ISTANBUL TECHNICAL UNIVERSITY ★ ENERGY INSTITUTE

TWO-PHASE FRICTION MULTIPLIER LOOK-UP TABLE FOR WATER-STEAM FLOWS IN VERTICAL TUBES

M.Sc. THESIS

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Energy Science and Technology Division

Energy Science and Technology Programme

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ ENERJİ ENSTİTÜSÜ

DÜŞEY BORULARDA SU-BUHAR AKIŞI İÇİN İKİ-FAZLI SÜRTÜNME ÇARPANI TARAMA TABLOSU

YÜKSEK LİSANS TEZİ

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Date of Submission : 4 May 2015
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To my mom, dad and brother,

FOREWORD

In this thesis, a two-phase friction multiplier (ϕ_{lo}^2) look-up table (LUT) for two-phase steam-water flows in vertical uniformly heated and adiabatic round tubes has been developed.

A new correlation has been derived from an experimental database with curve fitting by using the least square method. This correlation has been used to establish the first table (called the skeleton ϕ_{lo}^2 table). The skeleton ϕ_{lo}^2 table has been updated with the reliable experimental database in order to increase its accuracy. To eliminate discontinuities and jumps among the updated entries and the entries are not updated in the updated ϕ_{lo}^2 table, a smoothing procedure has been employed for a few times on the updated ϕ_{lo}^2 table until the final ϕ_{lo}^2 LUT has been obtained. Assessment of the table has been performed at each stage by checking the root-mean-square (RMS) and mean errors.

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May 2015

Bekir ÖZTÜZÜN
Mechanical Engineer

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ABBREVIATIONS

CHF : Critical Heat Flux
LUT **:** Look-Up Table
NVG **:** Net Vapor Generation
ONB **:** Onset of Nucleate Boiling
RMS **:** Root-Mean Square

Symbols

A_{x-s} : Cross-sectional flow area (m^2)
B_{Ch} : Chisholm parameter
C_o : Distribution parameter
c_p : Specific heat capacity (J/kg/K)
D : Tube inner diameter (m)
f : Friction-factor
g : Acceleration due to gravity (m/s^2)
G : Mass flux ($kg/m^2/s$)
H : Heat transfer coefficient ($W/m^2/K$)
h : Enthalpy (J/kg)
h_{fg} : Latent heat of vaporization (J/kg)
k : Thermal conductivity (W/m/K)
k_s : Surface roughness (m)
L : Length (m)
m : Constant
n : A constant
P : System pressure (kPa)
T : Temperature (^oC)
q : Power (W)
q'' : Heat flux (W/m^2 or kW/m^2)
u_{ga} : Drift velocity (m/s)
x : Thermodynamic quality
x_a : Actual (dynamic) quality
z : Axial distance (m)
Fr : Froude number
Pe : Peclet number
Re : Reynolds number
We : Weber number
α : Void fraction

ΔP	: Pressure drop (kPa)
Γ	: Chisholm property index
μ	: Viscosity (kg/m/s)
ϕ_{lo}^2	: Two-phase friction multiplier
ρ	: Density (kg/m^3)
σ	: Surface tension (N/m)

Subscripts

a	: Actual vapour mass
acc	: Acceleration
$adia$	: Adiabatic flow
ave	: Average
b	: At bulk conditions
f	: At saturated-liquid conditions
frc	: Friction
g	: At saturated-vapour conditions
go	: Total flow as vapour
gra	: Gravitation
h	: Heated section
in	: Inlet
l	: Liquid
lo	: Total flow as liquid
m	: Two-phase mixture
NVG	: Onset of net vapour generation
ONB	: Onset of nucleate boiling
out	: Outlet
sat	: At saturation
th	: Thermodynamic
w	: Near-wall condition
1ϕ	: Single-phase
2ϕ	: Two-phase
@	: At

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TWO-PHASE FRICTION MULTIPLIER LOOK-UP TABLE FOR WATER-STEAM FLOWS IN VERTICAL TUBES

SUMMARY

Two-phase (2ϕ) steam-water flows are extensively used in many industrial facilities, especially in nuclear power plants. The limits and capacity of these facilities are determined based on some critical parameters of the two-phase flows, such as critical heat flux (CHF) and two-phase friction multiplier (ϕ_{lo}^2), and the thermophysical properties of the working fluid, i.e., steam-water.

Many experimental measurements have been performed to determine the pressure drops (ΔP); and many studies are published in the form of experimental measurement results, as well as correlations and mathematical methods basing on the results of these measurements in order to determine the ϕ_{lo}^2 for different flow conditions. However, these studies cover only limited ranges of the flow conditions and thermophysical properties, such as heat flux (q''), pressure (P), mass flux (G) and actual (dynamic) quality (x_a). In addition, different results take place for the pressure drops depending on the experimental set-up and conditions such as geometry, type of fluid etc. and also evaluation methods and resulting correlations in these studies. Additionally, experimental data and correlations do not exist in some cases in the literature, where it is hard to reach the required flow conditions with an experimental set-up in addition to the scarce numbers of data.

On the other hand, it becomes widespread to construct and utilize look-up tables (LUTs) for selected critical parameters of the 2ϕ flows based on some approved methods, correlations and experimental database and using these LUTs in the computer codes in literature.

The applications and use of LUTs have some advantages. Their major advantages may be listed as:

- providing significantly accurate results without iterations,
- covering wide ranges of parameters or working conditions,
- having reliable parametric and asymptotic trends and
- being easy to update with new experimental data and correlations,
- hence improving their accuracy by updating them

In this thesis, a two-phase friction multiplier look-up table (ϕ_{lo}^2 LUT) for 2ϕ steam-water flows in vertical uniformly heated and adiabatic round tubes has been developed.

For this purpose, the following steps have been carried out:

- Initially, around 10,000 experimental 2ϕ-flow ΔP data for steam-water flows in vertical uniformly heated and adiabatic round tubes have been compiled to obtain a reliable experimental database. Assessment of the ΔP

database has been done by the comparison of experimental data with selected correlations and homogenous model after their selection in accordance with the data selection criteria by Durmayaz et al. (2012).

- A new correlation of ϕ_{lo}^2 based on *q"*, *P*, *G* and x_a has been derived from the reliable experimental ΔP database with curve fitting by using the least square method.
- This correlation has been used to establish the first 4-dimensional (based on *q"*, *P*, *G* and x_a) table (called the skeleton ϕ_{lo}^2 table).
- The skeleton ϕ_{lo}^2 table has been updated with the reliable experimental database to increase its accuracy.
- To eliminate discontinuities and jumps among the updated entries and the unupdated entries in the updated ϕ_{lo}^2 table, a smoothing procedure has been employed for a few times on the updated ϕ_{lo}^2 table until "*the final* ϕ_{lo}^2 *LUT*" has been obtained.
- Assessment of these ϕ_{lo}^2 tables has been performed at each stage by checking the root-mean-square (RMS) and the mean errors.

DÜŞEY BORULARDA SU-BUHAR AKIŞI İÇİN İKİ-FAZLI SÜRTÜNMEÇARPANI TARAMA TABLOSU

ÖZET

Günlük yaşamda ve endüstriyel işletmelerde kullanılan bir çok mühendislik uygulamalarında iki-fazlı (2ϕ) sıvı-buhar akışları ile karşılaşılmaktadır. Tez kapsamında incelenen iki-fazlı sürtünme çarpanı (ϕ_{lo}^2) ısı değiştiricilerinin, soğutucuların ve enerji tesislerinde yer alan çeşitli sistemlerin tasarımında, boru içi akışkan akışı termal-hidrolik hesaplamalarında basınç ve hız dağılımının ve uygulama sınırlarının belirlenmesinde, diğer çeşitli mühendislik uygulamalarında ve özellikle nükleer reaktörlerin güvenlik analizlerinde kullanılan önemli bir 2ϕ akış parametresidir.

Literatürde ϕ_{lo}^2'yu elde edebilmek için deneysel çalışmalarla elde edilmiş bir çok basınç düşümü (ΔP) ölçüm verisi ve bu verilere dayalı olarak geliştirilmiş bir çok korelasyon bulunmasına rağmen, literatürde mevcut olan bu ΔP korelasyonları ancak deneysel çalışmaların gerçekleştirildiği akış parametrelerinin belirli aralıklarında uygun sonuç verebilmektedir, yani kullanım alanları sınırlıdır. Ayrıca, mevcut ϕ_{lo}^2 korelasyonları basınç (P), kütlesel akı (G) ve dinamik kuruluk derecesi (x_a) ile bunlara bağlı olarak hesaplanan yoğunluklar, viskoziteler gibi termodinamik özeliklerin veya Reynolds sayısı, Weber sayısı, Froude sayısı gibi boyutsuz sayıların fonksiyonu olarak geliştirilmiştir. Bu korelasyonlarla elde edilen ϕ_{lo}^2'ya ısıtmanın etkisi ise, ısı akısını içeren Tarasova vd. (1966) gibi korelasyonların ϕ_{lo}^2 için düzeltme çarpanı olarak kullanılmasıyla dahil edilmektedir. Yani ısı akısı, basınç, kütlesel akı ve kuruluk derecesi parametrelerinin tümüne bağlı olarak düşey borularda sıvı su-buharın 2ϕ akışı için geliştirilmiş bir ϕ_{lo}^2 korelasyonu literatürde araştırılmış ancak bulunamamamıştır.

Ayrıca, bazı mühendislik uygulamalarında kullanılan kritik parametreler için tarama tabloları (TT) literatürde mevcuttur. Özellikle, yaklaşık yirmi yıldır geliştirilmesi sürmekte olan Kritik Isı Akısı Tarama Tablosu (CHF LUT) şu anda RELAP5/MOD3 nükleer reaktör güvenlik bilgisayar kodu gibi yazılım programlarında uygulamalarda literatürdeki mevcut korelasyonlar yerine kullanılmaktadır.

Öte yandan, düşey borularda sıvı su-buharının 2ϕ akışı için geliştirilmiş ϕ_{lo}^2 TT ise literatürde bulunmamaktadır.

Tarama tablolarının

- geniş veri aralığına sahip olma,
- hassas sonuç elde edilebilme,
- yeni veriler ve korelasyonlarla kolayca güncellenilebilme,
- iterasyonsuz sonuç verme ve ayrıca,

- daha doğru parametrik ve asimptotik davranışlara sahip olabilmesi gibi avantajları bulunmaktadır.

Bu tez kapsamında, sıvı su-buharın düşey borularda yukarı yönlü akışı için geliştirilmiş korelasyonlar ve deneysel verilerden istifade edilerek ısı akısı (q''), P, G ve x_a parametrelerine bağlı olarak bir iki-fazlı akış sürtünme çarpanı ϕ_{lo}^2 TT oluşturulması amaçlanmıştır. ϕ_{lo}^2 TT, aslen ısı akısı, basınç, kütlesel akı ve kuruluk derecesi parametrelerinin geniş bir aralığında bu bağımsız değişkenlerin fonksiyonu olarak sunulan bir veri matrisidir. ϕ_{lo}^2 TT, dış yüzeyinden sabit değerde üniform ısı akısıyla ısıtılmış borulardaki suyun iki-fazlı akışı için Alessandrini vd. (1963), Peterlongo vd. (1964), Leung (1994), Olekhnovitch (1997) ve Olekhnovitch vd. (2004)'den elde edilen deneysel ölçüm verilerinin belli kriterler çerçevesinde elendikten sonra güvenilir olarak geriye kalanlarıyla ve bu deneysel veriler kullanılarak Chisholm (1973) ve Tarasova vd. (1966) korelasyonlarının kombinasyonundan esinlenilerek bu tez kapsamında geliştirilen bir korelasyon kullanılarak oluşturulmuştur. Ayrıca ısı akısı, basınç, kütlesel akı ve kuruluk derecesi parametrelerinin igili aralığında literatürde deneysel çalışmalar yapılmamış bölgelerdeki iki-fazlı akış sürtünme çarpanının değerleri ise ϕ_{lo}^2'nin bu parametrelere bağlı değişimindeki genel fiziksel trendleri gözetilerek interpolasyonlar ve ekstrapolasyonlardan yararlanılarak elde edilmiştir. Bu tez kapsamında q'', P, G ve x_a'ya bağlı yani 4 boyutlu olarak oluşturulan ϕ_{lo}^2 TT'sinin geliştirilmesinde daha önce için P, G ve x'e bağlı yani 3 boyutlu olarak geliştirilmiş Kritik Isı Akısı TT'sinin algoritmasından esinlenilmiştir.

Bu tez çalışması kapsamında ϕ_{lo}^2 TT oluşturulması şu işlemlerle gerçekleştirilmiştir:

1- Literatürden tedarik edilen deneysel verilerle bir veri tabanı oluşturulmuştur (Odabaş, 2011; Durmayaz vd., 2012; Odabaş vd., 2013).

2- ϕ_{lo}^2 TT geliştirilmesinde kullanılacak deneysel verilerin belirlenmesi sırasında, önceden belirlenmiş kabul kıstasları dikkate alınarak, veri tabanından bu kıstaslara uygun (dolayısıyla güvenilir) olanları seçilmiş, bu kıstasları sağlamayan veriler ise elenmiştir (Odabaş, 2011; Durmayaz vd., 2012; Odabaş vd., 2013).

3- ΔP ölçüm sonucu deneysel verilerinden hesaplanalarak elde edilen ϕ_{lo}^2 değerleriyle, literatürdeki mevcut korelasyonlar kullanılarak elde edilen ϕ_{lo}^2 değerlerinin karşılaştırılmaları yapılmış, bu aşamada da kabul kıstasları dikkate alınarak, bu kriterleri sağlamayan bazı veriler elenmiştir (Odabaş, 2011; Durmayaz vd., 2012; Odabaş vd., 2013).

4- Eldeki mevcut deneysel veriler ışığında bu tez çalışması kapsamında geliştirilecek ϕ_{lo}^2 TT'sinde kullanılacak parametreler yani (q'', P, G ve x_a) bağlı, bu parametrelerin genel dağılımı incelenerek, onların ϕ_{lo}^2 TT için uygun olan aralıkları ve ϕ_{lo}^2 TT içerisindeki değerleri belirlenmiştir.

5- Deneysel veriler ϕ_{lo}^2 TT parametrelerinin mevcut deneysel verilerin elde edildiği x_a aralıklarında, Chisholm (1973) ve Tarasova vd. (1966) korelasyonlarının kombinasyonundan esinlenilerek ve en küçük kareler yöntemi kullanılarak eğri uydurma yöntemiyle bu tez kapsamında yeni bir ϕ_{lo}^2 korelasyonu geliştirilmiştir. Bir, ϕ_{lo}^2 TT için geliştirilen bu korelasyonun geliştirilmesi aşamasında, fark karelerinin ortalamasının karekökü (RMS hataya) dayalı hata analizleri yapılarak en az RMS hatası içereni belirlenmiştir.

6- Yeni geliştirilen ϕ_{lo}^2 korelasyonu kullanılarak ϕ_{lo}^2 TT'sinin ilk hali oluşturulmuş ve bu tablo ϕ_{lo}^2 İskelet Tablosu olarak adlandırılmıştır.

7- ϕ_{lo}^2 İskelet Tablosu interpolasyon ve ekstrapolasyonlardan yararlanılarak deneysel verilerle güncellenmiştir. Güncellenmiş ϕ_{lo}^2 TT elde edilirken deneysel veri bulunmayan tablo değerleri için fiziksel trendler gözetilmiş, interpolasyonlar ve ekstrapolasyonlardan yararlanılmıştır.

8- ϕ_{lo}^2 TT'sinde kullanılan ϕ_{lo}^2 değerleri arasında düzgün geçişler sağlanması amacı 4 boyutlu bir ϕ_{lo}^2 TT için düzgünleştirme bilgisayar programı geliştirilmiştir.

9- Her aşamada RMS hataya dayalı hata analizleri yapılarak en az RMS hatası içeren hali nihai ϕ_{lo}^2 TT olarak bu tez çalışmasında sunulmuştur.

1. INTRODUCTION

In this chapter, purpose of the thesis, a literature review and an outline of the thesis are introduced.

1.1 Purpose of Thesis

The purpose of this thesis is to focus on constructing and deriving a look-up table for prediction of the two-phase frictional multiplier for steam-water flows in vertical uniformly heated and adiabatic round tubes, which covers wide range and supersedes correlations and mathematical methods in literature.

1.2 Literature Review

A well-known LUT has been developed and employed for critical heat flux (CHF) which is the most important parameter to limit the heat transfer capability from the surface of a fuel element in a nuclear reactor. More than 1000 prediction methods including models and correlations to predict CHF have been proposed for tubes cooled by water over the past 60 years, but they have not been sufficient to cover overall ranges of conditions or parameters (Groeneveld et. al. 2007). The CHF LUT is made of a normalized CHF data base obtained from vertical-upward flow of steam-water for 8-mm inner diameter (ID) round tubes and classified as one of the empirical methods for the prediction of CHF.

Doroshcuk et al. (1975) generated the first CHF LUT basing on relatively small (approximately 5000) experimental data.

The improved CHF LUTs have been developed in the subsequent researches in the University of Ottawa; Groeneveld et al. (1986) derived their first CHF LUT based on a data bank including more than 15,000 experimental data.

Groeneveld et al. (1995) was developed their subsequent CHF LUT by using 23,114 experimental data accepted for LUT derivation. The up-to-date CHF LUT was published as “the 2006 CHF look-up table” (Groeneveld et. al. 2005; Groeneveld et.

al. 2007) employing a data base containing 24,781 selected CHF experimental data. It consists of entirely 11,592 normalized CHF values of entries providing CHF values at 24 pressures, 21 mass fluxes and 23 critical thermodynamic qualities (i.e., 24×21×23=11,592), comprising a pressure range of 0.1– 21 MPa, a mass flux range of 0 – 8000 $kg.m^{-2}s^{-1}$ and a quality range from -0.5 to 1 for water in 8-mm ID vertical round tubes. Currently, the 2006 CHF LUT has been implemented in some widely used nuclear reactor safety codes (such as RELAP5/MOD3, THERMALHYDRAULIK, CATHARE and CATHENA) for nuclear fuel design as well as the evaluation of fuel channel operating and safety margins (Tanase, 2007).

Durmayaz et al. (2004), Groeneveld et al. (2004) and Shan et al. (2005) also presented some studies used in the derivation of the 2006 CHF LUT.

El Nakla et al. (2004) and Groeneveld et al. (2005) also developed "the film boiling LUT" at ACR conditions of interest.

Recently, Zahlan et al. (2011) derived a "trans-critical heat transfer LUT" for water-cooled tubes, covering a wide range of flow conditions, therefore overcoming the range-of-validity-limitation associated with existing correlations. In addition, this trans-critical LUT includes the high-pressure subcritical region and thus provides the transition from the subcritical into the supercritical region.

Lately, a "look-up table for two-phase (2ϕ) frictional pressure drop multiplier (ϕ_{lo}^2)" covering various flow regimes has been established for adiabatic flows by (Alsurakji, 2012). Main parameters of their ϕ_{lo}^2 LUT were selected as density ratio, Reynolds Number and mass (actual, dynamic) quality. A skeleton table was constructed based on the homogeneous model, the Chisholm (1973) correlation, the Muller-Steinhagen and Heck (1986) correlation, the Grönnerud (1972) correlation, and the Friedel (1979) correlation. Updating of the skeleton table with 940 selected experimental data (after 1177 data removal in totally 2091 data), smoothing and error assessment were performed on ϕ_{lo}^2 LUT (Alsurakji, 2012; El Nakla, et al. 2013).

Approximately 10,000 experimental 2ϕ-flow pressure drop (ΔP) data for steam-water in uniformly heated and adiabatic vertical tubes have been complied and used to assess the prediction accuracy of ϕ_{lo}^2 correlations selected from the literature and the homogeneous-flow model for steam-water flow in vertical tubes. The predictions

of the correlations have also been compared to the predictions of the homogeneous-flow model. The assessments are based on comparisons of the average and the RMS errors. Screening criteria was employed on these experimental 2ϕ-flow ΔP data covers (i) 2ϕ-flow ΔP data compilation for water in vertical heated tubes and (ii) the 2ϕ-flow ΔP data in vertical adiabatic tubes. The two-phase friction multiplier predictions of the homogeneous model (McAdams et al. 1942) with and without the Tarasova et al. (1966) heating effect correction were compared with the two-phase friction multipliers extracted from the diabatic database of their study. Additionally, ΔP components (due to gravitation, acceleration and friction) have been examined for vertical heated tubes based on the databases. These ΔP components were also compared with those presented by Klausner et al. (1990) (Odabaş, 2011; Durmayaz et. al. 2012; Odabaş et al. 2013).

Widely used experimental and empirical researches on the 2ϕ-frictional ΔP for steam-water flow in vertical tubes were reviewed and categorized by Leung (1994) as follows:

- Flow boiling studies and correlations (not heating effect)
    - Muller-Steinhagen and Heck (1986) correlation
    - Lombardi ve Pedrocchi (1972) correlation (CISE)
- Flow boiling study and correlation (strong heating effect)
    - Tarasova et al. (1966) correlation
- Adiabatic-flow studies
    - Chisholm (1973) correlation
    - Friedel (1979) correlation
- Graphical Method
    - Thom (1964) method
- Modeling Study
    - Homogeneous-flow model of two-phase frictional multiplier

Aubé (1996), Idsinga (1975), Leung et al. (2004), Olekhnovitch (1997), Olekhnovitch et al. (2004), Peterlongo (1964) and Rohsenow (1951) were also presented some experimental and empirical researches on the 2ϕ-frictional ΔP.

Öztüzün et al. (2013) reviewed the methodology of look-up table development for two-phase flow parameters,

1.3 Outline

A literature review on the prediction methods and correlations of 2ϕ-flows, constructing and developing of look-up tables has been accomplished in in Chapter 1.

Selected correlations and mathematical methods from the literature are briefly described for 2ϕ-frictional ΔP for steam-water flow in vertical tubes in Chapter 2.

Chapter 3 describes main steps of constructing the ϕ_{lo}^2 LUT, which consists of compiling reliable database, constructing skeleton table, updating, smoothing, and error assessment.

The results (main findings) of this study and conclusions are summarized and recommendations are stated for subsequent studies in Chapter 4.

The ϕ_{lo}^2 LUT derived by this thesis has been presented in Appendix C.

2. REVIEW OF THE SELECTED METHODS AND CORRELATIONS OF THE TWO-PHASE FRICTIONAL MULTIPLIER

In this chapter, selected prediction methods and correlations of two-phase frictional multiplier (ϕ_{lo}^2) for uniformly heated liquid–water and vapor flows inside vertical pipes are reviewed.

2.1 Description of Two-Phase Frictional Multiplier

The two-phase frictional multiplier is a widely used parameter to determine the frictional ΔP in 2ϕ-flows. The parameter simply expresses "***the ratio of frictional pressure drop (or gradient) in*** 2ϕ***-flows to that in*** 1ϕ***-flow***" (Leung,1994). In literature, there exist different definitions for two-phase frictional multiplier. Their widely used ones in literature may be described as follows

$$\phi_{lo}{}^2 = \frac{(\Delta P/\Delta z)_{2\phi}}{(\Delta P/\Delta z)_{lo}} \qquad \textbf{(2.1)}$$

$$\phi_{go}{}^2 = \frac{(\Delta P/\Delta z)_{2\phi}}{(\Delta P/\Delta z)_{go}} \qquad \textbf{(2.2)}$$

$$\phi_{l}{}^2 = \frac{(\Delta P/\Delta z)_{2\phi}}{(\Delta P/\Delta z)_{l}} \qquad \textbf{(2.3)}$$

$$\phi_{g}{}^2 = \frac{(\Delta P/\Delta z)_{2\phi}}{(\Delta P/\Delta z)_{g}} \qquad \textbf{(2.4)}$$

In this study, Equation (**2.1**) is considered and used.

2.2 Methods and Correlations of Two-Phase Frictional Multiplier for Uniformly Heated Liquid–Water and Vapor Flows inside Vertical Pipes

In this section, selected prediction methods and correlations of ϕ_{lo}^2 for uniformly heated liquid–water and vapor flows inside vertical pipes, which are widely used in literature, are described.

2.2.1 Homogeneous-flow model of two-phase frictional multiplier

In accordance with the homogenenous-flow model, both phases of the 2ϕ-flows are assumed to have the same velocity in the tube. In that case, the two-phase frictional multiplier may be determined by

$$\phi_{lo}^2 = \frac{f_{2\phi}}{f_l}\left(1+x_a\left(\frac{\rho_l}{\rho_g}-1\right)\right) = \left(1+x_a\left(\frac{\rho_l}{\rho_g}-1\right)\right)\left(1+x_a\left(\frac{\mu_l}{\mu_g}-1\right)\right)^{-0.25} \quad \textbf{(2.5)}$$

where the friction factors and Reynolds numbers for 2ϕ-flows and 1ϕ-flow, respectively, and, the 2ϕ-mixture viscosity are defined as

$$f_{2\phi} = 0.316\,\mathrm{Re}_{2\phi}^{-0.25} \quad \textbf{(2.6)}$$

$$f_l = 0.316\,\mathrm{Re}_l^{-0.25} \quad \textbf{(2.7)}$$

$$\mathrm{Re}_{2\phi} = \frac{G\,D}{\mu_{2\phi}} \quad \textbf{(2.8)}$$

$$\mathrm{Re}_l = \frac{G\,D}{\mu_l} \quad \textbf{(2.9)}$$

$$\mu_{2\phi} = \left(\frac{x_a}{\mu_g}+\frac{1-x_a}{\mu_l}\right)^{-1} \quad \textbf{(2.10)}$$

by (McAdams et al., 1942).

2.2.2 Thom (1964) method

ϕ_{lo}^2 values based on the Thom (1964) method can be obtained from his tabulated values given in the literature (Thom, 1964).

2.2.3 Chisholm (1973) correlation

Chisholm (1973) correlation is presented in the literature applying to both smooth and rough pipes, which do not indicate any effect of heat flux on the frictional pressure drop as follows:

$$\phi_{lo}^2 = 1+\left(\Gamma^2-1\right)\left(B_{Ch}\,x_a^{(2-n)/2}\left(1-x_a\right)^{(2-n)/2}+x_a^{2-n}\right) \quad \textbf{(2.13)}$$

$$\Gamma = \left(\frac{\rho_l}{\rho_g}\right)^{0.5} \left(\frac{\mu_g}{\mu_l}\right)^{n/2} \qquad \textbf{(2.14)}$$

where Γ is a property index. Leung (1994) recommends to use n = 0.25 since it provides good results.

Table 2.1 : Coefficient, B_{Ch}, used in the Chisholm (1973) correlation to fit curves of Baroczy (1965) for general applications in smooth tubes (Leung, 1994; Quiben, 2005).

Property Index Γ	**Mass Flux** G (kg m^{-2} s^{-1})	**Coefficient** B_{Ch}
≤ 9.5	≤ 500	4.8
	$500 < G < 1900$	$2400/G$
	≥ 1900	$55/G^{0.5}$
$9.5 < \Gamma < 28$	≤ 600	$520/(\Gamma G^{0.5})$
	> 600	$21/\Gamma$
≥ 28		$15\ 00/(\Gamma^2 G^{0.5})$

2.2.4 Friedel (1979) correlation

Friedel (1979) correlation was selected as the best correlation among 14 correlations after these correlations were assessed with 12,868 experimental data (Leung,1994). It is expressed as

$$\phi_{lo}^2 = C_{F1} + \frac{3.24\, C_{F2}}{Fr^{0.045}\, We^{0.035}} \qquad \textbf{(2.15)}$$

where

$$C_{F1} = (1-x_a)^2 + x_a^2 \left(\frac{\rho_f}{\rho_g}\right) \cdot \left(\frac{f_{go}}{f_{lo}}\right) \qquad \textbf{(2.16)}$$

$$C_{F2} = x_a^{0.78}(1-x_a)^{0.24} \left(\frac{\rho_f}{\rho_g}\right)^{0.91} \left(\frac{\mu_g}{\mu_f}\right)^{0.19} \left(1-\frac{\mu_g}{\mu_f}\right)^{0.7} \qquad \textbf{(2.17)}$$

$$Fr = \frac{G^2}{g\, D\, \rho_{2\phi}^2} \qquad \textbf{(2.18)}$$

$$We = \frac{G^2\, D}{\rho_{2\phi}\, \sigma} \tag{2.19}$$

$$\rho_m = \left(\frac{x_a}{\rho_g} + \frac{1 - x_a}{\rho_f} \right)^{-1} \tag{2.20}$$

$$f = 64/\mathrm{Re} \tag{2.21}$$

for laminar flow ($\mathrm{Re} \langle 1055$) and

$$f = \left(0.86859 \ln \left(\frac{\mathrm{Re}}{(1.964 \ln \mathrm{Re} - 3.8215)} \right) \right)^{-2} \tag{2.22}$$

for turbulent flow **(** $\mathrm{Re} \geq 1055$ **).**

2.2.5 Muller-Steinhagen and Heck (1986) correlation

Muller-Steinhagen and Heck (1986) presented a straightforward correlation being depended on a linear interpolation between single-two phase pressure drops of liquid and of gas with respect to quality over the quality range of 0 to 0,7 is derived for prediction of the tw-phase pressure drop for various fluids (Leung,1994). Muller-Steinhagen and Heck (1986) correlation is as

$$\left(\frac{dP}{dz}\right)_{frc,2\phi} = \left(\left(\frac{dP}{dz}\right)_{frc,lo} + 2\, x_a \left(\left(\frac{dP}{dz}\right)_{frc,go} - \left(\frac{dP}{dz}\right)_{frc,lo} \right) \right) (1 - x_a)^{1/3} + x_a{}^3 \left(\frac{dP}{dz}\right)_{frc,go} \tag{2.23}$$

where

$$\left(\frac{dP}{dz}\right)_{frc,lo} = f_{lo} \frac{G^2}{2\, \rho_l\, D} \tag{2.25}$$

$$\mathrm{Re}_{lo} = \frac{G\, D}{\mu_l} \tag{2.26}$$

$$\mathrm{Re}_{go} = \frac{G\, D}{\mu_g} \tag{2.27}$$

For ($\mathrm{Re}_{lo},\ \mathrm{Re}_{go} \le 1187$),

$$f_{lo} = \frac{64}{\mathrm{Re}_{lo}} \qquad \textbf{(2.28)}$$

$$f_{go} = \frac{64}{\mathrm{Re}_{go}} \qquad \textbf{(2.29)}$$

Otherwise,

$$f_{lo} = \frac{0.3164}{\mathrm{Re}_{lo}^{1/4}} \qquad \textbf{(2.30)}$$

$$f_{go} = \frac{0.3164}{\mathrm{Re}_{go}^{1/4}} \qquad \textbf{(2.31)}$$

It is recommended that this correlation is restricted to use where Re_{lo} is greater than 100. Also, a correction factor should be applied on Equation (**2.23)** for flow boiling.

2.2.6 Lombardi ve Pedrocchi (1972) correlation (CISE studies)

Considering to 1400 2ϕ - ΔP data obtained in vertical tubes, annuli and rod bundles within heated and unheated steam-water flows, ΔP was calculated with energy balance. Then, homogenous-flow model was used to calculate acceleration and gravitional ΔP . Lombardi ve Pedrocchi (1972) correlation (CISE studies) is

$$\phi_{lo}^2 = \frac{1.66\,\sigma^{0.4}\,\rho_l}{f\,\rho_m^{0.86}\,G^{0.6}\,D^{0.2}} \qquad \textbf{(2.32)}$$

f and ρ_m equations are given in the Friedel (1979).

2.2.7 Tarasova et al. (1966) correlation

Tarasova et al. (1966) were examined heating effect at some local conditions for high pressure steam-water flows inside tubes and annuli. Their correlations are used to correct the predictions of other correlations for heating effect in subcooled-boiling and saturated-boiling regions. They are expressed as

for saturated boiling

$$\frac{\phi_{lo,heated}^2}{\phi_{lo,unheated}^2} = 1 + 0.0044\left(\frac{q''}{G}\right)^{0.7} \qquad \textbf{(2.32)}$$

for subcooled boiling

$$\frac{\phi_{lo,heated}^2}{\phi_{lo,unheated}^2} = 1 + \frac{20Z}{1.315 - Z}\left(\frac{q''}{h_{fg}G}\right)^{0.7}\left(\frac{\rho_f}{\rho_g}\right)^{0.08} \quad \textbf{(2.33)}$$

3. METHODOLOGY FOR THE DERIVATION OF TWO-PHASE FRICTIONAL MULTIPLIER LOOK-UP TABLE

Selection of an appropriate method among different available ones to predict any engineering parameter depends on some criteria such as

i. accuracy,
ii. range of validity,
iii. easyness to apply in a software,
iv. versatility.

A Look-Up Table (LUT) is basically "*a matrix of normalized experimental data collected from different sources* and then *processed*" to be ready to use in the LUT. LUTs are utilized as one of the best methods comparing to the other prediction models and empirical methods. The main advantages of using LUTs can be summarized as follows:

i. The accuracy of the LUT is generally better than the other prediction methods since its derivation combines different prediction methods basing on a large data base and different correlations.

ii. The LUT covers wide ranges of validity and applications combining the ranges of individual prediction methods.

iii. The use of LUTs is straightforward and does not need extensive computer programming hence it is computationally efficient.

iv. The LUT is easy to update hence its prediction accuracy can be improved once new data is available.

v. The LUT offers more correct parametric trends.

ϕ_{lo}^2 is a critical parameter for 2ϕ-flows on which many experimental and theoretical studies have been performed and published in literature. However, these studies have to be employed in very limited ranges of related parameters because most of them

are generated with database including limited experimental data. Moreover, verification of the experimental data could not be observed sufficiently.

In this thesis, a ϕ_{lo}^2 LUT having 20,736 LUT entries at 8 heat flux (q''), 12 pressure (P), 18 mass flux (G) and 12 actual (dynamic) quality (x_a) values (i.e., 8×12×18×12=20,736 entries), has been derived for steam-water flows in vertical-round tubes by inspiring from the CHF LUT derivation method in order to compile the experimental data in a unique normalized database and to overcome the difficulty of empirical studies being valid in restricted ranges of parameters. In the subsequent subsections, stages to construct the ϕ_{lo}^2 LUT are described.

3.1 Methodology

The steps given below may be followed to construct a ϕ_{lo}^2 LUT:

1- Searching literature and compiling ΔP data from reliable sources for steam-water flows in vertical uniformly heated and adiabatic round tubes.

2- Removing duplicate data, significantly scattered data (outliers) and the data not satisfying the other selection criteria, and completing the compilation of a reliable experimental ΔP database.

3- Developing a software program to generate a ϕ_{lo}^2 database by estimating the ϕ_{lo}^2 values from the ΔP database.

4- Assessing the accuracy of different prediction methods for calculating ϕ_{lo}^2 with the comparison of ϕ_{lo}^2 database and determining an appropriate method among them.

5- Deciding the dimensions, parameters and ranges of ϕ_{lo}^2 LUT in accordance with the ϕ_{lo}^2 database.

6- Selecting or generating an appropriate model, correlation or their combination to provide the initial estimate of (to generate) the skeleton table (first table) ϕ_{lo}^2 values which is the statring point of the derivation of the ϕ_{lo}^2 LUT.

7- Derivation of the skeleton table by using the selected or generated model(s) or correlation(s).

8- Updating the skeleton table by the contribution of ϕ_{lo}^2 database.

9- Eliminating discontinuities and jumps among the updated entries and the unupdated entries in the updated ϕ_{lo}^2 table by employing a smoothing procedure.

10- Assessment (error analysis to determine the accuracy) of the ϕ_{lo}^2 LUT.

3.2 Compilation of the Experimental Pressure Drop and the Two-Phase Frictional Multiplier Databases

Experimental studies performed on steam-water vertical tubes were reviewed and compilation of the experimental pressure drop and the two-phase frictional multiplier databases were performed by (Odabaş, 2011; Durmayaz et. al. 2012). These compilation studies were carried out considering the construction of a new ϕ_{lo}^2 LUT in the subsequent studies (as it is done in this study) for specifically the experimental data obtaining from steam-water vertical tubes and now are presented again in the following subsections by (Odabaş, 2011; Durmayaz et. al. 2012).

3.2.1 Experimental 2ϕ-flow ΔP data collecting from the literature and their compilation

Two types of data sets have been compiled; (i) 2ϕ-flow ΔP experimental database, called "*diabatic database*", for water in vertical uniformly heated tubes (Alessandrini et al., 1963; Peterlongo, et al., 1964, Olekhnovitch, 1997; Olekhnovitch, et al., 2004; Leung, 1994), and (ii) 2ϕ-flow ΔP experimental database, called "*adiabatic database*", for water in vertical adiabatic tubes (Alessandrini et al.). In this thesis, for this purpose, it is utilized from the studies by (Odabaş, 2011; Durmayaz et. al. 2012).

These 2ϕ-flow ΔP experimental databases for water in vertical uniformly heated or adiabatic tubes are briefly presented in Table 3.1 and Table 3.2, respectively. To compare the conditions of databases in these tables, exit pressure and

thermodynamic quality values are recalculated by the aid of an energy-balance equation considering the pressure drops by (Odabaş, 2011; Durmayaz et. al. 2012).

Table 3.1 : Experimental 2ϕ-flow ΔP data conditions for steam-water flows in uniformly heated vertical tubes (Odabaş, 2011; Durmayaz et. al. 2012).

Researcher	D (m)	L_h (m)	P (kPa)	G (kg/m^2s)	x -	q'' (kW/m^2)	ΔP (kPa)	Number of data
Alessandrini et al. (1963)	0,0152-0,0249	2,45	4992-5090	1080-3890	0,022-0,515	1680-2710	30,40-136,32	48
Peterlongo et al. (1964)	0,0151	4,02	4982-5050	1070-3940	0,145-0,608	1080-2260	72,56-279,49	79
Leung (1994)	0,0055	2,50	5020-9730	1130-9980	-0,20-0,563	113-3220	0,00-1553,80	1141
Olekhnovitch (1997)	0,0080	1,00-3,50	507-4036	977-6122	0,047-0,760	523-5550	56,00-1742,00	479
Olekhnovitch et al. (2004)	0,0080-0,0157	1,00-3,00	980-3993	910-6140	-0,06-0,648	57-4663	10,00-926,70	8574

Table 3.2 : Experimental 2ϕ-flow ΔP data conditions for steam-water flows in adiabatic vertical tubes (Odabaş, 2011; Durmayaz et. al. 2012).

Researcher	D (m)	L_h (m)	P (kPa)	G (kg/m^2s)	x -	q'' (kW/m^2)	ΔP (kPa)	Number of data
Alessandrini et al. (1963)	0,0152-0,0249	0,82-2,49	4913-5992	1080-3890	0,016-0,648	0	11,86-242,52	208

3.2.2 Pressure drop data screening

The degree of reliability and accuracy of LUT is directly affected by the reliability and accuracy of the experimental data. Therefore, ΔP data screening is one of the most critical steps for constructing the ϕ_{lo}^2 LUT. Unreliable data is removed from the compiled database if they are out of selection criteria listed in Table 3.3. N/A (not applicable) is indicated where criteria must not be applied on adiabatic database (Odabaş, 2011; Durmayaz et. al. 2012).

Table 3.3 : Data selection criteria for ϕ_{lo}^2 calculations (N/A: not applicable) (Durmayaz et. al. 2012).

Parameter	Selection criteria	# of data removed due to the selection criteria						
		Leung (1994) diabatic	Alessandrini et al.(1963) adiabatic	Alessandrini et al.(1963) diabatic	Peterlongo et al.(1994) diabatic	Olekhnovitch et al.(1997) diabatic	Olekhnovitch et al.(2004) diabatic	Total
# of data in the database		1141	208	48	79	479	8574	**10 529**
D (mm)	$3 < D < 25$	-	-	-	-	-	-	-
P (kPa)	$100 < P < 21000$	-	-	-	-	-	-	-
G (kg/m^{2}s)	$0 < G < 8000$	-	-	-	-	-	-	-
Inlet temperature (^{o}C)	$T_{in} \geq 0.01$	-	-	-	-	-	-	-
Heat balance	Error $<$ %5	-	NA	-	-	-	-	-
L/D for $x_{in} > 0$	$L/D \geq 100$	-	-	-	-	-	-	-
L/D for $x_{in} < 0$	$L/D > 50$ for $x_{out} > 0$ otherwise $L/D > 25$	-	-	-	-	-	-	-
x_{out}	$x_{out} \leq 1$	-	-	-	-	-	-	-
ΔP	$\Delta P > 0$	64	-	-	-	-	-	64
x_{out}	$x_{ONB} < x_{out}$	103	-	-	-	-	49	152
q''	$q'' > 0$	60	NA	-	-	-	-	60
ϕ_{lo}^2	$\phi_{lo}^2 > 1$	86	10	-	-	-	587	683
Average error	Average error $> \pm 100\%$ at least for two correlations	-	-	-	-	-	247	247
Total # of removed data		313	10	-	-	-	883	**1206**
Number of data accepted		828	198	48	79	479	7691	**9323**

Regarding by Leung (1994) database, 313 experimental data was removed. The reasons for their removal are: (i) 64 data were recorded that ΔP was zero, (ii) single-phase (1ϕ-) flow exists for 103 data, (iii) 60 data was performed in adiabatic conditions, (iv) value of calculated ϕ_{lo}^2 was smaller than 1, or at least one parameter of ϕ_{lo}^2 was smaller than 0 for 86 data.

Regarding by Alessandrini et al. (1963) adiabatic database, 10 experimental data was removed because value of the calculated ϕ_{lo}^2 was smaller than 1, or at least one parameter of ϕ_{lo}^2 was smaller than 0 for 10 data.

Regarding by Olekhnovitch et al. (2004) database, 883 experimental data was removed. The reasons for their removal are: (i) 1ϕ-flow exists for 49 data, (ii) value of calculated ϕ_{lo}^2 was smaller than 1, or at least one parameter of ϕ_{lo}^2 was smaller than 0 for 587 data, (iii) average error calculated was greater than ±100%.at least two different correlations of ϕ_{lo}^2 for 247 data.

As a result, **9323 data** is accepted as reliable in accordance with the data selection criteria in order to use in deriving the ϕ_{lo}^2 LUT.

3.2.3 Method of calculation of two-phase frictional multiplier from the experimental pressure drop data

The critical thermodynamic quality in CHF studies, which is considered at the exit of the tube, for uniformly heated round tube is calculated by the energy balance equation as

$$x = 4\left(\frac{q''}{G\,h_{fg@Pin}}\right)\left(\frac{L}{D}\right)-\left(\frac{\Delta h_{in}}{h_{fg@Pin}}\right) \qquad \textbf{(3.1)}$$

neglecting the effect of ΔP in the tube where q'' is equal to critical heat flux in the literature (Durmayaz et. al. 2012; Odabaş, 2011).

In accordance with Figure 3.1, when the ΔP is considered in the uniformly heated round tube, the heat balance equation is modified as

$$x = x_{@P} = 4\left(\frac{q''}{G\,h_{fg@P}}\right)\left(\frac{L}{D}\right) - \left(\frac{h_{f@P} - h_{@Pin,Tin}}{h_{fg@P}}\right) = \frac{h_{out@P} - h_{f@P}}{h_{fg@P}} \quad \textbf{(3.2)}$$

by Durmayaz et. al. (2012), where the power provided for heating the tube is $q = q''\pi DL$, enthalpy and pressure at the tube inlet are $h_{@Pin,Tin}$ (where value of h is at P_{in} and T_{in} condition) and P_{in} which are obtained at $P + \Delta P$, respectively. The enthalpy and pressure at the outlet of tube are depicted as $h_{@P}$ and P, respectively.

Similarly, energy balance between onset of nucleate boiling (ONB) and the tube outlet yields

$$L_{2\phi} = \frac{GD}{4q''}\left(h_{out@P} - h_{@(P,T)_{ONB}}\right) = \frac{GDh_{fg@P}}{4q''}\left(x - \frac{h_{@(P,T)_{ONB}} - h_{f@P}}{h_{fg@P}}\right) \quad \textbf{(3.3)}$$

where

$$h_{out@P} = h_{@P} = h_{f@P} + xh_{fg@P} \quad \textbf{(3.4)}$$

and

$$x_{ONB} = \left(\frac{h_{@(P,T)_{ONB}} - h_{f@P_{ONB}}}{h_{fg@P_{ONB}}}\right) \quad \textbf{(3.5)}$$

ΔP covers the value of pressure drops for two regions through the heated length: (i) 1ϕ-liquid flow and (ii) 2ϕ-flow regions; therefore, total heated length of tube constitutes of both of these two regions. They are calculated separately.

$$L_h = L_{1\phi} + L_{2\phi} \quad \textbf{(3.6)}$$

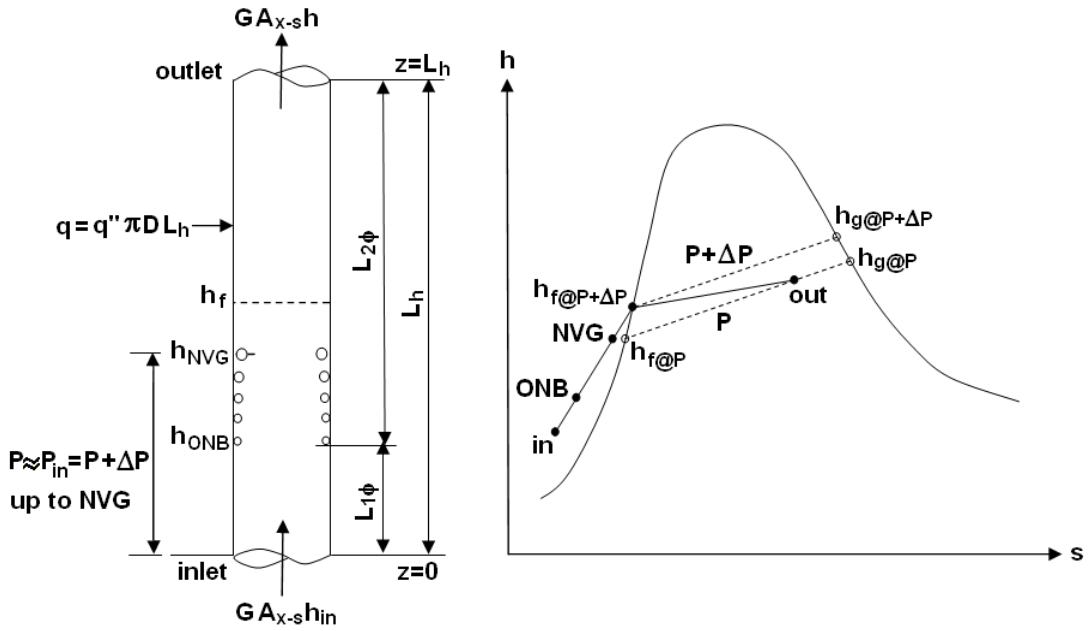


Figure 3.1 : Heat balance and *h-s* diagram for the uniformly heated tube when the ΔP is considered (Durmayaz et. al. 2012).

For calculating $L_{2\phi}$, first the wall temperature at the point of onset of nucleate boiling (ONB) ($T_{w@ONB}$) is calculated by Davis and Anderson (1966). Davis and Anderson (1966) equation is given as

$$(\Delta T)_{sat,ONB} = (T_w - T_{sat})_{ONB} = \left(\frac{8\,\sigma\, q''\, T_{sat}}{h_{fg}\, k_f\, \rho_g} \right)^{0.5}_{ONB} \qquad \textbf{(3.7)}$$

After that, the bulk temperature of the fluid at the point of ONB is calculated from $T_{ONB} = T_{w@ONB} - (q''/H)$ where H is the heat transfer coefficient. Then, the enthalpy at ONB ($h_{@\,Pin,T_{ONB}} \cong h_{@(P,T)_{ONB}}$) is substituted for $h_{@\,Pin,Tin}$ in Equation (**3.2**) to obtain $L_{2\phi}$ given in Equation (**3.3**).

$\Delta P_{frc,adia,1\phi@L_{2\phi}}$ is the 1ϕ-liquid flow frictional pressure drop over $L_{2\phi}$ if total mass flow rate of both phases as saturated liquid only through $L_{2\phi}$ inside the tube which is calculated by using the D'Arcy-Weisbach equation as

$$\Delta P_{frc,adia,1\phi@L_{2\phi}} = f\frac{L_{2\phi}G^2}{2D\rho_{@(P,T)_{ONB}}} \qquad \textbf{(3.8)}$$

Friction factor is determined by Chen (1979) equation as

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{(k_s/D)}{3.7065} - \frac{5.0452}{\mathrm{Re}}\log\left(\frac{(k_s/D)^{1.1098}}{2.8257} + \frac{5.8506}{\mathrm{Re}^{0.8981}}\right)\right] \qquad \textbf{(3.9)}$$

$\Delta P_{frc,1\phi@L_{2\phi}}$ in a heated tube over $L_{2\phi}$ is calculated in accordance with Sieder et. al (1936) given as

$$\Delta P_{frc,1\phi@L_{2\phi}} = \Delta P_{frc,adia,1\phi@L_{2\phi}}\left(\mu_b/\mu_w\right)^m \qquad \textbf{(3.10)}$$

where m = –0.28, as it is used by (Leung, 2004).

To calculate ΔP in the 1ϕ-liquid flow region for the length between the inlet of the tube and the ONB point, frictional pressure drop ($\Delta P_{frc@L_{1\phi}}$) is calculated similarly. ΔP due to acceleration can be neglected. The gravitational ΔP in the 1ϕ-liquid flow region is calculated by $\Delta P_{gra@L_{1\phi}} = \rho_{@P_{in},T_{ave}}\, g\, L_{1\phi}$ using the density at the average temperature through the 1ϕ-liquid flow region.

After that, $\Delta P_{@L_{1\phi}}$ in the 1ϕ-liquid flow region is deducted from the total ΔP in the heated length of the tube in order to obtain $\Delta P_{@L_{2\phi}}$ in the 2ϕ-flow region over length $L_{2\phi}$.

Considering an average density over $L_{2\phi}$, the 2ϕ-flow gravitational ΔP through $L_{2\phi}$ is calculated as

$$\begin{aligned}
\Delta P_{gra@L_{2\phi}} &= \rho_{ave@L_{2\phi}}\, g\, L_{2\phi} \\
&\cong \frac{1}{2}\left\{\left[\alpha\,\rho_{g@P} + (1-\alpha)\,\rho_{f@P}\right] + \rho_{f@P_{ONB}}\right\} g\, L_{2\phi} \\
&\cong \left\{\left[\frac{\alpha}{2}\rho_{g@P_{ave,2\phi}} + (1-\frac{\alpha}{2})\,\rho_{f@P}\right] + \rho_{f@P_{ave,2\phi}}\right\} g\, L_{2\phi}
\end{aligned} \qquad \textbf{(3.11)}$$

The 2ϕ-flow ΔP due to acceleration through $L_{2\phi}$ is calculated as

$$\Delta P_{acc@L_{2\phi}} = G^2 \left\{ \left(\frac{x_a^2}{\alpha\, \rho_{g@P}} \right) + \left(\frac{(1-x_a)^2}{(1-\alpha)\, \rho_{f@P}} \right) - \left(\frac{1}{\rho_{f@P_{ONB}}} \right) \right\} \qquad \textbf{(3.12)}$$

$P_{ONB} \cong P_{in}$ is considered when 1ϕ-liquid flow exists at the inlet of $L_{2\phi}$. In Equation (**3.11**) and (**3.12**), the flow dynamic quality (x_a) and the void fraction (α) are assumed at the outlet of the heated length used.

