A HEALTHCARE INVENTORY PROBLEM WITH BOTH RELIABLE AND UNRELIABLE SUPPLY CHANNELS

A Thesis

by

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A HEALTHCARE INVENTORY PROBLEM WITH BOTH RELIABLE AND UNRELIABLE SUPPLY CHANNELS

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To my family,

and

all of my friends,

Without whom none of my success would be possible

ABSTRACT

In this study, we investigate the inventory review policy for a healthcare facility to minimize the impact of inevitable drug shortages when an alternative reliable supplier is present. A continuous-time stochastic process is used to calculate optimal inventory levels for the primary (unreliable) and secondary (reliable but costly) suppliers. We present optimal strategies for tractable instances, provide insights through supervised learning tools, and highlight how these results can be generalized. In particular, we provide business rules for inventory managers that would simultaneously minimize average inventory and secondary supplier usage.

Keywords: OR in health services, supply disruption, inventory management, Markov chain, machine learning, dual sourcing.

ÖZETÇE

Bu çalışmada, ilaç kıtlığı durumunu azaltmak için alternatif tedarikçiye sahip sağlık tesisleri için stok yönetimi araştırılmak istenmektedir. İlk (güvenilmez) tedarikçi ve ikinci (güvenilir ama maliyetli) tedarikçinin optimal stok seviyesini hesaplamak için sürekli stokastik süreçler analizi kullanılmıştır. Çözülebilir örnekler için ideal stratejiler sunulmuş, denetimli öğrenme araçları aracılığıyla fikir verilmiş ve bu sonuçların nasıl genelleştirilebileceğine dikkat çekilmiştir. Özellikle ortalama stok seviyelerini düşürmek isteyen ve iki tedarikçi ile çalışan stok yöneticileri için işlerini kolaylaştıracak yöntemler sunulmuştur.

Keywords: Sağlık Servisleri için Yöneylem Araştırması , Tedarik Kıtlığı, Stok Yönetimi, Markov Zinciri, Yapay Zeka ile öğrenme, İkili Kaynak Kullanımı.

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CHAPTER I

INTRODUCTION

Pharmaceuticals' effective supply chain management, especially inventory management, is gaining importance considering its effect on the quality of care. According to [1], in the United States, pharmaceuticals also compose a large portion (nearly 10%) of annual healthcare expenditures. Considering these costs and risks associated with inefficient inventory management, novel mathematical models that potentially increase the availability of pharmaceuticals are starting to receive attention from the healthcare community.

Healthcare facilities aim to improve patients' well-being, yet they operate with associated risks. Medicine shortages, lack of stock visibility between hospitals and suppliers, non-delivery risk of medicines, unexpected peaks in demand, warehouse capacity issues, forecasting errors, and stock holding problems are some of these risks, which directly or indirectly deteriorate the quality of care provided to patients. Among these risks, drug shortages are one of the most frequently encountered issues in the last decade. As of 16 February 2016, [2] reports 66 ongoing shortages, with another 88 medicine unavailability recently resolved, and 56 drugs withdrawn from the market in the United States. Shortages may lead to serious issues such as delays in surgeries, irreversible health issues, even mortality.

Most of the healthcare facilities we observe in the United States and Turkey can be classified as reactive instead of proactive in responding drug shortages. In practice, when a national shortage or a supply disruption occurs for a drug, hospitals typically try and find the drug or its substitute through alternative channels, which might be another hospital or warehouse. This approach is risky for the hospital and calls for additional effort and man-hours. Our aim in this study is to avoid excessive costs associated with conventional approaches and propose a proactive inventory management scheme in the presence of an unreliable supplier. For the sake of mathematical simplicity we assume there is one mainstream (unreliable) supply channel and one alternative channel that is always available but costly. We do not propose a new inventory management mechanism. However, we conjecture a careful selection of inventory levels would lead to significantly less alternative channel searches. Meanwhile, average number of items on inventory is to be minimized that, in turn, reduce inventory holding costs.

Healthcare facilities, particularly hospitals, bill patients (or insurance companies) for all drugs that are used in treatments. This unique aspect of healthcare supply chains make problems theoretically different than those found in manufacturing industry for the following reasons:

- High risk supply disruption: Drug shortages lead to lack of medicine, which means that patients suffer from this disruption and it directly affects their life or can result in death.

- Stockouts: When demand is uncertain, we can not predict it exactly in advance. Since this causes damage to patients, it has critical importance in a healthcare facility.

- Perishability: Pharmaceuticals are perishable items and have a fixed lifetime that requires to keep low level stocks.

- Zero ordering costs: The contract between a hospital and a supplier typically allows orders to be placed in any frequency and quantity. The trade off exists between inventory level and quality of care, which differentiates this study from the aforementioned works.

– Care Quality: Quality of care is a significant issue in healthcare. Patients are at risk due to the unavailability of a drug. It is hard to quantify of its impacts.

– Alternative Channel: Quality of care is directly proportional to the drug availability. Quality of care may decrease if we seek for alternative channel and it can also be more costly.

– Zero lead time: Deliveries are made daily and overnight deliveries are possible when the item is accessible, so lead time is negligible for drugs.

However, the notion of holding cost still exists as the items are stored for a duration, which cannot be billed. Therefore, we focus on *two main objectives* considering all these issues: minimizing the average inventory level and minimizing purchases through the alternative channel. We deliberately avoid the cost figures as they are hard to estimate in practice and *observe the set of dominating solutions*. Looking beyond the healthcare literature, a multiple objective model that considers disruptions on one channel has not been introduced yet. We adopt the widely-used (Q, r) inventory control policy, consider a single-item model, and compute separate order and reorder quantity values for mainstream and alternative channels. While average inventory level is simultaneously minimized, we investigate two cases where *we either minimize* i. *alternative supply total purchases (expected quantity ordered) or* ii. *number of times alternative supply is used (expected usage frequency).*

This thesis is organized as follows: Chapter 2 provides a literature review on healthcare inventory management and general inventory problems with disruption. Chapter 3 presents the problem description where the mathematical model and solutions are explained. Chapter 4 involves computational results and insights obtained. Conclusion chapter summarizes our research findings and offers directions for future research.

CHAPTER II

LITERATURE SURVEY

There exist several articles on continuous review inventory models without supply disruptions. [3] utilizes a (Q, r) policy in a two player framework with asymmetric information. (Q, r) policy model is extended by adding a two echelon system with a supplier and a buyer. These two opposite inventory models are analyzed based on the use of consignment stock in their functions. [4] focuses on a multi echelon inventory system with a single supplier and multiple retailers under (Q, r) policy. The article proposes a centralized replenishment policy for the system. The policy adopts an inventory position for the retailer independently. Furthermore, an echelon policy is proposed and the supplier's stock policy is investigated with risk-pooling and information sharing.

There are studies that consider inventory policies in the presence of supply shortages. A recent survey by [5] discusses supply chain disruptions in depth through relevant articles on different areas of supply chain management. Most of the existing models focus on inventory systems with zero lead time (e.g., [6] and [7]), but there are also studies that consider constant lead time (e.g., [8] and [9]). [10] examines the continuous review stochastic inventory problem under the standard (Q, r) policy assumption where the supplier's availability is changeable. [8] studies supply disruption in a continuous (Q, r) inventory system with unreliable suppliers which might be either available or unavailable. These available or unavailable periods are exponentially distributed and examined under two different situations for zero and nonzero (constant) lead times. [6] consider the case of Erlang-k inter-failure times and generally distributed recovery times for suppliers in an EOQ framework. [9] focuses on a continuous-review lost-sales (s, Q) inventory system with Poisson demand and constant lead time where the unreliable supplier's on/off periods are hyper-exponentially distributed. This study is extended by addition of Erlang-k lead times in [11]. [12] target to provide an optimal base stock level to minimize expected inventory related costs, for retailer under deterministic demand and zero lead time. Considered product is perishable, retailer is open to supply disruptions, and disruption processes are modeled as a discrete time Markov chain. Except for [6] that considers multiple suppliers, most supply disruption studies such as [8] and [7] study single supplier problems. [7] present a single unreliable supplier model using (S, s) inventory policy. Similar to ours, a continuous-time Markov chain is utilized in their article. For this reason, we do not use this model in our thesis.

The study in this thesis considers *dual sourcing under supply disruptions*, which differentiates it from the aforementioned works. One of the most popular studies in this area is introduced by [13], with a periodic review inventory policy for a reliable supplier, an unreliable supplier, and one retailer. Backlogging part of the demand is allowed, and lead times of suppliers are constant. Failure and repair periods of the unreliable supplier are modeled in a discrete time Markov chain, which presents the main difference between this model and ours. With a reliable and an unreliable supplier, [14] examine gathering two types of risks together; disruptions and delays. Using a newsvendor policy for single period, utilizations for both suppliers under deterministic demand are observed. Later, this study is extended by including a periodic review policy for an infinite time horizon in [15]. In addition, [16] adapts the newsvendor model under deterministic demand for the case of one reliable and one unreliable supplier, where unreliable supplier's disruption probability is dependent on order quantity. Retailer's risk aversion is analyzed through an exponential utility function.

[17] develop a dynamic programming approach for one reliable and one unreliable supplier under stochastic demand and finite time horizon with discrete periods, in which unreliable supplier's disruption rate is estimated using a Bayesian update model. [18] measure the performance of dual sourcing strategy for single buyer, single supplier case, where every supplier can observe breakdown for uncertain amount of time, and suppliers' lead times are stochastic. Uncertainty in parameters of this study is modeled as a Semi-Markov decision process. A sourcing strategy is introduced to minimize buyer's expected total costs for a long time horizon. In addition, with three interrelated studies from the same group (i.e., [19], [20] and [21]) a newsvendor approach for dual sourcing is constituted. In these studies, there are one retailer and two suppliers, where both suppliers are unreliable and open to disruptions, and the aim is to maximize total weighted expected profits for a single period inventory system.

The studies that are closest to our inventory management framework are [22], [23] and [24]. [22] focus on a case study of long term humanitarian emergency relief operation by developing a stochastic inventory control model with two suppliers under (Q, r) policy and lead times of suppliers are constant. An emergency order is never placed before a normal order and backorders are allowed. We extend this problem to a healthcare setting using a continuous time Markov chain with zero lead time. In [22], emergency orders are placed due to demand explosion and there exists backorder cost, whereas in our work, alternative supplier is used during disruption of the mainstream supplier to prevent shortage cost. Dissimilarly, we do not use neither back order cost nor shortage cost in our model. [23] work on an inventory system consisting of two substitutable and perishable products with separate demands, where supply disruption is not included. They provide a steady state analysis for a continuous review inventory policy. [24] consider a healthcare supply chain with warehouse capacity, multiple commodities, and possible alternative drug shortages. Every commodity on hand has a more expensive alternative, where both regular and alternative supplies are open to disruptions. Similar to ours, a continuous-time Markov chain is utilized, but order quantities and reorder levels for all commodities are obtained using a stochastic optimization approach with a single objective based on cost minimization.

Next, we present our assumptions, details of our mathematical model, and solution approach.

CHAPTER III

MODEL

3.1 Problem Description

In this section, we present our assumptions to model the inventory control problem of a healthcare facility with a realistic yet tractable approach.

3.1.1 A Single Reliable Supplier Model

We aim to investigate the (Q, r) inventory policy with one regular supply channel for single item.

The proposed model is developed under the following assumptions: Regular supply channel is always available. Patients arrive at the hospital of interest in accordance with a Poisson process having rate λ and orders are placed with fixed cost. Variable ordering costs are disregarded. Our real-life observations suggest that variable costs are reflected on patients, thus have no effect on our decisions. There is no lead time for the supply channel. The hospital has unlimited capacity. A continuous-review inventory control policy is adopted where an order of size Q will be placed through the regular channel when inventory level is at reorder level r and regular supply is available.

Note that in this model the objective function is to minimize the expected total costs of the system.

3.1.2 Two-Supply Channel Model with a Reliable and an Unreliable Channel

We aim to investigate the (Q, r) inventory policy with one regular and one alternative supply channel for single item. The alternative supply channel can either be (i) a less-preferred substitute drug or (ii) an alternative supplier. In the former case, the substitute drug is typically more expensive or may not be clinically ideal. In the case of alternative supplier, a more expensive supplier is welcome if mainstream channel is unreliable, i.e., supply can be disrupted. In practice, when such disruptions occur, hospitals seek for another supplier or hospital to supply a drug, which typically is costly. See Fig. 1 for an illustration of our framework.

