# PARAMETRIC AND NONPARAMETRIC MODELS FOR NEXT DAY OPERATING ROOM SCHEDULING

A Thesis

by

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Submitted to the
Graduate School of Sciences and Engineering
In Partial Fulfillment of the Requirements for
the Degree of

Master of Science

in the
Department of Industrial Engineering

Özyeğin University
January 2018

# PARAMETRIC AND NONPARAMETRIC MODELS FOR NEXT DAY OPERATING ROOM SCHEDULING

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*To my family*

*and*

*the fallen soldiers of our nation*

# ABSTRACT

Operating rooms are the resources that generate the most part of the revenue of hospitals. On the other hand, they generate the most part of the expenses, as well. Because of the uncertainty of surgery durations, scheduling operating rooms are very difficult. But their impact on the finances of a hospital makes it vital for the planners to carry out scheduling as best as they can. Another problem that lies in the way of fine operating room scheduling is limited surgery data available for use. Uncertainty and diversity of surgeries that may take place in a given operating room makes it difficult to obtain sufficient amount of surgery duration data. In this study we describe a stochastic optimization model for computing OR schedules that are effected by the uncertainty in surgery durations. We focus on scheduling start times. We show that our model can be used to generate substantial reductions in OR team waiting, OR idling, overtime costs. The model in this study is studied with 3 solution approaches: (i) parametric approach, (ii) non parametric approach, (iii) a simple but practical heuristic. Considering all scenarios in this study, parametric approach manages to perform 6,18% close to optimal solution, whereas non parametric approach performs 7,66% and heuristic approach performs 78,17% close to optimal solution. When compared to non parametric approach, parametric approach performs better when number of historical surgery duration sample size is small. In contrast, when the number of historical surgery duration sample size is large, non parametric approach starts performing better. All three solution approaches provide meaningful results, where parametric approach performs better in most cases when compared to other solution approaches.

# ÖZETÇE

Hastanelerin karlarının büyük kısmını ameliyathaneler oluştumaktadır. Diğer yandan, hastanelerin harcamalarının büyük bir kısmı da ameliyathanelerde gerçekleşmektedir. Ameliyat sürelerindeki belirsizlik sebebiyle, ameliyathane planlaması zor bir süreçtir. Fakat, ameliyathanelerin hastanelerin kar ve zararları üzerindeki etkisi, ameliyathane planlamasını gerçekleştiren personelin bu işlemi en etkin şekilde uygulamalarına neden olmaktadır. İyi bir ameliyathane planlamasının önündeki diğer bir engel ise kullanılmaya müsait verinin sınırlı olmasıdır. Bir ameliyathanede gerçekleşecek ameliyatlarn belirsizliği ve çeşitliliği gerekli miktarda ameliyat süresi verisi elde etmeyi zorlaştırmaktadır. Bu çalışmada, ameliyat sürelerindeki belirsizlikten fazlaca etkilenen ameliyathane programlamalarını inceleyen rassal bir optimizasyon modeli sunmaktayız. Modelimizde planlama başlangıç zamanlarına odaklanıyoruz. Modelimizin, ameliyathane boş kalma zamanı, ameliyathane ekip bekleme zamanı, fazla mesai zamanı ve bir ameliyathane gününün beklenenden erken bitme sürelerini önemli ölçüde azalttığını gösteriyoruz. Modelimizi üç farklı çözüm yöntemiyle incelemekteyiz: (i) parametrik yaklaşım, (ii) non parametrik yaklaşım, (iii) yalın ama pratik bir heuristic. Bu çalışmadaki bütün senaryolar değerlendirildiğinde, parametrik yaklaşım optimal çözüme 6,18% yakın performans göstermektedir. Bu oran non parametrik yaklaşımda 7,66%, heuristic yaklaşımda 78,17% olmuştur. Geçmiş ameliyat süreleri örnek sayıları düşük olduğunda, parametrik yaklaşım non parametrik yaklaşıma göre daha iyi sonuçlar vermektedir. Tam tersi olarak, non parametrik yaklaşım geçmiş ameliyat süreleri örnek sayıları yüksek olduğunda daha iyi sonuçlar vermeye başlamaktadır. Her üç çözüm yöntemi anlamlı sonuçlar vermektedir, fakat parametrik yaklaşım diğer çözüm yöntemleri ile karşılaştırıldığında daha iyi sonuçlar vermektedir.

# ACKNOWLEDGEMENTS

First of all, I would like to express my profound gratitude to my advisor Asst. Prof. Enis Kayış for his endless support, guidance and patience not only during my research, but throughout my whole graduate school life. With him believing in me, all of this work was able to see the light of day. Also, I would like to thank all my professors in the faculty for helping me develop my skills as a researcher.

I would like to express my sincere appreciation to my friends and officemates, especially Cem Deniz Cağlar Bozkır, Hamed Shourabizadeh, Görkem Emirhüseyinoğlu and Tonguç Yavuz, who supported me with their invaluable friendships, comments and encouragement. And I would like to express my heart-felt gratitute to my best friend in the faculty and my wife Büşra Sevindik for everything she has done to help me find my way during my research.

Finally, I would like to express my gratitude to my mom, İnci Cenik, and my little sister Dilek Sevindik, for supporting me in my darkest days. Without them, I never would have been able to make the choices in life that make me who I am today.

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# CHAPTER I

# INTRODUCTION

Operating rooms (ORs) generate the most part of a hospital's expenses. However, they also generate more than 40% of a hospital's total revenues [1]. Use of well managed ORs would yield good results for the hospital administration and provide a good experience in rather unpleasant stays for the patients that seek to get well. On the other hand, obstacles that generally show in the way to a well managed ORs would cause hospitals to lose revenue and cause patients to experience discomfort along with their health problems. The role of an operating room makes it a very interesting and productive field for researchers.

An effective guide for scheduling ORs have the capability to make the work-life of OR planners and hospital board members easier. Hospitals are able to generate more revenue using an effective OR planning policy, and in addition have satisfied patients. An effective OR scheduling also helps improve OR teams' (surgeons, nurses, anesthesiologists) working conditions. A small example would be overtimes that OR teams must face because of ineffective OR scheduling policies.

Operating room scheduling can be executed in several ways. It can be strategic planning. An example of strategic planning would be assigning specialties to specific operating rooms. Another example is staffing problems such as nurse staffing or anesthesiologist staffing problems. In this article we study an extremely operational stochastic OR scheduling problem. The sequence of the surgeries is known in advance. We want to determine surgery durations of these surgeries and solve the problem in a data driven manner. We assume we have limited historical surgery durations data at hand. We try to find out if estimation is enough for a good solution or if we need further tools of optimization to make good use of limited data we have. We aim to increase the OR efficiency and present managerial insights.

The current status of operating rooms paves the way for studies for the purpose of overcoming the difficulties regarding both the revenue and expense factors of ORs. However, these efforts are still far from being very effective. The purpose of this article is to present results from a study of a stochastic optimization model for daily scheduling of a single OR in the hope to contribute to these efforts.

It is difficult to manage operating rooms mainly because they involve expensive resources such as human and technical resources. These two resources should be at the same place and time, and at the correct time. Any mistake in the planning prevents these expensive resources from being used effectively, resulting with challenges for the hospital administration. The main factors that effect OR efficiency are variable surgery durations and the limited data. What we would like to present in this paper is an efficient operating room model. We define efficiency by waiting time, idle time and overtime. Waiting time is the time that a patient waits because the length of the operation that is scheduled before, exceeds its scheduled length. Idle time defines the lost time for an OR which is incurred by the early finish of scheduled surgery. Overtime incurs when the actual total OR time extending beyond the planned length. We consider an additional idle time caused by the early finish of the last surgery of a given OR day. An efficient OR schedule is expected to decrease waiting time, idle time and overtime.

Variable surgery durations stem from uncertain surgery durations. Uncertain surgery durations effect OR performance measures. For example, uncertainty would contain a longer sample surgery duration than the planned duration, resulting in waiting time and overtime. Similarly, uncertainty would cause shorter sample surgery durations that results in idle time for operating rooms. Uncertain surgery duration distributions are also hard to obtain. Generally, OR planers have limited surgery duration data that is not sufficient to obtain such distributions. Since the sample size is small, making very good surgery duration and distribution estimations is quite challenging.

There are different solution approaches for OR scheduling problems. In this paper, we use three different methods. First one is the parametric method. In parametric method we fit probability distributions to sample data then we compute associated costs. In this method, we implement sample average approximation. Parametric approach is a two step approach; estimation and optimization. In the estimation step, we carry out estimation processes for the parameters of fitted distribution to generate samples. In the optimization step, we compute related costs that are used to evaluate the performance of the parametric method. The second method is the non parametric method. In this method we are not interested with any probability distribution information. Without distribution information, we carry out optimization in a single step. We compute the cost for non parametric approach to measure how a particular case performs. There are advantages and disadvantages for both methods. If the distribution is fitted correctly and we feed this information to the parametric method, parametric method makes a head start. This is when parametric method is expected to perform better. On the other hand, non parametric method is rather a free method, because it does not execute any estimation. Since there is no possibility to make wrong estimations regarding probability distributions, non parametric method is not effected by such inaccuracy. However, since non parametric method lacks this possible advantage, it can perform worse. The last method in this study is a simple heuristic called the expectation heuristic. This method is simple but practical. It does not entail long probability distribution estimation and sample generation processes, yet provides average results among the three solution methods.

In this paper, similar to the cited literature in the next section, we investigate the performance of our three different solution approaches. In addition, after evaluation of the numerical results, we would like to find out which one of the three solution methods is best for use in different planning environments. We want to deliver managerial insights for OR planners in a way that they could follow according to the capabilities they have present at their command. The OR planner may not have a fine optimization tool to optimize the planned surgery durations for a given OR day. Using our model, OR planners will be able to find out how well they are going to perform. In addition, in the process of setting up a new planning structure for hospitals, hospital administrations may use our model to adapt their OR scheduling. Using our model they can decide whether they need optimization tools for their planning environment, or implementing good statistics is sufficed for their institution.

The remainder of the thesis is organized as follows. In the next section we provide a brief review of the literature relating to OR scheduling. Next, in Section 3, we present and describe the structural properties of our model. In Section 4, we present our solution methods to the problem. In Section 5, we present the numerical study of our model. Finally, in Section 6, we conclude our findings and point out future research directions.

# CHAPTER II

# LITERATURE REVIEW

The costs of ORs represent the area with the highest potential for cost minimization. Even small improvements in the efficiency means significant cost savings to hospital administration and benefits to the patients. The gains of well structured OR optimization systems offer promising research topics to the researchers. This has been the reason why literature on OR optimization problems are growing extensively over the past decades.

