

# A MATHEMATICAL MODEL FOR CUSTOMER LIFETIME VALUE BASED OFFER MANAGEMENT

A Thesis

by

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# A MATHEMATICAL MODEL FOR CUSTOMER LIFETIME VALUE BASED OFFER MANAGEMENT

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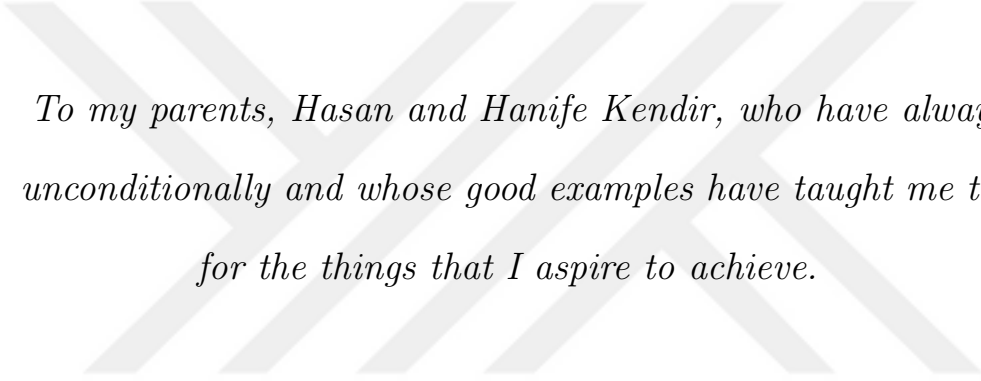
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*To my parents, Hasan and Hanife Kendir, who have always loved me unconditionally and whose good examples have taught me to work hard for the things that I aspire to achieve.*

*This thesis work is also dedicated to my twins Esin Duru and Atahan Doruk and my husband, Selim, who has been a constant source of support and encouragement during the challenges of graduate school and life. I am truly thankful for having them in my life.*

## ABSTRACT

Customers with prepaid lines possess higher attrition risk compared to postpaid customers, since prepaid customers do not sign long-term obligatory contracts and may churn anytime. For this reason, mobile operators have to offer engaging benefits to keep prepaid subscribers with the company. Since all such offers incur additional cost, mobile operators face an optimization problem while selecting the most suitable offers for customers at risk. In this study, an offer management framework targeting prepaid customers of a telecommunication company is developed. Proposed framework chooses the most suitable offer for each customer through a mathematical model, which utilizes customer lifetime value and churn risk. Lifetime values are estimated using logistic regression and Pareto/NBD models.

## ÖZETÇE

Ön-ödemeli hat müşterileri uzun süreli taahhütlü kontrat yapmadıkları ve herhangi bir zamanda operatörden ayrılacakları için faturalı müşterilere göre daha yüksek gitme riskine sahiptirler. Bu sebepten mobil operatörler ön-ödemeli hat müşterilerini ellerinde tutmak için bağlayıcı teklifler sunmalıdır. Müşterilere sunulacak her bir teklifin maliyeti olacağından, mobil operatörler riskli müşterilerine en uygun teklifi seçme aşamasında optimizasyon problemi ile karşı karşıya kalmaktadırlar. Bu çalışmada, ön-ödemeli hat müşterilerine en uygun teklifin hesaplanabilmesi için özel bir teklif yönetim modeli geliştirilmiştir. Yaşam boyu değeri ve ayrılma riski bilgilerinden de faydalanarak önerilen yapı her bir müşteri için en uygun teklifi matematiksel model aracılığıyla seçer. Yaşamboyu değeri Pareto/NBD ve lojistik regresyon modelleri ile tahminlenmiştir.

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# TABLE OF CONTENTS

<b>DEDICATION</b> . . . . .	<b>iii</b>
<b>ABSTRACT</b> . . . . .	<b>iv</b>
<b>ÖZETÇE</b> . . . . .	<b>v</b>
<b>ACKNOWLEDGEMENTS</b> . . . . .	<b>vi</b>
<b>LIST OF TABLES</b> . . . . .	<b>ix</b>
<b>LIST OF FIGURES</b> . . . . .	<b>x</b>
<b>I INTRODUCTION</b> . . . . .	<b>1</b>
1.1 Subscriber Analysis In Mobile Industry . . . . .	1
1.2 Mobile Churn with Numbers in Turkey . . . . .	5
<b>II BACKGROUND</b> . . . . .	<b>7</b>
2.1 CLV . . . . .	8
2.2 Recency Frequency Monetary (RFM) Analysis . . . . .	11
2.3 Pareto/NBD Model Assumptions . . . . .	12
2.4 Logistic Regression . . . . .	16
2.5 Offer Management . . . . .	17
<b>III RELATED WORK</b> . . . . .	<b>21</b>
<b>IV DATA PREPARATION</b> . . . . .	<b>25</b>
4.1 RFM Data . . . . .	25
4.2 Features Selection for the Model . . . . .	27
4.3 Exploratory Data Analysis . . . . .	28
4.4 Benchmarking Pareto/NBD Model with Logistic Regression . . . . .	30
4.5 Pareto/NBD Model & CLV . . . . .	33
4.6 Proposed Mathematical Model for Offer Management . . . . .	34
<b>V CONCLUSION</b> . . . . .	<b>42</b>
<b>Appendices</b> . . . . .	<b>44</b>

APPENDIX A	— PARETO/NBD R CODE . . . . .	45
APPENDIX B	— GAMS CODE FOR MATHEMATICAL MODEL	54
APPENDIX C	— FEATURES FOR LOGISTIC REGRESSION .	56
REFERENCES	. . . . .	66





## LIST OF TABLES

1	CLV Models . . . . .	10
2	A small sample of the RFM Data from Refill Transactions . . . . .	26
3	The Number of Distribution . . . . .	27
4	Basic properties of the selected data sets. . . . .	28
5	Basic statistics of the selected data sets. . . . .	29
6	The distributions of the data sets. . . . .	29
7	Values of the days between transactions. . . . .	29
8	Benchmark of Data with Logistic Regression . . . . .	32
9	Pareto/NBD Model Results . . . . .	34
10	Offers . . . . .	39
11	Offer Model Scenarios . . . . .	40
12	Churn Rate Breakdown . . . . .	40
13	Accepted Offer Distribution . . . . .	40
14	Calculated CLV . . . . .	41

## LIST OF FIGURES

1	Postpaid Prepaid Subscriber Trends in Turkey Published By BTK . . .	2
2	Churn and CLV Relationship . . . . .	3
3	3-Level Offer Management Process . . . . .	4
4	Mobile Subscriber Penetration in Turkey . . . . .	6
5	The Flow For This Study . . . . .	8
6	Linear Regression & Logistic Regression Comparison . . . . .	16
7	Offer Management Cycle . . . . .	18
8	Offer Management Framework . . . . .	19
9	ARPU trend for Postpaid & Prepaid Subscribers . . . . .	20
10	Subscriber Distribution Based on Age . . . . .	28
11	Flow of Benchmarking . . . . .	31
12	Tenure Groups . . . . .	33
13	Pareto/NBD Model Results . . . . .	35
14	Offer Model . . . . .	38

# CHAPTER I

## INTRODUCTION

### *1.1 Subscriber Analysis In Mobile Industry*

Under today's challenging market conditions, competitions become more and more important for companies. The attention of a customer is disturbed by the competitors. Therefore the companies must be proactively analyze their customer behavior based on their CRM and behavioral data and offer the customer the best product or service to keep their attraction. Satisfaction with the product or service improves the loyalty of the customer with the brand. Customer loyalty encourages customer to spend more money with the company's product and services, thus the revenue of the firm grows.

In mobile sector the churn rate of the customers are more dynamic than the other sectors. Especially predicting the behavior of the prepaid subscribers are more difficult than postpaid subscribers. Usually, it is accepted that prepaid subscribers individually generate less revenue than postpaid subscribers, as a result of those, operators mostly focus on the postpaid subscribers. However, in Turkey in last years, proportionally the volume of the prepaid subscribers' number converges to the postpaid subscribers' number so the prepaid revenue cannot be ignored. The trends of the postpaid and prepaid subscribers can be seen in Figure 1. This market data is published quarterly to report the market trends in mobile sector by BTK known as Bilgi Teknolojileri Kurumu which is the Governmental Organization of Information Technologies.

Telecommunication operators have to adapt to digital evolution of the customer services. Smart phones has changed the market conditions. Customers can experience

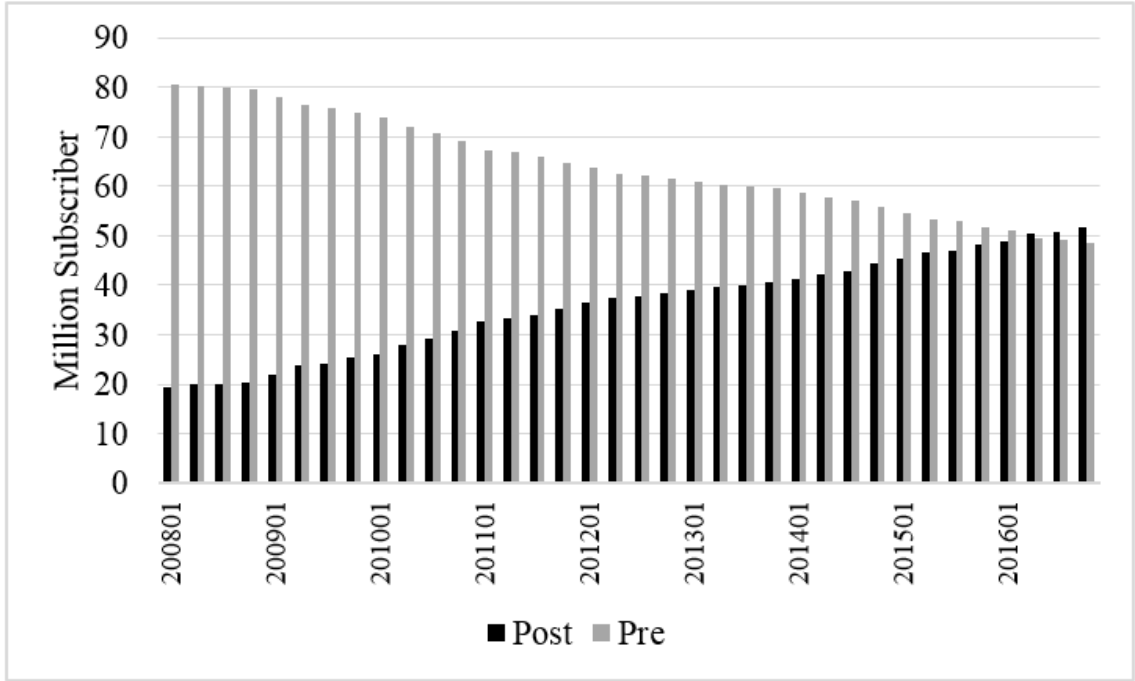


Figure 1: Postpaid Prepaid Subscriber Trends in Turkey Published By BTK

hundreds of digital services and applications, easily, with their smart phones. Therefore, telecommunication operators cannot be solely seen as communication providers anymore. Operators have to provide profitable services to survive in the business and to improve the customer experience to prevent customer churn.

The service and products of the operator should comply with the customer’s needs. If this is not the case, it is highly probable that customers may leave and never come back or may never subscribe for a service of the operator. Customer experience management starts with potential customers perception about the company brand. Starting with first impression, customer experience management aims to understand the desires of customers and to provide them with easy, simple and seamless experience with the offered products and services. The higher the quality in customer experience, the more customer will be attracted to and stay with the company. Retaining customers has always been a costly challenge. Companies have developed different strategies to fight against customer attrition. Most of these efforts include

proposing offers in varying forms, such as discounts, extra benefits etc. Almost in all of these efforts, increasing the amount of customer specific information pays off. In the digitalization era, the amount of customer specific data is more than ever, which yields numerous opportunities for developing granular and customized prediction models. When customer retention is considered, the challenge is turning this massive amount of data (and models built on the data) into business insights and eventually generate actions that decrease the retention cost, while increasing profit.

Churn analysis is one of the most fundamental customer behavior analysis in identifying silent, unhappy customers. The main output of churn analysis is a risk score, which quantifies the likelihood of customer churn. The output of churn analysis is best utilized when it is supported with extra customer specific features such as customer lifetime value (CLV). The relationship between churn and CLV is depicted in Figure 2. which is based on the idea in the [1]

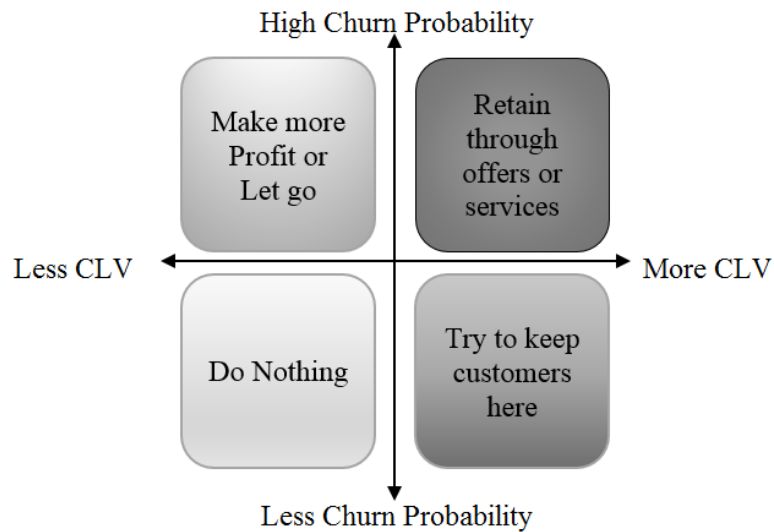


Figure 2: Churn and CLV Relationship

As can be seen in Figure 2, the ideal position of customers for a company is the bottom right, where customers produce the highest profit with lowest churn risk. It is recommended that companies should give special attention (provide offers or new exciting services) to the customers falling in the upper right region. Since, if retained

in the company, these are the high profit promising customers. Bottom left and upper left regions are the regions, where less profitable customers reside. If possible, companies should try to increase profit made out of these customers. On the other hand, if the customer is not promising higher profit and possessing high churn risk, companies may simply let these customers go and try to acquire new customers, who fall into high CLV, low churn risk region.

Companies usually identify risky customers according to the lift of their favorite churn-prediction model. In a typical setting, they try to give the best offer to the customers possessing highest risk. In this scenario, managing the campaign is simple but its efficiency (in terms of cost) is questionable. Because, in such an execution, decision maker does not know whether offer receiving customers are worth the effort or not. On the other hand, integrating CLVs into the offer management process helps to increase return of the campaign by targeting, and possibly retaining, customers with higher CLV.

