THE TWO-PERSON GUESSING GAME WITH ASYMMETRIC PLAYERS

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I hereby certify that this thesis meets all the requirements of the Graduate School of Social Sciences for a Master's degree.

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

İbrahim Çiçekli

ABSTRACT

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In this thesis, we theoretically and experimentally investigate the asymmetric players case in the two-person guessing game. We multiply one of the players' chosen number by a positive parameter $k > 1$. Here, the target number is some proportion (p) of the "weighted" average of the two numbers. The theoretical solution of the model depends on the value of p and k.

In the experimental sessions, we have one symmetric and two asymmetric cases in which the theoretical equilibrium is (0,0). We find that the chosen numbers in the asymmetric cases do not differ from those in the symmetric case. However, the speed of convergence toward equilibrium is slower in the asymmetric cases than that in the symmetric one.

Keywords: Guessing Game, Asymmetry, Convergence.

ÖZET

ASİMETRİK İKİ OYUNCULU TAHMİN OYUNU

ÇİÇEKLİ, İbrahim

Yüksek Lisans, Ekonomi Bölümü

Tez Yöneticisi: Yrd. Doç. Dr. Ayça Özdoğan

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Bu tezde iki-oyunculu tahmin oyununda "asimetrik oyuncular" durumu teorik ve deneysel olarak incelenmektedir. Bu modelde oyunculardan birinin seçtiği sayı pozitif bir parametre, $k > 1$, ile çarpılmaktadır. Burada hedef sayı seçilen iki sayının "ağırlıklı" ortalamasının p katına eşittir ($0 < p < 1$). Modelin teorik incelenmesi sonucunda dengenin belirlenen p ve k değerlerine bağlı olarak değiştiği ortaya konmuştur.

Deneysel oturumlarda teorik dengenin (0,0) olduğu bir simetrik iki de asimetrik durum incelenmiştir. Simetrik ve asimetrik durumlarda seçilen sayıların birbirlerinden farklı olmadığı, fakat asimetrik durumda seçilen sayıların (0,0) noktasına daha yavaş yakınsadığı gösterilmiştir.

Anahtar Kelimeler: Tahmin Oyunu, Asimetri, Yakınsama.

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CHAPTER 1

INTRODUCTION

1.1 The Standard Guessing Game

In the standard N-player ($N \geq 2$, $N \neq \infty$) guessing game (also known as p-Beauty Contest Game), players simultaneously choose a number from a closed interval, generally [0,100]. The player whose number is the closest to the target number (T) wins the game. The target number (T) is calculated as follows:

$$T = \left(\frac{1}{N} \sum_{i=1}^{N} n_i \right) p$$

where n_i is the player i's chosen number, $0 < p < 1$, and p is common knowledge. The winner of the game receives a pre-determined fixed prize, and other players get nothing. If there is a tie, the prize is equally divided among the winners.

The standard N-player guessing game is dominance-solvable if it is assumed that all players are rational and this rationality is common knowledge. Without loss of generality, if we assume that the strategy space is [0, 100], the target number cannot exceed $100p$. Thus, any number that is higher than

$100p$ is weakly dominated for all players. It means that no player will choose a number above $100p$, which in turn implies that the target number cannot exceed $100p^2$. Then, the players will not choose a number above $100p^2$. It is straightforward to see that if the above iteration process, called as Iterated Elimination of Weakly Dominated Strategies (IEWDS), goes on infinitely, the only undominated strategy will be zero. Hence, given $0 < p < 1$, all players choosing zero is the only Nash equilibrium of the game. By using IEWDS method, one can easily see that for $p > 1$, all players choosing 100 is the unique solution. However, we do not have a unique Nash equilibrium point if $p = 1$. In this case, every strategy profile in which all players choose the same number can be a Nash equilibrium.

1.2 Literature Review

In Nagel's first experiment (1995), there are three treatments in which only p values are different (p=2/3, p=1/2 and p=4/3). It is observed that in each treatment, most of the chosen numbers in the first period are far from the game theoretical solution (0 or 100, depending on the value of p). In p=2/3 and p=1/2 sessions, no subjects choose zero in the first period and 6% of the subjects choose a number below 10. In p=4/3 sessions, 10% of the subjects choose 99 or 100. Using Mann-Whitney U test, she rejects the null hypothesis that chosen numbers in p=1/2 and p=2/3 sessions are from the populations with the same distributions. The null hypothesis that choices in p=2/3 and p=4/3 sessions have the same distribution is also rejected. Then, Nagel concludes that the first period choices depend on the parameter, p. In the first period, no subject has any information about other subjects' choices and they choose their number based on their n^{th} order beliefs. She finds that most of the subjects in these sessions have second order beliefs. That is, while choosing their numbers in the first period, they make only two iterations. Upon this result, Nagel proposes the theory of boundedly rational behaviour which states that the depth of reasoning of the players is limited. Moreover, Nagel observes that after the first period, there is a gradual convergence to the equilibrium. She tests this observation by using simple learning-direction theory, and the theory predicts that the players tend to converge to the equilibrium over time.

After Nagel (1995), Stahl (1996) finds that subjects learn according to rule learning, and in contrast to Nagel, he shows that there is an increasing depth of reasoning. The results of Camerer et al. (1998) are similar to Nagel's findings. Weber (2000) investigates the feedback effect in learning. In one treatment, he gives feedback about previous period's results and in the other treatment he gives no feedback. Then, he finds that in both cases there is a convergence toward equilibrium, but in the treatment with no-feedback, the convergence is slower.

Sutter (2005) investigates how team size affects decision making and outcome in a guessing game. He finds that teams with 4 subjects earn more than both the teams with 2 subjects and single subjects. However, there is no significant difference between the teams with 2 subjects and single subjects. Thus, he concludes that teams perform better than individuals, but only if the team size is big enough. Kocher et al. (2006) state that more than 60% of individuals prefer to be in a team since they expect a higher payoff. The results of their experiment also reveal that teams earn more than individuals.

Kocher et al. (2007) study the effect of historical data (others' choices in the past) and advice on subjects' choices. In their game, the subjects in the control treatment fill an advice card which contains suggested number for the first period, the reason to choose that number and the strategy that should be followed in later periods. Then, these cards are used in the subsequent treatments. They find that if the subjects are provided advice or can access historical information, they earn more in the first period than the subjects with no access to advice and historical information. However, in the long run, they find only advice has a positive effect on earning. Sbriglia (2008) investigates the effect of information. At the end of each period, she announces the identity and chosen number of the winner. She finds that non-winners (even the sophisticated ones) imitate the winners in later periods.

Güth et al. (2002) examines the effect of being in a heterogeneous group on subjects' decisions. In their experiment, there are two groups of subjects: the homogeneous group in which all subjects have the same p value (1/2), and the heterogeneous group in which some subjects have p=2/3 and the others have p=1/3. Thus, in the heterogeneous group, while some subjects try to guess 2/3 of the mean, the others try to guess 1/3 of the mean to maximize their payoffs. The payoff scheme is continuous in the game, i.e., each player can earn some money depending on how close her number to her group's target. They find that the subjects in the heterogeneous group spend more time to decide and earn less than the subjects in the homogeneous groups.

Nagel et al. (2002) conducts a newspaper experiment in Spain, UK and Germany to see the effect of large groups on decisions. 7900 people participated the experiments and they find that the pattern of behaviours are common across countries. Also, this pattern is very similar to the lab experiments.

Grosskopf and Nagel (2008) run a two-person guessing game experiment with p=2/3. This two-player version of the game differs from its N-player counterparts since the iterative reasoning is unnecessary in this form. Here, the subject who chooses a smaller number wins the game. Thus, zero is always the winning number. Because of this basic structure of the game, they expect more subjects to choose zero than in N-player case. They run the experiment in two different subject pools: students and sophisticated subjects (game theorists). They find that only 9% of students and 36.92% of game theorists choose zero. According to the authors, one of the reasons is that the players cannot realize their own influence on the mean. Another reason is that some students try to choose half of the other subject's choice to find a fixed point. Finally the third reason is that the experienced subjects, who played the N-player version of the game before, transfer their experience negatively into this game and cannot realize that the situation is different here.

1.3 The Purpose of the Research

There is a similarity between the players' strategies in guessing games and the investors' strategies in stock markets. In stock markets, the direction of the stock prices is affected by investors' decisions. A rational investor forms a belief about other people's possible reactions, and act in accordance with this belief in an investment decision. Similarly, the target number in the guessing game is affected by all players' guesses about the target itself. Thus, a rational player would take her beliefs about others' possible reactions into consideration when deciding. From this perspective, the guessing game is a very simple model of stock markets.

In stock markets, investors differ from each other in terms of their market power. Big investors have the financial power to affect the price direction more than the small players. In the guessing game literature, to the best of our knowledge, there is neither theoretical nor experimental study investigating this asymmetric case which is more realistic. Thus, by introducing the asymmetric players, this research aims to contribute to the literature.

In this paper, we theoretically and experimentally investigate the two-person guessing game with asymmetric players. The players in our model are asymmetric in terms of their power level to influence the target number. Here, we multiply only one of the players' chosen number by a positive constant, k, which is greater than one. This parameter creates the asymmetry in the two person group. The target number in our model is some proportion (p) of the "weighted average" of the two choices. Thus, the big player -the player whose number is multiplied by k- has more power to affect the target number. The theoretical solution of our model depends on the value of k and p.

In the experimental sessions we have one p value (p=1/2), and three different k values (k=1,2,9) for three different treatments. In all of these three treatments, the equilibrium strategy for both players is zero as in the standard game. Notice that the treatment with k=1 is the same as the standard guessing game in which the players are symmetric. In the treatments with k=2 and k=9, however, the players are asymmetric (i.e. one big and one small player). In the experimental sessions of each treatment we asked the subjects to play the game 10 rounds. Our aim is to answer the following questions:

i) How do the choices differ across the symmetric and the asymmetric cases?

ii) Do the choices in each of the symmetric and the asymmetric cases converge to the equilibrium over time?

iii) If convergence exists, are there any differences in terms of speed of convergence across the symmetric and the asymmetric cases?

When we analyse the data, we find that there is no significant difference between the choices in the symmetric and the asymmetric cases. In each of these cases, we observe convergence toward equilibrium over time. However, choices in the symmetric case converge faster than those in the asymmetric case.

