BACKTESTING THE MODIFIED VaR and EXPECTED SHORTFALL METHODS: FOR NON-LINEAR PORTFOLIOS WITHIN BASEL ACCORDS

Submitted to the

Graduate School of Engineering
In Partial Fulfillment of the Requirements for
the Degree of

Master of Science
in the
Financial Engineering

Özyeğin University
May 2018

BACKTESTING THE MODIFIED VaR and EXPECTED SHORTFALL METHODS: FOR NON-LINEAR PORTFOLIOS WITHIN BASEL ACCORDS

Approved by:

Dr. Muzaffer Akat, Advisor,
Department of Financial Engineering and Risk Management
Özyeğin University

Assoc.Prof. Atakan Yalçın,
Department of Financial Engineering and Risk Management
Özyeğin University

Assoc. Prof. Giray Gözgör,
Department of International Relations
Istanbul Medeniyet University

Date Approved: 11 May 2018

To my family..

ABSTRACT

The banks have to measure the market risk daily for the calculation of their capital adequacy. According to the Fundamental Review of Trading Book (FRTB) market risk revision, which was released in 2016 by the Basel Committee on Banking Supervision (BCBS), the expected shortfall (ES) will replace the value-at-risk (VaR) approach in order to capture the tail risks. In this paper, various risk management methodologies have been compared based on their performances using both the VaR and the ES. The data are based on three different currencies (USD/TRY, EUR/TRY, and EUR/USD) for the period from Jan 2nd, 2007 to Jan 4th, 2017. The methodologies have been applied to several portfolios of assets, ranging from a linear one (pure FX Position) to highly non-linear one (complex derivative securities on FX). The binomial backtest method is used for comparing backtesting performance and the empirical results indicate that the ES method, in lieu of the VaR methods, ensures the significant reduction in the capital adequacy for the semi-parametric models. In addition, the ES yields a considerable capital adequacy reduction compared to the VaR in linear portfolios. The reduction in loses strengths as the portfolios get more non-linear. These findings mainly highlight the importance of the convexity and the subadditivity features of the non-linear portfolios.

ÖZET

Sermaye yeterliliği hesaplanmasında günlük piyasa riskini ölçmek için Parametrik (Variance-Covariance),Tarihsel ve Monte Carlo simülasyon yöntemlerinin kullanımı tavsiye edilmektedir. Özellikle Subprime kriz sonrası finansal kurumlar, geriye dönük test aşım sayılarındaki anormal artış, likidite sorunları nedeniyle riske maruz değer(VaR) yöntemlerini upgrade etme arayışına girdiler. Modified olan bu yöntemler (Filtered Historical Simulation,Age-Weighted Historical Simulation,Copula Monte Carlo, ES(Expected Shortfall),EVT vb.) son günlere ağırlık veren volatilite ve korelasyonu dikkate alan bu modeller birçok avantaj ile ön plana çıktı.2016'da BCBS tarafından yayınlanan FRTB Market Risk revizyonu ile Expected Shortfall kuyruk riskleri dikkate almasından dolayı VaR'ın yerine dikkate alınmaya başlamıştır.Bu çalışmada,farklı ES yöntemleri 02.01.2007-04.01.2017 arasında 3 farklı dayanak varlık üzerinde(USD/TRY,EUR/TRY ve EUR/USD) Linear ürünler ve Opsiyon portföylerinin backtesting performansları Binomial Backtest yöntemine göre karşılaştırılmış ve yapılan analizde Semi-Parametric modellerde VaR metodları yerine ES yönteminin sermaye yeterliliğinde önemli derecede azaltma sağladığı görülmüştür. Ayrıca portföy lineer den non-lineer e geçtikçe bu azaltım gücünün azaldığı görülmektedir. Bu bulgular , portföyün non-lineer oldukça sub-additivity ve convexity özelliklerinin önemini ortaya koymaktadır.

ACKNOWLEDGMENTS

I would like to thank my master thesis supervisor Assist. Prof. Dr. Muzaffer Akat, for all the great support and advices during the work with this thesis. I would also like to thank my friends who Giray Gözgör and Fatih Güven, for their supports and helps during the work of the thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

The use of Parametric (Variance-Covariance), Historical, and Monte Carlo simulation methods is recommended in measuring the daily market risk for the calculation of capital adequacy.

Especially following the Subprime crisis, financial institutions have been in quest of upgrading their value at risk (VaR) methods due to an abnormal increase in their backtesting exceedance and measuring liquidity of the risk factor problems. Such modified methods (Filtered Historical Simulation, Age-Weighted Historical Simulation, Copula Monte Carlo, ES(Expected Shortfall), EVT, etc.) that take into account the volatility and correlation that concentrate on the last days have come to the forefront thanks to the many advantages they offer.

1.1 Banks and Basel Regulations

Banks have a key role in the financial system since they provide the financial instruments or the funding to facilitate the cash flow obligations of the institutions or the individual customers. Problems that arise in banks can devastate the economy by yielding an inadequate liquidity, even if all the institutions and the customers of the banks fulfill their obligations on time up until that point. Hence, we can say that the risk of banks being incapable of managing their own cash flow is a systematic risk.

The systematic risk can be defined as a sudden shock that harms the entire financial system that could damage or even the economic activities. It has been observed that the systematic risk in the banking system could lead the countries' economies and even the global economy into the recession. In that regard, Lehman's bankruptcy and the ensuing crisis has been a phenomenal case study for the banking world. To contain the damages of this crisis, the countries tried to save the banks that were in trouble about their capital adequacy. The countries have supported the banks during the crisis to ensure that the crisis does not have a "Spillover effect" and that the banks continue their financial operations.

A systematic risk mainly occurs for two reasons:

- The panic behavior of the depositors or the investors.
- Interruptions in the payment systems.

To avoid this kind of a systematic risk, the central bank executives of the G-10 countries worked together with international agencies and financial authorities at the end of 1974 and ultimately established the Basel Committee on Banking Supervision (BCBS). The Committee aims to reach its targets by setting the minimum standards for the regulation and the supervision of banks. It aims to set a common standard in the financial markets by establishing a common set of rules, techniques and approaches for risk management.

Since the meeting of the BCBS Committee in 1975, the annulment has been held regularly three to four times a year. The BCBS, consisting of the representatives of some 30 countries, including Turkey, has the aim of strengthening the harmonization and the financial stability of member countries. The BCBS Committee shares its proposals, called the Basel Agreement, with the authorized representatives of the countries. Basel's first set of regulations was developed in 1988, and the second regulation was developed between 2004 and 2009. Last Basel regulation, mainly known as Basel 3, started to be developed in 2010.

1.1.1 Basel I :the Basel Capital Accord

One of the main goals of the Basel I, was to provide consensus on preventing capital erosion and the capital adequacy alignment in member banking systems. It was decided that the banks' on-balance sheet and off- balance sheet positions should be governed predominantly by a proper risk measurement technique. So, the minimum capital ratio of 8% for the risk weighted assets has been implemented until the end of 1992. This rate has started being used not only in the member countries but also in all countries where the international banks are located.

The first revision’s (1991) focus was on the Credit Risk. In 1995, a new statement was published. In this report, the credit risk and the additional risk factors were clarified, especially for the positions in derivative products. In 1996, the Committee removed the Market Risk Amendment which was enacted at the end of 1997.

The fundamental changes in this regulation, made in 1996,

- The risk of the instruments based on the equity and the interest-rate products were included in the trading book. The definition of the trading book, as amended by the document International Convergence of Capital Measurement and Capital Standards: A Revised Framework (Basel Committee, 2005)

- The banks has the currency risk and the commodity risk. The capital charges for foreign exchange risk and for commodities risk of the bank will be applied to total currency and commodity positions, which are some discretion to except foreign exchange positions. It is understood that some of these positions will be reported and hence calculated at the market value, but some of them may be reported and evaluated at the book value.

The changes divide the bank assets into two main categories: the Banking Portfolio and the Trading Portfolio. The banking portfolio consists of financial instruments that are held to maturity and valued based on historical cost.

The Trading portfolio represents the bank's short-term market-to-market portfolio such as the Bonds, the Forwards, the Options and the other Structured Products.

1.1.2 Basel II : the New Capital Framework

The Committee published a new capital adequacy regulation recommendation in 1999 as an update of the 1988 agreement. This was followed by another revision in 2004. The revised regulation, known as Basel II, consisted of three blocks.

