

**SIMULTANEOUS LOT SIZING, SCHEDULING  
WORKFORCE, OVERTIME AND SHIFT PLANNING  
MIP MODEL INCLUDING SETUP TIMES AND  
BACKLOGGING DECISIONS**

A Thesis

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*To my dear "Dream"*

## **ABSTRACT**

Studies focusing on simultaneous lot sizing and scheduling problems, discard the effect of workforce constraints and tactical level decisions such as overtime and selected number of shifts. Generally, in production planning, workforce and shift decisions are given first and respect to these decisions, the scheduling and lot sizing decisions are given. This study focuses on the extensions of the simultaneous lot sizing and scheduling MIP models in literature by the overtime, shift decisions and available workforce constraints including production environments of parallel non identical sets of machines using multiple sets of non-identical tools attached and sequence dependent/independent setups between the tools occur. Developed MIP models are based on the Capacitated Lot Sizing Problem with Sequence Dependent Setups (CLSD) models. In addition to the CLSD models sequence independent versions Capacitated Lot Sizing Problem with Sequence Independent Setups (CLSI) are also presented. Later a MIP based decomposition technique will be presented to solve industry size problems. The developed models are tested in a TV manufacturer in Europe, Vestel Electronics's production planning of the plastic injection plant. The results show that the developed heuristics solve the large size problems in a reasonable time.

Key words: Overtime and Shift Planning, Inventory, Backlogging and Production Decisions, Mixed Integer Linear Programming, Sequence Dependent and Independent Setup Time, Continuous Time Scheduling, Operation Planning, Simultaneously Lot Sizing and Scheduling, Tool and Machine Interaction

## ÖZET

Eşzamanlı parti büyüklüğü belirleme ve çizelgeleme modellerinde yapılan güncel çalışmalar işgücü bağımlı kısıtları ve fazla mesai ya da vardiya planları gibi kararların etkilerini göz ardı etmektedir. Üretim planlamadaki genel uygulama, işgücü ve vardiya saygılarının belirlenmesinden sonra sıralama ve parti büyüklüğü belirlenmesi şeklindedir. Bu çalışmada, eşzamanlı sıralama ve parti büyüklüklerinin belirlenmesini sağlayan karma tam sayılı programlama modellerinin, paralel ve farklı makine tipleri, makinalarda kullanılan paralel ve birbirinden farklı ekipmanları, sıralama bağımlı/bağımsız kurulum süreleri bulunan üretim birimlerinde, işgücü planlama, vardiya belirleme, fazla mesai kararlarını kapsayacak şekilde genişletilmektedir. Çalışmadaki karma tam sayılı programlama modelleri, literatürde bulunan sıralama bağımlı kurulumu göz önüne alan kapasitelendirilmiş parti büyüklüğü belirleme modelleri kullanılarak geliştirilmiştir. Karşılaştırma yapabilmek amacıyla, geliştirilen modellerin sıralama bağımsız versiyonları da çalışmada sunulmaktadır. Çalışmada endüstriyel boyuttaki problemlerin çözümü için karma tam sayılı programlama tabanlı parçalara ayırma tekniği kullanılmıştır. Geliştirilmiş olan modeller TV üreticisi olan Vestel Elektronik'in plastik enjeksiyon fabrikasının üretim planlamasında denenmiştir. Alınan sonuçlar geliştirilen modellerin ve parçalara ayırma yöntemlerinin büyük problemleri kısa sürede çözebildiği görülmüştür.

Anahtar kelimeler: Fazla Mesai ve Vardiya Planlaması, Stok, Geciktirilmiş Üretim, Üretim Kararları, Karma Tam Sayılı Programlama, Sıralama Bağımlı ve Sıralama Bağımsız Kurulum, Eşzamanlı Sıralama ve Parti Büyüklüğü Belirleme, Kalıp Makine Eşleşmesi

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## INTRODUCTION

Production planning problem has been studied analytically for more than a century (Toledo et al. 2015). Based on the length of the planning horizon, the production planning is divided into three groups: (i) strategic, (ii) tactical, (ii) operational. The strategic level plans are made for long term where the decisions of the investments to increase or decrease available installed capacity is given. The supplier selection, plant location or production system selections are the strategic long term decisions. The tactical level plans are mid-term plans where the production levels of the facilities are decided. The personnel recruitment decisions or shift decisions are the examples of tactical decisions. The operational level planning concentrates on the short term decisions such as the lot-sizing, product scheduling and available workforce planning.

One of the concerns of operational level planning is product scheduling and lot sizing. The scheduling and lot sizing are made to assign and schedule the resources, such as products, machines and tools to the production tasks (Urrutia, Aggoune, and Dauzère-Pérès 2014). Operating a machine or tool usually requires a worker; therefore, the available capacity of the plants is closely related to the available workforce. Considering this neither the shifts nor the overtimes can be decided without considering the workforce planning. Similarly, operational level decisions cannot be taken without considering capacity utilization decisions brought by the tactical level plans. So that the lot sizing, scheduling, shift planning, overtimes and workforce decisions should be made simultaneously.

The lot sizing and scheduling decisions should be considered simultaneously in order to achieve optimal and feasible solutions especially when sequence dependent

setup times and costs are present (Copil et al. 2016). One special case where the sequence dependent setups occur is the production technologies which use tools by the machines where setups occur due to the interchange of the tools. Another case where sequence dependent setups occur is the minor revisions on the tools to produce different versions of the products or the raw material changes such as the plastic injection production method. In this work simultaneous lot sizing and scheduling problems in the literature is studied and the models in the literature have been extended to cover the tool – machine – product interactions, overtime and shift planning decisions.

The rest of the thesis is organized as follows: Section 2 describes the simultaneous lot sizing and scheduling problem including the tool product interaction with overtime or shift planning extensions and workforce planning. Section 3 concentrates on the literature review on the field. In Section 4 the selection of methodology based on the literature review will be presented. In section 5 mathematical models with tool product interaction, overtime and shift decisions and workforce planning extensions are developed. In Section 6 the numerical study is made on the developed models. Section 7 concentrates on possible decomposition methods and heuristics for the industry size problems. Sections 8 and 9 are reserved for conclusions, managerial insight and future research possibilities.

## **PROBLEM DEFINITION**

Operational production planning task of the production plant, where tools are attached to the machines to produce the products using different raw materials and the operators are required for the machines, includes multiple decision making processes in tactical and operational level. The shift plans considering the available workforce is an example for the tactical level decisions that should be considered together with the operational production plans. The total daily capacities of the machines are determined by the total number of the shifts made on the machine. The workforce is the single input to determine the shift plans. Different types of machines capable to produce different products may require extra shifts during the planning horizon so the shift plans should be considered simultaneously with the operational level lot sizing and scheduling decisions.

Vestel Electronics produces TV sets. The TV sets requires various plastic parts and these parts are produced within the plastic injection plant. The final assembly production plant operates on the accepted orders. According to the bill of material of the TV sets and due dates of the orders the plastic injection factory production planning is made. The plastic injection process requires the tools (molds) machines (plastic injection machine) and operators as resources. The interchange of the molds brings sequence dependent setups to the studied problem. In order to be able to reduce the sequence dependent setup times the planners give the decisions of inventories. Contrarily if the capacities of the machines are not enough to produce the demanded products the planners are able to give backlogging decisions with penalty costs. The production planners are able decide to make three shifts at maximum considering the available workforce, in the high demand seasons.

The organization of this chapter is as follows. The first section concentrates on the simultaneous lot sizing and scheduling decisions made on single stage parallel sets of non-identical machines production environments. The second section explains the overtime and shift planning connections with the operational production planning problem. The third section is reserved for the workforce planning relations with the operational level production plans. Finally, specific characteristics of the studied problem are explained in the fourth section.

## ***2.1 Lot Sizing and Scheduling Problem***

The lot sizing is made to determine the production quantity of a product at a period of the production plant (Urrutia, Aggoune, and Dauzère-Pérès 2014). If the resources of the problem have a finite capacity the lot sizing problem becomes capacitated lot sizing problem. In a capacitated lot sizing problem, the production capacity is limited so backlogging and inventory decisions should be made when demand exceeds the available capacity in a period within the planning horizon.

Scheduling can be defined as assigning tasks to resources. The resources could be the personnel in a service industry such as operators in a call center or the machines in a production facility such as the injection machines in a plastic injection factory. In the review of (Harjunoski et al. 2014) scheduling task is explained by four basic decisions made; (i) selection of the tasks to execute, (ii) assigning the tasks to resources, (iii) sequencing the tasks and (iv) timing of the tasks. The tasks and resources considered in a scheduling problem can be classified according to different fundamental needs of different manufacturing or service enterprises. In a scheduling problem there are different constraints such as the capability of the resources to accomplish different

tasks, setup requirements of the resources to continue with other tasks, storage constraints of the products or production capacity of the resources.

If the setups are sequence dependent, the setup times cannot be determined solely by the number of setups made within the period. In a sequence dependent setup case the setup times are interrelated with the scheduling and sequencing of the tasks. So in a sequence dependent setup problem the lot sizing and scheduling decisions should be made simultaneously.

In a plastic injection production environment injection molds are used on the injection machines and the molds has to be interchanged in order to produce a different product by the machine. Due to different sizes of molds and machines with different attributes, the setups become sequence dependent in a plastic injection production plant. Furthermore, the molds may include different versions so that minor setups can be made to make production of different products when the same mold is assembled on the machine as well. Because of the sequence dependent setups, the production plans studied in this thesis should include simultaneous lot sizing and scheduling decisions.

## ***2.2 Overtime and Shift Planning***

The lot sizing and scheduling decisions are interrelated with the available workforce and the tactical decisions such as overtime and shift. The decisions of lot sizes and linked backlogs or inventory levels are constrained by the available capacity. Although the capacities may be presented as the production rates of the specified machines and tools attached, the available working time of the particular machine is closely related with the available workforce for the shifts and possible overtimes.



The planners are allowed make the tactical decisions such as putting extra shifts or making overtime. One shift in the period is eight hours with one hour of lunch break and the thirty minutes breaks given in the morning and the afternoon. If necessary, the planner may add one or two shifts in one period (one day) which increases the total capacity of the machine or the mold three times.

The overtime decisions on the other hand, can be given when required but the unit time labor cost increases by 50%. An overtime decision adds 2 hours to the available working time of the machine. The total time of extra work which can be done by a worker is limited legally. A worker can make maximum 270 hours of extra work in a fiscal year. Considering this the overtime decisions planners should consider that there is available workforce at the specific shift which are able to make overtime.

The shifts and required labor quantity during that shift is decided at the beginning of the planning horizon. Although the shifts are set for a frozen period such as two weeks or one month the planners may have the freedom to make extra shifts when required during the planning horizon. For instance, an increased demand for a specific product which requires a machine or tool with a limited capacity, urges the planners to make extra shifts on that specific machine. So the shift plans and overtime decisions should be made simultaneously with the operational level production plans.

### ***2.3 Workforce Planning***

The machines in the studied problem are operated by one operator. In a plastic injection factory, the operators are making the tasks of quality check of the products, extra operations on the products such as applying protection films and placing the ready products to the containers.

The requirement of labor on the machines brings major constraints on the studied problem. In tactical level production plans the decisions such as the overtime and extra shifts should be made considering the labor capacity and constraints due to the labor sanctions about overtime and shift. The workforce also has an impact on the capacity of the plant. Total number of available operators defines the total number of the machines that can be used for production.

The tactical decisions like overtime and shift can be planned within the planning horizon but it depends on the total number of the operators contracted and the suitable number of operators which can make overtime. Due to these facts workforce planning should be made simultaneously with the operational level production plans.

## ***2.4 Characteristics of the Problem***

In this section different characteristics of the Vestel Electronics's plastic injection plant production planning problem is explained in detail. The first section explains the time characteristic which includes the planning horizon. The demand characteristics are explained in detail in the second section. The machine characteristics section concentrates on the difference of the machines and the impact of these differences on the studied problem. The fourth and fifth section explains the tool machine interaction and the setup characteristics of the problem. Sixth section explains the capacity of the plastic injection plant and the parameters affecting this capacity. The final section elaborates on the parameters which generates cost in the planning problem.

### **2.4.1 Time Characteristics**

In Vestel Electronics the operational level production plans are made for the so called frozen period. The orders for the frozen period are fixed and the production plans for all parts are made according to the frozen period demands. This makes the planning horizon of the problem fixed with the frozen period which is usually one or two weeks (7-14 days).

The fixed orders for the products have special sub assembly plastic parts which require single stage production by the molds and machines. In tactical perspective the period length which is defined as one shift of the day and the capacity can be changed. The details of overtime and shifts will be presented in overtime and shift planning section.

### **2.4.2 Demand Characteristics**

The demand for the specific parts is calculated through the Bill of Materials (BOM) structure of the products. The part demand for the production is either produced in-house or supplied through contracted suppliers. The demand for the products may resemble seasonality features. The long term capacity plans on the products are usually done according to this demand feature.

Production plants such as Vestel Electronics, being an OEM supplier, the designs are consolidated within the size and product families. The differentiation of the products is achieved through the cosmetic parts and versions of the structural mechanical parts suitable for different components. By this consolidation long term demand forecasts for the preproduction of common parts can be done. The demand on

the differentiating parts can only be fixed by the orders so the production plans for these parts should be done on daily basis in the frozen periods.

### **2.4.3 Machine Characteristics**

The main characteristics of the plastic injection plant are that it is using several parallel clusters of non-identical machines capable of producing different parts. The machines and the injection mold should be compatible with each other in order to be able to make the production. The machines should be set up to correct mold and raw material in order to make production of a specified product.

The machines are mainly classified according to their sizes usually defined as the maximum available clamping force capability and material feeding system. The size of the parts and stamping molds dedicated to the parts requires a minimum clamping force in order to make the production.

In plastic injection plant there is similarly raw material feeding systems to the machines and each machine is set up for the specific raw material. Each plastic part has a specific raw material so each dedicated mold to the parts should be set up on the machines which are already set up for that raw material. Some of the cosmetic parts require an additional steam generator installed on the plastic injection machines. This additional feature on the machines makes it possible to produce steam injection requiring parts.

#### **2.4.4 Tool and Product Characteristics**

The product can only be produced by the tools capable to produce the products assigned to machines. For instance, a 32” TV back cover can only be produced by the mold designed for it.

The molds can be used in the capable machines. There can be multiple duplications of the molds to produce the same product. The duplication decisions of the molds are made by the long term plans. A high runner product capacity is increased by procuring multiple tools assigned to produce the specified product.

Every tool assigned to a machine requires a setup time and cost associated. The setup times between the interchange of the tools in the machines reduce the available working time within the periods. Cost and setup characteristics of these setups will be explained later in the dedicated sections.

In the special case the product variations can also be produced on the same mold. In this case the setup times and costs are minimal compared to the complete mold interchanges. These setups are called as the minor setups.

#### **2.4.5 Setup Characteristics**

As explained earlier, the tools (i.e. the molds) can produce one specific product when used on the production machine (i.e. the injection machine). The capacities and the quantities of the tools are decided by the long term strategic plans. The tools may have to be changed on the production machines according to the demand on different parts. This can be done by setups on the machines. There is an associated setup cost and time for the setup operations.

The setups can be done by dedicated labor to interchange the tools on the machines. During the setup operations the tools that are being interchanged and the machine should be idle. This brings the constraint that neither the tools nor the machines can be used during the setups. Setup times depend on the tool and machine sizes and features. In each setup the calibration of the machine and the mold should be done according to the quality specifications. A larger machine with a larger tool assigned to it requires more time to finalize the setup. These make our problem a sequence dependent setup time and cost problem. The setups in which the tools are interchanged on the machine will be called as macro setups in this work.

The plastic parts are designed so that the same mold is capable to produce different product requirements. This is achieved by adding versions on the molds. In addition to the setups occurring due to the interchange of the molds, the version changes on the molds are also causing setup times in the studied problem. The version changes on the molds will be called as minor setups in this work. The minor setups are assumed to be strictly less than the major setups.

The planning horizon of the studied problem is divided into macro periods. The setups at the beginning of the macro period are assumed not to consume time. The setup labor is assumed to make the necessary setup operations if needed before the start time of the first shift.

#### **2.4.6 Capacity Characteristics**

The production capacity of the problem depends on various constraints. For a plastic injection plant the available plastic injection machines, the amount of plastic

injection molds, the available workforce and the tactical decisions such as the overtime and shift quantities defines the capacity.

As explained earlier each injection mold can produce a specific product with a defined cycle time. The molds can be used by the machines which are set up to the suitable raw material, has enough clamping force (size of the machine) and if required suitable to produce steam injection parts. The cycle times of the molds does not change according to the machine types it is produced.

#### **2.4.7 Cost Characteristics**

There are several operations and decisions that can create costs in the problem. The production machines create costs according to the size of the machine. The labor costs are made in order to operate the machines. The setup operations generate costs according to the total time of the setup operation. The overtime decisions increase the labor cost of the production. The inventories being hold by one period creates the inventory holding costs and the backlogged quantities create a backlogging penalty cost in the plans.

In the machine characteristics of the problem it has been pointed that there are different sizes of plastic injection or metal stamping machines in the production plant. The size of the machine and unit production cost are proportional to each other. So the planners are urged to make the plans with the minimum size possible machines with minimum unit production costs in order to reduce the total production costs.

Each machine being used in the periods requires an operator with a daily salary. Whenever an overtime decision is made, the unit labor cost increases by 50% per unit time.

Since the production plant has a large product portfolio the plans should include the setups within the periods. The setups are made by the professionals with fixed salaries. The setup times are differentiating between the molds being interchanged. A large mold requires more time to make the setup whereas a smaller mold can be set up easier with less setup time. Considering these facts, the setup costs are given proportional to the setup times in the problem.

The problem includes lot sizing decisions as well. Sometimes production to the inventory would be necessary either to balance the capacity before the demand is high or to decrease the fixed daily cost of the machines, operators and setups. Since the production cost or the volume occupied by each product differs in the problem, the inventory costs are defined for each product. It is assumed that the inventory holding costs occur if the product is hold in inventory at the end of one period. To sum up the costs of the problem can be presented as below:

- Machine operating cost
- Operator salary
- Setup cost
- Overtime costs
- Inventory holding costs
- Backlogging costs



## LITERATURE REVIEW

In the literature review section, the papers that have mathematical models related to lot sizing and scheduling problem workforce planning, overtime decisions and shift planning, its classifications and solution methods will be explained. The literature review is conducted on the relevant papers including the topics; lot sizing, scheduling, set up times, set up costs, tactical decisions.

The purpose of the literature review is to collect the mathematical models ready in the literature covering the subject of simultaneous lot sizing and scheduling problems, the extensions made to cover the tool machine interaction and the combinations of tactical level and operational level decisions. This thesis deals with the single stage production systems with parallel sets of non-identical machines with sequence dependent setups.

In the first section the generic model of lot sizing and scheduling problem is introduced. The next four sub sections are dedicated to the classified lot sizing and scheduling problems and its extensions. In these sections the general mathematical formulations will be presented and a review of the related articles will be made. The second and third sections; workforce, overtime and shift planning extensions in the literature is discussed respectively. Finally, in the last section the literature review is summarized considering the requirements of the problem defined.

### ***3.1 The Lot Sizing and Scheduling Problem***

The lot sizing problem is mainly divided into large-bucket and small-bucket problems in the literature. The nature of lot sizing problem is characterized by the macro time periods for which the production quantities and inventory levels are

decided (large-bucket problem). Contrarily the scheduling problem concentrates on the sequence of the productions and the timing of the production decisions within the macro periods (small-bucket problem) (Babaei, Mohammadi, and Fatemi Ghomi 2014).

Recently, simultaneous lot sizing and scheduling problem was reviewed by (Copil et al. 2016) in their article Simultaneous lot sizing and scheduling problems: a classification and review of models. The problem is formulated as a generic lot sizing and scheduling problem (gGLSP) and further classified by different features and solution methods used (Copil et al. 2016). They offered generic mathematical model of GLSP (gGLSP) in order to classify different approaches to simultaneous lot sizing and scheduling problems. The model is presented below:

**DATA:**

$i, k$	Product indices, $i, k = 0, 1, \dots, K$ , the value 0 is the natural state
$s$	Index of micro periods, $s = 1, 2, \dots, S$
$t$	Index of macro periods, $t = 1, 2, \dots, T$
$S_t$	Set of micro periods within a macro period
$sc_{ik}$	Setup costs for a change over from product $i$ to product $k$
$hc_k$	Holding costs for product $k > 0$ (per unit and per macro period)
$pc_k$	Standby costs for preserving the setup state of product $k$ on the production resource (per time unit)
$a_k$	Production time per unit of product $k$ ( $a_0 = 1$ )
$st_{ik}$	Setup time for a changeover from product $i$ to product $k$

$C_t$	Capacity of the production resource in macro period t (time)
$I_{k0}$	Initial inventory of product $k > 0$ at the beginning of planning (units)
$d_{kt}$	Demand of product k in macro period t (units)
$w_{k0}$	$w_{k0} = 1$ indicates that the production resource is set up for product k at the beginning of planning ( $w_{k0} = 0$ , otherwise)
$q_k^{\min}$	Minimal production quantity of product $k > 0$ (units); minimal time for neutral state $k = 0$

### VARIABLES:

$q_{ks} \geq 0$	Production quantity of physical product $k > 0$ (units) in micro period s; time spent in neutral state if $k = 0$ , respectively
$\bar{q}_{ks} \geq 0$	Duration (time) for which the setup state of product k is preserved on the production resource in micro period s ( $\bar{q}_{0s} = 0$ ).
$I_{kt} \geq 0$	Inventory (units) of product $k > 0$ at the end of macro period t
$w_{ks} \in \{0,1\}$	Setup state variable; $w_{ks} = 1$ indicates that the production resource is set up for product k in micro period s (0 otherwise)
$z_{iks} \in \{0,1\}$	Changeover variable; $z_{iks} = 1$ indicates a change over from product i to product k in micro period s (0 otherwise)

### OBJECTIVE FUNCTION:

$$\text{MIN: } \sum_{s=1}^S \sum_{i=0}^K \sum_{k=0}^K sc_{ik} z_{iks} + \sum_{k=1}^K \sum_{t=1}^T hc_k I_{ikt} + \sum_{k=1}^K \sum_{s=1}^S pc_k \bar{q}_{ks} \quad (1)$$

### SUBJECT TO:

$$\sum_{k=0}^K \sum_{s \in S_t} (a_k q_{ks} + \bar{q}_{ks}) + \sum_{i=0}^K \sum_{k=0}^K \sum_{s \in S_t} st_{ik} z_{iks} = C_t \quad (2)$$

$$I_{kt} = I_{k,t-1} + \sum_{s \in S} q_{ks} - d_{kt} \quad \forall t, k > 0 \quad (3)$$

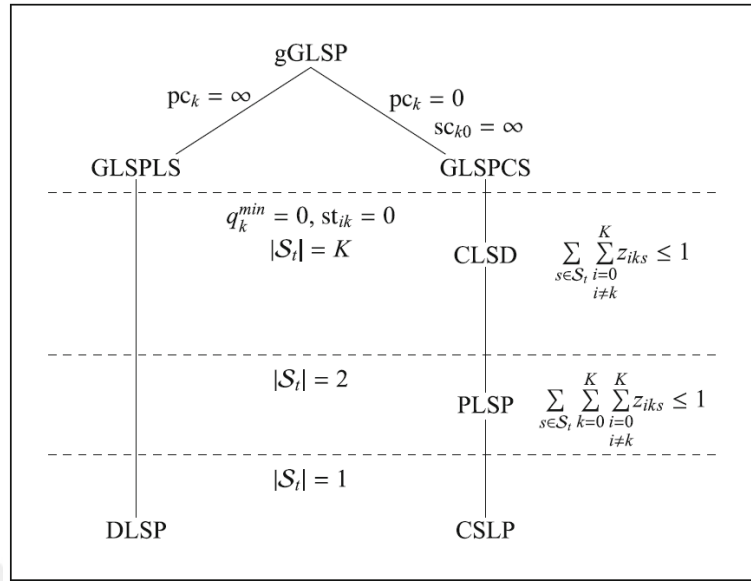
$$\sum_{k=0}^K w_{ks} = 1 \quad \forall s \quad (4)$$

$$a_k q_{ks} + \bar{q}_{ks} \leq C_t w_{ks} \quad \forall k, t, s \in S_t \quad (5)$$

$$z_{iks} \geq w_{i,s-1} + w_{ks} - 1 \quad \forall i, k, s \quad (6)$$

$$q_{ks} \geq q_k^{\min} (w_{ks} - w_{k,s-1}) \quad \forall k, s \quad (7)$$

In the g(GLSP) model the objective function (1) is to minimize the set up costs, inventory costs and set up preservation costs. Equation (2) limits the production to exceed the total available time in the macro period. Equation (3) is the inventory balance equation. Equation (4) ensures that in each micro period one product is set up, including the natural stage. Equation (5) links the set up decision variable to production quantity decision variable. If either the continuous variables  $q_{ks}$  or  $\bar{q}_{ks}$  is positive the set up decision variable  $w_{ks}$  should be set to 1. Equation (6) ensures that if the setup decision variable  $z_{iks}$  is 1 then the  $w_{ks}$  and  $w_{k,s-1}$  both should be 1. Equation (7) ensures that the minimum lot sizes are produced if the model decides to produce product k.