General studies regarding operating room scheduling can be found in Cayirli and Veral [2], Gupta [3] and Cardoen et al. [4].Our paper is related to two streams of literature. Papers in the first stream are related to next day OR scheduling, including with limited data. Papers in the second stream are related to surgery duration estimations using parametric and non parametric approaches.

In the first stream of literature, several authors propose stochastic optimization models for determining OR scheduling models. It is known that surgeries that are to be planned in a operating room involves uncertainty. And stochastic problems are harder to solve with regard to their deterministic counterparts and it is not always possible to obtain exact solutions. Weiss [5] studies the problem where the sequence of the surgeries are known by the OR manager and the manager should decide on the estimated start times of the surgeries. Lamiri et al. [6] proposes a model of elective and emergency surgeries in an identical multiple OR setup, where only emergency surgery durations are stochastic. Gerchak et al. [7] study the OR environment that includes uncertain elective procedures, as well as uncertain emergency surgeries.

Since stochastic optimization is very challenging in OR scheduling problems, the sample average approximation is often used. The basic idea of the method is that a random sample is generated and the expected value function is approximated by the corresponding sample average function (SAA). Given the fact that random samples are obtained using SAA, there is no uncertainty left in the problem. After generation of a set of random samples, a cost is computed. This process is repeated for a necessary amount of cases. After generation and cost computation processes are carried out, it is possible to evaluate the different cases of a given problem. By computing the average of a desired number of cases, minimization of the costs in objective function is executed. Jebali et al. [8] highlights the the robustness of stochastic OR scheduling and uses SAA to solve their problem. Min et al. [9] studies a stochastic optimization problem that includes elective surgery scheduling while considering capacity constraints. They implement SAA to minimize the total costs. Begen et al. [10] also studies SAA. They focus on the number of necessary samples if SAA is desired to be executed. Difference from the literature of their paper is that their approach is not standard stochastic programming methodology. They implement discrete convexity to solve the SAA in polynomial time for potentially correlated surgery durations under very mild conditions. They also develop distribution-free bounds on necessary number of samples to make sure that the optimal SAA solution is arbitrarily close to the optimal solution that could be obtained if the duration distributions are known. Since the bounds are distribution-free, required samples are relatively high. For 2 jobs, 0,95 accuracy level and 0.90 confidence level, their model suggests using more than one million samples.

Mancilla et al. [11] address a finite set of jobs with stochastic processing times. They used sample average approximation to approach the problem, which the authors call the scheduling problem. The authors develop algorithms for a single-resource stochastic appointment scheduling problem with waiting time, idle time, and overtime costs. In Denton et al. [12] and Mancilla et al [11], problems have been modeled as a two-stage stochastic programs, whereas in this study our model is an one-stage stochastic program.

One challenge in OR scheduling is how to carry out planning with limited data. To make use of distribution fitting, which is often used in OR scheduling, planners need large amount of data [13]. As an alternative for the need for large data sets, Delage and Ye [14] present a model that studies uncertainty in the distribution form and moments information of the durations. In the case of limited information of the data, they propose making use of mean and covariance matrix of the random vector. In addition to the supporting results of limited data performance in [14], Mak, Rong and Zhang [15] assume only moments information of durations to study models without distributions.

Papers in the second stream of literature are related to surgery duration estimation. OR scheduling problems involve uncertainties due to series of factors such as interruptions, communication failures, team familiarity and unplanned operations [16]. The decision is to try and find a suitable probability distribution and fit this distribution to a given set of surgeries. The literature to this day, mostly considers continuous processing durations with full probability characterization, meaning that the probability distribution of a problem is given as an input to the problem. Strum, Vargas, and May [17] describe an application of a news-vendor model as a heuristic for determining the planned OR schedule duration to allocate for surgical sub specialties. They fit probability distributions to historical patterns of surgical demand and combine them with the news vendor model to minimize costs associated with underutilization and over utilization of OR time.

Dexter and Ledolter [18] use weighted average of mean duration of past cases and scheduled durations where weights are calculated based on all cases, which Dexter et al. [19] improved further. Kayis et al. [20] provided another approach, that adjusts the scheduled duration using operational, temporal and surgical team effects.

Papers that examine the goodness fit of known distributions for estimating procedure durations, generally focus on the normal distribution and log normal distribution [21], [22], [23]. In May et al. [24] and Strum et al. [25] a comparison is made between log normal and normal distributions and log normal distribution is determined to represent the highly uncertain surgery durations.

In our paper, we fit the suitable distributions, log-normal and normal distribution, to generate samples using the parameters that we obtain by maximum likelihood estimation. In addition we exploit the structure of the problem and implement 3 solution methods that will be studied in section 4 of this paper. First method is a parametric method, second method is a non parametric method and the last one is a simple heuristic. After implementing the mentioned methods we will evaluate and compare each method with the other methods in means of cost minimization in our OR optimization problem. Introduction of our non parametric approach is our main contribution to the cited literature. In addition, our comparison between parametric, non parametric, heuristic approaches provide meaningful managerial insights for OR managers to be used especially in data-poor OR settings.

# CHAPTER III

# MODEL

Next-day OR scheduling starts with a given list of surgeries to be performed tomorrow, which is determined earlier. The OR manager, along with relevant hospital staff, decides on the assignment of surgeries to available ORs, the sequence of the surgeries to be completed and the OR time assigned to each surgery. During this decision making process the team aims to meet certain criteria: minimize the idle time of the OR, keep the amount of time OR has to stay beyond planned open time (i.e., overtime) as small as possible, start as many cases as possible on-time, and hold the time between planned and realized time of a surgery as low as possible.

In our model we assume that the quality of a schedule can be measured as a weighted sum of the expectation of four variables: waiting time, idling time, overtime. We let $N$ denote the number of surgeries to be scheduled in a given day. Random surgery durations are denoted by $t_i$, where the subscript $i$ indexes the $N$ surgeries in a given day. We assume that the first surgery start at time zero. $D_i$ denotes the scheduled duration for surgery $i$, in other words the start time of the next surgery in line. $H_i^s$ denotes the historical surgery duration of a surgery $i$, where superscript $s$ ($s = 1, 2, .., S_i$) indexes each historical duration for this surgery type. In our model we assume stochastic surgery durations, one operating room and a fixed sequence of the surgeries.

In this section we present a stochastic programming model for determining the optimal surgery durations and minimizing the cost incurred by several penalty costs. These penalty costs incur by (i) idle time, (ii) waiting time, (iii) overtime. Idle time ($s_i$) represents the time that an operating room becomes idle, meaning that the surgery $i$ finished before the planned finish time. As a result of this additional idle time, an idling cost ($\alpha_1$) incurs. This idle time occurs when the last surgery of a given OR day finishes early such that there is an idle time at the end of an entire OR day. Waiting time ($w_i$) represents the time that the actual finish time of a given surgery exceeds the planned surgery time, so that the next surgery can not begin, in other words next surgery runs late. As a result of waiting time, waiting cost ($\alpha_2$) incurs. We assume that the first surgery starts on time, in other words $w_1 = 0$. Overtime ($o_j$) represents the actual finish time of the last surgery of a given OR day, that exceeds the total planned time of an operating room. As a result of overtime, overtime cost ($\alpha_3$) incurs. As a result of earliness, an idling cost ($\alpha_1$) incurs. In Figure 1, we present the formation of idle time, waiting time and overtime for a 3 surgery case. To provide an example; we assume 3 surgeries for the next day with scheduled surgery durations, 3, 2.5 and 4 hours, respectively. We assume that in the next day, first sample surgery duration lasts for 4 hours, resulting with 1 hour of patient waiting time for the second surgery. Second sample surgery duration lasts for 1 hour, resulting with half hour idle time for second surgery. Third sample surgery duration lasts for 5 hours, resulting with 1 hour overtime for the given OR day.

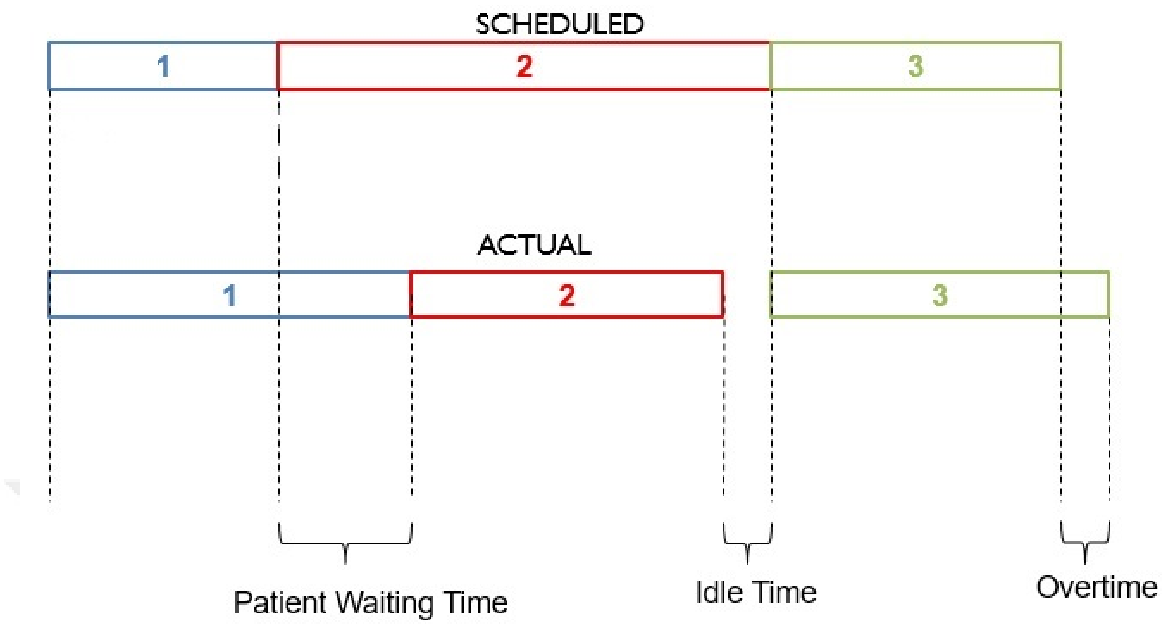


Figure 1: Formation of Idle Time, Waiting Time and Overtime

Given these definitions of our model, the stochastic optimization problem can be written as minimization of the weighted sum of the expectation of waiting, idling and overtime, as follows:

$$Z = \min_{D_i}\left\{\left(\sum_{i=1}^{N}\big(\alpha_1 E[s_i] + \alpha_2 E[w_i]\big) + \big(\alpha_3 E[o]\big)\right)\right\} \tag{1}$$

The fact that expectations in Equation 1 are over multiple random variables makes it challenging to evaluate the objective function in the scope of computation. The challenge of obtaining the optimal values is even greater. Since this is a hard problem to obtain an exact solution, we are going to use *sample average approximation*, hence we are going to solve the LP version of this stochastic problem.