The rest of the paper is organized as follows. In Chapter 2 we introduce our model and its theoretical solution. In Chapter 3 we describe the experimental procedures in detail. In Chapter 4 we present the experimental results. Finally, in Chapter 5 we conclude with a brief discussion about the possible reasons of the experimental results.

CHAPTER 2

THE MODEL AND THEORETICAL SOLUTION OF THE MODEL

2.1 The Model

In this thesis, we introduce a two-person guessing game with asymmetric players in which one of the players has more power to determine the target number. In this design, the target number (T) differs from the one in the standard guessing game. It is calculated as follows:

$$T = \left(\frac{n_s + kn_b}{k+1} \right) p$$

where $0 < p < 1$ and n_s, n_b are the chosen numbers of small and big players (We do not classify players as "big" and "small".), respectively. The parameter $k > 1$ is a positive integer and represents the power level of the big player (when $k = 1$, this boils down to the standard guessing game described in Chapter 1). All the parameters in the game are common knowledge. As in the standard guessing game, players simultaneously guess a number from the closed interval [0, 100]. The player with the closest guess to the target number wins the game, and if there is a tie, the prize is equally divided among the players. Mathematically,

$$
\begin{aligned}
|n_s - T| - |n_b - T| < 0 &\;\Rightarrow\; \text{the small player is the winner.} \\
|n_s - T| - |n_b - T| > 0 &\;\Rightarrow\; \text{the big player is the winner.} \\
|n_s - T| - |n_b - T| = 0 &\;\Rightarrow\; \text{there is a tie.}
\end{aligned}
$$

For the sake of simplicity, players are allowed to choose only integers in our model. Lopez (2001) states that when calculating the target number, the experimenter must use decimal approximation. For this reason, he calls the game as "beauty contest decimal game". In his paper, Lopez also proves that the beauty contest decimal game is equivalent to the beauty contest integer game. Thus, he concludes that any experimental guessing game is equivalent to its integer restricted version. This finding is also one of our motivations to make subjects choose only integers.

2.2 Theoretical Solution of the Model

In contrast to standard guessing game described in Chapter 1, the equilibrium in our model is not unique for all $p < 1$. The following lemmas characterize the equilibrium in detail. All proofs are available in Appendix A.

Lemma 1. *Let $k > 1$. If $p < \frac{k+1}{2k}$, then*

i) the player whose number is smaller wins the game.

ii) (0,0) strategy pair is the unique Nash Equilibrium of the game.

Lemma 1 states that although we have an asymmetry with $k > 1$, the game dynamics are the same with the symmetric case if we also have $p < \frac{k+1}{2k}$. The player with a smaller number wins the game as in the standard guessing game.

Lemma 2. *Let $k > 1$. If $p > \frac{k+1}{2k}$, then*

i) small player wins only when $an_b < n_s < n_b$ where $a = \frac{2pk-k-1}{k+1-2p} < 1$,

ii) there is at least one mixed strategy Nash Equilibrium.

Lemma 2 states that if we have $k > 1$ and $p > \frac{k+1}{2k}$, choosing a smaller number is not enough for small player to win. She must also choose a number that is greater than some proportion of the number of the big player. That is, small player wins the game only if she mimics the big player.

Lemma 3. *Let $k > 1$. If $p = \frac{k+1}{2k}$,*

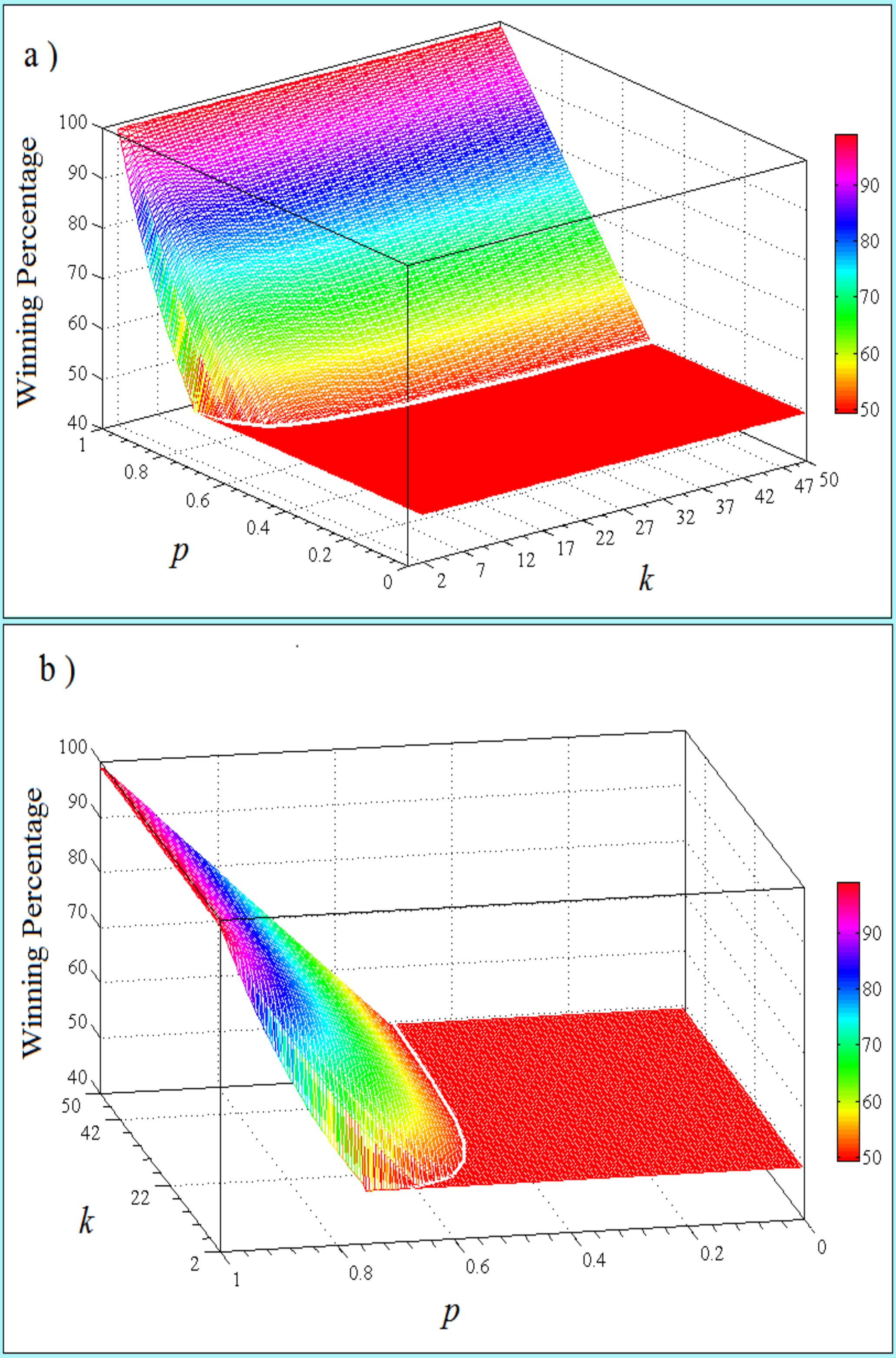
i) and if $n_s = 0$, there will be a tie regardless of the value of n_b.

ii) and if $n_s \neq 0$, the player whose number is smaller wins the game.

iii) there is at least one mixed strategy Nash Equilibrium.

Lemma 3 indicates that the small player guarantees to share the prize if she chooses zero. However, if she wants to win the game, she must choose a number that is greater than zero but smaller than the big player's number.

Since the strategy space for each player is the set of integers in [0, 100], we have a 101×101 finite game whose payoff matrix has $101^2 = 10201$ cells. Thus, given p and k, it is easy to figure out the number of cells (in percentage) at which the big player wins. Figure 1 shows this winning percentage of the big player as a function of p and k. For example, if $k = 7$ and $p = 0.74$, then the winning percentage is % 67.91. This means that when $k = 7$ and $p = 0.74$, the big player is the winner approximately in % 68 of all cells in the payoff matrix. The 3-D shapes in Figure 1a and 1b are the same except that they are captured from different angles.



Note: $0 < p < 1$. k is an integer and $k > 1$. k values above 50 are not included since there is no significant change in the figure's shape for $k > 50$.

Figure 1 Winning Percentage of the Big Player as a Function of p and k

Notice that there are mainly two regions in the Figure 1a: the flat region and the steeper region. The two regions are separated by a white curve which corresponds to the points satisfying $p = \frac{k+1}{2k}$. The flat region below the white curve corresponds to the points satisfying $p < \frac{k+1}{2k}$, and the steeper region above the white curve corresponds to the points satisfying $p > \frac{k+1}{2k}$.

Remember that if $p < \frac{k+1}{2k}$, the player whose number is smaller wins the game (See Lemma 1). Then, once $p < \frac{k+1}{2k}$ is satisfied, the structure of the payoff matrix does not change. That is why the surface below the white curve is flat. In this flat region, the winning percentage (about 49%) of the big player does not change with the changes in p and k.

However, if $p > \frac{k+1}{2k}$, we know that for small player to win, she should play her strategy, n_s, such that $an_b < n_s < n_b$ where $a = \frac{2pk-k-1}{k+1-2p}$ (See Lemma 2). Since the parameter a is a function of p and k, the structure of the payoff matrix changes with the changes in p and k. Therefore, we have a non-flat region above the white curve.

There are two observations for the steeper region of the Figure 1b:

1. For any fixed $p > \frac{k+1}{2k}$ and lower values of k, as k increases, the surface firstly becomes steeper and then its slope becomes constant in the direction of k axis. This implies that for any fixed $p > \frac{k+1}{2k}$, increasing the power level causes the winning percentage of the big player to rise, but to some extent.

2. For any fixed $k > 1$ with $p > \frac{k+1}{2k}$, as p increases, the winning percentage of the big player rises gradually and reaches its maximum, 99% .

The following propositions summarize above observations formally.

Proposition 1. *Let $z \in \mathbb{Z}$. Consider a $z \times z$ game with $p > \frac{k+1}{2k}$. There exists a $k^* > 1$ such that for all $k < k^*$, as k increases, the range in which the big player wins gets larger and for no $k > k^*$ the game structure changes.*

Proof. See Appendix A. $\square$

Proposition 2. *Let $z \in \mathbb{Z}$. Consider a $z \times z$ game with $p > \frac{k+1}{2k}$. Then, for any $k > 1$, there exists a $p^* < 1$ such that for all $p > p^*$ the big player wins the game if $n_s \neq n_b$.*

Proof. See Appendix A. $\square$

CHAPTER 3

EXPERIMENTAL DESIGN

The experiment was conducted in June 2013 at the experimental laboratory of the TOBB University of Economics and Technology (TOBB ETU), in Turkey. Computers in the experimental laboratory were placed around its perimeter and were isolated so that the subjects could not see the other subjects' screen. Subjects were also not allowed to communicate with each other during the sessions. The experiment was programmed and conducted with z-Tree (Fischbacher 2007).