**Figure 1-** Classification Lot Sizing Scheduling Problem (Copil et al. 2016)

In Figure 1 one can find the general classification of GLSP's. The models have been divided into two main versions according to the conservation of set-up stage after the idle periods. Namely the general lot sizing and scheduling problem with loss of setup stage (GLSPLS) and the general lot sizing and scheduling problem with conservation of set up stages (GLSPCS) (Fleischmann and Meyr 1997).

In the generic model if preserving cost of setup stage  $pc_k$  is set to  $\infty$  the model rejects the idle time of the machine. This makes the model to make setups in each micro period even the set up stage is preserved. In GLSPCS case, if  $pc_k$  is set to 0 and by  $sc_{i0}$  to  $\infty$  the model can conserve the set up stage between the periods.

The model is further classified according to the allowed setup decisions within the micro periods. The GLSPCS models are divided into three sub models namely the capacitated lot sizing problem with sequence dependent setups (CLSD),

proportional lot sizing and scheduling problem (PLSP) and continuous setup lot sizing problem (CSLP) (Copil et al. 2016).

The special case of GLSPCS, CLSD models limits the maximum number of micro periods to the number of products ( $|S_t| = K$ ). The set up time for the changeovers are set to zero ( $st_{ik} = 0$ ) and the minimum production quantity is neglected ( $q_k^{\min} = 0$ ). This model allows the production of the products at most once per macro period (8). These additional features are added to the model by below additional inequalities.

$$\sum_{s \in S_t} \sum_{\substack{i=0 \\ i \neq k}}^K z_{iks} \leq 1 \quad \forall t, k > 0 \quad (8)$$

The PLSP models allow at most once set up ( $|S_t| = 2$ ) and at most two different products can be produced per macro period (9). The below inequalities can be added to the gGLSP model in order to convert to PLSP model.

$$\sum_{s \in S_t} \sum_{k=0}^K \sum_{\substack{i=0 \\ i \neq k}}^K z_{iks} \leq 1 \quad \forall t \quad (9)$$

The CSLP model at most one product can be produced in the macro period ( $|S_t| = 1$ ) and being a GLSPCS model the setup is conserved between the macro periods. The DLSP models are able to produce at most one product per macro period as it is in CSLP models but the model does not allow the conservation of set ups. The DLSP models does not allow the idle stages in the micro periods so the model produces single product with full capacity or does not produce at all.

### **3.1.1 Capacitated Lot Sizing Problem with Sequence Dependent Setups (CLSD)**

The first CLSD model was developed by (Haase 1996) in his paper at 1996 (Copil et al. 2016). The model offered was capable of making the lot size and inventory decisions together with the sequence of the production within the macro periods. The model considers single machine and single production stage. Model offered by (Haase 1996) minimizes the total setup and inventory costs in the planning horizon. Decision variable for the setups was modelled as arcs from product  $i$  to product  $j$  (TSP). The decision variable indicates the sequence setups between product  $i$  to product  $j$  in the sub period but contrary to the original TSP the tour of production starts with the dummy product 0 and can end the tour at any product. The sub tours in the problem was eliminated by limiting the production decision of any product to once per macro period. The total number of setups was limited by limiting the total number of sub periods in the model. In the paper the solution method offered is backward heuristics. To note that the model does not consider multiple machines and setup times.

(A. R. Clark and Clark 2000) has offered another CLSD model which includes multiple parallel machines with multiple products. The model minimizes the inventory costs, backorder costs, setup costs and production costs at the end of the planning horizon. The decision variables for the setups are given for each parallel machine. The binary variable takes the value of 1 if machine  $l$  changes the setup from product  $i$  to product  $j$  at the  $n$ 'th sub period. The solution method is rolling horizon approach for the model. (A. R. Clark 2003) was able to solve the

problem with multiple production stages. This model also considers the capability of the work centers to produce different products.

(Gupta and Magnusson 2005) have studied single machine, single production stage problem and used the TSP approach to model the setup sequences. Setup times, initial and final setup products are included into the model compared to (Haase 1996). Heuristic approach is studied to solve the model.

(Quadt and Kuhn 2005) is concentrated on the flexible flow shops in which multiple parallel machines are used to produce different production stages of the products. The model suggests three stage hierarchical approaches to solve the problem. In the first stage model schedules the production minimizing the total cost of setup, inventory and backorder costs considering the bottleneck stage of the problem. The decision variable of setups is like the TSP. In the second stage the model minimizes the flowtime of the schedule. The aim of the second stage is not to alter inter period inventories or production quantities during the periods. In the first and second stages the product families are scheduled. Finally, in the third stage the model decides on the individual product level slot assignments keeping the flowtime and reducing the total setup costs.

(Almada-lobo et al. 2007) modeled single machine single stage CLSD. The model uses the TSP notation in order to state the changeovers of the production stage during the macro period. In the model product  $i$  which is setup at the beginning of the period can be produced multiple times in order to maximize the utilization of the machine. This feature adds the setup carry over feature to the model. The products other than the initial setup is made cannot be produced



multiple times during the period which is achieved by sub tour eliminations in the mathematical model. The heuristic solution method is offered in the paper.

(Almada-Lobo, Oliveira, and Carravilla 2008) has modelled multiple machine multiple product with different attributes CLSD model. The model was used in the glass container industry. In this industry glass plates are produced through the furnaces and idleness of the furnaces are not allowed. Different molds on the production lines produces different glass containers and the color setups should be handled in order to change the color of the glass containers during the periods. The decisions of the color changes on the parallel machines are handled by the TSP like decision variables. The model includes the capabilities of the machines to produce the products. The model uses variable neighborhood search (VNS) method to solve the problem.(Almada-Lobo and James 2010) has introduced variable neighborhood descend (VND) and taboo search (TS) methods to solve the problem.

(Almeder and Almada-Lobo 2011) has studied parallel non-identical machines with secondary resources like the tools. In the paper two types of MIP models has been developed and tested in the commercial optimization package CPLEX. The first model is developed as GLSP and the second model is developed according to CSLP. The time perspective of the model is constructed as macro period and smaller micro periods assigned to the macro period. The objective of the both models is to minimize the inventory, backorder, setup costs for changing the secondary resources (i.e. Tools).

(James and Almada-Lobo 2011) has proposed to solve the multiple machine multiple product CLSD model by dividing the original problem into pieces and solve the problem iteratively. The solution method is described as the MIP-based iterative neighborhood search heuristic starting with a relax and fix construction heuristic (INSRF)

(Kwak and Jeong 2011) has proposed two stage hierarchical approach to single machine multi product CLSD. In the first stage capacitated lot sizing problem (CLD) solved. In the second stage, fixing the production quantities coming from the CLD solution, the lower level scheduling problem is solved minimizing the make span of the production.

(Mohammadi 2010) has developed a model for flow shops with multiple stages of production. The model uses two level time structure as the macro period and the predefined number of micro periods. The MIP model has been formulated to decide the production, setup and idle micro periods. The model minimizes the total cost of production, inventory and setups. The problem is solved by rolling horizon and relax & fix heuristics.

(Giglio, Paolucci, and Roshani 2017) has studied manufacturing and remanufacturing job shop scheduling and lot sizing model. The model minimizes the Manufacturing, remanufacturing, holding inventory, setup and backlogging costs. Setup times are not considered in the model. Relax and fix heuristics are used to solve the model.

(Nejati et al. 2016) has studied multiple production stage multiple non-identical machine job shop production systems. The model minimizes the

completion time of the jobs. Setup times are sequence dependent and setups costs are not considered. Genetic algorithms and simulated annealing is used to solve the model.

### **3.1.2 Proportional Lot Sizing and Scheduling Problem (PLSP)**

(C Suerie 2005) has made extensions of campaign planning on the classical PLSP model. The model considers the demand is fulfilled when required numbers of products (batches) are ready. The model is structured to minimize the set up and inventory costs. The set up decisions are made according to set up times and set up costs which are sequence dependent. Single machine with single production stages are considered in the model. The model also uses minimum resource utilization constraints. The model uses MIP as the solution technique.

In the standard PLSP model there can be production of the products if the machine is set up at the beginning of the period or set up operation is done during the period. (Christopher Suerie 2006) has developed the model that is capable to produce a third product within the period. The related constraints allow the model to produce another product if set up time requirement is fulfilled within the period. Furthermore, period overlapping setups are possible in the model. The model considers single machine with single production stages of the products. The solution method chosen is MIP solver.

(Tempelmeier and Buschkühl 2008) has considered the common resource of set up operators. The standard formulation of PLSP is used to model the problem. Extra decision variables that keep the start time and finish time of the setups are used to assign the set up operator to the machines. Multiple machine and multiple

products with single production stage is considered in the model. MIP solver is used to solve the problem.

(Kaczmarczyk 2011) has proposed a PLSP model on multiple product parallel machines. Flow variables of amount of machines used and shared capacities to produce the products before and after the set up operations in periods are added to the standard one machine multiple products PLSP model. The model is using sequence independent set up costs and times and the model's objective is to minimize the set up and inventory holding costs. MIP solver is used to solve the model.

(Stadtler 2011) has proposed a multilevel single machine PLSP model with the extensions of period overlapping set up times and batch flow constraints. The period overlapping set up times extension of the model enables to solve the problems when set up operations start in the one period and finish in the next period. The batch flow constraints enable to model to handle the production that should be served in batches (i.e. tubs, tanks ovens). The model keeps the track of inflow inventories to solve the batch constraints. The model minimizes the inflow and final product inventory holding and set up costs. Branch and bound solution method is used to solve the model.

(Stadtler and Sahling 2013) has further developed (Stadtler 2011) by adding zero lead time constraints. The model can solve the PLSP models with multilevel production characteristics. The standard model proposed in (Stadtler 2011) is capable of handling the multilevel production but the model can make production if the required sub level products are ready at the beginning of the periods. (Stadtler

and Sahling 2013) is capable to make the production of higher level products even the sub level components of the product is produced within the same period. The objective is again to minimize the inflow and final product inventory holding and setup costs. Fix and optimize heuristics is used to solve the problem.

### **3.1.3 Continuous Lot Sizing and Scheduling Problem (CSLP)**

(Gopalakrishnan 2000) has studied the CSLP model in order to decide on setup carryovers. This model has additional decision variables to keep the first and last production lots and the decision variable to carryover the setup to the next macro period.

(Pochet Y 1991) has developed added cutting planes to the standard CSLP model. The model uses multi item multistage production planning in single machines. The objective is to minimize setup and inventory holding costs.

(Ghosh Dastidar and Nagi 2005) has developed a CSLP model and has studied injection molding operations. As in our case the model considers the tool machine and tool product interactions. The proposed solution method is to divide the problem into subgroups considering the capabilities of the tools (i.e. work centers) to produce parts. The critical parts that can be produced by scarce number of tools are grouped together with ascending order of the tools. The problem is solved for each sub groups in ascending order and the backorder quantities are added as demand for the next subgroup problem. The model objective is to minimize the setup, inventory holding and backorder costs. The set ups times and costs are sequence dependent. MIP solver is used to solve the problem.

(Silvio A. de Araujo, Arenales, and Clark 2008) has studied furnace scheduling in foundries in which the furnace is set up for a specific alloy and it can produce multiple different products using the alloy in single period. The problem studied is a single stage multi product model. The setups are sequence independent. The objective is to minimize the setup, inventory holding and backloging costs. The model is solved by rolling horizon heuristics.

(Gaglioppa, Miller, and Benjaafar 2008) has developed a multistage CSLP model that a single machine can produce different BOM levels of a product executing multiple tasks. The setup cost and times are sequence independent. The authors have proposed valid inequalities to the model and made numerical tests on MIP solver. The objective of the model is to minimize setup, inventory holding and backloging costs.

(Almada-Lobo et al. 2010) Has studied glass industries in which a single furnace produces melt glass and parallel lines of machines are casting the glass into the products. The furnace can be idle with the furnace idleness cost. The setup times and costs are sequence dependent. Being a CSLP model each casting machines can produce only one type product per period. The objective is to minimize the setup, furnace idleness and inventory holding costs. The model is solved by MIP solver and Lagrangian heuristics.

(Silvio Alexandre Araujo and Clark 2013) has offered a priori formulations to the standard CSLP model. Contrary the formulation characteristics of CSLP the time is modelled as days and sub periods within the days. The rolling horizon heuristics are formulated for each day. Since in each sub period only one product

can be produced the solved model in the RH heuristics is a CSLP. The model minimizes the inventory holding, backorder and setup costs. The setup costs and times are sequence dependent. The production is made in single stage and by a single machine.

(Motta Toledo et al. 2013) has extended the formulation of (Almada-Lobo et al. 2010) to a multi machine problem in glass industry. The model has been solved by genetic algorithms. As it is in (Almada-Lobo et al. 2010) casting machines can produce only one type product per period. The objective is to minimize the setup, furnace idleness and inventory holding costs.

### **3.1.4 Discrete Lot Sizing and Scheduling Problem (DSLSP)**

(Lasdon and Terjung 1971) has developed one of the first DLSP models in the literature. The model does not consider setups and minimizes the total production cost and inventory holding costs.

(Jans and Degraeve 2004) has developed DLSP model including the technological constraints about the capability of the tools and machines to produce the products. The model minimizes the setup, inventory holding and backlogging costs.

(Persson et al. 2004) studied crude oil refinery production scheduling problem. The model includes the sequence dependent setup costs, startup costs and inventory holding costs. Tabu search heuristics was used to solve the model.

(Céline Gicquel 2008) and (C. Gicquel et al. 2009) has developed valid inequalities to the standard DLSP model and proposed cutting plane generation

techniques. The model minimizes sequence dependent setup costs and inventory costs.

(Pahl, Voß, and Woodruff 2010) has developed a DLSP model for single machine problem. The model considers sequence dependent setup costs, inventory holding costs and perishability of the products (spoilage costs). Lifetime of products has been introduced to the model as parameters and spoiled quantity of the items in inventory is tracked in the model.

(Supithak, Liman, and Montes 2010) considered tardiness and earliness costs and developed a GA to solve the problem. The algorithms consider the demand as a set of orders which can be about the same or similar product at the same period. Considering this the demand data is not clustered originally. Preprocessing is done on the data in order to make the clusters of demand before the solution is done. The model later uses genetic algorithms in order to make the sequence of the production to minimize the earliness, tardiness, setup and inventory costs.

(Neidigh and Harrison 2017) has studied nonlinear production rates (i.e. learning curves are considered). The model developed minimizes production, inventory holding and setup costs. Setup times are not considered and setup costs are sequence independent. Heuristics are developed to solve the model.

(Claassen et al. 2016) has studied the DLSP models considering the decay of the products in time. The model minimizes the inventory holding and setup costs. The setup costs and times are sequence dependent. Relax and fix heuristics is used to solve the model.



### ***3.2 Overtime and Shift Planning***

(Tavaghof-gigloo, Minner, and Silbermayr 2016) is one of the papers which study the effect of overtimes and shift plans on the cost of production plans and the available capacity. The study concentrates on multiple facility multiple production stage production plans. The cost is generated by the holding cost, shift cost, cost of changing shifts and overtime costs. The authors discuss the effect of flexible shift and overtime deviations on the overall performance of the production plans and gives relevant data of improvement in the total production costs.

(Hulst, Hertog, and Nuijten 2017) has studied robust shift generation in air traffic controllers planning problem. The shift generation is optimized considering the cost of different shift type openings based on the workload data through the planning horizon. The jobs in the problem require a single skill so the workforce skill constraints are not included to the model.

### ***3.3 Workforce Planning Decisions***

In the review of (Bruecker et al. 2015) the workforce planning decisions are summarized as the workforce hiring or dismissing decisions and allocation of the labor on the tasks based on their skillsets, availability and cost. The problem discussed in this thesis has a single skill operation (i.e. packing the products produced by single stage machines) so the problem does not require the management of the skillset.

As it is in all production or service industry the availability of the labor and allocation of the labor to spontaneous tasks that should be done within the planning

horizon is an important issue to be handled. The management of the workforce should be done efficiently so that the production plans are secured to the deviations of sudden changes due to technical or labor related reasons.

As far as our knowledge no study is found considering the workforce availability decisions except for (Tempelmeier and Buschkühl 2008). The authors have studied the workforce requirements of the setup operators in simultaneous lot sizing and scheduling models. They developed a mathematical model to solve the plastic injection plant production plans considering common setup operators. The model includes additional constraints to prevent simultaneous setup operations when there is a single setup operator. However, this research does not cover the overall workforce plans working on the individual machines and the shift or overtime decisions.

### ***3.4 Conclusions About the Literature Review***

In the literature review different methods to model a simultaneous lot sizing and scheduling problems has been provided namely CSLP, CLSD, PLSP and DLSP. CSLP, CLSD and PLSP models are conserving the setup stage. That means in case of an idle period on the machine the setup stage is conserved in the next periods in which the machines are utilized. On the other hand, the DLSP approach is made on the all or nothing assumption where the setups are not conserved when an idle period occurs. That means in every period the machines are assumed to require a setup. All or nothing assumptions also refers that if the machine is utilized in a period full capacity is used to produce the product.

The literature review is basically made on five attributes namely the production stage, machines per stage, BOM levels, setup attributes, objective function and solution method. The model extensions of tool-product-machine interaction, workforce planning, overtime and shift decisions that will be elaborated in this thesis are also classified in the literature review and findings will be presented later in this section.

Some of the workforce related tactical decisions that should be considered in a production plan is the overtime and shift planning decisions. The overtime decision considering the available workforce may increase the efficiency of the production plans. Also the shift plans made together with the production plans simultaneously gives an opportunity to the planner to have free labor which can be used in extra spontaneous workforce requiring jobs. To the best of our knowledge there is no model which covers lot sizing, scheduling, workforce planning, shift planning and overtime decisions simultaneously.

### **3.4.1 Production Stages**

The production stages are important when there are multiple operations in a single product or several parts are required in the assembly of the final product.(Giglio, Paolucci, and Roshani 2017), (Nejati et al. 2016), (Stadtler and Sahling 2013), (Mohammadi 2010), (Gaglioppa, Miller, and Benjaafar 2008), (Persson et al. 2004) and (A. R. Clark 2003) has worked on multiple production stage problems. (Giglio, Paolucci, and Roshani 2017) considered remanufacturing extensions to the problem. (Stadtler and Sahling 2013) studied the zero lead time inventories.

(Neidigh and Harrison 2017), (Claassen et al. 2016), (Almeder and Almada-Lobo 2011), (Kwak and Jeong 2011), (Kaczmarczyk 2011) and (Pahl, Voß, and Woodruff 2010) are examples of multiple production stage problems. Some of the model extension examples are concentrating on perishability of products, secondary resources and different attributes of the products.

### **3.4.2 Machines per Stage**

The classification types on the literature about the machines are namely single, parallel identical and parallel non-identical. Recent papers in the literature review for single machine problems are (Neidigh and Harrison 2017), (Claassen et al. 2016), (Stadtler and Sahling 2013). The multiple machine problems are classified as the parallel identical and parallel non-identical problems. (Almeder and Almada-Lobo 2011), (Kaczmarczyk 2011) and (Mohammadi 2010) examples of parallel identical machine problems. Reader is referred to (Giglio, Paolucci, and Roshani 2017), (Nejati et al. 2016), (Motta Toledo et al. 2013) and (Almada-Lobo, Oliveira, and Carravilla 2008) for parallel non-identical problems.

### **3.4.3 Bill of Material (BOM) Structure**

The BOM structure of the products in the literature has been classified as single and multiple. Single BOM is referred if the product does not contain any sub processes or sub products. Multiple BOM structure refers to the products where multiple sub parts or sub production stages are required to manufacture a final product.