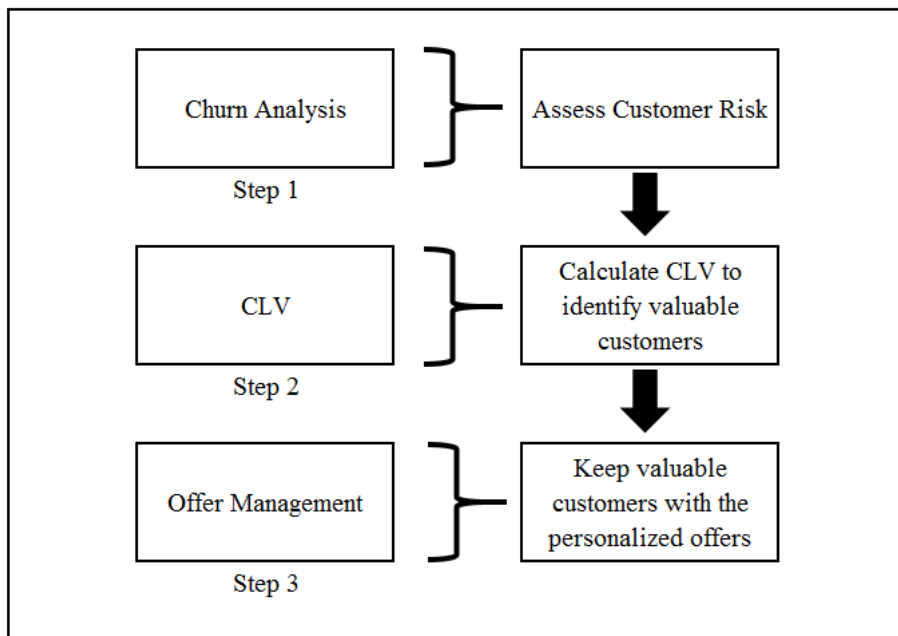


Figure 3: 3-Level Offer Management Process

Figure 3. shows our proposed 3-level offer management framework. The first step

calculates churn probability of customers, and the second step estimates CLVs. Without CLV calculation an operator churn management may cost higher than included with CLV calculation.

In study [2] the focus is on prepaid subscribers of a mobile operator. Mobile operators would prefer having postpaid subscribers more than prepaid subscribers. Because it is easier to manage the postpaid subscribers behavior and the revenue generation is more predictable than the prepaid subscribers. But in today's digital marketing ecosystem telecommunication operators have to act not only a call service provider (CSP), but also act like a full service provider (OTT Over the top provider) who offers many interactive social service products for their customers. Thus, keeping a customer is more valuable than before which means more users for the OTT services. Thus analyzing the churn of a customer must cover the behavioral effects of the people profile like their segment and tenure of the customer. Segment is the grouping of customers which gives indication about the behavior of the customer like his spending behavior or the age group of the customer. Beside segment customer tenure can also be used to classify customer behavior. Tenure can be defined as the age of the customer which gives the customer aliveness duration in time for the company.

The aim of the study is to analyze the churn behavior of the people according to their experience both with the operator and their social position. Therefore, we include both the segment and the tenure information of the customer in our data set.

## ***1.2 Mobile Churn with Numbers in Turkey***

Every year, millions of mobile subscribers churn from their mobile operator to the competitor operator. According to the 2017 2nd quarter report of the BTK (Information Technologies Organization), there is 76.6 million mobile subscribers in Turkey and mobile equipment penetration is 108% excluding the 0-9 ages population that can be seen in Figure 4 [3].

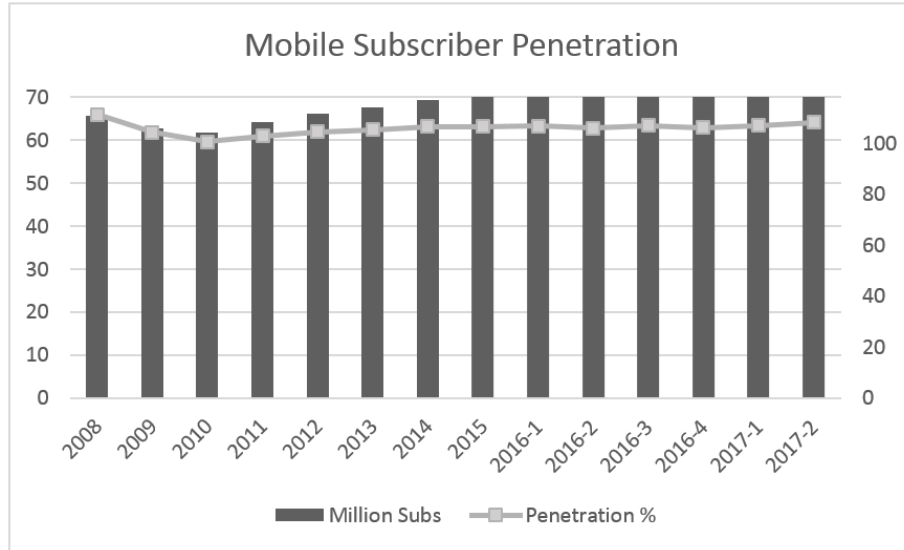


Figure 4: Mobile Subscriber Penetration in Turkey

The numbers in the same report also represented that there were 2.270.425 number portability transactions with 2.28% decrease comparing to the previous 3 months period. The number of the transactions were 106.540.208 at total in the period from the beginning of the number portability in November of 2008 to May of 2015.

In the 2nd quarter of the 2017, there was an increase in the subscriber numbers of Vodafone, Avea and Turkcell by 0.36%, 0.27% and 2.27% respectively.

The ARPU amount for Turkcell, Vodafone and Avea is 28.5, 28.2 and 27.9 respectively. Simply, if the operators lose their subscribers, for example, 100.000 subscriber would change their operator in a month, the revenue of Turkcell, Vodafone and Avea would decrease 285.000, 282.000 and 279.000 in amount.

Moreover, SAC (Subscriber Acquisition Cost) is 35.5 TL according to the Turkcell 2015, 1st Quarter IFRS Interim Report. Keeping the existing customers, will prevent the mobile operators to lose revenue [4].



## CHAPTER II

### BACKGROUND

Prepaid subscribers have to first make top-up before making calls. Usage behavior of prepaid subscribers differs from each other. The credit is purchased at any time whenever the subscriber decides to. In Turkish mobile market if the mobile subscribers does not make any top-up, the line contract is terminated by the mobile operator. The period for the cancellation of the subscriber line contract is 270-days after the last purchase. However, the operator never knows when the subscriber would do the last credit purchase. Hence, the subscriber behavior can be easily defined with only recency/frequency data and Pareto/NBD model can give the probability of a customer being alive and the expected number of transactions for a customer. Based on these valuable data a mobile operator can calculate the Customer Life Time Value and monetary value of a customer [5].

The main goal of this study is proposing to the subscribers the most appropriate offers by using the results of a defined mathematical model which uses customer life time value of each customer and their probability of churn as the result of the Pareto/NBD model.

The Pareto/NBD model is mostly preferable for sales transactions which have discrete purchase cycle among customers and non-contractual relationship with the company. The Pareto/NBD model and RFM data are studied in detail in the literature. [5], [6]. However, in this study, a benchmark with logistic regression is also included to show the power of the model and data.

The flow for this thesis is simply illustrated in the following figure 5.



Figure 5: The Flow For This Study

## 2.1 *CLV*

### 2.1.1 CLV Basics

CLV can be simply described as net the present value of the future cash flows associated with a customer [7]. The money has a time value which is represented by discount (interest) rate. In other words, future value of a money has a present value with a discount (interest) rate. This can be explained in the proceeding simple example. How much does one invest to get 100\$ amount money at the end of each year for 3 years? It can be easily seen that the value of the money is not 100\$. There has to be a rate between present and the future value. The present value of the money can be calculated if we accept the rate 5%;

$$TodayPrice = \frac{100}{(1 + 0.05)} + \frac{100}{(1 + 0.05)^2} + \frac{100}{(1 + 0.05)^3} \quad (1)$$

This definition can be generalized by the following equation

$$PV = \sum_{t=0}^n \frac{FV_t}{(1 + r)^t} \quad (2)$$

- PV = Present Value
- FV = Future Value
- r = Discount Rate
- t = Time

This formula is the basis for the CLV calculation. The basic CLV calculation can be given in the basic formula below:

$$CLV = CM_i \left( \frac{R_r}{1 + \delta - R_r} \right) - AC_i \quad (3)$$

Where

- $CM_i$  = Customer Contribution Margin;  $CM_i = Rv_i - VC_i$  Where  $Rv_i$  = Customer Revenue and  $VC_i$  = Customer Variable Cost
- $R_i$  = Retention Rate for Customers
- $(\delta)$  = Discount Rate (Cost of capital)
- $AC_i$  = Customer Acquisition Cost

In detailed, retention rate is the proportion of customers who stay with the company. It can be simply calculated by the following formula;

$$CustomerRetentionsRate = ((E - N)/S) * 100$$

Where

- E : Number of customer at the end of a period.
- N : Number of new customers acquired during that period.
- S : Number of customers at the start of that period.

On the other hand, customer acquisition cost in mobile industry which is also called subscriber acquisition cost (SAC) simply means the price of acquiring a new customer for the first time. In mobile industry it includes typically the cost of sales and marketing, and handset subsidies and simcard costs.

CLV can be used to calculate the value of the customers of the company and allows company to manage their total costs for their customer relationships.

### 2.1.2 CLV In This Study

If a mobile operator wants a profitable customer management they have to calculate the CLV of a subscriber [1]. Loyal customers can mean more profit for the company. They tend to use more product and service of the company and can spread positive word of mouth about the brand if they feel satisfied with the product and/or services. The studies on customer life time value in literature can be classified in four main categories as in Table 1 from [8];

Table 1: CLV Models

Focus	Category	Model
Structural Model	Customer Unit	Individual model
		Segment (Customer base) model
	Prediction data	Retrospective model
		Prospective model
	Transaction	Contractual model
		Non-Contractual model
	Purchase cycle	Discrete model
		Continuous model
Strategic Model	Strategic use of CLV in management	
Normative Model	Relationship between Duration and cost	
Analytic Model	Resource allocation (Budget allocation) / Pricing	

The structural model is mostly focused on calculation the CLV based on the customer and customer groups. It is divided into 8 sub models. The CLV can be calculated based on each individual customer or a customer group can be selected to calculate the CLV of this segment. For the data set, historic transaction data or predicted future data can be used for the calculation. In mobile industry prepaid subscribers sign a subscription contract with the operator, but they do not promise a regular payment to the mobile operator. They can do top-up action whenever they want and the mobile operator does not know when the customer will defect. This model is defined as non-contractual model in CL calculation. The top-up action of the subscriber is also discrete action. There is maximum time limit for this top-up action,

but within this duration subscriber can do anytime the purchase action. In this study we focused on both Non-Contractual model and Discrete model for purchase cycle which is based on Pareto/NBD model. In [5], the assumption is that monetary value is independent of the underlying transaction process. Two CLV calculation is used in the offer management model. One of them is CLV base which is calculated by the multiplication of the expected transaction count and the last refill amount of the subscriber :

$$CLV_{base} = (R)X(T) \quad (4)$$

where;

- R : Revenue of Refill amount
- T : Expected transaction count from the Pareto/NBD model

After accepting the offer, modified CLV is calculated which is;

$$CLV_{Modified} = (R)X(D)X(P) \quad (5)$$

where;

- R : Revenue of Refill amount
- D : Discount rate for offer
- P : Probability of aliveness

## ***2.2 Recency Frequency Monetary (RFM) Analysis***

RFM data includes the transactional data of the customers. Recency can be calculated by the date of the customers' last transaction data, frequency can be calculated by the count of the customers' transaction which fall into between the first transaction date and last transaction date of the subscribers' lifecycle and the monetary

value gives the transaction amount of the subscriber. This is the simplest way which can be used to define the non-contracted customers' behavior.

The RFM values can be also used to segment customers to identify the customers who have the highest probability to respond to campaigns. By using the RFM calculation, each subscriber would have a value score assigned to them.

The score is calculated first by dividing the customers in quintiles then the recency, the frequency and the monetary value of the customers are scored beginning from 5 to 1 in descending order. The customer who has the most recent value is scored with 5 then the less recent customers are scored with the following numbers 4,3,2,1. The same method than applied to the frequency and the monetary values of each customer. At the end, each customer has a score from 555 to 111. Totally, there could be 125 buckets in which the customers are segmented. The customers in the RFM bucket which is 555, could have the highest probability to respond the campaigns [9].

### ***2.3 Pareto/NBD Model Assumptions***

The Pareto/NBD model is defined by SMC (Schmittlein, Morrison, and Colombo) to model the repeat purchase for a non-contractual customers [6]. SMC states the model has several assumptions regarding customers: Individual customer;

- Poisson Purchases: While alive, each customer makes purchases according to a Poisson process with rate  $\lambda$ .
- Exponential Lifetime: Each customer remains alive for a lifetime which has an exponentially distributed duration with death rate  $\mu$ .

Heterogeneity across customers;

- Individuals' purchasing rates are distributed following Gamma distribution with rate  $\lambda$ .

- Customers is distributed according to a gamma distribution across the population of customers (NBD distribution).
- Death rates also follow a Gamma distribution with rate  $\mu$ , and customers have different gamma distribution across (Pareto distribution).
- Rates  $\lambda$  and  $\mu$  are independent: The purchasing rates  $\lambda$  and the death rates  $\mu$  are distributed independently of each other.

By using these distributions on the basic RFM data, SMC derived expressions for [10]

- The probability of a customer is still alive,
- The expected number of future transaction for a customer.

RFM data as a base data for Pareto/NBD model can be used by the mobile operator to offer their products or services after calculating the CLV and using this value as an input to a mathematical offer model. The Pareto/NBD model is defined by SMC (Schmittlein, Morrison, and Colombo) to define the repeat purchase for a non-contractual customers [6]. SMC states the model assumptions as below:

Individual customer;

1. Poisson Purchases: While alive, each customer makes purchases according to a Poisson process with rate  $\lambda$ .

$$P(X(t) = x|\lambda) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}, x = 0, 1, 2, \dots \quad (6)$$

This is equivalent to assuming that the time between transactions is distributed exponential with transaction rate  $\lambda$ ,

$$f(t_j - t_{j-1}|\lambda) = \lambda e^{-\lambda(t_j - t_{j-1})}, t_j > t_{j-1} > 0 \quad (7)$$

2. Exponential Lifetime: Each customer remains alive for a lifetime which has an exponentially distributed duration with death rate  $\mu$

$$f(\tau|\mu) = \mu e^{-\mu\tau} \quad (8)$$

Heterogeneity across customers;

3. Heterogeneity in transaction rates across customers follows a gamma distribution with shape parameter  $r$  and scale parameter  $\alpha$ .

$$g(\lambda|r, \alpha) = \frac{\alpha^r \lambda^{r-1} e^{-\lambda\alpha}}{\Gamma(r)} \quad (9)$$

4. Heterogeneity in dropout rates across customers follows a gamma distribution with shape parameter  $s$  and scale parameter  $\beta$ .

$$g(\mu|s, \beta) = \frac{\beta^s \mu^{s-1} e^{-\mu\beta}}{\Gamma(s)} \quad (10)$$

5. The transaction rate  $\lambda$  and the dropout rate  $\mu$  vary independently across customers
6. Death Rates Distributed Gamma: The customer's death rates  $\mu$  are distributed according to a different gamma distribution across customers. (Pareto distribution).
7. Rates  $\lambda$  and  $\mu$  are Independent: The purchasing rates  $\lambda$  and the death rates  $\mu$  are distributed independently of each other.

