbances in (4.55) having a stationary and invertible ARMA process given as again with the null of $\theta_S = 1$:

$$\phi(L)\Delta_S y_t = \theta(L)\varepsilon_t = \theta^*(L)(1 - \theta_S L^S)\varepsilon_t , \qquad (4.61)$$

(in that case z_t in (4.55) will be equal to $\phi(L)^{-1}\theta^*(L)\varepsilon$) and with a consistent estimator under both null and alternative hypotheses, the asymptotic distribution becomes unaffected by the use of a parametric correction which depends on the estimates of $\phi(L)$ and $\theta^*(L)$ (Ghysels & Osborn, 2001, pp. 37-39).

CHAPTER V

INTRODUCTION TO SEASONAL UNIT ROOT PROCESSES

Time series models with unit roots are of great importance in terms of understanding the responses of economic systems to shocks. First suggestion for univariate unit root tests was realized by Fuller (1976) and Dickey and Fuller (1979). The paper of Hylleberg et al. (1990) deals with tests for unit roots at seasonal frequencies which have a modulus of one. In this study, besides having the modulus of one the interest is on the root which is precisely one and therefore corresponding to a zero frequency peak in the spectrum[2]. Since many economic time series display substantial seasonality, it is very likely to have unit roots at seasonal frequencies.

5.1. Seasonal Time Series Processes

There are a lot of possible models to take seasonality into consideration that could differ across economic time series with crucial seasonal components. A seasonal series is a series with a spectrum having distinct peaks at the seasonal frequencies $w_s = \frac{2\pi j}{s}$, $j = 1, ..., s/2$ where s is the number of time periods in a year supposing that s is an even number. For example, if we are dealing with quarterly data, s is equal to 4 and for monthly data s is then 12.

According to Hylleberg et al. (1990), there are three classes of time series models prevalently used in order to model seasonality as follows:

a) Purely deterministic seasonal processes
b) Stationary seasonal processes
c) Integrated seasonal processes

The first type of process is one generated by seasonal dummy variables (as mentioned in subsection (4.2.1.) and could be expressed in the case of quarterly series in

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[2] The spectrum of a time series is the distribution of variance of the series as a function of frequency and the spectral analysis aims to estimate the spectrum. Actually, the mathematical computation of spectrum is possible through transformation of the auto covariance function (acvf). While spectrum contains information on the variance in the frequency domain, the latter summarizes this information in the time domain (“Spectrum”, 2015).

the following form:

$$x_t = m_0 + m_1 S_{1t} + m_2 S_{2t} + m_3 S_{3t}, \qquad (5.1)$$

The second type of process can be expressed in an autoregression form of $\varphi(B)x_t = \varepsilon_t$ with ε_t i.i.d. and all roots of $\varphi(B) = 0$ lying outside the unit circle. For quarterly data, this stationary process is represented as

$$x_t = \rho x_{t-4} + \varepsilon_t \qquad (5.2)$$

with a peak at both $w_1 = \frac{\pi}{2}$ (one cycle per year) and $w_2 = \pi$ (two cycles per year) frequencies as well as at zero frequency (zero cycles per year).

On the other hand, a series x_t has a third type of process if a seasonal unit root takes place in its AR representation and generally this integrated process is denoted as $x_t \sim I_\theta(d)$ with integration order d at θ frequency. The study of Hylleberg et al. (1990) examines the case of d=1. An example for the quarterly integrated process at π frequency is

$$x_t = -x_{t-1} + \varepsilon_t \qquad (5.3)$$

and at π/2 frequency is

$$x_t = -x_{t-2} + \varepsilon_t \qquad (5.4)$$

Box and Jenkins (1970) have proposed a very-well recognized seasonal differencing operator. Subsequent to them, Grether and Nerlove (1970) and Bell and Hillmer (1984) have made use of this operator as a seasonal process. For quarterly case it can be factorized as,

$$(1-L^4)x_t = (1-L).(1+L+L^2+L^3).x_t$$

$$= (1-L).(1+L).(1+L^2)..x_t = (1-L).(1+L).(1-iL).(1+iL).x_t$$

$$= (1-L).S(L).x_t \qquad (5.5)$$

where $S(L) = (1+L).(1+L^2)$ and i represents an imaginary part of a complex number such that $i^2 = -1$.

According to this factorization, there are four roots with modulus of one for quarterly stochastic seasonal unit root process: one is $(1-L)$ denoting zero frequency which removes the trend. Amongst other three roots which are $(1+L), (1-iL)$ and $(1+iL)$ and which eliminate the seasonal form, the first root is at 2 cycles per year and the other two roots are complex pairs at 1 cycle per year (Charemza & Deadman, 1997, p. 108).

Since in (5.5), $(1-L)..x_t$ implies zero frequency root and the stochastic difference equations of (5.3) and (5.4) express the roots at seasonal frequencies which are $\pi/2$ and π; the homogenous solutions to equations $(1-L).x_t = \varepsilon_t$, (5.3) and (5.4) become respectively,

$$s_{1t} = \sum_{j=0}^{t-1} \varepsilon_{t-j}\ , \quad \text{for zero frequency root,}$$

$$s_{2t} = \sum_{j=0}^{t-1} (-1)^j \varepsilon_{t-j}\ , \quad \text{for the two cycle per year root (}\pi\text{ frequency),}$$

$$s_{3t} = \sum_{j=0}^{\mathrm{int}[(t-1)/2]} (-1)\Delta\varepsilon_{t-2j}\ , \quad \text{for the one cycle per year root (}\pi/2\text{ frequency),} \qquad (5.6)$$

where $\Delta = 1-L$ and $\mathrm{int}[z]$ means the largest integer in z.

The variances of the s_{1t}, s_{2t} and s_{3t} series are all the same with $t\sigma^2$ (that is, linearly increasing variances) given as

$$V(s_{1t}) = V(s_{2t}) = V(s_{3t}) = t\sigma^2 \qquad (5.7)$$

and therefore it is valid for all unit roots that the variance has an inclination to go infinity with evolving process. In the case of being stimulated by the same $\{\varepsilon_t\}$ and when t is divisable by four, all the covariances of the series become zero. For other values of t, the covariances are at most σ^2, thus the series are asymptotically uncorrelated as well as being uncorrelated in finite samples for entire years of data.

A more general case of linear time series models which may exhibit complex forms of seasonality as a combination of seasonally integrated, deterministic or stationary seasonals can be written as:

$$d(B).a(B).(x_t - \mu_t) = \varepsilon_t \qquad (5.8)$$

where the first term $d(B)$ represents an integrated seasonal process in which all roots of $d(z) = 0$ lie on the unit circle, $a(B)$ includes stationary seasonality and other stationary elements of x with all roots of $a(z) = 0$ lying outside the unit circle and deterministic seasonal component is incorporated into μ_t when there are no seasonal unit roots in $d(B)$ (implying that there is no seasonal unit root in AR representation of x_t) (Hylleberg et al., 1990, pp. 215-220).

5.2. Testing Seasonal Integration

As well known, a series generated by a unit root process can wander widely over time not having any inclination to return to its underlying mean value and thus not having any tendency to return to a deterministic pattern. In that case, with the values wandering to a great extent for the seasons, any basic relationships between the expected values for the different seasons remain beside the point in practice. It is already outlined in the subsection (4.3.5.) that in the presence of seasonal unit roots, summer may become winter. From this point of view, in this section, a number of testing procedures will be mentioned in order to test the null hypothesis of seasonal integration and thus the implications of seasonal unit root processes will be handled in more detail.

Definition: The nonstationary stochastic process y_t, observed at S equally spaced time intervals per year, is said to be seasonally integrated of order d, denoted $y_t \sim SI(d)$, if $\Delta_S^d y_t$ is a stationary, invertible ARMA process.

Here Δ_S denotes the seasonal differencing filter. The implication of the definition is that if y_t becomes a stationary and invertible process after annual differencing, then $y_t \sim SI(1)$. Generally the case of $d > 1$ is not observed prevalently in practice. In subsection (4.3.3.), we had discussed the equivalance between deterministic seasonality and seasonal unit root process which requires seasonal differencing (in the special case of $\theta_S = 1$). However, the implication of this equivalence is not that the deterministic seasonal process is seasonally integrated. The underlying reason is that applying seasonal differencing to a deterministic seasonal process prompts the existence of first order annual differencing operator Δ_S in the MA operator and this will lead to non-invertibility of MA operator. Therefore, a deterministic seasonal process and a seasonally integrated process are not identical processes (Ghysels & Osborn, 2001, pp. 42-43).

Another definition for a seasonally integrated series is a simplified version of the definition given by Engle, Granger and Hallman (1989) for a seasonally integrated series can be given as:

Definition: A nonstationary series is said to be seasonally integrated of order (d, D), denoted $SI_s(d, D)$, If it can be transformed to a stationary series by applying s-differences D times and then differencing the resulting series d times using first differences.

In a simple manner, a seasonal difference is the difference between an observation and its value for the corresponding season one year before. If the series is measured s times per annum (for quarterly data, $s = 4$ and for monthly data $s = 12$) and it displays a seasonal pattern, then the differencing to remove seasonality should be s rather than one. So, the type of operator to be applied here is $x_t - x_{t-s}$ (representing seasonal difference) instead of $x_t - x_{t-1}$. Here, the transaction to get these variables is called seasonal differencing or s-differencing. Generally, it is very rare to use s-differencing more than once in order to remove seasonality. Taking seasonal differences transforms a linear trend with an additive seasonal effect to a constant (that is, to a variable with no trend or seasonal pattern). If this transaction is applied to a quadratic trend (where the trend is nonlinear) with additive seasonality, it brings about a series still including a trend component but with no seasonal pattern. So, in order to make such a series is stationary, first differencing of the s-differences may be required (Charemza & Deadman, 1992, pp. 53, 129-130). Seasonal differencing may be in additive or multiplicative form. An additive form of a seasonal difference at a seasonal lag – such as $(1 - L^s)$ – can be expressed as $(1 - L^4)y_t = C + e_t$. On the other hand, as implied by its name a multiplicative form of seasonal difference requires the multiplication of the nonseasonal by the seasonal differencing factors. So, in this form, getting a stationary series requires the multiplication of the first regular (nonseasonal) factor by the seasonal factor to obtain the differencing for the series. A multiplicative differencing in a multiplicative SARIMA model is expressed in the form of $(1 - L^d)(1 - L^s)y_t = C + e_t$, where y_t is the undifferenced series variable, d is the order of regular differencing and s is the order of seasonal differencing (Yaffee & McGee, 2000, pp. 161-162). As mentioned before, a clear definition of a multiplicative SARIMA process is available in subsection 3.5.

Ilmakunnas (1990) has tried to illustrate a testing sequence in order to test the appropriate order of differencing in quarterly data. Introducing this testing sequence requires two alternative definitions of seasonal integration. According to the first definition which is the one defined by Osborn et al. (1988), a time series is said to be integrated of order (d, D), denoted $I(d, D)$ if the series becomes stationary subsequent to first-differencing d times and seasonally differencing D times. In other saying; if $(1-L)^d(1-L^S)^D x_t = \Delta^d \Delta_S^D x_t$ becomes stationary, x_t is said to be $I(d,D)$. In the paper proposed by Ilmakunnas (1990), since the focus is on the quarterly time series $(s=4)$, it is concerned with the case where $I(1,1)$ is the maximum order of integration. The second alternative definition for seasonal integration comes from Engle, Granger and Hallman (1989) that has already been mentioned above. To this definition; if $(1-L)^d S(L)^D x_t = \Delta^d S(L)^D x_t$ is stationary, x_t is said to be seasonally integrated of orders d and D expressed as $SI(d,D)$ where $S(L)$ is a seasonal filter used in transforming the variables to moving sums. In the case of quarterly data, seasonal filter is stated as $S(L) = 1 + L + L^2 + L^3$ and it takes place in the decomposition of $\Delta_4 = (1-L)S(L) = (1-L)(1+L)(1+iL)(1-iL)$. Since $\Delta\Delta_4$ is decomposed as $(1-L)^2 S(L)$ or $(1-L)[(1-L)S(L)] = (1-L)(1-L^4)$, $SI(2,1)$ and $I(1,1)$ are the same. In the same manner, $SI(1,0)$ is the same as $I(1,0)$ and also $SI(1,1)$ and $I(0,1)$ are the same.

To illustrate the testing sequence for quarterly data, starting point is taken as the maximum order of seasonal integration, i.e. the case $SI(2,1)$. This testing sequence is shown as follows:

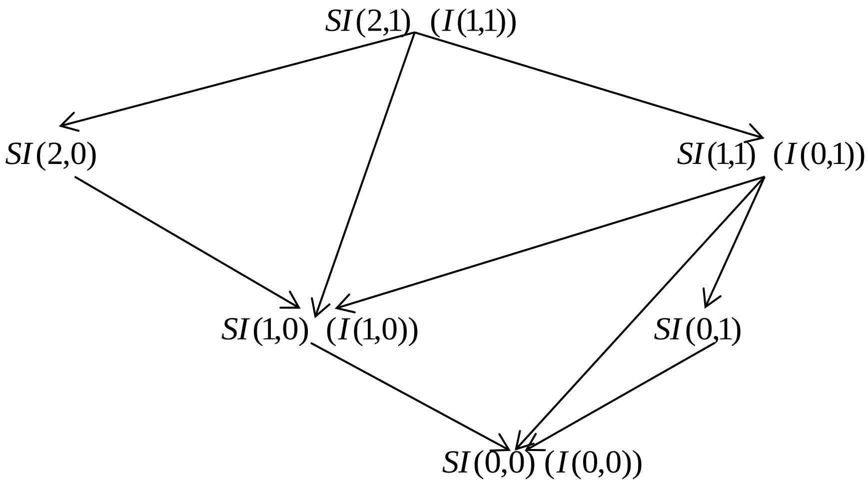


Figure 4. The testing sequence for determining the appropriate seasonal integration order in quarterly data
(Source: Ilmakunnas, 1990).

The representation in Figure 4 pursues the view proposed by Dickey and Pantula (1987). According to their view, if it is mentioned about multiple unit roots, the best thing is to start the testing sequence from the maximum number of unit roots in hand and in this case the nominal test size is preserved. Therefore, it can be expressed that determining the suitable integration order is based on the starting point of the testing sequence (Ilmakunnas, 1990, pp. 79-81). Ilmakunnas (1990) mentions about how to handle unit root testing in a seasonal context considering the initial test of the $SI(2,1)$ null hypothesis. In the study, it is expressed that $SI(2,1)$ is tested against $SI(2,0)$, $SI(1,1)$ and $SI(1,0)$ alternatives using the HEGY test regression applied to ΔX_t rather than to X_t as will be shown in Table 5. In case we reject the null hypothesis in favour of either $SI(1,1)$ or $SI(1,0)$ alternatives, we have to check the presence of zero frequency unit root against $SI(0,1)$ or $SI(0,0)$ processes, respectively continuing for testing against lower orders of integration (Ghysels & Osborn, 2001, p. 76).

In Table 5, it is shown that which hypotheses can be tested with each given test in the testing sequence:

Table 5
Seasonal Integration Tests for Different Hypotheses

Description of the tests	***Null Hypothesis***	***Alternative Hypothesis***	***Remarks***
ADF: *t*-statistics of β in $\Delta X_t = \beta X_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta X_{t-j} + u_t$	*SI(1,0)*	*SI(0,0)*	
ADF for Δ series: *t*-statistics of β in $\Delta^2 X_t = \beta \Delta X_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta^2 X_{t-j} + u_t$	*SI(2,0)*	*SI(1,0)*	
ADF for Δ_4 series: *t*-statistics of β in $\Delta\Delta_4 X_t = \beta \Delta_4 X_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta\Delta_4 X_{t-j} + u_t$	*SI(2,1)*	*SI(1,1)*	
ADF for $S(L)$ series: *t*-statistics of β in $\Delta_4 X_t = \beta S(L) X_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta_4 X_{t-j} + u_t$	*SI(1,1)*	*SI(0,1)*	
DHF: *t*-statistic for β in $\Delta_4 X_t = \beta Z_{t-4} + \sum_{j=1}^{p} \alpha_j \Delta_4 X_{t-j} + u_t$ where $Z_t = X_t - \sum_{j=1}^{p} \theta_j X_{t-j}$ and θ_j is the coefficient of $\Delta_4 X_{t-j}$ from a regression of $\Delta_4 X_t$ on its p lagged values.	*SI(1,1)*	*SI(0,0)*	

Table 5 (*Continued)*

Description of the tests			
DHF for Δ series: *t*-statistic for β in $\Delta\Delta_4 X_t = \beta Z^*_{t-4} + \sum_{j=1}^{p} \alpha_j \Delta\Delta_4 X_{t-j} + u_t$ where $Z_t = \Delta X_t - \sum_{j=1}^{p} \theta_j \Delta X_{t-j}$ and θ_j is the coefficient of $\Delta\Delta_4 X_{t-j}$ from a regression of $\Delta\Delta_4 X_t$ on its p lagged values.	*SI(2,1)*	*SI(1,0)*	
HEGY: *t*-statistics for π_1 and π_2 and F-statistic for testing $\pi_3 = \pi_4 = 0$ (or *t*-statistics sequentially for π_4 (two-sided test) and π_3) in	*SI(1,1)*	*SI(1,0)*	π_2, π_3, π_4 tested, $\pi_1 = 0$
$\Delta_4 X_t = \pi_1 Z_{1,t-1} + \pi_2 Z_{2,t-1} + \pi_3 Z_{3,t-2} + \pi_4 Z_{3,t-1} + \sum_{j=1}^{p} \alpha_j \Delta_4 X_{t-j} + u_t$	*SI(1,0)*	*SI(0,0)*	π_1 tested; $\pi_2, \pi_3, \pi_4 \neq 0$
where $Z_{1t} = S(L)(X_t - \sum_{j=1}^{p} \theta_j X_{t-j})$,	*SI(1,1)*	*SI(0,0)*	$\pi_1, \pi_2, \pi_3, \pi_4$ tested
$Z_{2t} = -(1 - L + L^2 - L^3)(X_t - \sum_{j=1}^{p} \theta_j X_{t-j})$,	*SI(1,1)*	*SI(0,1)*	π_1 tested; $\pi_2 = \pi_3 = \pi_4 = 0$
$Z_{3t} = -(1 - L^2)(X_t - \sum_{j=1}^{p} \theta_j X_{t-j})$ and θ_j are obtained as in DHF.	*SI(0,1)*	*SI(0,0)*	π_2, π_3, π_4 tested, $\pi_1 \neq 0$
HEGY for Δ series: *t*-statistics for π_1 and π_2 and F-statistic for testing $\pi_3 = \pi_4 = 0$ (or *t*-statistics sequentially for π_4 (two-sided test) and π_3) in	*SI(2,1)*	*SI(2,0)*	π_2, π_3, π_4 tested, $\pi_1 = 0$
$\Delta\Delta_4 X_t = \pi_1 Z^*_{1,t-1} + \pi_2 Z^*_{2,t-1} + \pi_3 Z^*_{3,t-2} + \pi_4 Z^*_{3,t-1} + \sum_{j=1}^{p} \alpha_j \Delta\Delta_4 X_{t-j} + u_t$	*SI(2,0)*	*SI(1,0)*	π_1 tested; $\pi_2, \pi_3, \pi_4 \neq 0$
where $Z^*_{1t} = (\Delta_4 X_t - \sum_{j=1}^{p} \theta_j \Delta_4 X_{t-j})$,	*SI(2,1)*	*SI(1,0)*	$\pi_1, \pi_2, \pi_3, \pi_4$ tested
$Z^*_{2t} = -(1 - L + L^2 - L^3)(\Delta X_t - \sum_{j=1}^{p} \theta_j \Delta X_{t-j})$, $Z^*_{3t} = -(1 - L^2)(\Delta X_t - \sum_{j=1}^{p} \theta_j \Delta X_{t-j})$ and θ_j are obtained as in DHF for Δ series.	*SI(2,1)*	*SI(1,1)*	π_1 tested; $\pi_2 = \pi_3 = \pi_4 = 0$

Table 5 (*Continued)*

Description of the tests			
OCSB: *t*-statistics for β_1 and β_2 in			
$\Delta\Delta_4 X_t = \beta_1 Z_{4,t-1} + \beta_2 Z_{5,t-4} + \sum_{j=1}^{p} \alpha_j \Delta\Delta_4 X_{t-j} + u_t$	*SI(2,1)*	*SI(1,0)*	β_2 tested; $\beta_1 = 0$
where $Z_{4t} = \Delta_4 X_t - \sum_{j=1}^{p} \theta_j \Delta_4 X_{t-j}$, $Z_{5t} = \Delta X_t - \sum_{j=1}^{p} \theta_j \Delta X_{t-j}$	*SI(2,1)*	*SI(1,1)*	β_1 tested; $\beta_2 = 0$
and θ_j are obtained as in DHF for Δ series.	*SI(1,0)*	*SI(0,0)*	β_1 tested; $\beta_2 \neq 0$
	SI(1,1)	*SI(0,0)*	β_2 tested; $\beta_1 \neq 0$

(Source: Ilmakunnas, 1990, pp. 82-83).

5.2.1. Dickey-Hasza-Fuller Test

One of the simplest testing procedures for seasonal integration possibly belongs to the one proposed by Dickey, Hasza and Fuller (1984) and modified by Osborn et al. (1988), denoted DHF. It can be regarded as a generalization of the Augmented Dickey Fuller test (ADF) and it is the first test of the null hypothesis $y_t \sim SI(1)$ .Using DHF test for seasonal integration is identical to testing for stochastic seasonality. Supposing that the process is known to be a SAR(1) $[y_t = \phi_s y_{t-s} + \varepsilon_t]$, then the DHF test can be parameterized as

$$\Delta_s y_t = \alpha_s y_{t-s} + \varepsilon_t \tag{5.9}$$

where $\alpha_s = -(1-\phi_s)$. Here the null hypothesis of seasonal integration is $\alpha_s = 0$ and the alternative of a stationary stochastic seasonal process implies $\alpha_s < 0$ (Baltagi, 2001, p. 661). Under the null hypothesis, *t* statistic becomes

$$t(\hat{\alpha}_s) = \frac{\frac{1}{T}\sum_{t=1}^{T} y_{t-s}\varepsilon_t}{\tilde{\sigma}\left[\frac{1}{T^2}\sum_{t=1}^{T} y_{t-s}^2\right]^{1/2}} \quad (and\ ,\ \frac{T}{S}\hat{\alpha}_s \Rightarrow \frac{\sum_{s=1}^{S}\int_0^1 W_s(r)dW_s(r)}{\sum_{s=1}^{S}\int_0^1 W_s^2(r)dr}) \tag{5.10}$$

and the asymptotic distribution of the DHF statistic is given by

$$t(\hat{\alpha}_s) \Rightarrow \left[\sum_{s=1}^{S}\int_0^1 W_s(r)dW_s(r)\right] / \left\{\left[\sum_{s=1}^{S}\int_0^1 [W_s(r)^2]dr\right]^{1/2}\right\} \tag{5.11}$$

which is nonstandard, but it has a similar type to the DF *t* distribution. It is very well known that the DF *t* statistic is not symmetric about zero. In terms of (5.11), the denominator is always positive and therefore $\Pr[\chi^2(S) < S]$ shows the probability that $t(\hat{\alpha}_s)$ is negative. Fuller (1996) comments that the asymptotically the probability of $\hat{\alpha}_1 < 0$ (that is, $\hat{\phi}_1 < 1$) is 0,68 for the nonseasonal random walk, since the probability of $\chi^2(1) < 1$ is 0,68. On the other hand, for a seasonal random walk with quarterly data, $\Pr[\chi^2(4) < 4] = 0{,}59$ and with monthly data, $\Pr[\chi^2(12) < 12] = 0{,}55$. Hence, it can be inferred from these values that the predominance of negative test statistics is expected to decrease as S increases. From this expression, it is apparent to see that the distribution for the DHF *t*-statistic depends on S which represents the frequency with which observations are made within each year. The limit distributions shown as functions of Brownian motions can also be found in Chan (1989), Boswijk and Franses (1996) and Osborn and Rodrigues (1998). Here the numerator involves the sum of S such terms that are mutually independent and therefore

$$\sum_{s=1}^{S}\int_{0}^{1} W_s(r)dW_s(r) = \frac{1}{2}\sum_{s=1}^{S}\{[W_s(1)^2 - 1] = \frac{1}{2}\left\{\chi^2(S) - S\right\} \tag{5.12}$$

which is half the difference between a $\chi^2(S)$ statistic and its mean of S (Baltagi, 2001, p. 662; Ghysels & Osborn, 2001, pp. 53-54). For more information about this distribution, see Appendix A4.

In Charemza and Deadman (1992), it is shown that for a series measured *s* times for each year, this test is build on the Student-t statistic for the OLS estimate of the parameter δ in the following regression:

$$\Delta_s y_t = \delta.z_{t-s} + \sum_{i=1}^{k}\delta_i.\Delta_s y_{t-i} + \varepsilon_t \tag{5.13}$$

where the variable z_{t-s} is constructed in that way: first, the regression of $\Delta_s y_t$ (where, $\Delta_s y_t = y_t - y_{t-s}$) is run on its own lagged values which are lagged up to k periods and the following equation is estimated:

$$\Delta_s y_t = \sum_{i=1}^{k}\lambda_i.\Delta_s y_{t-i} + \xi_t \tag{5.14}$$

Then, use the OLS estimates of $\lambda_1, \lambda_2, ..., \lambda_k$ (denoted as $\hat{\lambda s}$) to create the variable z_t from $y_t, y_{t-1}, ..., y_{t-k}$ as:

$$z_t = y_t - \sum_{i=1}^{k} \hat{\lambda}_i . y_{t-i} \tag{5.15}$$

and substitute the lagged value of z_t expressed as z_{t-s} into (5.13), estimate the equation and compute the Student-t statistic for δ (however it should be noted that here instead of $\Delta_s z_t$ proposed actually by Dickey, Hasza and Fuller (1984), Charemza and Deadman (1992) covered the view adopted by Osborn et al. (1988) and used $\Delta_s y_t$ as the dependent variable in equation (5.13). The critical values for the test are available in Dickey, Hasza and Fuller (1984). Here, the null hypothesis implies the presence of a seasonally integrated process and the alternative hypothesis says about either absence or nonexistence of stochastic seasonality which can be removed by using s-differences. In the case of significantly negative estimate of δ, the null hypothesis may be rejected in favour of the alternative hypothesis. If it is not rejected, we need to consider the order of nonseasonal differencing required for achieving stationarity; since it is not common to face with higher orders of seasonal differencing and general expectations for most economic series are in the direction of that they are $I(0,0), I(0,1)$ or $I(d,1)$ so that using s-differences once at most is expected to eliminate seasonal nonstationarity. Therefore, if we cannot reject the null hypothesis [$(\delta = 0)$ in (5.13)] saying that the variable is $I(0,1)$ (or $SI(1,1)$), for the next step we need to consider whether the variable is $I(1,1)$ (or $SI(2,1)$), instead of $I(0,1)$ with the former standing for the new null hypothesis and the latter the new alternative one. For these new hypotheses, the model which should be established and estimated like ADF test is given as:

$$\Delta\Delta_s y_t = \delta.\Delta_s y_{t-1} + \sum_i \delta_i .\Delta\Delta_s y_{t-i} + \varepsilon_t , \tag{5.16}$$

Here in the same way whether δ is significantly negative or not is examined. So, if the null that the variable is $I(1,1)$ cannot be rejected, then this expression becomes the next alternative hypothesis for the null which then says that the variable is $I(2,1)$ (or $SI(3,1)$), for the following equation:

$$\Delta\Delta\Delta_s y_t = \delta.\Delta\Delta_s y_{t-1} + \sum_i \delta_i .\Delta\Delta\Delta_s y_{t-i} + \varepsilon_t , \tag{5.17}$$

and so on. It should be noted that the constructed z_t variable is used only for the DHF test, so not used for testing the order of nonseasonal integration.

As a simple version of the DHF test, DF or ADF tests can be generalized. y_{t-s} can take the place of the constructed variable z_{t-s} in (5.13) and if the assumption is that all the $\delta_i s$ are equal to zero, the test turns into the *Dickey-Fuller seasonal integration test (DFSI)* examining again the significant negativity or otherwise of parameter δ for the following regression:

$$\Delta_s y_t = \delta . y_{t-s} + \varepsilon_t \tag{5.18}$$

Otherwise it becomes the *Augmented Dickey-Fuller seasonal integration test (ADFSI)* based on the following regression:

$$\Delta_s y_t = \delta . y_{t-s} + \sum_i^k \delta_i . \Delta_s y_{t-i} + \varepsilon_t \tag{5.19}$$

and the critical values for the DFSI and ADFSI tests are the same as for the DHF test (Charemza & Deadman, 1992, pp. 136-140)).

5.2.2. Testing a Unit Root of -1

It is necessary to handle how to test "nonstandard" unit roots. Recall that the factorization of $\Delta_s = (1 - L^s)$ operator which is shown in (5.5) for quarterly data enables us to handle tests for a unit root of -1 depending on $(1+L)$ and for pairs of complex unit roots depending on $(1+L^2)$. Here the discussion will be on a root of -1 and complex unit roots will be taken place in the next subsection. For a detailed discussion regarding such tests, see Ahtola and Tiao (1987), Chan and Wei (1988) or Chan (1989).

The case of a unit root of -1 can be covered through the process:

$$y_t^* = -y_{t-1}^* + v_t \tag{5.20}$$

The generalization of this process with starting value $y_0^* = 0$ becomes

$$y_t^* = \sum_{j=0}^{t-1} (-1)^j v_{t-j} \tag{5.21}$$

For these equations, our assumption is that v_t ~ i.i.d. $(0, \sigma^2)$. Now, a test of the unit root can be applied as a test of the null hypothesis $\alpha^* = 0$ against $\alpha^* > 0$ in

$$(1+L) y_t^* = \alpha^* y_{t-1}^* + v_t, \quad t = 1, ..., T \tag{5.22}$$

Estimating by OLS procedure, the usual t ratio for $\hat{\alpha}^*$ under the null hypothesis becomes as:

$$t(\hat{\alpha}^*) = \left(\sum_{t=1}^{T} v_t y_{t-1}^* \right) / \left\{ \tilde{\sigma} \left[\sum_{t=1}^{T} (y_{t-1}^*)^2 \right]^{1/2} \right\} \tag{5.23}$$

Obviously, (5.20) is no longer a random walk process. However, there exists a "mirror image" relationship between the two processes [see, e.g. Fuller (1996) or Chan & Wei (1988)]. When we consider the process y_t^* of (5.20) and the random walk process $y_t = y_{t-1} + \varepsilon_t$ in the case of $\varepsilon_t = (-1)^t v_t$, this "mirror image" relationship implies that

$$\sum_{t=1}^{T} v_t y_{t-1}^* = -\sum_{t=1}^{T} \varepsilon_t y_{t-1}, \tag{5.24}$$

Obtaining this relationship is possible through using (5.21) in order to substitute for y_{t-1}^* and therefore, also substituting for v_t in terms of ε_t,

$$\begin{aligned}\sum_{t=1}^{T} v_t y_{t-1}^* &= \sum_{t=1}^{T} v_t \sum_{j=0}^{t-2} (-1)^j v_{t-j-1} = \sum_{t=1}^{T} (-1)^t \varepsilon_t \sum_{j=0}^{t-2} (-1)^j (-1)^{t-j-1} \varepsilon_{t-j-1}, \\ &= -\sum_{t=1}^{T} \varepsilon_t \sum_{j=0}^{t-2} \varepsilon_{t-j-1} = -\sum_{t=1}^{T} \varepsilon_t y_{t-1}\end{aligned} \tag{5.25}$$

(5.24) is of great significance in terms of expressing that as long as ε_t and v_t are symmetrically distributed around zero, they are identically distributed. Therefore, with reference to testing a unit root of -1; $\sum_{t=1}^{T} v_t y_{t-1}^*$ has the same distributional properties with $-\sum_{t=1}^{T} \varepsilon_t y_{t-1}$ when y_t displays a random walk process.

Hence, when the unit root of -1 is taken into consideration (4.27) is replaced by

$$T^{-1} \sum_{t=1}^{T} v_t y_{t-1}^* \Rightarrow -\sigma^2 \int_0^1 W(r) dW(r) \tag{5.26}$$

Notice also that the variables $(y_t^*)^2 = \left[\sum_{j=0}^{t-1} (-1)^j v_{t-j} \right]^2$ and $(y_t)^2 = \left[\sum_{j=0}^{t-1} \varepsilon_{t-j} \right]^2$ have the same distributional properties because any sign change is unrelated to squaring. As a result, (4.28) continues to be valid for a process with unit root of -1. When both numerator and denominator of (5.23) are scaled by division by T, then it becomes

$$t(\hat{\alpha}^*) \Rightarrow -\left[\int_0^1 W(r) dW(r) \right] \Big/ \left\{ \left[\int_0^1 [W(r)]^2 dr \right]^{1/2} \right\} \tag{5.27}$$

which is the mirror image of the familiar DF *t* distribution. The implication of this conclusion is that in the case of not including a drift term, with a simple change of sign the DF tables can also be used while testing a unit root of -1 (Ghysels & Osborn, 2001, pp. 54-56).

5.2.3. Testing Complex Unit Roots

Before examining the procedure proposed by Hylleberg et al. (1990), it will be beneficial to mention about testing complex unit roots. The simplest process including a pair of complex roots is:

$$y_t^* = -y_{t-2}^* + v_t \tag{5.28}$$

with v_t ~ i.i.d. $(0, \sigma^2)$. So, the complex unit root case can be considered as a seasonal process with S=2 seasons per year and the process can be equivalently written as:

$$y_{s\tau}^* = -y_{s,\tau-1}^* + v_{s\tau}, \quad s = 1,2. \tag{5.29}$$

Here, notice that the seasonal patterns reverse each year. With starting values $y_0^* = y_{-1}^* = 0$, the process can be generalized to

$$y_{s\tau}^* = \sum_{j=0}^{\tau-1} (-1)^j v_{s,\tau-j} \tag{5.30}$$

It should be noted that $y_{s\tau}^*$ $(s = 1,2)$ are two independent nonstationary processes. In a similar fashion to the DHF test, testing the unit root process given in (5.28) is possible through the computed *t* ratio for $\hat{\alpha}_2^*$ in

$$(1 + L^2) y_t^* = \alpha_2^* y_{t-2}^* + v_t \tag{5.31}$$

where the null hypothesis is $\alpha_2^* = 0$ with the alternative of stationarity implying $\alpha_2^* > 0$. Then with the double subscript notation, under the null hypothesis,

$$t(\alpha_2^*) = \left(\sum_{s=1}^{2} \sum_{\tau=1}^{T\tau} v_{s\tau} y_{s,\tau-1}^* \right) / \left\{ \tilde{\sigma} \left[\sum_{s=1}^{2} \sum_{\tau=1}^{T_\tau} (y_{s,\tau-1}^*)^2 \right]^{1/2} \right\}. \tag{5.32}$$

After scaling both numerator and denominator by T_τ^{-1}, it follows that

$$t(\hat{\alpha}_2^*) \Rightarrow -\left[\sum_{s=1}^{2} \int_0^1 W_s(r) dW_s(r) \right] / \left\{ \left[\sum_{s=1}^{2} \int_0^1 [W_s(r)]^2 dr \right]^{1/2} \right\} \tag{5.33}$$

where $W_s(r)$ again represents standard Brownian motion processes. (5.33) has a very significant implication that with a simple change of sign, the DHF tables with S=2 seasons per year are also applicable for testing $\alpha_2^* = 0$ in (5.31) as in the case of a unit root -1 discussed above.