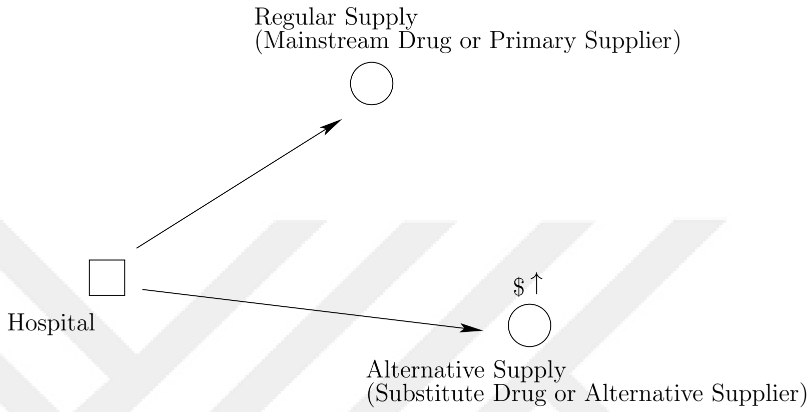

Figure 1: Two-supply channel model with a regular and an alternative channel

Notice that in this model the objective function is not minimize the expected total costs of the system. There are two main objectives of this study which consider different characteristics that can not be brought together in a single objective. One of them takes costs into account, whereas the other one does not include any calculations regarding costs. The first objective is *minimizing expected inventory level*, whereas the second objective is increasing quality of care through *minimization of expected alternative supply usage frequency* or and *order quantity*. We did not restrict ourselves to a better defined objective as the alternative channel usage can lead to a number of different issues that may need to be addressed differently. For instance an alternative channel usage might imply an uncertain (unstructured) leadtime or loss of manpower in a hospital during the search for an alternative supplier, a higher (per item) cost of supply, a higher cost of care due to medical consequences of using a non-ideal drug etc. Therefore, we propose a comprehensive multiobjective framework that sheds a light on most of these possible scenarios. The proposed model is developed under following assumptions:

- Regular supply channel observes shortages, but alternative supply channel is always available.
- Patients arrive at the hospital of interest in accordance with a Poisson process having rate λ.
- Supply disruptions on the regular supply channel occur according to a Poisson process with rate μ.
- Supply disruption periods are exponentially distributed with rate α.
- There is no lead time for both supply channels.
- The hospital has unlimited capacity.
- Fixed and variable ordering costs are disregarded. Our real-life observations suggest that fixed costs are actually zero and variable costs are reflected to patients, thus have no effect on our decisions.
- Drugs are delivered from either regular or alternative channel—order splitting is not allowed.
- A continuous-review inventory control policy is adopted where an order of size Q_1 will be placed through the regular channel when inventory level is at reorder level r_1 and regular supply is available.
- During a disruption period, an order of size Q_2 will be placed through the alternative channel when inventory level hits reorder level r_2.

Next, we define the mathematical model using a continuous time Markov chain that features these assumptions mentioned above.

3.2 Mathematical Model

In this section, we firstly present *a single reliable supplier model.*

In the light of our assumptions, the continuous time Markov chain for given values of Q and r is shown in Fig. 2. Using that the summation of limiting probabilities for

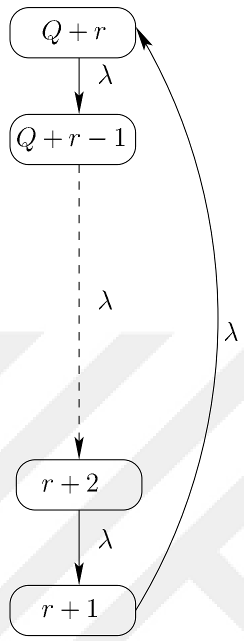

Figure 2: Transition rate diagram in a single reliable supplier model

all states is 1, i.e.,

$$\sum_{j=0}^{Q-1} P_{Q+r-j} = 1$$

$$\sum_{k=r+1}^{Q+r} P_k = 1$$

Let us solve the last equation and obtain P_k.

$$P_k = \frac{1}{Q} \qquad k = r+1, ..., r+Q$$

Secondly, we present *two-supply channel model with a reliable and an unreliable channel.*

A stochastic process that accounts for separate reorder point and order quantities for two supplier as well as disruption status for the regular supply channel is to be modeled. States are denoted by pairs where first value represents the number of drugs on hand and second value A or U indicate if regular supply channel is available or unavailable, respectively. In the light of our assumptions, the continuous time Markov chain for given values of Q_1, Q_2, and r_1 is shown in Fig. 3.

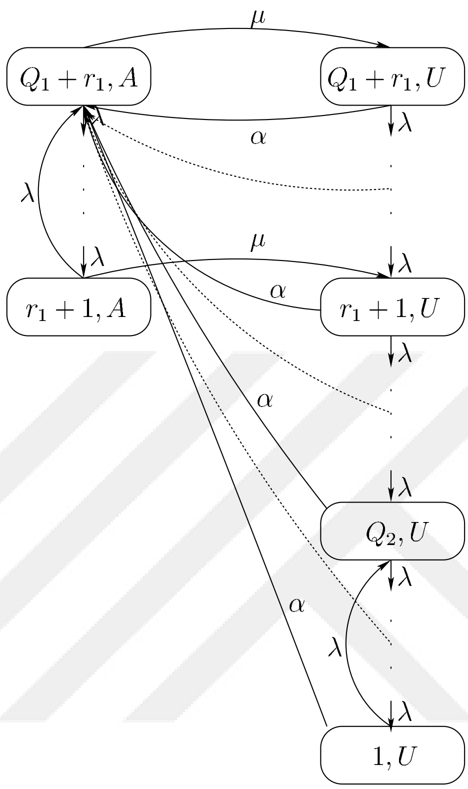

Figure 3: Transition rate diagram for the inventory model

It should be noted that this Markov chain assumes

- $r_2 = 0$,
- $Q_2 \leq r_1$.

First assumption is trivial; alternative supply is always available and minimum reorder point is preferred to ensure a lower stock level. Second assumption is for mathematical tractability purposes and ensures a certain structure on the Markov chain. Furthermore, real-life examples we have seen suggest that the alternative channel purchases are typically in smaller quantities due to unwillingness of other hospitals or warehouses in sharing drugs in larger quantities if a disruption is known to be present. In practice, the items that are available in those disruption periods hardly ever help hospitals achieve a higher level of inventory than their usual levels. Thus it is not unreasonable to assume the quantity Q_2 is a decision variable, but with a natural upper bound r_1, considering the possible scarcity of that drug.

The limiting probability $P_{Q_1+r_1,A}$ can be stated as follows:

$$\alpha \sum_{j=0}^{Q_1+r_1-1} P_{Q_1+r_1-j,U} + \lambda P_{r_1+1,A} = (\lambda+\mu) P_{Q_1+r_1,A} \tag{1}$$

To do generalization, we extract $P_{Q_1+r_1,U}$ and $P_{1,U}$ from the summation in equation (1)

$$\alpha \left(P_{Q_1+r_1,U} + P_{1,U}\right) + \alpha \sum_{k=2}^{Q_1+r_1-1} P_{k,U} + \lambda P_{r_1+1,A} = (\lambda+\mu) P_{Q_1+r_1,A}$$

Let us suppose that $\sum_{k=r_1+1}^{Q_1+r_1-1} P_{k,U} = SU_1$, $\sum_{k=Q_2+1}^{r_1} P_{k,U} = SU_2$, $\sum_{k=2}^{Q_2} P_{k,U} = SU_3$ and substitute these equalities in the last equation to obtain

$$\alpha \left(P_{Q_1+r_1,U} + P_{1,U} + SU_1 + SU_2 + SU_3\right) + \lambda P_{r_1+1,A} = (\lambda+\mu) P_{Q_1+r_1,A} \tag{2}$$

The limiting probabilities between $P_{Q_1+r_1-1,A}$ and $P_{r_1+1,A}$ can be stated as follows:

$$P_{Q_1+r_1-1-j,A} = \left(\frac{\lambda}{\lambda+\mu}\right)^{j+1} P_{Q_1+r_1,A} \qquad j = 0, ..., Q_1-2 \tag{3}$$

Let us substitute $Q_1 - 2$ for j in equation (3) to obtain

$$P_{r_1+1,A} = \left(\frac{\lambda}{\lambda+\mu}\right)^{Q_1-1} P_{Q_1+r_1,A} \tag{4}$$

To do generalization, we take the sum of geometric series in equation (3)

$$\sum_{j=0}^{Q_1-2} P_{Q_1+r_1-1-j,A} = \sum_{k=r_1+1}^{Q_1+r_1-1} P_{k,A} = \left[\frac{\lambda\left((\lambda+\mu)^{Q_1-1} - \lambda^{Q_1-1}\right)}{\mu(\lambda+\mu)^{Q_1-1}}\right] P_{Q_1+r_1,A} \tag{5}$$

Suppose that $\sum_{k=r_1+1}^{Q_1+r_1-1} P_{k,A} = SA$

$$SA = \left[\frac{\lambda\left((\lambda+\mu)^{Q_1-1} - \lambda^{Q_1-1}\right)}{\mu(\lambda+\mu)^{Q_1-1}}\right] P_{Q_1+r_1,A} \tag{6}$$

The limiting probability $P_{Q_1+r_1,U}$ can be stated as follows:

$$\mu\left(P_{Q_1+r_1,A}\right) = (\lambda+\alpha)\left(P_{Q_1+r_1,U}\right) \tag{7}$$

The limiting probabilities between $P_{Q_1+r_1-1,U}$ and $P_{r_1+1,U}$ can be stated as follows:

$$\mu\left(P_{Q_1+r_1-1-j,A}\right) + \lambda\left(P_{Q_1+r_1-j,U}\right) = (\lambda+\alpha)\left(P_{Q_1+r_1-1-j,U}\right) \tag{8}$$

$j = 0, ..., Q_1 - 2$

To do generalization, we take the sum of geometric series in the equation (8)

$$\frac{\mu}{\alpha}\sum_{j=0}^{Q_1-2} P_{Q_1+r_1-1-j,A} + \frac{\lambda}{\alpha}\left(P_{Q_1+r_1,U} - P_{r_1+1,U}\right) = \sum_{j=0}^{Q_1-2} P_{Q_1+r_1-1-j,U} \tag{9}$$

$$\sum_{k=r_1+1}^{Q_1+r_1-1} P_{k,U} = \frac{\mu}{\alpha}\sum_{k=r_1+1}^{Q_1+r_1-1} P_{k,A} + \frac{\lambda}{\alpha}\left(P_{Q_1+r_1,U} - P_{r_1+1,U}\right)$$

We can rearrange the last equation using equation (5) to obtain

$$\sum_{k=r_1+1}^{Q_1+r_1-1} P_{k,U} = \frac{\lambda}{\alpha}\left[\left(\frac{(\lambda+\mu)^{Q_1-1} - \lambda^{Q_1-1}}{(\lambda+\mu)^{Q_1-1}}\right) P_{Q_1+r_1,A} + P_{Q_1+r_1,U} - P_{r_1+1,U}\right] \tag{10}$$

Let us substitute the equality $\sum_{k=r_1+1}^{Q_1+r_1-1} P_{k,U} = SU_1$

$$SU_1 = \frac{\lambda}{\alpha}\left[\left(\frac{(\lambda+\mu)^{Q_1-1} - \lambda^{Q_1-1}}{(\lambda+\mu)^{Q_1-1}}\right) P_{Q_1+r_1,A} + P_{Q_1+r_1,U} - P_{r_1+1,U}\right] \tag{11}$$

The limiting probabilities between $P_{Q_1+r_1,A}$ and $P_{r_1+1,U}$ can be stated with the help of limiting probabilities between $P_{Q_1+r_1,U}$ and $P_{r_1+1,U}$ as follows:

From equation (7)

$$P_{Q_1+r_1,U} = \frac{\mu}{(\lambda+\alpha)} P_{Q_1+r_1,A} \tag{12}$$

From equation (8)

$$P_{Q_1+r_1-1,U} = \left[\left(\frac{\mu}{\lambda+\alpha}\right) P_{Q_1+r_1-1,A} + \left(\frac{\lambda}{\lambda+\alpha}\right) P_{Q_1+r_1,U}\right] \tag{13}$$