Assumptions 1 and 3 gives the NBD model;

$$P(X(t) = x|r, \alpha) = \frac{\Gamma(r+x)}{\Gamma(x)x!} \left(\frac{\alpha}{\alpha+t}\right)^r \left(\frac{t}{\alpha+t}\right)^x \quad (11)$$

2 and 4 gives Pareto model;

$$f(\tau|s, \beta) = \frac{s}{\beta} \left(\frac{\beta}{\beta+\tau}\right)^{s+1} \quad (12)$$



$$F(\tau|s, \beta) = 1 - \left( \frac{\beta}{\beta + \tau} \right)^s \quad (13)$$

Likelihood function for a randomly-chosen individual with purchase history

$$(X, t_x, T)$$

:

$$L(r, \alpha, s, \beta|x, t_x, T) = \frac{\Gamma(r+x)\alpha^r\beta^s}{\Gamma(r)} \left\{ \frac{1}{(\alpha+T)^{r+x}(\beta+T)^s} + \left( \frac{s}{r+s+x} \right) A_0 \right\} \quad (14)$$

where  $\alpha \geq \beta$

$$A_0 = \frac{F_1(r+s+x, s+1; r+s+x+1; \frac{\alpha-\beta}{\alpha+t_x})}{(\alpha+t_x)^{r+s+x}} - \frac{F_1(r+s+x, s+1; r+s+x+1; \frac{\alpha-\beta}{\alpha+T})}{(\alpha+T)^{r+s+x}} \quad (15)$$

where  $\alpha \leq \beta$

$$A_0 = \frac{F_1(r+s+x, s+1; r+s+x+1; \frac{\beta-\alpha}{\beta+t_x})}{(\beta+t_x)^{r+s+x}} - \frac{F_1(r+s+x, s+1; r+s+x+1; \frac{\beta-\alpha}{\beta+T})}{(\beta+T)^{r+s+x}} \quad (16)$$

The four Pareto/NBD model parameters  $(r, \alpha, s, \beta)$  can be estimated via the method of maximum likelihood. Suppose we have a sample of  $N$  customers, where customer  $i$  had  $x_i$  transactions in the period  $(0, T_i]$ , with the last transaction occurring at  $t_{x_i}$ . The sample log-likelihood function is given by;

$$LL(r, \alpha, s, \beta) = \sum_{i=1}^N \ln[L(r, \alpha, s, \beta|x_i, t_{x_i}, T_i)] \quad (17)$$

This can be maximized using standard numerical optimization routines [11]. With the derivations given above the aliveness probability of a customer can be obtained from the model in the following equation;

$$P(\text{alive}|r, \alpha, s, \beta, x, t_x, T) = \left\{ 1 + \left( \frac{s}{r+s+x} \right) (\alpha+T)^{r+x}(\beta+T)^s A_0 \right\} \quad (18)$$

$A_0$  is given in the previous equations (15) and (16).

## 2.4 Logistic Regression

Logistic regression measures the relationship between categorical dependent variable and the independent variables. The independent variables can be one or more. The function which is used to calculate the probability of the relation between the dependent and independent variables is logistic function. Logistic regression is one of the most powerful methods to calculate the probability of an event.

In this paper logistic regression is used to benchmark the Pareto/NBD model. First logistic regression is applied to the based RFM data and then applied to other calculated variables which will be explained in the following section.

Logistic regression model is a prediction between a predictor variable (independent variable) and response variables (dependent variables). The difference from the linear regression is the predictor is transformed using a non-linear function called the logistic function. It is simply non-linear transformation of linear regression. It is illustrated by the following simple graph Figure 6.

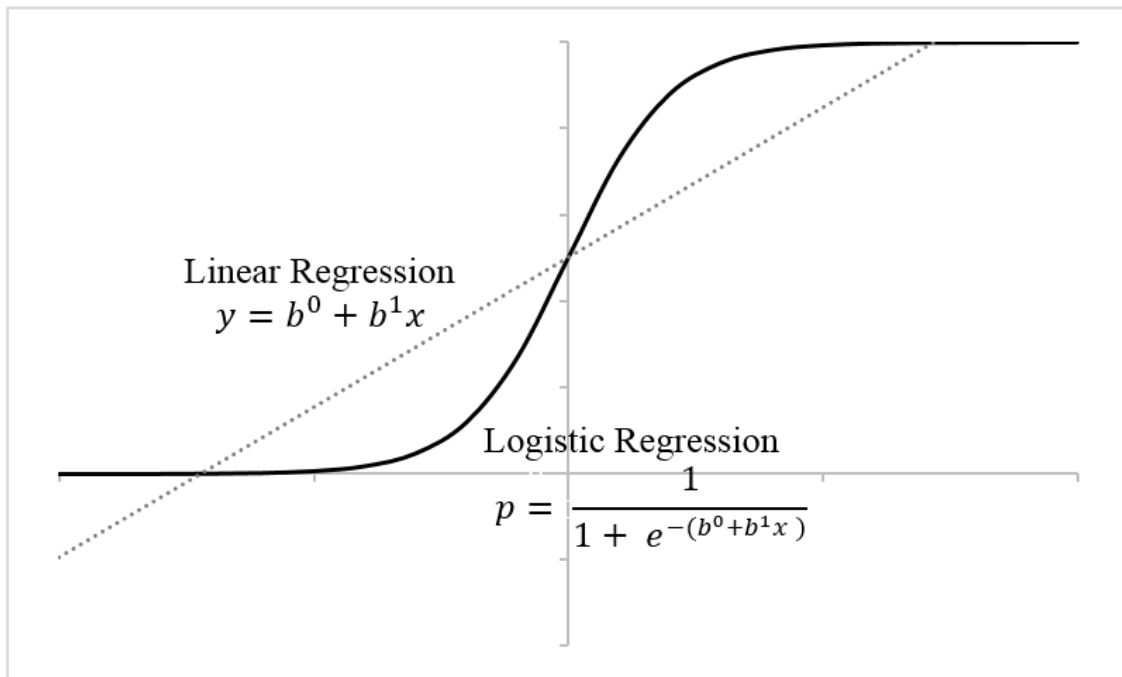


Figure 6: Linear Regression & Logistic Regression Comparison

There are three types of logistic regression.

- Binary Logistic Regression : Used when the response is binary like in our model if the subscriber has churned or is still active.
- Nominal Logistic Regression : Used when there are nominal categories and there is no order between these categories.
- Ordinal Logistic Regression : Used when there are ordinal categories and the categories can be sorted.

Binary logistic regression is mostly used with two outcomes scenarios like in predicting churn of subscribers. Unlike linear regression a link function is used to predict the related outcomes.

$$t = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \epsilon \quad (19)$$

The link function for the logistic regression;

$$t = g(\theta) = \log \left( \frac{\theta}{1 - \theta} \right) \quad (20)$$

Inverse of link function;

$$\theta = \frac{e^t}{(1 + e^t)} \quad (21)$$

The link function transforms the probabilities (between 0 and 1) into regression scores (between  $-\infty$  and  $+\infty$ )

## ***2.5 Offer Management***

In the first years of mobile communication, there was very limited number customer services that was only voice and short message services(SMS) were available. As mobile phone technology evolved, GSM service providers could not only provide voice or SMS services. They have to adapt to the rapidly changing technology more than other industries. Because the subscribers can easily leave the mobile operator, if the

mobile operator does not propose a value to the customer. Therefore during the years they generated many product and services for their subscribers to keep them loyal with their brand. The mobile operators created their own product lines in a very broad perspective. One can easily describe each of the products as offer.

Offers includes marketing activities to have the attraction of the customers over any kind of channel. Campaign management is to contact with the right customers at the right time over the right channel and with the right offer Figure 7.

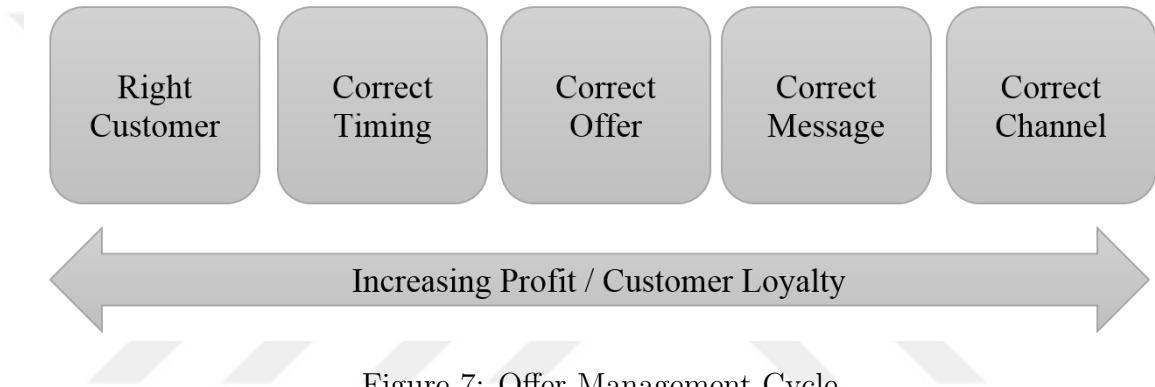


Figure 7: Offer Management Cycle

If a mobile operator can achieve a good campaign management, then has the probability to increase their revenue and also decrease their churn rate. We said that a mobile operator could not do anymore only churn analysis. It has to be adapted in a campaign management system. We can define an offer management framework for a mobile operator like below Figure 8;

The Average Revenue per User in Turkish Lira (TL) (ARPU-TL) and subscriber trend is shown in Figure 5 and Figure 6, respectively. This market data is published quarterly to report the market trends in mobile sector in Turkey by Bilgi Teknolojileri Kurumu (BTK) which is the Governmental Organization of Information Technologies. It can be seen that ARPU of the postpaid subscribers is higher than the prepaid subscribers and the time shows that the number of the postpaid and prepaid subscribers converge to each other. It is also known that keeping a subscriber is cheaper than acquiring a new customer. If a mobile operator calculate the CLV of the prepaid

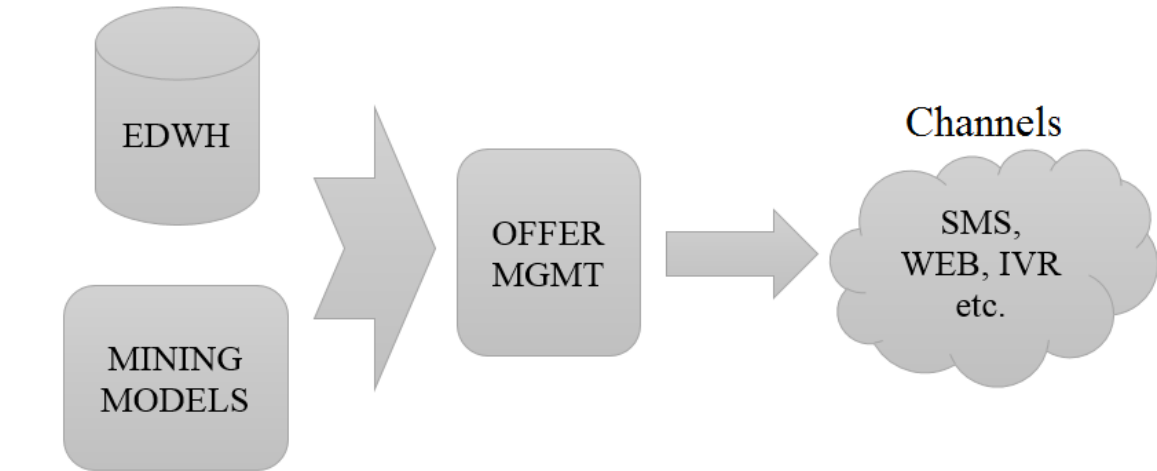


Figure 8: Offer Management Framework

subscriber and propose them the right offer than it is obvious that the operator can increase their revenue Figure 9.

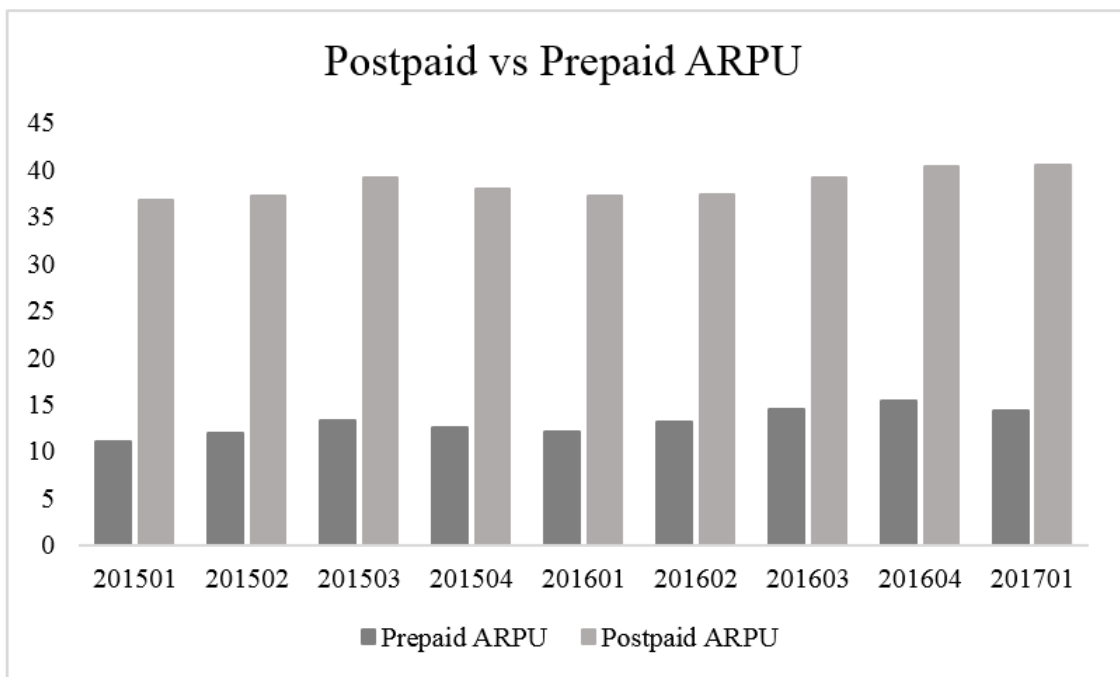


Figure 9: ARPU trend for Postpaid & Prepaid Subscribers

## CHAPTER III

### RELATED WORK

The activation of the prepaid subscribers can be defined as non-contractual process. They stay alive while continuing to purchase from the mobile operator. There are some purchasing actions that resets their 270-day period to zero day and the subscriber continues to generate revenue. Otherwise, they become inactive at the end of the 270-day period.

The churn rate of the prepaid subscribers is three times higher than the postpaid subscribers. In section 1.2, the penetration rate in Turkey Mobile Market has been given. It can be seen that the penetration rate is higher than the population which %108 at the end of the 2017-Q2. Therefore, increasing the customer loyalty or offering new product services to the subscribers will give the control to the mobile operator rather than the customer. As a result, the mobile operator can increase their revenue.

There are a lot of modelling works with postpaid subscribers in mobile sector. However, defining and managing prepaid subscribers' future value is harder than postpaid ones because of their unobserved behavioral format.

There is a publication about this behavior of prepaid subscribers [12]. They use decision tree algorithm to segment the prepaid subscribers to define who are going to churn based on CDR and SIM data.