Under the DGP given in (5.28), it is also likely to apply to testing the null hypothesis concerning with the omitted one-period lag, namely $\alpha_1^* = 0$ against the alternative of $\alpha_1^* \neq 0$ with the test regression given in (5.34):

$$(1+L^2)y_t^* = \alpha_1^* y_{t-1}^* + v_t \tag{5.34}$$

This test is not a unit root test in a strict manner, because the implication of the unit coefficient on L^2 in (5.34) is that the process has two roots of modulus one, regardless of the value of α_1^*. Now, express the roots of quadratic $1-\alpha_1^* L + L^2$ as $\kappa_1 \pm \kappa_2 i$ for the case of complex roots. κ_1 and κ_2 are affected by the value of α_1^*. However, the modulus of the pair of the roots is not influenced by it since ${\kappa_1}^2 + {\kappa_2}^2 = 1$. The values of κ_1 and κ_2 also give rise to the spectral frequency connected to the complex unit root process. $\kappa_1 = 0$ and $\kappa_2 = 1$ values are related to the frequency $\pi/2$ by yielding the roots $\pm i$. Therefore, the test of $\alpha_1^* = 0$ is a test of the null hypothesis that the unit root process occurs at spectral frequency $\pi/2$. That is, the process includes a half-cycle every S=2 periods and therefore a full cycle every four periods. Also, because we do not have a priori information about the periodicity of the process under the alternative, the suitable alternative hypothesis becomes two-sided. For the test regression (5.34),

$$t(\hat{\alpha}_1^*) = \frac{\sum_{t=1}^{T} v_t y_{t-1}^*}{\tilde{\sigma}\left[\sum_{t=1}^{T} (y_{t-1}^*)^2\right]^{1/2}} = \frac{\sum_{\tau=1}^{T_\tau}\left[v_{1\tau} y_{2,\tau-1}^* + v_{2\tau} y_{1\tau}^*\right]}{\tilde{\sigma}\left\{\sum_{\tau=1}^{T_\tau}\left[\left(y_{2,\tau-1}^*\right)^2 + (y_{1\tau}^*)^2\right]\right\}^{1/2}} \tag{5.35}$$

In order to deal with this case, we need some generalizations of (4.27) and (4.28). These generalizations are

$$T_\tau^{-1}\sum_{\tau=1}^{T_\tau} v_{s\tau} y_{q,\tau-1} \Rightarrow \sigma^2 \int_0^1 W_q(r)dW_s(r);\ q,s = 1,\ldots\ldots\ldots,S \tag{5.36}$$

$$T_\tau^{-2}\sum_{\tau=1}^{T_\tau} y_{s\tau} y_{qt} \Rightarrow \sigma^2 \int_0^1 W_s(r)W_q(r)dr;\ q,s = 1,\ldots\ldots\ldots,S \tag{5.37}$$

respectively [see, e.g., Hamilton (1994)]. By dividing numerator and denominator of (5.35) by T_τ^{-1}, using $y_{1\tau}^* = -y_{1,\tau-1}^* + v_{1\tau}$ and also taking the generalizations given above into consideration, (5.35) follows that

$$t(\hat{\alpha}_1^*) \Rightarrow \frac{\int_0^1 W_2(r)dW_1(r) - \int_0^1 W_1(r)dW_2(r)}{\left\{\sum_{s=1}^{2}\int_0^1 [W_s(r)]^2 dr\right\}^{1/2}} \tag{5.38}$$

Because $W_1(r)$ and $W_2(r)$ are identically distributed, $t(\hat{\alpha}_1^*)$ is symmetrically distributed around zero.

On the other hand, the results for the distributions of the test statistics given in (5.31) and (5.34) also remain valid for the following test regression:

$$(1+L^2)y_t^* = \alpha_1^* y_{t-1}^* + \alpha_2^* y_{t-2}^* + v_t \tag{5.39}$$

since the regressors y_{t-1}^* and y_{t-2}^* are asymptotically orthogonal. For more details, see Ahtola and Tiao (1987) or Chan and Wei (1988)) (Baltagi, 2001, pp. 663-666; Ghysels & Osborn, 2001, pp. 51, 56-58).

5.2.4. HEGY Test

As mentioned before, Dickey, Hasza and Fuller (1984) have followed the work suggested by Dickey and Fuller for the zero frequency unit root case. However, one main disadvantage of this test is that it does not take into account unit roots at some but not all of seasonal frequencies and the alternative is that all the roots have the same modulus (Hylleberg et al., 1990, p. 221). Since many time series display substantial seasonality, the presence of unit roots corresponding to other frequencies (like seasonal ones) rather than zero is highly possible. The analysis of seasonal unit roots is fundamentally conducted with the most popular approach developed by Hylleberg et al. (1990) called HEGY by working with different models that include trends, constants and seasonal dummies in order to determine the type of seasonality. Contrary to the Dickey, Hasza and Fuller (1984), Hylleberg et al. (1990) suggest a general testing strategy looking at unit roots at all seasonal frequencies as well as at the zero frequency. So, one apparent advantage of HEGY procedure over DHF is that it enables to test for unit roots at each frequency separately without maintaining that there are unit roots at some or all other frequencies (Ghysels, Lee & Noh, 1994a, p. 416). Hylleberg et al. (1990) have introduced a factorization of the seasonal differencing polynomial $\Delta_4 = (1-L)^4$ for quarterly data using lag operator L, where $L^j y_t = y_{t-j}$ and developed a testing procedure for seasonal unit roots that could be estimated by OLS in the following way:

$$\Delta_4 y_t = \sum_{i=1}^{4} \alpha_i D_{i,t} + \sum_{i=1}^{4} \pi_i Y_{i,t-1} + \sum_{i=1}^{k} c_i \Delta_4 y_{t-i} + \varepsilon_t \tag{5.40}$$

where k is the number of lagged terms included to ensure that residuals are white noise, the $D_{i,t}$ are seasonal dummy variables and the $Y_{i,t}$ variables are constructed from the series on y_t as:

$$Y_{1,t} = (1+L)(1+L^2).y_t = y_t + y_{t-1} + y_{t-2} + y_{t-3} \quad (5.41)$$

$$Y_{2,t} = -(1-L)(1+L^2).y_t = -y_t + y_{t-1} - y_{t-2} + y_{t-3} \quad (5.42)$$

$$Y_{3,t} = -(1-L)(1+L).y_t = -y_t + y_{t-2} \quad (5.43)$$

$$Y_{4,t} = -(L)(1-L)(1+L).y_t = Y_{3,t-1} = -y_{t-1} + y_{t-3} \quad (5.44)$$

(Charemza & Deadman, 1992, p. 141).

The HEGY regression in the most general and a more explicit form could be written as follows:

$$\Delta_4 y_t = \alpha + \beta t + \sum_{i=1}^{3} \alpha_i D_{i,t} + \pi_1 Y_{1,t-1} + \pi_2 Y_{2,t-1} + \pi_3 Y_{3,t-2} + \pi_4 Y_{3,t-1} + \sum_{i=1}^{k} c_i \Delta_4 y_{t-i} + \varepsilon_t \quad (5.45)$$

We mostly apply seasonal differencing to remove nonstationarity in seasonal data, so that we should use $\Delta_4 y_t = y_t - y_{t-4}$ in quarterly data.

In equation (5.45), the choice of lag parameter k could be done using a variety of lag selection criteria. For instance, while Osborn (1990) deals with the significance of LM test in order to choose the 'best' model, Lee and Siklos (1991) use the most popular AIC and Schwarz information criterion (SIC). According to Engle et al. (1993), the power and size of the unit root tests depend on the 'right' augmentation that will be used.

Ghysels et al. (1994a) point out to that DHF testing procedure seems unable to separate unit root at zero frequency or at one of seasonal frequencies of data generating processes with nonstationarity induced by the (1 – L^4) factor and therefore HEGY is a more advantageous procedure. However, when looked at the results of their Monte Carlo studies, it is seen that there exist some problems with available seasonal unit root tests regarding near-cancellation problem of a unit root in the AR polynomial with an MA root. That is, in seasonal time series models, this problem is said to be very common and to lead to adverse size distortions. Even if there are no size distortions, Monte Carlo study results indicate the weak power properties of DHF and HEGY tests especially in the case of absence of seasonal dummies.

The null hypothesis of the HEGY test is that the variable in question is seasonally integrated. Hence, if the null hypothesis of stochastic seasonality is true rather than deterministic seasonality, in this case in equation (5.40) all the $\alpha_i s$ will be equal to each other and all the $\pi_i s$ will be equal to zero. In the case of different $\alpha_i s$ and at least one of the $\pi_i s$ that is nonzero, there exists a combination of both deterministic and stochastic seasonality. The interpretation of each negative π_i is different from each other. Let's say, only π_1 is negative, in this case there is no non-seasonal stochastic stationary component. If only π_2 is negative, then there exists no bi-annual cycle. On the other hand, π_3 and π_4 are related to the annual cycle and testing them jointly is possible. Critical values of these tests are provided in the Hylleberg et al. (1990) paper.

The factorization of the expression $\Delta_4 = (1-L)^4$ could say somethings relating to roots: $(1-L)^4 = (1-L)(1+L)(1+L^2) = (1-L)(1+L)(1-i \cdot L)(1+i \cdot L)$ where i is an imaginary part of a complex number such that $i^2 = -1$. When looked at this factorization, it is seen that a quarterly stochastic seasonal unit root process has four roots of modulus one. One root $(1-L)$ described as being at 'zero frequency' (in the case of $\pi_1 = 0$) removes the trend. The other three roots which remove the seasonal structure imply stochastic cycles of biannual and annual periodicity. An elegant introduction to complex numbers and complex number dynamics could be found in Dhrymes (1970) (Charemza & Deadman, 1992, pp. 141-142). In this case, the unit roots are 1, - 1, i, and -i which correspond to zero frequency, $\frac{1}{2}$ cycle per quarter or 2 cycles per year, and $\frac{1}{4}$ cycle per quarter or one cycle per year. The last root, -i, is identical to the one at i with quarterly data and therefore it is also interpreted as the annual cycle. Now we can test the following hypotheses:

1)	2)	3)	
H0: $\pi_1 = 0$	H0: $\pi_2 = 0$	H0: $\pi_3 = \pi_4 = 0$	
H1: $\pi_1 < 0$	H1: $\pi_2 < 0$	H1: $\pi_3 \neq \pi_4 \neq 0$	(5.46)
(t statistic)	(t statistic)	(F statistic)	

Here, $H_A : \pi_1 = 0$ → the existence of nonseasonal unit root
$H_B : \pi_2 = 0$ → the existence of biannual unit root
$H_C : \pi_3 = \pi_4 = 0$ → the existence of annual unit root

As seen in (5.46), the first two hypotheses H_A and H_B are tested by using one-sided *t* tests against the hypothesis that $\pi_i < 0$. The other hypothesis which is H_C is tested with an F test. For a series to include no seasonal unit roots, both $\pi_2 = 0$ and the joint F test which is $\pi_3 = \pi_4 = 0$ should be rejected. That is, π_2 and either π_3 or π_4 should be different from zero. On the other hand, in conclusion to find out that a series is stationary and thus includes no unit roots at all (including at zero frequency), we must establish that each of the π's is different from zero (in other words, each of the *t* test of $\pi_1 = \pi_2 = 0$ and the joint F test of $\pi_3 = \pi_4 = 0$ should be rejected in order to have a stationary series) (Hylleberg et al., 1990, pp. 221-223).

In Table 6, a summary of long-run and seasonal frequencies has been presented for quarterly data:

Table 6

Long Run and Seasonal Frequencies for Seasonal Unit Root Tests in Quarterly Data

Frequency	**Period**	**Cycles/year**	**Root**	**Filter**	**Tested hypothesis** H_0: Unit Root
0 **Long run**	∞	0	1	$(1-L)$	$\pi_1 = 0$
$\frac{\pi}{2}, \frac{3\pi}{2}$ **Annual**	$4; \frac{4}{3}$	1; 3	$\pm i$	$(1+L^2)$	$\pi_3 \cap \pi_4 = 0$
π **Semiannual**	2	2	-1	$(1+L)$	$\pi_2 = 0$

Note. The information on first five columns have been obtained from Dıaz-Emparanza & López-de-Lacalle (2006, p.7).

In equation (5.40), $\alpha_i s$ represent a deterministic structure while $\pi_i s$ represent a stochastic structure. In order to test whether a series follows a deterministic or stochastic seasonal pattern, the hypotheses to be constructed are the null hypothesis H_0 which implies the presence of stochastic seasonality and the alternative hypothesis H_1 which implies the presence of deterministic seasonality. There are two conditions for the acceptance of stochastic seasonality: The first condition is the acceptance of the hypothesis in which all α coefficients are equal to each other and the second condition is the acceptance of the hypothesis in which all π coefficients are equal to zero. Thus, the null and alternative hypotheses can be expressed in the following way:

1st Condition: $H_0: \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$ 2nd Condition: $H_0: \pi_1 = \pi_2 = \pi_3 = \pi_4 = 0$

$H_1: \alpha_1 \neq \alpha_2 \neq \alpha_3 \neq \alpha_4$ H_1: at least one of π_i s $\neq 0$

In order to be able to test the first condition, α coefficients are tested in doubles and those following six hypotheses are tested:

$H_{01}:\alpha_1=\alpha_2$, $H_{02}:\alpha_1=\alpha_3$, $H_{03}:\alpha_1=\alpha_4$, $H_{04}:\alpha_2=\alpha_3$, $H_{05}:\alpha_2=\alpha_4$, $H_{06}:\alpha_3=\alpha_4$.
These hypotheses are tested by *t*-test with degrees of freedom $(n-k)$ and test statistic is given as:

$$t_{(\alpha_i-\alpha_j)}=\frac{(\hat{\alpha}_i-\hat{\alpha}_j)-E(\hat{\alpha}_i-\hat{\alpha}_j)}{\sqrt{Var(\hat{\alpha}_i)+Var(\hat{\alpha}_j)-2Cov(\hat{\alpha}_i\hat{\alpha}_j)}}$$

The second condition is tested by using Q statistic that has an F-distribution with degrees of freedom $(p,(n-k-1))$ and Q statistic is calculated in the following way:

$$Q=\frac{RSS-URSS}{URSS}.\frac{(n-k-1)}{p}$$

where RSS is the residual sum of squares of restricted regression, URSS is the residual sum of squares of unrestricted regression, p is the number of restrictions, n is the number of observations and k is the number of independent variables. If these two conditions mentioned above are satisfied, it is concluded that stochastic seasonality exists in the series in question. In case the presence of stochastic seasonality is not accepted, whether the series follows a deterministic seasonality or not is investigated. In other saying, frequencies corresponding to seasonal unit roots are tested. In order to be able to detect at which frequencies seasonal unit root exists, the hypotheses given in (5.46) should be tested for the necessary auxiliary HEGY regressions that will be just mentioned (Ayvaz, 2006, pp. 74-75).

There are five auxiliary regressions to be run in order to decide about the choice of a proper HEGY regression. These are (Mert & Demir, 2014, p. 14):

1) regression with no deterministic component (no intercept, no seasonal dummy, no trend):

$$\Delta_4 y_t=\pi_1 Y_{1,t-1}+\pi_2 Y_{2,t-1}+\pi_3 Y_{3,t-2}+\pi_4 Y_{3,t-1}+\sum_{i=1}^{k}c_i\Delta_4 y_{t-i}+\varepsilon_t \quad (5.47)$$

2) regression with only intercept (no seasonal dummy, no trend):

$$\Delta_4 y_t=\alpha+\pi_1 Y_{1,t-1}+\pi_2 Y_{2,t-1}+\pi_3 Y_{3,t-2}+\pi_4 Y_{3,t-1}+\sum_{i=1}^{k}c_i\Delta_4 y_{t-i}+\varepsilon_t \quad (5.48)$$

3) regression with intercept and seasonal dummy (no trend):

$$\Delta_4 y_t=\alpha+\sum_{i=1}^{3}\alpha_i D_{i,t}+\pi_1 Y_{1,t-1}+\pi_2 Y_{2,t-1}+\pi_3 Y_{3,t-2}+\pi_4 Y_{3,t-1}+\sum_{i=1}^{k}c_i\Delta_4 y_{t-i}+\varepsilon_t \quad (5.49)$$

4) regression with intercept and trend (no seasonal dummy):

$$\Delta_4 y_t=\alpha+\beta t+\pi_1 Y_{1,t-1}+\pi_2 Y_{2,t-1}+\pi_3 Y_{3,t-2}+\pi_4 Y_{3,t-1}+\sum_{i=1}^{k}c_i\Delta_4 y_{t-i}+\varepsilon_t \quad (5.50)$$

5) regression with intercept, seasonal dummy and trend:

$$\Delta_4 y_t = \alpha + \beta t + \sum_{i=1}^{3} \alpha_i D_{i,t} + \pi_1 Y_{1,t-1} + \pi_2 Y_{2,t-1} + \pi_3 Y_{3,t-2} + \pi_4 Y_{3,t-1} + \sum_{i=1}^{k} c_i \Delta_4 y_{t-i} + \varepsilon_t \quad (5.51)$$

Because HEGY test is easily affected by the inclusion of deterministic components, the most appropriate model selection amongst five models given above is based on the significance of the deterministic components (Habibullah, 1998, p. 119). For such models, Chan and Wei (1988) examined the asymptotic distribution of *t* statistics.

As in the ADF test, adding augmentation terms (in order to make sure about that the residuals are white noise) has no effect on the distribution of the statistics and the critical values that will be used in the augmentation case are not different from the case of without augmentation (Charemza & Deadman, 1997, p. 109).

Hylleberg et al. (1990) show how the limiting distributions relate to the standard unit root tests: testing for π_1=0 in the case of $\pi_2 = \pi_3 = \pi_4$=0 will have the familiar DF distribution. Because the model can be expressed as

$$Y_{1t} = (1 + \pi_1) Y_{1,t-1} + \varepsilon_t \quad (5.52)$$

In a similar manner, testing for a root of -1 when the other π's are zero will have the mirror image of the DF distribution. So, if Y_{2t} is regressed on $-Y_{2,t-1}$ as follows:

$$Y_{2t} = -(1 + \pi_2) Y_{2,t-1} + \varepsilon_t \quad (5.53)$$

the standard DF distribution will be suitable and third test can be written as

$$Y_{3t} = -(1 + \pi_3) Y_{3,t-2} + \varepsilon_t \quad (5.54)$$

with an assumption of $\pi_4 = 0$. So, by the same logic in (5.53) testing for biannual seasonality has the mirror image of Dickey-Hasza-Fuller distribution.

The distribution of the test statistics will not be influenced by the addition of a variable with a zero coefficient which is orthogonal to the added variables. For instance; in the case of testing $\pi_1 = 0$ assume that $\pi_2 = 0$, however Y_2 is still contained in the regression. In this situation, Y_1 and Y_2 will be asymptotically uncorrelated because of having unit roots at different frequencies and also both of them will be asymptotically uncorrelated with lags of $\Delta_4 y$ that is stationary. Therefore, irrespective of whether Y_2 is incorporated into the regression, the limiting distribution to test for $\pi_1 = 0$ will be unchanged. This can be generalized to other cases with similar arguments. However; apart from these when deterministic components are available in the regression model although not included in the data, the limiting distributions change. The intercept and trend components affect only the distribution of π_1. The reason for this is that they have all their spectral mass at zero frequency. In spite of the fact that the remaining three seasonal dummies do not have any influence on the limiting distribution of π_1 when the intercept is included in the regression, the distributions of π_2, π_3 and π_4 are influenced by the seasonal dummies.

The Monte Carlo values for the one sided *t* tests on π_1, π_2 and π_3 and the joint F test on $\pi_3 \cap \pi_4 = 0$ which are very close to the Monte Carlo values from Dickey-Fuller and Dickey-Hasza-Fuller are presented in Hylleberg et al. (1990) (see, pp. 226-227). It is seen that when seasonal dummies are not included in the auxiliary regression, the distribution of the *t* statistic is very akin to a standard normal. In the same manner, the distribution for the F statistic looks like an F distribution. However; if seasonal dummies are present in the regression, for both *t* and F statistics distributions will be fatter-tailed[3].

5.2.4.1. Extensions of the HEGY Procedure

Recall that the DHF test statistic deals with the testing for the null of unit roots at 0, $\pi/2$ and π frequencies jointly. A similar HEGY-type test corresponds to an F statistic on π_1, π_2, π_3 and π_4. Following the work by Engle, Granger, Hylleberg and Lee (1993), the derivation of the asymptotic distribution for this test is feasible. For the simplest HEGY regression given as $\Delta_4 y_t = \pi_1 Y_{1,t-1} + \pi_2 Y_{2,t-1} + \pi_3 Y_{3,t-2} + \pi_4 Y_{3,t-1} + \varepsilon_t$, we have the following F statistic:

$$F_{1234} \to \frac{1}{4}\left\{ \frac{(\int_0^1 W_1(r)dW_1)^2}{\int_0^1 W_1(r)^2 dr} + \frac{(\int_0^1 W_2(r)dW_2)^2}{\int_0^1 W_2(r)^2 dr} + \frac{(\int_0^1 W_3(r)dW_3 + \int_0^1 W_4(r)dW_4)^2 + (\int_0^1 W_3(r)dW_4 - \int_0^1 W_4(r)dW_3)^2}{\int_0^1 W_3(r)^2 dr + \int_0^1 W_4(r)^2 dr} \right\} \tag{5.55}$$

where $\to$ denotes weak convergence in distribution and $W_i(r)$ for $i = 1,\ldots,4$ stands for independent standard Brownian motions. It is remarkable to call attention to that F_{1234}

[3] The probability of extreme events (higher probability at the tail ends) –i.e. events that fall on the tail ends of a statistical distribution and are the most likely not to occur– cannot always be accurately described by the bell shaped curve . This kind of activity is usually described using fat-tailed distributions (Mello, (n.d.), p.1).

statistic in (5.55) and the sum of the squared t statistics for π_i $(i=1,...,4)$ have the same asymptotic distributions.

On the other hand, testing for presence of unit roots at all seasonal frequencies jointly without regarding the zero frequency is associated with the F statistic for the null hypothesis of $\pi_2=\pi_3=\pi_4=0$ and the asymptotic distribution of this statistic can be expressed as follows:

$$F_{234} \to \frac{1}{3}\left\{\frac{(\int_0^1 W_2(r)dW_2)^2}{\int_0^1 W_2(r)^2\,dr} + \frac{(\int_0^1 W_3(r)dW_3 + \int_0^1 W_4(r)dW_4)^2 + (\int_0^1 W_3(r)dW_4 - \int_0^1 W_4(r)dW_3)^2}{\int_0^1 W_3(r)^2\,dr + \int_0^1 W_4(r)^2\,dr}\right\} \quad (5.56)$$

This test statistic has also the same limiting distribution as the sum of the corresponding squared t statistics (Ghysels et al., 1994a, pp. 418-419). For the proofs of distributions given above and the critical values for F_{1234} and F_{234}, see Appendix A and Appendix C respectively in Ghysels et al. (1994a).

5.2.4.2. Testing for Seasonal Unit Roots in Monthly Data

Franses (1990) makes an extension of HEGY procedure to monthly data. In this case, the differencing operator Δ_{12} will have 12 roots lying on the unit circle ($1-L^{12}=0$) such that

$$\begin{aligned}1-L^{12} = (1-L)(1+L)(1-iL)(1+iL) &\times[1+(\sqrt{3}+i)L/2][1+(\sqrt{3}-i)L/2]\\ &\times[1-(\sqrt{3}+i)L/2][1-(\sqrt{3}-i)L/2]\\ &\times[1+(\sqrt{3}+i)L/2][1-(\sqrt{3}-i)L/2]\\ &\times[1-(\sqrt{3}+i)L/2][1+(\sqrt{3}-i)L/2]\end{aligned} \quad (5.57)$$

where all terms except (1–L) define the seasonal unit roots (Maddala & Kim, 1998, p. 368). Note that this factorization of $(1-L^{12})$ can also be expressed as $(1-L)(1+L+L^2+........+L^{11})$. However; on the purpose of being more practical in test equation, the factorization in (5.57) is preferred.

Beaulieu and Miron (1992b) also examine the HEGY testing procedure for monthly data. As mentioned in the quarterly case, assume that y_t is the series of interest having a DGP with a general autoregression form given as

$$\varphi(L)y_t = \varepsilon_t \quad (5.58)$$

where $\varphi(L)$ is a polynomial in the lag operator form and ε_t represents the usual white noise process. For simplicity, it is supposed that deterministic terms are not available in the y_t process. It is already known that in the case of $\alpha = \frac{2\pi j}{S}$ $(j = 1,......., S-1)$ which shows the frequency associated with a particular root, a root is seasonal (S is the number of observations per year). For monthly data, the seasonal unit roots are given as follows:

$$-1;\ \pm i;\ -\frac{1}{2}(1 \pm \sqrt{3}i);\ \frac{1}{2}(1 \pm \sqrt{3}i);\ -\frac{1}{2}(\sqrt{3} \pm i);\ \frac{1}{2}(\sqrt{3} \pm i) \tag{5.59}$$

These roots are associated with 6,3,9,8,4,2,10,7,5,1 and 11 cycles per year respectively and the corresponding frequencies for these roots are $\pi, \pm\frac{\pi}{2}, \mp\frac{2\pi}{3}, \pm\frac{\pi}{3}, \mp\frac{5\pi}{6}$ and $\pm\frac{\pi}{6}$ respectively. Notice that the root 1 does not take place in (5.59). Since it is not a seasonal unit root, rather it defines the long run or zero frequency unit root. Here, what is tried to find out is that whether the polynomial $\varphi(L)$ has roots that are equal to one in absolute value at the zero or seasonal frequencies.

With this HEGY procedure developed for monthly case, the polynomial $\varphi(L)$ is linearized around the zero frequency unit root plus the S–1 unit roots given in (5.59) and so, $\varphi(L)$ is expressed as

$$\varphi(L) = \sum_{k=1}^{S} \lambda_k \Delta(L) \frac{1-\delta_k(L)}{\delta_k(L)} + \Delta(L)\varphi^*(L) \tag{5.60}$$

where

$$\delta_k(L) = 1 - \frac{1}{\theta_k}L, \qquad \lambda_k = \frac{\varphi(\theta_k)}{\prod_{j \neq k} \delta_j(\theta_k)}, \qquad \Delta(L) = \prod_{k=1}^{S} \delta_k(L),$$

$\varphi^*(L)$ is a polynomial associated with roots that are outside the unit circle and the θ_k are the zero frequency unit root plus the S–1 seasonal unit roots. As seen obviously from the definition of λ_k, the polynomial $\varphi(L)$ will have a root at θ_k if and only if the corresponding λ_k is equal to zero (Franses, 1991, p. 96). Now, if we substitute (5.60) into (5.58), it becomes

$$\varphi(L)^* y_{13t} = \sum_{k=1}^{12} \pi_k y_{k,t-1} + \varepsilon_t \tag{5.61}$$

where,

$$y_{1,t} = (1 + L + L^2 + L^3 + L^4 + L^5 + L^6 + L^7 + L^8 + L^9 + L^{10} + L^{11})y_t$$

$$y_{2,t} = -(1 - L + L^2 - L^3 + L^4 - L^5 + L^6 - L^7 + L^8 - L^9 + L^{10} - L^{11})y_t$$

$$y_{3,t} = -(L - L^3 + L^5 - L^7 + L^9 - L^{11})y_t$$

$$y_{4,t} = -(1 - L^2 + L^4 - L^6 + L^8 - L^{10})y_t$$
$$y_{5,t} = -\frac{1}{2}(1 + L - 2L^2 + L^3 + L^4 - 2L^5 + L^6 + L^7 - 2L^8 + L^9 + L^{10} - 2L^{11})y_t$$
$$y_{6,t} = \frac{\sqrt{3}}{2}(1 - L + L^3 - L^4 + L^6 - L^7 + L^9 - L^{10})y_t$$
$$y_{7,t} = \frac{1}{2}(1 - L - 2L^2 - L^3 + L^4 + 2L^5 + L^6 - L^7 - 2L^8 - L^9 + L^{10} + 2L^{11})y_t$$
$$y_{8,t} = -\frac{\sqrt{3}}{2}(1 + L - L^3 - L^4 + L^6 + L^7 - L^9 - L^{10})y_t$$
$$y_{9,t} = -\frac{1}{2}(\sqrt{3} - L + L^3 - \sqrt{3}L^4 + 2L^5 - \sqrt{3}L^6 + L^7 - L^9 + \sqrt{3}L^{10} - 2L^{11})y_t \qquad (5.62)$$
$$y_{10,t} = \frac{1}{2}(1 - \sqrt{3}L + 2L^2 - \sqrt{3}L^3 + L^4 - L^6 + \sqrt{3}L^7 - 2L^8 + \sqrt{3}L^9 - L^{10})y_t$$
$$y_{11,t} = \frac{1}{2}(\sqrt{3} + L - L^3 - \sqrt{3}L^4 - 2L^5 - \sqrt{3}L^6 - L^7 + L^9 + \sqrt{3}L^{10} + 2L^{11})y_t$$
$$y_{12,t} = -\frac{1}{2}(1 + \sqrt{3}L + 2L^2 + \sqrt{3}L^3 + L^4 - L^6 - \sqrt{3}L^7 - 2L^8 - \sqrt{3}L^9 - L^{10})y_t$$
$$y_{13,t} = (1 - L^{12})y_t$$

(Beaulieu & Miron, 1992b, pp. 2-4).

The test equation for the presence of seasonal unit roots given in (5.61) takes a somewhat different form in Franses (1991) as follows

$$\varphi^*(L)y_{8,t} = \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-1} + \pi_4 y_{3,t-2} + \pi_5 y_{4,t-1} + \pi_6 y_{4,t-2} + \pi_7 y_{5,t-1} + \pi_8 y_{5,t-2}$$
$$+ \pi_9 y_{6,t-1} + \pi_{10} y_{6,t-2} + \pi_{11} y_{7,t-1} + \pi_{12} y_{7,t-2} + \mu_t + \varepsilon_t \qquad (5.63)$$

where $\varphi^*(L)$ is some polynomial function of L, μ_t represents the deterministic component which may include a constant, seasonal dummies or a trend, and

$$y_{1,t} = (1 + L)(1 + L^2)(1 + L^4 + L^8)y_t = (1 + L + L^2 + ... + L^{11})y_t$$
$$y_{2,t} = -(1 - L)(1 + L^2)(1 + L^4 + L^8)y_t$$
$$y_{3,t} = -(1 - L^2)(1 + L^4 + L^8)y_t$$
$$y_{4,t} = -(1 - L^4)(1 - \sqrt{3}L + L^2)(1 + L^2 + L^4)y_t$$
$$y_{5,t} = -(1 - L^4)(1 + \sqrt{3}L + L^2)(1 + L^2 + L^4)y_t \qquad (5.64)$$
$$y_{6,t} = -(1 - L^4)(1 - L^2 + L^4)(1 - L + L^2)y_t$$
$$y_{7,t} = -(1 - L^4)(1 - L^2 + L^4)(1 + L + L^2)y_t$$

$y_{8,t} = (1 - L^{12})y_t$ (Franses, 1991, p. 100; Maddala & Kim, 1998, p. 368).

It is remarkable to say that in order to make the residuals white noise, augmented lagged values of $y_{8,t}$ should be used in (5.63). With these transformations (y_i, s) of y_t

in (5.64), the seasonal unit roots are excluded at given frequencies while they are preserved at remaining frequencies. To give an example, consider the y_{1t} transformation. While it eliminates the seasonal unit roots, it preserves the long-run or zero frequency unit root. In table 7, the outline of long run and seasonal frequencies has been presented.

Table 7

Long Run and Seasonal Frequencies for Seasonal Unit Root Tests in Monthly Data

Frequency	**Period**	**Cycles/year**	**Root**	**Filter**	**Tested hypothesis** H_0 : Unit Root
0 **Long run**	∞	0	1	$(1-L)$	$\pi_1 = 0$
$\frac{\pi}{6}, \frac{11\pi}{6}$ **Annual**	12; 1.09	1; 11	$\frac{1}{2}(\sqrt{3} \pm i)$	$(1-\sqrt{3}L + L^2)$	$\pi_{11} \cap \pi_{12} = 0$
$\frac{\pi}{3}, \frac{5\pi}{3}$ **Semiannual**	6; 1.2	2; 10	$\frac{1}{2}(1 \pm \sqrt{3}i)$	$(1-L+L^2)$	$\pi_7 \cap \pi_8 = 0$
$\frac{\pi}{2}, \frac{3\pi}{2}$	4; $\frac{4}{3}$	3; 9	$\pm i$	$(1+L^2)$	$\pi_3 \cap \pi_4 = 0$
$\frac{2\pi}{3}, \frac{4\pi}{3}$ **Quarterly**	3; 1.5	4; 8	$-\frac{1}{2}(1 \pm \sqrt{3}i)$	$(1+L+L^2)$	$\pi_5 \cap \pi_6 = 0$
$\frac{5\pi}{6}, \frac{7\pi}{6}$	2.4; 1.7	5; 7	$-\frac{1}{2}(\sqrt{3} \pm i)$	$(1+\sqrt{3}L + L^2)$	$\pi_9 \cap \pi_{10} = 0$
π **Bimonthly**	2	6	**-1**	$(1+L)$	$\pi_2 = 0$

Note. The information on first five columns have been obtained from Dıaz-Emparanza & López-de-Lacalle (2006, p.7).

Applying OLS procedure to (5.63) gives estimates of the π_i. By the same logic in quarterly case; if π_2 through π_{12} are significantly different from zero (the case in which the null hypothesis of stochastic seasonality is not true), then there will be no seasonal unit roots and the pattern that the data display becomes deterministic or constant seasonal. Therefore, in this situation the dummy variable representation can be applied for modelling this pattern. The implication of the statement just given is that if there are seasonal unit roots, the corresponding π_i are zero. Due to the fact that pairs of complex unit root are conjugates, these roots will exist only in case pairs of $\pi's$ are jointly equal to zero. For instance, the roots i and $-i$ are only present if π_3 and π_4 are simultaneously equal to zero. If π_1 through π_{12} are all unequal to zero, we experience a stationary seasonal pattern and seasonal dummy variables can be used to model such a pattern. At the same time, when the coefficient for a given π is statistically not different from zero, then it can be said that data have a varying seasonal pattern. If $\pi_1 = 0$, we cannot reject the presence of root 1 with long-run frequency and if all π_i are equal to zero, it becomes suitable to apply the $(1 - L^{12})$ filter. If only some pairs of $\pi's$ are zero, the relevant operators can be used. In Abraham and Box (1978), it is exemplified that sometimes these operators may be adequate.

Either *t* tests or F tests can be employed in order to test for seasonal unit roots at the pertinent seasonal frequencies. The *t*-ratios corresponding to the estimates of π_1 and π_2 which represent long-run and semi-annual frequencies respectively track the DF distributions. All critical values of the test have a non-standard distribution. So, critical values are generated by Monte Carlo simulations (Franses, 1991, p. 101; Maddala & Kim, 1998, p. 370; Sørensen, 2001, p. 77).

On the other hand, Beaulieu and Miron (1992b) explain the testing hypotheses about unit roots in their paper. They implement the HEGY procedure as different from Franses (1990) in that the set of regressors in (5.61) are mutually orthogonal and this leads to the derivation of the asymptotic distribution to become easier. In (5.61), for frequencies 0 and π, the null hypothesis that is associated with the relevant *t* statistic becomes $\pi_k = 0$ while the alternative one says that $\pi_k < 0$. For the other roots, the alternative of testing the null of $\pi_k = 0$ where k is even becomes a two-sided test. Thus, the even coefficient may be positive or negative. If $\pi_k = 0$ cannot be rejected, then one tests $\pi_{k-1} = 0$ against the alternative of $\pi_{k-1} < 0$. Here, depending on the sensible alternative saying that the series has a root lying outside the unit circle, the test becomes one-sided rather than two-sided. Since, as known the true coefficient is less than zero under the stationarity condition. In addition, applying to an F statistic for testing $\pi_{k-1} = \pi_k = 0$ is another strategy. In case there is no unit root at any seasonal frequency, π_k must not be equal to zero for $k = 2$ and for at least one member of each of the sets $\{3,4\},\{5,6\},\{7,8\},\{9,10\},\{11,12\}$ (Beaulieu & Miron, 1992b, pp. 4-5).

5.2.4.3. Testing for Seasonal Unit Roots in Bimonthly Time Series

Franses (1992a) examines seasonal unit roots in bimonthly time series in his paper. Therefore, it is required to consider the operator $(1-L^6)$ corresponding to bimonthly time series. Franses (1992a) presents the decomposition of this polynomial as follows:

$$\begin{aligned}1-L^6 &= (1-L^2)(1-L+L^2)(1+L+L^2) = (1-L^2)(1+L^2+L^4)\\ &= (1-L)(1+L+L^2+L^3+L^4+L^5)\\ &= (1-L)(1+L)(1-\frac{1}{2}(\sqrt{3}i+1)L)\times(1+\frac{1}{2}(\sqrt{3}i-1)L)(1+\frac{1}{2}(\sqrt{3}i+1)L)\\ &\qquad \times(1-\frac{1}{2}(\sqrt{3}i-1)L) \end{aligned} \tag{5.65}$$

and the test equation in order to test seasonal unit roots in this type of data is given as

$$\varphi^*(L)y_{5,t} = \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \pi_5 y_{4,t-2} + \pi_6 y_{4,t-1} + \mu_t + \varepsilon_t \tag{5.66}$$

where

$$\begin{aligned} y_{1,t} &= (1+L)(1+L^2+L^4)y_t\\ y_{2,t} &= -(1-L)(1+L^2+L^4)y_t\\ y_{3,t} &= -(1-L^2)(1+L+L^2)y_t\\ y_{4,t} &= -(1-L^2)(1-L+L^2)y_t\\ y_{5,t} &= (1-L^6)y_t \end{aligned} \tag{5.67}$$

The tables for critical *t*-values of the individual π_i and for F-tests of $\pi_3 = \pi_4 = 0$ and $\pi_5 = \pi_6 = 0$ in this testing procedure can be obtained from Appendix part in Franses (1992a). As in the quarterly data; the tests for π_1 and π_2 are one-sided, the tests for π_4 and π_6 are two- sided and for π_3 and π_5 they are one-sided (Franses, 1992a, p. 411). For more, see Franses (1992a).

