

**ISTANBUL TECHNICAL UNIVERSITY ★ ENERGY INSTITUTE**

**CHAOTIC ANALYSIS OF WIND REGIME**

**M.Sc. THESIS**

**Mngereza Mzee MIRAJI**

**Energy Science and Technology Division**

**Energy Science and Technology Program**

**MAY 2015**



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**İSTANBUL TEKNİK ÜNİVERSİTESİ ★ ENERJİ ENSTİTÜSÜ**

**RÜZGAR REJİMİNİN KAOTİK ANALİZİ**

**YÜKSEK LİSANS TEZİ**

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**MAYIS 2015**



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**Date of Defense : 21MAY 2015**





*I whole-heartedly dedicate this Master to my late beloved father who passed-away  
on 16 January 2014 (May his soul rest in peace..ameen).*



## **FOREWORD**

I am extremely grateful to my supervisor, Prof Burak Barutcu. His trust in my potential capabilities in completing my Masters studies have been crucial to me in building confidence and hard-working. He allowed me a great deal of flexibility in choosing my course of work. He gave me some ideas on where and how to start digging and come back to check on me periodically with wisdom and a bit of humor. He provided me with invaluable advice and productive discussions at different stage of my thesis on how to frame my work and stay on track. Beyond the academic accomplishments, I have learned tremendously from him to be a great mentor, close friend and respectable scholar.

I would also like to thank the jury committee for their fruitful contributions in my thesis. Their comments and advices have made my work stronger and stronger. I would not have been able to accomplish my Master studies without the advice from many great faculty members. Dr Aslihan Albostan, Prof Filiz Baytas, Prof Ahmet Durmayaz. Also, I would like to extend my gratitude to my friends, Osman Urper and Ahmet Gultekin.

May 2015

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## **ABBREVIATIONS**

<b>MIT</b>	: Massachusetts Institute of Technology
<b>PSR</b>	: Phase Space Reconstruction
<b>FNN</b>	: False Nearest Neighbour
<b>AMIF</b>	: Average Mutual Information Function
<b>TISEAN</b>	: Time Series Analysis
<b>RPS</b>	: Reconstructed Phase Space
<b>RMSE</b>	: Root Mean Square Error
<b>R<sup>2</sup></b>	: Correlation Coefficient
<b>GPA</b>	: The Grassberger-Procaccia Algorithm
<b>SVR</b>	: Support Vector Regression





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## CHAOTIC ANALYSIS OF WIND REGIME

### SUMMARY

The usage of Wind is the fastest growing energy source among renewable energy sources. Usage of wind energy is getting wider with its competitive cost of production compared with other traditional means. Wind energy highly depends on wind speed. Wind speed is the most important parameter in the design of wind energy systems. According to the algorithm, which is used to calculate the power obtained from wind; the power is proportional to the cube of wind speed. Therefore, the analysis of wind speed is very important not only for better designing more effective and efficient wind power plants, but also for better understanding the underlying dynamical mechanisms. For this aim, it is crucial to investigate the inner dynamical structure of wind speed time series .

At current implementations, variability of wind is the major challenge of integrating wind power into electric systems. Understanding the dynamics of geophysical phenomena such as wind speed is a subject that has attracted scientific interest due to many technological applications as well as due to its impact in human life. Therefore, analyzing the chaotic characteristics of the wind speed time series can reveal the internal mechanism of wind speed changes in nature, but also can help to understand the action mechanism of the wind speed.

This study covers implementations of methods to investigate the chaotic characteristic of wind speed data. As the first step of chaotic analysis, phase space system parameters; delay time (T) and embedding dimension (m) were determined to reconstruct the phase space. Delay time (T) was calculated by using Average Mutual Information (AMI) function which is a nonlinear form of autocorrelation function. Embedding dimension (m) was calculated by using False Nearest Neighbor (FNN) algorithm. After the reconstruction of phase space, the dimension of the attractor occurred on the phase was calculated by using Correlation Dimension algorithm. The package program TISEAN 3.0.1 (Time Series Analysis) was used. The program which was written by Hegger et. al. (1999), is the most popular program in literature. The results signify the chaotic behavior of the observed system that all of the data set have fractal dimensions.

Lyapunov exponents method is another reliable criteria to determine the chaotic behavior. Rosenstein et. al. (1993) algorithm was used for calculating Lyapunov exponents. Although the exponents have a very small amount (less than 1), a positive exponent is considered to be enough to determine the chaotic character (Khatibi, 2012). Thus, the data sets in the study were proven to exhibit the chaotic behavior. In the last part of the case study, Local Approximation Method were examined to predict the data. The method based on the chaotic dynamics of the systems.

The successful results obtained from the study have convinced us that the chaotic analysis is very useful to determine main characteristics of the Wind . It also provides a better understanding and modeling of the underlying dynamics of the natural systems. After all, it is believed this study will be a novel approach for further studies.

## RÜZGAR REJİMİNİN KAOTİK ANALİZİ

### ÖZET

Rüzgar enerjisi yenilenebilir enerji kaynaklar içinde kullanımını en hızlı artan enerji kaynaklarından. Rüzgar santrallerinin enerji üretim maliyetlerinin de diğer enerji üretim metotlarıyla boyölçüshebilir düzeylere düşmesiyle bu artış son yıllarda hızlanmıştır. Kinetik enerji ifadelerine bakıldığı zaman rüzgardan üretilebilecek enerjinin, rüzgarın geçtiğı kesit alanı (rüzgar türbinlerinde: rotor süpürme alanı) ve havanın yoğunluğuyla doğru orantılı olmasının yanında rüzgar hızının küpüyle doğru orantılı olduğu görülmektedir. Dolayısıyla rüzgar hızı rüzgardan enerji üretimi açısından en önemli parametredir. Bir bölgede ortalama rüzgar hızı, rüzgar hızının değışkenliğı gibi parametrelerin yanında rüzgar hızı öngörüsünde ulaşılabilen doğruluk da rüzgar enerjisi açısından çok büyük önem taşımaktadır. Rüzgardan üretilebilecek enerji miktarının artırılması, rüzgar santralleri tarafından üretilen elektriğın şebekeye verilmesi sırasında enerji kalitesinin korunması, rüzgar türbinlerinde daha yüksek verim sağlayacak işletim algoritmalarının oluşturulması gibi faydalarının yanında atmosferik davranışların daha iyi anlaşılması açısından da rüzgar hız serilerinin iç dinamiklerinin incelenmesi hayati önem taşımaktadır.

Günümüz rüzgar teknolojisi açısından bakıldığında, rüzgar enerjisi santrallerinde rüzgar hızının değışkenliğı rüzgar türbinleri tarafından üretilen elektrik enerjisinin şebekeye arzı açısından en önemli problemi oluşturmaktadır. Elektrik enerjisinin kalitesinde en önemli parametrelerden bir olan frekansın sabit kalması için şebekede enerji arz ve talebinin dengeli olması gerekir. Eğer şebekeye, talep edilen enerjiden daha fazlası verilirse frekans yükselir. Aksi durumda yani şebekeye verilen enerji talebin altındaysa frekans düşer. Rüzgar santrallerinin enerji üretimi rüzgar hızına bağlıdır. Rüzgar hızı çok değışken olduğu için özellikle kısa dönemli rüzgar hızı öngörüsü şebekede enerji arz-talep dengesinin korunabilmesi için büyük önem taşımaktadır. Dolayısıyla rüzgar santrallerinden şebekeye verilecek enerji miktarı ve zamanının yüksek doğrulukla tahmini şebeke işletimi açısından hayati bir önem taşımaktadır. Rüzgar hızı tahmini başarımının artması elektrik şebekesinde rüzgar santrallerinin payının da artmasını sağlayacağından gerek rüzgar enerjisi sektörü gerekse yenilenebilir ve temiz enerji kaynaklarının enerji üretiminde kullanım payının artması açısından önemli bir katkı sağlamanın yanısıra şebeke işletiminde yaşanan en önemli problemlerden biri olan enerji kalitesinin korunmasına da yardımcı olacaktır.

Rüzgar hızı serileri incelendiğinde rastlantısal karakterin yanında kaotik karakter de taşımakta oldukları görülür. Dolayısıyla rüzgar hızının öngörüsünde sadece lineer olmayan öngörü metotlarının kullanılması yeterli değıldir. Rüzgar hızının daha yüksek doğrulukla öngörülmesi için kaotik öngörü metotlarının kullanılması gereklidir.

Bu tez çalışmasında rüzgar hız değışiminin kaotik karakteristikleri incelenmiş ve örnek bir öngörü çalışması yapılmıştır. Çalışmada TAV İstanbul Atatürk Havalimanı'nda bulunan Otomatik Hava Gözlem İstasyonu (AWOS: Automatic

Weather Observation Station) tarafından yerden 10 m yükseklikte, 1 dakika örnekleme periyoduyla örneklenmiş rüzgar hızı işareti kullanılmıştır. Örneklenmiş işaretler örnekleme işlemi nedeniyle gürültü içeriği fazla işaretlerdir. Ele alınan sistemin faz uzayının yeniden oluşturulması sırasında bu gürültü bileşeni sorun oluşturur. Kaotik sistemlerde işaret içindeki enformasyon da arka-plan gürültüsü de geniş bantlı bileşenler olacağından gürültü, işaretten klasik metotlarla (Fourier Analizi vb) ayrılamaz.

Kaotik analizin ilk aşaması olarak faz uzayının yeniden oluşturulabilmesi için, faz uzayının parametreleri olan Gecikme Zamanı (T: Delay Time) 45 dakika ve Gömme Boyutu (m: Embedding Dimension) 20 olarak belirlenmiştir. Gecikme zamanının belirlenmesi için Öz-ilişki fonksiyonunun (Auto-correlation Function) lineer olmayan bir formu olan Ortalama Müşterek Bilgi Fonksiyonu (Average Mutual Information Function) kullanılmıştır. Gömme boyutu, Yanlış En Yakın Komşular algoritması (False Nearest Neighbor (FNN) Algorithm) kullanılarak belirlenmiştir. Çekicinin (Attractor) gürültüden arındırılması için yersel sabit yaklaşımlı (Locally Constant Approximation) non-lineer gürültü azaltma yaklaşımı uygulanarak gerekli şartları sağlayan çekici elde edilmiştir. Faz uzayının (Phase Space) yeniden oluşturulmasından sonra İlişki Boyutu Algoritması (Correlation Dimension Algorithm) kullanılarak çekici boyutu hesaplanmıştır. Örneklenmiş işaretin fraktal boyutlara sahip olması kaotik karakter taşıdığına bir göstergesidir.

Tez çalışması sırasında kullanılan algoritmaların uygulanması için TISEAN 3.0.1 (Non-Linear Time Series Analysis: Non-Linear Zaman Serisi Analizi) paketinden yararlanılmıştır. Hegger ve diğ. (1999) tarafından, zaman serilerinin, Non-Linear Deterministik Dinamik Sistem Teorisi veya diğer bir isimle Kaos Teorisi üzerine kurulu metotlarla analizi için geliştirilmiş bir yazılım projesi olan TISEAN paket programı literatürde kaotik analiz için kullanılan en popüler programdır.

Kaotik davranışın belirlenmesinde bir diğer önemli kriter olan Lyapunov Üstelleri (Lyapunov Exponents) metodu da işaretin kaotik karakter taşıdığına gösterilmesi için kullanılmıştır. Lyapunov Üstellerinin hesaplanması için Rosenstein et. al. (1993) algoritması kullanılmıştır. Hesaplanan Lyapunov Üstelinin değeri çok küçük bile olsa ( $< 1$ ) pozitif olması kaotik karakterin olduğunu göstermek için yeterli kabul edilmektedir (Khatibi, 2012).

Lyapunov üstellerinin yanısıra dinamik sistemin faz uzayındaki davranışını (periyodik, yarı-periyodik, kaotik vs) belirlemek için Poincaré kesitlerinden (Poincaré Section, Poincaré Map) de yararlanılabilir. Bu amaçla tez çalışmasında ele alınan rüzgar işaretinin yeniden oluşturulan faz uzayında seçilen bir düzlem üzerinde Poincaré kesiti de bulunmuştur.

Gerek pozitif Lyapunov üsteline sahip olması gerekse Poincaré kesitinin dağınık yapısı, tez çalışmasında ele alınan rüzgar hızı serisinin kaotik karakter taşıdığını göstermektedir.

Serinin kaotik karakter taşıdığı gösterildikten sonra öngöründe kaotik bir öngörü metodunun kullanılmasının önemini göstermek için son olarak da bir adım ileri öngörü yapmak için dinamik sistemlerin kaotik davranışları üzerine kurulmuş bir öngörü metodu olan Yersel Kestirim Metodu (Local Approximation Method) uygulanmıştır.

Tez çalışmasında ele alınan rüzgar işareti üzerinde uygulanan öngörü çalışması sonucunda yapılan öngörünün Karesel Ortalama Hatası (Root Mean Square Error)

RMSE = 0.1670 m/s ve Determinasyon Katsayısı  $R^2 = 0.9986$  olarak bulunmuştur. Açıkça görüldüğü gibi her iki kriter de öngörü başarımın yüksek olduğunu göstermektedir. Bu da rüzgar hızı öngörüsünde sistemin kaotik davranışını gözönüne alan bir öngörü metodunun kullanılmasının önemine işaret etmektedir.

Çalışma sırasında elde edilen başarılı sonuçlar rüzgarın karakteristiğinin incelenmesinde ve rüzgar hızı öngörüsü yapılmasında kaotik analiz metotlarının kullanımının büyük yararlar sağlayacağını göstermektedir. Yine kaotik yaklaşımların kullanımı meteorolojik olayların altında yatan dinamiklerin daha iyi anlaşılmasında ve hava sisteminin modellenmesinde büyük gelecek vaad etmektedir. Rüzgar hız serilerinde her zaman rastlantısal görünen karakterin altında kaotik bir karakterin varlığı gözönüne alınmalıdır. Salt rastlantısal karakter üzerine kurulu istatistiksel metotlar rüzgar serilerinin modellenmesinde ve öngörüsünde yetersiz kalacaktır. Bu tez çalışması sırasında varılan sonuçlar nedeniyle bu alandaki çalışmalara yeni bir bakış açısı getirdiğimizi ummaktayız.

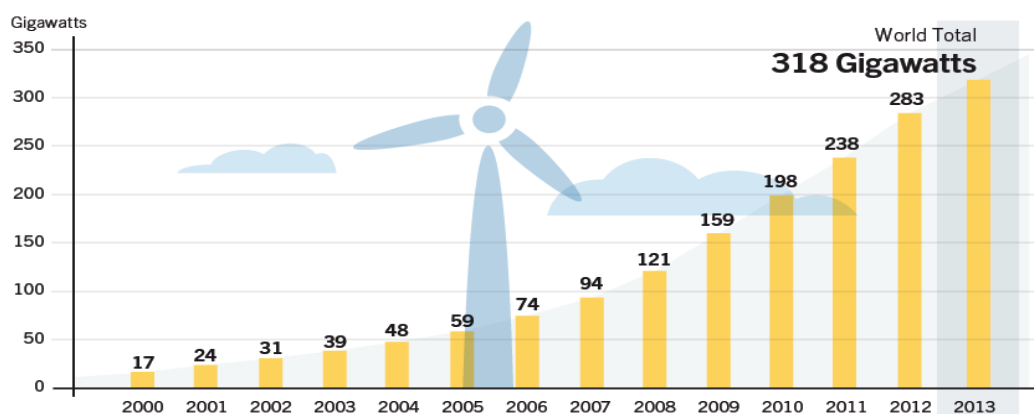




## 1. INTRODUCTION

### 1.1 The Potential of Wind Energy

Full use of renewable energy is an effective solution for the energy crisis and push for environmental protection at present. Wind power is one of the most potential and popular renewable resources. Over the past few years, capital costs of wind power have declined, primarily through competition, while technological advances—including taller towers, longer blades, and smaller generators in low wind speed areas—have increased capacity factors.



**Figure 1.1:** Wind power total world capacity, 2000-2013[1].

These developments have lowered the costs of wind-generated electricity, improving its cost competitiveness relative to fossil fuels. Onshore wind-generated power is now cost competitive, or nearly so, on a per kWh basis with new coal- or gas-fired plants, even without compensatory support schemes, in several markets (including Australia, Brazil, Chile, Mexico, New Zealand, South Africa, Turkey, much of the EU, and some locations in India and the United States). By one estimate, global levelised costs per MWh of onshore wind fell about 15% between 2009 and early 2014[1].