There is a similar work as mentioned above. The data set in this study [13] includes prepaid subscribers model variables. The churn prediction is done by using the logistic regression, linear regression and Fisher linear discriminant analysis and decision trees. There are other studies for mobile operator churn prediction. However, most of them does not directly focus on prepaid subscribers' behavior [14]. In this

study, specifically churn prediction is studied based on CDR data without specifying any customer base. They work on feature selection methods and supervised learning algorithms to predict the churn score for the subscribers. [15], [16], [17] are again the studies that focus on postpaid subscribers churn propensities. Another paper [18] again analysis the churn prediction but does not define a customer base.

As stated at the beginning, prepaid subscribers can be accepted non-contractual subscribers. In marketing non-contractual based data can be widely analyzed with RFM data [9], [19]

In this paper [20], RFM analysis, logistic regression and decision trees are used to compare with each other based on the accuracy of the data.

Some other publications also focused on the performance of the churn prediction models. [21] applies many data mining methods to a mobile operator data. But the main focus is to improve the model accuracy. [22] used hybrid churn prediction model for prepaid subscribers and compared the accuracy of the data mining models. [23] proposed a hybrid model for churn prediction.

Although there are many churn prediction publications, there are a few with mobile prepaid subscribers data. Most of them focuses on the performance improvement of the data mining methods. In this work, the power of RFM data is used to predict prepaid subscribers behavior.

This study uses refill history of prepaid subscribers which can be accepted as a purchase order thus can be defined easily with RFM data. RFM data is a simple but a strong data to analyze valuable customers. RFM data is reviewed in the study [24]. In this paper it is emphasized that there are a lot of advantage especially to use this data which can boost company revenue in a short term despite there are also disadvantages some of which are;

- It only focuses on best customers. If customer do not buy often or spend little or do not generate any transaction lately, it provides little information about



RFM scoring.

- It only focuses on limited variables. It is better to take into account other customer relation variables when using RFM data.

Although some of the disadvantage of the RFM data, the simplicity of the data variables and the preparation of the data makes it still a powerful and useful data. Specifically for mobile company. The RFM data can find for itself many kinds of applicable areas [25]. Furthermore RFM data can be used for calculating CLV of customers. The calculation of customer based CLV is not a common study area for a mobile operator company. There are a lot of churn analysis in the literature but mostly, they do not directly focus on customer value. Only doing churn analysis does not give the right customers all the times. Because there are millions of subscribers, a single subscriber based small cost can produce a high cost for the company if you apply your analysis results in a thousands of subscribers. Therefore a mobile operator has to improve their churn analysis with the power of the CLV calculation. There are some studies which covers CLV calculation for telco industry. But the customer base is not prepaid mobile subscribers. The study [8] deals with wireless telecommunication subscribers and calculate the CLV based on a new approach of Markov chain model. The study [26] is another study on land-line churn prediction. The study [27] is focused on contracted telecommunication services (land-line or mobile phone or internet line). Some hypothesis are defined in terms of validation and calculation of the CLV and customer equity values. CLV calculation cannot be ignored in today's challenging marketing. In the big data era, making descriptive analysis on historical data is not enough for improving customer loyalty. The competition conditions in any market compelled the companies to analyze the future behavior of their customer to improve the customer experience with their product. Thus, the CLV calculation is a must in today's market. Especially, like mobile market where the prepaid subscribers are dominated, the mobile companies have to take care on their prepaid subscribers

which has more complicated behavior than the postpaid subscribers.

In this study we also use the CLV for the best offer calculation for the subscriber to retain them with the company. One can base their best offer calculation just on the churn probabilities of the customers. In this case, the model false positives rates becomes very important because if you offered wrong customer the churn offer, you would probably wasting your budget on wrong customers which would not be leave your company. The study [28] is about cost-sensitive approach of offer management for churners.

The RFM data was extended by [5] for the CLV calculation. As stated in the paper, the study focused on using only the RFM data for the CLV calculation in customer base. The customer base data is the transactions for the non-contractual setting. This CLV calculation based on RFM data can be easily adopted to prepaid subscribers in mobile sector. However, there is no application of this method for mobile prepaid subscribers, we can give some other applications of CLV calculation based on RFM data. This method is used widely in many industries, there are many case studies like retail in these papers [29], [30].

In [5], the Pareto/NBD model is used. Pareto/NBD model is used to define the non-contractual purchase transactions for customers who you will never know when they leave your brand. There are many other models that can be used instead of Pareto/NBD model. In [30] a comparison for different models are made.

Offer management is also another aspect of predicting churn of subscribers. A mobile operator have to manage their costs for their customer loyalty. In [1], it is emphasized that companies cannot just only focus on profitable customer but also finding the loyal customers is more important than profitable customers. It is also important that companies has to solve the optimization problems for the promotions and calculate the maximum profit from their offer campaigns [31].

## CHAPTER IV

### DATA PREPARATION

In offer management model, the results are used from the base model which is Pareto/NBD. As stated in the thesis flow in Section 2, a benchmark is done for the RFM data and the extra subscriber features data. For this reason two data set is prepared for ;

1. RFM data
2. Features for the logistic regression

#### ***4.1 RFM Data***

Pareto-NBD model uses RFM data. In this thesis, we analyze the active mobile prepaid subscribers. The data is provided by one of the mobile operators in Turkey. If prepaid mobile subscribers do not make any top-up, their contract will end in a defined period of time, 270 days in Turkish market. But you never know when they will stop making top-up so when they will end their contract. In this thesis the RFM data set includes prepaid mobile subscribers who make their first activation with the prepaid charging method and do not change their charging method in the selected time interval. The selected time interval is 2 year period from 1st of February, 2015 to 31th of January, 2017.

The number of distinct subscribers is 386K. The refill amount and refill count is calculated based on the refill transactions. The number of transactions is 2.7M. The subscribers with no transactions are eliminated from the base subscriber set after elimination the observer subscribers with refill transactions are 327K. The refill

Table 2: A small sample of the RFM Data from Refill Transactions

Subscriber Id	Refill Date	Refill Amount (TL)
132047392	20160812	15
132047392	20160901	20
...	...	...
132054290	20150328	25
132054290	20150416	12

transactions for the same subscriber on the same date are merged into one record so the 2.700.349 record has become 2.578.681 distinct transactions.

For the simplicity of the model, not all the transactions are fed into the PARETO/NBD model, the data is split into 50 buckets with ORA-HASH function. The size of the one of the bucket has approximately 55K refill transaction and the distinct subscriber is 6500. The model is run with 6 sets of this data. Each transaction contains a Subscriber Id which uniquely defines the customers, refill date the date of the transaction, refill amount the amount of refill transaction in TL in other words the monetary value of the refill transaction. The data is order by subscriber id and refill date in ascending. A small sample is presented in Table 2.

Prepaid subscribers do refill action when they decide. The lifecycle period for the prepaid subscriber is 270-days. If they do not do any refill action, after 270-days the subscription will be ended by the mobile operator. But this period will be reset by refill transaction and another 270-days new period starts. The total subscriber used in this study is 6480 which were activated two year period starting from 1.2.2015 to 31.1.2017. Refill transactions are also for the same period for the activated subscribers. The subscribers are chosen among prepaid subscribers who did not do any charging method change and remains prepaid subscriber for their lifetime.

In this thesis we also add segment information of the subscribers to the subscriber set. We want to group the subscribers in to the same segment which usually has the same behavior for creating the transactions. Our segment information includes

Table 3: The Number of Distribution

<b>Segment Information</b>	<b>OTHER</b>	<b>YOUTH</b>	<b>MASS</b>	<b>Total</b>
# of Subscriber	1,063	1,352	4,065	6,480
# of Refill Transaction	6,290	12,620	34,338	53,248

YOUTH, MASS, and OTHER respectively which mean the subscribers under age 26, all existed subscribers and not segmented new subscribers that the mobile operator do not have any enough information about them. The number of distribution can be seen in Table 3 below. For the RFM data, the refill transactions belong to again the same two year period with the subscribers. For recency parameter we have taken the refill date, for the monetary value we have taken the refill amount in TL. The number of rows is 53.249. The number of distribution can be seen in Table 3.

#### ***4.2 Features Selection for the Model***

For model benchmarking with logistic regression the defined variables were prepared. Usage (Data, Voice, SMS) behaviors,

- Usage statistics which includes if the subscriber has any usage in the last 3 months.
- Refill behaviors which holds the sum of the last 12 months refill transaction amounts. The refill transactions include both voice and data separately.
- Package usage properties which hold if the subscriber make any package refill in the last 12 months.
- ARPU (average revenue per user) properties.
- Call center and online interaction transaction variables were also included.

The subscriber set for this variable is the same with RFM data set. The variables are calculated in monthly bases. The selected month is the last month before the calibration date which is January of 2016.

Table 4: Basic properties of the selected data sets.

Transaction Count	~50K
Distinct Subscriber Count	~6.500
Minimum Refill Date	01.03.2015
Maximum Refill Date	31.01.2017
Days Between Dates	708

### 4.3 Exploratory Data Analysis

Six buckets of the transaction data is used. 15% of the subscribers, which is 59.810 subscribers, never made a top-up during the selected period. They are removed from the used data set. The basic properties of the data sets are shown in Table 4.

The refill subscriber's behavior cannot be estimated beforehand like postpaid subscribers. The demographic information for the prepaid subscribers usually differs from the postpaid ones. Mostly young people prefer to use prepaid lines which is shown in the Figure 10.

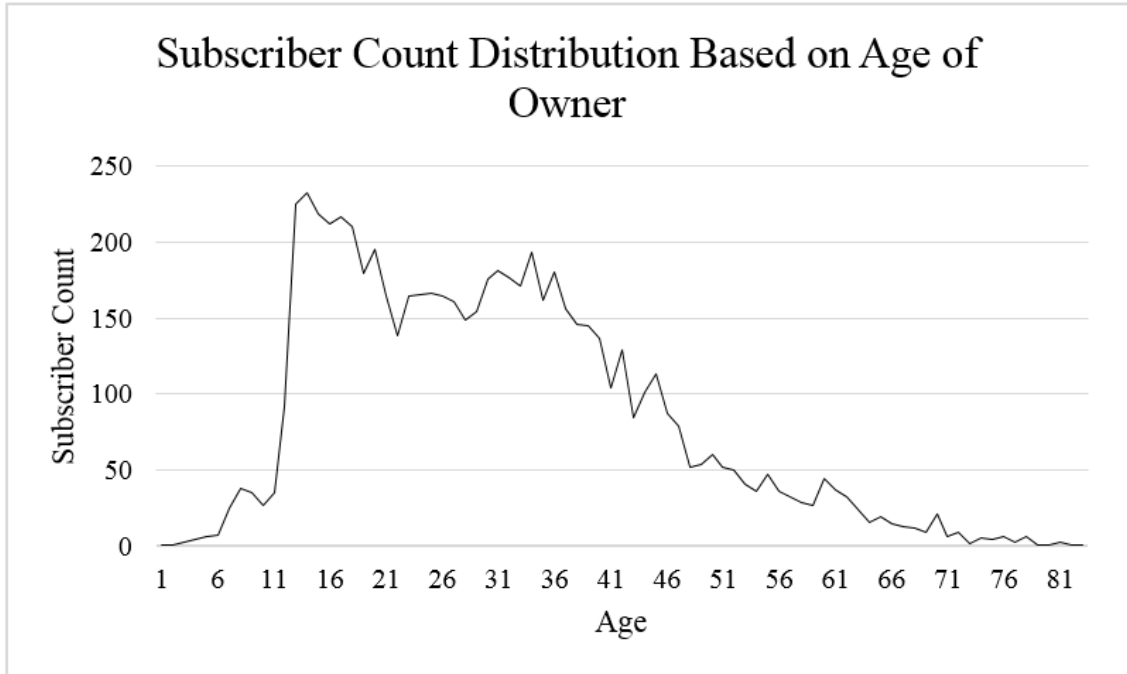


Figure 10: Subscriber Distribution Based on Age

The basic statistics of the sets are given below in Table5. The statistics are

Table 5: Basic statistics of the selected data sets.

Set	Trans	Min	Max	Mean	Var	Stddev
Set 1	53,320	2	180	24	122	11
Set 2	52,683	2	360	24	123	11
Set 3	53,091	1	180	24	123	11
Set 4	51,920	1	180	24	124	11
Set 5	53,900	1	360	25	130	11
Set 6	53,248	2	360	24	147	12

Table 6: The distributions of the data sets.

Set	Q5	Q25	Median	Q75	Q95
Set 1	10	19	25	30	40
Set 2	10	19	25	30	40
Set 3	10	19	25	30	40
Set 4	10	19	25	30	45
Set 5	10	19	25	30	45
Set 6	10	19	25	30	40
Main	10	19	25	30	40

generated based on the Refill Amount (TL).

The statistics for the subsets are nearly the same as the main set which has 2.7M transactions. This ensures that we can use one of the subsets to create the model. The 5th, 25th, 50th, 75th and 95th percentiles of the sets are listed in Table 6.

According to Table 7 it can be easily seen that the sets are shown similar distributions. To see the distribution of the subscriber behavior for the days between transactions, we can see there is positive skewness for the distribution of the days.

Table 7: Values of the days between transactions.

Set	Min	1st.Qu.	Median	Mean	3rd.Qu.	Max
Set 1	1	9	24	31.72	34	593
Set 2	1	9	24	31.93	34	515
Set 3	1	10	26	32.74	35	590
Set 4	1	10	25	32.45	35	545
Set 5	1	9	24	31.45	34	462
Set 6	1	9	25	32.21	35	536

#### ***4.4 Benchmarking Pareto/NBD Model with Logistic Regression***

There are 395 features selected for the logistic regression benchmark. The set includes different type of features which are;

- Call or usage behavior, includes incoming or outgoing calls and the types of communication like voice, data or short message service (sms).
- Network metrics, includes call drop rates.
- Refill metrics, includes the behavior of the refill transactions of the subscriber like refill amount and also usage type based refill utilization like if the subscriber uses mostly sms, data or voice.
- ARPU, includes the average revenue of per user in TL amount.
- # of lines, includes if the subscriber has other lines, or has one or more line.
- Age, gives the age of the subscriber.
- VAS usage / subscription, includes the subscriber usage behavior of value added service or if the subscriber has any subscription for the value added services.
- Pre-calculated SNA metrics, the features also include social network analysis metrics which are calculated in another product and taken as input for the logistic regression
- Network types (2g, 3g, 4g), includes the network type of the subscriber
- Equipment types, give if the subscriber has a smartphone or other basic types.

Features set elimination is done as following;

- 395 features is analyzed with statistical methods.



- First the features whose minimum and maximum values are equal are eliminated. 383 features are left.
- Correlation analysis is made and 308 features are left.
- With domain knowledge only 50 variables are selected.
- Stepwise regression with backward elimination is run over the 50 variables and 10 features are selected.