Examples of the single BOM structure in the literature review are (Neidigh and Harrison 2017), (Claassen et al. 2016), (Motta Toledo et al. 2013), (Almeder and Almada-Lobo 2011) and (Kwak and Jeong 2011). The multiple BOM structure can be reviewed in the following papers; (Giglio, Paolucci, and Roshani 2017), (Silvio A. de Araujo, Arenales, and Clark 2008), (Stadtler and Sahling 2013) and (Gaglioppa, Miller, and Benjaafar 2008).

#### **3.4.4 Setup**

Setups are one of the main features of the simultaneous lot sizing and scheduling problems. As explained earlier GLSP problems are first classified as the conservation and loss of setup stages. DLSP models are examples of lost setup stages. Reader can refer to CLSD, CSLP and PLSP models for the conserved setup problems.

The second classification can be done as the setup time and setup cost considerations. The models reviewed used setup costs or times as the requirement of the specified problems. The models are also further classified as the sequence dependent setups. (Nejati et al. 2016), (Claassen et al. 2016) and (Motta Toledo et al. 2013) are examples of the models considering sequence dependent setup times. (Stadtler and Sahling 2013), (Kaczmarczyk 2011), (Stadtler 2011) and (Tempelmeier and Buschkühl 2008) are examples are sequence independent setup time models.

The sequence dependent cost of setup models can be found in the following papers; (Claassen et al. 2016), (Motta Toledo et al. 2013) and (Gupta and Magnusson 2005). Sequence independent setup cost model examples are (Giglio,

Paolucci, and Roshani 2017), (Neidigh and Harrison 2017), (Supithak, Liman, and Montes 2010) and (C. Gicquel et al. 2009).

### **3.4.5 Objective Function**

In the reviewed papers two types of objective functions has been identified. One of which is the cost minimization and the other is the models considering the due dates or completion times of the jobs (orders). (Nejati et al. 2016) has used an objective function to minimize the sum of the weighted completion times.(Kwak and Jeong 2011) has used a hierarchical approach to solve a CSLP model. In the first step the lot sizes are determined considering the cost of production and inventories. In the second step scheduling is done to minimize the maximum completion time of the jobs.

(Almada-Lobo, Oliveira, and Carravilla 2008) has considered an objective function to minimize the weighted sum of sequence dependent setup times, average inventory levels, and number of stock outs. Rest of the reviewed work is using objective function to minimize the cost including production, setup and inventory holding costs.

### **3.4.6 Solution Method**

There is a wide variety of solution approaches in the literature. The solution methods can be classified as the exact methods (MIP), MIP based heuristic methods and the meta-heuristics.

The exact methods are studied by the researches and valid inequalities are used to make the large size problems solvable by standard MIP solvers however

these methods do not guarantee the optimality. (Celine Gicquel, Minoux, and Dallery 2011) and (A. Clark, Mahdieh, and Rangel 2014) are examples of these kind of studies.

Another method used by the researchers is the MIP based approaches such as fix and relax, fix and optimize and rolling horizon. In these approaches the standard MIP problem is divided into easier sub problems and the sub problems are solved by MIP solvers sequentially. Examples of MIP based heuristics are (Giglio, Paolucci, and Roshani 2017), (Claassen et al. 2016), (Stadtler and Sahling 2013) and (Mohammadi 2010).

Final solution approach for the combinatorial problems like the simultaneous lot sizing and scheduling models is the meta-heuristic algorithms. The meta-heuristics used in the reviewed paper are simulated annealing, genetic algorithms and tabu search etc. The reader may refer to the following articles using meta-heuristics; (Neidigh and Harrison 2017), (Nejati et al. 2016), (Motta Toledo et al. 2013) and (Supithak, Liman, and Montes 2010)

### **3.4.7 Tool, Product and Machine Interaction**

Tool, product and machine interaction is also classified in the literature review. This classification is referring mainly to the secondary resources that should be used on the machines. As an example the molds used with the plastic injection machines are secondary resources that should be considered in the production plans.

(Almeder and Almada-Lobo 2011) has studied this attribute in CLSD and CSLP model formulations.(Ghosh Dastidar and Nagi 2005) worked on the plastic

injection factory production plans. Developing a PLSP model the available tools to be used on the injection machines are considered as a capacity and technological constraint in the model. (Silvio A. de Araujo, Arenales, and Clark 2008) focused on the glass production plants in which parallel identical machines with specified dies are used to produce glass containers.





**Table 1** – Literature review classification of CLSD models

AUTHORS	PROBLEM FORMULATION	PRODUCTION STAGE	MACHINES PER STAGE	BOM STRUCTURE	SETUP				EXTENSIONS	OBJECTIVE FUNCTION	SOLUTION METHOD	NOTES
					S.D. TIME	S.ID TIME	S.D. COST	S.ID COST				
Haase, Knut (1996)	CLSD	+	+	+	+	+	+	+	+	BH	Backward heuristics TSP like model	
Clark, Alistair R. Simon J Clark (2000)	CLSD	+	+	+	+	+	+	+	+	RH	Rolling horizon, no allowance for setups, backlogging	
Clark, Alistair R. (2003)	CLSD	+	+	+	+	+	+	+	+	RF		
Gupta, Diwakar Magnusson, Thorkell (2005)	CLSD	+	+	+	+	+	+	+	+	ISI HEURISTICS	Decomposition reformulation	
Quadt, D Kuhn, H (2005)	CLSD	+	+	+	+	+	+	+	+	MIP BASED DECOMPOSITION		
Almada-lobo, Bernardo Klabjan, Diego Carravilla, Maria Antónia Oliveira, José F. (2007)	CLSD	+	+	+	+	+	+	+	+	MIP SOLVER		
Almada-Lobo, Bernardo Oliveira, José F. Carravilla, Maria Antónia (2008)	CLSD	+	+	+	+	+	+	+	+	VNS	Secondary resources minimize the weighted sum of sequence dependent setup times, average inventory levels, and number of stock outs	
Nejati, Mohsen Mähdavi, Iraj Hassanzadeh, Reza Mähdavi-Amini, Nezam (2016)	CLSD	+	+	+	+	+	+	+	+	GA SA		
Giglio, Davide Paolucci, Massimo Roshani, Abdolreza (2017)	CLSD	+	+	+	+	+	+	+	+	RF	Remanufacturing	

**Table 2 – Literature review classification of PLSP models**

AUTHORS	PROBLEM FORMULATION		PRODUCTION STAGE	MACHINES PER STAGE	BOM STRUCTURE	SETUP	EXTENSIONS	OBJECTIVE FUNCTION	SOLUTION METHOD		NOTES
	SINGLE	MULTIPLE							MIP SOLVER	F&O	
Suerie C (2005)	+		+		+			+	MIP SOLVER	Campaign planning, Minimum resource utilization	
Suerie C (2006)	+		+		+			+	MIP SOLVER	More than 2 products can be produced within the micro period contrary to standard PLSP model. Period overlapping set ups are possible	
Tempelmeier, Horst Buschkühl, Lisbeth (2008)	+		+		+			+	MIP SOLVER	Single resource set up operator constraints added to the original PLSP model.	
Kaczmarczyk, Waldemar (2011)	+		+		+			+	MIP SOLVER	Flow integer variables to speed up MIP branch and bound method	
Stadler, Hartmut (2011)	+		+		+			+	MIP SOLVER	Period overlapping set ups, Batch production, Campaign planning	
Stadler, Hartmut Sahling, Florian (2013)	+		+		+			+	F&O	Zero lead time inflow inventory	

**Table 3** – Literature review classification of CSLP models

AUTHORS	PROBLEM FORMULATION	PRODUCTION STAGE	MACHINES PER STAGE	BOM STRUCTURE	SETUP	EXTENSIONS	OBJECTIVE FUNCTION	SOLUTION METHOD		NOTES
								SINGLE	MULTIPLE	
Pochet Y, Wolsey LA (1991)	CSLP	+	+	+	+	+	+	+	MIP	Cutting planes
Gopalakrishnan, Mohan (2000)	CSLP									
Ghosh Dastidar, Satyaki Nagi, Rakesh (2005)	CSLP	+	+	+	+	+	+	+	MIP SOLVER	Product grouping and solving separate problems, Backlogging secondary resources
Gaglioppa, Francesco Miller, Lisa A. Benjaafar, Saif (2008)	CSLP	+	+	+	+	+	+	+	MIP SOLVER	
de Araujo, Silvio A. Arenales, Marcos N. Clark, Alistair R. (2008)	CSLP	+	+	+	+	+	+	+	RF	
Gicquel, Céline (2008)	DLSP	+	+	+	+	+	+	+	MIP SOLVER	
Almada-Lobo, Bernardo James, Ross J.W. (2010)	CSLP	+	+	+	+	+	+	+	VNS TS	
Almeder, Christian Almada-Lobo, Bernardo (2011)	CSLP	+	+	+	+	+	+	+	MIP SOLVER	Secondary Resources
Kwak, Ik-soon Jeong, In-jae (2011)	CSLP	+	+	+	+	+	+	+	MIP SOLVER	Hierarchical solution of lot sizing and scheduling
Motta Toledo, Claudio Fabiano Da Silva Arantes, Márcio De Oliveira, Renato Resende Ribeiro Almada-Lobo, Bernardo (2013)	CSLP	+	+	+	+	+	+	+	GA	

**Table 4** – Literature review classification of DLSP models

AUTHORS	PROBLEM FORMULATION	PRODUCTION STAGE		MACHINES PER STAGE			BOM STRUCTURE		SETUP				EXTENSIONS				OBJECTIVE FUNCTION	SOLUTION METHOD	NOTES	
		SINGLE	MULTIPLE	PI	PN	SINGLE	MULTIPLE	S.D. COST	S.ID COST	SINGLE	MULTIPLE	S.D. TIME	S.ID TIME	OVERTIME	WORKFORCE	SECONDARY RESOURCES				DUE DATE, COMPLETION TIME
Jans, Kai Degraeve, Zeger (2004)	DLSP	+				+		+		+								+	LP Relax. DP	
Göthe-Lundgren, Maud Lundgren, Jan T. Gendron, Bernard (2004)	DLSP		+		+				+									+	TS	
Lasdon, L S Terjung, R C (1971)	DLSP							+										+	Dantzig- Wolfe Decomposition	
Rau, Julia Voß, Stefan Woodruff, David L (2010)	DLSP							+										+	MIP	Perishability of products
Intagli, Robert O. Harrison, Terry P. (2017)	DLSP							+										+	Heuristics	learning curves
Supriatna, wisnu Liman, Surya D. Montes, Elliot J. (2010)	DLSP							+										+	BA GA	
Cluassen, U. D. H Gerdessen, J. C. Hendrix, E. M. T van der Vorst, J. G. A. J (2016)	DLSP																	+	RF	Perishability of products
Gicquel, C. Miègeville, N. Minoux, M. Dallery, Y. (2009)	DLSP																	+	MIP	Different attributes of the products

## OBJECTIVE AND METHODOLOGY

The objective of this work is to extend the MIP models in the literature in order to cover below decisions simultaneously and be able to solve Vestel Electronics's plastic injection factory of production plans:

- Simultaneous lot sizing and scheduling decisions
- Tool, product and machine based constraints
- Overtime decisions
- Shift planning
- Workforce planning

The assumptions made during the development of the MIP models are listed below:

- Demand is deterministic
- The setup times are deterministic
- Planning horizon is 7 days
- Available workforce does not change during the planning horizon
- Each mold should be used at most once in a macro period
- Minor setup times do not exceed major setup times

In order to solve the production plans of Vestel Electronics plastic injection plant, MIP based decomposition methods and heuristics are developed.

## MATHEMATICAL MODEL

In this section different CLSD and capacitated lot sizing with sequence independent setup (CLSI) mathematical model formulations will be presented. First a base model for multiple parallel machines with sequence dependent setup times and costs with CLSD formulations considering molds as the secondary resources will be developed. Later the workforce planning, shift and overtime decision extensions will be added to the models.

The CLSD<sub>a</sub><sup>S/OT</sup> formulation the setup changeover decisions are formulated as arcs from product  $i$  to product  $j$ . This methodology omits the usage of the predefined micro periods in the model. CLSD<sub>b</sub><sup>S/OT</sup> formulation used predefined number of micro periods for the allocation of the tools in the machines.

For the computational considerations the CLSI<sub>a</sub><sup>S/OT</sup> formulation considers the sequence independent setups to omit the setup changeover decision variables. In this formulation the setup times of each tool is independent of the sequence and can take different values.

Based on CLSI<sub>a</sub><sup>S/OT</sup> if the setup times are independent of the tool type but depends on the machine the tool is used CLSI<sub>b</sub><sup>S/OT</sup> is proposed. This type of formulation can be used on the metal stamping lines where the setup times are dependent on the stamping machine properties. The performance of each model will be studied in the Numerical Study section.

## 5.1 Base Model for $CLSD_a^{S/OT}$ Formulation

### DATA

$MO^{\max}$	Total number of molds
$T^{\max}$	Total number of periods
$L^{\max}$	Total number of machines
$MO$	Set of product types (molds)
$T$	Number of periods
$L$	Set of machines
$C$	Capacity of the production resource in one shift (time)
$M$	A big number
$D_{mt}$	Demand of product type $m$ at period $t$ (units) ( $\forall m \in MO; \forall t \in T$ )
$ML_{ml}$	1 indicates that machine $l$ is capable to produce product type $m$ , 0 otherwise ( $\forall m \in MO; \forall l \in L$ )
$LT_l$	The type of the machine $l$ ( $\forall l \in L$ )
$PC_l$	Operating production cost of machine $l$ ( $\forall l \in L$ )
$HC_m$	Variable unit inventory holding cost of product type $m$ ( $\forall m \in MO$ )
$BC_m$	Variable unit backlogging cost of product type $m$ ( $\forall m \in MO$ )
$LC$	Daily salary of an operator without overtime (labor cost)
$CT_{ml}$	Cycle time for product type $m$ on machine $l$ ( $\forall m \in MO; \forall l \in L$ )
$ST_{mn}$	Set up time from product type $m$ to product type $n$ ( $\forall m \in MO; \forall n \in MO$ )

$SC$	Set up cost per unit time
$B_m^0$	Initial backlog for product type $m$ ( $\forall m \in MO$ )
$S_m^0$	Initial inventory for product type $m$ ( $\forall m \in MO$ )
$MQ_m$	Mold quantities for different product types ( $\forall m \in MO$ )

### BINARY VARIABLES

$z_{mnl} \in \{0,1\}$	1 iff machine $l$ produces product types $m$ and $n$ consecutively at period $t$ ( $\forall m = 0 \dots MO; \forall n = 0 \dots MO; \forall l \in L; \forall t \in T$ )
$w_{ml} \in \{0,1\}$	1 iff machine $l$ produces product type $m$ at period $t$ ( $\forall m \in MO; \forall l \in L; \forall t \in T$ )
$sd_{lt}' \in \{0,1\}$	1 iff machine $l$ is opened for one shift (requires an operator) at period $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ )

### INTEGER VARIABLES

$q_{ml} \in \mathbb{N}^+$	Production quantity for product type $m$ produced at machine $l$ at period $t$ ( $\forall m \in MO; \forall l \in L; \forall t = 1 \dots T^{MAX}$ )
$s_{mt} \in \mathbb{N}^+$	Inventory level for product type $m$ at the end of period $t$ ( $\forall m \in MO; \forall t \in T$ )
$b_{mt} \in \mathbb{N}^+$	Backlogging level for product type $m$ at the end of period $t$ ( $\forall m \in MO; \forall t \in T$ )
$it_{li} \in \mathbb{N}^+$	Idle time of machine $l$ at period $t$ ( $\forall l \in L; \forall t \in T$ )
$u_{ml} \in \mathbb{N}^+$	Additional variable for sub tour eliminations



$$(\forall m \in MO; \forall l \in L; \forall t \in T)$$

## MILP MODEL

The objective function is to minimize the total inventory cost, production cost, setup cost, labor cost and backlogging penalty cost at the end of planning horizon.

$$[\text{MIN}]Z = \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} PC_l CT_{ml} q_{mlt} + \sum_{m \in MO} \sum_{n \in MO} \sum_{l \in L} \sum_{t=1}^{T^{\text{MAX}}} ST_{mn} z_{nm lt} SC + \\ \sum_{l \in L} \sum_{t=1}^{T^{\text{MAX}}} LC_s d_{lt}^l + \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} BC_m b_{mt}$$

Equation (1) is the inventory balance equation. The production of product type  $m$ , the stock coming from previous period and the backlogged quantity is equal to the sum of demand, stock at period  $t$  and backlogged quantity from the previous period.

$$D_{mt} + s_{mt} + b_{m,t-1} = s_{m,t-1} + b_{mt} + \sum_{m \in MO} \sum_{l \in L} q_{mlt} \quad (\forall m \in MO; \forall t \in T) \quad (1)$$

Equation (2) defines initial backlogged quantities of the product types.

$$b_{m,t=0} = B_m^0 \quad (\forall m \in MO) \quad (2)$$

Equation (3) defines initial stock quantities of the product types.

$$s_{m,t=0} = S_m^0 \quad (\forall m \in MO) \quad (3)$$

Equation (4) ensures that the production quantity of product type  $m$  to be zero if there is no production decision of mold  $m$  on machine  $l$  at period  $t$ .

$$w_{mlt} M \geq q_{mlt} \quad (\forall m \in MO; \forall l \in L; t = 1 \dots T^{\text{MAX}}) \quad (4)$$

By equation (5) Total available production time cannot be exceeded.

$$\sum_{m \in MO} CT_{ml} q_{mlt} + \sum_{m \in MO} \sum_{n \in MO} ST_{mn} z_{nm lt} \leq C \quad (\forall l \in L; t = 1 \dots T^{\text{MAX}}) \quad (5)$$

Equation (6) ensures that product type  $m$  is used at most once in a period.

$$\sum_{l \in L} w_{mlt} \leq MQ_m \quad (\forall m \in MO; t = 1..T^{\max}) \quad (6)$$

Equation (7) ensures that if machine is not opened there can be no production decision on the machine at period t.

$$w_{mlt} \leq sd_{lt}^l \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\max}) \quad (7)$$

Equation (8) ensures that if machine  $l$  is not capable to produce product type  $m$  there can be no production decision on the machine at period t.

$$w_{mlt} \leq ML_{ml} \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\max}) \quad (8)$$

Equation (9) defines the idle times of the machine  $l$  at period  $t$ .

$$it_{lt} = Csd_{lt}^l - \sum_{m \in MO} CT_{ml} q_{mlt} - \sum_{m \in MO} \sum_{n \in MO} ST_{mn} z_{nm} \quad (\forall l \in L; t = 1..T^{\max}) \quad (9)$$

Equations 10 through 15 are the setup sequencing constraints. Equation (10) states that if there is a setup to mold  $m$  then there should be a setup from mold  $m$ . Equation (11) and (12) ensures that if there is no production decision of mold  $m$  on machine  $l$  at period  $t$  then there cannot be sequence of setup on mold  $m$  on the respective machines and periods. Equations (13) and (14) states that if a machine  $l$  is opened in period  $t$  then there should be setup sequence to and from the dummy mold 0. Equations (15) and (16) are the sub tour elimination constraints.

$$\sum_{x=0}^{MO^{\max}} z_{mxlt} = \sum_{y=0}^{MO^{\max}} z_{ymlt} \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\max}) \quad (10)$$

$$\sum_{n=0}^{MO^{\max}} z_{nm} = w_{mlt} \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\max}) \quad (11)$$

$$\sum_{n=0}^{MO^{\max}} z_{nm} = w_{mlt} \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\max}) \quad (12)$$

$$\sum_{n=0}^{MO^{\max}} z_{n0t} \leq sd_t^I \quad (\forall l \in L; t = 1..T^{\max}) \quad (13)$$

$$\sum_{n=0}^{MO^{\max}} z_{0nt} \leq sd_t^I \quad (\forall l \in L; t = 1..T^{\max}) \quad (14)$$

$$u_{mlt} \leq MO^{\max} \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\max}) \quad (15)$$

$$u_{mlt} - u_{nlt} + MO^{\max} z_{mnl} \leq MO^{\max} - 1$$

$$(\forall m \in MO; \forall n \in MO | m \neq n; \forall l \in L; t = 1..T^{\max}) \quad (16)$$

Equation (17) sequences the same type of machines by ascending order so that the symmetrical solutions in parallel identical machines are reduced.

$$sd_t^l \leq sd_{kt}^l \quad (l \in L; \forall k \in L | k > l \& LT_l = LT_k; t = 1..T^{\max}) \quad (17)$$

### 5.1.1 Overtime and Shift Extensions to the Model $CLSD_a^{S/OT}$

The original problem would be extended by the overtime and shift decisions in order to increase the total available time of the machine per period. Following data are added to the base formulation.

$COT^T$  Time coefficient of overtime - Percentage of increase in capacity per one period

$COT^C$  Cost coefficient of overtime - Percentage of increase in labor cost in case of overtime decision is made

Following binary decision variables are added to the base formulation.

$sd_t^{II} \in \{0,1\}$  1 iff machine  $l$  is opened for second shift (requires an operator) at period  $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ )

$sd_t^{III} \in \{0,1\}$  1 iff machine  $l$  is opened for third shift (requires an operator) at

period  $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ )

$ot_{lt} \in \{0,1\}$  1 iff machine  $l$  is makes overtime at period  $t$ , 0 otherwise

( $\forall l \in L; \forall t \in T$ )

The labor cost statement in the objective function should be revised as follows:

$$[\text{MIN}] Z = \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} PC_l CT_{ml} q_{mlt} + \sum_{m \in MO} \sum_{n \in MO} \sum_{l \in L} \sum_{t=1}^{T^{\text{MAX}}} ST_{nm} z_{nmlt} SC \\ + \sum_{l \in L} \sum_{t=1}^T LC(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COT^C ot_{lt}) + \sum_{m \in MO} \sum_{t=1}^T BC_m b_{mt}$$

Adding the extra decision variables for shifts and overtimes the equations (5) and (9) should be revised as below. The total available time is revised including the additional shifts and overtimes.

$$\sum_{m \in MO} CT_{ml} q_{mlt} + \sum_{m \in MO} \sum_{n \in MO} ST_{nm} z_{nmlt} \leq C(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COT^T ot_{lt}) \\ (\forall l \in L; t = 1..T^{\text{max}}) \quad (5.b)$$

$$it_{lt} = C(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COT^T ot_{lt}) - \sum_{m \in MO} CT_{ml} q_{mlt} - \sum_{m \in MO} \sum_{n \in MO} ST_{nm} z_{nmlt} \\ (\forall l \in L; t = 1..T^{\text{max}}) \quad (9.b)$$

Below additional constraints should be added to the model in order to maintain relations between the newly added decision variables.