Wind power generation has greatly promoted the development of the wind energy industry as shown (in the Figure 1.1) above. In the majority of cases we have access only to several measurable quantities which depend on the underlying and usually

unknown dynamics of (geophysical phenomena such as wind, rainfall etc) the physical system.

During the previous decades theories for the dynamical systems and the advent of chaos theory have significantly influenced the way geophysical phenomena are treated. Complex phenomena such as the geophysical ones (wind speed, temperature, pressure, rainfall etc) may result from relatively simple systems which however present nonlinear behavior with sensitivity to initial conditions. Such systems are generally known as 'deterministic chaotic systems' and the corresponding theory as 'chaos theory'.

## **1.2 Introduction to Chaos Theory**

Chaos theory is mathematical fields of study, which states that non-linear dynamical systems that are seemingly random are actually deterministic from much simpler equations. Chaos is apparently noisy aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions [2].

The phenomenon of chaos theory was introduced to the modern world by Lorenz E.N.1963: 'Deterministic non-periodic flow'. As the chaos theory was developed by various inputs of mathematicians and scientists, it found applications in large number of scientific field.

Behavior of weather, behavior of airplane in flight, behavior of car in clustering on an expressway, cardiac arrhythmias, behavior traffic flow pattern, behavior of urban development decay, behavior of oil flowing in an underground pipe, epidemics and the behavior of people in group: Any idea what holds all of these systems together?

The only systems that could be understood in the past were those that could be believed to be linear, that's, the system that follow patterns and arrangements like linear equations, linear functions, linear algebra, linear programming etc. But there were some systems that couldn't be explained like weather patterns, ocean currents, or the actions of a cells. The answer to the above posed question is chaos!

Nature is highly complex, and the only prediction you can make is that she is unpredictable. The amazing unpredictability of nature is what Chaos Theory looks at. Why? Because instead of being boring and translucent, nature is marvelous and mysterious. Chaos Theory has managed to somewhat capture the beauty of the

unpredictable and display it in the most awesome patterns. Nature, when looked upon with the right kind of eyes, presents herself as one of the most fabulous works of art ever wrought [3].

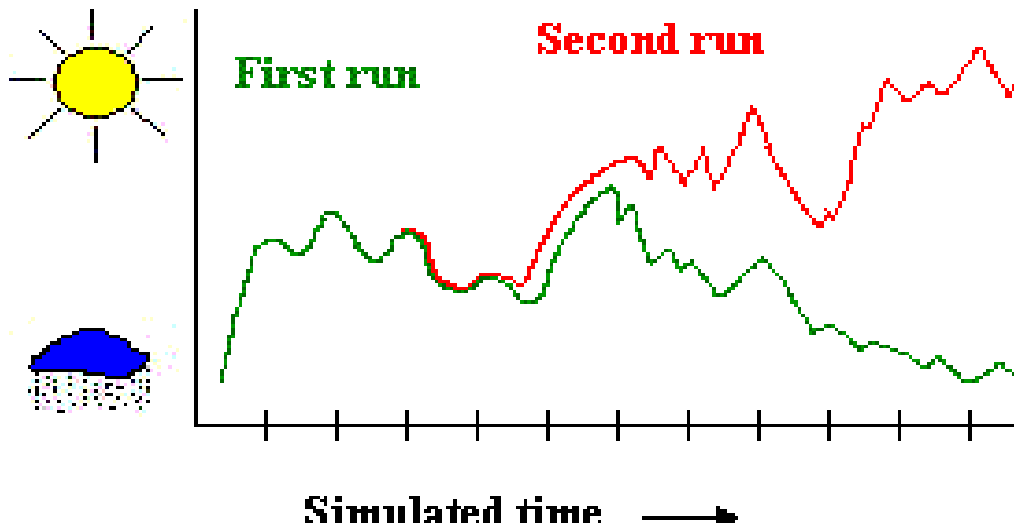
To chaos researchers, mathematics has become an experimental science, with computer replacing laboratories full of test tubes and microscopes. Traditional time series and trajectories in phase space are two ways of displaying and gaining a picture of a system's long-term behavior.

Now seeing the work done by his predecessors, one might infer that Lorenz did not do anything of any particular great importance, But Lorenz can be claimed to have "discovered chaos" because he put all the elements together being sensitive dependence on initial conditions with strange (fractal) attractors, and the resulting erratic dynamics and he saw them all together. He is the one who derived the mathematical understanding of chaos so as it becomes easy for others to study after him.

### **1.2.1 How chaos theory was born and why?**

It all started to dawn on people when in 1960 a man named Edward Lorenz created a weather-model on his computer at the Massachusetts Institute of Technology. Lorenz' weather model consisted of an extensive array of complex formulas that kicked numbers around like an old pigskin. Clouds rose and winds blew, heat scoured or cold came creeping up the breeches [3].

Colleagues and students marveled over the machine because it never seemed to repeat a sequence; it was really quite like the real weather. Some even hoped that Lorenz had built the ultimate weather-predictor and if the input parameters were chosen identical to those of the real weather howling outside the Maclaurin Building, it could mimic earth's atmosphere and be turned into a precise prophet. But then one day Lorenz decided to cheat a little bit. A while earlier he had let the program run on certain parameters to generate a certain weather pattern and he wanted to take a better look at the outcome. But instead of letting the program run from the initial settings and calculate the outcome, Lorenz decided to start half way down the sequence by inputting the values that the computer had come up with during the earlier run.



**Figure 1.2:** Lorenz's first experimental graph.

The computer that Lorenz was working with calculated the various parameters with an accuracy of six decimals. But the printout gave these numbers with a three decimal accuracy. So instead of inputting certain numbers (like wind, temperature and stuff like that) as accurate as the computer had them, Lorenz settled for approximations; 5.123456 became 5.123 (for instance), that puny little inaccuracy appeared to amplify and cause the entire system to swing out of whack [3].

### 1.2.2 Chaos theory before and after Lorenz

Chaos theory before Lorenz:

While Lorenz discovery has achieved wide spread attention that it didn't initially receive upon initial publication a well – deserved outcome largely being initially read by Meteorologists and Climatologists who didn't fully appreciate its broader mathematical implications. Henri Poincaré 1890 comes with non-periodic orbits while studying about three body problems.

In 1927, Van der Pol observed chaos in radio circuit. In 1960, Lorenz come with butterfly effect later, He lay the foundation of chaos theory in his paper Deterministic non-periodic flow [4].

Chaos theory after Lorenz:

In 1971, David Ruelle and Floris Takens described the phenomenon they called a strange attractor (today called chaotic attractor). In 1974-5 Jim Yorke an applied mathematician coined the term 'chaos' at University of Maryland .

Another major contributor to chaos theory is Mitchell Feigenbaum. A physicist at the theoretical division of the Los Alamos National Laboratory starting in 1974, Feigenbaum dedicated much of his time researching chaos and trying to build mathematical formulas that might be used to explain the phenomenon[5].

Logistic map studied by Robert May (1976) after his famous work on developing model that addressed how insect birthrate varied with food supply in 1970's. Benoit Mandelbrot (1982) found the piece of the chaos puzzle that put all things together, published a book 'Fractal geometry of Nature' [6].

In addition, the mathematical aspect of chaos and dynamical systems, several studies has been conducted including Eckmann & Ruelle (1985) and Devaney (1989). Numerical implementations are discussed in Parker & Chua (1989). Chaos has attracted attention of the statistical community including Nychka et al. (1997), Smith (1992), and Casdagli (1992)[7].

By the mid-1980s, chaos was a buzzword for the fast-growing movement reshaping scientific establishments, and conferences and journals on the subject were on the rise. Universities sought chaos "specialists" for high-level positions. A Center for Nonlinear Studies was established at Los Alamos, as were other institutes devoted to the study of nonlinear dynamics and complex systems. A new language consisting of terms such as fractals, bifurcations, and smooth noodle maps was born[5].

### **1.2.3 The foundation of chaos theory**

Chaos theory is the study of complex, non-linear dynamics systems. It deals with the systems that appear to be orderly but in fact harbor chaotic behavior, it also deals with the systems that appear to be chaotic but in fact have underlying order.

Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions, an effect that popularly referred as BUTTERFLY EFFECT. Small difference in initial conditions ( such as those due to rounding errors in numerical computations) yield widely diverging outcomes for chaotic systems rendering long-term prediction impossible in general. Chaos theory is most commonly attributed to the work of Edward Lorenz. He laid the foundation of chaos theory in his famous paper Deterministic non-periodic flow [4].

Technically, chaos models are based on "state space," improved versions of the Cartesian graphs used in calculus. In calculus, speed and distance can be represented on a Cartesian graph as  $x$  and  $y$ . Chaos models allow the plotting of many more variables in an imaginary space, producing more complex imaginary shapes. Even this model assumes, however, that all variables can be graphed, and may not be able to account for situations in the real world where the number of variables changes from moment to moment [5].

### **1.3 Scientific Application of Chaos Theory**

The mechanical engineer may be concerned with the regular oscillation of an out of balance drive shaft; the civil engineer with the potentially disastrous structural vibrations induced by vortex shedding on a bridge deck; the electrical engineer with the oscillatory output from nonlinear circuits; the chemist/chemical engineer with the regular cycling of a chemical reaction; the geologist/geophysicist with earthquake tremors; the biologist with the cycles of growth and decay in animal populations; the cardiovascular surgeon with the regular. Beating of the human heart; the economist with the boom—bust cycles of the stock market; the physicist with the oscillatory motion of a driven pendulum; the astronomer with the cyclical motion of celestial bodies; and so on [8]. (The list is extensive and diverse!)

In addition, only nonlinear systems are capable of a most fascinating behaviour known as chaotic motion, or simply chaos, whereby even simple nonlinear systems can, under certain operating conditions, behave in a seemingly unpredictable manner.

### **1.4 Literature Review**

The systematic study of chaos is of recent date, originating in the 1960s. One important reason for this is that linear techniques, so long dominant within applied mathematics and the natural sciences, are inadequate when considering chaotic phenomena. Furthermore, computers are a necessary tool for studying such systems. As a result, the amazingly irregular behaviour of some non-linear deterministic systems was not appreciated and when such behaviour was manifest in observations, it was typically explained as stochastic. Chaos has been identified in simple experiments such as a water-dripping faucet, simple electric circuits, and in situations

involving near turbulent flow such as the Couette-Taylor experiment. Outside the laboratory, chaotic behaviour has been claimed in climatic time series, astrophysics [9,10], hydrodynamics [4,11,12,13], economics [7], medicine [14] and several other fields.

The recently interesting review about the detection of chaos in geophysical phenomena includes, Chaotic analysis on river discharge times series [15], SVR-based prediction of evaporation combined with chaotic approach[9], investigating chaos in river stage and discharge time series [16],detection low-dimensional deterministic chaos in wind time series[17] and Identification of chaos in rainfall temporal disaggregation[18].

Research on chaotic behavior of the wind speed time series can be of great help to understand the mechanisms and characteristics, moreover provides useful clues for the inner structure of wind speed data.

Work concerning wind velocity time series is relatively restricted as many work focuses on Forecasting [19-23] and modelling [24-26] of wind energy and speed, where other focuses on wind speed persistence [27-29].

In 1988, The first study on presence of chaos in wind velocity was conducted by Tsonis and Elsner, who studied 10 s averages of vertical wind velocity recorded 10 m above the ground at the National Oceanic and Atmospheric Administration (NOOA) in Boulder, USA. Using the correlation dimension method by Grassberger and Procaccia [48], they reported on the presence of chaos in wind speed data.

In the year 2008, Karakasidis and Charakopoulos reported on the existence of low-dimensional deterministic chaos in wind time series measured from the New Anchialos (Greece) Air Base measurement station. The surrogate data test and the corresponding results provided significant evidence for the existence of low-dimensional chaotic dynamics underlying the wind time series [17].

Wind data observed at three wind farms experiencing different climatic conditions from 2006 to 2008 in Taiwan, where wind speed distribution can be properly classified to high wind season from October to March and low wind season from April to September. The variations of fractal dimensions among different wind farms are analyzed from the viewpoint of climatic conditions. The results show that the wind speeds studied are characterized by medium to high values of fractal

dimension; the annual dimension values lie between 1.61 and 1.66. This was reported in the paper published in 2011 named by Fractal dimension in wind speed time series at Elsevier[30].

The saturated correlation dimension algorithm with the surrogate data method (an indirect identification method) proposed to analyze the chaotic characteristics of the near-surface wind speed time series. The results published 2012 in IEEE conference publication concluded for the existence of chaotic characteristics in wind speed time's series [31].

Surrogate data method and spectral analysis were utilized to detect the nonlinearity and aperiodicity of the time series of wind waves in southern, central and northern regions of the Caspian Sea[32]. The false nearest neighbor analysis along with the correlation dimension estimation indicate the presence of high dimensional chaotic behavior with dimensions of the chaotic attractors at the range of 5.91, 6.30 and 7.55 for the significant wave height series and 6.59, 7.64 and 7.87 for the wave period for the southern, central and northern parts of the Sea. Positive Lyapunov exponents obtained for all 9 data series prove the exponential divergence of the trajectories which support the presence of chaos in the wind-wave characteristics. The results were published in March 2015 at Elsevier.

In this study we analyse the behavior of wind speed time series obtained at the Ataturk International Airport located in Istanbul, Turkey. In an attempt for seeking chaotic behavior in wind speed data as reported by several researchers, an important step is determining the presence of chaotic behavior in time series data. In order to identify such a behavior it is necessary to employ appropriate methods based on the theory of dynamical systems and chaos.

## **1.5 Characterization of Chaos**

The traditional methods for chaos identification can be divided into two types: direct methods and indirect methods. The direct methods can also be divided into the qualitative methods [33] and quantitative methods [34]. The classic qualitative methods is the technique of phase diagram and the power spectrum which can be used to distinguish roughly whether the test signals have chaotic properties by comparing the experimental signals with the stable signals and periodic signals



respectively. Note that those qualitative methods are concise and intuitive techniques. The limitation of qualitative methods is that they cannot distinguish between large periodic motion and chaotic motion. Usually, the quantitative methods calculate the characteristic values of strange attractor (e.g., Lyapunov exponent, saturated correlation dimension and Kolmogorov entropy) and determine chaos according to the corresponding results of the characteristic value[31].

In determining the largest Lyapunov exponent [35], commonly we have method proposed by Rosenstein [36] and Sato[37]. The Grassberger-Procaccia algorithm (GPA) [38] appears to be the most popular method used to quantify chaos. This is probably due to the simplicity of the algorithm and the fact that same intermediate calculations are used to estimate both dimension and entropy.

However, the GPA is sensitive to variations in its parameters, e.g., number of data points, embedding dimension, reconstruction delay, and it is usually unreliable except for long, noise-free time series. Hence, the practical significance of the GPA is questionable, and the Lyapunov exponents may provide a more useful characterization of chaotic systems. The algorithms due to Kantz [39] and due to Rosenstein et al. [36] allow one to establish a difference between periodic and chaotic motion, presenting no noise sensitivity. Hence, this study considers these algorithms in order to estimate Lyapunov exponents.

Phase space reconstruction (PSR) is an important aspect in visualizing trajectories complexities. Prior to phase space reconstruction based on Embedding Theorem [40], embedding parameters such as embedding dimension and time delay should be determined from the time series [41].

In literature, there are various methods such as Grassberger–Procaccia (GP)[38] and False Nearest Neighbour (FNN) algorithm to find the embedding dimension. Although these methods give similar results, application of the FNN is more practical when compared the other methods in finding embedding dimension.

Many researchers have addressed the problem of the selection of an appropriate delay time and proposed various methods. Well known among these are the Autocorrelation function method, ACF, the mutual information method, MIF and the correlation integral method, CI. The autocorrelation function method measures the linear dependence between successive points and, thus, may not be appropriate for

nonlinear dynamics. They suggested the use of the local minimum of the mutual information, which measures the general dependence between successive points. The mutual information method is a more comprehensive method of determining proper delay time values that is why MIF was used in this study [13].

Poincaré section as the powerful tool for the verification of dynamics complexities in particular to identify chaotic patterns in times series [42, 43] were also applied, as our goal is to gain insight into the geometrical structure of the attractor drawn. Two-dimensional Poincaré maps are the maps of minimum dimension, which are capable to explain the dynamics of three-dimensional attractors properly. Occurrence of one-dimensional Poincaré map is a rare phenomenon as it may be obtained by a very special choice of Poincaré section [10]. In fact, one-dimensional Poincaré map [44, 45] is unable to explain the proper chaotic dynamics of the three dimensional attractor in most of the cases.

