# SERVICE LEVEL SATISFACTION DURING SHORTAGES FOR HEALTH NETWORKS: BASE STOCK INVENTORY AND TRANSSHIPMENT POLICIES

A Thesis

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*To my family...*

# ABSTRACT

In this study, we consider a system of healthcare providers, which face the same uncertain supply disruptions (e.g., regional, nationwide, or worldwide drug shortages). Each hospital observes a stochastic demand and if demanded drug is unavailable, patients leave and receive care in another hospital system. As these unavailabilities hurt the brand value, hospital systems look for inventory sharing mechanisms among hospitals to mitigate the effect of uncertain supply disruptions. We explore reactive and proactive inventory sharing approaches by investigating how inventory related parameters affect service levels.

# ÖZETÇE

Bu çalışmada, sağlık hizmeti sağlayan kurumlardan oluşan ve ilaç tedarik süreçlerinde bölgesel, ulusal veya global ölçekte kıtlıklar gözlemlenen bir sistem ele alınmaktadır. Hastanelerdeki tedavi süreçleri kapsamında ortaya çıkan rassal ilaç talepleri, hastane sistemi dahilinde mevcut değilse, hastalar mevcut sistemden ayrılıp başka bir hastane zincirinde taleplerini karşılamaktadırlar. İlaçların talep edildiği anda elde bulunmama durumu hastane zincirlerinin marka değerlerine zarar verdiğinden dolayı, hastaneler ilaç kıtlıklarının etkilerini hafifleştirmek maksadıyla hastane zinciri içerisinde stok paylaşımı mekanizmalarını etkin kullanmayı arzulamaktadırlar. Bu çalışma dahilinde önetkin ve tepkisel stok paylaşımı politikaları altında envanter yönetimine dair parametrelerin sistemin servis seviyesi üzerindeki etkileri incelenmektedir.

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# Chapter I

# INTRODUCTION

Healthcare sector aims to improve society's well-being, in contrast to the commercial supply chains that aim to maximize profits. Impacts being significant, health expenditures constitute around 10% of countries' GDPs on average, with a market value of $8.7 trillion [1]. Ranging from 26% to 53%, hospitals have the largest share in healthcare expenditures [2]. It is estimated that hospitals spend between 10% and 18% of their budgets on inventory related investments [3], which makes it one of the most important cost components for hospitals.

On the other hand, number of customers served, i.e., service level, is another measure that is at least as critical as the cost metric mentioned above for hospitals. A high level of service is crucial in a healthcare system, because it represents the proportion of people who is able to receive timely treatment. Moreover, drug shortages are one of the obvious issues for healthcare systems, crippling the services provided to incoming patients, and hospitals need to learn living with this fact. As of April 2018, there are about 100 reported drug shortages in the U.S. [4]. These shortages plague the hospital system in various procedures such as antibiotic treatments, chemotherapy, pain medications in surgeries, etc. Some of the shortages have lasted from a few weeks to several months and some have stocks completely depleted whereas others brought inventories to critically low levels. Clearly, patients are able to receive drugs one way or the other, but being unable to provide drugs in a timely manner within a system has a negative impact on a healthcare system in terms of hospital chain's brand value. Stock transfers are the way to maintain a required service level while lowering the inventory level, or increasing the number of customers served with the same amount of items kept in the inventory of a system [5]. Therefore, hospitals in a healthcare system desiring to maintain at least a certain service level can achieve their target service level by allowing transshipments.

However, hospitals' inventory management practices are not in line with the stated goals. It is known that hospitals manage their inventories not using sophisticated methodical approaches, but simple procedures based on mostly intuition instead. Our motivation for this study comes from the fact that any improvement in efficiency within this area will have a direct effect on a human's life in terms of improved services, and on the healthcare system with the increase in number of people treated and a better image in the public eye.

# Chapter II

# LITERATURE REVIEW

We conducted an extensive literature review on inventory management in humanitarian operations. A total of 45 studies in the literature are classified into pre-disaster and post-disaster categories according to the time horizon considered in proposed studies. Our main findings are summarized in Tables 1, 2, 3, 4, 5, and 6. Interested readers can find further information on this study in [6].

| | | Demand | | | Facilities | | | |
|---|---|---|---|---|---|---|---|---|
| **Article** | **Network, Case Study (Disaster Type)** | **Randomness** | **Number of items** | **Perishability** | **Capacitated** | **Damage possibility** | **Stakeholders** | **Number of tiers** |
| Balcik and Beamon [7] | Global, Case Study (Earthquake) | Stochastic | $> 1$ | - | + | - | HO, beneficiaries | 2 |
| Bozkurt and Duran [8] | Global, Case study (Sudden onset disasters) | Stochastic | $> 1$ | - | - | - | HO, suppliers, beneficiaries | 3 |
| Bozorgi-Amiri et al. [9] | Regional, Case Study (Earthquake) | Stochastic | $> 1$ | - | + | + | HO, suppliers, beneficiaries | 3 |
| Campbell and Jones [10] | Hypothetical | Stochastic | 1 | - | - | + | suppliers, beneficiaries | 2 |
| Chakravarty [11] | Hypothetical | Stochastic | 1 | - | - | - | HO, suppliers, beneficiaries | 3 |
| Döyen et al. [12] | Hypothetical (Earthquake) | Stochastic | $> 1$ | - | + | - | HO, beneficiaries | 3 |
| Duran et al. [13] | Global, Case study (Sudden onset disasters) | Stochastic | $> 1$ | - | + | - | HO, suppliers, beneficiaries | 3 |
| Galindo and Batta [14] | Case Study (Hurricane) | Stochastic | 1 | - | + | + | HO, beneficiaries | 3 |
| Garrido et al. [15] | Hypothetical, Case Study (Flood) | Stochastic | $> 1$ | - | + | - | HO, supplier, beneficiaries | 3 |
| Hong et al. [16] | Case Study (Hurricane) | Stochastic | 1 | - | + | - | HO, beneficiaries | 2 |
| Klibi et al. [17] | Regional, Case Study (Natural disasters) | Stochastic | $> 1$ | - | + | - | HO, suppliers, beneficiaries | 3 |
| Lodree and Taskin [18] | Hypothetical (Sudden onset disasters) | Stochastic | 1 | - | - | - | HO, beneficiaries | 2 |
| Manopiniwes et al. [19] | Country, Case Study (Flood) | Deterministic | $> 1$ | - | + | - | HO, beneficiaries | 2 |
| Mete and Zabinsky [20] | City, Case Study (Earthquake) | Stochastic | $> 1$ | - | + | - | HO, beneficiaries | 2 |
| Mohammadi et al. [21] | City, Case Study (Earthquake) | Stochastic | $> 1$ | - | + | - | HO, beneficiaries | 2 |
| Noyan [22] | Regional, Case Study (Hurricane) | Stochastic | $> 1$ | - | + | + | HO, beneficiaries | 2 |
| Paul and MacDonald [23] | City, Case Study (Earthquake) | Stochastic | 1 | - | + | + | HO, beneficiaries | 2 |
| Rabbani et al. [24] | Country, Case Study (Earthquake) | Stochastic | $> 1$ | + | + | - | HO, beneficiaries | 2 |
| Rawls and Turnquist [25] | Regional, Case Study (Hurricane) | Stochastic | $> 1$ | - | + | + | HO, beneficiaries | 2 |
| Rawls and Turnquist [26] | Regional, Case Study (Hurricane) | Stochastic | $> 1$ | - | + | + | HO, beneficiaries | 2 |
| Renkli and Duran [27] | City, Case Study (Earthquake) | Deterministic | $> 1$ | - | + | - | HO, beneficiaries | 2 |
| Tofighi et al. [28] | City, Case Study (Earthquake) | Stochastic | $> 1$ | - | + | + | HO, beneficiaries | 3 |
| Van Hentenryck et al. [29] | Hypothetical (Hurricane) | Stochastic | 1 | - | + | - | HO, beneficiaries | 2 |

**Table 1:** Problem aspects in pre-disaster inventory management.

With this literature review, we establish that there does not exist a study which considers transshipment in a service level based approach in humanitarian operations, which is a similar field to healthcare in terms of supply and demand characteristics.

There exist a number of studies in the area of inventory management for systems considering transshipments. A review of these studies are presented in [5] and [52].

| Article | Metrics/Objectives |
|---|---|
| Balcik and Beamon [7] | [Max] Total expected demand covered<br>Cost of transporting supplies and locating facilities |
| Bozkurt and Duran [8] | [Min] Expected average response time |
| Bozorgi-Amiri et al. [9] | [Min] Expected total costs (facility, ordering, transportation, transshipment, holding, shortage)<br>[Min] variability of the total cost<br>[Min] Sum of maximum shortage across items |
| Campbell and Jones [10] | [Min] Expected total costs (ordering, restocking/disposal, transportation, shortage) |
| Chakravarty [11] | [Max] Buyer's expected surplus<br>[Min] Expected ordering, holding, and transportation costs<br>[Min] Expected response time |
| Döyen et al. [12] | [Min] Expected total costs (facility, holding, transportation, shortage)<br>Response time |
| Duran et al. [13] | [Min] Expected average response time |
| Galindo and Batta [14] | [Min] Expected total costs (facility, transportation, destroyed supplies) |
| Garrido et al. [15] | [Min] Expected total costs (holding, transportation)<br>Probability of demand satisfaction |
| Hong et al. [16] | [Min] Expected total costs (facility, ordering)<br>Probability of satisfying demand within a target response time<br>Equity in demand satisfaction (in terms of fraction of demand satisifed) |
| Klibi et al. [17] | [Min] Total shortage cost<br>Cost of locating facilities and holding inventory |
| Lodree and Taskin [18] | [Min] Expected total cost (ordering, holding, shortage) |
| Manopiniwes et al. [19] | [Min] Expected total costs (facility, holding, transportation)<br>Response time |
| Mete and Zabinsky [20] | [Min] Expected total costs (facility, ordering, transportation, shortage) |
| Mohammadi et al. [21] | [Max] Total expected demand coverage<br>[Min] Expected total costs (facility, supply, transportation)<br>[Min] Maximum difference between demand nodes' satisfaction rates (equity) |
| Noyan [22] | [Min] Expected total costs (facility, holding, transportation, shortage) |
| Paul and MacDonald [23] | [Min] Expected total costs (facility, ordering, fatalities) |
| Rabbani et al. [24] | [Min] Expected total costs (facility, ordering, transportation, shortage, spoilage) |
| Rawls and Turnquist [25] | [Min] Expected total costs (facility, holding, transportation, shortage) |
| Rawls and Turnquist [26] | [Min] Expected total costs (facility, holding, transportation, shortage)<br>Probability of satisfying demand within a target response time |
| Renkli and Duran [27] | [Min] Total weighted response time<br>Probability of satisfying demand |
| Tofighi et al. [28] | [Min] Expected total costs (facility, ordering, holding, transportation)<br>[Min] Total weighted response time<br>[Min] Maximum weighted response time (equity)<br>[Min] Shortage and surplus |
| Van Hentenryck et al. [29] | [Min] Expected total costs (facility, inventory, transportation, shortage) |

**Table 2:** Performance measures in pre-disaster inventory management studies.

| Article | Model Type | Solution Approach |
|---|---|---|
| Balcik and Beamon [7] | Two-stage stochastic programming | Exact; Optimization solver |
| Bozkurt and Duran [8] | Two-stage stochastic programming | Exact; Optimization solver |
| Bozorgi-Amiri et al. [9] | Robust optimization, two-stage stochastic programming; Multi-objective optimization | Exact; Optimization solver |
| Campbell and Jones [10] | Stochastic programming | Exact; Numerical techniques; Heuristic |
| Chakravarty [11] | Stochastic optimization | Exact; Numerical techniques |
| Döyen et al. [12] | Two-stage stochastic programming | Exact; Optimization solver; Heuristic; Lagrangean relaxation |
| Duran et al. [13] | Two-stage stochastic programming | Exact; Optimization solver |
| Galindo and Batta [14] | Stochastic optimization | Exact; Optimization solver |
| Garrido et al. [15] | Stochastic programming; Chance-constraints | Heuristic; Sample Average Approximation; Optimization solver |
| Hong et al. [16] | Two-stage stochastic programming; Chance-constraints | Exact; Optimization solver; Gale-Hoffman inequalities; Combinatorial pattern-based method |
| Klibi et al. [17] | Two-stage stochastic programming | Exact; Optimization solver; Sample average approximation |
| Lodree and Taskin [18] | Newsvendor model | Exact; Numerical techniques; Mathematical computing software |
| Manopiniwes et al. [19] | Mixed integer programming | Exact; Optimization solver |
| Mete and Zabinsky [20] | Two-stage stochastic programming | Exact; Optimization solver |
| Mohammadi et al. [21] | Two-stage stochastic programming; Multi-objective optimization | Heuristic; Particle Swarm Optimization |
| Noyan [22] | Two-stage stochastic programming; Conditional Value-at-Risk | Exact; Optimization solver; Benders-decomposition algorithm |
| Paul and MacDonald [23] | Stochastic optimization | Heuristic; Evolutionary Algorithm; Newsvendor approach |
| Rabbani et al. [24] | Two-stage stochastic programming | Exact; Optimization solver |
| Rawls and Turnquist [25] | Two-stage stochastic programming | Exact; Optimization solver; Lagrangian L-shaped method |
| Rawls and Turnquist [26] | Two-stage stochastic programming; Chance-constraints | Exact; Optimization solver |
| Renkli and Duran [27] | Mixed integer programming; Chance-constraints | Exact; Optimization solver |
| Tofighi et al. [28] | Two-stage possibilistic-stochastic programming; Multi-objective optimization | Exact; Optimization solver; Heuristic; Tailored Differential Algorithm |
| Van Hentenryck et al. [29] | Two-stage stochastic programming | Exact; Optimization solver |