First of all, it expands the standardized rules outlined in the 1988 Agreement. The capital adequacy has been defined as a sum of three factors: the market risk, the credit risk and the operational risk. Secondly, the inspection of the capital adequacy of an institution and its internal evaluation process of the supervision has been updated. In particular, the local supervisors were equipped with the authority of defining the risk categories of the banks and taking more comprehensive measures where specific problems were identified. Finally, it has strengthened the market discipline and promoted sound banking practices. The aim of this discipline is to allow the market surveillance mechanism to work. According to this update, the banks are supposed to provide information for public on their risk management activities, on the risk assessment processes and on their risk distribution.

1.1.3 Basel 2.5 and Basel III

Since the beginning of 2007, the liquidity-related shortcomings have brought a fundamental change in Basel regulations. After the Lehman crisis, it has come to light that the liquidity risk has not been managed and analyzed effectively. The Basel Committee has made arrangements to measure the liquidity risks of positions that cannot be followed by securitization transactions and off-balance sheet accounts in 2008 and 2009 under Basel 2.5. Some modifications have been made on the calculation of the Trading portfolio. Most importantly, the Stressed VaR calculation has been introduced under the scope of the Tail VaR.

In 2010, the Basel III settlement adopted various regulations on the measurement, the monitoring and the reporting of the liquidity risk. In this revision, the measures such as the leverage ratio, the systematic risk, the minimum capital ratio and the counterparty credit risk have been updated. The new set of rules started to be executed in 2016 in some member countries and it is expected to be fully executed in 2019 in all member countries.

Table 1-Overview of the Market Risk Regulation

Basel paper	Overview of the Market Risk regulation	
BCBS 1996	Supervisory framework for the use of 'backtesting' in conjunction with the internal models approach to market risk capital requirements	https://www.bis.org/publ/bcbs22.pdf
BCBS 1996	Amendment to the capital accord to incorporate market risks	https://www.bis.org/publ/bcbs24.pdf
BCBS 1997	Modifications to the market risk amendment	https://www.bis.org/publ/bcbs24a.pdf
BCBS 2005	International convergence of capital measurement and capital standards: A revised framework.	http://www.bis.org/publ/bcbs107.pdf
BCBS 2009	Revisions to the Basel II market risk framework.(Stressed VaR,Effect of securitization transactions in banking portfolio on capital adequacy)	http://www.bis.org/publ/bcbs158.pdf
BCBS 2012	Fundamental review of the trading book (consultative paper 1)	http://www.bis.org/publ/bcbs219.pdf
BCBS 2013	Fundamental review of the trading book: A revised market risk framework (consultative paper	http://www.bis.org/publ/bcbs265.pdf
BCBS 2014	Analysis of the trading book hypothetical portfolio exercise.	http://www.bis.org/publ/bcbs288.pdf
BCBS 2015	Fundamental review of the trading book: Outstanding issues (consultative paper 3).	http://www.bis.org/publ/bcbs305.pdf
BCBS 2015	Instructions for Basel III monitoring - Version for banks providing data for the trading book part of the exercise.	https://www.bis.org/bcbs/qis/biiiimplmoninstr_feb15.pdf
BCBS 2015	Instructions: Impact study on the proposed frameworks for market risk and cva risk	http://www.bis.org/bcbs/qis/instr_impact_study_jul15.pdf.
BCBS 2016	Standards - Minimum capital requirements for market risk.(Expected Shortfall,Arbitrage disaggregation of Banking and Trading portfolio)	http://www.bis.org/bcbs/publ/d352.pdf
BCBS 2018	Revisions to the minimum capital requirements for market risk	https://www.bis.org/bcbs/publ/d436.pdf

Source: BIS

1.2 Value at Risk

Value-at-risk is a measurement of the maximum potential loss that could be realized within a specific period for a given confidence level. It is a monetary value that could easily be interpreted. (Jorion, 2006)

Value-at-risk is useful in bringing together the types of risks induced by different factors such as equity risk, currency risk, interest-rate risk, commodity risk, etc. and representing all of these risks within a single number. In the Technical Document that J.P. Morgan published, where they suggested the RiskMetrics Model the first time, the concept of VaR has been defined as the answer to the question of "how much can I lose at worst with probability x% in a given period?" (Benninga & Wiener, 1998).

VaR is an important part of the risk measurement and risk management processes in a financial institution. In risk management applications, (often times) the scenario analyses and the results of the stress tests are used as supplementary measurements for the VaR calculations. The most important reason for this is the VaR calculations neglect the loss for a portfolio at the time of the worst case scenarios or namely the extreme market situations. Although, it is not very likely, there is always a probability for the tail events to happen in the financial markets.

The three main disadvantages of a generic VaR model are neglecting the loss level in the worst case scenario, the assumption of the lack of change in positions within the entire backtest period, and inability of recommending a future position for the portfolio.

VaR methodologies differ from each other by the different probability distributions and the volatility models that they are used.

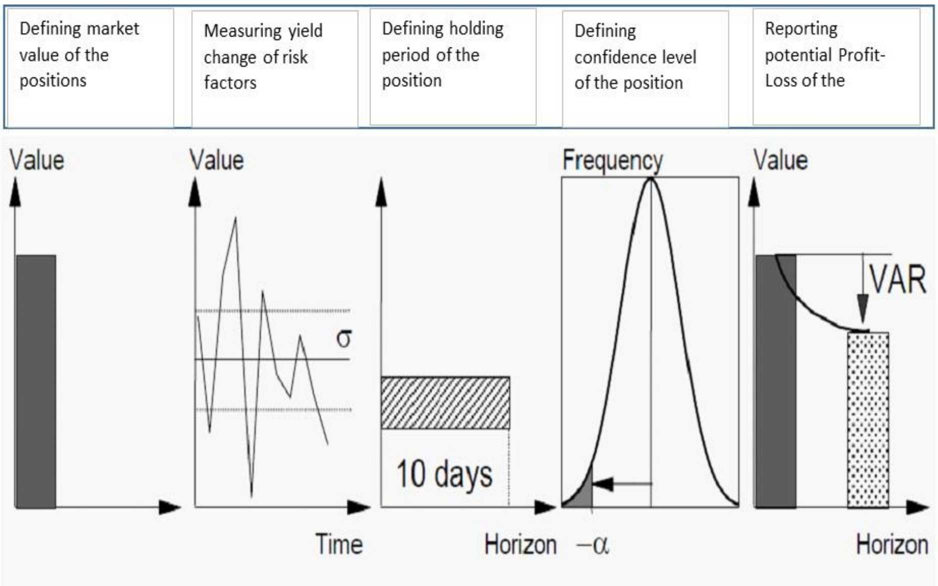
All VaR methods measure the level of risk of a portfolio as a smaller level than the sum of the risk levels of each factor in the portfolio due to the correlation effect among the risk factors. Despite all these disadvantages, the regulatory capital levels are still based on the results of the VaR analyses. Besides all these legal reports, VaR calculations are also used for the distribution of the financial resources and risk adjusted return on capital (RAROC) calculations (Jorion, 2006).

In 1996 the Basel Committee allowed the banks to use their internal models to calculate VaR levels. Hence, banks were able to calculate the adequate capital level based on their internal models instead of the Standard Model if they are authorized by the supervising organization. However, the Basel Committee requires all banks to use 10 days as holding period and the 99% for the confidence level within those internal models.

The VaR techniques have been utilized in the last 20 years in practice despite their disadvantages (BIS, 2013). The main reason for the switch Expected Shortfall from VaR is that the VaR techniques do not give any information about what happens beyond the 99% risk level. At first, the 99% level seems quite high and more than enough, but the regulators decided that these VaR methods are inadequate for risk measurements in extreme events.

The trading strategies and the product choices are constantly changing for most of the financial institutions. This makes the checks of exceeding the risk limits of the financial institutions even harder for the regulators. Hence, the regulators announced their plans on switching to ES models for the required risk level calculations with the FRTB document in January 2016.

Figure 1-VaR Calculation Steps



Source: Risk Measurement, Jorion

1.3 Expected Shortfall

Expected Shortfall is the expected value of the loss beyond the given confidence threshold. In contrast to VaR, ES only uses the values beyond the confidence level. (Jorion, 2006). Different terminologies have been used for ES such as the expected tail loss, the tail VaR or the conditional VaR.

The first step in the ES calculation is computing the VaR level. After that, the expected tail loss is computed. Hence, the uncertainty for the ES is more compared to the VaR. The ES calculations are recommended by the FRTB document to overcome most of the disadvantages of the VaR calculations.

Neiting (2011) proves ES and VaR methodologies should only be compared in terms of risk levels but not in terms of the returns of the portfolios. Because, the ES considers the average value of the risk in the entire tail whereas the VaR only takes into account a single value on the distribution.