The flow dynamic quality is calculated by Kroeger and Zuber (1968) equation as

$$x_a = \frac{x - x_{NVG} \exp\left(\frac{x}{x_{NVG}} - 1 \right)}{1 - x_{NVG} \exp\left(\frac{x}{x_{NVG}} - 1 \right)} \qquad \textbf{(3.13)}$$

x_{NVG} is the thermodynamic quality at the point of net vapor generation (NVG). It is calculated by Saha et al. (1974) as

$$x_{NVG} = -0.0022 \frac{q'' D\, c_{p,l}}{h_{fg}\, k_l} \qquad \text{for } Pe < 70\,000 \qquad \textbf{(3.14)}$$

and

$$x_{NVG} = -154 \frac{q''}{G\, h_{fg}} \qquad \text{for } Pe > 70\,000 \qquad \textbf{(3.15)}$$

Peclet number is calculated as

$$Pe = \frac{G\, D\, c_{p@(P,T)_{ONB}}}{k_{@(P,T)_{ONB}}} \qquad \textbf{(3.16)}$$

The void fraction at the outlet of the heated length is obtaned by Zuber and Findlay (1965) as

$$\alpha = \left(x_a\, \rho_{f@P}\, G \right) / \left\{ G\, C_o \left[x_a\, \rho_{f@P} + (1-x_a)\, \rho_{g@P} \right] + \rho_{f@P}\, \rho_{g@P}\, u_{gj} \right\} \qquad \textbf{(3.17)}$$

u_{gj} where the distribution parameter $C_o = 1.13$ is used as it is suggested by Zuber and Findlay (1965).

Then, the frictional pressure drop ($\Delta P_{frc,2\phi@L_{2\phi}}$) for 2ϕ-flow is plainly calculated by extracting the total ΔP due to gravity and acceleration from the total ΔP in the 2ϕ-flow region.

Eventually, the two-phase friction multiplier is obtained as

$$\phi_{lo}^2 = \frac{\Delta P_{frc,2\phi@L_{2\phi}}}{\Delta P_{frc,1\phi@L_{2\phi}}} \qquad \textbf{(3.18)}$$

where $\Delta P_{frc,1\phi@L_{2\phi}}$ is based on total flow as 1ϕ-liquid (Durmayaz et. al. 2012).

3.2.4 Comparisons of the two-phase friction multipliers based on experimental pressure drop data with those of selected correlations and homogeneous-flow model

In this section, the two-phase friction multipliers based on experimental ΔP data is compared to those of selected correlations and homogeneous-flow model for the conditions of the uniformly heated tube and adiabatic databases. Comparison depends on error analysis of correlations and homogenous-flow model.

Homogenous flow model is particularly selected to be compared with the selected correlations because Equation (**3.18**) is possible to be formed as the homogeneous-flow model to calculate ϕ_{lo}^2 by using ($\rho_{2\phi}^{-1} = \rho_g^{-1} x_a + \rho_l^{-1}(1-x_a)$) for the density and two-phase mixture viscosity ($\mu_{2\phi}^{-1} = \mu_g^{-1} x_a + \mu_l^{-1}(1-x_a)$) by (McAdams et al., 1946).

The error statics are tabulated in Table 3.4 (Durmayaz et al. 2012) for the selected ϕ_{lo}^2 correlations and for the homogeneous-flow model applied to the experimental databases, comparing to Leung's database (Leung, 1994), where error definitions are based on

$$Error = \frac{\phi_{lo,\,correlation}^2 - \phi_{lo,\,database}^2}{\phi_{lo,\,database}^2} \qquad \textbf{(3.19)}$$

$$Error = \frac{\phi_{lo,\,hom.model}^2 - \phi_{lo,\,database}^2}{\phi_{lo,\,database}^2} \qquad \textbf{(3.20)}$$

for the correlations and for the homogeneous-flow model, respectively.

The error statics are also tabulated in Table 3.5 (Durmayaz et al. 2012) for the selected ϕ_{lo}^2 correlations when compared to the homogeneous-flow model predictions for the conditions of the uniformly heated and adiabatic tube database where the error analysis depends on

$$Error = \frac{\phi_{lo,correlation}^2 - \phi_{lo,hom.model}^2}{\phi_{lo,hom.model}^2} \qquad \textbf{(3.21)}$$

Error histograms are depicted the error distribution of homogenous-flow model (1942), two of the selected correlations having the lowest RMS error, which are" the Muller-Steinhagen and Heck et. al. (1986) correlation" and "the Muller-Steinhagen and Heck et. al. (1986) correlation (for $x \leq 0.25$) + the Lombardi and Pedrocchi (1972) correlation (for $x > 0.25$)" for database of uniformly heated tube in accordance with Table 3.4.

From Table 3.3, Table 3.4 and Figure 3.2, it is concluded by Durmayaz et al. (2012) that:

(i) The Muller-Steinhagen and Heck (1986) correlation yields the best prediction for the database of uniformly heated tube. However it is not sufficiently accurate for adiabatic database.

(ii) The Chisholm (1973) correlation gives the best prediction for the database of adiabatic tube.

Table 3.4 : Error statistics of the predictions of the selected ϕ_{lo}^2 correlations and the homogeneous-flow model applied to the uniformly heated and adiabatic tube databases and the results reported by Leung,(1994) (Durmayaz et. al. 2012).

Source of experimental data ====>	**RESULTS (%) by (Durmayaz et al. 2012)**		**RESULTS (%) by (Leung, 1994).**
	9125 data for **heated tubes** in the diabatic database	198 data for **adiabatic tubes** in the adiabatic database	5085 data Leung's database
CORRELATIONS			
RMS error: Muller-Steinhagen and Heck + Lombardi and Pedrocchi	**20.93**	53.33	-
RMS error: Muller-Steinhagen and Heck	**23.00**	77.26	-
RMS error: Chisholm	32.08	**18.51**	20.09
RMS error: Lombardi and Pedrocchi	32.99	31.18	21.69
RMS error: Homogeneous-flow model	33.62	**20.38**	28.04
RMS error: Friedel	38.46	36.38	33.18
RMS error: Thom	46.65	46.32	31.74
Average error.: Muller-Steinhagen and Heck + Lombardi and Pedrocchi	**-7.19**	-4.76	-
Average error: Muller-Steinhagen and Heck	**-8.36**	-75.72	-
Average error: Chisholm	-11.65	**9.37**	4.69
Average error: Lombardi and Pedrocchi	5.11	26.45	-13.76
Average error: Homogeneous-flow model	-19.21	**-2.38**	11.58
Average error: Friedel	7.24	28.06	19.06
Average error: Thom	-40.00	34.56	14.04

Table 3.5 : Comparison of the error statistics of the selected ϕ_{lo}^2 correlations when compared to the predictions of the homogeneous-flow model for the conditions of the uniformly heated and adiabatic tube databases (Durmayaz et. al. 2012).

	RESULTS (%)	
	9125 data for **heated tubes** in the diabatic database (Alessandrini et al., 1963; Peterlongo, et al., 1964, Olekhnovitch, 1997; Olekhnovitch, et al., 2004; Leung, 1994)	198 data for **adiabatic tubes** in the adiabatic database (Alessandrini et al., 1963)
CORRELATIONS		
RMS error: Muller-Steinhagen and Heck	**26.15**	76.86
RMS error: Thom	28.76	39.31
RMS error: Friedel	34.38	32.85
RMS error: Muller-Steinhagen and Heck + Lombardi and Pedrocchi	36.02	56.46
RMS error: Chisholm	38.78	**23.54**
RMS error: Lombardi and Pedrocchi	51.40	38.55
Average error: Muller-Steinhagen and Heck	**17.80**	-75.71
Average error: Thom	-26.28	37.76
Average error: Friedel	32.42	32.05
Average error: Muller-Steinhagen and Heck + Lombardi and Pedrocchi	21.45	0.50
Average error: Chisholm	13.77	**14.53**
Average error: Lombardi and Pedrocchi	36.06	32.64

3.3 Selecting Dimensions, Parameters and Ranges of the Look-Up Table

When it is reviewed deeply, it is realized that equations in the literature are on the basis of 4 main parameters which are heat flux (q"), average pressure through the two-phase flow region (P_{ave}), mass flux (G) and actual (flow) quality (x_a). However, there is no equation containing these parameters altogether in the literature. Therefore, ϕ_{lo}^2 LUT is constructed as the function of q", P, G and x_a as

$$\phi_{lo}^2 = f(q'', P, G, x_a) \tag{3.22}$$

The ranges of these parameters in the ϕ_{lo}^2 LUT are determined based on the observations of experimental database given in Table 3.3, the boundaries of compiled experimental database tabulated in Table 3.6 and the distributions of the experimental database also shown in Figures A.1-A.11. As Figures A.1-A.11,

- The experimental data was obtained intensively at low thermodynamic quality, low outlet pressure, low heat flux conditions.
- In the areas of high thermodynamic quality, high outlet pressure, high heat flux conditions, there is no experimental data.
- The trends of data which was obtained in the same experimental set-up are almost plainly distinguished.

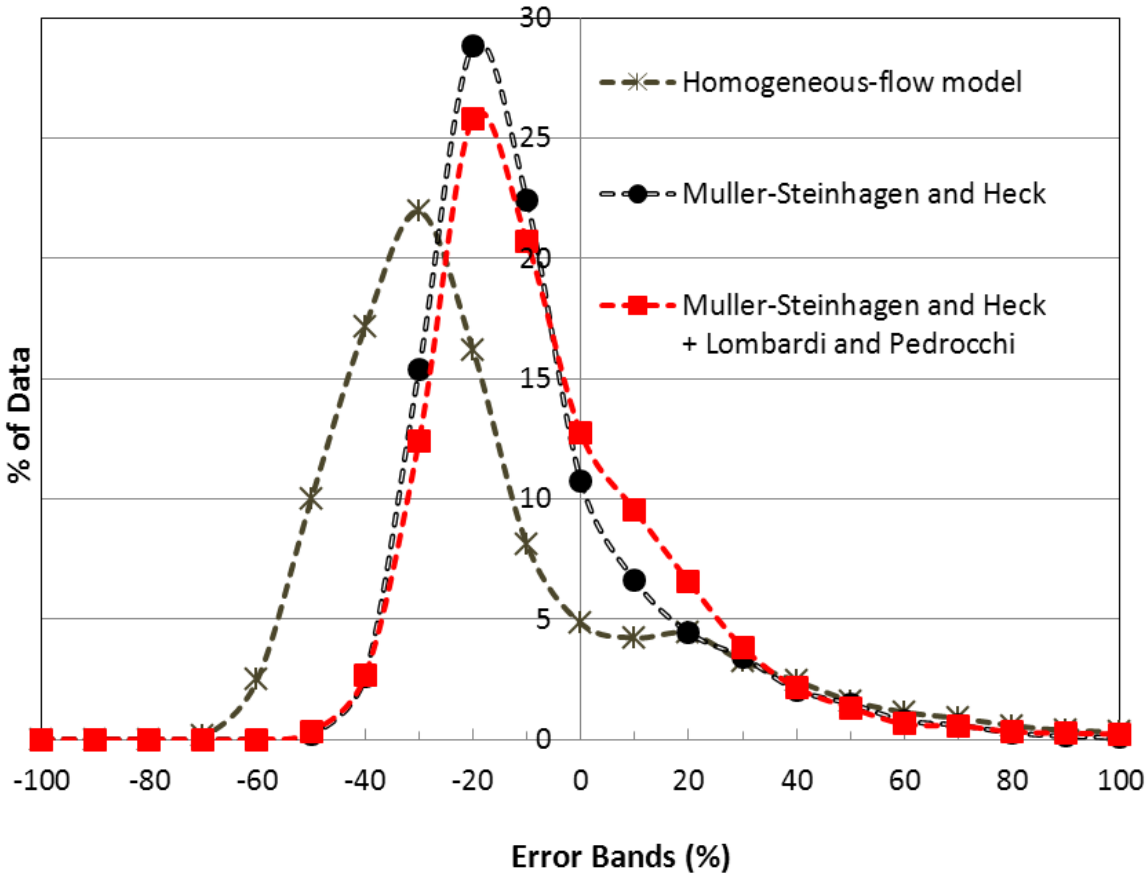


Figure 3.2 : Error histograms of "the Muller-Steinhagen and Heck (1986) correlation" and "the Muller-Steinhagen and Heck (1986) correlation (for $x \leq 0.25$) and the Lombardi and Pedrocchi (1972) correlation (for $x > 0.25$)" and also "the homogeneous-flow model" for the diabatic database. (Durmayaz et. al. 2012).

Table 3.6 : The minimum and maximum values of compiled databases

	q''	P	P_{ave}	G	x	$x_{a,ave}$
Unit	kW/m^2	kPa	kPa	kg/m^{2}s	-	-
Minimum Value	0	507	982	906.9	0.001	0.003
Maximum Value	5550	9720	10,327	9981	0.799	0.4

Thus, based on the existing experimental database, ranges and LUT entry values of parameters in the LUT are selected as follows, respectively:

- **Heat flux:** $0 \leq q'' \leq 6000$ kW/m^2,
- **Average Pressure:** $500 \leq P_{ave} \leq 11{,}000$ kPa ,
- **Mass flux:** $500 \leq G \leq 10{,}000$ kg/m^{2}s and
- **Average actual (flow) quality:** $0.0 \leq x_{a,ave} \leq 1.0$

- q'' : 0, 10, 1000, 2000, 3000, 4000, 5000, 6000 kW/m^2
- P_{ave} : 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10000, 11,000 kPa
- G : 500, 1000, 1500, 2000, 2500, 3000, 3500, 4000, 4500, 5000, 5500, 6000, 6500, 7000, 7500, 8000, 9000, 10,000 kg/m^{2}s
- $x_{a,ave}$: 0.00, 0.05, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 1.0

3.4 Developing a New Two-Phase Frictional Multiplier Correlation based on Heat Flux, Mass Flux, Pressure and Actual Quality

A new two-phase frictional multiplier correlation based on heat flux, pressure, mass flux and actual (dynamic) quality (at any point in the tube from the definition of ϕ_{lo}^2) has been developed to construct the skeleton table as

$$\phi_{lo}^2 = 0.09 + 1.106\left(\frac{q''}{G}\right)^{0.75} + \left(\frac{3.79\,(\Gamma^2-1)\left(1.2-x_a\right)^{0.7} x_a{}^{0.7}\right)}{G^{0.3}} + \frac{4.29\left(\frac{q''}{G}\right)^{0.7}(\Gamma^2-1)\left(1.2-x_a\right)^{0.7} x^{0.7}\right)}{g^{0.3}} + 2.6\,(\Gamma^2-1)\;x_a{}^{1.75} + 0.017\left(\frac{q''}{G}\right)^{0.75}(\Gamma^2-1)\;x_a{}^{1.75} \qquad \textbf{(3.23)}$$

Equation (**3.23)** follows general trends of existing databases quite well. "Γ" is a property index given in Equation (**2.14**) which was introduced by Chisholm (1973) as

$$\Gamma = \left(\frac{\rho_l}{\rho_g}\right)^{0.5}\left(\frac{\mu_g}{\mu_l}\right)^{n/2}$$

The validity (the ranges of use) of this correlation is

- $0 \le q'' \le 5550$ kW/m^2,
- $507 \le P \le 10{,}327$ kPa and

- $906.9 \le G \le 9981 \text{ kg/m}^2 s$
- $0.003 \le x_a \le 0.4$

as it is is given in Table 3.6 for the minimum and maximum values of compiled database.

The skeleton table yields the RMS and mean errors as:

- 38.89% and -36.85% for 198 adiabatic data by (Alessandrini et al. (1963);
- 30.67% and 7.87% for the 9125 data for steam-water flows in uniformly heated vertical tubes by (Alessandrini et al., 1963; Peterlongo, et al., 1964, Olekhnovitch, 1997; Olekhnovitch, et al., 2004; Leung, 1994).
- 30.86% and 6.92% for totally 9323 data for steam-water flows in both uniformly heated and adiabatic vertical tubes compiled from the literature, respectively, .

These errors of this correlation can easily be compared with those of other correlations presented in Table 3.4.

The reasons to develop a new correlation during this study are as follows:

- There is no experimental data to match the ϕ_{lo}^2 values of LUT entries.
- Correlations and mathematical models existing in literature are either not considered heating effect, hence they can be used for only adiabatic ϕ_{lo}^2 calculations or they require more parameters apart from q", P, G and x_a to calculate ϕ_{lo}^2.

3.5 Constructing the Skeleton Look-Up Table

First (skeleton) table is the basic table of the final LUT. The deriving and updating applications are implemented on the skeleton table. The skeleton table can be constructed with (i) experimental data, (ii) empirical correlations, (iii) mathematical models, (iv) previous studies and (v) any combination of them. (Shan et al. 2005)

In this study, a software program has been developed to generate 4-D skeleton look-up table by using the correlation in Equation (**3.23**). Results have been assigned to the relevant table entries in LUT.

3.6 Updating the Skeleton Look-Up Table

To increase the accuracy of the skeleton look-up table and to approach the table values to the experimental results, LUT is updated with reliable experimental database. In Figure 3.3, experimental data is surrounded by matrix conditions, which represent generic neighbor LUT entries.

The weighted experimental data depending on the distance to each neighbor matrix point are calculated and matrix point are replaced by them.

Using the known trends of ϕ_{lo}^2 with heat flux, pressure, mass flux and flow quality from the skeleton table, the experimental ϕ_{lo}^2 values were extrapolated to the surrounding matrix conditions (to obtain extrapolated ϕ_{lo}^2) as follows

$$\phi_{lo}^2(x_{a,i}, G_j, P_k, q''_m) = \phi_{lo}^2(x_{a,0}, G_0, P_0, q''_0) + \Delta\phi_{lo}^2 \qquad \textbf{(3.24)}$$

where

$$\Delta\phi_{lo}^2 = \frac{\partial\phi_{lo}^2}{\partial x_a}\left(x_{a,i} - x_{a,0}\right) + \frac{\partial\phi_{lo}^2}{\partial G}(G_j - G_0) + \frac{\partial\phi_{lo}^2}{\partial P}(P_k - P_{0)}) + \frac{\partial\phi_{lo}^2}{\partial q''}(q''_m - q''_0) \qquad \textbf{(3.25)}$$

3.7 Weight Functions for Two-Phase Frictional Multiplier Look-Up Table

Weighting factors were calculated and assigned to each experimental ϕ_{lo}^2 value, extrapolated to each of the surrounding 16 points. The idea is that the table point closest to the experimental conditions should receive more weight than the more distant ones during the updating process.

For example, weight factor of experimental data on the table point at $\phi_{lo}^2(x_{a,i}, G_j, P_k, q''_m)$ is calculated by

$$w = \left[\left(x_{a,i} - x_{a,0}\right)\cdot(G_j - G_0)\cdot(P_k - P_{0)})\cdot(q''_m - q''_0)\right]^{1/3} \qquad \textbf{(3.26)}$$

Weight factors for all matrix conditions are calculated separately, then, they are normalized as total of 16 weight factor equals to "1".

Experimental data is extrapolated at the table entry by using the weight factors as follows:

$$\phi^2_{lo(x_i,G_j,P_k,q''_m)'} = \frac{\sum_{n=1}^{n=m} (w_{i,j,k,m})_n \cdot \phi^2_{lo}(x_{a,i}, G_j, P_k, q''_m)_n}{\sum_{n=1}^{n=m} (w_{i,j,k,m})_n} \qquad \textbf{(3.27)}$$

The weighted averaged (thus calculated) ϕ^2_{lo} value is replaced with the old table ϕ^2_{lo} value at matrix conditions adjacent to the experimental ϕ^2_{lo} values. In the regions of the ϕ^2_{lo} table where matrix conditions do not have updated data points, the original ϕ^2_{lo} table value is maintained (Tanase, A. 2007).

3.8 Smoothing the Updated Look-Up Table

The updated ϕ^2_{lo} table is not smooth and displays an irregular variation (without any physical basis) in each parametric ranges: heat flux, pressure, mass flux and dynamic quality. Hence, smoothing is employed on LUT by the aid of spline functions to eliminate discontinuities or jumps which interrupt general trends of LUT**.**

A spline function is a combination of a large number of polynomials with continuous transition from one to the other. A polynomial is fitted to a specific number of neighboring data points surrounding a table entry of interest. Then, the table entry is recalculated using the polynomial. The filter consists of several polynomial functions fitted to selected data points on each parametric direction (i.e., q'' P, G and x_a). The polynomials intersect at a table entry. The filter has some adjustable parameters like the degree of the polynomials, the number of data points involved in the regression and the weighting coefficient. The difficulty of smoothing multi-dimensional tabulated data like the ϕ^2_{lo} LUT lies in the fact that parameter trends for all independent variables must be simultaneously taken into account when a table entry is to be adjusted (Huang and Cheng, 1994).

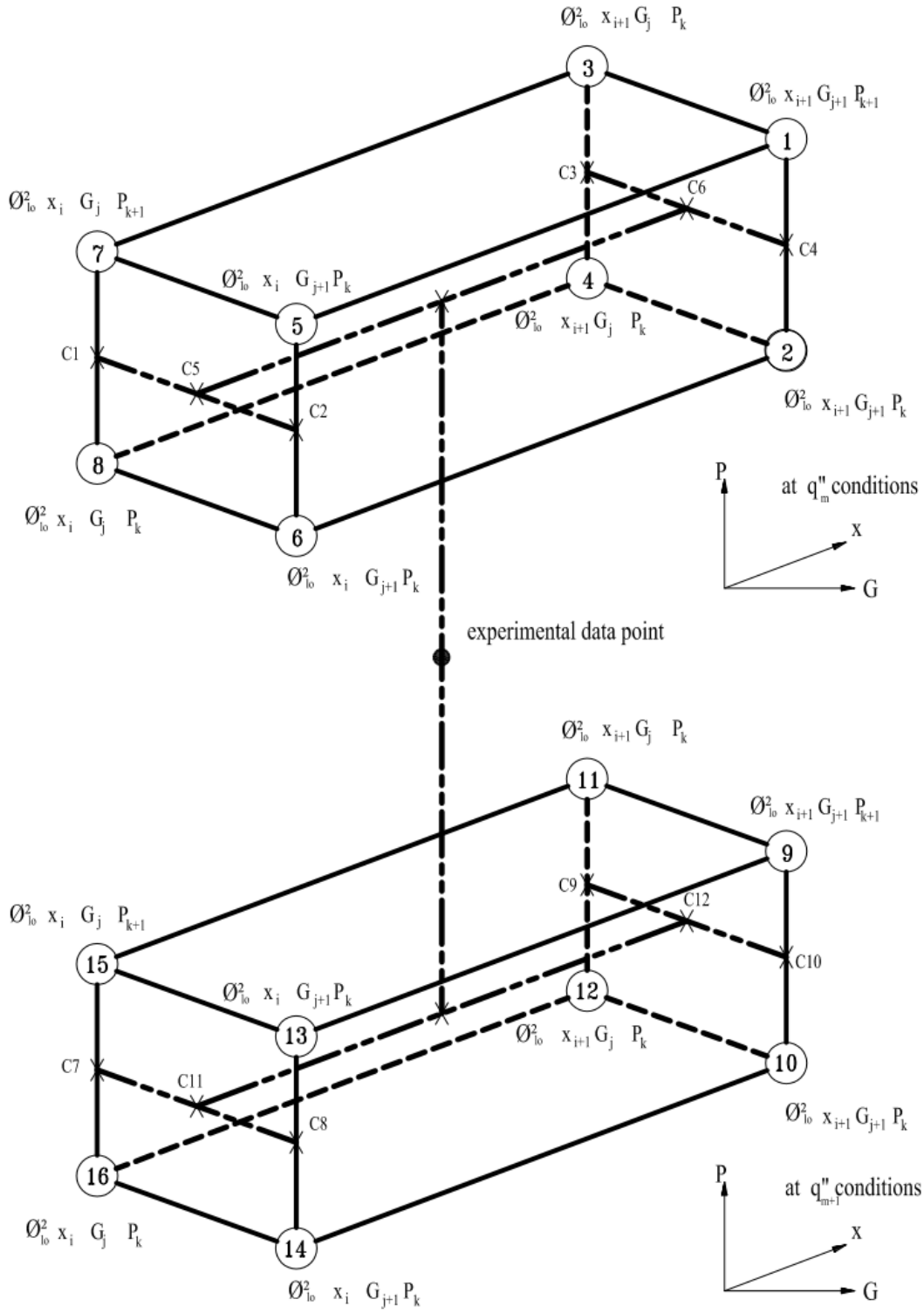


Figure 3.3 : Matrix conditions surrounding the experimental data point $(x_{a,0},\ G_0,\ P_0,\ q_0")$

3.9 Assesment (Error Analysis) of the Look-Up Table

Error of LUT table was assessed in accordance with the local condition approach. It means that the prediction of ϕ_{lo}^2 for each experimental data is done by using direct interpolation between matrix values of q'' P, G and x. Error and RMS are calculated by

$$Error = \frac{Predicted\ \phi_{lo}^2}{Measured\ \phi_{lo}^2} - 1 \qquad \textbf{(3.28)}$$

$$RMS\ Error = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(Error)_i^2} \qquad \textbf{(3.29)}$$

Figure 3.11 illustrates error histograms of skeleton table, updated table and smoothed tables. "Smoothed ϕ_{lo}^2 LUT" is the final form of the table.

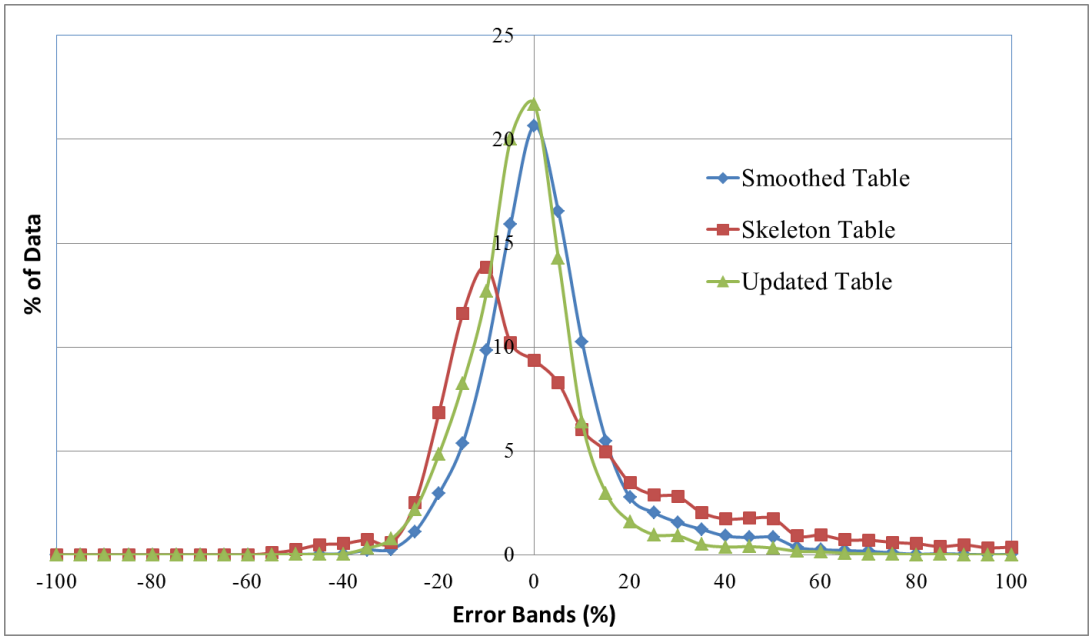


Figure 3.4 : Comparison of the Error Histograms of Skeleton Table, Updated Table and Smoothed (Final form of) Table

4. RESULTS, CONCLUSIONS AND RECOMMENDATIONS

In this chapter, the final form of the two-phase frictional multiplier look-up table is presented as the results of this study. Also, trends observed, procedure of using the two-phase frictional multiplier look-up table, an example on how to use the two-phase frictional multiplier look-up table, conclusions of this thesis and recommendations for future studies are presented.

4.1 Results: Final Form of the Two-Phase Frictional Multiplier Look-Up Table

Experimental data distribution is illustrated in Figures A.1 – A.11 in Appendix A.

The skeleton, updated and smoothed Look-Up Tables and the numbers of experimental data to update the table (distribution of experimental data to contribute the table) at q"=1000 kW/m^2, P=500kPa are presented in Tables B.1 - B.4 in Appendix B.

Final form of "the Two-Phase Friction Multiplier Look-Up Table" having an **RMS error** of **15.43%** is presented in Table C.1 in Appendix C.

Figures D.1 - D.8 illustrate 3-D graphics of selected divisions of ϕ_{lo}^2 LUTs in Appendix D. Each figure includes the same divisions from skeleton, updated and smoothed tables. Therefore, it is easy to compare trends in these regions of these LUTs.

Graphics in Figure D.2 in Appendix D are drawn by using the same data presented in Tables B.1 - B.4 in Appendix B in order to compare the tabulated values visually by the aid of graphics.

The ϕ_{lo}^2 LUT in Table C.1 in Tables B.1 - B.3 and in Appendix C covers 3 regions with different colors (the color codes are presented in Table B.5) as:

- **Tan cells indicate the regions where ϕ_{lo}^2 entries are obtained by using the** correlation presented in Equation (**3.23**).
- Yellow cells indicate the regions where ϕ_{lo}^2 entries are updated with experimental data.
- Orange cells indicate regions in which flow is in saturated liquid phase only. Hence, "1" is assigned to ϕ_{lo}^2 at orange cells.

Errors of the different tables are presented in Table 4.1 for Two-Phase Frictional Multiplier Look-Up Table.

Table 4.1 : Errors of the Two-Phase Frictional Multiplier Look-Up Table.

Table	Database	Number of Data	Mean Error (%)	RMS Error (%)
Skeleton Table	Adiabatic database	198	-36.85	38.89
	Diabatic database	9125	7.87	30.67
	All database	**9323**	**6.92**	**30.86**
Updated Table	Adiabatic database	198	0.72	10.57
	Diabatic database	9125	-1.87	12.91
	All database	**9323**	**-1.81**	**12.87**
Smoothed Table	Adiabatic database	198	-11.50	15.65
	Diabatic database	9125	3.22	15.42
	All database	**9323**	**2.90**	**15.43**

4.2 Trends Observed in the Two-Phase Frictional Multiplier Look-Up Table

The following trends are observed from Figures D.1-D.9:

- ✓ In Figure D.1, ϕ_{lo}^2 is increased by increasing x_a, and slightly decreases by increasing G at $q'' = 0$ and at low P values (such as at P =500 kPa).
- ✓ In Figure D.2, trend in the variation of ϕ_{lo}^2 with x_a is as same as trend in Figure D.1, although decrease in ϕ_{lo}^2 is much more than that in Figure D.1

with increasing G at low heat flux values (such as at $q'' = 1000$ kW/m^2) and at low pressure values (such as at P =500 kPa).

- ✓ In Figure D.3 and Figure D.4, increasing variation trend of ϕ_{lo}^2 with x at high heat flux values (such as at $q'' = 5000$ - 6000 kW/m^2) is the same as it is (but a little bit steeper) at low heat flux values (such as at $q'' = 1000$ kW/m^2 as shown in Figure D.1). It is also seen that decreasing trend of ϕ_{lo}^2 at high heat flux values (such as at $q'' = 5000$ - 6000 kW/m^2) is more than that at low heat flux values (such as at $q'' = 1000$ kW/m^2) when G is increasing especially at high x values.
- ✓ Comparing the trends in Figure D.5 with that in Figure D.3 and Figure D.4, ϕ_{lo}^2 dramatically (approximately 20 times) decreases with the increase in P (such as from P = 500 kPa to P = 11,000 kPa).
- ✓ In Figure D.6, water is assumed to be in liquid phase only; hence ϕ_{lo}^2=1 is assigned at x_a =0.0 from the definition of ϕ_{lo}^2.
- ✓ In Figure D.7, ϕ_{lo}^2 sharply decreases while P is increasing at low mass flux values (such as at G = 500 kg/m^2 s) and low flow quality values (such as at x_a = 0.05).
- ✓ Comparing trends in Figure D.7 to the trends in Figure D.8, ϕ_{lo}^2 decreases with some increase in G (such as from G = 500 kg/m^2 s to G =1000 kg/m^{2}s) and large increase in x_a (such as from x_a=0.05 to x_a=1.0). However, dramatically decreasing trend is still prevailing while P is increasing especially at higher flow quality values.
- ✓ Moreover, Figure D.7 shows that smooth transition trend has been provided and discontinuity between "LUT entries updated with experimental data" and "LUT entries calculated by Equation (**3.23**) has been prevented by smoothing process.

4.3 Procedure of Using the Two-Phase Frictional Multiplier Look-Up Table

One of the advantages of the look-up tables is that it is easy to use. Providing the flow conditions (q'', P, G, x_a) is enough to determine the requested ϕ_{lo}^2. After that, LUT allows to make interpolation to find ϕ_{lo}^2 if the flow condition is within the ranges of LUT.

4.4 Example on How to Use the Two-Phase Frictional Multiplier Look-Up Table

Result of an example ϕ_{lo}^2 value obtained from ϕ_{lo}^2 LUT by interpolation is compared with calculated ϕ_{lo}^2 value as its calculation method is explained in Section 3.2.3.

The calculation was executed by Odabaş (2011) for a selected experimental data given in Table 4.2 reported originally by Leung (1994).

Table 4.2 : An experimental data reported by Leung (1994) (Odabaş, 2011).

Data Code	Date	P (MPa)	T_{in} ($°C$)	T ($°C$)	T_{env} ($°C$)	W (g/s)	q (kW)	ΔP (kPa)
DPB74310	27/09/89	7.05	255.65	288.35	23.26	101.84	50.16	345.24

ϕ_{lo}^2 was calculated from the experimental data as ϕ_{lo}^2 = **2.42** ($x_{a,ave}$ was also calculated from the experimental data as $x_{a,ave}$ =0,223397 since x_a is not reported by Leung (1994)) by (Odabaş, 2011) ϕ_{lo}^2 LUT value is determined as **2.539** by linear interpolation having an RMS error of **4.92%** by using the ϕ_{lo}^2 LUT. Comprasions are presented in Table 4.3.

Table 4.3 : Calculated ϕ_{lo}^2 vs. result of LUT

q''	$P_{ave} = P + \Delta P/2$	G	$x_{a,ave} = x_a/2$	ϕ_{lo}^2		*Error*
kW//m^2	kPa	kg/m^2 s		**Calculated**	**LUT**	**%**
1171	7050	4365	0,112	**2.42**	**2.539**	**4.92**

4.5 Conclusions

- A new Two-Phase Frictional Multiplier Look-Up Table (ϕ_{lo}^2 LUT) is introduced based on heat flux (q"), average pressure through the two-phase flow region (P_{ave}), mass flux (G) and actual (flow) quality (x_a).
- The ϕ_{lo}^2 LUT enables us not only to generate the ϕ_{lo}^2 values but also to observe trends in the variation of ϕ_{lo}^2 with q", P, G and x_a clearly.

The main conclusions about the trends in the variation of ϕ_{lo}^2 with q", P, G and x_a observed from the ϕ_{lo}^2 LUT and Figures D.1-D.8 can be summarized as follows:

- ✓ Increase in x_a causes to increase in ϕ_{lo}^2.
- ✓ Increase in P causes to decrease in ϕ_{lo}^2 at the same heat flux values.
- ✓ Increase in q'' causes to increase in ϕ_{lo}^2 at low G and low P values.
- ✓ Effect of heating gets smaller while G increases and then almost disappears at high G values.

4.6 Recommendations for Future Studies

1- ϕ_{lo}^2 LUT table should be updated with enough number of additional data in order to reflect better asymptotic trends.

2- Experimental data obtained with 2ϕ-flows of other fluids (apart from water) may be scaled to water or experienced directly for improving the ϕ_{lo}^2 LUT.

3- In this LUT, average quality is used through 2ϕ-length of the tube during calculation. This study may be improved by using a database ranging at higher x_a values and also by using sufficient number of additional adiabatic flow data.

REFERENCES

Alsurakji I. H. (2012)**,** Look-up table for two-phase frictional pressure drop multiplier. King Fahd University of Petrolium and Minerals, Dahran, Saudi Arabia, *Thesis for the degree of Master of Science*.

Alessandrini, A., Peterlongo, G., Ravetta, R. (1963). R-86. Large scale experiments on heat transfer and hydrodynamics with steam-water mixtures: Critical heat flux and pressure drop measurements in round vertical tubes at the pressure of 51 kg/cm^2 abs.. CISE Report, No. 056.62.1-RDI, Milano.

Baroczy, C. J. (1965). A Systematic Correlation for Two-Phase Pressure Drop. Paper Presented at the 8th National Heat-Transfer Conference, Los Angeles, California, August 8-11, AIChE Preprint 37.

Aubé, F. (1996). Étude de la distribution axiale de la pression dans les écoulements diphasiques. *Thèse de doctorat*, École Polytechnique de Montréal, Canada.

Chen, N. H. (1979). An explicit equation for friction factor in pipe. *Industrial and Engineering Chemistry Fundamentals*, Vol. ***18,*** no. 3, pp. 296-297.

Chisholm, D. (1973). Pressure gradients due to friction during the flow of evaporating two-phase mixtures in smooth tubes and channels. *Int. J. Heat Mass Transfer*, ***16***, 347-358.

Colebrook, C. F. (1939). Turbulent flow in pipes with particular reference to the transition region between smooth and rough pipe laws. *J. of Inst. Civil Eng.*, ***11***, 133.

Davis E. S., and Anderson, G. H. (1966). The ıncipience of nucleate boiling in forced convective flow, *AIChE J.*, Vol. ***12,*** no.4, pp. 774-780.

Doroshchuk,V.E., Levitan, I.L., Lantzman, F.P. (1975). Investigation into Burnout in Uniformly Heated Tubes. ASME Publication, 75-WA/HT-22.

Durmayaz, A., Groeneveld, D.C., Cheng, S.C. (2004). Assessment of critical heat-flux look-up tables, experimental data and selected correlations. *Proceedings of the Sixth International Conference on Simulation Methods in Nuclear Engineering*, October 12–15, Montreal, Canada

Durmayaz, A., Odabas, İ.A., Groeneveld, D.C., Gengec, N.E. (2012).Assessment of selected two-phase friction multiplier correlations for steam-water flows in vertical heated and unheated tubes, *The 20th International Conference on Nuclear Engineering*, (Paper: ICONE20POWER2012-54938), American Nuclear Society, Anaheim, CA, USA, July 30-August 3.

El Nakla, M., Al-Sarkhi, A., Alsurakji, I. (2013). A look-up table for two-phase frictional pressure drop multiplier. *Nuclear Engineering and Design* ***265***, 450– 468.

El Nakla, M., Cheng, S.C., Groeneveld, D.C. (2004). Improvements of prediction accuracy and parametric trends of the film boiling look-up tables at ACR conditionsof interest. Technical Report for Atomic Energy of Canada Limited, UO-MCG-TH-2004-001-Rev. 1, March.

Friedel, L. (1979). Improved friction pressure drop correlations for horizontal and vertical two-phase pipe flow. *European Two-Phase Flow Group Meeting*, Ispra, Italy, paper E2.

Grönnerud, R. (1972). Investigation of liquid hold-up, flow-resistance and heat trans-fer in circulation type evaporators. Part iv: Two-phase flow resistance in boiling refrigerants. Bull. De l'Inst. Du Froid 1 (Annexe).

Groeneveld, D.C., Cheng, S.C., Doan, T. (1986). AECL-UO critical heat flux lookup table. Heat Transfer Eng. ***7***, 46–62.

Groeneveld, D.C., Leung, L.K.H., Guo, Y.J., Vasic, A.Z., El Nakla, M., Peng, S.W., Yang,J., Cheng, S.C. (2005). Look-up tables for predicting CHF and film boiling heat transfer: past, present and future. *Nucl. Technol.* ***152***, 87–104.

Groeneveld, D.C., Leung, L.K.H., Kirillov, P.L., Bobkov, V.P., Smogalov, I.P., Vinogradov, V.N., Huang, X.C., Royer, E. (1996). The 1995 look-up table fo critical heat flux in tubes. Nucl. Eng. Des. 163, 1–23.

Groeneveld, D. C., Shan, J. Q., Vasic, A. Z., Leung, L. K.-H., Durmayaz, A., Yang, J., Cheng, S. C. and Tanase, A. (2007). The 2006 CHF look-up table. *Nuclear Engineering and Design*, ***237***, 1909-1922.

Groeneveld, D.C., Shan, J.Q., Vasić , A.Z., Leung, L.K.H., Durmayaz, A., Yang, J., Cheng S.C., Tanase, A. (2005). The 2005 CHF look-up table. *The 11th International Topical Meeting on Nuclear Reactor Thermal-Hydraulics (NURETH-11)* Paper:166, Popes' Palace Conference Center, Avignon-France, October 2-6.

Groeneveld, D.C., Vasić, A.Z., Durmayaz, A., Shan J.Q., Leung, L.K.H., Peng, S.W. Yang, J., Cheng S.C. (2004). An improved CHF look-up table, *Proceedings of the Sixth International Conference on Simulation Methods in Nuclear Engineering*, Montréal-Québec, Canada, October 13-15.

Huang, X.C., Cheng, S.C. (1994). Simple method for smoothing multidimensional experimental data with application to the CHF and post dryout look-upt ables. Numer. Heat Transfer, Part B Vol. ***26*** no. 4, pp. 425–438.

Idsinga, W. (1975). An assessment of two-phase pressure drop correlations for steam-water systems. Massachusetts Institute of Technology, USA, *Master of Science thesis.*

Klausner J.F., Chao, B.Ti Soo, S.L. (1990). An improved method for simultaneous determination of frictional pressure drop and vapor volume fraction in vertical flow boiling. *Experimental Thermal and Fluid Science,*. ***3***, 404-415.

Kroeger, P. G. and Zuber, N. (1968). An analysis of the effects of various parameters on the average void fractions in subcooled boiling. *Int. J. Heat Mass Transfer*, ***11***, 211-233.

Leung, L. K. H. (1994). A model for predicting the pressure gradient along a heated channel during flow boiling. University of Ottawa, Canada, *Thesis for the degree of Doctor of Philosophy*.

Leung, L. K.-H., Groeneveld, D. C., Teyssedou, A. and Aubé, F. (2004). Pressure drop for steam and water flow in heated tube. *Nuclear Engineering and Design*, ***235***, 53-65.

Lombardi, C. and Pedrocchi, E. (1972). A pressure drop correlation in two-phase flow. *Energia Nucleare*, Vol. ***19***, no**.** 2, pp. 91-99.

McAdams, W. H., Woods, W. K. and Heroman, L. C. (1942). Vapourisation ınside horizontal tubes, 2: Benzene-oil mixtures. *Transactions of the ASME*, ***64***, 193-200.

Muller-Steinhagen, H. and Heck, K. (1986). A Simple friction pressure drop correlation for two-phase flow in pipes. *Chemical Engineering Progress*, Vol. ***20,*** no. 6**,** pp. 297-308.

Odabaş, İ.A. (2011). Analysis of pressure drop measurements, two-phase friction multiplier correlations and pressure drop components for water-steam flows in heated and unheated vertical tubes, *Master of Science thesis*, Istanbul Technical University Energy Institute, İstanbul, Turkey.

Odabaş, İ.A., Gengeç, N.E., Öztüzün, B., Durmayaz, A. (2013). Analysis of pressure drop measurements, two-phase friction multiplier correlations and pressure drop components for water-steam flows in heated and unheated vertical tubes. *19th National Thermal Sciences and Techniques Congress*, Ondokuzmayıs University and Association of the Turkish Thermal Sciences and Techniques, Samsun-Turkey, September 9-12, (Paper no: ULIBTK'13-049) (in Turkish) pp. 60-66.

Olekhnovitch, A. (1997). Étude de flux de chaleur critique à des pressions faibles. *Thèse de doctorat*, École Polytechnique de Montréal, Canada.

Olekhnovitch, A., Teyssedou, A., Tye, P. and Felisari, R. (2004). An empirical correlation for calculating steam-water two-phase pressure drop in uniformly heated vertical round tubes. *International Journal of Multiphase Flow*, ***31***, 358-370.

Öztüzün, B., Odabaş, İ.A., Durmayaz, A. (2013). Methodology of look-up table development for two-phase flow parameters, *19th National Thermal Sciences and Techniques Congress*, Ondokuzmayıs University and Association of the Turkish Thermal Sciences and Techniques, Samsun-Turkey, September 9-12, (Paper no: ULIBTK'13-049) (in Turkish) pp. 60-66.

Peterlongo, G., Ravetta, R., Riva, B., Rubiera, L. and Tacconi, F. A. (1964). R-122: Large scale experiments on heat transfer and hydrodynamics with steam-water mixtures: Further critical power and pressure drop measurements in round tubes with and without internal obstacles. CISE Report, **No. 056.62.1-RDI,** Milano.

Roshenow, W. M. and Clark, J. A. (1951). *Heat transfer and pressure drop data for high heat flux densities to water at high subcritical pressures*. Heat Transfer and Fluid Mechanics Institute, Stanford University Press, Stanford, California.

Saha, P. and Zuber, N. (1974). Point of net vapour generation and vapour void fraction in subcooled boiling. *Proceedings of the 5th Int. Heat Transfer Conference*, Tokyo, ***4***, 175-179.

Shan J.Q., Groeneveld, D.C., Vasić, A.Z., Yang, J., Durmayaz, A., Peng, S.W., Cheng S.C. (2005). Methodology of CHF look-up table derivation, *The 13th International Conference on Nuclear Engineering*, (Paper: ICONE13-50010), Chinese Nuclear Society, Beijing-China, May 16-20.

Sieder, E. N. and Tate, G. E. (1936). Heat transfer and pressure drop of liquids in tubes. *Industrial and Engineering Chemistry*, ***28***, 1429-1435.

Quiben, J. M. (2005). Experimental and analytical study of two-phase pressure drops during evaporation in horizontal tubes, *Ph.D. thesis*. École Polytechnique Fédérale de Lausanne, Lausanne.

Tanase, A. (2007). Improved methodology for deriving the critical heat flux look-up table. *Thesis for the degree of Master of Applied Science*. Mechanical Engineering Ottawa-Carleton Institute for Mechanical and Aerospace Engineering, University of Ottawa, Ottawa, Canada.

Tarasova, N. V., Leont'ev, A. I., Hlopuskin, V. I. and Orlov, V. M. (1966). Pressure drop of boiling subcooled water and steam-water mixture flowing in heated channels. *Proc. of the 3rd International Heat Transfer Conference*, Chicago, USA, Vol. ***4***, 178-183.

Thom, J. R. S. (1964). Prediction of pressure drop during forced circulation boiling of water. *International Journal of Heat and Mass Transfer*, ***7***, 709-724.

Zahlan, H., Groeneveld, D.C. and Tavoularis, S. (2011). Derivatıon of a look-up table for trans-critical heat transfer for water-cooled tubes. *The 14th International Topical Meeting on Nuclear Reactor Thermal-Hydraulics (NURETH14-042)*, Toronto, Ontario, Canada, September 25-30.

Zuber, N. And Findlay, J. A. (1965). Average volumetric concentration in two-phase flow systems. *J. Heat Transfer*, ***87***, 453-468.