**Table 3:** Methodological approaches in pre-disaster inventory management studies.

| | | | Demand | | | | | Facilities | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Article** | **Network, Case Study (Disaster Type)** | **Randomness** | **Backorder/Lost** | **Number of items** | **Perishability** | **Regular/Emergency** | **Capacitated** | **Damage possibility** | **Stakeholders** | **Number of tiers** |
| *Post-Disaster* | Beamon and Kotleba [30] | Case Study (Complex emergency) | Stochastic | backorder | 1 | - | - | - | - | HO, supplier, beneficiaries | 2 |
| | Beamon and Kotleba [31] | Case Study (Complex emergency) | Stochastic | backorder | 1 | - | - | - | - | HO, supplier, beneficiaries | 2 |
| | Das and Hanaoka [32] | Case Study (Earthquake) | Stochastic | lost | $> 1$ | - | - | - | - | HO, beneficiaries | 2 |
| | McCoy and Brandeau [33] | Case Study (Complex emergency) | Stochastic | lost | 1 | - | - | - | - | HO, supplier, beneficiaries | 2 |
| | Natarajan and Swaminathan [34] | Hypothetical | Stochastic | backorder | 1 | - | - | - | - | HO, beneficiaries | 2 |
| | Noyan et al. [35] | City, Case Study (Earthquake) | Stochastic | lost | 1 | - | - | + | - | HO, beneficiaries | 3 |
| | Ozbay and Ozguven [36] | Hypothetical | Stochastic | lost | 1 | - | - | + | | HO, supplier, beneficiaries | 2 |
| | Ozguven and Ozbay [37] | Case Study (Hurricane) | Stochastic | lost | $> 1$ | + | - | + | - | HO, supplier, beneficiaries | 2 |
| | Rabbani et al. [24] | Case Study (Earthquake) | Stochastic | backorder | $> 1$ | + | - | + | - | HO, beneficiaries | 2 |
| | Roni et al. [38] | Hypothetical | Stochastic | lost | 1 | - | + | - | - | HO, supplier, beneficiaries | 2 |
| | Roni et al. [39] | Hypothetical | Stochastic | lost | 1 | - | + | - | - | HO, supplier, beneficiaries | 2 |
| | Rottkemper et al. [40] | Country, Case Study (Earthquake) | Stochastic | backorder | 1 | - | - | - | - | HO, beneficiaries | 2 |
| | Shen et al. [41] | Case Study (Anthrax attack) | Deterministic | lost | 1 | + | - | - | - | Government, manufacturer | 2 |
| | Yadavalli et al. [42] | Hypothetical | Stochastic | lost | $> 1$ | + | - | + | - | HO, supplier, beneficiaries | 2 |
| *Post-Warning* | Davis et al. [43] | Regional, Case Study (Hurricane) | Stochastic | lost | 1 | - | - | + | + | Multiple HOs, beneficiaries | 2 |
| | Lodree and Taskin [44] | Case Study (Hurricane) | Stochastic | lost | 1 | - | + | - | - | Manufacturer or retailer, supplier | 2 |
| | Lodree [45] | Case Study (Hurricane) | Deterministic | lost | 1 | - | + | - | - | Retailer, supplier | 2 |
| | Lodree et al. [46] | Regional, Case Study (Hurricane) | Stochastic | lost | 1 | - | - | - | - | Manufacturer, retailer | 2 |
| | Pacheco and Batta [47] | Hypothetical (Hurricane) | Stochastic | lost | 1 | - | - | + | + | HO, beneficiaries | 3 |
| | Rawls and Turnquist [48] | Regional, Case Study (Hurricane) | Stochastic | lost | $> 1$ | - | - | + | - | HO, beneficiaries | 2 |
| | Salas et al. [49] | Case Study (Hurricane) | Stochastic | lost | 1 | + | - | - | - | HO, beneficiaries | 2 |
| | Taskin and Lodree [50] | Case Study (Hurricane) | Stochastic | lost | 1 | - | + | - | - | Manufacturer or retailer, supplier | 2 |
| | Taskin and Lodree [51] | Case Study (Hurricane) | Stochastic | lost | 1 | - | + | - | - | Retailer, supplier | 2 |

**Table 4:** Problem aspects in post-disaster/post-warning inventory management.

The literature can be classified into two categories according to timing of transshipments. *Proactive* transshipments allow the redistribution of stocks between retailers, whereas *reactive* stock transfers occur only when a stockout is observed. We categorize the studies similar to ours according to these streams of literature.

Managing transshipments proactively is a widely employed mechanism of handling stock transfers within a system. Nasr et al. [53] consider a production environment consisting of two locations, and random supply disruptions may be observed. Transshipments are used in a way that when a shortage occurs in supplier, retailers adjust their safety stock levels instantaneously once per shortage period by inventory sharing to hedge against the risk of stockout, and they aim to determine these safety stock levels. van Wijk et al. [54] analyze fractions of demand satisfied directly from a retailers own stock, using transshipments, and emergency ordering, for a spare part inventory system adopting continuous review policies and transshipment thresholds,

| Article | Metrics/Objectives |
|---|---|
| Beamon and Kotleba [30] | [Min] Expected total costs (ordering, holding, backorder) |
| Beamon and Kotleba [31] | [Min] Expected total costs (ordering, holding, backorder)<br>Maximum proportion of emergency order cycles<br>Response time |
| Das and Hanaoka [32] | [Min] Expected total cost (fixed and variable ordering, inventory, shortage)<br>Expected relief shortage |
| Davis et al. [43] | [Min] Expected total costs (transportation, shortage, supply loss)<br>Equity in demand satisfaction (in terms of fraction satisifed)<br>Response time |
| Lodree and Taskin [44] | [Min] Expected total costs (ordering, holding, shortage) |
| Lodree [45] | [Min] Maximum total costs (ordering, holding) |
| Lodree et al. [46] | [Min] Expected total costs (production, pre- and post- hurricane transportation, holding, shortage) |
| McCoy and Brandeau [33] | [Min] Expected total response penalty (based on units of unsatisfied demand)<br>Inventory and shipment costs |
| Natarajan and Swaminathan [34] | [Min] Expected total costs (ordering, holding, backorder) |
| Noyan et al. [35] | [Max] Expected total accessibility<br>Maximum response time (equity)<br>Equity in demand satisfaction (in terms of fraction of demand satisifed) |
| Ozbay and Ozguven [36] | [Min] Expected total cost (inventory, shortage, surplus)<br>Probability of meeting demand |
| Ozguven and Ozbay [37] | [Min] Expected total cost (inventory, shortage, surplus)<br>Probability of meeting demand |
| Pacheco and Batta [47] | [Min] Expected total costs (facility, transportation, shortage, acquisition, destroyed supplies) |
| Rabbani et al. [24] | [Min] Expected total costs (ordering, holding, backorder, shortage) |
| Rawls and Turnquist [48] | [Min] Expected total costs (facility, holding, transportation, shortage)<br>Probability of demand satisfaction within a target response time |
| Roni et al. [38] | [Min] Expected total costs (ordering, holding, shortage) |
| Roni et al. [39] | [Min] Expected total costs (ordering, holding, shortage) |
| Rottkemper et al. [40] | [Min] Expected total costs (facility, ordering, transportation, shortage) |
| Salas et al. [49] | [Min] Expected total costs (ordering, variable, holding, shortage, disposal) |
| Shen et al. [41] | [Min] Expected total costs (ordering, holding, variable, salvage) |
| Taskin and Lodree [50] | [Min] Expected total costs (variable, holding, shortage) |
| Taskin and Lodree [51] | [Min] Expected total costs (variable, holding, shortage) |
| Yadavalli et al. [42] | Average number of satisfied demands<br>Average number of lost demands<br>Average inventory level<br>Average number of substituted demands |

**Table 5:** Performance measures in post-disaster/post-warning inventory management studies.

| Article | Model Type / Policy | Solution Approach |
|---|---|---|
| Beamon and Kotleba [30] | Multi-supplier (Q, r) policy | Exact; Numerical techniques |
| Beamon and Kotleba [31] | Multi-supplier (Q, r) policy | Exact; Numerical techniques; Heuristic; Silver Meal; Simulation |
| Das and Hanaoka [32] | (Q, r) policy | Exact; Numerical techniques |
| Davis et al. [43] | Two-stage stochastic programming | Exact; Optimization solver |
| Lodree and Taskin [44] | Newsvendor policy; Optimal stoping with Bayesian updates | Exact; Dynamic programming algorithm; Mathematical computing software & postwarning |
| Lodree [45] | EOQ policy | Exact; Numerical techniques & postwarning |
| Lodree et al. [46] | Two-stage stochastic programming | Exact; Optimization solver |
| McCoy and Brandeau [33] | Base stock policy | Exact; Dynamic programming; Numerical techniques; Simulation |
| Natarajan and Swaminathan [34] | Base stock policy | Exact; Numerical techniques |
| Noyan et al. [35] | Two-stage stochastic programming | Exact; Optimization solver; Benders decomposition |
| Ozbay and Ozguven [36] | Hungarian Inventory Control Model; Stochastic Programming | Heuristic; pLEPs algorithm |
| Ozguven and Ozbay [37] | Hungarian Inventory Control Model; Stochastic Programming | Heuristic; pLEPs algorithm; Mathematical computing software |
| Pacheco and Batta [47] | Stochastic programming | Exact; Optimization solver |
| Rabbani et al. [24] | (Q, r) policy | Exact; Numerical techniques; Fuzzy ranking method |
| Rawls and Turnquist [48] | Two-stage stochastic programming | Exact; Optimization solver |
| Roni et al. [38] | (Q, r) policy | Exact; Optimization solver; Mixed Integer Programming; Level crossing method |
| Roni et al. [39] | (Q, r) policy | Heuristic; Tabu Search; Level crossing method |
| Rottkemper et al. [40] | Mixed Integer Programming | Exact; Optimization solver |
| Salas et al. [49] | Newsvendor policy; Stochastic programming | Exact; Optimization solver & postwarning |
| Shen et al. [41] | Modified Economic Manufacturing Quantity policy | Exact; Unconstrained optimization; Numerical techniques |
| Taskin and Lodree [50] | Multi-stage stochastic programming | Exact; Optimization solver; Heuristic (scenario reduction) |
| Taskin and Lodree [51] | Newsvendor policy with Bayesian updates | Exact; Mathematical computing software & postwarning |
| Yadavalli et al. [42] | Multi-item joint (s, S) replenishment policy | Exact; Numerical techniques; Steady state analyses |

**Table 6:** Methodological approaches in post-disaster/post-warning inventory management studies.

also considering the recovery rate for failed parts. Cheong [55] introduce a newsvendor model for multiple demand locations where proactive transshipments may occur immediately, and perishability of products is included. Feng et al. [56] develop a periodic review base stock policy for a multi-location problem that considers perishability, where proactive transshipments are also permitted, and model it as a Markov decision process. Hochmuth and Köchel [57] consider a similar setting, where retailers adopt periodic review $(s, S)$ policies for ordering, and continuous review policies for transshipment decisions. They aim to find optimal variables for policies using simulation. Zhao et al. [58] propose a queueing theory approach for a system consisting of multiple retailers. Inventory level can both increase due to production and decrease because of demand occurrence, both with Poisson rates, and retailers can request transshipments even when they have a positive inventory. They aim to find optimal maximum inventory levels and thresholds for requesting transshipments and accepting transshipment requests. Tagaras and Vlachos [59] evaluates the performance of a system with two locations, which replenish their inventories periodically, with respect to redistribution of stock between the two locations, which may happen only once in each of the replenishment periods.

There also exist studies that consider transshipments with a reactive approach. Archibald et al. [60] introduce a periodic review policy using discrete time Markov decision process for two retailers and one supplier, where they allow transshipments and emergency orders at any time in a period. They extend this study to a multi-location setting in [61]. Axsäter [62] defines a decision rule for transshipments between retailers considering future costs and outstanding orders, of which arrival times are known, within a system consisting of multiple retailers which utilize continuous review policies. Minner and Silver [63] study a continuous review $(R, Q)$ inventory system with two identical retailers subject to Poisson demands, which can use transshipments in case of a stockout until replenishments arrive. Ramakrishna et al. [64] develop a Markov decision process to decide on periodic review policy parameters and whether to accept a transshipment request according to time remaining for the replenishment.