In BCBS consultation paper (Basel Committee on Banking, 2014), the Basel Committee recommended the use of the ES models for the capital requirement calculations in the banks' internal models instead of the VaR models, however it did not recommend any changes for the backtesting methods. In addition, with the announcement of FRTB in 2016, the ES clearly became the standard model for the risk measurement calculations as opposed to the VaR methods.

In this document, the Global ES is defined to be the average of the diversified ES and the undiversified ES for the identified risk categories.

The main arrangements designed by the regulators for the ES models in FRTB can be classified into two groups:

For the Daily required capital calculations, the Global ES has to be utilized in the banks' internal models. Furthermore, the ES should be calculated separately for every trading desk included in the internal model.

The confidence level of 99% of VaR has been modified to 97.5% for the ES calculations. (BIS, 2016).

LITERATURE REVIEW

Mainly, the prediction of the financial risks is based on the predictions of the distributions of the financial assets or the portfolios using their historical returns. Measuring the risk of a financial asset is based on predicting the return, the volatility and therefore the distribution of the financial asset for time t+1 at time t. Hence, modelling the volatility and determining the parameters of the model is very important in the risk management models.

Although, first academic papers on VaR models started appearing in 1990's, the mathematical models used in those models go to the earlier years. For example, Markowitz' portfolio selection theory could be considered as an early study that points out the importance of risk management in financial portfolio analysis. A regulatory capital was calculated for the first time by the SEC in the year 1980. The historical return data has been used to calculate the potential loss of the financial institutions for a holding period of 30 days and at a confidence level of 95%. The haircut levels are adjusted according to these calculations. This has been the first step towards the calculation of the capital adequacy from the risk management point of view.

Angelidis & Degiannakis (2007) models are categorized in three main classes: the Parametric Models, the Semi-Parametric Models and the Non-Parametric Models. We are going to adopt a similar categorization with a small change that will be clarified later in this section.

2.1 Parametric Models

Parametric VaR methods were developed first by JP Morgan in 1994, using the variance-covariance matrix of the risk factors computed from different asset classes. They named this methodology as 'Riskmetrics' in 1996 in their famous work of the "Riskmetrics Technical Document." (Morgan, 1996)

One of the main advantages of using the non-parametric models is the fact that we do not need to use the probability distribution functions of the risk factors (Cheung & Powell, 2013). The problem of calibrating the probability distribution is particularly hard under volatile market conditions.

In the study we mainly focus on the applications of semi-parametric and non-parametric models due to the reasons mentioned above. It becomes even more crucial in the case where our portfolio includes non-linear derivative products.

2.2 Semi-Parametric Models

The Standard Historical Simulation (HS) method was first offered by Hendricks in (Hendricks, 1996). In that paper, they analyzed the oil price historical return data by classifying them in two groups as the positive and the negative returns. Then, they computed the VaR values at the 99% confidence levels in both directions. This method assumes that the distribution in the observed period will remain the same in the upcoming holding period.

Dowd (1998) finds evidence that the historical simulation method offered by Maude (1997) gives better results compared to the parametric models. HS methodology is considered to be the simplest technique among the full-valuation methods (Manfredo & Leuthold, 1998).

Although a lot of different VaR techniques have been developed in the academic literature so far, only three of those methods have been adopted by the Basel Committee.

Another HS method is the 'Age Weighted Historical Simulation' model that has been suggested by Richardson, Boudoukh, & Whitelaw (1998). This HS method also uses the historical return data. However, it gives more importance to the more recent values, and less importance to the less recent values by introducing a time decay factor. One of the main disadvantages of this method is the assumption of the volatility is stationary. This could lead to a misrepresentation of the market conditions when there happens a sudden change (Dowd, 2005). On the other hand, it leads to lower capital requirements in the periods of lower volatility for the P&L values of the risk factors (Pritsker, 2006)

Another HS method is the Volatility –Weighted HS Model that has been found by Hull & White (1998). This method focuses on the recent changes in the volatility level. Hull and White find evidence towards Volatility-Weighted HS beats the Age-Weighted HS both in terms of P&L and backtest performances. Sinhua & Chamu (2005) compare all these three HS methods that we have introduced so far for the Mexican Financial Markets in a very high volatility period. They also concluded that the Volatility-Weighted HS gives the best results in this horse race.

One last HS method appeared in the academic literature is the Filtered Historical Simulation (FHS) method which was suggested by (Pritsker, 2001). This paper claims that FHS performs better than the standard HS. Also Pritsker (2001) claims that the standard and the Age-Weighted HS methods can only be used when the portfolios under consideration do not have fat tails.

In the modern financial World, the HS methods appear to be most popular VaR methods in use. For instance, Pérignon & Smith (2010) pointed out that 73% of the commercial banks use in of these HS methods.

2.3 Non-Parametric Models

Woller (1996) claims that the Monte Carlo (MC) Simulation method is the most efficient method for pricing the complex derivative securities. Monte Carlo Simulation technique also assumes that the historical returns are normally distributed. The interaction between the volatility factors are modelled based on this assumption (Linsmeier & Pearson, 1996) . Caflisch (1998) confirmed Joy's claim that the MC method is the most efficient model in pricing complex derivative securities although it turns out to be rather slow. Larcher & Leobacher (2005) show that this MC methodology can also be used for VaR calculations.

All of the HS methods mentioned above and the MC method fall under the category of the Semi-parametric and non-parametric models. (Angelidis & Degiannakis, 2007). The main advantage of all these models is that they do not need to assume anything about the distributions of the risk factors (Cheung & Powell, 2013).

Figure 2-History of VaR Models

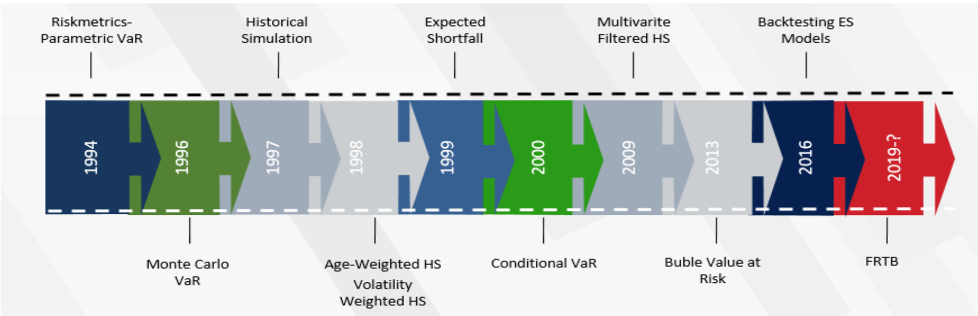


Table 2-Comparison of computation of risk measures

Factor	Parametric Variance/Covariance	Full-Valuation Procedures: Historical Simulation	Full-Valuation Procedures: Monte Carlo Simulation
Able to capture the risks of portfolios which include options?	No, except when computed using a short holding period for portfolios with limited or moderate options content.	Yes, regardless of the options content of the portfolio.	Yes, regardless of the options content of the portfolio.
Easy to implement?	Yes, for portfolios restricted to instruments and currencies covered by available off-the-shelf software. Otherwise reasonably easy to moderately difficult to implement, depending on the complexity of the instruments and availability of data.	Yes, for portfolios for which data on the past values of the market factors are available.	Yes, for portfolios restricted to instruments and currencies covered by available off-the-shelf software. Otherwise moderately to extremely difficult to implement.
Computations performed quickly?	Yes.	Yes.	No, except for relatively small portfolios.
Easy to explain to senior management?	No.	Yes.	No.
Produces misleading value at risk estimates when recent past is atypical?	Yes, except that alternative correlations or standard deviations may be used.	Yes.	Yes, except that alternative estimates of parameters may be used.
Easy to perform "What if" analyses to examine effect of alternative assumptions?	Easily able to examine alternative assumptions about correlations or standard deviations. Unable to examine alternative assumptions about the distribution of market factors, i.e., distributions other than the normal.	No.	Yes.

Source:Adapted from Linsmeier and Pearson(1996)

METHODS FOR MODELING RISK MEASURES

3.1 Parametric VaR Models

In 1994, JP Morgan developed the VaR method by using the variance-covariance matrix calculated over the risk factors of different asset classes. The assumptions of this method, given by the Riskmetrics method name, are disclosed in the "Riskmetrics Technical Document". The current value of the risk factors of the portfolio represents the total current value of the portfolio. The total volatility of the portfolio is used to calculate the value at risk value. Therefore the volatility of the risk factors do not have any significance. The assumptions of this model are given below;

-The calculation of method is supported by products where only the delta is linear. By the way, changes in portfolio value are linearly related to changes that may occur in risk factors.