Equation (18) ensures that the second shift cannot be decided if first shift is not made on machine  $l$  at period  $t$ .

$$sd_{lt}^I \geq sd_{lt}^{II} \quad (\forall l \in L; t = 1..T^{\text{max}}) \quad (18)$$

Equation (19) ensures that the third shift cannot be decided if second shift is not made on machine  $l$  at period  $t$ .

$$sd_t^{II} \geq sd_t^{III} \quad (\forall l \in L; t = 1..T^{\max}) \quad (19)$$

Equation (20) ensures that overtime decision cannot be made if first shift is not made on machine  $l$  at period  $t$ .

$$sd_t^I \geq ot_t \quad (\forall l \in L; t = 1..T^{\max}) \quad (20)$$

Equation (21) ensures that overtime decision cannot be made if all three shifts are decided on machine  $l$  at period  $t$ .

$$sd_t^I + sd_t^{II} + sd_t^{III} + ot_t \leq 3 \quad (\forall l \in L; t = 1..T^{\max}) \quad (21)$$

### 5.1.2 Workforce Planning Extensions to the Model CLSD<sub>a</sub><sup>S/OT</sup>

As explained in the problem definition section the number of available operators is limited for the planning horizon. There is a limit for the number of operators which may make overtime during the planning horizon. The other characteristics of the problem is that if an operator is decided to work in specified shift he or she should continue working on the shift throughout the planning horizon.

In order extend the model for workforce planning following constants are added to the model.

$AO^s$	Number of available operators for the shifts per day
$AO^{ot}$	Number of available operators to make overtime in the planning horizon

The following decision variables are added to the model for the extension:

$osd^I \in N^+$	Number of operators to work in first shift
$osd^{II} \in N^+$	Number of operators to work in second shift

$osd^{III} \in N^+$       Number of operators to work in third shift

$osf \in N^+$       Number of free operators during the planning horizon

The objective function should be revised according to the dedicated work force for shifts. If a worker is assigned to a shift even the machines are not utilized during the period a labor cost occurs during the planning horizon. The revised objective function is shown below:

$$[\text{MIN}]Z = \sum_{m \in MO} \sum_{t=1}^{T^{MAX}} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{t=1}^{T^{MAX}} PC_l CT_{ml} q_{mlt} + \sum_{m \in MO} \sum_{n \in MO} \sum_{l \in L} \sum_{t=1}^{T^{MAX}} ST_{mn} z_{nmlt} SC \\ + LC(osd^I + osd^{II} + osd^{III}) + \sum_{l \in L} \sum_{t=1}^T LCCOT^C ot_{lt} + \sum_{m \in MO} \sum_{t=1}^T BC_m b_{mt}$$

In order to satisfy the previously explained limitations of the workforce planning following extra constraints should be added to the model:

Equation (22) ensures that the total number of operators on all three shifts is not exceeded.

$$osd^I + osd^{II} + osd^{III} + osf = AO^s \quad (22)$$

Equation (23) ensures that the number of opened machines in first shift of period t does not exceed the decided total number of operators to work in first shift.

$$\sum_{l \in L} sd_t^I \leq osd^I \quad (t = 1..T^{max}) \quad (23)$$

Equation (24) ensures that the number of opened machines in second shift of period t does not exceed the decided total number of operators to work in second shift.

$$\sum_{l \in L} sd_t^{II} \leq osd^{II} \quad (t = 1..T^{max}) \quad (24)$$

Equation (25) ensures that the number of opened machines in third shift of period t does not exceed the decided total number of operators to work in third shift.

$$\sum_{l \in L} sd_t^{III} \leq osd^{III} \quad (t = 1..T^{\max}) \quad (25)$$

Equation (26) ensures that the total number of overtime decisions in all machines and all periods cannot exceed the total number of operators that can make overtime in the planning horizon.

$$\sum_{l \in L} \sum_{t=1}^{T^{\max}} ot_t \leq AO^{ot} \quad (26)$$

### 5.1.3 Minor Setup Revisions for CLSD<sub>a</sub><sup>S/OT</sup> Formulation (CLSD<sub>a</sub><sup>S/OT/MS</sup>)

The sequence dependent setups for the tools may be used for the tool based setup costs and times however the formulation presented cannot be used for the minor setups where the tools are kept on the machine but minor revisions are made to produce a different product version.

The model should be revised so that if the mold  $m$  is assigned to a machine  $l$ , all products capable to be produced by the mold  $m$  can be produced on the machine  $l$ . In order to do that all the  $m$  and  $n$  (product type indices) indices on the decision variables  $z_{mnl}$ ,  $w_{ml}$ ,  $q_{ml}$  and  $u_{ml}$  should be revised to  $i$  and  $j$  (product indices) respectively.

The revised decision variables are shown below:

$$z_{ijl} \in \{0,1\} \quad \begin{array}{l} 1 \text{ iff machine } l \text{ produces product } i \text{ and } j \text{ consecutively at period } t \\ (\forall i = 0..I^{MAX}; \forall j = 0..I^{MAX}; \forall l \in L; \forall t \in T) \end{array}$$

$$w_{il} \in \{0,1\} \quad \begin{array}{l} 1 \text{ iff machine } l \text{ produces product } i \text{ at period } t \\ (\forall i = I; \forall l \in L; \forall t \in T) \end{array}$$

$q_{ilt} \in \mathbb{N}^+$  Production quantity for mold  $m$  produced at machine  $l$  at period  $t$   
 $(\forall i \in I; \forall l \in L; \forall t = 1..T^{MAX})$

$u_{ilt} \in \mathbb{N}^+$  Additional variable for sub tour eliminations  
 $(\forall i \in I; \forall l \in L; \forall t \in T)$

Similarly below data should be revised as well:

$CT_{il}$  Cycle time for product  $i$  on machine  $l$  ( $\forall i \in I; \forall l \in L$ )

$ST_{ij}$  Set up time from product  $i$  to product  $j$  ( $\forall i \in I; \forall j \in I$ )

Below data should be added to the model

$I^{MAX}$  Maximum number of products

$I$  Set of products

$MI_{mi}$  1 indicates that mold  $m$  is capable to produce product  $i$ , 0 otherwise  
 $(\forall m \in MO; \forall i \in I)$

In order to assure that a product  $i$  cannot be produced if a mold  $m$  capable to produce product  $i$  is not installed to machine  $l$  at period  $t$  below decision variable should be added to the model:

$wm_{mlt}$  1 iff mold  $m$  is setup at machine  $l$  at period  $t$   
 $(\forall m \in MO; \forall l \in L; t = 1..T^{max})$

Based on the revised decision variables and data, the objective function of CLSD<sub>a</sub><sup>S/OT</sup> formulation becomes:



$$\begin{aligned}
[\text{MIN}]Z = & \sum_{i \in I} \sum_{t=1}^{T^{\text{MAX}}} HC_i s_{it} + \sum_{l \in L} \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} PC_{il} q_{ilt} + \sum_{i \in I} \sum_{j \in I} \sum_{l \in L} \sum_{t=1}^{T^{\text{MAX}}} ST_{ij} z_{ijlt} SC + \\
& + LC(osd^I + osd^{II} + osd^{III}) + \sum_{l \in L} \sum_{t=1}^T LCCOT^C ot_t + \sum_{i \in I} \sum_{t=1}^T BC_i b_{it}
\end{aligned}$$

The constraints (1), (5), (9), (10), (11), (12), (13), (14), (15), (16) should be revised based on the revised decision variables and constraints as shown below:

Equation (1) is the inventory balance equation. The production of product  $i$ , the stock coming from previous period and the backlogged quantity is equal to the sum of demand, stock at period  $t$  and backlogged quantity from the previous period.

$$D_{it} + s_{it} + b_{it-1} = s_{it-1} + b_{it} + \sum_{l \in L} q_{ilt} \quad (\forall i \in I; \forall t \in T) \quad (1.c)$$

By equation (5) Total available production time cannot be exceeded.

$$\begin{aligned}
\sum_{i \in MO} CT_{il} q_{ilt} + \sum_{i \in I} \sum_{j \in I} ST_{ij} z_{ijlt} \leq C(sd_t^I + sd_t^{II} + sd_t^{III} + COT^T ot_t) \\
(\forall l \in L; t = 1..T^{\text{MAX}}) \quad (5.c)
\end{aligned}$$

Equation (9) defines the idle times of the machine  $l$  at period  $t$ .

$$\begin{aligned}
it_{it} = C(sd_t^I + sd_t^{II} + sd_t^{III} + COT^T ot_t) - \sum_{i \in MO} CT_{il} q_{ilt} - \sum_{i \in I} \sum_{j \in I} ST_{ij} z_{ijlt} \\
(\forall l \in L; t = 1..T^{\text{MAX}}) \quad (9.c)
\end{aligned}$$

Equations 10 through 15 are the setup sequencing constraints. Equation (10) states that if there is a setup to product  $i$  then there should be a setup from product  $i$ . Equation (11) and (12) ensures that if there is no production decision of product  $i$  on machine  $l$  at period  $t$  then there cannot be sequence of setup on product  $i$  on the respective machines and periods. Equations (13) and (14) states that if a machine  $l$  is opened in period  $t$  then there should be setup sequence to and from the dummy

product 0. Equations (15) and (16) are the the sub tour elimination constraints.

$$\sum_{x=0}^{I^{\max}} z_{ixlt} = \sum_{y=0}^{I^{\max}} z_{jmlt} \quad (\forall i \in I; \forall l \in L; t = 1..T^{\max}) \quad (10.b)$$

$$\sum_{j=0}^{j^{\max}} z_{ijlt} = w_{ilt} \quad (\forall i \in I; \forall l \in L; t = 1..T^{\max}) \quad (11.b)$$

$$\sum_{j=0}^{j^{\max}} z_{jilt} = w_{ilt} \quad (\forall i \in I; \forall l \in L; t = 1..T^{\max}) \quad (12.b)$$

$$\sum_{j=0}^{j^{\max}} z_{j0lt} \leq sd_{lt}^l \quad (\forall l \in L; t = 1..T^{\max}) \quad (13.b)$$

$$\sum_{j=0}^{j^{\max}} z_{0jlt} \leq sd_{lt}^l \quad (\forall l \in L; t = 1..T^{\max}) \quad (14.b)$$

$$u_{ilt} \leq I^{\max} \quad (\forall i \in I; \forall l \in L; t = 1..T^{\max}) \quad (15.b)$$

$$u_{ilt} - u_{jlt} + I^{\max} z_{ijlt} \leq I^{\max} - 1 \quad (\forall i \in I; \forall j \in I | i \neq j; \forall l \in L; t = 1..T^{\max}) \quad (16.b)$$

Equation (27) should be added to the model in order to ensure if a mold  $m$  is not set up at machine  $l$  at period  $t$  any product  $i$  that is capable to be produced by mold  $m$ , cannot be produced on machine  $l$  at period  $t$ .

$$\sum_{m \in MO} MI_m w_{mlt} \geq w_{ilt} \quad (\forall m \in MO; \forall i \in I; \forall l \in L; t = 1..T^{\max}) \quad (27)$$

## 5.2 Base Model for $CLSD_b^{S/OT}$ Formulation

Another solution approach for the CLSD formulation is to introduce a micro period index and link the sequence dependent setup decision variables to the sequentially allocated molds on the machines in the micro periods.

### DATA

$MO^{\max}$	Total number of product types
$T^{\max}$	Total number of periods
$L^{\max}$	Total number of machines
$P^{\max}$	Total number of micro periods
$MO$	Set of product types
$T$	Number of macro periods
$L$	Set of machines
$P$	Set of micro periods
$C$	Capacity of the production resource in one shift (time)
$M$	A big number
$D_{mt}$	Demand of product type $m$ in period $t$ (units) ( $\forall m \in MO; \forall t \in T$ )
$ML_{ml}$	1 indicates that machine $l$ is capable to produce product type $m$ , 0 otherwise ( $\forall m \in MO; \forall l \in L$ )
$LT_l$	The type number of the machine $l$
$PC_{ml}$	Variable unit production cost of machine $l$ using product type $m$ ( $\forall m \in MO; \forall l \in L$ ).
$HC_m$	Variable unit inventory holding cost of product type $m$ ( $\forall m \in MO$ ).

$BC_m$	Variable unit backlogging cost of product type $m$ ( $\forall m \in MO$ )
$LC$	Daily salary of an operator without overtime
$CT_{ml}$	Cycle time for product type $m$ on machine $l$ ( $\forall m \in MO; \forall l \in L$ )
$ST_{mn}$	Set up time from product type $m$ to product type $n$ ( $\forall m \in MO; \forall n \in MO$ )
$SC$	Set up cost per unit time.
$B_m^0$	Initial backlog for product type $m$ ( $\forall m \in MO$ )
$S_m^0$	Initial inventory for product type $m$ ( $\forall m \in MO$ )
$MQ_m$	Mold quantities for different product types ( $\forall m \in MO$ )

#### **BINARY VARIABLES**

$z_{mnlpt} \in \{0,1\}$	1 iff machine $l$ makes setup from product type $m$ to product type $n$ at micro period $p$ and macro period $t$ , 0 otherwise ( $\forall m \in MO; \forall n \in MO; \forall l \in L; \forall p \in P; \forall t \in T$ )
$w_{mlpt} \in \{0,1\}$	1 iff machine $l$ produces product type $m$ at micro period $p$ and macro period $t$ , 0 otherwise ( $\forall m \in MO; \forall l \in L; \forall p \in P; \forall t \in T$ )
$sd_l^t \in \{0,1\}$	1 iff machine $l$ is opened for one shift (requires an operator) at period $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ )

#### **INTEGER VARIABLES**

$q_{mlt} \in \mathbb{N}^+$	Production quantity for product type $m$ produced at machine $l$ at period $t$ ( $\forall m \in MO; \forall l \in L; \forall t \in T$ ).
$s_{mt} \in \mathbb{N}^+$	Inventory level for product type $m$ at the end of period $t$ ( $\forall m \in MO; \forall t \in T$ )

$b_{mt} \in \mathbb{N}^+$  Backlogging level for product type  $m$  at the end of period  $t$   
 $(\forall m \in MO; \forall t \in T)$

$it_{it} \in \mathbb{N}^+$  Idle time of machine  $l$  at period  $t$  ( $\forall l \in L; \forall t \in T$ )

### MILP Model

The objective function is to minimize the total inventory cost, production cost, setup cost, labor cost and backlogging penalty cost at the end of planning horizon.

$$[\text{MIN}] Z = \sum_{m \in MO} \sum_{t \in T} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} PC_{ml} q_{mlt} + \sum_{m \in MO} \sum_{n \in MO} \sum_{l \in L} \sum_{p \in P} \sum_{t=1}^{T^{\text{MAX}}} ST_{mn} z_{mnlpt} SC + \sum_{l \in L} \sum_{t=1}^{T^{\text{MAX}}} LCs d_{it}^l + \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} BC_{mt} b_{mt}$$

Equation (1) is the inventory balance equation. The production of product type  $m$ , the stock coming from previous period and the backlogged quantity is equal to the sum of demand, stock at period  $t$  and backlogged quantity from the previous period.

$$D_{it} + s_{it} + b_{it-1} = s_{it-1} + b_{it} + \sum_{m \in MO} \sum_{l \in L} MI_{mi} q_{mlt} \quad (\forall i \in I; t = 1 \dots T^{\text{max}}) \quad (1)$$

Equation (2) defines initial backlogged quantities of the product types.

$$b_{m,t=0} = B_m^0 \quad (\forall m \in MO) \quad (2)$$

Equation (3) defines initial stock quantities of the product types.

$$s_{m,t=0} = S_m^0 \quad (\forall m \in MO) \quad (3)$$

Equation (4) prevents the production quantity to be positive if there is no production decision of product type  $m$  on machine  $l$  at period  $t$ .

$$\sum_{p \in P} w_{mpl} M \geq q_{mlt} \quad (\forall m \in MO; \forall l \in L; t = 1 \dots T^{\text{max}}) \quad (4)$$

Equation (5) prevents that the available production time is not exceeded.

$$\sum_{m \in MO} CT_{ml} q_{mlt} + \sum_{m \in MO} \sum_{n \in MO} \sum_{p \in P} ST_{nm} z_{nmplt} \leq C \quad (\forall l \in L; t = 1..T^{\max}) \quad (5)$$

Equation (6) ensures that product type  $m$  can only be used at most once in a micro period  $p$ .

$$\sum_{p \in P} \sum_{l \in L} w_{mlpt} \leq MQ_m \quad (\forall m \in MO; t = 1..T^{\max}) \quad (6)$$

Equation (7) prevents that more than one mold is assigned to micro period  $p$  in machine  $l$  at macro period  $t$ .

$$\sum_{m \in MO} w_{mlpt} \leq 1 \quad (p \in P; \forall l \in L; t = 1..T^{\max}) \quad (7)$$

Equation (8) ensures that if machine is not opened there can be no production decision on the machine  $l$  at period  $t$ .

$$w_{mlpt} \leq sd_{lt}^l \quad (\forall m \in MO; \forall l \in L; p \in P; t = 1..T^{\max}) \quad (8)$$

Equations (9), (10) and (11) links the variables  $z_{nmplt}$  and  $w_{mlpt}$ . If product type  $m$  and product type  $n$  are produced sequentially at period  $t$  and  $t-1$  then the variable  $y_{nmplt}$  takes the value of 1 and 0 otherwise.

$$z_{nmplt} + 1 \geq w_{nlpt} + w_{ml(p-1)t} \quad (\forall m \in MO; \forall n \in MO; \forall l \in L; p = 2..T^{\max}; t = 1..T^{\max}) \quad (9)$$

$$z_{nmplt} \leq w_{nlpt} \quad (\forall m \in MO; \forall n \in MO; \forall l \in L; p = 2..T^{\max}; t = 1..T^{\max}) \quad (10)$$

$$z_{nmplt} \leq w_{ml(p-1)t} \quad (\forall m \in MO; \forall n \in MO; \forall l \in L; p = 2..T^{\max}; t = 1..T^{\max}) \quad (11)$$

Equation (12) is the ordering constraint for the micro periods. The micro periods with lower index should be used first. This constraint is used to break the symmetry when the used molds in the machine  $l$  are less than the available number of micro periods.

$$\sum_{m \in MO} w_{ml(p-1)t} \geq \sum_{m \in MO} w_{mlpt} \quad (\forall m \in MO; \forall l \in L; p = 2..P^{\max}; t \in T) \quad (12)$$

Equation (13) ensures if machine  $l$  is not capable to produce product type  $m$  there can be no production decision on the machine at macro period  $t$ .

$$w_{mlpt} \leq ML_{ml} \quad (\forall m \in MO; \forall l \in L; p \in P; t = 1..T^{\max}) \quad (13)$$

Equation (14) defines idle time of the machine  $l$  at period  $t$ .

$$it_{it} = Csd_{it}^I - \sum_{m \in MO} CT_{ml} q_{mlt} - \sum_{p \in P} \sum_{m \in MO} \sum_{n \in MO} ST_{nm} z_{nm|pt} \quad (\forall l \in L; t = 1..T^{\max}) \quad (14)$$

Equation (15) sequences the same type of machines by ascending order so that the symmetrical solutions in parallel identical machines are reduced.

$$sd_{it}^I \leq sd_{kt}^I \quad (l \in L; \forall k \in L | k > l \& LT_l = LT_k; t = 1..T^{\max}) \quad (15)$$

### 5.2.1 Overtime and Shift Extensions to the Model $CLSD_b^{S/OT}$

Below data should be added to the model:

$COT^T$  Time coefficient of overtime - Percentage of increase in capacity per one period

$COT^C$  Cost coefficient of overtime - Percentage of increase in labor cost in case of overtime decision is made

Similar to  $CLSD_a^{S/OT}$  model below decision variables should be added to  $CLSD_b^{S/OT}$  model.

$sd_{it}^{II} \in \{0,1\}$  1 iff machine  $l$  is opened for second shift (requires an operator) at period  $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ ).

$sd_{it}^{III} \in \{0,1\}$  1 iff machine  $l$  is opened for third shift (requires an operator) at period  $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ ).

$ot_{it} \in \{0,1\}$  1 iff machine  $l$  is makes overtime at period  $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ ).

The labor cost statement in the objective function should be revised as follows:

$$[\text{MIN}] Z = \sum_{m \in MO} \sum_{t \in T} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{t=1}^{T^{MAX}} PC_{ml} q_{mlt} + \sum_{m \in MO} \sum_{n \in MO} \sum_{l \in L} \sum_{p \in P} \sum_{t=1}^{T^{MAX}} ST_{mn} z_{nmlpt} SC + \sum_{l \in L} \sum_{t=1}^{T^{MAX}} LC(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COTot_{lt}) + \sum_{mi \in MO} \sum_{t=1}^{T^{MAX}} BC_m b_{mt}$$

Adding the extra decision variables for shifts and overtimes the equations (5) and (14) should be revised as follows:

Equation (5.b) is revised by the introduction of shift and overtime variables. The total available time is revised by including the additional shifts and overtimes.

$$\sum_{m \in MO} CT_m q_{mlt} + \sum_{m \in MO} \sum_{n \in MO} \sum_{p \in P} ST_{mn} z_{nmlpt} \leq C(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COT^T ot_{lt}) \quad (\forall l \in L; t = 1..T^{\max}) \quad (5.b)$$

Equation (14.b) is revised so that the total available time is defined by the shift and overtime decisions

$$it_{it} = C(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COT^T ot_{lt}) - \sum_{m=1}^{MO^{MAX}} CT_m q_{mlt} - \sum_{p=2}^{P^{MAX}} \sum_{m=1}^{MO^{MAX}} \sum_{n=1}^{MO^{MAX}} ST_{mn} z_{nmlpt} \quad (\forall l \in L; t = 1..T^{\max}) \quad (14.b)$$

Below additional constraints should be added to the model in order to maintain relations between the newly added decision variables.