5.2.4.4. Testing for Weekly Seasonal Unit Roots

When we take a series including a weekly seasonal component into consideration, the assumed DGP belonging to such a type of series is $(1-L^7)y_t = \varepsilon_t$ ~ i.i.d. $(0,\sigma_\varepsilon^{\ 2})$; $t = 1,......,T$. Here, the characteristic polynomial $(1-L^7)$ can be decomposed as $(1-L)(1+L+.........+L^6)$ where the second factor represents the SMA filter. The auxiliary regression for testing weekly roots is constructed through the expansion of the characteristic polynomial just given as follows:

$$\Delta_7 y_t = \alpha + \beta t + \sum_{j=2}^{7}\alpha_j D_{jt} + \sum_{j=2}^{7}\gamma_j D_{jt} t + \sum_{j=1}^{7}\pi_j z_{j,t-1} + \sum_{r=1}^{p}\phi_r \Delta_7 y_{t-r} + \varepsilon_t \tag{5.68}$$

$\varepsilon_t \sim$ i.i.d. $(0, \sigma_\varepsilon{}^2)$

where D_{jt} is a zero/one seasonal dummy variable corresponding to the j-th day of the week and the regressors $z_{j,t}$ are described as

$$z_{1,t} = \sum_{j=1}^{7} \cos(0j) L^{j-1} y_t = (1 + L + ... + L^6)\ y_t$$
$$z_{2k,t} = \sum_{j=1}^{7} \cos(kjw) L^{j-1} y_t \qquad (5.69)$$
$$z_{2k+1,t} = -\sum_{j=1}^{7} \sin(kjw) L^{j-1} y_t \ ;$$

Since we are concerned with the weekly seasonal unit roots, this requires taking seven roots into consideration. Hence, the assumption under the null hypothesis is that the series includes one unit root at the zero frequency and three pairs of complex roots at the seasonal frequencies $\frac{2\pi k}{s}$, for $k = 1,2,3$ and $s = 7$ where k is the number of cycles per week of each frequency.

The most general specification for testing weekly roots under the alternative hypothesis of stationarity is given in (5.68) with the deterministic components given as a drift, a linear time trend, deterministic seasonal variables and seasonal drifts. As an alternative to this specification, different combinations of these deterministic components can be incorporated into the auxiliary regression (5.68). The correct determination of this specification is of great importance with regard to affecting the power of the test. Also, another point is that in the similar manner to the ADF test procedure, augmented lagged values of $\Delta_7 y_t$ are included in (5.63) to remove serial correlation in the error term.

A noteworthy characteristic of the HEGY procedure is the representation of the series y_t as a linear combination of the regressors $z_{j,t}$, $j = 1,........,7$ and when a linear filtering process is applied to y_t, all unit roots excluding the one associated with the specific frequency of the relevant one of these regressors are removed. To give a simple example, $z_{1,t}$ is the result of applying the SMA filter $(1 + L + ... + L^6)$ to y_t. In that case, all the seasonal unit roots are subtracted and only the long-run zero-frequency unit root becomes available in this regressor.

The asymptotically mutually orthogonality feature of the regressors enables the results of testing the unit root hypothesis in a given frequency and the ones in the remaining frequencies not to be influenced by one another. So, this also implies that the HEGY procedure enables to test whether there is a unit root in some, all or none of the frequencies analysed.

The estimation of each π_j parameters is possible when OLS procedure is applied to (5.68) or to any other alternative model specification of it including some deterministic components or none and if the null hypothesis of $\pi_1 = 0$ is not rejected against the alternative of $\pi_1 < 0$, then it is said that the series contains a long-run zero frequency unit root. The distribution of this test statistic follows the DF distribution. In addition, the null hypothesis for complex unit roots on each seasonal frequency implies the equivalence of two test statistics belonging to the same seasonal frequency to zero and is expressed as $\pi_{2k} = \pi_{2k+1} = 0$ that is a joint F test. As an alternative, testing this complex unit root hypothesis is also feasible via a two-sided *t* test for π_{2k+1}. If $\pi_{2k+1} = 0$ cannot be rejected, then one tests $\pi_{2k} = 0$ against the alternative of $\pi_{2k} < 0$ which implies a one-sided *t* test. However, generally the first testing approach is chosen because of better statistical properties (Ghysels et al., 1994a). Since, the critical values of the *t* statistics and F statistics have a non-standard distribution, they are generated by Monte Carlo simulations for different sample sizes and the distributions differ depending on which combinations of deterministic components are incorporated into the auxiliary regression (Rubia, 2001, pp. 7-9). For the critical values and more see Rubia (2001).

5.2.4.5. Testing for Seasonal Unit Roots in Semi-Annual Data

Feltham and Giles (1999) examine the properties of HEGY procedure on the semi-annual data. As in usual way, let y_t be the series of interest displaying a stochastic seasonal process in the autoregression form of (5.58). For the semi-annual case, now the polynomial $\varphi(L)$ in (5.60) is expressed with $s = 2$ as

$$\varphi(L) = \sum_{k=1}^{2} \lambda_k \Delta(L) \frac{1-\delta_k(L)}{\delta_k(L)} + \Delta(L)\varphi^*(L) \tag{5.70}$$

where

$$\delta_k(L) = 1 - \frac{1}{\theta_k} L \quad (k = 1,2), \qquad \lambda_k = \frac{\varphi(\theta_k)}{\prod_{j \neq k} \delta_j(\theta_k)}, \qquad \Delta(L) = \prod_{k=1}^{S} \delta_k(L),$$

By looking at the roots of $\varphi(L)=0$, one can detect whether the series is stationary or not. Here the θ_k s are the zero frequency unit root plus the S–1 seasonal unit roots. Therefore, in the case of semi-annual data, we have one zero-frequency unit root which is $\theta_1=1$ and one seasonal unit root (S–1 = 2–1 = 1) which is $\theta_2=-1$. So, $\delta_1(L)=(1-L)$ and $\delta_2(L)=(1+L)$. In that case, the difference operator $\Delta(L)$ becomes $\Delta(L)=(1-L)(1+L)=(1-L^2)$. As a consequence of substituting the expressions just given into (5.70), we get

$$\varphi(L)=\lambda_1(L)(1+L)+\lambda_2(-L)(1-L)+(1-L^2)\varphi^*(L) \qquad (5.71)$$

Then, let $\pi_1=-\lambda_1$ and $\pi_2=-\lambda_2$. Substituting the right hand side of (5.71) into the autoregression equation $\varphi(L)y_t=\varepsilon_t$ gives

$$-\pi_1(L)(1+L)y_t-\pi_2(-L)(1-L)y_t+(1-L^2)\varphi^*(L)y_t=\varepsilon_t \qquad (5.72)$$

This expression can be rewritten in the form of testing equation for the presence of semi-annual unit roots as

$$\varphi^*(L)y_{3,t}=\pi_1 y_{1,t-1}+\pi_2 y_{2,t-1}+\varepsilon_t \qquad (5.73)$$

where

$$y_{1,t}=(1+L)y_t=y_t+y_{t-1}$$
$$y_{2,t}=-(1-L)y_t=-(y_t-y_{t-1})$$

(5.74)

$$y_{3,t}=(1-L^2)y_t=y_t-y_{t-2} \qquad (t=1,2,3,......,n).$$

In (5.73) for simplicity it is assumed that the DGP y_t is free of any deterministic components. In order to obtain the estimates of π_1 and π_2, the OLS procedure is applied to (5.73). For testing a zero-frequency unit root, the null hypothesis becomes $\pi_1=0$ against the alternative of stationarity $\pi_1<0$. In a similar manner, the presence of a unit root at the π frequency is tested with the null hypothesis of $\pi_2=0$ against the alternative one that is $\pi_2<0$. Furthermore, for testing if the series is seasonally integrated (implied as the presence of unit roots at both frequencies concurrently), the F-statistic for $\pi_1=\pi_2=0$ may be used in order to test whether there are unit roots at both frequencies simultaneously (Feltham & Giles, 1999, pp. 3-4). The critical values for the nonstandard *t* and F statistics, asymptotic null distributions and more are given in Feltham & Giles (1999).

5.2.5. Kunst Test

Kunst (1997) suggests a general $AR(s)$ model in order to test $(1-L^s)$ and it is described as

$$y_t - y_{t-s} = \theta_1 y_{t-1} + \theta_2 y_{t-2} + ... + \theta_s y_{t-s} + \varepsilon_t \quad (5.75)$$

For testing the null hypothesis of $y_t - y_{t-s} = \varepsilon_t$, after applying to OLS procedure in (5.75) and estimating each θ, Kunst has made a comparison of the F-statistic for the test of $\theta_1 = \theta_2 = ... = \theta_s = 0$ with the critical value obtained through conducted Monte Carlo simulations. Kunst's test bears resemblance to the DHF test in that it only detects the presence of all seasonal unit roots. However, under the Kunst's alternative hypothesis which is more general than the alternative of the DHF test, the series displays any $AR(s)$ model except the model under the null while the alternative of DHF is the presence of s roots in the series all having the same modulus bigger than one. In addition, Osborn and Rodrigues (2002) have indicated that the Kunst F-test statistic and the HEGY overall F-statistic have the same asymptotic distributions.

The DHF regression model in (5.9) can be regarded as the reduced form of the Kunst model. The reason for this is that there is only one lag variable (y_{t-s}) in the DHF test while s lagged terms are available $(y_{t-1},......., y_{t-s})$ in Kunst's model and under the null hypothesis, the asymptotic distributions of y_{t-s} obtained from both models differ extremely. On the other hand, the augmented Kunst model can be expressed as

$$\Delta_s y_t = \theta_1 y_{t-1} + \theta_2 y_{t-2} + ... + \theta_s y_{t-s} + \kappa_1 \Delta_s y_{t-1} + ... + \kappa_p \Delta_s y_{t-p} + \varepsilon_t \quad (5.76)$$

When $s = 4$ (that is, for quarterly time series), the Kunst test regression is given as

$$\Delta_4 y_t = \alpha_1 y_{t-1} + ... + \alpha_3 y_{t-3} + \delta y_{t-4} + \varepsilon_t, \quad (t = 1,......,T) \quad (5.77)$$

which is an F-type test given as

$$F^*_{\hat{\alpha}_1,....\hat{\alpha}_3,\hat{\delta}} = (T-4)(\hat{\varepsilon}_0'\hat{\varepsilon}_0 - \hat{\varepsilon}'\hat{\varepsilon})/(\hat{\varepsilon}'\hat{\varepsilon}) \quad (5.78)$$

where $\hat{\varepsilon}_0$ and $\hat{\varepsilon}_1$ vectors represent the estimated residuals under the null $H_0 : \alpha_1 = ... = \alpha_3 = \delta = 0$ and alternative hypotheses respectively (El Montasser, 2011, p. 27; Zhang, 2008, pp. 9-10).

5.2.6. OCSB Test

Osborn, Chui, Smith and Birchenhall (OCSB) (1988) have modified the Hasza and Fuller (1982) test framework to detect the presence of multiplicative differencing filter $\Delta_1\Delta_s$. That is, the OCSB test investigates whether $(1-L)$ or $(1-L^s)$ operators or both of them or none of them should be applied to data. The OCSB regression model in the original form is expressed as

$$\Delta_1\Delta_s y_t = \beta_1\Delta_s y_{t-1} + \beta_2\Delta_1 y_{t-s} + \varepsilon_t \tag{5.79}$$

and this model is used to test whether $\Delta_1\Delta_s$ is a factor of $\varphi(L)$ in (5.58). This model can be generalized with deterministic components as follows:

$$\eta(L)\Delta_1\Delta_s y_t = \mu_t + \gamma_1\Delta_s y_{t-1} + \gamma_2\Delta_1 y_{t-s} + \varepsilon_t \tag{5.80}$$

where $\eta(L)$ is an AR polynomial (lag polynomial with roots outside the unit circle), $\Delta_s = (1-L^s), \Delta_1 = (1-L)$ and

$$\mu_t = \alpha_0 + \sum_{s=1}^{S-1}\alpha_s D_{s,t} + \beta_0 t + \sum_{s=1}^{S-1}\beta_s D_{s,t} t \tag{5.81}$$

Here, t is a deterministic trend. In the original study, the seasonal trend is not given place in μ_t i.e. $\beta_s = 0$ for $\forall$ s. However, Franses and Koehler (1998) suggest the model (5.80) with the β parameters not being equal to zero in μ_t so that the test becomes applicable to y_t series showing increasing seasonal variation. In order to find out which filter is suitable for y_t, the significances of γ_1 and γ_2 are tested. When both γ_1 and γ_2 are equal to zero ($\gamma_1 = \gamma_2 = 0$), using $\Delta_1\Delta_s$ filter is suitable. When $\gamma_1 = 0$ and $\gamma_2 \neq 0$, Δ_1 filter should be selected; when $\gamma_1 \neq 0$ and $\gamma_2 = 0$, Δ_s filter is suitable. If both γ_1 and γ_2 are unequal to zero ($\gamma_1 \neq \gamma_2 \neq 0$), in that case no differencing filter is required.

By the same logic just given above, in the case of quarterly data OCSB testing regression is given as

$$\Delta_1\Delta_4 y_t = \alpha_0 + \alpha_1 D_{1,t} + \alpha_2 D_{2,t} + \alpha_3 D_{3,t} + \gamma_1\Delta_4 y_{t-1} + \gamma_2\Delta_1 y_{t-4} + \sum_{i=1}^{k}\phi_i\Delta_1\Delta_4 y_{t-i} + \varepsilon_t \tag{5.82}$$

The necessary joint hypothesis about the usefulness of the $\Delta_1\Delta_4$ operator is $\alpha_1 = \alpha_2 = \alpha_3 = \gamma_1 = \gamma_2 = 0$. If $\gamma_2 = 0$ with $\gamma_1 < 0$, the Δ_4 filter is needed and if $\gamma_1 = 0$

and $\gamma_2 < 0$, the Δ_1 filter should be applied to data. For these three hypotheses, critical values are available only for a sample size of 136 in Osborn (1990) (Franses, 1998, p. 563; Maddala & Kim, 1998, p. 366; Zhang, 2008, p. 11; Platon, 2010, pp. 2-3).

5.3. Seasonal Cointegration

The concept of seasonal cointegration is valid for models including stochastic seasonals just as the concept of cointegration showing itself in models including stochastic trends (Maddala & Kim, 1998, p. 362). As mentioned before, one advantage of HEGY test procedure is that it enables to test for unit roots at each frequency separately. So, concerning quarterly data including the four roots which are $1,\ -1,\ \pm i$; Engle et al. (1993) propose different levels of seasonal cointegration. Assume that y_t and z_t series are seasonally cointegrated so that $\Delta_4 y_t$ and $\Delta_4 z_t$ are stationary. When these two series have a common non-seasonal unit root (that is, they are cointegrated at long-run zero frequency – at root 1), we have the error term

$$u_t = (1 + L + L^2 + L^3) y_t - \alpha_1 (1 + L + L^2 + L^3) z_t \tag{5.83}$$

which is stationary. If seasonal cointegration exists at frequency ½ corresponding to unit root –1, we have

$$v_t = (1 - L + L^2 - L^3) y_t - \alpha_2 (1 - L + L^2 - L^3) z_t \tag{5.84}$$

which is stationary (so, it does not require $(1 + L)$ filter to be stationary) and finally if seasonal cointegration exists at frequency ¼ corresponding to unit roots $\pm i$ and $(1 - L^2)$ filter we have

$$w_t = (1 - L^2) y_t - \alpha_3 (1 - L^2) z_t - \alpha_4 (1 - L^2) y_{t-1} - \alpha_5 (1 - L^2) z_{t-1} \tag{5.85}$$

is stationary. In case all three series u_t, v_t and w_t are stationary, the seasonal cointegration model is represented in a simple form as

$$\begin{aligned} \Delta_4 y_t &= \beta_{11} u_{t-1} + \beta_{21} v_{t-1} + \beta_{31} w_{t-2} + \beta_{41} w_{t-3} + \varepsilon_{1t} \\ \Delta_4 z_t &= \beta_{21} u_{t-1} + \beta_{22} v_{t-1} + \beta_{32} w_{t-2} + \beta_{42} w_{t-3} + \varepsilon_{2t} \end{aligned} \tag{5.86}$$

where βs represent the error correction terms. In addition; constant, seasonal dummies and trend variables can be incorporated into these equations. This method with two-step proposed by Engle et al. (1993) is similar to the Engle-Granger approach applied for nonseasonal time series: in the first step, equations (5.83) to (5.85) are estimated by OLS procedure and in the second step ADF unit root tests are applied to $\hat{u}_t$, $\hat{v}_t$ and $\hat{w}_t$

(in other words, this transaction allows us to check if estimated residuals $\hat{u}_t$ to $\hat{w}_t$ are stationary). The tests for $\hat{u}_t$ and $\hat{v}_t$ have the same critical values as those in Engle and Granger (1987). However, critical values for testing $\hat{w}_t$ are different. For this case, the critical values are tabulated in Engle et al. (1993). In her comments on the paper of Engle et al. (1993), Osborn discusses the implication of (5.86) to be varying equilibrium relations between y_t and z_t series depending on the lag (that is, the long-run relation at time $(t-1)$ differs from that at time $(t-2)$). She considers a more reasonable model which has changing coefficients with seasons and this results in the periodic cointegration model which will not be discussed here (Maddala & Kim, 1998, pp. 375-376).

5.3.1. Seasonal Cointegration-Single Equation

As mentioned above, subsequent to estimating α_1 to α_5 by OLS for bivariate time series involving y_t and z_t, the stationarity condition is checked for estimated residuals $\hat{u}_t$ to $\hat{w}_t$. This is executed by using the following auxiliary regressions:

$$
\begin{aligned}
(1-L)\hat{u}_t &= \pi_1 \hat{u}_{t-1} + \sum_{i=1}^{l_1} \gamma_i (1-L)\hat{u}_{t-i} + \varepsilon_t \\
(1+L)\hat{v}_t &= \pi_2 (-\hat{v}_{t-1}) + \sum_{i=1}^{l_2} \gamma_i (1+L)\hat{v}_{t-i} + \varepsilon_t \qquad (5.87) \\
(1+L^2)\hat{w}_t &= \pi_3 (-\hat{w}_{t-2}) + \pi_4 (-\hat{w}_{t-1}) + \sum_{i=1}^{l_3} \gamma_i (1+L^2)\hat{w}_{t-i} + \varepsilon_t
\end{aligned}
$$

(Löf, 2001, p .10).

As seen here, the lagged dependent variables may be added to these auxiliary regressions given above. To detect the cointegration at the zero and semi-annual frequencies, *t*-statistic values of π_1 and π_2 should be compared to the critical values in the paper of Engle and Yoo (1987) and the null hypotheses of no cointegration at zero frequency and no cointegration at ½ frequency should be tested for the first two auxiliary regressions in (5.87). On the other hand, for ¼ (and ¾ frequencies), F($\pi_3 = \pi_4 = 0$) test statistic value should be compared to the critical values which take place in the paper of Engle et al. (1993) and here the null hypothesis should be constructed as H_0 : No cointegration at ¼ (and ¾) frequencies for the third auxiliary regression given in (5.87) (Mert & Demir, 2014, p. 16).

5.3.2. Seasonal Cointegration-Multiple Equations

Lee (1992) presents a testing procedure for seasonal cointegration using an extension of Johansen approach with the ML estimator. Assuming that ε_t are i.i.d. n-dimensional Gaussian random vectors with zero mean and a variance-covariance matrix Ω whereas Y_t is an n-dimensional vector of I(1) variables with $(y_{1t}, y_{2t}, ..., y_{nt})$ and assume that the process Y_t can be described by the VAR process. With the lag length p , for quarterly data the estimation model can be written in the form of

$$\Delta_4 Y_t = \Pi_1 Y_{1,t-1} + \Pi_2 Y_{2,t-1} + \Pi_3 Y_{3,t-2} + \Pi_4 Y_{3,t-1} + \Gamma_1 \Delta_4 Y_{t-1} + ... + \Gamma_{p-4} \Delta_4 Y_{t-p+4} + \varepsilon_t \qquad (5.88)$$

where

$$\begin{aligned} Y_{1,t} &= (1 + L + L^2 + L^3) Y_t \\ Y_{2,t} &= -(1 - L + L^2 - L^3) Y_t \\ Y_{3,t} &= -(1 - L^2) Y_t \end{aligned} \qquad (5.89)$$

(5.88) looks like (5.47) except that the lower case y implies univariate processes while the capital letter Y implies multivariate processes. Since the coefficient matrices $\Pi_1, ..., \Pi_4$ convey information about the long-run behaviour of the series, it is of great importance to analyse their characteristics in depth. The ranks of the matrices Π_1, Π_2 and Π_3 determine the number of cointegrating vectors at zero, ½ and ¼ frequencies. If the matrix Π_k has full rank, then at the relevant frequency all series considered are stationary. In case Π_k has a zero rank, there is no seasonal cointegration among the variables at the corresponding frequency. On the other hand, the implication of the case of $0 < rank(\Pi_k) = r < n$ is that at the relevant frequency a linear combination of non-stationary variables becomes stationary.

In order to put the seasonal cointegration tests into practice, Lee suggests four tests pertinent to the rank of Π_k. That is to say, a cointegration test at frequency $w = 0$, $w = 1/2$, $w = 1/4$ and the joint test of $w = 1/4$ and $w = 3/4$. Lee draws attention to that the distribution of the ML cointegration test statistics and the asymptotic distributions belonging to these statistics may extremely differ. As a matter of fact, our tendency is generally in the direction of rejecting the true null hypothesis more often than implied by the asymptotic distribution. However, this bias decreases in very large samples. The regularity conditions for using the Johansen approach necessitates that DGP does not have unit roots other than the zero frequency. Contrary to this, Ghysels et al. (1994a) express in their paper that seasonal unit roots do not give rise to a complication in the use of the Johansen approach (Huang & Shen, 1999, pp. 114-115 ; Maddala & Kim, 1998, pp. 376-377).

5.4. An Extension of Seasonal Cointegration

By following equations (5.41), (5.42) and (5.43) in HEGY testing procedure, the given polynomials are shown in the following notations:

$$Z_1 = (1+L)(1+L^2) = (1+L+L^2+L^3) \qquad (5.90)$$

$$Z_2 = -(1-L)(1+L^2) = -(1-L+L^2-L^3) \qquad (5.91)$$

$$Z_3 = -(1-L)(1+L) = -(1-L^2) \qquad (5.92)$$

When HEGY (1990) procedure is applied to the time series y_t, HEGY testing equation can be expressed as

$$(1-L^4)y_t = \alpha_1 D_{1,t} + \alpha_2 D_{2,t} + \alpha_3 D_{3,t} + \alpha_4 D_{4,t} + \delta.t + \pi_1 Z_1 y_{t-1} + \pi_2 Z_2 y_{t-1} + \pi_3 Z_3 y_{t-2} + \pi_4 Z_3 y_{t-1}$$
$$+ \sum_{i=1}^{p} \phi_i (1-L^4) y_{t-i} + \varepsilon_{it} \qquad (5.93)$$

With the polynomial filters defined above, seasonal cointegration at seasonal cycles for quarterly data can be expressed in the following ways:

Definition 1: *Cointegration at the single period cycle*

y_t is cointegrated at the long run (corresponding to the root of 1 with the factor of $(1-L)$) if there is a cointegrating vector α_1 such that the residuals u_t from

$$\alpha_1' Z_1 y_t = u_t \qquad (5.94)$$

are stationary.

Definition 2: *Cointegration at the two period cycles*

y_t is cointegrated at the two period (or biannual) cycle (corresponding to the root of -1 with the factor of $(1+L)$) if there is a cointegrating vector α_2 such that the residuals v_t from

$$\alpha_2' Z_2 y_t = v_t \qquad (5.95)$$

are stationary.

It is very hard to establish the cointegration at the four period (or annual) cycle because of the fact that the effects of complex roots are indistinguishable in quarterly case. According to Yoo (1986), the cointegration should be constructed on the basis of lags in any vector (namely, a polynomial cointegrating vector) which tries to decrease the order of integration at the annual cycle.

Definition 3: *Cointegration at the four period cycles*

y_t is cointegrated at the four period (or annual) cycle (corresponding to the complex roots of $+i$ and $-i$ with the factor of $(1+L^2)$) if there is a cointegrating vector $\alpha_3 + \alpha_4.L$ such that the residuals w_t from

$$(\alpha_3' + \alpha_4' L) Z_3 y_t = w_t \tag{5.96}$$

are stationary.

In order to establish an error correction model including all these cointegration cases at different cycles, there are two criteria that must hold. First, a term corresponding to all the various possible cases of cointegration mentioned briefly above must be available in an error correction model. Second, all the variables which take place in the final error correction equation should be integrated of order zero (that is, $I(0)$). To satisfy this criterion, pre-filtered data Z_{it} (not the original vector time series) should be used in the specification of the terms in the error correction equation.

The general form of the error correction representation in which all existing terms are stationary and all possible cases of cointegration at different cycles are included is developed by Hylleberg et al. (1990) and Engle, Granger, Hylleberg and Lee (1990) and the equation is given as

$$\phi(L)(1-L^4) y_t = \gamma_1 u_{t-1} + \gamma_2 v_{t-1} + (\gamma_3 + \gamma_4.L) w_{t-1} + \varepsilon_t \tag{5.97}$$

where γ_i and the cointegrating parameters, α_i may be different at different frequencies. Both the α and γ coefficients should be estimated to estimate equation (5.97). If there are specific values for the cointegrating parameters, α_i proposed by an economic theory in interest, the estimation of (5.97) becomes easy to handle. Otherwise, Hylleberg et al. (1990) and Engle et al. (1990) suggest a generalisation of the two stage procedure proposed by Engle and Granger (1987) (Hurn, 1993, pp. 313-315).

5.4.1. Three Bivariate Error Correction Models

An application of seasonal cointegration may be useful in the context of monetary policy. To present an application concerning seasonal cointegration, Hurn (1993) uses South African monetary data in this context. To predict the future path of nominal income, it is focused on the ability of monetary aggregates. Therefore, the monetary aggregate is counted as the leading indicator of nominal income.

Consider the two variables case of (5.97) with nominal income y_t, and a monetary aggregate, m_t with the normalization with respect to the former:

$$(1-L^4)y_t = \sum_{i=1}^{m}\beta_i(1-L^4)y_{t-i} + \sum_{i=0}^{n}\delta_i(1-L^4)m_{t-i} + \gamma_1(Z_1y_{t-1} - \alpha_{12}Z_1m_{t-1}) + \gamma_2(Z_2y_{t-1} - \alpha_{22}Z_2m_{t-1})$$

$$+(\gamma_3 + \gamma_4.L)(Z_3y_{t-1} - \alpha_{32}Z_3m_{t-1} - \alpha_{41}Z_3y_{t-2} - \alpha_{42}Z_3m_{t-2}) + \varepsilon_{1t} \quad (5.98)$$

This equation represents the full seasonal error correction model. If there is cointegration at all cycles by the same cointegrating parameter or in other saying if the following restrictions hold

$$\alpha_{12} = \alpha_{22} = \alpha_{32} = \alpha\,,\;\; \alpha_{41} = \alpha_{42} = 0 \quad (5.99)$$

the error correction model reduces to the simple error correction representation (Engle & Granger, 1987; Granger, 1986) specified in terms of the original variables. In Hurn (1993) it is also expressed that the model draws apart from the original version of the error correcting equation when the error correcting term $(y_{t-1} - \alpha m_{t-1})$ may be included in the equation up to a maximum of four lags to capture the four unit roots to be removed (Hylleberg et al., 1990). The estimable equation becomes in the following way:

$$(1-L^4)y_t = \sum_{i=1}^{m}\beta_i(1-L^4)y_{t-i} + \sum_{i=0}^{n}\delta_i(1-L^4)m_{t-i} + \sum_{i=0}^{3}\gamma_i L^i(y_{t-1} - \alpha m_{t-1}) + \varepsilon_{2t} \quad (5.100)$$

Another practical type of the general model (5.98) is realized when cointegration exists at the single period cycle by filtered (seasonally adjusted) variables and at all other cycles by one cointegrating parameter. The restrictions on the $\alpha's$ are given as

$$\alpha_{12} = \alpha\,,\;\; \alpha_{22} = \alpha_{32} = \alpha_s\,,\;\; \alpha_{41} = \alpha_{42} = 0 \quad (5.101)$$

and the equation to be estimated becomes

$$(1-L^4)y_t = \sum_{i=1}^{m}\beta_i(1-L^4)y_{t-i} + \sum_{i=0}^{n}\delta_i(1-L^4)m_{t-i} + \gamma_1(Z_1 y_{t-1} - \alpha Z_1 m_{t-1})$$

$$+\sum_{i=0}^{2}\gamma_i L^i[(1-L)y_{t-1} - \alpha_s(1-L)m_{t-1}] + \varepsilon_{3t} \qquad (5.102)$$

Since the common error correcting relation in terms of the differenced variables appears as a maximum lag of three in the equation (5.102), there are three coefficients to be estimated on the seasonal error correction term. It is apparent from the general error correction model (5.98) and this specific restricted model that the aim in using seasonal cointegration is to augment the short-run dynamics of the model and the long-run solution remains the same as in the original simple error correction model. To sum up, the equations (5.98), (5.100) and (5.102) form three seasonal error correction models and the estimation of these models may be feasible by making use of the Engle-Granger two-step procedure (Hurn, 1993, pp. 315-317).

CHAPTER 6

ECONOMETRIC APPLICATIONS RELATED TO ECONOMIC TIME SERIES

6.1. Modelling Monthly Inflation Rates in Turkey

In terms of policy makers, it is of great importance to have a reliable inflation rate forecast. In this context, the most suitable model should be accessed using SARIMA. Since SARIMA models reveal more effective results in terms of handling the seasonal component of the series apart from the non-seasonal one when compared to the traditional ARIMA models. In this application, it has been aimed to find the best model for monthly inflation rates and therefore monthly (not seasonally adjusted) CPI data have been utilized for Turkish economy over the period 1995:01-2015:03 (Index 2010=1.00). Data have been obtained from Organization for Economic Co-operation and Development. This application has been carried out at the R Project for Statistical Computing-version 3.1.3. by using “forecast” and “uroot” packages. Since inflation is measured by the percentage change in CPI, inflation rates have been calculated by using the following transformation:

$$INF = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}}.100$$

where INF denotes inflation rate, CPI_t denotes consumer price index at time *t* and CPI_{t-1} denotes consumer price index at time *t*-1.

In modelling monthly inflation rates that are very crucial to design effective economic strategies, choosing a suitable seasonal ARIMA model which includes both seasonal and non-seasonal behaviours is not an easy task. Since such models give point to the recent past rather than distant past, primarily they are convenient for short term forecasting and this implies that long-term forecasts from ARIMA models are less reliable than short term forecasts (Aidoo, 2010, p. 3). The graph of inflation data has been presented in Figure 5:

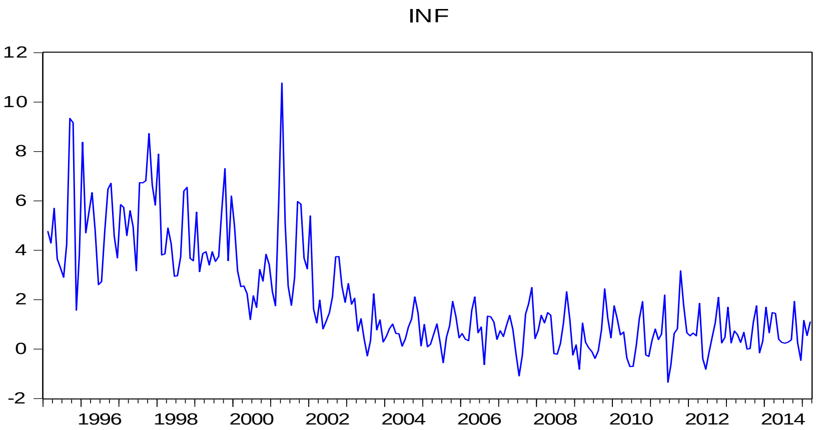


Figure 5. Graph of inflation series against time

It is very apparent to see from Figure 5 that inflation data are nonstationary with a non-constant mean and unsteady variance and follow some seasonal pattern. For this reason, first of all the series should be checked for seasonal unit roots at all seasonal frequencies and if INF series includes all seasonal unit roots, seasonal differencing operator has to be applied to this series. If INF series has seasonal unit roots only at some frequencies, filters corresponding to available unit roots at each given frequency have to be applied. Briefly, before constructing a suitable ARIMA model for our seasonal series, we should make a data transformation in a way to make the series stationary by taking Box-Jenkins methodology into consideration (see Appendix B for Box-Jenkins technique).

This study has mainly focused on searching for the best-fitted SARIMA model for the monthly inflation rates in order to provide the best forecast. Therefore, following the Box-Jenkins approach, at first model identification and estimation of parameters will be presented. Subsequent to this, diagnostic checking results based on the residuals of the possible model will be given place in order to make certain about the white-noise characteristic of residuals which becomes a vital assumption for a good ARIMA model.

Before the model identification, in order to detect at which frequencies INF series has unit roots and to decide about the appropriate order of differencing filter, we should recourse to HEGY monthly seasonal unit root test apart from CH test. As expressed in chapter 4 and 5, the null hypotheses differ for CH and HEGY tests. In the former, the null hypothesis implies the stationarity case at all seasonal cycles while the latter implies the presence of seasonal unit root (nonstationarity case).

Figure 6 and Figure 7 show the ACF and PACF of the original inflation series for maximum lag numbers of 48 respectively. When looked at the correlogram of series in

Figure 6, the autocorrelation coefficient is seen to decline very slowly towards zero with increasing lag length implying that the series is nonstationary. On the other hand,

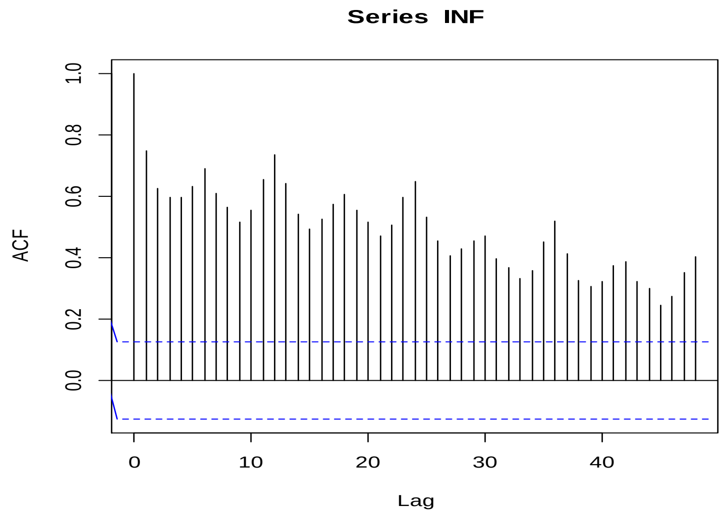


Figure 6. ACF of inflation series (for lag.max=48)

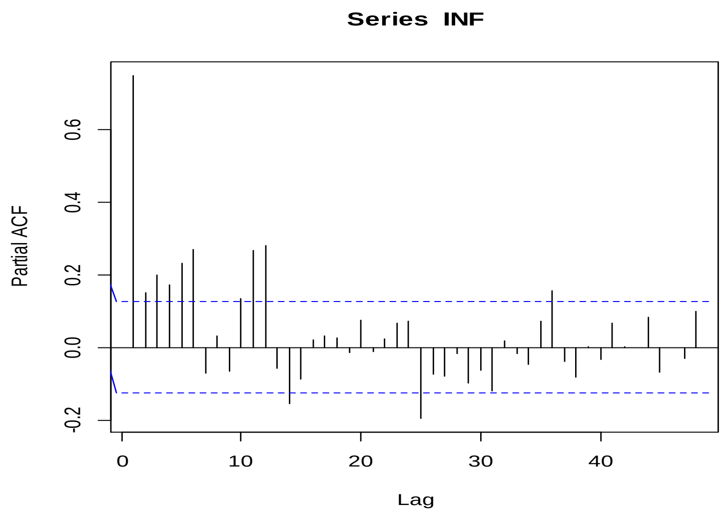


Figure 7. PACF of inflation series (for lag.max=48)

seasonal lags (12 24, 36,48) are clear to be significant. Thus, the presence of any seasonal unit root other than a zero (long-run) frequency unit root has to be detected.