-Returns of assets are assumed to be normal distributions.

As the asset return variances are assumed to be normal distributions, portfolio returns are assumed to be normal distributions as well.

Parametric VaR inputs can be defined as;

PV=Present Value of the Portfolio

σ= Volatility of the total portfolio

T= Holding period

Z= Standard normal distribution value in a given confidence level such as %99

Parametric VaR calculation can be calculated as;

Parametric VaR= $PV * \sigma * \sqrt{t} * Z$

The distribution or volatility model can be differentiated with adjustment on risk factors.

Gumbel VaR, Log-Normal VaR, t-VaR, and Laplace VaR are examples of models in which the distribution of the parametric VaR method changes. These models are the methods that accept the distribution of the tail distribution, in particular the normal distribution of the portfolio distribution.

EWMA (Exponentially Weighted Moving Average) method is used in general to calculate portfolio of the volatility. Moreover, methods such as Multivariate-GARCH, Dynamic Conditional Correlation (DCC), Regime Switching Models, and Stochastic Volatility Approaches are examples of models in which the volatility estimation method of the Parametric VaR .The Cornish Fisher method is used, which is based on adjusting the value of the coefficient z of the normal distribution. In this method, the VaR value is obtained by calibrating the values of the normal distribution coefficients (0,3) by calculation of skewness and kurtosis of the risk factors. In the thesis, no comparison will be made on parametric methods since non-linear products are used as portfolios.

In summary, Parametric Models can be used only in the linear portfolio even if the volatility and distribution are calibrated to capture the extreme events in the markets effectively. At the same time, since the use of these methods cannot be used especially in the case of complex products and different derivative products, the model usage area will be limited. Since it is not practically possible for all the risk factors of the portfolio to fit into a single volatility and distribution parameter, the usage of modelling is more favorable for standalone portfolios.

3.2 *Semi-Parametric VaR Models*

Non-parametric models are the ones that takes into account the empirical distribution of the risk factors or the portfolios while measuring VaR. They are based on the assumption that the previous return distribution will be repeated without any other statistical assumption. In short, they assume that the past behavior will be a good guide for the future behavior.

3.2.1 Standard Historical Simulation

The most simple and the most practical VaR method is the Historical Simulation (HS) (Dowd, 2005). The main modelling technique in this methodology is that all the possible returns realized in the past gives the distribution for the returns for the upcoming period.

Cabedo & Moya (2003) analyzed the oil price historical return data by classifying them in two groups as the positive and the negative returns. Then, they computed the VaR values at the 99% confidence levels in both directions.

Acerbi & Tasche (2002) applied the HS technique by keeping a fixed window of historical returns to determine the future probability distribution of the risk factors.

In HS simulation methods, one of the most important things is the choice of the time range that will be used for the historical data. Since the weights for all the past data are equal, the choice of this time window plays a crucial role and effects the results very significantly (Goorbergh, Vlaar, & Bank, 1999). Although, there has been many additions, many modifications, and many improvements as a result, to this methodology, the essence of the technique remains mostly the same. Most popular modified HS methods are the ones that considers uneven weights for the historical data (age-weighted) and uneven weights for volatilities of the historical data (volatility weighted) VaR methods.

The Application of Historical Simulation in Practice

As mentioned earlier HS is the most common VaR computing technique being used by the banks and other financial institutions. The reasons for this can be summarized as follows:

The positons in the Trading Books of most banks are of hundreds and thousands of different types. Consequently, the risk factors that needs to be measured are also coming from hundreds of different areas. Measuring the risk for all these factors are easier in the standard HS methodology compared to its modified peers. Because, while modifying the technique for the individual risk technique, one also has to modify the volatility and correlation structures (Cube Volatility, Cube Rates and Forward Rates Simulations) of those risk factors. This turns out to be a rather hard issue for the practitioners. As a result, parametric models carry both modelling risk and estimation risk.

When computing VaR and required levels for the capital adequacy, banks, naturally, do not methods that changes too much from one day to another. Since, the HS method merely reflects the past experience into the near future, it does not offer much room for big surprises. Hence, it proves to be a suitable technique for most of the banks and other financial institutions.

Standard HS methods also allows for aggregating the risk factors after computing the P&L values. This offers a great comfort for presenting the VaR level.

Limitations of the Historical Simulation

Main disadvantage of the HS method is that the results heavily depend on the training (Dowd, 2005). It makes the model rather incapable of predicting the future.

Volatility of the previous period could be misrepresenting the volatility of the upcoming period. For instance, if the returns are less volatile in the previous year, then it would lead to strong deviations for the backtesting of the method. In particular, the model would be highly incapable of capturing the behavior of the positions and the portfolios under extreme conditions.

The extreme changes in the risk factors that appears in the portfolio under examination could effect the tail of the P&L distribution (Christofferson, 2012).

An improbable event in the market conditions that appeared in the recent past would particularly influence the ES values until the occurrence date of the improbable event drops from the sample period.

The top three extreme movements in the past year would not change the VaR value at all since we use the 99% confidence level. In other words, we consider the third most extreme change for the HS method, and the tenth most extreme change for the Monte Carlo Simulation method where we use 1,000 as our sample size. Considering the update of the confidence level to 97.5% by Basel 3, the sixth most extreme change would effect the VaR level. Namely, the twenty-fifth most extreme change for the Monte Carlo Simulation method where we use 1,000 as our sample size.

3.2.2 Semi-Parametric Historical Models

In the standard version of the HS method the weights of the changes in the return values are evenly distributed among all the days in the training period. This yields to situations like an extreme event occurring six months ago having an impact on the risk analysis of the portfolio as if it happened yesterday. This even distribution of weights on one hand suppresses the recent changes in returns and on the other hand amplifies the effects of the changes in the past. As a result the risk measured based on this method could be underestimating or overestimating the risk level depending on the scenario. Especially, in the existence of extreme events this imbalance will keep on appearing until the extreme change drops off the training period. This phenomena will be even more obvious if we were to use the ES instead of VaR.

It is a known fact that the extreme shocks in the past has a memory effect. For this reason, the risk measuring methods that puts on more weight to the more recent data became more popular. The most popular versions of those weighted HS simulation methods have been explained in detail below.

3.2.2.1 Age-Weighted Historical Simulation

This approach has been introduced by (Richardson et al., 1998). This method suggests to adjust the weights of the changes in returns based on the dates. The older a value in the data set the smaller weight it has, and vice versa. One can even provide a specific formulation for the weights assigned to the i th observation of each risk factor in the data set. For a particular value of the so called decay factor λ, which is between 0 and 1, the weight of the i th observation is given by

$$\omega(i) = \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}$$

. One can easily verify here that the total weight is always 1 and the most recent observation has the biggest weight regardless of the value of the decay factor λ.

If one were to choose the decay factor to be 1, this would lead to a value of 1/n for all the observations and the method would reduce to the regular HS method. The calculation of Age Weighted VaR begins after the VaR and PL distribution has been established by the Standard Historical Simulation. Let's assume that the portfolio is sorted by ascending order and PL levels of the risk factors are aggregated according to the standard historical simulation method. The third smallest value in this distribution gives the VaR value according to Standard Historical Simulation. The Age-Weighted method gives a weighted value by the given formula above for each observation. Then, these weights are cumulatively aggregated based on the unordered profit / loss values. Finally, the level of VaR is obtained via a linear interpolation technique for the cumulative weights using a chosen level of 1% or 2.5%.

The method of Age-Weighted HS has 4 crucial features. First, the observations are weighted based on how recent they occurred which is explained in great detail above. Secondly, for a suitable choice of the decay factor, in case of a big P&L observation, the VaR or the ES calculation would react and adjust itself much quicker. Thirdly, the events in the past becomes less and less important as they go back. This plays a self-filtering and regulating role for the extreme events that appeared in the past but still has a high impact on the risk level calculations. Finally, the changes would be observed more smoothly due to the decaying effect and hence one is less likely to observe sudden changes in the VaR or ES calculations.

On the other hand, the decaying feature also reduces the efficient sample size. Pritsker (2001) also point out that the Age-Weighted HS method still fails to react quickly enough for the rapid changes in the volatility levels.

3.2.2.2 Volatility-Weighted Historical Simulation

This approach has been suggested by Hull and White in 1998. The main feature of the method can be described as the most recent in the volatility has the biggest weight in the calculation of the portfolio risk. Hull and White also finds evidence on the better performance of the Volatility - Weighted method compared to Age-Weighted method.