Announcement of the experiment was made by e-mail and participants were registered to one of the experimental sessions. There were totally 6 sessions with 14-18 participants in each. A subject participated in only one session which lasted approximately 40 minutes. We collected data from 104 subjects who were graduate and undergraduate students from TOBB ETU.

In all sessions, the strategy space for each player was the set of integers in [0, 100]. Thus, the game in the experiment was a 101×101 finite guessing game. For the sake of simplicity, we set $p = \frac{1}{2}$ in all sessions. Hence, (0,0) was the unique pure strategy Nash equilibrium throughout the whole experiment (See Lemma 1).

In the experiment, there were three treatments with different k values:

$k = 1$, $k = 2$ and $k = 9$. We will refer these treatments as $k1$, $k2$ and $k9$ treatments. In $k1$ treatment, subjects played the standard two-player guessing game in which playing zero is the unique pure strategy Nash equilibrium. We designed the $k1$ treatment as a baseline treatment to compare the results of $k2$ and $k9$ treatments. There were 2 sessions (36 subjects) for $k1$ treatment, 2 sessions (32 subjects) for $k2$ treatment and 2 sessions (36 subjects) for $k9$ treatment.

When subjects arrived at the laboratory, they were placed randomly to separate computer stations. Then, all subjects were given an instructions sheet in which the general rules and the rules of the game were written (see Appendix B). Instructions were read aloud and questions related to the instructions were answered. Then, subjects were given a multiple choice quiz in the computer environment. The quiz consisted of 5 questions related to the calculation of the target number and the payment scheme (see Appendix B).

After the quiz, the software randomly assigned player types to the subjects and formed our two-person groups that did not change throughout a session (Random matching in each round was also possible. By this way, we could prevent subjects not to choose zero strategically, i.e., choosing a number greater than zero in order for the rival not to realize that zero is the winning strategy.). In each group, we had a small player and a big player. Before starting the first round, the subjects saw their types on the computer screen for 15 seconds. We did not use the terms "small" and "big" while informing the subjects about their types. Instead, it was written on the screen "You are Player A and your number will be multiplied by 1." or "You are Player B and your number will be multiplied by k." depending on the subject's type. Subjects were also told that their types and groups were assigned randomly by the software, and their rival in the group will be the same person throughout the whole session.

After informing players about their types, the game was started and played for 10 rounds. In each round, the formula of the target number and the type of the player (A or B) were displayed on the screen. Subjects were given 45 seconds for each round to decide and enter their numbers. When they both entered their numbers, they were provided feedback about their own number, the number of the other player in the group, the calculated target number and the winner.

At the end of the tenth round, subjects were given a cognitive reflection test (CRT) which contains 5 standard questions (see Appendix B). They had 3 minutes to answer these questions. Afterwards, they were asked to fill out a short survey including some demographic questions and some possible strategies that might be implemented in the game. They were also provided a space to write their strategies with their own words. In the survey part, subjects were also asked whether they played or they heard about a number guessing game before that contains calculation of a target.

After subjects completed the survey, the software randomly chose 3 rounds out of the 10 rounds and calculated the subjects' earnings in those rounds. For each chosen round, the winner in each group earned 5 TL, and the other player got nothing. If there was a tie, we paid 2,5 TL to each player in the group. In order to avoid a possible wealth effect, we did not pay for each of the 10 rounds. We informed subjects about this payment scheme at the beginning of the session. In total, a subject was paid 5 TL show up fee plus the amount from the selected 3 rounds. Hence, a subject had a chance to earn 5 to 20 TL (\$2.70 - \$10.8) from the experiment.

CHAPTER 4

RESULTS

In the survey given at the end of the experimental sessions, 15 subjects reported that they played a number guessing game before that contains a target calculation. Also, 20 subjects reported that they heard but not played. We classified those 35 subjects in total as "experienced" and grouped the experimental data into two parts: the data set of all subjects and the data set of the inexperienced subjects. The latter was obtained by removal of experienced subjects from the whole data set. In k1, k2 and k9 treatments, the number of experienced subjects was almost equal (12 in k1 treatment, 11 in k2 treatment and 12 in k9 treatment). In k2 treatment, 5 out of 11 experienced subjects played as the small player and the remaining 6 subjects played as the big player. However, the distribution of the experienced players over player roles was quite different in k9 treatment. There were 4 experienced small players and 8 experienced big players in k9 treatment.

4.1 The First Round Behaviour

In this section, we investigate whether the chosen numbers in the first round differ across treatments. We first compare proportion of zero choices, and then compare all choices in the first round.

4.1.1 Comparison of Proportions of Zero Choices in the First Round

Table 1 presents the proportion of the subjects choosing zero in the first round. We compare these proportions pair by pair using the "one-sided Fisher's Exact test" since the sample size is very small for some comparison of proportions in the table (i.e. the inexperienced big players in k2 treatment and in k9 treatment). For each comparison, we provide the Fisher's exact value -*p*- in brackets.

Table 1 Proportion of Subjects that Choose Zero in the First Round

TR	ALL SUBJECTS						INEXPERIENCED SUBJECTS					
	All Players		Small Players		Big Players		All Players		Small Players		Big Players	
k1	**41.7%**	(15/36)	-	-	-	-	**45.8%**	(11/24)	-	-	-	-
k2	**37.5%**	(12/32)	**31.3%**	(5/16)	**43.8%**	(7/16)	**42.9%**	(9/21)	**36.4%**	(4/11)	**50.0%**	(5/10)
k9	**19.4%**	(7/36)	**33.3%**	(6/18)	**5.6%**	(1/18)	**25.0%**	(6/24)	**35.7%**	(5/14)	**10.0%**	(1/10)
Total	**32.7%**	(34/104)	**32.4%**	(11/34)	**23.5%**	(8/34)	**37.7%**	(26/69)	**36.0%**	(9/25)	**30.0%**	(6/20)

Note: **TR** denotes treatment.

In the first round, 32.7 percent (34 out of 104) of all subjects choose zero. We observe the highest percentage (41.7%) in k1 treatment and the lowest percentage (19.4%) in k9 treatment. It is 37.5% in k2 treatment. By comparing above proportions, we find that the subjects in k1 treatment choose zero significantly more often than the subjects in k9 treatment (p=0.036). Also, the proportion of zero choices in k2 treatment is marginally significantly higher than that of k9 treatment (p=0.083). However, the proportion of zero choices in k1 and k2 treatments are not significantly different from each other (p=0.460). On the other hand, the proportion of "inexperienced" subjects who choose zero in the first round is slightly higher than that of all subjects. 37.7 percent (26 out of 69) of the inexperienced subjects choose zero in the first round. This proportion is 45.8% in k1 treatment, 42.9% in k2 treatment and 25% in k9 treatment. When we compare these proportions treatment by treatment, we find no significant difference between one and the other (• comparison of k1 and k9 treatments, p=0.114; • comparison of k2 and k9 treatments, p=0.171; • comparison of k1 and k2 treatments, p=0.54).

In k2 treatment, 31.3% of the small players and 43.8% of the big players choose zero. The Fisher's Exact test show that these proportions are not significantly different (p=0.358). However, the same test for k9 treatment reveal that the proportion of the small players choosing zero (33.3%) is significantly greater than 5.6%, which is the proportion of the big players choosing zero (p=0.044). For the "inexperienced" players, however, the test reveal that there is no significant difference between the proportions of zero choices of small and big players in both k2 and k9 treatments (• comparison in k2 treatment, p=0.425; • comparison in k9 treatment, p=0.171).

We also compare the same type of players in k2 and k9 treatments. We find that there is no significant difference in terms of the proportion of zero choices between the small players of k2 treatment (31.3%) and that of k9 treatment (33.3%) (p=0.594). We obtain the same result for "inexperienced" small players in k2 and k9 treatments, either (p=0.648). In contrast to the comparison result of the small players, the test show that the big players in k2 treatment choose zero significantly more often (43.8%) than the big players in k9 treatment (5.6%, the lowest in the table) (p=0.012). The comparison result for the "inexperienced" big players, however, is not the same. The test show that the proportion of zero choices of "inexperienced" big players in k2 treatment (50%, the highest in the table) is marginally significantly higher than that of "inexperienced" big players in k9 treatment (10%) (p=0.07).

We observe that the results for all subjects differ from the results for the "inexperienced" subjects in terms of proportion of choices of zero. In general, when we obtain a significant difference from a comparison within all subjects, we see that the same difference from the same comparison is not significant within the inexperienced subjects. The only common significant difference is that the "big" players in k9 treatment differ significantly from the players in k1 treatment in terms of proportion of choices of zeros in the first round. The big players in k9 treatment choose zero significantly less often (5.6% for all subjects and 10% for inexperienced subjects) than the players in k1 treatment (41.7% for all subjects and 45.8% for inexperienced subjects) (• comparison for all players, p=0.005; • comparison for the inexperienced players, p=0.05).

4.1.2 Comparison of All Choices in the First Round

Table 2 presents the means and the medians of the first round choices for all treatments. While the k1 treatment has the lowest mean and median values, the k2 treatment has the highest values.

Table 2 Means & Medians of the First Round

	MEAN			MEDIAN		
SUBJECTS	**k1**	**k2**	**k9**	**k1**	**k2**	**k9**
All	**15.33**	**22.72**	**20.97**	**5**	**12.5**	**7.5**
Inexperienced	**13.33**	**20.33**	**20.04**	**1**	**10**	**5.5**

We compare the chosen numbers in the first round by using two-sided Kolmogorov Smirnov test for equality of distribution functions. Our aim is to show whether there are differences in chosen numbers of different groups of players in the experiment. Figure 2 shows the cumulative frequency of chosen numbers in k1, k2 and k9 treatments.