In this paper we assume a discrete finite set of scenarios, $(j = 1, ..., K)$ and these scenarios represent the realizations of surgery durations. Given these discrete set of scenarios, we can write the deterministic equivalent of the equation (1) as the following.

$$Z = min\left\{\frac{1}{K}\left(\sum_{i=1}^{N}\sum_{j=1}^{K}\left(\alpha_1 s_{ij} + \alpha_2 w_{ij}\right) + \sum_{j=1}^{K}\left(\alpha_3 o_j\right)\right)\right\} \tag{2}$$

s.t.

$$-w_{ij} + w_{i+1j} - s_{ij} + D_i = t_{ij}, \qquad \forall j, i \in 1, .., (N-1) \tag{3}$$

$$-w_{Nj} + o_j - s_{Nj} + D_N = t_{Nj}, \qquad \forall j \tag{4}$$

$$t_i \geq 0, \qquad \forall i \tag{5}$$

$$w_{ij} \geq 0 \qquad \forall (i, j) \tag{6}$$

$$s_{ij} \geq 0, \qquad \forall (i, j) \tag{7}$$

$$o_j, \geq 0, \qquad \forall j \tag{8}$$

*Variables*

- $w_{ij}$: waiting time for surgery $i$ in scenario $j$.
- $s_{ij}$: idle time for surgery $i$ in scenario $j$.
- $O_j$: overtime for scenario $j$
- $D_i$: scheduled duration for surgery $i$, start time of the next surgery in line.

*Parameters*

- $t_{ij}$: sample surgery duration of surgery $i$ in scenario $j$.
- $H_i^S$: historical surgery duration for surgery $i$.
- $\alpha_1$: idling cost.
- $\alpha_2$: waiting cost.

- $\alpha_3$: overtime cost.

*Indices and sets*

- $i$: surgery to be scheduled $i = 1, ..., N$,
- $j$: scenario in a given day $j = 1, ..., K$,
- $s$: each historical duration for surgery $i$. ($s$ = 1, 2, ... , $S_i$),
- $S_i$: total number of historical surgery duration sample for surgery $i$,
- $N$: number of surgeries to be scheduled in a given day.
- $K$: total number of scenarios in a day.

*Constraints*

Equation (3) represents the relationship between waiting and idling times with respect to the actual and scheduled duration for a given surgery in a given scenario. Equation (4) represents the relationship between overtime and earliness based on the completion time of the last surgery and the scheduled duration, $D$ for a given scenario. Equations (5,6,7,8) are constraints that provide non negativity for the model.

The important part in our model is to generate samples, specifically sample surgery durations. In the beginning we do not have the sample surgery durations, $t_{ij}$, however we have historical surgery durations, $H_i^S$. The question is how to make the connection between these durations. In parametric approach, we generate a derived distribution by using historical surgery durations. After obtaining derived distribution, we generate sample surgery durations from this derived distribution. In non parametric approach, we do not carry out an estimation process. Instead, we combine historical surgery durations and have sample surgery durations. In our model, using the historical data at hand, we obtain solution by (i)the parametric approach, (ii) non parametric approach, and (iii) a simple but practical heuristic. All three approaches have the same model. Parametric approach is dependent on a probability distribution, however, nonparametric approach does not depend any probability distribution information. These three methods will be studied in detail in the next section.

# CHAPTER IV

# SOLUTION METHODS

For the solution methods, we offer 3 approaches; (i) Parametric approach, (ii) Non parametric approach and (iii) a simple but practical heuristic. Results from these three approaches are compared with sample distribution cost which is studied in the following benchmark section.

## *4.1 Benchmark*

Most of the cases in this study assumes log normal distribution for sample distribution. We present one additional 6 surgery case with normal distribution. We assume that sample surgery durations, $t_{ij}$, form a distribution that we refer as the sample distribution. We generate samples using this sample distribution. After generation of samples, we solve our model with sample average approximation and obtain scheduled surgery durations. After obtaining sample durations, we use these samples in our model, and obtain scheduled surgery durations and associated idle time, waiting time and overtime with corresponding sample distribution cost. We call different versions of created samples as scenarios. Since it is challenging to carry out stochastic optimization and obtain exact solutions, we incorporate the average value of the cost of each scenario to approximate the stochastic optimization results.

For instance, for a specific 2 surgery example, let us assume parameters with $ln(3, 1)$ and $ln(2, 0.5)$. Let us have resulting sample surgery durations 2, 2.5 and 4 hours for the first surgery and 1.5, 2, 2.25 hours for the second surgery. A pair of $(2, 2.25)$ or combination of these sample durations represent one scenario. A total of 8 scenarios from 2 surgeries with 3 sample durations are used in the model. A sample distribution cost is obtained from each scenario. Then, an average of cost of these 8 scenarios is computed as the sample average cost. In addition to sample distribution cost, we obtain scheduled surgery durations for two surgeries based on each scenario, e.g. 3 hour for the first surgery and 2.5 hours for the second surgery. Moreover, idle times and waiting times for surgeries and overtime for a given OR day are obtained from the model.

Sample distribution cost makes it possible to know the associated cost if we knew the sample distribution. We use sample distribution as our benchmark, which we implement to compare the associated costs resulting from each solution method. The following represents the sample distribution cost:

$$O(D_i(F_i^T), F_i^T) \tag{9}$$

Scheduled surgery durations that are obtained from three solution approaches studied in the following sections are implemented in sample distribution to find out how each solution approach performs.

## 4.2 Parametric Approach

In parametric approach we assume that we know the family of the probability distribution of surgery durations, which is assumed to be log-normal or normal distribution based on the literature review.

In parametric approach, we have historical surgery duration samples, $H_i^S$. We derive a distribution in order to generate samples that are to be used in sample average approximation. We carry out a distribution fitting to the historical surgery duration samples. After distribution fitting, we use the parameters of this distribution to generate samples. The question is how to obtain the parameters of the distribution fitted samples. Assuming we know the family of the distribution, we use maximum likelihood approach to estimate the parameters of the fitted distribution. Maximum likelihood parameter estimators $\hat{\mu}$ and $\hat{\sigma}^2$ are obtained using the following:

$$\hat{\mu} = \frac{\sum ln(H_i^S)}{S_i} \tag{10}$$

$$\hat{\sigma^2} = \frac{\sum(ln(H_i^S) - \hat{\mu})^2}{S_i} \tag{11}$$

After executing maximum likelihood estimation, we obtain sample surgery durations, $t_{ij}$. Having the sample cost from the benchmark, we need to find parametric approach scheduled surgery duration, $D_i^P$, in order to compute the cost of parametric approach. Parametric approach cost is represented as the following:

$$O(D_i^P(F_i^D), F_i^T) \tag{12}$$

After computing the parametric approach scheduled surgery durations and costs, we are able to find out how this approach performs. Performance analyzation will be done using $GAP_P$ values, which will be studied in the next chapter of this study.

## *4.3 Non Parametric Approach*

In this approach we would like to find out how well an OR manager may perform OR scheduling with the absence of probability distribution information. In other words, we assume that we do not know the family of the probability distribution of the historical surgery duration samples nor we do not desire to carry out such estimation process. There are $m$ surgery duration samples for each surgery. In the non parametric approach, we use the combination of historical surgery durations as sample surgery durations $t_{ij}$. We treat historical surgery duration samples as non parametric empirical distribution samples.

Having the sample cost from the benchmark, we need to find parametric approach scheduled surgery duration, $D_i^{NP}$, in order to compute the cost of non parametric approach. Non parametric approach cost is represented as the following:

$$O(D_i^{NP}(F_i^D), F_i^T) \tag{13}$$

Having computed the non parametric cost, we are able to observe how non parametric approach performs using $GAP_{NP}$ values which will be studied in the next chapter.

The difference between parametric and non parametric approach is how to combine sample surgery duration with historical surgery durations.

## 4.4 A Simple Heuristic

As the third and the last approach we implement a simple heuristic, called expectation heuristic which is common in literature. In the heuristic approach, the sample surgery durations are the expected value of the historical surgery durations. Actual surgery duration are computed as follows:

$$t_{ij} = \frac{\sum_{J=1}^{m}(H_i^S)}{S_i} \tag{14}$$

After computing sample surgery durations, we need to find simple heuristic approach scheduled surgery duration, $D_i^H$, in order to compute the heuristic cost. The following represents the cost of our simple heuristic:

$$O(D_i^H(F_i^D), F_i^T) \tag{15}$$

Having computed the cost of our simple heuristic we compute $GAP_H$ value to evaluate the performance of our simple heuristic. Computation and evaluation of $GAP_H$ values are studied in the next chapter.

# CHAPTER V

# NUMERICAL STUDY

In this section we present numerical study to analyze and compare our three solution approaches. There are computations that three solution approaches have in common. These computations include the sample distribution related sample surgery durations and sample distribution costs.

There are 2 to 6 surgeries, inclusive, to be planned in a given OR day. Idling cost is either 0.1, 0.3, 0.5, 0.7 or 0.9, and:

$$\alpha_2 = \alpha_3 = 1 - \alpha_1 \tag{16}$$

We have 80 instances that are studied for each number of surgeries. In our model, we form our instances such that, scenarios from the same instance have the same surgery duration parameters and penalty costs $(\alpha_1, \alpha_2, \alpha_1)$. For each number of surgeries, we have different number of historical surgery duration samples (5, 10, 25, 50 and 100 samples). Our 80 instances are replicated for different number of times. A replication is the process of solving a given case with different parameters which are random. For 2 surgeries we replicate our instances 30 times and for 3, 4, 5 and 6 surgeries we replicate the solution process 10 times. The resulting number of scenarios will be 12000 for 2 surgery scenarios and 4000 for 3, 4, 5 and 6 surgery scenarios. After each surgery model is studied with associated number of replications, we compute the expectation of a given case. The reason behind these replications is to limit the effect of the high standard deviations. Different results with different mean and standard deviation values are stemmed from the random parameters such as sample surgery duration that are generated randomly in each replication. If we do not use replications, we may have results with low mean values, but high standard deviations which may be misleading for analysis. We refer to the samples in sample average approximation as scenarios to provide clarity against the possible confusion with historical surgery duration samples. We use 50000 scenarios for parametric and heuristic approach. For non parametric approach number scenarios change according to the following:

$$K = (S_i)^N \tag{17}$$

For cases in which the number of scenarios are smaller than 50000, we use the combination of historical surgery durations. The number of scenarios increase with increasing number of historical surgery duration samples and surgeries to be planned. For this reason, if the number of scenarios are larger than 50000, we put an upper bound of 50000 scenarios for the sake of computation time. This means for a given case, if the number of scenarios are larger than 50000, we limit this number of scenarios to 50000. We carry out this limitation by using sampling without replacement. In such cases where number of scenarios are larger than 50000, we generate all sample surgery durations from sample distribution. However, we randomly select 50000 of these durations for each surgery to form a matrix of size 50000 x $N$.

