A REAL LIFE OPERATING ROOM SCHEDULING PROBLEM

A Thesis

by

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Submitted to the
Graduate School of Sciences and Engineering
In Partial Fulfillment of the Requirements for
the Degree of

M.Sc.

in the
Department of Industrial Engineering

Özyeğin University
August 2015

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Dedicated to my family . . .

ABSTRACT

A real life operating room scheduling problem is studied using a data set from a leading hospital, Hospital X, in Turkey. After analyzing the real data, we solve daily operating room scheduling problems by mixed integer linear programming models. Various objective functions and performance metrics are analyzed including minimizing the waiting time of patients while maximizing fairness between operating rooms. We examine operation delays and incorporate an important delay type, operation durations, by a heuristic method embedded in the mathematical models. In addition, a simple heuristic that does not utilize optimization is introduced. We conclude that our methods perform better than the Hospital X's current schedules, especially with respect to fairness of operating rooms' usage. However, we measure the performance of schedules (computed via mixed integer linear programming models) under randomly generated scenarios and such schedules perform worse than the schedules computed via the simple heuristic. Extensive computational results demonstrate that Hospital X can adapt any of the proposed schedules and realize progress in their schedules.

ÖZETÇE

Türkiye'deki özel bir hastanenin, Hastane X, verileri kullanılarak gerçek hayat operasyon odası çizelgelemesi yapılmaktadır. Gerçek datayı inceledikten sonra, karma tamsayılı doğrusal programlama modelleri ile operasyon odası çizelgelemesi günlük çözülmektedir. Hastaların bekleme sürelerini minimize etmek için ve operasyon odaları arasındaki eşit dağılımı sağlamak için çeşitli amaç fonksiyonları ve performans ölçütleri analiz edilmektedir. Operasyonlardaki gecikmeler, gecikmelerin türleri ve operasyon süreleri incelenmektedir ve bu bilgiler matematiksel modellerin içine sezgisel metotla entegre edilmektedir. Ayrıca, optimizasyon yapılmadan kullanılan basit sezgisel yöntem tanıtılmaktadır. Özellikle operasyon odaları arasındaki eşit dağılımı sağlamada kullanılan metotlarımız, hastanenin mevcut çizelgelemesine göre daha iyi sonuç vermektedir . Karma tamsayılı doğrusal programlama modelleri kullanarak yaptığımız raslantısal senaryolar basit sezgisel yönteme göre daha kötü sonuç vermektedir. Kapsamlı hesaplama sonuçları göstermektedir ki, önerilen metotlarımız daha iyi çizelgelemeye sahip olabilmek için Hastane X'in sistemine entegre edilmelidir.

ACKNOWLEDGEMENTS

Firstly, I would like to express my sincere gratefulness to my advisor Assistant Professor Elvin Coban Gokturk for the continuous support of my thesis, her patience and motivation. Her guidance helped me throughout my research and thesis. I could not have imagined having a better advisor and mentor for my thesis. Besides my advisor, I cannot express enough thanks to my committee members Assistant Professor Enis Kayis and Assistant Professor Ibrahim Muter for their insightful comments and support.

My sincere thanks also goes to TUBITAK that awards scholarship throughout my master degree.

I also would like to thank Tugce Pinar who helps me contacting the hospital administration. Without her precious support, it would not be possible to conduct this research.

I would like to thank my family. I am very grateful to my father Huseyin Alper, my mother Turkan Alper, my brother Volkan Akin and my sister Nur Ugus Akin for supporting me throughout my all life and especially spiritually throughout writing this thesis. My nephew Arda Efe Akin brings joy to my life and I would like to thank him as he always reminds me that life is beautiful and amazing. Last but not the least, I would like to thank my fiance, Eray Tas. I may not finish this thesis without his love, support and inspiration. I always feel very lucky to have him in my life.

There are many people who deserve thanks, and I thank them with all my heart. All errors, etc. are mine alone.

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CHAPTER I

INTRODUCTION

Healthcare is very vital in today's world for not only its effect on the quality of patient life but also for its increasing impact on economy. One of the major goals is to maximize service quality. According to Turkish Investment Support and Promotion Agency's report, the Economist Intelligence Unit forecasts show that the healthcare sector in Turkey is set to grow as healthcare spending per capita will increase at a Compound Annual Growth Rate of 5.6 % between 2013 and 2017 and it will reach 5.6% of Turkey's gross domestic product (GDP) in Turkey (2014). TUİK mentions that total healthcare expenditure is 84.390 million TL and proportion of total health expenditure to GDP (2013) is 5.4% (2013).

While increasing service quality, maximizing resource utilization becomes very critical even though the main aim is not to minimize any cost related objective because a limited budget and limited available resources exist. Hence, a well managed healthcare system should operate aiming to maximize its service level while keeping its cost related spending as low as possible. According to Gupta et al., operating room (OR) is one of the important issues of a hospital administration and they show that OR is the most expensive resources with more than 10 % of the planned operating income of the hospital (2006). Motivated by this expensive resource, we study a real life operating room assignment and scheduling problem for elective patients.

In this thesis, we first analyze the data of a leading, private Turkish hospital, Hospital X. Then, we propose mathematical models: deterministic and stochastic methods to solve operation assignment and scheduling problem with various objective functions. We even propose a simple heuristics for Hospital X to utilize.

Hospital X has 7 ORs with same equipments, 141 beds and 54 intensive care units. Data are from June, 2013 to December, 2014. There are 7754 operations belonging to 9 different departments. Some operations are tried to be scheduled at certain times, such as, 90 % of pediatric operations are scheduled before 13:00. Moreover, some departments prefer operating at certain ORs, 80 % of orthopedic operations are scheduled at OR 1, but OR 4 is rarely used. Utilization of ORs seems low in actual data since surgeons are also busy with regular consultation.

After analyzing 594 days (7754 operations), Hospital X's problems are as follows:

- Scheduling of operations faces fairness problem. Although some ORs' utilization is high, some of them are very low. For example, OR 4 is nearly idle for most of its time (0.002 % as its utilization).

- Operations also face delays in operation durations. 2327 operations out of 7754 operations (30 %) are delayed. 53 % of pediatric operations are delayed and most of pediatric operations are scheduled consequently.

- Predicted operation duration (POD) is overestimated in comparison with actual operation duration (AOD).

Scheduling problem is a very challenging problem. In our thesis, we propose several Mixed Integer Linear Models (MILPs) to compute operations schedule. The objective functions studied in this work are: minimizing overtime, makespan, tardiness, delay probabilities and unfairness between OR usage. We first use predicted operation durations then, actual durations are used. Both of these solutions are compared to the hospital's actual objective function value. After solving the deterministic models, we generate random operation durations with the same actual operation types for each day. In other words, how our schedule performs under uncertain data is one of the main goals of this comparison. Lastly, a simple heuristic is proposed that does not utilize any optimization tool.

Our models have contributions to both real life implementation and literature as follows: We propose operation scheduling methods for each operation, OR and time periods. Results show that our mathematical models perform better than Hospital X's objective function values according to calculated actual schedule. Hence, we successfully implement fairness into Hospital X's problem. Although our mathematical models' solutions do not perform well under randomly generated scenarios, still Hospital X can improve its schedule. Even now, Hospital X can improve its schedule if the surgeons can better approximate the actual duration of an operation. Also, Hospital X may be interested in not utilizing an optimization tool. Hence, our thesis has potential contributions to the academic literature.

This thesis is divided into 5 chapters. In Chapter 2, literature review is summarized. In Chapter 3, empirical analysis of Hospital X's data is explained. In Chapter 4, our scheduling models are represented and we show our computation results of deterministic and simple heuristic approaches. Finally, conclusion and possible further works are introduced in Chapter 5.

CHAPTER II

LITERATURE REVIEW

Healthcare is vital in today's world and healthcare providers aim to maximize service quality while utilizing their resources, including operating rooms (ORs). Chaabane et al. explain that approximately 15 % of gross domestic product of the United States is consumed by healthcare sector (2006).

A major goal is to minimize cost; however this aim contradicts with increasing service quality. In addition a healthcare manager wants to manage available resources efficiently and one of the important resources is ORs. Gupta et al. show that ORs are the most expensive resources with more than 10 % of the planned operating income of the hospital (2006).

Magerlein et al. explain that scheduling and planning ensure that each patient is assigned an operation date by checking availability of ORs (1978). The scheduling problem may become hard to solve since there are various constraints, such as, preventing overlapping operations in addition to uncertainty prevalent in real life OR scheduling problem. In the literature, OR scheduling has been widely studied and there are many different solutions or evaluation methods, such as, mathematical programming, simulation or heuristics. These methods can be classified as deterministic and stochastic.

In scheduling problems, one of the important features is patient types. Cardoen et al. categorize patient types into two main groups: elective and non-elective patients (2010). In elective patient type, the operation can be well planned in advanced. There are two main subgroups of elective patients: inpatient and outpatient. Patients are hospitalized and stay overnight if they are inpatient patients, where as, for outpatient patients, they enter and leave the hospital without hospitalization. Cardoen et al. describe that in many cases, elective patients are focused and non-elective patients are ignored (2010). Furthermore, many works in the literature ignore types of patients.

Deterministic methods are very common in the literature where either uncertainty is ignored. One of the deterministic methods is to model the problem by utilizing a mathematical programming approach. For example, Mixed Integer Linear Programming (MILP) is one of the popular approaches to solve OR planning and scheduling problems. Adan et al. use MILP to model scheduling problem of orthopedics operations (2002). They use tactical planning to derive a weekly OR plan. They also consider beds, ORs, staff and intensive care beds' capacities. In their model, they identify the cyclic number and the mix of patients. In their case, outpatients are treated as inpatients that stay one day at hospital, hence, and there is no need for specialized resources such as intensive care beds. Furthermore, Blake et al. formulate a MILP in order to evaluate each OR types that have to be scheduled during the weekdays with limited funding, patient demand, and limited number of staff (2002a, 2002b). Blake et al. analyze the model by considering a heuristic in order to minimize the underallocation of OR (2002b). Where their problem is a timetabling problem with number of ORs, available hours of ORs, and patient priority. Cardoen et al. develop a multiple objective function in order to minimize waiting time of high priority patients, the stay in recovery and peak number of bed space (2009a). They consider a MILP model and find results with small average solution gaps for post anesthesia care unit (PACU).

Chaabane et al. studied a single performance criterion: waiting time for patients while developing a two-step solution approach (2006). The first step is to minimize the gap between the total supply and the weekly requests of the operation specialty by applying block scheduling method. The second is to schedule within each block according to step 1's solution aiming to minimize the sum of patient operating costs which equals the sum of overtime costs and patient waiting costs. Another two step approach is proposed by Jebali et al.(2006). The first step is the assignment of operations to ORs, whereas, the second step solves sequences for the assigned operations by considering available resources. Pure sequencing and sequencing with some re-assignment are predetermined. They conclude that good performance of the operations sequencing without predetermined the assignment problem performs as good as the assignment step in terms of patients' selection while minimizing total cost. Kharraja et al. consider two different methods by using block scheduling (2006). The first one is about individually scheduling of all operations and the second one is scheduling operations with respect to departments using MILP with elective patients. Next, Roland et al. solve an OR planning optimization to minimize the sum of OR and overtime cost by considering resources: OR, human resources and financial working budget (2006) for a short time horizon. They aim to minimize total cost for elective patients ignoring emergency cases, by genetic algorithm. In their data, they are 7 ORs, 19 operations, 12 surgeons, and 3 types of resources: (i) anesthetists, (ii) nurses, and (iii) OR nurses. They assume that if an OR is used for an operation, a fixed cost, 2040 € is charged while overtime costs 2000 € per hour. They conclude that the optimal solution is to utilize 5 ORs instead of 7 ORs. Santibanez et al. also determine a MILP model in order to optimize operation block schedule for each OR considering post-operation resources, such as, bed capacity, wait list, and surgeon's booking privileges (2007). Reallocating operation in the block schedule results in decreases in post-operation resource requirements needed for elective patients. They conclude that resource requirements can be reduced by reallocating the operating specialties in the block schedule. Moreover, Zhang et al. focus on a finite-horizon MILP model in order to minimize inpatients and outpatients' cost computed based on as their length of stay (2006). There are two main constraints: (i) patient priority grouped under emergency and non-emergency cases and (ii) clinical constraints to maximize number of hours allocated to each department. Then, they use the computed optimal solution as an input of their simulation model. They assume that only one OR is used for emergency operations every day. They also incorporate optimization and simulation to the hospital's existing information system.

Dexter et al. (2002, 2002b), Mulholland et al. (2005) utilize Linear Programming (LP) models to find the optimal solution of OR scheduling problem. Dexter et al. determine elective case scheduling to determine patients' and surgeons' preferred operations times (2002). Their aim is to maximize the productivity, use of OR available time, by determining two different factors: (i) latest starting time, and (ii) earliest starting time. Earliest starting time forces schedule should be planned at ORs as early as possible. Latest start time forces operations to be scheduled as late as possible but still avoiding tardiness. Thus, they maximize the productivity of an OR by minimizing the sum of underutilized OR hours multiplied with underutilization cost and overutilized OR hours multiplied with overutilization cost. They conclude that earliest starting time provides better than latest starting time to minimize waiting time of patients and doctors. Moreover, Dexter et al. introduce a LP model to maximize the variable costs because they want to determine the worst case scenario (2002b). In their paper, outpatients, patients entering and leaving the hospital on the same day, are considered. They examine how adequate planning and scheduling can contribute to revenue when variable costs are subtracted. Mulholland et al. study a LP to optimize mix of inpatients, outpatients, and emergent patients' financial outcome: profit of PACU, intensive care unit (ICU), holding cost, and ward (2005).

In addition to IP and LP models, goal programming is another common method planning to solve OR and scheduling problem (Arenas et al.,2002; Ogulata et al., 2003; Rohleder et al.and 2005). Common goals aimed are as follows: maximizing OR utilization (Arenas et al.,2002;Ogulata et al., 2003) , minimizing elective patient waiting time (Ogulata et al., 2003), minimizing ORs' underutilization (Ogulata et al., 2003; Rohleder et al., 2005) and minimizing overtime (Ogulata et al., 2003).

OR scheduling problems are very similar to machine scheduling problems where jobs (operations) are scheduled at several machines (ORs). Lio et al., 2003; Kellerer et al., 2003; Breit et al., 2001; Biskup et al., 2008; Lin et al., 2013; Gokhale et al., 2012; Kaczmarczyk et al., 2011; Ozturk et al., 2012; and Ağpak et al., 2015 mention mathematical model with parallel machines that is equal to ORs scheduling. For example, Lio et al. analyze two uniform parallel machine problems and their aim is to minimize makespan (2003). They show that their new scheduling model can find the optimal solution for large-sized problems. Biskup et al. try to minimize total tardiness given the number of jobs and identical parallel machines in their scheduling problem (2008).

Column generation is also a common tool to solve OR scheduling problems. Fei et al. analyze an operating case assignment problem to minimize total unexploited or overtime operating cost (2008). In their problem, they assume that all ORs are used for any operating types. Human, material and recovery room's beds are relaxed and finally emergency cases are not evaluated (i.e., they just focus elective patients). They use branch and price algorithm based on decomposition technique and determine 160 different operating cases. Furthermore, Fei et al. evaluate a schedule of endoscopy center by using a column generation without considering emergency cases (2006). Their aim is to prevent overtime and to assume unlimited capacity of the recovery room and operation materials. The performance criteria are OR overutilization, underutilization and makespan. Fei et al. also minimize the cost of overtime and maximize the utilization the OR by two different cases: the planning case and the scheduling case (2006b). They first use column-generation based heuristic to solve their tactical planning model in planning case and then they determine a daily scheduling by using Ganzalez-Sahni algorithm in scheduling case. As they schedule the operating cases assigned at the planning stage. They concluded that the extended earliest starting time approach has the best performance and they again focus elective patients. Belien et al. consider both nurse and OR scheduling by using a standard dynamic programming approach, a state of the art mixed integer programming and the column generation technique with inpatient patient (2008). They also generate two different scenarios: a hard and a flexible constraints scenario. In the flexible constraint scenario, the system lets nurses to change between different types of shifts, although the hard scenario is preventing such changes and their performance criterion is leveling capacity of ward. Finally, Perdomo et al. consider the operation scheduling with resources of OR and recovery beds (2006). Moreover, they consider cleaning time of OR and their aim is to minimize completion times of both operation and set up times for elective patients by using a Lagrangian relaxation method that divides their original problem into two different sets as good and bad constraints. Although good constraints can be solved easily, bad constraints cannot be solved quickly. Thus, bad constraints are the ones whose violations are penalized at the objective function.

Even if some of the OR scheduling problems can be solved to optimality, some of them can only be solved by heuristics as the problem becomes very hard, for example; due to large instances. Belien et al. (2007), Belien et al. (2009) utilize simulated-annealing. Fei et al. use tabu search algorithm (2006b). Genetic algorithm is used by Fei et al. (2006b) and Roland et al. (2006).

Constraint-programming is a novel method that are recently been utilized to solve multi-objective OR scheduling problems (Meskens et al. ,2013). They minimize makespan and overtime hours and maximize relation as material resource, staff and their affinities into a block scheduling. They use priority of operations. For example, children are preferred to be operated at the beginning of the day since they have the highest priority. They use three levels: high, medium and low. If operation has a low priority, it should start at the end of the day. They also introduce the earliness and the latest starting times for each operation for the comfort and safety of the patient.

In their model, they also do not consider emergency case.

In addition to the deterministic methods summarized so far, stochastic methods incorporate uncertainty to the problem. Patient / surgeon arrival and operation duration uncertainty are two well-known uncertainties addressed in the literature. Simulation is a common, for instance, Testi et al. consider a three-phase approach in order to schedule OR weekly when operation durations and arrival of patients are uncertain (2007). They aim to minimize overtime and waiting time for elective patients. In the fist-phase, they choose the number of sessions for each ward using a bin packing model. After that, they find optimal timetables by using blocked booking method. In block booking, they assign operations (corresponding patients) wards and ORs. Then, they use simulation in order to show different sequencing of operations. In their schedule, they have some priorities, such as, the longest waiting time, processing time and the shortest processing time. Moreover, they describe that the simulation permits to determine what extent beds might be reduced to exploit the productivity gain in OR utilization. Their performance criteria are throughput, overutilization of OR, minimizing number of patient deferrals or refusals and preferences. They apply their method to a case study improve OR productivity as they increase the number of operations and reduce overrun hours and shifted operations. Furthermore, Belien et al. (2007), Zhang et al.(2006), Belien et al.(2009), Lamiri et al.,(2008) and Lamiri et al.(2007) analyze stochastic operation durations and patients' arrivals. A three step approach is introduced by Lamiri et al. (2007). The first step is target planning to determine the number of hours to be assigned to each operation. In the second step, master operation scheduling decides the assignment of OR blocks to operation teams and the last step is to schedule, in other words, patient selection to be scheduled at each OR block. Belien et al. consider a number of models to determine operation scheduling with using bed capacity (2007). Patients' arrival and operation durations are uncertain. There are two types of constraints: (i) demand constraint and (ii) capacity constraint. Demand constraint deals with scheduling of OR blocks and capacity constraint restricts the available blocks on each day. They assume that all durations have multinomial distribution. They solve the problem by utilizing a MILP model combined with a metaheuristic. Their aim is to minimize expected total bed shortage for elective patients. They find that metaheuristic approach gives the best result. Their performance criterion is leveling capacity of ward.

Belien et al. describe a multi-objective linear and quadratic problem by minimizing three different objective functions with elective patients (2009). The first objective is to level the resulting ward occupancy; the second is to assign surgeon(s) to ORs. The last objective is to find schedule. After that, they expand a decision support system to progress master operation schedules. They conclude that by using decision support system, the built-in algorithms generally succeed well in generating schedules with leveled resulting bed occupancy and when the room objective is added, computation becomes difficult. Moreover, Lamiri et al. consider another OR planning model with uncertain operating times (2007). They consider both elective patient and emergency cases. Their performance matrix is to show utilization of OR.

Stochastic operation durations are also studied by Denton et al.(2003), van Oostrum et al.(2008) and Denton et al.(2007). Denton et al. develop a two-stage stochastic linear programming method for elective patients (2003). Sequences of operations and operation durations are uncertain. They also use the single server appointment schedule and consider the optimal starting times for operations. Their performance criteria are patient and operation waiting time, OR idle time and overtime of ORs. van Oostrum et al. study a master operation scheduling problem for each OR-day combination of the planning cycle for recurring operation types (2008). Hence, a cyclic master schedule is developed in order to divide the available amount of OR capacity to operations or patient types. They decide individual elective patients or patient types. They also assume operation procedure types are given. Their performance criteria are leveling bed capacity and the use of additional capacity of specific resources such as the number of OR openings. Denton et al. show that if average operation duration is underestimated, it may cause late starts in all schedules (2007). Then, there is some extra cost as overtime staffing and direct cost and they analyze sequencing affection of waiting time, OR idle time, operation waiting time and OR overtime. Thus, they use three different performance measures as patient waiting time, OR idle time and OR overtime. They use a stochastic model with heuristics that is a two-stage mixed integer programming with elective patients.

Furthermore, Sorino analyzes two at a time appointment system with intervals set equal to twice the mean consultation time (1966). They also compare the steady-state waiting time distribution functions of individual and multiple block and fixed interval and assume that patients' arrivals are deterministic and operation durations are distributed with gamma distribution. Furthermore, Persson et al. consider stochastic arrival times and mention a rule about Sweden Hospitals (2006). According to a Swedish law, a patient should not wait more than 90 days before they execute the operation. It means that, the hospital should organize their operations such that each patient should be operated within 90 days or should be assigned to another hospital. Simulation including optimization, helps them to decrease average waiting time for elective patients. Their objective function is to minimize the cost of using extra beds for post-operative care at the OR and the cost overtime that is divided into single and double overtime. Single overtime refers to the first less expensive hours of working overtime (i.e, two hours), double overtime refers to working additional overtime with a higher cost. They have different performance criteria such as the demand for extra capacity of bed in wards, patient's waiting time, OR overtime and patient deferral or refusal.

As mentioned before, there are many different objective functions to compare the results. The first one is to decrease waiting time, such as, waiting time of patients or surgeons. For example, Jebali et al. (2006), Chaabane et al. (2006), Persson et al. (2006), Santibanez et al. (2007), Blake et al.(2002), Ogulata et al.(2003), Zhang et al. (2006) and Arenas et al.(2002) analyze waiting time as one of their objectives. The second objective is utilization. In generally, the utilization rate of an OR is the most popular criterion in the recent researches. For example, Adan et al. (2002) and Vissers et al. (2005) consider their objective as to prevent underutilization and overutilization of ORs, ward and ICU. When utilization is maximized to prevent losing money, the OR is close to fully planned. Hence, there is not any buffer. If there is disruption in the plan, such as, longer operation duration, it may be necessary to reschedule all operations. Moreover, leveling of resources is another objective to provide smooth resources occupancies without peaks. Santibanez et al. study leveling bed capacity and throughput that is related to patient waiting time and they focus under throughput on increasing the number of treated patients that leads indirectly to shorter waiting time (2007). A common objective is to decrease makespan that is the completion time of the last patient's recovery. Perdomo et al.(2006) and Fei et al. (2006b) use makespan as their objective function. In addition, Persson et al. minimize patient deferral or refusal that is decreasing number of canceled elective operations (2006). Furthermore, objective function of Dexter et al. is relying on costs (2002).

For a more complete review of recent literature on OR scheduling we refer to the works of Cardoen et al. (2010), Gupta et al. (2008), Erdogan et al. (2011), Guerriero et al. (2011), Smith-Daniels et al. (1988), Blake et al. (1997), Przasnyski et al. (1986) and Magerlein et al. (1978).

We contribute to this literature by proposing several MILP models for a real life OR assignment and scheduling problem. Our case is an original case, and we use many objective function combinations. While scheduling daily operations, we also incorporate delay probabilities which are computed from Hospital X's data. We also evaluate our schedules with respect to uncertain scenarios randomly generated again using Hospital X's data. In addition a simple heuristic is proposed based on sorting without using any optimization tools.

CHAPTER III

EMPIRICAL ANALYSIS

Turkey has many private hospitals that may operate different than public hospitals in terms of service utilization, types of operating rooms. In this study, a real life hospital's data are analyzed. This hospital's name is referred as Hospital X in the rest of the thesis due to confidentiality concerns. This empirical analysis has following goals:

- Compute utilization
- Identify possible problems, especially delays
- Analyze whether there is any pattern in the data
- Compute statistics about the processes, especially operation durations

After analyzing the data, we utilize some of the findings in Section 4 while solving the assignment and scheduling problems.

3.1 Attributes of Hospital X

There are nine different departments in the hospital: (i) neurosurgery (Neuro), (ii) pediatric operation (Pedia), (iii) general operation (Gen), (iv) gynecology (Gyn), (v) cardiology (Card), (vi) ear-nose-throat (ENT), (vii) orthopedic operation (Orth), (viii) plastics operation (Plas), and (ix) urology (Uro). This hospital is a medium sized hospital according to Sjetne et al., 2007. There are 141 beds, 54 intensive care units, 7 operating rooms (ORs). According to the hospital administration, bed capacities and intensive care units are enough. However, they face several problems in ORs, such as, computing an optimal or close to optimal OR schedule and delays in OR schedules.