According to the Kolmogorov Smirnov (K-S) test, the first round choices in k1, k2 and k9 treatments are not significantly different from each other (two-sided K-S: • comparison of k1-k2, p=0.687; • comparison of k1-k9, p=0.337;

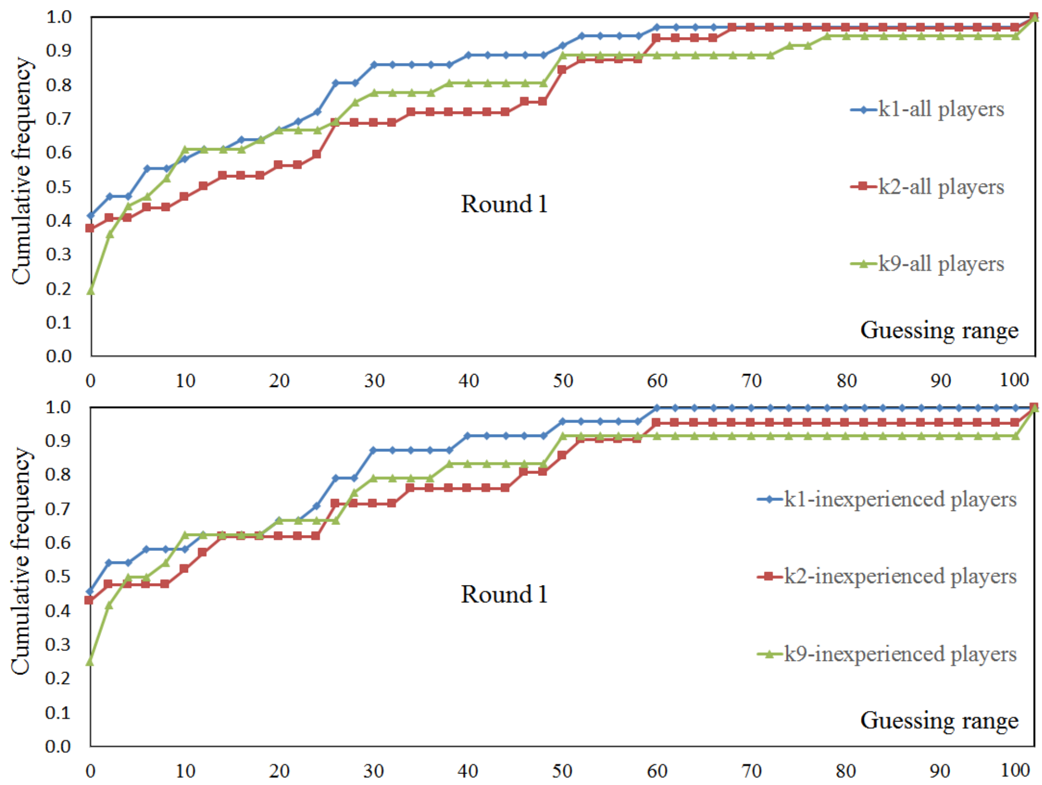


Figure 2 Cumulative Frequency of Choices in the First Round

• comparison of k2-k9, p=0.639). When we apply the test to the inexperienced players' data, we again find no significant difference (two-sided K-S: • comparison of k1-k2, p=0.934; • comparison of k1-k9, p=0.675; • comparison of k2-k9, p=0.867). In Figure 2, we see that cumulative distribution function of chosen numbers in k1 treatment lies above the other two distribution functions. For this reason, we expect chosen numbers in k1 treatment to be lower than in k2 and k9 treatments, but the test does not support this.

Figure 3 demonstrates the cumulative frequency of the first round choices of small and big players in k2 and k9 treatments. We find that there is no significant difference between the choices of small players and that of big players (two tailed K-S test, comparison of small and big players: • in k2 treatment, p=0.941; • in k9 treatment, p=0.131). This result is also valid for the inexperienced subjects (two tailed K-S test, comparison of inexperienced small and big players: • in k2 treatment, p=0.950; • in k9 treatment, p=0.308).

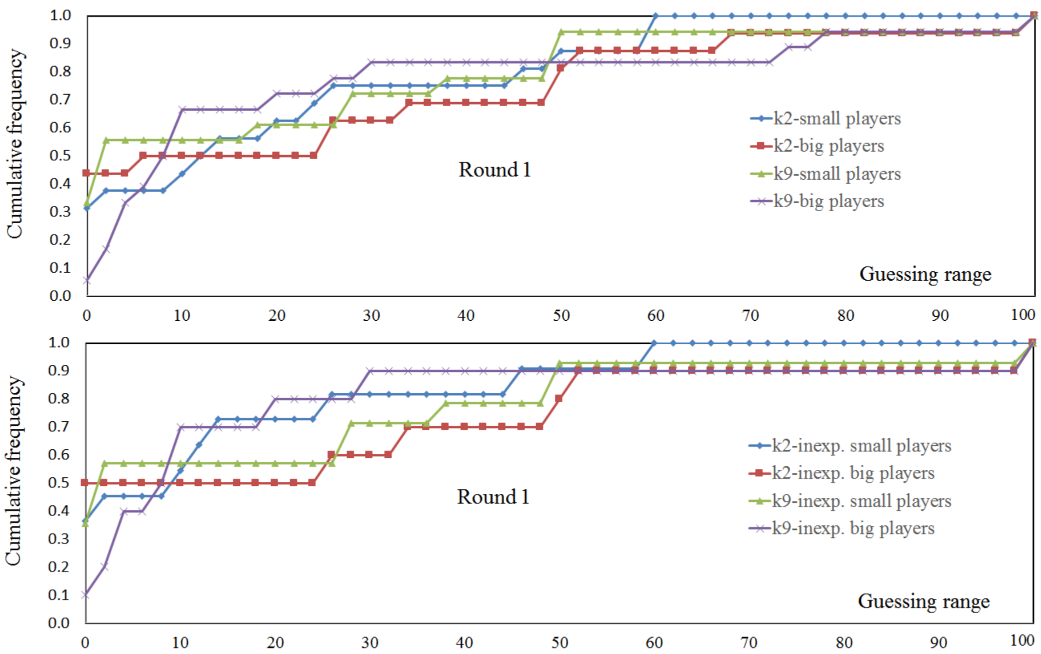


Figure 3 Cumulative Frequency of First Round Choices of Small and Big Players

We also compare the same type of players in different treatments. The K-S test show that choices of small players in k2 and k9 treatments are not significantly different from each other (two tailed K-S test: • comparison of small players in k2 and k9 treatments, p=0.945; • comparison of inexperienced small players in k2 and k9 treatments, p=0.847). The big players' choices are not significantly different in k2 and k9 treatments, either (two tailed K-S test: • comparison of big players in k2 and k9 treatments, p=0.169; • comparison of inexperienced big players in k2 and k9 treatments, p=0.400).

To sum up, analysis of data for all subjects and for the inexperienced subjects shows that the first round choices are not significantly different from each other. Thus, introducing asymmetry to the two-person guessing game does not cause the first round choices to differ.

4.2 The Behaviour in Later Rounds

Table 3 and Table 4 present the means and medians of chosen numbers for each round from 1 to 10 for all subjects and for the inexperienced subjects, respectively.

Table 3 Means & Medians of Rounds 1-10 (All Subjects)

	MEAN				MEDIAN			
ROUND	k1	k2	k9	k2+k9	k1	k2	k9	k2+k9
1	15.33	22.72	20.97	21.79	5	12.5	7.5	9.5
2	6.67	16.47	19.25	17.94	0.5	10	7.5	9.5
3	6.61	11.47	11.33	11.40	0	3.5	1	2.5
4	5.53	12.5	4.33	8.18	0	1.5	0.5	1
5	7.25	7.09	7.28	7.19	0	0	0	0
6	2.14	4.56	8.03	6.40	0	0	0	0
7	1	1.81	4.06	3	0	0	0	0
8	0.42	7.28	2.56	4.78	0	0	0	0
9	0.17	1.63	3.83	2.79	0	0	0	0
10	2.89	0.47	2.33	1.46	0	0	0	0

Table 4 Means & Medians of Rounds 1-10 (Inexperienced Subjects)

	MEAN				MEDIAN			
ROUND	k1	k2	k9	k2+k9	k1	k2	k9	k2+k9
1	13.33	20.33	20.04	20.18	1	10	5.5	7
2	7.5	15.67	15.79	15.73	0	2	4.5	4
3	6.79	10.33	12.88	11.69	0	4	1	1
4	6.63	11.71	3.92	7.56	0	2	0	1
5	10.08	3.52	8.75	6.31	0	0	0	0
6	2.08	2.90	5.04	4.04	0	0	0	0
7	1.42	0.81	4.33	2.69	0	0	0	0
8	0.63	5.95	0.08	2.82	0	0	0	0
9	0.08	1	0.13	0.53	0	0	0	0
10	4.29	0.29	0	0.13	0	0	0	0

Notice that in both tables the median of choices in all treatments becomes zero after round 4. For this reason, we think that comparing the numbers in rounds 2, 3 and 4 is sufficient. Using the two-tailed K-S test, we find some significant results for all subjects. However, we observe that all these significant results are not significant for the inexperienced subjects (**comparison of k1-k2:** • in round 2, all subj. p=0.04, inexp subj. p=0.176; • in round 3, all subj. p=0.033, inexp subj. p=0.108; • in round 4, all subj. p=0.085, inexp subj. p=0.064; **comparison of k1-k9:** • in round 2, all subj. p=0.043, inexp subj. p=0.577; • in round 3, all subj. p=0.083, inexp subj. p=0.342; • in round 4, all subj. p=0.257, inexp subj. p=0.837; **comparison of k2-k9:** • in round 2, all subj. p=1, inexp subj. p=0.546; • in round 3, all subj. p=0.449, inexp subj. p=0.476; • in round 4, all subj. p=0.342, inexp subj. p=0.253).

We also compare the chosen numbers of all the small players with the chosen numbers of all the big players. Nevertheless, we find no significant difference from this comparison, either (comparison of small and big players: • in round 2, all subj. p=0.375, inexp subj. p=0.937; • in round 3, all subj. p=0.796, inexp subj. p=0.999; • in round 4, all subj. p=0.954, inexp subj. p=0.937).

Above results show that there is no significant difference in chosen numbers. Thus, we conclude that introducing asymmetry into two-person guessing game does not lead to the chosen numbers to be significantly different from each other.

4.2.1 Existence of the Convergence

In Figure 4-6, we present the 3-D histograms of frequencies of choices in each of the 10 rounds. It is obvious that the proportion of zero choices increases over time. Only in the figure for k2 treatment, there is a considerable decrease from round 1 to round 2 (from 33% to 28% for all subjects, and from 43% to 33% for the inexperienced subjects). This is because 3 subjects in k2 treatment slightly increase their choices in the second round after choosing zero in the first round. Actually, there is one such player in each of the treatments k1 and k9, but their effect on the proportion is not observable in the figures since the number of new subjects choosing zero in the second round is greater than one.