In the beginning of our computation process, we find sample distribution cost. In order to find the sample cost, we need to implement sample average approximation. Sample average approximation is known to approximate better with more samples available for use. After implementing sample average approximation, we obtain the sample surgery durations and sample cost. As stated in the previous chapter, sample distribution cost is our benchmark. It is important to note that the sample surgery durations that are computed in our benchmark is different from the sample surgery durations that are to be computed within each solution method. In fact, sample surgery durations obtained in the sample distribution in benchmark is only used to compute the sample distribution cost.

When the process of parameter estimation and generation is next in solution process, the type of assumed distribution becomes important. As stated earlier, major part of this study focuses on log normal distribution with one exception of a 6 surgery case with normal distribution. In log normal distribution sample, our model assumes log normal distribution for parameter generation, when sample distribution has log normal distribution. However in the normal distribution case, our model assumes normal distribution for parameter generation when in fact sample distribution has log normal distribution.

After computation of sample distribution cost is complete, we begin our work on our solution methods. Largest difference in three solution methods is how to obtain the actual duration. Actual surgery durations are obtained differently in each solution approach. In parametric approach, we implement distribution fitting to generate the parameters of the probability distribution. Using this fitted distribution, we generate sample surgery durations. In non parametric approach, we combine historical surgery duration sample in order to generate an empirical non parametric samples, which are to be used as sample surgery durations. In heuristic approach, we carry out an expectation heuristic to obtain sample surgery durations. After obtaining the actual durations, we continue with the computation of approach costs. In this study, we define a performance measure $GAP$ value to find out how a particular solution method performs. $GAP$ values represent the level of performance when compared to sample costs. $GAP_P$ represents the performance level of parametric approach. $GAP_{NP}$ represents the performance level of non parametric approach. And finally, $GAP_H$ represents the the performance level of heuristic approach. The smaller the gap value, the closer we get to the sample distribution cost. For the optimization tool, we use IBM ILOG CPLEX Optimization Studio version 12.5 on a computer with a i7 processor, 8 gigabytes of physical memory.

In the following three subsections, we present the numerical results of the solution methods with respect to the selected parameters of our model, which are idling cost, number of historical samples and the number of surgeries. After presenting the numerical results for each method, we provide when each method should be used when compared to other solution methods. In the last subsection of this chapter, we present the comparison of the methods which includes the interpretations of numerical results presented in previous subsections.

## 5.1 *Parametric Approach*

To examine the results of parametric approach and find out how well it performs, we use the following gap value;

$$GAP_{NP} = \frac{O(D_i^P(F_i^D), F_i^T) - O(D_i^P(F_i^T), F_i^T)}{O(D_i^P(F_i^T), F_i^T)} \tag{18}$$

As stated in the previous chapter, to find out how well the parametric approach performs when compared to sample distribution, we need to find the performance level of this solution approach, $GAP_P$. In order to do so, we first find the sample distribution cost. Then we calculate parametric approach cost, in order to find out how our model would perform if implemented in the sample distribution. Computing the formulation above, we obtain $GAP_P$, which is a percentage of performance difference to sample distribution cost of scheduled surgery durations that are obtained from parametric approach if utilized in sample distribution. We present the average $GAP_P$ values in Table 1 that includes the numerical results of $GAP_P$ with respect to $\alpha_1$. For each number of surgery column, the average percentage represents the expected value for all results with a fixed idle cost.

From Table 1, it is possible to say that the parametric approach does not perform the best when the idling cost is lower. This worsened performance is also valid for higher idling costs. At average values of $\alpha_1$ (0.3, 0.5 and 0.7), parametric approach yields better results.

The reason behind presenting numerical results with respect to only $\alpha_1$ is that we can conclude other results from $\alpha_1$. $\alpha_2$ and $\alpha_3$ can be evaluated by studying the

Table 1: The Effect of $\alpha_1$ on $GAP_P$ (In %)

| | N | | | | |
|---|---|---|---|---|---|
| $\alpha_1$ | 2 | 3 | 4 | 5 | 6 |
| 0.1 | 11,07 | 11,55 | 11,34 | 12,92 | 12,80 |
| 0.3 | 5,11 | 5,75 | 6,29 | 6,67 | 6,74 |
| 0.5 | 3,90 | 4,06 | 4,40 | 4,63 | 5,20 |
| 0.7 | 3,91 | 3,81 | 3,79 | 3,67 | 3,98 |
| 0.9 | 6,37 | 4,91 | 4,57 | 4,21 | 4,68 |

results of $\alpha_1$ since they $\alpha_1$ yields just the opposite results as $\alpha_2$ and $\alpha_3$. We present the average $GAP_P$ values in Table 2 that includes the relationship between parametric approach and number of historical samples.

Table 2: The Effect of $S_i$ on $GAP_P$ (In %)

| | N | | | | |
|---|---|---|---|---|---|
| $S_i$ | 2 | 3 | 4 | 5 | 6 |
| 5 | 17.14 | 16.37 | 17.06 | 18.35 | 19.081 |
| 10 | 7.82 | 8.47 | 7.82 | 8.31 | 8.77 |
| 25 | 3.04 | 2.81 | 3.23 | 3.09 | 3.19 |
| 50 | 1.48 | 1.56 | 1.45 | 1.49 | 1.67 |
| 100 | 0.73 | 0.798 | 0.799 | 0.78 | 0.85 |

When we look at Table 2, we see that the gap values do not radically change with the increasing number of surgeries. What we can clearly see is that as the number of historical surgery duration samples increase, gap values decrease. In Table 2 we can note that $GAP_P$ values are highest when the number of surgeries are largest. Average values may change within replications due to randomness in our model. We desire to find out the significance of these changes. Since the average values may be misleading, we present the standard deviation of $GAP_P$ values in Table 3. In Table 3, each standard deviation percentage represents the expected value of standard deviations for a given number of surgery, including all replications.

Table 3: The Standard Deviation of $GAP_P$ With Respect to $S_i$ (In %)

| | N | | | | |
|---|---|---|---|---|---|
| $S_i$ | 2 | 3 | 4 | 5 | 6 |
| 5 | 20,21 | 15,83 | 14,68 | 14,69 | 14,56 |
| 10 | 9,19 | 7,81 | 6,43 | 6,71 | 6,60 |
| 25 | 3,48 | 2,43 | 2,82 | 2,35 | 2,64 |
| 50 | 1,64 | 1,37 | 1,17 | 1,09 | 1,22 |
| 100 | 0,81 | 0,70 | 0,62 | 0,59 | 0,55 |

In Table 3, we note that with increasing number or samples, slope of standard deviations of performance levels decrease dramatically. Apart from consistent decrease in standard deviations, decrease in transitions to higher sample sizes are greater when sample sizes are smaller such as 5 samples. In fact, improvement of performance in standard deviation from 5 samples to 10 samples are 5 time greater than the transition from 50 samples to 100 samples. When sample size reaches a level, such as 25 samples, scheduled durations are adjusted close to each other due to low variance of generated samples. Decrease in standard deviations support the validity of performance level average values in Table 2. Implementing parametric approach with larger sample sizes the sample sizes would not be beneficial because of computation times. This implicates that it would be wiser to execute parametric approach when the sample sizes are low.

## 5.2 Non Parametric Approach

To examine the results of non parametric approach and find out how well it performs, we use the following gap value;

$$GAP_{NP} = \frac{O(D_i^{NP}(F_i^D), F_i^T) - O(D_i^{NP}(F_i^T), F_i^T)}{O(D_i^{NP}(F_i^T), F_i^T)} \tag{19}$$

To find out how well the parametric approach performs when compared to sample distribution, we need to find the performance level of this solution approach, $GAP_{NP}$. In order to do so, we first find the sample distribution cost. Then we calculate non parametric approach cost, in order to find out how our model would perform if implemented in the sample distribution. Computing the formulation above, we obtain $GAP_{NP}$, which is a percentage of performance difference to sample distribution cost of scheduled surgery durations that are obtained from non parametric approach if utilized in sample distribution. We present the average $GAP_{NP}$ values in Table 4 that includes the numerical results of $GAP_{NP}$ with respect to $\alpha_1$. For each number of surgery column, the average percentage represents the expected value for all results with a fixed idle cost. From Table 4, it is possible to comment that non parametric

Table 4: The Effect of $\alpha_1$ on $GAP_{NP}$ (In %)

| | N | | | | |
|---|---|---|---|---|---|
| $\alpha_1$ | 2 | 3 | 4 | 5 | 6 |
| 0.1 | 13,71 | 14,34 | 16,35 | 16,34 | 17,87 |
| 0.3 | 6,68 | 6,96 | 7,38 | 8,23 | 8,23 |
| 0.5 | 5,12 | 5,28 | 5,04 | 5,52 | 6,35 |
| 0.7 | 5,09 | 4,76 | 4,71 | 4,72 | 4,70 |
| 0.9 | 6,62 | 5,36 | 5,15 | 4,75 | 5,23 |

approach behaves similar to parametric approach when idle cost changes. Similarly, $GAP_{NP}$ values are maximum when idle cost is lowest. The $GAP_{NP}$ values decrease when idle cost is average (0.3, 0.5, 0.7). We present the average GAP values of non parametric approach in Table 5 that includes the numerical results between GAP values and number of historical samples.

Table 5: The Effect of $S_i$ on $GAP_{NP}$ (In %)

| | N | | | | |
|---|---|---|---|---|---|
| $S_i$ | 2 | 3 | 4 | 5 | 6 |
| 5 | 20.02 | 19.21 | 20.97 | 21.35 | 23.15 |
| 10 | 9.86 | 10.22 | 10.05 | 10.91 | 11.66 |
| 25 | 4.19 | 3.86 | 4.44 | 4.12 | 4.54 |
| 50 | 1.97 | 2.23 | 1.99 | 1.97 | 2.18 |
| 100 | 1.018 | 1.040 | 1.045 | 1.029 | 1.092 |

When we look at Table 5, a increase is observed in the performance, as the historical surgery duration sample size is increased. Main pillar of non parametric approach is the number of historical samples. Since there is no interest in probability distribution, improvement in performance relies solely on larger number of historical samples. With increased number of samples, non parametric approach is able to create more sample surgery durations, resulting with less variance of sample surgery durations. In addition, with more historical samples at hand, non parametric approach provides scheduled surgery durations that are close to those of sample distribution, resulting with smaller $GAP_{NP}$ and increased performance. Average values may change within replications due to randomness in our model. We desire to find out the significance of these changes. Since the average values may be misleading, we present the standard deviation of $GAP_{NP}$ values in Table 6. In Table 6, each standard deviation percentage represents the expected value of standard deviations for a given number of surgery, including all replications.