ORs are available from 8:30 to 17:30. After each operation, OR is sterilized and this sterilization period is generally estimated as 10 minutes (Santibanaz et al., 2007).

A week before the operation, predicted operation durations (POD) and starting time of the operations are entered to Hospital X's system by surgeons. Then, according to availabilities, these appointments are approved, hence operations are scheduled) and patients are required to arrive earlier (mostly an hour ago) than their appointments for tests. Each operation has the following data: (i) predicted starting time, r_j and (ii) POD, p_j. Predicted ending time can be found using (i) and (ii): $r_j + p_j$.

Waiting time of an operation is defined as the nonnegative difference between the computed starting time of the operation and the predicted starting time of the operation (i.e., r_j) as represented as in Figure 1, whereas, tardiness is defined as the nonnegative difference between the computed finishing time of the operation and the deadline of the operation as in Figure 2. Waiting time and/or tardiness may also be realized due to uncertainty. For instance, operation j is computed to start at r_j, however, as the previous operation is delayed, operation j's starting time should be shifted by the delay amount of the previous operation (in case rescheduling is not allowed). Therefore, the patient of operation j realizes waiting time.

Considering the PODs and available OR days, the hospital administration computes the schedule manually. Due to manual computation, their schedules do not perform well. Hence, as mentioned before, one of the goals of this project is to optimize schedules with respect to first a completely deterministic environment considering PODs, second partially stochastic environment considering delay probabilities computed with respect to operation types. To solve the scheduling problem, Hospital X's processes are summarized in Section 3.2. Next, detailed data analysis is performed

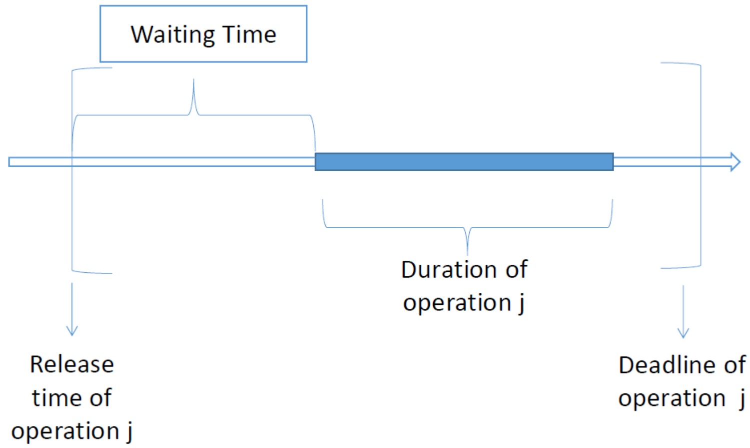

Figure 1: Time windows of a operation.

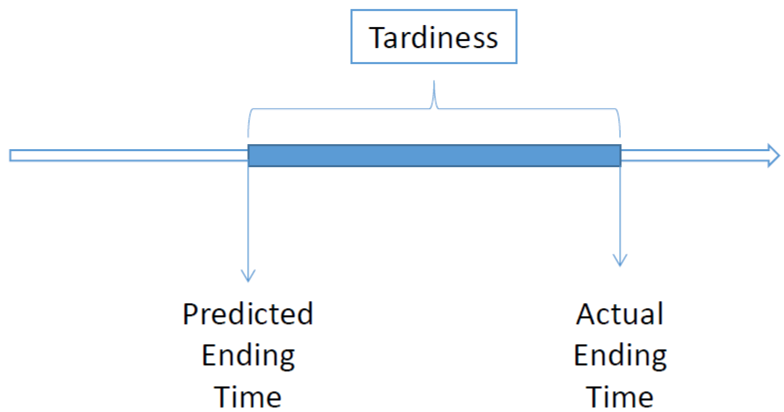

Figure 2: Tardiness.

in Section 3.3. Then, delays and corresponding reasons are represented in Section 3.4. Lastly, past operation schedules of Hospital X are analyzed and conclusion is in Section 3.6.

3.2 General Flow On The Day of Operation

On the day of the scheduled appointment, patient is expected to arrive usually an hour earlier than her/his appointment time. As in the Figure 3, the patient is first hospitalized. After hospitalization step, the patient starts to wait in ward for consultation and required tests before her/his operation. As mentioned before, emergency cases are not considered in this project. Next, the patient is transferred to an operating room and operation is performed. After the operation, the patient waits in recovery room then she/he is transferred to a room. Depending on the condition of the patient after the operation, the surgeon(s) finalizes the number of days that the patient should stay in the hospital and type of the room that the patient stays (such as, intensive care unit or ward). Patient's condition is regularly checked by the surgeon(s) and doctors. Finally, when the patient recovers, she/he is discharged from the hospital.

Figure 3: Flow chart of patients on the day of the operation.

3.3 *Analysis of Data*

The section describes detailed data analysis using the operating room data of 594 days. First, time-based flow on the day of operation is analyzed in Section 3.3.1. Next, general features of operations are shown in Section 3.3.2, such as the number of operations and their corresponding departments. Then, data about ORs are analyzed in Section 3.3.3. Finally, operation durations are analyzed in detail in Section 3.3.4.

3.3.1 Steps on The Day of Operations

Data about the detailed flow of patients, starting from their hospitalization to their discharge, represented in Figure 3 are provided from Hospital X except waiting times in ward. In Tables 1 - 4 show the statistical analysis of each step.

Maximum, minimum, average (Ave), median, standard deviation (SD), and number of operations for the waiting time in the recovery room are shown in Table 1.

Maximum waiting time in the recovery room is 86 minutes belonging to a plastic operation as depicted in Table 1. Waiting time in the recovery room analysis should be utilized by the nurse(s) who is not involved during the operation but is responsible to transfer the patient to her/his room.

Table 2 shows the transfer times between patient's room and OR before the operation. Even if the maximum transfer time lengths vary with respect to each department, averages are very close to each other in addition to averages, medians and standard deviations being close to each other. Transfer times with small variations are good since varying transfer times may become one of the major reasons causing waiting times as the next step after transferring the patient is performing the operation. Lastly, difference between each operation's starting time and corresponding

	Max	**Ave**	**Median**	**SD**
Neuro	92	8.84	7	9.6
Pedia	29	4.83	3	4.89
Gen	60	6.94	5	7.02
Gyn	73	8.67	7	7.37
Card	20	2.44	1.5	3.31
ENT	56	5.81	4	6.31
Orth	52	7.76	5	7.69
Plas	86	11.09	7	13.49
Uro	51	6.43	5	6.27

Table 1: Statistical information about waiting time in terms of minutes in the recovery and minimum waiting times for each department are zero.

	Max	**Ave**	**Median**	**SD**
Neuro	38	11.88	11	5.90
Pedia	82	10.74	9	6.71
Gen	45	10.99	10	5.50
Gyn	55	10.25	9	5.11
Card	26	11.06	10	5.75
ENT	42	10.62	10	4.62
Orth	46	11.58	11	5.71
Plas	72	11.93	11	6.29
Uro	45	11.04	10	5.59

Table 2: Statistical information about transfer time in terms of minutes.

	Max	Ave	Median	SD
Neuro	15704	192.65	154	1392.21
Pedia	24216	709.56	95	2524.17
Gen	33214	294.28	105	1943.38
Gyn	25565	189.05	111	803.93
Card	9320	1670.81	1115	2237.76
ENT	10138	119.68	86	494.39
Orth	16653	356.94	140	1937.83
Plas	11309	304.92	116	1016.17
Uro	11575	365.23	143	1401.47

Table 3: Statistical information about difference between operation starting times and hospitalization times in terms of minutes.

patient's hospitalization time, and difference between hospitalization and discharge are analyzed in Tables 3 and 4, respectively. Cardiology operations require longer preparations, such as, required tests performed before the operations last longer than the rest of the operations as in Table 3. In addition to longer preparations, cardiology operations have also the longest average and median difference between discharge and hospitalization (i.e., the total length of stay in Hospital X) as depicted in Table 4. However, there are operations belonging to other departments with longer total length of stay in Hospital X. Hence, if Hospital X's resources become limited in future, such as limited bed capacities, the hospital administration may first target analyzing such operations and their corresponding types (such as, cardiology) since similar patients may require similar length of stay and forecasting such operations' frequency may help the hospital administration to schedule their remaining operations to the remaining available times during which more beds may be available.

3.3.2 Operations

Operations are analyzed based on departments and time intervals.

Department-based Analysis: 7754 operations that are performed between January 1, 2013 and December 17, 2014 are utilized in our project. Figure 4 represents the percentage distribution of operations with respect to departments and the total

	Max	Min	Ave	Median	SD
Neuro	1172	0	53.60	27	79.57
Pedia	1168	1	43.19	5	141.12
Gen	736	0	43.89	26	59.73
Gyn	201	0	42.38	47	15.36
Card	291	4	96.11	71	91.45
ENT	1680	0	20.73	22	52.82
Orth	917	0	46.61	26	60.66
Plas	1320	0	33.90	10	93.04
Uro	1006	1	42.43	25	62.24

Table 4: Statistical information about difference between hospitalization time and discharge time in terms of hours.

number of operations of every department. Gynecology has the highest percentage distribution and it is 33 percent of total number of operations. Cardiology has also the lowest one and it is 50 operations out of 7754 operations. Moreover, the average number of daily operations is 13.06 (Figure 5).

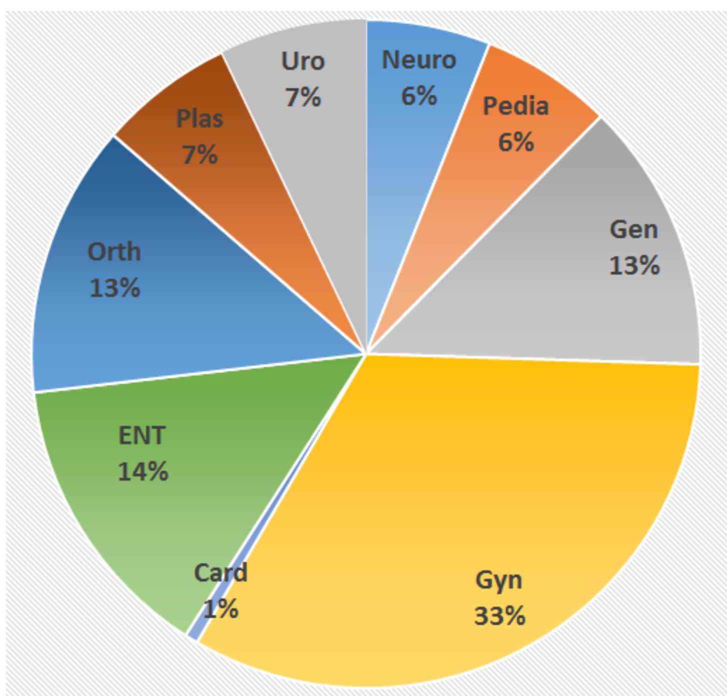

Figure 4: Pie chart representing each department's percentage divisions and total # of operations is represented inside parenthesis.

Time-based Analysis: The data also include scheduled and actual starting and

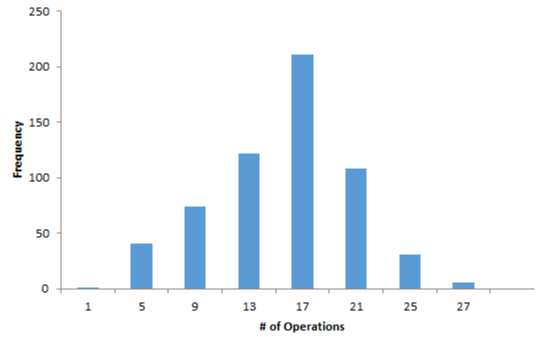

Figure 5: Histogram of operations.

ending time of operations. According this information, operations are aim to be scheduled from 8:30 to 17:30. We define a day by two intervals: (1) 8:30 - 13:00 and (2) 13:00 - 17:30 as seen in Table 36. We calculate the percentage of scheduled operations for each department with respect to the time intervals. 59 percent of all operations are scheduled during time interval (1). 90 percent of pediatric operations (445 operations out of 495 pediatric operations) are scheduled during time interval (1) because children have high priority for operation. Another important observation the common interval for the urology department (351 operations out of 551 urology operations): time interval (2). To the best of our knowledge, since there is not any underlying medical reason, we believe that it may be operation's preference. Table 5 shows also number of scheduled operations with respect to time for each department. In this table, we define a day by four intervals: (1a) 8:30-11:00, (2a) 11:00-13:00, (3a) 13:00-15:00 and (4a) 15:00-17:30. We find that 2703 operations out of 7754 operations are scheduled during time interval (1a) and 1929 operations are scheduled during time interval (3a).

Furthermore, performing an operation on Saturday is preferred pediatric operation as seen in Table 6. 26 percent of pediatric operations are scheduled on Saturday. For the other departments, we couldn't observe any patterns.

	08:30-11:00	11:00-13:00	13:00-15:00	15:00-17:30	Total
Neuro	122	61	203	82	**468**
Pedia	345	100	42	8	**495**
Gen	359	274	219	162	**1014**
Gyn	746	753	668	388	**2555**
Card	37	10	3	0	**50**
ENT	625	215	158	93	**1091**
Orth	342	204	283	194	**1023**
Plas	106	129	177	95	**507**
Uro	58	142	179	172	**551**
Total	**2703**	**1878**	**1929**	**1194**	**7754**

Table 5: # of scheduled operations with respect to time for each department.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Total
Neuro	0.19	0.18	0.19	0.18	0.18	0.08	**468**
Pedia	0.11	0.12	0.17	0.16	0.19	0.26	**495**
Gen	0.22	0.19	0.17	0.16	0.16	0.10	**1014**
Gyn	0.22	0.17	0.19	0.18	0.21	0.03	**2555**
Card	0.22	0.26	0.18	0.12	0.20	0.02	**50**
ENT	0.19	0.19	0.17	0.21	0.18	0.06	**1091**
Orth	0.07	0.29	0.24	0.25	0.09	0.05	**1023**
Plas	0.16	0.26	0.09	0.22	0.16	0.12	**507**
Uro	0.13	0.20	0.21	0.17	0.21	0.08	**551**
Total	**0.18**	**0.20**	**0.18**	**0.19**	**0.18**	**0.07**	**7754**

Table 6: Percentage of each operation's assignment during weekdays.

3.3.3 ORs

The data include seven different ORs and each department has different OR preferences (Table 8). For example, 80 percent of orthopedic operations are scheduled at OR # 1 and 54 percent of general operations are scheduled also at OR # 6. Furthermore, table 7 describes that generally two empty ORs are left to have buffer for state of emergency and the average number of OR used is 4.96 (Figure 6). Figure 7 describes the number of ORs used each day. Except 28 days out of 594 days, at least one OR is not used.

Furthermore, we also analyze doctors' OR preferential for each departments as seen in Table 37.

Moreover, we calculate the OR's utilization by using hospital target with AOD and POD (Table 9). We set that total daily incision to seture time is equal 8 hours

	OR Id						
	1	**2**	**3**	**4**	**5**	**6**	**7**
Neuro	0.29	0.04	0.12	0.00	0.13	**0.33**	0.09
Pedia	0.08	0.15	0.24	0.01	0.11	0.16	**0.26**
Gen	0.06	0.09	0.10	0.01	0.08	**0.54**	0.12
Gyn	0.06	0.15	0.22	0.00	0.12	0.13	**0.31**
Card	0.00	0.00	0.00	**0.96**	0.04	0.00	0.00
ENT	0.04	0.20	**0.30**	0.00	0.13	0.12	0.21
Orth	**0.80**	0.04	0.04	0.00	0.03	0.05	0.04
Plas	0.07	0.09	0.15	0.00	0.09	0.19	**0.40**
Uro	0.11	0.11	0.22	0.00	0.15	0.16	**0.25**
Total	**0.18**	**0.12**	**0.18**	**0.00**	**0.10**	**0.19**	**0.22**
Total # of Operations	**1363**	**928**	**1403**	**29**	**803**	**1474**	**1704**

Table 7: Percentage of each department's assignment in ORs.

	Preferential OR
Neuro	6-1-5-7-3-2-4
Pedia	7-3-6-2-5-1-4
Gen	6-7-2-3-5-1-4
Gyn	7-3-2-5-6-1-4
Card	4-5
ENT	3-7-2-5-6-1-4
Orth	1-6-7-2-3-5-4
Plas	7-6-3-5-2-1-4
Uro	7-3-6-5-1-2-4

Table 8: Preferential OR of each department.

per OR. Table 9 shows that OR 4's utilization is very low with respect to AOD and POD. OR 6's utilization is 55 percent with respect to AOD and 70 percent with respect to POD. It is one of the highest used capacity in all ORs. Utilization is not so high because of some reasons such as doctors' OR preferential as seen in Table 37 and doctors' time interval preferential as seen in Table 5. Doctors are not always available for operations, and we ignore examination of doctors. Because of these reasons, we can not change the schedule. Moreover, it is real data and there are some emergency cases that are integrated of data and release time of operation comes into prominence.

	AOD		POD	
# of ORs	**Used Capacity**	**Unused Capacity**	**Used Capacity**	**Unused Capacity**
1	0.55	0.45	0.65	0.35
2	0.27	0.73	0.35	0.65
3	0.43	0.57	0.57	0.43
4	0.04	0.96	0.04	0.96
5	0.27	0.73	0.34	0.66
6	0.55	0.45	0.70	0.30
7	0.56	0.44	0.68	0.32

Table 9: OR utilization calculated by using AOD and POD.

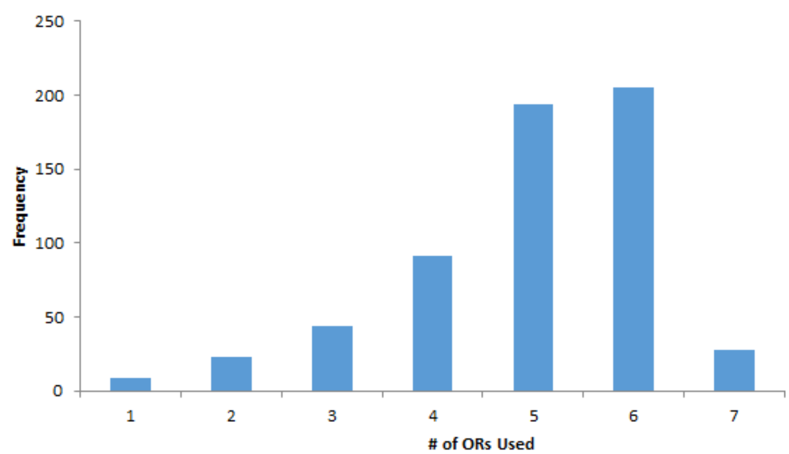

Figure 6: Histogram of ORs used #.

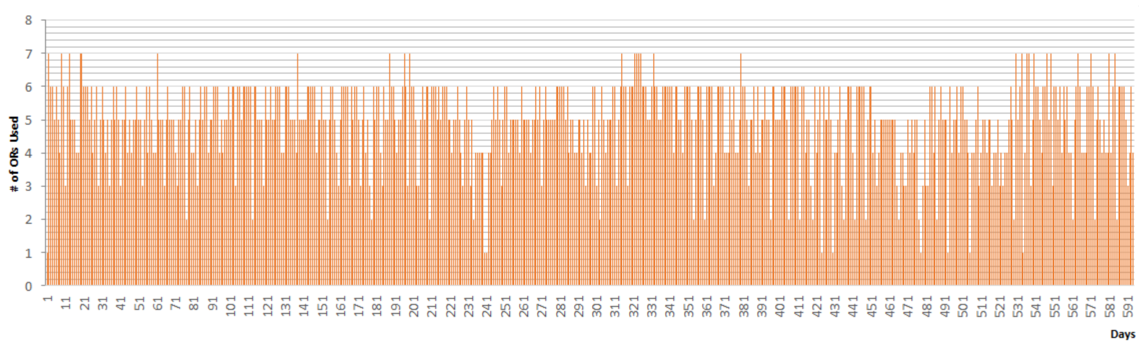

Figure 7: # of ORs used.

3.3.4 Operation Durations

There are two different operation durations. The first is about AOD and second is also POD. In this subsection, we analyze operation duration with respect to departments, days and ORs. Then, we try to show comparison of AOD and POD.

3.3.4.1 Departments

We firstly analyze statistical information about AOD and POD. Neurosurgery has the maximum of average AOD (149.96 minutes) and POD (194.22 minutes) and pediatric operation has the lowest average AOD (43.20 minutes) because of lower operation duration of circumcision feast as seen in Table 10.

We also find minimum AOD and POD with respect to different percentage the procedures have a time duration in terms of minutes. For example, if we analyze the minimum AOD with respect to 75 percent the procedures, pediatric operation has 48 minutes and it is the lowest duration. In this case, cardiology has the highest duration and it is 218 minutes. In POD cases, the minimum POD with respect to 75 percent the procedures, neurology has the highest duration (270 minutes) and pediatric and gynecology operation has the lowest duration (90 minutes).

In Figure 8, we plot both operation duration of pediatric operation and generally POD is higher than AODs. Since the hospital solves OR scheduling problem by using

	AOD				POD			
	Max	Min	Ave	Median	Max	Min	Ave	Median
Neuro	1125	0	149.96	126	510	25	194.22	180
Pedia	374	0	43.20	28	510	10	82.94	60
Gen	2749	0	86.06	67	510	10	99.37	90
Gyn	1007	0	64.89	54	300	15	80.91	60
Card	299	21	145.46	133	270	30	163.00	150
ENT	1109	7	83.19	67	390	23	124.76	120
Orth	1138	4	103.31	87	450	10	116.25	120
Plas	1118	9	112.81	91	510	15	144.46	120
Uro	1026	5	76.46	58	490	15	101.65	90

Table 10: Statistical information about AOD and PODs in terms of minutes.

PODs, we may end up underutilization of ORs due to overestimation of operation duration.

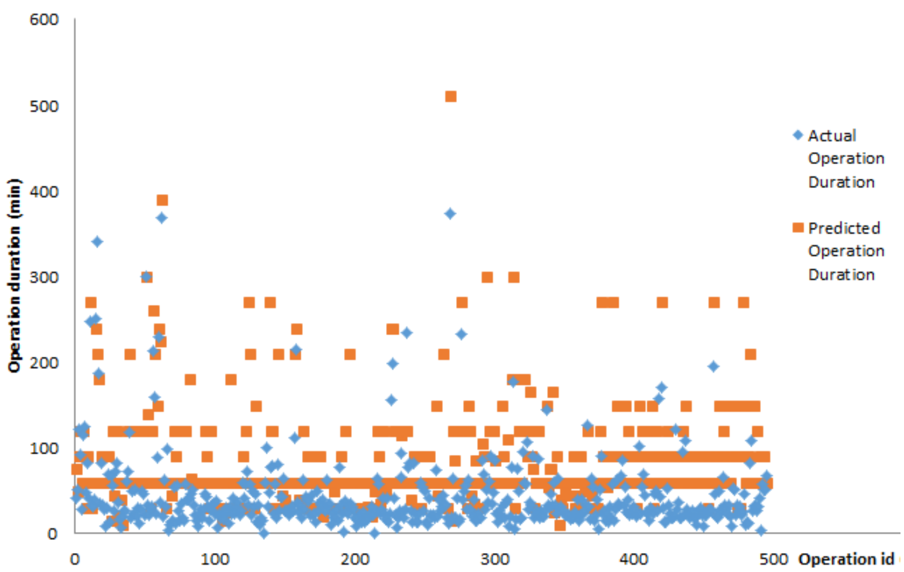

Figure 8: Pediatric operations's AOD and POD in terms of minutes.

Furthermore, we calculate goodness fit of each department's AOD and POD using EasyFit software as seen in Appendix C. We used 10 different distributions as Beta, Chi-Squared, Erlang, Exponential, Gamma, Lognormal, Normal, Triangular, Uniform and Weibull. We use two different tests, the first one is Kolmogorov Smirnov Test and the second one is Anderson Darling Test. We show the rank of each distribution for just one example, in Appendix Part. For AOD, Erland distribution gives the highest p-values in Kolmogorov Smirnov Test. It means that Erland distribution is the best fit for neurosurgery's AOD. Strum et al. (2000) show the distribution type of operation duration that is normal distribution. We find that distribution of operation duration is changeable according to department of operation and just normal or log-normal distribution may not be the best fit.

We show the each operation's AOD and POD with the operation's characteristics. An operation type is created by surgeon to show information about operation duration and equipment. We categorize firstly 5 main operation characteristic that are most popular operations for each operation types and remains are named by "Others" as seen in Appendix D. If we check the process of creating operating types, firstly, surgeon assigns the operation duration and heshe creates operation types to determine the operation's characteristics. That's why we also categorize operation types to show meaningful operation duration for operations.