Equation (16) ensures that the second shift cannot be decided if first shift is not made on machine l at period t.

$$sd_{lt}^I \geq sd_{lt}^{II} \quad (\forall l \in L; t = 1..T^{\max}) \quad (16)$$

Equation (17) ensures that the third shift cannot be decided if second shift is not made on machine l at period t.

$$sd_{lt}^{II} \geq sd_{lt}^{III} \quad (\forall l \in L; t = 1..T^{\max}) \quad (17)$$



Equation (18) ensures that overtime decision cannot be made if first shift is not made on machine  $l$  at period  $t$ .

$$sd_l^I \geq ot_t \quad (\forall l \in L; t = 1..T^{\max}) \quad (18)$$

Equation (19) ensures that overtime decision cannot be made if all three shifts are decided on machine  $l$  at period  $t$ .

$$sd_l^I + sd_l^{II} + sd_l^{III} + ot_t \leq 3 \quad (\forall l \in L; t = 1..T^{\max}) \quad (19)$$

### 5.2.2 Workforce Planning Extensions to the Model $CLSD_b^{S/OT}$

Similar to  $CLSD_a^{S/OT}$  model in order to extend the model for workforce planning following constants are added to the model:

$AO^s$             Number of available operators for the shifts per day

$AO^{ot}$            Number of available operators to make overtime in the planning horizon

The following decision variables are added to the model for the extension:

$osd^I \in N^+$     Number of operators to work in first shift

$osd^{II} \in N^+$    Number of operators to work in second shift

$osd^{III} \in N^+$    Number of operators to work in third shift

$osf \in N^+$         Number of free operators during the planning horizon

As it is done in the  $CLSD_a^{S/OT}$  model the labor cost should be modified according to the workers selected for the shifts. The modified objective function is shown below:

$$[\text{MIN}] Z = \sum_{m \in MO} \sum_{t \in T} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{t=1}^{T^{\max}} PC_{ml} q_{mlt} + \sum_{m \in MO} \sum_{n \in MO} \sum_{l \in L} \sum_{p \in P} \sum_{t=1}^{T^{\max}} ST_{mn} z_{mnlpt} SC + LC(osd^I + osd^{II} + osd^{III}) + \sum_{l \in L} \sum_{t=1}^T LCCOT^c ot_t + \sum_{mi \in MO} \sum_{t=1}^{T^{\max}} BC_m b_{mt}$$

In order to satisfy the previously explained limitations of the workforce planning

following extra constraints may be added to the model:

Equation (20) ensures that the total number of operators on all three shifts is not exceeded.

$$osd^I + osd^{II} + osd^{III} + osf = AO^s \quad (20)$$

Equation (21) ensures that the number of opened machines in first shift of period  $t$  does not exceed the decided total number of operators to work in first shift.

$$\sum_{l \in L} sd_t^I \leq osd^I \quad (t = 1 \dots T^{\max}) \quad (21)$$

Equation (22) ensures that the number of opened machines in second shift of period  $t$  does not exceed the decided total number of operators to work in second shift.

$$\sum_{l \in L} sd_t^{II} \leq osd^{II} \quad (t = 1 \dots T^{\max}) \quad (22)$$

Equation (23) ensures that the number of opened machines in third shift of period  $t$  does not exceed the decided total number of operators to work in third shift.

$$\sum_{l \in L} sd_t^{III} \leq osd^{III} \quad (t = 1 \dots T^{\max}) \quad (23)$$

Equation (24) ensures that the total number of overtime decisions in all machines and all periods cannot exceed the total number of operators that can make overtime in the planning horizon.

$$\sum_{l \in L} \sum_{t=1}^{T^{\max}} ot_t \leq AO^{OT} \quad (24)$$

### 5.3 Base Model for $CLSI_a^{S/OT}$ Formulation

The  $CLSD_a^{S/OT}$  and  $CLSD_b^{S/OT}$  models are capable to sequence the production in the machines but have the disadvantage of in terms of the binary variables in large data instances. The  $z_{milt} \in \{0,1\}$  decision variables in  $CLSD_a^{S/OT}$  and  $z_{mmlpt} \in \{0,1\}$  decision variables in  $CLSD_b^{S/OT}$  quantity increase by the mold quantities quadratically (MOxMO).

The sequencing decisions is required especially in the sequence dependent setup time and cost constraints inherited in the problem. Using the symmetry within the sequence dependent setup time and cost of the molds this variable can be eliminated. Usually the setup time is dependent on the duration of removing a mold from a machine and loading the new mold onto the machine. This brings an inherited symmetry to the model.

A disadvantage of this approach is that the minor setups such as the version changes in the tools cannot be considered in this model. These changes are strongly sequence dependent since in a major setup the tool is completely disassembled from the machine. However, considering the relatively short times for the minor setups this disadvantage may be omitted in real life problems such as the plastic injection or metal stamping production facilities.

Considering the cost and available capacity, the problem can be decomposed to the sequence independent setup times and costs. Assuming molds a, b and c are produced sequentially in a machine the decomposition of sequence dependent setup times are shown below:

$$ST_{ab} = ST_a^{removal} + ST_b^{load}$$

$$ST_{bc} = ST_b^{removal} + ST_c^{load}$$

$$ST_b^{SIN} = ST_b^{removal} + ST_b^{load}$$

On the other hand the sequence of the molds on the machines should be known on the scheduling perspective as well. Due to this fact the  $CLSD_b^{S/OT}$  model approach will be used to decompose the setups. The  $CLSI_a^{S/OT}$  model is presented below.

#### DATA

$MO^{max}$	Total number of molds
$T^{max}$	Total number of periods
$L^{max}$	Total number of machines
$P^{max}$	Total number of micro periods
$MO$	Set of molds
$T$	Number of macro periods
$L$	Set of machines
$P$	Set of micro periods
$C$	Capacity of the production resource in one shift (time)
$M$	A big number
$D_{mt}$	Demand of product type $m$ at macro period $t$ ( $\forall m \in MO; t = 1..T^{max}$ )
$ML_{ml}$	1 indicates that machine $l$ is capable to produce product type $m$ , 0 otherwise. ( $\forall m \in MO; \forall l \in L$ )
$LT_l$	The type of machine $l$ ( $\forall l \in L$ )

$PC_l$	Variable unit production cost of machine $l$ ( $\forall l \in L$ )
$HC_m$	Variable unit inventory holding cost of product type $m$ ( $\forall m \in MO$ )
$BC_m$	Variable unit backlogging cost of product type $m$ ( $\forall m \in MO$ )
$LC$	Daily salary of an operator without overtime
$CT_{ml}$	Cycle time for product type $m$ on machine $l$ ( $\forall m \in MO; \forall l \in L$ )
$ST_m^{SIN}$	Decomposed set up time for product type $m$ ( $\forall m \in MO$ )
$SC$	Set up cost per unit time
$B_m^0$	Initial backlog for product type $m$ ( $\forall m \in MO$ )
$S_m^0$	Initial inventory for product type $m$ ( $\forall m \in MO$ )
$MQ_m$	Mold quantities for product type $m$ ( $\forall m \in MO$ )

### **BINARY VARIABLES**

$w_{mlpt} \in \{0,1\}$	1 iff machine $l$ produces product type $m$ at micro period $p$ and macro period $t$ , 0 otherwise ( $\forall m \in MO; \forall l \in L; \forall p \in P; t = 1..T^{\max}$ )
$sd_{lt}^l \in \{0,1\}$	1 iff machine $l$ is opened for one shift (requires an operator) at period $t$ , 0 otherwise ( $\forall l \in L; t = 1..T^{\max}$ )

### **INTEGER VARIABLES**

$q_{mlpt} \in \mathbb{N}^+$	Production quantity for product type $m$ produced at machine $l$ at macro period $t$ ( $\forall m \in MO; \forall l \in L; \forall p \in P; t = 1..T^{\max}$ ).
$s_{mt} \in \mathbb{N}^+$	Inventory level for product type $m$ at the end of period $t$ ( $\forall m \in MO; t = 1..T^{\max}$ )
$b_{mt} \in \mathbb{N}^+$	Backlogging level for product type $m$ at the end of period $t$

$$(\forall m \in MO; t = 1..T^{\max}).$$

$$it_{it} \in N^+ \quad \text{Idle time of line } l \text{ at period } t (\forall l \in L; t = 1..T^{\max})$$

### MILP Model

The objective function is to minimize the total inventory cost, production cost, setup cost, labor cost and backlogging penalty cost at the end of planning horizon.

$$[\text{MIN}] Z = \sum_{m \in MO} \sum_{t=1}^{T^{\max}} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{p \in P} \sum_{t=1}^{T^{\max}} PC_{ml} q_{mlpt} + \sum_{m \in MO} \sum_{l \in L} \sum_{p=1}^{P^{\max}} \sum_{t=1}^{T^{\max}} ST_m^{SIN} w_{mlpt} SC + \sum_{l \in L} \sum_{t=1}^{T^{\max}} LCs d_{it}^l + \sum_{m \in MO} \sum_{t=1}^{T^{\max}} BC_m b_{mt}$$

Equation (1) is the inventory balance equation. The production of product type  $m$ , the stock coming from previous period and the backlogged quantity is equal to the sum of demand, stock at period  $t$  and backlogged quantity from the previous period.

$$D_{mt} + s_{mt} + b_{mt-1} = s_{mt-1} + b_{mt} + \sum_{m \in MO} \sum_{l \in L} \sum_{p \in P} q_{mlpt} \quad (\forall m \in MO; t = 1..T^{\max}) \quad (1)$$

Equation (2) defines the initial backlogging quantity.

$$b_{m0} = B_m^0 \quad (\forall m \in MO) \quad (2)$$

Equation (3) defines the initial inventory quantity.

$$s_{m0} = S_m^0 \quad (\forall m \in MO) \quad (3)$$

Equation (4) prevents the production quantity to be positive if there is no production decision of product type  $m$  on machine  $l$  at period  $t$ .

$$\sum_{p \in P} w_{mlpt} M \geq q_{mlpt} \quad (\forall m \in MO; \forall l \in L; p = 1..P^{\max}; t = 1..T^{\max}) \quad (4)$$

Equation (5) ensures that total available production time is not exceeded.

$$\sum_{m \in MO} CT_{ml} q_{mlt} + \sum_{m \in MO} \sum_{p=1}^{P^{\max}} ST_m^{SIN} w_{mlpt} \leq C \quad (\forall l \in L; t = 1..T^{\max}) \quad (5)$$

Equation (6) ensures that product type  $m$  can be assigned at period  $t$  only once.

$$\sum_{p \in P} \sum_{l \in L} w_{mlpt} \leq MQ_m \quad (\forall m \in MO; t = 1..T^{\max}) \quad (6)$$

Equation (7) ensures that only one product type is assigned to micro period  $p$  in line  $l$  at micro period  $t$ .

$$\sum_{m \in MO} w_{mlpt} \leq 1 \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\max}) \quad (7)$$

Equation (8) ensures that if line is not opened there can be no production decision on the line at period  $t$ .

$$w_{mlpt} \leq sd_{lt}^I \quad (\forall m \in MO; \forall l \in L; p \in P; t = 1..T^{\max}) \quad (8)$$

Equation (9) is the ordering constraint for the micro periods. The micro periods with lower index should be used first. This constraint is used to break the symmetry when the used molds in the line are less than the available number of micro periods.

$$\sum_{m \in MO} w_{ml(p-1)t} \geq \sum_{m \in MO} w_{mlpt} \quad (\forall m \in MO; \forall l \in L; p = 2..P^{\max}; t = 1..T^{\max}) \quad (9)$$

Equation (10) ensures that if line  $l$  is not capable to produce product type  $m$  there can be no production decision on the line at period  $t$ .

$$w_{mlpt} \leq ML_{ml} \quad (\forall m \in MO; \forall l \in L; p \in P; t = 1..T^{\max}) \quad (10)$$

Equation (11) defines the idle time of the machine  $l$  at period  $t$ .

$$it_{it} = Csd_{lt}^I - \sum_{m \in MO} \sum_{p \in P} CT_{ml} q_{mlpt} - \sum_{m \in MO} \sum_{p=2}^{P^{\max}} ST_m^{SIN} w_{mlpt} \quad (\forall l \in L; t = 1..T^{\max}) \quad (11)$$

Equation (12) sequences the same type of machines by ascending order so that the symmetrical solutions in parallel identical machines are reduced.

$$sd_{lt}^I \leq sd_{kt}^I \quad (l \in L; \forall k \in L | k > l \& LT_l = LT_k; t = 1..T^{\max}) \quad (12)$$

### 5.3.1 Overtime and Shift Extensions to the Model $CLSI_a^{S/OT}$

Below data should be added to the model:

$COT^T$  Time coefficient of overtime - Percentage of increase in capacity per one period

$COT^C$  Cost coefficient of overtime - Percentage of increase in labor cost in case of overtime decision is made

As done in previous CLSD models below decisions variables can be added to the base formulation.

$sd_{lt}^{II} \in \{0,1\}$  1 iff machine  $l$  is opened for second shift (requires an operator) at period  $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ ).

$sd_{lt}^{III} \in \{0,1\}$  1 iff machine  $l$  is opened for third shift (requires an operator) at period  $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ ).

$ot_t \in \{0,1\}$  1 iff machine  $l$  is makes overtime at period  $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ )

The labor cost statement in the objective function should be revised as follows:

$$[\text{MIN}]Z = \sum_{m \in MO} \sum_{t=1}^{T^{MAX}} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{p \in P} \sum_{t=1}^{T^{MAX}} PC_{ml} q_{mlpt} + \sum_{m \in MO} \sum_{l \in L} \sum_{p=1}^{P^{MAX}} \sum_{t=1}^{T^{MAX}} ST_m^{SIN} w_{mlpt} SC + \sum_{l=1}^L \sum_{t=1}^T LC(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COTot_{lt}) + \sum_{m \in MO} \sum_{t=1}^T BC_m b_{mt}$$

Adding the extra decision variables for shifts and overtimes the equations (5) and (11) should be revised as follows:

Equation (5.b) prevents total available production time to be exceeded. The total available time is revised by including the additional shifts and overtimes.



$$\sum_{m \in MO} \sum_{p \in P} CT_{ml} q_{mlpt} + \sum_{m \in MO} \sum_{p=1}^{p^{MAX}} ST_m^{SIN} w_{mlpt} \leq C(sd_t^I + sd_t^{II} + sd_t^{III} + COT^T ot_t) \quad (\forall l \in L; t = 1..T^{\max}) \quad (5.b)$$

Equation (11.b) defines the idle time of the machine  $l$  at period  $t$ . The total available term is revised as  $C(sd_t^I + sd_t^{II} + sd_t^{III} + COT^T ot_t)$  similarly.

$$it_{it} = C(sd_t^I + sd_t^{II} + sd_t^{III} + COT^T ot_t) - \sum_{m \in MO} \sum_{p \in P} CT_{ml} q_{mlpt} - \sum_{m \in MO} \sum_{p=1}^{p^{MAX}} ST_m^{SIN} w_{mlpt} \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\max}) \quad (11.b)$$

Below additional constraints should be added to the model in order to maintain relations between the newly added decision variables.

$$sd_t^I \geq sd_t^{II} \quad (\forall l \in L; t = 1..T^{\max}) \quad (13)$$

Equation (13) ensures that the second shift cannot be decided if first shift is not made on machine  $l$  at period  $t$ .

$$sd_t^{II} \geq sd_t^{III} \quad (\forall l \in L; t = 1..T^{\max}) \quad (14)$$

Equation (14) ensures that the third shift cannot be decided if second shift is not made on machine  $l$  at period  $t$ .

$$sd_t^I \geq ot_t \quad (\forall l \in L; t = 1..T^{\max}) \quad (15)$$

Equation (15) ensures that overtime decision cannot be made if first shift is not made on machine  $l$  at period  $t$ .

$$sd_t^I + sd_t^{II} + sd_t^{III} + ot_t \leq 3 \quad (\forall l \in L; t = 1..T^{\max}) \quad (16)$$

Equation (16) ensures that overtime decision cannot be made if all three shifts are decided on machine  $l$  at period  $t$ .

### 5.3.2 Workforce Planning Extensions to the Model $CLSI_a^{S/OT}$

Similarly, in order to extend the model for workforce planning following constants are added to the model.

$AO^s$             Number of available operators for the shifts per day

$AO^{ot}$             Number of available operators to make overtime in the planning horizon

The following decision variables are added to the model for the extension:

$osd^I \in N^+$     Number of operators to work in first shift

$osd^{II} \in N^+$     Number of operators to work in second shift

$osd^{III} \in N^+$     Number of operators to work in third shift

$osf \in N^+$         Number of free operators during the planning horizon

The objective function should be modified as it is done for the  $CLSD_a^{S/OT}$  and  $CLSD_b^{S/OT}$  formulations considering the workforce assigned to the shifts. The modified objective function is shown below:

$$[\text{MIN}]Z = \sum_{m \in MO} \sum_{t=1}^{T^{MAX}} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{p \in P} \sum_{t=1}^{T^{MAX}} PC_{ml} q_{mlpt} + \sum_{m \in MO} \sum_{l \in L} \sum_{p=1}^{P^{MAX}} \sum_{t=1}^{T^{MAX}} ST_m^{SIN} w_{mlpt} SC + LC(osd^I + osd^{II} + osd^{III}) + \sum_{l \in L} \sum_{t=1}^T LCCOT^c ot_{lt} + \sum_{m \in MO} \sum_{t=1}^T BC_m b_{mt}$$

In order to satisfy the previously explained limitations of the workforce planning following extra constraints may be added to the model:

$$osd^I + osd^{II} + osd^{III} + osf = AO^s \tag{17}$$

Equation (17) ensures that the total number of operators on all three shifts is not exceeded.

$$\sum_{l \in L} sd_t^I \leq osd^I \quad (t = 1 \dots T^{\max}) \quad (18)$$

Equation (18) ensures that the number of opened machines in first shift of period  $t$  does not exceed the decided total number of operators to work in first shift.

$$\sum_{l \in L} sd_t^{II} \leq osd^{II} \quad (t = 1 \dots T^{\max}) \quad (19)$$

Equation (19) ensures that the number of opened machines in second shift of period  $t$  does not exceed the decided total number of operators to work in second shift.

$$\sum_{l \in L} sd_t^{III} \leq osd^{III} \quad (t = 1 \dots T^{\max}) \quad (20)$$

Equation (20) ensures that the number of opened machines in third shift of period  $t$  does not exceed the decided total number of operators to work in third shift.

$$\sum_{l \in L} \sum_{t=1}^{T^{\max}} ot_t \leq AOot \quad (21)$$

Equation (21) ensures that the total number of overtime decisions in all machines and all periods cannot exceed the total number of operators that can make overtime in the planning horizon.

## 5.4 Base Model for $CLSI_b^{S/OT}$ Formulation

The  $CLSI_a^{S/OT}$  model has the property of solving the problems where setup times may vary in different tools and machines. However in real world instances the setup times may not vary according to the tools but it can solely depend on the machine type like metal stamping machines. This property gives the opportunity to further reduce the binary variables of the  $CLSI_a^{S/OT}$  model. The decision variable  $w_{mpt}$  takes the consideration of the setup times of each tool produced on machine  $l$  at period  $t$ . Using the property of machine dependent setup times the micro period index  $p$  can be cancelled from the model. The  $CLSI_b^{S/OT}$  model is presented below:

### DATA

$MO^{\max}$	Total number of molds
$T^{\max}$	Total number of periods
$L^{\max}$	Total number of machines
$P^{\max}$	Total number of micro periods
$MO$	Set of molds
$T$	Number of macro periods
$L$	Set of machines
$P$	Set of micro periods
$C$	Capacity of the production resource in one shift (time)
$M$	A big number
$D_{mt}$	Demand of product type $m$ at macro period $t$ ( $\forall m \in MO; t = 1..T^{\max}$ )

$ML_{ml}$	1 indicates that machine $l$ is capable to produce product type $m$ , 0 otherwise. ( $\forall m \in MO; \forall l \in L$ )
$LT_l$	The type of machine $l$ ( $\forall l \in L$ )
$PC_l$	Variable unit production cost of machine $l$ ( $\forall l \in L$ )
$HC_m$	Variable unit inventory holding cost of product type $m$ ( $\forall m \in MO$ )
$BC_m$	Variable unit backlogging cost of product type $m$ ( $\forall m \in MO$ )
$LC$	Daily salary of an operator without overtime
$CT_{ml}$	Cycle time for product type $m$ on machine $l$ ( $\forall m \in MO; \forall l \in L$ )
$ST_m^{SIN}$	Decomposed set up time for product type $m$ ( $\forall m \in MO$ )
$SC$	Set up cost per unit time
$B_m^0$	Initial backlog for product type $m$ ( $\forall m \in MO$ )
$S_m^0$	Initial inventory for product type $m$ ( $\forall m \in MO$ )
$MQ_m$	Mold quantities for product type $m$ ( $\forall m \in MO$ )

### **BINARY VARIABLES**

$w_{mlt} \in \{0,1\}$	1 iff machine $l$ produces product type $m$ at macro period $t$ , 0 otherwise ( $\forall m \in MO; \forall l \in L; t = 1..T^{\max}$ )
$sd_{lt}^l \in \{0,1\}$	1 iff machine $l$ is opened for one shift (requires an operator) at period $t$ , 0 otherwise ( $\forall l \in L; t = 1..T^{\max}$ )

### **INTEGER VARIABLES**

$q_{mlt} \in \mathbb{N}^+$	Production quantity for product type $m$ produced at machine $l$ at macro period $t$ ( $\forall m \in MO; \forall l \in L; t = 1..T^{\max}$ ).
----------------------------	--

$s_{mt} \in \mathbb{N}^+$  Inventory level for product type  $m$  at the end of period  $t$

$$(\forall m \in MO; t = 1..T^{\max})$$

$b_{mt} \in \mathbb{N}^+$  Backlogging level for product type  $m$  at the end of period  $t$

$$(\forall m \in MO; t = 1..T^{\max}).$$

$it_{it} \in \mathbb{N}^+$  Idle time of line  $l$  at period  $t$  ( $\forall l \in L; t = 1..T^{\max}$ )

### MILP Model

The objective function is to minimize the total inventory cost, production cost, setup cost, labor cost and backlogging penalty cost at the end of planning horizon.

$$[\text{MIN}] Z = \sum_{m \in MO} \sum_{t=1}^{T^{\max}} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{p \in P} \sum_{t=1}^{T^{\max}} PC_{ml} q_{mlpt} + \sum_{m \in MO} \sum_{l \in L} \sum_{t=1}^{T^{\max}} ST_m^{SIN} w_{mlpt} SC + \sum_{l \in L} \sum_{t=1}^{T^{\max}} LCs d_{it}^l + \sum_{m \in MO} \sum_{t=1}^{T^{\max}} BC_m b_{mt}$$

Equation (1) is the inventory balance equation. The production of product  $i$ , the stock coming from previous period and the backlogged quantity is equal to the sum of demand, stock at period  $t$  and backlogged quantity from the previous period.

$$D_{it} + s_{it} + b_{it-1} = s_{it-1} + b_{it} + \sum_{m \in MO} \sum_{l \in L} q_{mlt} \quad (\forall m \in MO; t = 1..T^{\max}) \quad (1)$$

Equation (2) defines the initial backlogging quantity.

$$b_{m0} = B_m^0 \quad (\forall m \in MO) \quad (2)$$

Equation (3) defines the initial inventory quantity.

$$s_{m0} = S_m^0 \quad (\forall m \in MO) \quad (3)$$

Equation (4) ensures the production quantity to be zero if there is no production decision of mold  $m$  on machine  $l$  at period  $t$ .

$$w_{mlt} M \geq q_{mlt} \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\max}) \quad (4)$$

Equation (5) ensures that total available production time is not exceeded.

$$\sum_{m \in MO} CT_{ml} q_{mlt} + \sum_{m \in MO} ST_m^{SIN} w_{mlt} \leq C \quad (\forall l \in L; t = 1..T^{\max}) \quad (5)$$

Equation (6) ensures that mold  $m$  can only be assigned once at period  $t$ .

$$\sum_{l=1}^{L^{\max}} w_{mlt} \leq MQ_m \quad (\forall m \in MO; t = 1..T^{\max}) \quad (6)$$

Equation (7) ensures that, if line is not opened there can be no production decision on the line at period  $t$ .

$$w_{mlt} \leq sd_{lt}^l \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\max}) \quad (7)$$

Equation (8) ensures that, if line  $l$  is not capable to produce mold  $m$  there can be no production decision on the line at period  $t$ .

$$w_{mlt} \leq ML_{ml} \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\max}) \quad (8)$$

Equation (9) defines the idle time of the machine  $l$  at period  $t$ .

$$it_{lt} = Csd_{lt}^l - \sum_{m \in MO} CT_{ml} q_{mlt} - \sum_{m \in MO} ST_m^{SIN} w_{mlt} \quad (\forall l \in L; t = 1..T^{\max}) \quad (9)$$

Equation (10) sequences the same type of machines by ascending order so that the symmetrical solutions in parallel identical machines are reduced.

$$sd_{lt}^l \leq sd_{kt}^l \quad (l \in L; \forall k \in L | k > l \& LT_l = LT_k; t = 1..T^{\max}) \quad (10)$$

#### 5.4.1 Overtime and Shift Extensions to the Model CLSI<sub>p</sub><sup>S/OT</sup>

Below data should be added to the model:

$COT^T$  Time coefficient of overtime - Percentage of increase in capacity per one period

$COT^C$  Cost coefficient of overtime - Percentage of increase in labor cost in case of overtime decision is made

As done in previous CLSD models below decisions variables can be added to the base formulation.