We can substitute $P_{Q_1+r_1-1,A}$ and $P_{Q_1+r_1,U}$ in terms of $P_{Q_1+r_1,A}$ in equation (13) by using equation (3) and equation (12) respectively to obtain

$$P_{Q_1+r_1-1,U} = \left(\frac{\mu}{\lambda+\alpha}\left(\frac{\lambda}{\lambda+\mu}\right) + \frac{\lambda}{\lambda+\alpha}\left(\frac{\mu}{\lambda+\alpha}\right)\right) P_{Q_1+r_1,A} \tag{14}$$

From equation (8)

$$P_{Q_1+r_1-2,U} = \left[\left(\frac{\mu}{\lambda+\alpha} \right) P_{Q_1+r_1-2,A} + \left(\frac{\lambda}{\lambda+\alpha} \right) P_{Q_1+r_1-1,U} \right] \tag{15}$$

We can substitute $P_{Q_1+r_1-2,A}$ and $P_{Q_1+r_1-1,U}$ in terms of $P_{Q_1+r_1,A}$ in equation (15) by using equation (3) and equation (14) respectively to obtain

$$P_{Q_1+r_1-2,U} = \left[\frac{\mu}{\lambda+\alpha} \left(\frac{\lambda}{\lambda+\mu} \right)^2 + \frac{\lambda}{(\lambda+\alpha)} \left(\frac{\mu}{\lambda+\alpha} \left(\frac{\lambda}{\lambda+\mu} \right) + \frac{\lambda}{\lambda+\alpha} \left(\frac{\mu}{\lambda+\alpha} \right) \right) \right] P_{Q_1+r_1,A}$$

$$P_{Q_1+r_1-2,U} = \frac{\mu\lambda^2}{(\lambda+\alpha)} \left[\frac{1}{(\lambda+\mu)^2} + \frac{1}{(\lambda+\alpha)(\lambda+\mu)} + \frac{1}{(\lambda+\alpha)^2} \right] P_{Q_1+r_1,A}$$

Generalization of equations can be stated as follows:

$$P_{Q_1+r_1-j,U} = \frac{\mu\lambda^j}{(\lambda+\alpha)} \left[\frac{1}{(\lambda+\alpha)^j} + \frac{1}{(\lambda+\alpha)^{j-1}(\lambda+\mu)^1} + \dots + \frac{1}{(\lambda+\alpha)^1(\lambda+\mu)^{j-1}} + \frac{1}{(\lambda+\alpha)^j} \right] P_{Q_1+r_1,A} \tag{16}$$

$j = 1, \dots, Q_1 - 1$

We can substitute $P_{r_1+1,A}$ in equation (16) to obtain

$$P_{r_1+1,U} = P_{Q_1+r_1,A} \times \frac{\mu\lambda^{Q_1-1}}{(\lambda+\alpha)} \left[\frac{1}{(\lambda+\alpha)^{Q_1-1}} + \frac{1}{(\lambda+\alpha)^{Q_1-2}(\lambda+\mu)^1} + \dots + \frac{1}{(\lambda+\alpha)^1(\lambda+\mu)^{Q_1-2}} + \frac{1}{(\lambda+\alpha)^{Q_1-1}} \right]$$

Let us rearrange this by using some mathematical formula

$$P_{r_1+1,U} = \frac{\mu\lambda^{Q_1-1}}{\lambda+\alpha} \left[\frac{\left(\frac{1}{\lambda+\mu}\right)^{Q_1} - \left(\frac{1}{(\lambda+\alpha)}\right)^{Q_1}}{\left(\frac{1}{\lambda+\mu}\right) - \left(\frac{1}{\lambda+\alpha}\right)} \right] P_{Q_1+r_1,A}$$

We solve the last equation and obtain $P_{r_1+1,U}$.

$$P_{r_1+1,U} = \frac{\mu}{(\lambda+\alpha)^{Q_1}} \left(\frac{\lambda}{\lambda+\mu} \right)^{Q_1-1} \frac{(\lambda+\alpha)^{Q_1} - (\lambda+\mu)^{Q_1}}{\alpha-\mu} P_{Q_1+r_1,A} \tag{17}$$

The limiting probabilities between $P_{r_1,U}$ and $P_{Q_2+1,U}$ can be stated as follows:

$$P_{r_1-j,U} = \left(\frac{\lambda}{\lambda+\alpha} \right)^{j+1} P_{r_1+1,U} \qquad j = 0, \dots, r_1 - Q_2 - 1 \tag{18}$$

We can set j equal to $r_1 - Q_2 - 1$ in equation (18) to obtain

$$P_{Q_2+1,U} = P_{r_1+1,U} \left(\frac{\lambda}{\lambda + \alpha} \right)^{r_1 - Q_2}$$

The limiting probability $P_{Q_2,U}$ can be stated as follows:

$$\lambda \left[P_{1,U} + P_{Q_2+1,U} \right] = (\lambda + \alpha) P_{Q_2,U}$$

We can substitute the equality for $P_{Q_2+1,U}$ in the last equation to obtain

$$\lambda \left[P_{1,U} + P_{r_1+1,U} \left(\frac{\lambda}{\lambda + \alpha} \right)^{r_1 - Q_2} \right] = (\lambda + \alpha) P_{Q_2,U} \tag{19}$$

To do generalization, we take the sum of geometric series in the equation (18)

$$\sum_{j=0}^{r_1 - Q_2 - 1} P_{r_1 - j,U} = \sum_{k=Q_2+1}^{r_1} P_{k,U} = \frac{\lambda}{\alpha} \left(\frac{(\lambda + \alpha)^{r_1 - Q_2} - \lambda^{r_1 - Q_2}}{(\lambda + \alpha)^{r_1 - Q_2}} \right) P_{r_1+1,U}$$

Let us substitute the equality $\sum_{k=Q_2+1}^{r_1} P_{k,U} = SU_2$, under the condition $r_2 = 0$.

$$SU_2 = \frac{\lambda}{\alpha} \left(\frac{(\lambda + \alpha)^{r_1 - Q_2} - \lambda^{r_1 - Q_2}}{(\lambda + \alpha)^{r_1 - Q_2}} \right) P_{r_1+1,U} \tag{20}$$

The limiting probabilities between $P_{Q_2,U}$ and $P_{2,U}$ can be stated as follows:

$$P_{2+j,U} = \left(\frac{\lambda + \alpha}{\lambda} \right)^{j+1} P_{1,U} \qquad\qquad j = 0, ..., Q_2 - 2 \tag{21}$$

We can substitute $P_{Q_2,U}$ in the last equation to obtain

$$P_{Q_2,U} = \left(\frac{\lambda + \alpha}{\lambda} \right)^{Q_2 - 1} P_{1,U} \tag{22}$$

To do generalization, we take the sum of geometric series in the equation (21)

$$\sum_{j=0}^{Q_2 - 2} P_{2+j,U} = \sum_{k=2}^{Q_2} P_{k,U} = \left[\frac{(\lambda + \alpha)^{Q_2} - (\lambda + \alpha) \lambda^{Q_2 - 1}}{\alpha \lambda^{Q_2 - 1}} \right] P_{1,U} \tag{23}$$

Let us substitute the equality $\sum_{k=2}^{Q_2} P_{k,U} = SU_3$ under the condition $r_2 = 0$.

$$SU_3 = \left[\frac{(\lambda + \alpha)^{Q_2} - (\lambda + \alpha) \lambda^{Q_2 - 1}}{\alpha \lambda^{Q_2 - 1}} \right] P_{1,U} \tag{24}$$

By using equation (19) and equation (22), we can establish the relationship between $P_{r_1+1,U}$ and $P_{1,U}$ in equation (25)

$$P_{1,U} = \frac{\lambda^{r_1}}{(\lambda+\alpha)^{r_1-Q_2}\left[(\lambda+\alpha)^{Q_2} - \lambda^{Q_2}\right]} P_{r_1+1,U} \qquad (25)$$

Using that the summation of limiting probabilities for all states is 1 as below.

$$P_{Q_1+r_1,A} + P_{r_1+1,A} + SA + (P_{Q_1+r_1,U} + P_{1,U} + SU_1 + SU_2 + SU_3) = 1$$

We can rearrange the last equation to state the limiting probabilities between $P_{Q_1+r_1,U}$ and $P_{1,U}$

$$P_{Q_1+r_1,U} + P_{1,U} + SU_1 + SU_2 + SU_3 = 1 - P_{Q_1+r_1,A} - P_{r_1+1,A} - SA$$

We can rewrite the right hand side in terms of $P_{Q_1+r_1,A}$ by using equation (4) and equation (6)

$$P_{Q_1+r_1,U} + P_{1,U} + SU_1 + SU_2 + SU_3 =$$
$$1 - P_{Q_1+r_1,A} - \left(\frac{\lambda}{\lambda+\mu}\right)^{Q_1-1} P_{Q_1+r_1,A} - \left[\frac{\lambda\left((\lambda+\mu)^{Q_1-1} - \lambda^{Q_1-1}\right)}{\mu(\lambda+\mu)^{Q_1-1}}\right] P_{Q_1+r_1,A} \qquad (26)$$

$P_{r_1+1,A}$ and SA is replaced in the equation above. By substituting the right hand side of the last equation in equation (2)

$$\alpha\left[1 - P_{Q_1+r_1,A} - \left(\frac{\lambda}{\lambda+\mu}\right)^{Q_1-1} P_{Q_1+r_1,A} - \left(\frac{\lambda\left((\lambda+\mu)^{Q_1-1} - \lambda^{Q_1-1}\right)}{\mu(\lambda+\mu)^{Q_1-1}}\right) P_{Q_1+r_1,A}\right]$$
$$+ \lambda P_{r_1+1,A} = (\lambda+\mu) P_{Q_1+r_1,A}$$

$P_{r_1+1,A}$ is replaced in the last equation with the aid of equation (4) to obtain

$$\alpha\left[1 - P_{Q_1+r_1,A} - \left(\frac{\lambda}{\lambda+\mu}\right)^{Q_1-1} P_{Q_1+r_1,A} - \left(\frac{\lambda\left((\lambda+\mu)^{Q_1-1} - \lambda^{Q_1-1}\right)}{\mu(\lambda+\mu)^{Q_1-1}}\right) P_{Q_1+r_1,A}\right]$$
$$+ \lambda\left(\frac{\lambda}{\lambda+\mu}\right)^{Q_1-1} P_{Q_1+r_1,A} = (\lambda+\mu) P_{Q_1+r_1,A}$$

Let us solve this to obtain $P_{Q_1+r_1,A}$.

$$P_{Q_1+r_1,A}=\frac{\mu\alpha\left(\lambda+\mu\right)^{Q_1-1}}{\mu\left(\lambda+\mu\right)^{Q_1-1}\left(\lambda+\mu+\alpha\right)+\mu\left(\alpha-\lambda\right)\lambda^{Q_1-1}-\alpha\lambda^{Q_1}} \tag{27}$$

$P_{Q_1+r_1,A}$ in equation (27) is replaced in equation (4) to obtain $P_{r_1+1,A}$.

$$P_{r_1+1,A}=\frac{\left(\frac{\lambda}{\lambda+\mu}\right)^{Q_1-1}\mu\alpha\left(\lambda+\mu\right)^{Q_1-1}}{\mu\left(\lambda+\mu\right)^{Q_1-1}\left(\lambda+\mu+\alpha\right)+\mu\left(\alpha-\lambda\right)\lambda^{Q_1-1}-\alpha\lambda^{Q_1}}$$

Rearranging this to obtain

$$P_{r_1+1,A}=\frac{\mu\alpha\lambda^{Q_1-1}}{\mu\left(\lambda+\mu\right)^{Q_1-1}\left(\lambda+\mu+\alpha\right)+\mu\left(\alpha-\lambda\right)\lambda^{Q_1-1}-\alpha\lambda^{Q_1}} \tag{28}$$

$P_{r_1+1,A}$ is found in equation (28).

$P_{Q_1+r_1,A}$ in equation (27) is replaced in equation (7) to obtain $P_{Q_1+r_1,U}$.

$$P_{Q_1+r_1,U}=\frac{\mu^2\alpha\left(\lambda+\mu\right)^{Q_1-1}}{\left(\alpha+\lambda\right)\left[\mu\left(\lambda+\mu\right)^{Q_1-1}\left(\lambda+\mu+\alpha\right)+\mu\left(\alpha-\lambda\right)\lambda^{Q_1-1}-\alpha\lambda^{Q_1}\right]} \tag{29}$$

$P_{Q_1+r_1,U}$ is found in equation (29).