## **2. THEORETICAL INFORMATION ABOUT CHAOS THEORY**

### **2.1 Dynamic Systems**

A dynamic system is a simplified model for time-varying behavior of an actual system. These systems are described using differential equations specifying the rates of change for each variable.

#### **2.1.1 A deterministic system**

A deterministic system is the system in which no randomness is involved in the development of future states of the system. Its said to be chaotic whenever its evolution sensitively depends on the initial conditions. This property implies that, two trajectories emerging from two close-by initial conditions separate exponentially in the course of time.

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#### **2.1.2 A linear system**

A linear system is a system in which all of the dependence of the current state on previous states can be expressed in terms of a linear combination.

#### **2.1.3 A nonlinear system**

A nonlinear system is a system in which the dependence of the current state on previous states cannot be expressed entirely as a linear combination; even if some of the dependence can be captured in a linear combination of the previous states, something extra is required to capture all of the dependence.

### **2.1.4 Complex systems**

Complex systems are systems that contain so much motion (so many elements that move) that computers are required to calculate all the various possibilities. That is why Chaos Theory could not have emerged before the second half of the 20th century.

Another reason that Chaos Theory was born so recently, and that is the Quantum Mechanical Revolution and how it ended the deterministic era! Complex systems often seek to settle in one specific situation. This situation may be static (Attractor) or dynamic (Strange Attractor).

## **2.2 Attractors**

Attractor is a set of points to which a dynamic system evolves after a long enough time. The attractor is stable, and non-periodic. It could never intersect itself, because if it did returning to a point already visited, from then on the motion would repeat itself in a periodic loop.

### **2.2.1 Types of attractors.**

There is a taxonomy of motion within dynamic systems which Schaffer and Kott (1985) have suggested can be understood in terms of different types of attractors. Chaos theorists have identified four fundamental types of attractors which describe the functioning of all systems:

#### **2.2.1.1 Point attractor**

This describes a system structured to move toward a single point, place or outcome. Crutchfield, Farmer, Packard, and Shaw (1986) define a point attractor as representing all systems that come to rest with the passage of time.

The typical physical representation of such a system is a basin, sinks in which objects or fluids move, flow toward the bottom, or plug hole. Psychologically this is a description of driven thinking and behavior.

#### **2.2.1.2 Limit cycle or Periodic attractor**

This describes a system, which functions by regular swings between two points, places or outcomes. The typical physical representation of such a system is a

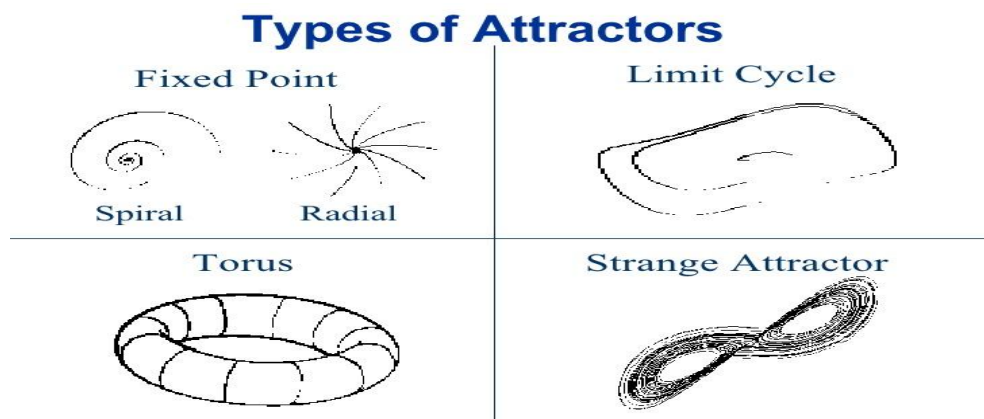
pendulum moving from one pole to the other pole of the swing passing through a vertical point in the process.

### 2.2.1.3 Torus attractor

This describes a system, which functions in a complex and predictable way. Such a system repeats itself either exactly or approximately over time. The typical physical representation of such a system is the maze in which there is only one way through and which leads eventually back to its beginning to start again. Once the solution is found each new time through the maze repeats previous routes to complete the task. This is an example of the attractor exactly repeating. The self-similar variant is akin to a long piece of wire being wrapped around a donut. As the wire circles around the dough, it describes a characteristic loop, which varies only it that each successive loop is a wire's width further around the donut. For most intents and purposes, the Torus Attractor can be thought of as exactly repeating for the minor differences are of little practical consequence in this system.

### 2.2.1.4 Strange attractor

This describes a system, which functions in complex but inherently unpredictable ways but which at the same self-organize into emergent order. It does so by establishing a pattern of functioning, which is bounded, self-similar but never exactly repeating. The typical (and historical) physical exemplar of such a system is the weather. Due to the multitude of interacting factors combining complexly the precise prediction of the weather beyond about a week in most parts of globe, is unreliable [46].



**Figure 2.2:** Types of attractors [47].

## 2.3 Lorenz Attractors

### 2.3.1 Theory

Edward Lorenz's first weather model exhibited chaotic behavior, but it involved a set of 12 nonlinear differential equations. Lorenz decided to look for complex behavior in an even simpler set of equations, and was led to the phenomenon of rolling fluid convection. The physical model is simple: place a gas in a solid rectangular box with a heat source on the bottom.

### 2.3.2 Application

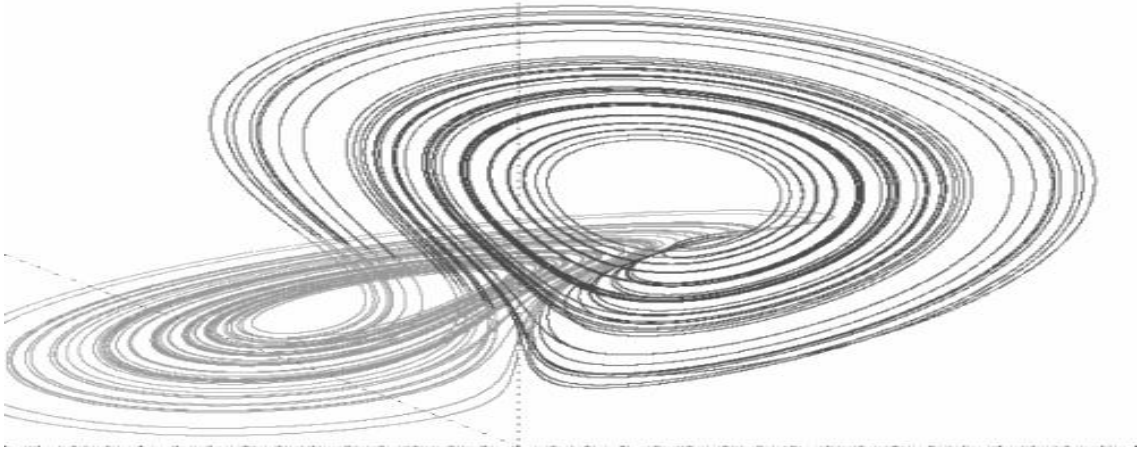
Lorenz simplified a few fluid dynamics equations (called the Navier-Stokes equations) and ended up with a set of three nonlinear equations:

$$\begin{aligned}\frac{dx}{dt} &= P(y - x) \\ \frac{dy}{dt} &= x(R - z) - y \\ \frac{dz}{dt} &= xy - Bz\end{aligned}\tag{2.1}$$

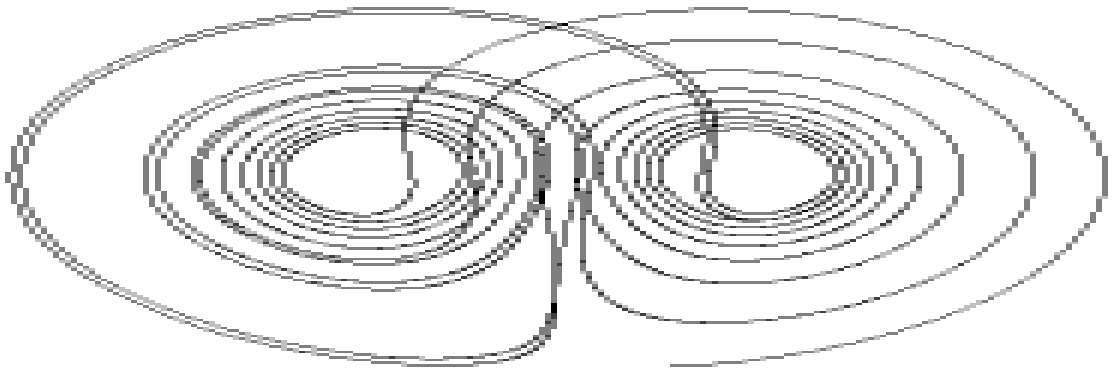
where  $P$  is the Prandtl number representing the ratio of the fluid viscosity to its thermal conductivity,  $R$  represents the difference in temperature between the top and bottom of the system, and  $B$  is the ratio of the width to height of the box used to hold the system. The values Lorenz used are  $P = 10$ ,  $R = 28$ ,  $B = 8/3$  [4].

On the surface, these three equations seem simple to solve. However, they represent an extremely complicated dynamical system.

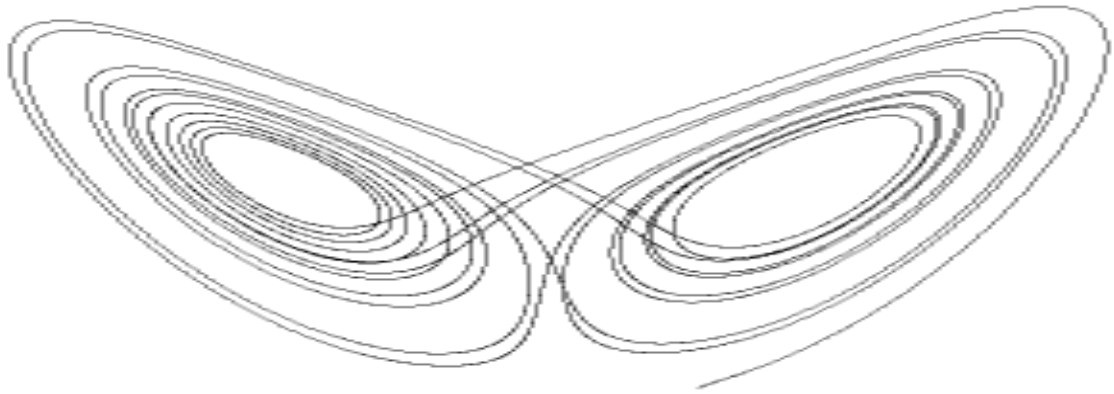
If one plots the results in three dimensions the following figures, called the Lorenz attractor is going to be seen, Figure 2.3



(a)



(b)



(c)

**Figure 2.3:** Projection of Lorenz attractors (a) Projection in three dimensions, (b) Projection on Y-Z plane, (c) Projection on X-Z plane.

## 2.4 Strange Attractors

### 2.4.1 Theory

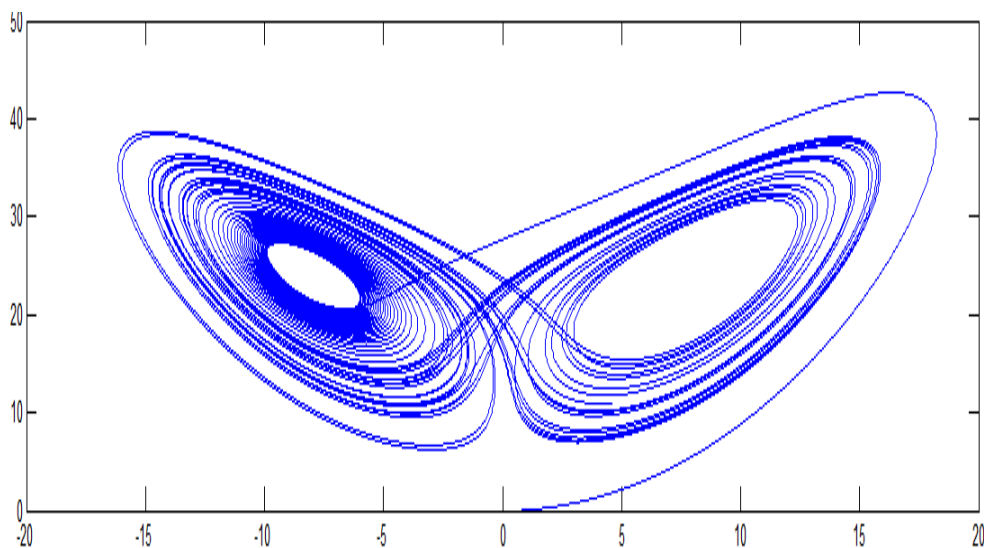
Attractor is strange if it has non integer dimension. Act strangely once the system is on the attractor, the near by states diverges from each other exponentially fast. Strange attractor was coined by David Ruelle and Floris Takens[40,48].

The Lorenz attractor is an example of a strange attractor. Strange attractors are unique from other phase-space attractors in that one does not know exactly where on the attractor the system will be. Two points on the attractor that are near each other at one time will be arbitrarily far apart at later times. The only restriction is that the state of system remains on the attractor.

### 2.4.2 Applications

The equations used are the same as those in (2.9). These equations are called the Lorenz Equations, and were derived from simplified equations of convection rolls rising in the atmosphere. On the surface these three equations seem simple to solve. However, they represent an extremely complicated dynamical system.

Where,  $P=a=10$ ,  $B=b=8/3$  and  $R=r=25$ ,  $n = 500000$ ,  $dt = 0.0001$ ;  $P$ ,  $r$ ,  $n$  and  $b$  are all constants ( $P$  represents the Prandtl number,  $r$  is the ratio of Rayleigh number to the critical Rayleigh number, number of points and  $B$  is ratio of width to height of the box), and  $x$ ,  $y$  and  $z$  are all functions of time [4]. Then, The equations were used numerically [49] and Matlab were used to look at trajectories.



**Figure 2.4:** The strange attractor.



These plots above are famous, and that butterfly-looking picture is known as a “strange attractor or the Lorenz attractor.” Strange attractors appear in phase spaces of chaotic dynamical systems. The “Buttery effect” was coined and described after studying the numerical solutions of these very equations.

The idea is that chaotic systems have a sensitive dependence on initial conditions if you were to play around with the initial conditions for  $x(t)$ ,  $y(t)$  and  $z(t)$  in these equations and plot phase space portraits for each solution set, you'd find that even the tiniest changes in initial conditions can lead to a crazy huge difference in position in phase space at some later time.

Strange attractors are also unique in that they never close on themselves — the motion of the system never repeats (non-periodic), in that way it demonstrates chaos. The motion we are describing on these strange attractors is what we mean by chaotic behavior. The Lorenz attractor was the first strange attractor, but there are many systems of equations that give rise to chaotic dynamics. There are several types of strange attractors including;

Chua systems in electric circuits, Duffing systems in non linear oscillators, Ikeda systems in the turbulence of trails of smoke, Lorenz systems in the atmospheric convection and Rossler systems in the chemical kinetics[50].

#### **2.4.1 The concept of butterfly effect**

Butterfly effect is the way of describing how, unless all factors can be counted for, large systems like weather remain impossible to predict with total accuracy because there are too many variables to track. It is also called a sensitive dependence on initial conditions. Lorenz coined the term and put forward the idea of “Butterfly effect” In his paper titled as “Predictability; Does the flap of butterfly’s wing in Brazil set off a Tornado in Texas?”

What does it mean, Simply it can be explained as The butterfly does not cause the Tornado. The flap of the wing is the part of initial conditions: one set of conditions lead to a typhoon while the other set of conditions does not. The flapping wings represents a small change in initial condition of the system, which cause a chain of events leading to a large-scale alterations of events[6].

A better analogy of Butterfly effect is an avalanche .It can be provoked with a small input (a loud noise, some burst of the wind), it's mostly unpredictable, and the resulting energy is huge.

Butterfly effect is a symbol of chaos. It's a simple and entertaining way of describing one component of the chaos theory, that's "Sensitive dependence on initial conditions". Essentially, the Butterfly effect explains how small changes at one point of non linear system can results into a larger differences to a later state.

Chaos theory itself is a much larger system of theorems and formulas for predicting and understanding the behaviors of complex, nonlinear system. Therefore, Chaos theory and Butterfly effect are not the same.