Logistic Regression is applied as in the following order and also shown in Figure 11 ;

- Result 1: Logistic Regression with RFM
- Result 2: Logistic Regression with 10 features
- Result 3: Logistic Regression with 10 features and RFM
- Result 4: Logistic Regression with 50 features
- Result 5: Logistic Regression with 50 features and RFM

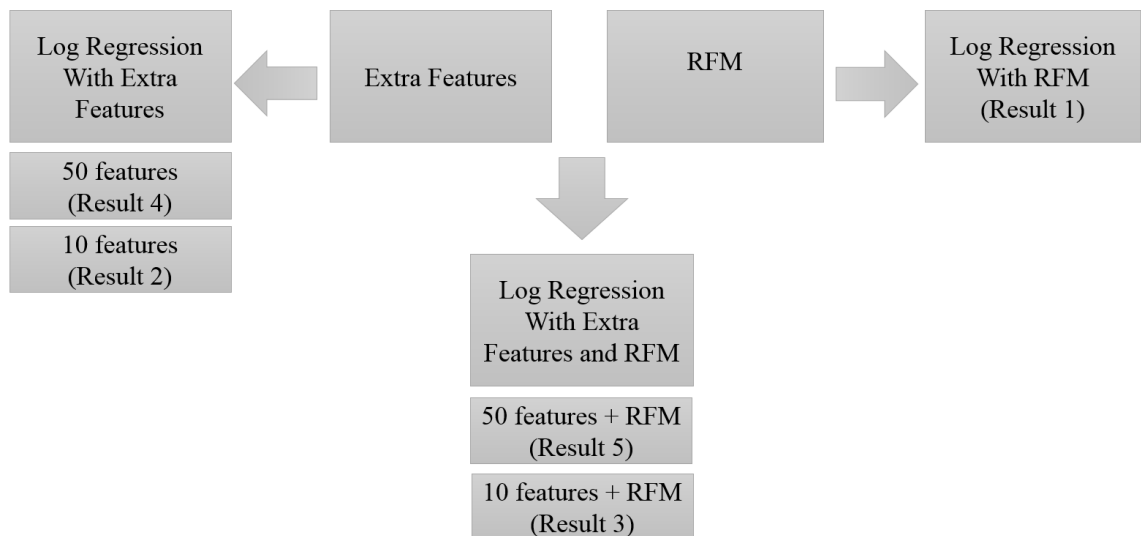


Figure 11: Flow of Benchmarking

The logistic regression benchmark for RFM and the calculated subscriber features results are shared in Table 8.

Table 8: Benchmark of Data with Logistic Regression

Cutoff	Result	Set Name	Accuracy	TP Rate	FP Rate	Precision
0.5	Result 1	RFM	95%	98%	24%	96%
	Result 2	Var 10	91%	100%	64%	90%
	Result 3	Var 10 + RFM	95%	100%	31%	95%
	Result 4	Var 50	92%	100%	56%	91%
	Result 5	Var 50 + RFM	96%	100%	28%	95%
0.6	Result 1	RFM	94%	97%	22%	96%
	Result 2	Var 10	92%	100%	50%	92%
	Result 3	Var 10 + RFM	96%	100%	24%	96%
	Result 4	Var 50	93%	99%	41%	93%
	Result 5	Var 50 + RFM	96%	100%	23%	96%
0.7	Result 1	RFM	93%	95%	20%	96%
	Result 2	Var 10	94%	99%	34%	94%
	Result 3	Var 10 + RFM	97%	100%	20%	97%
	Result 4	Var 50	95%	99%	30%	95%
	Result 5	Var 50 + RFM	97%	100%	18%	97%
0.8	Result 1	RFM	92%	93%	17%	97%
	Result 2	Var 10	94%	98%	29%	95%
	Result 3	Var 10 + RFM	97%	99%	15%	97%
	Result 4	Var 50	95%	98%	24%	96%
	Result 5	Var 50 + RFM	97%	99%	14%	98%
0.9	Result 1	RFM	86%	86%	13%	97%
	Result 2	Var 10	95%	98%	24%	96%
	Result 3	Var 10 + RFM	98%	99%	12%	98%
	Result 4	Var 50	95%	98%	20%	97%
	Result 5	Var 50 + RFM	98%	99%	11%	98%

#### 4.5 Pareto/NBD Model & CLV

Firstly, we only focused on subscriber refill transactions and did not include any tenure or segment information of the subscriber in the transaction data. We found out that without any tenure or segment information the model failed in the test period while showing a good performance in the training period.

Our segment information includes YOUTH, MASS, and NOT SEGMENTED respectively which mean the subscribers under age 26, all existed subscribers and new subscribers that the mobile operator do not have any enough information about them. First we generated the results based on the segment information then included 8 tenure groups which were based on the 3-months sub period information. The tenure was calculated according to the first refill action date. This is illustrated in Figure 12.

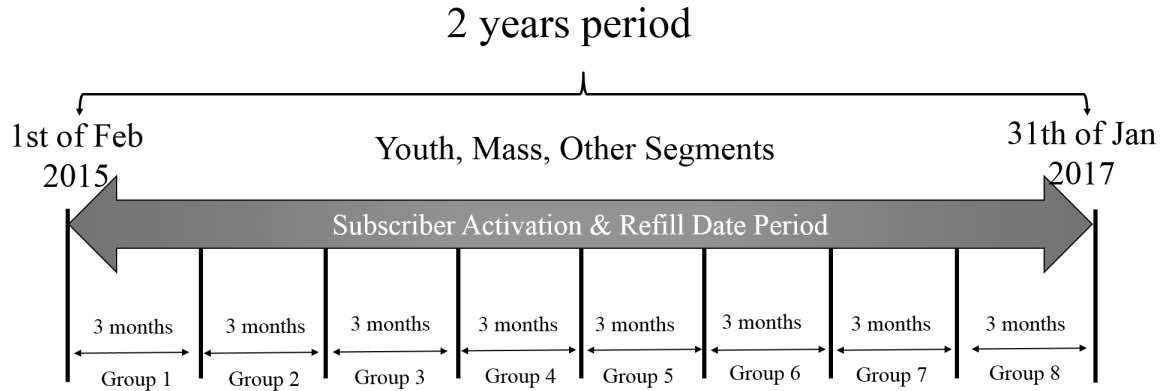


Figure 12: Tenure Groups

The Pareto/NBD model was run based on these data sets which is described above. The results are shared in the Table 9. The Pareto/NBD model results is defined as;

- Result 6: Pareto/NBD model is run with RFM without segment and tenure information
- Result 7: Pareto/NBD model is run with RFM with segment information

Table 9: Pareto/NBD Model Results

Cutoff	Result	Set Name	Accuracy	TP Rate	FP Rate	Precision
0.5	Result 6	w/o Segment + Tenure	88%	98%	69%	89%
	Result 7	Segment	95%	99%	29%	95%
	Result 8	Segment + Tenure	94%	98%	29%	95%
0.6	Result 6	w/o Segment + Tenure	88%	97%	64%	90%
	Result 7	Segment	96%	99%	18%	97%
	Result 8	Segment + Tenure	96%	98%	18%	97%
0.7	Result 6	w/o Segment + Tenure	89%	97%	58%	91%
	Result 7	Segment	96%	98%	13%	98%
	Result 8	Segment + Tenure	96%	97%	13%	98%
0.8	Result 6	w/o Segment + Tenure	90%	96%	46%	92%
	Result 7	Segment	97%	98%	10%	98%
	Result 8	Segment + Tenure	96%	97%	10%	98%
0.9	Result 6	w/o Segment + Tenure	91%	93%	18%	97%
	Result 7	Segment	97%	97%	6%	99%
	Result 8	Segment + Tenure	96%	96%	6%	99%

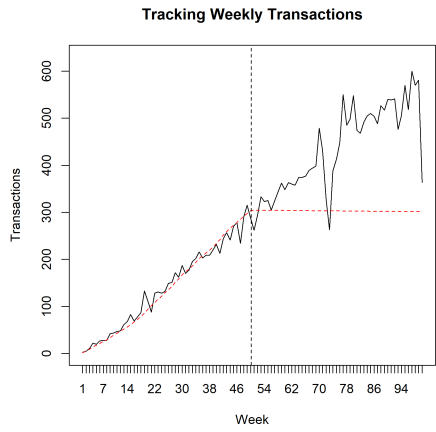
- Result 8: Pareto/NBD model is run with RFM with segment and tenure information

The model performance is given in the figures below (Figure 13) for the all set, mass, youth and not segmented sets. The segment & tenure based data sets have the best performance for the Pareto/NBD model.

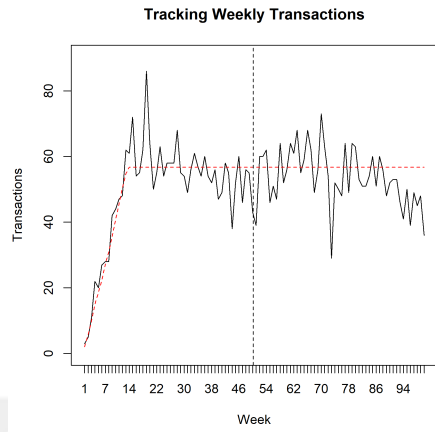
In preceding study since we did not get any good performance in the test period, we could not calculate the CLV value of the subscribers. In this study by adding the tenure and segment information we have got a good performance for the model in test period for that the expected number of transactions and probability of alive of each subscriber are calculated to be used in the offer model.

#### ***4.6 Proposed Mathematical Model for Offer Management***

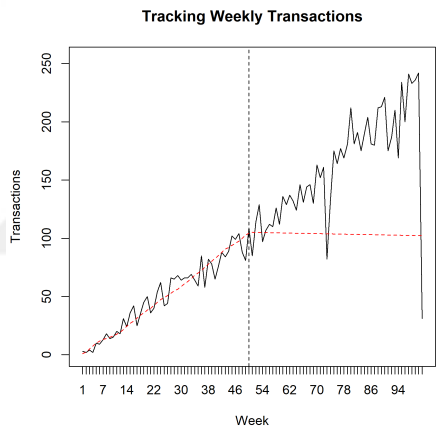
After identifying valuable customers to prevent them from churn, a best offer selection mathematical model is proposed. The model calculates output for each customer an eligible offer. Furthermore, the model maximizes the total CLV by proposing the



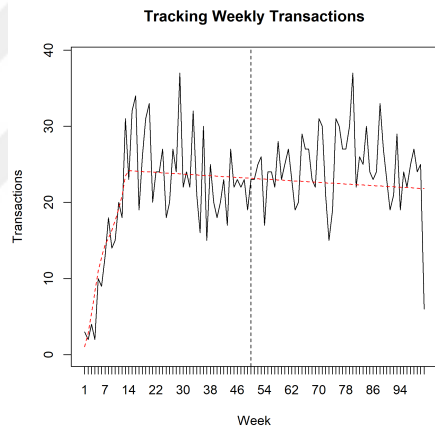
(a) Mass Segment



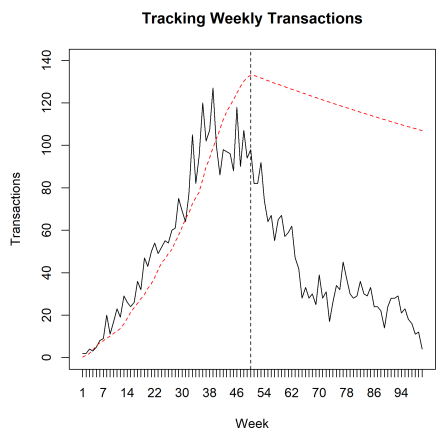
(b) Mass Segment with Tenure



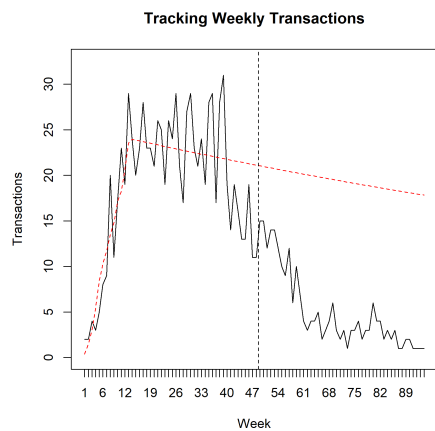
(c) Youth Segment



(d) Youth Segment with Tenure



(e) Others



(f) Others with Tenure

Figure 13: Pareto/NBD Model Results

customers with the eligible offers. Base CLV is calculated by the Pareto/NBD model which is based on the RFM data.

The model is defined as;

Sets;

- $i$  = Customer index,  $i \in I$
- $j$  = Offer index,  $j \in J$

Parameters;

- $clvbase_i$ : Base Customer Lifetime Value of customer  $i$ . This amount should be interpreted as the expected present net worth of customer  $i$  for the following cases:  $i$ ) customer receives no offer or  $ii$ ) customer refuses the proposed offer.
- $p_i$ : Churn probability of customer  $i$  to company before receiving the offer,
- $o_i$ : Upper bound of discount for customer  $i$ ,
- $w_j$ : Upper bound of offer  $j$ ,
- $c_j$ : Discount rate of offer  $j$ ,
- $f_i$ : Monthly payment of customer  $i$  to company before receiving the offer,
- $f_{avg}$ : Monthly average payment of customers,
- $\beta_{ij}$ : Probability of accepting offer  $j$  by customer  $i = p_i^{\frac{c_j f_{avg}}{f_i}}$
- $p_{ij}$ : Churn probability of customer  $i$  after taking offer  $= p_i(1 - p_i^{\frac{c_j f_{avg}}{f_i}})$ ,
- $clv_{ij}$ : Modified Customer Lifetime Value of customer  $i$ ,
- $t$ : Monthly budget for discount,
- $M$ : Large positive number.

Decision Variables;

$$\bullet x_{ij} = \begin{cases} 1, & \text{if customer } i \text{ get offer } j \\ 0, & \text{"o/w"} \end{cases}$$

$$\max z = \sum_i [\sum_j x_{ij} [\beta_{ij} clv_{ij} + (1 - \beta_{ij}) clv_{base_i}] + (1 - \sum_i x_{ij}) clv_{base_i}] \quad (22)$$

$$\sum_i \sum_j f_i c_j x_{ij} \leq t \quad (23)$$

$$\sum_i f_i c_j x_{ij} \leq o_i, \forall i \quad (24)$$

$$\sum_i x_{ij} \leq w_j, \forall j \quad (25)$$

$$x_{ij} \in \{0, 1\}, \forall i, j \quad (26)$$

The objective function aims to maximize expected customer lifetime value while increasing retention of customers by giving the right offer. In other words, the expected CLV depends on whether the customer is received an offer or not, and if the customer is received an offer, it also uses the probability of the accepting the offered or rejecting the offer by the customer. The model is simply illustrated in Figure 14.

Constraint 23 states that discounts made cannot exceed the total budget. Constraint 24 indicates the given offer cannot be predetermined discount limit for each customer. However, it occurs some infeasibilities for some customers. Constraint 25 ensures that number of offers given cannot exceed the specified limit.