$P_{Q_1+r_1,A}$ in equation (27) is replaced in equation (17) to obtain $P_{r_1+1,U}$.

$$P_{r_1+1,U}=\frac{\left(\lambda+\mu\right)^{Q_1-1}\left(\frac{\lambda}{\lambda+\mu}\right)^{Q_1-1}\mu^2\alpha\left[\left(\lambda+\alpha\right)^{Q_1}-\left(\lambda+\mu\right)^{Q_1}\right]}{\left(\alpha+\lambda\right)^{Q_1}\left(\alpha-\mu\right)\left[\mu\left(\lambda+\mu\right)^{Q_1-1}\left(\lambda+\mu+\alpha\right)+\mu\left(\alpha-\lambda\right)\lambda^{Q_1-1}-\alpha\lambda^{Q_1}\right]}$$

Rearranging this to obtain $P_{r_1+1,U}$ in equation (30).

$$P_{r_1+1,U}=\frac{\lambda^{Q_1-1}\mu^2\alpha\left[\left(\lambda+\alpha\right)^{Q_1}-\left(\lambda+\mu\right)^{Q_1}\right]}{\left(\alpha+\lambda\right)^{Q_1}\left(\alpha-\mu\right)\left[\mu\left(\lambda+\mu\right)^{Q_1-1}\left(\lambda+\mu+\alpha\right)+\mu\left(\alpha-\lambda\right)\lambda^{Q_1-1}-\alpha\lambda^{Q_1}\right]} \tag{30}$$

$P_{r_1+1,U}$ in equation (30) is replaced in equation (25) to obtain $P_{1,U}$.

$$P_{1,U}=\left(\frac{\lambda^{r_1+Q_1-1}}{\left[\left(\lambda+\alpha\right)^{Q_2}-\lambda^{Q_2}\right]\left(\lambda+\alpha\right)^{r_1-Q_2+Q_1}\left(\alpha-\mu\right)}\right)\left(\frac{\mu^2\alpha\left[\left(\lambda+\alpha\right)^{Q_1}-\left(\lambda+\mu\right)^{Q_1}\right]}{\mu\left(\lambda+\mu\right)^{Q_1-1}\left(\lambda+\mu+\alpha\right)+\mu\left(\alpha-\lambda\right)\lambda^{Q_1-1}-\alpha\lambda^{Q_1}}\right) \tag{31}$$

$P_{1,U}$ is found in equation (31).

$P_{1,U}$ in equation (31) is replaced in equation (22) to obtain $P_{Q_2,U}$.

$$P_{Q_2,U} = \left(\frac{\left(\frac{\lambda+\alpha}{\lambda}\right)^{Q_2-1} \lambda^{r_1+Q_1-1}}{\left[(\lambda+\alpha)^{Q_2} - \lambda^{Q_2}\right](\lambda+\alpha)^{r_1-Q_2+Q_1}(\alpha-\mu)} \right) \left(\frac{\mu^2\alpha\left[(\lambda+\alpha)^{Q_1} - (\lambda+\mu)^{Q_1}\right]}{\mu(\lambda+\mu)^{Q_1-1}(\lambda+\mu+\alpha) + \mu(\alpha-\lambda)\lambda^{Q_1-1} - \alpha\lambda^{Q_1}} \right)$$

Let us rearrange the last equation to obtain $P_{Q_2,U}$ in the equation (32)

$$P_{Q_2,U} = \left(\frac{\lambda^{r_1+Q_1-Q_2}}{\left[(\lambda+\alpha)^{Q_2} - \lambda^{Q_2}\right](\lambda+\alpha)^{r_1-2Q_2+Q_1+1}(\alpha-\mu)} \right) \left(\frac{\mu^2\alpha\left[(\lambda+\alpha)^{Q_1} - (\lambda+\mu)^{Q_1}\right]}{\mu(\lambda+\mu)^{Q_1-1}(\lambda+\mu+\alpha) + \mu(\alpha-\lambda)\lambda^{Q_1-1} - \alpha\lambda^{Q_1}} \right) \quad (32)$$

The limiting probabilities for states $P_{Q_1+r_1,A}$, $P_{r_1+1,A}$, $P_{Q_1+r_1,U}$, $P_{r_1+1,U}$, $P_{Q_2,U}$ and $P_{1,U}$ can be formulated as follows:

$$P_{Q_1+r_1,A} = \frac{\mu\alpha(\lambda+\mu)^{Q_1-1}}{\mu(\lambda+\mu)^{Q_1-1}(\lambda+\mu+\alpha) + \mu(\alpha-\lambda)\lambda^{Q_1-1} - \alpha\lambda^{Q_1}} \quad (33)$$

$$P_{r_1+1,A} = \frac{\mu\alpha\lambda^{Q_1-1}}{\mu(\lambda+\mu)^{Q_1-1}(\lambda+\mu+\alpha) + \mu(\alpha-\lambda)\lambda^{Q_1-1} - \alpha\lambda^{Q_1}} \quad (34)$$

$$P_{Q_1+r_1,U} = \frac{\mu^2\alpha(\lambda+\mu)^{Q_1-1}}{(\alpha+\lambda)\left[\mu(\lambda+\mu)^{Q_1-1}(\lambda+\mu+\alpha) + \mu(\alpha-\lambda)\lambda^{Q_1-1} - \alpha\lambda^{Q_1}\right]} \quad (35)$$

$$P_{r_1+1,U} = \frac{\lambda^{Q_1-1}\mu^2\alpha\left[(\lambda+\alpha)^{Q_1} - (\lambda+\mu)^{Q_1}\right]}{(\alpha+\lambda)^{Q_1}(\alpha-\mu)\left[\mu(\lambda+\mu)^{Q_1-1}(\lambda+\mu+\alpha) + \mu(\alpha-\lambda)\lambda^{Q_1-1} - \alpha\lambda^{Q_1}\right]} \quad (36)$$

$$P_{Q_2,U} = \left(\frac{\lambda^{r_1+Q_1-Q_2}}{\left[(\lambda+\alpha)^{Q_2} - \lambda^{Q_2}\right](\lambda+\alpha)^{r_1-2Q_2+Q_1+1}(\alpha-\mu)} \right) \left(\frac{\mu^2\alpha\left[(\lambda+\alpha)^{Q_1} - (\lambda+\mu)^{Q_1}\right]}{\mu(\lambda+\mu)^{Q_1-1}(\lambda+\mu+\alpha) + \mu(\alpha-\lambda)\lambda^{Q_1-1} - \alpha\lambda^{Q_1}} \right) \quad (37)$$

$$P_{1,U} = \left(\frac{\lambda^{r_1+Q_1-1}}{\left[(\lambda+\alpha)^{Q_2} - \lambda^{Q_2}\right](\lambda+\alpha)^{r_1-Q_2+Q_1}(\alpha-\mu)} \right)$$
$$\left(\frac{\mu^2\alpha\left[(\lambda+\alpha)^{Q_1} - (\lambda+\mu)^{Q_1}\right]}{\mu(\lambda+\mu)^{Q_1-1}(\lambda+\mu+\alpha) + \mu(\alpha-\lambda)\lambda^{Q_1-1} - \alpha\lambda^{Q_1}} \right) \tag{38}$$

Next, we construct the objective functions, given the limiting probabilities.

3.3 Objective Functions

In *a single reliable supplier model*, the objective is to minimize expected annual total cost. Let T(Q,r) denote expected annual total cost which is summation of expected holding cost and fixed cost defined as respectively H(Q,r) and K(Q,r). In this formulation h denotes holding cost, and k denotes fixed cost per item where $T(Q,r) = H(Q,r) + K(Q,r)$.

$$T(Q,r) = h\left[\sum_{j=0}^{Q-1}(Q+r-j)\,P_{Q+r-j}\right] + \frac{\lambda k}{Q} \tag{39}$$

Let us substitute the equality $P_{Q+r-j} = \frac{1}{Q}$ in equation (39) to obtain

$$T(Q,r) = h\left[\sum_{j=0}^{Q-1}(Q+r-j)\,\frac{1}{Q}\right] + \frac{\lambda k}{Q}$$

Let us solve the last equation and obtain

$$\begin{aligned} T(Q,r) &= \frac{h}{Q}\left[rQ + \frac{Q(Q+1)}{2}\right] + \frac{\lambda k}{Q} \\ &= h\left(r + \frac{Q+1}{2}\right) + \frac{\lambda k}{Q} \end{aligned} \tag{40}$$

Let us take its derivative and equalize to zero, which provides to reach Q^*.
Let us solve the equation $0 = \frac{\partial T(Q,r)}{\partial Q}$.

$$\frac{\partial T(Q,r)}{\partial Q} = \frac{h}{2} - \frac{\lambda k}{Q^2}$$
$$\frac{\lambda k}{Q^2} = \frac{h}{2}$$
$$Q^2 = \frac{2k\lambda}{h}$$
$$Q^* = \sqrt{\frac{2k\lambda}{h}}$$

Let us take derivative the expression (40) which provides to reach r^*.

$$\frac{\partial T(Q,r)}{\partial r} = h$$
$$r^* = 0$$

it is clear that r^* is equal to 0.

The objectives in *two-supply channel model with a reliable and an unreliable channel* are minimizing expected inventory level and minimizing alternative supply purchases. Besides, we investigate alternative supply order quantity and usage frequency separately. Despite the fact that multiple items share the same warehouse space in practice, we try to find the ideal levels in this model and understand a set of ideal behaviours for single item, thus we study an uncapacitated model.

Let $I(Q_1;Q_2;r_1)$, $SF(Q_1;Q_2;r_1)$, $SQ(Q_1;Q_2;r_1)$ denote expected inventory level, expected alternative supply monthly usage frequency and expected alternative supply monthly order quantity, respectively. Instead of a month, last two objectives can be computed annually, or with respect to another time unit that is appropriate considering the input. It should also be noted that expected inventory level is to be the same regardless of the time units and can be computed as follows:

$$I(Q_1;Q_2;r_1) = \sum_{j=0}^{Q_1-1} (Q_1 + r_1 - j)\, P_{Q_1+r_1-j,A} + \sum_{j=0}^{Q_1-1} (Q_1 + r_1 - j)\, P_{Q_1+r_1-j,U}$$
$$+ \sum_{j=0}^{r_1-Q_2-1} (r_1 - j)\, P_{r_1-j,U} + \sum_{j=1}^{Q_2} j P_{j,U} \qquad (41)$$

that can be rewritten as

$$I(Q_1;Q_2;r_1) = (Q_1+r_1)\,P_{Q_1+r_1,A} + \sum_{j=0}^{Q_1-2}(Q_1+r_1-1-j)\,P_{Q_1+r_1-1-j,A}$$
$$+\,(Q_1+r_1)\,P_{Q_1+r_1,U} + \sum_{j=0}^{Q_1-2}(Q_1+r_1-1-j)\,P_{Q_1+r_1-1-j,U}$$
$$+\sum_{j=0}^{r_1-Q_2-1}(r_1-j)\,P_{r_1-j,U} + \sum_{j=0}^{Q_2-2}(j+2)\,P_{j+2,U} + P_{1,U}. \tag{42}$$

To provide easy formulation, we rearrange the summation in equation (42) with probabilities $P_{Q_1+r_1-1-j,A}$, $P_{Q_1+r_1,U}$, $P_{Q_1+r_1-1-j,U}$, $P_{r_1-j,U}$ and $P_{j+2,U}$ by using equations (3), (12), (9), (18) and (21)

$$= \left[(Q_1+r_1) + (Q_1+r_1-1)\sum_{j=0}^{Q_1-2}\left(\frac{\lambda}{\lambda+\mu}\right)^{j+1} - \left(\frac{\lambda}{\lambda+\mu}\right)\sum_{j=0}^{Q_1-2} j\left(\frac{\lambda}{\lambda+\mu}\right)^{j}\right] P_{Q_1+r_1,A}$$
$$+\left[\left(\frac{\mu}{\lambda+\alpha}\right)(Q_1+r_1)\,P_{Q_1+r_1,A} + \left(\frac{\mu}{\alpha}\right)\sum_{j=0}^{Q_1-2}(Q_1+r_1-1-j)\,P_{Q_1+r_1-1-j,A}\right]$$
$$+\left[r_1\sum_{j=0}^{r_1-Q_2-1}\left(\frac{\lambda}{\lambda+\alpha}\right)^{j+1} - \left(\frac{\lambda}{\lambda+\alpha}\right)\sum_{j=0}^{r_1-Q_2-1} j\left(\frac{\lambda}{\lambda+\alpha}\right)^{j}\right] P_{r_1+1,U}$$
$$P_{1,U}\left[\left(\frac{\lambda+\alpha}{\lambda}\right)\sum_{j=0}^{Q_2-2} j\left(\frac{\lambda+\alpha}{\lambda}\right)^{j} + 2\sum_{j=0}^{Q_2-2}\left(\frac{\lambda+\alpha}{\lambda}\right)^{j+1} + 1\right]$$