According to this method, unlike Standard Historical VaR method, volatility adjustment is applied to the return changes of the risk factors. In practice, recent volatility is calculated based on then data for the last fifteen day period. Then the long term volatility is calculated based on the data for the last year. Finally, the weights of the return rates are adjusted by the ratio of the long term volatility over the recent volatility. The length of the recent period (15 days, 30 days, 90 days etc.) is the same for all the risk factors since the adjustment is calculated for the entire portfolio. If the calculated volatility in the last period (ex. 15 days) is 2% and the volatility value is 1.5% in the last 1 year, the volatility of the yield changes will increase by 33%.

One of EWMA or GARCH volatility models can be used for weighting.

$$r_{t,i*} = \left(\frac{\sigma_{T,i}}{\sigma_{t,i}}\right) r_{t,i}$$

This methodology does not change anything in the steps for the calculation of P&L values or the VaR and ES levels. It only adjusts the weights of the changes in return rates based on their observed volatility ratio of the last year over the last period.

The advantages of the Volatility-Weighted HS method compared to the previously cited HS methods can be summarized as follows:

Standard HS method does not capture the impact of the return rates that belong to the highly volatile periods. Volatility-Weighted HS yields to higher levels of VaR or ES for the high-volatile periods as one should expect. On the other hand, Age-Weighted method only makes adjustment in terms of how recent the observed return rate is whereas the Volatility-Weighted method also considers the impact of high-volatile and low-volatile periods.

The Volatility-Weighted HS accommodates the time effect in two ways: either by including the EWMA or GARCH type of models that could put more weight on the recent correlation in the dataset or combining the weighting methodology with the Age-Weighted method.

3.2.2.3 Filtered Historical Simulation

The methodology of Filtered Historical Simulation (FHS) has been developed by (Giovanni, Giannopoulas, & Vosper, 1999). In the same work Baron-Adesi finds evidence on the better performance of FHS over the standard HS.

This method is particularly handy when used along with the GARCH models that try to model the conditional variance of the given data. The VaR values are computed via simulation after bootstrapping the error terms. The particular GARCH model that is suggested by Baron-Adesi is the so-called asymmetric GARCH

(AGARCH) for the FHS model. The explanation for this is the ability to capture the jump effect of the recently observed volatility values.

In the second step the new return series is constructed by standardizing the realized return rates, i.e. dividing them by the predicted asymmetric volatility values. Then the newly created return series is bootstrapped. The scenarios are generated based on the standardized return series and the P&L values are computed as a result. The same procedure is repeated for each risk factor. In case of a portfolio that contains various types of assets one should use a multivariate GARCH or AGARCH model.

FHS method has almost all the advantages of the above mentioned HS methods. It brings together the semi-parametric methods with the GARCH type of models. The size of the portfolio, or the large number of the risk factors is not an issue as far as the speed of the method is concerned. It captures the impact of the high-volatile periods on the VaR or ES levels. The correlation between the risk factors and the autocorrelation models for each risk factor can easily be integrated into the method.

3.3 Non-Parametric VaR Models

Monte Carlo Simulation has first been used in 1940 in the area of nuclear physics. Afterwards, it has been used in numerical studies for a lot if different areas. Since late 1970's it has also been used as a tool in the pricing of the derivative securities and for the risk measurement calculations. It is particularly useful when we do not have analytical formulas available for whatever calculation we would like to make.

Woller (1996) claims it to be the most efficient method for pricing of the highly complex derivative securities. Caflisch (1998) also describes the Monte Carlo method as the most effective numerical technique for pricing of the complex financial securities although it is not the fastest. Glasserman (2003) confirms that the Monte Carlo simulation has become a common and important tool for the pricing of the derivative securities and measuring the risk levels of the portfolios. Larcher & Leobacher (2005) introduces the Monte Carlo simulation techniques to VaR calculations. Vergara & Ochoa (2009) creates a stock share synthetically that trades in the Columbian Stock Exchange and finds evidence on the better performance of the Monte Carlo Simulation over the parametric and semi-parametric VaR models based on this synthetically created stock.

The methodology behind the Monte Carlo method is the simulation a financial asset or portfolio by the assumption of random processes. The first step is defining a stochastic model. Then, factors such as the probability distributions, the correlation, and the volatility, which are calculated on the return changes of the risk factors, are measured and assigned to the parameters in the statistical model. We can give a specific example of the application of these steps for the Monte Carlo simulation as follows:

- The logarithmic return change function is applied to the historical data of risk the factors.
- Correlation matrix is calculated from the last 1 year return changes of risk factors.
- If the standard Monte Carlo method is used, the cholesky matrix is obtained from the covariance matrix. If the copula is calculated from the normal Monte Carlo, the cholesky matrix is obtained from the correlation matrix.

- New simulated values are obtained by correlating the value of the risk factors on the portfolio day with the correlated return changes.
- The Present value of the portfolios are simulated by the correlated risk factors.
- The portfolio present value is subtracted from each simulated present value.
- After the PL valuess are aggregated, they are sorted in ascending order. The lowest 10th value yields the VaR value for the 99% confidence level.

The use of the Monte Carlo method would be more useful to pricing a portfolio of Non-Linear or complex derivative-structured products.The Monte Carlo simulation can be used to measure risk for a single risk factor or as a multiple risk factor.

Single Risk Factor MC Simulation

The single risk factor Monte Carlo simulation does not require correlation of simulated values or cholesky values. The return of the asset from the Brownian Motion movement is obtained and the volatility value is obtained. Uncorrelated return change and volatility value are correlated. The return value is simulated by correlating the spot value with the portfolio day's portfolio value. After the portfolio's simulated values are obtained, the KZ values are obtained. The volatility value EWMA can be modeled by using GARCH models. The function used for Brownian Motion model motion is described below.

$S_t = S_0 \exp[\left(\mu - \frac{\sigma 2}{2}\right) t + \sigma\varepsilon\sqrt{t}]$

S_0=Spot price of the asset

μ= Average return of the asset

σ=Volatility of the asset returns

t= Year fraction of the time

Multi Risk Factor MC Simulation

Suppose that there is more than one risk factor in our portfolio. Random numbers up to T * N are generated for each risk factor of the portfolio. Using the Brownian Motion model, the risk factors are simulated up to the determined path. The uncorrelated return changes of the risk factors are obtained. On the portfolio day, the correlation matrix of the risk factors is obtained by using the last 1-year return changes.

The Cholesky matrix is obtained from the correlation matrix.

The values are correlated by multiplying the Cholesky matrix by the yield changes of the data in the last path of the simulation.

The price simulation is made on the correlated change of returns. By obtaining the simulated PV and PL values with the obtained prices, then VaR value is obtained.

Different correlation and volatility models can be applied in Monte Carlo Simulation method. EWMA method is used as volatility method because it is a weighted method for last days and it is easier to apply. Copula methods are preferred because it gives weight to last days and better reflects the non-linearity of portfolio.

RESEARCH METHODS AND PROCEDURES

4.1 Mathematical Comparison of VaR and ES

In theory, any plausible risk measurement techniques have the following five properties: Normalization, monotonicity, convexity, positive homogeneity, and subadditivity (Hult, Lindskog, Hammarlid, & Rehn, 2012). In terms of these features there are two more theoretical reasons for switching to the ES from the VaR. First, one should assume that the returns of the underlying assets are normally distributed to guarantee the subadditivity of the VaR measurement. One does not need such an assumption for the ES measure (Embrechts & Wang, 2015). It has to be kept in mind that this assumption gets further and further away from reality as the portfolio of interest contains more and more complex derivative securities. Secondly, a similar argument can also be constructed against the convexity of the VaR measurement. Again, the convexity becomes a more important issue as the portfolio of interest gets more non-linear.

The early versions of the Basel regulations do not focus on this phenomena since at the time the amount of highly complex derivative securities in the actual portfolios of the banks are not on a visible level. It becomes undeniably important in the subprime crisis period. As a result, Basel 2.5 (2009) wanted to introduce VaR + Stressed VaR to handle this phenomenon. Since, this does not solve the problem entirely, FRTB (2016) decided to make the switch to the ES from the VaR.

There are five crucial features of risk modelling.(Hult et al., 2012)

Normalization:

Normalization means there is no risk if there is no position i.e $\rho(0) = 0$.

Monotonicity:

Assuming that the portfolio X_1 is always larger than the X_2 portfolio, we can say that the portfolio X_1 is less risky than the portfolio X_2 based on the assumption that the volatility remains the same.

$X2 \leq X1$, $\rho(X1) \leq \rho(X2)$

Convexity:

Diversification effect can be defined that investing in different asset groups does not increase the total risk but may reduce it.