Herer and Rashit [65] evaluate a two-location single-period newsvendor model with joint and fixed replenishment costs, and transshipments when required. Later in [66], this study is extended by modeling it as a network flow problem. Paterson et al. [67] adopt the policy presented in [62] to derive a transshipment rule that behaves every reactive transshipment as a chance for proactive reallocation of current inventory in the system. Glazebrook et al. [68] also consider the hybrid transshipment policy in a periodically replenished system. In Grahovac and Chakravarty [69], a base stock policy with a transshipment threshold is proposed, but transshipments are only allowed when an emergency order cannot be received in time to meet the observed demand. Olsson [70] propose a $(Q, R)$ policy with reactive transshipments for a two location system under exponentially distributed lead times.

The closest studies in the literature to our approach are papers proposing models which consider service level in a system. Axsäter [71] proposes a continuous review replenishment policy for a network consisting of multiple retailers, and transshipments between retailers is allowed only in one direction. Under normally distributed demand, a non-linear approximation technique based on different fill rates for the sources of demand satisfaction is developed for the problem, and their models results are compared with a simulation study. Olsson [72] model the observed demand in [71] as a Poisson process, and propose two models based on $(S-1, S)$ and $(Q, R)$ policies. Kukreja and Schmidt [73] investigate a system with multiple retailers at demand locations and single supplier. Each of the retailers use continuous review $(s, S)$ policies, and complete inventory pooling between retailers is allowed. Under stochastic demand parameters for retailers, they aim to measure the performance of the system using simulation when inventory sharing is allowed while order quantities are set as EOQ and reorder levels are adjusted iteratively.

It can be observed from aforementioned articles that among transshipment studies, the most dominant metric is the cost incurred by considered systems, which consists of different components such as holding, transshipment, backordering, emergency ordering, and lost sales costs. However, due to unique nature of healthcare systems, the achieved service level is at least as much important as the operating cost of the system. Therefore, with this study, we address this gap in the literature, and aim to make meaningful observations regarding the service level of the system while keeping the total cost incurred at a reasonable level.

To the best of our knowledge, there is no study that conducts service level based analyses of proactive and reactive transshipment policies for continuous review base stock inventory systems. There are a number of studies with uncertain lead times, however, their results cannot be adapted for a system that observes shortages leading back-and-forth. Our objective in this study is to investigate the effect of different *partial pooling* approaches on the service level for a healthcare system observing shortages, while deciding on optimal *transshipment threshold* levels of each hospital.

The rest of this chapter is organized as follows: The considered system is described in further detail in Chapter 3. Proactive and reactive transshipment policies are introduced in Chapter 4 and Chapter 5, respectively. A numerical study is presented in Chapter 6 to measure the performance of our proposed methodology and to provide managerial insights. We make our concluding remarks in Chapter 7.

# Chapter III

# SYSTEM PROPERTIES AND KEY PERFORMANCE INDICATORS

We consider the inventory of a healthcare system with multiple hospitals and a single supplier. Hospitals' inventories are replenished from the supplier according to a continuous review $(S, S-1)$ base stock policy. We assume zero lead time for replenishments, a commonly made assumption due to healthcare systems' frequent replenishment cycles (see e.g., [3], [74]). In general, hospitals are visited at least once a day by suppliers for replenishments, if necessary. In other words, when a hospital places an order, the delivery is usually made the same day (or next day in the worst case), if the supplier has drugs in stock. However, the supplier is open to disruptions that are stochastic in both duration and frequency, and hospitals need to hedge against the risk of stock-out while a shortage for the supplier exists. Zero lead time helps us ensure that at the beginning of a shortage period, hospitals' inventory levels are exactly equal to their order-up-to levels. Moreover, we assume that transshipments occur instantaneously, which is widely accepted in the literature (see e.g., [75], [53]).

In practice, hospitals in a healthcare network do not determine their base stock levels by utilizing complex mathematical inventory policies. Since a higher service level is more desirable than operating with a lower holding cost in healthcare networks, hospitals neglect the amount of holding cost they pay, and they stock as much drugs as their inventory capacities can handle. Therefore, throughout this study, we do not consider holding cost in our analyses, and we do not aim to make any observations or conclusions regarding base stock policies. Neglecting holding cost helps us to establish decision rules that consider the trade-off between movement of inventory on hand and hedging against the risk of potential lost sales.

In the literature, there are different approaches for inventory pooling, i.e., retailers' allowed way of making stock transfers, namely complete and partial pooling policies. As it can be understood from the names of these policies, these policies are defined whether retailers share all or a part of their stocks. Complete pooling is a more adequate policy for systems where holding and backordering costs are significantly effective than transshipment costs [5], which would actually be very similar to the case for the considered system if replenishments could be made regularly. However, when shortages that are unknown in length and frequency are present in the system, there is a trade-off for hospitals between assisting the hospital requesting the stock transfer to satisfy its current demand and keeping inventory for its own potential demand occurrences.

What hospitals actually desire is to keep infinitely many items in stock, so that they never tranship and never lose a customer, but this is virtually impossible. In terms of service level, sharing inventory at its fullest is optimal, yet there are costs associated with inventory pooling, and hospitals aim to keep related costs or amount of stock transfers at least at a certain level. Therefore, for healthcare systems observing shortages, it becomes a strategic decision to define the levels until which transshipments are efficient for ensuring a certain service level within the system, and this is the reason why we do not consider complete pooling in our study.

## *3.1 Sharing Mechanism*

From this point on, we refer the levels that hospitals reject the transshipment requests and keep their remaining inventories for their own potential demand occurrences as *transshipment thresholds*. In addition, we assume that hospitals' inventory levels are totally visible to each other, so they exactly know which ones can make transshipments when requesting a stock transfer. In line with our zero lead time replenishments, we assume transshipments can also be performed instantaneously as these times are usually negligible in reality.

Within the scope of this study, we investigate two different partial pooling assumptions:

- A **proactive** approach where hospitals request a stock transfer when they hit their transshipment threshold. The request is granted by another hospital that is above its transshipment threshold. That means, individual inventories above transshipment thresholds are pooled, acting as a single entity. For prolonged shortages, this ensures there exists a point in time where all hospitals are at their thresholds. From that point on hospitals do not share inventory.
- A **reactive** approach in which hospitals request a stock transfer only when they are out of stock. The request is granted by another hospital that is above its transshipment threshold.

In the literature, studies considering supply unavailability and demand uncertainty mostly utilize Markov chains, due to their interest in steady-state behavior of the systems. However, in this study, we are interested in the service levels during shortage periods that are non-recursive. Therefore, we use properties of Poisson processes and exponential distributions to derive these probabilities.

## *3.2 Service Levels for Hospitals*

In this study, we assume each patient arrival implies a demand. There is no arrival placing a batch order as in a commercial supply chain. Therefore, the term *service level* in this study indicates fill (service) *rate*. We focus on the following two types of service levels in the sequel.

### 3.2.1 Type I Service Level

For a hospital system, we define the Type I service level as the fill rate of the entire system during shortage. That can be translated as the probability of satisfying a patient demand during shortage. Formally, *Type I service level during shortage* is defined as

$$\alpha_S = P(\text{demand during shortage} \leq \text{inventory on hand at the beginning of shortage period}) \tag{1}$$

### 3.2.2 Type II Service Level

For a hospital system, we aim to describe the proportion of total demand that is delivered without delay from stock on hand during shortage. As there is no lead time for transshipments, we formally define the *Type II service level during shortage* as

$$\beta_S = \alpha_S - \frac{E[\text{transshipments during shortage}]}{E[\text{demand during shortage}]} \tag{2}$$

It should be noted that Type I and II service levels during shortage can be generalized in a fairly straightforward way for the system in general. Fraction of shortage durations can be computed depending on shortage and recovery rates. As both service levels are 100% during a non-shortage period, general Type I ($\alpha$) and Type II ($\beta$) service levels can be computed using conditioning as follows:

$$\alpha = \frac{\alpha_S \times \text{shortage rate} + \text{recovery rate}}{\text{shortage rate} + \text{recovery rate}} \tag{3}$$

$$\beta = \frac{\beta_S \times \text{shortage rate} + \text{recovery rate}}{\text{shortage rate} + \text{recovery rate}} \tag{4}$$

# Chapter IV

# PROACTIVE SHARING POLICY

For this part of the study, we concentrate on the proactive approach that allows hospitals to make stock adjustments when they hit their individual transshipment thresholds. Occurrence of a transshipment depends on two factors:

- The hospital requesting a transfer has an inventory level of exactly its transshipment threshold. In other words, a hospital makes a transfer request to ensure a predetermined amount of safety stock on hand.
- There exists a hospital in the system with an inventory level above its threshold to serve the hospital requesting the transfer.

In other words, hospitals in the network make transshipments to consume the partially pooled inventory together at the beginning. When all of their inventory levels become equal to their transshipment thresholds, they begin satisfying the observed demand from their own inventories, if possible. Any unsatisfied demand becomes lost, that is backordering is not allowed.

Let us introduce the following notation:

$\mathcal{B}$: total base stock,

$B_i$: base stock level at hospital $i$,

$N$: number of hospitals in the system,

$\Omega$: total safety stock,

$\gamma_i$: proportion of total safety stock for hospital $i$, $\sum \gamma_i = 1$

$\omega_i$: transshipment threshold for hospital $i$, $\omega_i = \gamma_i \Omega$

$\Phi$: total amount of pooled inventory,

$\phi_i$: amount of inventory reserved for pooling at hospital $i$,

$\lambda_i$: demand rate for hospital $i$,

$\mu$: recovery rate for shortage occurrences.

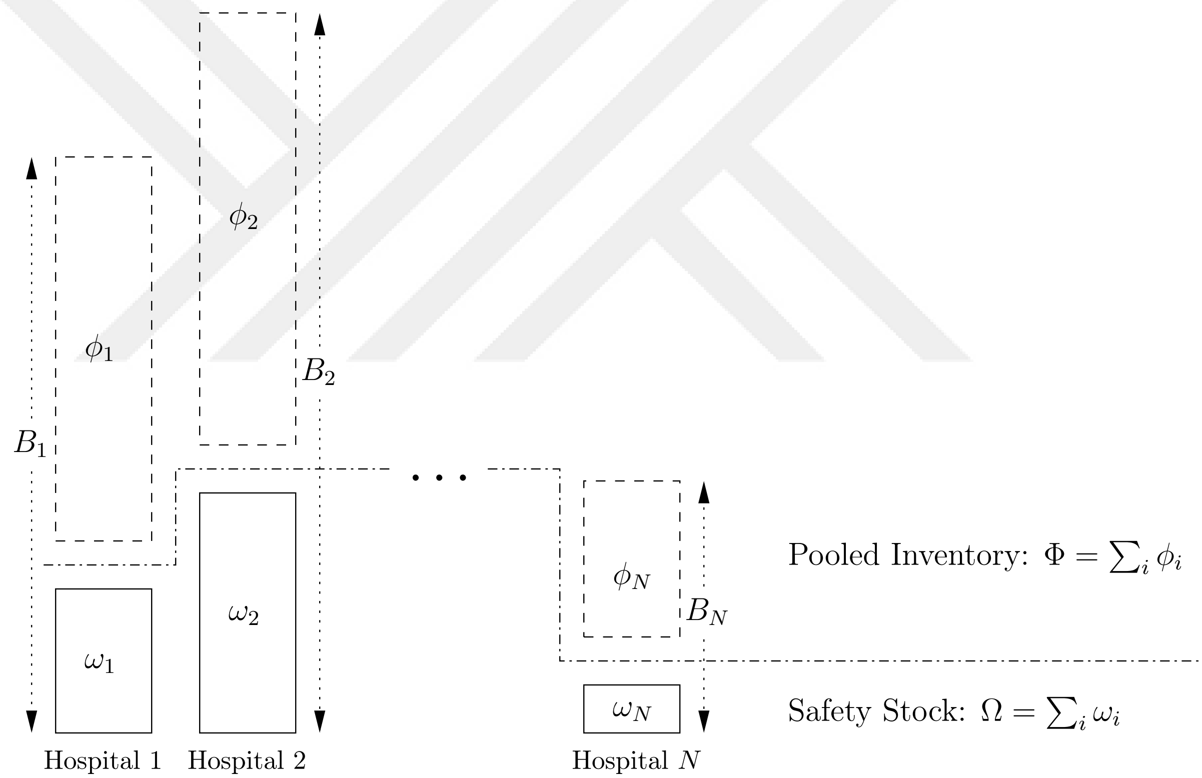


**Figure 1:** Allocation of pooled inventory and safety stocks.

## *4.1 Inventory Parameters that Maximize Type I Service Level*

We consider a system of $N$ hospitals, each observing a Poisson distributed demand with rate $\lambda_i$, and shortage durations are exponentially distributed with rate $\mu$. Hospitals regularly stock a total of $\mathcal{B}$ items, and their inventories are continuously replenished according to a base stock policy. When a shortage is present, they reserve a total of $\Omega$ items, and $0 \leq \gamma_i \leq 1$ is the proportion of safety stock held by hospital $i$, where $\sum_i \gamma_i = 1$. During shortages, they consume the system's inventory together by making transshipments when needed, until all of their inventory levels become their transshipment threshold, i.e., $\gamma_i \Omega, \forall i$. When all hospitals' inventories reaches respective thresholds, hospitals begin to satisfy any observed demand from their own inventories. Any unsatisfied patient demand is assumed lost. This happens when safety stocks are depleted as no more transshipments are allowed.