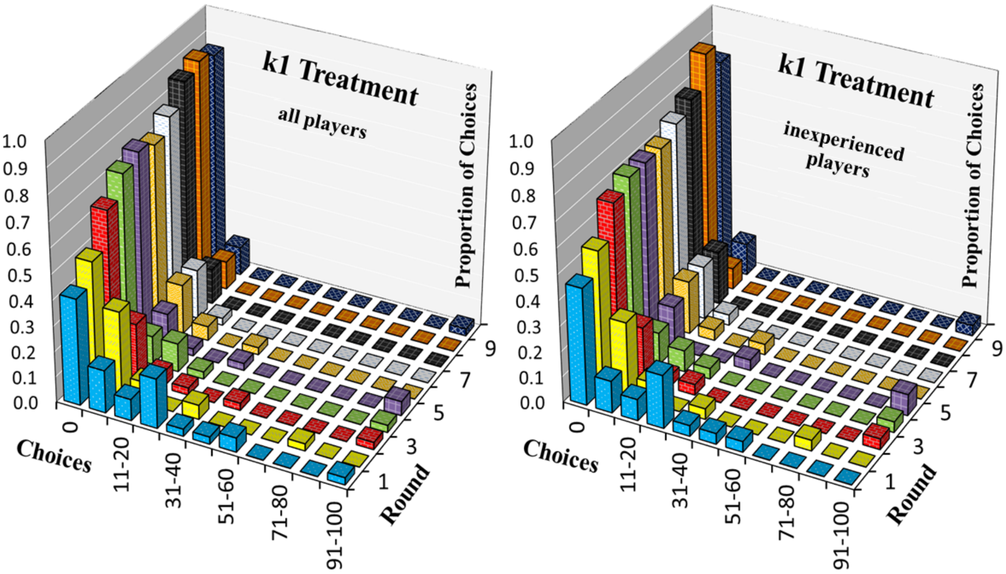


Figure 4 Proportion of Choices of k1-Subjects over Time

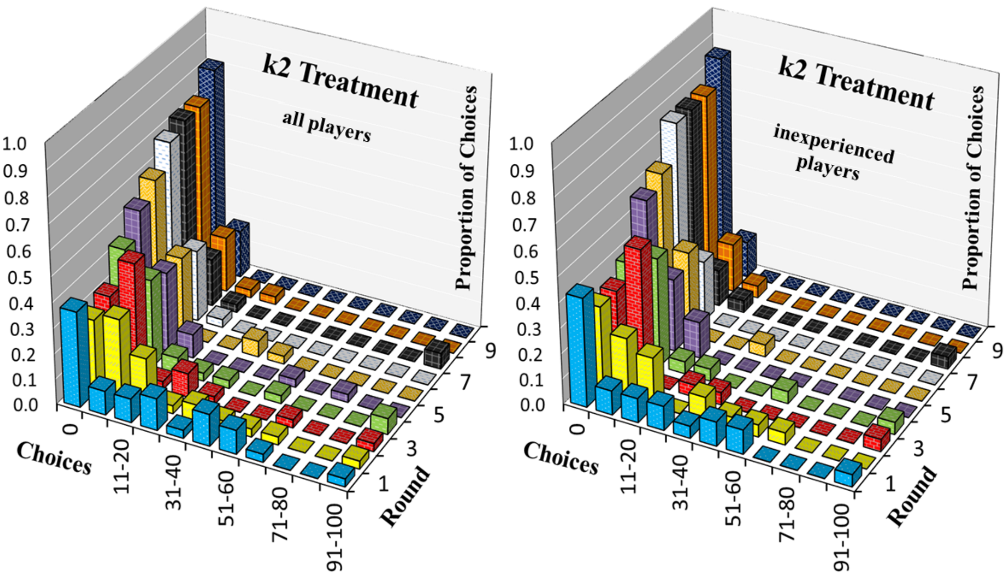


Figure 5 Proportion of Choices of k2-Subjects over Time

Notice that the magnitude of the bars located in [1,10] interval increases or stays fixed in early rounds. This implies that after the first round, some players decide to choose lower numbers but not realize that zero is always the

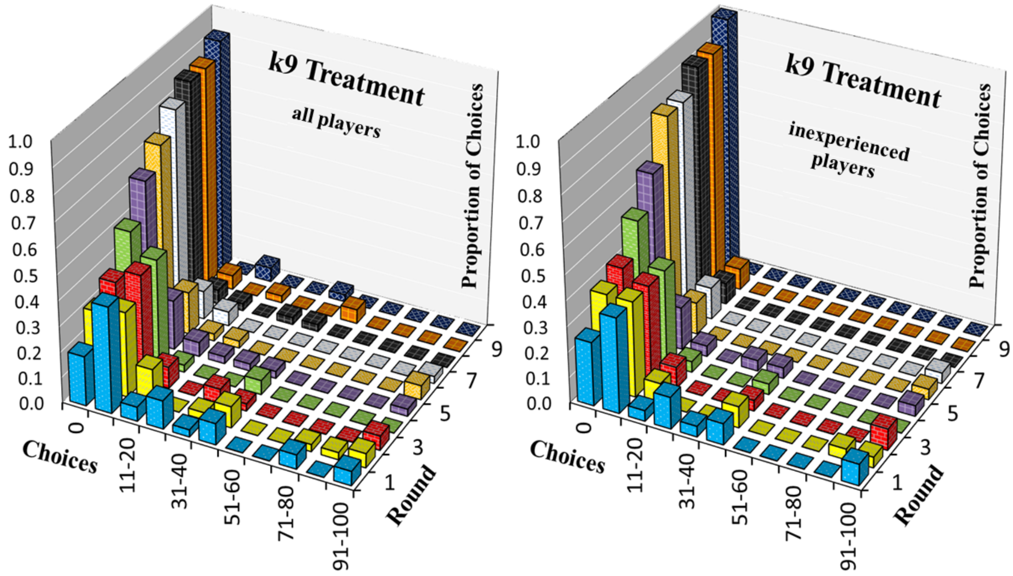


Figure 6 Proportion of Choices of k9-Subjects over Time

winning number. In addition, we observe that after choosing zero in early rounds some subjects increase their choices, but most of them return to zero again in later rounds.

Table 5 shows the proportion of zero choices from round 1 to round 10 for different treatments.

Table 5 Proportion of Subjects Choosing Zero over Time

	ALL SUBJECTS			INEXPERIENCED SUBJECTS		
ROUND	**k1 (36)**	**k2 (32)**	**k9 (36)**	**k1 (24)**	**k2 (21)**	**k9 (24)**
1	41.7%	37.5%	19.4%	45.8%	42.9%	25.0%
2	50.0%	28.1%	30.6%	54.2%	33.3%	37.5%
3	63.9%	31.3%	36.1%	66.7%	33.3%	41.7%
4	72.2%	43.8%	50.0%	70.8%	38.1%	54.2%
5	75.0%	53.1%	63.9%	70.8%	57.1%	66.7%
6	72.2%	59.4%	72.2%	70.8%	61.9%	83.3%
7	77.8%	68.8%	80.6%	75.0%	76.2%	83.3%
8	86.1%	71.9%	86.1%	79.2%	76.2%	91.7%
9	88.9%	71.9%	86.1%	91.7%	76.2%	91.7%
10	86.1%	81.3%	91.7%	83.3%	85.7%	100.0%

Note: The numbers in brackets are the total number of subjects in related treatments.

In the table, the proportion of zero choices in k1 and k2 treatments nearly doubles from round 1 to round 10. Also, that proportion in k9 treatment increases dramatically. This increase in proportions of choices of zero in all treatments provides an evidence for the convergence toward zero. As an additional evidence, the following Figure 7 shows the cumulative frequencies of choices in round 1 and round 5. In the figure, cumulative frequency of choices in round 5 lies above the cumulative frequency of choices in round 1 for all treatments. This demonstrates that frequency of lower choices is higher in round 5 than in round 1 which in turn means that chosen numbers decline over time (Wilcoxon signed-rank test results for the comparison of choices in round 1 and that in round 5: • comparison for k1 treatment, p=0.0012; • comparison for k2 treatment, p=0.0009; • comparison for k9 treatment, p=0.0006).

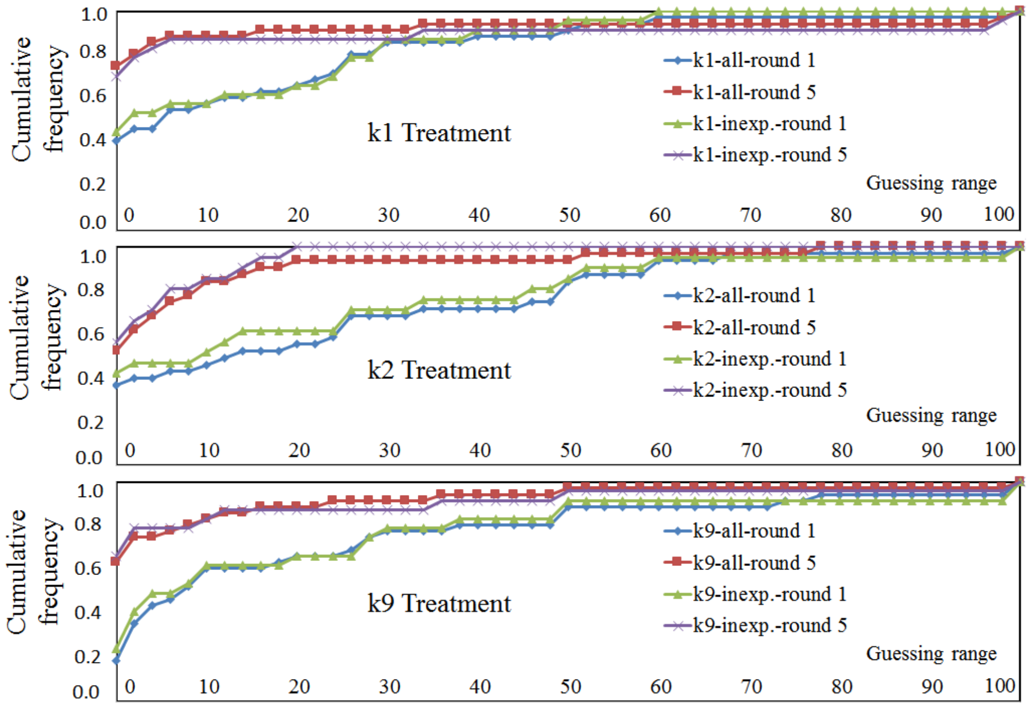


Figure 7 Cumulative Frequency of Choices by Treatment (Round 1 & 5)

Thus, we conclude that subjects in all treatments revise their choices towards the equilibrium in the first 4-5 rounds. The next section examines the speed of convergence across treatments.