$\rho\ (\lambda X1 + (1 - \lambda)X2) \leq \lambda\rho(X1) + (1 - \lambda)\rho(X2$

Positive homogeneity:

If a portfolio is twice as big, then it has twice the risk.

$\rho(\lambda X) = \lambda\rho(X)$ for all $\lambda>0$

Subadditivity:

The combined risk of total portfolios must be equal to or less than the sum of their individual risks.

$\rho(X1 + X2) \leq \rho\ (X1) + \rho\ (X2)$

Aggregation of risks under subadditive feature in ES can be summarized;

$$ES_\alpha\left(\sum_{i=1}^{n} L_i\right) \leq \sum_{i=1}^{n} ES_\alpha(L_i)$$

Despite that VaR has superadditive feature on risk aggregation.

$$VaR_\alpha\left(\sum_{i=1}^{n} L_i\right) > \sum_{i=1}^{n} VaR_\alpha(L_i)$$

4.2 Research methodology of VaR and ES

The data used is daily data starting from Jan 2th, 2007 up to Jan 4th, 2017. The period is chosen intentionally to include the subprime crisis years. There are many windows with different volatility patterns within the chosen time period. This allows the researcher a chance to compare the VaR versus the ES in different market conditions. The data used for the interest rates, the currency, and the volatility levels have been obtained from the Bloomberg EOD API service. For the required VaR and the ES calculations, we use the above mentioned data set starting from a year earlier. Hence, these items start from Jan 2nd, 2006 and end at Jan 6th, 2017. The currency data are taken from the free market database of the Bloomberg. The interest rate data come from the yield curves obtained via the Nelson-Siegel method, which is applied to the deposit market rates. The volatility data are the implied volatility data that have been constructed from the option prices using the Vanna-Volga technique.

The currencies appearing in the portfolios of this study are chosen to be the most commonly used ones in the local markets, i.e. the USD/TRY and the EUR/USD currencies. For linear portfolios a simple FX position that consists of 1M in the USD/TRY and 1M in the EUR/TRY is created. As we go to more non-linear portfolios, vanilla type at-the-money options have been synthetically created. To accommodate highly non-linear portfolios the paper makes use of barrier options where all the barriers are designed to be up-and-in options; the levels of the barriers are put to be 0.1 bps above the spot rate; and the strike levels are chosen to be 0.02 bps above the spot rate, which closely follows the at-the-money-forward rate. The type of the options is created by the call options on the USD/TRY and the put options on the EUR/USD. The maturities used for all the options are 1M, 3M, and 6M.

Linear and Non-Linear Portfolio Details

Since all the options in the study are synthetically created, a model needs to be used for each pricing. The model picked for pricing the vanilla type FX options (Garman & Kohlhagen, 1983), the model used for pricing the single barrier options (Rubinstein & Reiner, 1991), and the model used for digital options (Rubinstein & Reiner, 1991). All these choices of models are listed together in Table 3 that provides the option pricing models of the option portfolios.

Table 3-Option Pricing Models

OPTION PRICING MODELS	
VANILLA FX OPTION	BS-GarmanKohlhagen (1983)
SINGLE BARRIER OPTION	BlackSholes-Merton&Rubinstein(1991)
VANILLA DIGITAL OPTION	Reiner&Rubinstein(1991)
BARRIER DIGITAL OPTION	Reiner&Rubinstein(1991)

While comparing the backtesting performances of the VaR and the ES techniques seven different types of portfolios have been utilized. The simplest portfolio consists of a pure FX position. The second type of portfolio includes only vanilla type options. The third portfolio only includes the barrier options. The study also considers portfolios that consist of vanilla digital options and barrier digital options. As one can see the portfolios chosen gets more and more non-linear as they included more and more complex derivative securities. The last two types of portfolios investigated include all the above mentioned derivative securities and all the above mentioned products. The portfolios studied are summarized in Table 4 that represents the portfolio definition of the products.

Table 4-Portfolio Definitions

PORTFOLIO DETAILS	
FX POSITION PORTFOLIO	FX POSITION
FX OPTION PORTFOLIO	FX OPTIONS
STANDART KI PORTFOLIO	SINGLE BARRIER OPTION
VANILLA DIGITAL PORTFOLIO	VANILLA DIGITAL OPTION
DIGITAL TOUCH PORTFOLIO	BARRIER DIGITAL OPTION
ALL DERIVATIVE PORTFOLIO	FX OPTION,SINGLE BARRIER,VANILLA DIGITAL,BARRIER DIGITAL
ALL PRODUCTS PORTFOLIO	FX POSITION,FX OPTION,SINGLE BARRIER,VANILLA DIGITAL,BARRIER DIGITAL

4.3 Finding and Observations

The backtesting performance of the VaR and the ES have been studied on four different models: the Standard Historical Simulation (HS), the Age-Weighted Historical Simulation (AWHS), the Volatility-Weighted Historical Simulation (VWHS), and Monte-Carlo Simulation (MC). The exceedance of the profit and loss (P&L) levels are calculated by the binomial backtesting method, which is described in detail in the Appendix I.

The following tables are colored in red and yellow and a yellow-colored cell represents a year where the number of days of exceedances of the minimum required level set by the Basel Committee is between 4 and 7. This can interpreted as the model needs attention. A red colored cell represents a year where the number of days of exceedances of the maximum level set by the Basel Committee is more than 7. This means that the risk measurement technique cannot be used. For the results of this comparison in detail are provided in Tables from 5 to 11. Table 5 represents the Backtesting exceedance table of all risk models as the FX position portfolio. Table 6 represents the backtesting exceedance table of all risk models as the currency option portfolio. Table 7 provides the backtesting exceedance table of all risk models as the single barrier option portfolio. Table 8 reports the backtesting exceedance table of all risk models as the vanilla digital option portfolio. Table 9 provides the backtesting exceedance table of all risk models as the binary digital option portfolio. Table 10 represents the backtesting exceedance table of all risk models as the derivative portfolio. Finally, Table 11 provides the backtesting exceedance table of all risk models as the "all products portfolio".

Table 5-Backtesting Exceedence table of all risk models as FX Position Portfolio

	FX POSITION							
	Historical Standart(VaR)	HS Age Weighted(VaR)	HS Volatility Weighted(VaR)	MC(VaR)	Historical Standart(ES)	HS Age Weighted(ES)	HS Volatility Weighted(ES)	MC(ES)
2007	6	8	8	1	6	3	8	0
2008	11	7	13	1	11	5	13	2
2009	0	1	0	0	0	0	0	0
2010	3	4	5	0	3	3	5	0
2011	2	4	5	0	2	3	4	0
2012	2	3	0	3	2	3	0	2
2013	3	7	4	2	2	4	5	2
2014	4	3	6	3	3	2	3	2
2015	4	5	7	4	3	3	6	3
2016	3	3	3	2	2	3	2	2
Total	38	45	51	16	34	29	46	13
					-10,53%	-35,56%	-9,80%	-18,75%

Table 6-Backtesting Exceedence table of all risk models as Currency Option Portfolio

	Currency Option							
	Historical Standart(VaR)	HS Age Weighted(VaR)	HS Volatility Weighted(VaR)	MC(VaR)	Historical Standart(ES)	HS Age Weighted(ES)	HS Volatility Weighted(ES)	MC(ES)
2007	7	9	6	1	8	5	7	2
2008	10	8	10	1	10	5	11	2
2009	0	2	0	0	0	0	0	0
2010	4	4	5	0	4	3	5	0
2011	2	4	2	0	2	3	2	0
2012	2	3	1	3	2	3	2	2
2013	3	8	3	2	2	3	2	2
2014	4	4	5	3	3	2	4	1
2015	5	5	7	3	5	4	6	3
2016	2	3	1	2	2	2	1	2
Total	39	50	40	15	38	30	40	14
					-2,56%	-40,00%	0,00%	-6,67%

Table 7-Backtesting Exceedence table of all risk models as Single Barrier Option Portfolio

	Single Barrier Option							
	Historical Standart(VaR)	HS Age Weighted(VaR)	HS Volatility Weighted(VaR)	MC(VaR)	Historical Standart(ES)	HS Age Weighted(ES)	HS Volatility Weighted(ES)	MC(ES)
2007	7	6	6	1	8	4	6	2
2008	6	7	8	1	6	6	7	1
2009	0	2	0	1	0	2	0	2
2010	2	3	3	1	2	3	3	2
2011	5	6	7	3	2	5	4	2
2012	0	2	0	0	0	0	0	0
2013	9	7	13	2	9	7	13	2
2014	3	4	3	2	3	4	4	2
2015	4	6	8	2	4	5	8	1
2016	4	3	5	1	3	3	4	1
Total	40	46	53	14	37	39	49	15
					-7,50%	-15,22%	-7,55%	7,14%