Detailed explanations for testing monthly seasonal unit roots have been given place in Chapter 5 and Table 7 has presented long-run and seasonal frequencies for monthly series in details. In this study, the monthly seasonal unit root analysis has been carried out by using three different lag order selection methods. First, significant lags have been added to the four deterministic regressions (with only constant; constant and trend; constant and dummies; constant, trend and dummies) and one regression with no deterministic components in order to make certain about that the residuals are white noise (that is, insignificant lags have been removed until all selected lags become significant). These test results have been given in Table 8 and subsequent to this, selected lags for HEGY regressions in Table 8 have been shown in Table 9. As mentioned before, the first two hypotheses which are $\pi_1 = 0$ and $\pi_2 = 0$ are tested by *t*-test and the other five joint hypotheses which are $\pi_3 = \pi_4 = 0$, $\pi_5 = \pi_6 = 0$, $\pi_7 = \pi_8 = 0$, $\pi_9 = \pi_{10} = 0$ and $\pi_{11} = \pi_{12} = 0$ are tested by F-test.

Table 8

HEGY Monthly Seasonal Unit Root Test Results for Inflation Series (by Using Significant Lags)

Auxiliary Regression Null Hypotheses	**Seasonal Frequency**	**Estimates for the Model with Constant**	**Estimates for the Model with Constant and Trend**	**Estimates for the Model with Constant and Dummies**	**Estimates for the Model with Constant, Trend and Dummies**	**Estimates for the Model with No Constant, No Trend and No Dummies**
$\pi_1 = 0$	0	**-1.537***	**-0.288***	**-1.294***	**-1.548***	-2.762
$\pi_2 = 0$	π	-2.348	-2.313	-3.588	-3.608	-2.347
$\pi_3 = \pi_4 = 0$	$\pi/2$	6.966	6.761	20.174	20.222	6.960
$\pi_5 = \pi_6 = 0$	$2\pi/3$	4.220	4.008	14.163	14.297	4.208
$\pi_7 = \pi_8 = 0$	$\pi/3$	**1.675***	**1.606***	9.036	9.132	**1.668***
$\pi_9 = \pi_{10} = 0$	$5\pi/6$	12.656	12.342	22.248	22.352	12.662
$\pi_{11} = \pi_{12} = 0$	$\pi/6$	5.461	5.236	14.104	14.524	5.435

Note. [1] * denotes insignificant estimates (*p>.05) at 5% significance level
[2] See Monthly HEGY Critical Values in Appendix C .

Table 9

Selected Lags Estimates for HEGY Monthly Seasonal Unit Test on Inflation Series (by Using Significant Lags)

Models	**Selected Lags**	**Estimate**	**Standard Error**	***t*-value**	**Prob ($>\lvert t \rvert$)**
C	Lag.12	-0.213	0.065	-3.304	0.001
C,T	Lag.12	-0.221	0.067	-3.302	0.001
C,D	-	-	-	-	-
C,D,T	-	-	-	-	-
-	Lag.12	-0.216	0.064	-3.377	0.001

Note. "C" denotes constant term, "T" denotes trend, "D" denotes seasonal dummy variables and "-" denotes no deterministic component.

It can be inferred from Table 9 results that no lagged variable has been added to C,D and C,D,T models. However, for other three models (C; C,T and -) 12^{th} lag has been added as significant lag. When looked at Table 8, the results for the hypothesis

$\pi_1 = 0$ have revealed that the presence of the zero (non-seasonal or long-run) frequency unit root is accepted depending on the non-rejection of the null hypothesis $\pi_1 = 0$ at all deterministic models (except no deterministic component model). Thus, original INF series is not stationary at zero frequency. Having examined the other hypotheses, all other hypotheses implying the presence of a unit root at seasonal frequency except the hypothesis $\pi_7 = \pi_8 = 0$ are seen to be rejected for all deterministic models and therefore it is concluded that there are no seasonal unit roots at $\pi, \pm\frac{\pi}{2}, \pm\frac{2\pi}{3}, \pm\frac{5\pi}{6}$ and $\pm\frac{\pi}{6}$ frequencies. In other saying, there are conjugate complex seasonal unit roots only at $\pm\frac{\pi}{3}$ frequencies corresponding to (2, 10) cycles per year for "Constant", "Constant and Trend" and "No Deterministic Components" models. From this point of view, seasonal cycles can be said to follow mostly a deterministic structure.

Table 10
HEGY Monthly Seasonal Unit Root Test Results for Inflation Series (by Using AIC for Lags)

Auxiliary Regression Null Hypotheses	**Seasonal Frequency**	**Estimates for the Model with Constant**	**Estimates for the Model with Constant and Trend**	**Estimates for the Model with Constant and Dummies**	**Estimates for the Model with Constant, Trend and Dummies**	**Estimates for the Model with No Constant, No Trend and No Dummies**
$\pi_1 = 0$	0	**-1.546***	**-0.579***	**-1.417***	**-0.935***	-2.542
$\pi_2 = 0$	π	-2.541	-2.515	-2.978	-2.991	-2.534
$\pi_3 = \pi_4 = 0$	$\pi/2$	4.938	4.905	18.391	18.360	4.937
$\pi_5 = \pi_6 = 0$	$2\pi/3$	3.373	3.310	7.305	7.267	3.359
$\pi_7 = \pi_8 = 0$	$\pi/3$	**1.212***	**1.197***	**5.727***	**5.756***	**1.207***
$\pi_9 = \pi_{10} = 0$	$5\pi/6$	14.009	13.633	20.506	20.451	13.975
$\pi_{11} = \pi_{12} = 0$	$\pi/6$	3.897	3.842	13.631	13.624	3.860

Note. [1] * denotes insignificant estimates (*p>.05) at 5% significance level
[2] See Monthly HEGY Critical Values in Appendix C .

Table 10 presents monthly HEGY seasonal unit root test results based on AIC. The results are almost the same as Table 8 with regard to statistical significance: Since the hypothesis $\pi_1 = 0$ could not be rejected at 5% significance level (meaning that non-rejection of the presence of root $+1$), the presence of the zero frequency (non-seasonal) unit root has been accepted. Thus, inflation series is nonstationary and seasonal unit roots have been detected only at $\pm\frac{\pi}{3}$ frequencies for all five models given in Table 10. In addition, estimate results regarding lagged values added to the auxiliary regressions have been presented in Table 11. Evaluating the results has revealed that 12th lag has been given place in all five auxiliary regression models.

Table 11
Selected Lags Estimates for HEGY Monthly Seasonal Unit Test on Inflation Series (by Using AIC for Lags)

Models	**Selected Lags**	**Estimate**	**Standard Error**	***t*-value**	**Prob ($>\|t\|$)**
C	Lag.2	0.107	0.065	1.639	0.103
	Lag.12	-0.210	0.064	-3.260	0.001
C,T	Lag.2	0.106	0.067	1.574	0.117
	Lag.12	-0.212	0.067	-3.161	0.002
C,D	Lag.6	-0.162	0.072	-2.251	0.026
	Lag.12	0.007	0.070	0.100	0.920
C,D,T	Lag.6	-0.157	0.073	-2.151	0.033
	Lag.12	0.014	0.072	0.188	0.851
-	Lag.2	0.105	0.065	1.615	0.108
	Lag.12	-0.213	0.064	-3.346	0.001

Note. "C" denotes constant term, "T" denotes trend, "D" denotes seasonal dummy variables and "-" denotes no deterministic component.

Table 12
HEGY Monthly Seasonal Unit Root Test Results for Inflation Series (by Using BIC for Lags)

Auxiliary Regression Null Hypotheses	**Seasonal Frequency**	**Estimates for the Model with Constant**	**Estimates for the Model with Constant and Trend**	**Estimates for the Model with Constant and Dummies**	**Estimates for the Model with Constant, Trend and Dummies**	**Estimates for the Model with No Constant, No Trend and No Dummies**
$\pi_1=0$	0	**-1.537***	**-0.288***	**-1.499***	**-1.315***	-2.762
$\pi_2=0$	π	-2.348	-2.313	-3.232	-3.278	-2.347
$\pi_3=\pi_4=0$	$\pi/2$	6.966	6.761	15.593	15.816	6.960
$\pi_5=\pi_6=0$	$2\pi/3$	4.220	4.008	9.534	9.773	4.208
$\pi_7=\pi_8=0$	$\pi/3$	**1.675***	**1.606***	6.756	6.956	**1.668***
$\pi_9=\pi_{10}=0$	$5\pi/6$	12.656	12.342	17.772	17.988	12.662
$\pi_{11}=\pi_{12}=0$	$\pi/6$	5.461	5.236	10.906	11.126	5.435

Note. [1] * denotes insignificant estimates (*p>.05) at 5% significance level
[2] See Monthly HEGY Critical Values in Appendix C .

Table 12 considers the results of monthly HEGY seasonal unit root test based on BIC (Bayesian Information Criterion) and which lags have been added to the five auxiliary regressions is shown in Table 13 with given estimate results of these lags**.** Table 12 and Table 8 results do not differ. In conclusion, three methods discussed in terms of lag criteria (Significant lags, AIC and BIC) have revealed only the presence of conjugate complex seasonal unit roots at $\pm\frac{\pi}{3}$ frequencies corresponding to (2, 10) cycles per year. The presence of all other seasonal unit roots with $\pi, \pm\frac{\pi}{2}, \pm\frac{2\pi}{3}, \pm\frac{5\pi}{6}$ and $\pm\frac{\pi}{6}$ has been rejected and it has been concluded that seasonal cycles mostly display a deterministic structure. Therefore, there is no need to take the seasonal difference of INF series. However, because of the presence of zero (non-seasonal) frequency unit root cannot be denied it has been needed to take the first difference of INF series. In that case, INF series is not seasonally integrated and thus applying the seasonal difference filter $(1-L^{12})$ to the series is not required. Beaulieu and Miron (1992b, p.18) have also explained more clearly why applying $(1-L^{12})$ filter to the series is not required in that way: "The appropriateness of applying the filter $(1-L^{d})$ to a series with a seasonal component, as advocated by Box and Jenkins (1970) depends on the series being integrated at zero and all of the seasonal frequencies". Briefly, this explanation holds since the presence of all seasonal unit roots has not been accepted and there is weak evidence of seasonal unit roots on monthly series.

Table 13
Selected Lags Estimates for HEGY Monthly Seasonal Unit Test on Inflation Series (by Using BIC for Lags)

Models	**Selected Lags**	**Estimate**	**Standard Error**	***t*-value**	**Prob ($>\lvert t\rvert$)**
C	Lag.12	-0.213	0.065	-3.304	0.001
C,T	Lag.12	-0.221	0.067	-3.302	0.001
C,D	Lag.12	-0.046	0.066	-0.697	0.486
C,D,T	Lag.12	-0.03	0.07	-0.422	0.673
-	Lag.12	-0.216	0.064	-3.377	0.001

Note. "C" denotes constant term, "T" denotes trend, "D" denotes seasonal dummy variables and "-" denotes no deterministic component.

Table 14
CH Test Results for Inflation Series

Tested Frequencies	L-Statistic	Critical Values		
		1%	5%	10%
$\frac{\pi}{6},\frac{\pi}{3},\frac{\pi}{2},\frac{2\pi}{3},\frac{5\pi}{6},\pi$	2.005	3.27	2.75	2.49

After applying to HEGY test, now Table 14 presents CH test results in order to make inference about the seasonal behaviour of INF series. As mentioned before, contrary to the HEGY test, the null hypothesis of CH is the stationarity of all seasonal cycles (indicating to the presence of deterministic seasonality) while the alternative hypothesis is the presence of seasonal unit root (indicating to the presence of stochastic seasonality). According to the results, since calculated L-statistic (2.005) is smaller than not only 5% critical value (2.75) but also 1% (3.27) and 10% (2.49) critical values, we fail to reject the null hypothesis saying that seasonal pattern is deterministic. Therefore it can be said that the result of CH test is consistent with the result of HEGY test and once again there is no need for seasonal differencing operator. However, there is one important thing that since the presence of only conjugate complex seasonal unit roots with $\pm\frac{\pi}{3}$ frequencies has been determined with the adoption of the hypothesis $\pi_7 = \pi_8 = 0$, INF series should be transformed by the necessary filters corresponding to these frequencies. Filters corresponding to all frequencies have been presented in Table 7 (in sub-section 5.2.4.2.). Therefore, the necessary filter corresponding to $\pm\frac{\pi}{3}$ frequencies has been expressed as $(1-L+L^2)$**.** On the other hand, as expressed before, since the series includes zero (non-seasonal) frequency unit root, the first difference operator $(1-L)$ should also be applied. So, the necessary transformation that will be made in INF series will be $(1-L)(1-L+L^2)$. More precisely, if the new series to be obtained is called " $f\inf$ " (meaning filtered inflation), $f\inf$ will be formed as follows:

$$f\inf = \Delta(1-L+L^2) = \Delta(INF - INF(-1) + INF(-2))$$
$$= INF - 2INF(-1) + 2INF(-2) - INF(-3)$$

The ACF function of the " $f\inf$ " series obtained after this transformation given above for maximum lags of 48 is given in Figure 8 and PACF function is given in Figure 9:

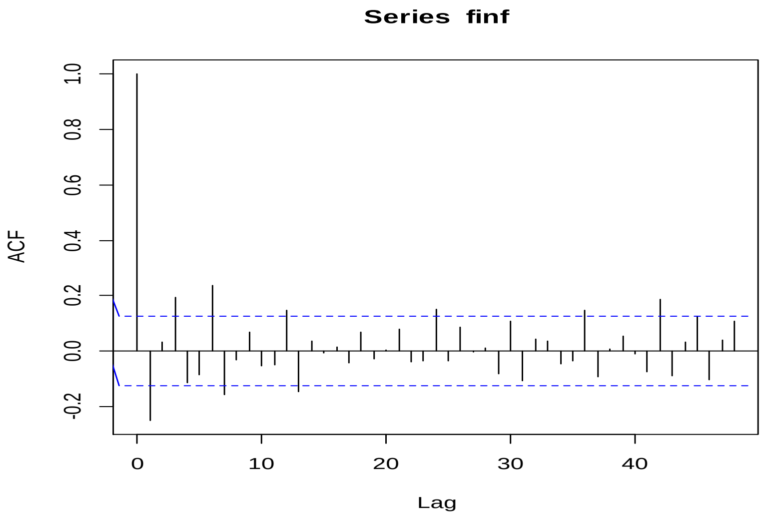


Figure 8. ACF of filtered inflation series ($f\inf$) for lag.max=48

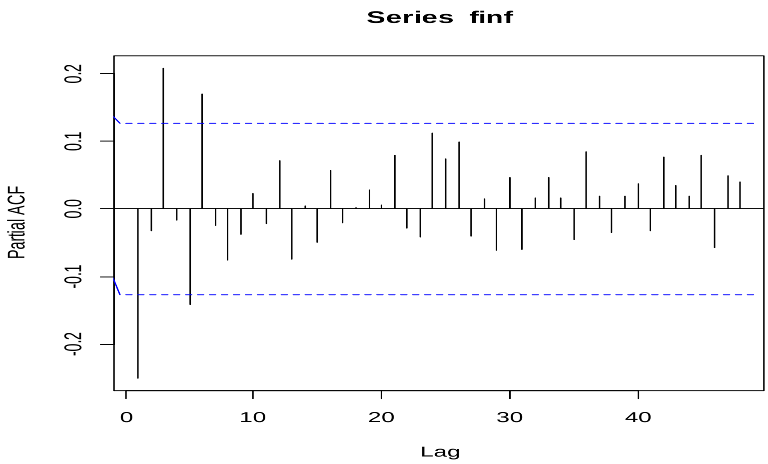


Figure 9. PACF of filtered inflation series ($f\inf$) for lag.max=48

As seen in Figure 8 and Figure 9, the significant spikes at lag 1 in both ACF and PACF suggest a non-seasonal MA(1) and non-seasonal AR(1) components. When looked at the PACF correlogram, there has been found no significant spikes at seasonal lags 12, 24, 36, 48. However, 6^{th} lag is seen to be significant. Therefore, it can be said again that series follows a semi-annual seasonal pattern (corresponding to the filter $(1-L+L^2)$ and thus to the hypothesis $\pi_7 \cap \pi_8 = 0$) as consistent with monthly seasonal unit root results and since three are no significant spikes at seasonal lags in PACF, once again it can be said that seasonal differencing is not required for the series.

"Forecast" package in R software offers us a very practical formula concerned with determining the order of both seasonal differencing and first-degree differencing benefiting from OCSB and CH tests. By running the following codes in "forecast" package of version R.3.1.3., we can compare the results that will obtained through these codes with the results described above:

Table 15

R Codes and Outputs for Determining the Order of Seasonal Differencing by Using OCSB and CH Tests

R Codes and Outputs
>nsdiffs(INF,12,test=”ocsb”)
[1] 0
>nsdiffs(INF,12,test=”ch”)
[1] 0

Note. [1]The function “nsdiffs” estimates the order of seasonal differencing in a series to satisfy stationarity condition. Here “12” indicates the length of seasonal period of the series and “test” expresses the kind of seasonal unit root test to be applied (OCSB or CH).
[2]For more information, see (Hyndman, 2015).
[3]For OCSB test, the null hypothesis is H_0 : Seasonal unit root exists while H_0 : Seasonal cycles are stationary (deterministic seasonality) for CH test.

As seen in Table 15, the result “[1] 0” reveals the number of seasonal differencing for inflation series as “0 (zero)” as a result of carrying out both OCSB test and CH test. Thus, there has been no need to take any seasonal difference. These results show consistency with the results expressed before. Now with the codes given in Table 16 similar to Table 15, let us verify that original inflation (INF) series is not stationary at zero frequency:

Table 16

R Codes and Outputs for Determining the Number of First Differences by Using KPSS and ADF Tests

R Codes and Outputs
>ndiffs(INF,test=”kpss”)
[1] 1
>ndiffs(INF,test=”adf”)
[1] 1

Note. [1] The function “ndiffs” estimates the number of first differences in order to make the series stationary. Here “test” expresses the kind of unit root test to be applied.
[2] For more information, see (Hyndman, 2015).
[3]For KPSS test, the null hypothesis implies the stationarity of series (or the absence of unit root) while the null of ADF test implies the non-stationarity case of series in interest at the non-seasonal level (or the presence of unit root).

The results of practical codes that take place in Table 16 tell us that INF series should be first-degree differenced. ADF and KPSS test results can be clearly given as follows:

Table 17
ADF Test Results for INF Series in Level Form with Constant Added

	***t*-statistic**	**Prob.***	**Critical Values**		
			1%	**5%**	**10%**
ADF Test Statistic	-1.695814	0.4321	-3.458719	-2.873918	-2.573443

Note. [1] Lag length has been chosen as 11 amongst max.lag=12 (based on SIC)
[2] * denotes MacKinnon (1996) one-sided p-values

Table 18
ADF Test Results for INF Series in Level Form with Constant and Trend Added

	***t*-statistic**	**Prob.***	**Critical Values**		
			1%	**5%**	**10%**
ADF Test Statistic	-1.585679	0.7959	-3.998457	-3.429484	-3.138243

Note. [1] Lag length has been chosen as 11 amongst max.lag=12 (based on SIC)
[2] * denotes MacKinnon (1996) one-sided p-values

Table 19
KPSS Test Results for INF Series in Level Form with Constant Added

	LM-Stat.	**Asymptotic Critical Values***		
		1%	**5%**	**10%**
Kwiatkowski-Phillips-Schmidt-Shin Test Statistic	1.721708	0.739000	0.463000	0.347000

Note. * Kwiatkowski-Phillips-Schmidt-Shin (1992).

Table 20
KPSS Test Results for INF Series in Level Form with Constant and Trend Added

	LM-Stat.	**Asymptotic Critical Values***		
		1%	**5%**	**10%**
Kwiatkowski-Phillips-Schmidt-Shin Test Statistic	0.422944	0.216000	0.146000	0.119000

Note. * Kwiatkowski-Phillips-Schmidt-Shin (1992).

Since ADF test statistics calculated at both constant model and constant-trend model in Table 17 and Table 18 have been found to be insignificant (that is, calculated ADF statistics lie outside the critical region) when compared to 1%, 5% and 10% critical values, we fail to reject the null hypothesis and it is concluded that INF series is not stationary at the zero frequency. On the other hand, Since KPSS test statistics calculated at both constant model and constant-trend model in Table 19 and Table 20 have been found to be significant (calculated KPSS statistics lie inside the critical region) when compared to 1%, 5% and 10% critical values, the null hypothesis saying that the original series is stationary has been rejected and it has been concluded that INF series is not stationary at the zero frequency.

Now, in order to show that the series should be first-degree differenced, let us test the first-degree difference of the series (In this case, the null hypothesis for KPSS will be the stationarity of the first-differenced series rather than the original series and the null for ADF will be non-stationarity of the first-differenced series):

Table 21
ADF Test Results for INF Series in First-Difference Form with Constant Added

	***t*-statistic**	**Prob.***	**Critical Values**		
			1%	**5%**	**10%**
ADF Test Statistic	-9.402136	0.0000	-3.458719	-2.873918	-2.573443

Note. [1] Lag length has been chosen as 10 amongst max.lag=12 (based on SIC)
[2] * denotes MacKinnon (1996) one-sided p-values

Table 22
ADF Test Results for INF Series in First-Difference Form with Constant and Linear Trend Added

	***t*-statistic**	**Prob.***	**Critical Values**		
			1%	**5%**	**10%**
ADF Test Statistic	-9.455364	0.0000	-3.998457	-3.429484	-3.138243

Note. [1] Lag length has been chosen as 10 amongst max.lag=12 (based on SIC)
[2] * denotes MacKinnon (1996) one-sided p-values

According to ADF test results with both "constant" added and "constant and linear trend" added in Table 21 and Table 22, the null hypotheses of nonstationarity of the first differenced series are rejected at 1%, 5% and 10% significance levels and therefore meaning that first-difference of the series is stationary.

Table 23
KPSS Test Results for INF Series in First-Difference Form with Constant Added

	LM-Stat.	Asymptotic Critical Values*		
		1%	**5%**	**10%**
Kwiatkowski-Phillips-Schmidt-Shin Test Statistic	0.154848	0.739000	0.463000	0.347000

Note. * Kwiatkowski-Phillips-Schmidt-Shin (1992).

Table 24
KPSS Test Results for INF Series in First-Difference Form with Constant and Linear Trend Added

	LM-Stat.	Asymptotic Critical Values*		
		1%	**5%**	**10%**
Kwiatkowski-Phillips-Schmidt-Shin Test Statistic	0.118458	0.216000	0.146000	0.119000

Note. * Kwiatkowski-Phillips-Schmidt-Shin (1992).

According to KPSS test results with both "constant" added and "constant and linear trend" added in Table 23 and Table 24, the null hypotheses of stationarity of the first differenced series cannot be rejected at 1%, 5% and 10% significance levels and therefore we conclude that to make the series stationary, the series should be in (non-seasonal) first-differenced form.

Another simple method in order to determine the optimal order of differencing comes from Box-Jenkins rule of thumb: The optimum order of differencing is the one with the smallest standard deviation (Akuffo & Ampaw, 2013, p. 15). In order to detect the optimal order, standard deviations corresponding to different orders of differencing are given in Table 25:

Table 25
Standard Deviations for Detecting the Optimal Order of Differencing by Box-Jenkins Rule of Thumb

Order of Differencing	Non	First	Second	Third
Standard Deviations	2.243578	**1.492247**	2.274733	3.806330

Hence, the minimum standard deviation is realized in first-degree differenced form with a value of 1.492247. Hence, once gain we have verified the optimum order as 1.

Now after the orders of seasonal and non-seasonal differences are determined in order to satisfy the stationarity condition of original series (since the series should be stationary for SARIMA modelling), we should determine AR, SAR, MA and SMA (seasonal moving average) orders to construct the best model.

In the model identification, possible best models have been tried to be discovered by “auto.arima” function in “forecast” package of R software. The method for selecting the best-fitted model is based on choosing AIC, AICc (Corrected Akaike Information Criterion) and BIC with minimum values. Mostly, the model that provides minimum AIC (or AICc) rather than BIC is a candidate to be selected as the best-fitted one. In Table 26, suggested ARIMA models by utilizing from OCSB and ADF tests have been presented with AICc and AIC information criteria given:

Table 26

AICc and AIC Values for Suggested ARIMA Models of INF Series by Using Stepwise Selection

Suggested ARIMA models	**AICc**	**AIC**
ARIMA(2,1,2)(1,0,1)[12] with drift	Inf	Inf
ARIMA(0,1,0) with drift	2560.113	2560.063
ARIMA(1,1,0)(1,0,0)[12] with drift	2494.328	2494.158
ARIMA(0,1,1)(0,0,1)[12] with drift	2466.34	2466.17
ARIMA(0,1,0)	2558.086	2558.069
ARIMA(0,1,1)(1,0,1)[12] with drift	Inf	Inf
ARIMA(0,1,1) with drift	2495.07	2494.969
ARIMA(0,1,1)(0,0,2)[12] with drift	2449.736	2449.481
ARIMA(1,1,1)(0,0,2)[12] with drift	2440.443	2440.084
ARIMA(1,1,0)(0,0,2)[12] with drift	2505.766	2505.511
ARIMA(1,1,2)(0,0,2)[12] with drift	2441.56	2441.079
ARIMA(0,1,0)(0,0,2)[12] with drift	2532.184	2532.015
ARIMA(2,1,2)(0,0,2)[12] with drift	2444.814	2444.194
ARIMA(1,1,1)(0,0,2)[12]	2440.654	2440.398
ARIMA(1,1,1)(1,0,2)[12] with drift	**2405.964**	**2405.484**
ARIMA(1,1,1)(1,0,1)[12] with drift	Inf	Inf
ARIMA(1,1,1)(0,0,1)[12] with drift	2453.309	2453.054
ARIMA(0,1,1)(1,0,2)[12] with drift	Inf	Inf
ARIMA(2,1,1)(1,0,2)[12] with drift	Inf	Inf
ARIMA(1,1,0)(1,0,2)[12] with drift	Inf	Inf
ARIMA(1,1,2)(1,0,2)[12] with drift	Inf	Inf
ARIMA(0,1,0)(1,0,2)[12] with drift	2498.705	2498.449
ARIMA(2,1,2)(1,0,2)[12] with drift	Inf	Inf
ARIMA(1,1,1)(1,0,2)[12]	Inf	Inf
ARIMA(1,1,1)(2,0,2)[12] with drift	Inf	Inf

As shown in Table 26, the best model under the stepwise-selection method among other models has been chosen as ARIMA(1,1,1)(1,0,2)[12] model with drift with the smallest AICc value 2405.964 and the smallest AIC value 2405.484. All the other models which have greater AIC values have been provided only for comparison purposes. After selecting the best model based on AIC and AICc, we need to estimate the significance of parameters:

Table 27

Estimates of Parameters for ARIMA (1,1,1)(1,0,2)[12] Model with Drift

	AR(1)	**MA(1)**	**SAR(1)**	**SMA(1)**	**SMA(2)**	**DRIFT**
Estimate	0.1750	-0.8857	0.8862	-0.7102	0.1813	-0.9323
Standard Error	0.0763	0.0375	0.0537	0.0847	0.0746	1.3789

Sigma^2 estimated: 1233 log likelihood: -1194.59 AIC: 2405.48 **AICc: 2405.96** BIC: 2429.88

As clearly seen in Table 27, the coefficients of *ARIMA (1,1,1)(1,0,2)[12] Model with Drift* are significantly different from zero.

Table 28

BIC Values for Suggested ARIMA Models of INF Series by Using Stepwise Selection

Suggested ARIMA models	**BIC**
ARIMA(2,1,2)(1,0,1)[12] with drift	Inf
ARIMA(0,1,0) with drift	2567.032
ARIMA(1,1,0)(1,0,0)[12] with drift	2508.098
ARIMA(0,1,1)(0,0,1)[12] with drift	2480.11
ARIMA(0,1,0)	2561.554
ARIMA(0,1,1)(1,0,1)[12] with drift	Inf
ARIMA(0,1,1) with drift	2505.423
ARIMA(0,1,1)(0,0,2)[12] with drift	2466.905
ARIMA(1,1,1)(0,0,2)[12] with drift	2460.993
ARIMA(1,1,0)(0,0,2)[12] with drift	2522.935
ARIMA(1,1,2)(0,0,2)[12] with drift	2465.473
ARIMA(0,1,0)(0,0,2)[12] with drift	2545.954
ARIMA(2,1,2)(0,0,2)[12] with drift	2472.072
ARIMA(1,1,1)(0,0,2)[12]	**2457.822**
ARIMA(1,1,1)(1,0,2)[12]	Inf
ARIMA(1,1,1)(0,0,1)[12]	2467.839
ARIMA(0,1,1)(0,0,2)[12]	2462.758
ARIMA(2,1,1)(0,0,2)[12]	2464.79
ARIMA(1,1,0)(0,0,2)[12]	2517.457
ARIMA(1,1,2)(0,0,2)[12]	2462.153
ARIMA(0,1,0)(0,0,2)[12]	2540.471
ARIMA(2,1,2)(0,0,2)[12]	2469.057

Table 28 presents BIC values for each suggested ARIMA model. If we take only BIC into account, the best model is seen to be ARIMA(1,1,1)(0,0,2)[12] model with a minimum value of 2457.822. The estimates of parameters of ARIMA(1,1,1)(0,0,2)[12] model are given in Table 29:

Table 29
Estimates of Parameters for ARIMA (1,1,1)(0,0,2)[12] Model

	AR(1)	MA(1)	SMA(1)	SMA(2)
Estimate	0.2412	-0.9183	0.2685	0.2295
Standard Error	0.0701	0.0249	0.0690	0.0569

Sigma^2: 1435 log-likelihood: -1219.39 AIC: 2448.78 AICc: 2449.03 BIC: 2466.2

If ARIMA(1,1,1)(1,0,2)[12] model with drift chosen by AIC (or AICc) in Table 26 and ARIMA(1,1,1)(0,0,2)[12] model chosen by BIC in Table 28 are compared, ARIMA(1,1,1)(1,0,2)[12] model with drift is chosen because of having smaller information criteria.

For selecting the best-fitted model (to find out how well the model fits the data), we need to continue with the examination of residuals diagnostics (or Diagnostic Checking) in order to find out whether the residuals display a White noise process which is a vital assumption of a good ARIMA model (zero mean, constant variance, no serial correlation). In this stage, first we will have a look at Box-Ljung Test results in order to make sure about residuals have no remaining autocorrelation. The null and alternative hypotheses are given respectively as follows:

H_0 : The residuals are random (independently distributed)

H_1 : The residuals are not random (not independently distributed, displaying serial correlation)

Table 30
Box-Ljung Test Results of ARIMA(1,1,1)(1,0,2)[12] Model with Drift at Seasonal Lags

Seasonal Lags	X-squared Statistics	p-value
12	10.6567	0.1543
24	21.996	0.2845
36	30.6726	0.4828
48	39.8145	0.6102

Table 30 presents the autocorrelation check results for the residuals of ARIMA(1,1,1)(1,0,2)[12] with drift model at seasonal lags and according to given results, we cannot reject the null hypothesis saying that residuals are independent and hence conclude about the absence of autocorrelation problem depending on the statistically insignificant chi-squared statistics (since p-values for Box-Ljung statistic are greater than 5% significance level for all seasonal lags 12,24,36,48). Therefore, this model can be said to fit the data well. This result is also verified by looking at the correlogram of residuals shown in Figure 10. All acf and pacf values in Figure 10 are within the significance limits and mean of the residuals seem to be randomly distributed around zero. Thus, the residuals appear to be White noise.

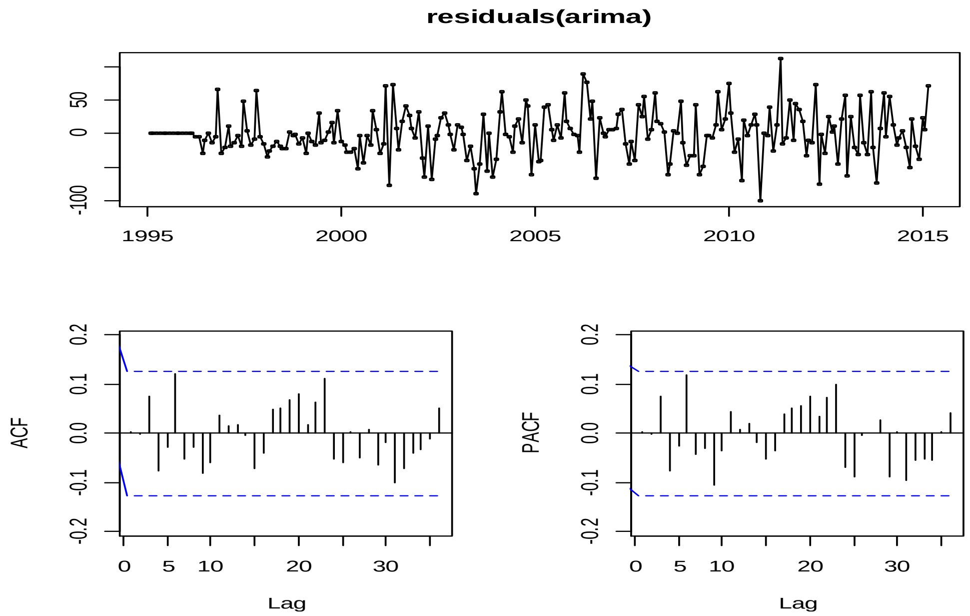


Figure 10. ACF and PACF plots of the residuals of ARIMA(1,1,1)(1,0,2)[12] model with drift

Now let us check the normality of ARIMA(1,1,1)(1,0,2)[12] model with drift residuals.

Table 31

Jarque-Bera Normality Test Results of ARIMA(1,1,1)(1,0,2)[12] Model with Drift

X-squared Statistic	**Asymptotic p-value**
3.2092	**0.201**

Table 31 shows the Jarque-Bera Test Results. As well known, the null hypothesis for the test is that residuals are normally distributed and the alternative hypothesis is that residuals are not normally distributed. Insignificant X-squared statistic in Table 31 with an asymptotic p-value of 0.201 that is greater than 5% significance level reveals that the null hypothesis cannot be rejected concluding that residuals are normally distributed.

Table 32
ARCH-LM Test Results of ARIMA(1,1,1)(1,0,2)[12] Model with Drift

Chi-squared	**p-Value**
14.7563	**0.255**

After checking the normality assumption, now ARCH-LM (Autoregressive Conditional Heteroscedasticity-Lagrange Multiplier) test results are presented in Table 32 to find out if there is a heteroscedasticity problem. For this test, the null hypothesis says that there are no ARCH (Autoregressive Conditional Heteroscedasticity) effects (indicating to the constant variance). From ARCH-LM test results with the number of lags chosen as 12, it can be inferred that since p-value (0.255) exceeds 5% significance level, the null hypothesis of no ARCH effect (homoscedasticity) in the residuals of ARIMA(1,1,1)(1,0,2)[12] with drift model cannot be rejected and therefore concluding that the residuals of ARIMA(1,1,1)(1,0,2)[12] with drift model are homoscedastic (that is, the residuals have constant variance). Briefly, it can be said that all assumptions regarding diagnostic checking (no serial correlation, normality of residuals, constant variance) hold for this model.

Table 33
Forecast Accuracy Measures for ARIMA(1,1,1)(1,0,2)[12] Model with Drift

ME	**RMSE**	**MAE**	**MPE**	**MAPE**	**MASE**
-0.3333779	34.08106	25.34299	-49.62708	70.50063	0.73495

Note. ME: Mean Error
RMSE: Root Mean Squared Error
MAE: Mean Absolute Error
MPE: Mean Percentage Error
MAPE: Mean Absolute Percentage Error
MASE: Mean Absolute Scaled Error
(For more information about the accuracy measures, see Ord & Fildes, 2013, chap. 2).

In Table 33, various forecast accuracy measures for ARIMA(1,1,1)(1,0,2)[12] with drift model that is chosen under the stepwise-selection method have been presented. Afterwards, these results will be compared to the model that will be chosen under the non-stepwise selection method.