We also find some statistical information about each department by using EasyFit software. The sample size, range, the mean and standard deviation, and the minimum and maximum values are determined and the mean duration of departments gives a better solution of operation duration. Then, we study more detail analysis as variance, standard error, coefficient of variance and skewness by calculating the three smallest and largest values as 5th, 10th, 25th (Q1), 50th (Median), 75th (Q3), 90th, and 95th percentiles. Range is difference of the maximum and minimum number and variance is a kind of measurement how far a group of number is spread out. Standard deviation is a measurement of the square root of the variance and coefficient of variation is normalized measure of dispersion of probability distribution. Standard error is also the standard deviation of the sampling distribution of a statistic. As known that skewness is a measure of the asymmetry of the probability distribution of a actual-valued random variable about its mean. If the distribution is symmetric, we can say

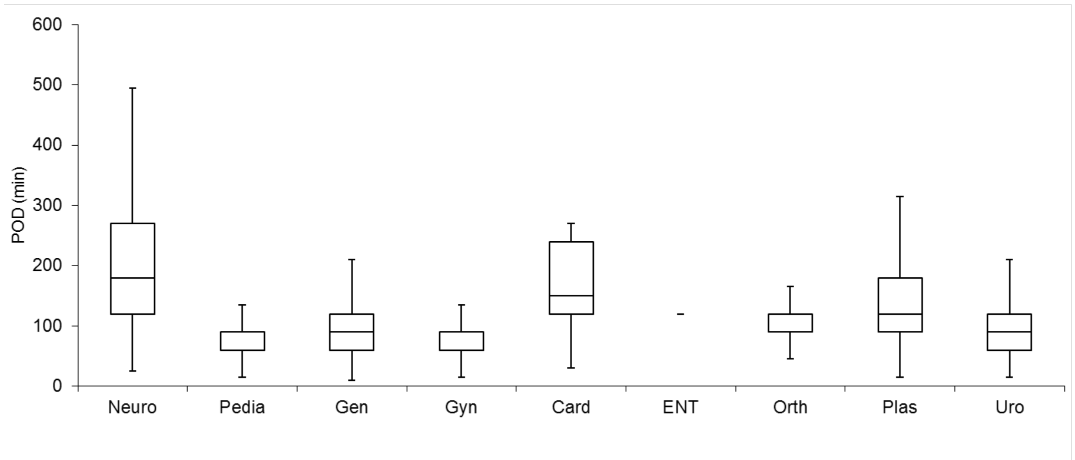

Figure 9: Box plot of POD.

the coefficient of skewness is about 0. In negative cases, we observe that the median is always bigger than the mean then the distribution is skewed left. Otherwise, the median is always smaller than mean and the distribution is skewed right. Excess Kurtosis is a kind of measure of the peakedness of the probability distribution of a actual-valued random variable. If the coefficient of kurtosis is small, the distribution is seen more normal. The normal distribution has a coefficient of kurtosis of 3 and it provides a convenient benchmark.

Figure 9 shows box plot of POD for each operations. To get sense of the general difference between operations, median values of POD can be determined. In general, neurosurgery has supreme median values: 180 minutes. Cardiology has approximate value that is 150 minutes. Ear-nose-throat, plastic operation and orthopedic operation contain same median values: 120 minutes. Median value of urology and general operation is 90 minutes. The minimum median is 60 minutes for pediatric operation and gynecology. Moreover, the boxes and whiskers demonstrate the same overall trend as the medians and they have more clues. Except urology and general operation, boxes of PODs are not symmetrical. Neurosurgery, cardiology and plastic operation generally have more closely upper regions and it means that distribution is skewed right. Pediatric operation and gynecology has equal Q1 and median values, orthopedic

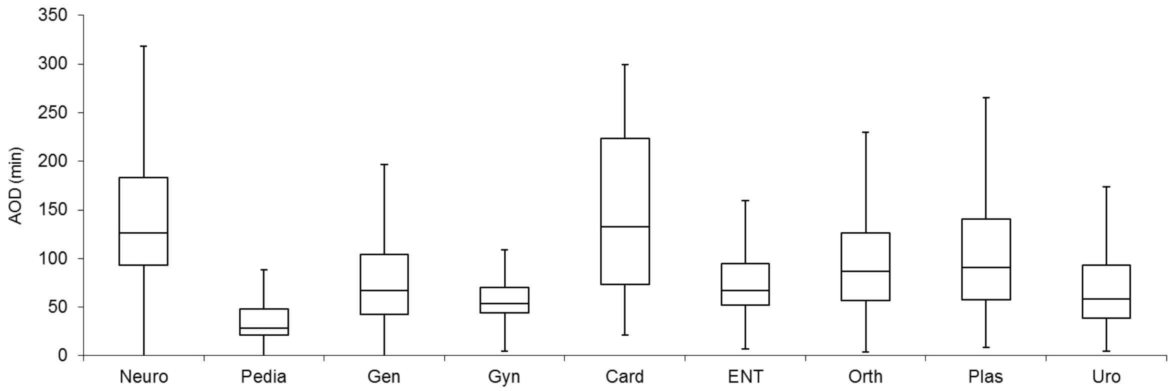

Figure 10: Box plot of AOD.

operation also has same median and Q3 values. Ear-nose-throat has same Q1, median and Q2 values. As is known that, the whiskers shows the minimum and maximum values for operations without outliers and neurology has wide whiskers.

Moreover, Figure 10 shows box plot of AOD for each operations and cardiology has supreme median values: 13 minutes. Then, neurology has the second biggest median time: 126 minutes. The minimum median is 21 minutes for pediatric operation. Any boxes are not symmetrical and distribution is skewed right. Neurology and cardiology have wide whiskers and pediatric operation and gynecology contain narrow whiskers. In both Figure 9 and Figure 10, we ignore outliers.

3.3.5 Days

In this part, we calculate average AOD and POD with respect to operation days in terms of minutes as seen Table 11. For AOD, both average operation duration is about 80 minutes, and average POD is over than 100 minutes. We also show total actual and predicted operation durations and Tuesday has the maximum total operation duration, Saturday is also lower the maximum total operation duration in both actual and predicted cases.

	Total AOD	Average AOD	Total POD	Average POD
Monday	**112926**	**82.01**	**144912**	**105.24**
Tuesday	**135418**	**88.86**	**169994**	**111.54**
Wednesday	**118380**	**83.31**	**151738**	**106.78**
Thursday	**127727**	**85.27**	**164370**	**109.73**
Friday	**108278**	**79.44**	**143074**	**104.97**
Saturday	**45375**	**78.91**	**58362**	**101.50**
Total	**648104**	**83.54**	**832450**	**107.30**

Table 11: Total and average operation duration with respect to operation days in terms of minutes.

3.3.6 ORs

Table 12 shows the average AOD and POD with respect to different ORs in terms of minutes. We find that there is not any balance between the schedule with respect to ORs.

OR id	Ave AOD	Ave POD
1	101.01	120.17
2	68.37	93.34
3	73.40	100.29
4	100.31	84.76
5	79.25	104.70
6	91.89	119.97
7	78.81	99.32
Total	**83.12**	**106.92**

Table 12: Average operation duration with respect to ORs in terms of minutes.

3.3.7 Compare Actual and Predicted Operation Duration

To study the effect of the difference between AOD and POD, we define a ratio 'RP'. RP is the ratio of AOD to POD as defined in (1). If RP ≥ 1, AOD lasts longer than the predicted one. Thus, the operation duration is underestimated. If RP < 1, the POD is longer than POD and it shows overestimation. We also find the average RP which is represented as $\overline{RP}$ as defined in (2). Table 13 represents $\overline{RP}$ when RP is 'less than 1' or 'greater than equal to 1'. When RP < 1, there are 5897 operations out of total operations and $\overline{RP}$ is 0.62. When RP ≥ 1, there are 1807 operations out of total operations and $\overline{RP}$ is 1.58.

$$RP_j = \frac{AOD_j}{POD_j} \tag{1}$$

$$\overline{RP} = \sum_{j \in J} \frac{RP_j}{\|J\|} \tag{2}$$

	$\overline{RP}\geq$ **1**	**Total # of RP≥1**	$\overline{RP}$ **<1**	**Total # of RP< 1**	$\overline{RP}$
Neuro	1.84	113	0.62	355	0.92
Pedia	1.44	54	0.46	441	0.57
Gen	1.68	288	0.65	726	0.94
Gyn	1.51	608	0.68	1947	0.88
Card	1.44	16	0.70	34	0.94
ENT	1.40	150	0.57	941	0.69
Orth	1.64	347	0.63	676	0.98
Plas	1.58	140	0.60	367	0.87
Uro	1.66	107	0.59	444	0.80
Total	**1.58**	**1807**	**0.62**	**5897**	**0.84**

Table 13: $\overline{RP}$ with respect to departments where $\overline{RP}$ is the average of RPs. RP is defined as the ratio of AOD to POD.

3.4 Delay and Delay's Reasons

The data show delay time of operations. If an operation finishes after its predicted ending time, it shows delay. For example if predicted ending time is 11:00 and the operation finishes 11:02, then, this operation is late by 2 minutes. Table 14 explains that the maximum number of delay occurs in pediatric operation. Another interesting point is that half of pediatric operations starts late (Table 14) and in general, 53 percent of operations are delayed. We also find statistical information about delay time in terms of minutes. Table 15 shows cardiology has the maximum average delay time although the percentage of delay of cardiology has the lowest value.

There are several reasons because of delay as inability of the pre-operation preparation's completion, equipment required is used for another operation, previous operation's duration is longer than POD, the patients arrival late, the longer transfer time, late hospitalization, doctors or nurses arrives late and specialty materials arrive late (Table 16 and Table 17). Table 16 shows that the most popular reason is previous operation's duration is longer than POD.

	# of Delayed	**Total # of**	**Percentage of Delayed**
Neuro	188	468	0.40
Pedia	261	495	0.53
Gen	198	1014	0.20
Gyn	776	2555	0.30
Card	2	50	0.04
ENT	272	1091	0.25
Orth	255	1023	0.25
Plas	182	507	0.36
Uro	193	551	0.35
Total	**2327**	**7754**	**0.30**

Table 14: Statistical information about delayed operation.

	Max	**Ave**	**Median**
Neuro	330	55.33	13
Pedia	162	48.98	19
Gen	127	38.05	4
Gyn	201	37.93	9
Card	121	90.50	1
ENT	230	34.49	7
Orth	319	63.27	5
Plas	249	56.90	9
Uro	133	35.27	11

Table 15: Statistical information about delay time.

3.5 Scheduling Analysis

In this section, we show the different analysis that is used predicted and actual data. We firstly calculate **tardiness** of each operation. Average total tardiness is 14.52 in terms of minutes.

Id	Reasons	Total
1	Pre-operation preparation's completion	**673**
2	Equipment required is used for another operation	**24**
3	Previous operation's duration is longer than predicted	**880**
4	The patients arrive late	**95**
5	The longer transfer time	**47**
6	Late hospitalization	**13**
7	Doctors arrive late	**573**
8	Nurses arrive late	**12**
9	Specialty materials arrive late	**8**
10	Others	**2**
	Total	**2327**

Table 16: Reasons of delays.

Id	Neuro	Pedia	Gen	Gyn	Card	ENT	Orth	Plas	Uro	Total
1	70	94	49	176	0	99	71	48	66	**673**
2	0	1	2	5	0	3	6	3	4	**24**
3	60	71	83	323	1	69	142	61	70	**880**
4	6	11	8	28	0	23	11	8	0	**95**
5	7	1	4	16	0	9	4	2	4	**47**
6	0	1	1	5	0	2	2	1	1	**13**
7	42	81	48	222	1	62	15	59	43	**573**
8	2	1	2	0	0	4	1	0	2	**12**
9	1	0	1	1	0	0	3	0	2	**8**
10	0	0	0	0	0	1	0	0	1	**2**
Total	**188**	**261**	**198**	**776**	**2**	**272**	**255**	**182**	**193**	**2327**

Table 17: Delays information for each department.

Moreover, in Table 18, there are maximum and average of tardiness information with respect to number of corresponding department operation and total number of early corresponding operation.

We also compare between two different schedules of a day by using MS Project software. The first part of figure is about predicted scheduling and second is also about actual scheduling of the day as seen in Figure 11. Also, another example is considered in Figure 12. In these two examples, we can see both earliness and tardiness by comparing actual and predicted operation starting and ending time.

We also find average **waiting time** in terms of minutes as seen in Table 19.

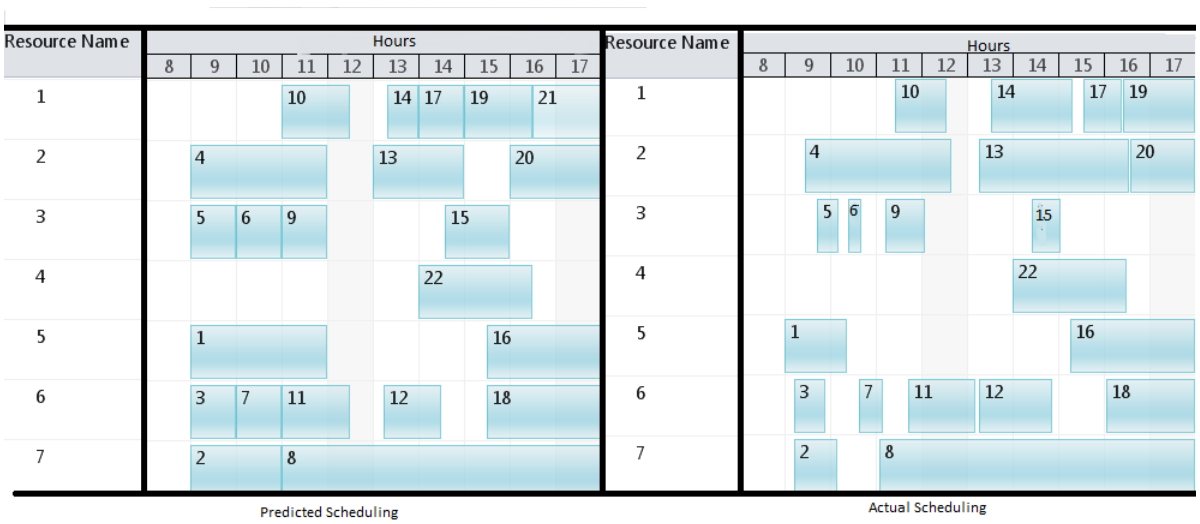

Figure 11: Example 1.

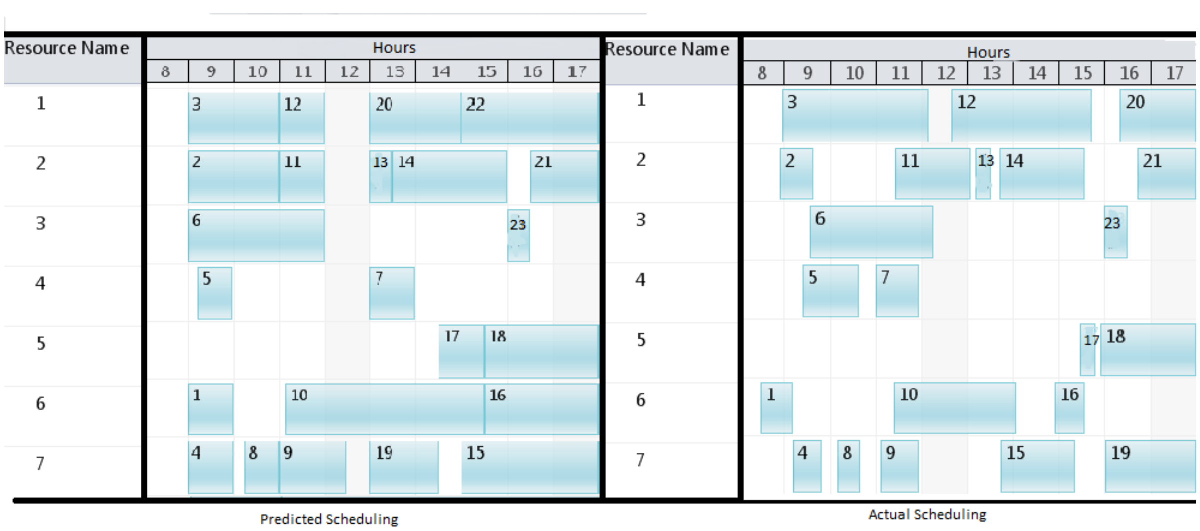

Figure 12: Example 2.

	Max	Ave Tardiness	
		With Respect to # of Corresponding Department Operation	With Respect to Total # of Tardy Corresponding Operation
Neuro	893	24.39	67.15
Pedia	225	9.46	26.32
Gen	1281	13.88	41.88
Gyn	1042	12.07	29.62
Card	156	11.98	49.92
ENT	998	6.45	40.89
Orth	957	24.97	66.00
Plas	1017	22.61	60.34
Uro	974	12.40	38.16

Table 18: Statistical information about total tardiness with respect to departments in terms of minutes.

Pediatric operation is waited 27.97 minutes with respect to number of corresponding department operation. It is the highest waiting time in all departments. Cardiology operation's waiting time is 7.90 minutes and it is the lowest one.

	Ave Waiting Time	
	With Respect to # of Corresponding Department Operation	With Respect to # of Hold Patient Corresponding Operation
Neuro	25.14	33.62
Pedia	27.97	37.01
Gen	10.49	18.03
Gyn	14.64	21.56
Card	7.90	15.80
ENT	11.88	18.82
Orth	18.42	32.66
Plas	22.93	35.33
Uro	15.91	21.07
Total	**20.35**	**29.29**

Table 19: Average waiting time in terms of minutes.

Table 20 shows the relationship between operation actual starting time (AST) and operation predicted starting time (PST) with earliness (E) and tardiness (T), the other word it is a kind of **under and overutilization reasons**' table. Following tables give more detail information about AST, PST and T with RP. We firstly determine two main groups, such as RP is greater or equal to 1 and RP is smaller than 1. Then we analyze relationship with AST and PST with earliness and tardiness case.

	AST < PST		AST= PST		AST > PST		
	E	**T**	**E**	**T**	**E**	**T**	**Total**
RP≥ 1	314	400	x	32	x	1063	**1809**
RP<1	1793	x	116	x	2795	1173	**5877**
Total	**2107**	**400**	**116**	**32**	**2795**	**2236**	**7686**

Table 20: Reasons of under or over estimation. If there is not any datum, x is used.

- Table 21 shows the statistical information about RP when RP is greater or equal to 1 and AST is smaller than PST with earliness case. In this cases, the operation starts early that's why there is not any delay.

	# of Operations	Max{RP}	Min{RP}	$\overline{RP}$
Neuro	11	7.2	1	1.73
Pedia	11	1.53	1.02	1.14
Gen	46	2.51	1	1.24
Gyn	85	2.93	1	1.17
Card	7	1.45	1.06	1.17
ENT	33	2.93	1	1.24
Orth	87	3.50	1	1.36
Plas	19	1.95	1	1.20
Uro	15	2.1	1	1.29
Total	314	7.2	1	1.26

Table 21: Statistical information about RP for cases when RP≥1 and AST<PST with Earliness.

- Table 22 shows statistical information about RP for cases when RP is greater or equal to 1 and AST is smaller than PST with tardiness case.

- Table 23 is another case where RP is smaller than 1 and AST is smaller than PST with earliness. There is not any kind of abnormality.

	# of Operations	Max{RP}	Min{RP}	$\overline{RP}$
Neuro	28	9.52	1.02	1.75
Pedia	6	4.53	1.04	1.93
Gen	67	4.63	1.04	1.55
Gyn	121	4.85	1.04	1.52
Card	2	1.35	1.11	1.23
ENT	43	2.93	1.06	1.36
Orth	73	17.18	1.06	1.95
Plas	38	6.33	1.02	1.68
Uro	21	3.43	1.06	1.54
Total	**400**	**17.18**	**1.02**	**1.62**

Table 22: Statistical information about RP for cases when RP$\geq$ 1 and AST<PST with Tardiness.

	# of Operations	Max{RP}	Min{RP}	$\overline{RP}$
Neuro	71	0.99	0.18	0.64
Pedia	89	0.95	0.00	0.48
Gen	279	0.99	0.10	0.65
Gyn	565	0.99	0.06	0.68
Card	15	0.98	0.18	0.66
ENT	300	0.99	0.17	0.58
Orth	269	0.99	0.13	0.63
Plas	111	0.99	0.12	0.60
Uro	93	1.00	0.13	0.62
Total	1793	1.00	0.00	0.63

Table 23: Statistical information about RP for cases when RP$<$ 1 and AST $<$ PST with Earliness.

- Table 24 mentions about RP for cases when RP is smaller than 1 and AST and PST are equal each other with earliness cases. There is underutilization situation because of the shortest actual duration of operation.

- Table 25 shows statistical information about RP when RP is greater than or equal to 1 and AST and PST are equal each other with tardiness case. In this situation, delay occurs because of tardiness.

- Table 26 mentions the statistical information about RP when RP again is

	# of Operations	Max{RP}	Min{RP}	$\overline{RP}$
Neuro	5	0.95	0.40	0.81
Pedia	13	0.92	0.07	0.53
Gen	25	0.98	0.22	0.62
Gyn	31	0.93	0.36	0.68
Card	1	0.98	0.98	0.98
ENT	23	0.95	0.26	0.59
Orth	7	0.91	0.23	0.66
Plas	6	0.91	0.60	0.73
Uro	5	0.71	0.25	0.52
Total	116	0.98	0.07	0.63

Table 24: Statistical information about RP for cases when RP<1 and AST=PST with Earliness.

	# of Operations	Max{RP}	Min{RP}	$\overline{RP}$
Neuro	2	2.43	1.53	1.98
Gen	5	2.00	1.02	1.27
Gyn	13	1.92	1.08	1.36
ENT	3	1.30	1.09	1.21
Orth	7	3.28	1.12	1.64
Plas	2	2.98	1.05	2.01
Total	32	3.28	1.02	1.47

Table 25: Statistical information about RP for cases when RP≥1 and AST=PST with Tardiness.

greater than or equal to 1 and AST is greater than or equal to PST with tardiness case. There are two reasons because of delay, late start time and wrong estimation of operation duration.

- Final analysis is for RP, when RP is smaller than 1 and AST is greater than or equal to PST with tardiness case. Table 27 shows that lateness of release time causes delay because the duration of actual operation is smaller than predicted duration.

Furthermore we analyze the **consecutive operations**. It may be another reason

	# of Operations	Max{RP}	Min{RP}	$\overline{RP}$
Neuro	71	29.20	1.00	1.90
Pedia	35	2.90	1.00	1.47
Gen	168	42.33	1.00	1.86
Gyn	384	38.72	1.00	1.59
Card	7	4.20	1.18	1.77
ENT	71	10.63	1.00	1.51
Orth	177	15.50	1.00	1.66
Plas	79	9.44	1.00	1.63
Uro	70	17.10	1.00	1.78
Total	1063	42.33	1.00	1.67

Table 26: Statistical information about RP for cases when RP≥1 and AST≥PST with Tardiness.

	# of Operations	Max{RP}	Min{RP}	$\overline{RP}$
Pedia	137	0.98	0.09	0.54
Gen	96	0.99	0.00	0.75
Gyn	523	0.99	0.08	0.78
Card	3	1.00	0.76	0.91
ENT	55	1.00	0.27	0.75
Orth	130	0.99	0.03	0.73
Plas	71	0.99	0.17	0.74
Uro	88	1.00	0.13	0.75
Total	1173	1.00	0.00	0.73

Table 27: Statistical information about RP for cases when RP<1 and AST≥PST with Tardiness.

for delay. Table 28 shows that 1293 different consecutive operations occurs. Gynecology has the highest consecutive operations, and it is 281 in different OR. If we check consecutive operations with in same OR, the solution is less than the different OR case. Table 29 shows that orthopedic operation has the highest consecutive operation and 138 operations are scheduled consecutively in same OR.

3.6 Conclusion

In this chapter, we analyze the healthcare data with different approaches. We have 9 different operational specialties and 7 ORs and ORs are available from 08:30

	Double	Triple	Quadruple	Total Consecutive Operations
Neuro	25	0	0	**50**
Pedia	64	6	1	**150**
Gen	77	2	1	**164**
Gyn	131	5	1	**281**
Card	4	0	0	**8**
ENT	96	6	1	**214**
Orth	91	5	0	**197**
Plas	47	2	0	**100**
Uro	53	5	2	**129**
Total	**588**	**31**	**6**	**1293**

Table 28: Consecutive operations.

	Double	Triple	Total Consecutive Operations Operations
Neuro	13	0	**26**
Pedia	50	2	**106**
Gen	56	0	**112**
Gyn	67	1	**137**
Card	3	0	**6**
ENT	58	1	**119**
Orth	66	2	**138**
Plas	32	1	**67**
Uro	31	1	**65**
Total	**376**	**8**	**776**

Table 29: Consecutive operations in same OR.

to 17:30. We do not consider emergency case. There are 7754 different operations and 2555 operations out of 7754 operations is gynecology operations. Furthermore, the average number of daily operation is 13.06. In general, 59 percent of all operations are scheduling during 08:30-13:00. We also analyze the ORs utilization and preferences. The data show that two ORs are left to have buffer for emergency and the daily average number of OR used is 4.96 ORs and the highest OR's utilization is 56 percent for AOD and 70 percent for POD. Moreover, we determine statistical information about waiting time in terms of minutes in the recovery, transfer time, difference between operation start time and hospitalization time and the difference between hospitalization time and discharge time. Then, we determine statistical information of AOD and POD for each department and neurosurgery has the highest duration and generally, POD is greater than AOD for each operation. We may end up underutilization of ORs due to overestimation of operation durations. We also consider goodness fit of each department using operations' AOD and POD. Then, we divide each operation in terms of the operation types. After that, we find average AOD and POD with respect to operation days and ORs in terms of minutes. Moreover, we try to make comparison between AOD and POD. We firstly find RP to take ratio of AOD and POD. We find that 5897 operations out of 7754 operations are overestimated. 2327 operations out of 7754 operations start late and the most popular reasons are previous operation's longer predicted duration and pre-operation preparation's completion. Then, we make scheduling analysis by using tardiness and waiting time information. Average total tardiness is 14.52 minutes, and average waiting time is 20.95 minutes. Then, we determine the relationship of AST, PST, T and RP. For example, we find 116 operations when RP is smaller than 1 and AST and PST are equal to each other with earliness cases. It is a kind of underutilization problem because of the POD is whacker than AOD. Another example can be considered when RP greater than or equal to 1 and AST is greater than PST with tardiness case. There are 2795 operations and they start late but they finish early because of short AOD. There are also late 1173 different operations because there are late start time and wrong estimation of operation durations. Then, we find consecutive operations with different ORs and same ORs. It may be end up another delay reasons.