Table 8-Backtesting Exceedence table of all risk models as Vanilla Digital Option Portfolio

	Vanilla Digital							
	Historical Standart(VaR)	HS Age Weighted(VaR)	HS Volatility Weighted(VaR)	MC(VaR)	Historical Standart(ES)	HS Age Weighted(ES)	HS Volatility Weighted(ES)	MC(ES)
2007	4	6	7	2	3	2	7	2
2008	11	8	11	2	10	8	13	3
2009	0	2	0	1	0	0	0	1
2010	5	5	3	0	5	3	3	0
2011	3	5	4	0	3	3	4	0
2012	2	3	1	3	2	3	2	3
2013	1	5	1	3	1	3	1	3
2014	2	3	3	3	3	2	3	3
2015	7	6	8	4	6	4	8	4
2016	1	4	1	4	1	1	0	3
Total	36	47	39	22	34	29	41	22
					-5,56%	-38,30%	5,13%	0,00%

Table 9-Backtesting Exceedence table of all risk models as Binary Digital Option Portfolio

	Binary_Digital							
	Historical Standart(VaR)	HS Age Weighted(VaR)	HS Volatility Weighted(VaR)	MC(VaR)	Historical Standart(ES)	HS Age Weighted(ES)	HS Volatility Weighted(ES)	MC(ES)
2007	7	6	6	2	8	3	6	2
2008	5	5	5	4	4	3	5	4
2009	0	3	0	2	0	1	0	3
2010	1	3	3	2	1	2	2	2
2011	3	7	4	3	3	3	3	3
2012	0	2	0	2	0	0	0	0
2013	7	8	11	2	9	5	12	2
2014	4	4	3	2	3	3	5	2
2015	2	6	1	3	3	3	2	3
2016	3	5	5	5	2	3	4	4
Total	32	49	38	27	33	26	39	25
					3,13%	-46,94%	2,63%	-7,41%

Table 10-Backtesting Exceedence table of all risk models as Derivative Portfolio

	DERIVATIVE PORTFOLIO							
	Historical Standart(VaR)	HS Age Weighted(VaR)	HS Volatility Weighted(VaR)	MC(VaR)	Historical Standart(ES)	HS Age Weighted(ES)	HS Volatility Weighted(ES)	MC(ES)
2007	4	5	6	0	4	5	5	0
2008	6	4	8	1	4	7	7	1
2009	0	1	0	0	0	0	0	0
2010	3	2	3	0	2	2	3	0
2011	0	0	0	0	0	2	0	0
2012	0	2	0	2	0	2	0	2
2013	1	2	1	2	1	2	1	2
2014	1	1	2	1	1	3	3	1
2015	1	2	4	2	1	6	4	2
2016	0	2	0	2	0	1	0	2
Total	16	21	24	10	13	30	23	10
					-18,75%	42,86%	-4,17%	0,00%

Table 11-Backtesting Exceedence table of all risk models as All Products Portfolio

	ALL PRODUCTS PORTFOLIO							
	Historical Standart(VaR)	HS Age Weighted(VaR)	HS Volatility Weighted(VaR)	MC(VaR)	Historical Standart(ES)	HS Age Weighted(ES)	HS Volatility Weighted(ES)	MC(ES)
2007	4	5	6	0	4	5	5	0
2008	6	4	8	1	4	7	7	1
2009	0	1	0	0	0	0	0	0
2010	3	2	3	0	2	2	3	0
2011	0	1	1	0	0	2	1	0
2012	0	3	0	2	0	2	0	2
2013	1	2	1	2	1	2	1	2
2014	1	1	2	1	1	3	3	1
2015	1	3	4	2	1	5	4	2
2016	0	2	0	2	0	1	0	2
Total	16	24	25	10	13	29	24	10
					-18,75%	20,83%	-4,00%	0,00%

It is clear from almost all these tables that the HS and the VWHS are unusable in the periods of high volatility such as the year 2008, regardless of the risk measurement technique. The other models, i.e. the AGHS and the MC, did not have any problem even in the year 2008. On the other hand, for the HS and the AWHS models, switching to the ES from the VaR almost all the time (with the exception of the year 2014, and only in the large portfolio) brings the significant reduction in terms of the number of critical years. Table 12 also represents the reduction of capital adequacy values provided by the VaR vs. the ES.

Table 12-Reduction of capital adequacy values provided by the VaR vs. the ES.

Product/Model	Historical Standart(ES)	HS Age Weighted(ES)	HS Volatility Weighted(ES)	MC(ES)
Overall_Currency	67,804.81	-23,738,231.91	-473,484.69	-1,290,172.08
Currency_Option	821,377.13	-11,942,348.24	776,912.19	-919,039.18
Single_Barrier_Option	611.10	-75,985.91	572.03	-5,704.53
Vanilla_Digital	14,652,021.99	-154,943,582.16	21,148,769.97	-14,706,479.38
Binary_Digital	-24,069,167.95	-90,993,323.35	-4,866,853.15	12,199,856.71
DERIVATIVE_PORTFOLIO	-8,595,157.94	-163,539,504.99	17,059,400.81	10,326,498.88
ALL_PORTFOLIO	-8,527,353.13	-179,618,551.79	16,585,916.12	11,472,684.54
Average Reduce PL in TRY	**-25,649,863.99**	**-624,851,528.35**	**50,231,233.28**	**17,077,644.96**

In addition, the required capital levels under these models turns out to be significantly less using the ES technique (See Table 12). However, the required capital level via the ES happens to be a little higher under the VWHS and the MC methods. Depending on the type of portfolio perspective, the reduction in the capital adequacy becomes more and more visible as the portfolios get more and more complex. Under the HS model, one starts seeing the reduction only after the portfolio gets complex. Under the AWHS although the reduction is always there, the amount of reduction increases as the portfolio gets more complex. This can be considered as the evidence that the rate of reduction in the capital adequacy violations is closely connected to how convex is the product under consideration. Table 13 reports the reduction of the backtesting exceedance rate provided by the VaR versus the ES.

Table 13-Reduction of the backtesting exceedance rate provided by the VaR versus the ES.

Portfolio/Model	Historical Standart(ES)	HS Age Weighted(ES)	HS Volatility Weighted(ES)	MC(ES)	Average
FX Portfolio	-10.53%	-35.56%	-9.80%	-18.75%	-18.66%
Currency Option	-2.56%	-40.00%	0.00%	-6.67%	-12.31%
Single Barrier Option	-7.50%	-15.22%	-7.55%	7.14%	-5.78%
Vanilla Digital	-5.56%	-38.30%	5.13%	0.00%	-9.68%
Binary Digital	3.13%	-46.94%	2.63%	-7.41%	-12.15%
Derivative Portfolio	-18.75%	42.86%	-4.17%	0.00%	4.99%
All Portfolio	-18.75%	20.83%	-4.00%	0.00%	-0.48%

The values show the percentage of the rate of reduction in the required capital levels as one goes from the VaR to the ES. The capital increase/decrease rates in the backtesting performance are summarized above in connection with the transition from the VaR models to the ES model with a product/model basis. The dominance of green cells is a strong indication of a higher performance of the ES over the VaR under various circumstances, portfolios and underlying models. Basically, using the ES approach instead of the VaR provides a higher efficiency in the capital adequacy. Interestingly, the efficiency of the ES approach increases as the portfolios get more non-linear due to the convexity and the subadditivity features of the non-linear portfolios. In short, our results indicate that banks and financial institutions can use our evidence of a less capital adequacy for providing an extra fund in other financial and non-financial activities.

CONCLUSION

The Basel Committee focused on market risk regulation in the last decade to calculate of the tail risk. Measuring tail risks have become much more important than ever as a consequence of the increase in the complexity of products used by the banks. In this paper, the backtesting performances of the VaR and the ES techniques are compared under four different models. All these models have been applied both on linear and non-linear products for a period of ten years, for the period from 2007 to 2017.

In terms of the models used for scenario analysis, switching from the VaR to the ES brings a lot of reduction in the number of days of exceedances of the critical levels and the amount of required capitals. The difference is particularly clear under the standard HS and the AWHS models, and the improvement gets clearer in the periods of high volatile regimes. In terms of the product types, a high rate of reduction in the number of capital adequacy violations has been achieved in transition from the VaR models to the ES models. However, this rate of reduction somewhat decreases as the product complexity intensifies, i.e. when the products become non-linear. For instance, from the VaR to the ES, the capital need has reduced at a higher rate in the vanilla type products, such as the vanilla currency and the vanilla digital options. On the other hand the reduction rate has been lower in more complex derivative securities, such as the single barrier and the binary digital options. Our paper also motivates that the rate of reduction in the level of the required capital is related to the convexity of the portfolio of interest. In other words, the rate of reduction decreases as the convexity of the products in the portfolio increases.