$sd_{lt}^{II} \in \{0,1\}$  1 iff machine  $l$  is opened for second shift (requires an operator) at period  $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ ).

$sd_{lt}^{III} \in \{0,1\}$  1 iff machine  $l$  is opened for third shift (requires an operator) at period  $t$ , 0 otherwise ( $\forall l \in L; \forall t \in T$ ).

$ot_{lt} \in \{0,1\}$  1 iff machine  $l$  makes overtime at period  $t$ , 0 otherwise  
( $\forall l \in L; \forall t \in T$ )

The labor cost statement in the objective function should be revised as follows:

$$[\text{MIN}] Z = \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{p \in P} \sum_{t=1}^{T^{\text{MAX}}} PC_{ml} q_{mlpt} + \sum_{m \in MO} \sum_{l \in L} \sum_{t=1}^{T^{\text{MAX}}} ST_m^{\text{SIN}} w_{mlt} SC + \sum_{l=1}^L \sum_{t=1}^T LC(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COT^c ot_{lt}) + \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} BC_m b_{mt}$$

Adding the extra decision variables for shifts and overtimes the equations (5) and (9) should be revised as follows:

Equation (5.b) ensures that total available production time cannot be exceeded. The total available time is revised including the additional shifts and overtimes.

$$\sum_{m \in MO} CT_{ml} q_{mlt} + \sum_{m \in MO} ST_m^{\text{SIN}} w_{mlt} \leq C(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COT^T ot_{lt}) - \sum_{m \in MO} \sum_{i \in I} DW_{it} MI_{mi} w_{mlt} \quad (\forall l \in L; t = 1..T^{\text{max}}) \quad (5.c)$$

(9.b) The total available term is revised as  $C(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COT^T ot_{lt})$  similarly.

$$it_{it} = C(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COT^T ot_{lt}) - \sum_{m \in MO} CT_{ml} q_{mlt} - \sum_{m \in MO} ST_m^{\text{SIN}} w_{mlt} \quad (\forall m \in MO; \forall l \in L; t = 1..T^{\text{max}}) \quad (9.b)$$

Below additional constraints should be added to the model in order to maintain relations



between the newly added decision variables.

(11) Ensures that the second shift cannot be decided if first shift is not made on machine  $l$  at period  $t$ .

$$sd_t^I \geq sd_t^{II} \quad (\forall l \in L; t = 1..T^{\max}) \quad (11)$$

(12) Ensures that the third shift cannot be decided if second shift is not made on machine  $l$  at period  $t$ .

$$sd_t^{II} \geq sd_t^{III} \quad (\forall l \in L; t = 1..T^{\max}) \quad (12)$$

(13) Ensures that overtime decision cannot be made if first shift is not made on machine  $l$  at period  $t$ .

$$sd_t^I \geq ot_t \quad (\forall l \in L; t = 1..T^{\max}) \quad (13)$$

(14) Ensures that overtime decision cannot be made if all three shifts are decided on machine  $l$  at period  $t$ .

$$sd_t^I + sd_t^{II} + sd_t^{III} + ot_t \leq 3 \quad (\forall l \in L; t = 1..T^{\max}) \quad (14)$$

#### 5.4.2 Workforce Planning Extensions to the Model $CLSI_b^{S/OT}$

Similarly, in order to extend the model for workforce planning following data are added to the model.

$AO^s$             Number of available operators for the shifts per day

$AO^{ot}$             Number of available operators to make overtime in the planning horizon

The following decision variables are added to the model for the extension:

$osd^I \in N^+$       Number of operators to work in first shift

$osd^{II} \in N^+$      Number of operators to work in second shift

$osd^{III} \in N^+$     Number of operators to work in third shift

$osf \in N^+$  Number of free operators during the planning horizon

The objective function should be modified as it is done for the  $CLSD_a^{S/OT}$  and  $CLSD_b^{S/OT}$  formulations considering the workforce assigned to the shifts. The modified objective function is shown below:

$$[\text{MIN}]Z = \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} HC_m s_{mt} + \sum_{l \in L} \sum_{m \in MO} \sum_{p \in P} \sum_{t=1}^{T^{\text{MAX}}} PC_{ml} q_{mlpt} + \sum_{m \in MO} \sum_{l \in L} \sum_{t=1}^{T^{\text{MAX}}} ST_m^{\text{SIN}} w_{mlpt} SC + LC(osd^I + osd^{II} + osd^{III}) + \sum_{l \in L} \sum_{t=1}^T LCCOT^C ot_{lt} + \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} BC_m b_{mt}$$

In order to satisfy the previously explained limitations of the workforce planning following extra constraints should be added to the model:

Equation (15) ensures that the total number of operators on all three shifts is not exceeded.

$$osd^I + osd^{II} + osd^{III} + osf = AO^s \quad (15)$$

Equation (16) ensures that the number of opened machines in first shift of period t does not exceed the decided total number of operators to work in first shift.

$$\sum_{l \in L} sd_{lt}^I \leq osd^I \quad (t = 1 \dots T^{\text{max}}) \quad (16)$$

Equation (17) ensures that the number of opened machines in second shift of period t does not exceed the decided total number of operators to work in second shift.

$$\sum_{l \in L} sd_{lt}^{II} \leq osd^{II} \quad (t = 1 \dots T^{\text{max}}) \quad (17)$$

Equation (18) ensures that the number of opened machines in third shift of period  $t$  does not exceed the decided total number of operators to work in third shift.

$$\sum_{l \in L} sd_t^{III} \leq osd^{III} \quad (t = 1 \dots T^{\max}) \quad (18)$$

Equation (19) ensures that the total number of overtime decisions in all machines and all periods cannot exceed the total number of operators that can make overtime in the planning horizon.

$$\sum_{l \in L} \sum_{t=1}^{T^{\max}} ot_t \leq AO^{ot} \quad (19)$$

## 5.5 Summary and Comparison of the Models

The existing models in the literature has been extended in order to cover the tool and machine interactions, workforce planning and the tactical decisions such as the shift plans and overtime. The so far presented models have different capabilities that can be used in different production environments. Main difference of the models is the setup properties of different production environments that use the tools with the machines to produce the products.

The  $CLSD_a^{S/OT}$  and  $CLSD_b^{S/OT}$  models has sequence dependent setup properties and can be used in the production environments where the setup sequence of high importance. Production facilities which include minor setup decisions such as the version changes in plastic injection molds may use the described models. In a minor change or version change in plastic injection process the tool should not be removed from the machine. The setup can be done on the tool which is generally shorter than a complete tool interchange.

The  $CLSI_a^{S/OT}$  model can be used in the production environments where the sequence dependence of the setups is not important. The setup times are dependent on the tools and machines in the model. This model can be used in the production environments where minor setup decisions are trivial and does not consume time compared to the tool interchange setups. The plastic injection plants can use the model.

The  $CLSI_b^{S/OT}$  model is also a sequence independent setup model. The main difference of  $CLSI_b^{S/OT}$  from  $CLSI_a^{S/OT}$  is that the setup times are defined on the

machines. The main area that the model can be used is the metal stamping machines where the setup times are highly dependent on the line types being worked on. The comparison of the models can be seen on below table:

**Table 5 –  $CLSD_a^{S/OT}$ ,  $CLSD_b^{S/OT}$ ,  $CLSI_a^{S/OT}$  and  $CLSI_b^{S/OT}$  model comparisons based on binary variables and setup properties**

MODEL	SETUP DEPENDENCE		Number of Binary Variables
	Sequence	Minor Setups	
$CLSD_a^{S/OT}$	+	+	-
$CLSD_b^{S/OT}$	+	+	--
$CLSI_a^{S/OT}$	-	-	+
$CLSI_b^{S/OT}$	-	-	++

## NUMERICAL STUDY

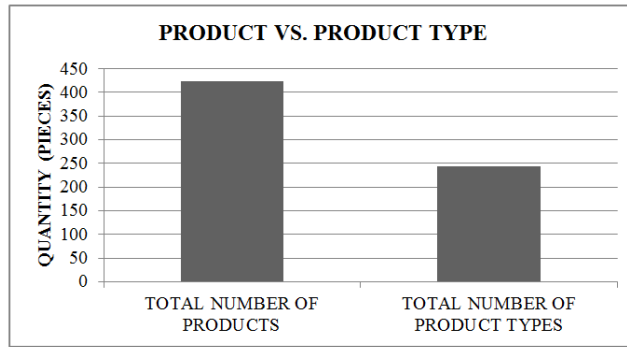
The numerical study of the developed models will be studied in two sections. In the first section, real life demand data of Vestel ELECTRONICS. final assembly line is analyzed and test data generation method is presented. In the second section the computational comparison of the developed models will be made in parallel identical machine problems.

### ***6.1 Test Data Generation***

Before the test data generation the attributes of Vestel ELECTRONICS.'s demand on plastic parts are studied. The attributes that will be presented are as follows; product and product type relations, compatibility of the machines to produce different product types, frequency of the orders within one week, mold quantity relations with the demand, demand quantities and cycle times. The studied data belongs to the demand of final assembly machines on 43'rd week (23-30 Oct) of 2017.

#### **Product vs Product Type:**

In the studied period the total number of product types and individual products are presented in below figure. Total number of demanded products is 411 and total number of products is 237.



**Figure 2-** Product and product type quantities in Vestel Electronics.

The details of product and product type relations can be found on below table. 163 product types cover only one product which is 40% of the total products and 69% of the total product types. 38 of the product types contains (two products total of 76 products) which is 16% of the product types and 18% of the products. 36 product types contain at least three products which makes total of 172 products so that % 17 of the product types contains three or more products.

**Table 6 –** Product type and product quantities in Vestel ELECTRONICS.

	NUMBER OF PRODUCT TYPES	TOTAL NUMBER OF PRODUCTS IN PRODUCT TYPE	TOTAL NUMBER OF PRODUCTS
	163	1	163
	38	2	76
	13	3	39
	10	4	40
	5	5	25
	3	6	18
	2	7	14
	1	10	10
	1	11	11
	1	15	15
<b>TOTAL</b>	<b>237</b>	<b>-</b>	<b>411</b>

### Machine – Product Type Compatibility:

Machines should be compatible to produce different product types. The compatibility features of the demanded products at Vestel Electronics case is shown in below table:

**Table 7** – Machine type and compatible product type quantities in Vestel Electronics.

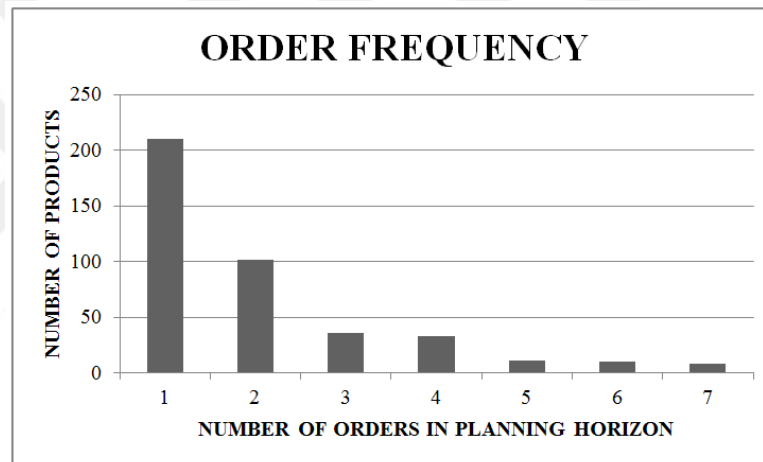
TONNAGE	MACHINE TYPES	NUMBER OF MACHINES		COMPATIBLE PRODUCT TYPE QUANTITY	
600-700	ES600-HE	13	35	63	106
	ES700-HE	9		103	
	NB700-HE	13		104	
850-1000	NB850-HE	9	19	101	128
	ES900-HE	9		113	
	NB1000-HE	1		41	
1200-1300	ES1200-HE	1	19	11	121
	NB1250-HE	10		103	
	NB1300-HE	6		118	
	ES1300-HE	2		97	
1500	ES1500-HE	9		100	
2000	ES2000-HE	1		24	
2700	ES2700-HE	1		12	
TOTAL		84		237	

Tonnage column in table above presents the range of machines clustered in different tonnage levels. The machine type represents the type of the machines in different tonnage levels. The number of product types compatible both to different tonnage levels and machine types are given. The overall compatibility considering the tonnage clusters are 47% and the overall compatibility for each machine type is 49%.



### Order Frequency:

Every product is not ordered at every period. The number of products with different order frequencies is presented below figure. The products can be classified as low frequency and high frequency products. The products with one day and two days frequencies are classified as high frequency products and the products with three and more days frequencies as low frequency products.



**Figure 3-** Order frequency distribution in Vestel Electronics.

Considering the classification defined above 312 of the products are classified as low frequency which has an average of 5,3 days between orders. 99 products which are classified as high frequency has an average of 1,7 days between orders. The overall average order frequency is 3,46 days.

**Table 8** – Order frequencies in Vestel Electronics.

	LOW FREQUENCY	HIGH FREQUENCY	OVERALL
NUMBER OF PRODUCTS	312	99	411
NUMBER OF ORDERS	414	418	832
TOTAL PRODUCTION HOURS	10906	3364	14270
ORDER FREQUENCY	5,3	1,7	3,5

### **Mold Quantities**

The high runner product types require more than one tool in order to be able to meet the capacity requirements. The product type mold quantity relation is presented in below table. 209 product types use single mold to produce the products which makes %82 of the product types use only one mold. The maximum total machine hour required per mold in 7 days for different mold quantities are presented in table below. The maximum machine hour in the molds are around 130,3 hours per 7 days which is almost the full capacity of the machines and the molds.

**Table 9** – Mold quantities and required machine hours per mold in Vestel Electronics.

NUMBER OF PRODUCT TYPES	MOLD QUANTITY	MAXIMUM REQUIRED MACHINE HOUR PER 7 DAYS
194	1	130,3
27	2	122,3
7	3	128,8
5	4	94,7
2	5	86,9
1	8	70,1
1	13	75,7

### Cycle Times:

Cycle times of the product types range from 24 second to 153,9 seconds. The cycle time distributions of the products fit a normal distribution ( $p < 0.005$ ) with an average of 73,2 seconds and standard deviation of 26,8 seconds.

### Used Parameters for Data Generation:

In order to be able to test and compare different models developed, the data is generated for macro setup case. The setup times are 3600 seconds both for sequence dependent and sequence independent models.

**Table 10** – Parameters for test data generation

Data	Definition	Value	Notes
$ST_{ml}, ST_l^{SIN}, ST_{ml}^{SIN}$	Setup time between molds	3600	Fixed for all product types
$C$	Production time per shift in a machine	25200	
$M$	A big number	1000000	
$LC$	Labor cost per one shift without overtime	150	
$SC$	Setup cost per unit time	0.2	
$COT^T$	Time coefficient for overtime	0.33	
$COT^C$	Cost coefficient for overtime	0.50	
$PC_i$	Operating cost of machines per unit time	0.02	Fixed for all machines
$HC_i$	Holding cost	0.1	Fixed for all products
$BC_i$	Backlogging cost	20	Fixed for all products
$CT$	Cycle Time	Unif. Dist (50;90)	
$CUT$	Capacity utilization rate	75%	
$f$	Order frequency	3	

## **6.2 Model Performance Comparisons for Parallel Identical Machines**

The  $CLSD_a^{S/OT}$  and  $CLSD_b^{S/OT}$  models use different approaches to the simultaneous lot sizing and scheduling problems. The  $CLSD_a^{S/OT}$  model does not use micro-periods but relies on the setup decision variables for the sequence of the production within the macro-periods. On the other hand, the  $CLSD_b^{S/OT}$  model uses micro-period formulation. The  $CLSI_a^{S/OT}$  and  $CLSI_b^{S/OT}$  formulations can be used in sequence independent setup conditions. The variable and constraint numbers for different problem sizes are shown below table:

**Table 11 –  $CLSD_a^{S/OT}$ ,  $CLSD_b^{S/OT}$ ,  $CLSI_a^{S/OT}$  and  $CLSI_b^{S/OT}$  model problem size comparison**

PROBLEM SIZE	$CLSD_a^{S/OT}$			$CLSD_b^{S/OT}$				$CLSI_a^{S/OT}$				$CLSI_b^{S/OT}$		
	Const.	Bin. Var.	Int. Var	Const.	Bin. Var.	Int. Var	P	Const.	Bin. Var.	Int. Var	P	Const.	Bin. Var.	Int. Var
L5/M10/T7	6133	4340	899	24778	8190	1249	3	3788	1190	1214	3	1583	490	549
L5/M20/T7	18893	15540	1759	63499	69640	3859	5	11438	3640	3824	5	2793	840	1059
L5/M30/T7	38653	33740	2619	394848	131390	5769	5	16848	5390	5834	5	4003	1190	1569

It can be seen that the  $CLSD_b^{S/OT}$  model has higher values for the variables and the constraints involved. The CLSI models which do not require sequence dependent setup decision variables have lower number of constraints and decision variables. The computation tests on the both models are made for the 10, 20 and 30 mold systems with 5 and 10 machines and 7 macro-periods where the capacity utilization rate is 80%. The comparison of proposed model performances on the generated data is shown in below table:

**Table 12** –  $CLSD_a^{S/OT}$ ,  $CLSD_a^{S/OT/MS}$ ,  $CLSD_b^{S/OT}$ ,  $CLSI_a^{S/OT}$  and  $CLSI_b^{S/OT}$  model comparison

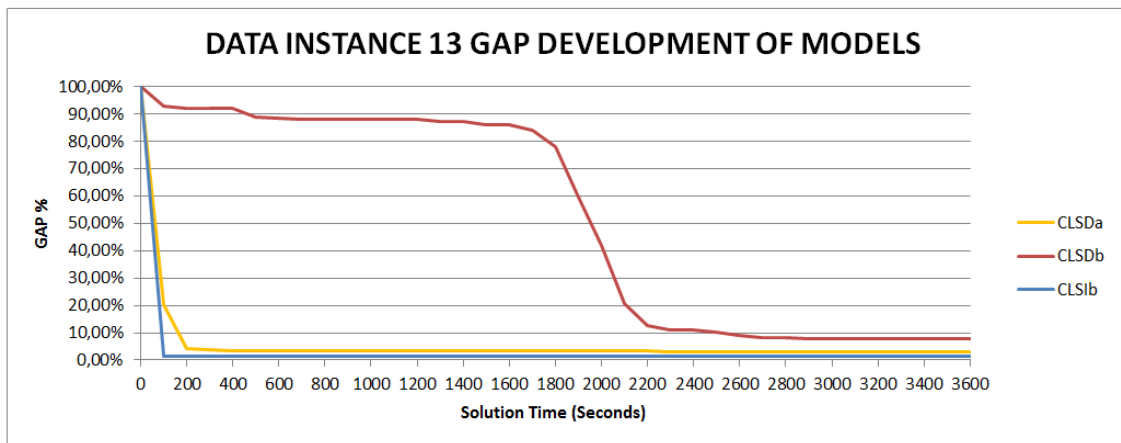
PROBLEM SIZE	DATA INSTANCE	$CLSD_a^{S/OT}$			$CLSD_a^{S/OT/MS}$			$CLSD_b^{S/OT}$			$CLSI_a^{S/OT}$			$CLSI_b^{S/OT}$		
		TIME	OBJ. VALUE	OPT. GAP	TIME	OBJ. VALUE	OPT. GAP	TIME	OBJ. VALUE	OPT. GAP	TIME	OBJ. VALUE	OPT. GAP	TIME	OBJ. VALUE	OPT. GAP
L5/M10/T7	1	121	38286	-	121	38286	-	3600	38286	0,17%	4	38286	-	28	38286	-
	2	12	40815	-	13	40815	-	3600	40815	0,49%	8	40815	-	12	40815	-
	3	15	39265	-	14	39265	-	3600	39265	4,98%	6	39265	-	5	39265	-
	4	3600	48425	0,11%	3600	48425	0,13%	3600	48425	0,16%	3600	48425	0,15%	3600	48425	0,18%
	5	3600	35634	0,14%	2979	35634	0,14%	3600	35634	0,14%	3600	35634	0,13%	3600	35634	0,13%
L5/M20/T7	6	386	35533	-	356	35533	-	3600	35533	7,09%	132	35533	-	104	35533	-
	7	3600	36207	0,90%	3600	36205	0,91%	3600	36222	9,64%	3600	36207	0,37%	278	36207	-
	8	40	36214	-	217	36214	-	1161*	36887	11,53%	87	36214	-	105	36214	-
	9	3600	36619	0,43%	3600	36619	0,43%	3361*	36971	13,89%	476	36619	-	58	36619	-
	10	3600	32900	0,36%	3600	32400	0,38%	3600	32930	2,91%	3600	32900	0,18%	3600	32900	0,42%
L5/M30/T7	11	3600	42808	2,17%	3600	42856	2,04%	1745*	44086	30,27%	3600	42594	1,64%	3600	42567	0,66%
	12	3600	38131	0,43%	3600	38131	0,45%	1652*	40037	23,64%	3600	38131	0,22%	78	38131	0,00%
	13	3600	42916	2,82%	1991*	42988	3,09%	3600	44066	30,17%	3600	42782	2,45%	3600	42818	1,08%
	14	3600	40444	2,55%	3600	40444	2,43%	2074*	42638	26,48%	3600	40444	2,18%	3600	40444	1,59%
	15	3600	41629	1,25%	3600	41629	1,18%	2555*	43326	29,45%	3600	41629	0,16%	658	41629	0,00%
L10/M30/T7	16	*	*	*	*	*	*	3238*	71535	2,46%	2903	71535	0,00%	660	71535	-
	17	*	*	*	*	*	*	3600	71912	2,94%	3600	70582	0,41%	3600	70288	0,44%
	18	*	*	*	*	*	*	3600	74022	7,83%	3600	73748	1,42%	3600	73748	1,37%
	19	*	*	*	*	*	*	2172*	80144	15,02%	3600	70912	0,66%	3600	70912	0,67%
	20	*	*	*	*	*	*	3600	72203	1,86%	3600	72111	0,39%	3600	72111	0,43%

\*Instances where the memory exceeded available 60 GB memory capacity and solution is aborted.