APPENDICES

APPENDIX A: Illustrations for Experimental Data Distribution

APPENDIX B: The Skeleton, Updated and Number of Experimental Data Look-Up Tables at q''=1000 kW/m^2, P=500 kPa

APPENDIX C: Final Form of "the Two-Phase Frictional Multiplier Look-Up Table"

APPENDIX D: 3-D illustrations of the Two-Phase Frictional Multiplier Look-Up Table

APPENDIX A

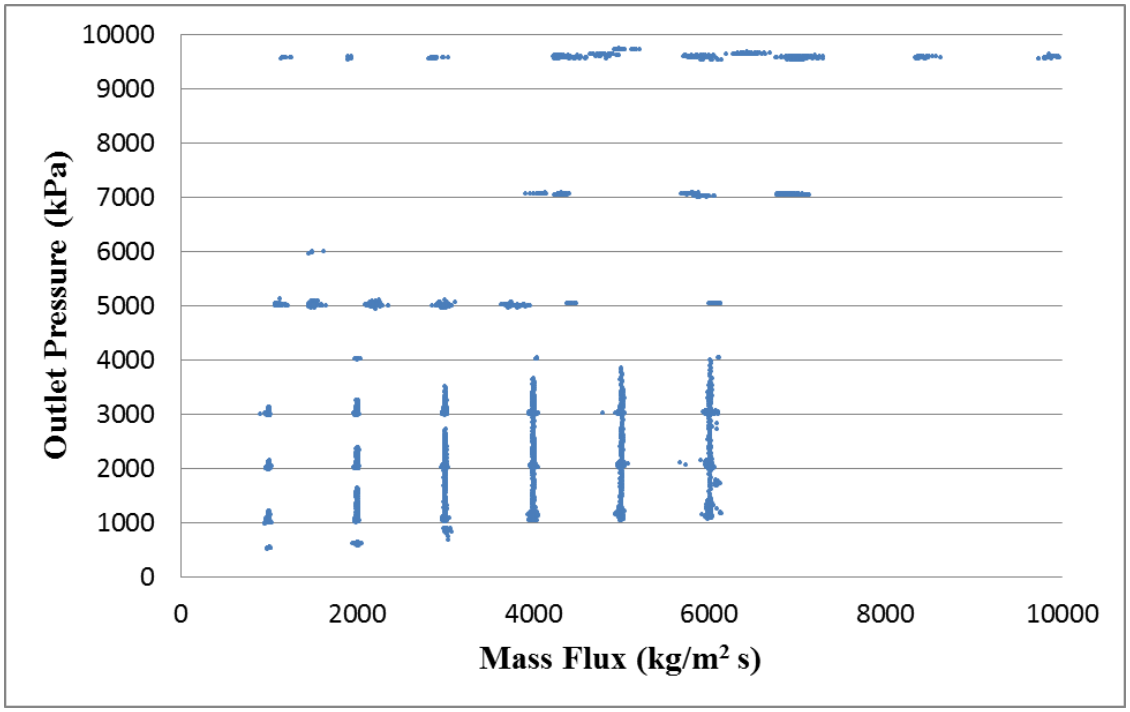


Figure A.1 : Outlet Pressure (*P*) vs. Mass Flux (*G*)

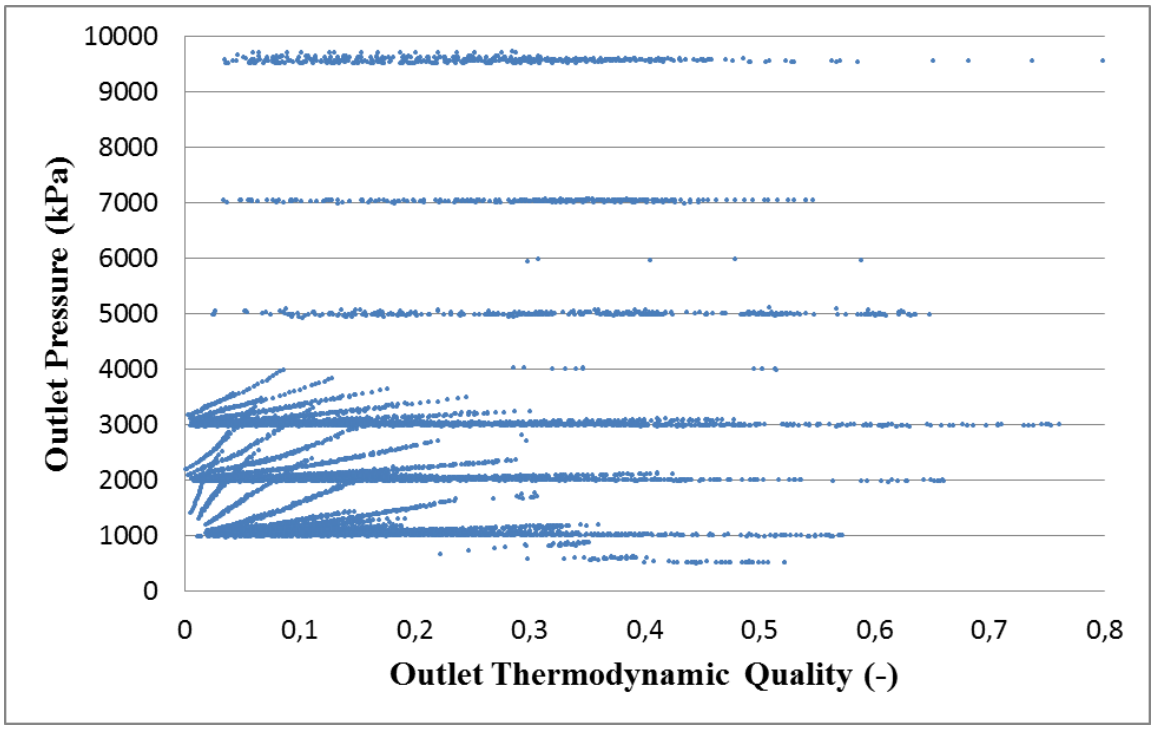


Figure A.2 : Outlet Pressure (*P*) vs. Outlet Thermodynamic Quality (*x*)

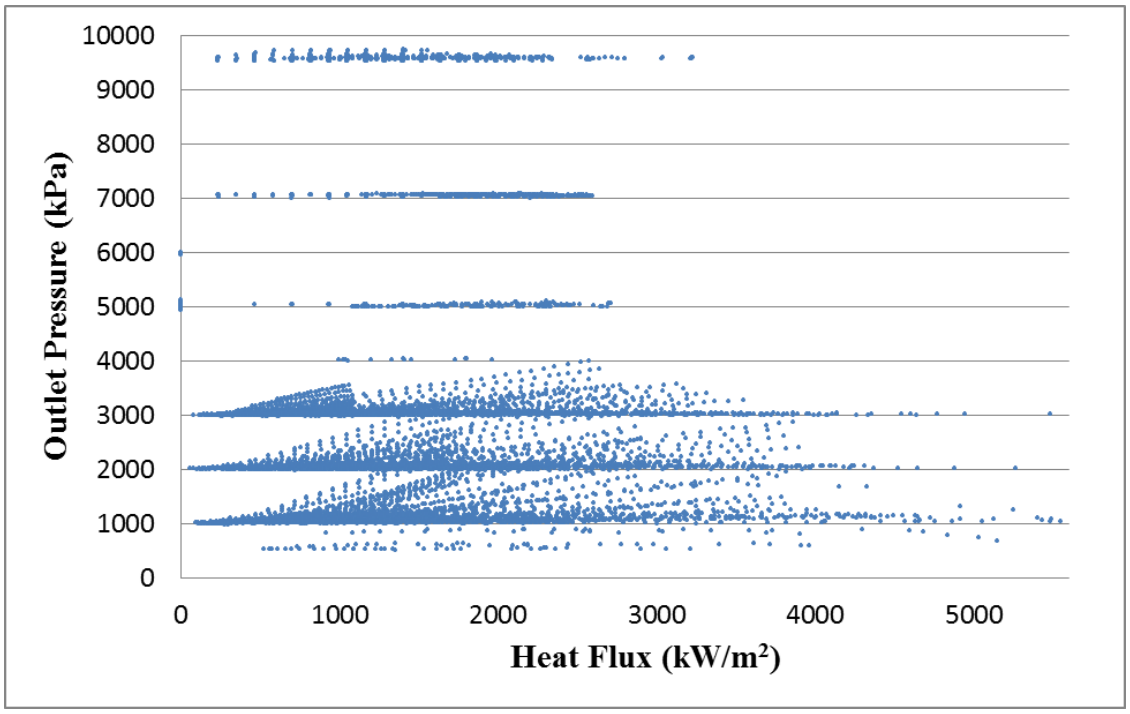


Figure A.3 : Outlet Pressure (*P*) vs. Heat Flux (*q*")

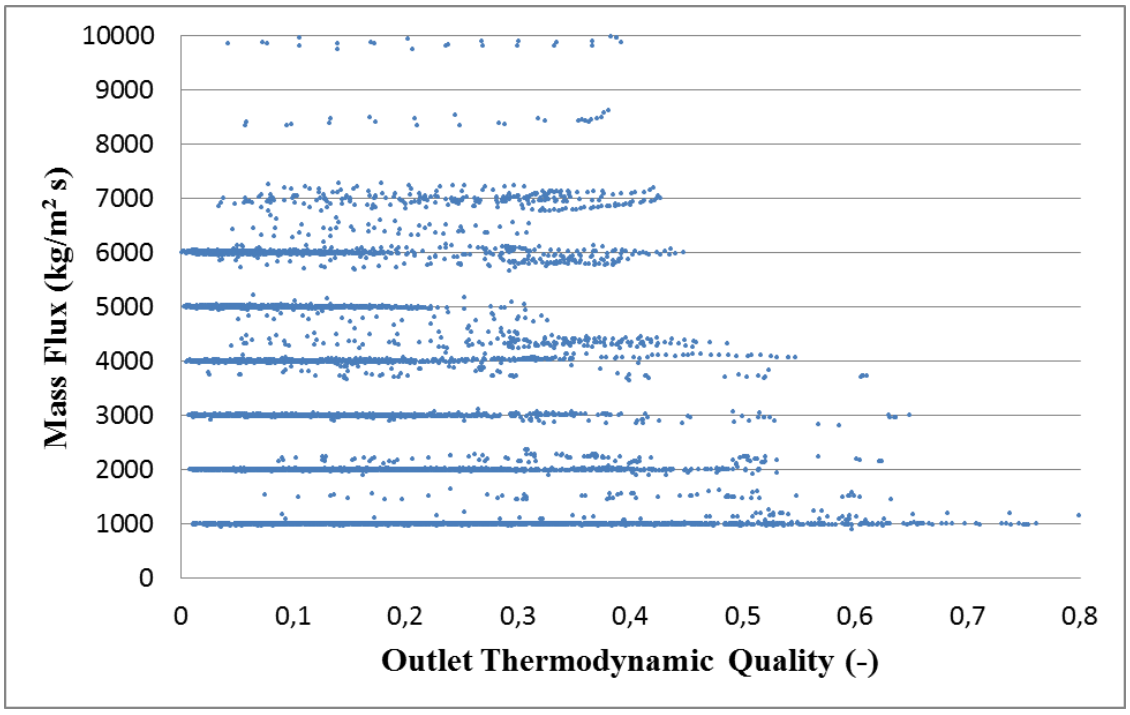


Figure A.4 : Mass Flux (*G*) vs. Outlet Thermodynamic Quality (*x*)

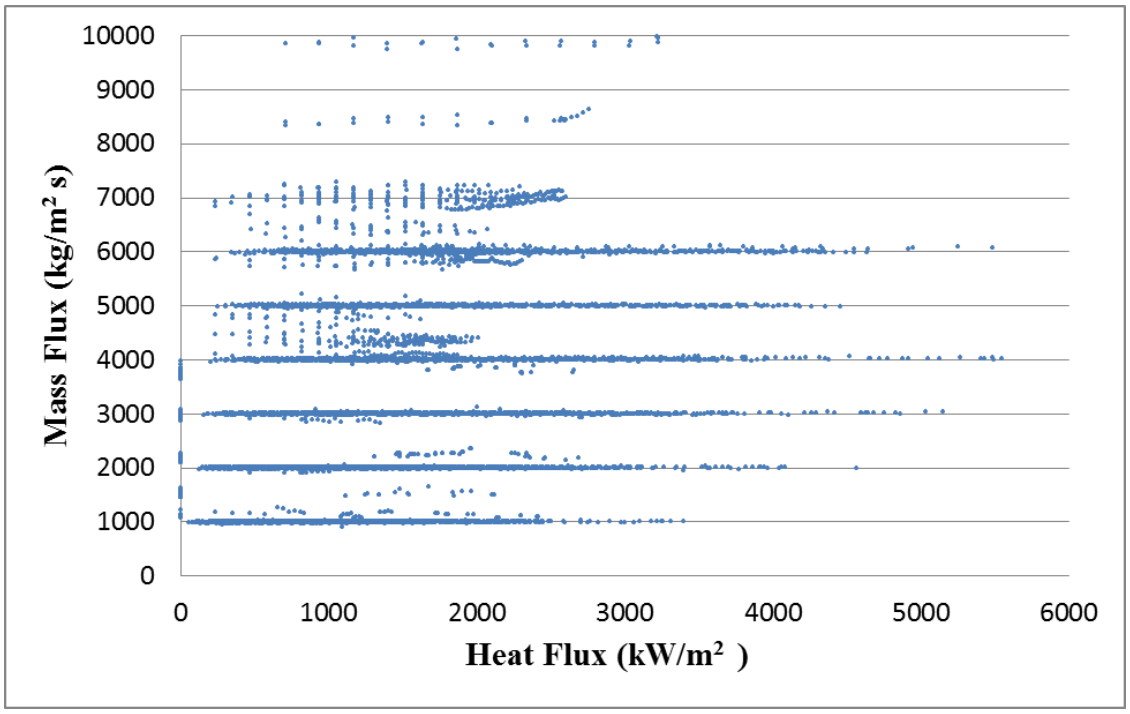


Figure A.5 : Mass Flux (G) vs. Heat Flux (q")

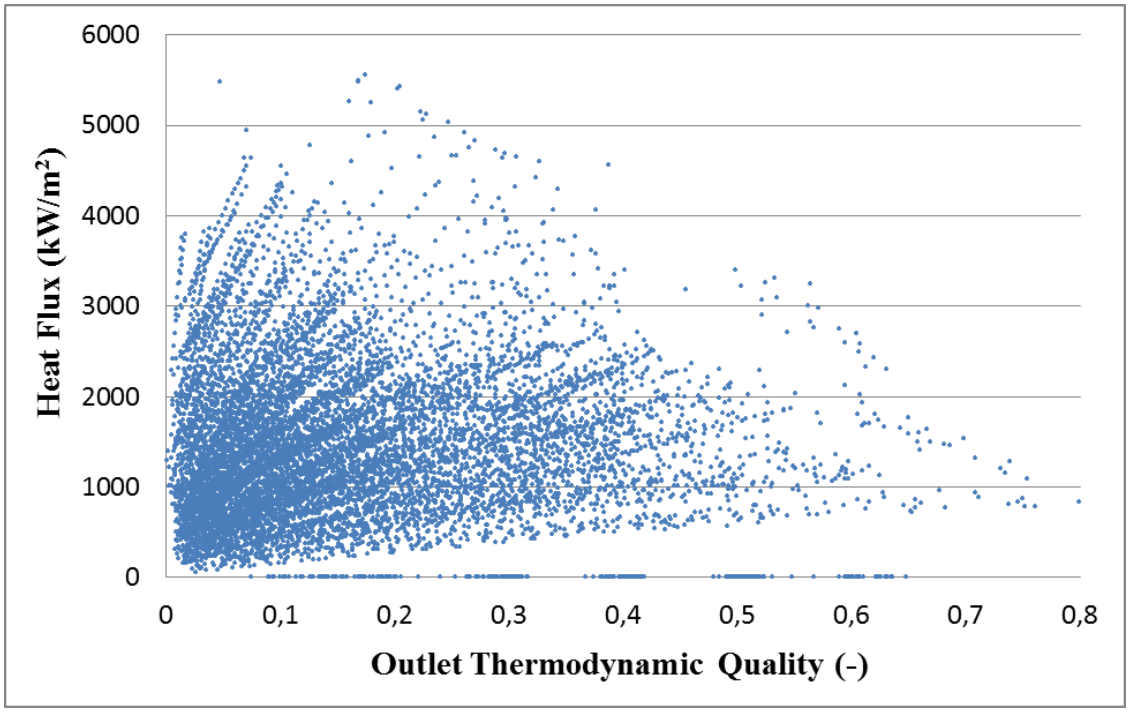


Figure A.6 : Heat Flux (q") vs. Outlet Thermodynamic Quality (x)

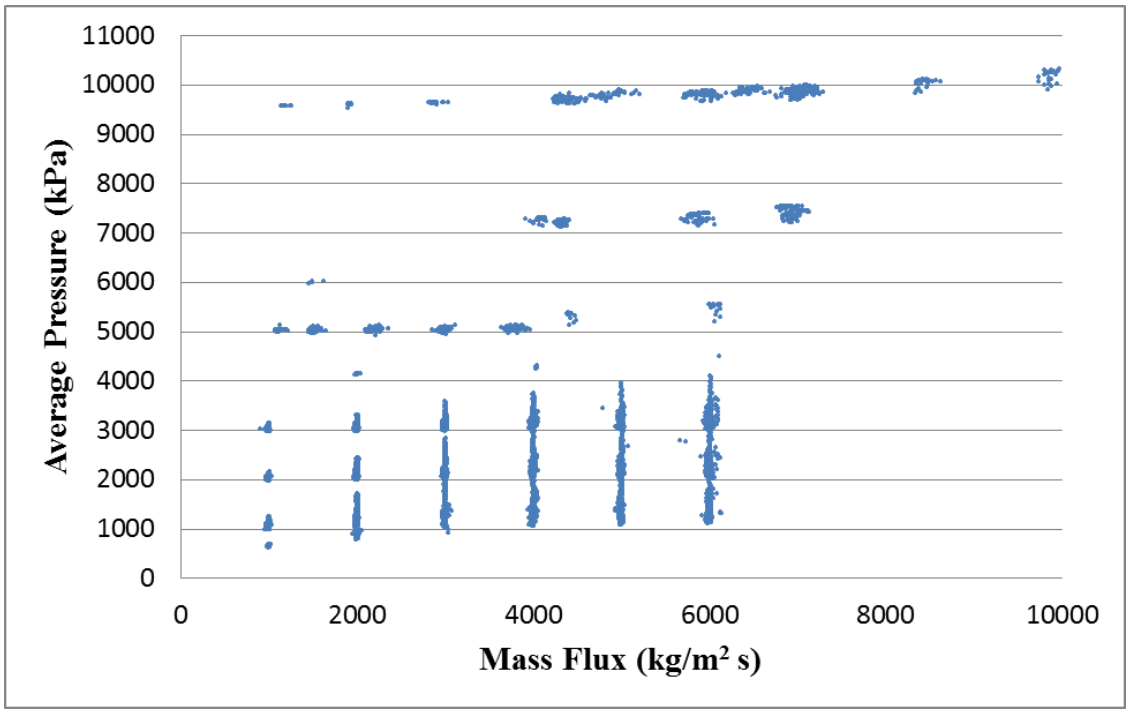


Figure A.7 : Average Pressure (P_{ave}) vs. Mass Flux (G)

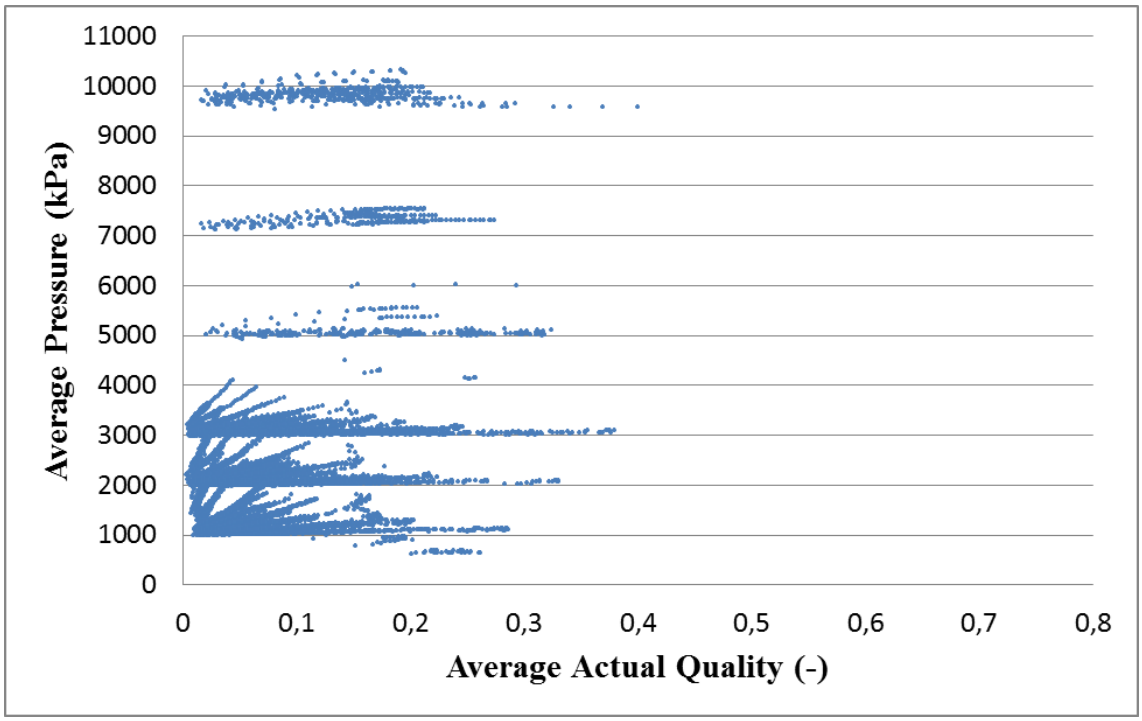


Figure A.8 : Average Pressure (P_{ave}) vs. Average Actual (Flow) Quality ($x_{a,ave}$)

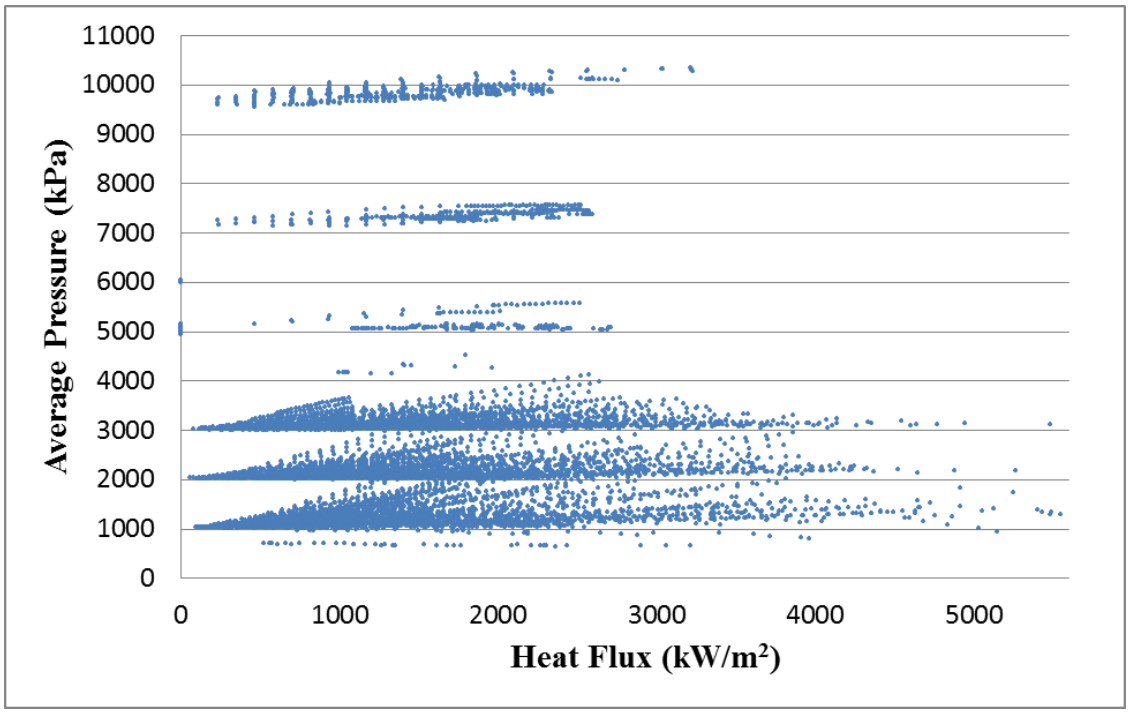


Figure A.9 : Average Pressure (P_{ave}) vs. Heat Flux (q")

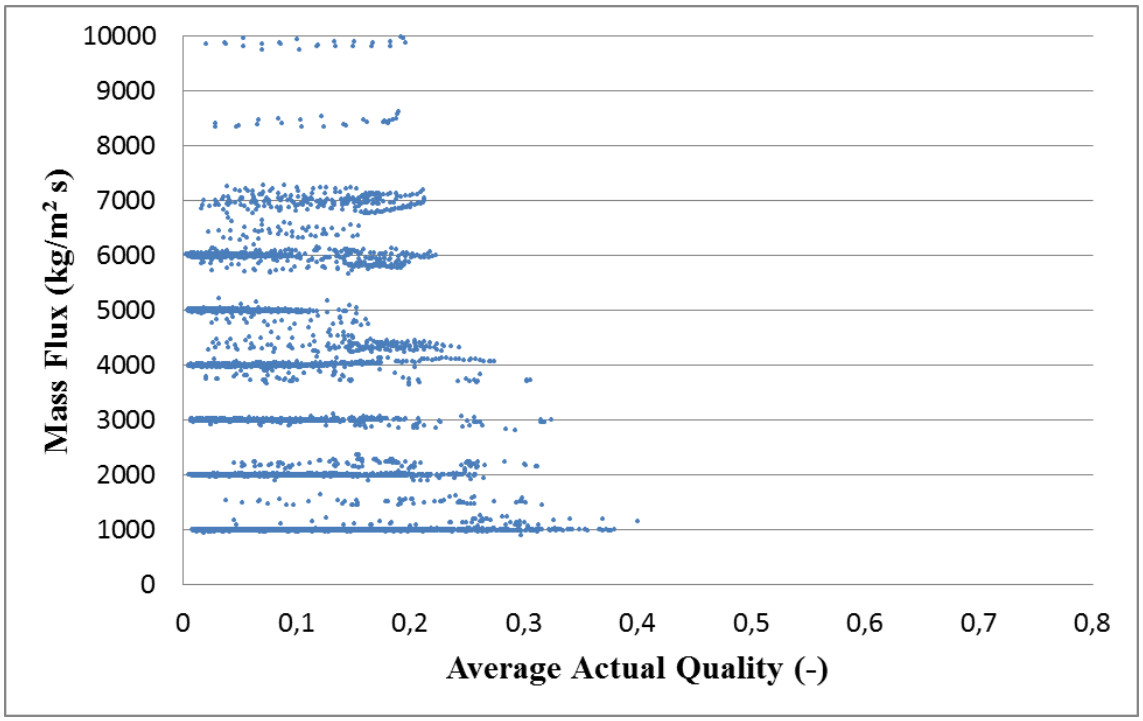


Figure A.10 : Mass Flux (G) vs. Average Actual Quality ($x_{a,ave}$)

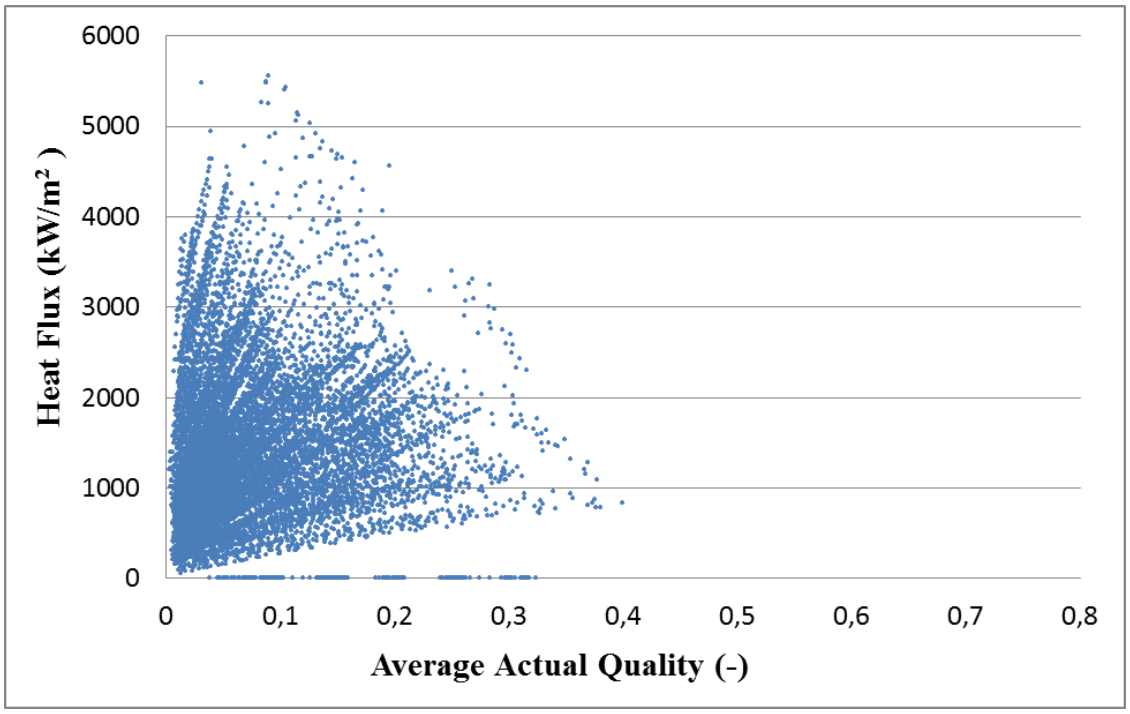


Figure A.11 : Heat Flux (q") vs. Average Actual Quality ($x_{a,ave}$)

APPENDIX B

Table B.1 : The Skeleton Look-Up Table at q"=1000 kW/m^2, P =500kPa (continued)

q"	P	G	x_a →											
			0	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1000	500	500	1	46.21	75.57	128.92	181.11	234.17	288.67	344.71	402.10	460.43	518.94	576.26
1000	500	1000	1	29.55	49.75	89.99	133.16	180.13	231.01	285.69	343.94	405.40	469.51	535.38
1000	500	1500	1	23.52	40.41	75.93	115.84	160.60	210.17	264.36	322.91	385.49	451.62	520.56
1000	500	2000	1	20.30	35.44	68.44	106.61	150.20	199.07	253.00	311.71	374.88	442.08	512.66
1000	500	2500	1	18.27	32.29	63.69	100.77	143.62	192.04	245.80	304.62	368.16	436.04	507.65
1000	500	3000	1	16.84	30.08	60.38	96.69	139.01	187.13	240.77	299.65	363.47	431.82	504.15
1000	500	3500	1	15.78	28.44	57.90	93.64	135.58	183.47	237.02	295.96	359.96	428.67	501.54
1000	500	4000	1	14.96	27.16	55.98	91.27	132.91	180.61	234.10	293.07	357.23	426.21	499.50
1000	500	4500	1	14.29	26.13	54.42	89.35	130.75	178.31	231.74	290.75	355.03	424.23	497.86
1000	500	5000	1	13.74	25.28	53.14	87.77	128.96	176.40	229.79	288.83	353.21	422.60	496.51
1000	500	5500	1	13.27	24.55	52.05	86.43	127.46	174.79	228.14	287.20	351.67	421.22	495.36
1000	500	6000	1	12.87	23.94	51.12	85.29	126.16	173.41	226.73	285.81	350.36	420.03	494.38
1000	500	6500	1	12.52	23.40	50.31	84.29	125.04	172.21	225.50	284.60	349.21	419.00	493.53
1000	500	7000	1	12.22	22.92	49.59	83.41	124.05	171.15	224.41	283.53	348.20	418.09	492.77
1000	500	7500	1	11.95	22.50	48.96	82.63	123.16	170.21	223.45	282.58	347.30	417.28	492.11
1000	500	8000	1	11.70	22.13	48.39	81.93	122.38	169.37	222.59	281.73	346.50	416.56	491.51
1000	500	9000	1	11.28	21.47	47.41	80.72	121.01	167.92	221.10	280.27	345.11	415.31	490.48
1000	500	10000	1	10.93	20.93	46.59	79.71	119.88	166.71	219.86	279.04	343.95	414.27	489.62

Table B.2 : Updated Look-Up Table at *q"*=1000 kW/m^2. *P*=500kPa

q"	P	G	x_a →											
			0	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1000	500	500	1	6.79	75.57	96.24	114.17	234.17	288.67	344.71	402.10	460.43	518.94	576.26
1000	500	1000	1	6.36	49.75	74.07	92.75	180.13	231.01	285.69	343.94	405.40	469.51	535.38
1000	500	1500	1	6.52	66.50	59.08	76.57	160.60	210.17	264.36	322.91	385.49	451.62	520.56
1000	500	2000	1	20.30	56.54	64.39	106.61	150.20	199.07	253.00	311.71	374.88	442.08	512.66
1000	500	2500	1	18.27	55.46	64.85	100.77	143.62	192.04	245.80	304.62	368.16	436.04	507.65
1000	500	3000	1	16.84	30.08	60.38	96.69	139.01	187.13	240.77	299.65	363.47	431.82	504.15
1000	500	3500	1	15.78	28.44	57.90	93.64	135.58	183.47	237.02	295.96	359.96	428.67	501.54
1000	500	4000	1	14.96	27.16	55.98	91.27	132.91	180.61	234.10	293.07	357.23	426.21	499.50
1000	500	4500	1	14.29	26.13	54.42	89.35	130.75	178.31	231.74	290.75	355.03	424.23	497.86
1000	500	5000	1	13.74	25.28	53.14	87.77	128.96	176.40	229.79	288.83	353.21	422.60	496.51
1000	500	5500	1	13.27	24.55	52.05	86.43	127.46	174.79	228.14	287.20	351.67	421.22	495.36
1000	500	6000	1	12.87	23.94	51.12	85.29	126.16	173.41	226.73	285.81	350.36	420.03	494.38
1000	500	6500	1	12.52	23.40	50.31	84.29	125.04	172.21	225.50	284.60	349.21	419.00	493.53
1000	500	7000	1	12.22	22.92	49.59	83.41	124.05	171.15	224.41	283.53	348.20	418.09	492.77
1000	500	7500	1	11.95	22.50	48.96	82.63	123.16	170.21	223.45	282.58	347.30	417.28	492.11
1000	500	8000	1	11.70	22.13	48.39	81.93	122.38	169.37	222.59	281.73	346.50	416.56	491.51
1000	500	9000	1	11.28	21.47	47.41	80.72	121.01	167.92	221.10	280.27	345.11	415.31	490.48
1000	500	10000	1	10.93	20.93	46.59	79.71	119.88	166.71	219.86	279.04	343.95	414.27	489.62

Table B.3 : Smoothed Look-Up Table at q''=1000 kW/m^2. P=500kPa

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1000	500	500	1	20.76	42.64	73.19	123.97	204.98	280.81	338.21	391.00	447.43	503.75	558.71
1000	500	1000	1	17.48	36.57	61.48	98.90	163.82	228.03	283.26	338.21	397.89	459.80	522.93
1000	500	1500	1	16.32	34.09	54.57	91.67	146.28	203.52	257.19	313.29	374.44	438.80	505.56
1000	500	2000	1	15.44	32.37	53.17	92.96	141.25	193.41	246.26	303.24	365.00	430.29	498.47
1000	500	2500	1	16.68	30.55	56.76	93.92	138.34	187.23	239.93	297.40	359.48	425.36	494.49
1000	500	3000	1	14.98	27.52	55.50	90.75	134.83	182.79	235.23	292.76	355.10	421.45	491.28
1000	500	3500	1	12.43	24.74	53.22	88.46	131.83	179.32	231.60	289.17	351.69	418.40	488.75
1000	500	4000	1	11.15	23.70	49.05	86.20	129.56	176.60	228.74	286.37	349.05	416.03	486.79
1000	500	4500	1	10.41	22.33	46.80	84.62	127.64	174.35	226.44	284.12	346.91	414.12	485.20
1000	500	5000	1	10.45	22.04	48.29	84.56	126.02	172.42	224.54	282.25	345.15	412.53	483.89
1000	500	5500	1	10.46	22.12	49.23	84.04	124.56	170.81	222.93	280.67	343.65	411.19	482.79
1000	500	6000	1	10.21	21.63	48.83	83.06	123.28	169.45	221.56	279.32	342.37	410.03	481.83
1000	500	6500	1	10.42	21.02	48.30	82.20	122.19	168.27	220.35	278.13	341.25	409.03	480.99
1000	500	7000	1	11.11	21.96	48.32	81.49	121.21	167.24	219.30	277.10	340.26	408.15	480.28
1000	500	7500	1	11.17	22.11	47.93	80.76	120.36	166.36	218.41	276.23	339.47	407.47	479.75
1000	500	8000	1	11.00	21.84	47.40	80.06	119.58	165.52	217.55	275.38	338.65	406.70	479.06
1000	500	9000	1	10.43	20.93	46.25	78.69	117.98	163.73	215.61	273.33	336.52	404.49	476.75
1000	500	10000	1	10.15	20.46	45.46	77.66	116.80	162.44	214.27	271.97	335.18	403.19	475.50

Table B.4 : Numbers of Experimental Data to Update theTable at *q"*=1000 kW/m^2 , *P*=500kPa

q"	P	G	x_a →										
			0	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1000	500	500	8	8	0	9	9	0	0	0	0	0	0	0
1000	500	1000	14	14	0	24	24	0	0	0	0	0	0	0
1000	500	1500	6	6	3	18	15	0	0	0	0	0	0	0
1000	500	2000	0	0	18	18	0	0	0	0	0	0	0	0
1000	500	2500	0	0	15	15	0	0	0	0	0	0	0	0
1000	500	3000	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	3500	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	4000	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	4500	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	5000	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	5500	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	6000	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	6500	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	7000	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	7500	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	8000	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	9000	0	0	0	0	0	0	0	0	0	0	0	0
1000	500	10000	0	0	0	0	0	0	0	0	0	0	0	0

Table B.5 : Color Codes used in the Look-Up Table

	Correlation based data region
	TPM = 1 assigned region
	Updated with experimental data region

APPENDIX C

Table C.1: The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	500	500	1	14.88	31.02	61.78	99.07	140.83	188.20	240.74	297.99	359.67	425.01	493.43
0	500	1000	1	14.05	27.74	51.34	79.39	126.93	179.54	231.91	288.83	351.18	417.75	487.90
0	500	1500	1	14.10	26.87	47.50	73.88	122.05	174.50	226.62	283.68	346.41	413.63	484.71
0	500	2000	1	14.07	27.39	50.19	79.75	122.85	171.17	223.20	280.75	343.68	411.19	482.77
0	500	2500	1	13.35	26.63	51.34	83.27	122.76	168.77	220.91	278.69	341.74	409.45	481.37
0	500	3000	1	11.43	23.53	48.64	81.70	121.24	167.16	219.26	277.04	340.19	408.08	480.25
0	500	3500	1	10.66	21.60	46.88	80.06	119.89	165.88	217.90	275.70	338.93	406.97	479.35
0	500	4000	1	10.54	21.15	46.47	79.27	118.86	164.77	216.77	274.60	337.89	406.05	478.60
0	500	4500	1	10.37	20.90	46.16	78.61	117.98	163.83	215.82	273.66	337.01	405.27	477.97
0	500	5000	1	10.07	20.45	45.64	77.95	117.22	163.01	214.99	272.84	336.25	404.59	477.42
0	500	5500	1	9.78	19.84	45.03	77.35	116.56	162.30	214.26	272.13	335.58	404.00	476.94
0	500	6000	1	9.65	19.62	44.62	76.82	115.96	161.67	213.62	271.50	334.99	403.47	476.51
0	500	6500	1	9.56	19.47	44.26	76.34	115.42	161.10	213.04	270.93	334.45	402.99	476.12
0	500	7000	1	9.51	19.34	43.91	75.91	114.94	160.59	212.51	270.42	333.97	402.57	475.78
0	500	7500	1	9.42	19.16	43.61	75.53	114.53	160.15	212.08	270.00	333.60	402.27	475.58
0	500	8000	1	9.32	18.98	43.31	75.17	114.12	159.72	211.63	269.57	333.18	401.88	475.22
0	500	9000	1	9.06	18.59	42.72	74.39	113.17	158.63	210.43	268.25	331.77	400.34	473.52
0	500	10000	1	8.89	18.29	42.25	73.80	112.49	157.89	209.64	267.45	330.97	399.55	472.73

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	1000	500	1	10.74	23.09	42.82	64.14	87.36	116.37	149.20	184.61	222.83	264.48	309.52
0	1000	1000	1	8.45	19.64	36.96	51.03	76.81	109.81	142.71	177.68	216.00	257.96	303.53
0	1000	1500	1	8.82	18.60	31.58	42.55	72.49	106.55	139.03	173.84	212.24	254.37	300.21
0	1000	2000	1	8.96	18.35	31.53	46.71	74.38	104.99	136.86	171.93	210.47	252.78	298.83
0	1000	2500	1	8.69	18.19	31.26	50.23	75.42	103.76	135.43	170.69	209.31	251.77	298.03
0	1000	3000	1	7.26	16.13	30.07	50.03	74.59	102.69	134.41	169.69	208.37	250.93	297.36
0	1000	3500	1	7.43	15.36	30.04	49.47	73.69	101.83	133.59	168.88	207.60	250.25	296.80
0	1000	4000	1	7.71	15.67	30.26	49.02	73.01	101.14	132.89	168.20	206.97	249.68	296.33
0	1000	4500	1	7.62	15.64	29.93	48.49	72.43	100.54	132.31	167.63	206.43	249.20	295.94
0	1000	5000	1	6.97	14.56	29.12	48.05	72.00	100.05	131.80	167.13	205.97	248.79	295.60
0	1000	5500	1	6.84	14.07	28.72	47.70	71.60	99.62	131.36	166.70	205.56	248.42	295.31
0	1000	6000	1	6.22	12.74	28.01	47.41	71.29	99.23	130.97	166.32	205.20	248.11	295.05
0	1000	6500	1	5.92	12.09	27.60	47.14	70.97	98.89	130.61	165.97	204.88	247.83	294.82
0	1000	7000	1	6.12	12.18	27.39	46.86	70.68	98.58	130.30	165.67	204.60	247.57	294.62
0	1000	7500	1	6.19	12.18	27.17	46.57	70.35	98.22	129.90	165.25	204.16	247.12	294.15
0	1000	8000	1	6.15	12.08	26.99	46.35	70.11	97.95	129.63	164.98	203.90	246.91	294.03
0	1000	9000	1	6.01	11.93	26.90	46.35	70.25	98.30	130.23	165.88	205.17	248.65	296.34
0	1000	10000	1	5.92	11.78	26.68	46.10	70.00	98.08	130.07	165.79	205.19	248.81	296.71
0	2000	500	1	7.31	13.80	24.97	32.53	39.27	49.47	62.94	77.83	93.74	110.50	127.95
0	2000	1000	1	4.78	9.80	20.68	25.96	32.50	45.92	60.84	75.75	91.76	108.76	126.76

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	2000	1500	1	4.77	9.51	16.70	20.45	30.07	44.79	59.64	74.50	90.64	107.83	126.07
0	2000	2000	1	4.84	10.39	15.63	19.50	30.75	44.83	58.83	73.66	89.93	107.28	125.58
0	2000	2500	1	5.25	11.53	15.58	20.78	31.79	44.68	58.18	73.07	89.41	106.84	125.20
0	2000	3000	1	4.60	9.98	15.38	22.00	32.12	44.20	57.67	72.65	89.01	106.48	124.91
0	2000	3500	1	4.23	8.75	14.94	22.15	31.90	43.79	57.31	72.30	88.68	106.19	124.68
0	2000	4000	1	4.26	9.02	14.61	21.58	31.54	43.52	57.02	72.02	88.42	105.95	124.49
0	2000	4500	1	4.39	9.44	14.27	21.14	31.28	43.28	56.78	71.77	88.19	105.75	124.32
0	2000	5000	1	4.30	8.78	13.61	20.91	31.11	43.08	56.56	71.56	87.99	105.57	124.18
0	2000	5500	1	4.82	9.82	14.00	20.85	30.91	42.88	56.37	71.38	87.82	105.42	124.06
0	2000	6000	1	3.68	7.36	13.02	20.72	30.83	42.73	56.20	71.21	87.66	105.28	123.94
0	2000	6500	1	2.89	5.69	12.38	20.64	30.74	42.58	56.05	71.06	87.52	105.16	123.84
0	2000	7000	1	3.08	5.63	12.18	20.51	30.61	42.45	55.91	70.93	87.40	105.05	123.76
0	2000	7500	1	3.25	5.77	12.10	20.39	30.51	42.35	55.82	70.85	87.33	105.00	123.74
0	2000	8000	1	3.24	5.74	12.02	20.30	30.40	42.23	55.70	70.72	87.21	104.89	123.62
0	2000	9000	1	3.13	5.61	11.85	20.04	30.07	41.83	55.22	70.18	86.59	104.20	122.88
0	2000	10000	1	3.09	5.53	11.71	19.87	29.88	41.60	54.98	69.92	86.32	103.92	122.59
0	3000	500	1	5.38	9.90	17.26	22.30	26.34	32.14	40.19	49.58	59.61	69.64	79.29
0	3000	1000	1	3.91	7.08	14.62	19.40	22.91	30.20	39.22	48.74	58.97	69.40	79.82
0	3000	1500	1	3.47	6.92	13.30	16.60	21.08	29.30	38.62	48.21	58.57	69.23	80.02
0	3000	2000	1	3.65	7.57	12.16	13.95	20.21	29.12	38.17	47.70	58.13	68.90	79.77

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a →										
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	3000	2500	1	3.97	8.38	11.79	13.74	20.38	28.98	37.77	47.32	57.79	68.60	79.50
0	3000	3000	1	3.67	7.41	11.43	14.74	20.82	28.69	37.43	47.05	57.53	68.37	79.31
0	3000	3500	1	3.32	6.75	10.88	14.87	20.80	28.44	37.18	46.82	57.32	68.18	79.16
0	3000	4000	1	3.07	6.55	9.95	14.24	20.58	28.30	37.00	46.64	57.15	68.03	79.04
0	3000	4500	1	3.02	6.53	9.50	13.92	20.46	28.18	36.85	46.48	57.00	67.90	78.94
0	3000	5000	1	3.11	6.33	9.23	13.81	20.37	28.05	36.71	46.34	56.87	67.79	78.85
0	3000	5500	1	3.31	6.48	9.22	13.72	20.25	27.93	36.59	46.23	56.76	67.69	78.77
0	3000	6000	1	2.93	5.67	8.89	13.67	20.17	27.82	36.48	46.12	56.66	67.60	78.69
0	3000	6500	1	2.33	4.36	8.46	13.64	20.11	27.72	36.38	46.02	56.57	67.52	78.62
0	3000	7000	1	2.35	4.00	8.18	13.55	20.04	27.63	36.28	45.93	56.48	67.44	78.56
0	3000	7500	1	2.46	4.06	8.12	13.49	20.00	27.61	36.27	45.93	56.51	67.51	78.70
0	3000	8000	1	2.46	4.04	8.06	13.44	19.94	27.55	36.21	45.88	56.47	67.47	78.65
0	3000	9000	1	2.38	3.95	7.85	13.09	19.45	26.90	35.39	44.88	55.25	65.89	76.51
0	3000	10000	1	2.35	3.87	7.73	12.94	19.26	26.67	35.12	44.56	54.88	65.44	75.91
0	4000	500	1	4.61	8.44	15.11	20.55	23.48	26.97	32.86	40.51	48.81	57.42	66.02
0	4000	1000	1	4.73	7.72	14.02	18.43	21.51	25.53	31.71	39.45	47.85	56.68	65.69
0	4000	1500	1	4.34	7.43	14.10	16.43	19.46	24.44	31.11	38.88	47.31	56.22	65.47
0	4000	2000	1	4.13	7.01	11.79	13.53	17.76	23.85	30.80	38.52	46.94	55.89	65.22
0	4000	2500	1	4.09	6.77	11.03	12.81	17.14	23.49	30.54	38.25	46.68	55.65	65.03
0	4000	3000	1	3.70	6.33	9.85	12.57	17.06	23.26	30.29	38.03	46.47	55.46	64.88

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	4000	3500	1	3.26	5.76	8.70	12.07	16.90	23.06	30.10	37.85	46.31	55.31	64.76
0	4000	4000	1	2.90	5.07	7.70	11.54	16.72	22.93	29.96	37.70	46.17	55.19	64.66
0	4000	4500	1	2.66	4.70	7.38	11.38	16.66	22.85	29.83	37.57	46.05	55.09	64.57
0	4000	5000	1	2.87	5.41	7.73	11.36	16.58	22.76	29.72	37.46	45.95	55.00	64.50
0	4000	5500	1	2.92	5.45	7.69	11.28	16.50	22.67	29.62	37.37	45.86	54.92	64.44
0	4000	6000	1	2.66	4.91	7.49	11.23	16.42	22.58	29.54	37.28	45.78	54.85	64.38
0	4000	6500	1	2.21	3.90	7.07	11.18	16.38	22.50	29.46	37.21	45.70	54.78	64.32
0	4000	7000	1	2.12	3.45	6.80	11.11	16.33	22.43	29.38	37.13	45.63	54.72	64.27
0	4000	7500	1	2.18	3.47	6.76	11.06	16.28	22.39	29.34	37.09	45.60	54.70	64.29
0	4000	8000	1	2.18	3.45	6.72	11.01	16.23	22.34	29.29	37.05	45.57	54.68	64.27
0	4000	9000	1	2.14	3.44	6.63	10.86	16.03	22.08	28.98	36.69	45.15	54.14	63.51
0	4000	10000	1	2.11	3.38	6.55	10.77	15.92	21.96	28.85	36.55	44.99	53.96	63.27
0	5000	500	1	3.38	5.91	10.63	14.78	18.10	21.68	26.76	32.98	39.66	46.78	54.24
0	5000	1000	1	3.97	6.78	12.33	15.94	17.90	20.58	25.55	31.92	38.68	45.91	53.51
0	5000	1500	1	3.88	6.59	12.06	14.98	16.91	19.91	25.01	31.36	38.14	45.42	53.11
0	5000	2000	1	3.56	5.82	10.16	13.04	15.86	19.65	24.81	31.04	37.84	45.16	52.90
0	5000	2500	1	3.35	5.24	8.97	11.78	15.04	19.30	24.62	30.83	37.63	44.96	52.75
0	5000	3000	1	3.24	5.05	8.11	10.96	14.39	18.93	24.45	30.68	37.47	44.80	52.63
0	5000	3500	1	3.10	4.80	7.29	10.26	13.87	18.61	24.30	30.56	37.34	44.67	52.54
0	5000	4000	1	2.64	4.10	6.59	9.76	13.56	18.43	24.20	30.45	37.23	44.56	52.46

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	5000	4500	1	2.25	3.57	6.13	9.47	13.51	18.46	24.12	30.34	37.13	44.49	52.39
0	5000	5000	1	2.23	3.69	6.06	9.32	13.53	18.47	24.04	30.24	37.05	44.42	52.33
0	5000	5500	1	2.22	3.69	6.01	9.27	13.48	18.39	23.96	30.16	36.97	44.36	52.27
0	5000	6000	1	2.12	3.55	5.96	9.25	13.43	18.32	23.89	30.10	36.91	44.30	52.23
0	5000	6500	1	1.95	3.18	5.79	9.21	13.38	18.26	23.83	30.04	36.85	44.25	52.19
0	5000	7000	1	1.94	3.02	5.69	9.14	13.31	18.20	23.77	29.98	36.80	44.20	52.15
0	5000	7500	1	1.97	3.03	5.66	9.08	13.26	18.15	23.72	29.93	36.75	44.16	52.11
0	5000	8000	1	1.96	3.01	5.62	9.03	13.21	18.11	23.67	29.88	36.71	44.12	52.08
0	5000	9000	1	1.93	2.98	5.58	8.97	13.14	18.03	23.60	29.81	36.64	44.07	52.05
0	5000	10000	1	1.91	2.95	5.53	8.91	13.08	17.96	23.52	29.74	36.58	44.01	52.01
0	6000	500	1	2.42	3.81	6.69	9.88	13.40	17.36	21.91	26.90	32.26	37.99	44.04
0	6000	1000	1	3.09	5.40	10.05	12.96	14.22	16.54	20.86	26.03	31.47	37.29	43.45
0	6000	1500	1	3.00	5.26	9.95	12.83	13.98	16.18	20.38	25.56	31.03	36.90	43.12
0	6000	2000	1	2.62	4.40	8.45	11.55	13.61	16.24	20.23	25.28	30.78	36.71	42.95
0	6000	2500	1	2.41	3.87	7.25	10.33	12.99	16.03	20.08	25.10	30.61	36.56	42.82
0	6000	3000	1	2.54	3.93	6.69	9.49	12.28	15.65	19.95	24.98	30.48	36.42	42.73
0	6000	3500	1	2.63	3.96	6.30	8.86	11.67	15.31	19.84	24.90	30.38	36.31	42.66
0	6000	4000	1	2.30	3.49	5.80	8.40	11.30	15.12	19.75	24.81	30.29	36.22	42.59
0	6000	4500	1	1.94	2.98	5.28	8.01	11.17	15.12	19.70	24.72	30.21	36.16	42.54
0	6000	5000	1	1.81	2.72	4.94	7.77	11.16	15.13	19.64	24.65	30.15	36.10	42.49

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	6000	5500	1	1.80	2.68	4.87	7.72	11.12	15.07	19.57	24.59	30.09	36.05	42.45
0	6000	6000	1	1.78	2.70	4.88	7.73	11.08	15.01	19.51	24.53	30.04	36.00	42.41
0	6000	6500	1	1.77	2.70	4.87	7.69	11.03	14.96	19.46	24.48	29.99	35.96	42.37
0	6000	7000	1	1.80	2.69	4.83	7.61	10.97	14.92	19.42	24.44	29.95	35.93	42.35
0	6000	7500	1	1.80	2.67	4.79	7.56	10.93	14.88	19.38	24.40	29.91	35.89	42.32
0	6000	8000	1	1.79	2.65	4.76	7.51	10.89	14.84	19.34	24.36	29.87	35.86	42.29
0	6000	9000	1	1.77	2.62	4.71	7.46	10.83	14.78	19.27	24.30	29.82	35.81	42.26
0	6000	10000	1	1.75	2.59	4.68	7.41	10.77	14.72	19.22	24.24	29.76	35.77	42.23
0	7000	500	1	2.15	3.19	5.36	7.98	11.04	14.52	18.32	22.43	26.86	31.58	36.56
0	7000	1000	1	2.30	3.90	7.37	9.77	11.22	13.77	17.52	21.73	26.21	31.01	36.10
0	7000	1500	1	2.17	3.70	7.18	9.62	10.98	13.41	17.13	21.35	25.85	30.69	35.84
0	7000	2000	1	1.92	3.14	6.20	8.89	10.91	13.41	16.94	21.12	25.65	30.52	35.69
0	7000	2500	1	1.83	2.85	5.43	8.15	10.62	13.30	16.81	20.97	25.51	30.40	35.59
0	7000	3000	1	1.95	2.96	5.12	7.58	10.15	13.09	16.70	20.86	25.40	30.30	35.51
0	7000	3500	1	2.07	3.13	5.08	7.29	9.79	12.90	16.62	20.77	25.31	30.21	35.45
0	7000	4000	1	1.97	2.96	4.86	7.07	9.60	12.78	16.54	20.70	25.24	30.14	35.39
0	7000	4500	1	1.80	2.68	4.53	6.82	9.50	12.74	16.48	20.63	25.17	30.09	35.35
0	7000	5000	1	1.70	2.46	4.30	6.65	9.43	12.70	16.42	20.57	25.12	30.04	35.31
0	7000	5500	1	1.66	2.40	4.26	6.64	9.40	12.65	16.37	20.52	25.07	30.00	35.27
0	7000	6000	1	1.65	2.40	4.24	6.63	9.37	12.60	16.32	20.47	25.03	29.96	35.24