Table 6: The Standard Deviation of $GAP_{NP}$ With Respect to $S_i$ (In %)

| | N | | | | |
|---|---|---|---|---|---|
| $S_i$ | 2 | 3 | 4 | 5 | 6 |
| 5 | 22,70 | 17,45 | 17,16 | 17,00 | 16,79 |
| 10 | 10,97 | 8,61 | 7,84 | 8,47 | 8,07 |
| 25 | 4,53 | 3,21 | 3,43 | 3,07 | 3,26 |
| 50 | 2,16 | 1,97 | 1,57 | 1,42 | 1,46 |
| 100 | 1,11 | 0,88 | 0,81 | 0,76 | 0,74 |

With supporting results from Table 6, it is possible to comment that when the number of surgeries to planned in a given OR day is minimum or maximum, the performance of non parametric approach decreases in a similar fashion to parametric approach.

## 5.3 Heuristic Approach

To find out how well the heuristic approach performs when compared to sample distribution, we need to find the performance level value of this solution approach, $GAP_H$ using the following:

$$GAP_H = \frac{O(D_i^H(F_i^D), F_i^T) - O(D_i^H(F_i^T), F_i^T)}{O(D_i^H(F_i^T), F_i^T)} \tag{20}$$

Computation of sample distribution cost and heuristic cost is introduced in the previous chapter. Similar to all gap values, the smaller $GAP_H$, the better performance for our heuristic approach. It is important to note that the results of heuristic approach yield average results as expected. The characteristics of this approach are easy implementation, less computation times, however average results when compared to other two solution approaches. We present the average performance of parametric approach in Table 7 that includes the numerical results of $GAP_H$ with respect to $\alpha_1$. For each number of surgery column, the average percentage represents the expected value for all results with a fixed idle cost.

Table 7: The Effect of $\alpha_1$ on $GAP_H$ (In %)

| | N | | | | |
|---|---|---|---|---|---|
| $\alpha_1$ | 2 | 3 | 4 | 5 | 6 |
| 0.1 | 168,10 | 238,74 | 263,97 | 302,55 | 366,81 |
| 0.3 | 30,25 | 51,40 | 62,98 | 72,42 | 94,73 |
| 0.5 | 5,81 | 12,12 | 17,30 | 20,89 | 31,19 |
| 0.7 | 11,78 | 6,43 | 6,44 | 5,80 | 7,86 |
| 0.9 | 107,43 | 66,37 | 56,49 | 47,04 | 34,72 |

From Table 7, we are not able to provide managerial insights about our heuristic approach, since it reflects the main characteristic of this approach, which is insensitivity to changes in $\alpha_1$. Since the number of sample surgery duration samples are smaller when compared to other approaches, we are not able to observe the effect of $\alpha_1$ very well. However, number of historical samples and number of surgeries have direct effects on $GAP_H$. We present the average GAP values of heuristic in Table

8 that includes the numerical results between GAP values and number of historical samples.

Table 8: The Effect of $S_i$ on $GAP_H$ (In %)

| | N | | | | |
|---|---|---|---|---|---|
| $S_i$ | 2 | 3 | 4 | 5 | 6 |
| 5 | 75,38 | 82,61 | 89,46 | 103,12 | 120,46 |
| 10 | 66,51 | 77,33 | 81,90 | 93,30 | 109,57 |
| 25 | 62,13 | 73,67 | 80,30 | 85,82 | 104,56 |
| 50 | 59,65 | 72,10 | 78,17 | 83,23 | 102,30 |
| 100 | 59,33 | 69,13 | 76,73 | 83,12 | 100,31 |

When we look at Table 8, a negative slope is observed in the performance, as the historical surgery duration sample size is increased as expected. Another interpretation of Table 8 is decreased performance of heuristic approach with increasing number of surgeries. For any number of historical surgery duration sample, heuristic approach performs better for 2 surgery cases when compared to 6 surgery cases. In heuristic approach, it is challenging to reach high performance values, when the complexity of model increases with increasing number of surgeries to be planned in a given OR day.

Average values may change within replications due to randomness in our model. We desire to find out the significance of these changes. Since the average values may be misleading, we present the standard deviation of $GAP_H$ values in Table 9. In Table 9, each standard deviation percentage represents the expected value of standard deviations for a given number of surgery, including all replications.

Table 9: The Standard Deviation of $GAP_H$ With Respect to $S_i$ (In %)

| | N | | | | |
|---|---|---|---|---|---|
| $S_i$ | 2 | 3 | 4 | 5 | 6 |
| 5 | 48,74 | 43,73 | 44,97 | 47,38 | 49,86 |
| 10 | 32,59 | 30,47 | 31,31 | 31,52 | 33,28 |
| 25 | 20,71 | 19,00 | 19,72 | 18,78 | 21,74 |
| 50 | 14,22 | 14,63 | 13,56 | 12,58 | 15,07 |
| 100 | 9,77 | 9,88 | 10,01 | 9,14 | 10,39 |

After carrying out analysis for heuristic approach, we can conclude that the number of historical samples and number of surgeries to be planned are the parameters that have direct effects on the performance of heuristic approach. This validates the fact that for OR environments that have more historical surgery duration samples and less number of surgeries, executing heuristic approach means better results, in contrast with OR environments with less historical surgery duration samples and more surgeries.

## *5.4 Comparison of Three Solution Methods*

In this section we combine our findings in numerical study to compare our three solution methods. Before beginning to analyze the numerical results of our three solution approaches, we should state that for comparison, the sample distribution cost for all three approaches are the same. The difference between each solution costs will stem from the planned surgery duration under each approach and the associated costs when each approach is implemented in sample distributions. At this point, we divide this section into two subsections in order to compare our solution methods under two probability distributions used in this study.

### 5.4.1 Log normal Distribution

Without taking a look in the numerical results of each three solution methods, one could argue that the parametric approach would yield the best results. The main advantage of parametric approach is the information of probability distribution. With this information parametric approach could perform better, provided that the probability distribution information is correct. When the information regarding the probability distribution is not correct, then other two approaches stand a chance of performing better than the parametric approach. In Figures 2 and 3 we present the comparison of performance levels of parametric and non parametric approach. In Figures 4 and 5 we present the comparison of performance levels of heuristic approach.

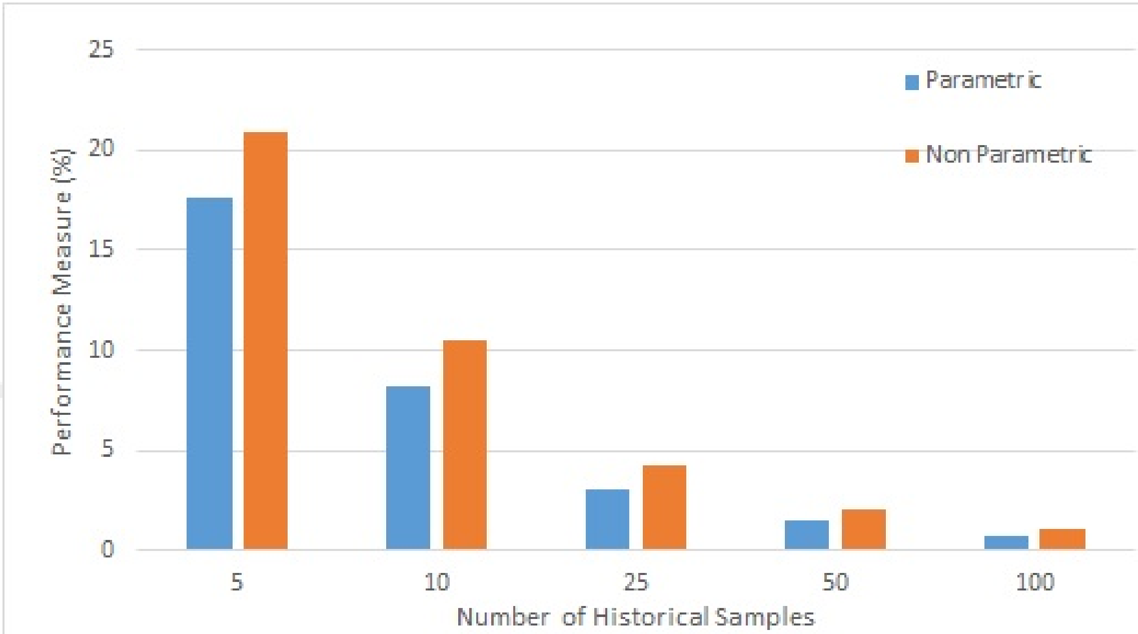


Figure 2: Performance of Parametric and Non Parametric Approach with Respect to Sample Sizes

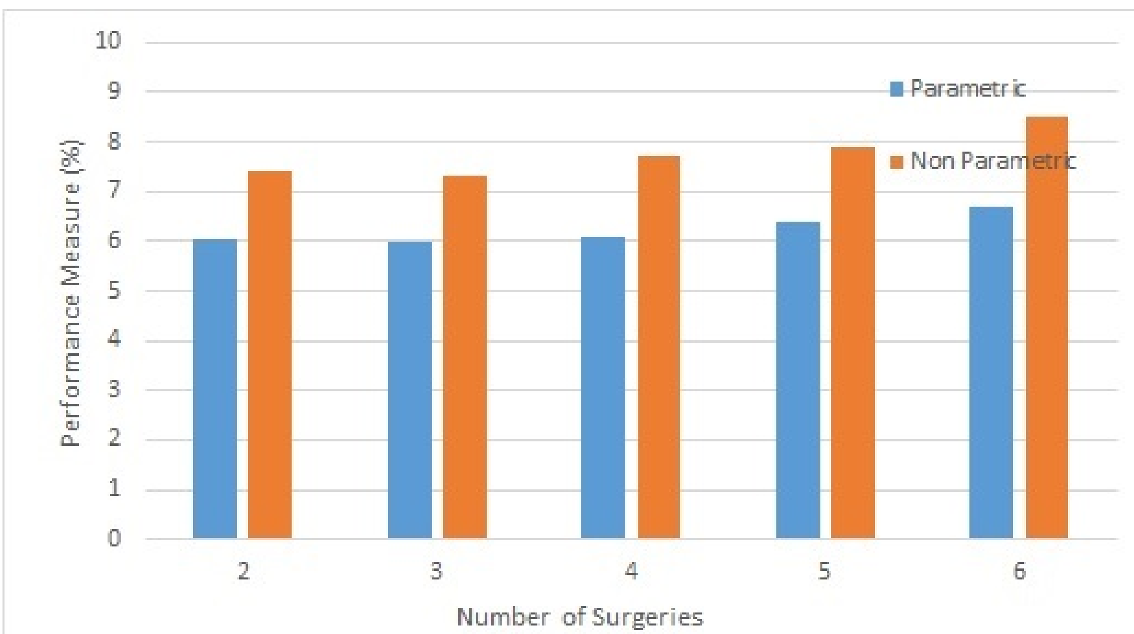


Figure 3: Performance of Parametric and Non Parametric Approach with Respect to Number of Surgeries