We use Pareto/NBD results for CLV and churn probabilities (Result 7). We calculate probability of accepting each offer by customers ( $\beta_{ij}$ ) depending on the customer's package refill payment ( $f_i$ ), churn probability ( $p_i$ ) and average package refill payment

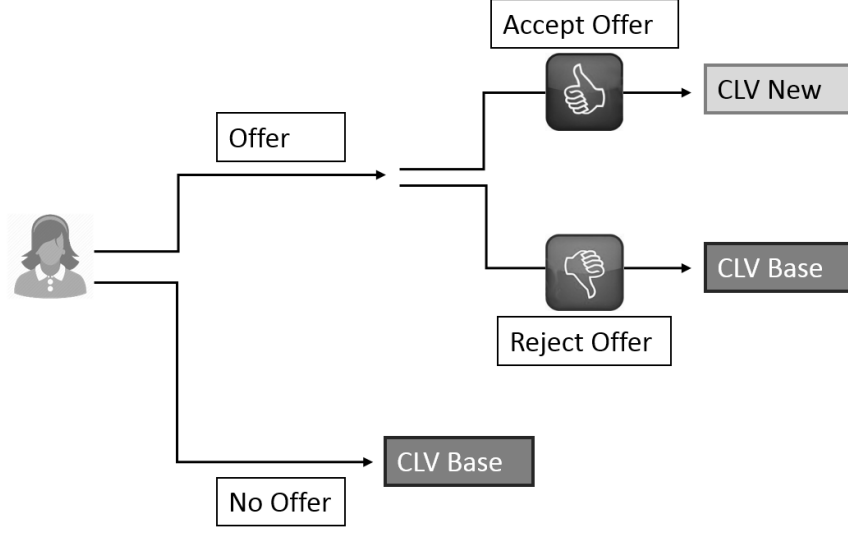


Figure 14: Offer Model

of all customers ( $f_{avg}$ ). Having lower churn probability, higher package refill payment from average and higher discount rate give higher probability of accepting an offer:

$$\beta_{ij} = p_i^{c_j \frac{f_{avg}}{f_i}}, \forall i, j \quad (27)$$

In the same way, we calculate new churn probabilities ( $\rho_{ij}$ ) after receiving each offer depending probabilities of accepting offer ( $\beta_{ij}$ ) and churn probabilities ( $p_i$ ). Having higher churn probability ( $\rho_i$ ) and higher probability of accepting an offer ( $\beta_{ij}$ ) gives lower new churn probability after receiving an offer:

$$\rho_{ij} = p_i(1 - \beta_{ij}^{c_j \frac{f_{avg}}{f_i}}), \forall i, j \quad (28)$$

Then, we use the new churn probabilities ( $\rho_{ij}$ ) to calculate modified CLVs ( $clv_{ij}$ ). Modified CLV is the expected present net worth of customer  $i$  assuming customer accepts offer  $j$ . A mixed integer based mathematical model for offer management is developed, and solved by using GAMS software with CPLEX Solver. We solved problem with 6480 customers and 5 offers in Table 10.

Different scenarios were run with the 6480 customers and the following results are



Table 10: Offers

<b>Offer</b>	<b>Discount %</b>
Offer 1	10
Offer 2	20
Offer 3	30
Offer 4	40
Offer 5	50

observed Table 11. We did not limit the customer set and used all the customers as input for the model.

- Scenario1, the budget was set to 10.000 and offer count was set to 300. There is no cut off value for the customer probability.
- Scenario2, the budget was set to 10.000 and no offer limit was applied. There is no cut off value for the customer probability.
- Scenario3, the budget was set to 5.000 and offer count was set to 300. There is no cut off value for the customer probability.
- Scenario4, the budget was set to 5.000 and no offer limit was applied. There is no cut off value for the customer probability.
- Scenario5, the budget was set to 1.000 and offer count was set to 300. There is no cut off value for the customer probability.
- Scenario6, the budget was set to 1.000 and no offer limit was applied. There is no cut off value for the customer probability.
- Scenario7, the budget was set to 10.000 and no offer limit was applied with 0.5 cut-off for the customer probability.

In table 14, the results show that when there is no limit for the offers, more customers accept the offer. The high-performance model is the "Scenario4". Because

Table 11: Offer Model Scenarios

	Budget	Offer Limit	Cutoff	No of Customers	No of Received
<b>Scenario1</b>	10,000	300	0	6,480	741
<b>Scenario2</b>	10,000	inf	0	6,480	3,398
<b>Scenario3</b>	5,000	300	0	6,480	532
<b>Scenario4</b>	5,000	inf	0	6,480	1,498
<b>Scenario5</b>	1,000	300	0	6,480	168
<b>Scenario6</b>	1,000	inf	0	6,480	168
<b>Scenario7</b>	10,000	inf	0.5	781	562

Table 12: Churn Rate Breakdown

	Churn Rates		Total Subs
	p<0.5	p>=0.5	
Scenario1	294	447	741
Scenario2	3,331	67	3,398
Scenario3	294	238	532
Scenario4	1,485	13	1,498
Scenario5	168		168
Scenario6	168		168
Scenario7		562	562

Table 13: Accepted Offer Distribution

	Discount Offers					Total Subs
	Offer1 %10	Offer2 %20	Offer3 %30	Offer4 %40	Offer5 %50	
<b>Scenario1</b>	300	57	39	60	285	741
<b>Scenario2</b>	3352	10	2	11	23	3398
<b>Scenario3</b>	300	22	13	35	162	532
<b>Scenario4</b>	1495	3				1498
<b>Scenario5</b>	168					168
<b>Scenario6</b>	168					168
<b>Scenario7</b>	121	57	39	60	285	562

Table 14: Calculated CLV

		CLV Calculated	CLV Base	CLV Diff	Churn Prob Base Avg = 12%
<b>Offer Limit = 300</b>	<b>Scenario1</b>	1,239,305	1,203,300	36,005	55%
	<b>Scenario3</b>	1,239,043	1,203,300	35,743	42%
	<b>Scenario3</b>	1,239,043	1,203,300	35,743	2%
<b>No Offer Limit</b>	<b>Scenario2</b>	1,267,980	1,203,300	64,680	3%
	<b>Scenario4</b>	1,257,194	1,203,300	53,894	2%
	<b>Scenario6</b>	1,228,630	1,203,300	25,330	2%

mostly Offer1 is accepted and the higher CLV is got from the model. Limiting total budget does not have much affect over the CLV maximization. In Scenario2, more customers accept the offer and higher CLV is got. However, comparing to Scenario4, with lowest churn probability and limmited budget, Scenario4 has the higher performance.

## CHAPTER V

### CONCLUSION

The mass amount of the mobile subscribers are still prepaid subscribers Figure 1. In years both the number of postpaid and prepaid subscriber has converged to each other as a result of data usage increasing due to 4g network upgrade by the mobile operators. In this study we have stated that life time value is important part of the churn prediction. Therefore a mobile operator should use the results in their offer management lifecycle. They have to optimize their cost for churn retention program and know the subscribers who will really value to propose an offer. Otherwise without calculating the CLV, the mobile operator has to depend on the results of the churn prediction model and accept the burden of the false positive subscriber sets effect over their cost.

In this study, the segment and tenure information is also included in the data set. It can be seen from the (Figure 13) that the model gave good results for the youth and mass segments in the test period. If the customer has other segment, the model failed in Figure 13e and Figure 13f.

For CLV calculation we have used the subscriber sets result with segment information which is Result 7 in 9. The selected set has the highest Accuracy among the other sets. Instead of proposing an offer to every churner, the mathematical model gives us the flexibility of the offering to the most valuable subscribers who has the higher potential to accept the offers.

Different scenarios were tested. The maximized CLV results are shared 14. As stated in the previous section 4.6, the highest performance is got from the Scenario4. This means that if no offer limit is applied most customers accept the proposed offer

and it also increases the retention rate which means more loyal customers.

On the other hand, parameter estimation has to be improved for the model. There are some different studies about the parameter estimation [32], [33] and [34] However we decided to focus on the data set to get the results for the CLV and offer management.

Lastly, we may try to test the framework and measure the real performance of the system by collecting also the response rates of the subscribers.





# Appendices

# APPENDIX A

## PARETO/NBD R CODE

```
1 #####
2 # INSTALL AND LOAD NEEDED PACKAGES
3 #####
4 install.packages(ggplot2)
5 install.packages(BTYD)
6 install.packages(reshape2)
7 install.packages(plyr)
8 install.packages(lubridate)
9 install.packages(xlsx)
10 install.packages(gsl)
11 install.packages(stringr)
12 library(ggplot2)
13 library(BTYD)
14 library(reshape2)
15 library(plyr)
16 library(lubridate)
17 library(xlsx)
18 library(gsl)
19 library(stringr)
20
21
22 #####
23 # USED BTYD PACKAGE IS FIXED ONE : https://github.com/theofilos/BTYD
24 #####
25 h2f1 <- function(a,b,c,z){
26   lenz <- length(z)
27   j = 0
28   uj <- 1:lenz
29   uj <- uj/uj
30   y <- uj
31   lsteps <- 0
32   while (lsteps<lenz){
33     lasty <- y
34     j <- j+1
```

```

35   uj <- uj*(a+j-1)*(b+j-1)/(c+j-1)*z/j
36   y <- y + uj
37   lsteps <- sum(y==lasty)
38 }
39 y
40 }
41
42 zc.pnbd.cbs.LL <- function(params, cal.cbs) {
43
44   dc.check.model.params(c("r", "alpha", "s", "beta"), params, "pnbd.cbs.LL")
45
46   tryCatch(x <- cal.cbs[, "x"], error = function(e) stop("Error in pnbd.cbs.LL: cal.
         cbs must have a frequency column labelled \"x\""))
47   tryCatch(t.x <- cal.cbs[, "t.x"], error = function(e) stop("Error in pnbd.cbs.LL:
         cal.cbs must have a recency column labelled \"t.x\""))
48   tryCatch(T.cal <- cal.cbs[, "T.cal"], error = function(e) stop("Error in pnbd.cbs.
         LL: cal.cbs must have a column for length of time observed labelled \"T.cal\"")
         )
49
50   if ("custs" %in% colnames(cal.cbs)) {
51     custs <- cal.cbs[, "custs"]
52   } else {
53     custs <- rep(1, length(x))
54   }
55   return(sum(custs * zehra.pnbd.LL(params, x, t.x, T.cal)))
56 }
57
58
59 zc.pnbd.LL= function (params, x, t.x, T.cal)
60 {
61   max.length <- max(length(x), length(t.x), length(T.cal))
62   if (max.length%%length(x))
63     warning("Maximum vector length not a multiple of the length of x")
64   if (max.length%%length(t.x))
65     warning("Maximum vector length not a multiple of the length of t.x")
66   if (max.length%%length(T.cal))
67     warning("Maximum vector length not a multiple of the length of T.cal")
68   dc.check.model.params(c("r", "alpha", "s", "beta"), params,
69     "pnbd.LL")
70   if (any(x < 0) || !is.numeric(x))
71     stop("x must be numeric and may not contain negative numbers.")

```



```

72  if (any(t.x < 0) || !is.numeric(t.x))
73    stop("t.x must be numeric and may not contain negative numbers.")
74  if (any(T.cal < 0) || !is.numeric(T.cal))
75    stop("T.cal must be numeric and may not contain negative numbers.")
76  x <- rep(x, length.out = max.length)
77  t.x <- rep(t.x, length.out = max.length)
78  T.cal <- rep(T.cal, length.out = max.length)
79  r <- params[1]
80  alpha <- params[2]
81  s <- params[3]
82  beta <- params[4]
83  maxab <- max(alpha, beta)
84  absab <- abs(alpha - beta)
85  param2 <- s + 1
86  if (alpha < beta) {
87    param2 <- r + x
88  }
89  part1 <- r * log(alpha) + s * log(beta) - lgamma(r) + lgamma(r + x)
90  part2 <- -(r + x) * log(alpha + T.cal) - s * log(beta + T.cal)
91  if (absab == 0) {
92    part2_times_F1_min_F2 <- ( (alpha+T.cal)/(maxab+t.x) )^(r+x) * (beta+T.cal)^s /
93      ((maxab+t.x)^s) -
94      ( (alpha+T.cal)/(maxab+T.cal) )^(r+x) * (beta+T.cal)^s /
95      ((maxab+t.x)^s)
96  }
97  else {
98    part2_times_F1_min_F2 = h2f1(r+s+x, param2, r+s+x+1, absab / (maxab+t.x)) *
99      ( (alpha+T.cal)/(maxab+t.x) )^(r+x) * (beta+T.cal)^s /
100      ((maxab+t.x)^s) -
101      h2f1(r+s+x, param2, r+s+x+1, absab / (maxab+T.cal)) *
102      ( (alpha+T.cal)/(maxab+T.cal) )^(r+x) * (beta+T.cal)^s /
103      ((maxab+t.x)^s)
104  }
105  return(part1 + part2 + log(1+(s/(r+s+x))*part2_times_F1_min_F2) )
106 }
107
108 zc_pnbd.EstimateParameters <- function(cal.cbs, par.start = c(1, 1, 1, 1),
109                                         max.param.value = 10000) {
110
111  dc.check.model.params(c("r", "alpha", "s", "beta"), par.start, "zehra.
      EstimateParameters")

```

```

112
113  ## helper function to be optimized
114  pnbdeLL <- function(params, cal.cbs, max.param.value) {
115    params <- exp(params)
116    #params[params > max.param.value] <- max.param.value
117    return(-1 * zc.pnbdcbs.LL(params, cal.cbs))
118  }
119  logparams <- log(par.start)
120
121  # results <- optim(logparams, pnbdeLL, cal.cbs = cal.cbs, max.param.value = max.
      param.value, method = "L-BFGS-B")
122  results <- optim(logparams, pnbdeLL, cal.cbs = cal.cbs, max.param.value = max.
      param.value, method = "L-BFGS-B", control=list(trace = TRUE))
123
124  estimated.params <- exp(results$par)
125  #estimated.params[estimated.params > max.param.value] <- max.param.value
126  return(estimated.params)
127
128 }
129
130 zc_ga.EstimateParameters <- function(cal.cbs, par.start = c(1, 1, 1, 1),
131                                     max.param.value = 10000) {
132
133   dc.check.model.params(c("r", "alpha", "s", "beta"), par.start, "zehra.
      EstimateParameters")
134
135   ## helper function to be optimized
136   pnbdeLL <- function(params, cal.cbs) {
137     params <- exp(params)
138     #params[params > max.param.value] <- max.param.value
139     return(-1 * pnbdcbs.LL(params, cal.cbs))
140   }
141   logparams <- log(par.start)
142
143   lst_CV_data <- cal.cbs
144   theta_min <- c(r = 0.1, alpha = 0.1, s = 0.1, beta = 0.1)
145   theta_max <- c(r = 9000, alpha = 9000, s = 9000, beta = 9000)
146
147   theta_min <-log(theta_min)
148   theta_max <-log(theta_max)
149

```

```

150 results <- ga(type = "real-valued", fitness = pnbd.eLL, lst_CV_data,
151             names = c("r", "alpha", "s", "beta"),
152             min = theta_min, max = theta_max,
153             popSize = 20, maxiter = 1000)
154
155 estimated.params <- exp(results@solution)
156
157 #estimated.params[estimated.params > max.param.value] <- max.param.value
158 return(estimated.params)
159
160 }
161
162 #####
163 # LOAD DATA
164 #####
165 refill_file <- file.choose()
166 refill_file_name <- substr(basename(refill_file), 1, str_locate(basename(refill_file)
167                       , ".CSV")[1]-1)
168
169 L <- readLines(refill_file, n = 1)
170 if (grepl(",", L))
171 {
172   elog <- read.csv(refill_file)
173 } else
174 {
175   print("Choose correct file format")
176 }
177
178 elog <- elog[,c(1,2,4)] # we need these columns ; customer id, transaction date, refill
179                       count
180 names(elog) <- c("cust","date","sales") # model functions expect these names
181
182 # format date
183
184 # DIVIDE DATA
185 #####
186 # into a calibration phase and a holdout phase , determine middle point for splitting
187 (end.of.cal.period <- min(elog$date)+as.numeric((max(elog$date)-min(elog$date))/2))
188