We rearrange this summation by substituting the summation $\sum_{k=r_1+1}^{Q_1+r_1-1} P_{k,A}$ in equation (5) and the probability $P_{j+2,U}$ in equation (23). Additionally, rest of the multiplications in this equation can be found with the aid of derivative of geometric series.

$$
\begin{aligned}
I(Q_1;Q_2;r_1) &= (Q_1+r_1)\,P_{Q_1+r_1,A} + (Q_1+r_1-1)\left[\frac{\lambda\left((\lambda+\mu)^{Q_1-1}-\lambda^{Q_1-1}\right)}{\mu\,(\lambda+\mu)^{Q_1-1}}\right]P_{Q_1+r_1,A} \\
&- \left[\frac{\lambda^2\left[\left((\lambda+\mu)^{Q_1-1}-\lambda^{Q_1-1}\right)-Q_1\lambda^{Q_1-2}\mu\right]}{\mu\,(2\lambda+\mu)\,(\lambda+\mu)^{Q_1-1}}\right]P_{Q_1+r_1,A} \\
&+ \left(\frac{\mu}{\lambda+\alpha}\right)(Q_1+r_1)\,P_{Q_1+r_1,A} + (Q_1+r_1-1)\left(\frac{\mu}{\alpha}\right)\left[\frac{\lambda\left((\lambda+\mu)^{Q_1-1}-\lambda^{Q_1-1}\right)}{\mu\,(\lambda+\mu)^{Q_1-1}}\right]P_{Q_1+r_1,A} \\
&- \left(\frac{\mu}{\alpha}\right)\left[\frac{\lambda^2\left((\lambda+\mu)^{Q_1-1}-\lambda^{Q_1-1}\right)-Q_1\lambda^{Q_1-2}\mu}{\mu\,(2\lambda+\mu)\,(\lambda+\mu)^{Q_1-1}}\right]P_{Q_1+r_1,A} \\
&+ r_1\left[\frac{\lambda}{\alpha}\left(\frac{(\lambda+\alpha)^{r_1-Q_2}-\lambda^{r_1-Q_2}}{(\lambda+\alpha)^{r_1-Q_2}}\right)\right]P_{r_1+1,U} \\
&- \left[\frac{\lambda^2\left[\left((\lambda+\alpha)^{r_1-Q_2}-\lambda^{r_1-Q_2}\right)-\alpha\,(r_1-Q_2)\,\lambda^{r_1-Q_2-1}\right]}{\alpha\,(2\lambda+\alpha)\,(\lambda+\alpha)^{r_1-Q_2}}\right]P_{r_1+1,U} \\
&+ P_{1,U}\left[\frac{(\lambda+\alpha)^2\left(\lambda^{Q_2-1}-(\lambda+\alpha)^{Q_2-1}+(Q_2-1)\,\alpha\,(\lambda+\alpha)^{Q_2-2}\right)}{-(2\lambda+\alpha)\,\lambda^{Q_2-1}}\right] \\
&+ P_{1,U}\left[2\left(\frac{(\lambda+\alpha)^{Q_2}-(\lambda+\alpha)\,\lambda^{Q_2-1}}{\lambda^{Q_2-1}}\right)+1\right] \qquad (43)
\end{aligned}
$$

$I(Q_1;Q_2;r_1)$ is found in equation (43).

Alternative supply purchase calculations and respective objective functions are relatively easier to compute. The number of times alternative supply channel is used is synonymous to finding the rate of traversing arc $(1,U)$ to (Q_2,U). That is equal to the limiting probability of node $(1,U)$ multiplied by the rate over the corresponding arc, i.e.,

$$SF(Q_1;Q_2;r_1) = \lambda P_{1,U}. \qquad (44)$$

For expected alternative supply order quantity, the equation in (44) should be multiplied by the order size as follows:

$$SQ(Q_1;Q_2;r_1) = \lambda\,(Q_2)\,P_{1,U} \qquad (45)$$

In the following section, we explain our solution and generalization approach for the mathematical model together with insights obtained from the numerical study.

CHAPTER IV

OPTIMIZATION STUDY

In this chapter, we first present data instances generated and how we solve those. Next, we use well-known data mining tools to learn from instances so that generalized business rules and insights are obtained. Also, we remind that we only focus on *two-supply channel model with a reliable and an unreliable channel* in this chapter.

Test problems for the optimization study are generated using all combinations of the parameter values shown in Table 1 and we end up with 12 instances. Note that rates used in our study are *monthly rates that are generated based on real data* [24] *considering slow/fast recovery cases, rare/frequent shortages, and high/med/low demands observed in critical drugs.* We enumerate all possible values of Q_1, Q_2 and r_1 considering the annual demand conditions mandated by the Markov chain, i.e., $Q_2 \leq r_1$. Upper bounds on order quantities and reorder point are annual demands of 12λ.

Our Study			**Saedi et al., 2016**		
Recovery Rate (α) (per month)	Demand Rate (λ) (per month)	Shortage Rate (μ) (per month)	Recovery Rate (α) (months)	Demand Rate (λ) (items/day)	Shortage Rate (μ) (shortages/year)
1/6 Slow recovery (lasts half a year on avg.)	**2** Low	**1/12** Rare occurence (once a year on avg.)	**10** Slow recovery	**0.06** Low	**0** Rare occurence
1 Fast recovery (lasts a month on avg.)	**12** Med **24** — High	**1/3** Frequent (once every four months on avg.)	**1** Fast recovery	**28.25** Med **152** High	**1** Medium **2** — Frequent occurence

Table 1: Rates used in the model with comparing real data

Here, we consider two multi objective problems: *(i) minimizing expected inventory level and expected alternative supply usage frequency, and (ii) minimizing expected inventory level and expected alternative supply order quantity.* As stated, we perform complete enumeration for all possible values of decision variables, that is inventory parameters. We identify the set of non-dominated solutions that constitute the *efficient frontier*. These can be seen in Appendix (A). The red points show the dominated solutions while the blue ones indicate the non-dominated solutions that are preferred. These non-dominated solutions show a nonlinear trade-off between two objectives considered. This nonlinear trade-off reflected by efficient frontiers' shapes in blue, vary depending on the input parameters. Note that the number of non-dominated solutions is relatively low compared to dominated solutions. Because it is not easy to derive a conclusion based on these figures (such as a set of decision variable values that are always non-dominated), we utilize classification techniques to come up with decision rules.

4.1 Approximating Objective Functions via Regression

Based on the experimental study of complete enumeration, we can say that there exists an exponential negative relationship between the two objective functions; which are expected inventory level and alternative supply expected usage frequency or alternative supply expected quantity ordered as shown in Appendix (A). Alternative supply expected usage frequency can be used instead of alternative supply expected quantity ordered as dependent variable and these individually could be a function of the inventory level. The expected inventory level and alternative supply expected usage frequency or order quantity are expected to be minimized because lower stock level and less use of alternative channel are desired. These two objectives conflict with each other as alternative channel is used when inventory is depleted during disruptions. Therefore, an efficient frontier is obtained that consist of non-dominated solutions. Different combinations of expected inventory level produce different levels of alternative supply expected usage frequency or order quantity. The efficient frontier represents the best of these combinations those that produce the minimum expected usage frequency for a given minumum level of inventory. The graphs show how many points have created the efficient frontier. The relation between the decision variables and the objective functions have been formulated with in equation (43), (44) and (45). At the same time, these equations express the objective functions as combination of the decision variables. We used regression analysis to explore whether the decision variables, i.e., Q_1, Q_2, r_1, may explain the objective functions as predictors. Moreover, we compare the explanatory power of regression models that contain different numbers of predictors by using adjusted R^2. That would also eliminate the necessity of using a number of highly nonlinear and nonconvex functions, thus might lead to closed form (approximate) solutions. Despite the fact that this approach is not theoretically interesting, it would be extremely useful from a practical standpoint. Therefore, we first performed a regression analysis to investigate this relation.

Simple linear regression is performed on R software [25] and decent adjusted R^2 values for our objective function regressors are obtained. In general, $0 \leq R^2 \leq 1$, and the larger the value of adjusted R^2, the better the model fits the data. Adjusted R^2 does not indicate whether a regression model is adequate. We can have a low adjusted R^2 value for a good model, or a high adjusted R^2 value for a model that does not fit the data. Determining adjusted R^2 value is to structure of the problem how is difficult to solution of problem. Thus, adjusted R^2 can be provided in terms of difficulties of problem. Consequently, threshold value have been chosen as %80 as approach in this study is readily soluble problem. Results for expected inventory level, alternative supply usage frequency, and alternative supply order quantity are shown in Tables 2, 4, and 5. Respective linear equations for these regression analyses are also shown in Tables 3 and 6. Due to weakness of the linear regression results for shortage frequency shown in Table 4, logarithmic and polynomial regression are also employed and equations with high adjusted R^2 values are presented in (46).

4.1.1 Expected Monthly Inventory Level Regressors

Instance	Recovery α	Demand λ	Shortage μ	Linear func. Adjusted R^2
1	1/6	2	1/12	**0.999**
2	1/6	2	1/3	**0.9945**
3	1	2	1/12	**0.9996**
4	1	2	1/3	**0.998**
5	1/6	12	1/12	**0.9988**
6	1/6	12	1/3	**0.9923**
7	1	12	1/12	**0.9995**
8	1	12	1/3	**0.9972**
9	1/6	24	1/12	**0.9988**
10	1/6	24	1/3	**0.9925**
11	1	24	1/12	**0.9997**
12	1	24	1/3	**0.9977**

Table 2: Linear Regression Adjusted R^2 Results for Expected Monthly Inventory Level

Linear regression results for all combinations of α, μ and λ are shown in Table 2, where the expected inventory level is the dependent variable. High adjusted R^2 values imply that the expected inventory level can be predicted from the independent variable, i.e., Q_1, Q_2 and r_1. Therefore, linear equations for all instances are meaningful, and are presented in Table 3.

4.1.2 Alternative Supply Expected Usage Frequency Regressors

Adjusted R^2 values for linear and polynomial regression for expected alternative supply usage frequency as the dependent variable are shown in Table 4. It can be seen that these values are high only for instances 1 and 2. This means that alternative supply usage frequency can be explained by Q_1, Q_2 and r_1 values and the dependent variable can be modelled as a second degree polynomial only for 2 cases. Nevertheless,

Instance	Linear Regression Function
1	$y_{inv} = -1.39939 + 0.54879Q_1 + 0.92244r_1 + 0.02702Q_2$
2	$y_{inv} = -3.92367 + 0.68680Q_1 + 0.86052r_1 + 0.05061Q2$
3	$y_{inv} = -0.0771070 + 0.5875389Q_1 + 0.9997017r_1 + 0.0003949Q_2$
4	$y_{inv} = -1.273317 + 0.783101Q_1 + 0.999284r_1 + 0.001114Q_2$
5	$y_{inv} = -10.47182 + 0.54517Q_1 + 0.92169r_1 + 0.02436Q_2$
6	$y_{inv} = -23.73671 + 0.67406Q_1 + 0.85224r_1 + 0.05476Q_2$
7	$y_{inv} = -2.373458 + 0.582625Q_1 + 0.999691r_1 + 0.000804Q_2$
8	$y_{inv} = -8.080 + 0.7743Q_1 + 0.9988r_1 + 0.00006.343Q_2$
9	$y_{inv} = -14.32181 + 0.50934Q_1 + 0.88989r_1 + 0.04116Q_2$
10	$y_{inv} = -32.72157 + 0.58475Q_1 + 0.79097r_1 + 0.08021Q_2$
11	$y_{inv} = -3.3643227 + 0.5614046Q_1 + 0.9991289r_1 + 0.0004094Q_2$
12	$y_{inv} = -12.500016 + 0.721169Q_1 + 0.997657r_1 + 0.001331Q_2$