4.2.2 The Speed of Convergence

We show in the previous section that there exist a fast convergence towards zero. In this section we examine whether the convergence speed differs across treatments. Table 6 presents the medians of rounds 1-10 and the rates of decrease in medians. We use the rate of decrease definition in Nagel (1995). The following formula shows how the rate of decrease is calculated.

$$w_{1-N}^{median} = \frac{median_{round=1} - median_{round=N}}{median_{round=1}}$$

where $w_{1\;N}^{median}$ denotes the rate of decrease from round 1 to round N.

Table 6 Medians of Rounds 1-10 and the Rates of Decrease in Medians

		ALL PLAYERS			INEXPERIENCED PLAYERS		
ROUND		k1	k2	k9	k1	k2	k9
1		5	12.5	7.5	1	10	5.5
2		0.5	10	7.5	0	2	4.5
3		0	3.5	1	0	4	1
4		0	1.5	0.5	0	2	0
5		0	0	0	0	0	0
6		0	0	0	0	0	0
7		0	0	0	0	0	0
8		0	0	0	0	0	0
9		0	0	0	0	0	0
10		0	0	0	0	0	0
Rate of Decrease	1 - 2	0.9	0.2	0	1	0.8	0.18
	1 - 3	1	0.72	0.87	1	0.6	0.82
	1 - 4	1	0.88	0.93	1	0.8	1
Average		0.97	0.6	0.6	1	0.73	0.67

For all treatments, the median is zero in round 5. Thus, the rate of decrease in median from round 1 to round 5 equals 1 for all treatments. For this reason, we analyse the speed of convergence for the first 4 rounds. We observe for all subjects and for the inexperienced subjects that rate of decrease in median in the first 4 rounds is generally higher in k1 treatment than in the other two treatments. However, when we compare the rate of decreases in k2 and k9 treatments, we observe that while the rate of decrease from round 1 to round 2 is higher in k2 treatment, the other two rates of decrease is higher in k9 treatment.

It seems that the speed of convergence in early rounds is relatively higher in k1 treatment than in k2 and k9 treatments. In order to test whether this observation is supported by the chosen numbers in different treatments, we define for each player the rate of decrease in the "choice" as follows:

$$w_{1-N}^{choice} = \frac{choice_{round=1} - choice_{round=N}}{max\{choice_{round=1}, choice_{round=N}\}}$$

where w_{1-N}^{choice} denotes the rate of decrease from round 1 to round N. Notice that the formula of the rate of decrease in "choice" is a little different from the formula of the rate decrease in "median". Here, we divide the difference by the maximum of the choice in round 1 and the choice in round N. By this way, we prevent the rate of decrease of some subjects not to be defined (i.e. the subjects who choose zero but increase their choice later). Also, we prevent the rate of decrease to take extreme negative values. Now all the rate of decrease values are between -1 and +1. We observe that most of the subjects that choose zero in the first round continue to choose zero in the rounds 2,3, and 4. Thus, the rate of decrease value is not defined for those subjects.

We calculate w_{1-2}^{choice}, w_{1-3}^{choice} and w_{1-4}^{choice} values for each subject (if defined). After that, using the two-tailed Wilcoxon rank-sum test we compare rate of decrease values defined above.

Comparison of k1 and k2 Treatments:

We find from the comparison of k1 and k2 treatments that w_{1-2}^{choice}, w_{1-3}^{choice} and w_{1-4}^{choice} values are significantly higher in k1 treatment than those in k2 treatment (**comparisons for all players:** • w_{1-2}^{choice}, p <0.001; • w_{1-3}^{choice}, p <0.001; • w_{1-4}^{choice}, p <0.001; **comparisons for the inexperienced players:** • w_{1-2}^{choice}, p <0.001; • w_{1-3}^{choice}, p <0.001; • w_{1-4}^{choice}, p <0.001). This means that the speed of convergence in the first 4 rounds is significantly higher in k1 treatment than that in k2 treatment.

Comparison of k1 and k9 Treatments:

We find for all subjects that w_{1-2}^{choice}, w_{1-3}^{choice} values in k1 treatment is significantly higher than those in k9 treatment. Also, w_{1-4}^{choice} values are marginally significantly higher in k1 treatment than in k9 treatment. The same comparisons for the inexperienced subjects yield very similar results except the comparison of w_{1-4}^{choice} values (**comparisons for all players:** • w_{1-2}^{choice}, p <0.001; • w_{1-3}^{choice}, p <0.001; • w_{1-4}^{choice}, p=0.0794; **comparisons for the inexperienced players:** • w_{1-2}^{choice}, p=0.0762; • w_{1-3}^{choice}, p=0.0525; • w_{1-4}^{choice}, p=0.7564). Thus, we can claim that the speed of convergence in the first 3 rounds is significantly higher in k1 treatment than that in k9 treatment.

Comparison of k2 and k9 Treatments:

Surprisingly, we find from the comparison of k2 and k9 treatments that w_{1-2}^{choice}, w_{1-3}^{choice} and w_{1-4}^{choice} values are significantly lower in k2 treatment than those in k9 treatment (**comparisons for all players:** • w_{1-2}^{choice}, p=0.0149; • w_{1-3}^{choice}, p <0.001; • w_{1-4}^{choice}, p <0.001; **comparisons for the inexperienced players:** • w_{1-2}^{choice}, p=0.001; • w_{1-3}^{choice}, p <0.001; • w_{1-4}^{choice}, p <0.001). That is, the speed of convergence in k2 treatment is significantly lower than that in k9 treatment.

We show in Section 4.1 that about 42 percent of the players in k1 treatment already choose zero in the first round. They are not included in the analysis of convergence speed since the rate of decrease formula is undefined for most of them. Despite this considerable data loss, the remaining players in k1 treatment converge to equilibrium faster than the players in k2 and k9 treatments. Thus, we claim that the speed of convergence in early rounds (i.e. rounds 2,3) is significantly faster in the symmetric case than that in the asymmetric case.

CHAPTER 5

CONCLUSION

In this paper, we introduce the two-person guessing game with asymmetric players. The game in our model differs from the standard two-person guessing game in terms of players' powers to determine the target number. We multiply one of the players' chosen number with a positive parameter k >1 which is common knowledge. The target number is the weighted average of the two chosen numbers multiplied by $p = \frac{1}{2}$.

In the experimental sessions, we have three treatments with different k values in which the theoretical equilibrium is (0,0). The first treatment is the baseline treatment (k =1, k1 treatment) which is the standard two-person guessing game. In the second treatment we multiply one of the players' chosen number by k =2 (k2 treatment). In the third treatment we set k =9 (k9 treatment). Clearly, the players in k2 and k9 treatments are asymmetric unlike the players in k1 treatment. The subjects play the game for 10 rounds. After each round, we provide feedback about the chosen numbers in the two-person group, the calculated target number and the winner. We aim to answer how asymmetry affects the first round behaviour. We also address the question how the speed of convergence -if exists- differs across the symmetric and the asymmetric cases. Before we analyse the data, we classified our subjects as "all subjects" and "the inexperienced subject" by detecting the experienced subjects at the survey part in the sessions. There were 35 experienced subjects who played or heard about the game before. Since the sample size of the experienced subjects is not big enough, we did not analyse their data in detail. When we obtained a "significant" result from the analysis of all subjects, we checked whether it is also significant for the inexperienced subjects. Thus, the following significant results are also significant for the inexperienced subjects unless otherwise noted.

In the first round, 32.7% of all subjects choose zero. We have the highest proportion (41.7%) in the baseline treatment, and the lowest one in the k9 treatment (19.4%). The proportion is between the two in k2 treatment (37.5%). However, the Fisher's Exact test shows that the above proportions are not significantly different from each other. The test reveals that only the proportion of zero choices in the baseline treatment (41.7%) and that of "big" players in k9 treatment (5.6%) are significantly different from each other (big players are the ones whose numbers are multiplied by $k > 1$). Thus, the test results show that if the degree of asymmetry -the value of k- is sufficiently high, the big players in the asymmetric case choose zero in the first round significantly less often than the players in the symmetric case. We think that this result should be further tested with greater values of k. A possible reason for the big players to choose zero rarely in the first round is that they feel themselves overconfident, since their numbers are multiplied by $k > 1$. In the formal debriefing part at the end of the sessions, some big players reported that since they were Player B (the big player), they felt themselves advantageous. Similarly some small players wrote that they were disadvantageous. At the time of the experiment, a small player in one of the k9 sessions raised his hand and told privately to the experimenter that the game is unfair since he has no chance to win. These reports show that asymmetry between the players affects players' feelings differently. While some big players feel overconfident, some small players lose their hopes to win.

The game in our baseline treatment is very similar to the two-person guessing game in Nagel et al. (2008). The only difference is that they multiply the mean of the two choices by $\frac{2}{3}$ in the calculation of the target number (in our k1 treatment it is multiplied by $\frac{1}{2}$). In their single round experiment, they observed that only 9.85% of students chose zero. This proportion is approximately one-quarter of the related proportion (45.8%) in our k1 treatment (within the inexperienced subjects). This considerable difference in proportions may result from the p-values ($\frac{2}{3}$ and $\frac{1}{2}$) of the two games, since the calculation of the target number is easier with $p = \frac{1}{2}$ in our k1 treatment. To the best of our knowledge, the proportion of zero choices in the first round in our baseline treatment is also the highest proportion ever observed in the literature. We think the reason is that the game in our baseline treatment is the simplest in the literature. In the standard two-person game, iterative reasoning is unnecessary since the zero is always the winning number. For this reason, the two-person guessing game is much simpler than its N-player versions. In our baseline treatment, we further simplify the game by multiplying the mean by $\frac{1}{2}$ instead of $\frac{2}{3}$, $\frac{3}{4}$ etc. In the formal debriefing part at the end of the sessions, some players reported that they consciously did not choose zero in the first round in order for their rival not to realize that zero is the winning strategy. Therefore, we think that the real proportion of zero choices in the first round is higher than observed. Furthermore, we expect almost all the experienced players choose zero in the first round, but only 23% of them choose zero. This proportion is lower than that of the inexperienced players. The guessing game that is thought and played in economics courses is the standard N-player guessing game. The winning numbers generally are not zero in the N-player guessing games. We think that most of the experienced players do not realize that the case is different here.