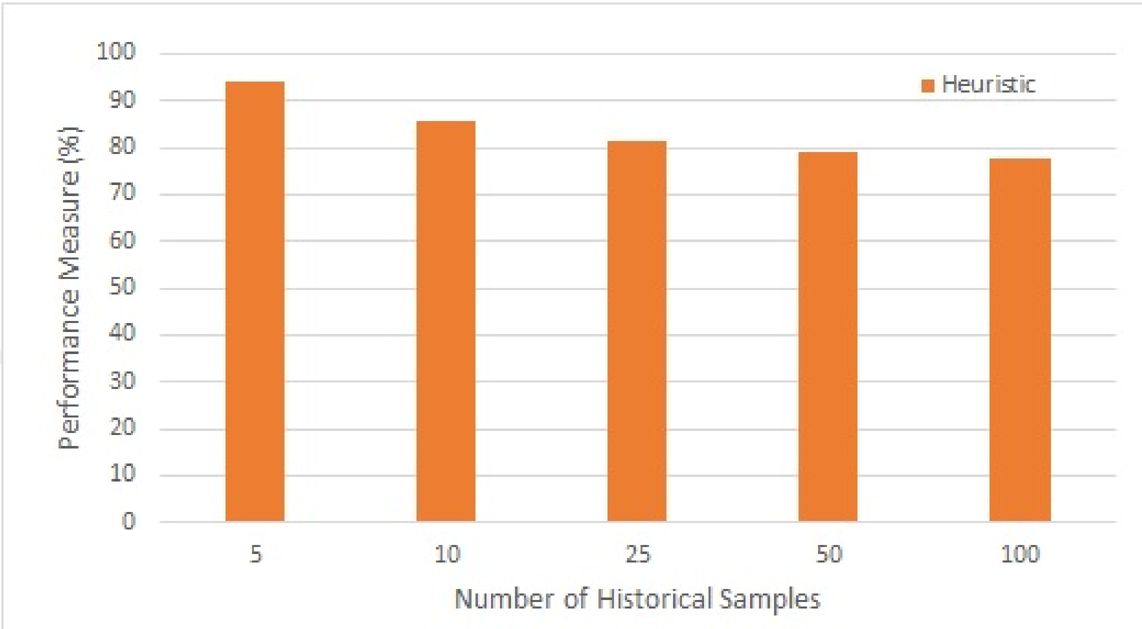


Figure 4: Performance of Heuristic Approach with Respect to Sample Sizes

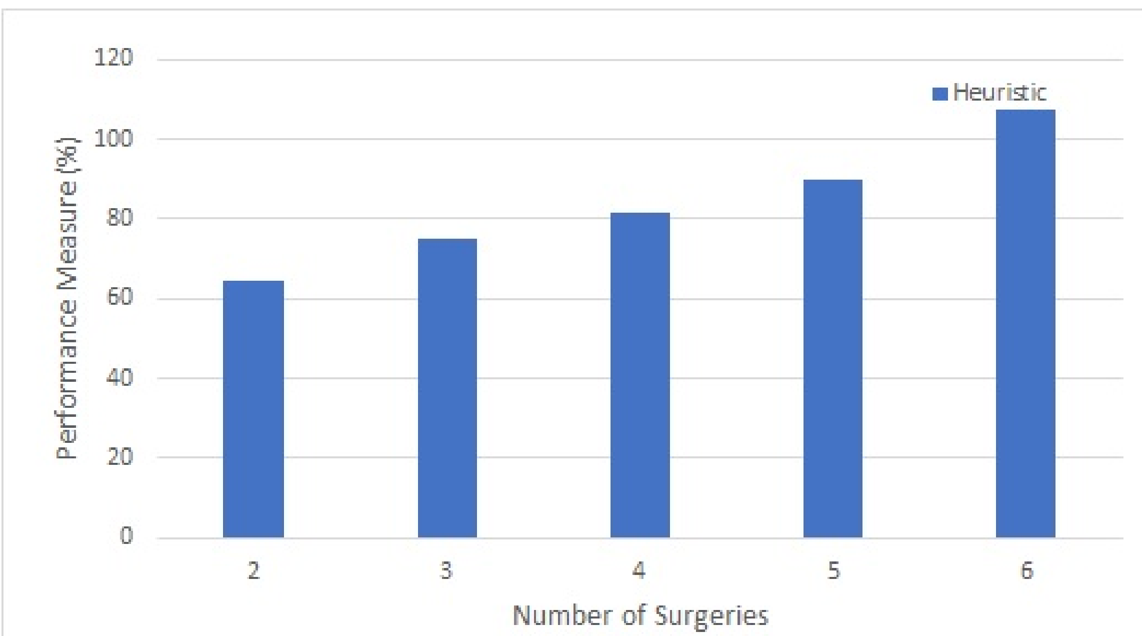


Figure 5: Performance of Heuristic Approach with Respect to Number of Surgeries

When we compare the numerical results, we can see that the parametric approach performed the best among other two solution approaches. The interesting point is not exactly parametric approach performing as the best, but non parametric approach performing almost as good as parametric approach when the number of historical surgery duration samples increase. In the non parametric approach, OR planner does not hold any information about the probability distribution of the surgeries. This may seem as a disadvantage at first glance, however, it also is a advantage, because non parametric approach does not make errors on estimating the probability distribution.

With small sample sizes such as 5 historical surgery duration samples, parametric approach performs 3 percent better than the non parametric approach. This better performance drops to 0.3 percent when the sample size is increased to 100. This implies that, with small numbers of historical data, parametric approach is advantageous. However, with large sample sizes, using parametric approach would not yield such great results. Implementing non parametric approach, an OR planner should conclude very good results and avoid all the necessary distribution estimation step of parametric approach.

We can clearly state that the parametric approach performs better than other approaches when: i) Correct assumption of probability distribution is carried out and, ii) Historical surgery duration sample size is small. Parametric approach performs better when sample size is small, because regardless of the sample sizes, parametric method could use many scenarios. Regarding a case where the sample size is 5 and the number of surgeries is 2, non parametric approach will have a scenario size of $5^2 = 25$, whereas the parametric approach will have 50000 generated scenarios. This advantage will hold for all cases, because for the sake of computation time, we assume that the scenario sizes can reach a maximum value of 50000. However, as the historical surgery duration sample size becomes larger,e.g. at least 25 samples, non parametric approach starts to perform almost as good as the parametric approach. And with an addition of wrong probability distribution assumption made by parametric approach, non parametric approach may even perform better. The probability of such case is small yet it exists. One of the main conclusions of our work states that the use of non parametric approach where there are at least 25 samples is more logical, whereas in cases of small sample sizes, use of non parametric approach is more beneficial since we can generate as much sample up to 50000 as we need.

The other solution approach that we presented in our model, expectation heuristic, may be taken for granted for average computations. Our proposed heuristic may seem to perform worse that other two methods, however, this heuristic is a simple but practical one. It does not entail long computation times, nor expensive optimization tools. Our heuristic is a very good alternative to the other two solution approaches. It may be implemented in the beginning of structuring new operating room environments. It may also serve as a buffer between transitions from one OR planning approach to an another. Our heuristic approach may be used to full extent by small hospitals, where very meticulous planning is not very required. The planners implementing the heuristic approach should note that this approach is robust to sample size. One advantage of implementing this approach is that the planner will know what the hospital loses in the absence of such fine optimization techniques.

After evaluating standard deviations of performance levels of each solution method, we note that the standard deviation of optimization gaps gets smaller values as the number of historical surgery duration samples increase. One conclusion that supports the previous results presented in this section is that the variance of optimization gap for the parametric method is the smaller when compared with the other two solution methods. Similarly, variance of optimization gap is largest in heuristic approach. We also note that, in contrast with other two methods, variance for heuristic method gets larger values as the number of surgeries increase.

#### *5.4.1.1 Study on the Parameters of Model*

In order to compare our solution aproaches, we study the effect of parameters in our model. This study allows us to make interpretations that help us comment on the advantages and disadvantages of our solution approaches, specifically parametric and non parametric approach. After such study, we are able to provide managerial insights regarding when and why to use parametric and nonparametric approach. In the following 3 subsections, we will study i) Idle time cost ii) Number of Historical Samples and iii) Number of Surgeries.

**Idle Time Cost** Idling cost may occur when a surgery finishes before its planned duration, and in addition it may occur for each scenario when total sample surgery time is less than total planned surgery duration for a given day. For this reason, as idling cost increases, we generally observe decreases in the GAP values, because our model forces to decrease the idling time while increasing waiting time and overtime accordingly.

When we examine the relationship between idle time cost and the performance of solution approaches, it is possible to state that the heuristic approach performs the worst (see Table 7). When it comes to the comparison between the parametric and the non parametric method, such statement is not so easy to make. What we may clearly see is that as idling cost increases from 0,1 to 0,9, GAP values for both approaches decrease. In addition, judging by the GAP values themselves, parametric approach perform better when compared to the non parametric approach. When the idling cost reaches the value 0.9, high idling cost becomes more effective such that we observe higher GAP values. However, these results are average values 28000 replications in total (12000 for 2 surgeries plus 4000 for 3-6 surgeries each), and they may be misleading. When the parametric approach outperforms non parametric approach, this happens more and with a big difference in the associated performance levels.

Whereas, when the non parametric approach outperforms parametric approach, this happens with slight differences in the performance levels. To support this intuition, we present the percentage of cases that the nonparametric approach outperforms the parametric approach in Table 10:

Table 10: Percentage of Cases Nonparametric Approach Outperforms Parametric Approach with Respect to $\alpha_1$

| | N | | | | |
|---|---|---|---|---|---|
| $\alpha_1$ | 2 | 3 | 4 | 5 | 6 |
| 0.1 | 18,60 | 18,21 | 16,52 | 18,56 | 15,99 |
| 0.3 | 19,95 | 20,60 | 19,90 | 19,14 | 20,28 |
| 0.5 | 19,37 | 18,21 | 21,08 | 20,58 | 21,27 |
| 0.7 | 19,97 | 19,90 | 20,80 | 19,93 | 20,28 |
| 0.9 | 22,11 | 23,07 | 21,70 | 21,80 | 22,19 |

The number of cases that the parametric approach outperforms non parametric approach is bigger than the opposite case. However, when we examine Table 10, we see that the non parametric approach performs better as idling cost increases such that it handles the rise in GAP values better when the idling cost is maximum at 0,9. So it may be recommended to use non parametric approach when both GAP values for parametric and non parametric approach increase at maximum idling cost, which is 0,9.