Table 3: Linear Regression Functions for Expected Monthly Inventory Level

Instance	Recovery α	Demand λ	Shortage μ	Linear Adjusted R^2	Logarithmic Adjusted R^2	Polynomial Adjusted R^2
1	1/6	2	1/12	0.6923	0.6957	**0.8719**
2	1/6	2	1/3	0.6581	0.7442	**0.8242**
3	1	2	1/12	0.2422	0.3033	0.4924
4	1	2	1/3	0.1956	0.2811	0.3797
5	1/6	12	1/12	0.3378	0.4186	0.5723
6	1/6	12	1/3	0.2992	0.2702	0.5194
7	1	12	1/12	0.1133	0.1764	0.2955
8	1	12	1/3	0.07121	0.211	0.1752
9	1/6	24	1/12	0.2342	0.1359	0.4079
10	1/6	24	1/3	0.2304	0.1936	0.4211
11	1	24	1/12	0.1153	0.185	0.2818
12	1	24	1/3	0.09722	0.1794	0.2297

Table 4: Regression Results for Alternative Supply Expected Usage Frequency

linear results are weaker than polynomial results. Therefore, corresponding polynomial regressors for alternative supply expected usage frequency are shown below:

$$\begin{aligned} y_{sf} =& 0.1601 - 0.00106Q_1 - 0.009066r_1 - 0.00636Q_2 \\ &+ 0.000007374(Q_1)^2 + 0.0002086(r_1)^2 + 0.000216(Q_2)^2 \\ y_{sf} =& 0.3093 - 0.002338Q_1 - 0.01714r_1 - 0.01243Q_2 \\ &- 0.000002816(Q_1)^2 + 0.0003978(r_1)^2 + 0.0004271(Q_2)^2 \end{aligned} \tag{46}$$

4.1.3 Alternative Supply Expected Quantity Ordered Regressors

Instance	Recovery α	Demand λ	Shortage μ	Linear Adjusted R^2	Logarithmic Adjusted R^2	Polynomial Adjusted R^2
1	1/6	2	1/12	**0.9282**	0.6229	**0.9473**
2	1/6	2	1/3	**0.9124**	0.6336	**0.9288**
3	1	2	1/12	0.3443	0.3885	0.5756
4	1	2	1/3	0.2767	0.3533	0.4459
5	1/6	12	1/12	**0.9178**	0.6354	**0.9407**
6	1/6	12	1/3	**0.8944**	0.6389	**0.9116**
7	1	12	1/12	0.2475	0.3529	0.4531
8	1	12	1/3	0.17	0.3475	0.3193
9	1/6	24	1/12	**0.9459**	0.6139	**0.9598**
10	1/6	24	1/3	**0.9396**	0.6173	**0.9506**
11	1	24	1/12	0.3575	0.4577	0.568
12	1	24	1/3	0.3065	0.434	0.4739

Table 5: Regression Results for Alternative Supply Expected Quantity Ordered

Next, alternative supply order quantity being the dependent variable, adjusted R^2 values for linear regression and polynomial regression for all the combinations of α, μ and λ are shown in Table 5. Results are high only for instances 1, 2, 5, 6, 9, and 10, which means that alternative supply order quantity can be explained by Q_1, Q_2 and r_1 values and the dependent variable can be modelled as a second degree polynomial only for these instances. We present linear equations in Table 6, because polynomial regression results are not highly better than linear regression results, and first degree equations are easier to interpret.

Instance	Linear Regression Function
1	$y_{sq} = 0.421452 - 0.006473Q_1 - 0.012949r_1 + 0.004496Q_2$
2	$y_{sq} = 0.813150 - 0.016132Q_1 - 0.023281r_1 + 0.008194Q_2$
5	$y_{sq} = 2.525766 - 0.006460Q_1 - 0.013313r_1 + 0.004407Q_2$
6	$y_{sq} = 4.970897 - 0.017093Q_1 - 0.024669r_1 + 0.008787Q_2$
9	$y_{sq} = 5.920876 - 0.009193Q_1 - 0.018360r_1 + 0.006890Q_2$
10	$y_{sq} = 11.72079 - 0.02291Q_1 - 0.03487r_1 + 0.01318Q_2$

Table 6: Linear Equations for Alternative Supply Expected Quantity Ordered

In addition to the results presented above, we investigate regressors for expected inventory level, alternative supply order frequency, and order quantity where inventory parameters (i.e., $Q_1, Q_2,\ r_1$) and α, μ and λ are all included as independent variables and all data instances are considered together. Adjusted R^2 values for linear, logarithmic and polynomial regression are presented for all regressors in Table 7.

Dependent Variable	**Linear Adjusted** R^2	**Logarithmic Adjusted** R^2	**Polynomial Adj.** R^2
Expected Monthly Inventory Level	**0.9819**	**0.8560**	**0.9821**
Alternative Supply Expected Usage Frequency	0.1824	0.0015	0.2498
Alternative Supply Expected Quantity Ordered	0.6093	0.1336	0.6107

Table 7: Regression Adjusted R^2 Results with All regressors for All Dependent Variable

Results are high only when expected inventory level is the dependent variable. In addition, polynomial results are marginally better than linear results. Therefore, linear equation with all instances is obtained for the expected inventory level as follows:

$$\begin{aligned} y_{inv} =& -12.72672 + 0.59724Q_1 + 0.90857r_1 + 0.03094Q_2 \\ &+ 28.46543\alpha - 0.67283\lambda - 12.90575\mu \end{aligned} \tag{47}$$

Next, alternative supply expected order quantity and frequency are not strongly predictable. However, alternative supply expected order quantity has a relatively better representation. It may be useful to note the linear regressor for alternative supply expected order quantity as follows:

$$\begin{aligned} y_{sq} =& 0.882484 - 0.008287Q_1 - 0.013637r_1 + 0.005205Q_2 \\ &- 2.305888\alpha + 0.186718\lambda + 2.423232\mu \end{aligned} \tag{48}$$

It is mentioned in the mathematical model description that alternative supply expected order quantity is obtained by multiplying alternative supply expected order frequency function by Q_2. Since linear regression analysis for order quantity results in better adjusted R^2 values, it can be stated that dividing linear regression functions for order quantity by Q_2 will obviously provide a decent measure for expected alternative supply order frequency.

4.2 Classification of Dominating and Non-dominating Solutions

The regression performance is quite high to approximate expected inventory level but poor on the alternative channel usage, therefore, in an effort to derive business rules we seek for an alternative approach. We employ classification tools to characterize a discriminating function between set of dominating and non-dominating solutions. For classification, we use Support Vector Machines (SVMs) [26] due to its appropriate structure for both linear and nonlinear analysis. Support Vector Machines which are developed by [27] is one of the simple and effective methods in classification techniques. If the training data are linearly separable, it is possible to seperate two groups in a hyperplane by drawing a boundary for classification. The place, which this boundary will be drawn, should be near both groups points where there is the highest distance between them. To begin with, all the datasets are *normalized* by dividing Q_1,Q_2 and r_1 values by the demand rate λ. With the help of this, more accurate datasets for linear and nonlinear classification are observed and we denote normalized respective inventory parameters with Q_1^N,Q_2^N and r_1^N.

4.2.1 Minimizing expected inventory level and expected alternative supply usage frequency

10-fold cross validation (CV) technique is used with a heavy penalty factor (C) of 1000 to avoid misclassifications, if possible. A **CV accuracy of 80.20%** is obtained, which is considered acceptable for generalization. We obtain the following classifier which provides a **training accuracy of 92.38%** for the entire dataset. Note that halfspace denoted by (49) contains instances that are **dominated** and (50) contains instances that are **non-dominated** (more desirable).

$$1.8162Q_1^N - 0.1279r_1^N - 0.2494Q_2^N - 3.7041\alpha + 0.1763\lambda - 0.6728\mu \geq 3.7927 \quad (49)$$

$$1.8162Q_1^N - 0.1279r_1^N - 0.2494Q_2^N - 3.7041\alpha + 0.1763\lambda - 0.6728\mu \leq 3.7927 \quad (50)$$

We performed polynomial classification using Kernel functions with a set of parameters that are presented together with the corresponding CV accuracies in Table 8. Soft margin parameter C is set to be 1000 as that value usually provided a better cross validation accuracy. Combinations of the following parameters are used for the Kernel function: Polynomial degree = $(2, 3)$, $\gamma = (1/6, 1, 6)$, Coefficient = $(0, 1, 2)$. As can be seen from the table, coefficients 1 and 2 with a degree of 2 results in relatively high accuracy compared to other parameter values and the classifier performances are quite insensitive to γ parameter values.

4.2.2 Minimizing expected inventory level and expected alternative supply order quantity

10-fold cross validation (CV) technique is used with a heavy penalty factor (C) of 1000 to avoid misclassifications, if possible. A **CV accuracy of 97.78%** is obtained, which is considered *excellent* for generalization. We obtain the following classifier which provides an extremely high **training accuracy of 98.16%** for the entire dataset. Note that halfspace denoted by (51) contains instances that are **dominated** and (52) contains instances that are **non-dominated** (more desirable).

$$0.5195Q_1^N - 0.1372r_1^N + 6.4336Q_2^N + 2.9536\alpha + 0.2068\lambda + 2.9179\mu \geq -7.4321 \quad (51)$$

$$0.5195Q_1^N - 0.1372r_1^N + 6.4336Q_2^N + 2.9536\alpha + 0.2068\lambda + 2.9179\mu \leq -7.4321 \quad (52)$$

Classifier	**Degree**	γ	**Coefficient**	**Accuracy**
1	2	1/6	0	87.5383
2	2	1	0	86.9113
3	2	6	0	86.3538
4	3	1/6	0	78.5251
5	3	1	0	75.6804
6	3	6	0	82.3887
7	2	1/6	1	**94.0916**
8	2	1	1	**94.43**
9	2	6	1	**93.6284**
10	3	1/6	1	89.6687
11	3	1	1	86.4767
12	3	6	1	**91.2825**
13	2	1/6	2	**93.9865**
14	2	1	2	**93.2152**
15	2	6	2	**93.9829**
16	3	1/6	2	84.0417
17	3	1	2	89.6117
18	3	6	2	**91.27**

Table 8: 10-fold cross validation results for SVM with polynomial kernel to discriminate dominating instances in for expected inventory level and expected alternative supply usage frequency

For the sake of completeness, we also performed polynomial classification using Kernel functions with a set of parameters that are presented together with the corresponding CV accuracies in Table 9. Soft margin parameter C is again set to 1000. Combinations of the following parameters are used for the Kernel function: Polynomial degree = $(2, 3)$, $\gamma = (1/6, 1, 6)$, Coefficient = $(0, 1, 2)$.

Next, we provide managerial insights based on our numerical analyses.

4.3 Insights

In this section, we summarize the key results obtained from this numerical study, which can be generalized for any real life problem that has the same framework. In the light of our study, if a hospital's inventory manager would like to simultaneously minimize its inventory level and number of items purchased through the alternative

Classifier	**Degree**	γ	**Coefficient**	**Accuracy**
1	2	1/6	0	**97.3548**
2	2	1	0	**93.4343**
3	2	6	0	**97.3976**
4	3	1/6	0	83.5714
5	3	1	0	66.3021
6	3	6	0	77.9996
7	2	1/6	1	**98.4058**
8	2	1	1	**98.4271**
9	2	6	1	**95.5949**
10	3	1/6	1	79.0114
11	3	1	1	85.6804
12	3	6	1	89.1539
13	2	1/6	2	**98.3719**
14	2	1	2	**98.2009**
15	2	6	2	**97.6826**
16	3	1/6	2	**96.5497**
17	3	1	2	80.8835
18	3	6	2	87.2444

Table 9: 10-fold cross validation results for SVM with polynomial kernel to discriminate dominating instances in for expected inventory level and expected alternative supply order quantity.

channel, we provide *strong evidence* that solutions that satisfy

$$0.5195Q_1 - 0.1372r_1 + 6.4336Q_2 + 2.9536\alpha\lambda + 0.2068\lambda^2 + 2.9179\mu\lambda \leq -7.4321\lambda$$

are more desirable where α, λ, and μ denote (monthly) recovery, demand, and shortage rates respectively. This implies that for about 4 units of increase in the reorder point, the order quantity from the main supplier can be increased by one. On the other hand, the order quantity from the alternative channel is quite resistant to increase, which is in line with our expectations, considering the objective functions.

We also provide *enough evidence* that shows solutions that satisfy

$$1.8162Q_1 - 0.1279r_1 - 0.2494Q_2 - 3.7041\alpha\lambda + 0.1763\lambda^2 - 0.6728\mu\lambda \leq 3.7927\lambda$$

are more desirable for a hospital where its inventory level and number of times an alternative channel are used to be simultaneously minimized. This change in the behavior of alternative channel order quantity can be explained by the change in the second objective from quantity to frequency. In order to reduce the alternative channel order frequency, higher Q_2 values are more desirable. This is coupled with a harder to increase Q_1 variable so that expected inventory levels are minimized as well.