CHAPTER IV

ASSIGNING AND SCHEDULING OPERATIONS

Hospital X aims to schedule their operations avoiding waiting time and satisfying fairness of OR usage. To achieve Hospital X's goals, we use mathematical models introduced in Table 32. In addition to Hospital X's goals, we study other objective functions. First, a deterministic setting is assumed where all data are known in advance. Next, Hospital X has delay probabilities of its operations which are incorporated to the proposed mathematical models by heuristics. Then, we compare these models' performance with what has actually happened (i.e., one scenario) and with what may happen (i.e., several scenarios). Lastly, we propose a simple heuristic to use optimization and analyze its performance. Table 30 represents our computational experiment goals and corresponding Hospital X Administration goals.

In this chapter, we introduce assumption of our models in Section 4.1, mathematical models in Section 4.2, and simple heuristic in Section 4.3 respectively. Then, computational results are summarized in Section 4.4.

4.1 Assumptions

Emergency cases are ignored in this study as the hospital administration first targets assigning and scheduling planned operations. A very common assumption is the nonpreemptive nature operations: if an operation starts in an OR, the operation cannot be interrupted. Next, material resources, such as sterilized medical trays, and human resources, such as nurses, are available during operations. Furthermore, all ORs are available starting from 08:30. In addition, all patients are assumed to be ready on their appointment time. Hence, we ignore uncertainties caused by patients' late arrivals. Lastly, clean-up time of an OR after each operation and induction time

Our Computational Experiment Goals	Hospital X Administration Goals
1) Solve deterministic mathematical models for assignment and scheduling of operations	**1)** Compute when the operations will start and which OR will be used
2) Solve mathematical models considering delay probabilities embedded in by heuristics	**2)** Improve our schedule given operations' delay data
3) Compute schedules via using mathematical models under possible scenarios	**3)** Whether adapting the schedules helps us improve current schedule problems
4) Propose a simple heuristic and measure its performance	**4)** Without complex models, whether we can use a simple rule to assign and schedule operations

Table 30: Comparison between our computational experiment goals and Hospital X Administration goals.

of each operation are included in operation durations.

4.2 Mixed Integer Linear Programming Formulation

In this section, we develop mathematical models to solve a real life operation scheduling problem. There are m operating rooms (ORs) where n operations will be scheduled. Each operation j has a time window defined by its release time (r_j) and deadline (d_j). Processing time of an operation is represented by p_j. The nomenclature of the problem is given in Table 31.

We formulate operation assignment and scheduling problem by a Mixed Integer Linear Programming (MILP) formulation . x_{ijt} is a binary decision variable representing whether operation j is scheduled to start at time t in OR i. If it is, x_{ijt} equals 1, otherwise, x_{ijt} equals to 0. The length of the time horizon is defined by set T where

Indices	
j	Operation
i	OR
t	Time
Sets	
T_{ijt}	Set of discrete times which operation j can start in OR i
Parameters	
p_j	Processing time of operation j
r_j	Release time of operation j
d_j	Deadline or due date of operation j
b	Percentage of upper bound tardiness for operations
ed	End of a day, such as, 540 represents 17:30
$\overline{P}$	Sum of operations' processing time divided by the number of OR
$\overline{PP}$	Sum of probability of operations divided by the number of OR
W_{OR}	Penalty of overtime
W_{PR}	Penalty of delayed operation
W_O	Weight of overutilization in the objective function
W_W	Weight of waiting time in the objective function
μ	Large number
Variables	
x_{ijt}	=1 if operation j starts in OR i at time t
M	Makespan
T_j	Tardiness of operation j
W_j	Waiting time of operation j
z_i	Overutilization of OR i
K_i	Overtime of OR i
prp_i	Probability of OR i's delay
yy_i	=1 if OR i is utilized

Table 31: Notation.

T={1,2,3,... $\|T\|$ }.

$$\texttt{Min [objective function]} \tag{3}$$

$$\texttt{subject to} \sum_{it} x_{ijt} = 1 \texttt{ for all j} \tag{4}$$

$$\sum_{j} \sum_{t' \in T_{ijt}} x_{ijt'} \leq 1 \texttt{ for all i,t} \tag{5}$$

$$x_{ijt} = 0 \texttt{ for all j,t with } t \leq r_j \tag{6}$$

$$x_{ijt} \in \{0,1\} \texttt{ for all i,j,t} \tag{7}$$

where

T_{ijt}={ $t' : t - p_j < t' \leq$ t }
is the set of discrete times repressing possible starting times for operation j in OR i.

Alternative objective functions can be introduced as (3) and such objective functions are introduced in Sections (4.2.1), (4.2.2), (4.2.3), (4.2.4), (4.2.5), (4.2.6) and (4.2.7) with details. Constraint (4) defines that every operation should be assigned exactly to an OR. Two or more operations are prevented to overlap for each OR by Constraint (5). Constraint (6) satisfies that each operation should start after its release time.

A list of mathematical models with different objective functions is introduced in Table 32: makespan, overtime, tardiness, waiting time, fairness, delay probability, and range. Model X-Y represents the model combination where X presents model number, 1, 3, and 5, and Y represents objective functions: overtime (O), makespan (M), tardiness (T) or makespan with deadline constraint (MD) as objective function. Models 2 and 4 are for Hospital X's current solution, hence, they are the same model except objective function calculations. Deterministic schedule is evaluated in Model 2, whereas, schedule with delay probabilities are studied by Model 4. However, even Models 2 and 4 seem to optimize the solution, we only compute the objective function values of Hospital X given their realized schedule. Models 1, 3, and 5 are for deterministic schedule, schedule with delay probability, schedule with delay probability when fairness is computed by a range model, respectively.

Z refers to the type of the operation durations in Model X-Y-Z notation: either predicted (P) or actual (A). However, we compute assignment and schedule with respect to P operation durations and the objective function of any model with notation Model X-Y-Z is using the sequence computed by using the predicted operation durations and inserting A operation durations given this computed sequence. For example, Model-1-O-P means that first model with overtime objective and predicted operation duration, whereas, Model-1-O-A means that first model with overtime objective is solved by predicted operation durations and this computed sequence is used to compute the objective function value given actual operation durations.

The aim of Model 1 is to minimize the total penalty that is computed by the waiting time of patients and unfairness between ORs (i.e., overutilized ORs) and either overtime, or makespan, or tardiness or makespan with each operation finished before its deadline. For example, Model 1-O aims to minimize the penalty waiting time and unfairness and either overtime (O), or makespan (M), or tardiness (T) or makespan where each operation finishes before its deadline (MD).

Model 1 ignores uncertainty prevalent in operations. To consider such uncertainty, two heuristics have been developed: (i) considering delay probabilities through linear summations, (ii) considering starting time weighed probabilities through linear summations. Heuristic (i) is formulated by Models 3 and 5, whereas, heuristic (ii) is formulated by Model-3' and 5'. Model-3 minimizes waiting time of patient, unfairness and delay probability and either overtime, or makespan, or tardiness, or makespan with each operation finished before its deadline. For example, Model-3-T aims to minimize tardiness, waiting time, unfairness and delay probability. In Model-3', weighing corresponding operation's delay probabilities is considered in the objective function. For example, Model-3'-MD minimizes makespan with deadline, waiting time, fairness and starting time weighed delay probability. Moreover, Model-5 aims to minimize

Model	Objective Function
1-O	Overtime + Waiting Time + Fairness
1-M	Makespan + Waiting Time + Fairness
1-T	Tardiness + Waiting Time + Fairness
1-MD	Makespan with Deadline + Waiting Time + Fairness
3-O	Overtime + Waiting Time + Fairness + Delay Probability
3-M	Makespan + Waiting Time + Fairness + Delay Probability
3-T	Tardiness + Waiting Time + Fairness + Delay Probability
3-MD	Makespan with Deadline + Waiting Time + Fairness + Delay Probability
3'-O	Overtime + Waiting Time + Fairness + Starting Time Weighed Delay Probability
3'-M	Makespan + Waiting Time + Fairness + Starting Time Weighed Delay Probability
3'-T	Tardiness + Waiting Time + Fairness + Starting Time Weighed Delay Probability
3'-MD	Makespan with Deadline + Waiting Time + Fairness + Starting Time Weighed Delay Probability
5-O	Overtime + Waiting Time + Range + Delay Probability
5-M	Makespan + Waiting Time + Range + Delay Probability
5-T	Tardiness + Waiting Time + Range + Delay Probability
5-MD	Makespan with Deadline + Waiting Time + Range + Delay Probability
5'-O	Overtime + Waiting Time + Range + Starting Time Weighed Delay Probability
5'-M	Makespan + Waiting Time + Range + Starting Time Weighed Delay Probability
5'-T	Tardiness + Waiting Time + Range + Starting Time Weighed Delay Probability
5'-MD	Makespan with Deadline + Waiting Time + Range + Starting Time Weighed Delay Probability

Table 32: Model Combinations.

waiting time, range and delay probability with either overtime, or makespan, or tardiness, or makespan with deadline. Difference between Model-3 and Model-5 is fairness calculation. In Model-5, a range model is adapted whereas, in Model-3 unfairness of each OR is summed. Similar to the relation between Models 3 and 3', Model-5 and 5' resemble each other except delay probabilities of operations are weighed by their starting times in Model-5"s objective function. Models are explained with details in the following sections. Note that the weights used in the objective functions are provided by Hospital X.

4.2.1 Makespan

Makespan is the total length of the operation schedule, in other words, it equals to the maximum of each operation's finishing time. Since MILP is utilized, Constraint (8) is added to Constraints (4) - (7) to linearize the maximum. In addition, we update our objective function as min M where M represents makespan. Constraint (8) is to define M as being greater than or equal to finishing time of each operation j.

$$M \geq \sum_{it} (t + p_j) x_{ijt} \texttt{ for all j} \tag{8}$$

In makespan with deadlines models, Constraint (9) is added to Constraint (4) - (8) to prevent an operation to finish later its deadline.

$$x_{ijt} = 0 \texttt{ for all j,t with } t > d_j - p_j \tag{9}$$

In addition, each OR's makespan can be computed by Constraint (10).

$$M_i \geq \sum_t (t + p_j) x_{ijt} \texttt{ for all i, j} \tag{10}$$

4.2.2 Overtime

To compute overtime, $\sum_i W_{OR} K_i$ is minimized in addition to satisfying Constraint with (4) - (7), (10) - (12). All constraints mentioned in Section 2.1 are also required for this model.

$$K_i \geq M_i - ed \texttt{ for all i} \tag{11}$$

$$K_i \geq 0 \tag{12}$$

4.2.3 Tardiness

Tardiness is the nonnegative difference between finishing time of an operation and its deadline. Constraints (13) and (14) formulate tardiness and they are added to Constraints (4) - (7) in addition to the objective function that also aims to minimize sum of T_js.

$$T_j \geq \sum_{it} (t + p_j) x_{ijt} - d_j \texttt{ for all j} \tag{13}$$

$$T_j \geq 0 \texttt{ for all j} \tag{14}$$

The hospital administration may consider an upper bound (UB) on each operation's tardiness such that based on each operation's duration they may tolerate duration length times the UB as tardiness.

$$T_j \leq p_j \text{ b } \texttt{ for all i,j} \tag{15}$$

Such tolerance to tardiness is formulated by Constraint (15) where b represents UB on the percentage.

4.2.4 Waiting Time

The operation waiting time problem minimizes $\sum_j W_W W_j$ subject to Constraints (4) - (7), where W_W is the weight of waiting time of operation j, W_j. To define the waiting time of operation j, Constraint (16) is introduced as the difference between the starting time and the release time of operation j.

$$
\begin{aligned}
W_j \geq \textstyle\sum_{it}(\text{t-}\ r_j)x_{ijt} \quad \texttt{for all j} \\
W_j \geq 0 \quad \texttt{for all j}
\end{aligned}
\tag{16}
$$

$W_j \geq 0$ can be omitted as Constraint (6) prevents operations to start before their release times, hence, waiting time is always nonnegative due to Constraint (6).

4.2.5 Fairness (Overutilization of ORs)

Hospital X defines fairness as balanced utilization of ORs and they aim to achieve this by assigning equal operation durations (if possible) to each OR. Fairness is very vital for Hospital X. Since they have already allocated resources to these ORs (such as, budget spent for constructing ORs or for the equipments), they focus on equivalent utilization.

To compute fairness, average operation durations are computed daily and deviations from this average are calculated for each OR. Constraints (17) and (18) define the unfairness and nonnegativity of unfairness. If an OR is overutilized, the total operation duration assigned to this OR is more than the corresponding daily average duration, hence, another OR should be underutilized. To avoid double penalizing unfairness, only overutilization is considered in the objective function and this is satisfied by Constraints (17) and (18).

$$z_i \geq (\sum_{jt}(p_j x_{ijt}) - \overline{P}) \texttt{ for all i} \quad (17)$$

$$z_i \geq 0 \texttt{ for all i} \quad (18)$$

where

$$\overline{P} = \frac{\sum_j p_j}{\texttt{number of OR}}$$

4.2.6 Range Model

Fairness can be modeled by minimizing the maximum range of operation durations' sum assigned to each OR. Then, the objective function becomes minimizing $W_O(qmax - qmin)$ and subject to Constraints (4) - (7) and (19) - (21). Constraint (19) defines the maximum total operation duration assigned to an OR, Constraint (20) provides the minimum total operation duration assigned to an OR except unutilized OR. Constraint (21) defines which OR is used with the help of yy_i variable which is a binary variable equal to 1 if OR i is utilized.

$$qmax \geq \sum_{jt}(p_i x_{ijt}) \texttt{ for all i,} \quad (19)$$

$$qmin \leq \sum_{jt}(p_i x_{ijt}) + \mu(1 - yy_i) \texttt{ for all i,} \quad (20)$$

$$x_{ijt} \leq \mu yy_i \texttt{ for all i,j and t} \quad (21)$$

4.2.7 Delay Probability

Operations' delay probabilities play an important role in scheduling as Hospital X Administration prefers assigning two or more operations with high probability of delay to distinct ORs. Hence, a major possible problem they try to avoid is to prevent delays caused with these two or more operations with high probability of delay. Hence, we aim to incorporate delay probabilities into the mathematical models by heuristics since scheduling integrated with delay probabilities is a very hard problem. For instance, assume that there are two consecutive operations and their delay probabilities are 0.6 and 0.5. If there is a gap between these two operations (i.e., positive difference between starting time of a successor operation and finishing time of its predecessor operation), delay probability of the predecessor operation may not result in delaying the successor operation since the gap between these two operations may be longer than the delay of the predecessor operation (i.e., preventing the successor operation's delay). However, each operation's delay probability should still be considered in the model as there may be no gaps between consecutive operations. Gaps are computed during the scheduling problem's optimization; one can either solve the deterministic scheduling problem ignoring the delay probabilities and integrate probabilities as a second step (or vice versa) or solve a deterministic problem where the delay probabilities have already been integrated. The second alternative, deterministic problem with integrated delay probabilities, is formulated via two proposed heuristics: (i) considering delay probabilities through linear summations and (ii) starting time weighed delay probabilities through linear summations.

$$prp_i \geq (\sum_{jt}(pr_j x_{ijt}) - \overline{PP}) \texttt{ for all i} \tag{22}$$

$$prp_i \geq 0 \tag{23}$$

where

$\overline{PP} = \frac{\sum_j pr_j}{\texttt{number of OR}}$

In heuristic (i), average delay probabilities are computed daily and deviations from this average is determined for each OR. Constraints (22) and (23) define deviation from the average delay probability and nonnegativity of the deviation, , respectively, whereas, in heuristic (ii) Constraints (24) and (25) provide that delay operations are scheduled as early as possible and nonnegativity, respectively.

$$prp_i \geq (\sum_{jt}(pr_j x_{ijt} t)) \texttt{ for all i} \tag{24}$$

$$prp_i \geq 0 \tag{25}$$

4.3 Sorting Heuristic

A simple heuristic is developed for Hospital X Administration to compute schedule without optimization. This simple heuristic is referred as *sorting heuristic* since it depends on sorting. Steps of sorting heuristic are as follows.

- Sort predicted starting times in ascending order.
- Sort predicted operation durations in descending order.
- Sort probability of delay in descending order.
- Start assigning operations from OR 1 to OR 7 and then from OR 7 to OR 1.

First step provides the earliest released operations to be scheduled first. Second step satisfies from the earliest released operations, operations with the shortest operation duration are given higher priority. The next step considers delay probabilities of operations and aims to schedule the ones with high delay probability before the ones with lower delay probability.

4.4 Computational Results

We aim to solve Hospital X's daily assignment and scheduling problem. Computing weekly schedules is not studied since time windows of operations are not wide enough to solve a weekly schedule and the computation time of weekly schedule is at least an order of magnitude slower than the daily schedule. Hence, Hospital X Administration is also interested in improving daily schedules first. Solving weekly or monthly schedules may be a future work.

Hospital X has 7 ORs with the same equipments, 141 beds and 54 intensive care units. Data represent the operations performed between June, 2013 and December, 2014. There are 7754 operations belonging to 9 different departments. 341 days out of 594 are chosen where the number of operations varies between 10 and 17. There are few data of the days with the number of operations less than 10 or more than 17 and we analyze them separately. Moreover, 180 days out of 341 days Hospital X's objective function values can be calculated. There are data errors at the remaining days, such as, double booking on the same time period. Hence, we clean such data entries and solve our models and compute Hospital X's current objective function values. We compare to our models and Hospital X's current objective function values by calculation average deviation.

Hospital X administration share the information that the weight of patient waiting time is 5 times the weight of tardiness, overtime, unfairness, and unfairness computed via range model. Moreover, assumptions made while solving the instances are summarized in Section 4.1 and models are introduced in Section 4.2, respectively. We also show the sorting heuristic in Section 4.3.

IBM ILOG CPLEX Optimization Studio 12.6 is used to run the mathematical models on a laptop with Inter core i5 processor 2.5 Ghz, 4 GB RAM and Windows 7 Professional operating system. Our computational experiment goals and their corresponding goals at Hospital X are summarized in Table 30. In Section 4.4.1, we summarize the results of Model-1-O, M, T, and MD and Model-2-O, M, T, and MD (Hospital X's objective functions). Then, Model-3 and 3'-O, M, T, and MD are mathematical models considering stochasticity by heuristics. Model-4 and 4'-O, M, T, and MD are Hospital X's objective functions that are compared between Model-3 and 3'. Model-5, 5',7 and 7' schedule computed via using mathematical models under some different calculation such as different fairness calculation and both of them are compared between Hospital X's objective functions. Then we create different scenarios to analyze performance of our mathematical models in real life. Next, in Section 4.4.2 , the results of the sorting heuristics are mentioned. Finally, Section 4.5 mentions about conclusion of results briefly.

4.4.1 MILP

We examine whether Hospital X's current schedule performs well given predicted operation durations. In Model-1, we optimize four different models ignoring uncertainty. First model is Model-1-O-P that aims to minimize the total penalty caused by overtime, waiting time and unfairness. Next model is Model-1-M-P that minimizes the total penalty of makespan, waiting time and unfairness, whereas, the third model is Model-1-T-P minimizing the total penalty caused by tardiness, waiting time and unfairness. The last model is Model-1-MD-P that is the same model as Model-1-M-P except an additional constraint: each operation has to be performed between its time windows (i.e., each operation has to finish before its deadline). Given these models we summarize main findings for 180 days as follows.

- As the number of operations increases each day,
  - objective function values do not change immensely for Model-1-M-P and Model-1-MD-P,
  - unfairness decreases since there are operations to be scheduled at each OR,
  - makespan increases.

  Since unfairness has the highest penalty value, objective function values stay nearly unchanged due to decreasing unfairness and increasing makespan.

- Objective function values of Model-1-O-P and Model-1-T-P decrease with increasing number of operations each day since unfairness decreases. Overtime and tardiness values are almost zero for most of the days, hence, unfairness directs our objective function values.

- Waiting times and tardy jobs are not the main problems of Hospital X's schedule considering the predicted operation durations since both are zero in most of the instances.

As mentioned before, we calculate Hospital X's objective function value when actual operation durations are used with the computed sequences using our optimized mathematical models. Since actual durations are not the same as the predicted duration, the objective function values deviate. We refer Hospital X's objective function values as Actual solution, whereas, Predicted solution is used to refer the optimized objective function value. Then, *deviation* is formulated as follows.

$$\left(\frac{\texttt{Actual Solution-Predicted Solution}}{\texttt{Predicted Solution}}\right) * 100 \tag{26}$$

If deviation percentage is positive and large, Actual solution's objective function value is larger than the predicted solution's. Hence, mathematical models underestimate the actual conditions.

- Model-1-O-P may perform bad under actual operation durations. For example, Model-1-O-A's objective function values are 9.21 % more than Model-1-O-P's and Model-1-T-A's objective function values are 315.77 % more than Model-1-T-P's. For these two models, reason of such bad performance of actual model is caused due to fairness and/or waiting. In most cases, operation durations are overestimated and satisfying fairness between ORs becomes a major problem. If an operation duration is underestimated, Actual solution may realize waiting times. As mentioned before, weight of waiting time is 5 and the rest of the weights is 1. Moreover, Model-1-M-P and Model-1-MD-P's objective function values are 1.95 % more than Model-1-M-A and Model-1-MD-A's. Although unfairness has lower values in predicted models, makespan is higher because of overestimated predicted operation durations.

We calculate Hospital X's objective function values. Model-2 models the same problem as Model-1, however, we use Hospital X's current operation sequence in Model-2. To compare these two models, we again define deviation similarly to the one formulated in (26); instead of using Actual solution (26), we use Hospital X's current operation sequences and calculate its objective function value given predicted operation durations.

- Figure 13 represents the box plot of deviation percentages of Model-2-O-P and Model-1-O-P. In the y-axis, deviation percentages are represented and in the x-axis number of operations are represented. Model-1-O-P performs better than Model-2-O-P since Hospital X's current sequence has fairness problems. For example, an outlier day is the 232^{th} day: there are 11 operations scheduled at 3 ORs. Moreover, operations are scheduled consecutively. Hospital X can improve its objective function value if more ORs are utilized instead of just utilizing 3 ORs. Note that also the smallest maximum deviation percentage is more than 500 % as represented in Figure 13.

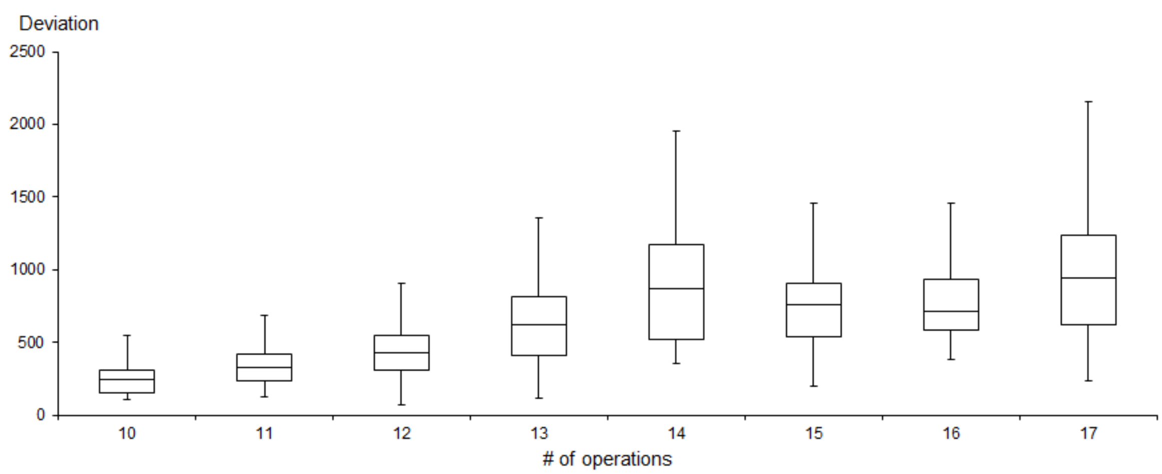

Figure 13: Deviation of Model-2-O-P and Model-1-O-P.

- Figure 14 shows deviation of Model-2-M-P and Model-1-M-P and Model-1-M-P performs better than Model-2-M-P. In addition, deviation percentages of

Model-1-M-P and Model-2-M-P are smaller than the deviation percentages of Model-1-O-P and Model-2-O-P depicted in Figure 13 due to very long overtime.