Because of the Butterfly effect, now is well understood that weather forecasts can be accurate only in short-term, and that long-term forecasts even made with the most sophisticated computers methods will always be no better than a guessing.

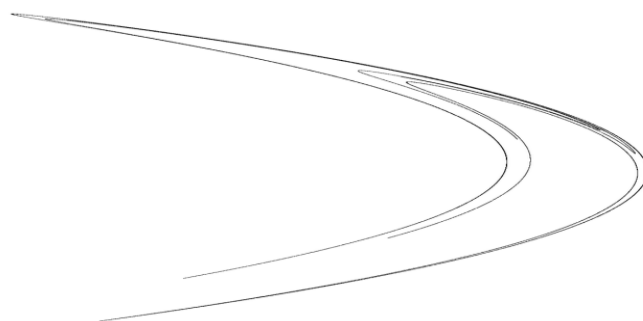
## 2.5 The Hénon Map

### 2.5.1 Theory

The Hénon map is a discrete-time dynamical system. It is one of the most studied examples of dynamical systems that exhibit chaotic behavior. The Hénon map takes a point  $(x_n, y_n)$  in the plane and maps it to a new point.

$$\begin{aligned}x_{n+1} &= 1 - ax^2 + y_n \\y_{n+1} &= bx_n\end{aligned}\tag{2.2}$$

The equations (2.2) were used , and the following plot or map(Figure 2.5) were obtained,



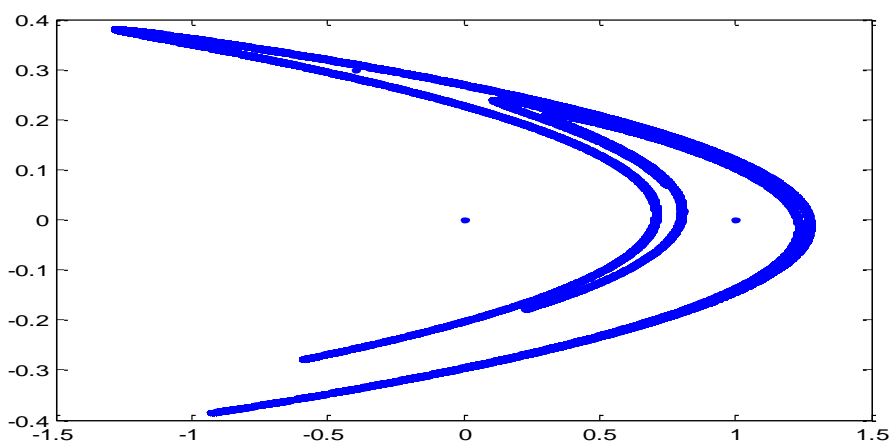
**Figure 2.5:** Classical Hénon attractor.

The map depends on two parameters,  $a$  and  $b$ , which for the classical Hénon map have values of  $a = 1.4$  and  $b = 0.3$ . For the classical values the Hénon map is chaotic. For other values of  $a$  and  $b$  the map may be chaotic, intermittent, or converge to a periodic orbit. An overview of the type of behavior of the map at different parameter values may be obtained from its orbit diagram. So that, it yields irregular solutions for many choices of  $a$  and  $b$ , for example, when  $a = 1.4$  and  $b = 0.3$ , a typical sequence of  $x$ , will not be periodic but chaotic [43].

The map was introduced by Michel Hénon as a simplified model of the Poincaré section of the Lorenz model. For the classical map, an initial point of the plane will either approach a set of points known as the Hénon strange attractor, or diverge to infinity [51].

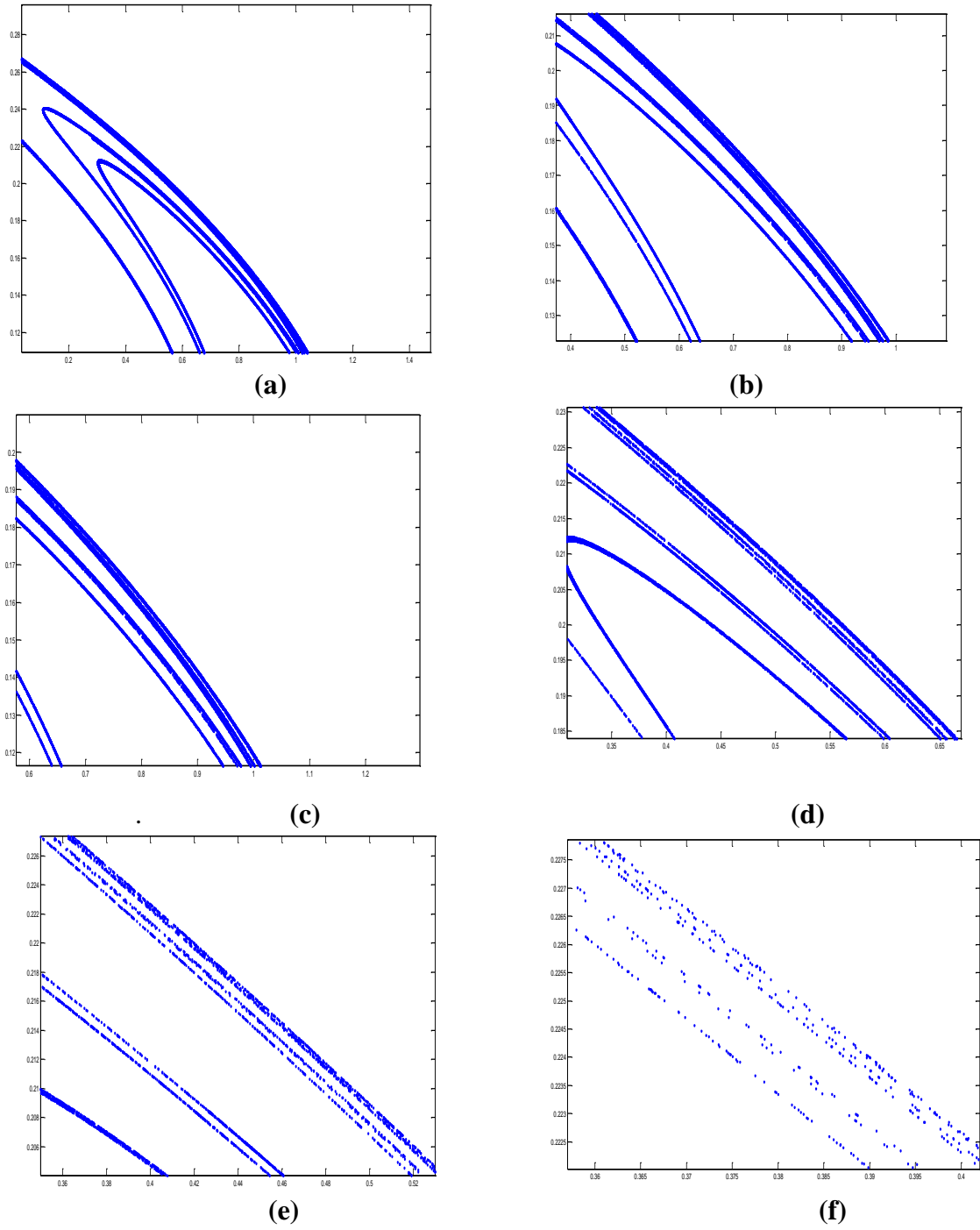
### 2.5.2 Applications

Hénon investigated a two-dimensional difference equation which was motivated by a hydrodynamical system of Lorenz. Numerically solving this equation indicated for certain parameter values the existence of a “strange attractor”, i.e., a region in the plane which attracts bounded solutions and in which solutions wander erratically. Hénon investigated a two-dimensional difference equation which was motivated by a hydrodynamical system of Lorenz. Numerically solving this equation indicated for certain parameter values the existence of a “strange attractor”, i.e., a region in the plane which attracts bounded solutions and in which solutions wander erratically.



**Figure 2.6:** The Hénon map.

100000 points were used to draw the Hénon attractor with initial conditions  $x(1)=y(1)=0$ . The points within an attractor do not flow continuously, but jumps from one point to another on a microscopic scale. The Hénon structure shows a great deal of fine structure with an infinite amount to be exact. For the clarity, several magnification steps were carried out and the following Figure 2.7 were obtained :



**Figure 2.7:** The Hénon map's magnifications: (a)-Magnified view of the Hénon map's upper arm,(b)-The thickest line yields other three lines, (c)-Third

magnifications, (d)-The Fourth magnifications, (e)-The Fifth magnification and (f)-The Sixth magnifications.

Taking a snapshot and magnified view of the upper arm of the Hénon structure, (figure 2.6). Three lines are seen (figure (a)), the upper line appear to be thickest followed by two embedded lines and one line at the bottom.

On another magnifications (figure (b)), the thickest line yields other three lines in the order of thickness from top to the bottom. If we can do a third magnifications we notes the upper line is made up of three lines, a second line made up of two lines and a single line at the bottom (see figure (c) and (d)).

The same phenomenon seen on the fourth magnifications, that, the upper line seems made of three lines, followed by two line at the middle and single line at the bottom. The clarity of the Hénon map is diminishes, (Figure (e) and figure (f)), due to finite number of points we took in our simulation. If we could have infinite number of points, we would have infinite number of sub structures or line-patterns in our Hénon maps.

These experiments illustrates that the structures within a Hénon maps repeats identically at each observation scale, so it has FRACTAL dimension, despite the flow of points being irregular and Chaotic.

## **2.6 Logistic equation.**

### **2.6.1 Theory**

One could think that chaotic systems need complicated formulae, but there are very simple functions which can lead not only chaos, but how this develops from “ordered” behaviour. The logistic function, used in population dynamics, is one of these functions, which we will describe in this section.

### **2.6.2 Applications**

Biologists had been studying the variability in populations of various species and they found an equation that predicted animal populations reasonably well. This equation was a simple quadratic equation called the logistic difference equation. On the surface, one would not expect this equation to provide the fantastically complex and chaotic behavior that it exhibits.

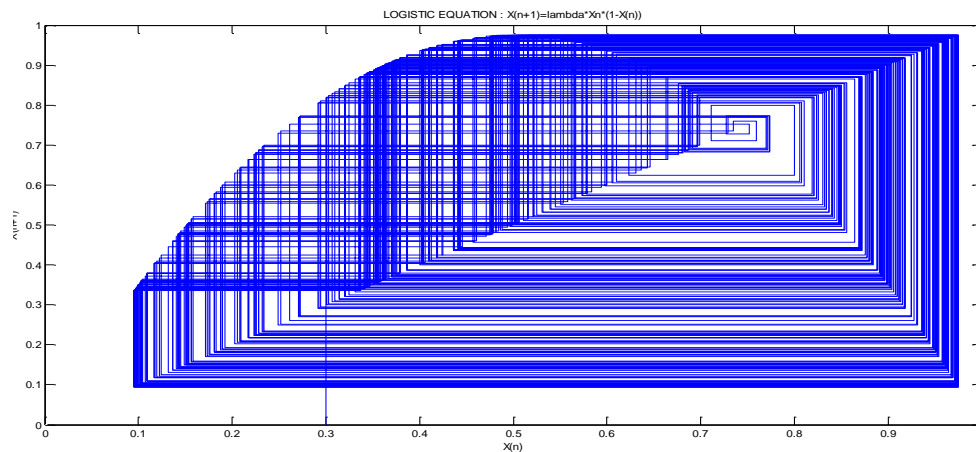
The logistic difference equation is given by

$$X_{n+1} = rX_n \left(1 - X_n\right) \quad (2.3)$$

where  $r$  is the so-called driving parameter.

The equation (2.3) is used in the following manner. Start with a fixed value of the driving parameter,  $r$ , and an initial value of  $x_0$ . One then runs the equation recursively, obtaining  $x_1, x_2, \dots, x_n$ . For low values of  $r$ ,  $x_n$  (as  $n$  goes to infinity) eventually converges to a single number. In biology, this number ( $x_n$  as  $n$  approaches infinity) represents the population of the species.

The logistic difference equation (2.3) were used and the Logistic map (Figure 2.8) were obtained as,

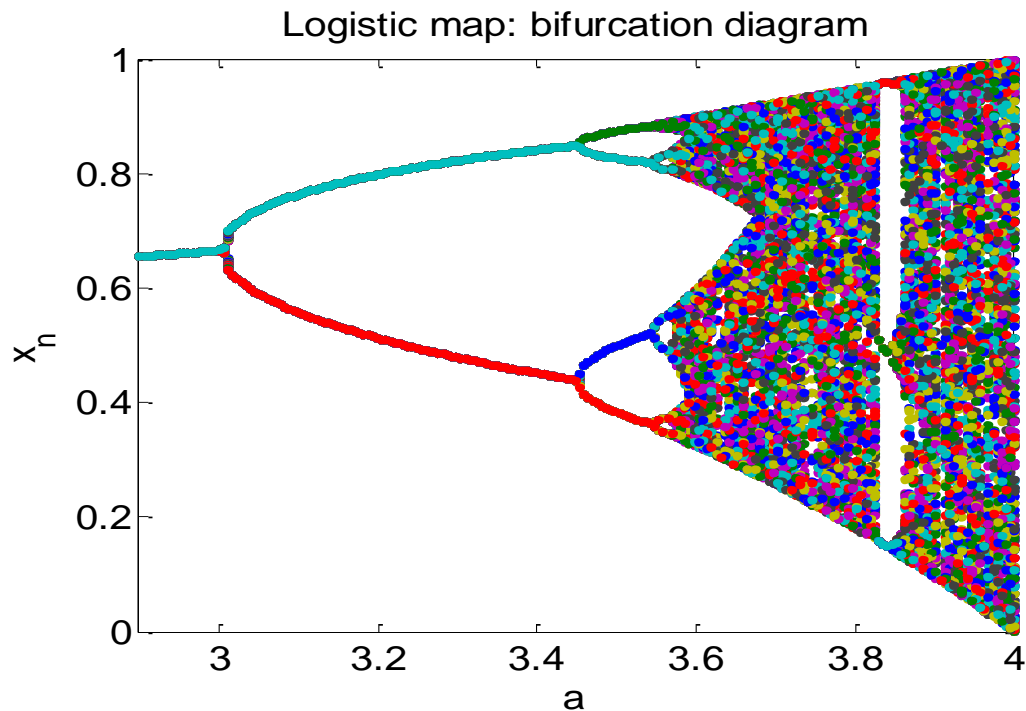


**Figure 2.8:** The cobweb diagram for logistic equation.

The equation were used in its simplified form, such that, Driving parameter  $r = \text{lambda} = 3.9 = \text{Fertility coefficient}$ , Number of iterations =  $\text{niter} = 500$ . The cobweb diagram [52,53] of the logistic map above were produced, showing Chaotic behavior for  $r = 3.9$ . A chaotic orbit would show a 'filled out' area, indicating an infinite number of non-repeating values.

The logistic difference equation (2.3) show how a small interval, under repeated application of the logistic map grows to cover the entire interval from 0 to 1, providing a graphical understanding of sensitive dependence on initial conditions under a chaotic map.

### 2.6.3 Logistic map-bifurcations diagram



**Figure 2.9:** Bifurcation diagram for logistic map.

Where,  $a$  = Fertility coefficient ;  $2.9 \leq a \leq 4$ . This plot obtained in the following way : For each value of  $r=a$  (recall equation 2.3 ), the logistic difference equation is iterated for  $n$  steps (starting from a random initial number) to attain stable behaviour (if there is any). Then a further  $m$  iteration steps are performed and  $X_n$  is plotted at the resulting time points  $n < t < n+m$ .

In the bifurcation diagram for logistic map, at  $a = 3.45$  ( see Figure 2.9), the first bifurcation (period doubling) as the solution begins to oscillate between two values for this value of " $a$ ". As " $a$ " increases further there are further period doublings. Eventually, for even larger values of " $a$ " the logistic differential equation shows chaotic behaviour, which means that the population behaviour cannot be predicted accurately for longer periods of time. Two trajectories starting from nearly identical values will diverge further and further away from each other. In this case, the routine to CHAOS starts with period doubling as seen in the above diagram.

## 2.7 Determination of Chaos

### 2.7.1 Lyapunov exponent theory

A number of researchers have developed methods which can be divided into two distinct approaches, direct methods and tangent space methods. Direct methods consist of searching the time series for neighbors at any given point and calculating expansion rates through comparison to these neighboring points. The first such method was that of Wolf[54].He developed a methodology in which one can calculate the largest positive Lyapunov exponent from a data set by following the long term evolution of one principal axis, a ‘fiducial trajectory’, progressively reorthonormalized maintaining phase space orientation.The method is highly sensitive to inputs, however, and can easily lead to an erroneous result [54].