The associated expected inventory level can be *closely* approximated using

$$\begin{aligned} y_{inv} = & -12.72672 + 0.59724Q_1 + 0.90857r_1 + 0.03094Q_2 \\ & + 28.46543\alpha - 0.67283\lambda - 12.90575\mu \end{aligned}$$

This function, despite being hard to foresee, is not counterintuitive. An increase in reorder point has one to one effect on the average inventory level. Order quantity from the regular supplier has a direct but relatively small effect, considering the constant demand on drugs. Alternative supply order quantity has a minimal effect, which can be explained by the relatively low fraction of time it is in effect.

Finally, the expected number of items purchased through the alternative channel during a month can be *loosely* approximated using

$$\begin{aligned} y_{sq} = & 0.882484 - 0.008287Q_1 - 0.013637r_1 + 0.005205Q_2 \\ & - 2.305888\alpha + 0.186718\lambda + 2.423232\mu \end{aligned} \tag{53}$$

which can also be divided into Q_2 to obtain number of times an alternative channel is used. Here, regular supply order quantity and reorder point has an inverse effect on the alternative channel usage.

We also provide evidence that, despite being slightly better, nonlinear classifiers and regressors are not significantly better than their linear counterparts, hence not worth investigating. Furthermore, they cannot be interpreted in closed form in the case of classification, which is a major drawback considering the insights that can be obtained from the aforementioned inequalities.

CHAPTER V

CONCLUSION

In this study, a novel stochastic model for hospitals' inventory management with a continuous review policy under supply disruption is presented. Supplies can be received from two suppliers, where regular supplier is open to disruptions and alternative supplier is always available. Shortage and recovery processes for the regular supplier and the inventory levels are modeled as a continuous time Markov chain. The two conflicting objectives in this model are minimization of expected inventory level and purchase of alternative supply, which is measured through expected alternative supply usage frequency and order quantity.

To the best of our knowledge, there exists no models considering supply disruption for one of the suppliers and includes the second supplier as a backup to prevent possible supply shortages. Due to highly stochastic nature of the problem, approximation of results are needed, thus regression techniques are employed to provide necessary analysis on the outcomes of model under different scenarios. Moreover, we employ classification tools to provide key functions for healthcare inventory managers that sheds a light on how inventory related parameters should be tuned.

There exists further directions for the proposed study. To begin with, problem setting can be made more realistic by extending current background into a two hospitals and two suppliers case, where transshipment between hospitals is also allowed. In addition, warehouse capacity constraints for suppliers can be introduced to the model. Moreover, analysis performed in this paper can be made for multiple items or with the existence of lead time under certain conditions.

APPENDIX A

FIGURES

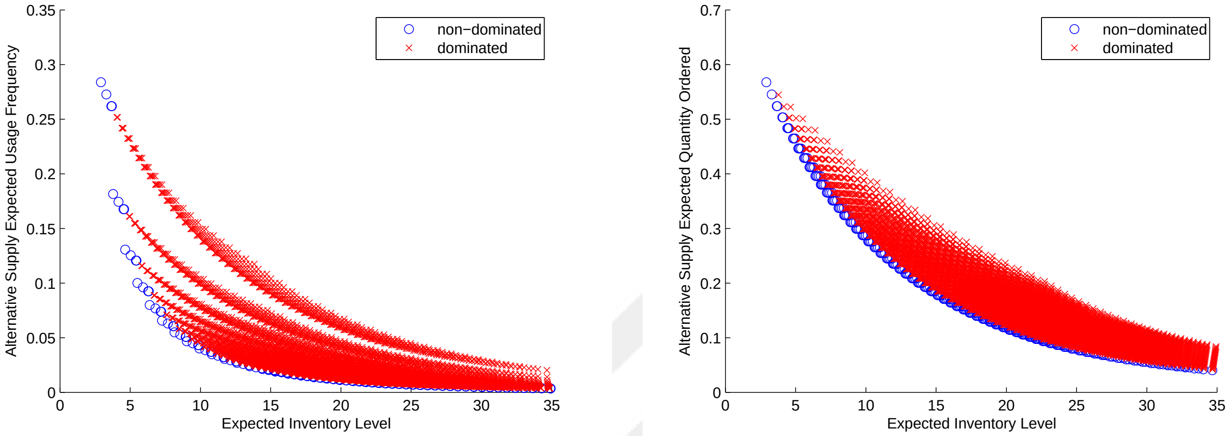

Figure 4: Illustration of solutions for $\alpha = 1/6$ $\lambda = 2$ $\mu = 1/12$.

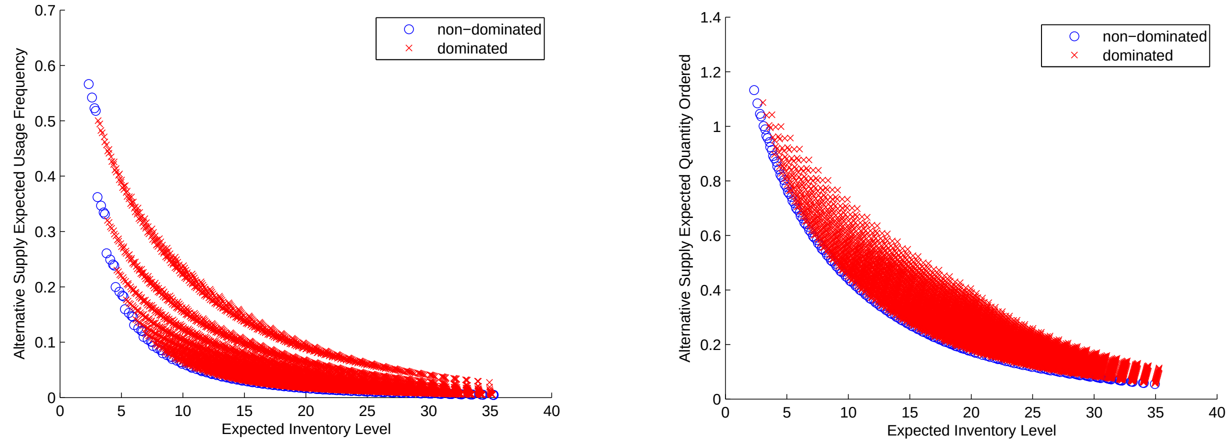

Figure 5: Illustration of solutions for $\alpha = 1/6$ $\lambda = 2$ $\mu = 1/3$

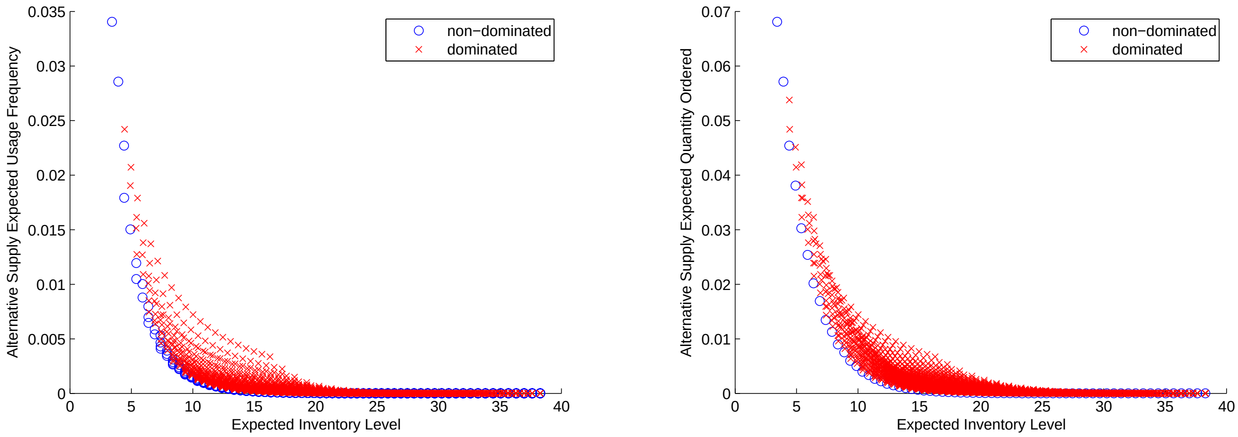

Figure 6: Illustration of solutions for $\alpha = 1$ $\lambda = 2$ $\mu = 1/12$

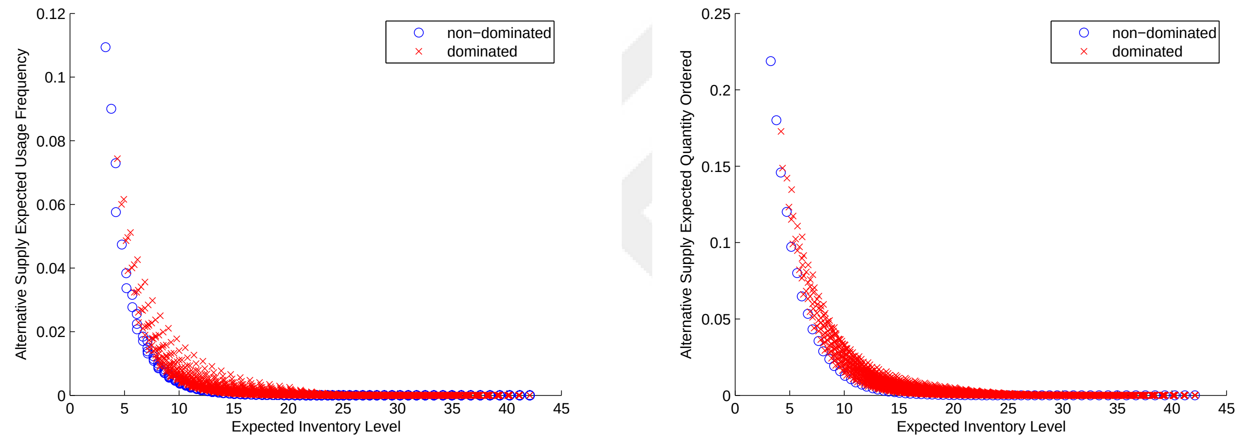

Figure 7: Illustration of solutions for $\alpha = 1$ $\lambda = 2$ $\mu = 1/3$

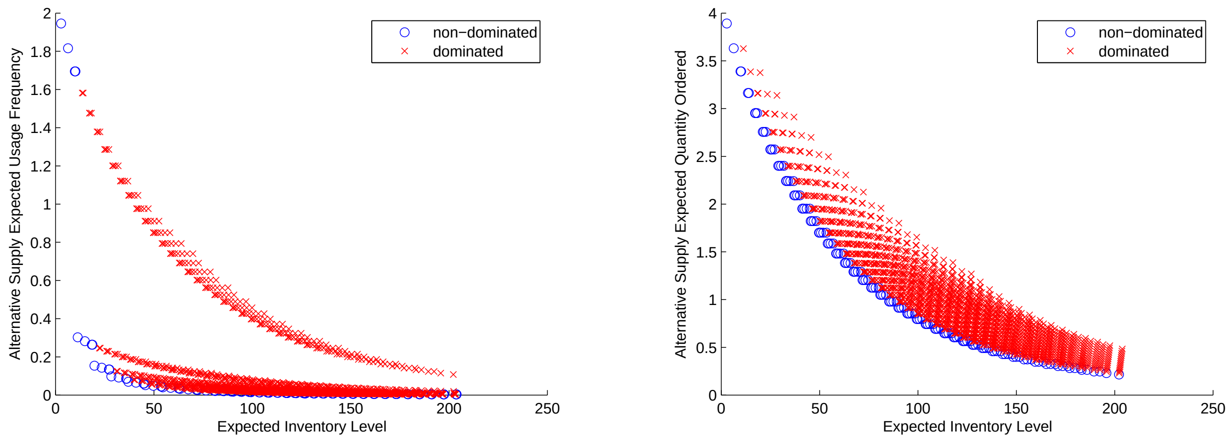

Figure 8: Illustration of solutions for $\alpha = 1/6$ $\lambda = 12$ $\mu = 1/12$