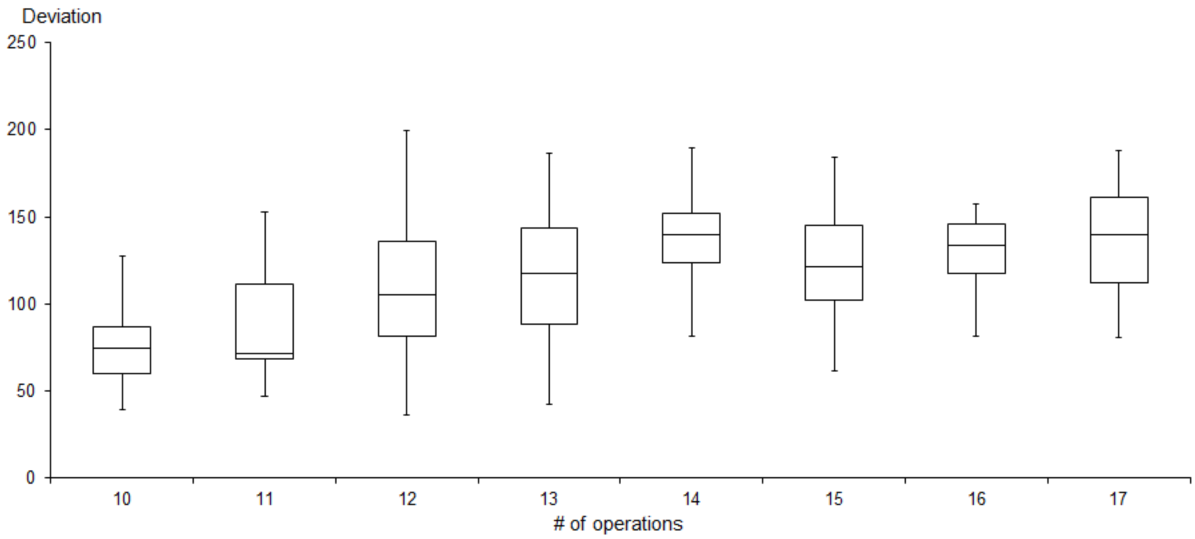

Figure 14: Deviation of Model-2-M-P and Model-1-M-P.

- Figure 15 shows deviation of Model-2-T-P and Model-1-T-P. Model-1-T-P performs worse than than Model-2-T-P. When the number of operations increases, median of deviation percentage shows an increasing pattern which shows that we can improve Hospital X's current schedule more by optimization when the number of operations is large.

- When deviation of Model-2-MD-P and Model-1-MD-P is computed, the boxplot is the same as depicted in Figure 14 since very job finished before its deadline, Model-MD and Model-M result in the same objective function values.

We also calculate the objective function values of Model-2-O-A, Model-2-M-A, Model-2-T-A and Model-2-MD-A (actual operation durations are used with Hospital X's current schedule).

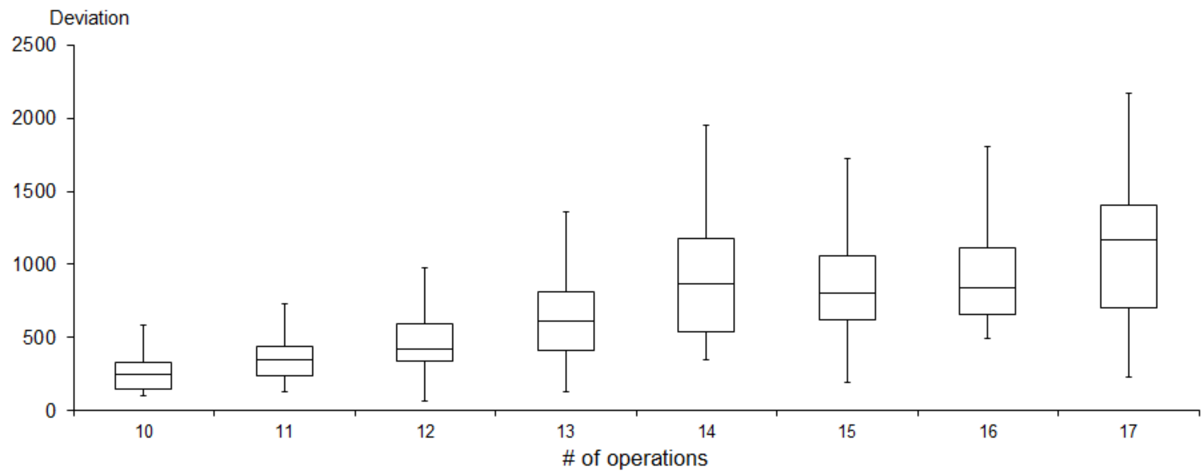

Figure 15: Deviation of Model-2-T-P and Model-1-T-P.

- Except Model-2-T, Model-2-Y-P's objective function values (using predicted operation durations for objective function Y) are larger than Model-2-Y-A. For instance, deviation percentage of Model-2-T-P and Model-2-T-A is 86.98 % for 232nd day. In other words, objective function value of Model-2-T-A is 86.98 % larger than Model-2-T-P's objective function value. Even if 32 % of the operation durations are overestimated for this day, remaining operation durations are underestimated. Hence, using the actual operation durations for Hospital X's current schedule increases the objective function value because of waiting time.

Computation time of Model-1 increases as the number of operations increases. Figure 16 illustrates Model-1-O-P's computation time. Remaining Model-1's also have similar patterns as shown in Appendix E.

In deterministic models, uncertainty is ignored as mentioned before. However, uncertainty is prevalent in Hospital X's data. To incorporate uncertainty, the proposed mathematical models are updated by delay probabilities. In Model 3, as mentioned in Table 32, delay probabilities are incorporated into mathematical model by penalizing deviation from the average delay probabilities computed by linear summation. Delay

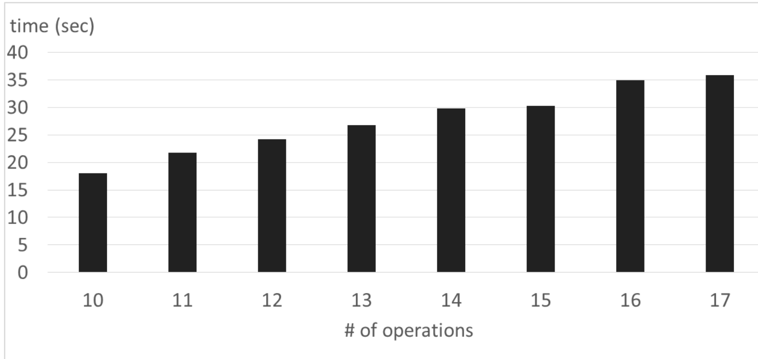

Figure 16: Computation time of Model-1-O-P.

probabilities can be very small numbers (such as, with two decimal digits), where as the other objectives in the objective function may have two or even three digits. Hence, we use 100 to weight the penalty of delay probability in the objective function.

We analyze Model-3-O-P that optimizes overtime, waiting time, fairness and delay probability. Then, the next model is Model-3-M-P that optimizes makespan, waiting time, fairness and delay probability. The third model is Model-3-T-P that optimizes tardiness, waiting time, fairness and delay. The last model is Model-3-MD-P determines makespan with deadline constraint, waiting time, fairness and delay probability. The objective function values of Model-3 mimics Model-1's objective function values because waiting time and unfairness penalization are the dominant values.

- When the number of operations increases, objective function values do not change for Model-3-M-P and Model-3-MD-P, similar to Model-1-M-P and Model-1-MD-P.

- Objective function values of Model-3-O-P and Model-3-T-P decrease by increasing number of operations because of increasing fairness and decreasing delay probability deviations. Overtime and tardiness values are generally zero while predicted operation durations are used; fairness and delay probabilities dominate the objective function values.

- Model 3-M-P's makespan increases as the number of operations increases, whereas, delay probability deviations and unfairness decrease.

- We again calculate deviations as in (26) similar to the once used for Model-1. For example, on average, Model-3-O-A's objective function values are 1.26 % more than Model-3-O-P's and Model-3-T-A's objective function values are 250.32 % more than Model-3-T-P's. Objective function values of Model-3-M-P is 8.32 % more than Model-3-M-A's on average. Finally, Model-3-MD-P's objective function values is 1.85 % more than Model-3-M-A on average. Such deviations show that except tardiness model, Model-3-Y-A and Model-3-Y-P objective function values are very close to each other. This result shows that our models can also provide good schedules under actual durations.

- Overtime and tardiness models' objective function values when predicted operation durations are used perform better than the case when actual operation durations are used. Note that delay probabilities are same for the cases of predicted operation durations and actual operation durations since the sequence is the same.

We also calculate Hospital X's objective function values. Model-4 has the same constraints as Model-3 with only one exception: we use Hospital X's current OR assignments in Model-4. In order to analyze these two models, we again calculate deviation as formulated in (26) with a minor change where Hospital X's objective function values are used instead of Actual solution (similar to the deviations defined for Model-1).

- Figure 17 represents deviation of Model-4-O-P and Model-3-O-P. Model-3-O-P performs better than Model-4-O-P as Hospital X has unfairness and delay probability problems. For example, one of the outlier days is 141^{st} day with 16 operations and 4 ORs. There are two operations that have long predicted operation durations (240 and 270 minutes). These operations are scheduled in the same OR and 2 ORs are empty. Hence, unfairness is realized. Moreover, pediatric and gynecology operations are scheduled at the same OR. Since these operations have the highest delay probabilities; deviation of delay probabilities is also high. Other outliers have similar patterns.

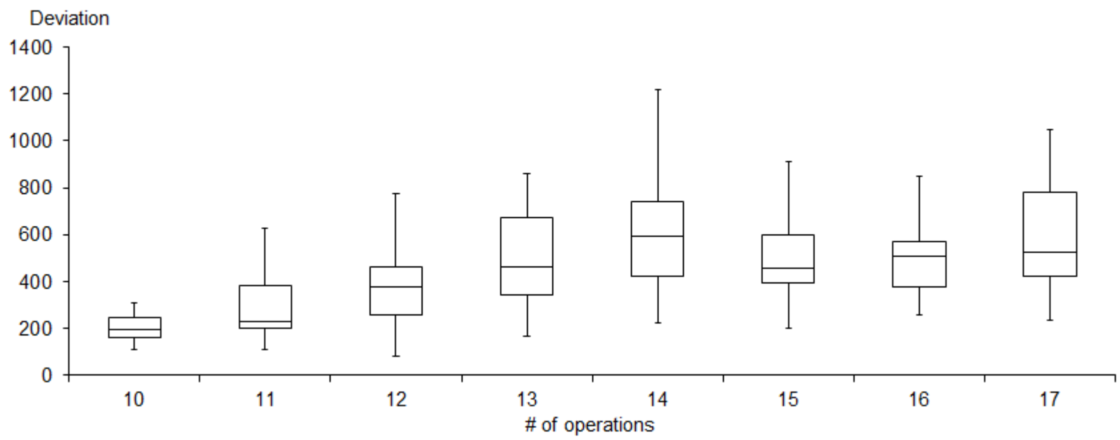

Figure 17: Deviation of Model-4-O-P and Model-3-O-P.

- Figure 18 shows deviation of Model-4-M-P and Model-3-M-P where Model-3-M-P performs better than Model-4-M-P due to decreasing unfairness and delay probability deviations.

- Figure 19 depicts deviation of Model-4-T-P and Model-3-T-P. Model-3-T-P performs better than Model-4-T-P. As the number of operations increases up to 15 operations, median of deviations keeps increasing as depicted in Figure 19. For days with more operations, the deviations do not vary much since fairness and delay probability deviations stay stable.

- Figure 19 shows deviation of Model-4-MD-P and Model-3-MD-P. Model-3-MD-P performs better than Model-4-MD-P because Model-4-MD-P's unfairness and delay probability deviations are high.

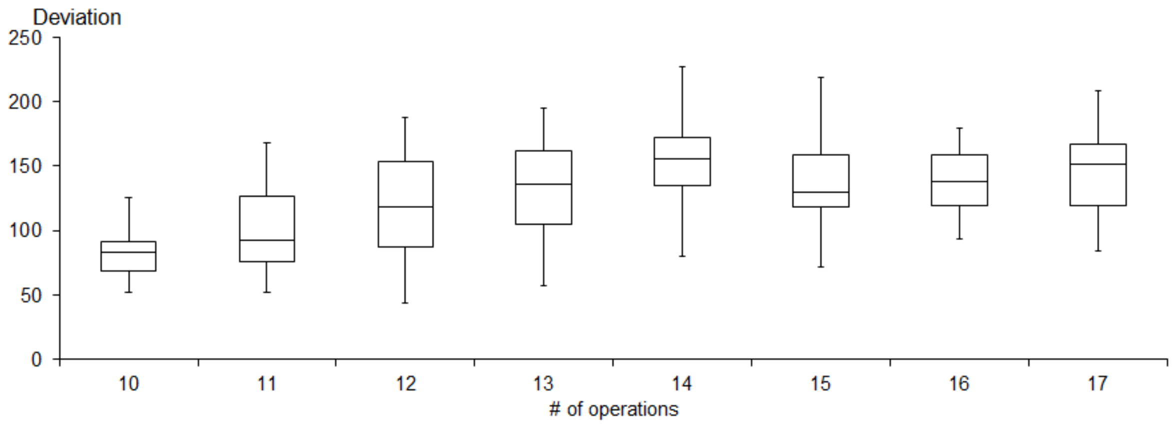

Figure 18: Deviation of Model-4-M-P and Model-3-M-P.

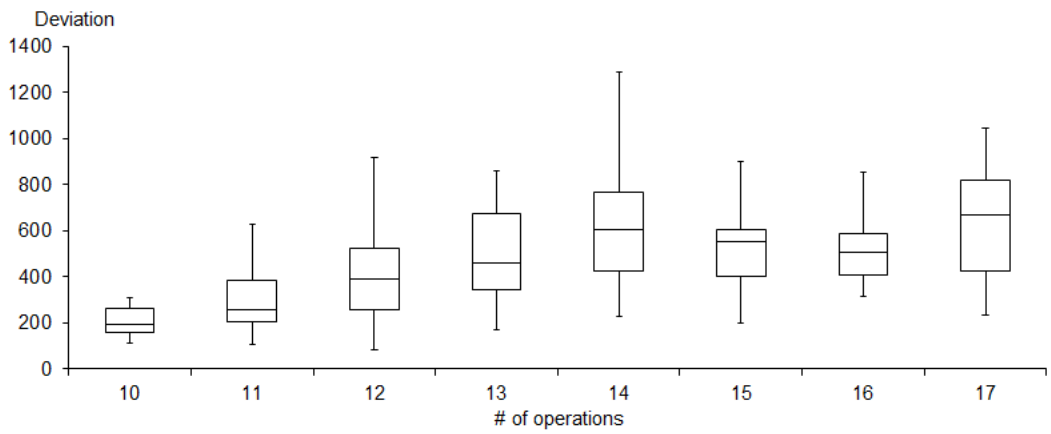

Figure 19: Deviation of Model-4-T-P and Model-3-T-P.

We also calculate Model-4-O-A, Model-4-M-A, Model-4-T-A and Model-4-MD-A when Hospital X's actual schedule and actual operation durations are used.

- Except the case minimizing tardiness , predicted models perform better than actual models. Objective function values of Model-4-T-A are 86.98 % larger than Model-4-T-P's. Model-4-O-P's objective functin values are 15.37 % less than Model-4-O-A's. Model-4-M-P and Model-4-MD-P's objective function values are 12.83 % less than Model-4-M-A's and Model-4-MD-A's as fairness and waiting time change. Note that delay probability deviations are the same since the same sequences are used.

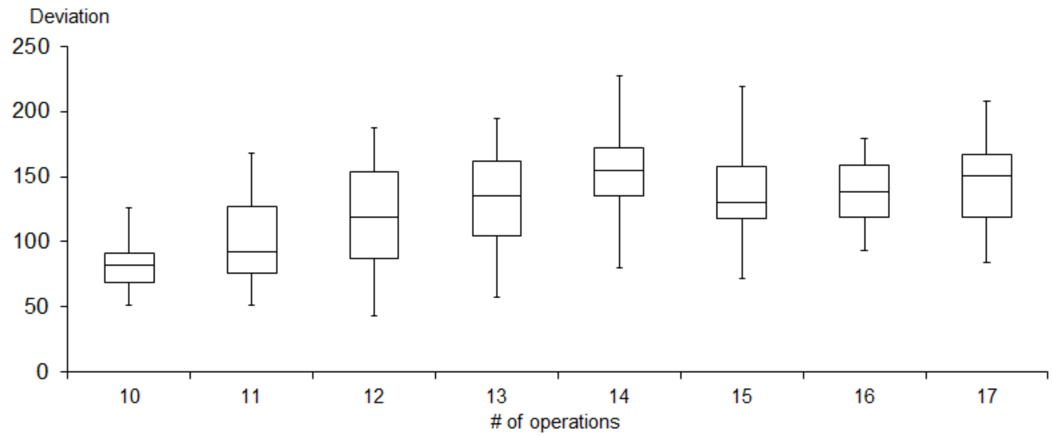

Figure 20: Deviation of Model-4-MD-P and Model-3-MD-P.

- When the number of operations increases, computation time increases as depicted in Figure 21. Other models' computation times have same patterns as shown in Appendix E.

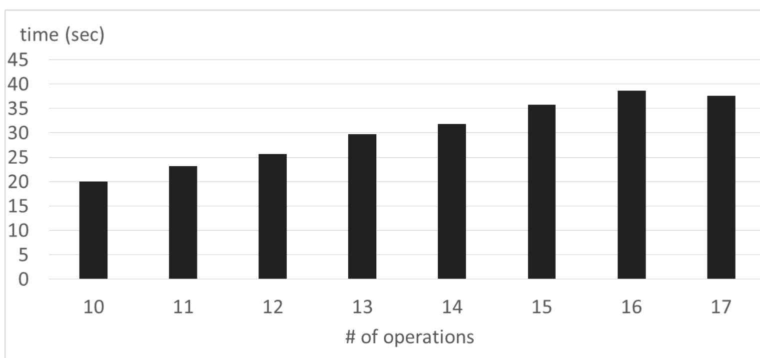

Figure 21: Computation time of Model-3-O-P.

All models are analyzed assuming that all ORs are utilized. However, Hospital X seems to leave an OR idle, most likely for emergency cases. We decrease the number of ORs to 6 and show the resulting statistics in Table 33. If 6 ORs are utilized instead of 7 ORs, scheduling objective function values may drop due to decreasing unfairness.

In Model-3', delay probabilities are incorporated to the deterministic models by using starting times as weights. In other words, since objective function is minimized,

	Total # of operations							
	10	**11**	**12**	**13**	**14**	**15**	**16**	**17**
Median	-30.85	-43.90	-29.30	-21.50	-22.08	-17.04	-28.65	-35.66

Table 33: Median deviation of Model-3-O-P when # of ORs decreases to 6 from 7.

operations with high delay probabilities will be scheduled as early as possible. Model-3'-O-P aims to minimize overtime, waiting time, unfairness and starting time weighed delay probabilities, whereas, Model-3'-M-P minimizes makespan, waiting time, fairness and starting time weighed delay probabilities. The third model is Model-3'-T-P minimizing tardiness, waiting time, fairness and starting time weighed delay probabilities. Finally, the last model is Model-3'-MD-P minimizing makespan with deadline constraint, waiting time, fairness and starting time weighed delay probabilities. The objective function values' pattern of Model-3' is different from Model-3 and general findings are as follows.

- In all Model-3' models, starting time weighed delay probabilities dominate the objective function values. In addition, as the number of operations increases, objective function values also increase due to increasing makespan (if it is Model-3'-M) and starting time weighed delay probabilities.

- To compare objective function values of the cases with predicted operation durations and actual operation durations, deviation formulated in (26) is used for each day. Then, average deviations are computed. Results show that deviations are very small on the average. For example, Model-3'-O-A's objective function values are 0.18 % more than Model-3'-O-P's and Model-3'-T-A's objective function values are 11.73 % more than Model-3'-T-P's. In addition, Model-3'-MD-A's objective function values are 0.03 % more than Model-3'-M-P. Finally, objective function values of Model-3'-M-P is 0.09 % more than Model-3'-M-A's.

- Starting time weighed delay probabilities do not vary the objective function values a lot because sequences of operation are the same for the models with actual operation durations and with predicted operation durations. We also calculate Hospital X's objective function values and Model-4' has the same constraints as in Model-3' and we use actual assignment ORs in Model-4'. In order to analyze two models, we again calculate deviation (Formula 26).

- Appendix F shows deviation of Model-4' and Model-3'. In both models, Model-3' performs better than Model-4' and their deviation values are nearly the same due to minimized unfairness and starting time weighed delay probabilities.

We also compute Model-4'-O-A, Model-4'-M-A, Model-4'-T-A and Model-4'-MD-A with Hospital X's current schedule and by utilizing actual operation durations.

- Except the case of tardiness, models with predicted operation durations perform better models with actual operation durations. Objective function values of Model-4'-T-A are 7.28 % larger than Model-4'-T-P's. Model-4'-O-P's objective function values are 1.86 % smaller than Model-4'-O-A's. Objective function values of Model-4'-M-P and Model-4'-MD-P are 2.06 % smaller than the cases with actual operation durations.

- When the number of operations increases, computation times generally increase. Figure 22 illustrates Model-3'-O-P's computation time. Remaining models' computation times are depicted in Appendix E.

In Model-5, calculation of fairness is done by a range model by which the difference between the minimum and the maximum fairness values is minimized. Furthermore, in Model-5', starting time weighted delay probabilities are used in combination with range model to minimize unfairness. However, range model makes the problem harder and its computation times take hours to reach near optimal solution. Hence, we generate heuristic models: Model-7 and Model-7'.

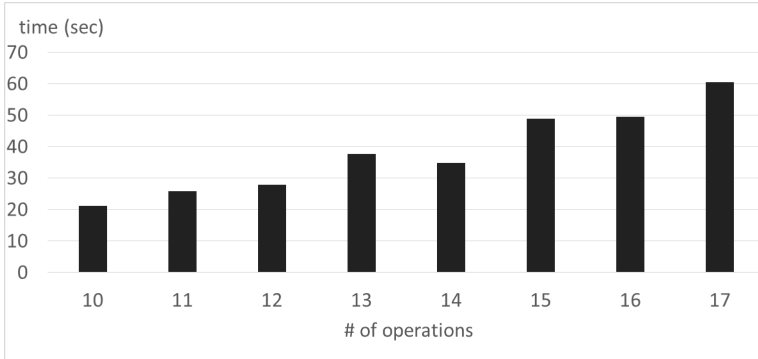

Figure 22: Computation times of Model-3'-O-P.

Sequences of Model-3's solution are used as given at Model-7, and then the range model approach is used. Similar to Model-7, we compute optimal sequences of Model-3' and use them at Model-7'. Then, we again use range model approach for fairness. However, this heuristic does not perform well generally. In other words, solutions of Model-3 and Model-3' are not good approximations as deviations are very large between the Model-5 and Model-7 (similarly, Model-5' and Model-7') for the cases which we can optimize the range model approach. Lastly, computation times of Model-7 are much shorter than Model-5.

Our next aim is to find how the deterministic models perform under uncertainty. To determine the performance of deterministic models (including the ones with delay probabilities) we generate random instances based on the real data of Hospital X. Each random instance is a scenario that consists of randomly generated operation durations based on two different distributions: lognormal and exponential distributions. Based on the mean and standard deviations of predicted operation durations computed from the real data, we generate 50, 250, 500, 2000, and 5000 scenarios. k refers to the number of scenarios generated by Matlab in Table 34. Sequences of operations computed via Model-3-O-P are used and given these sequences for each day, generated

	Total # of operations							
k	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
50	142	271	309	440	612	926	889	1290
250	149	263	310	427	610	930	872	1348
500	143	271	313	437	607	922	873	1356
2000	93	199	262	372	441	751	768	1026
5000	92	107	261	369	437	722	767	1019

Table 34: Lognormal distribution's median deviation for varying k, # of scenarios.

scenarios objective functions are computed. Then, deviation is calculated as follows.

$$(\frac{\texttt{Random Solution-Deterministic Solution}}{\texttt{Deterministic Solution}}) * 100 \qquad (27)$$

where Deterministic Solution refers to the objective function value optimized by using Model-3-O-P.

- Figures 23- 27 represent deviations when operation durations have lognormal distribution for k=50 - 5000, respectively. As k increases, deviation is expected to decrease, however, even if the deviations converge, there is not any obvious decreasing pattern represented in Figures 23- 27. Median deviation values over all scenarios are summarized in Table 34.

- We also calculate exponential cases as seen in Figure 28, Figure 29 and Figure 30. Deviation shows that lognormal distribution gives better solution than exponential distribution and we stop calculation of exponential cases not to find converge values (Table 35).

We repeat the same random scenario analysis when operation durations are distributed exponentially and Table 35 represents the median deviations with respect to varying number of scenarios.

- For both lognormal and exponential distributions, when the number of operations increases each day, deviation median also increases.