Similarly low idling costs favor the use of parametric approach, resulting in increase of difference in GAP values between parametric and non parametric approach. For any number of surgery scenarios, the parametric approach yields better results when the idling cost is minimum rather than it is maximum. These findings suggest that if the idling cost is low, using parametric approach may give better results.

When idling cost is low, model tends to allocate more time to surgeries, because allocating more time does not effect total planned surgery duration and GAP values that much as it would when the idling cost is high, in other words model tends to have less restriction and more freedom. This fact results in high variability of

OR day when the idling cost is low. In contrast, when the idling cost is average ($\alpha_1$ = 0.3,0.5,0.7), model performs more strictly by decreasing the derived surgery durations and total planned surgery time, resulting in less cost and lower GAP values in all our three approaches. We observe the same high variability of GAP values with maximum idling cost ($\alpha_1$ = 0,9) scenarios, with the outcomes of increased GAP values. For these reasons, OR planners should be aware of the impact of low idling and high waiting and overtime costs. It is safe to say that average idling costs yield less variable OR days resulting in less OR time, OR costs and GAP values. When we examine the cases where non parametric approach outperforms parametric approach, we find supporting results. As the idling cost increases, the chances increase for non parametric approach to perform close or even better than parametric approach. Using nonparametric approach for OR environments that contains high idling costs would be the better option. On the contrary, when idling cost is low, difference in performance measures between parametric and non parametric is high, making the use of parametric approach more appropriate.

**Number of Surgeries** When examining the performance levels with respect to number of surgeries, there is no clear trend between the $GAP_P$, $GAP_{NP}$ and the number of surgeries. The results of parametric and non parametric approach are within margin of error (see Table 2).Randomness of these two approaches is also a factor of the absence of such trend. Variance of a specific scenario may be too high or too low depending on the different random samples. This is why results for some cases may be higher or lower than anticipated.

For the heuristic approach, as the number of surgeries increase $GAP_H$ increase. Complexity of the model increases due to additional parameters and variables fed to our model (e.g. scheduled duration of the additional surgery for variable and sample surgery duration parameters of the additional surgery for parameters). In Table 11, we present the number of times and associated percentage of the performance our two approaches when;

1) They perform close to each other within %1,

2) Parametric approach performs better by at least %1,

3) Non parametric approach performs better by at least %1.

We do not include 2 surgery cases in summations as 2 surgery cases have 30 replications whereas all other cases have 10 replications.

Table 11: Performance Comparison of Parametric and Non Parametric Approach

| N | Equal Performance | | $GAP_P$ is better | | $GAP_{NP}$ is better | |
|---|---|---|---|---|---|---|
| | # | % | # | % | # | % |
| 2 | 5674 | 47,28 | 4201 | 35,0083 | 2125 | 17,708 |
| 3 | 1968 | 49,2 | 1411 | 35,28 | 621 | 15,53 |
| 4 | 2010 | 50,25 | 1400 | 35 | 590 | 14,75 |
| 5 | 2091 | 52,28 | 1350 | 33,75 | 559 | 13,98 |
| 6 | 2113 | 52,83 | 1404 | 35,1 | 483 | 12,075 |
| total | 8182 | 51,14 | 5565 | 34,78 | 2253 | 14,08 |

From Table 11, our previous conclusion about the performance decrease of non parametric approach may be seen. As the number of surgeries increase the number of cases in which the non parametric approach outperforms parametric approach decreases. However, there is no such conclusion about the performance of parametric approach when the number of surgeries is increased. One interesting result is that even tough non parametric approach starts to perform worse with increasing number of surgeries, the number of cases in which two approaches perform close to each other increase as number of surgeries increase. Given the fact that non parametric approach performs worse with increased amount of surgeries, using parametric approach when there are high number of surgeries to be planned would be the safe choice for OR planners.

**Number of Historical Samples** Through our work, we noted that for all scenarios of all number of surgeries, our solution methods performed better as the sample size increased in terms of GAP values and associated costs (see Tables 2, 5 and 8). With increased number of samples, non parametric and heuristic approaches are able to create more sample surgery durations, resulting with less variance of sample surgery durations. In addition, with more historical samples at hand, non parametric approach provides scheduled surgery durations that are close to those of sample distribution, resulting with smaller $GAP_{NP}$ and increased performance. On the other hand, during maximum likelihood estimation of parameters, parametric approach is able to generate parameters more accurately due to increased number of historical samples. Since parameters are more accurate, scheduled surgery durations are close to sample surgery durations, and the resulting cost of parametric approach is closer to the sample distribution cost.

Although standard deviation of our results may not be as conclusive as the average values, it still holds important information to help analyze the results with the average values. One example would include the cases with small number of historical samples. The averages of our three solution approaches may be close to each other, however without looking into the effect of standard deviation, these results may be misleading. The average of a given scenario may be low, while the standard deviation is high, resulting in undesired cases.

In this study, we are interested in how much effect the number of historical samples holds in cases when non parametric approach outperforms or performs close to parametric approach. To carry out such analysis, we filtered our results such that the difference between $GAP_NP$ and $GAP_P$ is less than or equal to 0,01. For 2 surgery scenarios, %65 of the cases, the difference in GAP values ($GAP_{NP}$ - $GAP_P$) is less than 0,01. This percentage is lowest at %49 with 5 sample scenarios, and highest at %87 with 100 historical surgery sample scenarios. For 6 surgery scenarios, %65 of the cases, the difference is less than or equal to 0,01. This percentage is lowest at %46 with 5 sample scenarios, and highest at %91 with 100 historical surgery sample scenarios. In addition, in Table 12, when we examine the cases where the parametric approach outperforms non parametric approach by at least %5, we note that total of %82 of all 3372 cases have 5 or 10 samples and only 20 of 3372 cases have 100 historical samples.

Table 12: Parametric Approach Performing Better Than Non Parametric Approach by at least %5 With Respect to Number of Historical Samples

| Number of Samples | Number of Cases | Percentage of Cases (in 28000 reps) |
|---|---|---|
| 5 | 1608 | 47,69 |
| 10 | 1163 | 34,49 |
| 25 | 457 | 13,55 |
| 50 | 124 | 3,68 |
| 100 | 20 | 0,59 |
| total | 3372 | 12,04 |

To support the notion of non parametric approach performing better as the number of historical samples increase, we carry out an additional study. In this study we examine three cases: 1) Parametric and non parametric approach performs close to each other. 2) Parametric approach performs better by at least 0,01. 3)Non parametric approach performs better by at least 0,01. We present these results in Table 13 and in Figure 6. Note that for 2 surgery scenarios, total number of scenarios is 12000, whereas for all other number of surgery scenarios, total number of scenarios are 4000 each.

Table 13: Effect of Number of Historical Samples on Relationship Between Parametric and Non Parametric Approach

| N | $S_i$ | Equal Performance | | P Performs Better | | NP Performs Better | |
|---|---|---|---|---|---|---|---|
| | | # | % | # | % | # | % |
| 2 | 5 | 395 | 16,458 | 1225 | 51,0416 | 780 | 32,5 |
| | 10 | 567 | 23,625 | 1165 | 48,5416 | 668 | 27,833 |
| | 25 | 1089 | 45,375 | 937 | 39,0416 | 374 | 15,583 |
| | 50 | 1605 | 66,875 | 565 | 23,5416 | 230 | 9,583 |
| | 100 | 2018 | 84,083 | 309 | 12,875 | 73 | 3,0416 |
| | Total | 5674 | 47,283 | 4201 | 35,008 | 2125 | 17,708 |
| 3 | 5 | 114 | 14,25 | 439 | 54,875 | 247 | 30,875 |
| | 10 | 224 | 28 | 373 | 46,625 | 203 | 25,375 |
| | 25 | 379 | 47,375 | 314 | 39,25 | 107 | 13,375 |
| | 50 | 551 | 68,875 | 203 | 25,375 | 46 | 5,75 |
| | 100 | 700 | 87,5 | 82 | 10,25 | 18 | 2,25 |
| | Total | 1968 | 49,2 | 1411 | 35,275 | 621 | 15,525 |
| 4 | 5 | 131 | 16,375 | 428 | 53,5 | 241 | 30,125 |
| | 10 | 217 | 27,125 | 395 | 49,375 | 188 | 23,5 |
| | 25 | 390 | 48,75 | 315 | 39,375 | 95 | 11,875 |
| | 50 | 566 | 70,75 | 187 | 23,375 | 47 | 5,875 |
| | 100 | 706 | 88,25 | 75 | 9,375 | 19 | 2,375 |
| | Total | 2010 | 50,25 | 1400 | 35 | 590 | 14,75 |
| 5 | 5 | 155 | 19,375 | 411 | 51,375 | 234 | 29,25 |
| | 10 | 178 | 22,25 | 430 | 53,75 | 192 | 24 |
| | 25 | 422 | 52,75 | 300 | 37,5 | 78 | 9,75 |
| | 50 | 614 | 76,75 | 146 | 18,25 | 40 | 5 |
| | 100 | 722 | 90,25 | 63 | 7,875 | 15 | 1,875 |
| | Total | 2091 | 52,275 | 1350 | 33,75 | 559 | 13,975 |
| 6 | 5 | 163 | 20,375 | 421 | 52,625 | 216 | 27 |
| | 10 | 174 | 21,75 | 463 | 57,875 | 163 | 20,375 |
| | 25 | 451 | 56,375 | 291 | 36,375 | 58 | 7,25 |
| | 50 | 603 | 75,375 | 163 | 20,375 | 34 | 4,25 |
| | 100 | 722 | 90,25 | 66 | 8,25 | 12 | 1,5 |
| | Total | 2113 | 52,825 | 1404 | 35,1 | 483 | 12,075 |