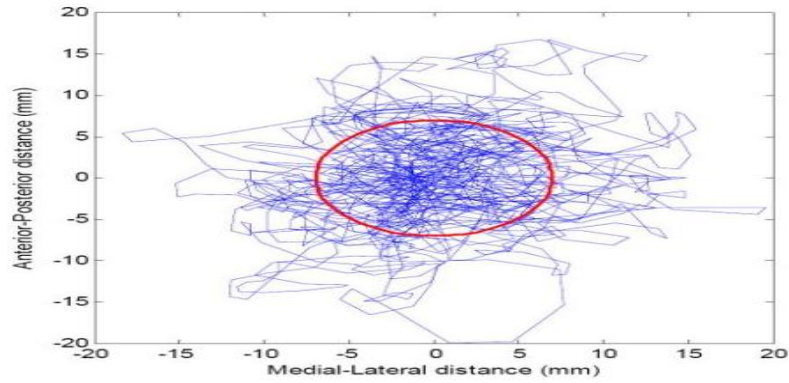
In the early 1990’s two separate research groups produced a new method. The approach eliminates Wolf requirements,imposes upon maintaining phase-space orientation stating it is unnecessary for calculating the largest Lyapunov exponent [36]. Additionally, rather than following one trajectory, the full data set is used, and in essence a trajectory for every pair of nearest neighbors is calculated.

Both methods are substantively similar. The Kantz algorithm (and similarly the Rosenstein algorithm) calculates the largest Lyapunov exponent by searching for all neighbors within a neighborhood of the reference trajectory and computes the average distance between neighbors and the reference trajectory as a function of time (or relative time scaled by the sampling rate of the data) [36].

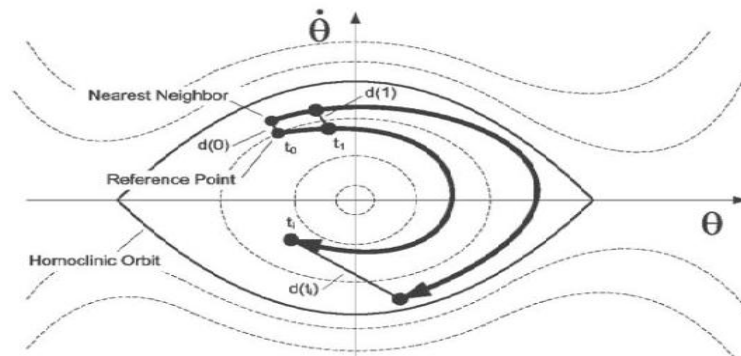
System stability is better quantified using stability diffusion analysis or maximum Lyapunov exponents. In both of these methods trajectories are followed for a finite time and the stability of the system determined by averaging the results over the time series[14].

Consider the following illustration for Lyapunov exponent method,

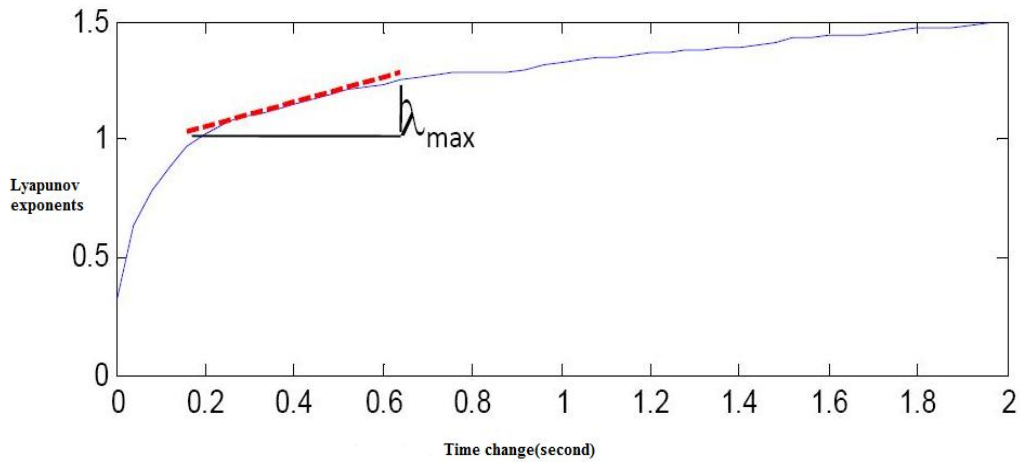




(a)



(b)



(c)

**Figure 2.10:** Trajectories are followed for a finite time (a) The attractor (b)The orbit involved, (c) Lyapunov exponent graph[14].

Detecting the presence of chaos in a dynamical system is an important problem that is solved by measuring the largest Lyapunov exponent (as shown in figure 2.10(c)). Lyapunov exponents quantify the exponential divergence of initially close state-space trajectories and estimate the amount of chaos in a system [35].

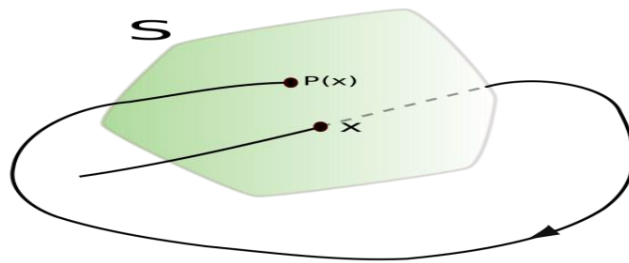
The signs of Lyapunov exponents demonstrate qualitative picture of a system's dynamics, as it was illustrated (Figure 2.10). One dimensional maps are

characterized by a single Lyapunov exponents which is positive for chaos, zero for marginally stable orbit and negative for periodic orbit.

## 2.7.2 Poincaré map

### Theory

Let  $(\mathbf{R}, M, \varphi)$  be a global dynamical system, with  $\mathbf{R}$  the real numbers,  $M$  the phase space and  $\varphi$  the evolution function. Let  $\gamma$  be a periodic orbit through a point  $p$  and  $S$  be a local differentiable and transversal section of  $\varphi$  through  $p$ , called Poincaré section through  $p$ , as seen in Figure 2.11 below,



**Figure 2.11:** Poincaré map  $P$  projects point  $x$  onto point  $P(x)$ [56].

Given an open and connected neighborhood  $U$  of  $p$ , a function,  $P : U \rightarrow S$  is called Poincaré map for orbit  $\gamma$  on the Poincaré section  $S$  through point  $p$  if

- $P(p) = p$
- $P(U)$  is a neighborhood of  $p$  and  $P:U \rightarrow P(U)$  is a diffeomorphism
- for every point  $x$  in  $U$ , the positive semi-orbit of  $x$  intersects  $S$  for the first time at  $P(x)$

In order to gain insight into the geometrical structure of the attractors, we use the Poincaré section technique. In Poincaré section all qualitatively interesting trajectories actually intersect the plane transversely [55].

Poincaré sections describe the intersection of phase space with a lower-dimensional plane or hyper plane. Often chaotic properties are easier analyzed on a lower-dimensional Poincaré section than on the original phase space. For example the property that an attractor is indeed a chaotic one is determined much easier with Poincaré sections. As I see it, the goal of a Poincaré section is to detect some sort of structure in the attractor[56]

The dynamical systems we study are of the form

$$\frac{d\bar{x}}{dt} = F(\bar{x}, t) \quad (2.4)$$

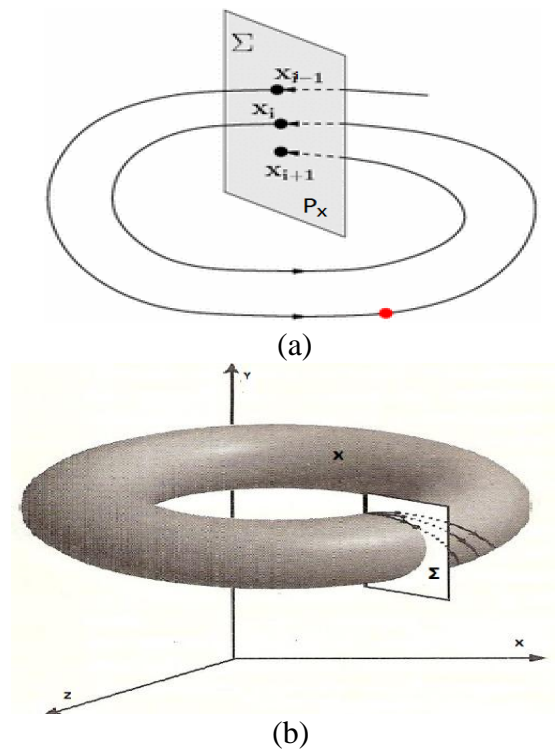
Systems of such equations describe a flow in phase space. The solution is often studied by considering the trajectories of such flows. But the phase trajectory is itself often difficult to determine, if for no other reason than that the dimensionality of the phase space is too large. Thus we seek a geometric depiction of the trajectories in a lower-dimensional space—in essence, a view of phase space without all the detail [14].

Consider the following mapping.

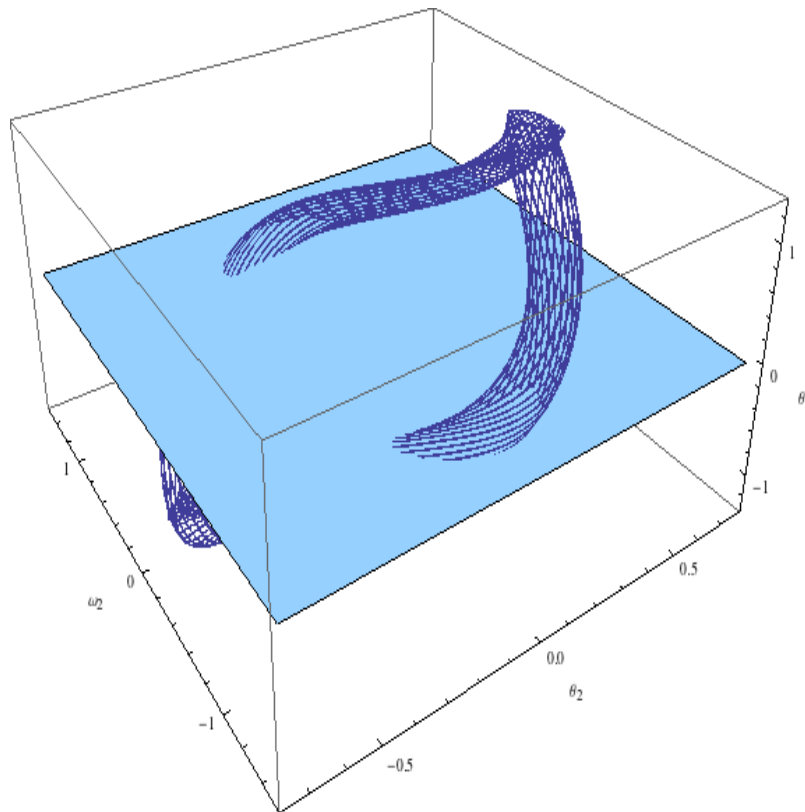
$$P : \Sigma \rightarrow \Sigma$$

$\Sigma$  is called a Poincaré surface of section.

$P$  is called a Poincaré first return map. It generates a new dynamical system with discrete time, as shown in Figure 2.12



**Figure 2.12:** (a) Poincaré mapping, (b) The section of torus.



(c)

**Figure 2.12** :(Continued) (c) Surface of section by Mathematica.

A quick way to look right at the surface of section plane (c) is by plotting the orbit but with a very restricted range in Mathematica: `PlotRange->{-0.001, 0.001}`[57].

We can simply say, Poincaré section, takes an attractor and an arbitrary plane, which cuts the attractor into two pieces. The orbits which comprise the attractor cross the plane many times. Plot the intersections of the orbits and the Poincaré plane, although only plot the intersections which occur in one direction (crossing from the "bottom" side to the "top" side for example). This is the Poincaré section, which can reveal structure of the attractor.

### 2.7.3 Dynamic and geometric measure of chaos

#### Dynamic measure chaos

This is time dependence which includes Lyapunov exponent,  $\lambda$  and Kolmogorov entropy.

Lyapunov exponents measure divergence of nearby trajectories. For chaotic system, divergence is exponential in time and it can simply be represented as

$$\lambda = \frac{\sum_{n=1}^N \lambda_n}{N} \quad (2.5)$$

If  $\lambda$  is zero system's trajectory is periodic, negative infers the system's trajectory to be stable periodic and positive the system's trajectory is chaotic.

Entropy, as a measure of the time rate of creation of information as a chaotic orbit evolves.

Shannon entropy(s) gives the amount of uncertainty concerning the outcome of the phenomenon,

$$S = \sum_{i=1}^{i=\infty} p_i \ln \left( \frac{1}{p} \right) \quad (2.6)$$

$0 \leq S \leq \ln r$ , where  $r$  is the number of events.

Kolmogorov-Sinai entropy rate  $K_n$  gives the rate of change of entropy as the system evolves ,

$$K_n = \frac{1}{\tau} \left( S_{n+1} - S_n \right) \quad (2.7)$$

### Geometric measure of chaos

Focuses on the geometrical aspects of attractors. Dimensionality of an attractor gives the actual degrees of freedom for the system.

Fractal dimension,

Dimensionality is the minimum number of variables needed to describe the state of the system. Chaotic system are of non integer dimension i.e Fractal dimension. Measured by box counting method.

Boxes of side length "R" to cover the space occupied by the object. Counting the minimum number of boxes ,N(R) needed to contain all the points of the geometric object. Then

Box counting dimension,

$$N(R) \approx \lim_{R \rightarrow 0} KR^{-D_b} \quad (2.8)$$

K-Constant.

$$D_b = \lim_{R \rightarrow 0} \frac{\log N(R)}{\log R} \quad (2.9)$$

For a point in 2D space,  $D_b = 0$ , For a line segment of length L,  $D_b = 1$  and For a surface of length L,  $D_b = 2$

Correlation dimension,

A simpler approach to the determination of dimension using correlation sum. It uses trajectory points directly.

Number of trajectory points lying within the distance, R of point i then the relative number of points will be,

$$P_i(R) = \frac{N_i(R)}{N-1} \quad (2.10)$$

Then Correlation dimension ( )

Correlation,

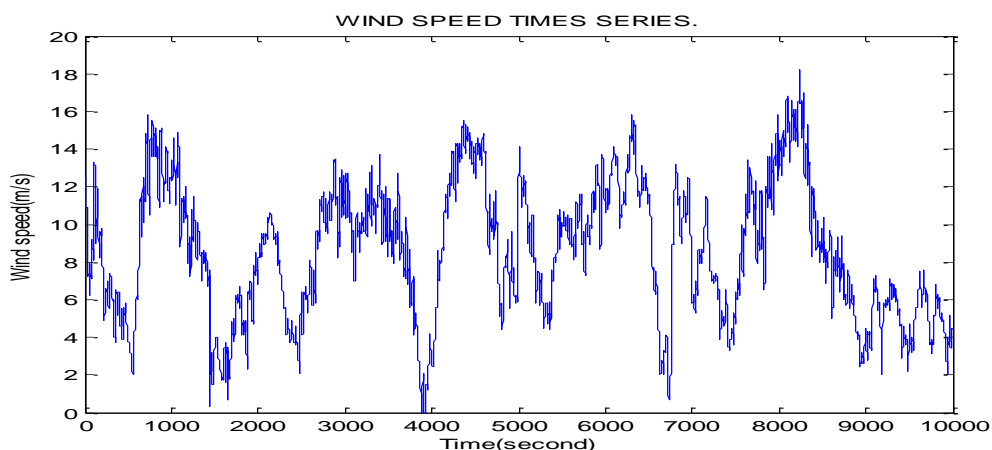
$$C(R) = \frac{1}{N \sum P_i(R)} \quad (2.11)$$

Thus,  $C(R) = 0$  No chaos,  $C(R) = 1$  Absolute chaos.

### 3. WIND SPEED DATA

#### 3.1 The Data Sets

The data sets available for the 5-year period from 1 January 2005 to 31 December 2009 with a sampling rate of 1 min at international aerodrome standards, were taken from an automatic weather observation station (AWOS) installed at a height of 10 m at Atatürk International Airport.