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	7000	6500	1	1.66	2.43	4.23	6.59	9.33	12.56	16.28	20.43	24.99	29.92	35.22
0	7000	7000	1	1.67	2.43	4.22	6.57	9.29	12.52	16.24	20.40	24.95	29.89	35.19
0	7000	7500	1	1.67	2.40	4.18	6.49	9.24	12.49	16.21	20.36	24.92	29.87	35.17
0	7000	8000	1	1.66	2.38	4.14	6.42	9.20	12.46	16.18	20.33	24.89	29.84	35.15
0	7000	9000	1	1.65	2.35	4.08	6.35	9.14	12.40	16.12	20.27	24.84	29.79	35.10
0	7000	10000	1	1.63	2.33	4.05	6.32	9.10	12.36	16.07	20.22	24.79	29.74	35.06
0	8000	500	1	2.03	2.97	4.84	7.04	9.56	12.42	15.58	19.04	22.77	26.74	30.93
0	8000	1000	1	1.86	2.77	4.86	6.82	8.90	11.74	14.99	18.46	22.22	26.25	30.54
0	8000	1500	1	1.73	2.54	4.48	6.41	8.50	11.40	14.67	18.14	21.92	25.98	30.32
0	8000	2000	1	1.65	2.35	4.09	6.17	8.51	11.30	14.47	17.95	21.75	25.83	30.19
0	8000	2500	1	1.61	2.28	3.91	6.06	8.49	11.21	14.34	17.83	21.63	25.73	30.11
0	8000	3000	1	1.64	2.33	3.90	5.95	8.36	11.11	14.24	17.73	21.53	25.65	30.04
0	8000	3500	1	1.73	2.52	4.04	5.94	8.27	11.03	14.16	17.64	21.46	25.58	29.99
0	8000	4000	1	1.76	2.58	4.08	5.96	8.24	10.96	14.09	17.58	21.40	25.52	29.94
0	8000	4500	1	1.68	2.43	3.92	5.88	8.19	10.90	14.03	17.52	21.34	25.48	29.90
0	8000	5000	1	1.60	2.27	3.77	5.78	8.12	10.85	13.98	17.47	21.30	25.44	29.87
0	8000	5500	1	1.57	2.22	3.76	5.77	8.09	10.81	13.94	17.43	21.26	25.40	29.84
0	8000	6000	1	1.57	2.22	3.75	5.74	8.05	10.77	13.90	17.39	21.22	25.37	29.81
0	8000	6500	1	1.57	2.22	3.73	5.71	8.02	10.74	13.87	17.36	21.19	25.34	29.79
0	8000	7000	1	1.57	2.21	3.73	5.72	8.00	10.70	13.83	17.33	21.16	25.31	29.77

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	8000	7500	1	1.56	2.19	3.70	5.66	7.96	10.68	13.81	17.30	21.13	25.29	29.75
0	8000	8000	1	1.56	2.17	3.66	5.59	7.91	10.65	13.78	17.27	21.11	25.27	29.73
0	8000	9000	1	1.54	2.15	3.61	5.51	7.86	10.61	13.73	17.22	21.06	25.23	29.70
0	8000	10000	1	1.53	2.13	3.59	5.49	7.82	10.57	13.69	17.18	21.02	25.19	29.67
0	9000	500	1	1.90	2.71	4.33	6.20	8.34	10.74	13.42	16.36	19.53	22.91	26.48
0	9000	1000	1	1.76	2.44	3.83	5.24	7.12	9.97	12.94	15.89	19.07	22.48	26.14
0	9000	1500	1	1.58	2.13	3.32	4.62	6.58	9.62	12.67	15.62	18.81	22.25	25.96
0	9000	2000	1	1.46	1.93	3.09	4.73	6.96	9.70	12.50	15.44	18.66	22.13	25.85
0	9000	2500	1	1.40	1.87	3.14	5.09	7.32	9.72	12.37	15.32	18.56	22.05	25.77
0	9000	3000	1	1.40	1.90	3.25	5.20	7.31	9.62	12.27	15.24	18.48	21.98	25.72
0	9000	3500	1	1.52	2.10	3.44	5.19	7.21	9.54	12.21	15.17	18.41	21.92	25.67
0	9000	4000	1	1.55	2.17	3.50	5.20	7.18	9.48	12.15	15.11	18.36	21.87	25.63
0	9000	4500	1	1.46	2.01	3.25	5.09	7.15	9.45	12.10	15.06	18.32	21.83	25.60
0	9000	5000	1	1.42	1.94	3.16	5.01	7.08	9.40	12.06	15.02	18.28	21.80	25.57
0	9000	5500	1	1.44	1.97	3.28	5.03	7.04	9.36	12.02	14.99	18.24	21.77	25.55
0	9000	6000	1	1.44	1.98	3.34	5.01	7.00	9.33	11.99	14.95	18.21	21.74	25.52
0	9000	6500	1	1.44	1.98	3.31	5.00	6.97	9.30	11.96	14.93	18.18	21.71	25.50
0	9000	7000	1	1.43	1.97	3.31	5.05	6.97	9.27	11.93	14.90	18.16	21.69	25.49
0	9000	7500	1	1.44	1.97	3.29	5.00	6.94	9.24	11.90	14.87	18.14	21.67	25.47
0	9000	8000	1	1.45	1.98	3.27	4.93	6.90	9.22	11.88	14.85	18.12	21.66	25.45

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	9000	9000	1	1.44	1.98	3.24	4.85	6.84	9.18	11.85	14.81	18.08	21.62	25.43
0	9000	10000	1	1.44	1.96	3.22	4.84	6.82	9.15	11.81	14.78	18.05	21.59	25.40
0	10000	500	1	1.78	2.49	3.89	5.51	7.34	9.38	11.68	14.21	16.93	19.84	22.91
0	10000	1000	1	1.71	2.31	3.39	4.57	6.22	8.70	11.27	13.80	16.53	19.46	22.61
0	10000	1500	1	1.54	2.01	2.93	4.00	5.72	8.39	11.03	13.57	16.31	19.26	22.45
0	10000	2000	1	1.41	1.82	2.77	4.15	6.09	8.49	10.89	13.41	16.18	19.16	22.35
0	10000	2500	1	1.34	1.75	2.86	4.55	6.47	8.52	10.77	13.31	16.09	19.09	22.29
0	10000	3000	1	1.35	1.77	2.97	4.68	6.47	8.42	10.69	13.24	16.02	19.03	22.24
0	10000	3500	1	1.45	1.93	3.11	4.65	6.36	8.34	10.63	13.18	15.97	18.98	22.20
0	10000	4000	1	1.46	1.96	3.13	4.62	6.31	8.29	10.58	13.13	15.92	18.94	22.17
0	10000	4500	1	1.37	1.82	2.90	4.51	6.29	8.26	10.54	13.09	15.88	18.90	22.14
0	10000	5000	1	1.36	1.79	2.84	4.45	6.24	8.23	10.51	13.05	15.85	18.87	22.12
0	10000	5500	1	1.38	1.84	2.97	4.46	6.19	8.19	10.48	13.02	15.82	18.85	22.09
0	10000	6000	1	1.38	1.85	3.03	4.44	6.15	8.16	10.45	12.99	15.79	18.82	22.08
0	10000	6500	1	1.38	1.84	3.00	4.44	6.13	8.14	10.42	12.97	15.77	18.80	22.06
0	10000	7000	1	1.38	1.83	2.99	4.48	6.14	8.11	10.40	12.95	15.75	18.78	22.04
0	10000	7500	1	1.39	1.85	2.97	4.44	6.12	8.09	10.38	12.93	15.73	18.77	22.03
0	10000	8000	1	1.39	1.85	2.96	4.39	6.08	8.07	10.36	12.91	15.71	18.75	22.02
0	10000	9000	1	1.37	1.84	2.95	4.33	6.03	8.04	10.33	12.88	15.68	18.73	22.00
0	10000	10000	1	1.38	1.83	2.92	4.31	6.01	8.01	10.30	12.85	15.65	18.70	21.98

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
0	11000	500	1	1.67	2.28	3.47	4.84	6.42	8.22	10.20	12.38	14.72	17.22	19.85
0	11000	1000	1	1.57	2.11	3.24	4.56	6.08	7.85	9.83	12.00	14.37	16.91	19.60
0	11000	1500	1	1.51	2.01	3.09	4.37	5.88	7.64	9.62	11.80	14.17	16.73	19.45
0	11000	2000	1	1.48	1.95	2.99	4.26	5.77	7.52	9.50	11.68	14.07	16.64	19.38
0	11000	2500	1	1.46	1.92	2.92	4.19	5.70	7.45	9.42	11.61	13.99	16.57	19.32
0	11000	3000	1	1.45	1.89	2.88	4.13	5.64	7.38	9.36	11.54	13.93	16.52	19.28
0	11000	3500	1	1.44	1.87	2.85	4.09	5.59	7.33	9.30	11.49	13.89	16.48	19.25
0	11000	4000	1	1.43	1.86	2.83	4.06	5.56	7.29	9.26	11.45	13.85	16.44	19.22
0	11000	4500	1	1.42	1.85	2.81	4.03	5.52	7.26	9.22	11.41	13.81	16.41	19.19
0	11000	5000	1	1.42	1.83	2.79	4.01	5.49	7.22	9.19	11.38	13.78	16.38	19.17
0	11000	5500	1	1.41	1.82	2.77	3.99	5.47	7.20	9.16	11.35	13.76	16.36	19.15
0	11000	6000	1	1.40	1.81	2.75	3.97	5.44	7.17	9.14	11.33	13.74	16.34	19.14
0	11000	6500	1	1.40	1.80	2.73	3.95	5.42	7.15	9.12	11.31	13.72	16.32	19.12
0	11000	7000	1	1.39	1.79	2.72	3.93	5.41	7.13	9.10	11.29	13.70	16.31	19.11
0	11000	7500	1	1.39	1.79	2.71	3.91	5.39	7.11	9.08	11.27	13.68	16.29	19.10
0	11000	8000	1	1.38	1.77	2.70	3.90	5.37	7.10	9.06	11.26	13.67	16.28	19.09
0	11000	9000	1	1.35	1.74	2.69	3.88	5.34	7.07	9.03	11.23	13.64	16.25	19.06
0	11000	10000	1	1.35	1.73	2.66	3.86	5.32	7.04	9.01	11.20	13.61	16.23	19.05
10	500	500	1	14.16	30.54	58.63	97.81	140.61	189.33	242.35	299.56	361.13	426.26	494.42

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
10	500	1000	1	13.93	28.14	50.24	75.98	124.79	179.90	233.14	289.76	352.01	418.45	488.48
10	500	1500	1	14.31	27.28	45.87	69.56	119.87	174.85	227.56	284.28	346.94	414.08	485.09
10	500	2000	1	13.95	27.79	49.54	77.02	121.70	171.63	223.80	281.18	344.08	411.56	483.06
10	500	2500	1	13.06	26.81	51.03	82.98	122.93	169.14	221.26	279.04	342.08	409.76	481.61
10	500	3000	1	10.36	23.47	48.18	81.89	121.61	167.45	219.54	277.34	340.48	408.34	480.46
10	500	3500	1	10.30	21.39	46.45	80.05	120.12	166.14	218.17	275.96	339.18	407.20	479.53
10	500	4000	1	10.47	21.08	46.27	79.30	119.05	165.00	217.00	274.83	338.12	406.25	478.77
10	500	4500	1	10.28	20.85	46.17	78.73	118.16	164.02	216.02	273.86	337.21	405.45	478.12
10	500	5000	1	9.91	20.24	45.71	78.10	117.38	163.19	215.18	273.03	336.43	404.76	477.56
10	500	5500	1	9.47	19.36	45.03	77.49	116.71	162.47	214.43	272.31	335.75	404.15	477.07
10	500	6000	1	9.38	19.23	44.63	76.96	116.10	161.82	213.77	271.66	335.14	403.61	476.63
10	500	6500	1	9.41	19.28	44.31	76.47	115.56	161.24	213.18	271.08	334.59	403.13	476.23
10	500	7000	1	9.51	19.36	43.99	76.02	115.06	160.72	212.65	270.56	334.10	402.69	475.88
10	500	7500	1	9.46	19.22	43.68	75.63	114.64	160.28	212.21	270.14	333.73	402.39	475.68
10	500	8000	1	9.36	19.04	43.39	75.26	114.23	159.84	211.76	269.69	333.30	401.99	475.31
10	500	9000	1	9.08	18.62	42.78	74.46	113.26	158.73	210.53	268.35	331.87	400.44	473.60
10	500	10000	1	8.91	18.32	42.30	73.86	112.57	157.97	209.73	267.54	331.06	399.63	472.80
10	1000	500	1	10.71	22.97	42.84	64.40	88.22	117.25	150.16	185.61	223.76	265.26	310.13
10	1000	1000	1	8.38	19.69	36.98	51.69	76.58	109.60	143.33	178.28	216.50	258.34	303.86
10	1000	1500	1	8.58	18.58	32.20	41.99	70.76	106.16	139.53	174.24	212.54	254.57	300.39

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
10	1000	2000	1	8.90	18.28	31.58	45.33	72.42	104.86	137.25	172.21	210.68	252.94	298.96
10	1000	2500	1	8.54	18.02	31.42	49.60	74.92	103.90	135.68	170.88	209.48	251.90	298.12
10	1000	3000	1	7.28	15.91	30.16	49.84	74.59	102.86	134.58	169.84	208.51	251.04	297.42
10	1000	3500	1	7.30	15.35	30.25	49.43	73.74	101.97	133.73	169.00	207.72	250.33	296.85
10	1000	4000	1	7.69	15.67	30.68	49.15	73.07	101.25	133.02	168.31	207.06	249.75	296.37
10	1000	4500	1	7.66	15.68	30.63	48.72	72.48	100.64	132.41	167.72	206.51	249.26	295.97
10	1000	5000	1	7.12	14.81	29.76	48.22	72.05	100.13	131.89	167.21	206.03	248.83	295.62
10	1000	5500	1	6.89	14.19	29.23	47.88	71.65	99.69	131.44	166.77	205.61	248.46	295.32
10	1000	6000	1	6.24	12.83	28.21	47.50	71.35	99.30	131.04	166.38	205.25	248.14	295.06
10	1000	6500	1	5.88	12.05	27.60	47.20	71.05	98.95	130.68	166.03	204.92	247.85	294.82
10	1000	7000	1	6.03	12.07	27.41	46.92	70.74	98.64	130.36	165.72	204.63	247.59	294.61
10	1000	7500	1	6.20	12.19	27.21	46.63	70.41	98.27	129.96	165.29	204.19	247.13	294.15
10	1000	8000	1	6.17	12.10	27.03	46.40	70.16	98.00	129.68	165.01	203.93	246.92	294.01
10	1000	9000	1	6.03	11.95	26.93	46.39	70.29	98.33	130.26	165.90	205.19	248.65	296.33
10	1000	10000	1	5.94	11.80	26.71	46.13	70.04	98.11	130.10	165.81	205.20	248.81	296.69
10	2000	500	1	7.25	13.84	24.97	32.81	39.99	50.32	63.46	78.29	94.17	110.87	128.24
10	2000	1000	1	4.53	9.80	20.59	26.34	32.45	45.77	61.07	76.10	92.03	108.93	126.94
10	2000	1500	1	4.46	9.39	16.85	20.17	29.50	44.45	59.83	74.78	90.82	107.92	126.20
10	2000	2000	1	4.81	10.32	15.84	19.15	29.87	44.54	59.02	73.85	90.07	107.36	125.69
10	2000	2500	1	5.12	11.44	16.71	20.58	31.26	44.67	58.35	73.19	89.52	106.93	125.29

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
10	2000	3000	1	4.61	10.09	15.76	22.24	32.10	44.30	57.78	72.74	89.10	106.57	124.98
10	2000	3500	1	4.24	8.74	15.28	22.48	32.03	43.87	57.38	72.38	88.76	106.26	124.74
10	2000	4000	1	4.23	8.96	15.23	21.99	31.61	43.56	57.09	72.09	88.49	106.02	124.54
10	2000	4500	1	4.43	9.45	15.13	21.48	31.29	43.33	56.84	71.84	88.25	105.81	124.37
10	2000	5000	1	4.60	9.75	14.43	21.07	31.10	43.12	56.62	71.62	88.05	105.63	124.23
10	2000	5500	1	5.11	10.28	15.34	21.16	30.89	42.91	56.43	71.44	87.87	105.47	124.10
10	2000	6000	1	3.84	7.66	13.75	20.90	30.84	42.76	56.25	71.26	87.71	105.33	123.98
10	2000	6500	1	2.95	5.77	12.47	20.69	30.78	42.62	56.10	71.11	87.57	105.20	123.88
10	2000	7000	1	2.96	5.47	12.17	20.54	30.65	42.49	55.96	70.98	87.44	105.09	123.79
10	2000	7500	1	3.23	5.76	12.12	20.43	30.55	42.39	55.86	70.89	87.37	105.05	123.78
10	2000	8000	1	3.24	5.74	12.03	20.32	30.44	42.27	55.74	70.77	87.25	104.93	123.65
10	2000	9000	1	3.15	5.63	11.87	20.06	30.10	41.86	55.26	70.21	86.62	104.23	122.90
10	2000	10000	1	3.10	5.54	11.73	19.89	29.90	41.63	55.01	69.95	86.35	103.94	122.62
10	3000	500	1	5.39	9.90	17.29	22.41	26.51	32.88	40.72	49.91	59.92	69.93	79.54
10	3000	1000	1	3.83	7.15	14.65	19.45	22.87	30.34	39.53	49.01	59.17	69.56	79.99
10	3000	1500	1	3.44	6.96	13.29	16.56	21.16	29.27	38.80	48.41	58.71	69.33	80.15
10	3000	2000	1	3.65	7.56	12.34	13.88	19.96	29.03	38.31	47.85	58.25	68.99	79.88
10	3000	2500	1	3.92	8.34	12.40	13.79	20.05	28.96	37.90	47.42	57.88	68.69	79.60
10	3000	3000	1	3.67	7.69	11.58	14.98	20.84	28.76	37.51	47.13	57.61	68.45	79.40
10	3000	3500	1	3.29	6.73	11.13	15.31	20.96	28.50	37.24	46.90	57.40	68.26	79.25

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
10	3000	4000	1	3.06	6.46	10.53	14.68	20.67	28.34	37.06	46.71	57.22	68.11	79.12
10	3000	4500	1	3.02	6.48	10.18	14.22	20.49	28.22	36.91	46.54	57.07	67.97	79.01
10	3000	5000	1	3.12	6.53	9.90	13.98	20.36	28.09	36.76	46.40	56.94	67.86	78.92
10	3000	5500	1	3.34	7.11	9.95	13.87	20.23	27.96	36.64	46.28	56.82	67.76	78.84
10	3000	6000	1	2.99	5.81	9.60	13.84	20.17	27.85	36.53	46.17	56.72	67.66	78.76
10	3000	6500	1	2.37	4.38	8.66	13.71	20.14	27.76	36.42	46.07	56.62	67.58	78.69
10	3000	7000	1	2.29	3.94	8.18	13.58	20.08	27.67	36.33	45.98	56.53	67.50	78.62
10	3000	7500	1	2.45	4.05	8.12	13.52	20.04	27.64	36.31	45.98	56.56	67.57	78.76
10	3000	8000	1	2.46	4.04	8.07	13.46	19.98	27.58	36.25	45.93	56.52	67.53	78.71
10	3000	9000	1	2.39	3.97	7.88	13.11	19.48	26.93	35.43	44.92	55.30	65.95	76.57
10	3000	10000	1	2.35	3.88	7.75	12.96	19.29	26.70	35.16	44.61	54.93	65.50	75.98
10	4000	500	1	4.58	8.28	14.90	20.71	23.77	27.51	33.22	40.75	49.05	57.67	66.21
10	4000	1000	1	4.74	7.73	13.92	18.17	21.40	25.76	31.94	39.59	47.99	56.82	65.81
10	4000	1500	1	4.30	7.41	14.02	16.39	19.39	24.52	31.23	38.97	47.40	56.32	65.55
10	4000	2000	1	4.09	6.95	12.28	13.12	17.54	23.87	30.89	38.60	47.02	55.96	65.29
10	4000	2500	1	4.03	6.66	11.30	12.82	16.97	23.48	30.61	38.32	46.75	55.70	65.09
10	4000	3000	1	3.69	6.41	9.99	12.75	17.07	23.28	30.35	38.08	46.53	55.51	64.93
10	4000	3500	1	3.24	5.74	8.91	12.29	16.98	23.12	30.15	37.90	46.35	55.36	64.81
10	4000	4000	1	2.85	5.17	7.99	11.67	16.78	22.99	30.00	37.74	46.21	55.23	64.70
10	4000	4500	1	2.66	4.75	7.65	11.46	16.69	22.90	29.87	37.61	46.09	55.13	64.62

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
10	4000	5000	1	2.85	5.54	8.30	11.50	16.58	22.79	29.76	37.50	45.98	55.03	64.54
10	4000	5500	1	2.98	5.69	8.34	11.42	16.49	22.69	29.66	37.40	45.89	54.95	64.48
10	4000	6000	1	2.71	5.00	7.98	11.36	16.43	22.60	29.57	37.31	45.81	54.88	64.41
10	4000	6500	1	2.24	3.96	7.24	11.23	16.40	22.53	29.49	37.24	45.73	54.81	64.36
10	4000	7000	1	2.08	3.44	6.81	11.13	16.35	22.46	29.41	37.16	45.67	54.75	64.31
10	4000	7500	1	2.17	3.46	6.76	11.07	16.30	22.41	29.36	37.12	45.63	54.74	64.32
10	4000	8000	1	2.18	3.44	6.72	11.03	16.25	22.36	29.31	37.07	45.59	54.71	64.30
10	4000	9000	1	2.15	3.46	6.65	10.87	16.04	22.10	29.01	36.72	45.18	54.17	63.54
10	4000	10000	1	2.12	3.38	6.56	10.78	15.94	21.98	28.87	36.57	45.02	53.99	63.31
10	5000	500	1	3.64	6.38	11.41	16.01	19.03	22.14	26.96	33.14	39.85	46.98	54.37
10	5000	1000	1	4.09	6.69	11.81	15.41	17.90	20.88	25.75	32.02	38.79	46.02	53.59
10	5000	1500	1	3.87	6.36	11.58	14.05	16.48	20.01	25.16	31.43	38.21	45.48	53.16
10	5000	2000	1	3.50	5.58	9.60	12.17	15.39	19.59	24.90	31.12	37.90	45.19	52.94
10	5000	2500	1	3.30	5.01	8.62	11.25	14.71	19.23	24.69	30.90	37.68	44.99	52.78
10	5000	3000	1	3.12	4.83	7.81	10.60	14.23	18.94	24.51	30.73	37.51	44.83	52.66
10	5000	3500	1	2.92	4.60	7.04	10.05	13.85	18.70	24.37	30.59	37.37	44.70	52.56
10	5000	4000	1	2.57	4.06	6.44	9.65	13.61	18.55	24.25	30.47	37.26	44.60	52.48
10	5000	4500	1	2.31	3.69	6.13	9.43	13.56	18.53	24.16	30.36	37.16	44.51	52.41
10	5000	5000	1	2.37	4.03	6.24	9.35	13.54	18.49	24.06	30.27	37.07	44.44	52.34
10	5000	5500	1	2.37	4.02	6.21	9.30	13.49	18.41	23.98	30.19	37.00	44.38	52.29

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
10	5000	6000	1	2.23	3.81	6.11	9.29	13.44	18.34	23.91	30.12	36.93	44.32	52.24
10	5000	6500	1	1.97	3.27	5.85	9.23	13.39	18.28	23.85	30.06	36.87	44.26	52.20
10	5000	7000	1	1.92	3.01	5.70	9.15	13.33	18.22	23.79	30.00	36.82	44.22	52.16
10	5000	7500	1	1.97	3.02	5.66	9.09	13.27	18.17	23.74	29.95	36.77	44.17	52.13
10	5000	8000	1	1.97	3.01	5.63	9.05	13.23	18.12	23.69	29.90	36.73	44.13	52.10
10	5000	9000	1	1.94	3.00	5.59	8.99	13.15	18.04	23.61	29.82	36.66	44.08	52.06
10	5000	10000	1	1.92	2.95	5.53	8.92	13.09	17.97	23.54	29.75	36.59	44.02	52.02
10	6000	500	1	2.57	4.10	7.13	10.35	13.78	17.58	22.07	27.07	32.43	38.13	44.15
10	6000	1000	1	2.87	4.87	8.84	11.72	13.70	16.64	21.05	26.14	31.56	37.37	43.52
10	6000	1500	1	2.72	4.59	8.53	11.31	13.24	16.17	20.55	25.64	31.09	36.95	43.17
10	6000	2000	1	2.39	3.90	7.33	10.41	12.99	16.12	20.32	25.35	30.83	36.73	42.98
10	6000	2500	1	2.22	3.49	6.50	9.57	12.56	15.91	20.14	25.17	30.66	36.58	42.86
10	6000	3000	1	2.31	3.54	6.09	8.94	12.01	15.62	20.01	25.03	30.52	36.44	42.76
10	6000	3500	1	2.39	3.64	5.88	8.53	11.57	15.37	19.89	24.93	30.41	36.33	42.68
10	6000	4000	1	2.19	3.34	5.55	8.22	11.31	15.21	19.80	24.83	30.32	36.25	42.61
10	6000	4500	1	1.96	2.99	5.19	7.94	11.21	15.18	19.73	24.74	30.24	36.18	42.55
10	6000	5000	1	1.86	2.82	4.95	7.77	11.18	15.15	19.66	24.67	30.17	36.12	42.50
10	6000	5500	1	1.82	2.76	4.89	7.74	11.14	15.09	19.59	24.61	30.11	36.07	42.46
10	6000	6000	1	1.78	2.75	4.89	7.74	11.10	15.03	19.53	24.55	30.05	36.02	42.42
10	6000	6500	1	1.75	2.71	4.88	7.70	11.05	14.98	19.48	24.50	30.01	35.98	42.39

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
10	6000	7000	1	1.79	2.69	4.84	7.63	10.99	14.93	19.43	24.45	29.96	35.94	42.36
10	6000	7500	1	1.81	2.68	4.80	7.57	10.94	14.89	19.39	24.41	29.92	35.91	42.33
10	6000	8000	1	1.80	2.66	4.77	7.53	10.90	14.85	19.35	24.37	29.89	35.87	42.30
10	6000	9000	1	1.77	2.62	4.72	7.47	10.84	14.79	19.29	24.31	29.83	35.82	42.27
10	6000	10000	1	1.76	2.60	4.68	7.42	10.78	14.73	19.23	24.25	29.77	35.78	42.24
10	7000	500	1	2.17	3.23	5.39	7.99	11.10	14.64	18.47	22.58	27.00	31.70	36.66
10	7000	1000	1	2.15	3.50	6.35	8.70	10.76	13.82	17.66	21.82	26.29	31.08	36.16
10	7000	1500	1	2.01	3.24	5.99	8.39	10.42	13.43	17.24	21.41	25.91	30.73	35.88
10	7000	2000	1	1.84	2.83	5.33	7.99	10.45	13.37	17.02	21.17	25.69	30.55	35.72
10	7000	2500	1	1.77	2.65	4.87	7.59	10.30	13.24	16.86	21.02	25.54	30.42	35.62
10	7000	3000	1	1.87	2.77	4.76	7.24	9.97	13.07	16.75	20.90	25.43	30.32	35.53
10	7000	3500	1	2.01	3.00	4.84	7.07	9.72	12.92	16.65	20.80	25.34	30.23	35.47
10	7000	4000	1	1.98	2.94	4.73	6.93	9.58	12.82	16.57	20.72	25.26	30.16	35.41
10	7000	4500	1	1.83	2.72	4.50	6.76	9.50	12.77	16.50	20.65	25.19	30.10	35.36
10	7000	5000	1	1.70	2.47	4.29	6.65	9.45	12.72	16.44	20.59	25.14	30.06	35.32
10	7000	5500	1	1.65	2.39	4.26	6.66	9.42	12.67	16.39	20.54	25.09	30.01	35.29
10	7000	6000	1	1.63	2.39	4.24	6.65	9.39	12.62	16.34	20.49	25.04	29.97	35.26
10	7000	6500	1	1.65	2.43	4.24	6.61	9.35	12.58	16.30	20.45	25.00	29.94	35.23
10	7000	7000	1	1.67	2.44	4.23	6.58	9.31	12.54	16.26	20.41	24.97	29.91	35.20
10	7000	7500	1	1.67	2.41	4.19	6.50	9.26	12.50	16.22	20.38	24.94	29.88	35.18

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
10	7000	8000	1	1.67	2.39	4.15	6.44	9.21	12.47	16.19	20.35	24.91	29.85	35.16
10	7000	9000	1	1.65	2.36	4.09	6.36	9.15	12.41	16.13	20.28	24.85	29.80	35.11
10	7000	10000	1	1.64	2.34	4.06	6.32	9.10	12.37	16.08	20.23	24.80	29.75	35.07
10	8000	500	1	2.07	3.01	4.89	7.10	9.66	12.54	15.71	19.16	22.89	26.85	31.02
10	8000	1000	1	1.88	2.75	4.64	6.53	8.74	11.76	15.07	18.54	22.29	26.31	30.59
10	8000	1500	1	1.74	2.48	4.19	6.04	8.29	11.39	14.72	18.19	21.96	26.02	30.35
10	8000	2000	1	1.64	2.29	3.88	5.92	8.38	11.32	14.52	17.99	21.78	25.86	30.22
10	8000	2500	1	1.60	2.24	3.80	5.95	8.46	11.24	14.38	17.86	21.66	25.76	30.13
10	8000	3000	1	1.64	2.33	3.86	5.93	8.37	11.14	14.27	17.75	21.56	25.67	30.06
10	8000	3500	1	1.80	2.60	4.06	5.93	8.27	11.04	14.18	17.67	21.48	25.60	30.00
10	8000	4000	1	1.84	2.67	4.10	5.93	8.23	10.98	14.11	17.60	21.42	25.54	29.96
10	8000	4500	1	1.71	2.45	3.90	5.84	8.19	10.92	14.05	17.54	21.36	25.49	29.92
10	8000	5000	1	1.59	2.25	3.73	5.75	8.13	10.87	14.00	17.49	21.31	25.45	29.88
10	8000	5500	1	1.54	2.19	3.74	5.77	8.10	10.83	13.95	17.44	21.27	25.41	29.85
10	8000	6000	1	1.55	2.20	3.75	5.76	8.07	10.79	13.91	17.40	21.23	25.38	29.82
10	8000	6500	1	1.56	2.22	3.74	5.72	8.03	10.75	13.88	17.37	21.20	25.35	29.80
10	8000	7000	1	1.56	2.21	3.74	5.73	8.01	10.72	13.85	17.34	21.17	25.32	29.78
10	8000	7500	1	1.56	2.19	3.71	5.67	7.97	10.69	13.82	17.31	21.14	25.30	29.76
10	8000	8000	1	1.55	2.17	3.67	5.60	7.93	10.66	13.79	17.28	21.12	25.28	29.74
10	8000	9000	1	1.54	2.15	3.62	5.52	7.87	10.61	13.74	17.23	21.07	25.23	29.70

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
10	8000	10000	1	1.53	2.13	3.59	5.50	7.83	10.57	13.70	17.19	21.03	25.20	29.67
10	9000	500	1	1.94	2.78	4.43	6.33	8.48	10.86	13.53	16.47	19.64	23.01	26.56
10	9000	1000	1	1.81	2.51	3.84	5.07	6.89	9.86	13.00	15.97	19.13	22.53	26.20
10	9000	1500	1	1.56	2.09	3.19	4.33	6.28	9.45	12.70	15.69	18.85	22.27	25.99
10	9000	2000	1	1.37	1.79	2.87	4.46	6.76	9.64	12.55	15.49	18.69	22.15	25.87
10	9000	2500	1	1.28	1.70	2.95	4.96	7.30	9.76	12.40	15.35	18.58	22.07	25.80
10	9000	3000	1	1.31	1.78	3.14	5.16	7.35	9.67	12.30	15.26	18.50	22.00	25.73
10	9000	3500	1	1.51	2.09	3.40	5.18	7.23	9.56	12.23	15.19	18.43	21.94	25.69
10	9000	4000	1	1.56	2.19	3.48	5.17	7.17	9.50	12.17	15.13	18.38	21.89	25.65
10	9000	4500	1	1.44	1.98	3.18	5.00	7.14	9.47	12.12	15.08	18.33	21.85	25.61
10	9000	5000	1	1.38	1.87	3.07	4.94	7.08	9.42	12.07	15.04	18.29	21.81	25.58
10	9000	5500	1	1.39	1.91	3.22	4.99	7.04	9.38	12.03	15.00	18.25	21.78	25.56
10	9000	6000	1	1.40	1.95	3.33	5.01	7.00	9.34	12.00	14.97	18.22	21.75	25.53
10	9000	6500	1	1.41	1.95	3.31	5.01	6.97	9.31	11.97	14.94	18.19	21.73	25.51
10	9000	7000	1	1.41	1.95	3.30	5.05	6.98	9.28	11.94	14.91	18.17	21.70	25.49
10	9000	7500	1	1.42	1.96	3.29	5.01	6.96	9.26	11.91	14.89	18.15	21.68	25.48
10	9000	8000	1	1.43	1.97	3.27	4.94	6.91	9.23	11.89	14.86	18.12	21.66	25.46
10	9000	9000	1	1.43	1.97	3.25	4.86	6.85	9.19	11.85	14.82	18.09	21.63	25.43
10	9000	10000	1	1.43	1.96	3.22	4.84	6.83	9.16	11.82	14.79	18.05	21.60	25.41

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
10	10000	500	1	1.83	2.58	4.01	5.65	7.48	9.50	11.78	14.30	17.03	19.92	22.98
10	10000	1000	1	1.74	2.36	3.45	4.47	6.01	8.58	11.32	13.88	16.59	19.50	22.66
10	10000	1500	1	1.50	1.95	2.85	3.77	5.44	8.21	11.06	13.63	16.35	19.27	22.48
10	10000	2000	1	1.29	1.64	2.56	3.90	5.90	8.42	10.93	13.45	16.21	19.18	22.38
10	10000	2500	1	1.20	1.55	2.66	4.42	6.44	8.55	10.80	13.33	16.11	19.11	22.31
10	10000	3000	1	1.23	1.61	2.83	4.62	6.50	8.47	10.71	13.26	16.04	19.05	22.26
10	10000	3500	1	1.40	1.86	3.03	4.62	6.38	8.37	10.65	13.20	15.98	18.99	22.22
10	10000	4000	1	1.44	1.93	3.09	4.59	6.31	8.31	10.60	13.15	15.94	18.95	22.18
10	10000	4500	1	1.34	1.77	2.82	4.43	6.28	8.29	10.56	13.10	15.90	18.92	22.15
10	10000	5000	1	1.31	1.73	2.76	4.38	6.24	8.25	10.52	13.07	15.86	18.88	22.13
10	10000	5500	1	1.34	1.79	2.91	4.42	6.19	8.21	10.49	13.04	15.83	18.86	22.10
10	10000	6000	1	1.35	1.82	3.01	4.44	6.16	8.17	10.46	13.01	15.80	18.83	22.08
10	10000	6500	1	1.35	1.81	2.99	4.44	6.14	8.15	10.43	12.98	15.78	18.81	22.07
10	10000	7000	1	1.35	1.81	2.98	4.48	6.15	8.12	10.41	12.96	15.76	18.79	22.05
10	10000	7500	1	1.37	1.83	2.97	4.44	6.13	8.10	10.39	12.94	15.74	18.77	22.04
10	10000	8000	1	1.37	1.84	2.97	4.40	6.09	8.08	10.37	12.92	15.72	18.76	22.02
10	10000	9000	1	1.36	1.83	2.95	4.33	6.04	8.05	10.33	12.88	15.69	18.73	22.00
10	10000	10000	1	1.37	1.82	2.93	4.32	6.02	8.02	10.30	12.85	15.66	18.71	21.98
10	11000	500	1	1.71	2.33	3.52	4.90	6.50	8.30	10.29	12.46	14.80	17.29	19.91
10	11000	1000	1	1.59	2.14	3.26	4.58	6.12	7.90	9.88	12.05	14.42	16.95	19.63

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
10	11000	1500	1	1.53	2.03	3.09	4.38	5.91	7.67	9.65	11.83	14.21	16.76	19.48
10	11000	2000	1	1.50	1.97	2.99	4.27	5.79	7.55	9.52	11.71	14.09	16.66	19.39
10	11000	2500	1	1.47	1.93	2.94	4.20	5.72	7.47	9.44	11.63	14.01	16.59	19.34
10	11000	3000	1	1.46	1.90	2.90	4.15	5.66	7.40	9.37	11.56	13.95	16.53	19.29
10	11000	3500	1	1.45	1.88	2.86	4.10	5.61	7.35	9.32	11.51	13.90	16.49	19.26
10	11000	4000	1	1.44	1.87	2.83	4.07	5.57	7.31	9.27	11.46	13.86	16.45	19.23
10	11000	4500	1	1.43	1.86	2.82	4.04	5.53	7.27	9.24	11.43	13.83	16.42	19.20
10	11000	5000	1	1.42	1.85	2.80	4.02	5.50	7.24	9.20	11.39	13.80	16.39	19.18
10	11000	5500	1	1.41	1.83	2.78	4.00	5.48	7.21	9.17	11.37	13.77	16.37	19.16
10	11000	6000	1	1.40	1.81	2.76	3.98	5.45	7.18	9.15	11.34	13.75	16.35	19.14
10	11000	6500	1	1.40	1.81	2.74	3.96	5.43	7.16	9.13	11.32	13.72	16.33	19.13
10	11000	7000	1	1.40	1.80	2.73	3.94	5.41	7.14	9.11	11.30	13.71	16.32	19.12
10	11000	7500	1	1.39	1.79	2.72	3.92	5.40	7.12	9.09	11.28	13.69	16.30	19.10
10	11000	8000	1	1.38	1.77	2.71	3.91	5.38	7.11	9.07	11.26	13.67	16.29	19.09
10	11000	9000	1	1.34	1.74	2.69	3.89	5.35	7.07	9.04	11.23	13.64	16.26	19.07
10	11000	10000	1	1.34	1.73	2.67	3.86	5.33	7.05	9.01	11.21	13.62	16.24	19.05
1000	500	500	1	20.76	42.64	73.19	123.97	204.98	280.81	338.21	391.00	447.43	503.75	558.71
1000	500	1000	1	17.48	36.57	61.48	98.90	163.82	228.03	283.26	338.21	397.89	459.80	522.93
1000	500	1500	1	16.32	34.09	54.57	91.67	146.28	203.52	257.19	313.29	374.44	438.80	505.56
1000	500	2000	1	15.44	32.37	53.17	92.96	141.25	193.41	246.26	303.24	365.00	430.29	498.47

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
1000	500	2500	1	16.68	30.55	56.76	93.92	138.34	187.23	239.93	297.40	359.48	425.36	494.49
1000	500	3000	1	14.98	27.52	55.50	90.75	134.83	182.79	235.23	292.76	355.10	421.45	491.28
1000	500	3500	1	12.43	24.74	53.22	88.46	131.83	179.32	231.60	289.17	351.69	418.40	488.75
1000	500	4000	1	11.15	23.70	49.05	86.20	129.56	176.60	228.74	286.37	349.05	416.03	486.79
1000	500	4500	1	10.41	22.33	46.80	84.62	127.64	174.35	226.44	284.12	346.91	414.12	485.20
1000	500	5000	1	10.45	22.04	48.29	84.56	126.02	172.42	224.54	282.25	345.15	412.53	483.89
1000	500	5500	1	10.46	22.12	49.23	84.04	124.56	170.81	222.93	280.67	343.65	411.19	482.79
1000	500	6000	1	10.21	21.63	48.83	83.06	123.28	169.45	221.56	279.32	342.37	410.03	481.83
1000	500	6500	1	10.42	21.02	48.30	82.20	122.19	168.27	220.35	278.13	341.25	409.03	480.99
1000	500	7000	1	11.11	21.96	48.32	81.49	121.21	167.24	219.30	277.10	340.26	408.15	480.28
1000	500	7500	1	11.17	22.11	47.93	80.76	120.36	166.36	218.41	276.23	339.47	407.47	479.75
1000	500	8000	1	11.00	21.84	47.40	80.06	119.58	165.52	217.55	275.38	338.65	406.70	479.06
1000	500	9000	1	10.43	20.93	46.25	78.69	117.98	163.73	215.61	273.33	336.52	404.49	476.75
1000	500	10000	1	10.15	20.46	45.46	77.66	116.80	162.44	214.27	271.97	335.18	403.19	475.50
1000	1000	500	1	14.24	30.11	52.83	81.86	125.35	172.24	211.08	244.81	279.93	316.29	353.91
1000	1000	1000	1	10.76	24.20	44.17	67.10	100.01	139.74	175.75	209.89	246.71	286.13	328.09
1000	1000	1500	1	10.01	22.34	38.93	60.40	89.56	125.23	159.16	193.60	231.20	271.91	315.66
1000	1000	2000	1	10.21	21.98	36.20	57.55	85.78	119.08	152.35	187.23	225.24	266.53	311.03
1000	1000	2500	1	10.62	22.26	36.70	54.22	82.42	115.32	148.42	183.55	221.79	263.46	308.54
1000	1000	3000	1	10.46	20.49	35.40	54.59	81.63	112.80	145.36	180.63	219.05	261.02	306.51

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
1000	1000	3500	1	9.00	18.63	34.87	55.34	81.37	110.78	142.99	178.38	216.94	259.13	304.91
1000	1000	4000	1	8.34	16.90	30.10	53.08	80.10	109.21	141.24	176.64	215.29	257.65	303.67
1000	1000	4500	1	8.20	16.37	29.83	52.02	78.71	107.80	139.83	175.24	213.96	256.45	302.67
1000	1000	5000	1	8.09	16.26	31.34	52.23	77.82	106.59	138.64	174.09	212.87	255.46	301.84
1000	1000	5500	1	7.89	15.73	31.55	52.08	77.03	105.60	137.63	173.11	211.94	254.62	301.15
1000	1000	6000	1	7.64	14.99	30.97	51.57	76.32	104.77	136.78	172.27	211.15	253.91	300.56
1000	1000	6500	1	7.64	14.56	30.65	51.08	75.64	104.03	136.04	171.54	210.46	253.29	300.04
1000	1000	7000	1	7.55	14.39	30.37	50.60	75.06	103.39	135.39	170.90	209.86	252.75	299.58
1000	1000	7500	1	7.35	14.13	30.07	50.16	74.47	102.73	134.69	170.18	209.13	252.03	298.89
1000	1000	8000	1	7.20	13.88	29.74	49.75	74.00	102.22	134.18	169.68	208.66	251.63	298.63
1000	1000	9000	1	6.99	13.60	29.39	49.42	73.79	102.21	134.42	170.24	209.62	253.09	300.69
1000	1000	10000	1	6.80	13.29	28.95	48.91	73.25	101.68	133.93	169.84	209.34	253.00	300.84
1000	2000	500	1	9.46	17.05	29.50	38.02	46.98	67.86	88.62	103.90	117.82	131.53	146.13
1000	2000	1000	1	6.13	11.84	23.94	33.67	42.72	59.39	75.40	89.64	104.53	120.02	136.47
1000	2000	1500	1	5.16	10.91	22.10	29.31	36.22	52.18	68.43	82.99	98.39	114.58	131.89
1000	2000	2000	1	5.36	12.30	20.99	26.47	35.67	50.13	65.33	80.00	95.81	112.46	129.96
1000	2000	2500	1	5.84	13.12	20.91	25.24	35.77	49.66	63.57	78.20	94.25	111.18	128.80
1000	2000	3000	1	6.01	12.39	19.65	25.85	35.95	48.57	62.09	76.93	93.06	110.11	127.91
1000	2000	3500	1	5.72	11.71	18.41	26.10	35.90	47.55	61.02	75.97	92.14	109.28	127.22
1000	2000	4000	1	5.26	11.16	15.84	24.78	35.10	46.85	60.28	75.21	91.43	108.63	126.68

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
1000	2000	4500	1	5.69	12.13	17.40	24.28	34.18	46.19	59.68	74.61	90.85	108.11	126.25
1000	2000	5000	1	5.69	11.09	15.89	23.29	33.54	45.69	59.18	74.11	90.37	107.68	125.89
1000	2000	5500	1	5.30	9.68	14.40	22.31	33.10	45.30	58.75	73.68	89.97	107.32	125.59
1000	2000	6000	1	4.93	9.02	13.84	22.01	32.81	44.95	58.38	73.31	89.62	107.01	125.32
1000	2000	6500	1	4.79	8.68	14.18	22.06	32.53	44.60	58.05	72.99	89.32	106.73	125.10
1000	2000	7000	1	4.44	7.95	13.81	21.92	32.32	44.30	57.76	72.72	89.05	106.50	124.91
1000	2000	7500	1	3.87	6.84	13.41	21.90	32.15	44.05	57.54	72.51	88.86	106.34	124.79
1000	2000	8000	1	3.71	6.53	13.23	21.74	31.94	43.82	57.29	72.27	88.63	106.11	124.57
1000	2000	9000	1	3.62	6.39	12.90	21.26	31.42	43.23	56.63	71.53	87.83	105.27	123.73
1000	2000	10000	1	3.52	6.23	12.68	20.99	31.08	42.86	56.23	71.12	87.40	104.85	123.33
1000	3000	500	1	7.81	13.50	22.08	27.81	32.20	45.45	57.36	66.14	74.57	82.26	89.54
1000	3000	1000	1	5.52	9.19	18.20	24.66	30.58	39.07	48.72	57.55	66.89	76.09	85.10
1000	3000	1500	1	4.59	8.22	16.26	21.71	25.86	34.61	44.40	53.52	63.29	73.13	82.95
1000	3000	2000	1	4.20	8.82	15.35	19.16	24.53	33.12	42.29	51.55	61.62	71.77	81.76
1000	3000	2500	1	4.31	9.48	14.71	17.87	24.12	32.48	41.08	50.39	60.58	70.89	80.97
1000	3000	3000	1	4.14	8.85	13.52	17.04	23.72	31.56	40.10	49.56	59.80	70.18	80.38
1000	3000	3500	1	4.07	7.95	12.37	16.87	23.49	30.90	39.41	48.93	59.20	69.64	79.93
1000	3000	4000	1	3.94	7.51	10.90	16.34	23.07	30.44	38.91	48.43	58.73	69.22	79.58
1000	3000	4500	1	4.02	7.74	11.53	16.07	22.50	29.99	38.52	48.03	58.35	68.87	79.30
1000	3000	5000	1	3.94	7.42	10.30	15.36	22.04	29.66	38.19	47.70	58.04	68.59	79.07

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
1000	3000	5500	1	3.79	6.88	9.14	14.45	21.64	29.40	37.92	47.43	57.77	68.35	78.87
1000	3000	6000	1	3.41	6.24	8.79	14.19	21.41	29.17	37.67	47.18	57.54	68.14	78.69
1000	3000	6500	1	3.21	6.07	9.15	14.32	21.23	28.93	37.46	46.97	57.34	67.96	78.54
1000	3000	7000	1	3.09	5.67	9.12	14.36	21.07	28.69	37.26	46.78	57.15	67.79	78.40
1000	3000	7500	1	2.90	4.85	8.99	14.45	20.98	28.56	37.17	46.72	57.12	67.81	78.51
1000	3000	8000	1	2.82	4.63	8.91	14.37	20.85	28.42	37.02	46.58	57.00	67.69	78.36
1000	3000	9000	1	2.73	4.46	8.48	13.77	20.16	27.56	35.95	45.30	55.47	65.75	75.78
1000	3000	10000	1	2.65	4.35	8.31	13.55	19.86	27.20	35.54	44.84	54.95	65.13	74.98
1000	4000	500	1	7.98	14.40	22.52	27.28	32.77	39.53	47.22	54.49	61.87	68.97	75.65
1000	4000	1000	1	6.56	10.71	18.72	23.96	27.50	32.25	39.27	46.91	54.89	62.96	70.91
1000	4000	1500	1	5.45	9.23	15.89	19.81	23.32	28.97	35.90	43.43	51.61	60.08	68.62
1000	4000	2000	1	4.70	8.58	13.67	16.31	20.77	27.47	34.48	41.99	50.23	58.83	67.58
1000	4000	2500	1	4.42	8.27	11.76	14.26	19.37	26.40	33.56	41.13	49.40	58.06	66.95
1000	4000	3000	1	4.08	7.47	10.80	13.67	19.07	25.73	32.79	40.44	48.75	57.48	66.46
1000	4000	3500	1	3.77	6.55	9.40	13.13	18.91	25.30	32.23	39.91	48.25	57.03	66.08
1000	4000	4000	1	3.48	6.06	8.65	12.42	18.38	24.90	31.83	39.50	47.86	56.68	65.78
1000	4000	4500	1	3.23	5.32	7.62	12.03	18.05	24.57	31.50	39.17	47.55	56.39	65.54
1000	4000	5000	1	3.04	4.85	7.00	11.83	17.96	24.32	31.21	38.90	47.29	56.16	65.35
1000	4000	5500	1	2.98	5.14	7.27	11.71	17.76	24.08	30.97	38.67	47.07	55.96	65.18
1000	4000	6000	1	2.77	4.90	7.12	11.51	17.53	23.88	30.78	38.47	46.88	55.79	65.04