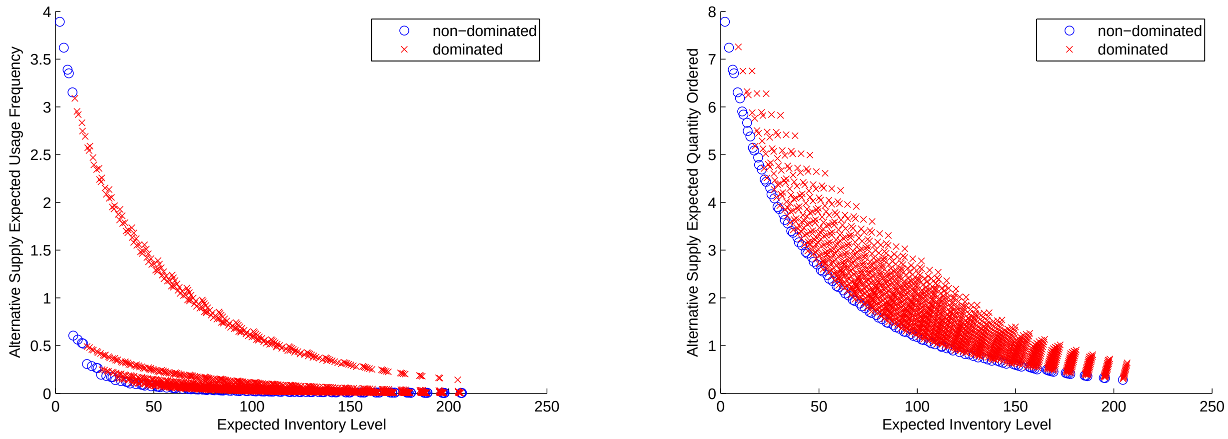

Figure 9: Illustration of solutions for $\alpha = 1/6$ $\lambda = 12$ $\mu = 1/3$

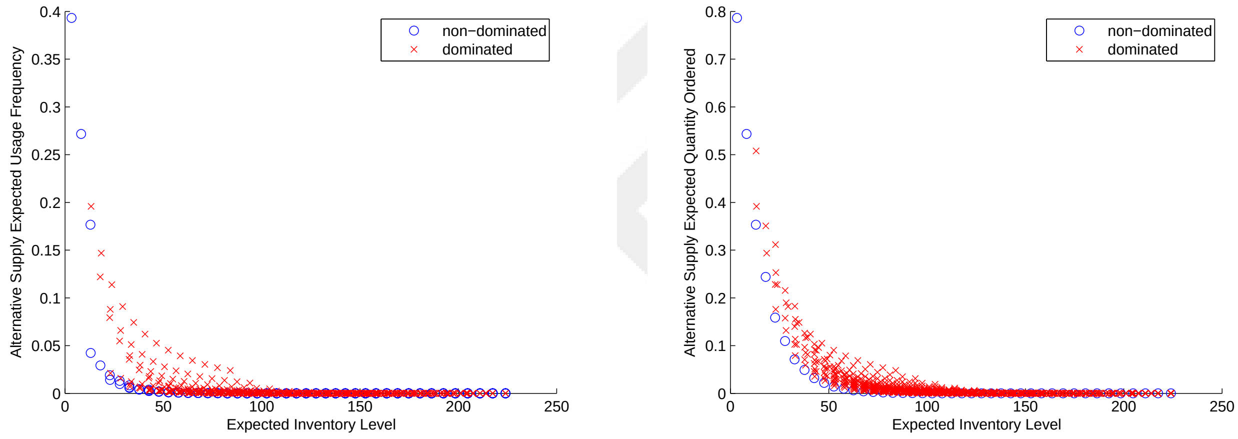

Figure 10: Illustration of solutions for $\alpha = 1$ $\lambda = 12$ $\mu = 1/12$

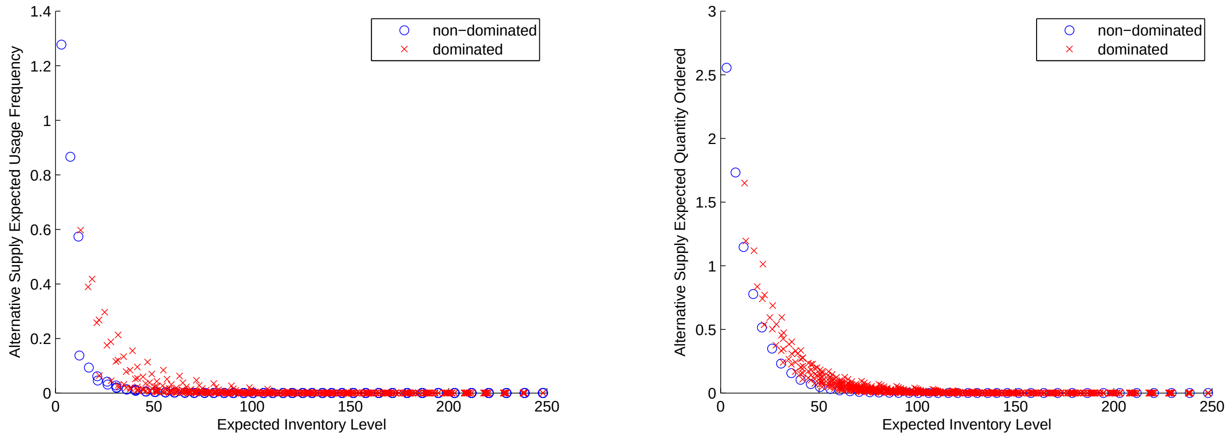

Figure 11: Illustration of solutions for $\alpha = 1$ $\lambda = 12$ $\mu = 1/3$

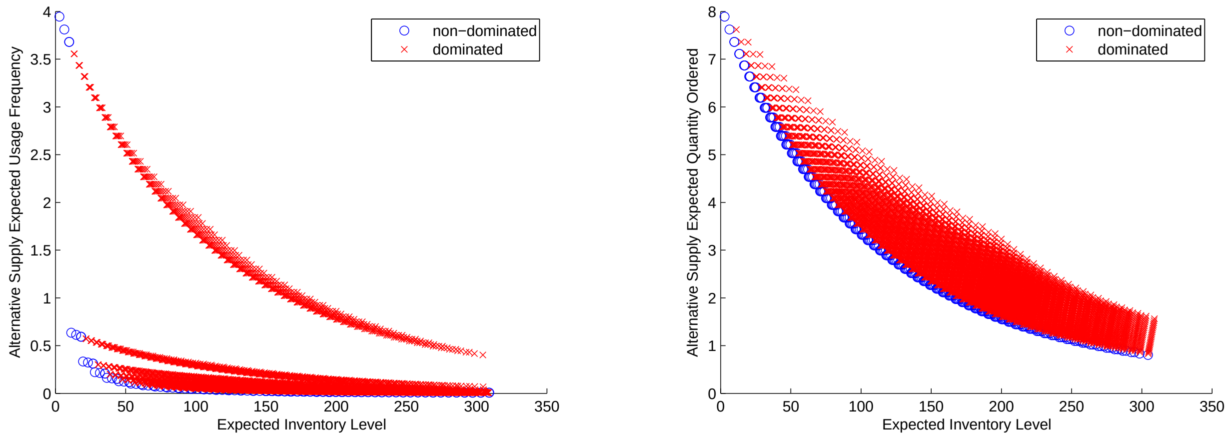

Figure 12: Illustration of solutions for $\alpha = 1/6$ $\lambda = 24$ $\mu = 1/12$

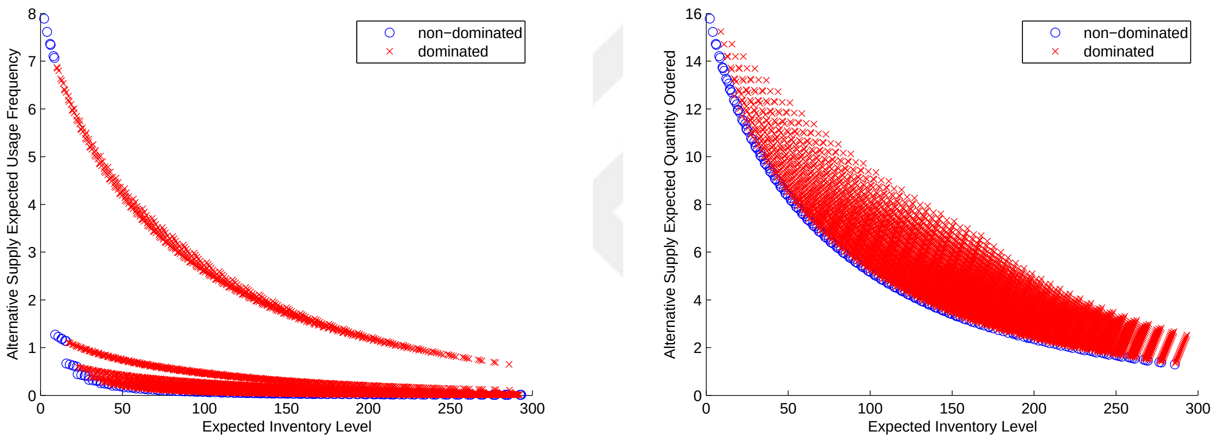

Figure 13: Illustration of solutions for $\alpha = 1/6$ $\lambda = 24$ $\mu = 1/3$

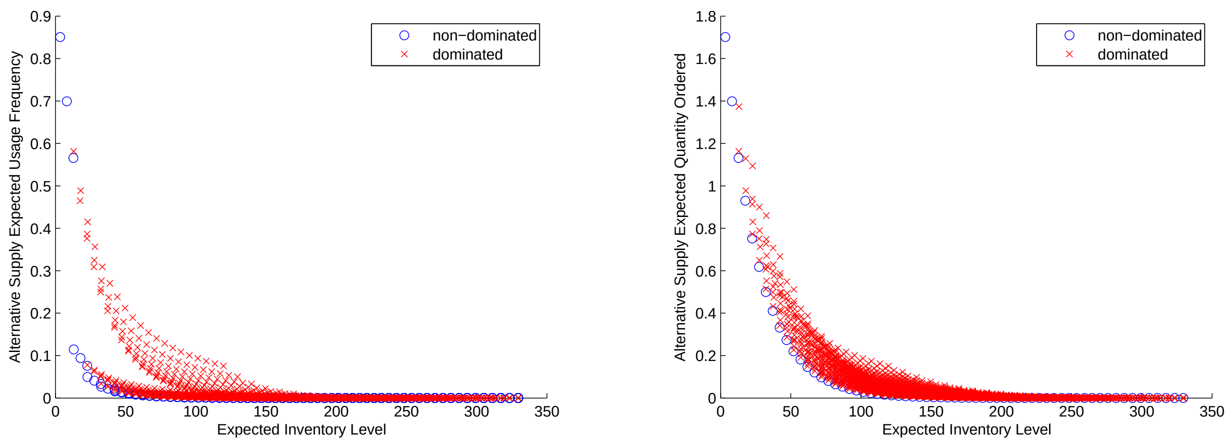

Figure 14: Illustration of solutions for $\alpha = 1$ $\lambda = 24$ $\mu = 1/12$

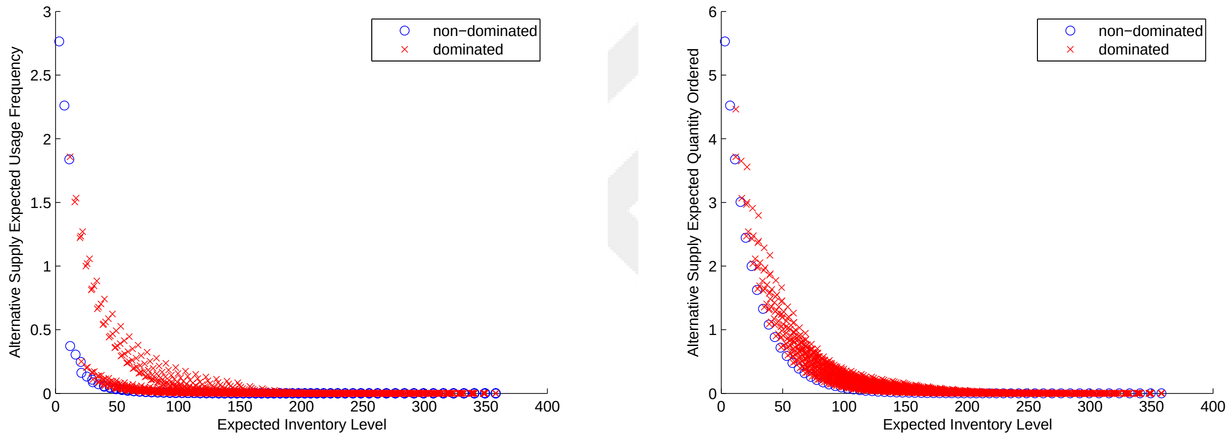

Figure 15: Illustration of solutions for $\alpha = 1$ $\lambda = 24$ $\mu = 1/3$

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