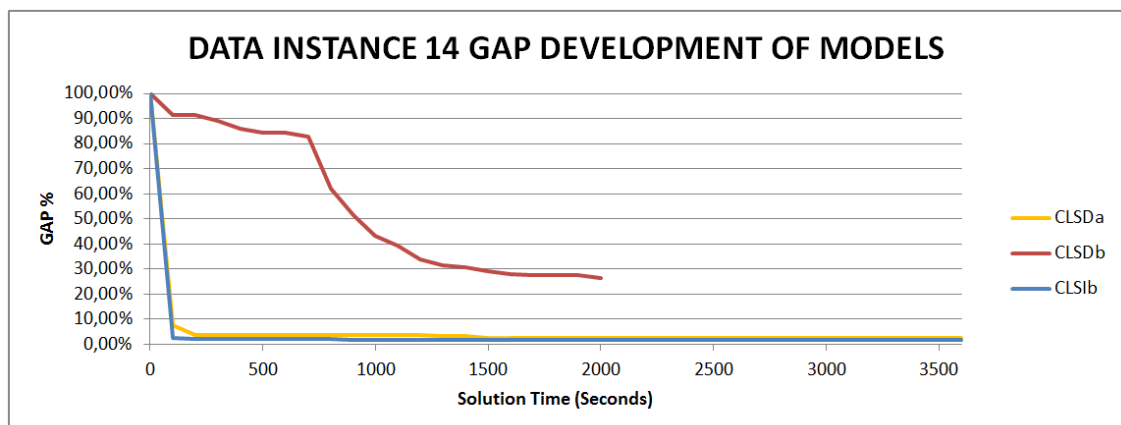
As it can be seen on above table the  $CLSD_a^{S/OT}$  model performs better then  $CLSD_b^{S/OT}$  model in terms of the computation time, objective value and lower bound. However for the 10 machines 30 molds 7 periods problem the  $CLSD_a^{S/OT}$  model could not generate a solution since the solver exceeded the available RAM storage limit. As it is expected the  $CLSI_b^{S/OT}$  gives the best performance over all

proposed models thanks to the reduced number of constraints and decision variables.

Moreover, considering the optimality of the solutions provided by four developed models, for data instances 13 and 14 none of the models could provide optimal solution within 3600 seconds. The comparison of the models is shown in below figures:

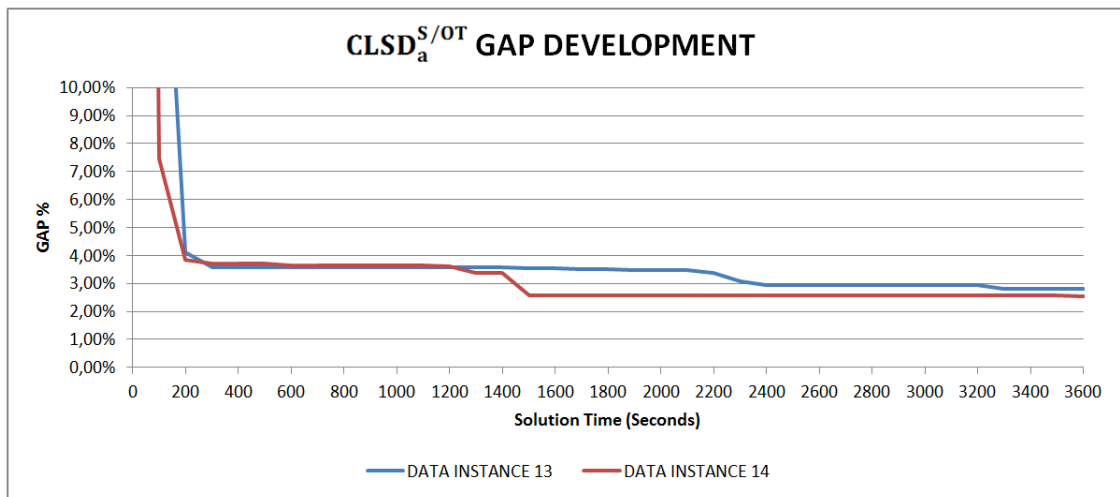


**Figure 4-** Optimality gap improvement chart for exact models on data instance 13



**Figure 5 -** Optimality gap improvement chart for exact models on data instance 14

As it is shown in above figures the computational performance of  $CLSD_b^{S/OT}$  model is worse than the  $CLSD_a^{S/OT}$  model. The  $CLSD_a^{S/OT}$  generates much better gaps in a short time. The gap improvement data for  $CLSD_a^{S/OT}$  model for the data instances 13 and 14 can be found on below figure. The optimality gap reduces below 4% in the first 400 seconds and it takes 3600 seconds to reach the gap just below 3% in both instances.



**Figure 6** - Optimality gap improvement chart for CLSDa model for Data Instances 13 and 14

## **DECOMPOSITION APPROACH FOR CLSD FORMULATIONS**

The production methods which require tool and machine interactions are common in mass production industries. As explained earlier the examples of these industries are the plastic injection and metal stamping factories. The presented formulations for simultaneous lot sizing and scheduling are NP hard problems. The complexity of the models increases exponentially by the increased number of tool. Due to this fact two different decomposition approaches are studied in this section.

The tool machine interaction in the metal stamping or injection molding production methods brings the opportunity to introduce a decomposition method based on the capability of the machines to use the molds. As explained in the problem definition section the machines should be capable to use the tools in order to make production. For instance, an injection mold that requires a 1000 MT clamping force should be produced in a machine which has the required specifications or a mold that requires a steam process should be produced on a steam injection machine.

In the first decomposition model of the parallel sets of non-identical machines is introduced. In this model the tools are assigned to the sets of different machines. Later the introduced CLSD/CLSI models can be used to make the production schedules for the decomposed problems as parallel identical machines. In the second section the results of the parallel nonidentical decomposition will be presented.



In the third section a MIP based heuristic approach for the solution of the real world problems is introduced. In the heuristic approach the decomposition models and CLSD/CLSI models are used sequentially to reach a solution close to the optimality. Final section is dedicated to the numerical study of the decomposition methods presented.

### 7.1 Decomposition for Parallel Non-Identical Machines

In a real world problem, the tools can be produced by a set of different machines which has certain attributes. In below figure the tool machine type capability is presented. The row and column indices indicate the tools and the machine types respectively.

TOOL / MACHINE TYPE	MT1	MT2	MT3	MT4
TOOL 1	1	0	0	0
TOOL 2	1	0	0	0
TOOL 3	1	1	0	0
TOOL 4	1	1	0	0
TOOL 5	1	1	0	0
TOOL 6	0	1	0	0
TOOL 7	0	1	0	0
TOOL 8	0	1	1	0
TOOL 9	0	1	1	0
TOOL 10	0	0	1	0
TOOL 11	0	0	1	0
TOOL 12	0	0	1	0
TOOL 13	0	0	1	1
TOOL 14	0	0	1	1
TOOL 15	0	0	0	1

TOOL / MACHINE TYPE	MT1	MT2	MT3	MT4
TOOL 1	1	0	0	0
TOOL 2	1	0	0	0
TOOL 3	1	0	0	0
TOOL 4	0	1	0	0
TOOL 5	0	1	0	0
TOOL 6	0	1	0	0
TOOL 7	0	1	0	0
TOOL 8	0	0	1	0
TOOL 9	0	0	1	0
TOOL 10	0	0	1	0
TOOL 11	0	0	1	0
TOOL 12	0	0	1	0
TOOL 13	0	0	1	0
TOOL 14	0	0	0	1
TOOL 15	0	0	0	1

**Figure 7 – Tool and Machine Type Decomposition Based on Machine Capability**

If the tools are only capable to be used on a single machine type it is easy to decompose the model in to sets of simpler sub problems. However, in real world problems the tools are capable to be used on different sets of machines. In this case the whole problem should be considered at once to reach a global optimum. However, the model can be decomposed by assigning the tools to a single type of

set of machines. One way is to make the decomposition, considering the cost and capacity requirements of the sets of the machines.

In the developed CLSD models the cost is generated by the production, inventory holding, backloging, setup, shift, labor and overtime decisions. The machine tool decomposition effects on the costs related with the capacity and the production cost of the selected machine types.

The total capacity of the decomposed machine types is interrelated by the number of workers attached to the machines and the total shift quantities made. Similarly, each machine type has a different operating cost so the production quantities of the products which are assigned to different machine types play an important role on the overall performance of the model. The machine related decision variables affecting the total cost of the production plan is listed below based on the objective function of the CLSI<sub>b</sub><sup>S/OT</sup> model:

$$[\text{MIN}]Z = \sum_{i \in I} \sum_{t=1}^{T^{\text{MAX}}} HC_i s_{it} + \sum_{l \in L} \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} PC_{ml} q_{mlt} + \sum_{l \in L} \sum_{t=1}^{T^{\text{MAX}}} ST_l^{\text{SIN}} SCst_{lt} + LC(osd^I + osd^{II} + osd^{III}) + \sum_{l \in L} \sum_{t=1}^T LCCOT^C ot_{lt} + \sum_{i=1}^I \sum_{t=1}^T BC_i b_{it}$$

$$\sum_{l \in L} \sum_{m \in MO} \sum_{t=1}^{T^{\text{MAX}}} PC_{ml} q_{mlt} \quad \text{- Production Cost}$$

$$\sum_{l \in L} \sum_{t=1}^{T^{\text{MAX}}} ST_l^{\text{SIN}} SCst_{lt} \quad \text{- Setup Cost}$$

$$\sum_{l=1}^L \sum_{t=1}^T LC(sd_{lt}^I + sd_{lt}^{II} + sd_{lt}^{III} + COTot_{lt}) \quad \text{- Labor Cost}$$

To wrap up the decomposition on the machine types should consider the total number of workers, the shift decision made, the operating cost of the machines and finally the estimated setups made. The model is presented below:

## DATA

$I^{MAX}$	Maximum number of product types
$MT^{MAX}$	Maximum number of machine types.
$T^{MAX}$	Total number of periods
$MO$	Set of product types
$MT$	Set of machine types
$T$	Set of periods
$NM_m$	Number of machines in the machine type $m$
$DI_i$	Total demand of product type $i$ in the planning horizon
$CT_{im}$	Cycle time for product type $i$ on machine type $m$ ( $\forall i \in I; \forall m \in MT$ )
$DW_{it}$	1 iff there is a demand for product $i$ at period $t$ ( $\forall i \in I; t = 1..T^{max}$ )
$ST_i^{SIN}$	Setup time for product type $i$ ( $\forall i \in I$ )
$IM_{im}$	1 iff machine type $m$ is capable to produce product type $i$ ( $\forall i \in I; \forall m \in MT$ )
$C$	Capacity of the production resource in one shift (time)
$SC$	Set up cost per unit time
$LC$	Daily salary of an operator without overtime
$AO^s$	Number of available operators for the shifts per day
$PC_m$	Variable unit production cost of machine type $m$ ( $\forall m \in MT$ )

$BC_m$  Variable unit backloging cost of product type  $i$  ( $\forall i \in I$ )

### **BINARY VARIABLES**

$d_{im} \in \{0,1\}$  1 iff product  $i$  is assigned to machine type  $m$ , 0 otherwise

$$(\forall i \in I ; \forall m \in MT)$$

### **INTEGER VARIABLES**

$w_m \in N^+$  Number of required labor to work on machine type  $m$  ( $\forall m \in MT$ )

$q_{im} \in N^+$  Total production of product type  $i$  on machine type  $m$

$$(\forall i \in I ; \forall m \in MT)$$

$b_i \in N^+$  Total backlog quantity of product type  $i$  on machine type  $m$

$$(\forall i \in I)$$

### **MILP MODEL**

The objective function is to minimize the production cost, labor cost and backloging cost at the end of planning horizon.

$$[\text{MIN}] Z = \sum_{i \in I} \sum_{m \in MT} PC_m CT_i q_{im} + \sum_{m \in MT} LCT^{MAX} w_m + \sum_{i \in I} BC_i b_i$$

Equation (1) states that the sum of backlogged and produced product quantity is equal to the total demand.

$$b_i + \sum_{m \in MT} q_{im} = DI_i \quad (\forall i \in I) \quad (1)$$

Equation (2) ensures that if product type  $i$  is not assigned to machine type  $m$ , machine type  $m$  cannot produce product type  $i$ .

$$q_{im} \leq M d_{im} \quad (\forall i \in I ; \forall m \in MT) \quad (2)$$

Equation (3) states that total working hours on machine type  $m$  is not exceeded

$$\sum_{i \in I} CT_i q_{im} + \sum_{i \in I} \sum_{t=1}^{T^{MAX}} ST_i^{SIN} DW_{it} d_{im} \leq CT^{MAX} w_m \quad (\forall m \in MT) \quad (3)$$

Equation (4) assigns each product type to a machine type.

$$\sum_{m \in MT} d_{im} = 1 \quad (\forall i \in I) \quad (4)$$

Equation (5) ensures that total number of operators is not exceeded

$$\sum_{m \in MT} w_m = AO^s \quad (5)$$

Equation (6) ensures that total number of operators does not exceed the total number of machines

$$w_m \leq 3NM_m \quad (\forall m \in MT) \quad (6)$$

Equation (7) ensures that product types are assigned to capable machine types

$$d_{im} \leq IM_{im} \quad (\forall i \in I; \forall m \in MT) \quad (7)$$

Equation (8) makes sure that available mold capacity is not exceeded

$$\sum_{m \in MT} CT_i q_{im} \leq 3CMQ_i T^{MAX} \quad (\forall i \in I) \quad (8)$$

## 7.2 Results of Decomposition

The decomposition results are shown in below table. It can be seen that the problem sizes after the non-identical machine decomposition exceeds the capability of the exact CLSD models. In order to be able to solve the CLSD models a hierarchical solution may be used. The hierarchical solution procedure is explained in the next section.

**Table 13** – Decomposition results in Vestel Electronics

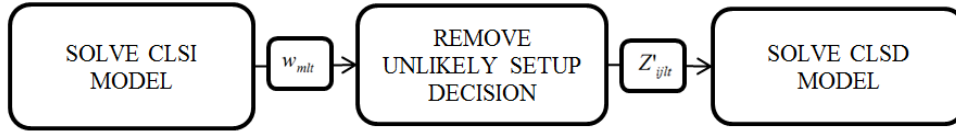
Machine Type	Number of Machines	Number of Products	Number of product Types
1	13	28	8
2	9	27	7
3	13	49	29
4	9	20	14
5	9	93	71
6	1	3	2
7	1	4	2
8	10	16	10
9	6	37	15
10	2	15	7
11	9	85	60
12	1	14	6
13	1	20	6
Total	84	411	237

### ***7.3 A Heuristic Approach for the Solution of Industry Size Problems***

In the numerical study section it has been shown that the CLSD models do not give reasonable solutions if the maximum number of products exceeds 30. After the decomposition of the Vestel Electronics problem the product quantities exceed this value for the machine types 1, 3, 5 and 11.

However, the CLSI models are still capable to solve the decomposed problems. The CLSI models are capable to solve the sequence independent major setup problems. The solution of the CLSI models gives the mold allocations to the machines. This data may be used to prune the sequence dependent setup decision variables on the CLSD models. The mold allocation solution of the CLSI models is used remove the unlikely setup decision variables on the CLSD models so that the

decomposed problems can be solved considering the sequence dependent major and minor setups. The solution procedure is shown in below figure.



**Figure 8** – Hierarchical solution procedure for large problems

$w_{mlt}$  represents the mold allocation decision variable of the CLSI models. The setup decisions on the CLSD models are constrained if the specified molds are not chosen to be used on the machines. Let the set of possible arcs from product  $i$  to product  $j$  on machine  $l$  at period  $t$  be  $Z_{ijlt}$ . Considering the  $w_{mlt}$  values the set of  $Z_{ijlt}$  can be reduced to  $Z'_{ijlt}$ . The procedure is shown in below pseudocode:

1. **for**  $m=1$  to  $MO^{MAX}$  **do**
2.   **if**  $w_{mlt} = 1$  **then**
3.     **for**  $i=1$  to  $I^{MAX}$  **do**
4.       **for**  $j=1$  to  $I^{MAX}$  **do**
5.         **if**  $MI_{mi} = 1$  and  $MI_{nj} = 1$  **then**
6.          $z_{ijlt} \in Z'_{ijlt}$
7.         **end if**
8.       **end for**
9.     **end for**
10.   **end if**
11. **end for**

The setup decision variables are given in below table after the heuristics is applied:

**Table 14** – Setup decision variable comparison of exact models and developed heuristics

MACHINE TYPE	PROBLEM SIZE	NUMBER OF SETUP DECISION ARCS	
		EXACT MODEL	HEURISTICS
1	I28/M8/L13/T5	71344	8624
2	I27/M7/L9/T7	45927	14670
3	I49/M29/L13/T7	218491	2730
4	I20/M14/L9/T7	25200	936
5	I93/M71/L9/T7	544887	2508
6	I3/M2/L1/T7	63	54
7	I4/M2/L1/T7	112	90
8	I16/M10/L10/T7	17920	1824
9	I37/M15/L6/T7	57498	3286
10	I15/M7/L2/T7	3150	542
11	I85/M60/L9/T7	455175	5122
12	I14/M6/L1/T7	1372	318
13	I20/M6/L1/T7	2800	296

#### ***7.4 Comparison of Exact vs Heuristic Solutions***

The results of the heuristics and exact models are presented for each machine type in below table. Exact model were not able to solve the problems 3, 5, 9 and 11 as expected. The results will be discussed in next section.



**Table 15** – Solution of exact model and developed heuristic

MACHINE TYPE	PROBLEM SIZE	HIERARCHICAL SOLUTION			EXACT SOLUTION							
		TIME	OBJ. VALUE	GAP	TIME	OBJ. VALUE	LB	GAP	TIME	OBJ. VALUE	LB	GAP
1	I28/M8/L13/T5	1500	1318601	1,3%	1500	1434365	1297096	9,6%	3600	1434365	1297096	9,6%
2	I27/M7/L9/T7	1500	301901	7,8%	1500	324096	241873	25,4%	3600	307624	241885	21,4%
3	I49/M29/L13/T7	1500	1589137	4,4%	*	*	*	*	*	*	*	*
4	I20/M14/L9/T7	1500	891302	0,0%	1500	904328	779530	13,8%	3600	895856	787368	12,1%
5	I93/M71/L9/T7	1500	1438524	0,2%	*	*	*	*	*	*	*	*
6	I3/M2/L1/T7	2	181519	-	3	181519	181519	-	-	-	-	-
7	I4/M2/L1/T7	2	288110	-	1	288110	288110	-	-	-	-	-
8	I16/M10/L10/T7	1500	935845	0,4%	1500	950677	890784	6,3%	2070*	949668	893353	5,9%
9	I37/M15/L6/T7	1500	522711	1,8%	*	*	*	*	*	*	*	*
10	I15/M7/L2/T7	905	97147	0,4%	1500	97714	88822	9,1%	3117*	97714	89447	8,5%
11	I85/M60/L9/T7	1500	899689	1,8%	*	*	*	*	*	*	*	*
12	I14/M6/L1/T7	11	279150	-	515	277385	277385	-	-	-	-	-
13	I20/M6/L1/T7	3	764113	-	13	763335	763335	-	-	-	-	-

\*Instances where the memory exceeded available memory capacity and solution is aborted.

In order to compare the accuracy of the heuristics below table is presented. The optimality gap in the below table is calculated considering the best lower bound found in the exact model.