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
1000	4000	6500	1	2.60	4.70	7.21	11.57	17.42	23.70	30.60	38.30	46.72	55.64	64.91
1000	4000	7000	1	2.57	4.45	7.37	11.84	17.34	23.50	30.43	38.15	46.57	55.50	64.79
1000	4000	7500	1	2.57	4.15	7.58	11.97	17.23	23.36	30.32	38.04	46.48	55.43	64.77
1000	4000	8000	1	2.54	4.04	7.55	11.91	17.12	23.25	30.20	37.93	46.38	55.35	64.70
1000	4000	9000	1	2.46	3.91	7.22	11.54	16.78	22.85	29.74	37.40	45.77	54.59	63.64
1000	4000	10000	1	2.40	3.83	7.12	11.41	16.59	22.63	29.50	37.15	45.51	54.29	63.27
1000	5000	500	1	7.50	12.72	19.20	23.85	28.73	33.16	38.57	44.66	50.85	57.03	63.06
1000	5000	1000	1	5.94	9.27	14.67	17.91	21.26	26.09	32.03	38.33	44.77	51.46	58.39
1000	5000	1500	1	4.84	7.42	11.30	13.06	17.81	23.44	29.22	35.35	41.92	48.87	56.16
1000	5000	2000	1	4.23	6.58	9.31	10.44	15.79	22.25	28.13	34.15	40.77	47.83	55.27
1000	5000	2500	1	3.87	5.90	7.77	9.36	14.96	21.54	27.40	33.43	40.09	47.21	54.75
1000	5000	3000	1	3.25	5.10	7.02	10.17	15.36	21.07	26.73	32.85	39.55	46.73	54.34
1000	5000	3500	1	2.91	4.53	6.25	10.23	15.36	20.68	26.25	32.41	39.14	46.36	54.02
1000	5000	4000	1	2.78	4.33	5.34	9.00	14.39	20.26	25.98	32.09	38.82	46.06	53.78
1000	5000	4500	1	2.60	3.78	5.14	8.66	14.06	20.00	25.72	31.82	38.56	45.82	53.58
1000	5000	5000	1	2.50	3.68	5.44	9.25	14.46	19.83	25.42	31.58	38.34	45.63	53.41
1000	5000	5500	1	2.41	3.87	5.87	9.77	14.62	19.64	25.19	31.39	38.16	45.47	53.27
1000	5000	6000	1	2.14	3.52	5.38	9.35	14.38	19.50	25.04	31.23	38.01	45.33	53.15
1000	5000	6500	1	2.01	3.29	5.16	9.20	14.23	19.37	24.90	31.09	37.87	45.21	53.04
1000	5000	7000	1	2.16	3.38	5.76	9.61	14.21	19.20	24.76	30.97	37.76	45.10	52.95

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
1000	5000	7500	1	2.33	3.59	6.37	9.89	14.12	19.05	24.65	30.85	37.65	45.00	52.87
1000	5000	8000	1	2.33	3.58	6.42	9.86	14.03	18.95	24.54	30.76	37.55	44.91	52.80
1000	5000	9000	1	2.21	3.42	6.12	9.61	13.86	18.80	24.38	30.59	37.40	44.78	52.69
1000	5000	10000	1	2.18	3.37	6.07	9.53	13.74	18.65	24.24	30.45	37.27	44.66	52.60
1000	6000	500	1	6.08	9.72	14.31	18.50	22.89	27.18	31.77	36.67	41.65	46.63	51.53
1000	6000	1000	1	4.29	6.19	8.40	10.76	15.28	20.97	26.50	31.49	36.61	41.96	47.62
1000	6000	1500	1	3.08	4.09	5.56	7.78	12.96	18.76	24.06	28.99	34.25	39.81	45.75
1000	6000	2000	1	2.48	3.26	5.07	7.78	12.83	18.24	23.05	27.94	33.28	38.97	44.99
1000	6000	2500	1	2.04	2.72	4.72	8.26	13.06	17.88	22.39	27.31	32.70	38.47	44.55
1000	6000	3000	1	1.87	2.59	4.65	8.65	13.19	17.42	21.86	26.84	32.26	38.06	44.21
1000	6000	3500	1	1.92	2.70	4.61	8.34	12.70	17.02	21.51	26.48	31.92	37.75	43.94
1000	6000	4000	1	2.07	2.83	3.90	6.71	11.50	16.68	21.32	26.21	31.65	37.50	43.73
1000	6000	4500	1	2.13	2.89	3.92	6.40	11.22	16.47	21.10	25.99	31.43	37.30	43.56
1000	6000	5000	1	2.18	3.09	4.66	7.45	11.81	16.31	20.82	25.79	31.25	37.14	43.42
1000	6000	5500	1	1.97	2.93	4.89	8.22	12.11	16.13	20.62	25.63	31.11	37.01	43.31
1000	6000	6000	1	1.59	2.37	4.16	7.77	11.92	16.03	20.50	25.50	30.98	36.89	43.21
1000	6000	6500	1	1.53	2.25	3.98	7.56	11.77	15.92	20.38	25.38	30.86	36.79	43.12
1000	6000	7000	1	1.83	2.68	4.61	7.86	11.72	15.78	20.27	25.28	30.77	36.70	43.04
1000	6000	7500	1	2.09	3.09	5.26	8.13	11.64	15.66	20.17	25.19	30.68	36.62	42.97
1000	6000	8000	1	2.13	3.17	5.40	8.18	11.58	15.57	20.09	25.10	30.60	36.54	42.91

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
1000	6000	9000	1	2.03	3.04	5.24	8.04	11.45	15.44	19.95	24.97	30.47	36.43	42.84
1000	6000	10000	1	2.00	2.98	5.15	7.94	11.34	15.32	19.83	24.85	30.36	36.33	42.76
1000	7000	500	1	5.16	7.96	11.58	15.18	18.97	22.86	26.81	30.83	34.92	39.01	43.03
1000	7000	1000	1	3.68	5.25	7.11	9.01	12.96	18.02	22.35	26.37	30.63	35.08	39.71
1000	7000	1500	1	2.79	3.74	5.14	7.07	11.09	16.00	20.21	24.26	28.63	33.25	38.12
1000	7000	2000	1	2.33	3.09	4.47	7.47	11.51	15.46	19.29	23.38	27.81	32.52	37.47
1000	7000	2500	1	2.04	2.75	4.29	7.88	11.71	15.06	18.73	22.86	27.32	32.08	37.09
1000	7000	3000	1	2.01	2.75	4.55	7.81	11.29	14.62	18.33	22.46	26.94	31.73	36.80
1000	7000	3500	1	2.03	2.75	4.20	6.78	10.52	14.32	18.07	22.16	26.65	31.46	36.57
1000	7000	4000	1	1.99	2.56	3.20	5.10	9.35	13.99	17.93	21.93	26.42	31.25	36.39
1000	7000	4500	1	1.93	2.47	2.99	4.82	9.14	13.82	17.75	21.74	26.23	31.08	36.24
1000	7000	5000	1	1.86	2.47	3.42	5.64	9.72	13.76	17.49	21.57	26.08	30.95	36.12
1000	7000	5500	1	1.54	2.16	3.79	6.66	10.10	13.60	17.30	21.43	25.96	30.83	36.02
1000	7000	6000	1	1.31	1.87	3.55	6.62	10.03	13.49	17.19	21.32	25.85	30.73	35.94
1000	7000	6500	1	1.37	1.94	3.49	6.40	9.88	13.39	17.09	21.22	25.75	30.64	35.86
1000	7000	7000	1	1.53	2.16	3.62	6.29	9.74	13.29	17.00	21.13	25.67	30.57	35.79
1000	7000	7500	1	1.76	2.49	4.00	6.49	9.76	13.20	16.92	21.05	25.59	30.50	35.74
1000	7000	8000	1	1.87	2.67	4.38	6.77	9.77	13.11	16.84	20.98	25.53	30.44	35.68
1000	7000	9000	1	1.93	2.80	4.69	6.95	9.70	12.97	16.71	20.86	25.40	30.32	35.58
1000	7000	10000	1	1.85	2.67	4.46	6.77	9.59	12.88	16.61	20.76	25.31	30.23	35.50

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
1000	8000	500	1	4.74	7.21	10.33	13.33	16.46	19.70	23.01	26.40	29.84	33.28	36.66
1000	8000	1000	1	3.42	5.06	7.25	9.25	11.94	15.55	19.10	22.52	26.12	29.86	33.75
1000	8000	1500	1	2.78	3.99	5.69	7.50	10.16	13.75	17.26	20.70	24.38	28.26	32.36
1000	8000	2000	1	2.50	3.50	4.95	7.12	10.02	13.24	16.50	19.94	23.67	27.62	31.79
1000	8000	2500	1	2.35	3.23	4.51	6.79	9.84	12.90	16.04	19.49	23.24	27.24	31.46
1000	8000	3000	1	2.27	3.09	4.28	6.29	9.38	12.56	15.70	19.14	22.91	26.94	31.20
1000	8000	3500	1	2.19	2.93	3.74	5.36	8.79	12.31	15.46	18.88	22.65	26.70	31.00
1000	8000	4000	1	1.98	2.49	2.66	3.98	7.83	12.01	15.35	18.68	22.45	26.52	30.84
1000	8000	4500	1	1.72	2.10	2.39	3.95	7.80	11.89	15.18	18.51	22.29	26.37	30.71
1000	8000	5000	1	1.57	1.98	2.67	4.64	8.30	11.83	14.95	18.36	22.16	26.26	30.61
1000	8000	5500	1	1.36	1.80	2.93	5.31	8.46	11.66	14.79	18.25	22.05	26.15	30.52
1000	8000	6000	1	1.30	1.76	2.91	5.25	8.33	11.55	14.70	18.15	21.96	26.06	30.44
1000	8000	6500	1	1.36	1.86	2.96	5.21	8.28	11.47	14.61	18.06	21.87	25.99	30.38
1000	8000	7000	1	1.39	1.93	3.04	5.29	8.32	11.40	14.52	17.98	21.80	25.92	30.32
1000	8000	7500	1	1.51	2.09	3.27	5.52	8.41	11.33	14.44	17.92	21.74	25.86	30.27
1000	8000	8000	1	1.63	2.28	3.69	5.86	8.45	11.25	14.37	17.86	21.68	25.81	30.22
1000	8000	9000	1	1.75	2.48	4.11	6.04	8.36	11.12	14.26	17.75	21.57	25.71	30.14
1000	8000	10000	1	1.70	2.39	3.91	5.89	8.28	11.04	14.17	17.66	21.49	25.63	30.07
1000	9000	500	1	4.42	6.64	9.30	11.81	14.45	17.20	20.01	22.89	25.82	28.75	31.63

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
1000	9000	1000	1	3.12	4.51	6.09	7.21	9.12	12.83	16.58	19.58	22.57	25.68	29.06
1000	9000	1500	1	2.45	3.31	4.18	5.07	7.31	11.23	14.99	17.98	21.03	24.26	27.82
1000	9000	2000	1	2.15	2.73	3.24	4.60	7.64	11.27	14.37	17.25	20.39	23.74	27.31
1000	9000	2500	1	2.02	2.43	2.60	4.21	7.83	11.30	14.00	16.82	20.01	23.42	27.01
1000	9000	3000	1	2.02	2.39	2.37	3.76	7.46	11.00	13.70	16.52	19.72	23.15	26.78
1000	9000	3500	1	2.12	2.57	2.47	3.46	7.14	10.73	13.45	16.29	19.49	22.94	26.60
1000	9000	4000	1	1.89	2.23	2.09	3.03	6.53	10.42	13.31	16.11	19.32	22.78	26.46
1000	9000	4500	1	1.42	1.62	2.02	3.60	6.98	10.41	13.14	15.96	19.18	22.65	26.34
1000	9000	5000	1	1.15	1.40	2.09	4.31	7.29	10.29	12.93	15.83	19.06	22.55	26.25
1000	9000	5500	1	1.27	1.53	2.12	3.98	6.86	10.05	12.84	15.74	18.97	22.45	26.17
1000	9000	6000	1	1.41	1.69	2.09	3.55	6.43	9.91	12.79	15.66	18.88	22.37	26.11
1000	9000	6500	1	1.38	1.71	2.22	3.79	6.61	9.87	12.69	15.58	18.81	22.30	26.05
1000	9000	7000	1	1.33	1.76	2.60	4.50	7.13	9.89	12.57	15.50	18.74	22.25	25.99
1000	9000	7500	1	1.27	1.71	2.64	4.89	7.40	9.85	12.48	15.44	18.69	22.20	25.95
1000	9000	8000	1	1.31	1.80	2.97	5.17	7.48	9.78	12.41	15.38	18.64	22.15	25.91
1000	9000	9000	1	1.41	1.92	3.32	5.15	7.28	9.66	12.33	15.29	18.55	22.07	25.84
1000	9000	10000	1	1.47	2.03	3.36	5.16	7.25	9.59	12.25	15.22	18.47	22.00	25.78
1000	10000	500	1	4.14	6.17	8.48	10.59	12.85	15.19	17.60	20.07	22.59	25.11	27.59
1000	10000	1000	1	2.92	4.13	5.36	6.23	7.84	11.16	14.56	17.16	19.70	22.36	25.29
1000	10000	1500	1	2.24	2.92	3.49	4.17	6.18	9.74	13.14	15.73	18.33	21.09	24.16

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
1000	10000	2000	1	1.93	2.34	2.64	3.72	6.52	9.86	12.61	15.06	17.75	20.62	23.69
1000	10000	2500	1	1.81	2.05	2.04	3.38	6.75	9.96	12.28	14.68	17.41	20.34	23.43
1000	10000	3000	1	1.83	2.06	1.86	3.07	6.49	9.69	12.01	14.41	17.15	20.10	23.22
1000	10000	3500	1	1.99	2.36	2.22	3.09	6.29	9.43	11.78	14.20	16.95	19.92	23.06
1000	10000	4000	1	1.83	2.15	2.16	3.05	5.91	9.13	11.62	14.05	16.80	19.77	22.93
1000	10000	4500	1	1.39	1.60	2.12	3.58	6.34	9.13	11.47	13.91	16.67	19.66	22.83
1000	10000	5000	1	1.13	1.39	2.10	4.09	6.52	9.03	11.31	13.79	16.57	19.56	22.75
1000	10000	5500	1	1.33	1.57	2.05	3.55	6.00	8.81	11.24	13.72	16.48	19.48	22.68
1000	10000	6000	1	1.47	1.73	2.00	3.09	5.57	8.67	11.19	13.64	16.41	19.40	22.62
1000	10000	6500	1	1.40	1.69	2.11	3.37	5.77	8.65	11.10	13.57	16.34	19.35	22.57
1000	10000	7000	1	1.34	1.71	2.45	4.09	6.31	8.68	10.98	13.50	16.29	19.30	22.52
1000	10000	7500	1	1.23	1.61	2.43	4.42	6.57	8.65	10.90	13.44	16.24	19.25	22.48
1000	10000	8000	1	1.17	1.54	2.52	4.54	6.63	8.61	10.85	13.40	16.19	19.21	22.44
1000	10000	9000	1	1.13	1.46	2.58	4.38	6.43	8.51	10.78	13.32	16.11	19.14	22.38
1000	10000	10000	1	1.25	1.67	2.84	4.51	6.41	8.43	10.70	13.25	16.05	19.08	22.33
1000	11000	500	1	3.84	5.67	7.67	9.50	11.44	13.46	15.54	17.66	19.83	21.99	24.12
1000	11000	1000	1	2.85	4.08	5.61	7.16	8.91	10.80	12.81	14.96	17.24	19.61	22.04
1000	11000	1500	1	2.37	3.30	4.61	6.04	7.69	9.51	11.50	13.67	16.00	18.45	21.02
1000	11000	2000	1	2.16	2.98	4.19	5.60	7.20	8.98	10.97	13.15	15.50	17.98	20.60
1000	11000	2500	1	2.04	2.79	3.96	5.35	6.91	8.68	10.67	12.85	15.21	17.72	20.37

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
1000	11000	3000	1	1.95	2.65	3.78	5.12	6.67	8.44	10.43	12.61	14.97	17.51	20.19
1000	11000	3500	1	1.89	2.54	3.60	4.89	6.46	8.26	10.24	12.42	14.79	17.34	20.04
1000	11000	4000	1	1.83	2.44	3.39	4.60	6.26	8.11	10.10	12.28	14.66	17.21	19.92
1000	11000	4500	1	1.78	2.35	3.27	4.52	6.15	8.00	9.98	12.16	14.54	17.11	19.83
1000	11000	5000	1	1.73	2.30	3.27	4.53	6.12	7.90	9.88	12.06	14.45	17.02	19.76
1000	11000	5500	1	1.68	2.22	3.20	4.51	6.07	7.82	9.79	11.98	14.37	16.95	19.69
1000	11000	6000	1	1.64	2.16	3.16	4.48	6.01	7.75	9.72	11.91	14.30	16.88	19.64
1000	11000	6500	1	1.63	2.12	3.14	4.43	5.95	7.69	9.66	11.85	14.25	16.83	19.59
1000	11000	7000	1	1.58	2.01	3.06	4.33	5.88	7.64	9.61	11.80	14.20	16.78	19.55
1000	11000	7500	1	1.50	1.93	2.88	4.24	5.85	7.60	9.56	11.75	14.15	16.74	19.51
1000	11000	8000	1	1.28	1.63	2.56	4.09	5.82	7.57	9.52	11.71	14.11	16.70	19.48
1000	11000	9000	1	1.04	1.29	2.22	3.90	5.75	7.52	9.45	11.63	14.04	16.64	19.42
1000	11000	10000	1	1.23	1.58	2.60	4.03	5.68	7.43	9.38	11.57	13.98	16.58	19.37
2000	500	500	1	44.77	76.88	109.84	162.75	258.23	344.58	403.45	454.94	508.37	559.10	605.38
2000	500	1000	1	27.67	50.40	83.68	132.60	198.29	261.28	316.66	371.96	430.23	489.33	548.12
2000	500	1500	1	23.34	44.51	76.30	118.62	169.15	223.27	277.29	334.01	394.37	457.02	521.19
2000	500	2000	1	19.68	39.85	73.57	111.91	156.89	208.44	262.12	319.31	380.43	444.43	510.69
2000	500	2500	1	20.90	36.33	71.59	105.71	149.34	199.80	253.34	310.74	372.30	437.17	504.78
2000	500	3000	1	15.85	31.71	65.41	99.14	144.28	193.60	246.52	304.00	365.93	431.49	500.07
2000	500	3500	1	13.79	28.02	58.58	95.15	140.39	188.74	241.30	298.87	361.08	427.13	496.43

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2000	500	4000	1	12.54	26.96	53.18	92.03	137.12	184.87	237.29	294.93	357.34	423.76	493.62
2000	500	4500	1	12.07	25.97	50.92	89.90	134.33	181.73	234.09	291.78	354.35	421.07	491.39
2000	500	5000	1	11.88	25.57	52.28	89.81	132.18	179.08	231.45	289.20	351.90	418.86	489.55
2000	500	5500	1	11.57	24.22	52.59	88.93	130.23	176.90	229.25	287.03	349.85	417.01	488.01
2000	500	6000	1	11.31	22.80	51.46	87.43	128.51	175.07	227.38	285.18	348.09	415.43	486.69
2000	500	6500	1	11.88	23.37	51.15	86.24	127.01	173.48	225.76	283.58	346.57	414.06	485.55
2000	500	7000	1	12.52	24.31	51.49	85.40	125.68	172.08	224.35	282.19	345.25	412.87	484.56
2000	500	7500	1	12.39	24.15	50.95	84.45	124.55	170.89	223.14	281.01	344.15	411.91	483.80
2000	500	8000	1	12.14	23.72	50.25	83.54	123.53	169.79	222.01	279.89	343.09	410.93	482.94
2000	500	9000	1	11.45	22.63	48.76	81.81	121.54	167.59	219.65	277.44	340.57	408.38	480.36
2000	500	10000	1	11.08	22.00	47.75	80.49	120.03	165.96	217.96	275.73	338.90	406.79	478.88
2000	1000	500	1	26.89	47.72	73.67	106.12	159.29	211.42	250.19	282.91	316.01	348.84	380.97
2000	1000	1000	1	18.81	33.04	57.28	85.79	121.19	160.03	195.31	229.34	265.11	302.64	341.65
2000	1000	1500	1	15.36	28.87	50.63	74.23	103.36	137.21	170.68	205.19	242.10	281.55	323.37
2000	1000	2000	1	12.54	26.95	47.93	68.61	95.78	128.03	161.20	196.02	233.43	273.68	316.66
2000	1000	2500	1	11.75	25.29	45.89	63.08	90.15	122.55	155.69	190.66	228.35	269.15	312.94
2000	1000	3000	1	10.99	22.92	41.74	60.82	87.38	118.73	151.40	186.47	224.40	265.60	309.98
2000	1000	3500	1	10.05	21.22	39.20	59.87	85.88	115.77	148.14	183.31	221.40	262.89	307.69
2000	1000	4000	1	9.28	19.29	34.00	57.21	84.09	113.54	145.67	180.87	219.09	260.81	305.93
2000	1000	4500	1	8.96	18.41	32.97	55.68	82.34	111.63	143.70	178.92	217.25	259.14	304.53

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2000	1000	5000	1	8.58	17.97	33.30	55.20	81.21	110.07	142.07	177.32	215.73	257.77	303.38
2000	1000	5500	1	8.60	17.13	33.43	54.66	80.08	108.74	140.71	175.98	214.46	256.62	302.41
2000	1000	6000	1	8.66	16.48	32.68	54.02	79.08	107.60	139.55	174.85	213.38	255.64	301.60
2000	1000	6500	1	8.72	15.81	32.20	53.33	78.18	106.62	138.56	173.87	212.45	254.80	300.89
2000	1000	7000	1	8.51	15.56	31.94	52.71	77.39	105.77	137.69	173.01	211.65	254.07	300.28
2000	1000	7500	1	8.12	15.31	31.75	52.15	76.63	104.92	136.80	172.11	210.74	253.20	299.44
2000	1000	8000	1	7.91	14.99	31.33	51.61	75.99	104.24	136.10	171.42	210.08	252.62	299.01
2000	1000	9000	1	7.63	14.63	30.81	51.07	75.55	103.97	136.08	171.70	210.77	253.84	300.91
2000	1000	10000	1	7.38	14.21	30.22	50.37	74.80	103.21	135.36	171.07	210.27	253.53	300.88
2000	2000	500	1	15.25	25.74	38.75	50.03	63.94	85.62	106.50	121.46	134.65	146.92	159.19
2000	2000	1000	1	10.53	17.60	30.71	41.63	50.05	67.81	84.57	98.94	113.62	128.46	143.68
2000	2000	1500	1	8.05	15.21	26.95	35.86	44.43	59.22	74.22	88.69	104.10	120.04	136.50
2000	2000	2000	1	7.49	14.97	25.36	32.61	42.80	55.71	69.80	84.51	100.30	116.72	133.62
2000	2000	2500	1	7.67	14.87	24.82	30.67	41.01	53.64	67.25	82.05	98.02	114.72	131.93
2000	2000	3000	1	7.17	13.86	23.03	29.63	39.37	51.62	65.30	80.23	96.28	113.14	130.61
2000	2000	3500	1	6.61	13.18	21.84	28.98	38.27	50.13	63.87	78.85	94.96	111.93	129.61
2000	2000	4000	1	6.19	12.90	20.39	27.75	37.23	49.15	62.80	77.77	93.94	111.01	128.83
2000	2000	4500	1	6.30	12.99	19.84	26.80	36.32	48.30	61.93	76.91	93.12	110.28	128.21
2000	2000	5000	1	6.20	12.71	17.23	25.02	35.45	47.67	61.24	76.21	92.45	109.67	127.70
2000	2000	5500	1	5.77	11.48	15.54	23.66	34.80	47.13	60.65	75.62	91.89	109.17	127.28

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2000	2000	6000	1	5.64	11.42	16.51	23.90	34.42	46.60	60.13	75.11	91.41	108.73	126.91
2000	2000	6500	1	5.45	9.54	14.81	23.39	34.06	46.17	59.69	74.68	90.99	108.35	126.59
2000	2000	7000	1	4.97	8.23	14.21	23.05	33.71	45.77	59.30	74.30	90.63	108.03	126.32
2000	2000	7500	1	4.29	7.36	14.25	23.01	33.42	45.43	58.99	74.00	90.36	107.80	126.15
2000	2000	8000	1	4.10	7.07	14.08	22.79	33.14	45.12	58.67	73.69	90.06	107.51	125.88
2000	2000	9000	1	3.98	7.00	13.70	22.21	32.49	44.41	57.87	72.81	89.11	106.52	124.88
2000	2000	10000	1	3.84	6.73	13.38	21.85	32.06	43.93	57.36	72.29	88.60	106.01	124.39
2000	3000	500	1	11.54	18.37	27.09	34.39	41.16	55.96	69.51	78.60	86.69	93.59	99.80
2000	3000	1000	1	7.61	12.69	22.86	31.88	37.22	46.30	55.49	64.26	73.65	82.69	91.16
2000	3000	1500	1	6.11	10.69	19.72	27.63	33.24	40.60	48.75	57.75	67.71	77.62	87.13
2000	3000	2000	1	6.08	10.44	17.95	23.59	30.04	37.44	45.65	55.04	65.21	75.40	85.28
2000	3000	2500	1	6.24	10.68	17.06	21.43	27.73	35.38	43.93	53.45	63.69	74.01	84.13
2000	3000	3000	1	5.40	9.60	15.53	19.53	26.22	33.90	42.64	52.23	62.53	72.96	83.25
2000	3000	3500	1	4.90	8.86	13.99	18.59	25.33	32.99	41.70	51.31	61.64	72.16	82.58
2000	3000	4000	1	4.66	8.46	13.16	18.04	24.73	32.32	40.97	50.59	60.96	71.55	82.06
2000	3000	4500	1	4.55	8.31	12.73	17.47	24.12	31.74	40.40	50.02	60.42	71.05	81.65
2000	3000	5000	1	4.40	7.97	11.05	16.44	23.51	31.30	39.93	49.55	59.97	70.65	81.31
2000	3000	5500	1	4.27	7.82	9.56	15.37	23.02	30.92	39.54	49.16	59.60	70.31	81.03
2000	3000	6000	1	3.91	7.54	9.97	15.38	22.73	30.57	39.20	48.82	59.27	70.02	80.78
2000	3000	6500	1	3.59	6.33	9.04	15.14	22.49	30.27	38.90	48.53	58.99	69.76	80.56

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2000	3000	7000	1	3.41	5.73	9.24	15.23	22.23	29.95	38.63	48.27	58.74	69.53	80.37
2000	3000	7500	1	3.22	5.27	9.69	15.37	22.04	29.75	38.47	48.13	58.64	69.48	80.41
2000	3000	8000	1	3.14	5.10	9.64	15.25	21.85	29.56	38.29	47.96	58.48	69.33	80.26
2000	3000	9000	1	3.02	4.95	9.13	14.58	21.13	28.68	37.21	46.70	57.00	67.51	77.87
2000	3000	10000	1	2.91	4.76	8.91	14.31	20.78	28.27	36.76	46.20	56.47	66.89	77.12
2000	4000	500	1	10.84	17.64	26.07	33.20	39.58	48.30	57.07	64.42	71.41	77.82	83.59
2000	4000	1000	1	7.80	13.12	22.58	28.48	35.11	39.06	44.69	52.09	60.03	67.94	75.34
2000	4000	1500	1	6.17	10.94	18.46	23.85	29.67	33.91	39.25	46.64	54.88	63.32	71.50
2000	4000	2000	1	6.05	10.02	15.67	20.26	24.93	30.61	37.01	44.60	52.84	61.36	69.93
2000	4000	2500	1	5.96	9.63	14.02	17.65	22.02	28.43	35.67	43.37	51.60	60.18	68.99
2000	4000	3000	1	5.02	8.29	12.17	15.53	20.91	27.48	34.65	42.36	50.64	59.31	68.25
2000	4000	3500	1	4.19	6.95	10.04	14.28	20.34	26.89	33.88	41.59	49.91	58.66	67.69
2000	4000	4000	1	3.82	6.18	9.01	13.38	19.62	26.32	33.31	41.00	49.35	58.15	67.26
2000	4000	4500	1	3.61	5.75	8.43	12.94	19.19	25.85	32.83	40.53	48.91	57.74	66.91
2000	4000	5000	1	3.40	5.16	7.28	12.52	19.03	25.50	32.43	40.14	48.54	57.41	66.63
2000	4000	5500	1	3.35	5.22	6.96	12.11	18.73	25.18	32.10	39.82	48.23	57.13	66.39
2000	4000	6000	1	3.05	4.84	6.77	11.63	18.25	24.90	31.84	39.54	47.96	56.89	66.18
2000	4000	6500	1	2.88	4.59	6.64	11.57	18.06	24.65	31.60	39.30	47.74	56.68	66.00
2000	4000	7000	1	2.84	4.59	7.45	12.29	18.11	24.39	31.36	39.10	47.53	56.49	65.84
2000	4000	7500	1	2.86	4.57	8.18	12.71	18.02	24.18	31.18	38.93	47.39	56.37	65.77

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2000	4000	8000	1	2.82	4.49	8.19	12.64	17.87	24.02	31.02	38.78	47.25	56.25	65.67
2000	4000	9000	1	2.70	4.28	7.69	12.10	17.44	23.60	30.54	38.24	46.65	55.54	64.75
2000	4000	10000	1	2.62	4.17	7.59	11.96	17.22	23.33	30.26	37.95	46.36	55.23	64.40
2000	5000	500	1	9.90	16.11	23.49	29.83	35.58	40.92	46.63	52.63	58.46	64.02	69.13
2000	5000	1000	1	7.01	11.08	17.03	20.11	26.44	31.16	36.36	42.54	48.85	55.28	61.69
2000	5000	1500	1	5.46	8.58	12.80	15.07	21.49	26.73	31.88	37.98	44.49	51.29	58.25
2000	5000	2000	1	4.83	7.52	11.18	13.40	18.60	24.45	30.14	36.22	42.78	49.70	56.90
2000	5000	2500	1	4.34	6.72	9.79	12.30	17.20	23.20	29.06	35.15	41.75	48.76	56.12
2000	5000	3000	1	3.46	5.56	8.04	11.89	16.96	22.50	28.17	34.31	40.96	48.04	55.50
2000	5000	3500	1	2.92	4.64	6.71	11.08	16.49	21.94	27.53	33.68	40.36	47.50	55.03
2000	5000	4000	1	2.86	4.29	5.76	9.49	15.19	21.38	27.13	33.21	39.90	47.07	54.67
2000	5000	4500	1	2.89	4.20	5.61	9.11	14.80	21.01	26.74	32.82	39.53	46.73	54.38
2000	5000	5000	1	2.85	4.11	5.67	9.60	15.21	20.75	26.34	32.49	39.22	46.45	54.14
2000	5000	5500	1	2.75	3.99	5.53	9.71	15.13	20.47	26.05	32.23	38.97	46.22	53.94
2000	5000	6000	1	2.46	3.40	4.56	8.24	14.14	20.22	25.90	32.01	38.75	46.02	53.77
2000	5000	6500	1	2.38	3.27	4.45	8.03	13.90	20.03	25.71	31.81	38.56	45.85	53.62
2000	5000	7000	1	2.51	3.69	5.67	9.30	14.48	19.87	25.46	31.63	38.40	45.70	53.49
2000	5000	7500	1	2.58	3.97	6.80	10.37	14.72	19.68	25.27	31.48	38.25	45.56	53.37
2000	5000	8000	1	2.56	3.96	6.94	10.50	14.66	19.53	25.13	31.34	38.12	45.45	53.27
2000	5000	9000	1	2.43	3.73	6.49	10.02	14.33	19.31	24.91	31.12	37.91	45.26	53.12

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →										
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2000	5000	10000	1	2.38	3.66	6.44	9.95	14.19	19.13	24.72	30.93	37.73	45.10	52.99
2000	6000	500	1	8.25	13.08	18.91	24.09	29.18	33.82	38.52	43.36	48.05	52.53	56.67
2000	6000	1000	1	4.98	7.24	10.23	13.26	18.59	24.54	30.13	35.09	40.07	45.17	50.44
2000	6000	1500	1	3.47	4.75	7.02	9.80	15.16	20.96	26.32	31.27	36.44	41.85	47.54
2000	6000	2000	1	2.76	4.01	6.66	9.92	14.77	19.96	24.80	29.71	34.99	40.56	46.40
2000	6000	2500	1	2.16	3.31	6.08	10.07	14.73	19.32	23.85	28.78	34.13	39.80	45.73
2000	6000	3000	1	1.86	2.90	5.48	9.84	14.42	18.64	23.11	28.09	33.47	39.19	45.21
2000	6000	3500	1	1.89	2.83	4.99	9.00	13.64	18.10	22.60	27.57	32.96	38.73	44.81
2000	6000	4000	1	2.18	3.00	4.15	6.97	12.16	17.65	22.30	27.17	32.58	38.37	44.50
2000	6000	4500	1	2.48	3.31	4.20	6.55	11.82	17.35	21.98	26.85	32.26	38.09	44.26
2000	6000	5000	1	2.61	3.62	4.97	7.54	12.36	17.10	21.62	26.57	32.01	37.85	44.05
2000	6000	5500	1	2.41	3.34	4.67	7.61	12.23	16.85	21.39	26.35	31.80	37.66	43.89
2000	6000	6000	1	2.11	2.67	3.29	6.04	11.27	16.64	21.27	26.17	31.62	37.49	43.74
2000	6000	6500	1	2.05	2.57	3.15	5.83	11.08	16.50	21.11	26.00	31.46	37.34	43.62
2000	6000	7000	1	2.22	3.01	4.13	6.85	11.61	16.37	20.90	25.85	31.32	37.22	43.51
2000	6000	7500	1	2.32	3.35	5.18	7.95	11.93	16.21	20.73	25.73	31.20	37.11	43.41
2000	6000	8000	1	2.33	3.45	5.65	8.48	12.05	16.08	20.60	25.61	31.09	37.01	43.32
2000	6000	9000	1	2.23	3.33	5.61	8.45	11.87	15.88	20.41	25.42	30.91	36.84	43.19
2000	6000	10000	1	2.18	3.24	5.48	8.30	11.73	15.73	20.24	25.26	30.76	36.71	43.08

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2000	7000	500	1	7.28	11.23	15.91	20.16	24.41	28.56	32.62	36.59	40.46	44.14	47.54
2000	7000	1000	1	4.64	6.64	9.13	11.97	16.09	21.12	25.47	29.49	33.65	37.89	42.20
2000	7000	1500	1	3.45	4.72	6.63	9.27	13.23	17.98	22.19	26.24	30.55	35.04	39.71
2000	7000	2000	1	2.86	3.97	5.89	9.17	13.10	16.98	20.84	24.93	29.31	33.93	38.73
2000	7000	2500	1	2.39	3.37	5.40	9.08	12.89	16.31	20.03	24.15	28.57	33.25	38.15
2000	7000	3000	1	2.26	3.22	5.32	8.70	12.28	15.69	19.44	23.56	28.01	32.73	37.70
2000	7000	3500	1	2.21	3.03	4.60	7.40	11.41	15.28	19.02	23.11	27.57	32.34	37.36
2000	7000	4000	1	2.19	2.80	3.33	5.31	10.03	14.88	18.79	22.77	27.24	32.03	37.09
2000	7000	4500	1	2.24	2.81	3.07	4.82	9.67	14.62	18.52	22.50	26.97	31.78	36.88
2000	7000	5000	1	2.30	2.94	3.33	5.16	9.95	14.46	18.21	22.26	26.76	31.58	36.70
2000	7000	5500	1	1.98	2.49	3.08	5.26	9.54	14.14	18.03	22.08	26.57	31.41	36.56
2000	7000	6000	1	1.79	2.15	2.56	4.68	9.08	13.96	17.90	21.92	26.42	31.27	36.43
2000	7000	6500	1	1.81	2.18	2.46	4.45	8.96	13.85	17.76	21.78	26.28	31.14	36.32
2000	7000	7000	1	1.84	2.25	2.53	4.46	8.87	13.70	17.63	21.66	26.16	31.03	36.23
2000	7000	7500	1	1.98	2.51	3.02	5.01	9.20	13.64	17.49	21.54	26.06	30.94	36.15
2000	7000	8000	1	2.08	2.83	4.04	6.21	9.79	13.57	17.33	21.44	25.97	30.86	36.07
2000	7000	9000	1	2.12	3.10	5.11	7.44	10.17	13.38	17.11	21.27	25.81	30.70	35.93
2000	7000	10000	1	2.03	2.93	4.76	7.08	9.93	13.24	16.98	21.13	25.67	30.58	35.82
2000	8000	500	1	6.73	10.24	14.19	17.72	21.22	24.70	28.11	31.45	34.70	37.80	40.67
2000	8000	1000	1	4.59	6.81	9.51	12.12	15.08	18.49	21.85	25.25	28.77	32.36	35.97

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2000	8000	1500	1	3.63	5.24	7.28	9.52	12.36	15.70	19.01	22.43	26.07	29.87	33.79
2000	8000	2000	1	3.21	4.56	6.29	8.57	11.52	14.66	17.87	21.31	25.00	28.88	32.92
2000	8000	2500	1	2.96	4.12	5.59	7.83	10.91	14.02	17.19	20.64	24.35	28.29	32.41
2000	8000	3000	1	2.77	3.82	5.12	7.15	10.31	13.52	16.68	20.12	23.86	27.83	32.01
2000	8000	3500	1	2.54	3.41	4.28	6.03	9.64	13.16	16.31	19.73	23.48	27.49	31.71
2000	8000	4000	1	2.24	2.80	2.81	4.18	8.47	12.80	16.10	19.43	23.19	27.22	31.47
2000	8000	4500	1	2.04	2.48	2.41	3.78	8.19	12.58	15.87	19.19	22.95	27.00	31.29
2000	8000	5000	1	1.96	2.39	2.51	4.09	8.46	12.45	15.59	18.98	22.76	26.83	31.13
2000	8000	5500	1	1.70	2.00	2.26	4.12	8.00	12.15	15.44	18.83	22.60	26.67	31.00
2000	8000	6000	1	1.63	1.89	2.06	3.74	7.63	11.98	15.33	18.69	22.47	26.55	30.89
2000	8000	6500	1	1.67	1.97	2.03	3.53	7.52	11.89	15.21	18.56	22.35	26.44	30.80
2000	8000	7000	1	1.64	1.91	1.87	3.36	7.33	11.75	15.10	18.46	22.25	26.34	30.71
2000	8000	7500	1	1.74	2.05	2.14	3.74	7.62	11.71	14.98	18.36	22.15	26.26	30.64
2000	8000	8000	1	1.86	2.40	3.20	5.03	8.32	11.66	14.82	18.26	22.07	26.19	30.57
2000	8000	9000	1	1.93	2.77	4.49	6.53	8.82	11.48	14.62	18.12	21.93	26.05	30.45
2000	8000	10000	1	1.87	2.62	4.13	6.14	8.57	11.37	14.51	18.00	21.82	25.95	30.36
2000	9000	500	1	6.24	9.39	12.75	15.73	18.70	21.65	24.54	27.39	30.15	32.79	35.23
2000	9000	1000	1	4.26	6.23	8.39	10.29	12.62	15.89	19.06	21.96	24.93	27.96	31.06
2000	9000	1500	1	3.39	4.71	6.00	7.42	9.98	13.45	16.58	19.48	22.54	25.75	29.11
2000	9000	2000	1	3.05	4.06	4.87	6.32	9.42	12.76	15.59	18.46	21.59	24.89	28.33

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2000	9000	2500	1	2.84	3.62	3.81	5.18	8.87	12.32	15.02	17.86	21.01	24.37	27.87
2000	9000	3000	1	2.69	3.35	3.25	4.46	8.33	11.88	14.57	17.40	20.57	23.96	27.52
2000	9000	3500	1	2.57	3.20	3.13	4.10	7.92	11.51	14.21	17.05	20.24	23.65	27.24
2000	9000	4000	1	2.24	2.68	2.28	3.13	7.02	11.12	13.99	16.79	19.98	23.41	27.03
2000	9000	4500	1	1.86	2.15	1.95	2.99	6.89	10.92	13.77	16.58	19.78	23.22	26.87
2000	9000	5000	1	1.52	1.79	2.09	3.74	7.34	10.82	13.51	16.39	19.61	23.06	26.73
2000	9000	5500	1	1.45	1.66	1.94	3.73	7.07	10.59	13.38	16.25	19.47	22.93	26.61
2000	9000	6000	1	1.56	1.72	1.68	3.02	6.46	10.40	13.30	16.13	19.34	22.81	26.51
2000	9000	6500	1	1.53	1.63	1.62	2.64	6.24	10.31	13.20	16.02	19.24	22.72	26.43
2000	9000	7000	1	1.48	1.60	1.57	2.81	6.27	10.21	13.09	15.93	19.15	22.63	26.35
2000	9000	7500	1	1.50	1.69	1.85	3.38	6.62	10.16	12.97	15.84	19.07	22.56	26.29
2000	9000	8000	1	1.53	1.88	2.52	4.35	7.29	10.15	12.83	15.76	19.00	22.49	26.23
2000	9000	9000	1	1.52	2.07	3.37	5.51	7.74	10.01	12.65	15.63	18.87	22.38	26.13
2000	9000	10000	1	1.60	2.14	3.32	5.29	7.52	9.90	12.56	15.52	18.77	22.29	26.04
2000	10000	500	1	5.82	8.70	11.60	14.13	16.68	19.20	21.68	24.12	26.50	28.77	30.87
2000	10000	1000	1	3.95	5.69	7.50	9.06	11.07	13.98	16.81	19.31	21.85	24.44	27.12
2000	10000	1500	1	3.12	4.24	5.25	6.41	8.68	11.80	14.59	17.08	19.70	22.45	25.34
2000	10000	2000	1	2.79	3.64	4.24	5.41	8.22	11.25	13.71	16.16	18.84	21.67	24.63
2000	10000	2500	1	2.60	3.23	3.23	4.33	7.75	10.89	13.21	15.62	18.33	21.21	24.22
2000	10000	3000	1	2.47	3.00	2.71	3.73	7.29	10.49	12.80	15.21	17.93	20.85	23.90

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2000	10000	3500	1	2.42	2.99	2.89	3.71	7.00	10.14	12.47	14.90	17.64	20.57	23.66
2000	10000	4000	1	2.18	2.64	2.44	3.20	6.35	9.76	12.25	14.67	17.41	20.35	23.47
2000	10000	4500	1	1.82	2.12	2.15	3.13	6.26	9.58	12.05	14.48	17.22	20.18	23.32
2000	10000	5000	1	1.45	1.75	2.22	3.77	6.66	9.50	11.83	14.31	17.07	20.04	23.19
2000	10000	5500	1	1.45	1.71	2.16	3.82	6.51	9.33	11.70	14.18	16.94	19.92	23.08
2000	10000	6000	1	1.56	1.73	1.89	3.11	5.91	9.15	11.63	14.08	16.83	19.81	23.00
2000	10000	6500	1	1.48	1.59	1.81	2.76	5.69	9.05	11.54	13.98	16.74	19.73	22.92
2000	10000	7000	1	1.44	1.59	1.88	3.03	5.80	8.97	11.44	13.89	16.66	19.65	22.85
2000	10000	7500	1	1.39	1.55	1.98	3.49	6.10	8.93	11.33	13.81	16.59	19.58	22.79
2000	10000	8000	1	1.24	1.43	2.06	3.97	6.53	8.94	11.22	13.74	16.52	19.53	22.74
2000	10000	9000	1	1.01	1.23	2.22	4.55	6.85	8.86	11.08	13.62	16.41	19.43	22.65
2000	10000	10000	1	1.23	1.54	2.49	4.48	6.67	8.74	10.99	13.53	16.32	19.35	22.58
2000	11000	500	1	5.43	8.08	10.61	12.75	14.93	17.11	19.25	21.35	23.39	25.34	27.14
2000	11000	1000	1	3.79	5.47	7.26	8.98	10.84	12.81	14.86	17.01	19.22	21.48	23.75
2000	11000	1500	1	3.00	4.22	5.68	7.22	8.93	10.80	12.82	14.99	17.28	19.66	22.12
2000	11000	2000	1	2.67	3.71	5.04	6.51	8.17	10.00	12.01	14.19	16.50	18.94	21.48
2000	11000	2500	1	2.47	3.41	4.67	6.11	7.73	9.54	11.54	13.72	16.05	18.52	21.11
2000	11000	3000	1	2.33	3.19	4.39	5.77	7.37	9.17	11.17	13.35	15.70	18.20	20.83
2000	11000	3500	1	2.21	3.01	4.14	5.47	7.08	8.90	10.89	13.07	15.43	17.94	20.60
2000	11000	4000	1	2.13	2.86	3.87	5.12	6.82	8.69	10.68	12.86	15.22	17.75	20.43

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
2000	11000	4500	1	2.05	2.74	3.69	4.93	6.64	8.52	10.50	12.68	15.05	17.59	20.29
2000	11000	5000	1	1.98	2.65	3.64	4.91	6.55	8.37	10.36	12.54	14.92	17.47	20.17
2000	11000	5500	1	1.92	2.55	3.49	4.79	6.44	8.25	10.24	12.42	14.80	17.36	20.08
2000	11000	6000	1	1.87	2.47	3.41	4.69	6.34	8.15	10.14	12.32	14.70	17.27	20.00
2000	11000	6500	1	1.84	2.38	3.39	4.61	6.24	8.07	10.05	12.23	14.62	17.19	19.92
2000	11000	7000	1	1.78	2.25	3.19	4.42	6.14	7.99	9.97	12.15	14.54	17.12	19.86
2000	11000	7500	1	1.61	2.03	2.80	4.31	6.12	7.94	9.90	12.09	14.48	17.06	19.81
2000	11000	8000	1	1.23	1.47	2.28	4.16	6.12	7.91	9.84	12.02	14.42	17.00	19.76
2000	11000	9000	1	1.00	1.00	1.77	3.95	6.08	7.85	9.73	11.92	14.32	16.91	19.68
2000	11000	10000	1	1.12	1.34	2.20	4.00	5.94	7.73	9.65	11.83	14.24	16.83	19.61
3000	500	500	1	70.55	116.91	162.11	215.31	307.85	393.61	453.16	504.80	555.89	602.09	641.55
3000	500	1000	1	42.62	75.04	115.30	162.38	225.41	287.14	342.94	398.46	455.52	512.31	567.66
3000	500	1500	1	34.72	61.17	99.36	137.08	185.62	238.85	293.35	350.22	409.86	471.12	533.23
3000	500	2000	1	29.97	48.00	93.83	125.99	168.31	220.17	274.63	331.79	392.36	455.31	520.01
3000	500	2500	1	26.67	43.03	86.65	117.15	158.88	209.68	263.74	321.08	382.22	446.26	512.62
3000	500	3000	1	20.82	38.63	73.62	108.19	152.52	202.01	255.24	312.71	374.31	439.19	506.75
3000	500	3500	1	17.36	33.78	64.50	102.06	147.36	196.00	248.82	306.38	368.30	433.79	502.23
3000	500	4000	1	15.86	30.96	59.61	97.97	143.05	191.23	243.91	301.54	363.71	429.65	498.77
3000	500	4500	1	15.24	29.79	57.62	95.31	139.55	187.40	240.01	297.69	360.05	426.35	496.02
3000	500	5000	1	13.38	28.59	57.50	94.56	136.94	184.22	236.80	294.54	357.06	423.65	493.77

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
3000	500	5500	1	13.46	26.71	56.35	93.01	134.62	181.60	234.13	291.91	354.56	421.40	491.89
3000	500	6000	1	13.28	25.12	54.42	91.02	132.55	179.40	231.87	289.67	352.44	419.48	490.29
3000	500	6500	1	13.47	25.65	53.97	89.54	130.73	177.49	229.92	287.74	350.61	417.83	488.90
3000	500	7000	1	13.61	26.14	54.01	88.45	129.15	175.81	228.22	286.07	349.01	416.39	487.71
3000	500	7500	1	13.32	25.71	53.28	87.32	127.80	174.38	226.76	284.64	347.67	415.21	486.76
3000	500	8000	1	12.99	25.17	52.44	86.24	126.57	173.08	225.42	283.32	346.41	414.06	485.76
3000	500	9000	1	12.26	23.97	50.73	84.21	124.27	170.54	222.72	280.53	343.60	411.26	483.00
3000	500	10000	1	11.79	23.19	49.52	82.67	122.50	168.63	220.75	278.54	341.67	409.44	481.35
3000	1000	500	1	41.35	69.69	105.08	141.74	193.79	243.10	280.57	313.33	345.00	375.05	402.77
3000	1000	1000	1	26.26	45.97	74.10	104.66	139.15	176.00	210.97	245.23	280.26	316.33	353.02
3000	1000	1500	1	20.88	38.00	62.11	86.77	114.19	146.42	180.06	214.81	251.28	289.79	330.14
3000	1000	2000	1	17.47	31.61	58.85	77.47	103.85	135.36	168.50	203.39	240.43	279.94	321.76
3000	1000	2500	1	16.50	28.66	53.94	72.28	97.89	128.79	161.71	196.70	234.08	274.26	317.09
3000	1000	3000	1	13.93	26.61	48.39	67.81	93.17	123.73	156.49	191.53	229.17	269.83	313.39
3000	1000	3500	1	12.31	24.77	44.67	64.67	89.90	119.94	152.54	187.63	225.46	266.47	310.56
3000	1000	4000	1	11.75	23.19	40.27	61.45	87.38	117.16	149.52	184.63	222.62	263.91	308.39
3000	1000	4500	1	11.29	22.26	38.25	59.40	85.32	114.86	147.10	182.25	220.36	261.86	306.67
3000	1000	5000	1	9.45	20.04	36.82	58.20	84.01	113.04	145.12	180.30	218.51	260.19	305.26
3000	1000	5500	1	9.74	18.96	36.05	57.15	82.65	111.46	143.47	178.67	216.97	258.79	304.07
3000	1000	6000	1	10.04	18.12	35.04	56.38	81.45	110.08	142.07	177.30	215.66	257.61	303.08

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
3000	1000	6500	1	9.78	17.45	34.25	55.54	80.38	108.90	140.87	176.12	214.54	256.59	302.23
3000	1000	7000	1	9.23	16.88	33.66	54.61	79.41	107.89	139.83	175.09	213.56	255.71	301.48
3000	1000	7500	1	8.71	16.31	33.17	53.85	78.51	106.90	138.79	174.04	212.52	254.71	300.55
3000	1000	8000	1	8.44	15.89	32.64	53.18	77.75	106.08	137.96	173.21	211.73	254.00	299.99
3000	1000	9000	1	8.15	15.47	32.00	52.48	77.10	105.58	137.68	173.23	212.15	254.96	301.64
3000	1000	10000	1	7.83	14.95	31.28	51.63	76.18	104.64	136.77	172.40	211.46	254.47	301.46
3000	2000	500	1	19.66	33.35	51.50	67.48	83.10	101.81	120.29	134.95	147.79	159.14	169.38
3000	2000	1000	1	13.45	24.46	39.29	53.37	62.83	76.52	91.40	105.98	120.69	135.18	149.27
3000	2000	1500	1	11.39	21.42	33.45	45.61	53.71	64.90	78.28	93.00	108.51	124.30	140.04
3000	2000	2000	1	10.91	18.16	30.44	39.15	48.52	59.70	73.09	87.99	103.75	119.98	136.42
3000	2000	2500	1	11.16	16.74	27.89	35.62	45.01	56.44	70.06	85.04	100.92	117.41	134.31
3000	2000	3000	1	8.87	15.73	26.23	33.17	42.02	53.88	67.79	82.78	98.74	115.42	132.66
3000	2000	3500	1	7.73	14.76	25.10	31.52	39.99	52.08	66.07	81.06	97.10	113.93	131.41
3000	2000	4000	1	7.63	14.46	23.17	29.69	38.71	50.88	64.74	79.72	95.83	112.79	130.44
3000	2000	4500	1	7.16	14.21	21.86	28.34	37.72	49.88	63.67	78.67	94.83	111.88	129.68
3000	2000	5000	1	6.28	13.36	18.07	26.20	36.81	49.16	62.82	77.80	94.01	111.14	129.05
3000	2000	5500	1	6.44	12.10	17.25	25.13	36.10	48.49	62.09	77.07	93.32	110.52	128.52
3000	2000	6000	1	6.34	12.07	17.51	25.32	35.67	47.85	61.46	76.46	92.73	109.99	128.07
3000	2000	6500	1	5.68	9.94	16.21	24.79	35.25	47.34	60.92	75.93	92.23	109.53	127.69
3000	2000	7000	1	5.10	8.64	15.29	24.17	34.77	46.87	60.46	75.47	91.79	109.14	127.35