**Figure 3.1:** Wind speed time series.

#### 3.2 The Concept of Time Series

One definition of time series is that of a collection of quantitative observations that are evenly spaced in time and measured successively. Examples of time series include the continuous monitoring of a person's heart rate, hourly readings of air temperature, daily closing price of a company stock, monthly rainfall data, and yearly sales figures.

Time series analysis is generally used when there are 50 or more data points in a series. If the time series exhibits seasonality, there should be 4 to 5 cycles of observations in order to fit a seasonal model to the data.

Time series (as seen in figure 3.1) are analyzed in order to understand the underlying structure and function that produce the observations. Understanding the mechanism

of a time series allows a mathematical model to be developed that explains the data in such a way that predictions, monitoring, or control can occur. Examples include prediction/forecasting, which is widely used in economics and business. Monitoring of ambient conditions, or of an input or an output, is common in science and industry. Quality control is used in computer science, communications and industry.

Discrete time series means that observations are recorded in discrete times – it says nothing about the nature of the observed variable. The time intervals can be annually, quarterly, monthly, weekly, daily, and hourly.

Continuous time series means that observations are recorded continuously -e.g. temperature and/or humidity in some laboratory. Again, time series can be continuous regardless of the nature of the observed variable. The most important feature of time series data is their intrinsic order and Time plot is extremely important.

### **3.3 Nonlinear Time Series**

The time variability of many natural and social phenomena is not well described by standard methods of data analysis. However, nonlinear time series analysis uses chaos theory and nonlinear dynamics to understand seemingly unpredictable behavior. The results are applied to real data from physics, biology, medicine, and engineering in this volume. Researchers from all experimental disciplines, including physics, the life sciences, and the economy, will find the work helpful in the analysis of real world systems [43].

### **3.4 The Predictability of a Time Series**

The predictability of a time series using phase space techniques can be considered as a test for the deterministic nature of the system. These prediction techniques have been based on the fact that nearby trajectories, either converge or do not diverge fast enough for small sample steps in the phase space.



## **4. DEMONSTRATION OF CHAOTIC CHARACTER IN WIND SPEED SERIES AND PREDICTION**

### **4.1 Determination of Lyapunov Exponents From Experimental Data**

A large volume of work has been dedicated to the problem of calculating Lyapunov exponents from experimental time series. Detecting the presence of chaos in a dynamical system is an important problem that is solved by measuring the largest Lyapunov exponent.

The method of Lyapunov characteristic exponents serves as a useful tool to quantify chaos. Specifically, Lyapunov exponents measure the rates of convergence or divergence of nearby trajectories. Negative Lyapunov exponents indicate convergence, while positive Lyapunov exponents demonstrate divergence and chaos. The magnitude of the Lyapunov exponent is an indicator of the time scale on which chaotic behavior can be predicted or transients decay for the positive and negative exponent cases respectively [54].

Lyapunov exponents are important because they provide a well defined signature for the existence of chaos within a time series. If the largest Lyapunov exponent in a time series is positive and the area of its attractor is bounded, then nearby trajectories diverge at an exponential rate and thus the system exhibits sensitive dependence on initial conditions.

#### **4.1.1 Largest lyapunov exponents**

If we have time series from a real dynamical system, which may contain noise, then it is more difficult to differentiate between the broadband components associated with noise and those associated with chaotic behavior. Thus, the frequency spectra are generally not enough to confirm the presence of chaos in an experimental signal which will inevitably contain an element of background noise.

Therefore, we require techniques which can differentiate between those times series which are random (noisy) and those which are Chaotic. This can be done either by

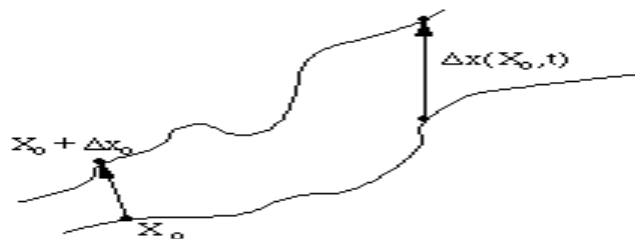
investigating its fractal structure or by measuring the exponential divergence of nearby trajectories on the attractor [8].

For a dynamical system, sensitivity to initial conditions is quantified by the Lyapunov exponents. For example; consider two trajectories with nearby initial conditions on an attracting manifold (figure 4). When the attractor is chaotic, the trajectories diverge, on average, at an exponential rate characterized by the largest Lyapunov exponent [58].

Lyapunov exponent's determination furnishes important indications with respect to chaotic patterns of dynamic systems. Lyapunov exponents describe the mean exponential increase or decrease of small perturbations on an attractor and are invariant with respect to diffeomorphic changes of the coordinate system [59].

On a practical framework to test chaotic dynamics, one may consider two points in a state space:  $x_0$  and  $(x_0 + \Delta x_0)$  as shown in figure 4, each of which will generate an orbit in that space using some equation or system of equations. These orbits can be thought as parametric functions of a variable, which is related to time [7].

If we use one of the orbits as reference orbit, then the separation between the two orbits will also be a function of time. This separation is also a function of the location of the initial value and has the form  $\Delta x(x_0, t)$ . The perturbation  $\Delta x_0$  created initially between  $x_0$  and  $(x_0 + \Delta x_0)$ , generates perturbed and unperturbed trajectories,



**Figure 4:** Two trajectories for Lyapunov exponent method derivation.

According to Saida [7] on his paper, "Using the Lyapunov exponent as a practical test for noisy chaos", In a more comprehensive form (from equation (2.5)) the difference between these two trajectories after "t" time steps is measured by:

$$\lambda = \lim_{t \rightarrow 0} \frac{1}{t} \ln \frac{|\Delta x(x_0, t)|}{|\Delta x_0|} \quad (4.1)$$

Where  $\lambda$  is called The Lyapunov exponent.

### 4.1.2 Classifications of Lyapunov exponents

The Lyapunov exponent,  $\lambda$ , measures the average exponential divergence (positive exponent) or convergence (negative exponent) rate between nearby trajectories within a time horizon that differ in initial conditions only by an infinitesimally small amount. Here, 3 cases of  $\lambda$  can be explained as:

$$\lambda < 0;$$

The orbit attracts to a stable fixed point or stable periodic orbit. Such systems exhibit asymptotic stability; the more negative the exponent, the greater the stability. Super-stable fixed points and super-stable periodic points have a Lyapunov exponent of  $\lambda = -\infty$ .

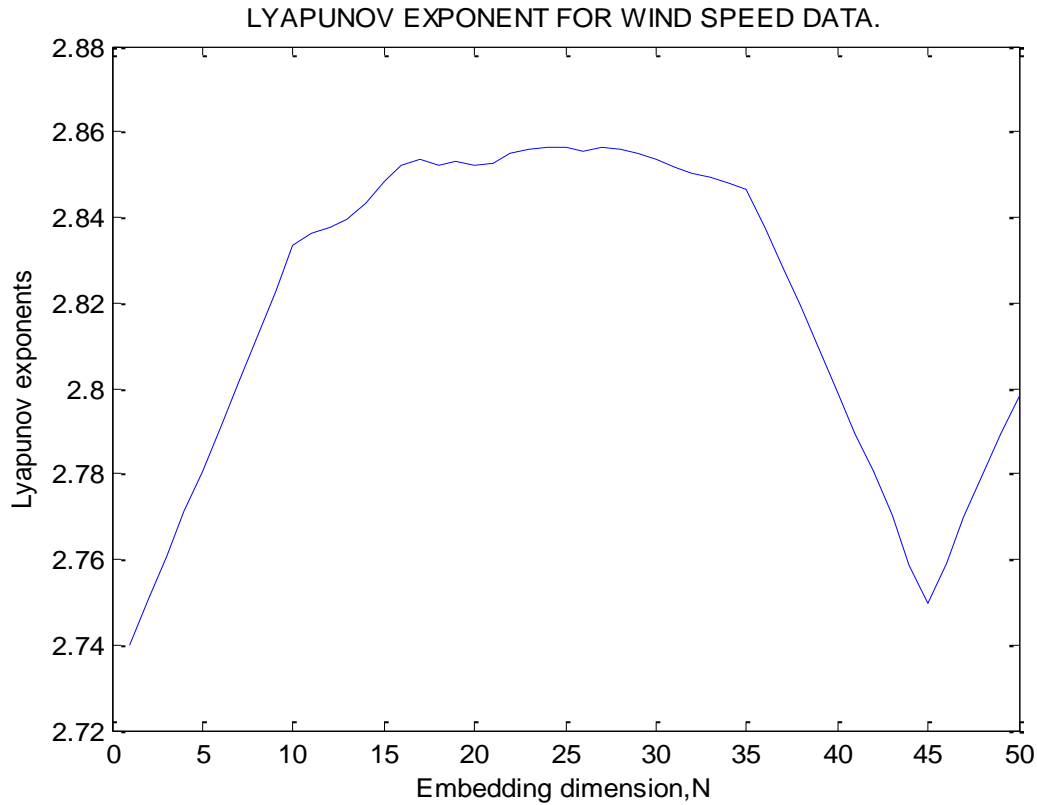
$$\lambda = 0;$$

The orbit is a neutral fixed point (or an eventually fixed point). A Lyapunov exponent of zero indicates that the system is in some sort of steady state mode. Such systems exhibit Lyapunov stability. A system with a zero Lyapunov exponent is near the “transition to chaos”.

$$\lambda > 0;$$

The orbit is unstable and chaotic. Nearby points, no matter how close, will diverge to any arbitrary separation.

We estimated the largest Lyapunov exponent  $\lambda$ , as a function of the embedding dimension and the results are summarized (in Figure 4.1) below. We obtain an estimated value around  $\lambda = 2.85 \pm 0.01$ . This positive value indicates the existence of chaotic behavior.



**Figure 4.1:** Largest Lyapunov exponent graph for wind speed data.

## 4.2 Phase Space Reconstruction

The analysis of nonlinear dynamical systems from time series involves state space reconstruction. There are two different methods for this aim: derivative coordinates and delay coordinates. The method of delay coordinates has proven to be a powerful tool to analyze chaotic behavior of dynamical system. Ruelle [48], Packard [60] and Takens [40] introduced the basic idea of this method and the main problem arising is the determination of the embedding parameters.

Prior to phase space reconstruction based on Embedding Theorem (Takens,1981) embedding parameters such as embedding dimension and time delay should be determined from the time series[41].

The time evolution of a phenomenon can be given by its trajectories in the phase space. Coordinates of this space are spanned by the variables, which are necessary to specify the time evolution of the system. Every point in a phase space shows a state of the system and every trajectory represents the time evolution of the system

corresponding to different initial conditions. Points or a set of points in a phase space compose a pattern which attract trajectories onto itself.

PSR is necessary in order to estimate the attractor's complexity of the system quantitatively and determine whether dynamic behaviours observed are complex or not [9].

By considering a time series of a single variable, it is assumed that the time series is generated from a chaotic dynamical system [40]. In addition, it is generated by a nonlinear dynamic system with  $d$  degrees of freedom. Therefore to have a better view, it is necessary to construct an appropriate series of state vectors  $X^d(t)$  with delay coordinates in the  $d$ -dimensional phase space as equation (4.2)

$$X^d = [X(t), X(t+T), \dots, X(t+(d-1)T)] \quad (4.2)$$

where,  $T$  is the delay time and,  $d$  is the the embedding dimension.

Phase Space Reconstruction is at the core of nonlinear time series analysis. In such reconstructed spaces, the deterministic features of systems can be identified and studied, such as empirical invariant measures, attractor dimensions, entropies, Lyapunov exponents, equations of motion, and short-term forecasts can be made.

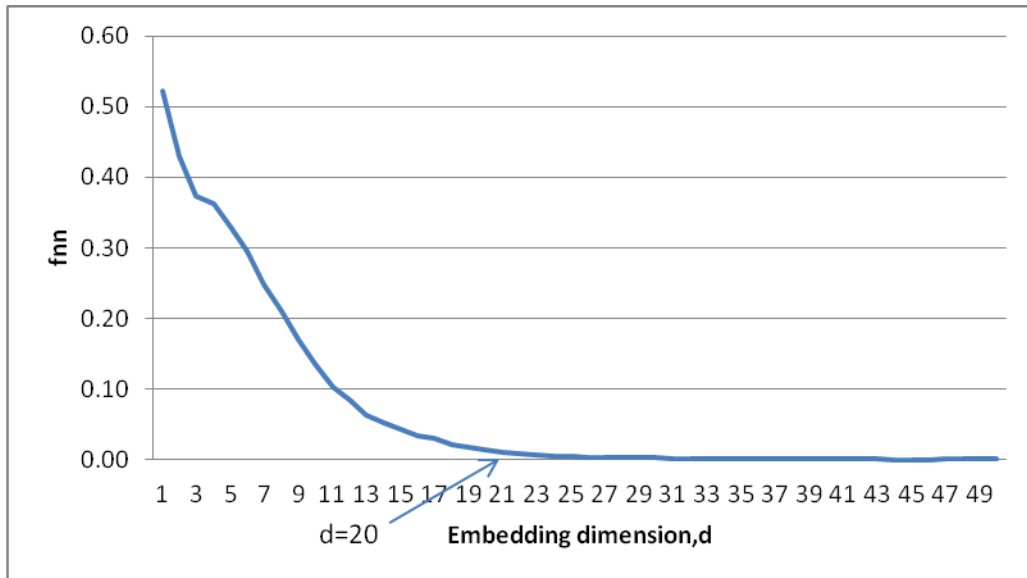
For a scalar time series  $X_t$ , where  $t = 1, 2, 3, \dots$ , the phase space can be reconstructed using the method of delays. The basic idea in the method of delays is that the evolution of any single variable of a system is determined by the other variables with which it interacts. Information about the relevant variables is thus implicitly contained in the history of any single variable. On the basis of this, an "equivalent" phase space can be reconstructed by assigning an element of the time series  $X_t$  and its successive delays as coordinates of a new vector time series [9, 41].

#### 4.2.1 False nearest neighbour

The false nearest neighbours procedure is a method to obtain the optimum embedding dimension for phase space reconstruction. By checking the neighbourhood of points embedded in projection manifolds of increasing dimension, the algorithm eliminates 'false neighbours': This means that points apparently lying close together due to projection are separated in higher embedding dimensions. The key idea is that as we enlarge the dimension of our vector, we tend to eliminate step by step the

intersections of orbits on the system attractor arising from our projection during the measurement process.

The implementations of FNN algorithm were carried out by using TISEAN package [61], as shown (from figure 4.2), percentage of FNN takes its minimum at  $d=20$ .

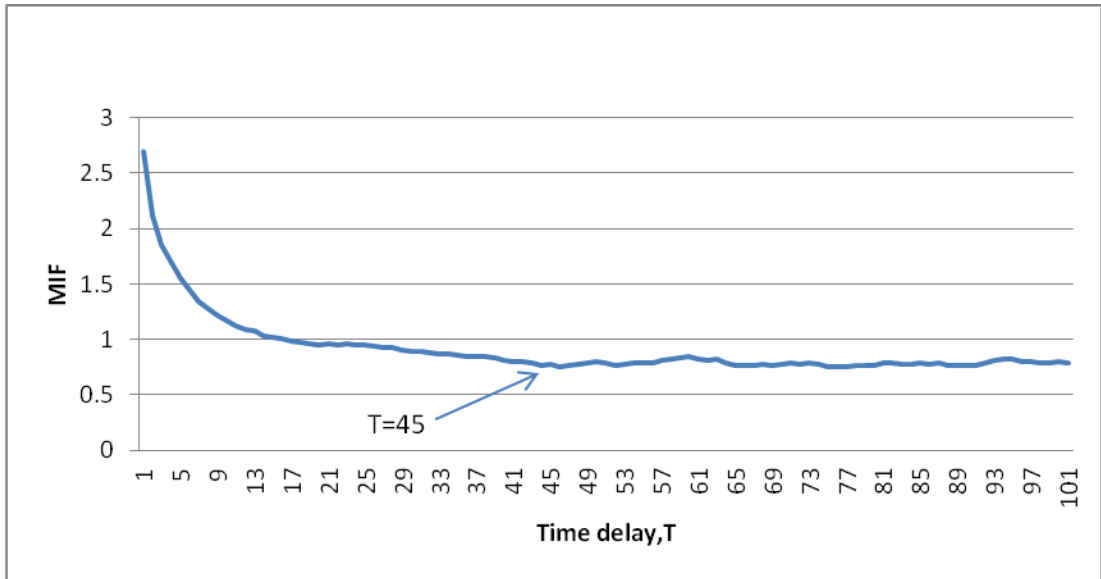


**Figure 4.2:** False nearest neighbours for wind speed data.

#### 4.2.2 Mutual information function

MIF measures how much one random variables tells us about another. High mutual information indicates a large reduction in uncertainty; low mutual information indicates a small reduction; and zero mutual information between two random variables means the variables are independent. These algorithms have been implemented by using TISEAN [61].