It is expected that the banks will encounter different risk capital results depending on their products in their portfolios, the model they use in transition from the VaR to the ES. Though one cannot claim that the ES technique is better than the VaR, but it is clear that there are many advantages of using the ES over the VaR under various circumstances. In the future papers, the backtesting performances should be tested not only in the currency risk category, but also in other common risk categories, such as the equity, the fixed-income, and the commodity risk. The tests also need to be enlarged to a wider set of financial securities.

APPENDIX A

Backtesting

Backtesting (model verification test) can be defined as being subject to a test for the purpose of testing the parameters and accuracy or the VaR model used. While conducting this test, the portfolio's PL values are compared with the then current VaR values. There are 2 types of approaches in the comparison of PL values. Marked to Market PL method that is based on obtaining the values of risk factors included in the portfolio by calculating their market values based on the just value, if any, or, if such a value is not available, based on the fair value approach. Marked to Model approach enables to obtain the Profit/Loss value by comparing a portfolio's theoretical present value with the same portfolio's theoretical present value on the next day. Basel (2016-FRTB) document sets forth that Liquid positions could be valued with Marked to Market model, while Illiquid positions with fair value or marked to model approaches. Below, we are explaining the model verification tests of Marked to Market or Marked to Model methods while there is a PL distribution.

Binomial Test (1993), or, Kupiec method is one of the statistical methods that can be used for the quantification of tail distribution in the Profit/Loss distribution. Binomial method can be defined as a frequency test of exceeding values. Binomial test is the basic backtest methodology where Profit/Loss values are compared with VaR and VaR exceeding is indicated in the most prominent way. Binomial Backtesting can be tested by means of the Binomial distribution or normal distribution. Where we define the daily calculated PL values as i, Hs= PLİ>VaRi the exceeding numbers during a year are determined. Total business days within 1 year are used in determining the value for average and deviation figures. The calculations are done as follows:

Average Deviation = $N*(1-\alpha)=Ad$

Standard Deviation = Squareroot $(\alpha*(1-\alpha)*N) = Sd$

$Z=(Hs- Ad)/ Sd$.

Table 14-Basel Additonal Capital Factor Table

Zone	**Number of exceptions**	**Increase in scaling factor**	**Cumulative probability**
Green Zone	0	0,00	8,11 %
	1	0,00	28,58 %
	2	0,00	54,32 %
	3	0,00	75,81 %
	4	0,00	89,22 %
Yellow Zone	5	0,40	95,88 %
	6	0,50	98,63 %
	7	0,65	99,60 %
	8	0,75	99,89 %
	9	0,85	99,97 %
Red Zone	10 or more	1,00	99,99 %

Source:BIS

BIBLIOGRAPHY

Acerbi, C., & Tasche, D. (2002). On the coherence of Expected Shortfall. *Journal of Banking and Finance*, *26*(7), 1487–1503.

Angelidis, T., & Degiannakis, S. (2007). Backtesting VaR models:a two-stage procedure. *The Journal of Risk Model Validation*, *1*(2), 27–48.

Basel Committee. (2005). *International Convergence of Capital Measurement and Capital Standards*. Basel.

Basel Committee on Banking. (2014). *Consultative Document - Fundamental Review of the Trading Book: a Revised Market Risk Framework* (No. 265). Basel.

Benninga, S., & Wiener, Z. (1998). Value-at-Risk (VaR). *Mathematica in Education and Research*, *7*(4), 1–8.

BIS. (2013). *Basel Committee on Banking Supervision Consultative Document Fundamental Review of the Trading Book* (No. 219). Basel.

BIS. (2016). *Basel Committee on Banking Supervision Minimum capital requirements for market risk* (No. 352).

Cabedo, J. D., & Moya, I. (2003). Estimating oil price “Value at Risk” using the historical simulation approach. *Energy Economics*, *25*(3), 239–253.

Caflisch, R. E. (1998). Monte Carlo and quasi-Monte Carlo methods. *Acta Numerica*, 1–49.

Cheung, Y. H., & Powell, R. J. (2013). Anybody can do Value at Risk : A NonparametricTeaching Study. *Australian Accounting,Business and Finance Journal*, *6*(1), 111–123.

Christofferson, P. F. (2012). *Elements of Financial Risk Management* (Second). 225 Wyman Street, Waltham, MA 02451, USA: Elsevier.

Dowd, K. (1998). *Beyond Value at Risk*. New York: John Wiley&Sons,LTD.

Dowd, K. (2005). *Measuring Market Risk* (Second). John Wiley&Sons,LTD.

Embrechts, P., & Wang, R. (2015). Seven Proofs for the Subadditivity of Expected Shortfall. *Dependence Modeling*, *3*(1), 1–23.

Garman, M. B., & Kohlhagen, S. V. (1983). Foreign currency option values. *Journal of International Money and Finance*, *2*(3), 231–237.

Giovanni, B.-A., Giannopoulas, K., & Vosper, L. (1999). VaR without correlations for portfolios of derivative securities. *The Journal of Futures Markets*, *19*(5), 583–602.

Glasserman, P. (2003). *Monte Carlo Methods in Financial Engineering*. New York: Springer.

Goorbergh, R. Van Den, Vlaar, P., & Bank, D. N. (1999). *Value-at-risk analysis of stock returns. historical simulation, variance techniques or tail index estimation? De Nederlandse Bank-Staff Report (1999).*

Hendricks, D. (1996). Evaluation of Value-at-Risk Models Using Historical Data. *Economic Policy Review*, *2*(1).

Hull, J., & White, A. (1998). Incorporating volatility updating into the historical simulation method for VaR. *Journal of Risk*, *1*(1), 5–19.

Hult, H., Lindskog, F., Hammarlid, O., & Rehn, C. J. (2012). *Risk and Portfolio Analysis*. Springer.

Jorion, P. (2006). *Value at Risk:The New Benchmark for Managing Financial Risk* (Third). New York: McGraw-Hill.

Larcher, G., & Leobacher, G. (2005). Quasi-Monte Carlo and Monte Carlo methods and their application in finance. *Surveys on Mathematics for Industry*, *11*(1), 95–30.

Linsmeier, T. J., & Pearson, N. D. (1996). Risk Measurement : An Introduction to Value at Risk. *Working Paper*, *6153*(July), 1–45. Retrieved from

http://www.casact.net/education/specsem/99frmgt/pearson2.pdf
Manfredo, M. R., & Leuthold, R. M. (1998). Agricultural Applications of Value-at-Risk Analysis: A Perspective. *SSRN Electronic Journal*, *5*(1).
Maude, D. J. (1997). Bank Capital and Value-at-Risk. *Journal of Derivatives*, *4*(3), 73–89.
Morgan, J. P. (1996). *RiskMetrics-JP Morgan Technical Document* (Fourth).
Neiting, T. G. (2011). Making and Evaluating Point Forecasts. *Journal of the American Statistical Association*, *106*(494), 746–762.
Pérignon, C., & Smith, D. R. (2010). The level and quality of Value-at-Risk disclosure by commercial banks. *Journal of Banking and Finance*, *34*(2), 362–377.
Pritsker, M. (2001). The Hidden Dangers of Historical Simulation. *Finance and Economics Discussion Series 27*. Washington D.C.
Pritsker, M. (2006). The hidden dangers of historical simulation. *Journal of Banking and Finance*, *30*(2), 561–582.
Richardson, M. P., Boudoukh, J. (Kobi), & Whitelaw, R. F. (1998). The Best of Both Worlds: A Hybrid Approach to Calculating Value at Risk. *SSRN Electronic Journal*, 1–13.
Rubinstein, M., & Reiner, E. (1991). Breaking Down the Barriers. *Risk Magazine*, *4*(8), 28–35.
Sinhua, T., & Chamu, F. (2005). *Comparing different methods of calculating Value-at-risk*.
Vergara, M. A., & Ochoa, C. M. (2009). *Structured Monte Carlo. Estimated value at risk in a stock portfolio in Colombia*. *Modellin Numero*. Colombia.
Woller, J., 1996. The basic of monte carlo simulation. [online]. Available at: <http://www.chem.unl.edu/zeng/joy/mclab/mcintro.html>