Using Kolmogorov-Smirnov test, we compare all chosen numbers in different treatments. We find that there is no significant difference between the choices in the symmetric case and those in the asymmetric cases.

In round 10, we observe that 87.5% of all subjects choose zero, and this proportion is above 80% in each of the treatments k1, k2 and k9. This "significant" increase in the proportion of zero choices from round 1 to round 10 proves that there exist a convergence toward equilibrium. We observe that the median choice in round 5 is zero in all treatments. By comparing round 1 and round 5 choices, we show that both the players in symmetric case and those in the asymmetric cases revise their choices toward the equilibrium even in the first 5 rounds.

To sum up, we analyse the speed of convergence in the first four rounds since the median is already zero in round 5. We define the formula of the rate of decrease in choice for each player. As expected, we find that subjects' rate of decrease values in k1 treatment tend to be higher than those of subjects in k2 and k9 treatments. Surprisingly, we also observe that the rate of decrease values in k9 treatment tend to be higher than those in k2 treatment. In this research, we find that when we introduce asymmetry to the two-person guessing game, the chosen numbers do not differ from those in the symmetric case. However, we observe that the speed of convergence to the equilibrium in early rounds (i.e. rounds 2,3) is faster in the symmetric case.

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APPENDIX A

Proof of Lemma 1. Assume $p < \frac{k+1}{2k}$.

i) Note that $p < \frac{k+1}{2k} < 1$ for any $k > 1$. Suppose $n_s < n_b$. Remember that the target number is weighted average of the two numbers multiplied by p, ($T = (\frac{n_s+kn_b}{k+1})p$). Therefore, it can never be equal to the greater number, n_b. Then, we have two possible cases: 1) $T \leqslant n_s < n_b$ 2) $n_s \leqslant T < n_b$. If $T \leqslant n_s < n_b$, clearly the small player wins. If $n_s \leqslant T < n_b$, the small player is the winner again, since

$$\begin{aligned} |n_s - T| - |n_b - T| &= 2T - n_s - n_b = \underbrace{(\frac{2p}{k+1} - 1)}_{<0} n_s + \underbrace{(\frac{2pk}{k+1} - 1)}_{<0} n_b \\ &< 0. \end{aligned}$$

Now suppose that $n_b < n_s$. If $T \leqslant n_b < n_s$, clearly the big player wins. If $n_b \leqslant T < n_s$, the big player is the winner again, since

$$\begin{aligned} |n_b - T| - |n_s - T| &= 2T - n_s - n_b = \underbrace{(\frac{2p}{k+1} - 1)}_{<0} n_s + \underbrace{(\frac{2p}{k+1} - 1)}_{<0} n_b \\ &< 0. \end{aligned}$$

ii) Given $p < \frac{k+1}{2k}$, we know from i that the player whose number is smaller wins the game. Iterated elimination of weakly dominated strategies implies that zero is a weakly dominant strategy for both players, since it is always the winning number. Thus, the unique Nash Equilibrium of the game is (0,0).

$\square$

Proof of Lemma 2. Assume $1 > p > \frac{k+1}{2k}$.

i) Suppose $n_s > n_b$. Remember that the target number is given by $T = (\frac{n_s + kn_b}{k+1})p$. Note that the weighted average is already closer to n_b than n_s. Now, it is easy to see that if the weighted average is multiplied by $p < 1$, the resulting number, (T), will be closer to n_b than n_s. Thus, for all $p < 1$ and $k > 1$, playing $n_s < n_b$ is necessary for the small player to win.

Now, suppose $n_s < n_b$. The required condition for the small player to win is:

$$\begin{aligned}
|n_s - T| - |n_b - T| < 0 \quad &\Rightarrow \quad 2T - n_s - n_b < 0 \\
&\Rightarrow \quad (\frac{2p}{k+1} - 1)n_s + (\frac{2pk}{k+1} - 1)n_b < 0 \\
&\Rightarrow \quad (\frac{2pk - k - 1}{k + 1 - 2p})n_b < n_s \\
&\Rightarrow \quad an_b < n_s \qquad\qquad (0 < a < 1)
\end{aligned}$$

Hence, for small player to win she should play her strategy such that $an_b < n_s < n_b$. In all other cases except $n_s = n_b$ that leads to a tie, big player wins.

ii) Suppose for a contradiction that (n_s^*, n_b^*) is a pure strategy Nash Equilibrium (PSNE).

a) If a winner exists at (n_s^*, n_b^*), the other player has always an incentive to deviate because she always has the chance to share the prize by choosing her opponent's strategy at (n_s^*, n_b^*). For this reason, if one of the players is the winner at (n_s^*, n_b^*), this point cannot be PSNE.

b) If there is a tie at (n_s^*, n_b^*) with $n_s^* \neq 0$, the big player has an incentive to deviate because she can win by choosing a number (n_b) lower than n_s^*. Hence, such a point cannot be a PSNE.

c) If there is a tie at $(n_s^*, n_b^*) = (0, 0)$, the big player has an incentive to deviate. Because, given $p > \frac{k+1}{2k}$, choosing the highest possible number (100 in this game) guarantees her to be the winner. Thus, (0,0) cannot be a PSNE, either.

Hence, the game has no PSNE. Since the game is finite, we have at least one mixed strategy Nash Equilibrium.

□

Proof of Lemma 3.

i) Assume $p = \frac{k+1}{2k}$ and $n_s = 0$. Then, $T = \frac{n_b}{2}$. Thus, $|n_s - T| - |n_b - T| = 0$.

ii) If $n_s \neq 0$ and $n_b < n_s$, the big player will be the winner, since we know from the proof of Lemma 2 *i*) that for all $p < 1$ and $k > 1$, the necessary condition for small player to win is $n_s < n_b$.

If $n_s \neq 0$ and $n_s < n_b$, the small player will be the winner, since $|n_s - T| - |n_b - T| = 2T - n_s - n_b = (\frac{1}{k} - n_s) < 0$.

iii) Suppose for a contradiction that (n_s^*, n_b^*) is a pure strategy Nash Equilibrium (PSNE).

a) See the proof of Lemma 2 *ii*) *a*).

b) See the proof of Lemma 2 *ii*) *b*).

c) If there is a tie at $(n_s^*, n_b^*) = (0, 0)$, the big player has an incentive to deviate because when $p = \frac{k+1}{2k}$ and $n_s^* = 0$, she is indifferent between her strategies. Thus, (0,0) cannot be a PSNE, either.

Hence, the game has no PSNE. Since the game is finite, we have at least one mixed strategy Nash Equilibrium.

$\square$

Proof of Proposition 1. Suppose $1 > p > \frac{k+1}{2k}$. Remember that $a = \frac{2pk-k-1}{k+1-2p}$. Then, for all $n_b \in \{0, 1, .., z\}$, $\lim_{k\to\infty} an_b = \lim_{k\to\infty} \frac{2pk-k-1}{k+1-2p} n_b = (2p-1)n_b$. Thus, for each $n_b \in \{0, 1, .., z\}$, there exists a $k_i^* > 1$, $i \in \{0, 1, .., z\}$, such that

$$\frac{2pk_i^* - k_i^* - 1}{k_i^* + 1 - 2p}\, n_b = a_i^* n_b = \lceil (2p-1)n_b - 1 \rceil.$$

Since $\frac{da}{dk} > 0$, for all $k > k_i^*$, $an_b \in (a_i^* n_b,\ (2p-1)n_b)$, which means that for each n_b, there exists $k_i^* > 1$ such that for all $k > k_i^*$ the range in which the small player wins does not change. Now, if we set $k^* = \max\{k_0^*, k_1^*, .., k_z^*\}$, then for all $n_b \in \{0, 1, .., z\}$, there exists a $k^* > 1$ such that for all $k > k^*$ the game structure does not change. Since $\frac{da}{dk} > 0$, it is clear that for $k < k^*$, as k increases the range in which the big player wins gets larger. $\square$

Proof of Proposition 2. Suppose $k > 1$, $p > \frac{k+1}{2k}$ and $n_s \neq n_b$. Remember that if $p > \frac{k+1}{2k}$, for small player to win she should play her strategy (n_s) such that $an_b < n_s < n_b$ where $a = \frac{2pk-k-1}{k+1-2p}$. Since $n_s, n_b \in \{0, 1, .., z\}$, if we have $an_b > n_b - 1$ for all n_b, the small player can never win the game. To prove the result, it is sufficient to find a $p^* < 1$ such that for all $p > p^*$, $an_b > n_b - 1$ for all n_b. Moreover, note that finding a p^* satisfying $az = z - 1$ is sufficient, since

$$\begin{aligned} an_b = n_b - 1 \ &\Rightarrow\ an_b - 1 = n_b - 2 \\ &\Rightarrow\ an_b - a > n_b - 2 \ (\textit{since } 0 < a < 1) \\ &\Rightarrow\ a(n_b - 1) > n_b - 2. \end{aligned}$$

Then,

$$\begin{aligned} az = z - 1 \;\; &\Rightarrow \;\; (\frac{2p^*k - k - 1}{k + 1 - 2p^*})z = z - 1 \\ &\Rightarrow \;\; p^* = 1 - \frac{1}{2}(\frac{kz + z - 1}{k + 1}) < 1. \end{aligned}$$

Since $\frac{da}{dp} > 0$, given any $k > 1$, for all $p > p^* = 1 - \frac{1}{2}(\frac{kz+z-1}{k+1})$, $an_b > n_b - 1$ is satisfied for all n_b. □

APPENDIX B

INSTRUCTIONS

(for k2 treatment, in Turkish)

Hoş geldiniz, katılımınız için teşekkür ederiz. Bu çalışmanın amacı kişilerin belli durumlarda nasıl kararlar aldıklarını anlamaktır. Bu andan itibaren başkalarıyla konuşmayınız veya bir şekilde iletişim kurmayınız. Lütfen cep telefonlarınızı kapalı tutunuz.