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
3000	2000	7500	1	4.57	7.85	14.99	23.87	34.39	46.48	60.08	75.10	91.45	108.84	127.13
3000	2000	8000	1	4.39	7.55	14.73	23.58	34.05	46.12	59.71	74.73	91.10	108.51	126.82
3000	2000	9000	1	4.28	7.46	14.32	22.95	33.32	45.30	58.80	73.76	90.06	107.44	125.74
3000	2000	10000	1	4.09	7.13	13.93	22.51	32.81	44.74	58.22	73.16	89.47	106.86	125.20
3000	3000	500	1	15.40	24.23	34.63	43.72	52.94	66.21	78.60	87.70	95.66	102.06	107.09
3000	3000	1000	1	9.80	16.34	28.12	38.25	44.75	52.07	60.03	69.09	78.56	87.46	95.29
3000	3000	1500	1	7.74	13.46	23.97	33.37	39.23	44.48	51.43	60.76	70.83	80.71	89.83
3000	3000	2000	1	8.19	12.43	20.50	28.14	33.69	39.90	47.94	57.54	67.69	77.79	87.47
3000	3000	2500	1	8.52	12.37	19.16	24.56	30.29	37.20	45.92	55.59	65.79	76.02	86.04
3000	3000	3000	1	7.00	11.19	17.53	22.05	28.05	35.51	44.42	54.07	64.33	74.70	84.93
3000	3000	3500	1	6.08	9.91	15.90	20.43	26.61	34.39	43.28	52.91	63.23	73.70	84.09
3000	3000	4000	1	5.56	9.34	14.54	19.25	25.78	33.58	42.38	52.02	62.38	72.94	83.44
3000	3000	4500	1	5.22	9.18	13.60	18.38	25.12	32.90	41.67	51.32	61.71	72.33	82.93
3000	3000	5000	1	5.13	8.96	11.37	17.06	24.41	32.37	41.10	50.74	61.15	71.83	82.51
3000	3000	5500	1	4.83	8.62	10.38	16.20	23.90	31.90	40.61	50.26	60.70	71.41	82.16
3000	3000	6000	1	4.26	7.98	10.52	16.25	23.61	31.49	40.19	49.84	60.30	71.05	81.85
3000	3000	6500	1	3.74	6.45	10.03	16.04	23.33	31.13	39.83	49.49	59.96	70.74	81.59
3000	3000	7000	1	3.53	5.91	10.04	16.04	23.00	30.77	39.50	49.17	59.66	70.46	81.35
3000	3000	7500	1	3.46	5.63	10.24	16.01	22.75	30.53	39.29	48.98	59.50	70.36	81.34
3000	3000	8000	1	3.38	5.47	10.13	15.84	22.53	30.30	39.07	48.77	59.30	70.18	81.17

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
3000	3000	9000	1	3.24	5.30	9.59	15.13	21.76	29.38	37.96	47.49	57.83	68.40	78.90
3000	3000	10000	1	3.11	5.07	9.34	14.82	21.36	28.92	37.47	46.95	57.26	67.77	78.16
3000	4000	500	1	14.41	22.59	31.03	38.91	46.95	56.26	64.75	71.97	78.72	84.61	89.45
3000	4000	1000	1	9.20	15.07	24.63	33.07	39.62	43.61	48.74	56.14	63.99	71.66	78.58
3000	4000	1500	1	7.13	11.93	20.44	28.32	33.72	36.87	41.63	49.17	57.38	65.69	73.57
3000	4000	2000	1	7.05	10.66	17.37	23.65	27.88	32.57	38.89	46.61	54.79	63.19	71.56
3000	4000	2500	1	6.97	10.86	16.67	20.29	24.26	30.02	37.27	45.06	53.24	61.71	70.38
3000	4000	3000	1	5.60	9.11	13.95	17.68	22.56	28.83	36.06	43.79	52.03	60.62	69.45
3000	4000	3500	1	4.76	7.52	11.39	15.78	21.52	28.04	35.13	42.83	51.12	59.80	68.74
3000	4000	4000	1	4.23	6.62	9.95	14.60	20.70	27.36	34.40	42.10	50.43	59.17	68.20
3000	4000	4500	1	4.03	6.36	9.38	14.06	20.19	26.79	33.81	41.52	49.87	58.66	67.77
3000	4000	5000	1	4.10	6.35	8.77	13.58	19.85	26.33	33.32	41.04	49.42	58.25	67.42
3000	4000	5500	1	4.07	6.06	8.08	13.07	19.48	25.94	32.92	40.65	49.04	57.90	67.12
3000	4000	6000	1	3.47	5.38	7.56	12.36	18.93	25.62	32.60	40.31	48.71	57.61	66.87
3000	4000	6500	1	3.21	4.98	7.47	12.19	18.67	25.33	32.31	40.02	48.43	57.35	66.65
3000	4000	7000	1	3.08	4.87	8.05	12.88	18.71	25.03	32.02	39.76	48.19	57.13	66.45
3000	4000	7500	1	3.07	4.88	8.63	13.24	18.59	24.78	31.80	39.56	48.00	56.97	66.34
3000	4000	8000	1	3.02	4.79	8.59	13.13	18.41	24.59	31.61	39.37	47.83	56.81	66.21
3000	4000	9000	1	2.90	4.59	8.08	12.54	17.93	24.12	31.08	38.79	47.20	56.09	65.32
3000	4000	10000	1	2.80	4.44	7.93	12.36	17.66	23.81	30.75	38.46	46.87	55.75	64.96

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
3000	5000	500	1	12.11	19.44	27.55	34.65	41.32	47.27	53.06	58.93	64.50	69.58	73.93
3000	5000	1000	1	7.52	12.07	18.73	24.77	30.67	34.85	39.80	45.89	52.11	58.33	64.33
3000	5000	1500	1	5.77	9.19	14.90	19.94	24.91	28.99	33.93	40.08	46.55	53.22	59.91
3000	5000	2000	1	5.14	8.22	13.26	17.20	21.39	26.16	31.70	37.87	44.38	51.19	58.21
3000	5000	2500	1	4.71	7.71	12.31	15.47	19.41	24.56	30.36	36.53	43.09	50.00	57.21
3000	5000	3000	1	3.86	6.40	10.12	13.85	18.39	23.62	29.32	35.49	42.09	49.10	56.43
3000	5000	3500	1	3.39	5.41	8.18	12.35	17.51	22.91	28.55	34.70	41.34	48.41	55.84
3000	5000	4000	1	3.27	4.90	7.21	10.99	16.43	22.29	28.00	34.10	40.77	47.88	55.39
3000	5000	4500	1	3.33	4.92	7.08	10.62	15.99	21.82	27.51	33.62	40.31	47.46	55.02
3000	5000	5000	1	3.34	4.95	7.26	10.98	16.06	21.44	27.06	33.22	39.93	47.12	54.72
3000	5000	5500	1	3.31	4.69	6.61	10.52	15.73	21.11	26.73	32.90	39.62	46.83	54.48
3000	5000	6000	1	2.97	4.10	5.38	8.76	14.63	20.82	26.53	32.63	39.35	46.58	54.27
3000	5000	6500	1	2.86	3.94	5.20	8.46	14.34	20.59	26.30	32.38	39.12	46.37	54.08
3000	5000	7000	1	2.82	4.13	6.26	9.77	14.94	20.39	26.00	32.17	38.92	46.19	53.92
3000	5000	7500	1	2.77	4.25	7.18	10.81	15.19	20.17	25.77	31.98	38.74	46.02	53.78
3000	5000	8000	1	2.72	4.21	7.28	10.92	15.12	20.00	25.60	31.82	38.58	45.88	53.66
3000	5000	9000	1	2.61	4.00	6.81	10.38	14.72	19.73	25.34	31.55	38.33	45.65	53.47
3000	5000	10000	1	2.53	3.90	6.73	10.28	14.55	19.50	25.10	31.32	38.11	45.44	53.29
3000	6000	500	1	9.96	15.76	22.63	28.58	34.13	39.06	43.89	48.67	53.15	57.22	60.76

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
3000	6000	1000	1	5.78	8.70	12.96	17.55	22.67	27.93	33.01	37.91	42.84	47.78	52.70
3000	6000	1500	1	4.20	6.19	9.68	13.58	18.17	23.15	28.10	33.04	38.19	43.51	48.98
3000	6000	2000	1	3.50	5.46	8.95	12.68	16.94	21.46	26.16	31.11	36.36	41.85	47.52
3000	6000	2500	1	2.90	4.72	8.12	12.00	16.17	20.43	25.00	29.97	35.27	40.86	46.68
3000	6000	3000	1	2.61	4.22	7.19	11.17	15.38	19.57	24.12	29.10	34.44	40.10	46.02
3000	6000	3500	1	2.56	3.95	6.37	10.15	14.55	18.93	23.47	28.44	33.81	39.52	45.52
3000	6000	4000	1	2.72	3.92	5.77	8.74	13.47	18.42	23.02	27.94	33.32	39.07	45.13
3000	6000	4500	1	2.95	4.14	5.84	8.37	13.07	18.03	22.62	27.54	32.94	38.72	44.82
3000	6000	5000	1	2.94	4.22	6.25	8.97	13.21	17.69	22.23	27.20	32.62	38.43	44.57
3000	6000	5500	1	2.78	3.86	5.42	8.19	12.72	17.41	21.97	26.93	32.36	38.18	44.36
3000	6000	6000	1	2.65	3.37	3.98	6.37	11.65	17.18	21.82	26.70	32.13	37.97	44.18
3000	6000	6500	1	2.59	3.29	3.83	6.15	11.43	16.99	21.62	26.50	31.94	37.80	44.03
3000	6000	7000	1	2.57	3.50	4.66	7.19	11.98	16.83	21.37	26.32	31.77	37.64	43.89
3000	6000	7500	1	2.51	3.61	5.49	8.26	12.31	16.64	21.17	26.16	31.62	37.51	43.77
3000	6000	8000	1	2.47	3.66	5.93	8.82	12.43	16.48	21.01	26.02	31.49	37.38	43.66
3000	6000	9000	1	2.39	3.57	5.90	8.77	12.22	16.24	20.78	25.79	31.27	37.18	43.50
3000	6000	10000	1	2.32	3.45	5.75	8.59	12.05	16.05	20.58	25.60	31.08	37.01	43.36
3000	7000	500	1	8.96	13.82	19.35	24.14	28.72	33.08	37.24	41.17	44.86	48.21	51.12
3000	7000	1000	1	5.60	8.23	11.57	15.37	19.50	23.82	27.94	31.96	36.05	40.15	44.19
3000	7000	1500	1	4.19	6.02	8.68	11.96	15.70	19.73	23.74	27.81	32.08	36.48	40.99

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
3000	7000	2000	1	3.54	5.18	7.80	11.04	14.55	18.22	22.07	26.17	30.51	35.05	39.73
3000	7000	2500	1	3.04	4.49	7.04	10.38	13.81	17.28	21.08	25.19	29.58	34.19	38.99
3000	7000	3000	1	2.84	4.20	6.58	9.74	13.08	16.54	20.32	24.44	28.86	33.53	38.42
3000	7000	3500	1	2.72	3.93	5.94	8.85	12.41	16.00	19.76	23.88	28.32	33.03	37.99
3000	7000	4000	1	2.69	3.72	5.04	7.33	11.49	15.60	19.37	23.44	27.90	32.65	37.65
3000	7000	4500	1	2.71	3.68	4.80	6.84	11.09	15.27	19.03	23.10	27.57	32.35	37.38
3000	7000	5000	1	2.66	3.62	4.87	7.01	10.98	14.97	18.72	22.82	27.30	32.09	37.16
3000	7000	5500	1	2.44	3.10	3.67	5.66	10.01	14.66	18.55	22.59	27.07	31.88	36.98
3000	7000	6000	1	2.33	2.78	2.94	4.73	9.40	14.46	18.40	22.39	26.87	31.70	36.83
3000	7000	6500	1	2.29	2.75	2.86	4.59	9.29	14.31	18.22	22.21	26.71	31.55	36.69
3000	7000	7000	1	2.20	2.70	2.87	4.64	9.17	14.11	18.06	22.06	26.56	31.41	36.57
3000	7000	7500	1	2.21	2.81	3.30	5.22	9.45	14.02	17.88	21.93	26.44	31.30	36.47
3000	7000	8000	1	2.27	3.09	4.28	6.43	10.08	13.93	17.70	21.80	26.32	31.19	36.38
3000	7000	9000	1	2.26	3.31	5.36	7.74	10.48	13.70	17.44	21.60	26.12	31.01	36.21
3000	7000	10000	1	2.17	3.13	5.01	7.35	10.20	13.54	17.28	21.43	25.96	30.86	36.08
3000	8000	500	1	8.31	12.66	17.27	21.21	25.01	28.68	32.16	35.47	38.58	41.40	43.84
3000	8000	1000	1	5.52	8.22	11.34	14.35	17.52	20.76	24.05	27.45	30.90	34.36	37.75
3000	8000	1500	1	4.23	6.17	8.60	11.17	14.09	17.18	20.40	23.83	27.43	31.15	34.93
3000	8000	2000	1	3.69	5.33	7.49	9.90	12.73	15.75	18.96	22.41	26.07	29.88	33.82
3000	8000	2500	1	3.36	4.81	6.76	9.08	11.89	14.90	18.10	21.56	25.25	29.13	33.17

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →										
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
3000	8000	3000	1	3.11	4.43	6.24	8.45	11.25	14.26	17.44	20.90	24.62	28.55	32.66
3000	8000	3500	1	2.88	4.05	5.63	7.75	10.71	13.78	16.95	20.41	24.14	28.11	32.28
3000	8000	4000	1	2.67	3.62	4.55	6.33	9.95	13.44	16.60	20.03	23.78	27.77	31.98
3000	8000	4500	1	2.52	3.36	4.13	5.84	9.59	13.15	16.31	19.73	23.49	27.50	31.74
3000	8000	5000	1	2.40	3.17	4.04	5.93	9.49	12.89	16.04	19.48	23.25	27.28	31.55
3000	8000	5500	1	2.18	2.64	2.92	4.68	8.56	12.62	15.90	19.28	23.05	27.09	31.39
3000	8000	6000	1	2.10	2.43	2.46	4.00	8.08	12.45	15.76	19.10	22.88	26.94	31.25
3000	8000	6500	1	2.06	2.41	2.36	3.78	7.92	12.31	15.61	18.95	22.73	26.80	31.13
3000	8000	7000	1	1.98	2.28	2.10	3.49	7.61	12.13	15.49	18.82	22.60	26.68	31.03
3000	8000	7500	1	2.02	2.38	2.40	3.87	7.82	12.06	15.34	18.70	22.49	26.58	30.93
3000	8000	8000	1	2.09	2.70	3.42	5.18	8.56	11.99	15.15	18.59	22.39	26.49	30.85
3000	8000	9000	1	2.10	3.00	4.71	6.81	9.12	11.77	14.91	18.41	22.22	26.33	30.71
3000	8000	10000	1	2.02	2.83	4.34	6.37	8.82	11.63	14.78	18.27	22.08	26.20	30.59
3000	9000	500	1	7.70	11.61	15.53	18.86	22.10	25.19	28.15	30.97	33.61	36.01	38.10
3000	9000	1000	1	5.15	7.58	10.26	12.78	15.46	18.23	21.01	23.90	26.83	29.78	32.66
3000	9000	1500	1	4.00	5.73	7.67	9.73	12.33	15.07	17.80	20.70	23.76	26.93	30.14
3000	9000	2000	1	3.53	4.98	6.59	8.43	11.07	13.81	16.53	19.44	22.55	25.80	29.15
3000	9000	2500	1	3.25	4.49	5.73	7.37	10.21	13.06	15.78	18.69	21.82	25.13	28.57
3000	9000	3000	1	3.02	4.12	5.17	6.73	9.63	12.50	15.19	18.11	21.27	24.61	28.11
3000	9000	3500	1	2.79	3.80	4.85	6.42	9.24	12.06	14.75	17.67	20.84	24.22	27.77

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
3000	9000	4000	1	2.59	3.42	3.98	5.32	8.60	11.73	14.43	17.33	20.52	23.92	27.50
3000	9000	4500	1	2.38	3.08	3.54	4.94	8.28	11.46	14.16	17.06	20.26	23.68	27.29
3000	9000	5000	1	2.13	2.75	3.54	5.36	8.44	11.25	13.91	16.84	20.05	23.48	27.11
3000	9000	5500	1	1.96	2.40	2.99	4.87	8.05	11.07	13.75	16.66	19.87	23.32	26.97
3000	9000	6000	1	1.97	2.29	2.52	4.03	7.44	10.90	13.64	16.51	19.72	23.17	26.84
3000	9000	6500	1	1.90	2.17	2.07	3.42	6.94	10.73	13.53	16.38	19.59	23.05	26.73
3000	9000	7000	1	1.85	2.08	1.85	3.09	6.63	10.57	13.43	16.26	19.48	22.94	26.64
3000	9000	7500	1	1.91	2.17	2.20	3.48	6.88	10.50	13.30	16.15	19.37	22.85	26.56
3000	9000	8000	1	1.88	2.31	2.82	4.50	7.52	10.46	13.13	16.05	19.29	22.77	26.48
3000	9000	9000	1	1.75	2.36	3.56	5.80	8.03	10.28	12.92	15.90	19.14	22.63	26.36
3000	9000	10000	1	1.77	2.34	3.41	5.44	7.75	10.15	12.80	15.77	19.01	22.52	26.26
3000	10000	500	1	7.17	10.74	14.13	16.97	19.75	22.40	24.93	27.35	29.62	31.68	33.48
3000	10000	1000	1	4.78	6.97	9.28	11.44	13.76	16.17	18.58	21.05	23.57	26.10	28.59
3000	10000	1500	1	3.73	5.26	6.93	8.69	10.95	13.34	15.69	18.18	20.81	23.53	26.29
3000	10000	2000	1	3.30	4.59	5.96	7.53	9.83	12.22	14.55	17.05	19.72	22.51	25.39
3000	10000	2500	1	3.03	4.15	5.15	6.51	9.05	11.56	13.88	16.38	19.07	21.91	24.86
3000	10000	3000	1	2.81	3.79	4.60	5.91	8.52	11.05	13.36	15.86	18.57	21.45	24.45
3000	10000	3500	1	2.63	3.54	4.40	5.74	8.19	10.65	12.96	15.46	18.19	21.09	24.14
3000	10000	4000	1	2.47	3.24	3.79	4.95	7.66	10.34	12.67	15.16	17.90	20.82	23.90
3000	10000	4500	1	2.29	2.94	3.41	4.64	7.39	10.09	12.43	14.92	17.67	20.60	23.70

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
3000	10000	5000	1	2.03	2.62	3.42	5.06	7.57	9.91	12.20	14.72	17.48	20.42	23.54
3000	10000	5500	1	1.89	2.39	3.18	4.97	7.44	9.75	12.04	14.56	17.32	20.28	23.41
3000	10000	6000	1	1.91	2.30	2.74	4.18	6.87	9.60	11.94	14.42	17.18	20.15	23.30
3000	10000	6500	1	1.84	2.12	2.33	3.57	6.37	9.44	11.85	14.30	17.06	20.04	23.20
3000	10000	7000	1	1.80	2.06	2.20	3.38	6.16	9.30	11.75	14.20	16.96	19.94	23.12
3000	10000	7500	1	1.78	2.04	2.33	3.65	6.36	9.25	11.64	14.10	16.87	19.86	23.04
3000	10000	8000	1	1.54	1.80	2.32	4.14	6.75	9.22	11.50	14.01	16.79	19.78	22.97
3000	10000	9000	1	1.14	1.37	2.31	4.80	7.14	9.12	11.32	13.87	16.66	19.66	22.86
3000	10000	10000	1	1.33	1.61	2.42	4.58	6.89	8.98	11.22	13.75	16.54	19.56	22.77
3000	11000	500	1	6.70	10.01	12.95	15.34	17.71	20.01	22.19	24.27	26.22	28.00	29.54
3000	11000	1000	1	4.54	6.58	8.58	10.42	12.39	14.42	16.51	18.64	20.81	22.98	25.12
3000	11000	1500	1	3.50	4.96	6.53	8.14	9.92	11.84	13.88	16.04	18.30	20.63	23.01
3000	11000	2000	1	3.07	4.30	5.72	7.24	8.95	10.82	12.85	15.02	17.31	19.71	22.18
3000	11000	2500	1	2.82	3.91	5.25	6.71	8.38	10.23	12.24	14.42	16.73	19.17	21.71
3000	11000	3000	1	2.62	3.62	4.88	6.30	7.93	9.76	11.77	13.95	16.28	18.75	21.34
3000	11000	3500	1	2.47	3.39	4.60	5.97	7.59	9.41	11.41	13.59	15.94	18.43	21.05
3000	11000	4000	1	2.35	3.21	4.36	5.70	7.32	9.14	11.14	13.32	15.68	18.18	20.83
3000	11000	4500	1	2.26	3.06	4.17	5.49	7.11	8.93	10.92	13.10	15.46	17.99	20.65
3000	11000	5000	1	2.18	2.94	4.03	5.34	6.95	8.75	10.74	12.92	15.29	17.82	20.51
3000	11000	5500	1	2.11	2.83	3.86	5.16	6.78	8.60	10.59	12.77	15.15	17.69	20.39

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
3000	11000	6000	1	2.06	2.75	3.76	5.03	6.65	8.48	10.46	12.65	15.02	17.57	20.28
3000	11000	6500	1	2.02	2.69	3.67	4.90	6.54	8.37	10.35	12.54	14.92	17.47	20.19
3000	11000	7000	1	1.97	2.57	3.46	4.70	6.41	8.28	10.26	12.44	14.82	17.39	20.12
3000	11000	7500	1	1.79	2.29	3.07	4.58	6.38	8.21	10.17	12.36	14.74	17.31	20.05
3000	11000	8000	1	1.40	1.69	2.51	4.41	6.38	8.17	10.09	12.28	14.67	17.25	19.98
3000	11000	9000	1	1.00	1.00	1.82	4.15	6.32	8.09	9.97	12.15	14.55	17.13	19.88
3000	11000	10000	1	1.14	1.32	2.05	4.07	6.16	7.96	9.86	12.04	14.44	17.03	19.79
4000	500	500	1	88.17	145.31	208.39	268.82	354.69	437.09	498.21	550.19	598.97	640.78	673.84
4000	500	1000	1	52.11	89.95	139.88	189.28	249.65	310.76	367.09	422.60	478.50	533.09	585.13
4000	500	1500	1	39.82	70.41	112.49	151.89	199.99	253.26	308.00	364.83	423.81	483.74	543.82
4000	500	2000	1	36.26	63.46	102.69	135.18	178.99	231.29	285.92	342.98	403.04	464.97	528.11
4000	500	2500	1	31.63	56.01	92.87	124.83	167.94	219.03	273.12	330.37	391.10	454.33	519.41
4000	500	3000	1	25.50	46.99	80.16	115.21	159.98	209.74	263.12	320.54	381.79	446.00	512.50
4000	500	3500	1	21.70	40.71	71.62	108.45	153.53	202.52	255.59	313.12	374.74	439.64	507.18
4000	500	4000	1	19.26	35.94	65.46	103.33	148.41	196.98	249.86	307.45	369.36	434.79	503.12
4000	500	4500	1	17.89	33.14	62.01	99.90	144.37	192.54	245.30	302.95	365.09	430.93	499.90
4000	500	5000	1	16.35	31.04	61.29	98.63	141.29	188.85	241.57	299.29	361.60	427.79	497.28
4000	500	5500	1	15.74	29.49	60.02	96.70	138.55	185.81	238.48	296.23	358.70	425.17	495.09
4000	500	6000	1	15.28	28.36	58.16	94.45	136.13	183.26	235.86	293.64	356.24	422.95	493.22
4000	500	6500	1	15.00	28.08	57.12	92.66	134.05	181.06	233.61	291.41	354.12	421.03	491.62

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
4000	500	7000	1	14.61	27.79	56.39	91.24	132.26	179.15	231.65	289.48	352.28	419.37	490.24
4000	500	7500	1	14.15	27.10	55.37	89.89	130.71	177.50	229.97	287.83	350.73	418.00	489.13
4000	500	8000	1	13.75	26.44	54.37	88.65	129.31	176.00	228.43	286.31	349.28	416.67	487.98
4000	500	9000	1	12.99	25.19	52.51	86.36	126.69	173.12	225.39	283.18	346.13	413.56	484.95
4000	500	10000	1	12.42	24.25	51.10	84.60	124.68	170.96	223.14	280.92	343.94	411.51	483.10
4000	1000	500	1	53.70	90.48	138.36	180.10	227.20	271.86	308.87	341.82	372.26	399.87	423.69
4000	1000	1000	1	32.43	56.83	91.12	122.72	156.21	191.29	225.95	260.32	294.73	329.58	364.31
4000	1000	1500	1	24.58	44.29	72.37	97.51	124.45	155.58	189.17	224.03	260.13	297.89	337.08
4000	1000	2000	1	21.61	37.09	66.36	87.10	111.71	142.29	175.56	210.50	247.25	286.19	327.14
4000	1000	2500	1	19.52	33.22	59.79	79.63	104.10	134.58	167.62	202.61	239.76	279.48	321.61
4000	1000	3000	1	17.04	31.27	53.93	72.99	97.92	128.56	161.49	196.52	233.97	274.25	317.24
4000	1000	3500	1	15.23	28.85	49.72	68.62	93.68	124.08	156.86	191.94	229.61	270.30	313.91
4000	1000	4000	1	14.51	26.72	44.82	64.71	90.62	120.81	153.32	188.42	226.28	267.29	311.36
4000	1000	4500	1	14.35	25.76	42.09	62.23	88.22	118.12	150.50	185.64	223.63	264.89	309.33
4000	1000	5000	1	12.35	23.68	40.47	60.83	86.68	115.99	148.19	183.37	221.47	262.94	307.68
4000	1000	5500	1	12.08	21.89	38.83	59.54	85.13	114.15	146.27	181.47	219.67	261.31	306.30
4000	1000	6000	1	11.91	20.93	37.69	58.55	83.73	112.56	144.65	179.88	218.16	259.94	305.15
4000	1000	6500	1	11.18	19.99	36.76	57.57	82.50	111.20	143.26	178.51	216.85	258.76	304.15
4000	1000	7000	1	10.10	18.55	35.46	56.41	81.41	110.03	142.05	177.32	215.73	257.74	303.28
4000	1000	7500	1	9.24	17.24	34.54	55.51	80.38	108.91	140.87	176.12	214.55	256.61	302.24

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
4000	1000	8000	1	8.89	16.66	33.88	54.74	79.51	107.97	139.92	175.17	213.64	255.80	301.60
4000	1000	9000	1	8.66	16.31	33.18	53.88	78.68	107.28	139.45	175.01	213.89	256.60	303.11
4000	1000	10000	1	8.25	15.66	32.32	52.90	77.61	106.18	138.38	174.02	213.04	255.97	302.81
4000	2000	500	1	24.15	41.14	66.19	86.69	101.51	116.55	132.60	147.00	159.59	170.13	178.40
4000	2000	1000	1	15.20	27.79	47.14	62.81	72.50	83.55	97.68	112.53	127.06	141.05	154.19
4000	2000	1500	1	12.62	24.39	38.41	50.55	58.45	68.48	82.08	97.11	112.45	127.87	143.09
4000	2000	2000	1	12.04	20.34	33.90	42.92	51.26	62.28	76.18	91.20	106.79	122.71	138.77
4000	2000	2500	1	12.74	18.45	30.42	38.03	46.65	58.65	72.75	87.70	103.44	119.69	136.27
4000	2000	3000	1	10.27	17.43	29.24	35.31	43.58	55.91	70.06	85.00	100.86	117.36	134.32
4000	2000	3500	1	8.95	16.17	27.77	33.54	41.62	53.91	68.00	82.97	98.93	115.60	132.84
4000	2000	4000	1	9.08	16.05	25.35	31.40	40.18	52.49	66.44	81.40	97.44	114.26	131.70
4000	2000	4500	1	9.38	16.50	23.91	29.93	39.02	51.30	65.18	80.17	96.26	113.20	130.80
4000	2000	5000	1	9.06	15.89	21.60	28.10	37.96	50.41	64.18	79.15	95.30	112.33	130.06
4000	2000	5500	1	8.29	14.69	19.80	26.59	37.16	49.66	63.33	78.31	94.50	111.60	129.45
4000	2000	6000	1	7.76	13.60	18.99	26.18	36.61	48.96	62.61	77.59	93.82	110.99	128.93
4000	2000	6500	1	6.53	11.93	17.89	25.65	36.12	48.36	61.99	76.98	93.23	110.45	128.48
4000	2000	7000	1	5.59	9.96	16.36	24.97	35.65	47.83	61.44	76.45	92.73	109.99	128.09
4000	2000	7500	1	4.83	8.35	15.63	24.64	35.24	47.38	61.00	76.02	92.33	109.64	127.81
4000	2000	8000	1	4.59	7.90	15.28	24.30	34.85	46.96	60.57	75.59	91.92	109.26	127.47
4000	2000	9000	1	4.55	7.88	14.88	23.59	34.02	46.04	59.56	74.52	90.79	108.10	126.32

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →										
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
4000	2000	10000	1	4.30	7.48	14.41	23.08	33.45	45.41	58.91	73.85	90.12	107.46	125.72
4000	3000	500	1	18.09	29.33	43.42	55.49	65.61	76.17	86.55	95.52	103.41	109.30	112.89
4000	3000	1000	1	10.86	18.77	32.60	43.32	49.10	55.43	64.07	73.55	82.83	91.28	98.53
4000	3000	1500	1	8.84	15.41	26.72	35.37	40.03	45.60	53.97	63.65	73.51	83.03	91.86
4000	3000	2000	1	9.24	14.46	22.53	28.43	34.16	41.22	50.09	59.73	69.74	79.59	89.02
4000	3000	2500	1	10.13	14.62	21.29	25.17	30.80	38.67	47.77	57.38	67.48	77.54	87.32
4000	3000	3000	1	8.23	12.75	20.07	23.48	28.94	36.88	45.95	55.57	65.76	75.98	86.01
4000	3000	3500	1	6.92	10.87	18.18	22.16	27.69	35.57	44.58	54.20	64.46	74.81	85.01
4000	3000	4000	1	6.35	10.25	16.04	20.45	26.71	34.63	43.53	53.16	63.46	73.91	84.25
4000	3000	4500	1	6.34	10.52	15.04	19.40	25.90	33.83	42.69	52.33	62.67	73.19	83.65
4000	3000	5000	1	6.54	10.83	13.85	18.42	25.16	33.19	42.01	51.65	62.02	72.61	83.15
4000	3000	5500	1	6.35	10.69	13.04	17.56	24.60	32.66	41.45	51.09	61.49	72.12	82.74
4000	3000	6000	1	5.53	9.56	12.33	17.11	24.24	32.21	40.96	50.60	61.03	71.70	82.38
4000	3000	6500	1	4.35	8.03	11.47	16.73	23.91	31.82	40.54	50.19	60.63	71.34	82.07
4000	3000	7000	1	3.91	6.80	10.81	16.59	23.59	31.42	40.17	49.83	60.28	71.02	81.80
4000	3000	7500	1	3.67	5.98	10.66	16.51	23.33	31.14	39.92	49.60	60.08	70.88	81.76
4000	3000	8000	1	3.56	5.73	10.49	16.32	23.08	30.88	39.65	49.34	59.84	70.67	81.56
4000	3000	9000	1	3.45	5.61	10.00	15.58	22.23	29.87	38.46	47.98	58.29	68.79	79.19
4000	3000	10000	1	3.27	5.33	9.67	15.21	21.79	29.36	37.91	47.38	57.66	68.11	78.41

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
4000	4000	500	1	16.11	25.81	36.60	45.80	54.96	63.73	71.60	78.79	85.32	90.68	94.55
4000	4000	1000	1	9.76	16.29	27.07	35.73	41.00	45.81	52.42	60.07	67.63	74.85	81.43
4000	4000	1500	1	8.06	13.17	22.07	29.00	33.14	37.42	43.93	51.70	59.66	67.63	75.37
4000	4000	2000	1	8.40	12.96	19.17	23.67	27.97	33.64	40.75	48.49	56.55	64.75	72.95
4000	4000	2500	1	8.78	13.13	18.39	20.87	25.02	31.47	38.85	46.56	54.69	63.05	71.53
4000	4000	3000	1	7.00	10.54	15.87	18.95	23.55	30.07	37.36	45.07	53.26	61.75	70.43
4000	4000	3500	1	5.62	8.54	13.36	17.41	22.58	29.05	36.23	43.94	52.19	60.78	69.59
4000	4000	4000	1	4.79	7.54	11.47	15.98	21.75	28.27	35.36	43.08	51.36	60.03	68.95
4000	4000	4500	1	4.60	7.37	10.71	15.22	21.11	27.61	34.68	42.39	50.71	59.44	68.44
4000	4000	5000	1	4.82	7.66	10.46	14.81	20.61	27.06	34.11	41.83	50.18	58.95	68.02
4000	4000	5500	1	4.84	7.78	10.18	14.37	20.17	26.60	33.64	41.37	49.73	58.54	67.68
4000	4000	6000	1	4.19	7.09	9.52	13.74	19.74	26.23	33.25	40.97	49.36	58.20	67.38
4000	4000	6500	1	3.78	6.24	8.93	13.39	19.45	25.90	32.90	40.63	49.03	57.90	67.12
4000	4000	7000	1	3.45	5.58	8.87	13.58	19.28	25.58	32.59	40.34	48.74	57.64	66.90
4000	4000	7500	1	3.27	5.17	8.97	13.63	19.06	25.32	32.35	40.10	48.52	57.45	66.76
4000	4000	8000	1	3.18	4.99	8.85	13.46	18.85	25.09	32.12	39.88	48.32	57.26	66.61
4000	4000	9000	1	3.08	4.88	8.45	12.95	18.36	24.56	31.53	39.24	47.63	56.48	65.65
4000	4000	10000	1	2.95	4.67	8.23	12.70	18.04	24.20	31.16	38.86	47.25	56.10	65.25
4000	5000	500	1	13.60	21.79	31.09	38.81	46.16	52.77	58.93	64.76	70.07	74.66	78.35
4000	5000	1000	1	8.41	13.56	20.94	27.55	32.62	37.34	43.00	49.11	55.15	61.08	66.78

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a →										
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
4000	5000	1500	1	6.62	10.50	16.48	21.77	25.90	30.33	35.92	42.11	48.45	54.94	61.46
4000	5000	2000	1	6.18	9.74	14.71	18.53	22.47	27.47	33.25	39.41	45.86	52.55	59.41
4000	5000	2500	1	5.89	9.26	13.79	16.67	20.55	25.82	31.65	37.81	44.32	51.15	58.22
4000	5000	3000	1	4.97	7.77	11.73	15.07	19.39	24.66	30.41	36.57	43.14	50.07	57.30
4000	5000	3500	1	4.28	6.67	10.03	13.84	18.52	23.78	29.47	35.64	42.25	49.25	56.59
4000	5000	4000	1	3.89	5.98	8.97	12.88	17.76	23.09	28.77	34.93	41.57	48.63	56.05
4000	5000	4500	1	3.73	5.72	8.56	12.40	17.24	22.53	28.19	34.36	41.03	48.13	55.62
4000	5000	5000	1	3.63	5.62	8.48	12.26	16.91	22.07	27.72	33.90	40.59	47.73	55.27
4000	5000	5500	1	3.62	5.55	8.17	11.88	16.53	21.69	27.33	33.51	40.22	47.39	54.98
4000	5000	6000	1	3.46	5.33	7.53	11.06	16.02	21.38	27.02	33.19	39.90	47.10	54.73
4000	5000	6500	1	3.31	5.02	7.21	10.76	15.74	21.11	26.74	32.91	39.63	46.85	54.52
4000	5000	7000	1	3.07	4.70	7.44	11.12	15.73	20.85	26.47	32.66	39.40	46.64	54.33
4000	5000	7500	1	2.94	4.51	7.56	11.25	15.61	20.62	26.24	32.45	39.19	46.45	54.16
4000	5000	8000	1	2.87	4.39	7.47	11.14	15.45	20.43	26.05	32.25	39.01	46.28	54.02
4000	5000	9000	1	2.77	4.26	7.16	10.76	15.11	20.12	25.73	31.94	38.71	46.00	53.79
4000	5000	10000	1	2.67	4.10	6.99	10.57	14.88	19.85	25.46	31.67	38.45	45.77	53.59
4000	6000	500	1	11.74	18.47	26.08	32.46	38.35	43.76	48.83	53.56	57.83	61.53	64.52
4000	6000	1000	1	7.32	11.23	16.11	21.11	25.99	30.81	35.67	40.53	45.39	50.17	54.78
4000	6000	1500	1	5.39	8.17	12.04	16.22	20.49	25.01	29.76	34.68	39.79	45.02	50.30
4000	6000	2000	1	4.53	6.97	10.63	14.39	18.41	22.78	27.46	32.41	37.62	43.02	48.56

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
4000	6000	2500	1	3.96	6.16	9.65	13.24	17.19	21.46	26.09	31.06	36.32	41.84	47.55
4000	6000	3000	1	3.65	5.65	8.70	12.26	16.21	20.44	25.04	30.02	35.33	40.93	46.77
4000	6000	3500	1	3.45	5.28	8.02	11.47	15.44	19.68	24.26	29.24	34.59	40.24	46.17
4000	6000	4000	1	3.34	5.01	7.61	10.81	14.77	19.08	23.67	28.65	34.01	39.72	45.71
4000	6000	4500	1	3.26	4.84	7.41	10.45	14.32	18.61	23.19	28.17	33.56	39.30	45.35
4000	6000	5000	1	3.09	4.60	7.23	10.31	14.05	18.22	22.79	27.78	33.19	38.95	45.05
4000	6000	5500	1	2.97	4.35	6.77	9.82	13.68	17.91	22.47	27.46	32.88	38.67	44.80
4000	6000	6000	1	2.92	4.19	6.20	9.08	13.22	17.65	22.21	27.19	32.61	38.43	44.59
4000	6000	6500	1	2.85	4.09	6.04	8.86	12.99	17.42	21.97	26.95	32.38	38.22	44.40
4000	6000	7000	1	2.76	4.03	6.21	9.10	12.97	17.21	21.74	26.74	32.19	38.03	44.25
4000	6000	7500	1	2.69	3.97	6.30	9.24	12.90	17.01	21.55	26.56	32.01	37.87	44.10
4000	6000	8000	1	2.63	3.91	6.31	9.26	12.80	16.85	21.39	26.40	31.86	37.73	43.98
4000	6000	9000	1	2.53	3.77	6.14	9.05	12.54	16.58	21.11	26.13	31.60	37.49	43.78
4000	6000	10000	1	2.44	3.65	5.98	8.86	12.33	16.36	20.89	25.91	31.39	37.30	43.61
4000	7000	500	1	10.61	16.40	22.62	27.83	32.70	37.25	41.48	45.37	48.90	51.95	54.39
4000	7000	1000	1	6.76	10.18	14.13	18.07	22.15	26.23	30.24	34.26	38.27	42.22	46.02
4000	7000	1500	1	4.99	7.39	10.48	13.83	17.47	21.27	25.18	29.26	33.48	37.80	42.15
4000	7000	2000	1	4.22	6.25	9.12	12.30	15.73	19.35	23.21	27.30	31.61	36.08	40.64
4000	7000	2500	1	3.73	5.54	8.23	11.34	14.67	18.21	22.03	26.14	30.50	35.06	39.77
4000	7000	3000	1	3.47	5.14	7.64	10.59	13.83	17.32	21.13	25.25	29.64	34.27	39.09

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
4000	7000	3500	1	3.28	4.82	7.19	10.00	13.19	16.66	20.46	24.58	29.00	33.67	38.57
4000	7000	4000	1	3.14	4.58	6.84	9.48	12.66	16.15	19.94	24.07	28.51	33.22	38.17
4000	7000	4500	1	3.01	4.37	6.57	9.11	12.27	15.75	19.53	23.66	28.11	32.86	37.85
4000	7000	5000	1	2.89	4.15	6.24	8.76	11.93	15.42	19.20	23.33	27.79	32.56	37.59
4000	7000	5500	1	2.76	3.85	5.53	7.93	11.45	15.16	18.94	23.05	27.52	32.31	37.37
4000	7000	6000	1	2.68	3.66	5.09	7.39	11.11	14.94	18.71	22.81	27.30	32.10	37.19
4000	7000	6500	1	2.61	3.58	5.00	7.27	10.94	14.74	18.50	22.61	27.10	31.92	37.03
4000	7000	7000	1	2.54	3.51	4.86	7.12	10.77	14.55	18.32	22.43	26.93	31.76	36.89
4000	7000	7500	1	2.48	3.48	5.02	7.34	10.74	14.39	18.15	22.27	26.78	31.62	36.77
4000	7000	8000	1	2.45	3.50	5.29	7.67	10.79	14.24	18.00	22.14	26.65	31.50	36.66
4000	7000	9000	1	2.37	3.45	5.45	7.85	10.69	13.99	17.75	21.90	26.42	31.28	36.46
4000	7000	10000	1	2.29	3.31	5.24	7.60	10.47	13.80	17.55	21.70	26.23	31.11	36.31
4000	8000	500	1	9.74	14.89	20.10	24.43	28.52	32.34	35.89	39.17	42.14	44.70	46.76
4000	8000	1000	1	6.32	9.45	12.92	16.13	19.43	22.75	26.10	29.49	32.87	36.19	39.39
4000	8000	1500	1	4.73	6.95	9.64	12.37	15.33	18.43	21.70	25.13	28.69	32.32	35.98
4000	8000	2000	1	4.08	5.94	8.35	10.90	13.71	16.74	19.98	23.43	27.05	30.80	34.65
4000	8000	2500	1	3.69	5.35	7.58	10.02	12.74	15.73	18.95	22.41	26.07	29.90	33.87
4000	8000	3000	1	3.39	4.89	7.01	9.35	12.01	14.95	18.16	21.63	25.32	29.21	33.26
4000	8000	3500	1	3.16	4.54	6.56	8.84	11.46	14.37	17.57	21.04	24.76	28.69	32.80
4000	8000	4000	1	2.98	4.27	6.18	8.38	11.02	13.93	17.12	20.59	24.32	28.29	32.45

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
4000	8000	4500	1	2.84	4.05	5.87	8.04	10.67	13.58	16.76	20.23	23.98	27.96	32.17
4000	8000	5000	1	2.72	3.84	5.53	7.67	10.36	13.29	16.46	19.94	23.69	27.70	31.93
4000	8000	5500	1	2.60	3.55	4.84	6.88	9.91	13.06	16.24	19.69	23.46	27.48	31.74
4000	8000	6000	1	2.52	3.38	4.47	6.42	9.63	12.87	16.04	19.48	23.26	27.30	31.58
4000	8000	6500	1	2.45	3.29	4.36	6.27	9.46	12.70	15.86	19.31	23.09	27.14	31.44
4000	8000	7000	1	2.37	3.18	4.12	5.98	9.25	12.54	15.70	19.15	22.94	27.00	31.31
4000	8000	7500	1	2.33	3.15	4.29	6.20	9.23	12.39	15.56	19.01	22.80	26.88	31.21
4000	8000	8000	1	2.30	3.18	4.62	6.61	9.32	12.26	15.41	18.89	22.69	26.77	31.11
4000	8000	9000	1	2.22	3.15	4.81	6.88	9.27	12.04	15.19	18.68	22.49	26.58	30.94
4000	8000	10000	1	2.14	3.02	4.59	6.63	9.07	11.87	15.02	18.51	22.32	26.43	30.80
4000	9000	500	1	9.03	13.64	18.08	21.73	25.21	28.46	31.47	34.26	36.79	38.97	40.73
4000	9000	1000	1	5.90	8.71	11.67	14.41	17.21	20.01	22.84	25.72	28.60	31.43	34.15
4000	9000	1500	1	4.45	6.44	8.73	11.06	13.57	16.19	18.95	21.87	24.90	27.99	31.10
4000	9000	2000	1	3.85	5.53	7.57	9.72	12.11	14.67	17.43	20.36	23.44	26.63	29.90
4000	9000	2500	1	3.49	4.98	6.87	8.91	11.23	13.77	16.51	19.45	22.57	25.83	29.20
4000	9000	3000	1	3.21	4.56	6.34	8.30	10.58	13.09	15.82	18.76	21.90	25.21	28.66
4000	9000	3500	1	2.99	4.23	5.93	7.85	10.09	12.57	15.29	18.24	21.40	24.74	28.25
4000	9000	4000	1	2.83	3.97	5.57	7.43	9.69	12.18	14.89	17.84	21.01	24.39	27.93
4000	9000	4500	1	2.70	3.77	5.29	7.13	9.38	11.86	14.57	17.52	20.71	24.10	27.67
4000	9000	5000	1	2.57	3.57	5.05	6.91	9.15	11.60	14.30	17.26	20.46	23.87	27.47

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
4000	9000	5500	1	2.44	3.32	4.63	6.51	8.89	11.40	14.09	17.04	20.24	23.67	27.29
4000	9000	6000	1	2.39	3.19	4.31	6.07	8.59	11.23	13.92	16.86	20.07	23.50	27.14
4000	9000	6500	1	2.33	3.08	4.02	5.67	8.33	11.07	13.76	16.70	19.91	23.36	27.02
4000	9000	7000	1	2.27	2.97	3.78	5.36	8.13	10.94	13.63	16.56	19.78	23.24	26.91
4000	9000	7500	1	2.23	2.94	3.89	5.51	8.11	10.81	13.50	16.44	19.66	23.13	26.81
4000	9000	8000	1	2.17	2.90	4.05	5.83	8.19	10.70	13.37	16.33	19.56	23.03	26.72
4000	9000	9000	1	2.01	2.72	3.96	5.96	8.16	10.51	13.18	16.14	19.38	22.86	26.57
4000	9000	10000	1	1.95	2.63	3.81	5.74	7.98	10.36	13.03	15.99	19.23	22.73	26.45
4000	10000	500	1	8.41	12.64	16.46	19.57	22.55	25.33	27.92	30.32	32.49	34.36	35.88
4000	10000	1000	1	5.53	8.08	10.64	12.99	15.40	17.81	20.24	22.71	25.18	27.61	29.95
4000	10000	1500	1	4.19	5.99	7.97	9.96	12.12	14.37	16.74	19.24	21.84	24.50	27.18
4000	10000	2000	1	3.63	5.16	6.92	8.76	10.81	13.00	15.37	17.88	20.53	23.27	26.08
4000	10000	2500	1	3.29	4.65	6.29	8.02	10.02	12.20	14.55	17.08	19.75	22.55	25.45
4000	10000	3000	1	3.03	4.26	5.79	7.46	9.43	11.59	13.93	16.46	19.15	22.00	24.96
4000	10000	3500	1	2.83	3.95	5.41	7.06	8.99	11.12	13.46	15.99	18.70	21.58	24.59
4000	10000	4000	1	2.69	3.72	5.08	6.67	8.62	10.77	13.09	15.63	18.35	21.25	24.29
4000	10000	4500	1	2.57	3.54	4.84	6.41	8.34	10.48	12.80	15.34	18.08	20.99	24.06
4000	10000	5000	1	2.45	3.36	4.66	6.26	8.15	10.25	12.57	15.11	17.85	20.78	23.87
4000	10000	5500	1	2.33	3.17	4.40	6.06	7.98	10.06	12.37	14.91	17.66	20.60	23.72
4000	10000	6000	1	2.29	3.05	4.12	5.66	7.70	9.90	12.21	14.74	17.50	20.45	23.58

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
4000	10000	6500	1	2.23	2.94	3.83	5.27	7.45	9.76	12.08	14.60	17.36	20.32	23.47
4000	10000	7000	1	2.18	2.84	3.64	5.03	7.28	9.64	11.96	14.48	17.24	20.21	23.37
4000	10000	7500	1	2.13	2.77	3.64	5.10	7.26	9.53	11.84	14.37	17.13	20.11	23.28
4000	10000	8000	1	1.99	2.57	3.49	5.18	7.30	9.44	11.73	14.27	17.04	20.02	23.20
4000	10000	9000	1	1.66	2.10	2.99	5.08	7.26	9.30	11.56	14.10	16.88	19.88	23.07
4000	10000	10000	1	1.64	2.08	2.94	4.91	7.09	9.17	11.42	13.96	16.75	19.75	22.96
4000	11000	500	1	7.85	11.78	15.10	17.71	20.27	22.67	24.90	26.96	28.82	30.43	31.73
4000	11000	1000	1	5.21	7.60	9.80	11.76	13.82	15.92	18.03	20.15	22.27	24.36	26.38
4000	11000	1500	1	3.95	5.62	7.32	8.99	10.84	12.79	14.85	17.01	19.24	21.52	23.83
4000	11000	2000	1	3.43	4.83	6.34	7.91	9.67	11.57	13.61	15.77	18.05	20.41	22.82
4000	11000	2500	1	3.12	4.37	5.78	7.28	8.99	10.86	12.89	15.06	17.36	19.77	22.26
4000	11000	3000	1	2.89	4.01	5.34	6.79	8.46	10.31	12.33	14.50	16.82	19.26	21.81
4000	11000	3500	1	2.70	3.73	5.01	6.42	8.06	9.89	11.90	14.08	16.41	18.88	21.47
4000	11000	4000	1	2.56	3.52	4.75	6.13	7.75	9.56	11.57	13.75	16.09	18.58	21.20
4000	11000	4500	1	2.45	3.35	4.54	5.90	7.50	9.31	11.31	13.49	15.84	18.35	20.99
4000	11000	5000	1	2.36	3.21	4.36	5.70	7.30	9.10	11.09	13.28	15.64	18.16	20.82
4000	11000	5500	1	2.28	3.09	4.21	5.53	7.12	8.92	10.92	13.10	15.46	17.99	20.67
4000	11000	6000	1	2.21	2.99	4.09	5.39	6.98	8.77	10.76	12.95	15.32	17.86	20.55
4000	11000	6500	1	2.16	2.92	3.99	5.28	6.85	8.64	10.63	12.82	15.19	17.74	20.44
4000	11000	7000	1	2.11	2.84	3.88	5.16	6.74	8.53	10.52	12.71	15.08	17.64	20.35