Subsequent to applying (faster) stepwise-selection method which provides a short-cut for selecting the best-fitted model, now let us try the same thing under the (slower) non-stepwise selection method which searches for all possible models. By benefiting from “auto.arima” function in “forecast” package of R as previously, the best choice under the nonstepwise-selection method has been determined to be ARIMA(1,1,1)(2,0,0)[12] with drift model for inflation series. The estimates of parameters of this new model are given in Table 34:

Table 34
Estimates of Parameters for ARIMA (1,1,1)(2,0,0)[12] Model with Drift

	AR(1)	MA(1)	SAR(1)	SAR(2)	DRIFT
Estimate	0.2202	-0.9273	0.2961	0.3136	-0.4393
Standard Error	0.0752	0.0336	0.0610	0.0633	0.5195
Sigma^2: 1270 log-likelihood: -1200.75 AIC: 2413.51 AICc: 2413.87 BIC: 2434.42					

As it is apparent in Table 34, the coefficients of *ARIMA (1,1,1)(2,0,0)[12] Model with Drift* are seen to be significant.

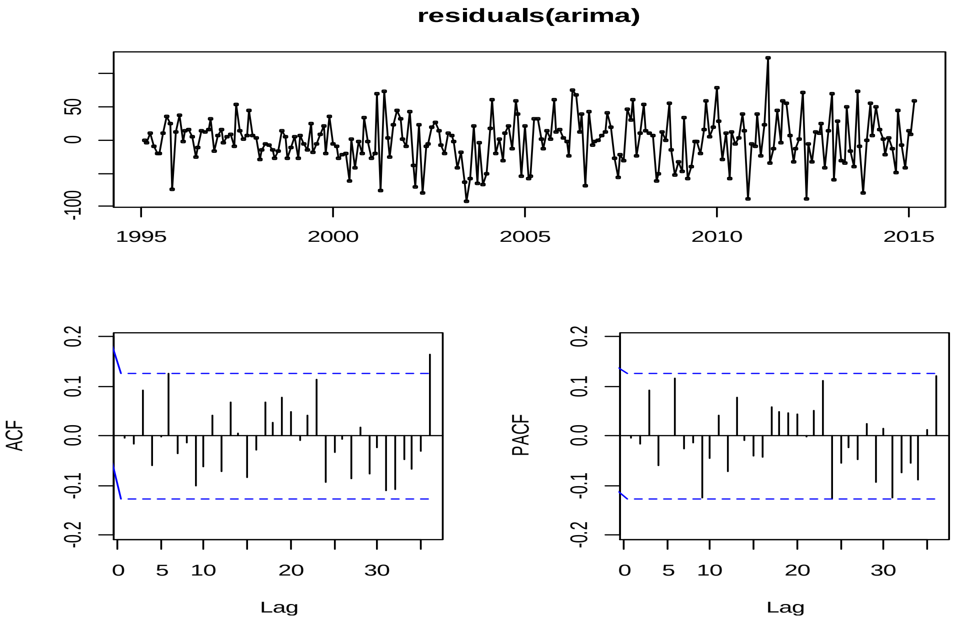


Figure 11. ACF and PACF plots of the residuals of ARIMA(1,1,1)(2,0,0)[12] model with drift

When looked at Figure 11, mean of the residuals of ARIMA(1,1,1)(2,0,0)[12] model with drift is seen to be distributed around zero. However, acf and pacf values are within the significance limits only up to 12 and 24 seasonal lags. Even though the absence of autocorrelation at seasonal lag 12 is sufficient to make a positive inference about no serially correlated residuals (since we are dealing with monthly inflation rates in which the length of seasonal period is 12), a spike is realized at 36th lag and therefore not all acf values are seen to take place within the significance limits because of this 36th lag. If ARIMA(1,1,1)(2,0,0)[12] model with drift is compared to ARIMA (1,1,1)(1,0,2)[12] model with drift that does not enable such a spike at 36th lag apart from other seasonal lags as observed in Figure 10, the latter (with stepwise-selection method) can be said to be a stronger model than the former (with non-stepwise selection method). Let us verify this with an examination on Box-Ljung test statistics at seasonal lags:

Table 35
Box-Ljung Test Results of ARIMA(1,1,1)(2,0,0)[12] Model with Drift at Seasonal Lags Based on the Non-stepwise Selection

Seasonal Lags	X-squared Statistics	p-value
12	12.6478	0.1246
24	25.7961	0.1727
36	46.7037	**0.04507**
48	58.2202	0.07392

Table 35 presents the autocorrelation check results for the residuals of ARIMA(1,1,1)(2,0,0)[12] with drift model at seasonal lags based on the non-stepwise selection. According to both the plot of ACF in Figure 11 and Table 35 results, no serial correlation has been detected except 36th lag with a probability value (p-value) of 0.04507 which is smaller than 5% significance level. Therefore p-values for Box-Ljung statistics at seasonal lags 12, 24, 48 are greater than 5% significance level indicating to the non-rejection of the null hypothesis of independently distributed residuals at these seasonal lags. Only 36th lag creates serially correlated residuals depending on the rejection of the null. Now let us check the normality of ARIMA(1,1,1)(2,0,0)[12] model with drift residuals:

Table 36
Jarque-Bera Normality Test Results of ARIMA(1,1,1)(2,0,0)[12] Model with Drift

X-squared Statistic	Asymptotic p-value
1.0074	**0.6043**

According to the Jarque-Bera test results given in Table 36, we fail to reject the null hypothesis saying that the residuals are normally distributed with an insignificant X-squared statistic having an asymptotic p-value of 0.6043 that is greater than 5% significance level.

Table 37

ARCH-LM Test Results of ARIMA(1,1,1)(2,0,0)[12] Model with Drift

Chi-squared	**p-Value**
15.6521	**0.2077**

From the ARCH-LM test results, it can be inferred that the null hypothesis of no ARCH effect (homoscedasticity) in the residuals of ARIMA(1,1,1)(2,0,0)[12] model with drift cannot be rejected and hence the residuals of this model are said to be homoscedastic. Briefly, all assumptions regarding normality of residuals, and constant variance hold for this model except autocorrelation check for 36th lag. Residuals of ARIMA(1,1,1)(2,0,0)[12] model with drift are independently distributed up to seasonal lags 12 and 24, however not independently distributed for seasonal lag 36.

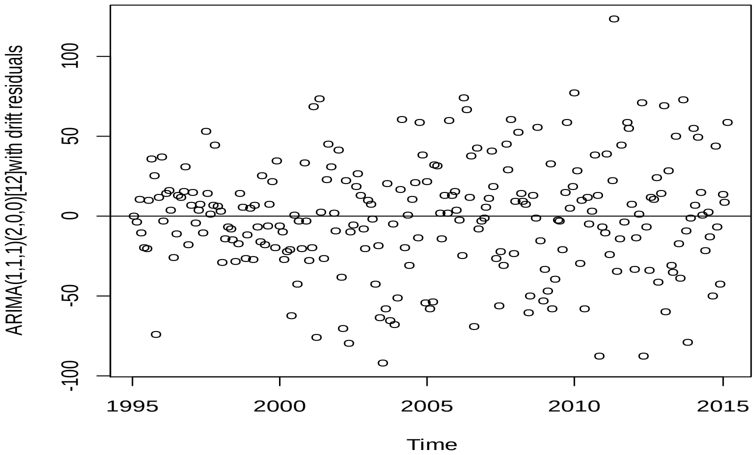


Figure 12. Plot of ARIMA(1,1,1)(2,0,0)[12] with drift residuals against time

Table 38

Forecast Accuracy Measures for ARIMA(1,1,1)(2,0,0)[12] Model with Drift

ME	**RMSE**	**MAE**	**MPE**	**MAPE**	**MASE**
-0.3060675	35.49517	27.12352	-60.00625	80.78677	0.7865855

Note. For more information about the accuracy measures, see Ord & Fildes, 2013, chap. 2.

In Table 38, forecast accuracy measures for ARIMA(1,1,1)(2,0,0)[12] with drift model that is based on the non-stepwise selection method have been presented.

Now that we have identified two models based on both stepwise and non-stepwise selection, we can provide a summary of final results: In this application, ARIMA(1,1,1)(1,0,2)[12] with drift model chosen by using (faster) stepwise selection method and ARIMA(1,1,1)(2,0,0)[12] with drift model chosen by using (slower) non-stepwise selection which seeks for all possible models have been compared. Although we expect the latter model with non-stepwise selection to be better (since, stepwise selection offers short-cuts in selecting the best model), the results have showed that the former model with stepwise-selection is better as the best-fitted SARIMA model. A summary of the comparison of both models are given in Table 39:

Table 39
Comparison of ARIMA(1,1,1)(1,0,2)[12] with Drift and ARIMA(1,1,1)(2,0,0)[12] with Drift Models

Models	**Accuracy Measures**	**Significancy of Coefficients**	**AICc**	**Normality**	**ARCH-LM**	**ACF of Residuals (Autocorrelation check for residuals)**
Model 1	RMSE: 34.08106 MAE: 25.34299 MAPE: 70.50063 MASE: 0.73495	All seasonal and non-seasonal AR and MA coefficients are significant.	2405.96	ok	ok	There is no spike (no autocorrelation at all seasonal lags 12,24,36,48.)
Model 2	RMSE: 35.49517 MAE: 27.12352 MAPE: 80.78677 MASE: 0.7865855	All seasonal and non-seasonal AR and MA coefficients are significant.	2413.87	ok	ok	There is a spike at 36^{th} lag (autocorrelation problem exists at 36^{th} lag).

Note. Model 1 represents ARIMA(1,1,1)(1,0,2)[12] with Drift.
Model 2 represents ARIMA(1,1,1)(2,0,0)[12] with Drift.

As seen in Table 39, forecast accuracy measures of model 1 are smaller than the ones of model 2. In the light of given information, it is possible to say that model 1 satisfies all the necessary assumptions (no serial correlation, constant variance and normality) and is better in all respects than model 2 with the smallest AICc, significant parameters, no spike at ACF etc. Therefore having satisfied all the model assumptions, model 1 can be regarded as the best-fitted model for forecasting monthly inflation rates in Turkish economy.

In order to verify once again that model 1 is the best model for forecasting, we can utilize from Diebold-Mariano test for predictive accuracy.

Table 40

Diebold-Mariano Test Results of ARIMA(1,1,1)(1,0,2)[12] with Drift and ARIMA(1,1,1)(2,0,0)[12] with Drift Models for Predictive Accuracy

Diebold-Mariano Test for Model 1 ARIMA(1,1,1)(1,0,2)[12] with drift (stepwise) and Model 2 ARIMA(1,1,1)(2,0,0)[12] with drift (non-stepwise)		
Tested Alternative Hypotheses	**DM Statistic**	**p-value**
Two-sided	-2.0771	0.03885
Less	-2.0771	0.01943
Greater	-2.0771	0.9806

The Diebold-Mariano test provides a comparison of the forecast accuracy of two forecast methods by using forecast errors from two models in interest. This test is tested using three different alternative hypotheses with the null hypothesis saying that the two methods have the same forecast accuracy. The alternative “less” says that method 2 is less accurate than method 1. The alternative “greater” says that method 2 is more accurate than method 1. The alternative “two-sided” says that method 1 and method 2 have different levels of accuracy (this alternative is expressed as the default hypothesis among others) (Hyndman, 2015, pp. 20-21).

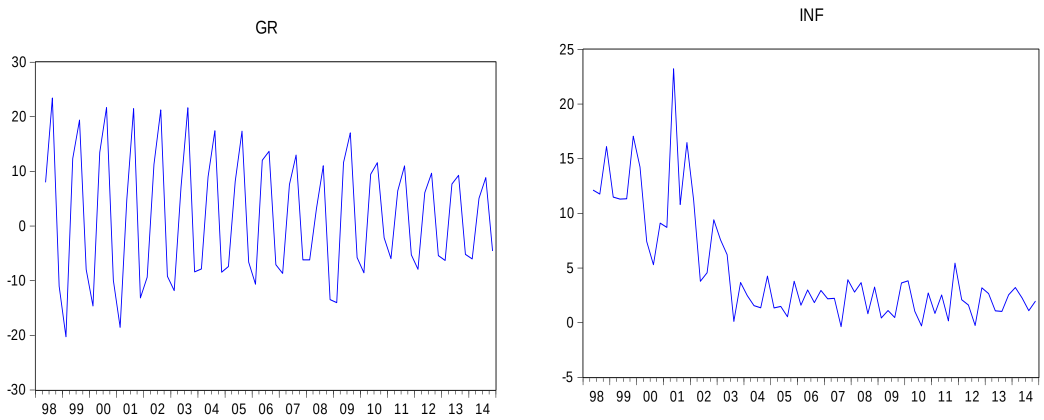
According to the results of two-sided alternative hypothesis in Table 40, it is clear to see that the null hypothesis is easily rejected at 5% significance level when looked at the prob value of 0.03885. Thus, method 1 and method 2 are considered to have different levels of accuracy. According to the results of the alternative hypothesis “less”, it is seen once again that the null hypothesis is rejected at 5% level with a p-value of 0.01943 concluding that method 2 is less accurate than method 1 (in other saying, method 1 - ARIMA(1,1,1)(1,0,2)[12] with drift (stepwise)- is more accurate). On the other hand, the results of the alternative hypothesis “greater” say that the null hypothesis fails to be rejected at 5% level with a p-value of 0.9806. The evaluation of the results given by the tested hypotheses reveals that except the alternative “greater”, two methods have different accuracy levels and method 1 (ARIMA(1,1,1)(1,0,2)[12] with drift (stepwise)) is more accurate than method 2 (ARIMA(1,1,1)(2,0,0)[12] with drift (non-stepwise)) (Apart from all necessary checks mentioned in this application, a good SARIMA model should also satisfy causality, stationarity and invertibility conditions. Of course, our ARIMA(1,1,1)(1,0,2)[12] with drift (stepwise) model satisfies these conditions. About this, see Appendix D).

6.2. Seasonal Integration Tests and a Different Look at Cointegration Relationship between Quarterly Inflation Rates and Growth

Inflation is one of the most important facts in our daily life referring to a sustained increase in consumer prices and it can be measured through CPI, producer price index (PPI) or GDP deflator. However, it is generally measured as a change in the harmonized index of consumer prices (HICP) that has been harmonized across all European Union member states. Holmes (2014) has presented the definition of HICP as “The HICP is the measure of inflation which the governing council uses to define and assess price stability in the Euro area as a whole in quantitave terms.” (p.16).

In this part, first seasonal integration tests will be applied in a unified approach for inflation rates and growth variables and after determining the seasonal integration orders of these variables, the cointegration relationship between them will be investigated. Inflation data have been derived through $INF = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}}.100$ as in section 6.1 and real gdp growth rates have been obtained by $GR = \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}.100$ transformation where INF denotes inflation rate, CPI_t denotes consumer price index at time *t* and CPI_{t-1} denotes consumer price index at time *t*-1, GR denotes real gdp growth rate and GDP denotes real GDP. For deriving inflation data, we have utilized from quarterly HICP data (with Index 2010=100) as CPI for Turkey and HICP data have been obtained from Organization for Economic Co-operation and Development. On the other hand, GDP data have been collected from Central Bank of the Republic of Turkey (CBRT). The separate graphs of GR and INF variables have been given in Figure 13. It is likely from the graphs to see the seasonal patterns clearly. In addition, in Figure 14 these two variables have been presented in the same graph in terms of giving a clue about their cointegrating relations. Since it is seen that they are moving together in the graph, they are highly possible to be cointegrated.

In this application, seasonal integration tests will be applied for quarterly data on the real gdp growth rates and inflation over 1998q1:2014q4 period by taking the study of Ilmakunnas (1990) as basis. When looked at the graphs in Figure 13, it is apparent to see the seasonal behaviours of both INF and GR variables. In ADF and HEGY test applications, constant term and seasonal dummies have been included in the regressions to be applied and seasonal means have not been removed in DHF and OCSB tests.



(a) (b)

Figure 13. Graphs of quarterly growth rates (a) and inflation rates (b) against time over 1998Q1-2014Q4 period

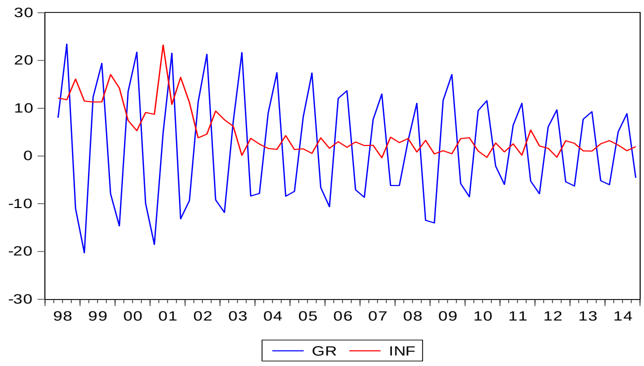


Figure 14. Graph of quarterly growth rates and inflation rates together

As seen in Figure 14, a decrease (increase) in gdp growth is generally matched by a corresponding increase (decrease) in inflation. As clear from graph (a) in Figure 13, seasonal movements in growth series are very marked and inflation series in graph (b) also displays some seasonal pattern. Depending on the clear seasonal patterns of these two series, we can recourse to seasonal differencing procedure in order to capture such patterns. Because two series have quarterly frequency, seasonally differenced variables have been obtained by using $(1-L^4)$ operator. Therefore our transformed series that will be called D4INF and D4GR respectively for inflation and growth can be expressed as $D4INF = INF_t - INF_{t-4}$ and $D4GR = GR_t - GR_{t-4}$. As a result of these transformations, D4INF and D4GR variables which are seasonally integrated of order $\mathrm{SI}(1,1)$ (or integrated of order $I(0,1)$) have been graphed together in Figure 15 (in order to see the difference between SI(d,D) and I(d,D), see subsection 5.2):

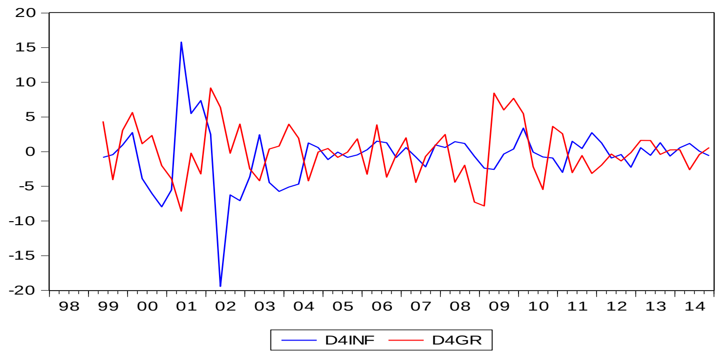


Figure 15. Graph of seasonally differenced growth rates and inflation rates together

Figure 15 also shows that these seasonally differenced two series are moving together, but at an opposite direction. Thus, it supports the idea that they seem to be cointegrated.

Table 41
Seasonal Integration Test Results for Inflation and Growth Series

Test	**Test Statistic for Variable**			
	GR	**Lag Length (p)**	**INF**	**Lag Length (p)**
ADF	-3.689032	4	-1.553273 * ** ***	4
ADF for Δ Series	-5.028365	7	-7.782215	3
ADF for Δ_4 Series	-6.711397	3	-2.826763 * **	4
ADF for $S(L)$ Series	-3.751048	1	-1.981335 * ** ***	5
DHF	-4.801168	1	-3.146821 * ** ***	5
DHF for Δ Series	-2.539052 * ** ***	5	-5.068007	9

Table 41
(Continued)

Test		Test Statistic for Variable			
		GR	Lag Length (p)	INF	Lag Length (p)
HEGY	π_1	-3.689032		-2.412879 * ** ***	
	π_2	-2.104948 * ** ***		-2.797232 * **	
	π_3	-3.082795 * ** ***	$p = 1$	-6.117571	$p = 3$
	π_4	-0.287342 * ** ***		-2.678279 *	
	$\pi_3 \cap \pi_4$	4.804014 * ** ***		22.65265	
HEGY (with $\pi_1 = 0$)	π_2	-1.916769 * ** ***		-5.872822	
	π_3	-1.491104 * ** ***	$p = 5$	-7.031611	$p = 1$
	π_4	-0.561135 * ** ***		-5.095386	
	$\pi_3 \cap \pi_4$	1.273491 * ** ***		37.99593	

Table 41 (*Continued*)

Test		**Test Statistic for Variable**			
		GR	**Lag Length (p)**	**INF**	**Lag Length (p)**
HEGY (with $\pi_2 = \pi_3 = \pi_4 = 0$)	π_1	*see ADF for $S(L)$ Series*			
HEGY for Δ Series (with $\pi_1 = 0$)	π_2	-1.081247 * ** ***		-2.582364 * ** ***	
	π_3	-1.126513 * ** ***	$p = 8$	-5.468436	$p = 4$
	π_4	0.980675 * ** ***		2.412156 * ** ***	
	$\pi_3 \cap \pi_4$	1.148816 * ** ***		20.04481	
HEGY for Δ Series (with $\pi_2 = \pi_3 = \pi_4 = 0$)	π_1	*see ADF for Δ_4 Series*			
HEGY for Δ Series	π_1	-5.028365		-7.782215	
	π_2	-1.916769 * ** ***		-5.872822	
	π_3	-1.447264 * ** ***	$p = 4$	-8.504117	$p = 0$
	π_4	0.639263 * ** ***		0.814304 * ** ***	
	$\pi_3 \cap \pi_4$	1.273491 * ** ***		37.99593	

Table 41 (*Continued*)

Test		Test Statistic for Variable			
		GR	Lag Length (p)	INF	Lag Length (p)
OCSB	β_1	-5.553677	$p=3$	-3.604473	$p=0$
	β_2	-1.798949 * **		-10.06257	
OCSB (with $\beta_1=0$)	β_2	*see DHF for* Δ *Series*			
OCSB (with $\beta_2=0$)	β_1	*see ADF for* Δ_4 *Series*			

Note: * denotes insignificant values at 1% significance level
** denotes insignificant values at 5% significance level
*** denotes insignificant values at 10% significance level.

Table 41 presents the results of different seasonal integration tests in order to decide about integration orders of both INF and GR variables. In this application, the selection of lags (p) has been made in a way not to have autocorrelation and heteroscedasticity problems apart from the examination of correlogram of residuals. First, it is necessary to choose appropriate integration orders for inflation and growth by utilizing from the given information in Table 41. In Table 41, the column GR presents the estimates of growth variable and the column for INF gives the estimates for inflation variable under the different regression models. The null and alternative hypotheses corresponding to different models have been mentioned in Table 5. Therefore, we have three (null) hypotheses that will be used as the starting point of testing sequence: starting point may be either SI(2,1), SI(1,1) or SI(1,0). As a conclusion of a thorough evaluation on Table 41, the results of these three cases are given in Table 42 along with the accepted hypotheses shown in bold type.

Table 42

A Summary of Null and Alternative Hypotheses to be Used under the Starting Points SI(2,1), SI(1,1) or SI(1,0)

Case 1: If the starting point is SI(2,1)	**Null Hypothesis**	**Alternative Hypothesis**
ADF for Δ_4	SI(2,1)	SI(1,1)
DHF for Δ	SI(2,1)	SI(1,0)
HEGY for Δ**:**		
while $\pi_1 = 0$, π_2, π_3, π_4 tested	SI(2,1)	SI(2,0)
$\pi_1, \pi_2, \pi_3, \pi_4$ tested	SI(2,1)	SI(1,0)
while $\pi_2 = \pi_3 = \pi_4 = 0$, π_1 tested	SI(2,1)	SI(1,1)
Case 2: If the starting point is SI(1,1)	**Null Hypothesis**	**Alternative Hypothesis**
ADF for $S(L)$	SI(1,1)	SI(0,1)
DHF	SI(1,1)	SI(0,0)
HEGY:		
while $\pi_1 = 0$, π_2, π_3, π_4 tested	SI(1,1)	SI(1,0)
$\pi_1, \pi_2, \pi_3, \pi_4$ tested	SI(1,1)	SI(0,0)
while $\pi_2 = \pi_3 = \pi_4 = 0$, π_1 tested	SI(1,1)	SI(0,1)
OCSB:		
$\beta_1 \neq 0$, β_2 tested	SI(1,1)	SI(0,0)
Case 3: If SI(1,0) is tested	**Null Hypothesis**	**Alternative Hypothesis**
ADF	SI(1,0)	SI(0,0)
HEGY:		
while $\pi_2, \pi_3, \pi_4 \neq 0$, π_1 tested	SI(1,0)	SI(0,0)
OCSB:		
while $\beta_2 \neq 0$, β_1 tested	SI(1,0)	SI(0,0)

Table 43

Accepted Hypotheses in Seasonal Integration Tests for INF and GR series

Case 1: If the starting point is SI(2,1),	**GR(growth)**	**INF (inflation)**
ADF for Δ_4	SI(1,1)	**SI(2,1)** may be accepted for 1% and %5 levels (and SI(1,1) may be accepted for 10% level).
DHF for Δ	**SI(2,1)**	SI(1,0)
HEGY for Δ:		
while $\pi_1 = 0$, iken π_2, π_3, π_4 tested	**SI(2,1)**	**SI(2,1)** can be accepted because of the presence of unit roots at π_2 and π_4.
$\pi_1, \pi_2, \pi_3, \pi_4$ tested	**SI(2,1)**	SI(1,0) may be accepted since there is no biannual and annual unit roots.
while $\pi_2 = \pi_3 = \pi_4 = 0$, π_1 tested	SI(1,1)	(See *ADF for* Δ_4)

*The results of the case "while $\pi_2 = \pi_3 = \pi_4 = 0$, π_1 tested" in HEGY test for Δ are the same as ADF for Δ_4 results. The results for two series are not certain if the starting point is SI(2,1). However in most cases the hypothesis SI(2,1) cannot be rejected for growth series and inflation series may be accepted as either SI(2,1) or SI(1,0).

Case 2: If the starting point is SI(1,1),	**GR(growth)**	**INF (inflation)**
ADF for $S(L)$	SI(0,1)	**SI(1,1)**
DHF	SI(0,0)	**SI(1,1)**
HEGY:		
while $\pi_1 = 0$, π_2, π_3, π_4 tested	**SI(1,1)**	SI(1,0)
$\pi_1, \pi_2, \pi_3, \pi_4$ tested	**SI(1,1)**	For %1 level, **SI(1,1)** may be accepted.
while $\pi_2 = \pi_3 = \pi_4 = 0$, π_1 tested	SI(0,1)	**SI(1,1)**
OCSB: $\beta_1 \neq 0$, β_2 tested	**SI(1,1)**	SI(0,0)

*The results of the case "while $\pi_2 = \pi_3 = \pi_4 = 0$, π_1 tested" in HEGY test are the same as ADF for $S(L)$ results. As it is seen obviously, the result of two variables may be in the form of SI(1,1) dominates.

Case 3: If SI(1,0) is tested,	**GR(growth)**	**INF (inflation)**
ADF	SI(0,0)	SI(1,0)
HEGY:		
$\pi_2, \pi_3, \pi_4 \neq 0$ iken π_1 tested	SI(0,0)	SI(1,0)
OCSB: $\beta_2 \neq 0$ iken β_1 tested	SI(0,0)	SI(0,0)

Note. [1] Bold expressions have been used to highlight mostly accepted hypotheses under the starting point in interest.

[2] For 1998Q1-2014Q4 period (that is, 68 observations), in most cases, N=100 (observations) has been taken as basis in critical values tables.

[3] ADF critical values have been considered as -3.51 for 1%, -2.89 for 5% and -2.58 for 10% significance level for the model with constant and no trend (N=100) (Critical values have been cited from Fuller (1976, p. 373)).

[4] DHF critical values have been cited from table 7 (percentiles, the studentized statistic for the seasonal means model) in Dickey et al.(1984, p. 362). For quarterly data, d has been considered as 4 and for DHF test, n=md (total number of observations) has been taken as 80 (seasonal means have not been removed). Percentiles of the studentized statistic for the seasonal means model are given as: -4.78 for 1%, -4.11 for 5% and -3.78 for 10%.

[5] Critical values have been obtained from Osborn et al. (1988, p. 376) for OCSB test (with no seasonal mean subtraction).

[6] Critical values for HEGY test have been taken from Hylleberg et al. (1990, pp. 226-227) for the model with intercept and seasonal dummies. See Appendix E.

Table 43 presents the accepted hypotheses of growth and inflation variables under the different forms of ADF, DHF, HEGY and OCSB tests. The second “GR” column gives the accepted hypotheses for this variable under the given tests and third column “INF” presents the accepted hypotheses for this variable under the given tests. In addition, the mostly accepted hypotheses for two variables when they are considered together are shown in bold type in Table 43 so that if the starting point is SI(2,1), mostly SI(2,1) has been accepted for two variables and if the starting point is SI(1,1), mostly SI(1,1) has been accepted.

As Ilmakunnas (1990) expressed, the conclusion on the appropriate order of integration depends on the starting point of testing sequence . If starting from the most general model (case 1 in Table 43), the result is that in most cases the growth variable is stationary after both first differencing and quarterly differencing (in most cases, the null of SI(2,1) is accepted against the other alternative hypotheses) and according to this starting point, it may be concluded that inflation series may be either SI(2,1) or SI(1,0) (given in “INF” column). If the starting point is case 2 in Table 43 (or quarterly differencing (that is, SI(1,1)), we cannot obtain accurate results for variables: While INF series may be accepted as SI(1,1) in most cases, GR series may be SI(0,1), SI(1,1) or SI(0,0).

When looked at the DHF test result in Case 2 where the null hypothesis is SI(1,1) and the alternative is SI(0,0), GR variable can be said to reach full stationarity with SI(0,0) seasonal integration order. The other tests apart from DHF in Case 2 imply that seasonal frequency unit roots clearly can be accepted (or cannot be rejected) for GR variable. However, the evidence is not certain for INF series (it may also be SI(1,0) or SI(0,0) other than SI(1,1) – in other words, it may not include seasonal unit roots).

It is worth mentioning about some equivalences between the tests when $\theta_j = 0$ (taking place in Table 5). In case the main hypothesis to be tested is the presence of seasonal frequency unit roots, i.e. $\pi_2 = \pi_3 = \pi_4 = 0$ in the HEGY test, the test regression does not differ from ADF test for seasonally averaged ($S(L)$) data. In a similar manner, in the case of $\pi_2 = \pi_3 = \pi_4 = 0$ in the HEGY test for first differenced data, the test regression is the same as the ADF test for seasonally differenced data. This is also the same as the OCSB test with $\beta_2 = 0$. At last, the OCSB test with $\beta_1 = 0$ is the same as the DHF test for first-differenced data (Ilmakunnas, 1990, p. 81).

One of the most important problems in applying integration tests is the appropriate choice of the value of lag length p to be used: too low a value gives rise to invalid statistics due to autocorrelation left in the residuals; on the other hand, the implication of an extremely high lag length is a reduction in power (Osborn et al., 1988, p. 365). In this application, in selecting the appropriate lag lengths, LM test statistics for residual autocorrelation have been calculated and examined up to order four for all test regressions. Lag lengths have been increased one by one until detecting no significant autocorrelations at the 5% level. All applications in this section have been carried out in R.3.1.3. version and Eviews 7.

Now we will have a different look at cointegration relationship between INF and GR series for growth equation in which dependent variable is economic growth (GR) and independent variable is inflation (INF). Table 44 shows the cointegration results for growth equation. Since there are two variables in our model, at most 1 cointegrating relation can be found. When the growth equation is taken into consideration, it can be said that the resulted statistics can be used to give a clue about whether the variables are cointegrated or not at seasonal frequencies. For the first three models in Table 44 which are given in level form, seasonally averaged form and seasonally differenced form, respectively; all tests of the residuals (DW, DF, ADF) strongly suggest that the variables are cointegrated (where the null hypothesis is H_0:no cointegration and the alternative one is H_1:cointegration exists) (in other saying, the evidence against no-cointegration is said to be very strong).

When we look at the first differenced (Δ) and twice differenced variables (Δ^2), it is seen that the evidence of cointegration is strong when differenced variables are considered with significant π_i estimates at seasonal frequencies. However in the twice differenced form, since π_3 and π_4 estimates regarding annual unit root are not significant, we cannot strictly say that twice differenced variables are cointegrated at seasonal frequencies even though only π_2 is significant.

Table 44

Cointegration Results for Growth Equation

Form of the variables in the regression (Dependent Variable=GR)					
Estimated Coefficients	**Levels**	**Seasonally Averaged** $S(L)$ ($S(L)=1+L+L^2+L^3$)	**Seasonally Differenced** (Δ_4)	**Differenced** (Δ)	**Twice Differenced** (Δ^2)
INF	-0.083274 (Constant+ Dummies Model) "Significant"	-0.049137 (Constant Model) "Significant"	-0.258978 (Model with No Deterministic Component)	-0.394322 (Constant+ Dummies Model)	-0.409318 (Constant+ Dummies Model)
Test of the Residuals Test Statistics					
DW	1.999220*	0.492387*	1.662716*		
DF	-8.059352*	-2.996607*	-6.732354*		
ADF(p)	-3.648481(4)*	-3.838019(1)*	-6.713961(3)*		
	%5Critical Values: -1.945823(for DF) -1.946161(for ADF(p))	%5Critical Values: -1.946072 (DF) -1.946161(ADF(p))	%5Critical Values: -1.946161(DF) -1.946447(ADF(p))		
HEGY Test Results					
π_1				-6.486017*	-7.636372*
π_2				-2.762810*	-2.464176*
π_3				-2.558021*	**-0.253216**
π_4				**1.536945**	**0.923592**
π_3 & π_4				4.772130*	**0.466443**
(Model with No Deterministic Component)				$(p=0)$	$(p=4)$

Note. [1] * denotes significant values at 5% level.
[2] Critical values for HEGY test have been obtained from Hylleberg et al. (1990, pp. 226-227).
[3] Critical values for DW statistic have been taken from Engle & Yoo (1987, p. 158) for N=2 variables.

Level form regression results show the existence of one cointegrating relation with significant residual test statistics which are Durbin-Watson (DW), DF and ADF test statistics. The seasonally averaged form results (S(L)) also support this result with significant ADF, DW statistics obtained for the residuals of given regressions.

Empirical results reveal that all forms of the variables except twice-differenced (Δ^2) form show the sign of cointegration. Therefore, this analysis in which GR is dependent variable and INF is independent variable has revealed that the variables in question are SI(1,1). Since seasonally averaged ($S(L)$) variables have been found to be cointegrated of order 1 at zero (non-seasonal) frequency and first differenced variables (Δ) have been found to be cointegrated at seasonal frequencies. Thus, it can be said that in growth-inflation model, it would be suitable to incorporate the variables in Δ_4 form into the regression.

At the core of this analysis, how different seasonal integration tests can be carried out in a unified approach lies. Seasonal integration results imply that growth and inflation variables may be either SI(2,1) or SI(1,1) in the dominant sense. Therefore we have taken five cointegration regressions in the level, seasonally averaged (S(L)), quarterly differenced (Δ_4), first differenced (Δ) and twice differenced (Δ^2) forms. In the level form, GR series has been regressed on INF series. In the level, differenced and twice differenced forms; a constant and three seasonal dummies have been included and in the seasonally averaged form, a constant has been added. In Table 44, “p” shows the necessary lag numbers that will be included in the regressions applied. Surely, the analysis reveals that both series in their level forms are cointegrated.

When the results of regression analyses are considered in terms of economic interpretation, the inflation-growth relationship in Turkey has been understood to be in an opposite direction. This has been confirmed by the negative sign of the coefficient of INF variable in any case. According to the results of regression analyses applied, each percentage point increase in inflation reduces economic growth over 1998Q1-2014Q4 period. This result indicates that primarily there should be further reductions in inflation in order to increase the average growth rate declining gradually in recent years.

Most empirical studies on the inflation-growth relationship show that these two series are negatively related. However, these studies are mostly based on the periodical cross section data of a group of countries and its validity is questionable. Some authors assert that in analyses executed by such data; country in interest, time considered and the selection of variables discussed in the given model affect the achieved results significantly. Therefore, it has been recommended that the relationship between inflation and growth should be investigated through the time series analysis. However, studies conducted by time series analysis have failed to reveal a clear result so far (Karaca, 2003, p. 254).

As well known, first of all the series in interest must be stationary in time series analysis. When worked with non-stationary time series, it may be faced with spurious regression problem. In this case the results obtained by the regression analysis do not reflect the actual relation. Because these test statistics do not display a standard distribution, they lose their validity. The regression analyses conducted with non-stationary time series can reflect the real relationship only if a cointegration relationship exists between the series in interest (Kızılgöl & Erbaykal, 2008, p. 355). To summarize, this application addresses the cointegrating relationship between inflation and growth from a different view by taking the concept of seasonality into consideration.