```

```

189 # split data into train(calibration) and test (holdout) and make matrices
190 data <- dc.ElogToCbsCbt(eelog, per="week", T.cal=end.of.cal.period, merge.same.date=
      TRUE, statistic = "freq") # which CBT to return
191
192 # extract calibration matrix
193 cal2.cbs <- as.matrix(data[[1]][[1]])
194
195 #####
196 # ESTIMATE PARAMETERS FOR MODEL
197 #####
198 #(params2 <- pnb.d.EstimateParameters(cal2.cbs))
199 (params2 <- zc_pnb.d.EstimateParameters(cal2.cbs))
200
201 # look at log likelihood
202 (LL <- pnb.d.cbs.LL(params2, cal2.cbs))
203
204 #####
205 # HOW WELL DOES MODEL DO IN HOLDOUT PERIOD?
206 #####
207
208 # get holdout transactions from dataframe data, add in as x.star
209 x.star <- data[[2]][[2]][,1]
210 cal2.cbs <- cbind(cal2.cbs, x.star)
211
212 holdoutdates <- attributes(data[[2]][[1]])[[2]][[2]]
213 holdoutlength <- round((as.numeric(max(as.Date(holdoutdates))-min(as.Date(holdoutdates
      )))/7)
214
215 # plot predicted vs actual by week
216
217 # get data without first transaction, this removes those who buy 1x
218 removedFirst.eelog <- dc.SplitUpElogForRepeatTrans(eelog)$repeat.trans.eelog
219 removedFirst.cbt <- dc.CreateFreqCBT(removedFirst.eelog)
220
221 # get all data, so we have customers who buy 1x
222 allCust.cbt <- dc.CreateFreqCBT(eelog)
223
224 # add 1x customers into matrix
225 tot.cbt <- dc.MergeCustomers(data.correct=allCust.cbt, data.to.correct=removedFirst.
      cbt)
226

```

```

227 lengthInDays <- as.numeric(max(as.Date(colnames(tot.cbt)))-min(as.Date(colnames(tot.
      cbt))))
228 origin <- min(as.Date(colnames(tot.cbt)))
229
230 tot.cbt.df <- melt(tot.cbt, varnames=c("cust", "date"), value.name="Freq")
231 tot.cbt.df$date<-as.Date(tot.cbt.df$date)
232 tot.cbt.df$week<-as.numeric(1+floor((tot.cbt.df$date-origin+1)/7))
233
234 transactByWeek <- ddply(tot.cbt.df,.(week), summarize, sum(Freq))
235 names(transactByWeek) <- c("week", "Transactions")
236
237 T.cal <- cal2.cbs[, "T.cal"]
238 T.tot <- round(as.numeric(max(eelog$date)-min(eelog$date))/7)
239
240 comparisonByWeek <- pnbld.PlotTrackingInc(params2, T.cal, T.tot, actual.inc.tracking.
      data=transactByWeek$Transactions)
241
242 #####
243 #GET PALIVES AND CONFUSSION MATRIX
244 #####
245 p.alives <- pnbld.PAlive(params2, cal2.cbs[, "x"], cal2.cbs[, "t.x"], cal2.cbs[, "T.cal"
      ])
246 ggplot(as.data.frame(p.alives), aes(x=p.alives))+ geom_histogram(colour="grey", fill="
      gray")+ylab("Number of Customers")+xlab("Probability Customer is 'Live'")+theme_
      minimal()
247 df.prob.alives <- data.frame(names(p.alives), as.vector(p.alives))
248 names(df.prob.alives) <-c("subscriberid", "probability")
249 write.xlsx(df.prob.alives, paste(refill_file_name, sep = "", "_PROB_OF_ALIVES.xlsx"))
250
251 #####
252 #SPEND - MONETARY
253 #####
254 elog_cal <- elog[which(eelog[, "date"] < end.of.cal.period),]
255 elog_cal$Week <- as.Date(cut(eelog_cal$date, breaks = "week", start.on.monday = TRUE))
256 cu.tracking <- aggregate(eelog_cal$cust, by=list(eelog_cal$Week), FUN=length) #ddply(
      elog, .(eelog$Week), transform, Cumulative.Sum = cumsum(sales))
257 est.params <- params2
258 m.x <- aggregate(eelog_cal$sales, list(eelog_cal$cust), mean)
259 cdnowSummary <- list(cal2.cbs, as.vector(cu.tracking[[2]]), as.vector(est.params), as
      .vector(m.x[[2]]))
260 m.x.vector <-as.vector(m.x[[2]])

```

```

261 names(m.x.vector) <- c("m.x")
262 calCBS <- cbind(m.x.vector, cal2.cbs)
263 names(cdnwSummary) <- c("cbs", "cu.tracking", "est.params", "m.x")
264 #str(cdnwSummary)
265 ave.spend <- cdnowSummary$m.x
266 tot.trans <- cdnowSummary$cbs[, "x"]
267
268 # Now we can estimate model parameters. spend.LL, which is used
269 # by spend.EstimateParameters, will give you warning if you pass
270 # it a customer with zero transactions. To avoid this, we can
271 # remove customers with zero transactions
272 ave.spend <- ave.spend[which(tot.trans > 0),]
273 tot.trans <- tot.trans[which(tot.trans > 0)]
274 params_spend <- spend.EstimateParameters(ave.spend, tot.trans)
275 spend.cal.expected <- spend.expected.value(params_spend, calCBS[, "m.x"], calCBS[, "x"
    ]) #-> abone bazinda matrix ya da dataframe olustur
276 names(spend.cal.expected) <- c("expected_value")
277 calCBS.expected.value <- cbind(calCBS, expected.value=as.vector(spend.cal.expected))
278 df.expected.value <- data.frame(calCBS.expected.value)
279 write.xlsx(df.expected.value, paste0(refill_file_name, "_EXOECTEDVALUE.xlsx"))
280 spend.plot.average.transaction.value(params_spend, ave.spend, tot.trans)
281
282 # Uses Pareto/NBD model parameters and a customer's past transaction behavior to
    return the number
283 # of transactions they are expected to make in a given time period.
284 # pnbld.ConditionalExpectedTransactions(params, T.star, x, t.x, T.cal)
285 # T.star <- holdoutlength
286 # predict<-pnbld.ConditionalExpectedTransactions(params2, T.star = holdoutlength, x
    = cal2.cbs[, "x"], t.x = cal2.cbs[, "t.x"], T.cal = cal2.cbs[, "T.cal"])
287
288 # pnbld.DERT Pareto/NBD Discounted Expected Residual Transactions
289 # Calculates the discounted expected residual transactions of a customer, given their
    behavior during the calibration period.
290 # pnbld.DERT(params, x, t.x, T.cal, d)
291
292 #####
293 # DATA EXPLORATION
294 #####
295 # plot predicted vs seen conditional freqs and get matrix 'comp' w values
296 T.star <- holdoutlength
297 censor <- 10 # Bin all order numbers here and above

```

```

298 comp <- pnbld.PlotFreqVsConditionalExpectedFrequency(params2, T.star, cal2.cbs, x.star,
      censor)
299 rownames(comp) <- c("act", "exp", "bin")
300 comp
301
302
303 #####
304 # PLOT GROUP DISTRIBUTION OF PROPENSITY TO PURCHASE, DROPOUT
305 #####
306 # par to make two plots side by side
307 par(mfrow=c(1,2))
308 # Plot the estimated distribution of lambda
309 # (customers' propensities to purchase)
310 pnbld.PlotTransactionRateHeterogeneity(params2, lim = NULL)
311
312 # lim is upper xlim
313 # Plot estimated distribution of gamma
314 # (customers' propensities to drop out).
315 pnbld.PlotDropoutRateHeterogeneity(params2)
316 # set par to normal
317 par(mfrow=c(1,1))
318
319
320 # look at days between orders
321 # model describes rates via a gamma distribution across customers
322 purchaseFreq <- ddply(eleg, .(cust), summarize, daysBetween = as.numeric(diff(date)))
323 ggplot(purchaseFreq, aes(x=daysBetween))+geom_histogram(fill="darkgrey")+xlab("Time
      between purchases (days)")+theme_minimal()

```

## APPENDIX B

### GAMS CODE FOR MATHEMATICAL MODEL

```
1 option optcr = 0.00001;
2 Sets
3 i customer
4 j offer
5
6 $call GDXXRW offer.xlsx dset=i rng=main!a2 rdim=1 dset=j rng=main!c2 rdim=1"
7 $gdxin offer.gdx
8 $load i j
9 $gdxin
10
11 display j;
12
13 Parameters
14 clv_base(i) The customer lifetime value of the customer i
15 p(i) The churn probability of the customer i
16 f(i) Monthly payment of customer i to company before taking the offer
17 o(i) The upper bound of offer monetary value for customer i
18 clv(i,j) The customer lifetime value of the customer i
19 bb(i,j)
20 pp(i,j)
21 c(j) The monetary value of offer j
22 ;
23
24 execute "GDXXRW offer.xlsx par=c rng=main!as2 Rdim=1 par=clv_base rng=main!K2 Rdim=1
      par=p rng=main!E2 Rdim=1 par=f rng=main!H2 Rdim=1 par=o rng=main!ap2 Rdim=1 par=
      b rng=main!aj2 Rdim=1 par=n rng=main!am2 Rdim=1"
25 execute_load 'offer.gdx' c clv_base p f o
26 display c ,clv_base, p, f, o;
27
28 execute "GDXXRW offer.xlsx par=bb rng=bb!a1 Rdim=1 Cdim=1 "
29 execute_load 'offer.gdx' bb
30
31 execute "GDXXRW offer.xlsx par=pp rng=pp!a1 Rdim=1 Cdim=1 "
32 execute_load 'offer.gdx' pp
```



```

33
34 execute "GDXXRW offer.xlsx par=clv rng=clv!a1 Rdim=1 Cdim=1 "
35 execute_load 'offer.gdx' clv
36
37 display bb,pp,clv;
38
39 Scalar
40 t Monthly budget for offers/
41 10000/;
42
43 Variable z;
44
45 Binary Variable
46 x(i,j) If customer i get offer j
47
48 Equations
49 obj
50 c3
51 c4(i)
52 c5(j);
53
54 obj.. z =e= sum(i, sum(j,x(i,j)*(bb(i,j)*clv(i,j)+(1-bb(i,j))*clv_base(i)))+(1-sum(j,
    x(i,j))*clv_base(i));
55 c3.. sum ((i,j), f(i)*x(i,j)*c(j)) =l= t;
56 c4(i).. sum (j, f(i)*x(i,j)*c(j)) =l= o(i) ;
57 c5(j).. sum (i, x(i,j)) =l= 300;
58
59 model offer_management /all/;
60 Solve offer_management using mip maximizing z;
61
62 file out2 /offer_ham-ek2.txt/;
63 put out2;
64 loop((i,j), put$(x.l(i,j)) i.t1 j.t1/);

```

## APPENDIX C

### FEATURES FOR LOGISTIC REGRESSION

- 1 Subscriber Activation Reason
- 2 Counter Slope in the last six months (Usage + Fee counters)
- 3 Trend of usage + fee credits used in last 6 months
- 4 Variance of credits used in last 6 months
- 5 End date of subscription
- 6 End reason of subscription
- 7 Average number of dropped calls for last six months
- 8 Average number of dropped calls for last month
- 9 Last package date
- 10 Last tariff group change year month
- 11 Minimum packet fill date
- 12 Minimum packet fill year month
- 13 Minimum sms fill date
- 14 Minimum sms fill year month
- 15 Last MNP port in year month
- 16 Last Portout Year Month
- 17 End Date of latest acquisition given in the last month
- 18 Start Date of latest acquisition given in the last month
- 19 Last activation date
- 20 Last Activation year month of subscriber
- 21 Contract Signed Year Month
- 22 First activation date
- 23 VAS Package purchase year month
- 24 Last month line count less than 2
- 25 OMO CC Incoming call flag
- 26 OMO CC outgoing call flag
- 27 ONNET CC call flag
- 28 Flag for ONNET prefix
- 29 4g flag
- 30 Gnctrkcll flag
- 31 Last equipment 4g support flag
- 32 Last Equipment Mobil TV Flag
- 33 Last Equipment screen dimensions
- 34 Last equipment 3G flag

35 Youth segment flag  
36 Subscriber Id  
37 Customer Id  
38 Average call revenue (usage + fee) for last six months  
39 Average number of incoming different B number(voice) for last six months  
40 Average number of incoming different onnet B number(voice) for last six months  
41 Average number of incoming voice calls for the last six months  
42 Average Incm Voice MOU for last six months for last six months  
43 Average Night MOU for last six months  
44 Average Offnet Night MOU for last six months  
45 Average Onnet Night MOU for last six months  
46 Average Not Roaming Incm Not Vas Mms OnnetCALL for last six months  
47 Not roaming incoming not vas voice OMO MoU in the last month  
48 Not roaming incoming not vas voice OnnetMoU in the last month  
49 Last month incoming TT mou  
50 Last month Not roaming Incm Not Vas SMS OnnetCall  
51 Average Not roaming Incm Not Vas SMS OnnetCall for last six month  
52 Average Not roaming Incm Not Vas Voice OnnetCall for last six month  
53 Average Not Roaming Incm Sms other operator 1 CALL for last six months  
54 Average Not Roaming Incm SMS CALL for last six months  
55 Average Not Roaming Incm Sms Int CALL for last six months  
56 Average Not Roaming Incm Not Vas Sms Nat CALL for last six months  
57 Average Not Roaming Incm Sms Omo CALL for last six months  
58 Average Not Roaming Incm Sms other operator 2 CALL for last six months  
59 Average Not Roaming Incm Voice other operator 1 CALL for last six months  
60 Average Not Roaming Incm Voice other operator 1 MOU for last six months  
61 Average Not Roaming Incm Voice CALL for last six months  
62 Average Not Roaming Incm Voice International CALL for last six months  
63 Average Not Roaming Incm Voice National CALL for last six months  
64 Average Not Roaming Incm Voice Omo CALL for last six months  
65 Average Not Roaming Incm Voice Onnet CALL for last six months  
66 Average Not Roaming Incm Voice other operator 2 CALL for last six months  
67 Average Not Roaming Incm Voice other operator 2 MOU for last six months  
68 Average Not Roaming Incm Voice CALL for last six months  
69 Average Not Roaming Incm Voice Int MOU for last six months  
70 Average Not Roaming Incm Voice MOU for last six months  
71 Average Not Roaming Incm Not Vas Voice Nat MOU for last six months  
72 Average Not Roaming Incm Voice Omo MOU for last six months  
73 Average Not Roaming Incm Voice TT MOU for last six months  
74 Average Not Roaming Incm VAS SMS Call for last six months  
75 Last month Not Roaming Not VAS Incm OMO SMS Call