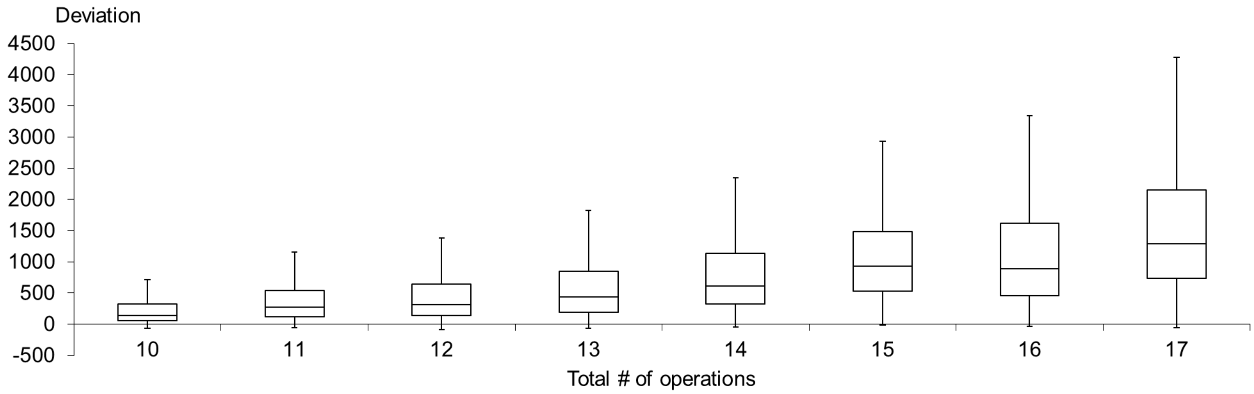

Figure 23: Deviation under k =50 random scenarios with lognormally distributed operation durations.

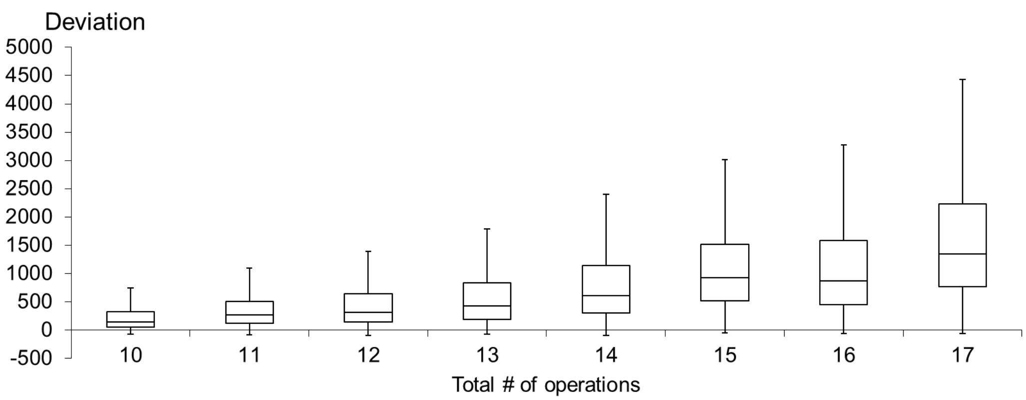

Figure 24: Deviation under k =250 random scenarios with lognormally distributed operation durations.

- Random operation durations perform worse than deterministic operation durations because of waiting time and unfairness. Scenarios include overestimated durations dominating these huge deviations since patients start to wait at several scenarios. Delay probabilities do not change as same sequences are used. Unfortunately, we can conclude that Hospital Administration should promote its surgeons to estimate their operation durations well, hence, randomly generated operation durations may not cause such huge deviations.

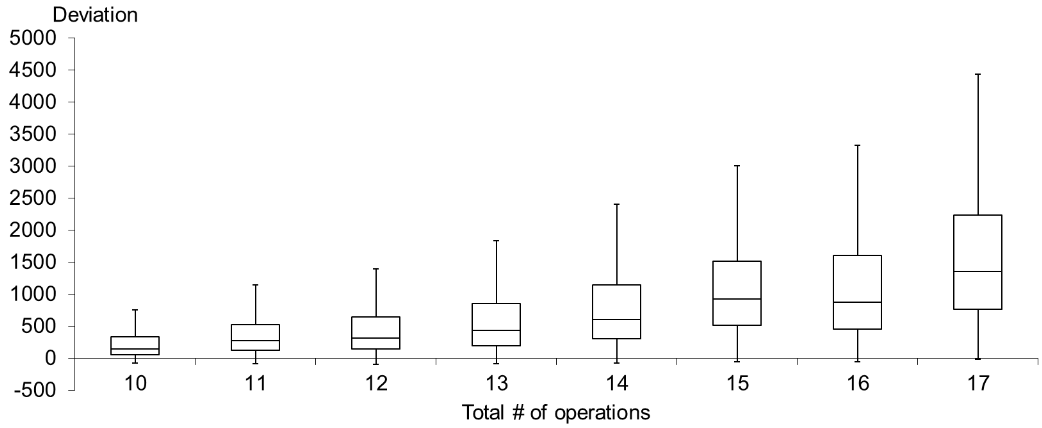

Figure 25: Deviation under k =500 random scenarios with lognormally distributed operation durations.

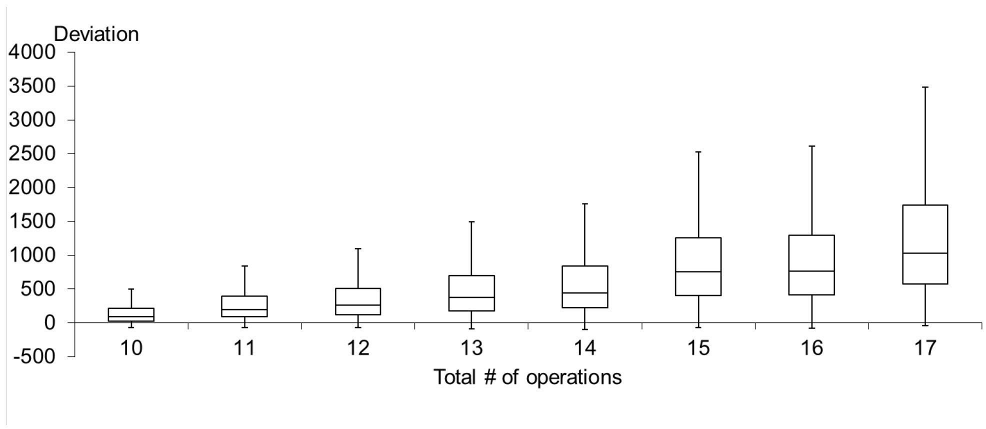

Figure 26: Deviation under k =2000 random scenarios with lognormally distributed operation durations.

4.4.2 Sorting Heuristic

Hospital Administration may not prefer buying an optimization tool. Therefore, all solutions proposed by optimizing mathematical models may not be realistic. To prevent manual scheduling, we aim to improve current schedule by a simple heuristic based on sorting. We find that our simple heuristic, sorting heuristic, performs better than Hospital X's and our random cases' objective function values. For example, we analyze 10 different examples for sorting heuristic as seen in Table 31. First

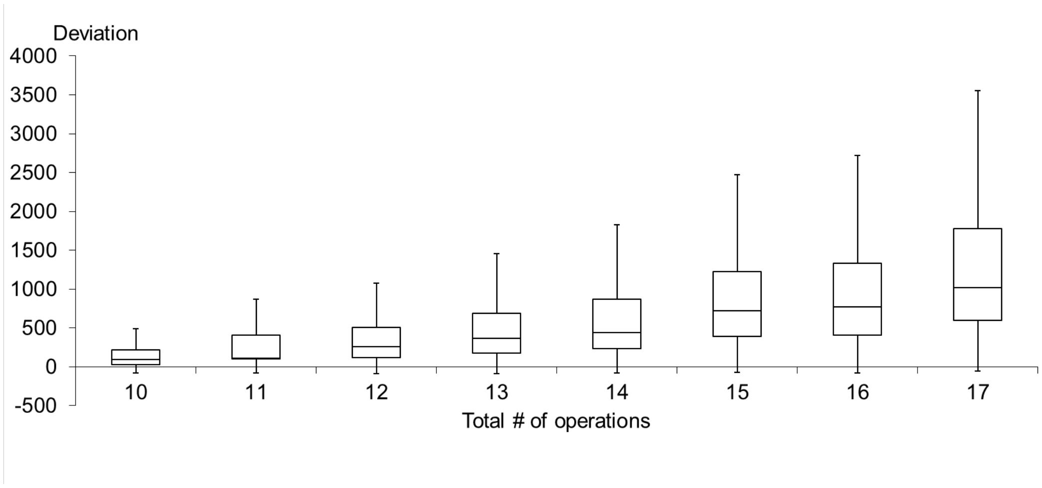

Figure 27: Deviation under k =5000 random scenarios with lognormally distributed operation durations.

	Total # of operations							
k	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
50	280	499	559	853	1142	1609	1618	2166
250	272	455	578	857	1116	1590	1608	2235
500	280	453	575	848	1126	1579	1676	2234

Table 35: Exponential distribution's median deviation for varying k, # of scenarios.

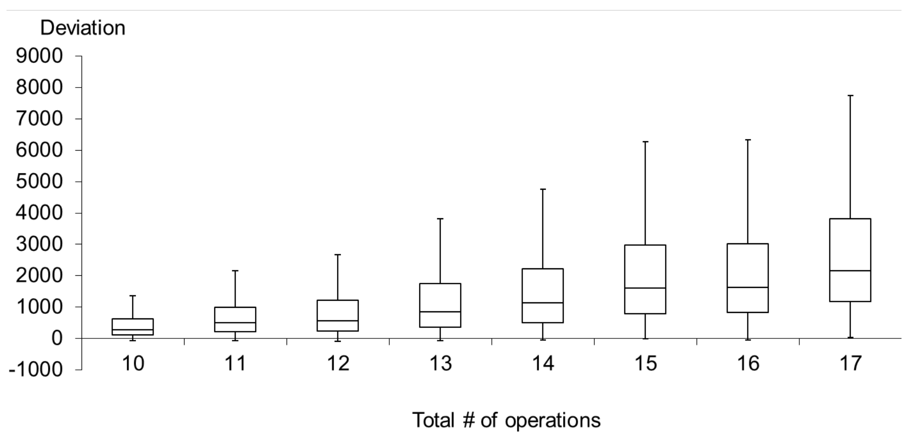

Figure 28: Deviation under k =50 random scenarios with exponentially distributed operation durations.

example shows that Hospital X's objective function value (Model-4-O-P) is 372.14, mathematical model's optimal objective function value (Model-3-O-P) is 225.98 and sorting heuristic's solution is 261.25. Example 1 shows that simple heuristic approach

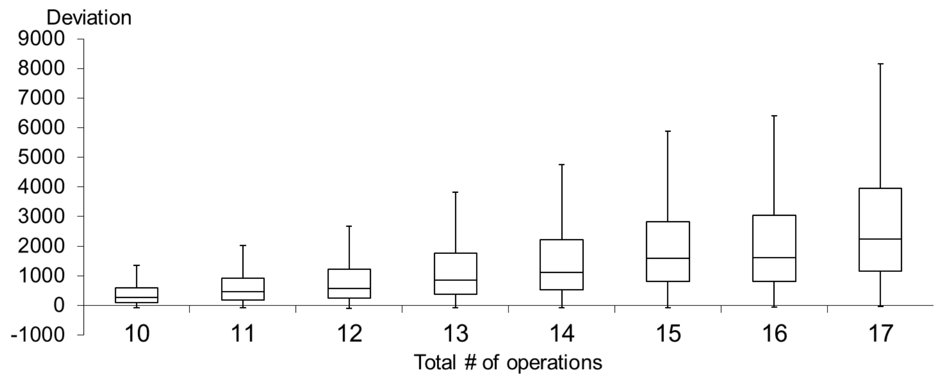

Figure 29: Deviation under k =250 random scenarios with exponentially distributed operation durations.

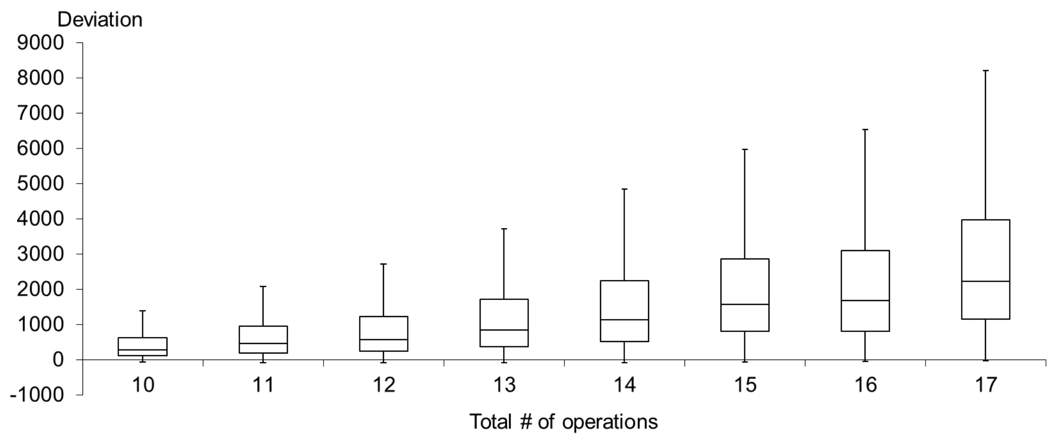

Figure 30: Deviation under k =500 random scenarios with exponentially distributed operation durations.

performs better than Model-4-O-P and worse than Model-3-O-P. Percentage of deviation between simple heuristic and deterministic model is 15.61 which may be a tolerable percentage by Hospital X.

Examples 2 and 4: deviation between sorting heuristic and Model-3-O-P is zero. In other words, sorting heuristic also finds the optimal solutions for these two examples. Moreover, deviation of example 5 is 8.89 %. Hospital's objective function value is 965.0 and this is approximately five times of sorting heuristic's objective function value. Although example 6 gives a very large deviation, 248.39 %, hospital's objective function value is 873, whereas, sorting heuristic's objective function value is 299.6. In general, Hospital X faces unfairness and delay in operation durations, hence, sorting heuristics seem to perform well compared to Hospital's actual schedule.

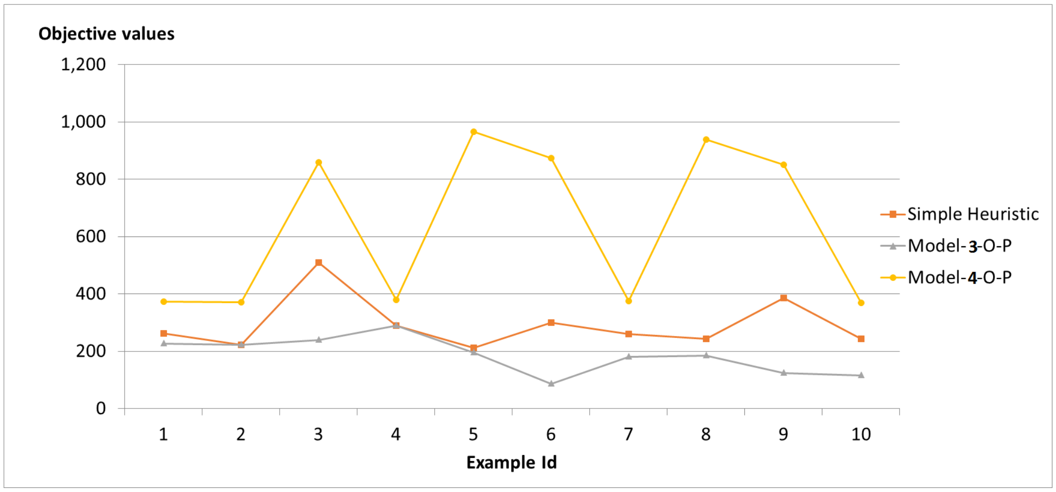

Figure 31: Examples of sorting heuristic method for Model-3-O-P.

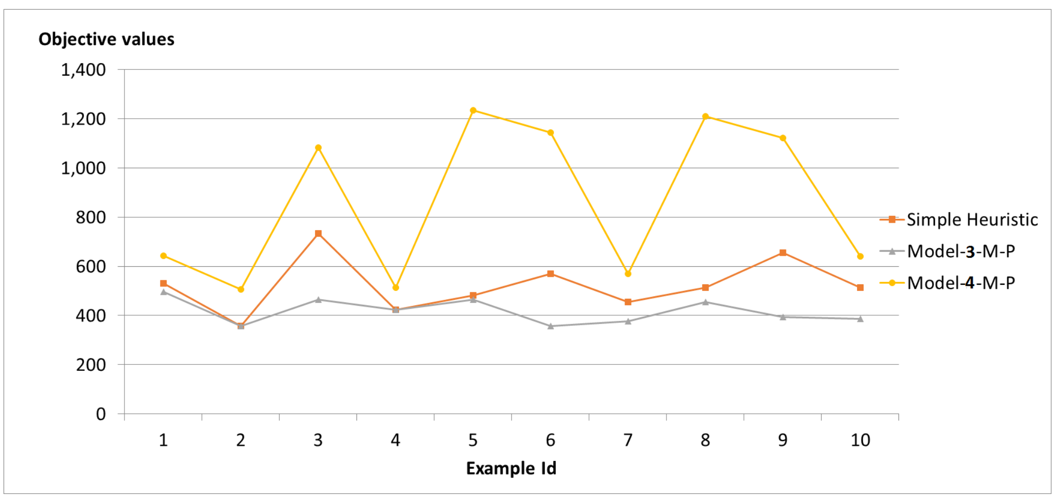

Figure 32: Examples of sorting heuristic method for Model-3-M-P.

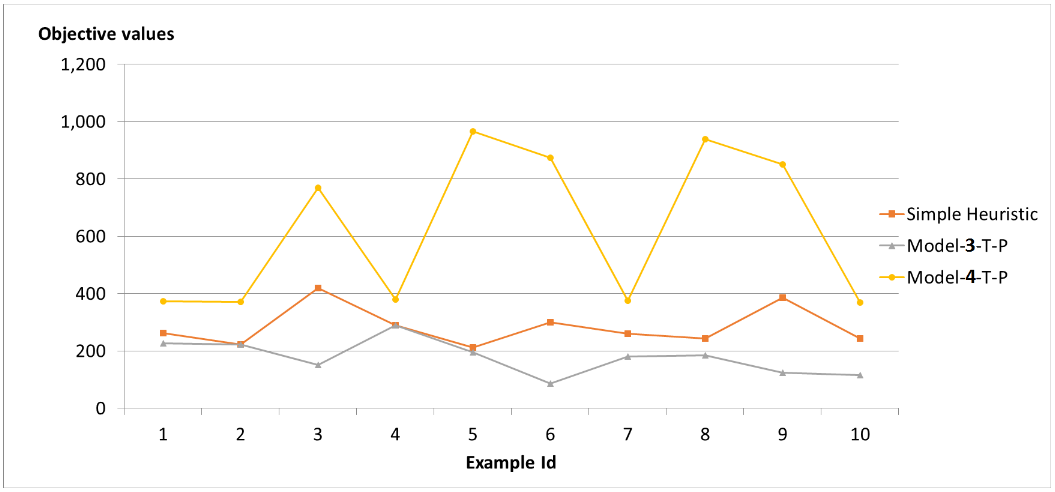

Figure 33: Examples of sorting heuristic method for Model-3-T-P.

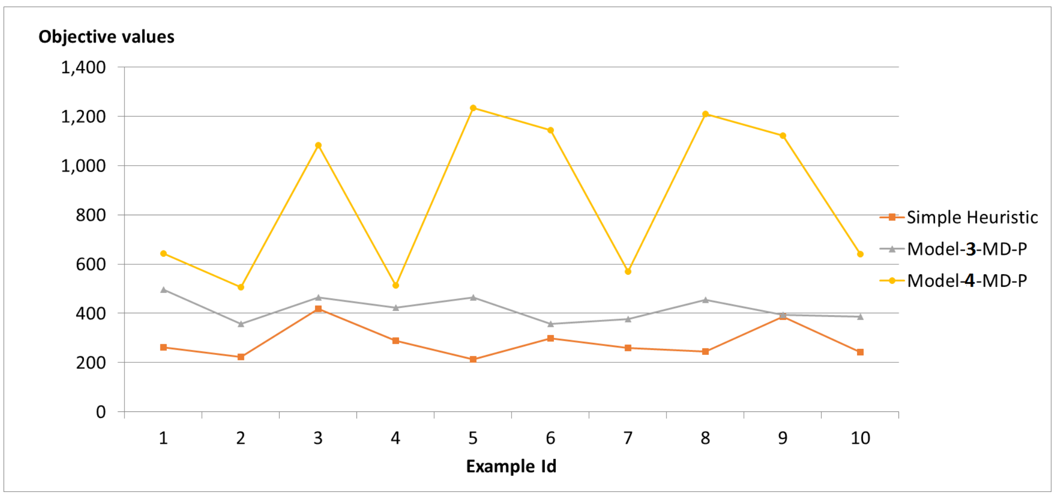

Figure 34: Examples of sorting heuristic method for Model-3-MD-P.

4.5 Conclusion

We study a real life operating room scheduling problem. First, we propose deterministic mathematical models formulating the assignment and scheduling problem. Then, inevitable uncertainty is incorporated to the deterministic models by two heuristics using delay probabilities. Next, Hospital X may not prefer buying an optimization tool, hence, a simple heuristic based on sorting is developed. All methods' performances are measured given predicted and actual operation durations. In addition, optimized schedules are evaluated under random operation durations.

As a future work, different objective functions can be studied and various heuristics can be proposed. In addition, uncertainty of the data should be carefully analyzed as the operation durations are mostly overestimated.

CHAPTER V

CONCLUSION

In this thesis, we study a real life operating room scheduling problem of Hospital X that is a leading, private Turkish hospital. We first determine empirical analysis of data. Then, we propose first deterministic models and then stochastic models to solve the scheduling problem.

There are 7 unique ORs, 141 beds and 54 intensive care units. 7754 operations belonging to 9 departments are analyzed. While considering data of hospital, we find that Hospital X faces two main problems: delay probabilities and unfairness between ORs usage. If an operation finishes after its predicted finishing time, a delay occurs. The main reason of delays is that pre-operation preparation is not finished as estimated. Other reasons are as follows: (i) previous operation's duration is longer than the estimated duration, (ii) surgeon or patient arrives late. We aim to include delays caused by processing durations in the mathematical models by two heuristics.

Average delay time is 25.14 minutes and 53 % of pediatric operations are delayed and generally pediatric operations are scheduled consecutively. The hospital administration should be careful while scheduling operations with high delay probabilities consecutively. Although some OR's utilization is high, some of them are so low. For example, OR 4 generally is not used and operations are scheduled consecutively ignoring the possible delays. Moreover, Hospital X overestimates the solutions by using predicted operation durations. According to this information, we have studied Hospital X's assignment and scheduling problems to achieve Hospital X's aim such as: avoiding waiting time and satisfying fairness of OR utilization. We use mathematical models and according to Hospital X's goals, we generate other objective functions.

We first assume that all data are known in advance for the deterministic setting. The second aim is to solve Hospital X's delay probabilities, then; we build models via incorporating delay probabilities by heuristics. Then we compare these models' performance with what has actually happened and with what may happen. We finally propose a simple heuristic to use optimization and analyze its performance without using modeling approach.

As mentioned before, we build mathematical models ignoring uncertainty. Model-1 optimizes waiting time, fairness and either overtime, or makespan, or tardiness, or makespan with deadline constraint. For Model-1, we find the optimum solution and sequences of operations, then, we use these sequences to find performance of our models with actual operation durations. Except tardiness cases, deviation of actual and predicted operation durations' solutions are very close to each other. To find Hospital X's objective values, we propose Model-2. In both cases, Model-1 performs better than Model-2 because of smaller unfairness values.

Model-1 does not consider uncertainty prevalent in operations. To analyze such uncertainty, two different heuristics are proposed. The first is Model-3 to consider delay probabilities through linear summations and the second is Model-5 considering delay probabilities by starting time weighted probabilities through linear summations. Model-3 minimizes waiting time of patient, unfairness, and delay probabilities and either overtime, or makespan, or tardiness, or makespan with deadline with each operation finished before its deadline. We find performance of Model-3 with actual operation duration and except tardiness model, deviation is very low. It means that our model performs well with actual operation durations. Model-4 computes Hospital X's objective function values and Model-3 performs better than Model-4 for both models because Model-3 faces less unfairness and incorporates delay probabilities. Moreover, Model-3' minimizes waiting time, unfairness and starting time weighed delay probabilities with either overtime, or makespan, or tardiness, or makespan with deadline. We also analyze actual operation duration in the deterministic models and we find that objective function values computed by using actual and predicted operations' durations are very close to each other except tardiness models. Model-4' also computes Hospital X's objective function values and Model-3' performs better than Model-4' because of less unfairness and incorporating delay probabilities.

When we decrease the number of ORs to 6 from 7, objective function values decrease due to decreasing unfairness. In addition, Model-5, 5',7 and 7' schedule operations computed via using mathematical models which compute fairness differently: In Model-5 and 5', a range model is adapted, whereas, in Model-3 and 3', overutilization of each OR is summed. Unfortunately, Model-5 and 5' cannot find optimum solution and it takes hours to reach near optimal solution. That's why, we generate other methods: Model-7 and 7' find schedule by getting sequences of Model-3 and 3', and we calculate range model for overutilization values. This method is not successful because deviation of Model-7 and Model-5 is quite high but completion time of Model-7 is smaller than Model-5.

We analyze the performance of deterministic models with respect to random scenarios. Firstly, we analyze predicted operation duration distributions and we generate random operation durations with lognormal and exponential distributions. According to actual operation types for each day, we first generate operations and corresponding random operation durations. For each day, 50, 250 and 500 scenarios are generated. We use Model-3-O-P's sequences and utilize randomly generated operation durations to find the objective function values of scenarios. Finally, we compare deterministic model's objective function with the random scenarios' objective function value. We find that, lognormal distribution gives lower deviations but convergence with respect to increasing number of scenarios cannot be observed. However, the deviations between the deterministic schedule and deterministic schedule with random operation durations keep decreasing with increasing number of scenarios. We also increase the number of scenarios to 2000 and then to 5000. When the number of operations increases, median of deviation also increases. Random model performs worse than the deterministic models because of higher unfairness and waiting time values.