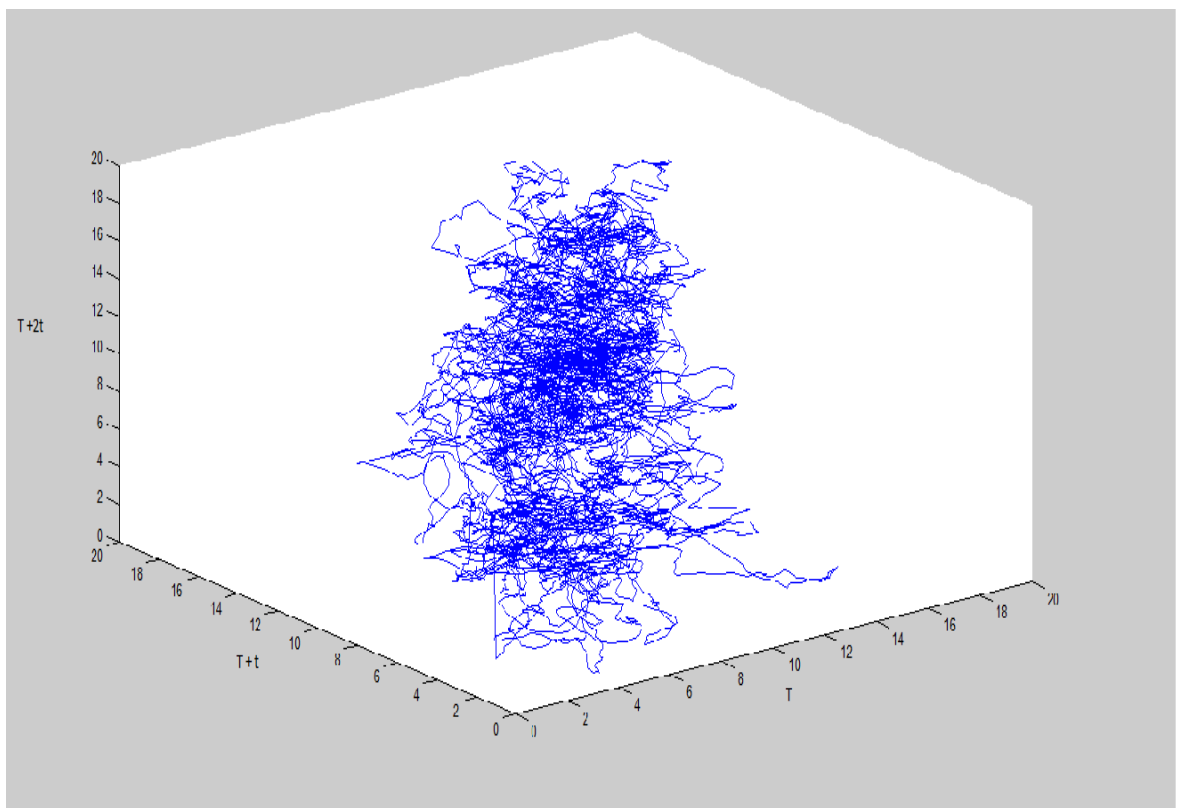
Time delay can be taken as  $\tau=45$  because it corresponds to the first minimum of the mutual information function as shown in Figure. 4.3



**Figure 4.3:** Mutual information for wind speed data.

### 4.3 The Attractor in the Reconstructed Phase Space

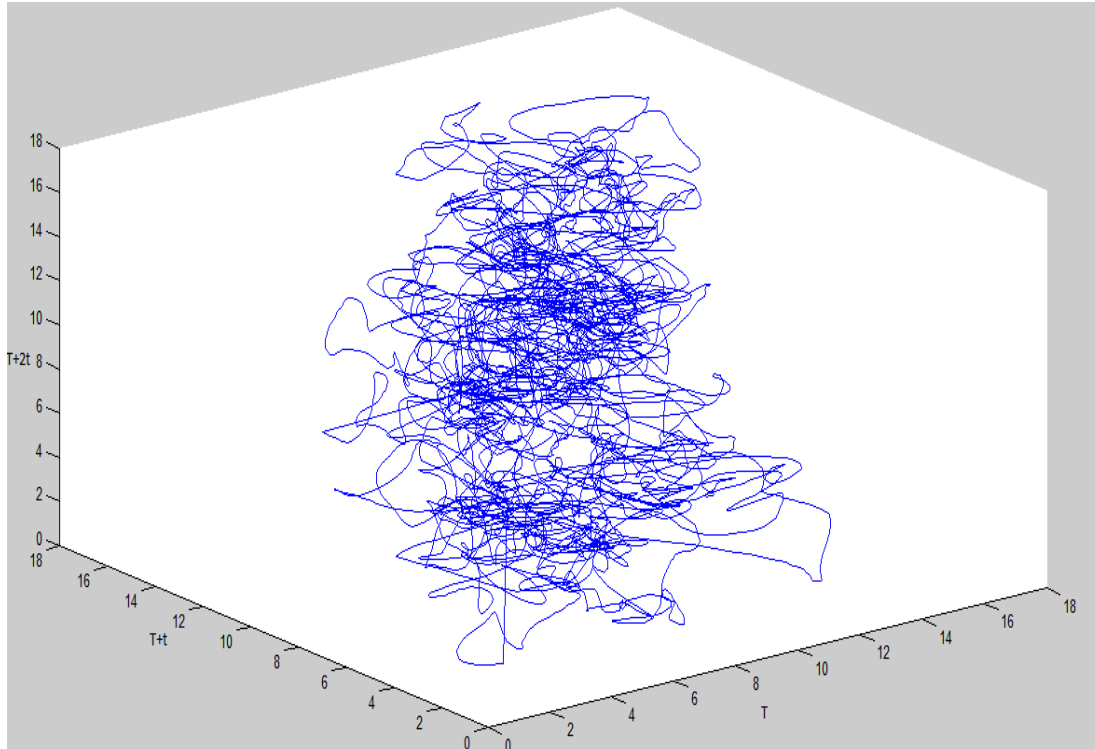
The wind speed data exhibit chaos, here the attractor were produced from the vector space obtained, as shown in the Figure 4.4 below,



**Figure 4.4:** The attractor in the RPS for wind speed data.

#### 4.4 The Denoised Attractor in the Reconstructed Phase Space

Later the data were denoised so as to provide clear sense that, lines are not intersected in the attractors, as shown in the Figure 4.5 below,



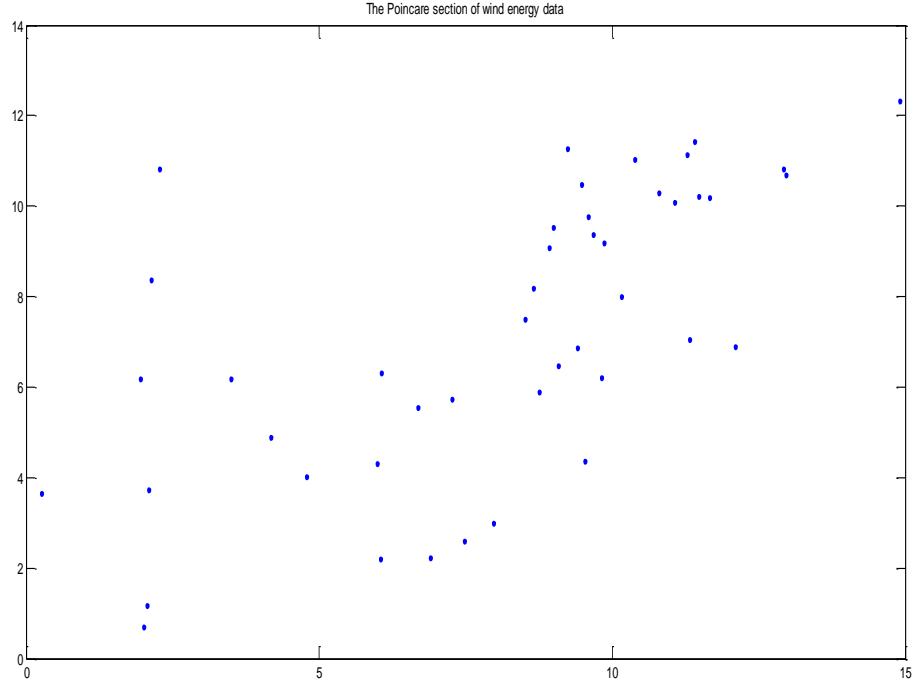
**Figure 4.5:** The denoised attractor in the RPS for wind speed data.

#### 4.5 The Poincaré Section of RPS.

The Poincaré section computation has been based on the work of Merkwirth [42] and Kantz [43] which proposes the section extracted directly from an embedded time series. The result is a set of  $(n-1)$ -dimensional vector points, used to perform an orthogonal projection.

When the plane cuts through the RPS, Figure 4.5, it plots the points where they intersect on the plane's surface as shown in the figure 4.6. The algorithms have been implemented by using TISEAN package [61].





**Figure 4.6:** The Poincaré section of RPS.

#### 4.6 Local Prediction Method

A correct PSR in a dimension  $m$  facilitates an interpretation of the underlying dynamics in the form of a  $m$ -dimensional map  $f_T$ . According to Eq (4.3)

$$Y_{j+T} = f_T(Y_j) \quad (4.3)$$

where  $Y_j$  and  $Y_{j+T}$  are vectors of dimension  $m$ , describing the state of the system at times  $j$  and  $j+T$  where they are current and future state respectively. Local approximation entails the subdivision of the  $f_T$  domain into many subsets (neighborhoods), in order to determine a proper value for  $f_T$ . In other words, the dynamics of the system is described step by step locally in the phase space. Before applying reconstruction procedure, it is necessary to have some information such as, embedding dimension and delay time. One of the independent coordinates mentioned above is taken as the time series itself. The remaining coordinates are formed by its  $(m+1)$  lagged time series shifted by  $(m+1)$  multiples of the correlation time  $T$ , at which correlation between coordinates become zero. It is assumed that the time series data are generated from a chaotic dynamical system in the  $v$ -dimensional space ( $v$  is the dimension of attractor). In this  $m$ -dimensional space, prediction is performed by estimating the change of  $X_i$  with time. Considering the relation

between the points  $X_t$  and  $X_{(t+T)}$  at time  $T$  later on the attractor is approximated by function  $F$  as in Eq (4.4);

$$X_{t+T} \cong F(X_t) \quad (4.4)$$

In this prediction method, the change of  $X_t$  with time on the attractor is assumed to be the same as those of nearby points,  $(X_T, h:1,2,\dots,n)$ . Herein,  $X_{(t+T)}$  is determined by the  $d^{th}$  order polynomial  $F(X_t)$  in Eq (4.5-4.10)[62];

$$X_{t+p} \cong f_0 + \sum_{k_1=0}^{m-1} f_{1k_1} X_{t-k_1T} + \sum_{k_2=k_1}^{m-1} f_{2k_1k_2} X_{t-k_1T} X_{t-k_2T} + \sum_{k_2=k_1}^{m-1} \dots \sum_{k_d=k_1}^{m-1} f_{dk_1k_2\dots k_d} X_{t=k_1T} \dots X_{t=k_dT} \quad (4.5)$$

$$x \cong Af \quad (4.6)$$

where;

$$x = (X_{t_1+T}, X_{t_2+T}, \dots, X_{t_n+T}) \quad (4.7)$$

$$f = (f_0, \dots, f_{10}, f_{11}, \dots, f_{1(n-1)}, f_{200}, \dots, f_{10d}, f_{(n-1)(n-1)}, \dots, f_{nl}) \quad (4.8)$$

$$A = n \times (n + d) / m!d! \quad (4.9)$$

And

$$A = \begin{bmatrix} X_{T_1} X_{T_1} \dots & X_{T_1} \dots & X_{T_1} \dots & X_{T_1}^2 \dots & X_{1-(n-1)T}^d \\ X_{T_2} X_{T_2} \dots & X_{T_2} \dots & X_{T_2} \dots & X_{T_2}^2 \dots & X_{2-(n-1)T}^d \\ \dots & \dots & \dots & \dots & \dots \\ X_{T_m} X_{T_m} \dots & X_{T_m} \dots & X_{T_m} \dots & X_{T_m}^2 \dots & X_{1-(n-1)T}^d \end{bmatrix} \quad (4.10)$$

In order to obtain a stable solution, the number of rows in the Jacobian matrix  $A$  must satisfy the relation in Eq(4.11):

$$n \geq \frac{(n + d)}{m!d!} \quad (4.11)$$

As stated by Porporato and Ridolfi even though in the case  $F$  are first degree polynomials, the prediction is nonlinear, because during the prediction procedure every point  $X(t)$  belongs to a different neighborhood and is therefore defined by different expressions for  $f$  [12],

## 4.7 Goodness of Fit

### 4.7.1 Root mean square error (RMSE)

The Root Mean Square Error (RMSE) is a frequently used measure of the difference between values predicted by a model and the values actually observed from the environment that is being modeled. These individual differences are also called residuals, and the RMSE serves to aggregate them into a single measure of predictive power. The RMSE of a model prediction with respect to the estimated variable  $X_{model}$  is defined as the square root of the mean squared error [41].

$$RSME = \sqrt{\frac{\sum_{i=1}^n (X_{OBSERVEDi} - X_{MODELi})^2}{n}} \quad (4.12)$$

As shown in Equation above,  $n$  is the number of point in the attractor,  $X_{obs}$  is observed values and  $X_{model}$  is modelled values at time/place  $i$ . The root-mean-square error (RMSE) statistics calculate the variance of the residual. The RMSE is always positive; the best value is zero; the higher the value, the poor the model performance.

### 4.7.2 Determination Coefficient ( $R^2$ )

The quantity ( $R^2$ ), called the linear correlation coefficient, measures the strength and the direction of a linear relationship between two variables. The value of  $R^2$  takes the value between the  $-1 < R^2 < +1$  and correlation greater than 0.8 is generally described as strong, whereas a correlation less than 0.5 is generally described as weak.

### 4.7.3 Results for local approximations

Taking into considerations the chaotic characteristic shown in figure 4.1, figure 4.4 and figure 4.5 then, local approximation method is applied to observe the prediction accuracy. The scatterplots of real and predicted value were plotted, figure 4.7 and the graph for the points distribution in predictions were drawn, figure 4.8. The results are as shown

below;

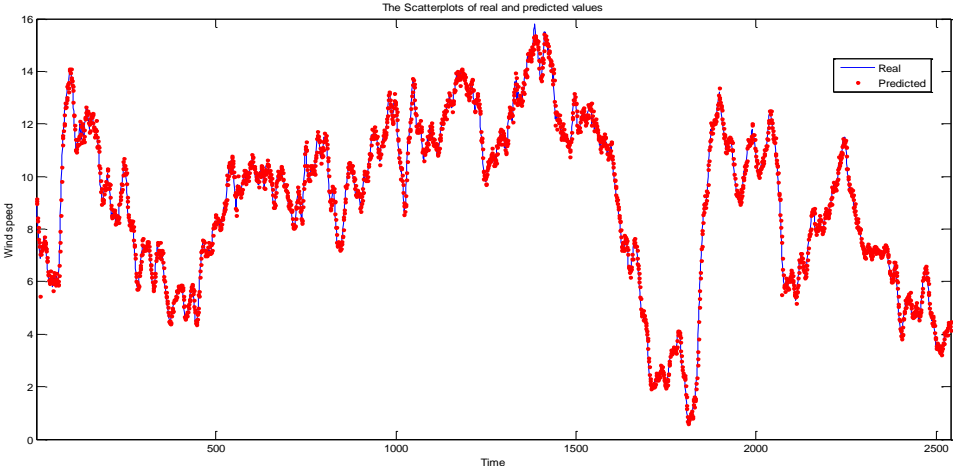


Figure 4.7: The scatterplots of real and predicted values.