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
4000	11000	7500	1	2.04	2.71	3.70	5.05	6.65	8.43	10.42	12.61	14.99	17.55	20.27
4000	11000	8000	1	1.86	2.40	3.29	4.86	6.59	8.36	10.33	12.52	14.90	17.47	20.19
4000	11000	9000	1	1.48	1.79	2.52	4.46	6.47	8.26	10.18	12.36	14.75	17.33	20.07
4000	11000	10000	1	1.48	1.81	2.52	4.36	6.34	8.13	10.05	12.24	14.63	17.22	19.97
5000	500	500	1	97.68	165.67	254.15	328.48	405.49	477.71	538.33	591.37	638.24	676.04	702.91
5000	500	1000	1	57.88	100.25	159.95	214.65	273.65	332.74	389.21	444.81	499.63	552.11	600.94
5000	500	1500	1	40.39	71.42	117.90	162.56	213.00	266.46	321.25	377.93	436.25	494.91	553.02
5000	500	2000	1	34.82	62.13	103.70	142.61	189.46	241.51	295.85	352.81	412.40	473.34	534.90
5000	500	2500	1	31.12	56.77	94.42	131.66	176.93	227.61	281.46	338.65	398.97	461.39	525.15
5000	500	3000	1	29.27	53.09	85.22	121.58	166.79	216.73	270.22	327.55	388.43	451.93	517.35
5000	500	3500	1	27.19	49.30	78.95	114.36	158.94	208.39	261.68	319.12	380.41	444.69	511.30
5000	500	4000	1	22.89	41.70	71.64	108.57	153.25	202.13	255.17	312.69	374.31	439.19	506.68
5000	500	4500	1	19.79	36.02	66.36	104.42	148.80	197.14	250.02	307.60	369.47	434.82	503.04
5000	500	5000	1	18.16	33.53	64.80	102.33	145.21	193.00	245.81	303.46	365.54	431.27	500.06
5000	500	5500	1	17.42	32.24	63.31	99.97	142.07	189.58	242.33	300.01	362.26	428.31	497.59
5000	500	6000	1	16.92	31.33	61.38	97.49	139.35	186.71	239.38	297.10	359.49	425.80	495.48
5000	500	6500	1	16.33	30.42	59.83	95.44	137.03	184.24	236.85	294.59	357.10	423.65	493.68
5000	500	7000	1	15.52	29.35	58.53	93.74	135.03	182.10	234.65	292.42	355.04	421.79	492.13
5000	500	7500	1	14.88	28.37	57.28	92.22	133.31	180.25	232.77	290.57	353.30	420.24	490.87
5000	500	8000	1	14.41	27.59	56.14	90.83	131.74	178.57	231.04	288.86	351.66	418.74	489.56

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
5000	500	9000	1	13.62	26.29	54.12	88.28	128.81	175.36	227.65	285.38	348.17	415.30	486.23
5000	500	10000	1	12.96	25.21	52.53	86.31	126.57	172.95	225.15	282.86	345.73	413.00	484.15
5000	1000	500	1	61.72	105.66	166.02	214.22	258.18	299.13	336.14	369.46	398.82	424.24	444.63
5000	1000	1000	1	36.80	64.31	103.06	137.11	171.22	206.24	240.98	275.35	309.25	343.09	376.21
5000	1000	1500	1	26.18	46.55	76.50	103.95	132.84	164.70	198.36	233.18	269.04	306.28	344.58
5000	1000	2000	1	22.51	38.84	67.98	91.82	118.41	149.31	182.63	217.62	254.22	292.79	333.09
5000	1000	2500	1	20.47	35.53	62.05	84.07	109.88	140.48	173.61	208.68	245.74	285.20	326.86
5000	1000	3000	1	19.97	35.74	58.91	77.80	102.71	133.45	166.62	201.75	239.14	279.23	321.88
5000	1000	3500	1	19.11	34.47	55.33	72.96	97.61	128.26	161.33	196.51	234.16	274.71	318.06
5000	1000	4000	1	17.87	31.11	48.62	67.85	94.01	124.57	157.31	192.51	230.35	271.27	315.15
5000	1000	4500	1	17.24	29.06	44.53	64.70	91.31	121.55	154.10	189.34	227.35	268.55	312.84
5000	1000	5000	1	14.58	26.10	42.51	63.23	89.50	119.09	151.48	186.76	224.90	266.33	310.96
5000	1000	5500	1	13.51	23.78	40.73	61.81	87.73	117.01	149.31	184.62	222.86	264.48	309.40
5000	1000	6000	1	13.49	23.15	39.56	60.43	86.08	115.22	147.48	182.81	221.14	262.92	308.08
5000	1000	6500	1	12.67	22.09	38.43	59.24	84.67	113.69	145.91	181.26	219.66	261.58	306.95
5000	1000	7000	1	10.99	19.91	36.99	58.17	83.49	112.36	144.55	179.92	218.39	260.42	305.97
5000	1000	7500	1	9.73	18.11	35.92	57.23	82.36	111.09	143.23	178.58	217.07	259.17	304.82
5000	1000	8000	1	9.28	17.41	35.17	56.38	81.39	110.05	142.16	177.53	216.07	258.28	304.12
5000	1000	9000	1	9.15	17.14	34.38	55.35	80.39	109.21	141.54	177.23	216.20	258.98	305.54
5000	1000	10000	1	8.65	16.37	33.41	54.26	79.20	107.97	140.33	176.10	215.22	258.24	305.14

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
5000	2000	500	1	28.13	47.86	75.78	97.96	114.36	129.07	144.05	158.09	170.23	179.85	186.46
5000	2000	1000	1	17.40	30.87	50.37	65.97	78.20	89.93	103.91	118.55	132.77	146.22	158.52
5000	2000	1500	1	13.82	25.07	39.41	50.95	60.64	71.88	85.84	100.73	115.85	130.88	145.61
5000	2000	2000	1	12.70	20.95	34.66	43.85	52.72	64.79	79.03	93.93	109.35	124.97	140.64
5000	2000	2500	1	12.72	18.95	31.47	39.23	48.11	60.83	75.10	89.97	105.56	121.57	137.82
5000	2000	3000	1	11.15	18.61	31.68	37.09	45.08	57.73	72.00	86.90	102.64	118.93	135.60
5000	2000	3500	1	10.53	17.88	30.23	35.11	42.95	55.45	69.66	84.59	100.43	116.92	133.91
5000	2000	4000	1	11.20	18.28	26.78	32.30	41.28	53.87	67.89	82.81	98.75	115.40	132.62
5000	2000	4500	1	11.88	19.04	25.03	30.62	40.00	52.54	66.47	81.41	97.41	114.19	131.59
5000	2000	5000	1	10.78	17.15	22.55	28.91	38.96	51.53	65.33	80.26	96.32	113.21	130.75
5000	2000	5500	1	9.11	15.27	20.27	27.36	38.13	50.68	64.37	79.31	95.42	112.39	130.06
5000	2000	6000	1	8.83	14.34	19.15	26.59	37.44	49.91	63.55	78.50	94.65	111.69	129.47
5000	2000	6500	1	7.65	13.09	18.32	26.09	36.86	49.23	62.85	77.81	93.99	111.09	128.96
5000	2000	7000	1	6.16	10.64	16.98	25.61	36.39	48.63	62.24	77.21	93.42	110.58	128.52
5000	2000	7500	1	5.03	8.71	16.20	25.31	35.97	48.13	61.74	76.72	92.97	110.18	128.20
5000	2000	8000	1	4.72	8.18	15.81	24.94	35.53	47.65	61.25	76.24	92.50	109.74	127.80
5000	2000	9000	1	4.81	8.28	15.37	24.13	34.61	46.65	60.16	75.08	91.29	108.51	126.61
5000	2000	10000	1	4.49	7.79	14.85	23.58	33.98	45.95	59.43	74.34	90.55	107.81	125.96
5000	3000	500	1	20.34	33.49	49.56	63.04	74.49	84.21	93.36	102.08	109.66	114.77	116.86

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
5000	3000	1000	1	12.22	20.70	33.88	44.19	51.56	58.95	67.87	77.19	86.19	94.11	100.56
5000	3000	1500	1	9.74	16.36	26.76	34.36	40.17	47.30	56.29	65.83	75.46	84.57	92.89
5000	3000	2000	1	9.69	15.17	23.00	28.46	34.60	42.65	51.80	61.31	71.14	80.68	89.63
5000	3000	2500	1	10.19	15.04	21.98	25.83	31.66	40.00	49.13	58.64	68.60	78.38	87.72
5000	3000	3000	1	8.55	13.21	21.22	24.48	29.81	37.95	47.05	56.59	66.64	76.61	86.23
5000	3000	3500	1	7.29	11.26	19.43	23.04	28.45	36.44	45.49	55.05	65.17	75.27	85.10
5000	3000	4000	1	6.92	10.90	17.11	21.19	27.34	35.38	44.31	53.86	64.04	74.26	84.24
5000	3000	4500	1	7.14	11.52	16.21	20.15	26.47	34.48	43.36	52.92	63.14	73.45	83.56
5000	3000	5000	1	7.18	11.52	14.88	19.16	25.72	33.77	42.60	52.16	62.41	72.79	83.00
5000	3000	5500	1	6.96	11.10	13.68	18.18	25.14	33.19	41.96	51.52	61.81	72.24	82.53
5000	3000	6000	1	6.27	10.06	12.74	17.61	24.70	32.69	41.41	50.98	61.29	71.77	82.13
5000	3000	6500	1	5.03	8.83	12.09	17.23	24.32	32.24	40.94	50.51	60.85	71.36	81.78
5000	3000	7000	1	4.31	7.29	11.35	17.02	23.99	31.82	40.52	50.11	60.45	71.00	81.48
5000	3000	7500	1	3.84	6.24	11.01	16.89	23.73	31.51	40.24	49.85	60.23	70.85	81.43
5000	3000	8000	1	3.68	5.93	10.79	16.67	23.45	31.21	39.94	49.55	59.95	70.59	81.18
5000	3000	9000	1	3.62	5.87	10.28	15.86	22.49	30.08	38.61	48.03	58.21	68.49	78.53
5000	3000	10000	1	3.40	5.53	9.90	15.44	21.99	29.51	37.99	47.36	57.51	67.72	77.62
5000	4000	500	1	17.72	28.77	41.45	51.70	61.59	70.14	77.77	84.89	91.14	95.89	98.75
5000	4000	1000	1	10.50	17.30	27.30	35.61	41.92	48.50	55.98	63.51	70.82	77.65	83.76
5000	4000	1500	1	8.47	13.55	21.35	27.50	32.65	38.77	46.15	53.82	61.60	69.30	76.75

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
5000	4000	2000	1	8.61	13.31	19.19	23.53	28.47	35.01	42.40	50.06	58.01	66.02	73.96
5000	4000	2500	1	8.95	13.45	18.69	21.45	26.12	32.84	40.19	47.85	55.90	64.12	72.35
5000	4000	3000	1	7.29	10.90	16.41	19.76	24.58	31.21	38.47	46.15	54.28	62.64	71.10
5000	4000	3500	1	5.85	8.86	14.16	18.27	23.46	29.99	37.18	44.86	53.05	61.53	70.14
5000	4000	4000	1	4.95	7.81	12.46	17.02	22.59	29.07	36.19	43.88	52.12	60.67	69.41
5000	4000	4500	1	4.75	7.64	11.71	16.25	21.87	28.31	35.40	43.11	51.38	60.00	68.83
5000	4000	5000	1	5.05	8.05	11.44	15.68	21.26	27.69	34.77	42.47	50.77	59.44	68.35
5000	4000	5500	1	5.16	8.24	11.07	15.17	20.76	27.18	34.24	41.95	50.27	58.98	67.96
5000	4000	6000	1	4.70	7.82	10.54	14.70	20.36	26.75	33.78	41.50	49.84	58.59	67.62
5000	4000	6500	1	4.30	7.12	10.03	14.34	20.02	26.37	33.40	41.12	49.47	58.25	67.33
5000	4000	7000	1	3.80	6.14	9.57	14.15	19.73	26.03	33.06	40.78	49.15	57.96	67.07
5000	4000	7500	1	3.44	5.43	9.29	13.98	19.48	25.76	32.78	40.51	48.90	57.74	66.92
5000	4000	8000	1	3.30	5.19	9.10	13.78	19.24	25.51	32.53	40.27	48.67	57.54	66.75
5000	4000	9000	1	3.24	5.13	8.77	13.30	18.70	24.89	31.85	39.53	47.88	56.63	65.61
5000	4000	10000	1	3.07	4.88	8.48	12.98	18.33	24.49	31.43	39.10	47.44	56.19	65.14
5000	5000	500	1	15.19	24.35	34.76	43.07	50.90	57.97	64.38	70.16	75.23	79.39	82.46
5000	5000	1000	1	9.30	14.88	22.21	29.06	34.47	40.05	46.06	52.08	57.98	63.70	69.09
5000	5000	1500	1	7.11	11.21	16.84	22.29	26.81	31.95	37.82	43.93	50.20	56.54	62.89
5000	5000	2000	1	6.60	10.32	15.03	19.19	23.54	28.83	34.68	40.81	47.21	53.80	60.51
5000	5000	2500	1	6.37	9.83	14.12	17.44	21.67	27.01	32.84	38.99	45.46	52.21	59.17

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
5000	5000	3000	1	5.46	8.35	12.26	15.83	20.35	25.66	31.42	37.57	44.11	50.98	58.11
5000	5000	3500	1	4.72	7.23	10.80	14.72	19.39	24.63	30.34	36.51	43.10	50.04	57.30
5000	5000	4000	1	4.23	6.52	10.00	14.11	18.73	23.83	29.51	35.70	42.32	49.33	56.68
5000	5000	4500	1	3.93	6.08	9.52	13.60	18.13	23.19	28.86	35.05	41.70	48.76	56.19
5000	5000	5000	1	3.74	5.83	9.21	13.11	17.58	22.67	28.34	34.53	41.20	48.30	55.79
5000	5000	5500	1	3.74	5.81	8.98	12.74	17.17	22.23	27.90	34.09	40.78	47.91	55.45
5000	5000	6000	1	3.76	5.94	8.83	12.50	16.90	21.87	27.51	33.72	40.42	47.59	55.17
5000	5000	6500	1	3.66	5.75	8.57	12.23	16.60	21.56	27.19	33.40	40.12	47.31	54.93
5000	5000	7000	1	3.31	5.16	8.19	11.91	16.27	21.28	26.92	33.13	39.85	47.06	54.71
5000	5000	7500	1	3.10	4.76	7.87	11.60	16.00	21.04	26.68	32.88	39.62	46.84	54.53
5000	5000	8000	1	3.00	4.59	7.68	11.40	15.79	20.83	26.46	32.66	39.41	46.65	54.36
5000	5000	9000	1	2.92	4.50	7.47	11.11	15.47	20.47	26.10	32.31	39.07	46.35	54.10
5000	5000	10000	1	2.79	4.30	7.24	10.85	15.18	20.18	25.80	32.01	38.78	46.08	53.88
5000	6000	500	1	13.34	21.03	29.42	36.22	42.46	48.20	53.41	58.09	62.18	65.54	68.02
5000	6000	1000	1	8.50	13.24	18.67	23.86	28.71	33.40	38.16	43.01	47.79	52.41	56.75
5000	6000	1500	1	6.20	9.55	13.72	18.06	22.29	26.64	31.29	36.21	41.28	46.41	51.52
5000	6000	2000	1	5.25	8.02	11.76	15.60	19.63	23.98	28.65	33.60	38.77	44.09	49.51
5000	6000	2500	1	4.69	7.15	10.61	14.16	18.11	22.44	27.10	32.07	37.30	42.75	48.37
5000	6000	3000	1	4.32	6.55	9.63	13.07	16.99	21.27	25.91	30.88	36.16	41.71	47.47
5000	6000	3500	1	4.03	6.10	9.02	12.39	16.19	20.39	25.00	29.99	35.31	40.92	46.78

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →										
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
5000	6000	4000	1	3.77	5.73	8.73	12.08	15.66	19.70	24.30	29.31	34.65	40.32	46.25
5000	6000	4500	1	3.50	5.32	8.35	11.66	15.15	19.16	23.76	28.77	34.13	39.84	45.84
5000	6000	5000	1	3.24	4.90	7.85	11.11	14.66	18.73	23.32	28.32	33.71	39.45	45.49
5000	6000	5500	1	3.12	4.68	7.56	10.78	14.32	18.37	22.95	27.95	33.36	39.12	45.21
5000	6000	6000	1	3.08	4.66	7.47	10.66	14.13	18.06	22.62	27.64	33.06	38.84	44.97
5000	6000	6500	1	3.01	4.57	7.31	10.45	13.88	17.80	22.35	27.37	32.80	38.61	44.76
5000	6000	7000	1	2.91	4.39	7.04	10.12	13.57	17.57	22.12	27.14	32.58	38.40	44.58
5000	6000	7500	1	2.85	4.25	6.79	9.81	13.32	17.37	21.92	26.94	32.38	38.22	44.42
5000	6000	8000	1	2.78	4.14	6.60	9.59	13.13	17.19	21.74	26.75	32.20	38.05	44.27
5000	6000	9000	1	2.65	3.97	6.38	9.32	12.84	16.89	21.43	26.45	31.91	37.79	44.06
5000	6000	10000	1	2.55	3.82	6.19	9.10	12.60	16.64	21.18	26.20	31.67	37.57	43.87
5000	7000	500	1	12.02	18.71	25.62	31.22	36.38	41.11	45.40	49.27	52.65	55.41	57.43
5000	7000	1000	1	7.70	11.78	16.25	20.30	24.40	28.40	32.40	36.41	40.35	44.16	47.74
5000	7000	1500	1	5.62	8.46	11.87	15.29	18.91	22.62	26.52	30.59	34.78	39.01	43.23
5000	7000	2000	1	4.75	7.07	10.11	13.31	16.74	20.38	24.25	28.34	32.62	37.01	41.48
5000	7000	2500	1	4.23	6.28	9.09	12.14	15.48	19.07	22.92	27.02	31.35	35.85	40.48
5000	7000	3000	1	3.91	5.80	8.43	11.32	14.53	18.06	21.89	26.00	30.37	34.95	39.70
5000	7000	3500	1	3.67	5.44	8.04	10.81	13.84	17.29	21.11	25.24	29.63	34.27	39.10
5000	7000	4000	1	3.46	5.17	7.88	10.66	13.44	16.69	20.50	24.65	29.07	33.75	38.64
5000	7000	4500	1	3.26	4.87	7.55	10.31	13.02	16.23	20.03	24.18	28.62	33.33	38.28

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
5000	7000	5000	1	3.09	4.56	7.03	9.74	12.56	15.86	19.66	23.80	28.25	32.99	37.98
5000	7000	5500	1	2.96	4.35	6.65	9.30	12.28	15.57	19.33	23.48	27.95	32.71	37.73
5000	7000	6000	1	2.86	4.19	6.43	9.08	12.07	15.30	19.06	23.21	27.69	32.47	37.52
5000	7000	6500	1	2.78	4.06	6.30	8.93	11.85	15.07	18.82	22.98	27.46	32.26	37.34
5000	7000	7000	1	2.72	3.99	6.12	8.69	11.64	14.87	18.62	22.78	27.27	32.08	37.18
5000	7000	7500	1	2.66	3.91	6.06	8.59	11.45	14.69	18.45	22.60	27.10	31.92	37.04
5000	7000	8000	1	2.60	3.79	5.88	8.36	11.23	14.53	18.30	22.45	26.95	31.79	36.92
5000	7000	9000	1	2.47	3.59	5.58	8.00	10.91	14.27	18.03	22.17	26.69	31.54	36.69
5000	7000	10000	1	2.38	3.47	5.44	7.83	10.72	14.05	17.80	21.95	26.47	31.34	36.52
5000	8000	500	1	11.01	16.94	22.74	27.41	31.76	35.73	39.34	42.59	45.44	47.76	49.46
5000	8000	1000	1	7.04	10.60	14.39	17.75	21.18	24.61	28.03	31.40	34.71	37.91	40.93
5000	8000	1500	1	5.19	7.67	10.57	13.41	16.41	19.58	22.90	26.32	29.84	33.40	36.95
5000	8000	2000	1	4.44	6.51	9.08	11.74	14.58	17.64	20.91	24.35	27.95	31.64	35.40
5000	8000	2500	1	3.98	5.83	8.25	10.78	13.51	16.50	19.74	23.19	26.83	30.62	34.51
5000	8000	3000	1	3.65	5.32	7.63	10.05	12.67	15.61	18.84	22.30	25.97	29.82	33.82
5000	8000	3500	1	3.41	4.97	7.23	9.58	12.07	14.94	18.15	21.62	25.32	29.22	33.29
5000	8000	4000	1	3.24	4.75	7.10	9.47	11.73	14.41	17.62	21.11	24.83	28.76	32.88
5000	8000	4500	1	3.09	4.52	6.82	9.18	11.37	14.01	17.20	20.70	24.43	28.39	32.56
5000	8000	5000	1	2.95	4.28	6.36	8.62	10.94	13.68	16.88	20.36	24.11	28.09	32.29
5000	8000	5500	1	2.84	4.09	5.99	8.21	10.69	13.43	16.59	20.08	23.84	27.84	32.07

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
5000	8000	6000	1	2.76	3.94	5.75	7.96	10.48	13.20	16.35	19.85	23.61	27.63	31.88
5000	8000	6500	1	2.67	3.81	5.61	7.81	10.29	12.99	16.15	19.64	23.41	27.45	31.72
5000	8000	7000	1	2.59	3.71	5.45	7.63	10.13	12.82	15.97	19.46	23.24	27.29	31.58
5000	8000	7500	1	2.52	3.62	5.44	7.60	9.97	12.66	15.81	19.31	23.09	27.15	31.46
5000	8000	8000	1	2.45	3.50	5.27	7.38	9.76	12.52	15.68	19.17	22.96	27.03	31.35
5000	8000	9000	1	2.33	3.31	4.97	7.00	9.46	12.29	15.45	18.93	22.73	26.81	31.15
5000	8000	10000	1	2.25	3.20	4.84	6.86	9.30	12.10	15.25	18.74	22.54	26.64	31.00
5000	9000	500	1	10.21	15.54	20.47	24.40	28.10	31.48	34.55	37.32	39.74	41.72	43.17
5000	9000	1000	1	6.57	9.76	12.97	15.83	18.72	21.64	24.57	27.44	30.26	32.98	35.55
5000	9000	1500	1	4.85	7.08	9.59	12.04	14.53	17.19	20.03	22.95	25.94	28.96	31.99
5000	9000	2000	1	4.15	6.01	8.30	10.62	12.93	15.47	18.26	21.20	24.25	27.40	30.59
5000	9000	2500	1	3.72	5.40	7.64	9.89	12.03	14.46	17.22	20.16	23.26	26.48	29.80
5000	9000	3000	1	3.41	4.93	7.10	9.26	11.29	13.66	16.41	19.37	22.50	25.77	29.17
5000	9000	3500	1	3.19	4.60	6.63	8.70	10.71	13.07	15.81	18.77	21.92	25.23	28.69
5000	9000	4000	1	3.04	4.39	6.46	8.53	10.39	12.62	15.34	18.31	21.47	24.82	28.33
5000	9000	4500	1	2.92	4.22	6.24	8.27	10.08	12.26	14.97	17.95	21.12	24.49	28.04
5000	9000	5000	1	2.84	4.06	5.88	7.78	9.66	11.96	14.68	17.65	20.83	24.22	27.80
5000	9000	5500	1	2.73	3.88	5.57	7.44	9.42	11.73	14.43	17.40	20.59	24.00	27.60
5000	9000	6000	1	2.64	3.73	5.36	7.22	9.24	11.53	14.21	17.19	20.39	23.81	27.43
5000	9000	6500	1	2.57	3.62	5.17	7.01	9.06	11.35	14.03	17.00	20.21	23.65	27.28

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
5000	9000	7000	1	2.50	3.51	5.01	6.82	8.93	11.20	13.87	16.85	20.06	23.50	27.15
5000	9000	7500	1	2.42	3.39	4.90	6.73	8.79	11.06	13.73	16.71	19.93	23.38	27.04
5000	9000	8000	1	2.35	3.28	4.76	6.56	8.59	10.93	13.61	16.58	19.81	23.27	26.94
5000	9000	9000	1	2.22	3.07	4.41	6.19	8.32	10.72	13.41	16.38	19.60	23.08	26.77
5000	9000	10000	1	2.13	2.94	4.28	6.06	8.17	10.55	13.23	16.20	19.44	22.92	26.63
5000	10000	500	1	9.51	14.40	18.67	21.99	25.15	28.06	30.70	33.07	35.15	36.86	38.11
5000	10000	1000	1	6.18	9.11	11.87	14.31	16.79	19.29	21.80	24.27	26.68	29.02	31.24
5000	10000	1500	1	4.58	6.63	8.78	10.89	13.01	15.28	17.71	20.22	22.79	25.39	27.99
5000	10000	2000	1	3.92	5.63	7.60	9.59	11.56	13.73	16.13	18.65	21.27	23.97	26.72
5000	10000	2500	1	3.53	5.07	7.03	8.96	10.76	12.82	15.19	17.73	20.39	23.15	26.00
5000	10000	3000	1	3.24	4.63	6.53	8.39	10.10	12.11	14.47	17.02	19.70	22.51	25.44
5000	10000	3500	1	3.03	4.31	6.05	7.84	9.57	11.58	13.93	16.48	19.18	22.03	25.00
5000	10000	4000	1	2.88	4.10	5.84	7.61	9.25	11.18	13.51	16.06	18.78	21.65	24.67
5000	10000	4500	1	2.77	3.94	5.64	7.37	8.97	10.86	13.18	15.73	18.46	21.36	24.40
5000	10000	5000	1	2.71	3.81	5.36	6.96	8.60	10.58	12.92	15.46	18.20	21.11	24.18
5000	10000	5500	1	2.61	3.65	5.11	6.68	8.36	10.37	12.69	15.24	17.98	20.91	24.00
5000	10000	6000	1	2.52	3.52	4.93	6.49	8.21	10.19	12.50	15.05	17.80	20.74	23.85
5000	10000	6500	1	2.46	3.42	4.74	6.30	8.05	10.03	12.33	14.88	17.64	20.59	23.71
5000	10000	7000	1	2.39	3.31	4.57	6.11	7.92	9.89	12.19	14.74	17.50	20.46	23.60
5000	10000	7500	1	2.32	3.20	4.45	6.01	7.80	9.77	12.06	14.61	17.38	20.35	23.50

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a →										
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
5000	10000	8000	1	2.26	3.08	4.29	5.86	7.64	9.65	11.95	14.50	17.27	20.24	23.41
5000	10000	9000	1	2.08	2.79	3.82	5.48	7.40	9.47	11.77	14.32	17.09	20.08	23.26
5000	10000	10000	1	1.98	2.65	3.69	5.35	7.27	9.32	11.61	14.16	16.94	19.94	23.13
5000	11000	500	1	8.86	13.41	17.14	19.93	22.63	25.14	27.41	29.45	31.24	32.70	33.77
5000	11000	1000	1	5.81	8.55	10.97	13.02	15.17	17.32	19.45	21.57	23.65	25.66	27.56
5000	11000	1500	1	4.35	6.24	8.05	9.79	11.68	13.67	15.75	17.90	20.11	22.35	24.58
5000	11000	2000	1	3.75	5.32	6.91	8.53	10.33	12.26	14.31	16.47	18.73	21.05	23.40
5000	11000	2500	1	3.39	4.79	6.28	7.81	9.55	11.45	13.48	15.65	17.94	20.32	22.76
5000	11000	3000	1	3.12	4.38	5.77	7.25	8.95	10.81	12.84	15.01	17.32	19.74	22.25
5000	11000	3500	1	2.91	4.06	5.39	6.82	8.48	10.33	12.35	14.52	16.85	19.30	21.85
5000	11000	4000	1	2.75	3.81	5.10	6.51	8.13	9.96	11.97	14.15	16.48	18.96	21.55
5000	11000	4500	1	2.62	3.62	4.86	6.25	7.85	9.66	11.67	13.85	16.20	18.69	21.30
5000	11000	5000	1	2.51	3.46	4.66	6.02	7.62	9.42	11.42	13.61	15.96	18.46	21.10
5000	11000	5500	1	2.42	3.32	4.50	5.85	7.42	9.22	11.22	13.40	15.76	18.28	20.94
5000	11000	6000	1	2.35	3.21	4.37	5.69	7.26	9.05	11.04	13.23	15.59	18.12	20.79
5000	11000	6500	1	2.28	3.11	4.24	5.56	7.12	8.90	10.89	13.08	15.45	17.99	20.67
5000	11000	7000	1	2.23	3.04	4.16	5.46	7.00	8.77	10.76	12.95	15.33	17.87	20.56
5000	11000	7500	1	2.20	2.99	4.10	5.37	6.89	8.66	10.65	12.84	15.22	17.76	20.47
5000	11000	8000	1	2.16	2.90	3.93	5.23	6.79	8.56	10.55	12.74	15.12	17.67	20.39
5000	11000	9000	1	1.97	2.59	3.44	4.92	6.63	8.41	10.37	12.56	14.95	17.51	20.24

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →										
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
5000	11000	10000	1	1.87	2.45	3.30	4.79	6.50	8.27	10.23	12.42	14.81	17.39	20.13
6000	500	500	1	101.91	179.41	287.89	375.20	450.45	517.35	577.83	631.21	676.05	709.91	730.83
6000	500	1000	1	60.41	106.88	176.30	237.99	296.71	354.12	410.87	466.47	520.15	570.52	616.19
6000	500	1500	1	40.33	71.90	123.58	174.50	225.83	278.70	333.65	390.26	447.91	505.29	561.48
6000	500	2000	1	32.95	59.33	103.82	150.73	199.55	250.72	304.93	361.89	420.97	480.89	540.86
6000	500	2500	1	30.70	55.56	94.44	137.44	184.96	235.53	289.35	346.45	406.31	467.85	530.29
6000	500	3000	1	31.14	56.00	88.70	126.79	173.04	223.43	276.99	334.17	394.65	457.41	521.71
6000	500	3500	1	29.94	53.80	83.62	119.35	164.27	214.08	267.44	324.74	385.69	449.32	514.93
6000	500	4000	1	24.75	45.13	76.05	113.45	158.05	207.05	260.17	317.58	378.88	443.17	509.78
6000	500	4500	1	20.82	38.13	70.21	108.92	153.09	201.49	254.44	311.93	373.50	438.32	505.71
6000	500	5000	1	19.29	35.59	67.98	105.88	148.94	196.91	249.78	307.33	369.13	434.36	502.40
6000	500	5500	1	18.36	34.28	66.15	103.04	145.41	193.12	245.92	303.51	365.50	431.08	499.65
6000	500	6000	1	17.69	33.06	63.99	100.34	142.42	189.94	242.65	300.28	362.42	428.30	497.32
6000	500	6500	1	17.03	31.87	62.08	98.05	139.87	187.22	239.85	297.50	359.78	425.92	495.32
6000	500	7000	1	16.23	30.64	60.51	96.12	137.66	184.85	237.43	295.11	357.50	423.86	493.60
6000	500	7500	1	15.58	29.61	59.12	94.43	135.75	182.82	235.35	293.07	355.58	422.15	492.21
6000	500	8000	1	15.07	28.76	57.87	92.88	134.01	180.95	233.43	291.16	353.76	420.47	490.72
6000	500	9000	1	14.18	27.32	55.66	90.09	130.79	177.43	229.70	287.33	349.91	416.66	487.02
6000	500	10000	1	13.46	26.13	53.91	87.93	128.33	174.77	226.94	284.55	347.21	414.11	484.69

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
6000	1000	500	1	66.76	116.04	182.95	236.35	283.86	326.45	364.34	397.56	425.86	449.23	466.46
6000	1000	1000	1	39.45	68.72	111.64	148.97	185.32	221.28	256.49	290.88	324.36	357.30	389.00
6000	1000	1500	1	26.43	46.21	78.29	109.05	140.80	173.73	207.65	242.53	278.24	315.09	352.70
6000	1000	2000	1	21.74	38.63	66.37	94.50	124.51	156.33	189.81	224.88	261.44	299.75	339.58
6000	1000	2500	1	20.24	36.57	62.06	86.93	115.24	146.52	179.84	215.03	252.08	291.40	332.74
6000	1000	3000	1	21.44	38.49	61.61	82.01	107.78	138.63	171.97	207.28	244.71	284.73	327.20
6000	1000	3500	1	21.92	38.58	58.71	77.22	102.20	132.77	166.02	201.39	239.10	279.64	322.89
6000	1000	4000	1	20.27	34.26	50.72	71.14	98.01	128.63	161.53	196.90	234.84	275.78	319.61
6000	1000	4500	1	18.46	30.31	45.65	67.40	94.90	125.24	157.95	193.36	231.47	272.73	317.02
6000	1000	5000	1	15.19	25.92	43.54	65.92	92.65	122.40	155.01	190.48	228.73	270.24	314.91
6000	1000	5500	1	12.93	23.05	41.93	64.39	90.58	120.04	152.59	188.09	226.45	268.17	313.16
6000	1000	6000	1	13.44	23.33	40.79	62.58	88.64	118.05	150.55	186.07	224.53	266.43	311.69
6000	1000	6500	1	13.27	22.83	39.64	61.11	87.03	116.35	148.81	184.34	222.89	264.93	310.42
6000	1000	7000	1	11.45	20.59	38.41	60.06	85.72	114.87	147.29	182.85	221.46	263.64	309.32
6000	1000	7500	1	10.22	19.00	37.37	59.03	84.46	113.47	145.82	181.37	220.01	262.25	308.06
6000	1000	8000	1	9.75	18.31	36.57	58.10	83.39	112.32	144.66	180.22	218.92	261.30	307.32
6000	1000	9000	1	9.61	17.96	35.62	56.93	82.27	111.35	143.92	179.81	218.97	261.93	308.67
6000	1000	10000	1	9.05	17.10	34.57	55.73	80.94	109.98	142.57	178.56	217.88	261.10	308.19
6000	2000	500	1	31.38	52.74	79.53	101.41	121.51	139.46	155.14	168.83	180.35	188.95	194.14
6000	2000	1000	1	18.94	31.56	49.09	64.36	79.69	95.19	110.04	124.36	138.19	151.05	162.60

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
6000	2000	1500	1	12.91	21.38	34.85	47.44	60.91	75.12	89.46	104.06	118.93	133.60	147.83
6000	2000	2000	1	10.63	18.06	30.13	41.53	54.06	67.57	81.65	96.36	111.61	126.95	142.19
6000	2000	2500	1	9.54	16.83	29.08	38.87	50.12	63.22	77.23	92.00	107.46	123.23	139.11
6000	2000	3000	1	10.03	17.54	30.37	37.76	46.95	59.63	73.77	88.61	104.22	120.29	136.66
6000	2000	3500	1	11.07	18.66	29.71	35.79	44.44	57.04	71.16	86.03	101.75	118.05	134.77
6000	2000	4000	1	12.66	19.89	26.24	32.52	42.49	55.27	69.20	84.05	99.87	116.35	133.32
6000	2000	4500	1	12.96	19.70	23.96	30.61	41.10	53.79	67.62	82.49	98.39	115.01	132.18
6000	2000	5000	1	9.52	15.29	21.40	29.42	40.12	52.60	66.34	81.22	97.18	113.91	131.25
6000	2000	5500	1	7.41	12.05	19.04	28.19	39.24	51.63	65.27	80.16	96.17	113.00	130.47
6000	2000	6000	1	8.33	12.92	18.55	27.26	38.35	50.75	64.38	79.27	95.32	112.23	129.82
6000	2000	6500	1	8.31	12.71	18.13	26.65	37.63	49.99	63.60	78.51	94.59	111.56	129.25
6000	2000	7000	1	6.38	10.49	17.36	26.30	37.10	49.34	62.93	77.85	93.96	110.99	128.77
6000	2000	7500	1	5.22	9.04	16.76	25.93	36.62	48.78	62.37	77.31	93.46	110.54	128.40
6000	2000	8000	1	4.90	8.58	16.37	25.51	36.13	48.26	61.83	76.76	92.93	110.05	127.95
6000	2000	9000	1	5.02	8.63	15.81	24.63	35.14	47.18	60.66	75.52	91.65	108.77	126.75
6000	2000	10000	1	4.65	8.09	15.26	24.03	34.44	46.41	59.86	74.71	90.84	108.00	126.06
6000	3000	500	1	21.75	35.56	52.10	66.18	78.54	89.47	99.22	107.76	114.78	118.82	119.34
6000	3000	1000	1	13.38	21.64	32.33	42.27	52.13	61.90	71.22	80.24	88.88	96.15	101.62
6000	3000	1500	1	9.47	15.21	23.29	31.37	40.08	49.14	58.21	67.47	76.85	85.52	93.12
6000	3000	2000	1	7.93	12.91	20.31	27.61	35.62	44.16	53.07	62.41	72.04	81.18	89.48

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
6000	3000	2500	1	7.05	11.54	18.86	25.55	33.00	41.28	50.13	59.50	69.26	78.67	87.39
6000	3000	3000	1	6.33	10.32	18.07	24.08	30.89	39.00	47.85	57.24	67.11	76.73	85.77
6000	3000	3500	1	6.05	9.89	17.25	22.74	29.27	37.30	46.12	55.53	65.47	75.24	84.52
6000	3000	4000	1	6.60	10.66	16.35	21.34	28.02	36.02	44.80	54.23	64.23	74.12	83.57
6000	3000	4500	1	6.88	11.01	15.71	20.44	27.05	35.00	43.76	53.20	63.24	73.22	82.81
6000	3000	5000	1	5.81	9.36	14.17	19.56	26.32	34.21	42.91	52.35	62.44	72.49	82.19
6000	3000	5500	1	5.16	8.17	12.74	18.68	25.71	33.55	42.20	51.65	61.77	71.89	81.68
6000	3000	6000	1	5.44	8.42	12.36	18.13	25.14	32.96	41.60	51.06	61.20	71.37	81.24
6000	3000	6500	1	5.25	8.09	12.03	17.73	24.67	32.46	41.09	50.55	60.71	70.92	80.86
6000	3000	7000	1	4.41	7.07	11.62	17.43	24.28	32.01	40.63	50.10	60.28	70.53	80.53
6000	3000	7500	1	4.00	6.50	11.32	17.18	23.98	31.70	40.33	49.82	60.04	70.38	80.50
6000	3000	8000	1	3.84	6.26	11.07	16.90	23.66	31.37	39.99	49.48	59.72	70.06	80.17
6000	3000	9000	1	3.72	6.03	10.46	16.02	22.58	30.07	38.47	47.73	57.71	67.63	77.06
6000	3000	10000	1	3.50	5.69	10.05	15.55	22.02	29.42	37.76	46.98	56.90	66.72	75.97
6000	4000	500	1	19.10	31.02	44.97	56.44	66.44	75.44	83.52	90.57	96.47	100.49	102.24
6000	4000	1000	1	11.73	18.81	27.58	35.63	43.75	51.67	59.26	66.63	73.76	80.21	85.69
6000	4000	1500	1	8.35	13.30	19.80	26.33	33.43	40.76	48.11	55.64	63.31	70.77	77.81
6000	4000	2000	1	7.11	11.36	17.14	23.04	29.63	36.58	43.82	51.40	59.27	67.08	74.65
6000	4000	2500	1	6.42	10.19	15.48	21.16	27.44	34.18	41.37	48.98	56.95	64.98	72.89
6000	4000	3000	1	5.57	8.77	13.81	19.47	25.70	32.34	39.46	47.09	55.14	63.34	71.50

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
6000	4000	3500	1	4.84	7.64	12.65	18.25	24.39	30.93	38.02	45.66	53.78	62.10	70.43
6000	4000	4000	1	4.30	6.96	12.12	17.49	23.37	29.84	36.91	44.57	52.73	61.14	69.62
6000	4000	4500	1	4.11	6.71	11.72	16.82	22.55	28.98	36.04	43.71	51.91	60.39	68.97
6000	4000	5000	1	4.20	6.69	11.15	16.17	21.89	28.29	35.33	43.00	51.23	59.77	68.44
6000	4000	5500	1	4.25	6.64	10.71	15.65	21.35	27.71	34.74	42.42	50.67	59.26	68.00
6000	4000	6000	1	4.17	6.46	10.41	15.25	20.88	27.22	34.24	41.92	50.20	58.82	67.63
6000	4000	6500	1	3.92	6.17	10.11	14.91	20.48	26.80	33.81	41.50	49.79	58.45	67.30
6000	4000	7000	1	3.71	5.90	9.86	14.60	20.14	26.43	33.44	41.12	49.43	58.12	67.02
6000	4000	7500	1	3.62	5.73	9.64	14.34	19.85	26.13	33.14	40.83	49.16	57.89	66.87
6000	4000	8000	1	3.53	5.59	9.44	14.10	19.59	25.85	32.86	40.56	48.90	57.66	66.66
6000	4000	9000	1	3.33	5.29	9.01	13.57	18.96	25.14	32.06	39.69	47.96	56.55	65.22
6000	4000	10000	1	3.19	5.07	8.71	13.21	18.55	24.69	31.58	39.20	47.47	56.03	64.63
6000	5000	500	1	16.80	27.05	38.52	47.59	55.74	63.11	69.67	75.41	80.26	84.03	86.50
6000	5000	1000	1	10.40	16.49	23.69	30.01	36.50	42.88	49.01	54.99	60.78	66.28	71.37
6000	5000	1500	1	7.42	11.62	16.96	22.12	27.81	33.67	39.60	45.68	51.88	58.10	64.27
6000	5000	2000	1	6.27	9.80	14.51	19.29	24.60	30.19	36.03	42.15	48.50	55.00	61.57
6000	5000	2500	1	5.62	8.77	13.07	17.67	22.75	28.20	33.99	40.13	46.57	53.24	60.08
6000	5000	3000	1	5.03	7.81	11.74	16.29	21.31	26.65	32.40	38.55	45.06	51.87	58.91
6000	5000	3500	1	4.56	7.05	10.79	15.27	20.20	25.47	31.20	37.36	43.92	50.82	57.99
6000	5000	4000	1	4.13	6.39	10.18	14.54	19.34	24.57	30.28	36.45	43.05	50.01	57.30

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
6000	5000	4500	1	3.85	5.95	9.68	13.93	18.66	23.86	29.55	35.73	42.35	49.38	56.75
6000	5000	5000	1	3.80	5.85	9.38	13.47	18.11	23.27	28.96	35.14	41.79	48.86	56.30
6000	5000	5500	1	3.72	5.71	9.12	13.08	17.65	22.79	28.46	34.65	41.32	48.43	55.92
6000	5000	6000	1	3.59	5.48	8.84	12.73	17.25	22.38	28.05	34.24	40.93	48.06	55.61
6000	5000	6500	1	3.44	5.29	8.59	12.43	16.92	22.03	27.69	33.88	40.58	47.75	55.33
6000	5000	7000	1	3.35	5.17	8.38	12.17	16.64	21.73	27.38	33.58	40.29	47.48	55.10
6000	5000	7500	1	3.27	5.04	8.19	11.96	16.40	21.46	27.10	33.30	40.03	47.23	54.88
6000	5000	8000	1	3.19	4.91	8.02	11.76	16.17	21.22	26.86	33.06	39.80	47.02	54.70
6000	5000	9000	1	3.02	4.68	7.73	11.42	15.80	20.83	26.46	32.67	39.42	46.68	54.41
6000	5000	10000	1	2.91	4.50	7.50	11.13	15.49	20.50	26.13	32.34	39.11	46.39	54.17
6000	6000	500	1	14.78	23.52	32.77	40.03	46.63	52.58	57.86	62.50	66.42	69.47	71.49
6000	6000	1000	1	9.24	14.43	20.35	25.44	30.58	35.68	40.64	45.47	50.15	54.60	58.72
6000	6000	1500	1	6.64	10.20	14.61	18.82	23.31	27.99	32.78	37.70	42.71	47.73	52.72
6000	6000	2000	1	5.60	8.56	12.42	16.34	20.59	25.07	29.79	34.74	39.87	45.11	50.42
6000	6000	2500	1	4.98	7.61	11.18	14.92	19.02	23.40	28.08	33.04	38.24	43.63	49.16
6000	6000	3000	1	4.52	6.90	10.22	13.81	17.78	22.09	26.74	31.72	36.97	42.47	48.16
6000	6000	3500	1	4.19	6.37	9.49	12.95	16.83	21.10	25.73	30.71	36.01	41.58	47.38
6000	6000	4000	1	3.94	5.95	8.89	12.25	16.09	20.35	24.96	29.95	35.28	40.90	46.79
6000	6000	4500	1	3.72	5.60	8.42	11.71	15.53	19.75	24.35	29.34	34.69	40.37	46.32
6000	6000	5000	1	3.54	5.33	8.12	11.33	15.08	19.26	23.85	28.85	34.22	39.93	45.94

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
6000	6000	5500	1	3.38	5.09	7.84	11.00	14.69	18.85	23.43	28.43	33.82	39.56	45.62
6000	6000	6000	1	3.25	4.88	7.58	10.70	14.35	18.50	23.08	28.09	33.49	39.25	45.35
6000	6000	6500	1	3.15	4.72	7.35	10.44	14.07	18.21	22.78	27.79	33.20	38.99	45.11
6000	6000	7000	1	3.06	4.59	7.17	10.22	13.84	17.95	22.52	27.53	32.95	38.76	44.91
6000	6000	7500	1	2.98	4.46	7.01	10.04	13.63	17.73	22.29	27.30	32.73	38.55	44.73
6000	6000	8000	1	2.91	4.35	6.86	9.88	13.45	17.53	22.08	27.09	32.53	38.37	44.57
6000	6000	9000	1	2.76	4.16	6.62	9.59	13.13	17.19	21.75	26.76	32.22	38.09	44.35
6000	6000	10000	1	2.66	4.00	6.42	9.35	12.87	16.92	21.46	26.48	31.95	37.84	44.14
6000	7000	500	1	13.30	20.90	28.54	34.49	39.94	44.85	49.22	53.05	56.29	58.78	60.39
6000	7000	1000	1	8.38	12.90	17.80	22.04	26.27	30.44	34.53	38.52	42.39	46.06	49.43
6000	7000	1500	1	6.08	9.19	12.83	16.34	20.02	23.86	27.81	31.87	36.02	40.16	44.26
6000	7000	2000	1	5.15	7.73	10.92	14.16	17.65	21.34	25.24	29.33	33.57	37.90	42.26
6000	7000	2500	1	4.60	6.88	9.84	12.91	16.28	19.90	23.77	27.87	32.17	36.62	41.16
6000	7000	3000	1	4.18	6.24	9.01	11.93	15.21	18.77	22.62	26.73	31.07	35.61	40.29
6000	7000	3500	1	3.86	5.74	8.34	11.16	14.38	17.92	21.75	25.86	30.24	34.84	39.62
6000	7000	4000	1	3.62	5.36	7.79	10.52	13.73	17.27	21.09	25.21	29.61	34.25	39.10
6000	7000	4500	1	3.43	5.06	7.39	10.06	13.23	16.75	20.56	24.68	29.11	33.79	38.69
6000	7000	5000	1	3.27	4.83	7.11	9.74	12.85	16.33	20.13	24.25	28.69	33.41	38.35
6000	7000	5500	1	3.13	4.62	6.90	9.50	12.53	15.97	19.77	23.90	28.35	33.09	38.07
6000	7000	6000	1	3.01	4.44	6.69	9.26	12.24	15.67	19.46	23.60	28.06	32.82	37.84

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →											
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
6000	7000	6500	1	2.92	4.29	6.48	9.03	12.00	15.42	19.20	23.34	27.81	32.59	37.63
6000	7000	7000	1	2.84	4.16	6.32	8.84	11.79	15.20	18.98	23.12	27.60	32.39	37.46
6000	7000	7500	1	2.77	4.04	6.15	8.65	11.61	15.00	18.78	22.92	27.41	32.21	37.30
6000	7000	8000	1	2.71	3.94	6.01	8.49	11.45	14.83	18.60	22.74	27.24	32.06	37.17
6000	7000	9000	1	2.57	3.77	5.81	8.25	11.18	14.54	18.30	22.44	26.95	31.78	36.91
6000	7000	10000	1	2.48	3.63	5.62	8.04	10.95	14.29	18.05	22.19	26.71	31.56	36.72
6000	8000	500	1	12.19	18.93	25.35	30.31	34.90	39.03	42.70	45.93	48.65	50.75	52.10
6000	8000	1000	1	7.74	11.76	15.87	19.43	22.98	26.48	29.91	33.27	36.52	39.60	42.45
6000	8000	1500	1	5.65	8.42	11.47	14.41	17.51	20.73	24.05	27.46	30.95	34.44	37.88
6000	8000	2000	1	4.81	7.10	9.77	12.47	15.41	18.52	21.79	25.23	28.80	32.44	36.11
6000	8000	2500	1	4.31	6.34	8.80	11.35	14.19	17.25	20.50	23.95	27.57	31.31	35.13
6000	8000	3000	1	3.92	5.75	8.04	10.48	13.25	16.26	19.49	22.95	26.60	30.42	34.36
6000	8000	3500	1	3.62	5.28	7.44	9.80	12.53	15.51	18.73	22.19	25.87	29.74	33.76
6000	8000	4000	1	3.39	4.91	6.93	9.22	11.94	14.94	18.15	21.61	25.31	29.22	33.30
6000	8000	4500	1	3.21	4.63	6.58	8.81	11.50	14.48	17.68	21.15	24.87	28.81	32.93
6000	8000	5000	1	3.07	4.42	6.34	8.55	11.18	14.11	17.30	20.77	24.51	28.47	32.64
6000	8000	5500	1	2.94	4.25	6.17	8.35	10.89	13.79	16.99	20.46	24.21	28.19	32.39
6000	8000	6000	1	2.84	4.09	5.99	8.14	10.64	13.53	16.72	20.20	23.95	27.95	32.18
6000	8000	6500	1	2.75	3.95	5.81	7.94	10.43	13.31	16.49	19.97	23.73	27.75	31.99
6000	8000	7000	1	2.67	3.83	5.67	7.78	10.24	13.11	16.29	19.77	23.54	27.57	31.84

Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q″ ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
6000	8000	7500	1	2.61	3.73	5.50	7.59	10.08	12.94	16.11	19.60	23.37	27.42	31.70
6000	8000	8000	1	2.55	3.64	5.37	7.45	9.94	12.79	15.96	19.44	23.22	27.28	31.58
6000	8000	9000	1	2.43	3.48	5.20	7.25	9.71	12.53	15.69	19.18	22.97	27.04	31.36
6000	8000	10000	1	2.34	3.35	5.03	7.06	9.50	12.32	15.47	18.96	22.76	26.84	31.18
6000	9000	500	1	11.29	17.37	22.84	27.02	30.91	34.43	37.55	40.29	42.61	44.40	45.56
6000	9000	1000	1	7.23	10.85	14.34	17.34	20.37	23.35	26.26	29.11	31.88	34.51	36.93
6000	9000	1500	1	5.31	7.80	10.38	12.85	15.51	18.25	21.07	23.97	26.94	29.91	32.84
6000	9000	2000	1	4.53	6.60	8.84	11.10	13.62	16.29	19.07	21.99	25.03	28.13	31.25
6000	9000	2500	1	4.07	5.89	7.94	10.07	12.53	15.15	17.92	20.85	23.93	27.11	30.37
6000	9000	3000	1	3.71	5.34	7.24	9.28	11.68	14.27	17.02	19.96	23.07	26.32	29.67
6000	9000	3500	1	3.44	4.92	6.72	8.70	11.05	13.61	16.34	19.28	22.42	25.71	29.13
6000	9000	4000	1	3.22	4.58	6.28	8.21	10.53	13.09	15.82	18.77	21.92	25.24	28.72
6000	9000	4500	1	3.04	4.31	5.95	7.83	10.14	12.69	15.41	18.36	21.52	24.87	28.39
6000	9000	5000	1	2.90	4.10	5.72	7.59	9.85	12.35	15.07	18.02	21.20	24.57	28.12
6000	9000	5500	1	2.79	3.94	5.54	7.39	9.60	12.07	14.79	17.74	20.93	24.32	27.89
6000	9000	6000	1	2.69	3.80	5.38	7.21	9.37	11.84	14.55	17.51	20.70	24.11	27.70
6000	9000	6500	1	2.61	3.67	5.24	7.05	9.19	11.64	14.34	17.30	20.50	23.92	27.54
6000	9000	7000	1	2.54	3.57	5.12	6.92	9.02	11.46	14.17	17.13	20.33	23.76	27.40
6000	9000	7500	1	2.48	3.47	4.99	6.76	8.87	11.31	14.01	16.97	20.18	23.62	27.27
6000	9000	8000	1	2.42	3.39	4.88	6.64	8.75	11.17	13.87	16.83	20.05	23.50	27.16

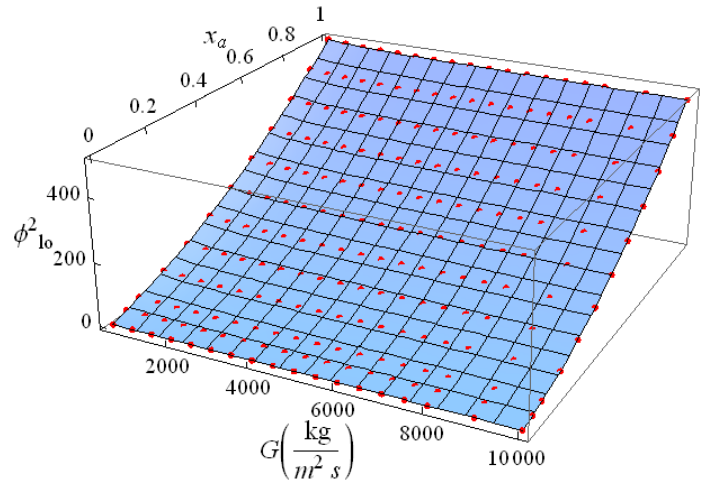
Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a → 0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
6000	9000	9000	1	2.31	3.26	4.73	6.46	8.54	10.94	13.63	16.60	19.82	23.29	26.97
6000	9000	10000	1	2.24	3.14	4.58	6.29	8.36	10.75	13.44	16.41	19.64	23.11	26.81
6000	10000	500	1	10.51	16.10	20.85	24.37	27.70	30.73	33.41	35.77	37.76	39.30	40.31
6000	10000	1000	1	6.78	10.12	13.14	15.68	18.27	20.84	23.34	25.79	28.17	30.43	32.52
6000	10000	1500	1	5.01	7.29	9.51	11.60	13.88	16.25	18.67	21.16	23.71	26.26	28.78
6000	10000	2000	1	4.28	6.17	8.09	10.02	12.18	14.47	16.86	19.37	21.98	24.64	27.33
6000	10000	2500	1	3.85	5.52	7.27	9.08	11.20	13.47	15.84	18.36	21.00	23.73	26.54
6000	10000	3000	1	3.52	5.02	6.63	8.37	10.44	12.68	15.04	17.56	20.23	23.02	25.91
6000	10000	3500	1	3.26	4.63	6.17	7.85	9.87	12.07	14.42	16.95	19.64	22.47	25.41
6000	10000	4000	1	3.06	4.32	5.78	7.43	9.41	11.61	13.95	16.48	19.19	22.05	25.03
6000	10000	4500	1	2.90	4.06	5.48	7.09	9.06	11.24	13.58	16.11	18.83	21.71	24.73
6000	10000	5000	1	2.76	3.86	5.25	6.86	8.79	10.94	13.27	15.81	18.54	21.44	24.49
6000	10000	5500	1	2.66	3.71	5.08	6.66	8.56	10.69	13.02	15.56	18.30	21.21	24.28
6000	10000	6000	1	2.57	3.57	4.93	6.49	8.36	10.48	12.81	15.35	18.09	21.02	24.11
6000	10000	6500	1	2.49	3.46	4.80	6.35	8.19	10.30	12.62	15.16	17.91	20.85	23.96
6000	10000	7000	1	2.42	3.36	4.70	6.23	8.04	10.13	12.46	15.00	17.76	20.70	23.83
6000	10000	7500	1	2.37	3.27	4.59	6.10	7.91	10.00	12.32	14.86	17.62	20.58	23.71
6000	10000	8000	1	2.32	3.20	4.49	5.99	7.79	9.88	12.19	14.74	17.50	20.46	23.61
6000	10000	9000	1	2.23	3.09	4.38	5.83	7.61	9.67	11.98	14.53	17.30	20.28	23.44
6000	10000	10000	1	2.15	2.99	4.25	5.68	7.44	9.49	11.81	14.36	17.13	20.12	23.30

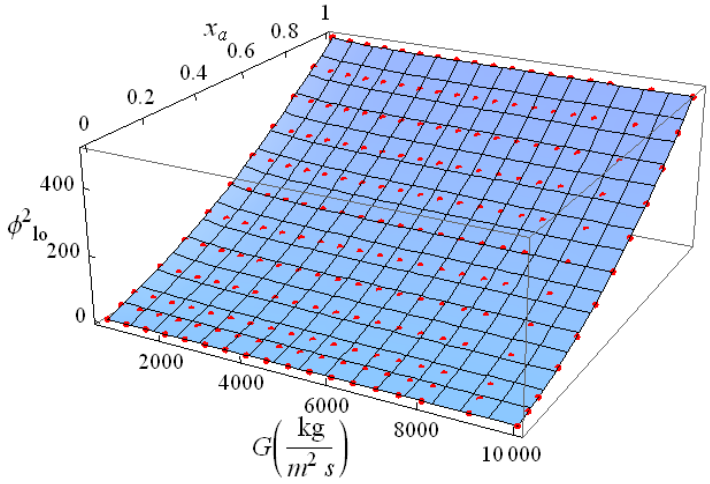
Table C.1(continued): The Two-Phase Frictional Multiplier Look-Up Table.

q'' ↓	P ↓	G ↓	x_a →										
			0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
6000	11000	500	1	9.78	15.00	19.19	22.12	24.95	27.56	29.87	31.89	33.60	34.92	35.78
6000	11000	1000	1	6.37	9.50	12.15	14.28	16.50	18.71	20.86	22.97	25.01	26.95	28.75
6000	11000	1500	1	4.71	6.85	8.79	10.57	12.50	14.53	16.61	18.76	20.95	23.14	25.31
6000	11000	2000	1	4.03	5.79	7.48	9.13	10.96	12.91	14.97	17.13	19.37	21.66	23.96
6000	11000	2500	1	3.64	5.20	6.77	8.33	10.10	12.02	14.06	16.23	18.50	20.85	23.25
6000	11000	3000	1	3.33	4.74	6.20	7.70	9.42	11.31	13.34	15.51	17.81	20.21	22.68
6000	11000	3500	1	3.10	4.37	5.76	7.21	8.90	10.76	12.79	14.96	17.27	19.70	22.23
6000	11000	4000	1	2.92	4.09	5.42	6.83	8.49	10.34	12.36	14.54	16.86	19.32	21.88
6000	11000	4500	1	2.77	3.87	5.15	6.53	8.17	10.01	12.02	14.20	16.54	19.01	21.61
6000	11000	5000	1	2.65	3.69	4.93	6.29	7.91	9.74	11.74	13.93	16.27	18.76	21.38
6000	11000	5500	1	2.55	3.54	4.74	6.09	7.70	9.51	11.51	13.70	16.05	18.56	21.19
6000	11000	6000	1	2.47	3.41	4.59	5.92	7.51	9.32	11.32	13.50	15.86	18.38	21.03
6000	11000	6500	1	2.40	3.29	4.45	5.77	7.35	9.15	11.15	13.34	15.70	18.23	20.90
6000	11000	7000	1	2.33	3.20	4.33	5.64	7.21	9.01	11.00	13.19	15.56	18.09	20.78
6000	11000	7500	1	2.27	3.11	4.22	5.53	7.09	8.88	10.88	13.06	15.44	17.98	20.67
6000	11000	8000	1	2.22	3.04	4.15	5.44	6.99	8.77	10.76	12.95	15.33	17.87	20.58
6000	11000	9000	1	2.14	2.95	4.08	5.30	6.80	8.58	10.57	12.76	15.14	17.70	20.42
6000	11000	10000	1	2.07	2.85	3.96	5.16	6.65	8.42	10.41	12.60	14.99	17.55	20.28

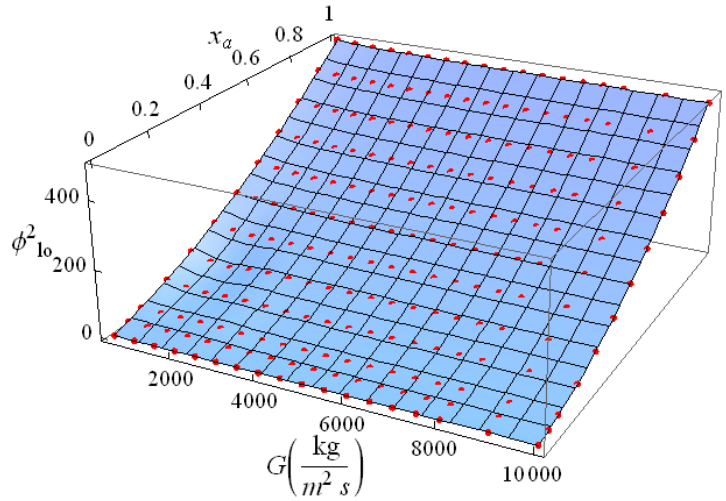
APPENDIX D



(a)

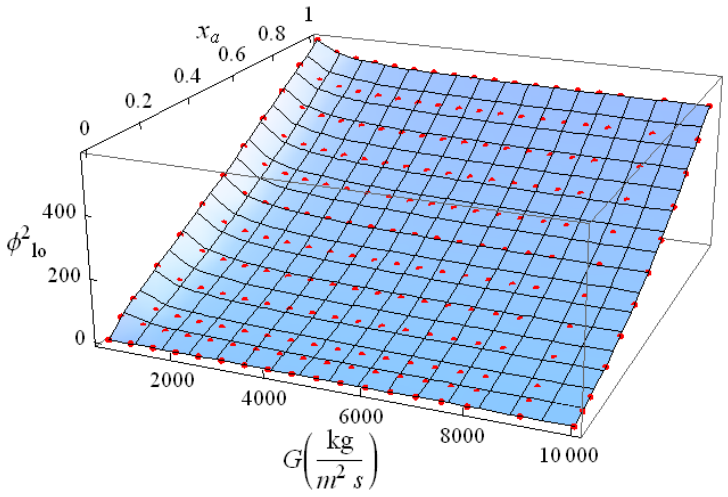


(b)

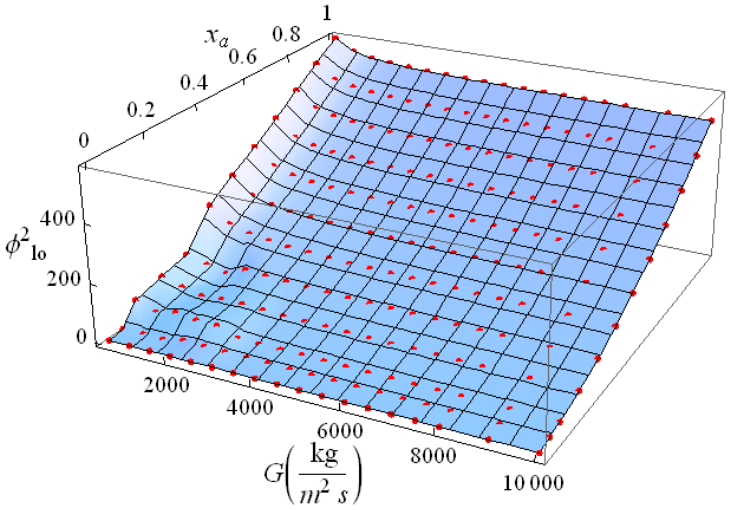


(c)

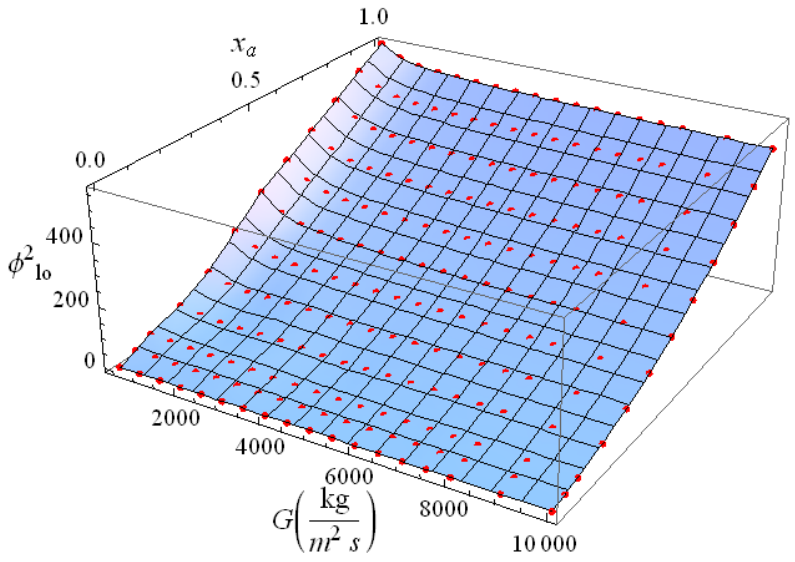
Figure D.1 : A 3-D illustration of the LUT at q"=0 kW/m^2 and P=500 kPa for (a) skeleton table (b) updated table, and (c) smoothed table



(a)

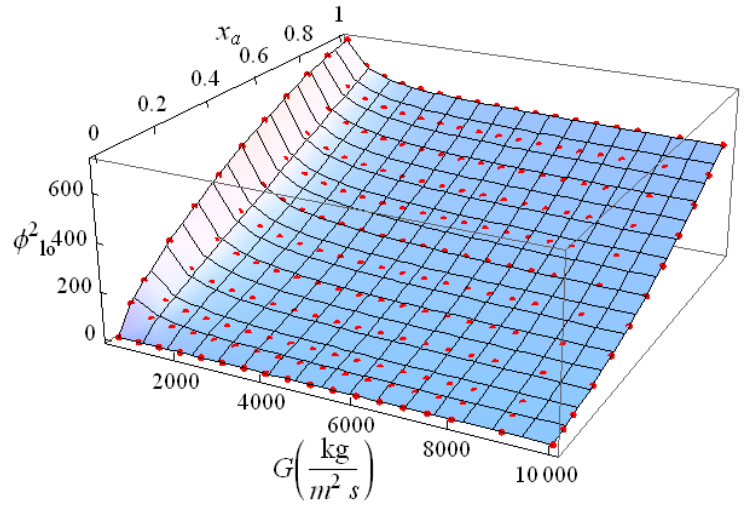


(b)

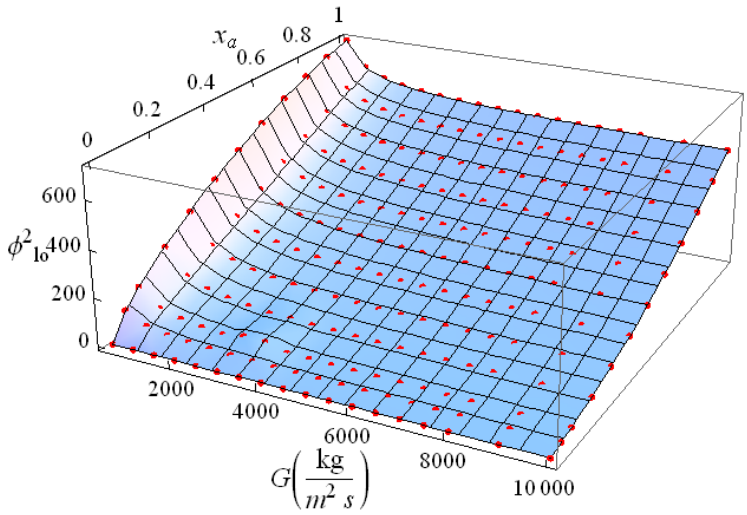


(c)

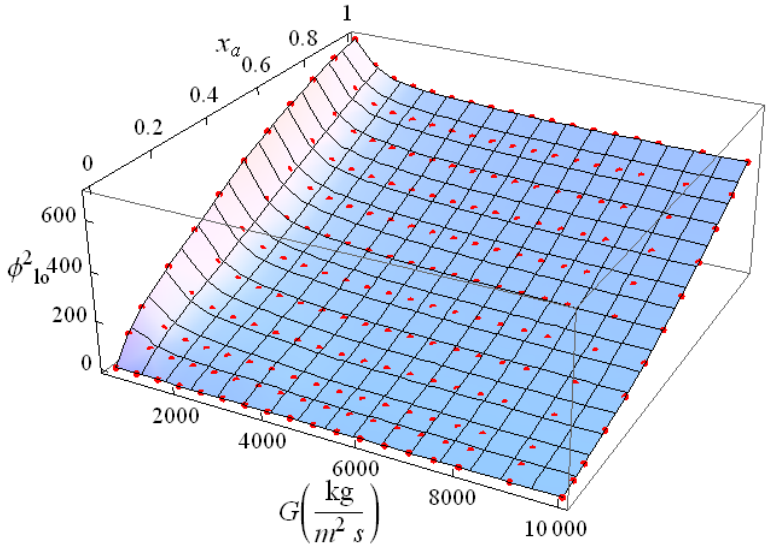
Figure D.2 : A 3-D illustration of the LUT at q"=1000 kW/m^2 and P=500 kPa for (a) skeleton table (b) updated table, and (c) smoothed table



(a)

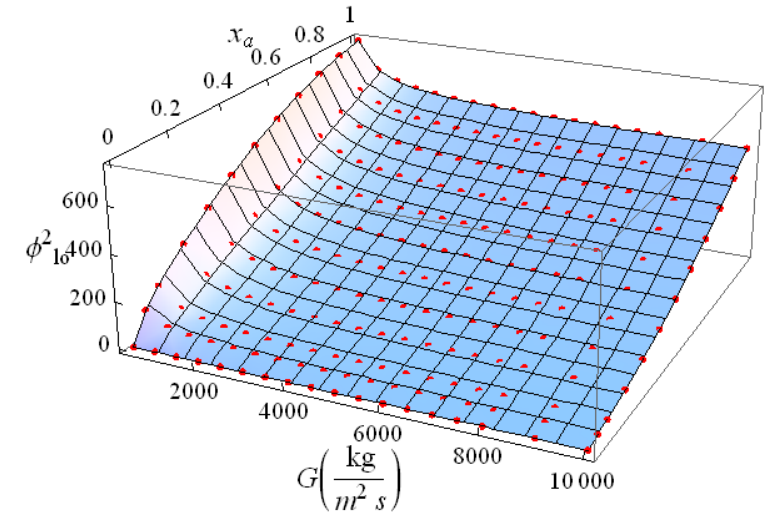


(b)

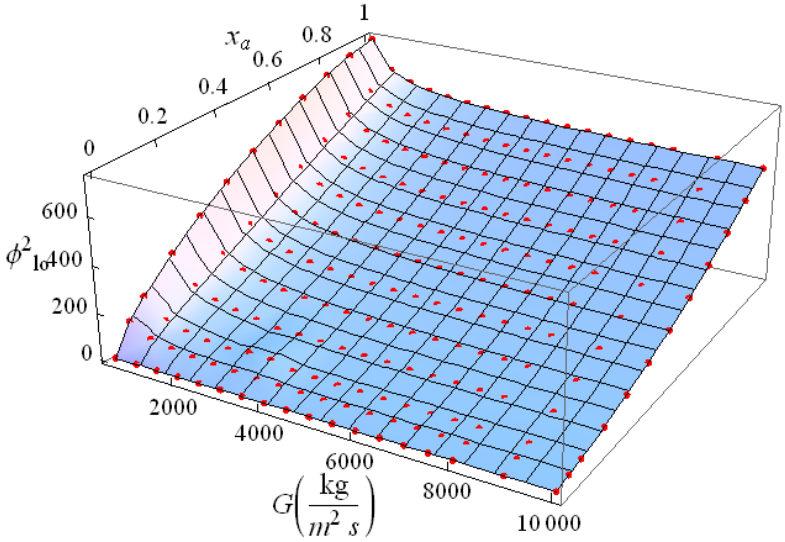


(c)

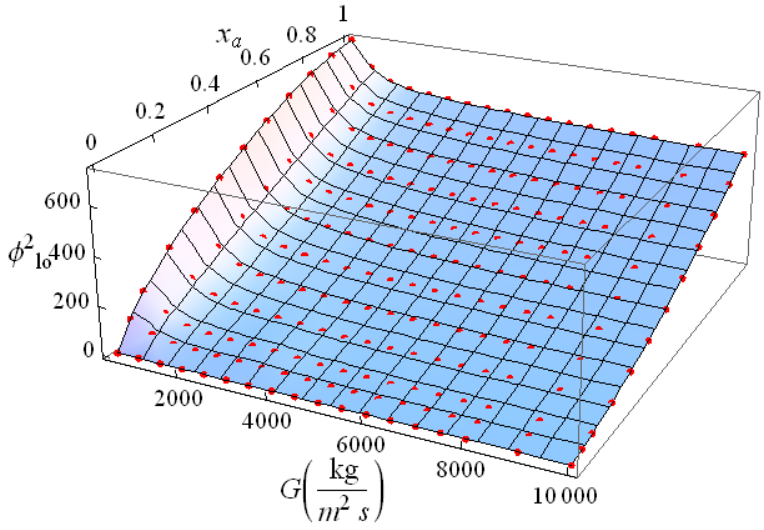
Figure D.3 : A 3-D illustration of the LUT at q"=5000 kW/m^2 and P=500 kPa for (a) skeleton table (b) updated table, and (c) smoothed table



(a)

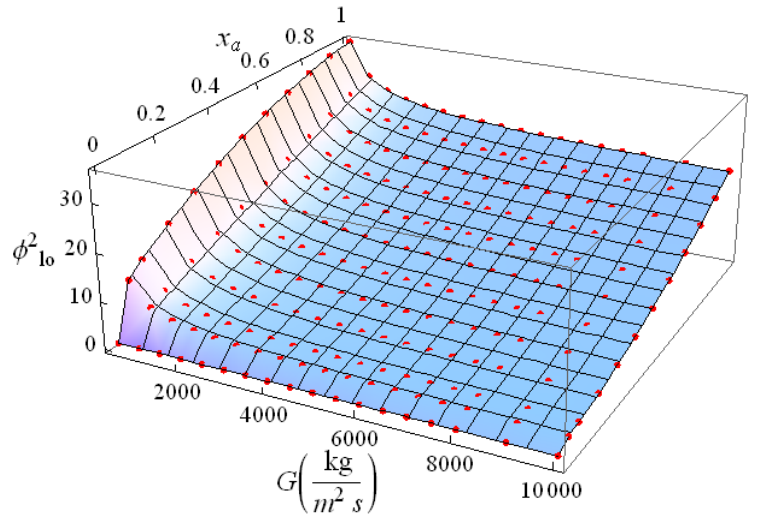


(b)

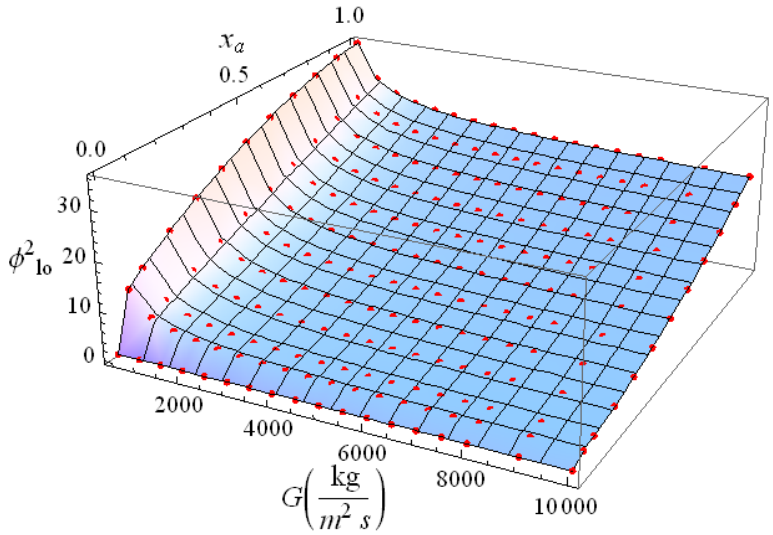


(c)

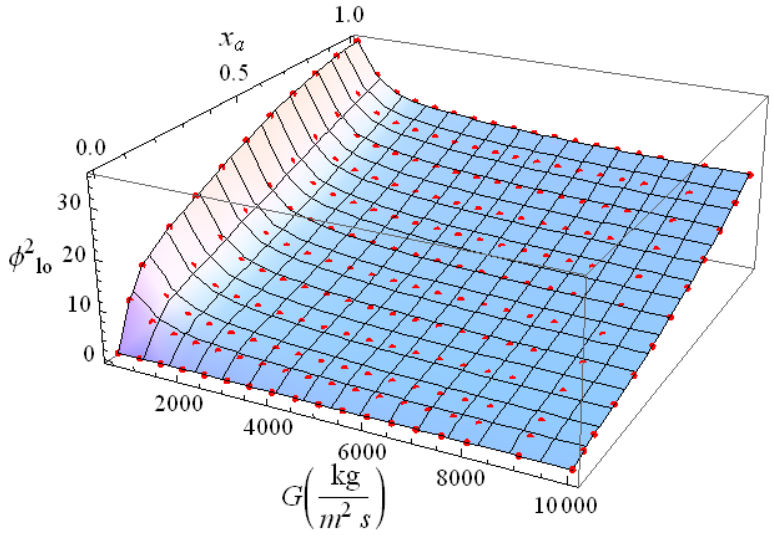
Figure D.4 : A 3-D illustration of the LUT at q"=6000 kW/m^2 and P=500 kPa for (a) skeleton table (b) updated table, and (c) smoothed table



(a)

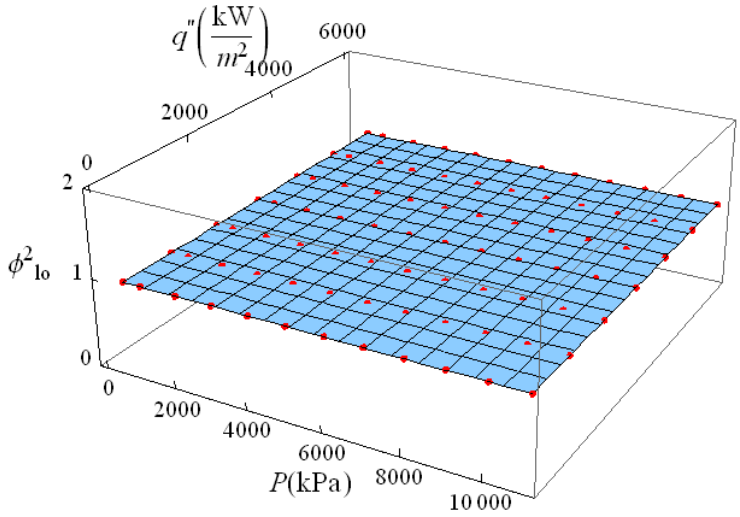


(b)

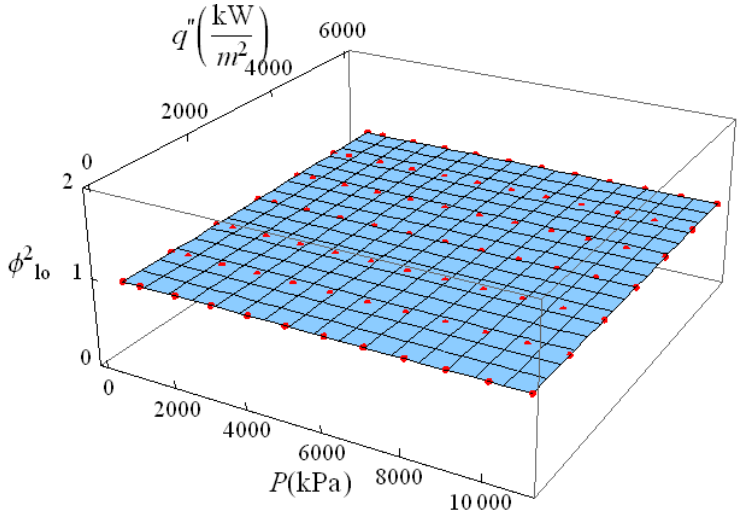


(c)

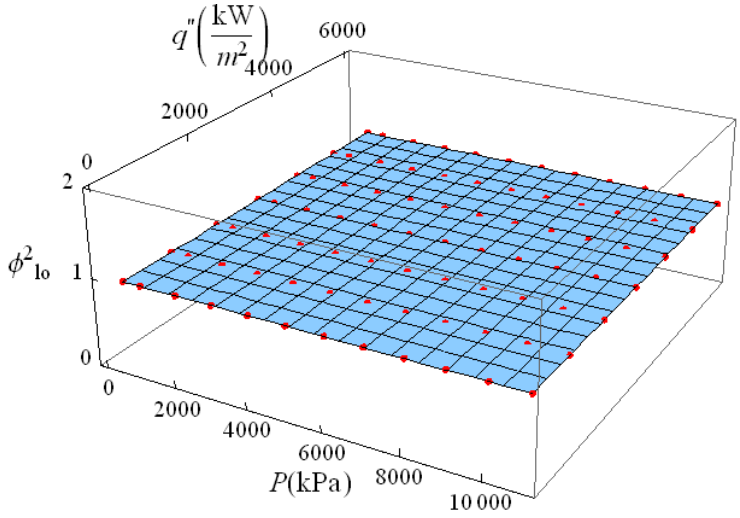
Figure D.5 : A 3-D illustration of the LUT at q"=6000 kW/m^2 and P=11.000 kPa for (a) skeleton table (b) updated table, and (c) smoothed table



(a)

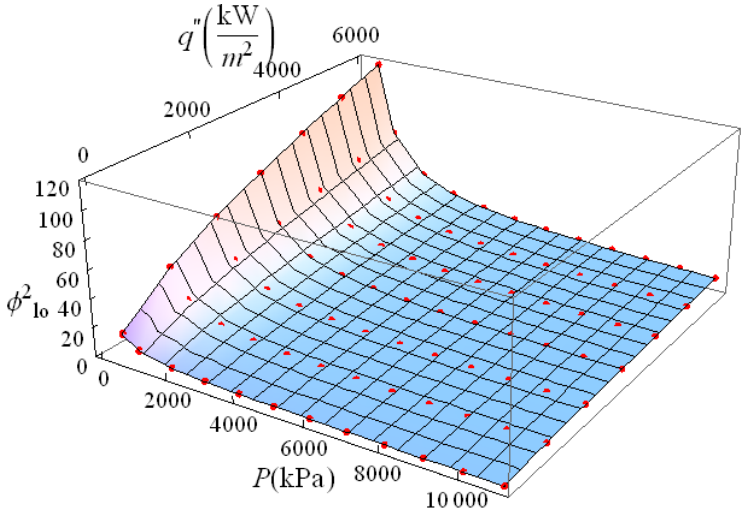


(b)

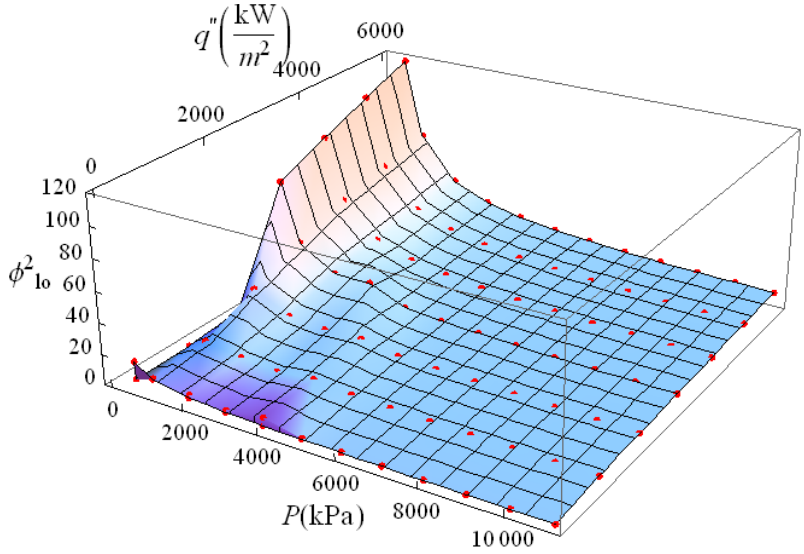


(c)

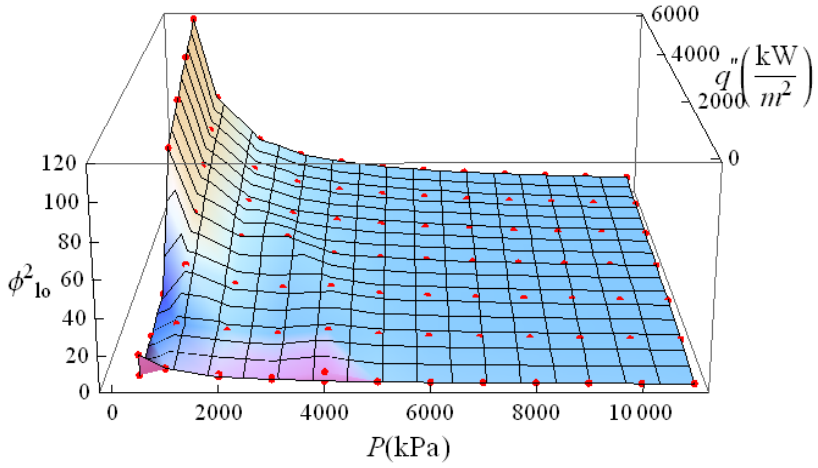
Figure D.6 : A 3-D illustration of the LUT at G=500 kg/m^2.s and x_a =0.0 for (a) skeleton table (b) updated table, and (c) smoothed table



(a)

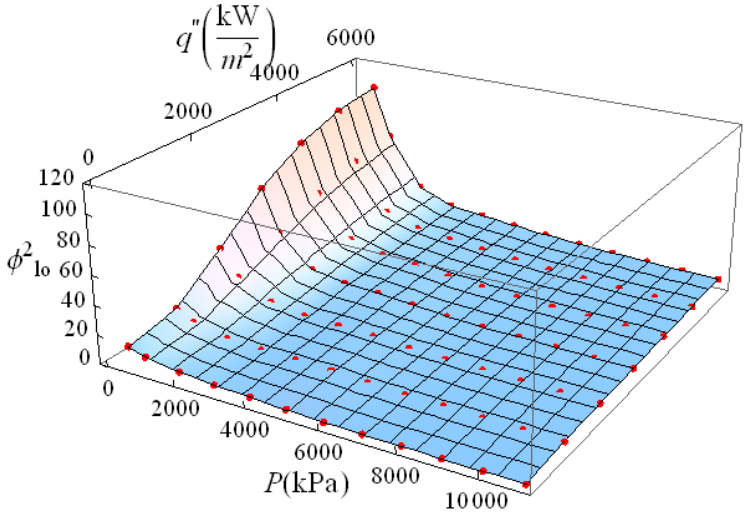


(b)

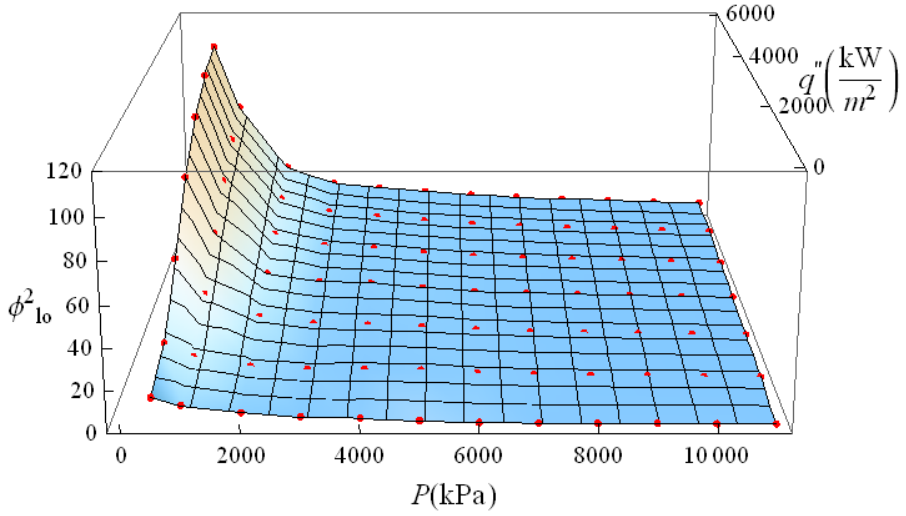


(c)

Figure D.7 : A 3-D illustration of the LUT at G=500 kg/m^2.s and x_a =0.05 for (a) skeleton table (b) and (c) updated table and (d) and (e) smoothed table (continued)

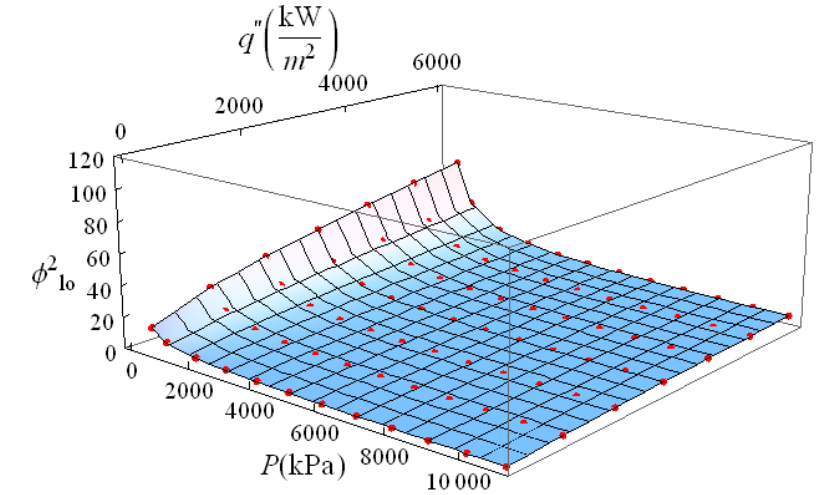


(d)

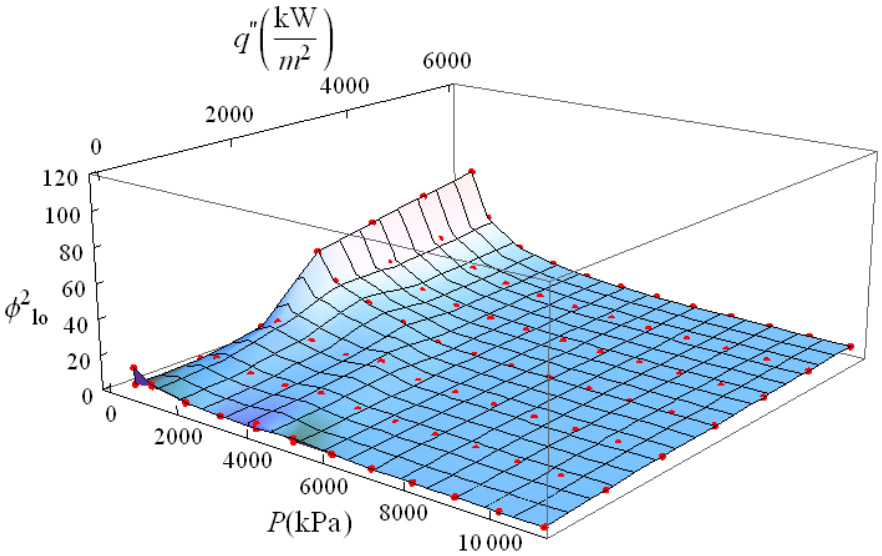


(e)

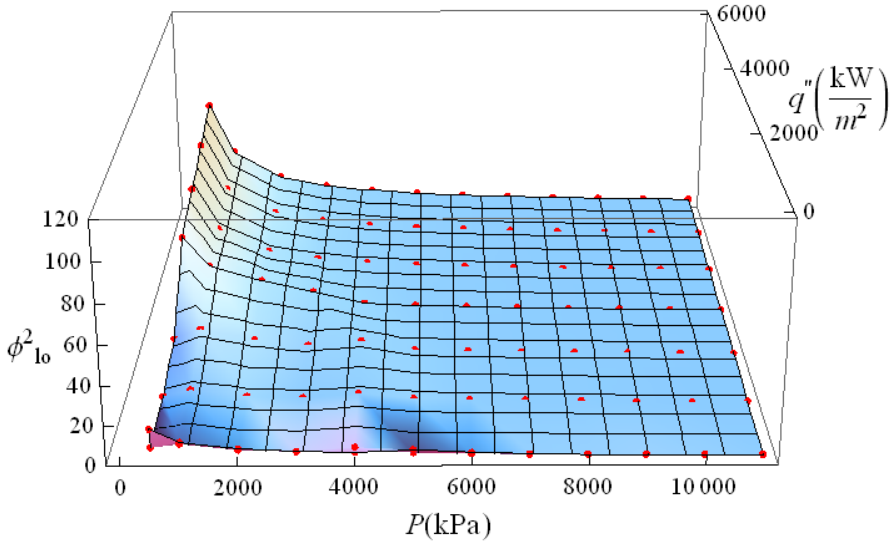
Figure D.7(continued) : A 3-D illustration of the LUT at G=500 kg/m^2.s and x_a=0.05 for (a) skeleton table (b) and (c) updated table, and (d) and (e) smoothed table



(a)

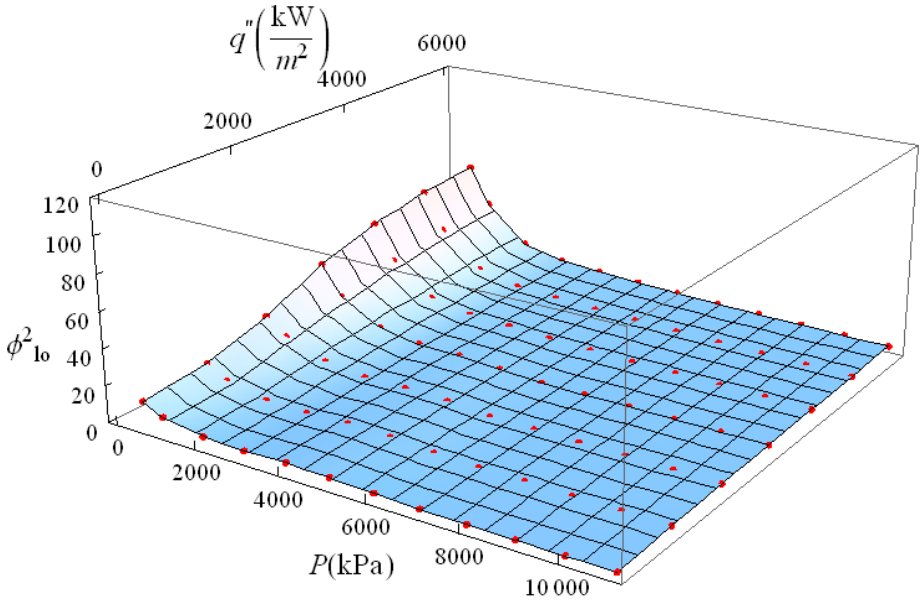


(b)

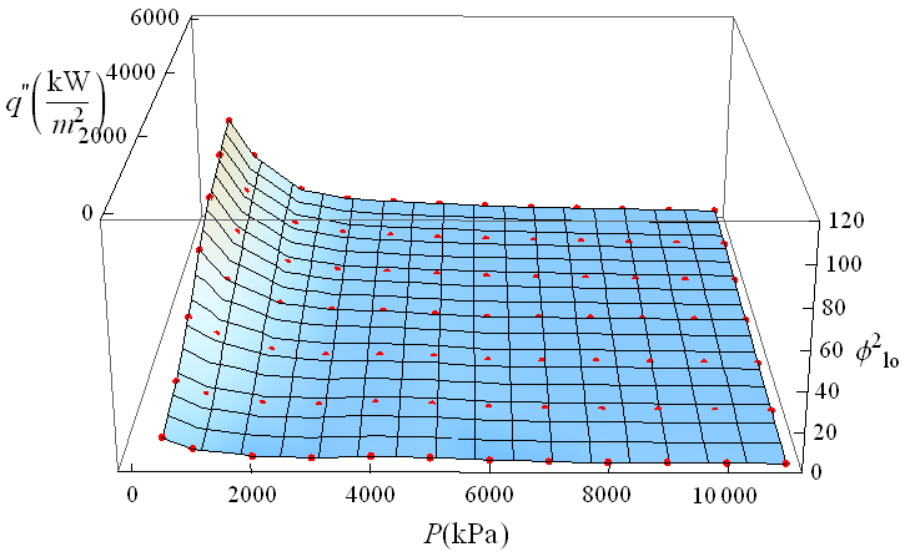


(c)

Figure D.8 : A 3-D illustration of the LUT at G=1000 kg/m^2 s and x_a=0.05 for (a) skeleton table (b) and (c) updated table, and (d) and (e) smoothed table

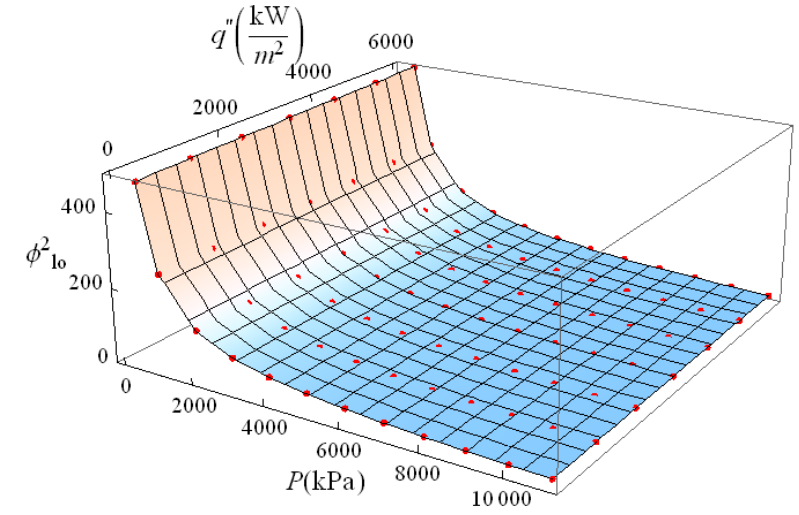


(d)

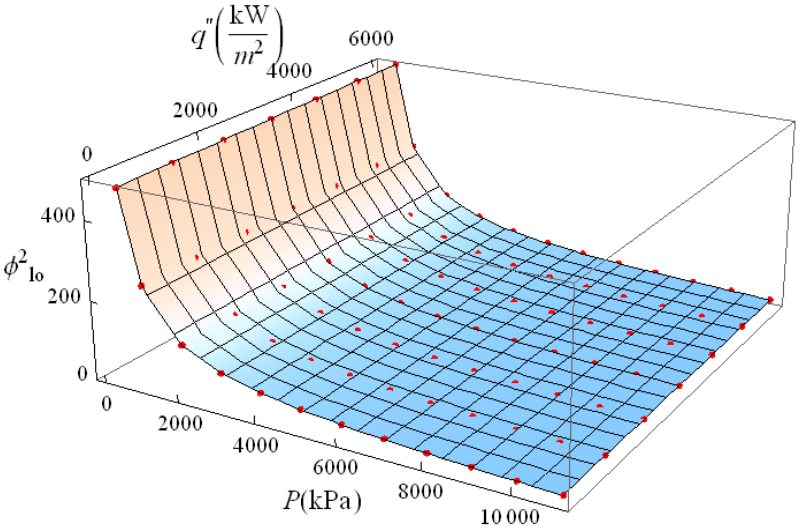


(e)

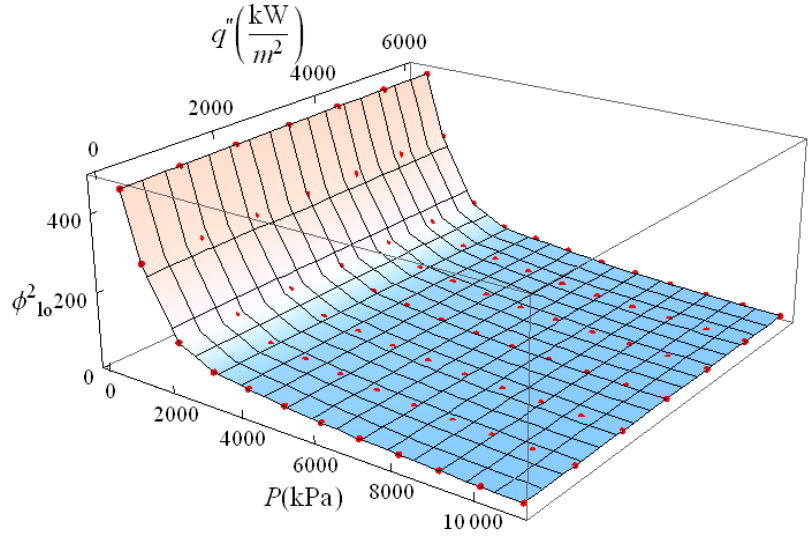
Figure D.8(continued) : A 3-D illustration of the LUT at G=1000 kg/m^2 s and x_a=0.05 for (a) skeleton table (b) and (c) updated table, and (d) and (e) smoothed table



(a)



(b)



(c)

Figure D.9 : A 3-D illustration of the LUT at G=1000 kg/m^2.s and x_a =1.0 for (a) skeleton table (b) updated table, and (c) smoothed table

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PUBLICATIONS. PRESENTATIONS AND PATENTS ON THE THESIS:

- **B. Öztüzün**. İ.A. Odabaş. A. Durmayaz. 2013: Methodology of look-up table development for two-phase flow parameters. *19th National Thermal Sciences and Techniques Congress*. Ondokuzmayıs University and Association of the Turkish Thermal Sciences and Techniques. *Samsun-Turkey. September 9-12. 2013* (Paper no: ULIBTK'13-049) (in Turkish) pp. 60-66.

- İ.A. Odabaş. N.E. Gengeç. **B. Öztüzün**. A. Durmayaz. 2013: Analysis of pressure drop measurements. two-phase friction multiplier correlations and pressure drop components for water-steam flows in heated and unheated vertical tubes. *19th National Thermal Sciences and Techniques Congress*. Ondokuzmayıs University and Association of the Turkish Thermal Sciences and Techniques. *Samsun-Turkey. September 9-12. 2013* (Paper no: ULIBTK'13-049) (in Turkish) pp. 53-59.