Deterministic models perform poorly under random scenarios. Hence, necessity of a simple heuristic becomes more important in addition to the case in which Hospital X may not plan to buy an optimization tool. This simple heuristic relies on sorting and we do not utilize any optimization tool. We find that our sorting heuristic performs better than Hospital X's objective function values under random scenarios but it performs worse than our deterministic models.

As mentioned before, we also solve weekly schedule but computing the optimal solution requires longer computation times than solving daily schedule. As a future direction of our thesis, weekly schedule or a shorter time period schedule (such as, 2 or 3 days) may be studied. Furthermore, we can improve performance by including uncertainty with another method: robust scheduling or chance constraints. While creating uncertainty data set polyhedrally, we can cover a large percent of data points. It has some advantages; the main ones are as follows. It is distribution free and tractable. Disadvantage is that we are unable to evaluate expectations including delay probabilities. We may also utilize chance constraints that are competitive tools for solving optimization problems under uncertainty for delay probabilities.

APPENDIX A

TIME INTERVALS

	08:30-13:00	13:00-17:30	Total
Neuro	183	285	**468**
Pedia	445	50	**495**
Gen	633	381	**1014**
Gyn	1499	1056	**2555**
Card	47	3	**50**
ENT	840	251	**1091**
Orth	546	477	**1023**
Plas	235	272	**507**
Uro	200	351	**551**
Total	**4581**	**3123**	**7754**

Table 36: Percentage of scheduled operations for each department with respect to time intervals.

APPENDIX B

OR PREFERENCE

	OR Id							
Id	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Total**
Neuro	**134**	**19**	**55**	**1**	**63**	**154**	**42**	**468**
1	95	13	36	1	44	110	30	329
2	39	6	19		19	44	12	139
Pedia	**41**	**72**	**119**	**3**	**54**	**78**	**128**	**495**
3	41	72	119	3	54	78	128	495
Gen	**65**	**93**	**103**	**6**	**79**	**544**	**124**	**1014**
4	12	7	11		16	19	21	86
5	46	76	83	3	42	138	92	480
6	7	10	9	3	21	387	11	448
Gyn	**163**	**380**	**558**	**9**	**315**	**327**	**803**	**2555**
7	32	43	56	1	32	57	52	273
8	38	86	137	1	85	88	198	633
9	29	73	117	4	72	81	184	560
10	23	105	149	1	74	60	263	675
11	13	33	35		12	12	28	133
12					1			1
13	22	37	52	1	30	20	62	224
14	1	1	6	1	1	2	4	16
15	5	2	6		8	7	12	40
Card				**48**	**2**			**50**
16				21				21
17				26				26
18				1	2			3
ENT	**40**	**219**	**331**	**5**	**138**	**129**	**229**	**1091**
19	15	36	57		29	32	39	208
20					2			2
21	14	66	98	1	34	41	90	344
22	11	117	176	4	73	56	100	537
Orth	**821**	**39**	**42**	**2**	**27**	**55**	**37**	**1023**
23	147	11	11		8	10	9	196
24	426	17	20	1	14	34	19	531
25	248	11	11	1	5	11	9	296
Plas	**37**	**48**	**75**	**2**	**46**	**97**	**202**	**507**
26	3	16	25		11	24	27	106
27	34	32	50	2	35	73	175	401
Uro	**62**	**58**	**120**	**1**	**81**	**90**	**139**	**551**
28	36	36	73	1	68	58	87	359
29	1		2			1	2	6
30	25	22	45		13	31	50	186
Total	**1363**	**928**	**1403**	**77**	**805**	**1474**	**1704**	**7754**

Table 37: Operations distribution with respect to surgeons' preferences*.

*Each number shows surgeons' ids.

APPENDIX C

DISTRIBUTION

		AOD				POD			
#	Distribution	Kolmogorov Smirnov Rank	Statistic	Anderson Darling Rank	Statistic	Kolmogorov Smirnov Rank	Statistic	Anderson Darling Rank	Statistic
1	**Beta**	0.14	3	23934.00	3	0.11	5	16734	6
2	**Chi-Squared**	0.43	8	928.53	8	0.41	10	958.13	10
3	**Erlang**	0.45	9	177.85	9	0.09	3	47187	3
4	**Exponential**	0.26	7	52181.00	6	0.32	9	68331	8
5	**Gamma**	0.16	5	21.93	4	0.09	1	45857	2
6	**Lognormal**	0.07	1	66167.00	1	0.11	4	6893	5
7	**Normal**	0.14	4	34254.00	5	0.11	6	55358	4
8	**Triangular**	Nofit				0.09	2	41003	1
9	**Uniform**	0.20	6	86051.00	7	0.19	8	23128	7
10	**Weibull**	0.08	2	12301.00	2	0.15	7	80474	9

Table 38: Goodness of fit test results of AOD and POD of neurosurgery.

APPENDIX D

OPERATION TYPES

Neurosurgery Operation Types	Total # of Operations	AOD			POD		
		Max	Min	Average	Max	Min	Average
1	248	876	0	123.48	510	30	181.45
2	72	310	53	149.94	510	50	201.67
3	64	1125	34	249.11	510	60	249.92
4	22	233	3	81.23	330	30	140.68
5	13	106	18	63.46	210	25	110.38
Others	49	1076	28	185.65	510	25	229.10
Total	**468**	**1125**	**0**	**149.96**	**510**	**25**	**194.22**

Table 39: Neurosurgery types.

Pediatric Operation Types	Total # of Operations	AOD			POD		
		Max	Min	Average	Max	Min	Average
1	311	101	4	26.32	300	10	62.50
2	34	108	7	46.88	270	50	98.97
3	22	341	10	78.50	270	30	107.05
4	19	125	47	87.63	270	25	130.00
5	18	145	28	53.00	150	60	102.22
Others	91	374	0	93.44	510	15	135.83
Total	**495**	**374**	**0**	**43.20**	**510**	**10**	**82.94**

Table 40: Pediatric operation types.

General Operation Types	Total # of Operations	AOD			POD		
		Max	Min	Average	Max	Min	Average
1	226	2749	33	96.97	330	35	102.61
2	111	123	3	38.31	270	25	70.09
3	84	193	57	122.00	510	30	132.20
4	79	108	26	66.96	360	25	80.25
5	63	151	15	51.37	140	25	72.06
Others	451	2540	0	105.14	510	10	120.04
Total	**1014**	**2749**	**0**	**86.06**	**510**	**10**	**99.37**

Table 41: General operation types.

Gynecology Operation Types	Total # of Operations	AOD			POD		
		Max	Min	Average	Max	Min	Average
1	1887	968	0	55.97	250	15	69.27
2	258	921	5	68.34	300	20	96.98
3	116	1007	4	125.29	270	45	137.49
4	60	217	24	121.45	240	85	143.92
5	58	173	45	94.66	270	60	125.07
Others	176	437	9	96.51	270	20	116.18
Total	**2555**	**1007**	**0**	**64.89**	**300**	**15**	**80.91**

Table 42: Gynecology operation types.

Cardiology Operation Types	Total # of Operations	AOD			POD		
		Max	Min	Average	Max	Min	Average
1	21	299	69	215.95	270	135	230.71
2	9	91	35	60.67	150	60	100.00
3	5	114	21	66.00	120	60	102.00
4	3	228	55	136.33	240	30	116.67
5	2	175	159	167.00	135	120	127.50
Others	10	223	48	112.86	210	60	128.57
Total	**50**	**299**	**21**	**145.46**	**270**	**30**	**163.00**

Table 43: Cardiology operation types.

Ear-nose-throat Operation Types	Total # of Operations	AOD			POD		
		Max	Min	Average	Max	Min	Average
1	431	329	26	93.21	390	30	135.30
2	227	144	23	65.07	240	40	112.69
3	128	87	26	54.67	240	30	107.15
4	55	288	15	96.09	270	60	139.18
5	41	257	49	98.98	240	60	141.46
Others	209	1109	7	89.02	1109	7	89.02
Total	**1091**	**1109**	**7**	**83.19**	**390**	**23**	**124.76**

Table 44: Ear-nose-throat operation types.

Orthopedic Operation Types	Total # of Operation	AOD			POD		
		Max	Min	Average	Max	Min	Average
1	203	239	6	65.82	330	20	103.00
2	133	1138	9	127.75	270	25	109.40
3	131	233	24	121.50	315	30	129.81
4	84	440	18	100.75	240	40	116.67
5	68	355	18	135.62	300	80	155.15
Others	404	1045	4	108.20	450	10	122.81
Total	**1023**	**1138**	**4**	**103.31**	**450**	**10**	**116.25**

Table 45: Orthopedic operation types.

Plastic Operation Types	Total # of Operation	AOD			POD		
		Max	Min	Average	Max	Min	Average
1	55	398	23	86.95	390	15	118.82
2	47	188	27	75.21	240	45	113.72
3	27	1118	30	144.67	390	30	137.41
4	25	134	18	58.20	330	30	121.20
5	22	345	40	113.77	390	60	144.77
Others	331	1068	9	144.93	510	25	167.08
Total	**507**	**1118**	**9**	**112.81**	**510**	**15**	**144.46**

Table 46: Plastic operation types.

Urology Operation Types	Total # of Operation	AOD			POD		
		Max	Min	Average	Max	Min	Average
1	130	1026	8	63.33	390	20	83.65
2	89	214	7	88.93	270	30	119.78
3	71	139	11	70.24	180	30	91.04
4	55	113	11	57.09	210	30	97.00
5	36	126	5	41.67	180	30	87.92
Others	170	976	7	124.05	490	15	125.28
Total	**551**	**1026**	**5**	**76.46**	**490**	**15**	**101.65**

Table 47: Urology operation types.

APPENDIX E

COMPUTATION TIMES

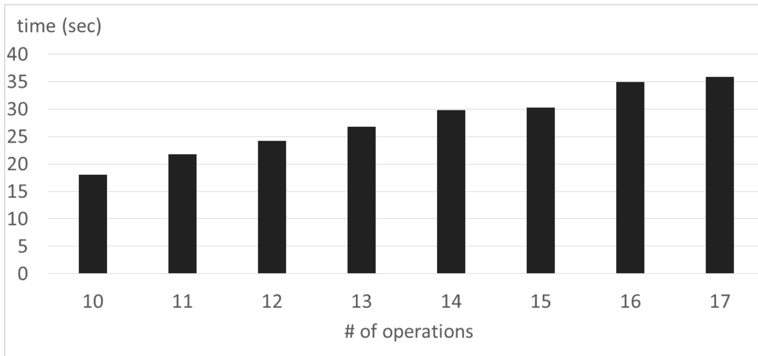

Figure 35: Computation times of Model 1-O-P.

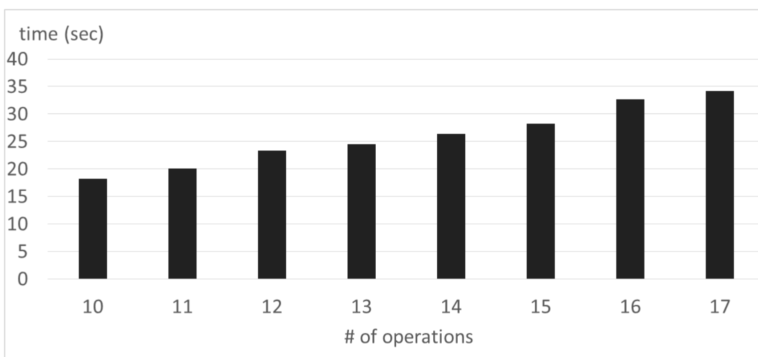

Figure 36: Computation times of Model 1-M-P.

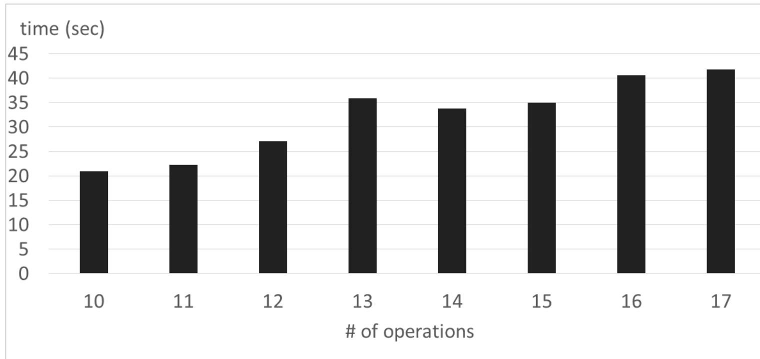

Figure 37: Computation times of Model 1-T-P.

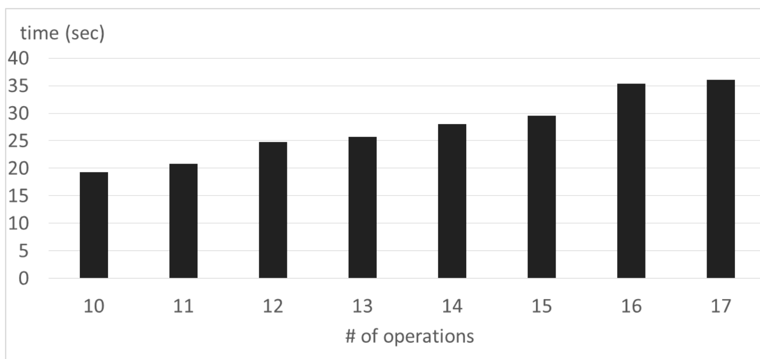

Figure 38: Computation times of Model 1-MD-P.

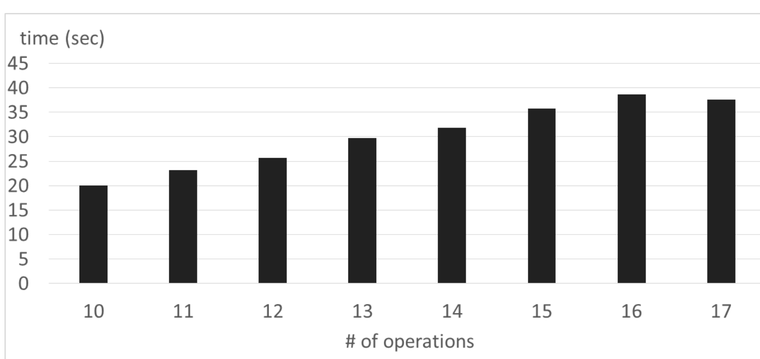

Figure 39: Computation times of Model-3-O-P.

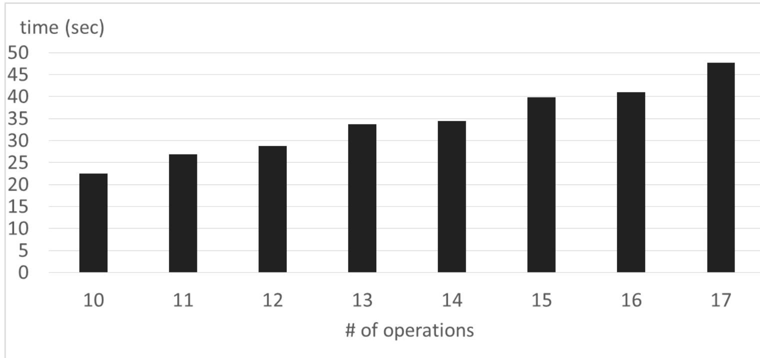

Figure 40: Computation times of Model-3-M-P.

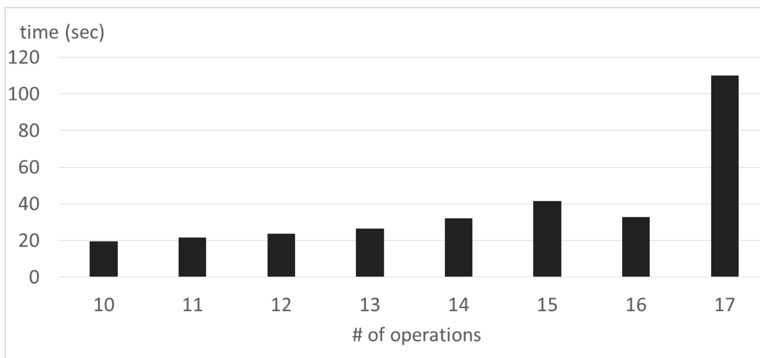

Figure 41: Computation times of Model-3-T-P.

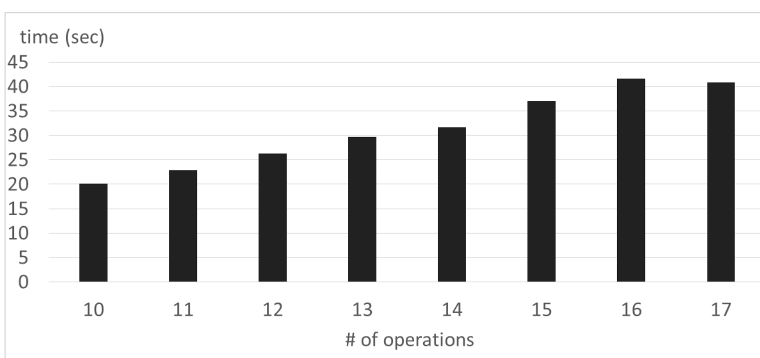

Figure 42: Computation times of Model-3-MD-P.

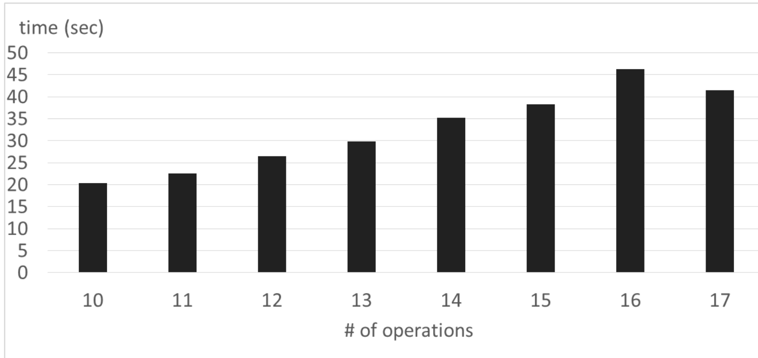

Figure 43: Computation times of Model-3'-O-P.

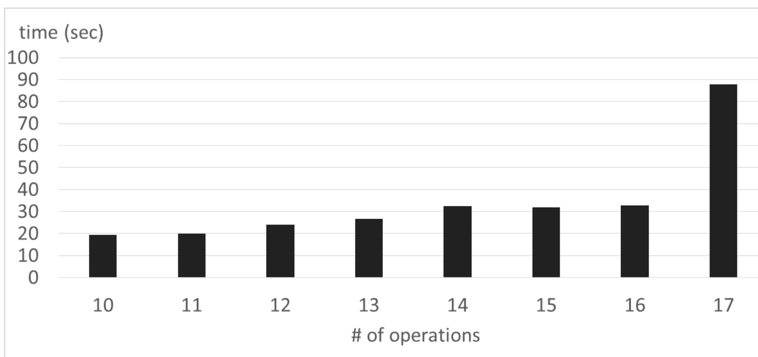

Figure 44: Computation times of Model-3'-M-P.

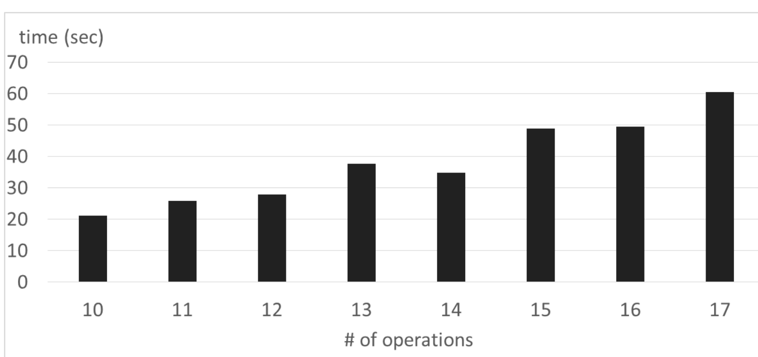

Figure 45: Computation times of Model-3'-T-P.

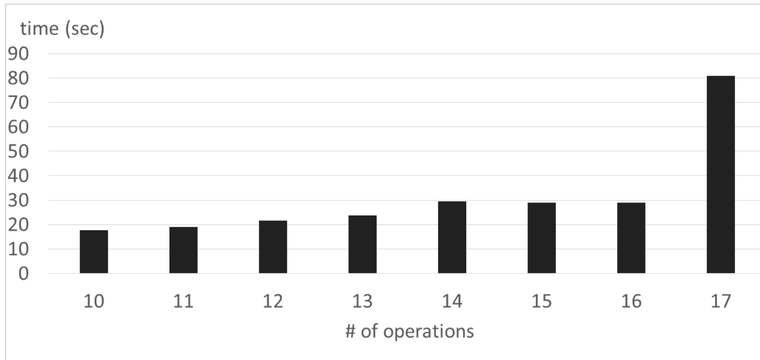

Figure 46: Computation times of Model-3'-MD-P.

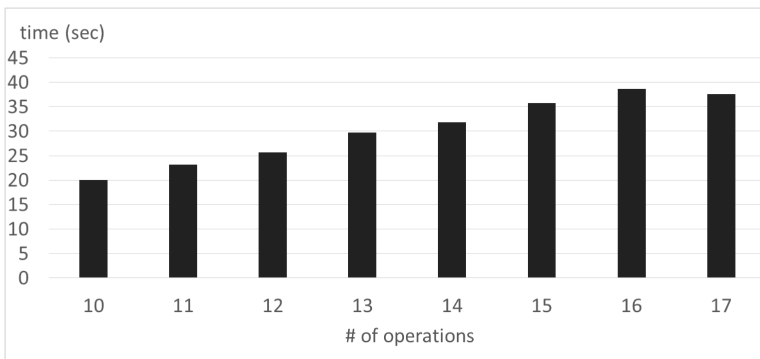

Figure 47: Computation times of Model-7-O-P.

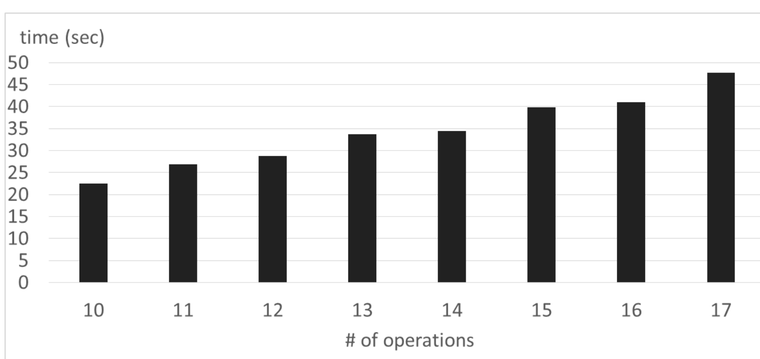

Figure 48: Computation times of Model-7-M-P.

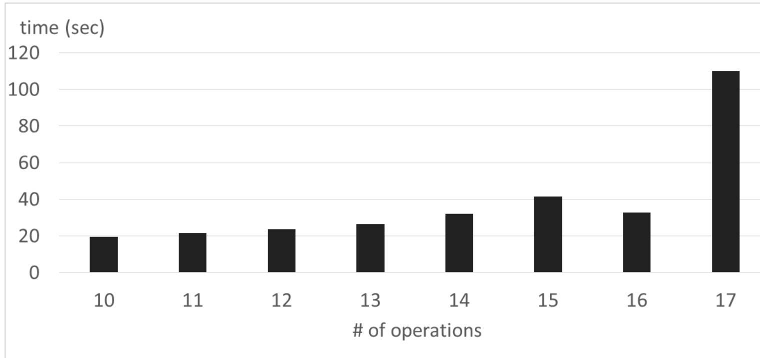

Figure 49: Computation times of Model-7-T-P.

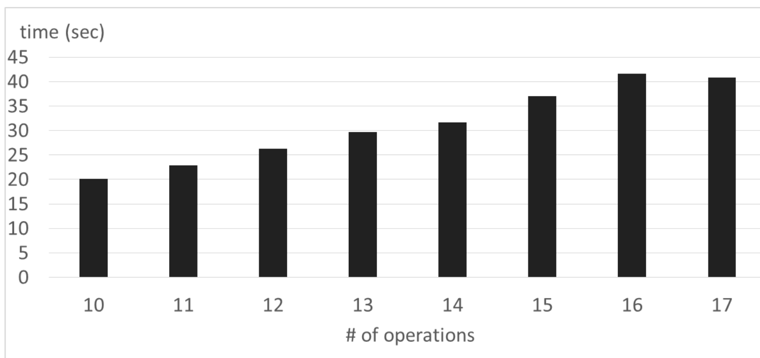

Figure 50: Computation times of Model-7-MD-P.