First, we formally define the equation for Type I service level as a function of inventory variables.

**Theorem 1.** *For a system of hospitals, given total base stock ($\mathcal{B}$), total threshold value ($\Omega$), and proportion of each hospital in the total threshold ($\gamma_i$), Type I service level is*

$$\alpha_S = 1 - \left( \frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j} \right)^{\mathcal{B}-\Omega} \sum_i \left( \frac{\lambda_i}{\lambda_i + \mu} \right)^{\gamma_i \Omega} \frac{\lambda_i}{\sum_j \lambda_j}.$$

It is easy to see that an increase in $\mathcal{B}$ increases the service level achieved. However, since we aim to keep transshipments between hospitals at a certain level, we require $\Omega$ to be a positive value. Then, the allocation of total threshold between hospitals can be determined by finding the value for each $\gamma_i$ that maximizes the service level equation.

**Corollary 1.** *As $\Omega$ decreases, for fixed $\mathcal{B}$, Type I service level increases.*

Proof of Corollary 1 is straightforward, and can be easily observed from Theorem 1. Therefore, it can be stated that optimal Type I service level is achieved when $\Omega$ is zero. However, from Theorem 1, a conclusion on Type II service level cannot be made.

**Corollary 2.** *For a total base stock level of $\mathcal{B}$, the best possible Type I service level is attained under complete pooling, where*

$$\alpha_S^{\max} = 1 - \left( \frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j} \right)^{\mathcal{B}}$$

.

It should also be noted that for a total base stock level of $\mathcal{B}$, the service level when no transshipments are allowed is

$$\alpha_S^{\text{No Transshipment}} = 1 - \sum_i \left(\frac{\lambda_i}{\lambda_i + \mu}\right)^{\gamma_i \mathcal{B}} \frac{\lambda_i}{\sum_j \lambda_j}.$$

Next, we present what optimal threshold levels should be for each individual hospital.

**Theorem 2.** *For a system of hospitals, given total threshold* $(\Omega)$, *in order to maximize Type I service level, proportions of transshipment threshold for any two hospitals* $k$ *and* $m$ *must hold the following equality:*

$$\gamma_m \ln\left(\frac{\lambda_m}{\mu + \lambda_m}\right) - \gamma_k \ln\left(\frac{\lambda_k}{\mu + \lambda_k}\right) = \frac{1}{\Omega} \ln\left(\frac{\lambda_k \ln\left(\frac{\lambda_k}{\mu+\lambda_k}\right)}{\lambda_m \ln\left(\frac{\lambda_m}{\mu+\lambda_m}\right)}\right)$$

.

From Theorem (2), proportions of total threshold for each hospital $i$, $\gamma_i$, can be found by solving a set of linear equations including $\sum_i \gamma_i = 1$.

## *4.2 Amount to Pool at Each Hospital that Maximize Type II Service Level*

After defining the service level for given base stocks and total threshold, the question comes to how to determine the amount of inventory to be pooled. Knowing the amounts to stock at each location while pooling inventory, allocation of total threshold among hospitals can be made using Theorem 2.

In practice, hospitals proactively share their inventories for a predefined time period from the beginning of a shortage occurrence. They begin to satisfy patient demand from their own safety stocks if the shortage still continues when the sharing period is over. Working it backwards, we can determine the number of items required for a given sharing duration from the properties of Poisson process.

Next, the allocation of pooled inventory among locations is to be decided in a way that minimizes the expected number of transshipments in the system to maximize efficiency. For this purpose, we first determine the expected total amount of demand satisfied in any system given an inventory level.

**Lemma 1.** *For a system with demand rate $\lambda$, recovery rate $\mu$, and current inventory level of $I$,*

$$E[\textit{demand satisfied by the system}] = \frac{\lambda}{\mu}\left(1 - \left(\frac{\lambda}{\mu + \lambda}\right)^{I}\right)$$

Using Lemma 1, given the amount of inventory reserved for sharing, we can find the total amount of demand satisfied by the system and by each hospital during sharing period. Assuming that a hospital that transfers an item cannot request a transshipment, we can calculate the expected total amount of demand satisfied by the system using transshipments.

**Theorem 3.** *For a system of $i$ hospitals, expected amount of demand satisfied with transshipments can be calculated as follows:*

$$E[\textit{satisfied with transshipment}] = \sum_i \frac{\lambda_i}{\mu}\left(\left(\frac{\lambda_i}{\mu + \lambda_i}\right)^{\phi_i} - \left(\frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j}\right)^{\Phi}\right).$$

**Theorem 4.** *The expected amount of demand satisfied from hospitals' own inventories, namely Type II service level, is calculated as follows:*

$$\beta = 1 - \left(\frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j}\right)^{\mathcal{B}-\Omega} \sum_i \left(\frac{\lambda_i}{\lambda_i + \mu}\right)^{\gamma_i \Omega} \frac{\lambda_i}{\sum_j \lambda_j} - \sum_i \frac{\lambda_i}{\sum_j \lambda_j}\left(\left(\frac{\lambda_i}{\mu + \lambda_i}\right)^{\phi_i} - \left(\frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j}\right)^{\Phi}\right).$$

Inventory levels in each location that minimize the expected number of transshipments can be found using Theorem (3).

**Theorem 5.** *For a system of hospitals, in order to maximize Type II service level, allocation of inventory pool at any two locations $k$ and $m$ must satisfy the equality:*

$$\phi_m \ln\left(\frac{\lambda_m}{\mu + \lambda_m}\right) - \phi_k \ln\left(\frac{\lambda_k}{\mu + \lambda_k}\right) = \ln\left(\frac{\lambda_k \ln\left(\frac{\lambda_k}{\mu + \lambda_k}\right)}{\lambda_m \ln\left(\frac{\lambda_m}{\mu + \lambda_m}\right)}\right).$$

Similar to allocation of safety stock among hospitals, inventory pool allocation can be determined by solving a set of linear equations derived from Theorem (5).

# Chapter V

# REACTIVE POLICY: WHAT LEVEL TO STOP TRANSFERS

In this chapter, our aim is to derive a rule from a reactive perspective that hospitals use while granting a transshipment request. For this part of the study, we assume that during shortages, hospitals satisfy their observed demands from their own inventory, until one of their stock levels becomes zero. After that point, the hospital with zero inventory follows a reactive approach to fulfill the demand when observed. It can either request a transshipment from the other hospital by incurring a penalty, or the demand will be lost with a higher penalty. However, this situation raises the question for the other hospital to share or not to share its inventory. If the request is granted and that hospital's inventory level also becomes zero before the current shortage period ends, it will also have to incur a penalty cost for any excess demand that is observed. In other words, the system would incur a higher amount of total penalty due to the additional penalty for transshipments. On the other hand, if transshipment request is not accepted and the hospital with stock has remaining inventory at the end of the shortage period, it means that the system could have operated with a lower amount of penalty. Therefore, there exist transshipment thresholds for hospitals, under which one of the hospitals' inventory sharing would actually increase the total penalty incurred.

The following notations regarding parameters of the system should be noted:

$\delta_L$: the amount of penalty incurred by the system when a patient is not served. This can be considered as dispatching a patient from the hospital chain, and associated loss of brand value.

$\delta_T$: the amount of penalty incurred when a patient demand is satisfied by a transshipment from another hospital within the chain. This can be the monetary cost of transshipment, loss of quality-adjusted life years (QALY) for the patient due to lead time, etc.

It is assumed that $\delta_L > \delta_T$. Hospital $i$ observes Poisson distributed demand, with rate $\lambda_i$. In addition, shortage durations are exponentially distributed with rate $\mu$.

**Theorem 6.** *Threshold to transship for hospital $i$ is*

$$\omega_i = \underset{\{x|x>\log\frac{\delta_L-\delta_T}{\delta_L}/\log\frac{\lambda_i}{\lambda_i+\mu}\}}{\arg\min} x.$$

One of the most significant results obtained by the above theorem is, the threshold for a hospital does not depend on the demand rate of the other hospitals. Moreover, it is not linearly proportional with the demand rate of the considered hospital. We provide the optimal values for a three hospital case under various $\delta_T/\delta_L$ ratios in Table 7.

**Table 7:** Hospitals' threshold values for reactive policy ($\lambda_1 = 500$, $\lambda_2 = 200$, $\lambda_3 = 100$)

| $\delta_T/\delta_L$ | $\omega_1$ | $\omega_2$ | $\omega_3$ |
|---|---|---|---|
| 0.03 | 3 | 1 | 0 |
| 0.1 | 13 | 5 | 2 |
| 0.2 | 28 | 11 | 5 |
| 0.3 | 44 | 18 | 9 |
| 0.4 | 64 | 25 | 13 |
| 0.5 | 86 | 35 | 17 |
| 0.6 | 114 | 46 | 23 |
| 0.7 | 151 | 60 | 30 |
| 0.8 | 201 | 81 | 41 |
| 0.9 | 288 | 116 | 58 |
| 0.97 | 440 | 177 | 89 |

As it can be observed from Table 7, the thresholds are in favor of the hospital with a larger demand rate when $\delta_T/\delta_L$ is small. However, as expected, when $\delta_T/\delta_L$ increases, the system becomes more protective of hospitals with smaller demand rates.

One other important insight is, when $\delta_T << \delta_L$, thresholds converge to zero. That is, hospitals do not keep any safety stock, and apply complete pooling. However, it can be stated with certainty that hospitals with larger demand rates incorporate a complete pooling strategy under a smaller $\delta_T/\delta_L$ ratio. This is also intuitive, due to the fact that hospitals with larger demand rates desire to keep at least a part of their inventories to their own, since they have a higher probability of observing a demand occurrence before the shortage period ends.

# Chapter VI

# A CASE STUDY FOR PROACTIVE APPROACH

To measure the effects of system parameters on key performance indicators, $\alpha_S$ and $\beta_S$, we obtained real life data about an antibacterial drug, Ceftriaxone, from Harris Health System which is based in Houston. In addition, we aim to make observations regarding amounts to be allocated as pooled inventories and transshipment thresholds under different scenarios for demand, shortage duration, and total amount of inventory.

According to the available data, the hospital chain consists of 2 hospitals and a small clinic, namely Lyndon B. Johnson Hospital, Ben Taub Hospital, and Monroe Clinic. These locations observe Poisson demand rates of 500, 200, and 100, respectively. Shortage durations are exponentially distributed with mean of three months, and the system generally carries an inventory level that is equal to the expected demand for one year. To be able to provide extensive analyses for the considered system, we evaluate the system under different values for each of the parameters related to demand rates, shortage duration, and total inventory level. We measure Type I and Type II service levels for different proportions of total amount of pooled inventory within the total amount of inventory held, from 0 to 1, increasing with an increment of 0.1. Scenarios for various combinations of parameters are obtained by both increasing and decreasing demand rates for each hospital by 0.1, and changing shortage duration rate and total amount of inventory held linearly. We present our results regarding how optimal allocations change with respect to changes in demand rates, shortage duration rate, and total amount of inventory in Appendix. With this study, we provide guidelines on determining the allocation of total inventory, allocation of pools, and thresholds among hospitals.

We present to our results with respect to various combinations of $\lambda_1$, $\lambda_2$, $\lambda_3$, $\mu$, years of inventory kept, and changing $\Phi/(\Phi+\Omega)$ combinations in Appendix B. First, a comment can be made on the behavior of $\alpha_S$ and $\beta_S$. As it can be clearly seen from the results, Type I service level increases with the amount of pooled inventory. On the other hand, Type II service level is the highest when complete pooling and no pooling policies are applied. Therefore, it can be concluded that for healthcare systems observing shortage the best proactively pooling solution can be observed by allowing hospitals to share their inventories its fullest in terms of both Type I and Type II service levels.

Our analyses imply that optimal allocation of total amount of pooled inventory and total amount reserved for thresholds are not linearly proportional with demand rates of hospitals. Even though the optimal values become proportional with demand rates when rounded for practical purposes, it cannot be stated that they are strictly proportional.

**Remark 1.** *Optimally allocated pooled inventories and safety stocks are not linearly proportional with demand rates when the system observes supply shortages.*

In addition, sensitivity of pool and threshold allocation differ with respect to changes in proportion of total inventory left for pooling, total amount of inventory held, and shortage duration parameters.