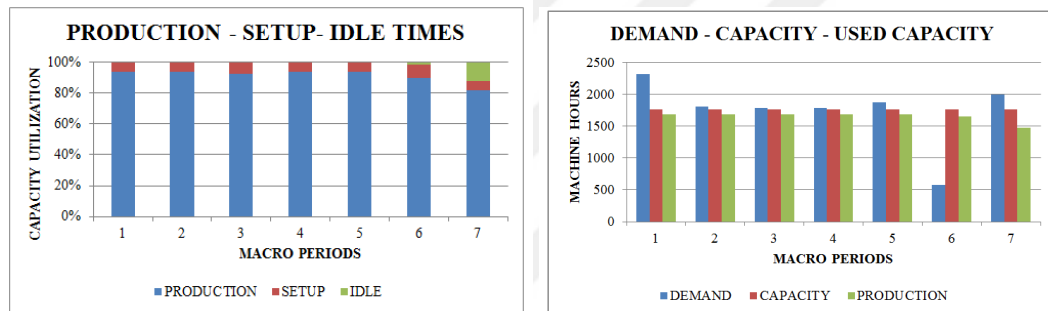
**Table 16** – Comparison of exact model and developed heuristic

PROBLEM SIZE	TIME	OBJ. VALUE			LOWER BOUND		
		HEURISTICS	EXACT	GAP	HEURISTICS	EXACT	GAP
I3/M2/L1/T7	2	181519	181519	0,0%	181519	181519	0,0%
I4/M2/L1/T7	2	288110	288110	0,0%	288110	288110	0,0%
I14/M6/L1/T7	11	279150	277385	-0,6%	279150	277385	0,6%
I20/M6/L1/T7	3	764113	763335	-0,1%	764113	763335	0,1%
I15/M7/L2/T7	905	97147	97714	0,6%	96720	88822	8,2%
I20/M14/L9/T7	1500	891302	904328	1,4%	891124	779531	12,5%
I16/M10/L10/T7	1500	935845	950677	1,6%	931727	890784	4,4%
I27/M7/L9/T7	1500	301901	324096	6,8%	278504	241873	13,2%
I28/M8/L13/T5	1500	1318601	1434365	8,1%	1301459	1297096	0,3%
I37/M15/L6/T7	1500	522711	*	*	513511	*	*
I49/M29/L13/T7	1500	1589137	*	*	1518897	*	*
I85/M60/L9/T7	1500	899689	*	*	883585	*	*
I93/M71/L9/T7	1500	1438524	*	*	1435935	*	*

\*Instances where the memory exceeded available memory capacity and solution is aborted.

## 7.5 Results of the Vestel Electronics's Problem

The capacity utilizations of the total 84 machines are presented in below figures. The utilizations are around 100% where the demand is high and reduces to around 90% in the last period. The demand vs production rates are also presented in below figure. The overall utilizations are uniformly distributed during the planning horizon. The reader can find the capacity utilizations of different machine types in the Appendix section



**Figure 9** – Demand, production time, setup time and idle times of complete problem

## CONCLUSIONS AND MANAGERIAL INSIGHT

Simultaneous lot sizing and scheduling models has been investigated in a multiple tool, multiple machine single stage production environment. Among the existent simultaneous lot sizing and scheduling models, the CLSD model formulations has been studied and the extensions of tool machine interaction, workforce planning, shift and overtime decisions are added to the model.

It has been shown that the capacity constraints are closely related to the available workforce and the tactical decisions like the shift planning and overtime decisions. Due to the effects of workforce capacity and tactical decisions the planning effort to decide the lot sizes and schedules the tactical decisions should be considered.

Although the exact MIP method can be used for small sized instances other techniques such as MIP based decomposition or heuristics should be used for large problems. Decomposition techniques and heuristics have been implemented to solve real world problems. The decomposition of the parallel sets of non-identical machines is based on the capacity requirements and costs of different machine types. The heuristics can be used for the reduction of the quantity of the setup variables and the large size problems become solvable.

The developed models and decomposition techniques can be used for production environments where tools and machines should be used together to produce the products, such as metal stamping or plastic injection factories. The models can be used as a decision support system for the professional planners. The developed heuristic algorithm can be used to make fast and agile production plans.

The other area that the developed models can be used is the final assembly factories where the parts are produced in separate production facilities. The overall production plan of the final assembly factory can be modified according to the results gained from the developed models. In this way the general efficiency of the whole system including the sub factories producing the parts can be increased.



## **FUTURE RESEARCH**

The possible future research on the area can concentrate on the heuristics that can give faster and better solutions. Some of the heuristics used so far for the simultaneous lot sizing and scheduling in the literature are relax and fix, fix and optimize, column generation and genetic algorithms. The developed decomposition methods can be used as initial solutions for the heuristic methods or they can be used as a part of the new heuristic approaches.

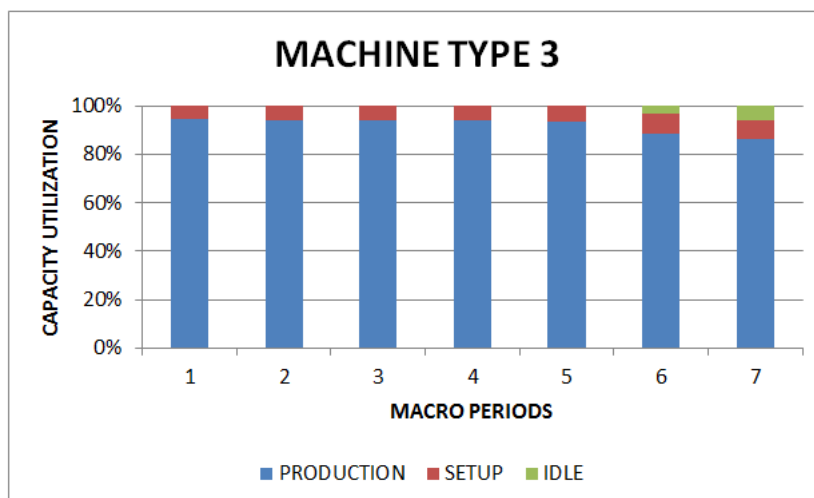
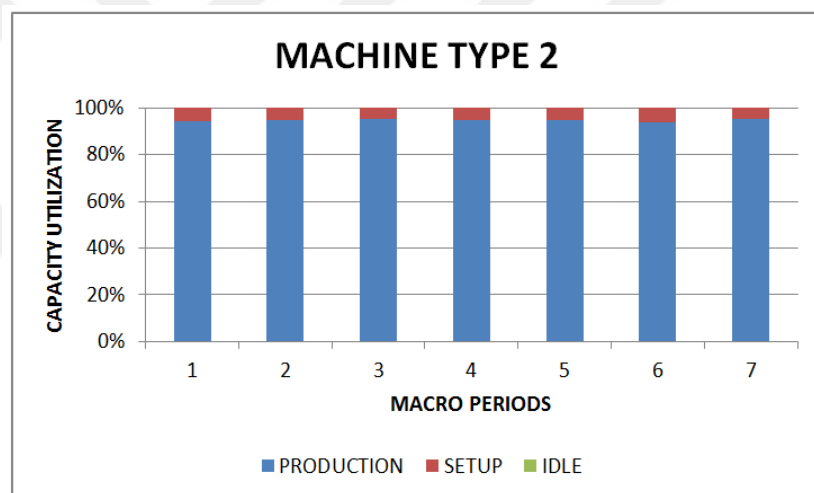
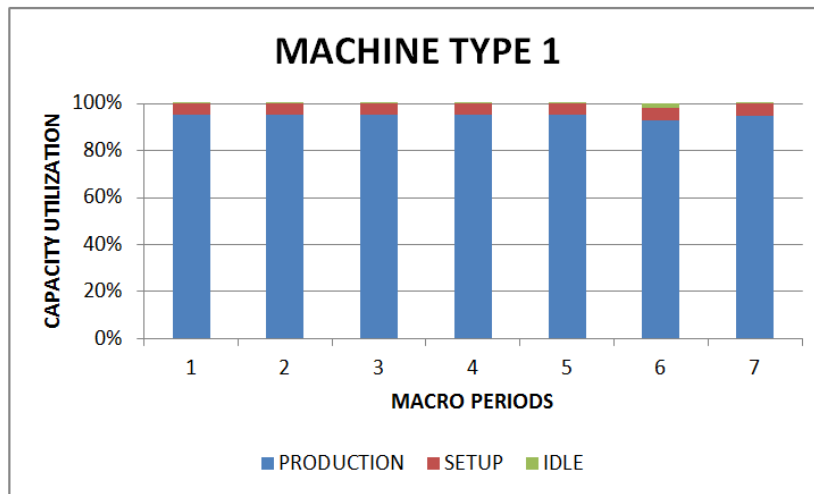
The other possible future research is to make the production plans which consider the inventory balance during the macro periods. The inventories obtained in the developed model should be hold at least for one period to meet the requirements of the next macro periods production demand. The modelling approach which considers the part flow during the macro period has the potential to decrease the inventories and inventory holding costs.

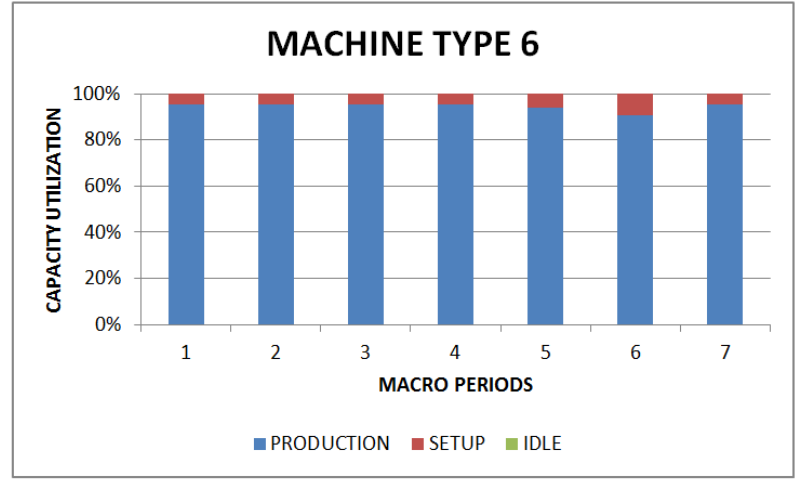
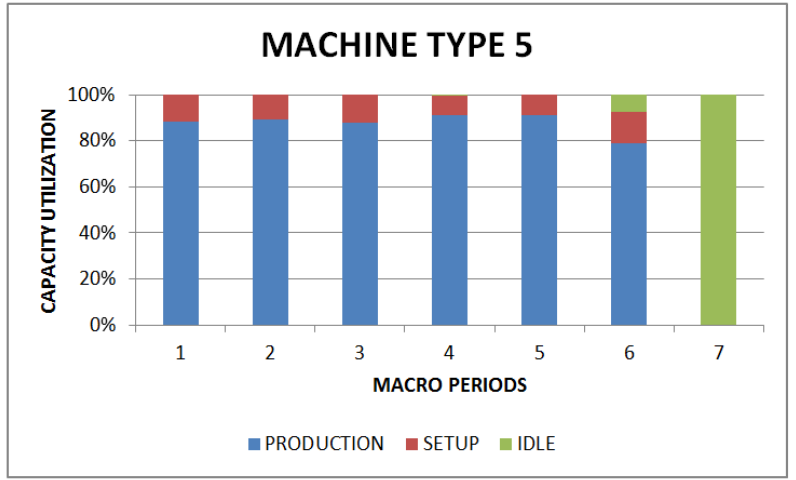
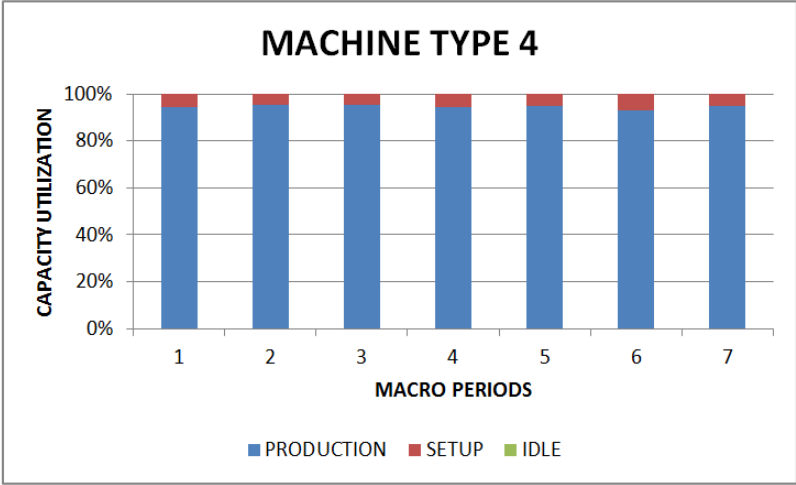
Although the developed deterministic models can give answers to multiple decisions such as shift plans, overtimes, schedules or lot sizes, the real world problems may have deviations in the given sets of parameters to the model. The setup time deviations, machine and tool breakdowns or labor absenteeism are some of the deviations that can affect the efficiency of the deterministic models. Further researches may concentrate on the stochastic, scenario based or robust optimization techniques to minimize the effects of the parameter deviations to the overall performance of the models developed.

To summarize heuristic approaches for large problems, in period inventory flow models and robust models to parameter deviations are the possible future research opportunities in the area.

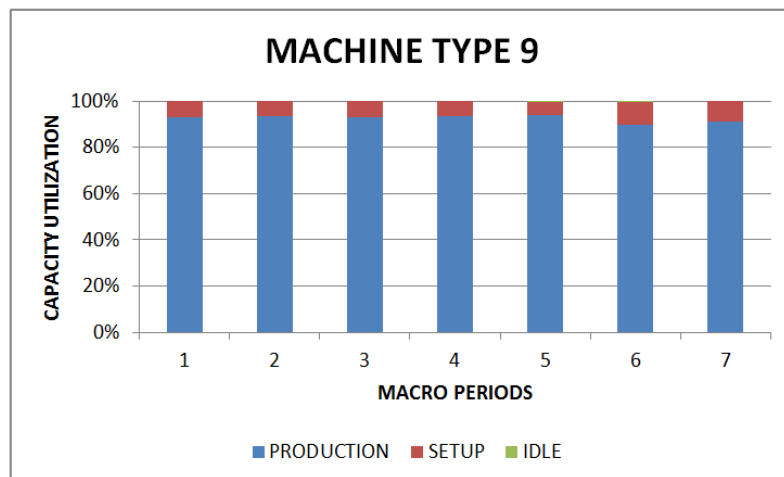
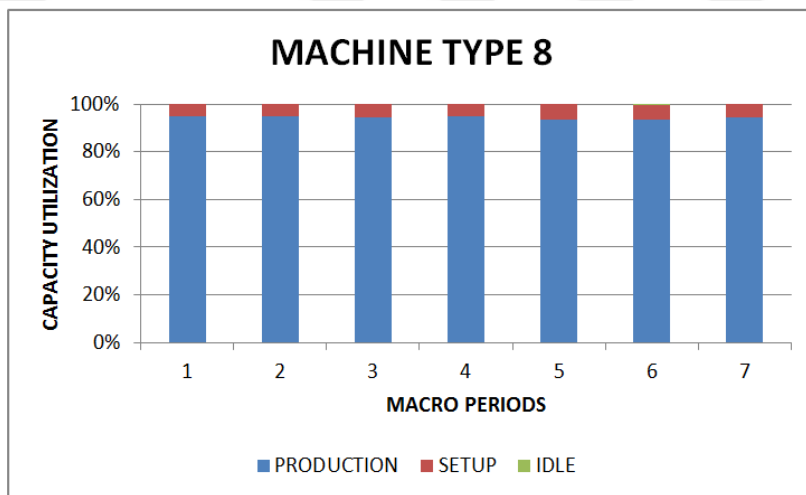
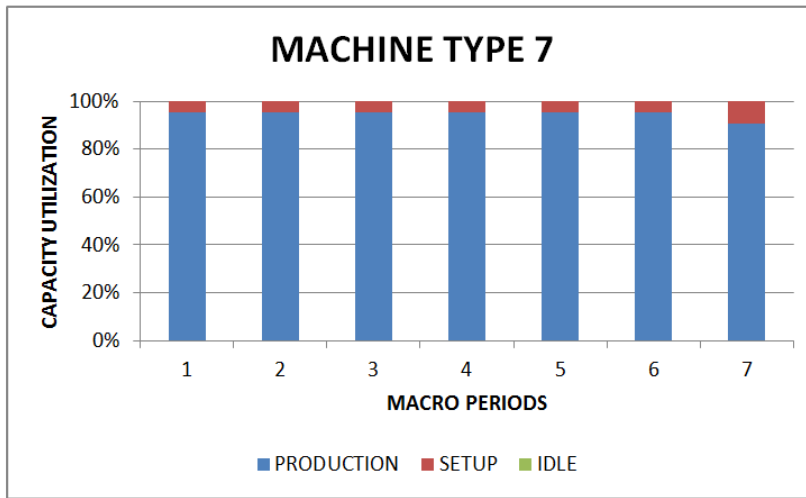


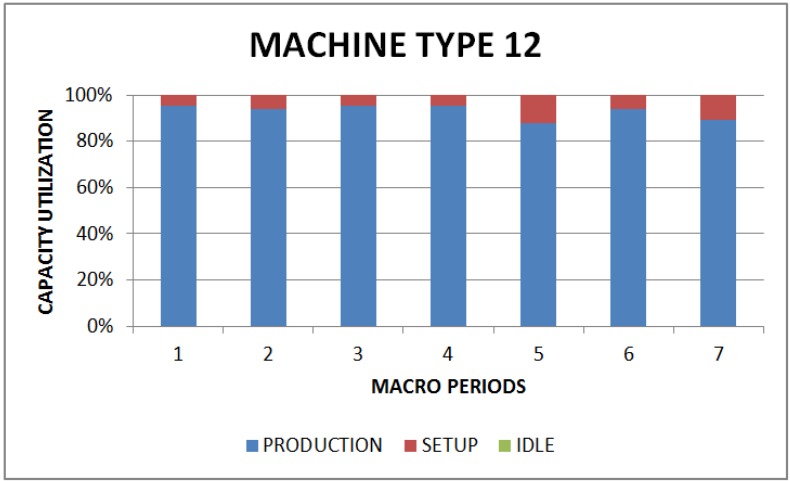
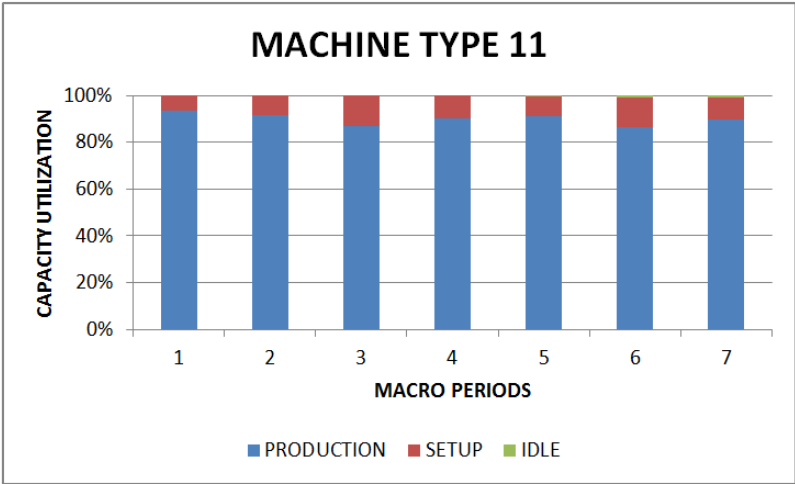
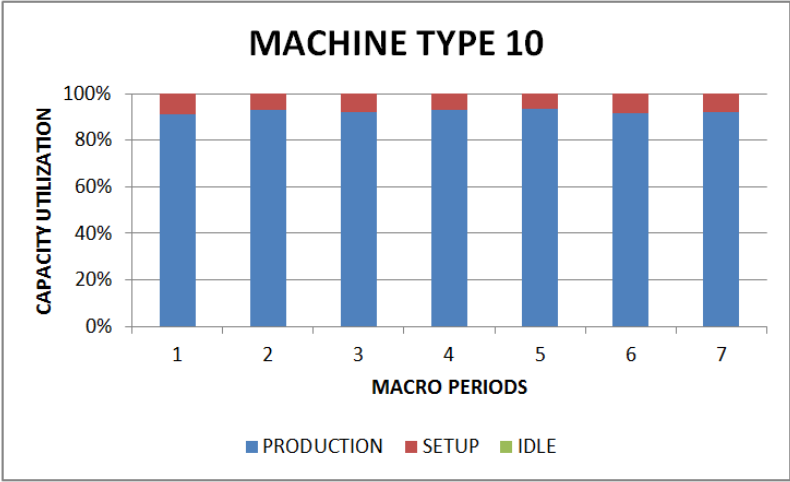
# APPENDIX

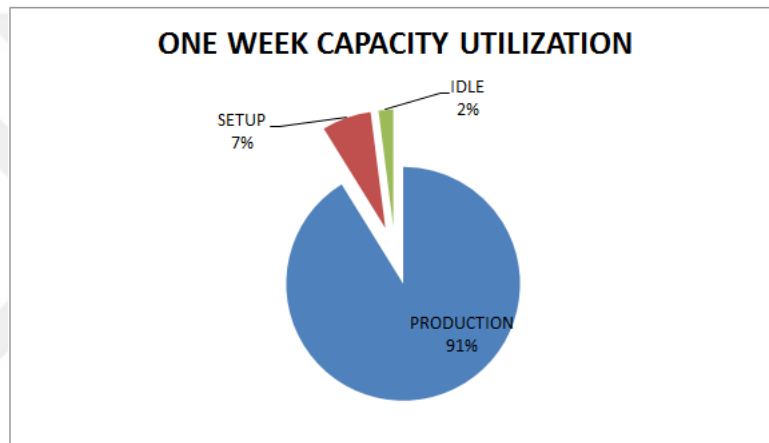
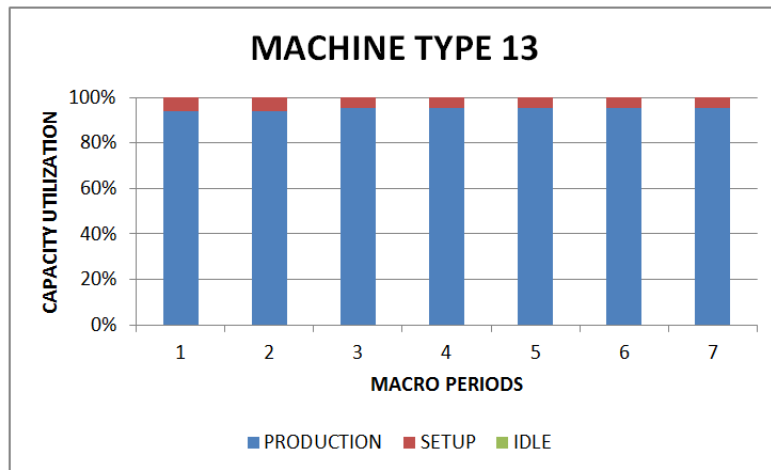












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