76 Total not roaming incoming not vas SMSOMO calls in the last month  
77 Last month outgoing OMO mou  
78 Not Roaming Outgoing Not Vas Last month outgoing Onnetmou  
79 Last month outgoing TT mou  
80 Average Not roaming Outg Not Vas SMS Yurtici Call for last six months  
81 Last month Not roaming Outg Not Vas SMS OnnetCall  
82 Average Not roaming Outg Not Vas SMS OnnetCall for last six months  
83 Average Not roaming Outg Not Vas Voice Yurtici Call for last six months  
84 Average Not roaming Outg Not Vas Voice Yurtici MOU for last six months  
85 Average Not roaming Outg Not Vas Voice OnnetCall for last six months  
86 Average Not roaming Outg Not Vas Voice OnnetMOU for last six months  
87 Average Not Roaming Outg Not Vas Voice CALL for last six months  
88 Average Not Roaming Outg Not Vas Voice MOU for last six months  
89 Average not roaming outgoing SMS other operator 1 calls in the last six months  
90 Average Not Roaming Outg SMS Call for last six months  
91 Average Not Roaming Outg SMS Int Call for last six months  
92 Average Not Roaming Outg SMS Omo Call for last six months  
93 Average not roaming outgoing SMS VDF calls in the last six months  
94 Average Not Roaming Outg Voice other operator 1 Call for last six months  
95 Average Not Roaming Outg Voice other operator 1 MOU for last six months  
96 Average Not Roaming Outg Voice Int Call for last six months  
97 Average Not Roaming Outg Voice Int MOU for last six months  
98 Average Not Roaming Outg Voice OMO Call for last six months  
99 Average Not Roaming Outg Voice Onnet Call for last six months  
100 Average Not Roaming Outg Voice Onnet MOU for last six months  
101 Average Not Roaming Outg Voice other operator 2 CALL for last six months  
102 Average Not Roaming Outg Voice other operator 2 MOU for last six months  
103 Average Not Roaming Outg Voice Omo MOU for last six months  
104 Average Not Roaming Outg Voice TT CALL for last six months  
105 Average Not Roaming Outg Voice TT MOU for last six months  
106 Average not roaming outgoing VAS SMS calls in the last six months  
107 Last 6 months sent and received sms average  
108 Last month Number of call forwarding  
109 Number of call forwarding for last 3 months  
110 Monthly Distinct Cell City Count for last three months  
111 Monthly Distinct Cell Site Count for last three months  
112 Total number of distinct top3 call for the last six months  
113 Average number of Night Sms Calls for last six months  
114 Average number of Offnet Night Sms Calls for last six months  
115 Average number of Onnet Night Sms Calls for last six months  
116 Number of tariff change in last 6 months

117 Sum of calls made to top1 OMO number for last six moths  
118 Last month Call Revenue excluding Package Fee for all call types  
119 Average number of outgoing different B number(voice) for last six months  
120 Average number of outgoing different OnnetB number(voice) for last six months  
121 Average total top1 mou for last six months  
122 Average total top1 calls for last six months  
123 Average total top(1+2+3)mou for last six months  
124 Average total top(1+2+3) call count for last six months  
125 Total Amount of Discounted MOU for the last six months  
126 Total number of discounted sms usage for the last six months  
127 Average Week End MOU for last six months  
128 Days since last CHURN campaign ACCEPTED  
129 Days since last offer campaign  
130 GPRS package count in last month  
131 Number of ACCEPTED churn campaigns for last 3 months  
132 Number of ACCEPTED campaigns for last 3 months  
133 Utilization of data package in last 3 months  
134 Utilization of sms package in last 3 months  
135 Utilization of voice package in last 3 months  
136 Contracted Packeg Fee in last month  
137 Voice package count in last month  
138 Number of MOC Status Changes for last twelve months  
139 Number Of Postpaid Lines (excluding deactive lines)  
140 Number Of Prepaid Lines (excluding deactivated lines)  
141 Total number of status change in the last twelve months  
142 Number of lines for the customer (excluding deactivated lines)  
143 Incoming TT count in last month  
144 Outgoing TT count in last month  
145 Incoming TT vo ce count in last month  
146 Outgoing TT voice count in last month  
147 Data cell count in last 3 month  
148 Change in postpaid line count  
149 Change in prepaid line count  
150 Number of SRs for the last month  
151 Number of SRs for six month  
152 Discounted Mou Average for last six months  
153 Last 6 month Average Amount of dedicated free counter earned  
154 Average amount of package sms calls for the last six months  
155 Total Number of SMS PACKAGE Calls for last 6 months  
156 Total number of refills excluding refills(event type refilland konbara) for last six  
moths

157 Total Number of Voice PAKET Mou for last 6 months  
158 Web usage count in last 3 month  
159 Web last usage in last 3 month  
160 Total number of port-ins  
161 Total number of port-outs  
162 Total OnnetTenure for port-in customers  
163 Top 3 incoming call subscriber port count in last month  
164 Top 3 outgoing call subscriber port count in last month  
165 Tenure since the last portin  
166 Total amount of refills for last six months  
167 Average of Daily Balance for the last six months  
168 Standard Deviation of SDP counter balance for last six months  
169 Total count of negative SDP balance days for last six months  
170 Advanced credit usage in last 6 months  
171 Average number of days between refills  
172 Maximum number of days between refills  
173 Minimum number of days between refills  
174 Standart deviation of day count between refills  
175 Number of days since last refill  
176 Total number of refills for last six months  
177 Sum of prepaid package purchases made for the last six months  
178 Count of NAR package in last 6 months  
179 Count of other package in last 6 months  
180 Average data utilization  
181 Maximum data utilization  
182 Minimum data utilization  
183 Average of other package utilization in last 6 months  
184 Average voice utilization  
185 Maximum voice utilization  
186 Minimum voice utilization  
187 Average SMS utilization  
188 Maximum SMS utilization  
189 Minimum SMS utilization  
190 Refill count from Shop in last 6 months  
191 Refill count other than POS/Shop/Bank channel in last 6 months  
192 Average utilization  
193 SNA Average offnet subscriber count in last six months  
194 SNA Change in offnet subscriber count in last six months  
195 SNA Standart deviation of offnet subscriber count in last six months  
196 SNA Trend of offnet subscriber in last six months  
197 SNA Average onnet subscriber count in last six month

198 SNA Change in onnet subscriber count in last six months  
199 SNA Standart deviation of onnet subscriber in last six months  
200 SNA Trend of onnet subscriber in last six months  
201 SNA Average postpaid subscriber count in last six months  
202 SNA Change in postpaid subscriber count in last six months  
203 SNA Standart deviation of postpaid subscriber count in last six months  
204 SNA Trend of postpaid subscribers  
205 SNA Average prepaid subscriber count in last six months  
206 SNA Change in prepaid subscriber count in last six months  
207 SNA Standart deviation of prepaid subscriber count in last six months  
208 SNA Trend of prepaid subscribers  
209 SNA upsell count in last month  
210 Last SNA Role of the subscriber  
211 Last acquisition sum in last month  
212 Last acquisition usage in last month  
213 Age  
214 ARPU Average for last six months  
215 ARPU Slope for last six months  
216 ARPU trend in the last six months  
217 Call Center Cost  
218 Usage + Fee Revenue Slope for last six months  
219 Usage + fee revenue trend in the last six months  
220 Total number of CSI logins for last six months  
221 Num of actions made on CSIMI in last 6 months  
222 Num of actions made on CSITS in last 6 months  
223 Num of actions made on CSIWS in last 6 months  
224 Day since last equipment changed  
225 Customer Small Business flag  
226 Final Cost  
227 Final Profit  
228 Interconnect Cost  
229 Network Cost  
230 Number of different equipment used in 6 months  
231 Outgoing MOU Slope for last six months  
232 Outgoing MOU Trend for last six months  
233 Outgoing different B number(voice)call Slope for last 6 months  
234 Outgoing diffB number trend in the last six months  
235 Subscriber General ARMU for previous month  
236 Soho Club flag  
237 Tenure in Months (Months after last portin for portin subscribers months after last  
activation for others )

238 Tax revenue  
239 Treasure Cost  
240 3g data usage ratio in last month  
241 Data usage between 08:00 - 19:00 (MB)  
242 Average Data Volume  
243 Standart deviation of Data Volume  
244 Data Volume Standard Deviation  
245 Data Volume Trend  
246 Data TL Average  
247 Data TL Change - StandardDeviation/Mean  
248 Data TL Standard Deviation  
249 Data TL Trend  
250 Data usage between 06:00 - 08:00 (MB)  
251 Incoming MOU Average  
252 Incoming MOU Change - StandardDeviation/Mean  
253 Incoming MOU Standard Deviation  
254 Incoming MOU Trend  
255 Incoming OMO MOU Average  
256 Incoming OMO MOU Change - StandardDeviation/Mean  
257 Incoming OMO MOU Standart Deviation  
258 Incoming OMO MOU Trend  
259 Incoming Onnet MOU Average  
260 Incoming Onnet MOU Change - StandardDeviation/Mean  
261 Incoming Onnet MOU Standart Deviation  
262 Incoming Onnet MOU Trend  
263 Incoming SMS Count Change - StandardDeviation/Mean  
264 Incoming SMS Trend  
265 Incoming Voice Count Change - StandardDeviation/Mean  
266 Incoming Voice Count - StandardDeviation  
267 Incoming Voice Count AVERAGE  
268 Incoming Voice Count Trend  
269 Last six months usage days  
270 Data usage between 23:00 - 07:00 (MB)  
271 Data usage between 19:00 - 23:00 (MB)  
272 Charged Outgoing Voice TL Average  
273 Charged Outgoing Voice TL Change - StandardDeviation/Mean  
274 Charged Outgoing Voice TL - StandardDeviation  
275 Charged Outgoing Voice TL Trend  
276 Discounted Outgoing Voice TL Average  
277 Discounted Outgoing Voice TL Change - StandardDeviation/Mean  
278 Discounted Outgoing Voice TL - StandardDeviation/Mean



279 Discounted Outgoing Voice TL Trend  
280 Last month last call date difference  
281 Last month last data usage date difference  
282 Outgoing MOU Average  
283 Outgoing MOU Change - StandardDeviation/Mean  
284 Outgoing MOU - StandardDeviation  
285 Outgoing MOU Trend  
286 Outgoing OMO MOU Average  
287 Outgoing OMO MOU Change - StandardDeviation/Mean  
288 Outgoing OMO MOU - StandardDeviation  
289 Outgoing OMO MOU Trend  
290 Outgoing OMO Voice Count Average  
291 Outgoing OMO Voice Count Change - StandardDeviation/Mean  
292 Outgoing OMO Voice Count - StandardDeviation  
293 Outgoing OMO Voice Count Trend  
294 Outgoing OMO Voice TL Average  
295 Outgoing OMO Voice TL Change - StandardDeviation/Mean  
296 Outgoing OMO Voice TL - StandardDeviation  
297 Outgoing OMO Voice TL Trend  
298 Outgoing Onnet MOU Average  
299 Outgoing Onnet MOU Change - StandardDeviation/Mean  
300 Outgoing Onnet MOU - StandardDeviation  
301 Outgoing Onnet MOU Trend  
302 Outgoing Onnet Voice Count Average  
303 Outgoing Onnet Voice Count Change - StandardDeviation/Mean  
304 Outgoing Onnet Voice Count Average  
305 Outgoing Onnet Voice Count Trend  
306 Outgoing Onnet Voice TL Average  
307 Outgoing Onnet Voice TL Change - StandardDeviation/Mean  
308 Outgoing Onnet Voice TL - StandardDeviation  
309 Outgoing Onnet Voice TL Trend  
310 Outgoing SMS Change - StandardDeviation/Mean  
311 Outgoing SMS Trend  
312 Outgoing Voice Count Average  
313 Outgoing Voice Count Change - StandardDeviation/Mean  
314 Outgoing Voice Count - StandardDeviation  
315 Outgoing Voice Count Trend  
316 Outgoing Voice TL Average  
317 Outgoing Not Roaming Voice TL Change - StandardDeviation/Mean  
318 Outgoing Voice TL - StandardDeviation  
319 Outgoing Not Roaming Voice TL Trend

320 Ratio of incoming and outgoing call  
321 Other operator 1 incoming call ratio in all calls  
322 Onnet incoming call ratio in all calls  
323 Other operator 2 incoming call ratio in all calls  
324 Other operator 1 outgoing call ratio in all calls  
325 Onnet outgoing call ratio in all calls  
326 Other operator 2 outgoing call ratio in all calls  
327 Total call in last month  
328 Total outgoing call to operator 1 in last month  
329 Total voice package excess in last month  
330 Total outgoing call to operator 2 in last month  
331 Vas 3g Tenure  
332 Data usage KB in the last month  
333 Average volume of data used for last six months  
334 VAS entertainment package lus for last month  
335 VAS entertainment package maximum for last month  
336 VAS entertainment package rec for last month  
337 VAS entertainment package slope for last month  
338 VAS Total amount of incoming credit transfer for last three months  
339 VAS Prepaid SMS package in last 12 months  
340 VAS Social media usage in last 12 months  
341 VAS Subscription count  
342 VAS mobile apps maximum usage in last 12 months  
343 VAS mobile apps record usage in last 12 months  
344 VAS mobile apps slope usage in last 12 months  
345 VAS Total TL revenue from MMS usage for last 3 months  
346 VAS Total TL revenue from SMS usage for last 3 months  
347 VAS Usage count for last 3 months  
348 Last economic package  
349 Last shared package  
350 Most frequent Cell City for last month  
351 Current Tariff Group  
352 Previous Tariff Group  
353 Subscriber to benefit based upon the most recent company name for the six months  
354 Operating System of the equipment  
355 Contract Type  
356 Most frequent Data Cell City  
357 Last Port-IN Operator  
358 Last portin operator  
359 Last Port-In Reason  
360 Last portout operator

361 Last portout reason  
362 Average refill card unit  
363 Last refill card unit  
364 Maximum amount of refill card unit  
365 Minimum amount of refill card unit  
366 Most frequently refilled card unit  
367 Standard Deviation of amounts of refill card unit  
368 Activation Status Reason of the Subscriber  
369 Activation Reason of the Subscriber  
370 Bill City for postpaid  
371 Activation dealer city  
372 Gender  
373 Subscriber Last Equipment Manufacturer Group  
374 Last Equipment Marketing Model  
375 Current Simcard Type  
376 Subscriber Subscription system (Postpaid/Prepaid)  
377 Last 3 months PSTN usage segmentation  
378 Last month End Of Month Contract status  
379 General Segment  
380 General Sub Segment  
381 Last equipment category  
382 Number of incoming other operator 1 different B numbers for last month  
383 Number of incoming other operator 1 different B numbers for previous month  
384 Number of incoming Onnet different B numbers for last month  
385 Number of incoming Onnet different B numbers for previous month  
386 Number of outgoing other operator 1 different B numbers for last month  
387 Number of outgoing other operator 1 different B numbers for previous month  
388 Number of outgoing Onnet different B numbers for last month  
389 Number of outgoing Onnet different B numbers for previous month  
390 Number of incoming other operator 2 different B numbers for last month  
391 Number of incoming other operator 2 different B numbers for previous month  
392 Number of outgoing Telsim different B numbers for last month  
393 Number of outgoing other operator 2 different B numbers for previous month  
394 Last Tariff Duration  
395 Year Month of the Variable pool data

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