**Remark 2.** *The allocation of inventory pool between hospitals is more sensitive to changes in proportion* $\Phi/(\Phi+\Omega)$ *and the total amount stocked, whereas thresholds divided between hospitals vary more with changing shortage duration rate.*

Moreover, it can be observed that deviations of optimal allocations also vary according to changes in shortage duration rate, demand rates, and total amount of inventory in the system. We first investigate how allocations vary with respect to shortage duration rates.

**Remark 3.** *An increase in shortage duration rate results in a higher deviation from linear allocation in terms of both pooled inventory and total safety stock.*

Then, we determine the deviations of pooled inventory and safety stocks from linear proportion with the demand rate according to increasing years of inventory held.

**Remark 4.** *Increasing the total amount of inventory held in the system results in a more deviated optimal distribution of total pool and total threshold than linear.*

We also analyse the relationship between deviation of pool and threshold from linear allocation with respect to increasing demand rates in hospitals.

**Remark 5.** *Increasing demand rate linearly with linearly proportional total inventory results in the allocations of both pool and threshold that are closer to a linear distribution.*

Furthermore, according to optimal allocations of inventory pool and total threshold, effects of changes in shortage duration rate, total amount of inventory held, and demand rates on Type I and Type II service levels can be observed. It can be noticed from Tables 8, 12, 13, 14, and 15 that a linear increase in $\mu$ produces exponential rises in both $\alpha_S$ and $\beta_S$. Likewise, as shown in Tables 8, 9, 10, and 11, as total amount of inventory increases linearly, Type I and Type II service levels also grow exponentially. These insights on $\mu$ and total amount of inventory can be observed from Figures 2 and 3, according to the best solution for each case.

**Remark 6.** *Linearly increasing shortage duration rate ($\mu$) increases Type II and Type II service levels logarithmically.*

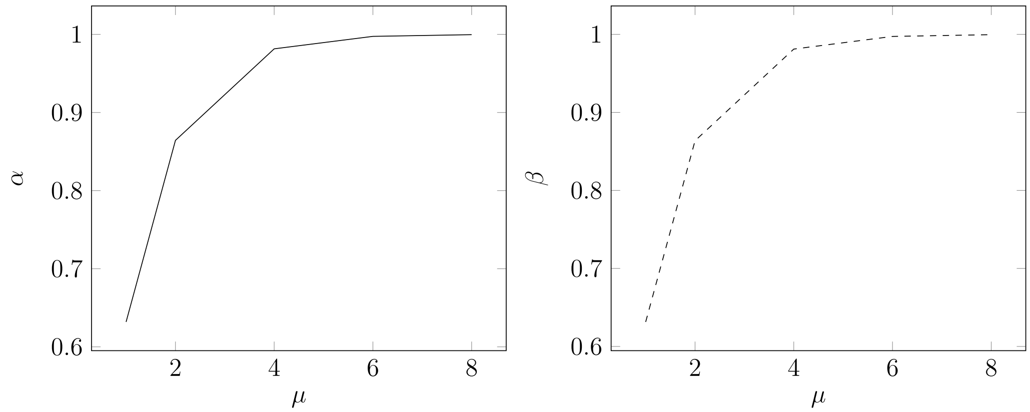


**Figure 2:** Type I and Type II service levels, respectively, with respect to shortage duration rate ($\mu$)

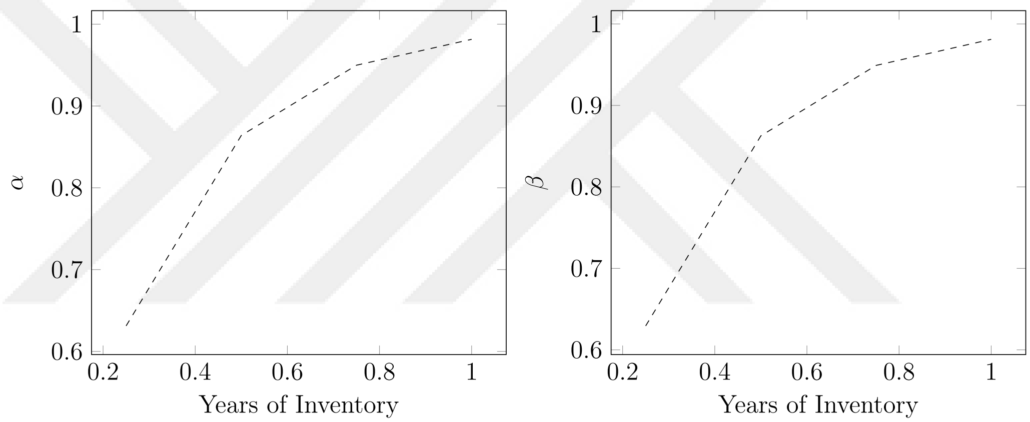


**Figure 3:** Type I and Type II service levels, respectively, with respect to years of inventory

**Remark 7.** *Linearly increasing total amount of inventory kept at hospitals increase Type I and Type II service levels logarithmically.*

On the other hand, as it can be seen from Tables 8, 16, and 17, increasing demand rates linearly generates linear increases in both of the service levels. The movements of Type I and Type II service levels with linearly increasing demand rates are presented in Figure 3

**Remark 8.** *Linear increases in demand rates together with total amount of inventory increase Type I and Type II service levels linearly.*

We can also explain the relationship between shorter shortage durations with a

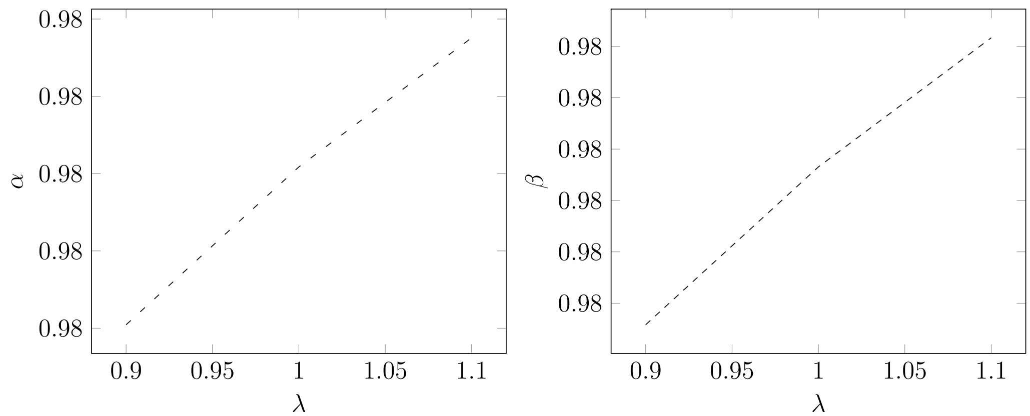


**Figure 4:** Type I and Type II service levels, respectively, with respect to yearly total demand rate ($\lambda$)

lower amount of total inventory and longer shortage durations with a higher total inventory from tables (9) and (12).

**Remark 9.** *Under a constant ratio between shortage duration ($\mu$) and total amount stocked, Type I and Type II service levels increase with higher $\mu$ and total inventory levels.*

Then, we focus on how optimal allocations for pool and threshold vary when hospitals demand rates are far from being linearly proportional. For this purpose, we compare the case presented in Table 8 with a new case in which $\lambda_1 = 1000$, $\lambda_2 = 100$, $\lambda_3 = 10$, $\mu = 4$, and the system keeps an inventory that is exactly equal to expected amount of yearly demand. Optimal allocations for the new case for complete pooling and no pooling policies are presented in Table 18. As it can be observed from these results, when the variance between hospitals' demand rates increases, optimal allocations for both pool and threshold in the system deviate more from linear. Even for integer values, hospitals begin not to keep linearly proportional inventory levels, and the system becomes more protective of hospitals with smaller demand rates.

Finally, we want to present a case that proves all of our remarks above. Therefore, we extend the newly introduced case by changing $\mu = 8$, the value of shortage duration rate which is expected to create the maximum deviation from linear allocation. Our results in Table 19 show that the highest deviation from linear allocation is achieved under the proposed case.

**Remark 10.** *An increase in the variance between hospitals' demand rates results in higher deviations from linear allocation.*

# Chapter VII

# CONCLUSION

In this study, we analyze a healthcare system's inventory allocation with respect to expected demand satisfaction percentage and expected number of transshipments during a shortage period that is uncertain in duration. We propose a proactive and a reactive policy for allowing transshipments within the system.

In the light of our analyses, we are able to prove that inventory level for each hospital is not directly proportional to the demand rate for a system observing supply shortages, in contrast to the commonly adopted conjectures. We provide a numerical study with real life data obtained from a hospital chain. As a result, it can be stated that completely pooled inventories should be the followed approach when there is a probability of shortage occurrence in a healthcare system, to maximize the percentage of satisfied demand and the percentage of demand satisfied from each hospital's own inventories. In addition, this case study helps us to provide managerial insights regarding sensitivity of inventory allocations and service levels obtained with respect to different demand, shortage occurrence rates and amounts of total inventory held in the system.

There exists potential research directions that requires further analyses to completely understand how such healthcare systems behave under shortages that are uncertain in length and frequency are present. Within the scope of this study, we assumed that transshipments occur one-by-one, but in reality, systems mostly desire transshipments in bulk quantities. Therefore, decisions on transshipment quantities in systems observing shortages is a point that is open to further investigation. Another issue that is worth to analyze deeper is the effect of transshipment lead time on performance metrics. The zero lead time assumption in this study can be extended and system behavior under non-zero transshipment lead time can be evaluated.

# Appendix A

# PROOFS OF THEOREMS AND LEMMAS

## *A.1 Proof for Theorem 1*

It is known that $N$ hospitals will consume $\mathcal{B} - \Omega$ items together first. Then, they will satisfy the observed demand from their own inventory. Any demand occurrence at each of the hospitals after each of them reaches an inventory level of zero becomes lost.

Random variable that denotes the time until next patient arrival in hospital $i$ is denoted by $T_i$, where $T_i \sim \text{Exponential}(\lambda_i)$. Random variable $T_d$ denotes the time until next demand occurrence in the system and $T_d \sim \text{Exponential}(\sum_j \lambda_j)$. $W$ denotes the time that it takes until the supplier becomes available and $W \sim \text{Exponential}(\mu)$.

We define $\Omega$ as the total amount of transshipment thresholds, and the threshold for hospital $i$ is defined as $\gamma_i\Omega$, where $\sum_i \gamma_i = 1$.

The total expected number of lost sales during a shortage can be written using conditioning as follows:

$$E[L] = P(T_d = \min(T_d, W))^{\mathcal{B}-\Omega} \sum_i P(T_i = \min(T_i, W))^{\gamma_i\Omega} E[\text{arrivals with rate } \lambda_i \text{ before shortage e}$$

$$= \left(\frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j}\right)^{\mathcal{B}-\Omega} \sum_i \left(\frac{\lambda_i}{\lambda_i + \mu}\right)^{\gamma_i\Omega} \frac{\lambda_i}{\mu} \tag{5}$$

Service level, denoted by $\alpha_S$, can be calculated as follows:

$$\alpha_S = 1 - \frac{E[L]}{E[\textit{total demand during shortage}]} = 1 - \frac{\left(\frac{\sum_j \lambda_j}{\mu+\sum_j \lambda_j}\right)^{\mathcal{B}-\Omega} \sum_i \left(\frac{\lambda_i}{\lambda_i+\mu}\right)^{\gamma_i\Omega} \frac{\lambda_i}{\mu}}{\frac{\sum_j \lambda_j}{\mu}} \tag{6}$$

$$= 1 - \left(\frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j}\right)^{\mathcal{B}-\Omega} \sum_i \left(\frac{\lambda_i}{\lambda_i + \mu}\right)^{\gamma_i\Omega} \frac{\lambda_i}{\sum_j \lambda_j} \tag{7}$$

## A.2 Proof of Theorem 2

We can get rid of the $\sum_i \gamma_i = 1$ constraint substituting $N$-th hospital's threshold with $\gamma_N = 1 - \sum_{j=1}^{N-1} \gamma_i$. Keeping in mind $0 \leq \gamma_i \leq 1$ for all $i$, Type I service level can be written as

$$\alpha_S = 1 - \left(\frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j}\right)^{\mathcal{B}-\Omega} \left(\sum_{i=0}^{N-1} \left(\frac{\lambda_i}{\lambda_i + \mu}\right)^{\gamma_i\Omega} \frac{\lambda_i}{\sum_j \lambda_j}\right) + \left(\frac{\lambda_N}{\lambda_N + \mu}\right)^{(1-\sum_{j=1}^{N-1}\gamma_i)\Omega} \frac{\lambda_N}{\sum_j \lambda_j} \tag{8}$$