Deneyde oynayacağınız oyunun kurallarını anladığınızı teyit etmek için deney başlamadan önce bilgisayar ortamında çok kısa bir quiz yapılacaktır. Lütfen bu açıklamayı dikkatlice takip ediniz.

Açıklamalar oldukça basittir. Dikkatli bir şekilde takip ederseniz belli miktarda para kazanabilirsiniz. Deney toplamda en fazla 1 saat sürecek olup, yapılacak ödeme miktarı deney esnasında vereceğiniz kararların sonuçlarına bağlıdır. Toplam kazancınız katılımınızdan dolayı kesin olarak alacağınız 5 TL ile deneyde kazanacağınız miktarın toplamı kadar olacaktır. Kazandığınız toplam para nakit olarak deney biter bitmez size ödenecektir. Bu deneyde toplanacak olan veri seti sadece bilimsel amaçlar için kullanılacak ve katılımcıların tüm bilgileri (kimlik, seçim vb.) tamamen gizli tutulacaktır.

Eğer sorunuz olursa lütfen elinizi kaldırıp bekleyiniz. Anlamadığınız hususlar hakkında soru sormaktan lütfen çekinmeyiniz, çünkü böyle bir çalışmada her katılımcının kuralları tam ve doğru şekilde anlaması çok önemlidir. Deney bilgisayar ortamında gerçekleştirilecek olup katılımcılar tüm kararlarını bilgisayar başında vereceklerdir.

Gruplar ve Roller:

Bu laboratuvardaki tüm deney katılımcıları iki kişilik gruplara ayrılmıştır. Birazdan sizden bir oyun oynamanız istenecektir. Her katılımcı bu oyunu kendi grubundaki diğer kişi ile oynayacaktır. Bu kişi, bu laboratuvardaki katılımcılardan rastgele seçilen biri olacaktır, fakat hiç kimse grubundaki diğer kişinin kim olduğunu bilmeyecektir. Gruplar tüm oyun süresince sabit kalacaktır.

Her grupta bir "A Oyuncusu" bir de "B Oyuncusu" olmak üzere iki çeşit oyuncu rolü vardır. 2-kişilik gruplar ve bu gruplardaki kişilerin hangi rolde olacağı bilgisayar tarafından rastgele belirlenir. Gruptaki kişiler ve rolleri tüm oyun süresince sabit kalacaktır.

Deneyde oynayacağınız oyunun kuralları ve size yapılacak olan ödeme miktarının nasıl belirleneceği aşağıda anlatılacaktır. Lütfen oyun anlatılırken sesli bir şekilde yorum yapmayınız ve oyunla ilgili düşüncelerinizi başkalarıyla paylaşmayınız. Oyunun kurallarına dair anlamadığınız bir husus olursa oyun anlatımının sonunda elinizi kaldırıp bekleyiniz.

OYUN

Bu oyun arka arkaya 10 tur oynanacaktır. Her turda A ve B oyuncularından 0 ile 100 arasında (0 ve 100 dâhil) bir tamsayı seçmeleri istenmektedir. Seçtiği sayı Hedef Sayı' ya daha yakın olan oyuncu o turda oyunu kazanmaktadır. Hedef Sayı (H) şu şekilde hesaplanmaktadır:

$$H = \left(\frac{S_A + 2S_B}{3}\right)\frac{1}{2}$$

S_A : A Oyuncusu' nun seçtiği sayı, S_B : B Oyuncusu' nun seçtiği sayı

Dikkat! Hedef Sayı (H) hesaplanırken A Oyuncusu' nun seçtiği sayı 1 ile, B Oyuncusu' nun seçtiği sayı ise 2 ile çarpılıp, çarpımların sonucunda elde edilen sayılar toplanıp 3' e bölünmekte, son olarak ortaya çıkan sayı (1/2) ile çarpılmaktadır.

Her turun sonunda ekranda seçtiğiniz sayı, grubunuzdaki diğer kişinin seçtiği sayı, hesaplanan Hedef Sayı ve kazanan oyuncu görülebilecektir. Oynadığınız 10 turdan rastgele seçilen 3 farklı turun her biri için kazanana 5 TL ödenecektir, kaybedene bir ödeme yapılmayacaktır. Şayet oyuncuların seçtiği sayılar Hedef Sayı' ya eşit yakınlıkta ise her bir oyuncuya 2.5 TL ödenecektir. Mesela, rastgele seçilen bu 3 turun hepsinde kazanan oyuncu sizseniz, deneyden toplam 15 TL kazanmış olacaksınız ve bu durumda grubunuzdaki diğer kişi deneyden hiçbir şey kazanamamış olacaktır. 10 turun her birinin seçilme ihtimali eşit olduğundan toplam kazancınızı en yüksek düzeyde tutmak için bütün turlara aynı ölçüde önem vermelisiniz!

Tamsayı seçiminizi yazacağınız ekranın sol üst köşesinde kaçıncı turda olduğunuz, sağ üst köşesinde ise seçim kararını vermeniz için kalan süre yazacaktır. Süre bittiği halde kararınızı vermediyseniz yine sağ üst köşede uyarı yazısı görülecektir. Verilen süre seçim kararını vermeniz için yeterlidir. Bütün gruplardaki oyuncular ilgili yere sayı seçimini yazıp "TAMAM" butonuna basmadan oyun bir sonraki tura geçmeyecektir. Bu nedenle verilen süreyi aşmamaya dikkat ediniz. Lütfen 0 ile 100 aralığı (0 ve 100 dahil) dışında veya küsuratlı bir sayı girmeyiniz.

Seçeceğiniz sayıya karar verirken varsa hesaplamalarınızı kendi kendinize yapınız ve sesli düşünmeyiniz. Kalem kâğıt çıkararak veya masanın üzerine yazarak hesaplamalar yapmayınız. Oyun süresince hiçbir katılımcı ile hiçbir konuda iletişim kurmayınız. Yanınızdaki kişinin ekranına bakmayınız. Bu kuralları ihlal eden katılımcılara hiçbir ödeme yapılmayacaktır ve deneyi terk etmeleri istenecektir.

Oynayacağınız 10 turun hepsinde sayı seçim ekranında hatırlatma amaçlı olarak yukarıdaki Hedef Sayı (H) formülü ve oyun süresince hiç değişmeyecek olan oyuncu rolünüz yazıyor olacaktır. Bu yazı ekranın üst kısmında çerçeve içerisinde olacaktır. Şayet hedef sayının nasıl hesaplandığını anladığınızdan eminseniz her turda bu bölüme bakmanıza gerek yoktur.

10 tur bittikten sonra sizden birkaç kısa soruya cevap vermeniz ve kısa bir anket doldurmanız istenecektir. Anket sonrasında göreceğiniz ekranda size ne kadar ödeme yapılacağı yazacaktır. Bu rakam rastgele seçilen 3 turdaki toplam kazancınız ve katılım ücreti olan 5 TL' nin toplamı olacaktır.Deney özetle şu şekilde ilerleyecektir:

- Önce, oynayacağınız oyunun kurallarına ve size nasıl ödeme yapılacağına ilişkin çoktan seçmeli quizi cevaplamanız istenecektir. Her sorudan sonra ekranda doğru cevap görülecektir.
- Quiz sonrasında ekranda bilgisayar tarafından rastgele seçilen oyuncu rollerinizi göreceksiniz. Oyuncu rolleriniz 15 saniye ekranda kaldıktan

sonra oyunun birinci turu başlayacaktır.

- Birinci turda sayınızı seçip TAMAM butonuna tıkladığınızda size o turla ilgili geribildirim verilecektir. Bu geribildirim 10 saniye ekranda kalacak ve ikinci tura geçilecektir. Geribildirim ekranı her turun sonunda görülecektir.

- 10. tur bittikten sonra size birkaç soru sorulacak ve bir anket doldurmanız istenecektir.

- Bu işlemleri yaptıktan sonra ekranda 10 tur arasından rastgele seçilen 3 turu ve bu turlardaki toplam kazancınızı göreceksiniz. Bu kazancınıza 5 TL katılım ücreti de eklenerek deneyden toplamda ne kadar kazandığınız belirlenecektir. Bu ekranı gördüğünüzde, çağrılmadan yerinizden kalkmayınız. Teker teker çağrılarak ödemeniz kişisel olarak yapılacaktır. Ödemeyle birlikte deney sona erecektir.

Sorusu olan var mı?

Katıldığınız için teşekkür ederiz.

QUIZ QUESTIONS

(for k2 treatment)

1 You will be paid for

A) all rounds.
B) randomly chosen 5 rounds.
C) randomly chosen 3 rounds.
D) randomly chosen 7 rounds.
E) no rounds.

2 For each chosen round, the winner gets ... TL, and the other player gets ... TL. If there is a tie, each player in the group gets ... TL.

A) 5, 3, 2.5   *B*) 5, 0, 2.5   *C*) 3, 5, 2.5   *D*) 1, 0, 2.5   *E*) 0, 0, 0

3 In the calculation of the target number, the chosen numbers of Player A and Player B are multiplied by ...... and ....., respectively.

A) 1 , 2   *B*) 2 , 1   *C*) 3 , 2   *D*) 4 , 2   *E*) 1 , 4

4 The chosen numbers of players will be multiplied by some coefficients and then added. Then, the resulting sum will be

A) divided by 3 and multiplied by 1/4.

B) divided by 3 and multiplied by 1/2.

C) divided by 2 and multiplied by 1/2.

D) divided by 3 and multiplied by 1/5.

E) divided by 4 and multiplied by 1/3.

5 Which one is the formula of the target number?

A) $\text{T} = [\ (n_A + 2n_B)\ /\ 2\ ] * (1/2)$
B) $\text{T} = [\ (n_A + 2n_B)\ /\ 3\ ] * (1/3)$
C) $\text{T} = [\ (n_A + 6n_B)\ /\ 4\ ] * (1/2)$
D) $\text{T} = [\ (n_A + 2n_B)\ /\ 3\ ] * (1/2)$
E) $\text{T} = [\ (n_A + 4n_B)\ /\ 5\ ] * (1/4)$

CRT QUESTIONS

1 A bat and a ball cost \$ 110 in total. The bat costs a dollar more than the ball. How much does the ball cost?

2 If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets?

3 In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?

4 All flowers have petals. Roses have petals. If these two statements are true, can we conclude from them that roses are flowers?

5 Jack is looking at Anne but Anne is looking at George. Jack is married but George is not. Is a married person looking at an unmarried person?
(A) Yes (B) No (C) Cannot be determined.