Whether the data contain seasonality or not plays an important role in determining integration orders of the series. Thus, if seasonality is present (this case is possible for our series, because they have been expressed in quarterly frequency), series should be referred to as seasonal integrated series. In this application, various seasonal integration tests have been carried out in order to detect the appropriate order of seasonal integration. As a result of the application, two series have been found to have the same degree of seasonal integration as SI(1,1). Thus, by knowing that the two series have the same integration order (both are SI(1,1)) and applying various tests (DW, DF, ADF, HEGY) to the residuals obtained from the regression equations formed by using difference operators and raw data, whether there is a long-term relationship between the series or not has been examined through the cointegration analysis. As a result, the presence of a cointegrating relationship has been determined between two variables and this means a real long-term relationship.

6.3. Seasonal Cointegration Test Application for Turkey

In this application, it has been aimed to investigate the existence of co-integration relationship between quarterly gross domestic product (GDP), final consumption expenditures of resident households (CONS), exports of goods and services (EXP), government final consumption expenditures (GOV) and private sector machinery-equipment (PRIEQ) series for the period 1998Q1-2014Q4. Data that are based on expenditure based GDP time series at fixed 1998 prices have been obtained from CBRT.

First, in order to linearize the exponential growth in these series, their logarithms have been taken. Since by taking logarithm, variance is stabilized and the effects

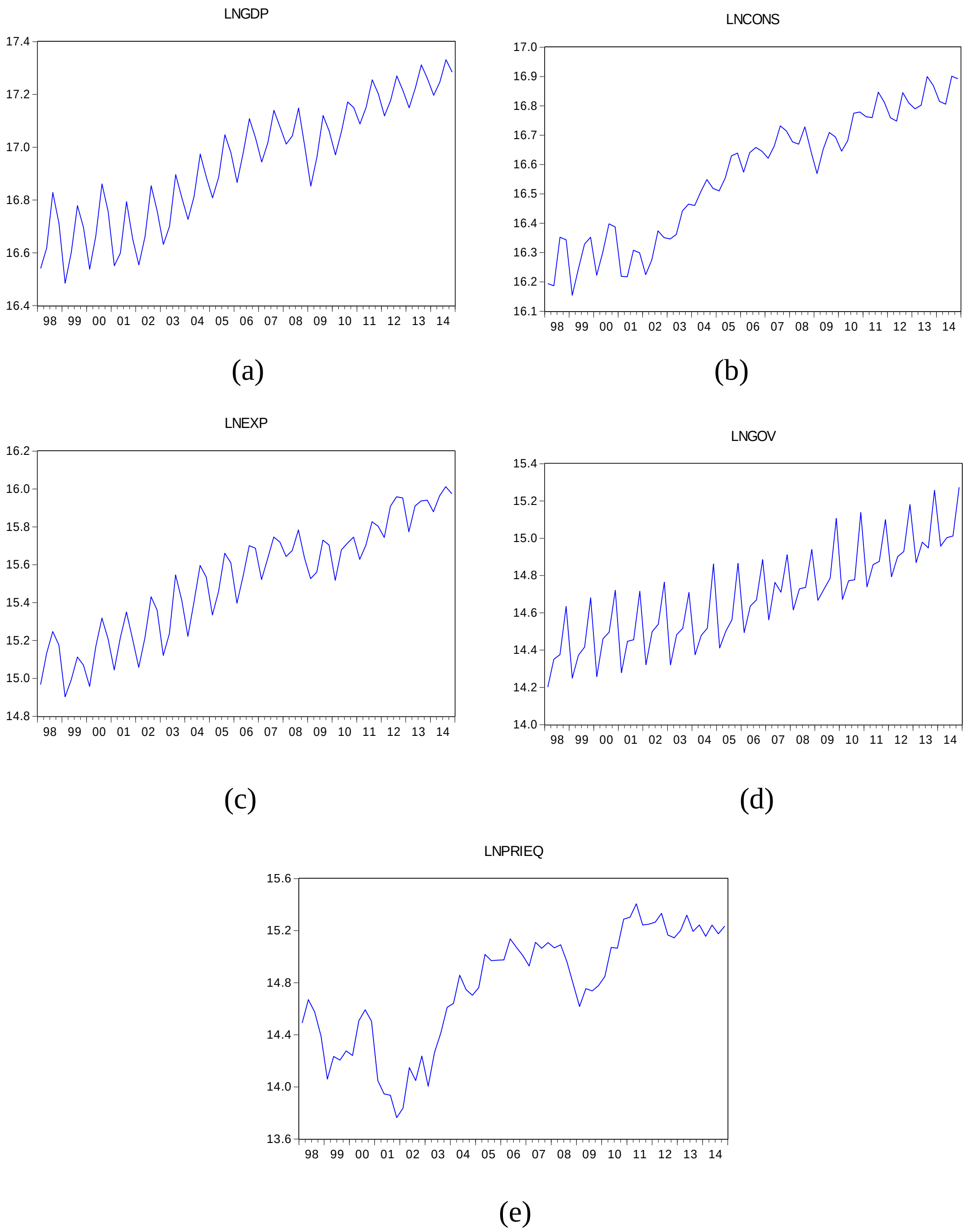


Figure 16. Graphs of logarithmic and seasonally unadjusted macroeconomic variables

of outliers are reduced (Türe & Akdi, 2005, p. 6). In Figure 16, graphs of logarithmic GDP, CONS, EXP, GOV and PRIEQ series have been presented respectively from (a) to (e).

Table 45

HEGY Seasonal Unit Root Test Results for Quarterly Macroeconomic Series

Variables	**Auxiliary Regressions**	**Lags**	$t(\pi_1)$	$t(\pi_2)$	$t(\pi_3)$	$t(\pi_4)$	**F** (π_3, π_4)
LNGDP	Intercept	2	-0.374639*	-1.658963*	-1.568273*	-1.650405*	2.681313*
	Intercept + Dummies	2	-0.352000*	-2.284324*	-2.049658*	-2.370408	5.446632*
	Intercept + Dummies + Trend	2	-2.528751*	-2.394737*	-1.809312*	-2.278266	4.629866*
LNCONS	Intercept	1	-1.108130*	-2.087701	-2.413501	-1.715589	4.330118
	Intercept + Dummies	2	-0.624776*	-2.256329*	-2.649649*	-3.690152	12.64432
	Intercept + Dummies + Trend	2	-2.329712*	-2.341915*	-2.498616*	-3.514822	11.17496
LNPRIEQ	Intercept	0	-1.048793*	-3.006195	-4.078979	-6.309589	45.08480
	Intercept + Dummies	1	-1.255175*	-4.758546	-2.938663*	-5.416084	19.21220
	Intercept+ Dummies + Trend	0	-2.739185*	-5.277844	-3.066718*	-5.066384	23.55738
LNGOV	Intercept	1	1.037847*	-0.672012*	-0.941324*	-0.406816*	0.522808*
	Intercept + Dummies	0	0.719595*	-3.616989	-3.364203*	-0.412796*	5.776124*
	Intercept + Dummies + Trend	0	0.745482*	-0.608322*	0.013417*	-0.235981*	6.278467*
LNEXP	Intercept	2	-1.033219*	-2.208915	-1.552280*	-0.795612*	1.551227*
	Intercept+ Dummies	0	-0.119803*	-3.880223	-2.968691*	-3.321427	12.04958
	Intercept + Dummies + Trend	2	-2.661086*	-3.188178	-1.732976*	-2.357943	4.688940*

Note. [1] * denotes insignificant values at 5% level.

[2] *t*-statistic for π_1 $t(\pi_1)$ shows whether there is a unit root or not at zero frequency $(H_0 : \pi_1 = 0)$. *t*-statistic for π_2 $t(\pi_2)$ tests the presence of the semi-annual unit root $(H_0 : \pi_2 = 0)$. F statistic for $\pi_3 \cap \pi_4$ $(F(\pi_3, \pi_4))$ tests whether there is a unit root at quarterly frequency or not.

[3] Critical values have been taken from HEGY (1990, pp. 226-227) for N=100 observations and 5% level. For zero frequency, critical values are -2.88, -2.95, -3.53 and for semi-annual frequency, they are -1.95, -2.94, -2.94 respectively for “intercept”, “intercept+dummies”,

"intercept+dummies+trend" models.

In order to determine which series have a cointegrating relationship, it is necessary to find out at which frequencies series are integrated of the same order (or at which frequencies unit roots are present). For each series, three different models including "constant (C)", "constant+dummies (C, D)" and "constant+dummies+trend (C, D, T)" have been constructed. Also, the lagged values of the dependent variable have been added into these models. Since the series discussed are at quarterly frequency, seasonal unit root test results of the series at $\theta = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ frequencies have been presented in Table 45.

In Table 45, lag augmentation by lagged values of the dependent variable has been made in the auxiliary regressions including deterministic components in order to make sure about the whitened residuals. As expressed in Engle et al. (1993), this augmentation does not affect the distribution under the null hypothesis as is the case with DF procedure; but, the power and size of the test may depend critically on the 'right' augmentation being used (p.279). In the application for seasonal unit root test, the appropriate lag length has been chosen in that way: Regression equation has been estimated first with Lag 1 and it has been investigated if first order and fourth order autocorrelations exist between residuals. For this investigation, it has been utilized from LM test statistics (thus for first order: LM(1) and for fourth order: LM(4)). If any one of the null hypotheses of H_0 : There is no 1st order autocorrelation and H_0 : There is no 4th order autocorrelation is rejected, lag length has been increased by one and LM test has been applied again. This process has been continued until the null hypothesis cannot be rejected for each order and homoscedastic residuals are obtained. LM(1) and LM(4) statistics results have been presented in Table 46:

Table 46

LM(1) and LM(4) Statistics for Quarterly Macroeconomic Series

VARIABLES	**AUXILIARY REGRESSIONS**	**LAGS**	**LM(1)**	**LM(4)**
LNGDP	Intercept	2	2.285563 (0.1306)	8.015350 (0.0910)
	Intercept + Dummies	2	0.728109 (0.3935)	4.011422 (0.4045)
	Intercept + Dummies + Trend	2	0.018672 (0.8913)	5.215494 (0.2659)
LNCONS	Intercept	1	1.470532 (0.2253)	5.879280 (0.2083)
	Intercept + Dummies	2	0.377129 (0.5391)	5.110070 (0.2762)
	Intercept + Dummies + Trend	2	0.461174 (0.4971)	6.520621 (0.1635)

Table 46 (*Continued*)

VARIABLES	AUXILIARY REGRESSIONS	LAGS	LM(1)	LM(4)
LNPRIEQ	Intercept	0	0.130705 (0.7177)	8.780591 (0.0668)
	Intercept + Dummies	1	0.468251 (0.4938)	1.220068 (0.8748)
	Intercept+Dummies + Trend	0	0.693561 (0.4050)	4.515437 (0.3407)
LNGOV	Intercept	1	0.364082 (0.5462)	4.615978 (0.3290)
	Intercept + Dummies	0	0.798644 (0.3715)	2.349286 (0.6718)
	Intercept + Dummies + Trend	0	0.294179 (0.5876)	1.192969 (0.8793)
LNEXP	Intercept	2	0.924163 (0.3364)	9.401415 (0.0518)
	Intercept+Dummies	0	0.875675 (0.3494)	4.337515 (0.3623)
	Intercept + Dummies + Trend	2	0.039849 (0.8418)	1.551871 (0.8174)

Note. LM(1) and LM(4) represent LM test statistics investigating the presence of 1^{st} and 4^{th} order autocorrelations and the values given in parentheses indicate the significance levels.

As is clear from Table 46 that for selected lags, there are no first order and fourth order autocorrelation problems for all macroeconomic series.

If looked at the Table 45 results, it is seen that the presence of a unit root at zero frequency has been accepted for all variables in all three auxiliary regression models. When $t(\pi_1)$, $t(\pi_2)$ and $F(\pi_3, \pi_4)$ columns are examined, it is concluded that the null hypotheses that there is a (non-seasonal) unit root at zero frequency and there are seasonal unit roots at other seasonal frequencies cannot be rejected for three auxiliary regression models of LNGDP series at 5% significance level. Thus, LNGDP series has a non-seasonal unit root at zero frequency and seasonal unit roots at semi-annual ($\frac{1}{2}$ frequency) and quarterly frequencies. While both LNCONS and LNPRIEQ series have the zero frequency unit root for three models with deterministic components given in Table 45, according to the results they both do not include any annual unit root (at quarterly frequency). For LNCONS series, the presence of semi-annual unit root has been accepted for two models except the “intercept” model. However, no semi-annual unit root has been detected in any model for LNPRIEQ series. When looked at the LNGOV and LNEXP series, both series are seen to include the zero frequency unit root.

However, while LNGOV series has a seasonal unit root at semi-annual frequency for two models except the "intercept+dummies" model, LNEXP series rejects the presence of the semi-annual unit root for all three models. Finally, while LNGOV series has annual unit roots at quarterly $\frac{1}{4}\left(\frac{3}{4}\right)$ frequencies for all deterministic models, LNEXP series has seasonal unit roots at quarterly frequencies for two models except only "intercept+dummies" model. In conclusion, cointegration relationship will be analysed at frequencies in which these series are both integrated at the same order. In this case, it is necessary to determine which series are integrated of the same order at which frequencies. In all series, the presence of the zero frequency unit root has been detected in common. LNGDP, LNCONS and LNGOV series have been found to include seasonal unit root at semi-annual frequency. On the other hand, it has been determined that LNGDP, LNGOV and LNEXP series include seasonal unit roots at the quarterly frequencies $\frac{1}{4}\left(\frac{3}{4}\right)$. The results of seasonal cointegration analyses of the series at 0, ½ and ¼ (and ¾) frequencies have been presented in Table 47, Table 48 and Table 49 respectively.

In this application, regression models obtained from the linear components of the variables that are integrated at the same frequency have been estimated through OLS procedure. Before applying to cointegration analysis, it is necessary to give the transformations of variables that will be used in cointegration models. As a matter of example, it will be sufficient to present only LNGDP series (the other series will be transformed in the same way with LNGDP):

$$LNGDP_{1t} = (1 + L + L^2 + L^3)LNGDP$$
$$LNGDP_{2t} = -(1 - L + L^2 - L^3)LNGDP$$
$$LNGDP_{3t} = -(1 - L^2)LNGDP$$
$$LNGDP_{4t} = (1 - L^4)LNGDP$$

Now let us mention about the cointegration models to be used. Seasonal cointegration has been mentioned in section 5.3 (also look at the sub-section 5.3.1). In addition, as also summarized by Ayvaz Kızılgöl (2011, p. 18), in cointegration analysis the regression model to be estimated for all variables that are integrated of the same order at the zero frequency is $Y_{1t} = \alpha_1 Z_{1t} + u_t$. The residuals ($u_t$) obtained from this cointegration model will be used in order to estimate auxiliary regression model at the zero frequency. For semi-annual (½) frequency, the cointegration model to be used is

$Y_{2t} = \alpha_2 Z_{2t} + v_t$ and for quarterly frequencies, it is $Y_{3t} = \beta_1 Z_{3t} + \beta_2 Z_{3,t-1} + w_t$. Also, the residuals obtained from these models (v_t and w_t) will be used for estimating auxiliary regressions at specified frequencies respectively.

Table 47

Cointegration Test Results at Zero (Long Run) Frequency

	Cointegration Analysis: LGDP and LCONS			**Auxiliary Regression**	**Tests for Unit Roots in Residuals**	
Regressand	**Coefficient Regressor $LGDP_{1t}$**	**Deterministic Components Included**	R^2	**Augmentation**	**DW**	***t* statistic** $t(\pi_1)$
$LCONS_{1t}$	0.981403 (80.95842)	C	0.990479	1, 4, 5	1.998160	-1.936136
$LCONS_{1t}$	0.981366 (79.04407)	C, D	0.990498	1	1.890828	-2.615745
$LCONS_{1t}$	1.006343 (16.64040)	C, D, T	0.990526	1	1.892945	-2.725444
	Cointegration Analysis: LGDP and LPRIEQ			**Auxiliary Regression**	**Tests for Unit Roots in Residuals**	
Regressand	**Coefficient Regressor $LGDP_{1t}$**	**Deterministic Components Included**	R^2	**Augmentation**	**DW**	***t* statistic** $t(\pi_1)$
$LPRIEQ_{1t}$	1.951163 (21.48804)	C	0.879940	1, 4, 5	2.139510	-3.405938
$LPRIEQ_{1t}$	1.951973 (20.98828)	C, D	0.880156	1, 2, 4	1.934590	-1.932861
$LPRIEQ_{1t}$	4.817088 (19.58387)	C, D, T	0.964776	1, 4, 5, 8	2.240764	-2.182390
	Cointegration Analysis: LGDP and LGOV			**Auxiliary Regression**	**Tests for Unit Roots in Residuals**	
Regressand	**Coefficient Regressor $LGDP_{1t}$**	**Deterministic Components Included**	R^2	**Augmentation**	**DW**	***t* statistic** $t(\pi_1)$
$LGOV_{1t}$	0.938623 (32.05764)	C	0.942239	1, 3, 4	1.969561	-1.531071
$LGOV_{1t}$	0.938442 (31.27638)	C, D	0.942281	1, 3	1.935207	-2.273428
$LGOV_{1t}$	0.096085 (1.020667)	C, D, T	0.976126	1	1.998045	-1.664541
	Cointegration Analysis: LGDP and LEXPORT			**Auxiliary Regression**	**Tests for Unit Roots in Residuals**	
Regressand	**Coefficient Regressor $LGDP_{1t}$**	**Deterministic Components Included**	R^2	**Augmentation**	**DW**	***t* statistic** $t(\pi_1)$
$LEXPORT_{1t}$	1.304336 (47.43188)	C	0.972760	1, 2, 4	1.982701	-3.079903
$LEXPORT_{1t}$	1.304245 (46.27101)	C, D	0.972768	1, 2, 4	1.989971	-3.085921
$LEXPORT_{1t}$	0.825112 (6.772424)	C, D, T	0.978622	1, 2	2.070897	-3.329934

Note. [1] These tests at zero frequency are based on the (ADF) auxiliary regression model $\Delta u_t = \pi_1 u_{t-1} + \sum_{j=1}^{k} b_j \Delta u_{t-j} + e_t$ (here without deterministic components) where u_t represents the residuals obtained from cointegration model that are used to estimate this auxiliary regression model. The distribution of 't' statistic is as characterized in Engel & Granger (1987) and Engle & Yoo (1987) (Engle et al., 1993, p. 289). As it is clear, the necessary significant lagged values of the dependent variable Δu_t have been added into auxiliary regression model in order to whiten the residuals (the lagged variables with insignificant coefficients at 5% significance level have been removed from the model).

[2] The values in parentheses are *t*-statistics.

[3] C, D and T denote constant, seasonal dummies and trend terms respectively.

[4] The basic hypothesis to be tested is H_0 : There is no cointegration at zero frequency $(\pi_1 = 0)$.

[5] Critical values have been obtained from Engle and Yoo (1987). See Appendix F.

As mentioned before, in order to detect the long-run equilibrium relationship between the series, first of all it is necessary to determine the stationarity order of the series. In this application, for investigating the presence of seasonal cointegration relationship between the series, firstly seasonal unit root test has been applied in order to make inference about at which frequencies there are unit roots if they exist. The series discussed here have quarterly frequencies. Therefore, HEGY seasonal unit root test which is developed by Hylleberg et al. (1990) has been applied in order to detect seasonal unit roots and general results have been presented in Table 45 for three models with deterministic components that are "C", "C,D", "C,D,T". Now, Table 47 presents the cointegration test results at zero frequency. As a result, when cointegration test results are evaluated at the zero frequency, although the explanatory variables that take place in the cointegrating regression have been found to be statistically significant, no cointegrating relationship has been found between LNGDP and LNCONS, LNGDP and LNPRIEQ, LNGDP and LNGOV, LNGDP and LNEXP at 5% significance level in the long-run.

LNGDP and LNCONS series have been found to be integrated of the same order for "C,D" and "C,D,T" models at ½ frequency. Also, LNGDP and LNGOV series have been found to be integrated of the same order for "C" and "C,D,T" models at ½ frequency. Therefore, cointegration analysis results at ½ frequency have been shown in Table 48 for LNGDP, LNCONS and LNGOV series.

Table 48
Seasonal Cointegration Test Results at Semi-Annual (½) Frequency

	Cointegration Analysis: LGDP and LCONS			Auxiliary Regression	Analysis of the residuals	
Regressand	**Coefficient Regressor LGDP$_{2t}$**	**Deterministic Components Included**	R^2	**Augmentation**	**DW**	***t* statistic** $t(\pi_2)$
LCONS$_{2t}$	0.734494 (5.536534)	C, D	0.339386	1, 4	1.926116	-1.649370
LCONS$_{2t}$	0.743244 (5.543788)	C, D, T	0.343757	1, 4	1.904166	-1.607737
	Cointegration Analysis: LGDP and LGOV			**Auxiliary Regression**	**Analysis of the residuals**	
Regressand	**Coefficient Regressor LGDP$_{2t}$**	**Deterministic Components Included**	R^2	**Augmentation**	**DW**	***t* statistic** $t(\pi_2)$
LGOV$_{2t}$	-3.577777 (-3.598662)	C	0.170511	1, 2, 4	1.993776	-1.179657
LGOV$_{2t}$	0.096323 (0.537327)	C, D, T	0.979684	0	2.073641	-3.411280

Note. [1] In lag augmentations, only significant lags have been added into the auxiliary regressions (insignificant lags have been removed) in order to get white noise residuals.
[2] These tests at semi-annual frequency are based on the auxiliary regression

$(\upsilon_t + \upsilon_{t-1}) = \pi_2(-\upsilon_{t-1}) + \sum_{j=1}^{k} b_j(\upsilon_{t-j} + \upsilon_{t-j-1}) + e_t$ (here without deterministic components) where υ_t represents the residuals obtained from cointegration model that are used to estimate the auxiliary regression models. The distribution of 't' statistic is as characterized in Engel & Granger (1987) and Engle & Yoo (1987) (Engle et al., 1993, p. 290). For critical values see Appendix F.

[3] The basic hypothesis to be tested is H_0 : There is no cointegration at semi-annual frequency $(\pi_2 = 0)$.

When Table 48 results are compared to the Engle and Yoo (1987) critical values for 5% significance level, no cointegration relationship has been found between LNGDP & LNCONS series and LNGDP & LNGOV series at ½ frequency. Thus, these series in interest do not seem to be cointegrated at the semi-annual frequency.

Table 49 presents seasonal cointegration test results at quarterly ¼ (¾) frequencies. According to the Table 49 results, it can be said that there has been found a cointegration relationship between LNGDP and LNGOV series at quarterly frequencies ¼ (and ¾) for only the model with constant and seasonal dummies ("C,D"). In other saying, the null hypothesis saying that there is no cointegration at quarterly frequencies has been rejected with a significant joint F statistic of 12.19361. On the other hand, no cointegration relationship has been detected for no models between LNGDP and LNEXP series at ¼ (and ¾) frequencies.

Table 49

Seasonal Cointegration Test Results at ¼ (¾) Frequencies

	Cointegration Analysis: LGDP and LGOV				Auxiliary Regression	Analysis of the residuals		
Regressand	**Coefficient Regressor LGDP$_{3t}$**	**Coefficient Regressor LGDP$_{3t-1}$**	**Deterministic Components Included**	R^2	**Augmentation**	***t* statistic** $t(\pi_3)$	***t* statistic** $t(\pi_4)$	***F* statistic** $\pi_3 \cap \pi_4$
LGOV$_{3t}$	0.786609 (11.53730)	0.903772 (13.53999)	C	0.841278	1	-2.712937	-1.646483	5.045242
LGOV$_{3t}$	0.532680 (4.450165)	-0.182431 (-1.536042)	C, D	0.949005	1, 2	**-4.866891***	0.559515	**12.19361***
LGOV$_{3t}$	0.780067 (11.52444)	0.896061 (13.55706)	-	0.839207	1, 2	-2.887014	-1.199424	5.244342

	Cointegration Analysis: LGDP and LEXPORT				Auxiliary Regression	Analysis of the Residuals 'HEGY' test		
Regressand	**Coefficient Regressor LGDP$_{3t}$**	**Coefficient Regressor LGDP$_{3t-1}$**	**Deterministic Components Included**	R^2	**Augmentation**	***t* statistic** $t(\pi_3)$	***t* statistic** $t(\pi_4)$	***F* statistic** $\pi_3 \cap \pi_4$
LEXPORT$_{3t}$	1.108250 (21.79906)	-0.007793 (-0.156570)	C	0.884680	1, 4, 6, 8	-2.497336	-1.769538	5.254946
LEXPORT$_{3t}$	0.901760 (5.869706)	-0.073642 (-0.483110)	C, D	0.890232	1, 4	-2.542985	**-2.241085***	5.692280
LEXPORT$_{3t}$	1.111528 (22.10056)	-0.003930 (-0.080016)	-	0.884001	1, 4, 5, 6	-2.312909	-1.327629	3.454144

Note. [1] These tests at ¼ (and ¾) frequencies are based on the auxiliary regression

$$(w_t + w_{t-2}) = \pi_3(-w_{t-2}) + \pi_4(-w_{t-1}) + \sum_{j=1}^{k} b_j (w_{t-j} + w_{t-j-2}) + e_t$$ (here without deterministic components)

where w_t represents the residuals obtained from cointegration model that are used to estimate the auxiliary regression models (Engle et al., 1993, p. 290).

[2] "C" denotes constant, "D" denotes seasonal dummies and "–" denotes no deterministic component.

[3] * denotes significant values at 5% significance level.

[4] Critical values have been obtained from Engle et al. (1993). See Appendix F for critical values.

[5] The basic hypothesis to be tested is H_0 : There is no cointegration at ¼ (and ¾) frequencies $(\pi_3 \cap \pi_4 = 0)$.

6.4. Determining the Type of Seasonality for Quarterly Turkish Unemployment Series

In this application, it has been tried to make inference about whether the seasonal pattern of quarterly total harmonized unemployment series (units: persons) for Turkey is deterministic or stochastic. The unemployment series (namely UNEMP here) will be examined over 1988Q1-2014Q4 period (108 observations) and the unemployment data have been obtained from Organisation for Economic Co-operation and Development as not-seasonally adjusted.

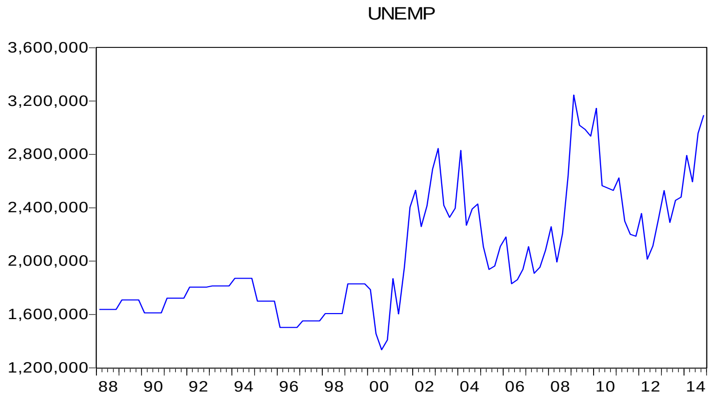


Figure 17. Graph of total harmonised unemployment series

In Figure 17, UNEMP series has been graphed against years. In order to obtain healthy results, the transaction of taking logarithm has been applied to the series and this new logarithmic UNEMP series (namely LOGUNEMP) has been given in Figure 18.

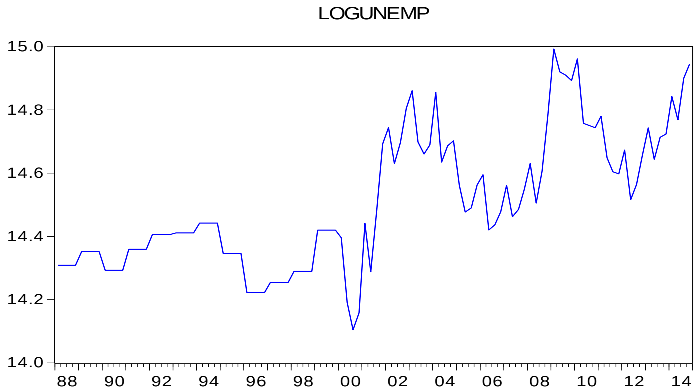


Figure 18. Graph of logarithmic unemployment series

It can be observed that logarithmic UNEMP series seems to display some seasonal pattern. In order to find out if UNEMP series displays deterministic or stochastic seasonality, *t*-statistics and Q-statistics which take place in subsection (5.2.4.) have been examined. As mentioned before, for testing if a series follows a deterministic or stochastic seasonal pattern, the hypotheses to be constructed are the null hypothesis H_0 which implies the presence of stochastic seasonality and the alternative hypothesis H_1 which implies the presence of deterministic seasonality. For the acceptance of stochastic seasonality, there are two conditions that should hold: the first one is the acceptance of the hypothesis saying that all α coefficients (in equation 5.40) are equal to each other and the second one is the acceptance of the hypothesis in which all π coefficients (in equation 5.40) are equal to zero. In case these two conditions are satisfied, it is concluded that there is stochastic seasonality in the series.

Table 50

HEGY Stochastic Seasonality Test for Unemployment Series

	Unrestricted Model Dependent Variable: $Y_{4t}(=y_t - y_{t-4})$		**Restricted Model: Dependent Variable** $Y_{4t}(=y_t - y_{t-4})$	
Variables	**Coefficient**	***t*-statistic**	**Coefficient**	***t*-statistic**
D1	1.085887	1.976383	0.006065	0.346905
D2	0.987717	1.793923	0.000907	0.052881
D3	1.069343	1.950722	0.008263	0.481820
D4	1.071505	1.952493	0.005868	0.341765
$Y_{1,t-1}$	-0.018080	-1.910056		
$Y_{2,t-1}$	-0.403887	-3.820225		
$Y_{3,t-2}$	-0.339509	-3.469135		
$Y_{3,t-1}$	-0.370895	-3.830075		
$Y_{4,t-1}$	0.266355	2.630320	0.818124	13.70320
Sum Squared Resid	0.486572		0.746470	
n	103		103	
Q	12.55231			
	$t_{\alpha_1-\alpha_2} = 3.816143$		$t_{\alpha_2-\alpha_3} = -3.18582$	
	$t_{\alpha_1-\alpha_3} = 0.639368$		$t_{\alpha_2-\alpha_4} = -3.29757$	
	$t_{\alpha_1-\alpha_4} = 0.550436$		$t_{\alpha_3-\alpha_4} = -0.08282$	

Note. [1] y_t represents logarithmic unemployment series.

[2] For $\alpha = 5\%$ significance level, critical *t*-value and F value have been taken as 1.984 and 2.46 respectively for T=100 observations.

[3] n shows the number of observations; D1, D2, D3 and D4 are seasonal dummies for quarterly series. The coefficients of $Y_{1,t-1}$, $Y_{2,t-1}$, $Y_{3,t-1}$ and $Y_{4,t-1}$ are $\pi_1, \pi_2, \pi_3, \pi_4$ values.

[4] Maximum lag number has been taken as 4 in lag augmentation and only significant lags among four lags have been included into the regressions and insignificant lags have been removed. In the models given in this table, only one lagged value of the dependent variable ($Y_{4,t-1}$) has been added.

Table 50 presents HEGY stochastic seasonality test results by using unrestricted and restricted models. In the table, equality of α coefficients has been tested by calculating $t_{\alpha_1-\alpha_2}, t_{\alpha_1-\alpha_3}, t_{\alpha_1-\alpha_4}, t_{\alpha_2-\alpha_3}, t_{\alpha_2-\alpha_4}, t_{\alpha_3-\alpha_4}$ statistics and restricted model has been formed by the equality $\pi_1 = \pi_2 = \pi_3 = \pi_4 = 0$. Since some *t*-values (testing α coefficients in doubles) are significant and some are not (briefly, not all of *t*-values are insignificant), the first condition saying that all α coefficients are equal to each other has not been satisfied for unemployment series. At the same time, since Q statistic (Q=12.55231) is greater than F critical value, the second condition saying that all π's are equal to zero does not hold. Therefore, according to the test results stochastic seasonality does not exist in this series (that is, the rejection of the null hypothesis).

In this case, we should detect the presence of deterministic seasonality and if there are seasonal unit roots or not. The test results for this examination have been given in Table 51 and Table 52.

Table 51

Unrestricted Models for Deterministic Seasonality in Unemployment Series

Unrestricted Models- Dependent Variable $Y_{4t}(=y_t - y_{t-4})$						
Variables	**Model 1**		**Model 2**		**Model 3**	
	Coefficient	***t*-statistic**	**Coefficient**	***t*-statistic**	**Coefficient**	***t*-statistic**
C	0.668284	1.058282	1.071505	1.952493	3.140335	3.861960
$Y_{1,t-1}$	-0.011401	-1.047051	-0.018080	-1.910056	-0.054956	-3.842289
$Y_{2,t-1}$	-0.186721	-2.128281	-0.403887	-3.820225	-0.359729	-3.548019
$Y_{3,t-2}$	-0.176010	-2.148737	-0.339509	-3.469135	-0.337346	-3.625700
$Y_{3,t-1}$	-0.169071	-2.042294	-0.370895	-3.830075	-0.299038	-3.161835
D1			0.014382	0.550283	0.017476	0.702829
D2			-0.083788	-3.295832	-0.072619	-2.975929
D3			-0.002161	-0.082726	-0.004297	-0.172932
t					0.001198	3.317023
$Y_{4,t-1}$	0.599778	6.079347	0.266355	2.630320	0.338956	3.433257
$Y_{4,t-4}$	-0.164306	-2.611528				
Sum Squared Resid	0.541177		0.486572		0.435097	
n	100		103		103	

Note. [1] t denotes trend term.

[2] Model 1 includes only constant term, Model 2 includes both constant and seasonal dummies and Model 3 includes constant, seasonal dummies and trend.

[3] First and fourth lagged values of the dependent variable have been added into Model 1 and only first lagged value of the dependent variable has been added into both Model 2 and Model 3.

Table 52

Restricted Models for Deterministic Seasonality in Unemployment Series

Restricted Models - Dependent Variable $Y_{4t}(=y_t-y_{t-4})$						
Variables	**Model 1**		**Model 2**		**Model 3**	
	Coefficient	*t*-statistic	Coefficient	*t*-statistic	Coefficient	*t*-statistic
C	0.645069	0.991028	1.367346	2.246100	3.831040	4.422681
$Y_{1,t-1}$	-0.010992	-0.979343	-0.023884	-2.279054	-0.067730	-4.456102
$Y_{2,t-1}$	-0.170174	-1.880700	-0.411364	-3.468801	-0.355139	-3.166997
D1			0.046257	1.756732	0.040051	1.619330
D2			-0.004276	-0.190925	-0.003297	-0.157027
D3			0.049303	1.879366	0.043377	1.760631
T					0.001470	3.779968
$Y_{4,t-1}$	0.795275	10.64539	0.626631	7.600825	0.670017	8.577214
$Y_{4,t-4}$	-0.213220	-3.407657				
Sum Squared Resid	0.590563		0.625524		0.543744	
N	100		103		103	
Q	4.243434		13.42195		11.6114	

Note. [1] First and fourth lagged values of the dependent variable have been added into Model 1 and only first lagged value of the dependent variable has been added into both Model 2 and Model 3.
[2] In this table, restricted models have been formed by assuming $\pi_3=\pi_4=0$.

Table 53

Decision Table for Unemployment Series

	π_1 : *t*--statistic	Critical value	π_2 : *t*-statistic	Critical value	Q statistic	Critical value
Model 1	-1.047051	-2.88	-2.128281	-1.95	4.243434	3.08
	H_0 : ACCEPT		H_0 : REJECT		H_0 : REJECT	
Model 2	-1.910056	-2.95	-3.820225	-2.94	13.42195	6.57
	H_0 : ACCEPT		H_0 : REJECT		H_0 : REJECT	
Model 3	-3.842289	-3.53	-3.548019	-2.94	11.6114	6.60
	H_0 : REJECT		H_0 : REJECT		H_0 : REJECT	

Note. Critical values have been obtained from Hylleberg et al. (1990, pp. 226-227) for 5% significance level and N=100.

t-values for π_1 and π_2 in Table 53 have been taken from the unrestricted models in Table 51 for three models in order to test the presence of non-seasonal (zero frequency) and semi-annual (½ frequency) unit roots, respectively. Q statistics that take place in Table 53 have been taken from the restricted models in Table 52 (In Table 52, models are called "restricted" because of assuming $\pi_3=\pi_4=0$ in order to test annual unit roots at quarterly frequencies $\frac{1}{4}\left(\frac{3}{4}\right)$ jointly). According to the Table 53, while the presence of nonseasonal unit root ($\pi_1 = 0$) in unemployment series is accepted for Model 1 (constant) and Model 2 (constant and seasonal dummies), no semi-annual and annual seasonal unit roots have been detected for no models. Hence, seasonal fluctuations in the series have not been able to emerge in the six-month and one-year intervals. In other saying, there is a non-seasonal unit root in the series.