To find the optimal $\gamma_i$ values, we find the Jacobian, where $i$-th entry is

$$\begin{aligned} \frac{\partial \alpha_S}{\partial \gamma_i} = - \left(\frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j}\right)^{\mathcal{B}-\Omega} \Bigg[ &\Omega \ln\left(\frac{\lambda_i}{\lambda_i + \mu}\right) \frac{\lambda_i}{\sum_j \lambda_j} \left(\frac{\lambda_i}{\lambda_i + \mu}\right)^{\gamma_i\Omega} \\ &- \Omega \ln\left(\frac{\lambda_N}{\lambda_N + \mu}\right) \frac{\lambda_N}{\sum_j \lambda_j} \left(\frac{\lambda_N}{\lambda_N + \mu}\right)^{(1-\sum_{j=1}^{N-1}\gamma_j)\Omega} \Bigg] \end{aligned} \tag{9}$$

Equating Jacobian to zero yields the following condition for critical points:

$$\lambda_i \ln\left(\frac{\lambda_i}{\lambda_i + \mu}\right) \left(\frac{\lambda_i}{\lambda_i + \mu}\right)^{\gamma_i\Omega} = \lambda_N \ln\left(\frac{\lambda_N}{\lambda_N + \mu}\right) \left(\frac{\lambda_N}{\lambda_N + \mu}\right)^{(1-\sum_{j=1}^{N-1}\gamma_j)\Omega} \quad \forall i \tag{10}$$

Therefore, for any two hospitals $k$ and $m$, at the critical point we have

$$\lambda_k \ln\left(\frac{\lambda_k}{\lambda_k + \mu}\right) \left(\frac{\lambda_k}{\lambda_k + \mu}\right)^{\gamma_k\Omega} = \lambda_m \ln\left(\frac{\lambda_m}{\lambda_m + \mu}\right) \left(\frac{\lambda_m}{\lambda_m + \mu}\right)^{\gamma_m\Omega} \tag{11}$$

which leads to

$$\gamma_m \ln\left(\frac{\lambda_m}{\mu + \lambda_m}\right) - \gamma_k \ln\left(\frac{\lambda_k}{\mu + \lambda_k}\right) = \frac{1}{\Omega} \ln\left(\frac{\lambda_k \ln\left(\frac{\lambda_k}{\mu+\lambda_k}\right)}{\lambda_m \ln\left(\frac{\lambda_m}{\mu+\lambda_m}\right)}\right). \tag{12}$$

Next we use the Hessian to prove the concavity of the Type I service level function, where

$$\frac{\partial^2 \alpha_S}{\partial \gamma_i^2} = -\left(\frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j}\right)^{\mathcal{B}-\Omega} \frac{\Omega^2 \left(\lambda_i \ln^2\left(\frac{\lambda_i}{\lambda_i+\mu}\right)\left(\frac{\lambda_i}{\lambda_i+\mu}\right)^{\gamma_i \Omega} + \lambda_N \ln^2\left(\frac{\lambda_N}{\lambda_N+\mu}\right)\left(\frac{\lambda_N}{\lambda_N+\mu}\right)^{(1-\sum_{j=1}^{N-1}\gamma_j)\Omega}\right)}{\sum_j \lambda_j} \tag{13}$$

and

$$\frac{\partial^2 \alpha_S}{\partial \gamma_i \partial \gamma_l} = -\left(\frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j}\right)^{\mathcal{B}-\Omega} \frac{\Omega^2 \left(\lambda_N \ln^2\left(\frac{\lambda_N}{\lambda_N+\mu}\right)\left(\frac{\lambda_N}{\lambda_N+\mu}\right)^{(1-\sum_{j=1}^{N-1}\gamma_j)\Omega}\right)}{\sum_j \lambda_j}. \tag{14}$$

It can be shown that at the critical point, all off-diagonal entries of the Hessian is a negative constant, whereas the each diagonal entry depends on the individual demand rates, yet definitely less than off-diagonal constants. It can be easily shown that all eigenvalues of this matrix are negative, guaranteeing Hessian is negative definite and $\alpha_S$ is concave. Thus, the critical point is the global maximizer for $\alpha_S$.

## *A.3 Proof of Lemma 1*

For a system with a demand rate of $\lambda$, a recovery rate of $\mu$, and a total inventory level of $\phi$, expected amount of demand satisfied can be written as:

$$\begin{aligned} E[\text{demand satisfied}] &= \sum_{j=0}^{\phi-1} j \times P(\text{Demand} = j) + \phi \times P(\text{Demand} \geq \phi) \\ &= \sum_{j=0}^{\phi-1} j \left(\frac{\lambda}{\mu+\lambda}\right)^j \frac{\mu}{\mu+\lambda} + \phi\left(1 - \sum_{j=0}^{\phi-1}\left(\frac{\lambda}{\mu+\lambda}\right)^j \frac{\mu}{\mu+\lambda}\right) \\ &= \frac{\mu\lambda}{(\mu+\lambda)^2} \sum_{j=0}^{\phi-1} j \left(\frac{\lambda}{\mu+\lambda}\right)^{j-1} + \phi - \phi\left(1 - \left(\frac{\lambda}{\mu+\lambda}\right)^{\phi}\right). \end{aligned} \tag{15}$$

Knowing that

$$\sum_{j=0}^{\phi-1} j p^{j-1} = \sum_{j=0}^{\phi-1} \frac{\partial p^j}{\partial p} = \frac{\partial}{\partial p}\sum_{j=0}^{\phi-1} p^j = \frac{\partial}{\partial p}\frac{1-p^{\phi}}{1-p} = \frac{-\phi p^{\phi-1}(1-p) + 1 - p^{\phi}}{(1-p)^2}, \tag{16}$$

expected amount of demand satisfied can be written as follows:

$$
\begin{aligned}
E[\text{demand satisfied}] &= \frac{\mu\lambda}{(\mu+\lambda)^2} \frac{-\phi\left(\frac{\lambda}{\mu+\lambda}\right)^{\phi-1}\left(\frac{\mu}{\mu+\lambda}\right)+1-\left(\frac{\lambda}{\mu+\lambda}\right)^{\phi}}{\left(\frac{\mu}{\mu+\lambda}\right)^2} + \phi - \phi\left(1-\left(\frac{\lambda}{\mu+\lambda}\right)^{\phi}\right) \\
&= \frac{\lambda}{\mu}\left(1-\left(\frac{\lambda}{\mu+\lambda}\right)^{\phi}\right) \qquad (17)
\end{aligned}
$$

## *A.4 Proof of Theorem 3*

Let us introduce following notation for this part of the study: Let us introduce the following notation:

$D$: amount of demand during shortage,

$P$: amount of demand satisfied from hospitals' pooled inventories, $\phi_i$, directly,

$T$: amount of demand satisfied with transshipments,

$S$: amount of demand satisfied from each hospital's safety stock, $\gamma_i\Omega$,

$L$: amount of unsatisfied demand.

For a system of hospitals, demand during shortage can be calculated as

$$E[\mathrm{D}] = E[P] + E[T] + E[S] + E[L].$$

Alternatively, we can use

$$E[D] = E[P] + E[T].$$

From Lemma (1), since we know that

$$E[D] = \frac{\sum_i \lambda_i}{\mu}\left(1-\left(\frac{\sum_i \lambda_i}{\mu+\sum_i \lambda_i}\right)^{\sum_i \phi_i}\right)$$

and

$$E[P] = \sum_i \frac{\lambda_i}{\mu}\left(1-\left(\frac{\lambda_i}{\mu+\lambda_i}\right)^{\phi_i}\right),$$

expected amount of demand satisfied with transshipment can be calculated as follows:

$$E[T] = \sum_i \frac{\lambda_i}{\mu} \left( \left( \frac{\lambda_i}{\mu + \lambda_i} \right)^{\phi_i} - \left( \frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j} \right)^{\sum_j \phi_j} \right). \tag{18}$$

## A.5 Proof of Theorem 5

Type II service level can be maximized when expected number of transshipments ($E[T]$) is minimized because demand during shortage is not affected by the allocation of pooled inventory. Thus, we can find the optimal allocation of inventory pool to minimize the number of transshipments within the system by taking the derivative of Equation (18) with respect to $\phi_i$.

$$\frac{\partial E[T]}{\partial \phi_i} = \frac{\lambda_i}{\mu} \ln \left( \frac{\lambda_i}{\mu + \lambda_i} \right) \left( \frac{\lambda_i}{\mu + \lambda_i} \right)^{\phi_i} - \frac{\sum_j \lambda_j}{\mu} \ln \left( \frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j} \right) \left( \frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j} \right)^{\sum_j \phi_j} \tag{19}$$

Using (19), obtained Jacobian can be set to zero, which yields

$$\lambda_i \ln \left( \frac{\lambda_i}{\mu + \lambda_i} \right) \left( \frac{\lambda_i}{\mu + \lambda_i} \right)^{\phi_i} = \sum_j \lambda_j \ln \left( \frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j} \right) \left( \frac{\sum_j \lambda_j}{\mu + \sum_j \lambda_j} \right)^{\sum_j \phi_j}, \tag{20}$$

where $\sum_j \phi_j = \Phi$. Therefore, for any two hospitals $k$ and $m$, critical points for allocated amounts of inventory from the pool to minimize expected number of transshipments must hold the equality below:

$$\begin{aligned} \lambda_k \ln \left( \frac{\lambda_k}{\mu + \lambda_k} \right) \left( \frac{\lambda_k}{\mu + \lambda_k} \right)^{\phi_k} &= \lambda_m \ln \left( \frac{\lambda_m}{\mu + \lambda_m} \right) \left( \frac{\lambda_m}{\mu + \lambda_m} \right)^{\phi_m} \\ \phi_m \ln \left( \frac{\lambda_m}{\mu + \lambda_m} \right) - \phi_k \ln \left( \frac{\lambda_k}{\mu + \lambda_k} \right) &= \ln \left( \frac{\lambda_k \ln \left( \frac{\lambda_k}{\mu+\lambda_k} \right)}{\lambda_m \ln \left( \frac{\lambda_m}{\mu+\lambda_m} \right)} \right) \end{aligned} \tag{21}$$

## A.6 Proof of Theorem 6

Assume that there are two hospitals in the system. For this part of the study, based on the assumption regarding holding cost, we assume that both of the hospitals have high levels of inventory at the beginning of the shortage period, which is stochastic in length. At some point during shortage period, one of the hospitals' inventory is totally consumed. From that point on, there begins the question of accepting the transshipment request as demand occurs in the hospital with no inventory on hand.

When one of the hospitals observes a stock-out, the other one currently has $X$ items in the inventory, and ongoing shortage does not leave the system as the next event occurrence, then there are two separate states to be analyzed, denoted by $\Phi(X, 0)$ and $\Phi^{\text{arr}}(X, 0)$.

$\Phi(X, 0)$: Expected total loss in quality of care for the system (e.g., cost, lead time etc.) if the next event in the system is demand occurrence for the first hospital, when there are $X$ items in first hospital's inventory.

$\Phi^{\text{arr}}(X, 0)$: Expected total loss in quality of care for the system if the next event in the system is demand occurrence at the second hospital, when there are $X$ items in first hospital's inventory.

Random variable that denotes the time until next patient arrival in hospital $i$ is denoted by $T_i$, where $T_i \sim \text{Exponential}(\lambda_i), i = 1, 2$. $W$ denotes the time that it takes until the drug becomes available through regular supplier and $W \sim \text{Exponential}(\mu)$.