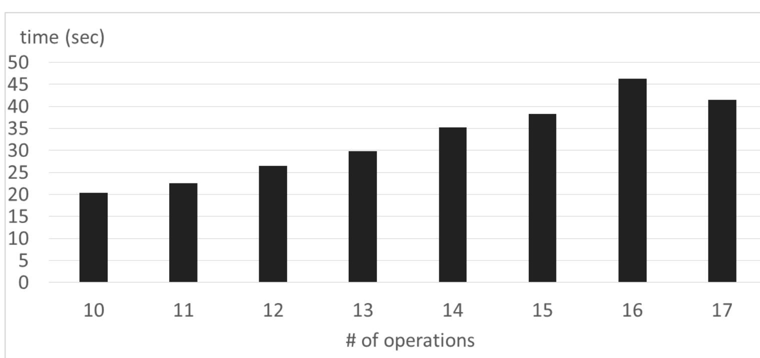

Figure 51: Computation times of Model-7'-O-P.

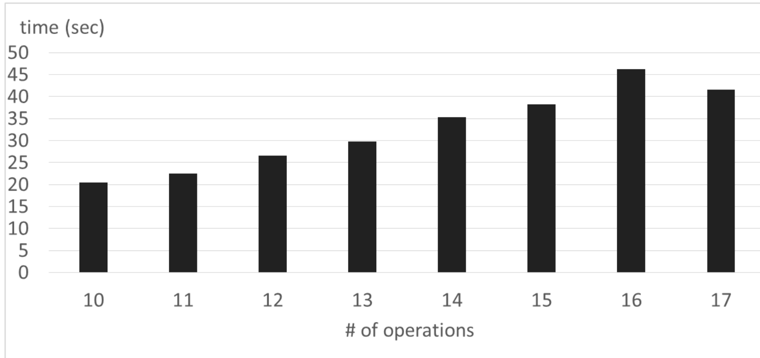

Figure 52: Computation times of Model-7'-M-P.

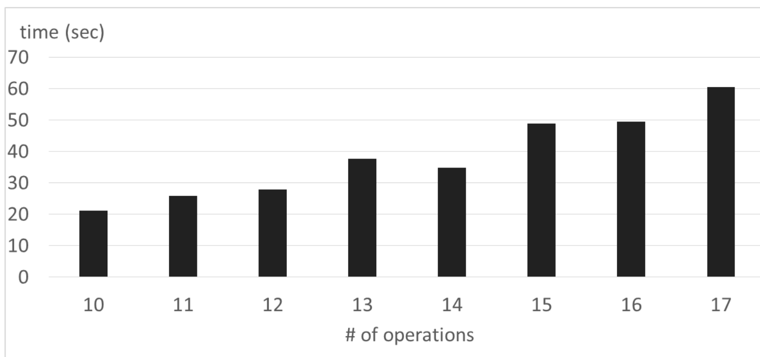

Figure 53: Computation times of Model-7'-T-P.

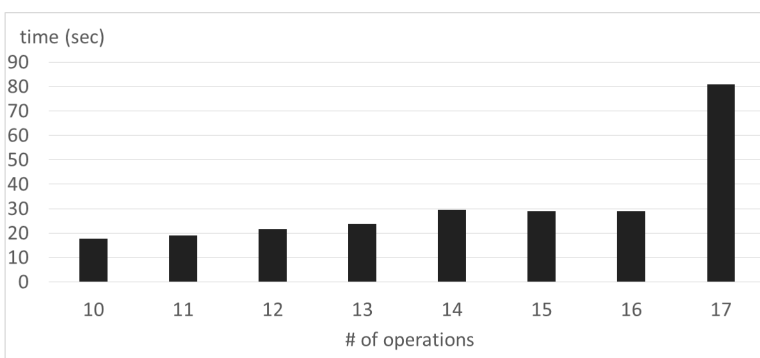

Figure 54: Computation times of Model-7'-MD-P.

APPENDIX F

COMPARISON OF MODEL-4' AND MODEL-3'

To compare objective function values of predicted and actual model, deviation is found :

$$(\frac{\texttt{Actual Solution-Predicted Solution}}{\texttt{Predicted Solution}}) * 100 \tag{28}$$

Where Actual solution comes from Model-4' and predicted solution comes from Model-3'.

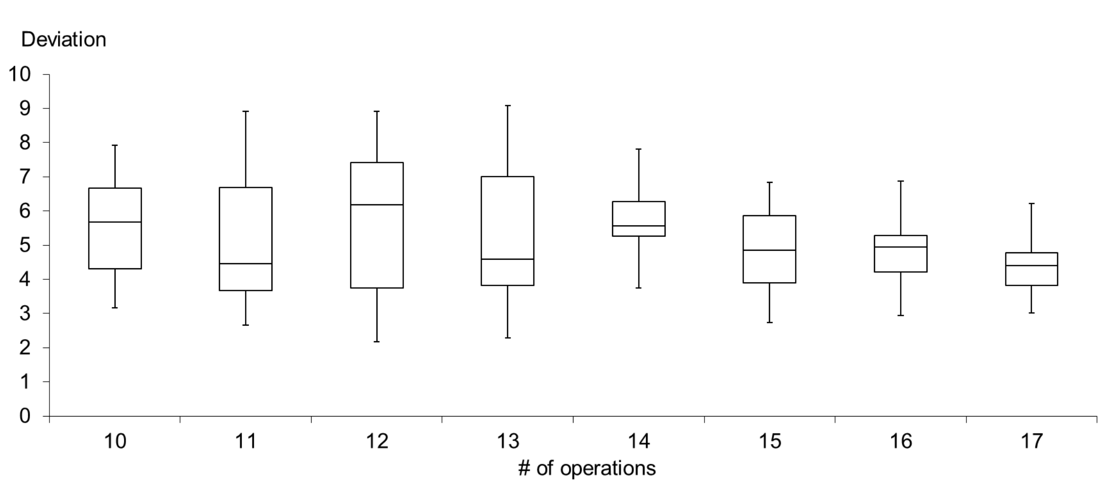

Figure 55: Deviation of Model-4'-O-P and Model-3'-O-P.

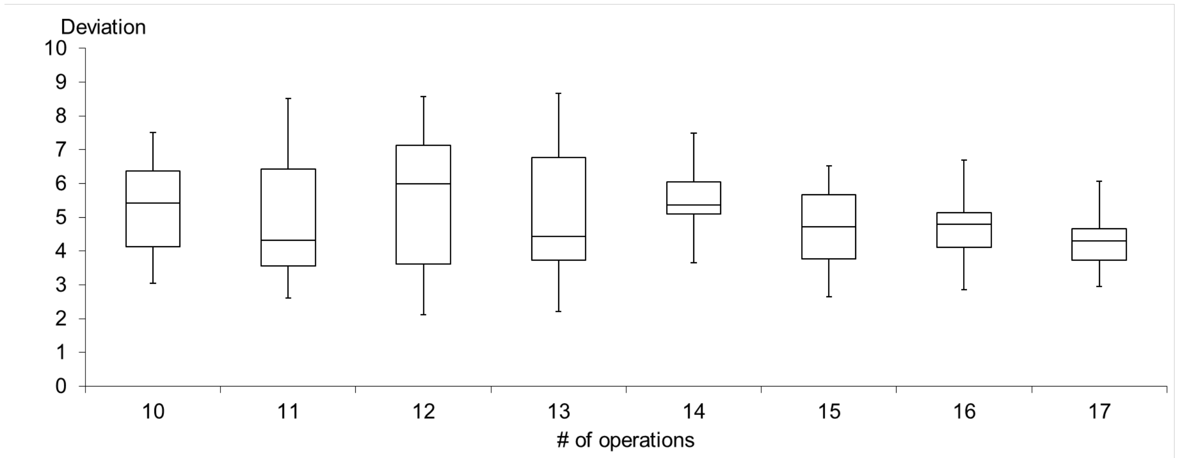

Figure 56: Deviation of Model-4'-M-P and Model-3'-M-P.

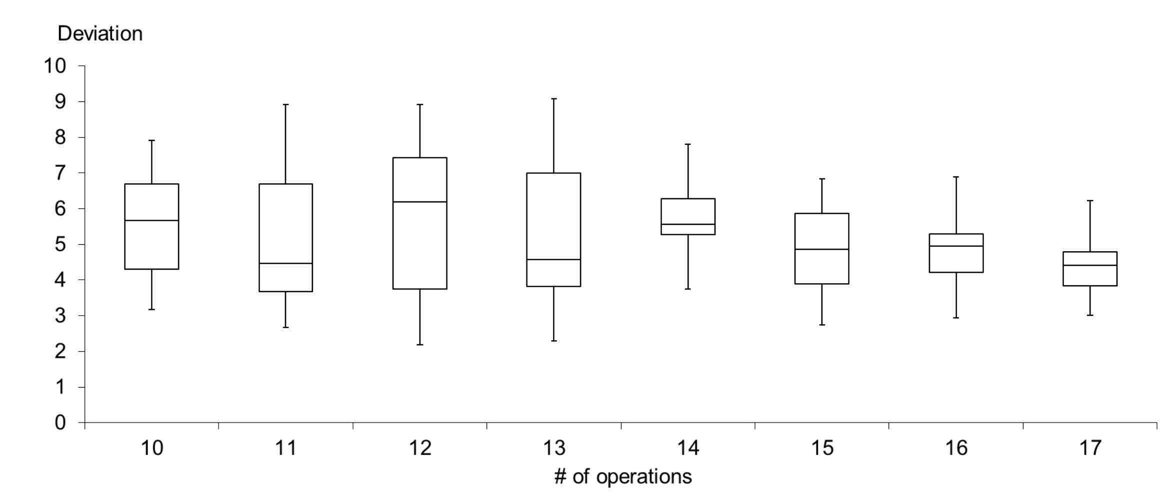

Figure 57: Deviation of Model-4'-T-P and Model-3'-T-P.

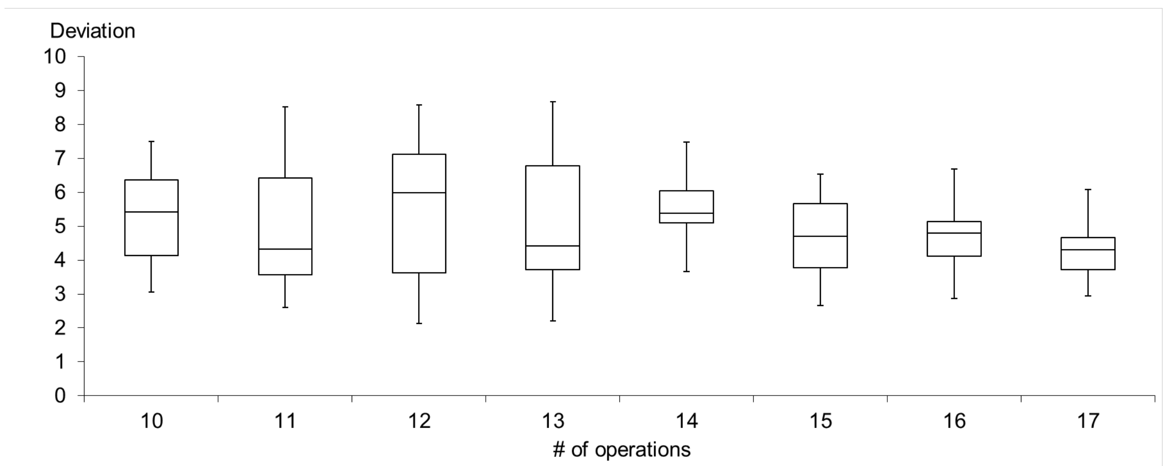

Figure 58: Deviation of Model-4'-MD-P and Model-3'-MD-P.

Bibliography

[1] I. Adan and J. Vissers, "Patient mix optimisation in hospital admission planning: A case study," *International Journal of Operations and Production Management*, vol. 22, no. 4, pp. 445–461, 2002.

[2] K. Agpak and S. Zolfaghari, "Mathematical models for parallel two-sided assembly line balancing problems and extensions," *International Journal of Production Research*, vol. 53, no. 4, pp. 1242–1254, 2015.

[3] M. Arenas, A. Bilbao, R. Caballero, T. Gomez, M. Rodriguez, and F. Ruiz, "Analysis via goal programming of the minimum achievable stay in surgical waiting lists," *Journal of the Operational Research Society*, no. 53, pp. 387–396, 2002.

[4] M. Basson and T. Butler, "Evaluation of operating room suite efficiency in the veterans health administration system by using dataenvelopment analysis," *American Journal of Surgery*, vol. 192, pp. 649–656, 2006.

[5] J. Belin and E. Demeulemeester, "Building cyclic master surgery schedules with leveled resulting bed occupancy," *European Journal of Operational Research*, vol. 176, no. 2, pp. 1185–1204, 2007.

[6] J. Belin and E. Demeulemeester, "A branch and price approach for integrating nurse and surgery scheduling," *European Journal of Operational Research*, vol. 189, no. 3, pp. 652–668, 2008.

[7] J. Belin, E. Demeulemeester, and B. Cardoen, "A decision support system for cyclic master surgery scheduling with multiple objectives," *Journal of Scheduling*, vol. 12, no. 2, pp. 147–161, 2009.

[8] D. Biskup, J. Herrmann, and J. N. Gupta, "Scheduling identical parallel machines to minimize total tardiness," *International Journal of Production Economics*, vol. 115, no. 1, pp. 134–142, 2008.

[9] J. Blake and M. Carter, "A goal programming approach to strategic resource allocation in acute care hospitals," *European Journal of Operational Research*, vol. 140, no. 541–561, 2002.

[10] J. Blake and M. Carter, "Surgical process scheduling: A structured review," *Journal of Health Systems*, vol. 5, no. 3, pp. 17–30, 1997.

[11] J. Blake, F. Dexter, and J. Donald, "Operating room managers use of integer programming for assigning block time to surgical groups: A case study," *Anesthesia and Analgesia*, vol. 94, pp. 143–148, 2002a.

[12] J. Blake and J. Donald, "Mount sinai hospital uses integer programming to allocate operating room time," *Interfaces*, vol. 32, pp. 63–73, 2002b.

[13] J. Breit, G. Schmidt, and V. A. Strusevich, "Two-machine open shop scheduling with an availability constraint," *Operations Research Letters*, vol. 29, no. 2, pp. 65–77, 2001.

[14] B. Cardoen, E. Demeulemeester, and J. Belin, "Operating room planning and scheduling: A literature review," *European Journal of Operational Research*, vol. 201, no. 3, pp. 921–932, 2010.

[15] B. Cardoen, E. Demeulemeester, and J. Belin, "Optimizing a multiple objective surgical case sequencing problem," *International Journal of Production Economics*, vol. 119, pp. 354–366, 2009a.

[16] B. Cardoen, E. Demeulemeester, and J. Belin, "Sequencing surgical cases in a day-care environment: An exact branch-and-price approach," *Computers and Operations Research*, vol. 9, no. 2009b, pp. 2660–2669, 36.

[17] M. W. Carter and S. D. Lapierre, "Scheduling emergency room physicians," *Health Care Management Science*, vol. 4, no. 4, pp. 347–360, 2001.

[18] T. Cayirli and E. Veral, "Outpatient scheduling in health care: a review of literature," *Production and Operations Management*, vol. 12, no. 4, pp. 519–549, 2003.

[19] S. Chaabane, N. Meskens, A. Guinet, and M. Laurent, "Comparison of two methods of operating theatre planning: Application in belgian hospitals," *Proceedings of the International Conference on Service Systems and Service Management*, 2006.

[20] B. Denton, J. Viapiano, and A. Vogl, "Optimization of surgery sequencing and scheduling decisions under uncertainty," *Health Care Management Science*, vol. 10, pp. 13–24, 2007.

[21] B. Denton and D. Gupta, "A sequential bounding approach for optimal appointment scheduling," *Transactions*, vol. 35, pp. 1003–1016, 2003.

[22] F. Dexter and R. D. Traub, "How to schedule elective surgical cases into specific operating rooms to maximize the efficiency of use of operating room time," *Anesthesia & Analgesia*, vol. 94, no. 4, pp. 933–942, 2002.

[23] F. Dexter, J. Blake, D. Penning, and D. Lubarsky, "Calculating a potential increase in hospital margin for elective surgery by changing operating room time allocations or increasing nursing staffing to permit completion of more cases: A case study," *Anesthesia and Analgesia*, vol. 94, pp. 138–142, 2002.

[24] F. Dexter, D. Lubarsky, and J. Blake, "Sampling error can significantly affect measured hospital financial performance of surgeons and resulting operating room time allocations," *Anesthesia and Analgesia*, vol. 95, pp. 184–188, 2002b.

[25] F. Dexter, J. Blake, D. Penning, B. Sloan, P. Chung, and D. Lubarsky, "Use of linear programming to estimate impact of changes in a hospitals operating room time allocation on perioperative variable costs," *Anesthesiology*, vol. 96, pp. 718–724, 2002.

[26] S. A. Erdogan, "Surgery planning and scheduling," *Wiley Encyclopedia of operations research and management science*, 2011.

[27] H. Fei, N. Meskens, and C. Chu, "An operating theatre planning and scheduling problem in the case of a block scheduling strategy," *Proceedings of the International Conference on Service Systems and Service Management*, 2006.

[28] H. Fei, C. Chu, N. Meskens, and A. Artiba, "Solving surgical cases assignment problem by a branch-and-price approach," *International Journal of Production Economics*, vol. 112, pp. 96–108, 2008.

[29] H. Fei, C. Combes, N. Meskens, and C. Chu, "Endoscopies scheduling problem: A case study," *Proceedings of the IFAC Symposium on Information Control Problems in Manufacturing*, 2006.

[30] L. V. Green, S. Savin, and B. Wang, "Managing patient service in a diagnostic medical facility," *Operations Research*, vol. 54, no. 1, pp. 11–25, 2006.

[31] F. Guerriero and R. Guido, "Operational research in the management of the operating theatre: a survey," *Health care management science*, vol. 14, no. 1, pp. 89–114, 2014.

[32] D. Gupta, "Performance metrics for advanced access," *Journal of Healthcare Management*, vol. 51, no. 4, p. 246, 2006.

[33] D. Gupta and B. Denton, "Appointment scheduling in health care: Challenges and opportunities," *IIE transactions*, vol. 40, no. 9, pp. 800–819, 2008.

[34] R. Gokhale and M. Mathirajan, "Scheduling identical parallel machines with machine eligibility restrictions to minimize total weighted flowtime in automobile gear manufacturing," *The International Journal of Advanced Manufacturing Technology*, vol. 60, no. 12, pp. 1099–1110, 2012.

[35] J. H. Iser, B. T. Denton, and R. E. King, "Heuristics for balancing operating room and post-anesthesia resources under uncertainty," *Simulation Conference*, 2008.

[36] A. Jebali, A. Alouane, and P. Ladet, "Operating rooms scheduling," *International Journal of Production Economics*, vol. 99, pp. 52–62, 2006.

[37] A. Jebali, A. Alouane, and P. Ladet, "Operating rooms scheduling," *International Journal of Production Economics*, vol. 99, pp. 52–62, 2006.

[38] G. C. Kaandorp and G. Koole, "Optimal outpatient appointment scheduling," *Health Care Management Science*, vol. 10, no. 3, pp. 217–229, 2007.

[39] W. Kaczmarczyk, "Proportional lot-sizing and scheduling problem with identical parallel machines," *International Journal of Production Research*, vol. 49, no. 9, pp. 2605–2623, 2011.

[40] S. Kharraja, P. Albert, and S. Chaabane, "Block scheduling: Toward a master surgical schedule," *Proceedings of the International Conference on Service Systems and Service Management*, 2006.

[41] H. Kellerer and V. A. Strusevich, "Scheduling problems for parallel dedicated machines under multiple resource constraints," *Discrete Applied Mathematics*, vol. 133, no. 1, pp. 45–68, 2003.

[42] L. Y. Kuei, "Fast lp models and algorithms for identical jobs on uniform parallel machines," *Applied Mathematical Modelling*, vol. 37, no. 5, pp. 3436–3448, 2013.

[43] M. Lamiri and X. Xie, "Operating room planning with uncertain operating times," *Ecole Nationale Suprieure des Mines de Saint Etienne France*, 2007.

[44] M. Lamiri, X. Xie, A. Dolgui, and F. Grimaud, "A stochastic model for operating room planning with elective and emergency demand for surgery," *European Journal of Operational Research*, vol. 185, pp. 1026–1037, 2008.

[45] M. Lamiri, X. Xie, and S. Zhang, "Column generation for operating theatre planning with elective and emergency patients," *IIE Transactions*, vol. 40, pp. 838–852, 2008.

[46] C.-J. Liao and C.-H. Lin, "Makespan minimization for two uniform parallel machines," *International Journal of Production Economics*, vol. 84, no. 2, 2003.

[47] J. Magerlein and J. Martin, "Surgical demand scheduling: A review," *Health Services Research*, vol. 13, pp. 418–433, 1978.

[48] N. Meskens, D. Duvivier, , and A. Hanset, "Multi-objective operating room scheduling considering desiderata of the surgical team," *Decision Support Systems*, vol. 55, no. 2, pp. 650–659, 2013.

[49] W. Mulholland, P. Abrahamse, and V. Bahl, "Linear programming to optimize performance in a department of surgery," *Journal of the American College of Surgeons*, vol. 200, no. 6, pp. 861–868, 2005.

[50] K. Muthuraman and M. A. Lawley, "Stochastic overbooking model for outpatient clinical scheduling with no-shows," *IIE Transactions*, vol. 40, no. 9, 2008.

[51] S. Ogulata and R. Erol, "A hierarchical multiple criteria mathematical programming approach for scheduling general surgery operations in large hospitals," *Journal of Medical Systems*, vol. 27, no. 3, pp. 259–270, 2003.

[52] I. Ozkarahan, "Allocation of surgeries to operating rooms by goal programming," *Journal of Medical Systems*, vol. 24, pp. 339–378, 2000.

[53] A. Ozen, Y. Marmor, T. Rohleder, H. Balasubramanian, and J. Huddleston, "Optimization and simulation in the orthopedic spine surgery practice at mayo clinic," *Submitted to MSOM Special Issue on Practice Focused Research*, 2014a.

[54] O. Ozturk, Espinouse, Marie-Laure, M. D. Mascolo, and A. Gouin, "Makespan minimisation on parallel batch processing machines with non-identical job sizes and release dates," *International Journal of Production Research.*, vol. 14, no. 3, pp. 6022–6035, 2012.

[55] V. Perdomo, V. Augusto, and X. Xie, "Operating theatre scheduling using lagrangian relaxation," *Proceedings of the International Conference on Service Systems and Service Management*, 2006.

[56] M. Persson and J. Persson, "Health economic modelling to support surgery management at a swedish hospital," *Working Paper, Blekinge Institute of Technology, Sweden*, 2006.

[57] M. Persson and J. Persson, "Optimization modelling of hospital operating room planning: Analyzing strategies and problem settings," *Proceedings of the 2006 Annual Conference of OR Applied to Health Services*, 2006.

[58] Z. Przasnyski, "Operating room scheduling: A literature review," *AORN Journal*, vol. 44, no. 1, pp. 67–79, 1986.

[59] D. Pham and A. Klinkert, "Surgical case scheduling as a generalized job shop scheduling problem," *European Journal of Operational Research*, vol. 185, pp. 1011–1025, 2008.

[60] T. Rohleder, D. Sabapathy, and R. Schorn, "An operating room block allocation model to improve hospital patient flow," *Clinical and Investigative Medicine*, vol. 28, no. 6, pp. 353–355, 2005.

[61] B. Roland, C. D. Martinelly, and F. Riane, "Operating theatre optimization: A resource-constrained based solving approach," *Proceedings of the International Conference on Service Systems and Service Management*, 2006.

[62] P. Santibanez, M. Begen, and D. Atkins, "Surgical block scheduling in a system of hospitals: An application to resource and wait list management in a british columbia health authority," *Health Care Management Science*, vol. 10, pp. 269–282, 2007.

[63] I. S. Sjetne, M. Veenstra, and K. Stavem, "The effect of hospital size and teaching status on patient experiences with hospital care: a multilevel analysis," *Medical care*, vol. 45, no. 3, pp. 252–258, 2007.

[64] V. Smith-Daniels, S. Schweikhart, and D. Smith-Daniel, "Capacity management in health care services: Review and future research directions," *Decision Sciences*, vol. 19, pp. 889–919, 1988.

[65] A. Soriano, "Comparison of two scheduling systems," *Operations Research*, vol. 14, no. 3, pp. 388–397, 1966.

[66] Y. Tan, T. ElMekkawy, Q. Peng, and L. Oppenheimer, "Mathematical programming for the scheduling of elective patients in the operating room department," *Proceedings of the 2007 CDEN and CCEE Conference*, 2007.

[67] A.Testi, E. Tanfani, and G. Torre, "A three-phase approach for operating theatre schedules," *Health Care Management Science*, vol. 10, pp. 163–172, 2007.

[68] J. van Oostrum, M. V. Houdenhoven, J. Hurink, E. Hans, G. Wullink, and G. Kazemier, "A master surgery scheduling approach for cyclic scheduling in operating room departments," *OR Spectrum*, vol. 30, no. 2, pp. 355–374, 2008.

[69] J. Vissers, I. Adan, and J. Bekkers, "Patient mix optimization in tactical cardiothoracic surgery planning: A case study," *IMA Journal of Management Mathematics*, vol. 16, pp. 281–304, 2005.

[70] J. D. Welch, T. Norman, and J. Bailey, "Appointment systems in hospital outpatient departments," *The Lancet*, vol. 259, no. 6718, pp. 1105–1108, 1952.

[71] B. Zhang, P. Murali, M. Dessouky, and D. Belson, "A mixed integer programming approach for allocating operating room capacity," *Working Paper, University of Southern California, USA,*, 2006.

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