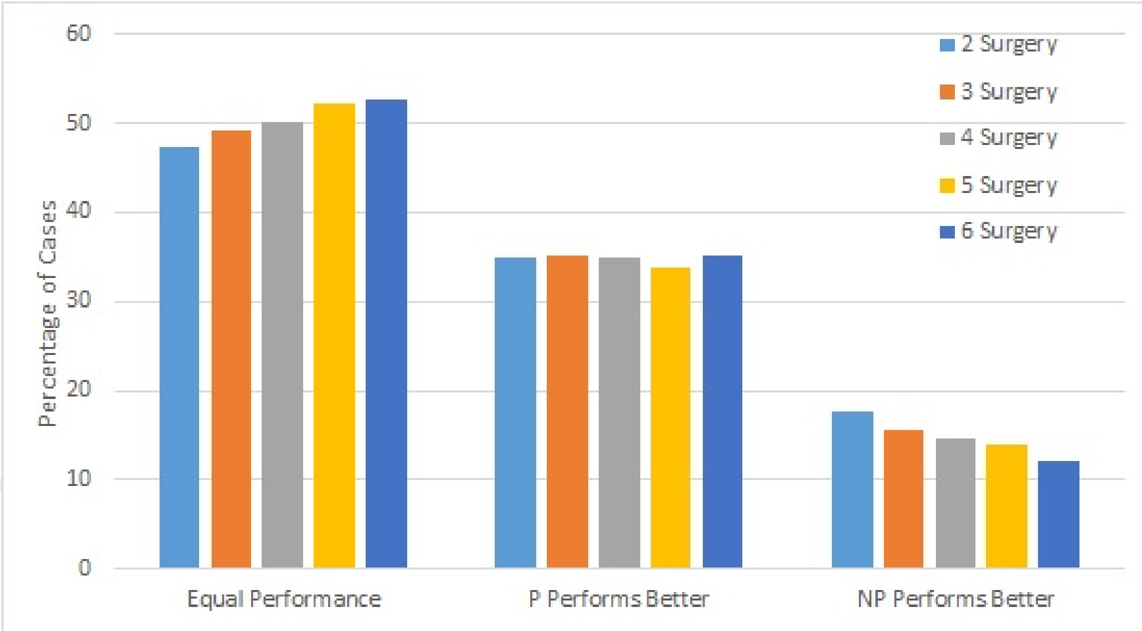


Figure 6: Effect of Number of Surgeries on the Performance of Parametric and Non Parametric Approach

After examining the results, it is possible to note that as the number of historical samples increase the number of cases in which both approaches perform close to each other increase in large quantities. The reason for such conclusion is that the non parametric approach performs better as the number of historical samples increase. From Figure 6, it is safe to say that increasing number of surgeries does not effect the performance of parametric approach. On the other hand, increases in the number of surgeries weaken the performance of non parametric approach. For all three approaches, as the number of historical samples increase, the variability of OR days decreases due to smaller variance of the increased sample sizes. Provided with smaller variance of our samples, the costs and associated GAP values decrease.

After presenting these informations, we believe it is safe to say that as the number of historical surgery samples increases, performance difference between non parametric approach and parametric approach declines.

### 5.4.2 Normal Distribution

All the computation and comparison to this point involves samples with log-normal distribution. In Table 14 and 15, we present the numerical results for a 6 surgery sample with a truncated normal distribution. We use truncated normal distribution with a lower bound, which represents the lowest possible surgery duration in hours. By selecting such duration, we prevent durations to take negative values. We apply this notion to the generations of both sample distribution samples and the historical surgery sample durations to assure the consistency of our model. In truncated normal distributed model, parametric approach assumes the probability distribution to be log-normal distribution whereas in reality, the model has truncated normal distribution. In Table 14, we present the performance levels of three solution approaches. Performance levels are expectations of particular GAP value with respect to a specific $\alpha_1$ value.

Table 14: Optimality Gaps in Normal Distributed Cases with Respect to $\alpha_1$ (In %)

| $\alpha_1$ | $GAP_P$ | $GAP_H$ | $GAP_{NP}$ |
|---|---|---|---|
| 0,1 | 16,72 | 448,29 | 16,27 |
| 0,3 | 7,48 | 114,02 | 7,21 |
| 0,5 | 5,54 | 38,64 | 5,57 |
| 0,7 | 4,44 | 9,57 | 4,80 |
| 0,9 | 5,45 | 27,90 | 5,29 |

From Table 14, we can observe high GAP values for all three solution approaches when the idling cost is low. For average values of $\alpha_1$ (e.g. 0.3, 0.5, 0.7), GAP values are low. However for maximum value of $\alpha_1$ (e.g. 0.9), all GAP values increase, which is similar to the behavior of cases with log normal distribution. In Table 15 we present performance levels of our solution approaches with respect to historical surgery duration sample sizes.

When we look at Table 15, we can see that the parametric approach performs

Table 15: Optimality Gaps in Normal Distributed Cases with respect to $S_i$ (In %)

| $S_i$ | $GAP_P$ | $GAP_H$ | $GAP_{NP}$ |
|---|---|---|---|
| 5 | 19,29 | 139,65 | 22,20 |
| 10 | 8,76 | 130,93 | 10,19 |
| 25 | 5,00 | 127,76 | 3,97 |
| 50 | 3,56 | 124,13 | 1,97 |
| 100 | 3,21 | 123,66 | 0,96 |

better than non parametric approach when the historical sample size is small (e.g. 5 or 10 samples). However, after 25 samples we note that the non parametric approach outperforms parametric approach. We previously stated in this work that parametric approach has two advantages: i) correct distribution assumption and ii) superior number of scenarios, especially for cases with small amount of historical samples. In our model with normal distribution, parametric approach assumes the probability distribution to be log normal distribution, whereas in reality it is normal distribution. Parametric approach loses its first advantage, which is correct distribution assumption making. And for cases that have more than or equal to 25 historical samples, parametric approach loses its second advantage, which is superior number of scenarios. Despite the fact that parametric approach may still have more scenarios than non parametric approach for some cases, non parametric approach performs better because it does not make errors in estimation and generation of scenarios since there is no such process in non parametric approach.

### 5.4.3 When Each Method Should Be Used?

After carrying out the numerical study of our model, we are able to present when each of our three solution approaches may be used. First of all, our third solution method, heuristic approach performs at undesired levels. However, if the OR environment is not a complex one, and it is not required to carry out detailed planning then heuristic approach emerges as simple but practical solution method. Our first solution method, parametric approach performs better than other two approaches. This happens with

i) lower and higher idling costs, ii) limited data at hand (historical surgery duration samples) and iii) larger number of surgeries in an OR day.

When the idling cost is lower, e.g. $\alpha_1 = 0.1$, one may suggest to adjust the derived surgery durations by allocating more time to the planned surgery durations to limit the waiting times and overtimes. On the contrary, when the idling cost is higher, e.g. $\alpha_1$= 0.9, one may suggest to adjust the derived surgery durations by allocating less time to the planned surgery durations to limit the idle times. When idling cost is average, e.g. $\alpha_1 = 0.1$, 0.3 or 0.5, this to-the-limits understanding of allocating times to planned surgery durations diminishes and the difference in $GAP_P$ and $GAP_{NP}$ values decrease. At the lower and higher idling costs, parametric approach performs better because estimation and generation of surgery scenarios enable parametric approach to generate derived surgery durations that are close to the surgery parameters. Derived surgery durations that are close to the historical surgery duration samples results in less $GAP_P$ values, thus better results in performance. When the idling cost is average, parametric approach no longer has an advantage that effect the performance difference very much, because surgery duration allocation is not as hard as they are when the idling cost is lower or higher.

Parametric approach performs better than non parametric approach when the number of historical surgery duration samples are lower, e.g. 5 or 10 samples. When the historical samples size lower, parametric approach takes advantage of scenario generation. In other words, there may be just 5 historical surgery sample at hand, but with the help of scenario generation and correct probability distribution assumption, parametric approach makes good use of this relatively small amount of data. When sample sizes increase, then non parametric approach starts to perform better to the point where advantages of parametric approach do not effect the results very much. Difference in performance, e.g. GAP values, decrease, and in some cases non parametric approach outperforms the parametric approach. This happens when available historical surgery duration sample size is rather large, e.g. 50 or 100 samples. In cases with large historical sample sizes, non parametric approach have enough data to enable it to allocate non parametric scheduled surgery durations that are close to the sample distribution sample surgery durations. And this results in decreased values of $GAP_{NP}$.

Parametric approach performs better when the number of surgeries in a OR day increase. As the number of surgeries increase, the complexity of the problem increase accordingly. Parametric approach uses its two distinctive advantage, estimation and generation of surgery scenarios for each individual surgery in an OR day. In other words, parametric approach has 50000 scenarios for each surgery that is to be planned. The variance of derived surgery durations and $GAP_P$ values are generally lower due to available data generated from 50000 scenarios. When the number of surgeries increase, non parametric approach may not yield GAP values close to the sample distribution cost, because non parametric derived surgery durations may not be close to the sample surgery duration when compared to cases with less surgery cases. When the non parametric derived surgery durations are not close to sample durations, the result is increased idle, waiting, overtime and earliness causing increased non parametric costs and increased $GAP_{NP}$ values. When the number of surgeries in a given OR day is lower, non parametric approach performs closer to the parametric approach because non parametric approach is able to provide non parametric derived durations that are close to the sample durations.

# CHAPTER VI

# CONCLUSIONS

In this paper, we present three solution approaches to a highly stochastic operating room model. Our goal is to investigate the three methods and according to the numerical results from each three solution approaches, present managerial insights, provided with comparisons.

From our numerical results, we find that for complex operating room environments with more than a few surgeries to be planned, parametric and non parametric approach yield better results. Considering all cases in this study, parametric approach manages to perform 6,18% close to optimal solution, whereas non parametric approach performs 7,66% close to optimal solution. The difference between these two approaches concentrate also on the capabilities of an OR planner and the hospital. If the hospital is a very large complex institution, with OR planners possessing fine optimization tools, we suggest using parametric approach and non parametric approach for such OR planning purposes. Implementing parametric and non parametric approach would cost more than other heuristic approach. In addition parametric approach may be time consuming in some cases due to the estimation and generation of scenarios. This is the reason why we suggest using non parametric approach, provided with large historical surgery duration samples. At this point, the difference of the performance between parametric and non parametric approaches begin to diminish. And lastly, if the planning environment is small or the hospital does not require very good planning outcome, then we suggest using our simple but practical heuristic. Our simple but practical heuristic yields average results, but it does not require employing complex statistics nor any expensive optimization tools. All scenarios considered, heuristic approach performs 78,17% close to optimal solution.

We believe that OR planners may use our model to incorporate planning techniques. Knowing the strengths and the weaknesses of each solution approach we presented in this paper, OR planners may obtain fine results without handling very big data sets.

In this study, we assume fixed sequence of surgeries in a single operating room environment with unlimited total surgery duration. In addition, we assume equal idle time cost for all surgeries for a given OR day. For future work, our study may be extended for multiple operating rooms with/without surgery specialty constraints. In other words, any surgery specialty may be assigned to any operating room or operating rooms belong to certain surgery specialties. Moreover, fixed idle time costs may be changed such that each surgery in sequence has different idle costs. In addition to time costs, costs that may be caused by the formation or experience of surgical teams may be considered as an extension to our work. For instance, such a cost may incur if a very experienced surgeon or a team of surgeons are assigned to a surgery that has lower idle time cost.

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# VITA

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