If the next event in the system is demand occurrence at the first hospital, the expected total cost from this point until the end of shortage period can be calculated as follows:

$$\begin{aligned}\Phi(X, 0) =& P(T_1 = \min(T_1, T_2, W)) \times \Phi(X - 1, 0) + P(T_2 = \min(T_1, T_2, W)) \times \Phi^{\text{arr}}(X, 0) \\ &+ P(W = \min(T_1, T_2, W)) \times 0 \\ =& \frac{\lambda_1}{\lambda_1 + \lambda_2 + \mu} \times \Phi(X - 1, 0) + \frac{\lambda_2}{\lambda_1 + \lambda_2 + \mu} \times \Phi^{\text{arr}}(X, 0) \qquad (22)\end{aligned}$$

If the next event in the system is demand occurrence at the second hospital, since its inventory level is currently zero, the observed demand is either satisfied with transshipments or lost. Minimum of these two options is selected as the action taken in this state, and expected total cost from this point until the end of shortage period can be written in terms of $\Phi(X,0)$, $\Phi^{\text{arr}}(X,0)$ can be written as follows:

$$\Phi^{\text{arr}}(X,0) = \min(\delta_T + \Phi(X-1,0), \delta_L + \Phi(X,0)) \tag{23}$$

Therefore, $\Phi(X,0)$ for the states in which transshipment is used to cover the demand of hospital with no inventory can be written as:

$$\begin{aligned}
\Phi(X,0) &= \frac{\lambda_1}{\lambda_1+\lambda_2+\mu}\Phi(X-1,0) + \frac{\lambda_2}{\lambda_1+\lambda_2+\mu}(\delta_T + \Phi(X-1,0)) \\
\Phi(X,0) &= \frac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2+\mu}\Phi(X-1,0) + \frac{\lambda_2}{\lambda_1+\lambda_2+\mu}\delta_T
\end{aligned} \tag{24}$$

On the other hand, $\Phi(X,0)$ for the states in which losing the observed patient demand in the hospital with zero inventory level can be written as:

$$\begin{aligned}
\Phi(X,0) &= \frac{\lambda_1}{\lambda_1+\lambda_2+\mu}\Phi(X-1,0) + \frac{\lambda_2}{\lambda_1+\lambda_2+\mu}(\delta_L + \Phi(X,0)) \\
\Phi(X,0) &= \frac{\lambda_1}{\lambda_1+\mu}\Phi(X-1,0) + \frac{\lambda_2}{\lambda_1+\mu}\delta_L
\end{aligned} \tag{25}$$

Expected total cost of the system after both of the hospitals' inventory levels reach zero, $\Phi(0,0)$, can be expressed as:

$$\begin{aligned}
\Phi(0,0) &= \delta_L \times E[\text{number of arrivals with rate } \lambda_1+\lambda_2 \text{ within } W] \\
&= \delta_L \frac{\lambda_1+\lambda_2}{\mu}
\end{aligned} \tag{26}$$

This is due to memoryless property of $T_i, i = 1,2$ and $W$ and the following:

$E[\text{number of arrivals with rate } \lambda_1+\lambda_2 \text{ within } W] = \dfrac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2+\mu}(1+E[\text{number of arrivals with rate}$

Computation of $\Phi^{\text{arr}}(0,0)$ is fairly easy as there is no decision to be made, but losing the patient demand, thus

$$\Phi^{\text{arr}}(0,0) = \delta_L + \Phi(0,0) = \delta_L \frac{\lambda_1+\lambda_2+\mu}{\mu}. \tag{27}$$

Using equation (27), $\Phi(0,0)$ can be verified as

$$\Phi(0,0) = \frac{\lambda_1}{\lambda_1+\lambda_2+\mu} \times (\delta_L + \Phi(0,0)) + \frac{\lambda_2}{\lambda_1+\lambda_2+\mu} \times \Phi^{\text{arr}}(0,0)$$

$$
\begin{aligned}
\Phi(0,0) &= \frac{\lambda_1}{\lambda_1+\lambda_2+\mu} \times (\delta_L + \Phi(0,0)) + \frac{\lambda_2}{\lambda_1+\lambda_2+\mu} \times (\delta_L + \Phi(0,0)) \\
\frac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2+\mu}\delta_L &= \frac{\mu}{\lambda_1+\lambda_2+\mu}\Phi(0,0) \\
\Phi(0,0) &= \frac{\lambda_1+\lambda_2}{\mu}\delta_L
\end{aligned}
$$

In fact, it can be assumed that if a hospital decides not to make a transshipment at an inventory level, it also will not accept any transshipment requests below that inventory level.

Since each of the states include the probability that shortage will be over before a demand occurs in hospitals one or two, states' costs can be seen as a decreasing function with a decreasing rate, when losing the patient demand is the chosen option. At some point, the cost will start to increase, so hospitals will begin to use transshipments from that point. We would like to find the transshipment threshold, $\Omega$, as the inventory level described above. In order to find the transshipment threshold, we use equation (24) for states where transshipment is preferred and (25) for states where lost demand is preferred. To summarize $\Omega$ should satisfy:

- The patient demand should be lost when $\Omega$ items are on hand
- Transshipment should be preferred when $\Omega + 1$ items are on hand

Therefore, for the transshipment threshold, the inequalities below must hold:

$$\frac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2+\mu}\Phi(\Omega-1,0) + \frac{\lambda_2}{\lambda_1+\lambda_2+\mu}\delta_T > \frac{\lambda_1}{\lambda_1+\mu}\Phi(\Omega-1,0) + \frac{\lambda_2}{\lambda_1+\mu}\delta_L \tag{28}$$

$$\frac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2+\mu}\Phi(\Omega,0) + \frac{\lambda_2}{\lambda_1+\lambda_2+\mu}\delta_T < \frac{\lambda_1}{\lambda_1+\mu}\Phi(\Omega,0) + \frac{\lambda_2}{\lambda_1+\mu}\delta_L. \tag{29}$$

Using inequalities (28) and (29), lower and upper bounds for $\Phi(\Omega-1,0)$ can be found, respectively. For the lower bound:

$$\frac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2+\mu}\Phi(\Omega-1,0) - \frac{\lambda_1}{\lambda_1+\mu}\Phi(\Omega-1,0) > \frac{\lambda_2}{\lambda_1+\mu}\delta_L - \frac{\lambda_2}{\lambda_1+\lambda_2+\mu}\delta_T$$

$$\Phi(\Omega-1,0) > \frac{\lambda_1+\mu}{\mu}(\delta_L-\delta_T) + \frac{\lambda_2}{\mu}\delta_L \tag{30}$$

For the upper bound:

$$\frac{\lambda_2}{\lambda_1+\mu}\delta_L - \frac{\lambda_2}{\lambda_1+\lambda_2+\mu}\delta_T > \frac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2+\mu}\Phi(\Omega,0) - \frac{\lambda_1}{\lambda_1+\mu}\Phi(\Omega,0)$$
$$\frac{\lambda_1+\mu}{\mu}(\delta_L-\delta_T) + \frac{\lambda_2}{\mu}\delta_L > \Phi(\Omega,0)$$

As we know losing patient demands is preferred when $\Omega$ items are on hand, we use (25) and obtain

$$\frac{\lambda_1+\mu}{\mu}(\delta_L-\delta_T) + \frac{\lambda_2}{\mu}\delta_L > \frac{\lambda_1}{\lambda_1+\mu}\Phi(\Omega-1,0) + \frac{\lambda_2}{\lambda_1+\mu}\delta_L$$
$$\frac{(\lambda_1+\mu)^2}{\lambda_1\mu}(\delta_L-\delta_T) + \frac{\lambda_2}{\mu}\delta_L > \Phi(\Omega-1,0) \tag{31}$$

Knowing that the hospital with an inventory level of $X$ will not share inventory below that level, a closed-form equation for $\Phi(X,0)$ can be written:

$$\begin{aligned}
\Phi(X,0) &= \frac{\lambda_1}{\lambda_1+\mu}\Phi(X-1,0) + \frac{\lambda_2}{\lambda_1+\mu}\delta_L \\
&= \frac{\lambda_1}{\lambda_1+\mu}\left(\frac{\lambda_1}{\lambda_1+\mu}\Phi(X-2,0) + \frac{\lambda_2}{\lambda_1+\mu}\delta_L\right) + \frac{\lambda_2}{\lambda_1+\mu}\delta_L \\
&= \left(\frac{\lambda_1}{\lambda_1+\mu}\right)^X \Phi(0,0) + \frac{\lambda_2}{\lambda_1+\mu}\delta_L \sum_{i=0}^{X-1}\left(\frac{\lambda_1}{\lambda_1+\mu}\right) \\
&= \left(\frac{\lambda_1}{\lambda_1+\mu}\right)^X \Phi(0,0) + \frac{\lambda_2}{\lambda_1+\mu}\delta_L \frac{1-\left(\frac{\lambda_1}{\lambda_1+\mu}\right)^X}{1-\left(\frac{\lambda_1}{\lambda_1+\mu}\right)} \\
&= \frac{\lambda_1+\lambda_2}{\mu}\delta_L\left(\frac{\lambda_1}{\lambda_1+\mu}\right)^X + \frac{\lambda_2}{\mu}\left(1-\left(\frac{\lambda_1}{\lambda_1+\mu}\right)^X\right)\delta_L \\
&= \frac{\lambda_1}{\mu}\delta_L\left(\frac{\lambda_1}{\lambda_1+\mu}\right)^X + \frac{\lambda_2}{\mu}\delta_L
\end{aligned} \tag{32}$$

Using (30), (31), and (32) the transshipment threshold $\Omega$ can be computed as the smallest integer that satisfies

$$\frac{\lambda_1+\mu}{\mu}(\delta_L-\delta_T) + \frac{\lambda_2}{\mu}\delta_L < \frac{\lambda_1}{\mu}\delta_L\left(\frac{\lambda_1}{\lambda_1+\mu}\right)^{\Omega-1} + \frac{\lambda_2}{\mu}\delta_L < \frac{(\lambda_1+\mu)^2}{\lambda_1\mu}(\delta_L-\delta_T) + \frac{\lambda_2}{\mu}\delta_L$$

which, after simple algebra, gives the following condition:

$$\frac{\delta_L - \delta_T}{\delta_L} < \left(\frac{\lambda_1}{\lambda_1 + \mu}\right)^{\Omega} < \frac{\lambda_1 + \mu}{\lambda_1} \times \frac{\delta_L - \delta_T}{\delta_L} \tag{33}$$

$$\omega_1 = \operatorname*{arg\,min}_{\{x | x > \log \frac{\delta_L - \delta_T}{\delta_L} / \log \frac{\lambda_1}{\lambda_1 + \mu}\}} x \tag{34}$$

Knowing that only $\lambda_i$ and $\mu$ effects the threshold value, it can be defined for each of the hospitals in a multi-hospital setting.

# Appendix B

## RESULTS FOR NUMERICAL STUDY

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 498.2747402 | 200.4951257 | 101.2301341 | 0.981132857 | 0.981132857 | 0 |
| 10 | 50.26309021 | 19.92533119 | 9.811578606 | 448.4473812 | 180.4455803 | 91.1070385 | 0.981169989 | 0.979856204 | 0.262757026 |
| 20 | 100.0905098 | 39.97485928 | 19.93463095 | 398.6200372 | 160.3960307 | 80.98393208 | 0.981207047 | 0.979435961 | 0.354217276 |
| 30 | 149.9179293 | 60.02438737 | 30.0576833 | 348.7927149 | 140.3464749 | 70.86081021 | 0.981244033 | 0.979457599 | 0.357286898 |
| 40 | 199.7453489 | 80.07391546 | 40.18073564 | 298.965425 | 120.2969098 | 60.73766517 | 0.981280946 | 0.979680189 | 0.320151381 |
| 50 | 249.5727685 | 100.1234436 | 50.30378799 | 249.138187 | 100.24733 | 50.61448304 | 0.981317786 | 0.979973374 | 0.268882491 |
| 60 | 299.400188 | 120.1729716 | 60.42684033 | 199.3110397 | 80.19772425 | 40.49123602 | 0.981354554 | 0.980270727 | 0.216765314 |
| 70 | 349.2276076 | 140.2224997 | 70.54989268 | 149.484074 | 60.14806679 | 30.36785923 | 0.98139125 | 0.980541828 | 0.169884386 |
| 80 | 399.0550272 | 160.2720278 | 80.67294502 | 99.65756207 | 40.09827995 | 20.24415798 | 0.981427874 | 0.980775774 | 0.130419985 |
| 90 | 448.8824467 | 180.3215559 | 90.79599737 | 49.83286545 | 20.04797563 | 10.11915892 | 0.981464428 | 0.980971647 | 0.098556064 |
| 100 | 498.7098663 | 200.371084 | 100.9190497 | 0 | 0 | 0 | 0.9815009 | 0.981133119 | 0.073556174 |

**Table 8:** $\lambda_1 = 500$, $\lambda_2 = 200$, $\lambda_3 = 100$, $\mu = 4$, Inventory=1 year

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 124.5707273 | 50.12319924 | 25.30607349 | 0.629382027 | 0.629382027 | 0 |
| 10 | 12.89252553 | 4.888185118 | 2.219289347 | 112.1141144 | 45.11074822 | 22.77513737 | 0.629564516 | 0.629131268 | 0.086649522 |
| 20 | 25.34938043 | 9.900567141 | 4.750052433 | 99.65756207 | 40.09827995 | 20.24415798 | 0.629746917 | 0.628951084 | 0.159166484 |
| 30 | 37.80623532 | 14.91294916 | 7.280815519 | 87.20109617 | 35.08578704 | 17.71311679 | 0.629929229 | 0.628843383 | 0.217169369 |
| 40 | 50.26309021 | 19.92533119 | 9.811578606 | 74.74475993 | 30.07325716 | 15.1819829 | 0.630111456 | 0.62879767 | 0.262757026 |
| 50 | 62.7199451 | 24.93771321 | 12.34234169 | 62.28863116 | 25.06066815 | 12.65070069 | 0.630293597 | 0.628804792 | 0.297760922 |
| 60 | 75.17679999 | 29.95009523 | 14.87310478 | 49.83286545 | 20.04797563 | 10.11915892 | 0.630475657 | 0.628856772 | 0.323776967 |
| 70 | 87.63365488 | 34.96247726 | 17.40386786 | 37.37782585 | 15.03507612 | 7.58709803 | 0.630657644 | 0.628946675 | 0.342193704 |
| 80 | 100.0905098 | 39.97485928 | 19.93463095 | 24.92460155 | 10.02165912 | 5.053739329 | 0.630839583 | 0.629068496 | 0.354217276 |
| 90 | 112.5473647 | 44.9872413 | 22.46539404 | 12.47863842 | 5.006172178 | 2.515189399 | 0.631021595 | 0.629217128 | 0.360893517 |
| 100 | 125.0042196 | 49.99962332 | 24.99615712 | 0 | 0 | 0 | 0.631202771 | 0.629387134 | 0.363127503 |