Apart from the *t*-statistics and Q statistics, it can also be looked at CH Test results in Table 54 to decide about the seasonal pattern of the series. As known, the null hypothesis of CH test is the stationarity of all seasonal cycles (indicating to the deterministic seasonality) and the alternative one is the presence of seasonal unit root (indicating to the presence of stochastic seasonality). The results reveal that since calculated L-statistic (1.496) is greater than all 1%, 5% and 10% critical values (known that the distribution of L-statistic is Von Mises distribution), the null hypothesis is rejected and thus it is concluded that seasonal pattern is not deterministic (indicating to the presence of seasonal unit root) which is completely a different conclusion when compared to the previous analysis which is based on the restricted and unrestricted models with *t* and Q statistics saying that there is no stochastic seasonality in the series.

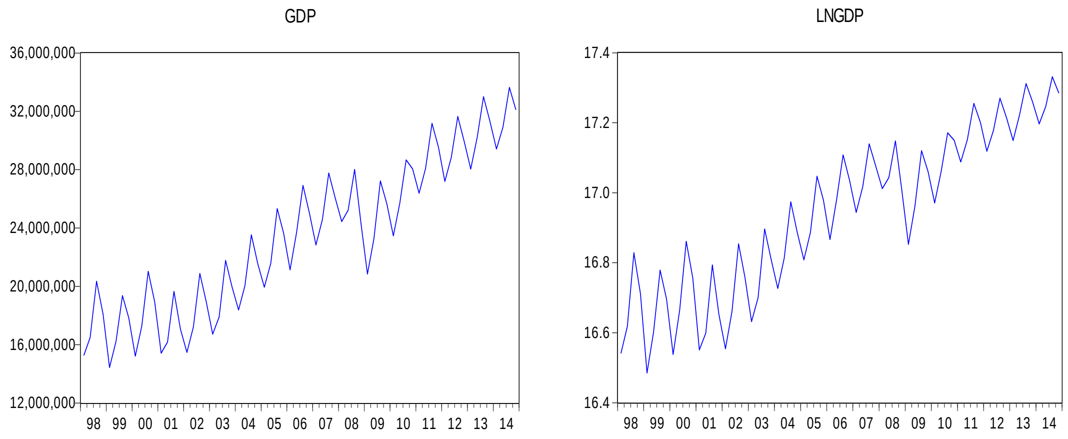
Table 54
CH Test Results for Unemployment Series

Tested Frequencies	**L-Statistic**	**Critical Values**		
		1%	**5%**	**10%**
$\frac{\pi}{2}, \pi$	1.496	1.35	1.01	0.846

As a result, it can be said that two methods say different things for detecting the type of seasonal pattern of the unemployment series. It is not certain to say only one type of seasonal behaviour for the series. Unemployment series may have both a deterministic and stochastic structure.

6.5. Modelling Quarterly Gross Domestic Product in Turkey

In this application, it has been aimed to decide about which seasonal pattern GDP series displays over 1998Q1-2014Q4 by recoursing to different tests. Quarterly Turkish real GDP series (expenditure based) has been taken in millions of national currency (at constant 1998 prices). Data for GDP have been obtained from CBRT. In order to linearize exponential growth, the logarithm of the series has been taken (namely, lngdp). The raw and logarithmic real GDP series have been graphed in Figure 19.



(a) (b)

Figure 19. Graphs of original (a) and logarithmic (b) gdp series

Logarithmic graph (b) in Figure 19 is the indicator for an upward trend implying that this series is not stationary (it includes a unit root) under the given period. In addition, the presence of seasonal components can be easily detected from this graph. In that case, in order to remove the growth trend from the series, the first difference of the logarithmic GDP series can be taken in the form of $\Delta \ln gdp = \ln gdp - \ln gdp(-1)$.

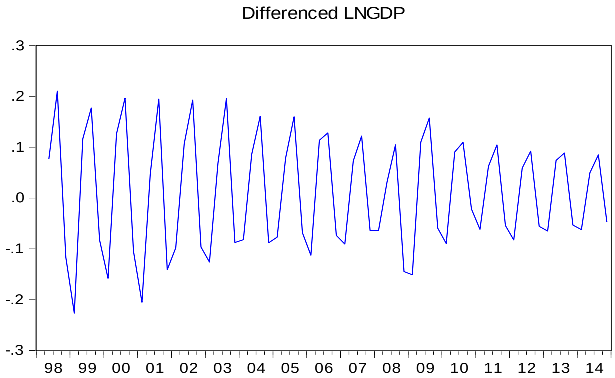


Figure 20. Graph of first-differenced lngdp series

The graph of first-differenced lngdp series in Figure 20 implies once again the presence of some seasonal pattern of Turkish real gdp series.

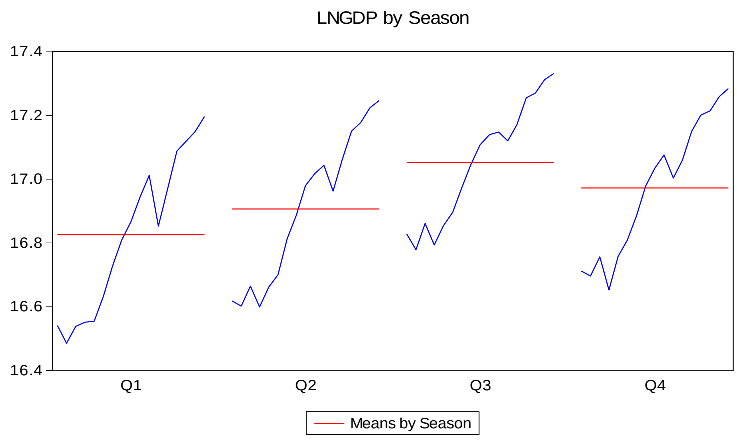


Figure 21. Seasonal means of lngdp series

We can also compare four seasons for lngdp series. It is clear to see from the Figure 21 that the seasonal peak is observed in the third quarter. Quarters two and four seem as if they yield approximately the same amount of output. The difference between the four seasons can be clearly seen from the Figure 21: Seasonal mean in quarter 1 is the lowest, while the mean for quarter 2 and 4 are in the middle and that of quarter 3 is the highest.

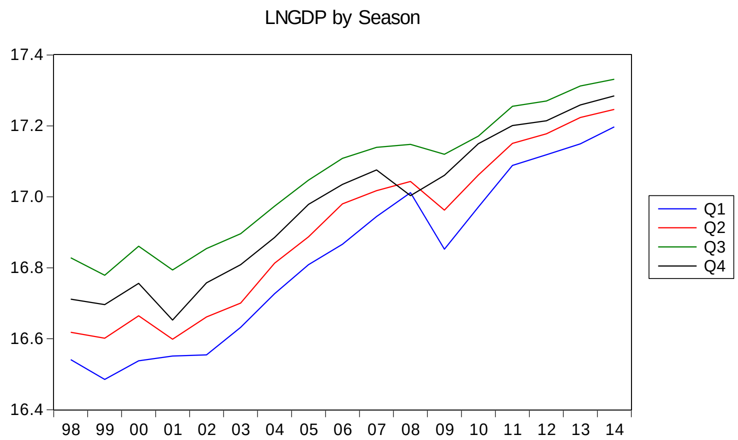


Figure 22. Graph of lngdp series

On the other hand, it can be said that Figure 22 implies that a seasonal deterministic model may seem not to be suitable for Turkish real GDP series over the given period because of not having a time constant mean for all of the four quarters.

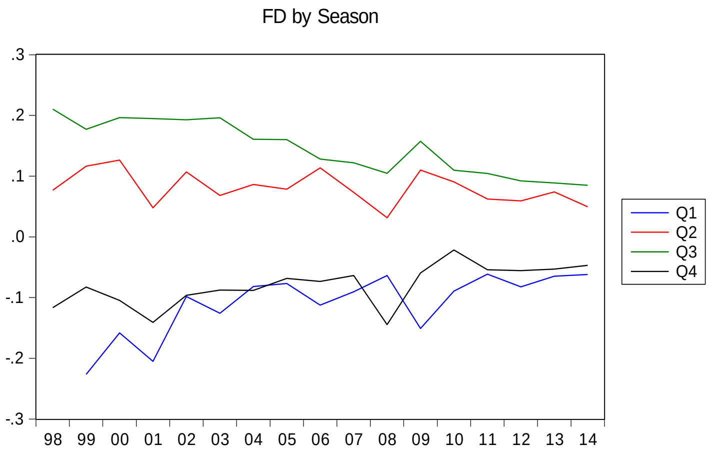


Figure 23. Graph of first-differenced lngdp series

In Figure 23, first-differenced lngdp series (namely, FD) has been graphed. For Figure 23, it can be said that quarterly means may be accepted as stationary and with this first-differencing, the growth trend effect has been removed from the gdp series. Depending on these, a seasonal deterministic model with time constant means for all of the four quarters may be accepted as a suitable one for the first differenced real gdp series. Thus, primarily it has been aimed to adopt a seasonal deterministic model for this transnformed gdp series.

As mentioned in Chapter 4.2, there are two representations of a deterministic seasonal model: Dummy variable representation and trigonometric representation. Firstly, by using the most frequently used dummy variable representation which takes place in subsection 4.2.1 and is shown as in Equation (4.1)

$$y_t = \sum_{s=1}^{S} \gamma_s \delta_{st} + z_t \quad , \quad t = 1,........,T_\tau \tag{4.1}$$

we are trying to investigate about the presence of deterministic seasonality. This analysis has been executed for the first-differenced real gdp series (dependent variable: dlngdp=lngdp-lngdp(-1)). Application results of (4.1) have been presented in Table 55. Accrding to the results in Table 55, all the seasonal dummy variables from D1 to D4 for each of the four quarters have been found to be highly significant. R-squared value of 0.884956 reveals that the explanatory power of the model is very good as a measure of goodness of fit since it is very close to 1. In addition, DW statistic (1.972045) that is close to 2 shows that there is almost no autocorrelation problem. Therefore, it can be concluded that a dummy variable representation as a seasonal deterministic model can be appropriate for Turkish GDP series.

Table 55

Dummy Variable Representation of GDP Series

		Dependent Variable: DLNGDP		
Variable	**Coefficient**	**Std. Error**	***t*-statistic**	**Prob.**
D1	-0.109426	0.009930	-11.01963	0.0000
D2	0.080682	0.009634	8.375044	0.0000
D3	0.145937	0.009634	15.14865	0.0000
D4	-0.079901	0.009634	-8.293962	0.0000
R-squared: 0.884956	Adjusted R-squared: 0.879478		DW stat: 1.972045	

Now, let us have a look at the trigonometric representation for GDP series. Recall that trigonometric representation had been given in Equation (4.6) as

$$y_t = \mu + \sum_{k=1}^{S/2}\left[\alpha_k \cos\left(\frac{2\pi kt}{S}\right) + \beta_k \sin\left(\frac{2\pi kt}{S}\right)\right] + z_t$$

and the relationship between the parameters of dummy variable and trigonometric representation can be associated as in Equation (4.11) as follows:

$$\gamma_1 = \mu + \beta_1 - \alpha_2$$
$$\gamma_2 = \mu - \alpha_1 + \alpha_2$$
$$\gamma_3 = \mu - \beta_1 - \alpha_2$$
$$\gamma_4 = \mu + \alpha_1 + \alpha_2$$

Equation (4.11) can also be represented in a different notation as in Equation (4.12):

$$\Gamma = R.B,$$

where $\Gamma = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)'$, $B = (\mu, \alpha_1, \beta_1, \alpha_2)'$ and $R = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 1 & -1 & 0 & 1 \\ 1 & 0 & -1 & -1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$.

Here $\Gamma = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)'$ matrix is composed of the seasonal means in the dummy variable representation for any season s.

By looking at the Table 55, Γ matrix which gives the seasonal means in the dummy variable representation can be expressed as,

$$\Gamma = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \end{pmatrix} = \begin{pmatrix} -0.109426 \\ 0.080682 \\ 0.145937 \\ -0.079901 \end{pmatrix}$$

Given R in quarterly case in (4.13), and the matrix of the parameters, B that is associated with the trigonometric representation can be calculated as:

$$B = \begin{pmatrix} \mu \\ \alpha_1 \\ \beta_1 \\ \alpha_2 \end{pmatrix} = R^{-1}\Gamma = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0 & -0.5 & 0 & 0.5 \\ 0.5 & 0 & -0.5 & 0 \\ -0.25 & 0.25 & -0.25 & 0.25 \end{bmatrix} . \begin{pmatrix} -0.109426 \\ 0..080682 \\ 0.145937 \\ -0.079901 \end{pmatrix} = \begin{pmatrix} 0.009323 \\ -0.0802915 \\ -0.1276815 \\ -0.0089325 \end{pmatrix}$$

Now, let us verify this result by calculating seasonal means matrix Γ:

$$\gamma_1 = \mu + \beta_1 - \alpha_2 = 0.009323 + (-0.1276815) - (-0.0089325) = -0.109426$$

$$\gamma_2 = \mu - \alpha_1 + \alpha_2 = 0.009323 - (-0.0802915) + (-0.0089325) = 0.080682$$

$$\gamma_3 = \mu - \beta_1 - \alpha_2 = 0.009323 - (-0.1276815) - (-0.0089325) = 0.145937$$

$$\gamma_4 = \mu + \alpha_1 + \alpha_2 = 0.009323 + (-0.0802915) + (-0.0089325) = -0.079901$$

And now we can verify the value of overall mean μ of y_t in (4.1). As mentioned before, the expected value of y_t had been given in the form of:

$$E(y_t) = \mu = \frac{1}{S}\sum_{s=1}^{S}\gamma_s$$

Thus, the overall mean is calculated as:

$$E(y_t) = \frac{1}{4}(-0.109426 + 0.080682 + 0.145937 - 0.079901) = 0.009323.$$

After all these, let us calculate the deterministic seasonal effect for season s which is denoted by m_s and is found by using the formula $m_s = \gamma_s - \mu$ and verify that summation of deterministic seasonal effects $\sum_{s=1}^{S} m_s = 0$ are zero:

$$m_1 = \gamma_1 - \mu = -0.109426 - 0.009323 = -0.118749$$

$$m_2 = \gamma_2 - \mu = 0.080682 - 0.009323 = 0.071359$$

$$m_3 = \gamma_3 - \mu = 0.145937 - 0.009323 = 0.136614$$

$$m_4 = \gamma_4 - \mu = -0.079901 - 0.009323 = -0.08922$$

With a summation of deterministic seasonal effects getting to zero that is shown as

$$m_1 + m_2 + m_3 + m_4 = -0.118749 + 0.071359 + 0.136614 - 0.08922 = 0,$$

these deterministic seasonal effects can be used to assess and verify the parameters α_1, α_2 and β_1 (which had been found in B matrix) in that way:

$$\alpha_1 = \frac{1}{2}\sum_{s=1}^{4} m_s \cos\left(\frac{s\pi}{2}\right) = \frac{1}{2}(-m_2 + m_4) = \frac{1}{2}(-0.071359 + (-0.08922)) = -0.0802915$$

$$\alpha_2 = \frac{1}{4}\sum_{s=1}^{4} m_s \cos(s\pi) = \frac{1}{4}(-m_1 + m_2 - m_3 + m_4)$$

$$= \frac{1}{4}(-(-0.118749) + 0.071359 - 0.136614 + (-0.08922)) = -0.0089325$$

$$\beta_1 = \frac{1}{2}\sum_{s=1}^{4} m_s \sin\left(\frac{s\pi}{2}\right) = \frac{1}{2}(m_1 - m_3) = \frac{1}{2}(-0.118749 - 0.136614) = -0.1276815.$$

Table 56

DHF Test Results for Quarterly GDP Series

Dependent Variable: D4Z				
Variable	**Coefficient**	**Std. Error**	***t*-statistic**	**Prob.**
D1	0.264860	0.378433	0.699884	0.4869
D2	0.251822	0.380409	0.661977	0.5108
D3	0.252065	0.383114	0.657936	0.5133
D4	0.262699	0.381165	0.689201	0.4936
LNGDP(-4)	-0.014497	0.022497	**-0.644416**	0.5220
D4Z(-1)	1.024267	0.126176	8.117793	0.0000
D4Z(-2)	-0.339303	0.123893	-2.738689	0.0083
R-squared: 0.645741	Adjusted R-squared: 0.607095	DW stat: 2.030687		

In Table 56, DHF test results have been presented for quarterly GDP series. As recalled from Chapter 5, DHF test can be parameterized as in Equation (5.9):

$$\Delta_s y_t = \alpha_s y_{t-s} + \varepsilon_t$$

Here the null hypothesis of seasonal integration is $\alpha_s = 0$ and the alternative of a stationary stochastic seasonal process implies $\alpha_s < 0$ (Baltagi, 2001, p. 661). Here, the dependent variable has been given as D4Z ($LNGDP_t - LNGDP_{t-4}$). LNGDP(-4) variable represents y_{t-4} in Equation (5.9). Dummy variables, first and second lagged values of the dependent variable D4Z (which are D4Z(-1) and D4Z(-2)) have been added into the DHF test regression as shown in Table 56 and lags have been determined in a way to get white noise residuals (firstly, it has been started from the Lag 1 and lags have been increased by one until the autocorrelation and heteroscedasticity problems are resolved). Here, critical *t*-value of the DHF test statistic has been taken as equal to the ADF test statistic. Thus, ADF critical value that is -1.95 has been used. According to this critical value, since *t*-value of LNGDP(-4) variable which is -0.644416 is very small in absolute value when compared to the critical value -1.95, it is concluded that the null hypothesis cannot be rejected (where the null hypothesis is $H_0 : y_t \sim SI(1)$ (Seasonal integration of order one, meaning that simultaneous existence of all four roots in quarterly series), while the alternative one is $H_1 :\ y_t$ is a stationary stochastic seasonal process). Therefore, DHF test results show that GDP series has a seasonal integration of order one process. Based on this result, it can be said that GDP series can also be modelled as a SARIMA model.

Table 57

HEGY Test Results for Quarterly GDP Series

		Dependent Variable: D4Z		
Variable	**Coefficient**	**Std. Error**	***t*-statistic**	**Prob.**
C	0.094024	0.345823	0.271886	0.7868
D1	0.003348	0.032161	0.104109	0.9175
D2	0.062682	0.040941	1.531031	0.1318
D3	0.088378	0.029407	3.005341	0.0041
Z11	-0.001789	0.005081	**-0.352000**	0.7263
Z21	-0.364257	0.159460	**-2.284324**	0.0265
Z31	-0.187522	0.091489	**-2.049658**	0.0455
Z41	-0.214609	0.090537	**-2.370408**	0.0215
D4Z(-1)	0.542410	0.175555	3.089690	0.0032
D4Z(-2)	-0.212251	0.125955	-1.685133	0.0980
R-squared: 0.729045	Adjusted R-squared: 0.682149	DW stat: 2.037153		

Table 57 presents HEGY test results for quarterly GDP series. As is seen clearly, first and second lagged values of the dependent variable have been added into the regression in order to get white-noise residuals. Here, the null hypothesis for HEGY test means that all four roots are simultaneously equal to zero (simultaneous existence of four roots, that is $\pi_1 = \pi_2 = \pi_3 = \pi_4 = 0$). The hypotheses to be tested in the HEGY test equation have been given in Equation (5.46). In Table 57, coefficients for Z11, Z21, Z31, Z41 give $\pi_1, \pi_2, \pi_3, \pi_4$ values. In order to decide about seasonal integration of order one, all of the four hypotheses ($\pi_1 = 0, \pi_2 = 0, \pi_3 = 0, \pi_4 = 0$) have to be accepted separately. For T=100 observations, critical HEGY values have been obtained from Hylleberg et al. (1990, pp. 226-227) for constant, seasonal dummies and no trend models at 5% significance level. These critical values are -2.95, -2.94, -3.44 and -1.96 respectively for π_1, π_2, π_3 and π_4. When *t*-statistics for π_1, π_2, π_3 and π_4 are compared to the critical values, it is concluded that $\pi_1 = 0, \pi_2 = 0, \pi_3 = 0$ hypotheses cannot be rejected among four hypotheses. Only $\pi_4 = 0$ hypothesis is rejected. In other saying, we can mention about the presence of unit roots at 0, ½ and ¼ frequencies**.** However, there is no ¾ frequency unit root in the series. Hence, since not all four unit roots exist according to the HEGY test results (the presence of all of the four roots is not accepted), it can be said that GDP series cannot be described by a seasonal integration of order one process. Therefore, the results for DHF test and HEGY test have differed.

When looked at the seasonal deterministic model representations and DHF and HEGY test results, the general result can be expressed as modelling first-differenced real GDP series as a seasonal deterministic model would be more suitable compared to a SARIMA model. Even though the results for dummy variable representation are positive for first-differenced GDP series, Figure 21 and Figure 22 imply that a seasonal deterministic model for GDP may not be suitable. Nevertheless, it is not certain to say about the seasonal pattern of GDP series, since DHF and HEGY test results also differ. According to the final results, it can be said that GDP series can be represented in both deterministic and stochastic structures depending on this uncertainty.

6.6. Monthly HEGY Seasonal Unit Root Test Application for Exports and Imports in Turkey

In this application, it has been aimed to detect at which frequencies seasonal unit roots exist for seasonally unadjusted monthly exports and imports series for 1975M1-2015M1 period. Data for the value of exports of goods and imports of goods have been obtained from IMF/IFS (International Monetary Fund/International Financial Statistics) and taken in units of Dollars.

Testing monthly seasonal unit roots have been summarized in Table 7 of Chapter 5. In this analysis, only significant lags have been added into the five auxiliary regression models (with only constant; constant and trend; constant and dummies; constant, trend and dummies; no deterministic components) to get white-noise residuals (that is, insignificant lags have been removed until all selected lags become significant).

Monthly HEGY seasonal unit root test results for exports series have been given in Table 58 and selected lags for HEGY regressions in Table 58 have been presented in Table 59. As well known, the hypotheses of $\pi_1 = 0$ and $\pi_2 = 0$ are tested by *t*-test and the other five hypotheses which are $\pi_3 = \pi_4 = 0$, $\pi_5 = \pi_6 = 0$, $\pi_7 = \pi_8 = 0$, $\pi_9 = \pi_{10} = 0$ and $\pi_{11} = \pi_{12} = 0$ are tested jointly by F-test.

Table 58

HEGY Monthly Seasonal Unit Root Test Results for the Value of Exports of Goods Series

Auxiliary Regression Null Hypotheses	Seasonal Frequency	Estimates for the Model with Constant	Estimates for the Model with Constant and Trend	Estimates for the Model with Constant and Dummies	Estimates for the Model with Constant, Trend and Dummies	Estimates for the Model with No Constant, No Trend and No Dummies
$\pi_1 = 0$	0	**-1.682***	**-2.150***	**-1.539***	**-1.988***	4.659
$\pi_2 = 0$	π	-4.861	-4.823	-5.541	-5.558	-4.831
$\pi_3 = \pi_4 = 0$	$\pi/2$	10.079	9.725	18.870	18.874	10.089
$\pi_5 = \pi_6 = 0$	$2\pi/3$	16.058	15.879	35.386	35.858	15.867
$\pi_7 = \pi_8 = 0$	$\pi/3$	14.574	14.960	42.536	43.278	14.418
$\pi_9 = \pi_{10} = 0$	$5\pi/6$	15.591	14.920	27.114	26.721	15.521
$\pi_{11} = \pi_{12} = 0$	$\pi/6$	8.334	8.543	9.236	9.392	8.267

Note. [1] * denotes insignificant estimates (*p>.01 and .05) at both 1% and 5% significance levels
[2] See Monthly HEGY Critical Values in Appendix G .

Table 59

Selected Lags for HEGY Monthly Seasonal Unit Root Test on Exports Series

Models	Selected Lags	Estimate	Standard Error	*t*-value	Prob (> $\|t\|$)
C	Lag.1	0.346	0.047	7.418	0.000
	Lag.2	0.148	0.046	3.230	0.001
	Lag.6	0.083	0.037	2.272	0.024
	Lag.11	-0.144	0.031	-4.654	0.000
C,T	Lag.1	0.345	0.047	7.416	0.000
	Lag.2	0.151	0.046	3.305	0.001
	Lag.6	0.095	0.037	2.561	0.011
	Lag.11	-0.130	0.032	-4.106	0.000
C,D	Lag.1	0.314	0.042	7.505	0.000
	Lag.6	0.119	0.036	3.353	0.001
	Lag.9	-0.086	0.041	-2.125	0.034
	Lag.11	-0.179	0.046	-3.897	0.000
	Lag.12	0.106	0.041	2.598	0.010
C,D,T	Lag.1	0.314	0.042	7.520	0.000
	Lag.6	0.127	0.036	3.544	0.000
	Lag.9	-0.080	0.041	-1.960	0.051
	Lag.11	-0.175	0.046	-3.804	0.000
	Lag.12	0.112	0.041	2.747	0.006
-	Lag.1	0.351	0.047	7.498	0.000
	Lag.2	0.150	0.046	3.258	0.001
	Lag.6	0.081	0.037	2.195	0.029
	Lag.11	-0.136	0.031	-4.432	0.000

Note. “C” denotes constant term, “T” denotes trend, “D” denotes seasonal dummy variables and “-” denotes no deterministic components.

If Table 58 is examined thoroughly, the results for the hypothesis $\pi_1 = 0$ show that the presence of the zero (non-seasonal) frequency unit root (the null hypothesis of $\pi_1 = 0$) cannot be rejected at 1% and 5% significance levels for all deterministic models (except no deterministic component model). Thus, it can be said that exports series is non-stationary at zero (long-run) frequency. When the results for other hypotheses except $\pi_1 = 0$ are examined, these hypotheses implying the presence of a unit root at seasonal frequency are seen to be rejected for all deterministic models for 1%, 5% and 10% significance levels and hence the conclusion is that there are no seasonal unit roots at $\pi, \pm\frac{\pi}{2}, \pm\frac{2\pi}{3}, \pm\frac{\pi}{3}, \pm\frac{5\pi}{6}$ and $\pm\frac{\pi}{6}$ seasonal frequencies.

Table 60

HEGY Monthly Seasonal Unit Root Test Results for the Value of Imports of Goods Series

Auxiliary Regression Null Hypotheses	Seasonal Frequency	Estimates for the Model with Constant	Estimates for the Model with Constant and Trend	Estimates for the Model with Constant and Dummies	Estimates for the Model with Constant, Trend and Dummies	Estimates for the Model with No Constant, No Trend and No Dummies
$\pi_1 = 0$	0	**-0.199***	**-3.338***	**-0.279***	**-3.246***	3.727
$\pi_2 = 0$	π	-3.681	-4.618	-4.740	-4.501	-3.679
$\pi_3 = \pi_4 = 0$	$\pi/2$	11.863	14.277	30.709	30.905	11.889
$\pi_5 = \pi_6 = 0$	$2\pi/3$	9.843	13.497	32.676	30.831	9.847
$\pi_7 = \pi_8 = 0$	$\pi/3$	13.570	15.494	51.398	51.756	13.583
$\pi_9 = \pi_{10} = 0$	$5\pi/6$	13.317	14.459	18.876	19.606	13.318
$\pi_{11} = \pi_{12} = 0$	$\pi/6$	18.707	25.713	39.387	38.754	18.728

Note. [1] * denotes insignificant estimates (*p>.01 and .05) at both 1% and 5% significance levels
[2] See Monthly HEGY Critical Values in Appendix G .

Monthly HEGY seasonal unit root test results for imports series have been presented in Table 60 and selected lags for HEGY regressions in Table 60 have been presented in Table 61. Table 60 results are the same as Table 58 results. It is clear to see that as in the case of exports series, once again the unit root hypothesis with zero frequency (that is, $\pi_1 = 0$) cannot be rejected for all deterministic models (except no deterministic component model). Thus, imports series is also not stationary and with an examination of other hypotheses except $\pi_1 = 0$ in Table 60, it is concluded that there are no seasonal unit roots at no seasonal frequency as in the exports series as a result of

F-calculated values being greater than the critical table values (therefore, the null hypotheses are rejected).

Table 61

Selected Lags for HEGY Monthly Seasonal Unit Root Test on Imports Series

Models	**Selected Lags**	**Estimate**	**Standard Error**	***t*-value**	**Prob ($> \|t\|$)**
C	Lag.1	0.151	0.048	3.129	0.002
	Lag.2	0.172	0.046	3.696	0.000
	Lag.12	-0.110	0.031	-3.602	0.000
C,T	Lag.1	0.169	0.048	3.543	0.000
	Lag.2	0.195	0.047	4.152	0.000
	Lag.7	-0.073	0.037	-1.969	0.050
C,D	Lag.1	0.106	0.047	2.244	0.025
	Lag.7	-0.094	0.032	-2.959	0.003
C,D,T	Lag.1	0.118	0.047	2.497	0.013
	Lag.7	-0.081	0.039	-2.063	0.040
	Lag.12	0.046	0.035	1.308	0.191
-	Lag.1	0.151	0.048	3.132	0.002
	Lag.2	0.172	0.046	3.694	0.000
	Lag.12	-0.111	0.030	-3.648	0.000

Note. "C" denotes constant term, "T" denotes trend, "D" denotes seasonal dummy variables and "-" denotes no deterministic components.

If we take both exports and imports series into consideration, it can be concluded that both series include a non-seasonal unit root and no seasonal unit roots. Based on this, seasonal differencing is not required for two series and since they are non-stationary, their first differences have to be taken. Briefly, there is no need to apply the seasonal difference filter $(1-L^{12})$ for two series and seasonal cycles are mostly in a deterministic structure.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

7.1. Conclusions

In this study, various seasonality analyses have been conducted for some macroeconomic variables. In the first part of the study, why the concept of seasonality is important, what the negative aspects of ignoring the presence of seasonality are, what the structural features of the series that are subject to seasonality are will be examined; the importance of detecting what kind of seasonality (deterministic or stochastic) exists in the data worked in modelling seasonality for conducted analyses, methods related to seasonal integration and seasonal unit root analyses will be presented. Since most time series display seasonality feature substantially and seasonal unit root analyses will be conducted through HEGY procedure which is the most popular approach of seasonal unit root analyses have been expressed.

In the second part of the study; various studies concerning seasonal patterns, seasonal cointegration and seasonal integration (seasonal unit roots), modelling seasonal behaviour of various macroeconomic series, the comparison of seasonal cycle and business cycle and the detection of deterministic and stochastic seasonality have been summarized.

In the third part of the study, the concept of time series has been introduced since seasonality is a component of time series and also what the seasonal adjustment is, various seasonality tests and theoretical structures of various seasonal processes (stochastic stationary seasonal processes, nonstationary unit root processes, SMA and SARIMA models and so on) have been explained.

In the fourth part; the concept of deterministic seasonality, its two representations which are dummy variable and trigonometric representations, deterministic seasonality tests which are CH test, Caner test and Tam Reinsel test, asymptotical features of seasonal random walk have been presented in details by considering deterministic and stochastic seasonality together.

In the fifth part; economic theory has been put forward in order to set light to the route to be followed in practice. Various tests concerning how seasonal integration orders will be determined have been presented by taking the study of Ilmakunnas (1990) as basis. In addition; various seasonal unit root tests (DHF, HEGY, KUNST, OCSB etc.) based on quarterly-monthly-bimonthly-weekly-biannual data, frequencies of seasonal unit roots and filters and tested hypotheses corresponding these frequencies and seasonal cointegration have been given place in details.

In the sixth part, various seasonality analyses have been applied to some macroeconomic variables. In the first application of this part, it has been tried to model monthly inflation rates by utilizing from SARIMA approach that considers both seasonal and nonseasonal behaviour in Turkey for 1995:1-2015:3 period. When looked at the ACF and PACF of the series, seasonal lags (12, 24, 36, 48) have been found to be significant. Depending on this, in order to find the best-fitted SARIMA model, the presence of seasonal unit roots has been checked for the nonstationary inflation series. For the monthly HEGY seasonal unit root test, three different lag order selection methods have been used (selection of significant lags, AIC, BIC). As a result, all three methods have showed only the presence of conjugate complex seasonal unit roots at $\pm\frac{\pi}{3}$ frequencies corresponding to (2,10) cycles per year and it has been concluded that seasonal cycles mostly display a deterministic structure. Hence it is not required to apply seasonal difference operator $(1-L^{12})$ to inflation series. Instead, depending on the $\pm\frac{\pi}{3}$ frequencies, inflation series has been transformed by the necessary filter corresponding to these frequencies which is $(1-L+L^2)$ and since it includes zero frequency unit root, transformation of the series has been $(1-L)(1-L+L^2)$. In addition, CH test results (with L statistic: 2.005) have also revealed that seasonal pattern is deterministic. This result is seen to be consistent with the HEGY result. According to the OCSB and CH tests, the order of seasonal differencing has been determined as zero and the number of first differences has been determined to be 1 according to KPSS and ADF test results. These results are consistent with the evidence given above. In the model identification, when all AICc, AIC and BIC criteria are taken into account together, the best model under the stepwise selection method among other suggested ARIMA models has been determined as ARIMA(1,1,1)(1,0,2)[12] model with drift with the smallest AIC value of 2405.484. Apart from the (faster) stepwise selection, the best model under the (slower) non-stepwise selection has been chosen as ARIMA(1,1,1)(2,0,0)[12] with drift model. For this model, all assumptions regarding normality of residuals and constant variance have been satisfied except the fact that residuals are not independently distributed for seasonal lag 36. When stepwise and non-stepwise results are compared with forecast accuracy measures, the model with stepwise selection has been regarded as the best-fitted model for forecasting monthly inflation rates in Turkey.

In the second application, different seasonal integration tests have been applied in a unified approach for inflation and growth series and after determining seasonal integration orders, the cointegration relationship between them has been examined. Based on the study of Ilmakunnas (1990), it has been expressed that the conclusion on the appropriate order of integration depends on the starting point of testing sequence. If starting point is SI(2,1), it can be said that growth variable becomes stationary mostly after 1st differencing and seasonal differencing (that is, the null of SI(2,1) is accepted) and inflation series can be accepted as either SI(2,1) or SI(1,0). If starting point is the case of quarterly differencing (that is, SI(1,1)), the results are not certain for both variables: while inflation may be regarded as SI(1,1) in most cases, growth may be accepted as SI(0,1), SI(1,1) or SI(0,0). So, according to the starting points, the results have differed. Also, the cointegration relationship for growth equation in which economic growth is dependent variable while inflation is independent has been investigated in a different manner by taking the concept of seasonality into consideration. Empirical results have revealed that all forms of the variables (level form, seasonally averaged form, seasonally differenced form, first differenced form) except twice-differenced form show the sign of cointegration with the significant residual test statistics which are DW, DF and ADF tests. Thus, with this analysis, the variables have been found to be SI(1,1). Since seasonally averaged ($S(L)$) variables have been found to be cointegrated of order 1 at zero frequency and first-differenced variables have been found to be cointegrated at seasonal frequencies. Therefore, it is concluded that it would be suitable to incorporate the variables in Δ_4 form into the regression.

In the third application, whether a cointegration relationship exists or not between quarterly GDP, CONS, EXP, GOV and PRIEQ series has been investigated. As a result of HEGY application, the presence of a zero frequency (nonseasonal) unit root has been detected for all series for the three models with “constant”, “constant+dummies” and “constant+dummies+trend”. LNGDP, LNCONS and LNGOV series have been found to include a seasonal unit root at semi-annual frequency. In addition, LNGDP, LNGOV and LNEXP series have been detected to have seasonal unit roots at quarterly $\frac{1}{4}\left(\frac{3}{4}\right)$ frequencies. It should be noted that cointegration analysis should be evaluated among the series having unit roots at the same frequency. When cointegration test results are evaluated thoroughly at the zero (long-run) frequency, there has been found no cointegrating relationship between LNGDP & LNCONS, LNGDP & LNPRIEQ, LNGDP & LNGOV, LNGDP & LNEXP at 5% significance level. Similarly, no cointegrating relationship has been detected between LNGDP&LNCONS series and LNGDP&LNGOV series at semi-annual (½) frequency. However, there has been found a cointegrating relationship between LNGDP & LNGOV series at quarterly $\frac{1}{4}\left(\frac{3}{4}\right)$ frequencies for only the model with "constant+dummies". On the other hand, no cointegrating relationship has been found between LNGDP & LNEXP series for no models at these quarterly frequencies.