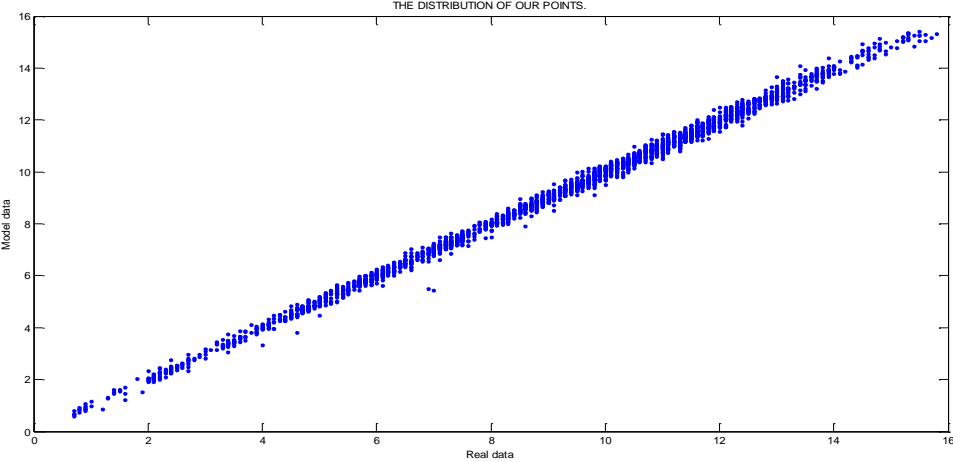


Figure 4.8: The graph for the points distributon in predictions.

## 5. RESULTS AND DISCUSSION

We implement phase diagram analysis in order to reconstruct an attractor. Firstly, the delay time and embedding dimension of the wind speed time series is calculated using the TISEAN [10] and results were  $T=45$  and  $d=20$  respectively. Then the Attractor without noise filtering (Figure 4.3) and denoised attractor (Figure 4.4) of the wind speed time series were obtained, in which three phase space components for  $x, y$  and  $z$  are  $T, T+t$  and  $T+2t$  respectively. So the three-dimensional phase diagrams of the wind speed time series were drawn.

The dimensions of the attractor were reduced to two dimensions by Poincaré sections technique, and the results are as shown. Poincaré map is neither a finite set of points (means periodic) nor a closed orbit (means quasi-periodic), which implies that the motion is chaotic. The wind speed time series is not a periodic sequence but a complex nonlinear time series.

Lyapunov exponent's method used to evaluate exponential divergence of nearby orbits and the positive exponent value obtained as in Figure 4.1. Positive Lyapunov exponents characterise the exponential divergence of nearby trajectories. This positive value,  $\lambda = 2.85 \pm 0.01$  indicates the existence of chaotic behavior. The fact that trajectories diverge directly implies a loss of information about their future position.

The local prediction model was also applied to evaluate its predictability performance. In this prediction model, the dynamics of the system are described systematically locally in the phase space. . In this prediction model, the dynamics of the system are described step by step locally in the phase space. The local predictor chooses a set of nearest neighbors, which evolves similarly in the reconstructed chaotic attractor.

The graph helps us to observe the prediction accuracy. It is a measure that allows us to determine how certain one can be in making predictions from a certain model/graph

The predicted values(Figure 4.7and Figure 4.8) are in good agreement with the observations by having high values of determination coefficient, $R^2 =0.9986$  and RMSE as 0.1670. The predictability of a time series using phase space techniques can be considered as a test for the deterministic nature of the system.

## 6. CONCLUSIONS

This study describes a series of analytic techniques for discerning and investigating chaotic behaviors in the wind speed times series data. The results obtained, Phase space reconstruction, Poincaré sections and Positive Lyapunov exponent for wind speed data shows that, there is exist a chaotic behavior in the wind speed data. Based on the above analysis, we can say that the wind speed time series has the chaotic evolution characteristics. Results give us enough confidence on our methodologies.

Certain deterministic non-linear systems may show chaotic behavior. Times series derived from such system seem stochastic when analysed with linear techniques, However uncovering the deterministic structure is important because it allows for construction of more realistic and better model and thus improve predictive capabilities[63].

This study demonstrated key Chaotic features that wind times series have such as positive Lyapunov exponent and Chaotic Poincaré section. The emphasis kept on state space reconstruction techniques that used to estimate these properties and displaying the chaotic behavior. Based on the above analysis, we can say that the wind speed time series has the chaotic evolution characteristics.

Also , it is worthwhile to note that many types of non linear equations may give rise to the chaotic behavior. Chaotic time's series data are observed routinely in experiments on physical systems and in observations in the field. Thus, if one is interested in non-linear system but not chaos *per' se*, the model or the system under study may still be chaotic in some parts (classical values) of parameter space [43]. To diagnose and understand or prevent such situations, knowledge of chaos is necessary.





## REFERENCES

- [1] **Renewable energy policy network for the 21 century.** (2014). *REN21, Renewables, Global status report*, Page 57-59.
- [2] **Inés, P.M.** (1999). *Synchronization and Control of Chaotic Systems. Spatio Temporal Structures and Applications to Communications*, (Msc Thesis), Universidad de Santiago de Compostela.
- [3] **Uittenbogaard, A.** (2000). *Introductions to Quantum Mechanics, Chaos Theory and Scripture Theory; Chaos Theory for Beginners (Chapter7)*. Date retrieved: 07.05.2015, address: <https://www.abarim-publications.com>
- [4] **Lorenz, E.N.** (1963). *Deterministic Nonperiodic Flow*, *Journal of Atmospheric Science*, **20**, 130-141.
- [5] **Mason, W.H.** (2015). “Reference for business. [Encyclopedia of Business](#)”, **2nd ed.** *Chaos Theory*. Date retrieved: 07.04.2015, address: [http:// www.Referenceforbusiness.com/management/Bun-Comp/Chaos-heory.html](http://www.Referenceforbusiness.com/management/Bun-Comp/Chaos-heory.html)
- [6] **Sardar Patel Institute of Technology** (2012). *Report on Chaos theory, Group C 2* (pp; 54-60)
- [7] **Saida, B.A.** (2004). *Using the Lyapunov exponent as a practical test for noisy chaos*, Department of Finance, Institut Supérieur de Gestion.Tunis.
- [8] **Addson, P.S.** (1997). *Fractal and Chaos an illustrated course*, page 22-24, 175-180.
- [9] **Baydaroglu, O., Kocak, K.** (2014). *SVR-based prediction of evaporation combined with chaotic approach*, *Journal of Hydrology*, **508**, 356–363
- [10] **Parker, T. S. and Chua, L.O.** (2000). *Practical Numerical Algorithms for Chaotic Systems*, Springer, New York
- [11] **Bublitz, A., and Allan, D.** (2007). *Chaos in a dripping Faucet*, University of Rochester. Date retrieved: 06.04.2015, address: <http://www.pas.rochester.edu/~advlab /ChaosFaucet.htm>
- [12] **Porporato, A. and Ridolfi L.** (1997). *Nonlinear analysis of river flow time sequences*, *Water Resources Research*, **33 (6)**, 1353–1367.
- [13] **Sivakumar, B.** (2000). *Chaos Theory in Hydrology: Important Issues and Interpretations*. *Journal of Hydrology*, **227**, 1-15
- [14] **Human performance and Biodynamics Laboratory** (2010). *Wake forest university school of medicine’s*, New York City. Date retrieved: 06.04.2015, address: [http://biodynamicslab.org/Analysis\\_Tools/Lyapunov.html](http://biodynamicslab.org/Analysis_Tools/Lyapunov.html)
- [15] **Albostan, A., Onoz, B.** (2015). *Implementation of Chaotic analysis on river discharge times series*, *Energy and Power Engineering*, **7**, 81-92.

- [16] **Khatibi, R., Sivakumar, B., Mohammad, A., Kisi, O., Koçak, K. and Zadeh, D.** (2012). Investigating Chaos in River Stage and Discharge Time Series. *Journal of Hydrology*, **414-415**, 108-117.
- [17] **Karakasidis, T.E., Charakopoulos, A.** (2009). Detection low-dimensional deterministic chaos in wind time series. *Chaos, Solitons and Fractals* **41** 1723–1732.
- [18] **Gaume, E., Sivakumar, B.** (2006). Identification of chaos in rainfall temporal disaggregation: Application of the correlation dimension method to 5-minute point rainfall series measured with a tipping bucket and an optical raingage, *Journal of Hydrology*, **328**, 56– 64.
- [19] **Daniel, A.R., Chen, A.A.** (1991). Stochastic simulation and forecasting of hourly average wind speed sequences in Jamaica, *Solar Energy*, **46**, 1-11.
- [20] **Kusiak, A., Zheng, H., Song, Z.** (2009). Short-term prediction of wind farm power: A data mining approach, *IEEE Transactions on Energy Conversion*, **24**, 1-12.
- [21] **Kani, S.A., Riahy, G. H.** (2008). A new ANN-based methodology for very short-term wind speed prediction using Markov chain approach, *IEEE Electrical power & energy conference*. Vancouver, BC, Canada, 06 - 07 Oct.
- [22] **Liu, H., Hong-qi,T., Di-fu, P., Yan-fei, L.** (2013). Forecasting models for wind speed using wavelet, wavelet packet, time series and Artificial Neural Networks, *Applied Energy*, **107**, 191–208.
- [23] **Cassola, F., Burlando, M.** (2012). Wind speed and wind energy forecast through Kalman filtering of numerical weather prediction model, *Applied Energy*, **99**, 154–166
- [24] **Castini, F., Festa, R., Ratto, C.F.** (1998). Stochastic modelling of wind velocities time series, *Journal of Wind Engineering and Industrial Aerodynamics*, **74—76**, 141—151.
- [25] **Negra, N.B., Holmstrøm., Bak-Jensen, O., B., Sørensen, P.** (2007). Model of a synthetic wind speed time series generator, *Wind Energy*, **11**, 193–209
- [26] **Czechowski, Z., Telesca, L.** (2013). Construction of a Langevin model from time series with a periodical correlation function: Application to wind speed data, *Physica A*, **392**, 5592–5603.
- [27] **Kocak, K.** (2002). A method for determination of wind speed persistence and its application, *Energy*, **27**, 967–973.
- [28] **Kocak, K.** (2007). Practical ways of evaluating wind speed persistence, *Energy*, **33**, 65–70.
- [29] **Kocak, K.** (2009). Examination of persistence properties of wind speed records using detrended fluctuation analysis, *Energy*, **34**, 1980–1985
- [30] **Chang, T., Ko, H., Chen, P., et al** (2012). Fractal dimension in wind speed times series, *Applied Energy*, **9**, 742–749

- [31] **Zeng, M., Jia, H., et al.** (2012). Non linear analysis of the near surface wind speed times series, *5th International Congress on Image and Signal Processing*, Sichuan, China, 16-18 Oct.
- [32] **Zounemat-Kermani, M., Kisi, O.** (2015). Time series analysis on marine wind-wave characteristics using chaos theory, *Ocean Engineering*, **100**, 46–53
- [33] **Peter, D.W.** (1967). The use of fast Fourier Transform for the estimation of power spectra: a method based on time averaging over short, modified period-grams. *IEEE Transactions on Audio and Electroacoustics*, **15(2)**: 70-73.
- [34] **Lai, Y. C and Lerner .D.** (1998). Effective scaling regime for computing the correlation dimension forms chaotic time series, *Physica D*, **115**, 1-18.
- [35] **Rosenstein, M.T., Collins, J.J., De Luca, C.J.** (1992). A practical method for calculating largest Lyapunov exponents from small data sets, *Physica D*, **65**, 117–134 .
- [36] **Rosenstein, M. T., Collins, J. J., Luca, C. J. D., and Michael, C.** (1993). A practical method for calculating largest Lyapunov exponents from small data sets, *Physica D*, **65**, 117–134.
- [37] **Sato, S., Sano, M., and Sawada, Y.** (1987). Practical methods of measuring the generalized dimension and largest Lyapunov exponent in high dimensional chaotic systems, *Progress of Theoretical Physics*, **77**, 1–5.
- [38] **Grassberger, P. and Procaccia.I.** (1983). Characterization of strange attractors, *Phys.Rev. Lett.*, **50**, 346.
- [39] **Kantz, H.** (1994). A robust method to estimate the maximal Lyapunov exponent of a time series, *Physics Letters A*, **185**, 77–87.
- [40] **Takens, F.** (1981). Detecting Strange Attractors in Turbulence. In: Rand, D.A. and Young, L.S., Eds., *Lectures Notes in Mathematics*, Springer-Verlag, New York, 366-381.
- [41] **Abarbanel, H.D.I.** (1996). *Analysis of Observed Chaotic Data.* (pp. 272). Springer- Verlag, New York.
- [42] **Merkwirth, C., Parlitz, U., and Lauterborn, W.** (1998). TSTOOL – A software package for nonlinear time series analysis, Katholieke Universiteit Leuven, Belgium, July 8–10.
- [43] **Kantz, H., Shreiber, T.** (2004). *Nonlinear Time Series Analysis*, Cambridge: Cambridge U. Press, 2nd ed.
- [44] **Houben, S.H.M.J., Maubach, J.M.L., Mattheij, R.M.M.** (2003). An accelerated Poincaré-map method for autonomous oscillators, *Applied Mathematics and Computation*, **140**, 191-216.
- [45] **Schuster, H.G.** (1989). *Reviews of Nonlinear Dynamics and Complexity*, 3th edition, pp 220-224, Wiley, Weinheim
- [46] **Robert, G. And Jim E.H.** (2007). Applying Chaos Theory to Careers: Attraction and attractors, *Journal of Vocational Behavior*, **71**, 375–400

- [47] **Sprott, J.C.** (1997). Strange attractors, from art to science Presented to the university of Wisconsin-Madison, Date retrieved: 06.04.2015, address: <http://sprott.physics.wisc.edu/lectures/Sacolloq/sld001.htm>
- [48] **Ruelle, D.** (1979). *Mathématique of the Institut des Hautes Études Scientifiques* **5**, 27.
- [49] **Giordano, N., Nakanishi, H.** (2012). *Computational physics using Matlab*, page 24
- [50] **Zmiers** (2008). Date retrieved: 06.04.2015, address: <http://www.slideshare.net/zmiers/choas-theory3-presentation>
- [51] **The Hénonmap.** *Wikipedia.* Date retrieved: 06.04.2015, address: <http://en.wikipedia.org/wiki/Hénon>
- [52] **Cobweb diagram,** *Wikipedia.* Date retrieved: 06.04.2015, address: [http://en.wikipedia.org/wiki/Cobweb\\_plot](http://en.wikipedia.org/wiki/Cobweb_plot)
- [53] **Stoop, R., Steeb, W.** (2006). Berechenbares Chaos in dynamischen Systemen [Computable Chaos in dynamic systems] Retrieved data August 2014.
- [54] **Wolf, A., Swift, J.B., Swinney, H.L.** (1984). Determining Lyapunov exponents from a time series, *Physica D*, **16**, 285–317
- [55] **Masoller, C., Sicardi Schifino, A.C., Romanelli, L.** (1995). Characterization of Strange Attractors of Lorenz Model of General Circulation of the Atmosphere, *Chaos, Solitons & Fractals*, **6**, 357–366
- [56] **Eric, R.W.** (1997). My adventures in Chaotic time series analysis. Retrived from the [http:// www.physics.emory .edu/faculty /weeks/](http://www.physics.emory.edu/faculty/weeks/)
- [57] **Martin, R.** (2013). Surface of surface plot, Department of Physics, PHY380 03, (Page,100-105) (Spring2013). Illinois State University.,
- [58] **Eckmann, J.P, Ruelle, D.** (1985). Ergodic theory of chaos and strange attractors. *Rev. Mod. Phys.* **57** 617.
- [59] **Parlitz, U.** (1998). Nonlinear time-series analysis, *Nonlinear Modeling-Advanced black-box Techniques*, edited by J. Suykens and J.Vandewalle, Kluwer Academic Publishers, Boston, pp. 209–239.
- [60] **Packard, N. J., Crutchfield, J. P., Fromer, J. D., and Shaw, R. S.** ( 1980). Geometry from a time-series, *Physical Review Letters*, **45**, 712–716.
- [61] **Hegger, R., Kantz, H., Schreiber.T.** (1999). Practical implementation of nonlinear time series methods: the TISEAN package, *Chaos*, **9**, 413–435.
- [62] **Itoh, K.** (1995). A method for predicting chaotic time-series with outliers, *Electronics and Communications in Japan*, **78(5)**, 44-53.
- [63] **Kugiumtzis, D., Lillekjendlie, B., Christophersen, N.** (1994). Chaotic Times series, Estimation on invariant properties in state space, *Modeling Identification and Control*, **15**, 205-224



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