In the fourth application, the kind of seasonality (whether it is deterministic or stochastic) has been tried to be determined for quarterly unemployment series over 1988Q1-2014Q4 period. According to *t* and Q statistics, since the first condition saying that α coefficients are equal to each other and the second one saying that all $\pi's$ are equal to zero do not hold, it is concluded that stochastic seasonality does not exist in the series. While there is a zero frequency unit root in the series for the models with "constant" and "constant+dummies", no biannual and annual unit roots have been found for no models. Hence, the general result is that seasonal fluctuations in the series have not been able to emerge in the six-month and one- year intervals. However, CH test result shows that seasonal pattern is not deterministic (indicating to the presence of seasonal unit root) and this result is completely different from the previous result which is based on *t* and Q statistics saying that there is no stochastic seasonality in the series. So, it can be said that two methods say different things regarding the type of seasonal pattern in the series and it is not certain to say that unemployment series displays only one type of seasonal behaviour. Thus, it may have both deterministic and stochastic structure.

In the fifth application, it has been tried to detect the kind of seasonality of quarterly GDP series in Turkey over 1998Q1-2014Q4 period. While DHF test results show that GDP series can be described by a seasonal integration of order 1 process (that is, GDP series can be modelled as a SARIMA model), HEGY test results say the opposite. On the other hand, all dummy variables have been found to be significant for dummy variable representation meaning that modelling first-differenced GDP series as a deterministic model can be more suitable. However, when all results are taken into account together, it is not certain to say that GDP series has only a deterministic structure. It can be said that the series can be represented in both deterministic and stochastic structures.

As a result, as summarized in fourth and fifth applications, generally it is possible to say that most macroeconomic series can display both a deterministic and stochastic structure.

In the sixth application, it has been tried to detect at which frequencies there are seasonal unit roots for monthly exports and imports series. According to the results, both series have been found to include a non-seasonal unit root and no seasonal unit roots. Depending on this evidence, it is concluded that seasonal differencing filter which is $(1-L^{12})$ is not required for two series (but they should be in first-differenced form because of the zero frequency unit root) and seasonal cycles are said to be mostly in a deterministic structure. It can be inferred from this application that even though the data are available on monthly basis, they may not include any seasonal unit roots. Thus, we cannot say that data which are collected at monthly or quarterly or any other basis include surely seasonal unit roots.

7.2. Recommendations

It is wrong to say that each time series collected at seasonal basis (quarterly, monthly etc.) includes seasonal unit roots. A series should be subject to the $(1-L^{s})$ filter where s is the length of the period only if it includes unit roots at all frequencies. Otherwise, if this filter is applied to the series in question in case there is a unit root at only one frequency, inaccurate results can be obtained. Briefly, as expressed in Beaulieu and Miron (1992b), “The appropriateness of applying the filter $(1-L^{d})$ to a series with a seasonal component, as advocated by Box and Jenkins (1970) depends on the series being integrated at zero and all of the seasonal frequencies” (p.18). Therefore, if a series has unit roots at which frequencies, filters corresponding to those frequencies should be applied to the series in interest in order to make it stationary.

For all analyses conducted so far, all series have been taken as seasonally unadjusted. Since in case seasonally adjusted data are used, there will be a bias in ADF and PP statistics toward non-rejection of the unit root. Therefore, it has been expected that unit root test are more powerful when worked with unadjusted data (Maddala & Kim, 1998, pp. 364-365).

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APPENDICES

APPENDIX A1: Brownian Motion

Botanist Robert Brown explained the motion of a pollen particle suspended in fluid in 1828 and the observations on these particle movements revealed that they move in an unsteady and random manner. In 1905, Albert Einstein derived the equations related to Brownian motion claiming that the movements are stemmed from bombardment of the particle by the molecules of the fluid. In 1900, Brownian motion was used by Louis Bachelier as a model for movement of stock prices and since the mathematical foundation of Brownian motion as a stochastic process is based on Norbert Wiener, it is also called the Wiener process (Klebaner, 2005, p. 56).

The concept of Brownian motion is closely related to the normal distribution. It is well known that a random variable has a normal distribution with mean μ and variance σ^2, where $\mu \in \mathrm{R}$ and $\sigma > 0$, if the probability density function of a random variable is $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, then it is said that it has a normal distribution with mean μ and variance σ^2, where $\mu \in \mathrm{R}$ and $\sigma > 0$.

Definition 1.1. A stochastic process $\{B(t): t \geq 0\}$ is said to be a Brownian motion process with a variance parameter $\sigma^2 > 0$ if:

(i) $B(0) = 0$

(ii) (independent increments) For each $0 \leq t_1 < t_2 < ...... < t_m$, $B(t_1), B(t_2) - B(t_1), ..., B(t_m) - B(t_{m-1})$ are independent random variables.

(iii) (stationary increments) For each $0 \leq s < t$, $B(t) - B(s)$ has a normal distribution with mean zero and variance $\sigma^2 (t - s)$.

This process operates as a basic model for the cumulative effect of pure noise. If $B(t)$ is the position of a particle at time *t*, then the displacement $B(t) - B(0)$ gives the effect of noise over time *t* (Klebaner, 2005, p. 56).

If $\sigma^2 = 1$, the stochastic process $B(t)$ becomes a standard Brownian motion and if $\{B(t): t \geq 0\}$ is a Brownian motion process with $\sigma^2 > 0$, then $\{\sigma^{-1} B(t): t \geq 0\}$ represents a standard Brownian motion (this concept will also be mentioned in Wiener process).

Theorem 1.1. Now let $\{B(t): t \geq 0\}$ be a standard Brownian motion. Then, the probability density function of $B(t)$ is given as:

$$f_{B(t)}(x) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}}$$

(http://home.cc.umanitoba.ca/~thavane/ASS305/5bm.pdf).

APPENDIX A2: Wiener Process

In mathematics, the Wiener process is a continuous-time stochastic process named in honour of Norbert Wiener. It is mostly called standard Brownian motion, after Robert Brown and it occurs frequently in pure and applied mathematics, economics, quantitative finance and physics. In applied mathematics, one of the reasons to use this process is to represent the integral of a Gaussian white noise process, and so is useful as a model of noise in electronics engineering (Wikipedia, 2010). This process is a kind of Markov stochastic process (The Markov stochastic process is a particular type of stochastic process where only the current value of a variable is relevant for predicting the future movement). Essentially, the Wiener process is a series of normally distributed random variables, and for later time points, the variances of these variables rise to bring as a consequence that it is more uncertain and therefore harder to predict the value of the process after a longer period (http://homepage.ntu.edu.tw/~jryanwang/course/Financial%20Computation%20or%20Financial%20Engineering%20(graduate%20level)/FE_Ch01%20Wiener%20Process.pdf).

Definition 2.1. A Gaussian, continuous parameter process characterized by mean value $m(t) = 0$ and covariance function $\varphi := \min\{s,t\} = s \wedge t$, for any $s,t \in [0,T]$, is called a Wiener process (denoted by $\{W_t\}$).

It is remarkable to say that the next definition can also provide a more intuitive description of the fundamental properties of a process of this kind.

Definition 2.2. A stochastic process $\{W_t\}_{t\geq 0}$ is called a Wiener process if it satisfies the properties expressed below:

1. $W_0 = 0$
2. The function $t \rightarrow W_t$ is almost surely continuous in t (with probability 1).

3. W_t has stationary, independent increments with $W_t - W_s \sim N(0, t-s)$ for $\forall s, t \in [0, T]$, with $0 \le s < t$ [that is, the variance of the change is equal to the distance between points]. This implies that taking s=0, $W_t - W_0$ has $N(0,t)$ distribution.

These two definitions are equivalent.

Proof:

$(2.1) \Rightarrow (2.2)$

The independence of increments is a usual consequence of the Gaussian structure of Wt. In actuality, W_t and $W_t - W_s$ can be expressed as a linear combination of Gaussian random variables,

$$(W_s, W_t - W_s) = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} . \begin{pmatrix} W_s \\ W_t \end{pmatrix}$$

Furthermore, they are not correlated and therefore independent:[4]

$$a_{ij} = a_{ji} = E[(W_{t_i} - W_{t_{i-1}})(W_{t_j} - W_{t_{j-1}})] = ... = t_i - t_{i-1} + t_{i-1} - t_i = 0$$

Now Wiener process can be written as:

$$W_t = (W_{\frac{t}{N}} - W_0) + (W_{\frac{2t}{N}} - W_{\frac{t}{N}}) + \ldots + (W_{\frac{tN}{N}} - W_{\frac{t(N-1)}{N}})$$

then,

$$W_{\frac{kt}{N}} - W_{\frac{(k-1)t}{N}} \sim N(0, \frac{T}{N})$$

$(2.2) \Rightarrow (2.1)$

On the other hand, the 'shape' of the increments distribution implies that a Wiener process is a Gaussian process. As expected, for any $0 < t_1 < t_2 < .... < t_n$, a random vector $(W_{t1}, W_{t2}, ..., W_{tn})$ has a normal probability distribution. Since it is a linear combination of the vector $(W_{t1}, W_{t2} - W_{t1}, ..., W_{tn} - W_{tn-1})$ whose components are Gaussian by definition (Caiaffa, 2011-2012).

Properties of a One Dimensional Wiener Process

Basic properties:

The unconditional probability density function at a fixed time *t* is:

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[4] It should be noted that under the assumption $s < t$, the relations $E(W_t) = m(t) = 0$. $Var(W_t) = \min\{t, t\} = t$, $E(W_t - W_S) = 0$ and $E[(W_t - W_S)^2] = t - 2s + s = t - s$ are valid.

$$f_{W_t}(x) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}}$$

*The expectation is: $E[W_t] = 0$

*The variance is: $Var(W_t) = E[W_t^{\,2}] - E^2[W_t] = E[W_t^{\,2}] - 0 = E[W_t^{\,2}] = t$

The results obtained for the mean and variance follow from the definition that increments have a normal distribution centered at zero. Therefore,

$W_t = W_t - W_0 \sim N(0,t)$.

*The covariance and correlation are:

$\mathrm{cov}(W_s, W_t) = \min(s,t)$,

$$cor(W_s, W_t) = \frac{\mathrm{cov}(W_s, W_t)}{\sigma w_s \sigma w_t} = \frac{\min(s,t)}{\sqrt{st}} = \sqrt{\frac{\min(s,t)}{\max(s,t)}}$$

The results obtained for the covariance and correlation follow from the definition that non-overlapping increments are independent (they are uncorrelated). Assume that $t_1 < t_2$.

$\mathrm{cov}(W_{t1}, W_{t2}) = E[(W_{t1} - E[W_{t1}]).(W_{t2} - E[W_{t2}])] = E[W_{t1}.W_{t2}]$

Substituting

$W_{t2} = (W_{t2} - W_{t1}) + W_{t1}$, we obtain:

$E[W_{t1}.W_{t2}] = E[W_{t1}.((W_{t2} - W_{t1}) + W_{t1})] = E[W_{t1}.(W_{t2} - W_{t1})] + E[W_{t1}^{\,2}]$.

Since we know $W(t_1) = W(t_1) - W(t_0)$ and $W(t_2) - W(t_1)$ are independent,

$E[W_{t1}.(W_{t2} - W_{t1})] = E[W_{t1}].E[W_{t2} - W_{t1}] = 0$ and therefore,

$\mathrm{cov}(W_{t1}, W_{t2}) = E[W_{t1}^{\,2}] = t_1$

(Wikipedia, 2010).

The Wiener Process as a Scaled Random Walk

Consider a simple random walk $\{X_n\}_{n \in N}$ on the lattice of integers Z:

$$X_n = \sum_{k=1}^{n} \xi_k$$

where $\{\xi_k\}_{k \in N}$ is a sequence of independently, identically distributed (i.i.d.) random variables with $P(\xi_k = \pm 1) = \frac{1}{2}$. According to the Central Limit Theorem (CLT),

$\frac{X_N}{\sqrt{N}} \to N(0,1)$ is a Gaussian variable with mean zero and variance one in distribution as $N \to \propto$. This suggests defining the piecewise constant random function W_t^N on $t \in [0, \propto)$ as follows:

$$W_t^N = \frac{X_{\lfloor Nt \rfloor}}{\sqrt{N}},$$

where $\lfloor Nt \rfloor$ stands for the largest integer less than Nt and in order to be compatible with general notations for stochastic processes, here t is written as a subscript, i.e. $W_t^N = W^N(t)$. It can be said that as $N \to \propto$, W_t^N converges in distribution to a stochastic process W_t denoting the Wiener process (https://www.cscamm.umd.edu/lectures/EVandenLectures_final.pdf).

APPENDIX A3: Brownian Bridge

Definition 3.1. A standard Brownian bridge is a Gaussian process X with continuous paths, mean zero and covariance function $Cov(X(s), X(t)) = s.(1-t)$ for $0 \le s \le t \le 1$.

A standard Brownian bridge over the interval $t \in [0,1]$ is a standard Brownian motion $W(.)$ given the condition that $W(1) = 0$. If expressed in a clear way:

$X_t := (W(t) / W(1) = 0)$.

The variance of the Brownian bridge is $t(1-t)$ which implies that the most uncertainty takes place in the middle of the bridge. Also, it should be noted that the increments in a Brownian bridge are not independent.

If $W(t)$ is a standard Brownian motion (for $t \ge 0$, normally distributed with expected value zero and variance *t*, and with stationary and independent increments), then Brownian bridge can be expressed as:

$X(t) = W(t) - tW(1)$.

Although a standard Wiener process satisfies $W(0) = 0$ and so it is “tied down” to the origin, a Brownian bridge process requires not only $X(0) = 0$, but also $X(1) = 0$ implying that this process is also “tied down” at time 1 to have the value zero. (Wikipedia, n.d.; Chang, 2007).

APPENDIX A4: Distribution Theory for Autoregressive Unit Root Tests

Consider the following simple autoregressive AR(1) process:

$y_t = \phi y_{t-1} + \varepsilon_t$ with $t = 1,........,T$ where $\varepsilon_t \sim WN(0,\sigma^2)$ is a white noise process. In order to test for a unit root, the null and alternative hypothesis will be as in the following way:

$H_0 : \phi = 1$ (implies the existence of unit root – nonstationarity case)

$H_1 : |\phi| < 1$ (implies the presence of no unit root – stationarity case)

The test statistic becomes $t_{\phi=1} = \dfrac{\hat{\phi}-1}{SE(\hat{\phi})}$ where $SE(\hat{\phi})$ denotes the standard error of least squares estimate $\hat{\phi}$ and OLS estimation of ϕ is given as $\hat{\phi} = \dfrac{\sum y_t y_{t-1}}{\sum y_{t-1}^2}$.

First, assume that $|\phi| < 1$, so the process is stationary. In that case, the asymptotic results for stationary AR(1) process take place in the standard framework used for the basic linear regression model given as:

$\sqrt{T}(\hat{\phi}-\phi) \xrightarrow{d} N(0,(1-\phi^2))$ (Hamilton, 1994, p. 216)

or

$\hat{\phi} \sim N(\phi, \frac{1}{T}(1-\phi^2))$ and it follows that $t_{\phi=\phi_0} \sim N(0,1)$.

In order to make inferences about the interested null hypothesis, a limiting distribution of a suitable standardized version of $\hat{\phi}$ should be available. However, there is a problem that under the unit root null, since y_t follows a nonstationary process (random walk) the basic assumptions underlying CLT fail to hold and therefore we have to rely on less-standard asymptotic theory based on the concepts like the Functional Central Limit Theorem (FCLT), the Continuous Mapping Theorem and Brownian motions in order to study the behavior of such a statistic. When $\phi = 1$, the variance of asymptotic result becomes zero which apparently does not make any sense ($\text{var}(\hat{\phi}) = 0$ and therefore $\sqrt{T}(\hat{\phi}-1) \to 0$). That is, $\hat{\phi}$ has no longer a standard distribution and the usual sample moments do not converge to fixed constants. Here it will be shown that the statistic $T(\hat{\phi}-1)$ features a convergent distribution for the null hypothesis of a random walk. Because the resulting asymptotic distribution is not standard, it requires the Brownian motion process (It is a zero-mean normally distributed continuous process with independent increments i.e., loosely speaking, the continuous version of the discrete random walk (Dolado, Gonzalo & Marmol, 1999)). So, Phillips (1987) showed that the sample moments of y_t converge to random functions of Brownian motions.

Replacing y_t given in $y_t = \phi y_{t-1} + \varepsilon_t$ in OLS estimation of ϕ given above,

$$\hat{\phi} = \frac{\sum y_t y_{t-1}}{\sum y_{t-1}^2} = \frac{\sum (\phi y_{t-1} + \varepsilon_t) y_{t-1}}{\sum y_{t-1}^2} = \frac{\phi \sum y_{t-1}^2 + \sum y_{t-1}\varepsilon_t}{\sum y_{t-1}^2}$$

$$\hat{\phi} = \phi + \frac{\sum y_{t-1}\varepsilon_t}{\sum y_{t-1}^2} \rightarrow under \quad \phi = 1, \; \hat{\phi} - 1 = \frac{\sum y_{t-1}\varepsilon_t}{\sum y_{t-1}^2}$$

we get,

$$T(\hat{\phi}-1) = \frac{(1/T)\sum y_{t-1}\varepsilon_t}{(1/T^2)\sum y_{t-1}^2}$$

Now, first consider the asymptotic behavior of the nominator part. Under the null hypothesis,

$$y_t^{\ 2} = (y_{t-1} + \varepsilon_t)^2 = y_{t-1}^2 + 2y_{t-1}\varepsilon_t + \varepsilon_t^{\ 2}$$

Then, $y_{t-1}\varepsilon_t$ becomes

$$y_{t-1}\varepsilon_t = \frac{1}{2}(y_t^2 - y_{t-1}^2 - \varepsilon_t^2)$$

Summing over t, dividing by T and also supposing that the initial value $y_0 = 0$

$$\sum_t y_{t-1}\varepsilon_t = \frac{1}{2}(\sum_t y_t^2 - \sum_t y_{t-1}^2 - \sum_t \varepsilon_t^2) = \frac{1}{2}(y_T^{\ 2} - y_0^{\ 2}) - \frac{1}{2}\sum_t \varepsilon_t^{\ 2}$$

$$= \frac{1}{T}\sum_t y_{t-1}\varepsilon_t = \frac{1}{2T} y_T^{\ 2} - \frac{1}{2T}\sum_t \varepsilon_t^{\ 2}$$

Let us have a look at the first term on the right side of the equation. Under the null hypothesis of $\phi = 1$, y_T ~ $N(0, \sigma^2 T)$ (since, y_T is a random walk process: $y_T = \varepsilon_1 + \varepsilon_2 + ... + \varepsilon_{T-1} + \varepsilon_T$ with $y_0 = 0$). So, $\frac{y_T}{\sigma\sqrt{T}}$ ~N(0,1).

Let us continue from the previous equation. Dividing it by σ^2, we get

$$\frac{1}{T\sigma^2}\sum_t y_{t-1}\varepsilon_t = \frac{1}{2}(\frac{y_T}{\sqrt{T}\sigma})^2 - \frac{1}{2\sigma^2} . \frac{1}{T}\sum_t \varepsilon_t^{\ 2}$$

Again look at the first term on the right side,

$$(\frac{y_T}{\sigma\sqrt{T}})^2 \sim \chi^2(1)$$

and the second term $\frac{1}{T}\sum \varepsilon_t^{\ 2} \rightarrow \sigma^2$ converges to σ^2 in probability (by the law of large numbers). Then the nominator converges to $\frac{1}{T\sigma^2}\sum_t y_{t-1}\varepsilon_t \xrightarrow{D} \frac{1}{2}(\chi_1^{\ 2} - 1)$.

Now in order to deal with the asymptotic behavior of a random process in the denominator part, we need to know about the Brownian Motions and FCLT.

Let { ε_t } be a sequence of i.i.d. random variables with mean zero and variance σ^2 . Define a partial sum process S_i as $S_i = \sum_{t=0}^{i} \varepsilon_t$. This partial sum process is a discrete process with values $i = 1,......,T$. Here the aim is to describe a standardized continuous process over the interval [0,1]. For this, the process should be redefined as a standardized discrete form over the points i/T as: $X(r = i/T) = \frac{S_i}{\sigma\sqrt{T}}$, $r = 0,1/T,..........,T/T$. So, we standardized the S_i process by its total variance and adjusted the time scale in a way to go from 0 to 1. Now, in order to describe a continuous process from this discrete form, we introduce a notation $[Tr]$ and let it be the integer part of Tr . Then, the continuous process can be expressed as:

$$X_T(r) = \sum_{t=0}^{[Tr]} \frac{\varepsilon_t}{\sigma\sqrt{T}} , \quad r \in [0,1]$$

This function satisfies the whole conditions of the FCLT.

Definition: Let $X_T(r)$ be the continuous process expressed above. Then, $X_T(r)$ converges to $W(r)$, a continuous time process known as standard Brownian motion.

This definition tells us that the process constructed by adding more points eventually converges to a well-defined process. So, $X_T(r) \Rightarrow W(r)$.

Now, in order to derive the asymptotic behaviour of denominator part $T^{-2}\sum y_{t-1}^2$, applying the definition of $Y_T(r)$ we can write y_{t-1} as follows:

$$y_{t-1} = Y_T(r)\sigma\sqrt{T}$$

and squaring and dividing this expression by T^2 , we get

$$(1/T^2)y_{t-1}^2 = Y_T^2(r)\sigma^2/T$$

Because of the constancy and continuity of this expression for r in the interval $[(t-1)/T, t/T]$, it can be rewritten as

$$(1/T^2)y_{t-1}^2 = \int_{(t-1)/T}^{t/T} Y_T^2(r)\sigma^2 dr$$

Since the denominator part has a summation, the summation form of this expression over *t* becomes

$$T^{-2}\sum_{t=1}^{T} y_{t-1}^2 = \sum_{t=1}^{T} \int_{(t-1)/T}^{t/T} Y_T^2(r)\sigma^2 dr = \int_0^1 Y_T^2(r)\sigma^2 dr$$

According to FCLT, as $t \to \propto$ $Y_T(r)$ converges to the Brownian motion $W(r)$. In conclusion, the asymptotic behaviour of the denominator can be given as

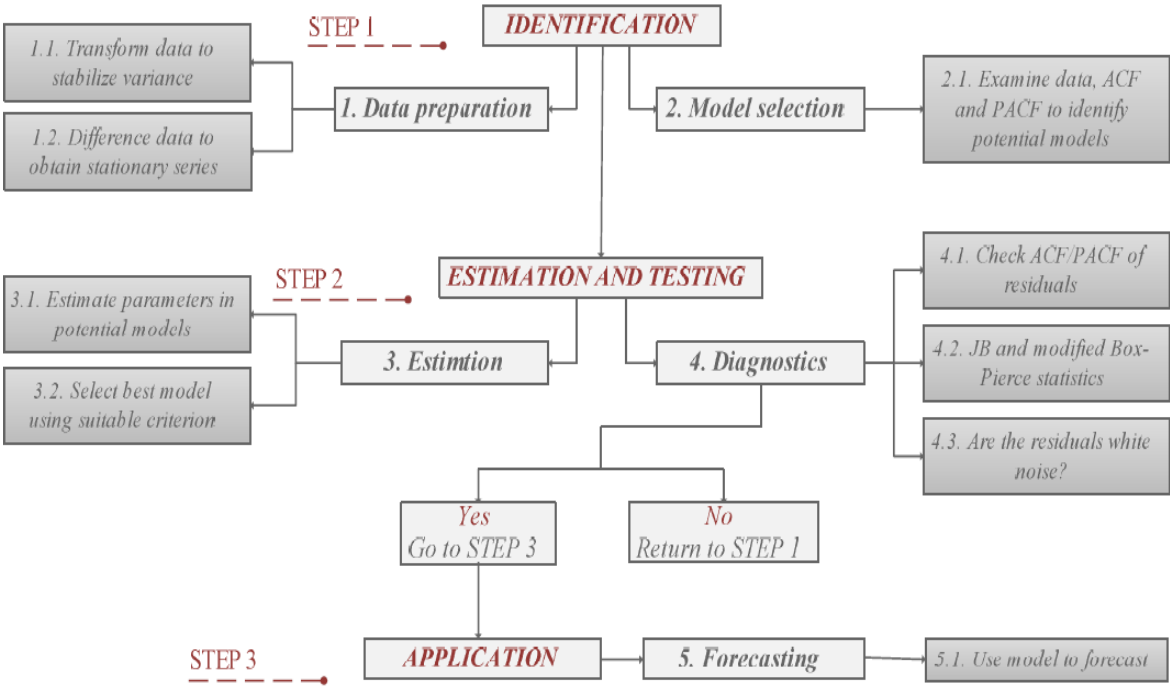
$$T^{-2}\sum_{t=1}^{T} y_{t-1}^2 \Rightarrow \sigma^2 \int_0^1 [W(r)]^2 dr$$

After cancelling σ^2, the limiting behaviour of $T(\hat{\phi}-1) = \dfrac{(1/T)\sum y_{t-1}\varepsilon_t}{(1/T^2)\sum y_{t-1}^2}$ can be expressed as follows:

$$T(\hat{\phi}-1) \Rightarrow \frac{(1/2)(W(1)^2-1)}{\int W(r)^2 dr} = \frac{\int W(r)dW(r)}{\int W(r)^2 dr} \text{ where } \int_0^1 W(r)dW(r) = \frac{1}{2}\{W(1)^2 - 1\}$$

It should be remembered that $W(1) = W(1) - W(0)$ depending on the assumption that the Brownian motion starts at zero and its square as a standard normal variable is a chi-square variable with one degree of freedom. As a conclusion, it can be inferred that although the OLS-like statistic is not a standard one in fact, its appropriate standardization has a limiting distribution including Brownian motions (Escudero, 2001; Unit Root Tests, n.d.)

APPENDIX B: Schematic Representation of the Box-Jenkins Methodology



Source: Bigović, 2012, p. 6.

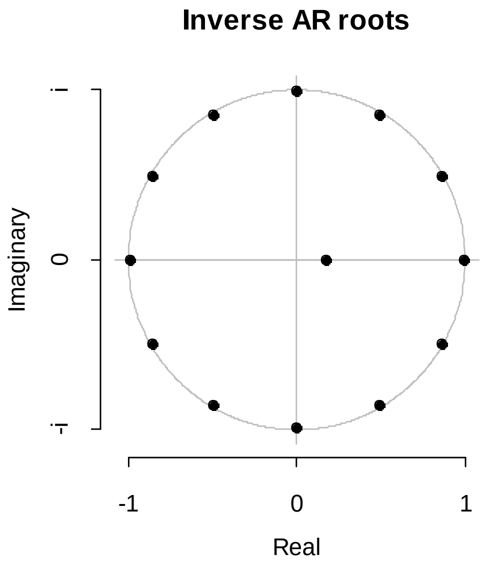
APPENDIX C : Monthly HEGY Critical Values for 240 Observations

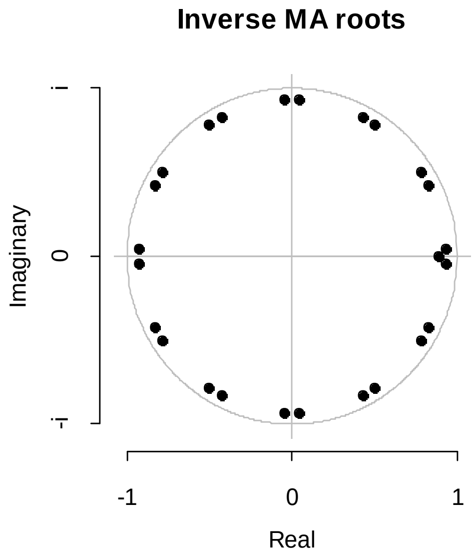
Models	$\pi_1 = 0$	$\pi_2 = 0$	$\pi_3 = \pi_4 = 0$	$\pi_5 = \pi_6 = 0$	$\pi_7 = \pi_8 = 0$	$\pi_9 = \pi_{10} = 0$	$\pi_{11} = \pi_{12} = 0$
C	-2.79	-1.88	3.03	2.99	3.02	3.04	3.06
C,T	-3.32	-1.88	3.01	2.96	3.02	3.02	3.03
C,D	-2.76	-2.76	6.27	6.28	6.21	6.22	6.21
C,D,T	-3.29	-2.76	6.24	6.26	6.18	6.20	6.20
-	-1.91	-1.88	3.05	3.01	3.05	3.06	3.09

Note. [1] Critical values have been obtained from Franses & Hobjin (1997) for S=12 and for 5% significance level (see pp. 29-33) for 20 years.

[2] c shows constant, t denotes trend, d denotes seasonal dummy variables and "-" shows no deterministic components (critical values have been searched for 20 years – that is 240 observations)

APPENDIX D: Checking Causality, Stationarity and Invertibility Conditions for ARIMA(1,1,1)(1,0,2)[12] Model with Drift





R Codes and Outputs for Checking Causality, Stationarity and Invertibility:

"plot.Arima" in forecast package works in order to plot characteristic roots from ARIMA model. This function produces a plot of the inverse AR and MA roots of an ARIMA model. Inverse roots outside the unit circle are shown in red (Hyndman, 2015, p. 53).

- polyroot(c(1,-0.1750)) **#For Non-seasonal AR#**
  [1] 5.714286+0i

  > Mod(polyroot(c(1,-0.1750)))>1

  [1] **TRUE**

- polyroot(c(1,-0.8857**)) #For Non-seasonal MA#**
  [1] 1.12905+0i

  > Mod(polyroot(c(1,-0.8857)))>1

  [1] **TRUE**

- polyroot(c(1,0,0,0,0,0,0,0,0,0,0,-0.8862)) #**Seasonal AR#**

```
[1]  0.5050593+0.8747883i -0.8747883+0.5050593i -0.5050593-0.8747883i
[4]  0.8747883-0.5050593i  0.0000000+1.0101186i -1.0101186-0.0000000i
[7]  0.0000000-1.0101186i  1.0101186+0.0000000i -0.5050593+0.8747883i
[10] -0.8747883-0.5050593i  0.5050593-0.8747883i  0.8747883+0.5050593i
```

> Mod(polyroot(c(1,0,0,0,0,0,0,0,0,0,0,-0.8862)))>1

[1] **TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE**

- polyroot(c(1,0,0,0,0,0,0,0,0,0,0,-0.7102,0,0,0,0,0,0,0,0,0,0,0,0.1813)) #S**easonal MA#**

```
[1]  0.5815120+0.9026442i -0.9549262+0.4909568i -0.5815120-0.9026442i
[4]  0.9549262-0.4909568i  0.4909568+0.9549262i -0.9026442+0.5815120i
[7] -0.4909568-0.9549262i  1.0724688-0.0522820i -0.0522820+1.0724688i
[10] -1.0724688+0.0522820i  0.0522820-1.0724688i  1.0724688+0.0522820i
[13]  0.0522820+1.0724688i -0.9549262-0.4909568i  0.4909568-0.9549262i
[16]  0.9549262+0.4909568i -0.4909568+0.9549262i -0.9026442-0.5815120i
[19]  0.5815120-0.9026442i  0.9026442+0.5815120i -0.5815120+0.9026442i
[22] -1.0724688-0.0522820i  0.9026442-0.5815120i -0.0522820-1.0724688i
```

> Mod(polyroot(c(1,0,0,0,0,0,0,0,0,0,0,-0.7102,0,0,0,0,0,0,0,0,0,0,0,0.1813)))>1

[1] TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE

[16] TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE

Here, ”True” means that roots have modulus which are greater than 1 (>1).

For ARIMA(1,1,1)(1,0,2)[12] with drift (stepwise) model, apart from all required checks, we need to check also the causality, stationarity and invertibility condition. For ARIMA(1,1,1)(1,0,2)[12] with drift (stepwise) model to be causal, stationary and invertible, all roots of the characteristic polynomial of AR, MA, SAR and SMA operators should be greater than 1 in absolute value.

A causal invertible model should have all the roots outside the unit circle. Equivalently, the inverse roots should lie inside the unit circle (Hyndman, 2014). Here, all inverse roots lie inside the unit circle as shown in the figures given above.

APPENDIX E: Quarterly HEGY(1990) Critical Values for Intercept and Seasonal Dummies Model (for N=100)

	π_1	
%1	%5	%10
-3.55	-2.95	-2.63

	π_2	
%1	%5	%10
-3.60	-2.94	-2.63

	π_3	
%1	%5	%10
-4.06	-3.44	-3.14

	π_4	
%1	%5	%10
-2.78	-1.96	-1.53

	F: $\pi_3 = \pi_4 = 0$	
%99	%95	%90
8.74	6.57	5.56

(Source: Hylleberg et al., 1990, pp. 226-227)

APPENDIX F: Critical Values for Seasonal Cointegration (for 100 Observations)

Table F1

Critical Values for Seasonal Cointegration at Zero and Semiannual Frequencies

Number of Variables (k=5, N=100)	π_1 **ve** π_2		
Significance Level	**1%**	**5%**	**10%**
Critical Value	5.18	4.58	4.26

Source: Engle & Yoo (1987, p. 157).

Table F2

Critical Values for Seasonal Cointegration at ¼ (and ¾) Quarterly Frequencies

N=100	π_3			π_4			$\pi_3 \cap \pi_4$		
Deterministic Components in Cointegrating Regression	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**	**99%**	**95%**	**90%**
-	-3.94	-3.30	-3.00	-3.01	-2.12	-	10.24	**7.21**	5.91
C	-3.86	-3.27	-2.95	-2.95	-2.08	-	10.15	**7.10**	5.83
C, D	-4.77	-4.12	-3.81	-3.02	-2.14	-	13.26	**10.12**	8.66

Source: Engle et al., 1993, p. 293.

APPENDIX G: Monthly HEGY Seasonal Unit Root Test Critical Values (For S=12 and 40 years, that is 480 observations)

Table G1

Monthly HEGY Critical Values for $t(\pi_1)$

Models	1%	5%	10%
No Constant, No Trend, No Dummies	-2.51	-1.93	-1.59
Constant	-3.40	-2.82	-2.52
Constant & Dummies	-3.40	-2.81	-2.51
Constant & Trend	-3.93	-3.37	-3.09
Constant, Trend & Dummies	-3.91	-3.35	-3.08

Source: Franses & Hobjin (1997), p. 29

Table G2

Monthly HEGY Critical Values for $t(\pi_2)$

Models	1%	5%	10%
No Constant, No Trend, No Dummies	-2.53	-1.94	-1.60
Constant	-2.54	-1.94	-1.60
Constant & Dummies	-3.34	-2.81	-2.51
Constant & Trend	-2.54	-1.94	-1.59
Constant, Trend & Dummies	-3.34	-2.81	-2.51

Source: Franses & Hobjin (1997), p. 30.

Table G3

Monthly HEGY Critical Values for $F(\pi_3 \cap \pi_4)$

Models	1%	5%	10%
No Constant, No Trend, No Dummies	4.74	3.07	2.36
Constant	4.72	3.07	2.36
Constant & Dummies	8.40	6.35	5.45
Constant & Trend	4.71	3.05	2.35
Constant, Trend & Dummies	8.38	6.35	5.45

Source: Franses & Hobjin (1997), p. 31.

Table G4

Monthly HEGY Critical Values for $F(\pi_5 \cap \pi_6)$

Models	1%	5%	10%
No Constant, No Trend, No Dummies	4.61	3.06	2.38
Constant	4.63	3.05	2.38
Constant & Dummies	8.58	6.48	5.46
Constant & Trend	4.60	3.05	2.38
Constant, Trend & Dummies	8.55	6.48	5.46

Note. Source: Franses & Hobjin (1997), p. 33.

Table G5

Monthly HEGY Critical Values for $F(\pi_7 \cap \pi_8)$

Models	1%	5%	10%
No Constant, No Trend, No Dummies	4.69	3.10	2.40
Constant	4.70	3.09	2.39
Constant & Dummies	8.39	6.33	5.32
Constant & Trend	4.69	3.08	2.39
Constant, Trend & Dummies	8.39	6.30	5.33

Source: Franses & Hobjin (1997), p. 33.

Table G6

Monthly HEGY Critical Values for $F(\pi_9 \cap \pi_{10})$

Models	1%	5%	10%
No Constant, No Trend, No Dummies	4.75	3.11	2.35
Constant	4.73	3.09	2.34
Constant & Dummies	8.56	6.41	5.46
Constant & Trend	4.73	3.08	2.34
Constant, Trend & Dummies	8.50	6.40	5.47

Source: Franses & Hobjin (1997), p. 33.

Table G7

Monthly HEGY Critical Values for $F(\pi_{11} \cap \pi_{12})$

Models	1%	5%	10%
No Constant, No Trend, No Dummies	4.65	3.11	2.41
Constant	4.65	3.10	2.40
Constant & Dummies	8.76	6.47	5.36
Constant & Trend	4.65	3.09	2.39
Constant, Trend & Dummies	8.75	6.46	5.36

Source: Franses & Hobjin (1997), p. 33.

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