**Table 9:** $\lambda_1 = 500$, $\lambda_2 = 200$, $\lambda_3 = 100$, $\mu = 4$, Inventory=0.25 year

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 249.138187 | 100.24733 | 50.61448304 | 0.86264229 | 0.86264229 | 0 |
| 10 | 25.34938043 | 9.900567141 | 4.750052433 | 224.2245982 | 90.22253142 | 45.55287035 | 0.862777521 | 0.861981688 | 0.159166484 |
| 20 | 50.26309021 | 19.92533119 | 9.811578606 | 199.3110397 | 80.19772425 | 40.49123602 | 0.862912619 | 0.861598834 | 0.262757026 |
| 30 | 75.17679999 | 29.95009523 | 14.87310478 | 174.3975244 | 70.17290476 | 35.4295708 | 0.863047584 | 0.861428699 | 0.323776967 |
| 40 | 100.0905098 | 39.97485928 | 19.93463095 | 149.484074 | 60.14806679 | 30.36785923 | 0.863182417 | 0.861411331 | 0.354217276 |
| 50 | 125.0042196 | 49.99962332 | 24.99615712 | 124.5707273 | 50.12319924 | 25.30607349 | 0.863317119 | 0.861501481 | 0.363127503 |
| 60 | 149.9179293 | 60.02438737 | 30.0576833 | 99.65756207 | 40.09827995 | 20.24415798 | 0.863451689 | 0.861665255 | 0.357286898 |
| 70 | 174.8316391 | 70.04915142 | 35.11920947 | 74.74475993 | 30.07325716 | 15.1819829 | 0.86358613 | 0.861877484 | 0.341729097 |
| 80 | 199.7453489 | 80.07391546 | 40.18073564 | 49.83286545 | 20.04797563 | 10.11915892 | 0.863720446 | 0.862119689 | 0.320151381 |
| 90 | 224.6590587 | 90.09867951 | 45.24226181 | 24.92460155 | 10.02165912 | 5.053739329 | 0.863854661 | 0.862378496 | 0.295233064 |
| 100 | 249.5727685 | 100.1234436 | 50.30378799 | 0 | 0 | 0 | 0.863988604 | 0.862644192 | 0.268882491 |

**Table 10:** $\lambda_1 = 500$, $\lambda_2 = 200$, $\lambda_3 = 100$, $\mu = 4$, Inventory=0.5 year

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 373.7063728 | 150.3712537 | 75.92237347 | 0.949092756 | 0.949092756 | 0 |
| 10 | 37.80623532 | 14.91294916 | 7.280815519 | 336.3358889 | 135.3340846 | 68.33002653 | 0.949167915 | 0.948082068 | 0.217169369 |
| 20 | 75.17679999 | 29.95009523 | 14.87310478 | 298.965425 | 120.2969098 | 60.73766517 | 0.949242964 | 0.947624079 | 0.323776967 |
| 30 | 112.5473647 | 44.9872413 | 22.46539404 | 261.59499 | 105.2597268 | 53.14528321 | 0.949317902 | 0.947513435 | 0.360893517 |
| 40 | 149.9179293 | 60.02438737 | 30.0576833 | 224.2245982 | 90.22253142 | 45.55287035 | 0.94939273 | 0.947606295 | 0.357286898 |
| 50 | 187.288494 | 75.06153344 | 37.64997255 | 186.8542756 | 75.18531635 | 37.96040805 | 0.949467447 | 0.947809924 | 0.331504615 |
| 60 | 224.6590587 | 90.09867951 | 45.24226181 | 149.484074 | 60.14806679 | 30.36785923 | 0.949542055 | 0.948065889 | 0.295233064 |
| 70 | 262.0296234 | 105.1358256 | 52.83455107 | 112.1141144 | 45.11074822 | 22.77513737 | 0.949616553 | 0.948338535 | 0.255603605 |
| 80 | 299.400188 | 120.1729716 | 60.42684033 | 74.74475993 | 30.07325716 | 15.1819829 | 0.949690943 | 0.948607116 | 0.216765314 |
| 90 | 336.7707527 | 135.2101177 | 68.01912959 | 37.37782585 | 15.03507612 | 7.58709803 | 0.949765231 | 0.948860484 | 0.180949337 |
| 100 | 374.1413174 | 150.2472638 | 75.61141885 | 0 | 0 | 0 | 0.949839374 | 0.949093462 | 0.149182495 |

**Table 11:** $\lambda_1 = 500$, $\lambda_2 = 200$, $\lambda_3 = 100$, $\mu = 4$, Inventory=0.75 year

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 499.5645785 | 200.1245476 | 100.3108738 | 0.63143208 | 0.63143208 | 0 |
| 10 | 50.39344319 | 19.88767289 | 9.718883921 | 449.608236 | 180.1120599 | 90.27970407 | 0.631477973 | 0.63136609 | 0.089506222 |
| 20 | 100.3498464 | 39.90014325 | 19.75001034 | 399.6519087 | 160.0995679 | 80.24852344 | 0.631523861 | 0.631320634 | 0.162581192 |
| 30 | 150.3062496 | 59.91261361 | 29.78113675 | 349.695603 | 140.0870697 | 70.21732733 | 0.631569743 | 0.631293548 | 0.220956011 |
| 40 | 200.2626529 | 79.92508397 | 39.81226317 | 299.7393298 | 120.0745622 | 60.18610799 | 0.631615619 | 0.631282163 | 0.266764976 |
| 50 | 250.2190561 | 99.93755433 | 49.84338958 | 249.7831087 | 100.0620398 | 50.15485147 | 0.631661489 | 0.631284153 | 0.301869467 |
| 60 | 300.1754593 | 119.9500247 | 59.874516 | 199.8269786 | 80.04949145 | 40.1235299 | 0.631707354 | 0.631297491 | 0.327890538 |
| 70 | 350.1318625 | 139.9624951 | 69.90564241 | 149.8710307 | 60.03689109 | 30.09207823 | 0.631753214 | 0.631320417 | 0.346237794 |
| 80 | 400.0882658 | 159.9749654 | 79.93676883 | 99.91553795 | 40.02416074 | 20.06030131 | 0.631799069 | 0.6313514 | 0.358134936 |
| 90 | 450.044669 | 179.9874358 | 89.96789524 | 49.96186622 | 20.01091039 | 10.02722338 | 0.63184492 | 0.631389117 | 0.364642371 |
| 100 | 500.0010722 | 199.9999061 | 99.99902166 | 0 | 0 | 0 | 0.631890754 | 0.631432407 | 0.366677194 |

**Table 12:** $\lambda_1 = 500$, $\lambda_2 = 200$, $\lambda_3 = 100$, $\mu = 1$, Inventory=1 year

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 499.1316477 | 200.2486606 | 100.6196917 | 0.863651611 | 0.863651611 | 0 |
| 10 | 50.34969193 | 19.90028551 | 9.750022556 | 449.2185982 | 180.2237617 | 90.55764018 | 0.863719253 | 0.863315678 | 0.161429762 |
| 20 | 100.2628021 | 39.92516715 | 19.81203072 | 399.3055638 | 160.1988584 | 80.49557785 | 0.863786861 | 0.863123322 | 0.265415621 |
| 30 | 150.1759123 | 59.95004878 | 29.87403889 | 349.392551 | 140.1739489 | 70.43350004 | 0.863854436 | 0.863038169 | 0.326506862 |
| 40 | 200.0890225 | 79.97493042 | 39.93604705 | 299.4795708 | 120.1490302 | 60.37139902 | 0.863921978 | 0.863029933 | 0.356817996 |
| 50 | 250.0021327 | 99.99981205 | 49.99805522 | 249.5666425 | 100.1240967 | 50.30926086 | 0.863989486 | 0.863075774 | 0.365484557 |
| 60 | 299.9152429 | 120.0246937 | 60.06006338 | 199.6538051 | 80.09913717 | 40.24705771 | 0.86405696 | 0.863158599 | 0.359344385 |
| 70 | 349.8283531 | 140.0495753 | 70.12207155 | 149.7411497 | 60.07412575 | 30.18472456 | 0.864124402 | 0.863265727 | 0.34346993 |
| 80 | 399.7414633 | 160.074457 | 80.18407971 | 99.82894904 | 40.04898454 | 20.12206642 | 0.864191811 | 0.863387853 | 0.32158316 |
| 90 | 449.6545735 | 180.0993386 | 90.24608788 | 49.91856747 | 20.02332418 | 10.05810835 | 0.864259191 | 0.863518246 | 0.296378136 |
| 100 | 499.5676837 | 200.1242202 | 100.308096 | 0 | 0 | 0 | 0.86432652 | 0.863652092 | 0.269771084 |

**Table 13:** $\lambda_1 = 500$, $\lambda_2 = 200$, $\lambda_3 = 100$, $\mu = 2$, Inventory=1 year

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 497.4295156 | 200.7392628 | 101.8312216 | 0.997351116 | 0.997351116 | 0 |
| 10 | 50.17766393 | 19.9501395 | 9.872196568 | 447.6866788 | 180.6653038 | 91.64801738 | 0.997362744 | 0.994954527 | 0.321095659 |
| 20 | 99.92056115 | 40.02408126 | 20.0553576 | 397.9438571 | 160.5913406 | 81.46480236 | 0.997374321 | 0.994709874 | 0.355259627 |
| 30 | 149.6634584 | 60.09802301 | 30.23851863 | 348.2010569 | 140.5173712 | 71.28157192 | 0.997385848 | 0.99518008 | 0.294102307 |
| 40 | 199.4063556 | 80.17196476 | 40.42167966 | 298.4582891 | 120.4433926 | 61.09831834 | 0.997397323 | 0.995775118 | 0.216294091 |
| 50 | 249.1492528 | 100.2459065 | 50.60484069 | 248.715573 | 100.3693993 | 50.91502774 | 0.997408749 | 0.996290541 | 0.149094338 |
| 60 | 298.89215 | 120.3198483 | 60.78800172 | 198.9729475 | 80.29538013 | 40.73167236 | 0.997420124 | 0.996680246 | 0.098650379 |
| 70 | 348.6350472 | 140.39379 | 70.97116275 | 149.2305032 | 60.22130942 | 30.54818741 | 0.99743145 | 0.996955529 | 0.063456129 |
| 80 | 398.3779444 | 160.4677318 | 81.15432378 | 99.48851172 | 40.14710974 | 20.36437853 | 0.997442725 | 0.997142852 | 0.039983068 |
| 90 | 448.1208417 | 180.5416735 | 91.33748481 | 49.7483318 | 20.07239423 | 10.17927397 | 0.997453953 | 0.997267963 | 0.024798639 |
| 100 | 497.8637389 | 200.6156153 | 101.5206458 | 0 | 0 | 0 | 0.997465127 | 0.997351197 | 0.015190664 |

**Table 14:** $\lambda_1 = 500$, $\lambda_2 = 200$, $\lambda_3 = 100$, $\mu = 6$, Inventory=1 year

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 496.5956778 | 200.9810962 | 102.423226 | 0.99962276 | 0.99962276 | 0 |
| 10 | 50.09338344 | 19.974713 | 9.931903563 | 446.9362245 | 180.882954 | 92.18082146 | 0.999625677 | 0.996134241 | 0.349143631 |
| 20 | 99.75289699 | 40.07283804 | 20.17426497 | 397.2767863 | 160.7848076 | 81.93840615 | 0.999628572 | 0.996455133 | 0.317343879 |
| 30 | 149.4124105 | 60.17096309 | 30.41662637 | 347.6173696 | 140.686655 | 71.69597545 | 0.999631444 | 0.997473167 | 0.21582771 |
| 40 | 199.0719241 | 80.26908814 | 40.65898778 | 297.9579852 | 120.5884932 | 61.45352164 | 0.999634295 | 0.998330283 | 0.130401176 |
| 50 | 248.7314376 | 100.3672132 | 50.90134918 | 248.2986524 | 100.4903167 | 51.21103088 | 0.999637123 | 0.998898661 | 0.073846243 |
| 60 | 298.3909512 | 120.4653382 | 61.14371059 | 198.63941 | 80.39211453 | 40.96847544 | 0.999639929 | 0.99923851 | 0.040141936 |
| 70 | 348.0504647 | 140.5634633 | 71.386072 | 148.9803484 | 60.29386093 | 30.72579064 | 0.999642714 | 0.999430582 | 0.021213247 |
| 80 | 397.7099783 | 160.6615883 | 81.6284334 | 99.32173878 | 40.19547877 | 20.48278245 | 0.999645477 | 0.999535666 | 0.010981087 |
| 90 | 447.3694918 | 180.7597134 | 91.87079481 | 49.66493692 | 20.0965824 | 10.23848068 | 0.999648219 | 0.999592265 | 0.005595441 |
| 100 | 497.0290054 | 200.8578384 | 102.1131562 | 0 | 0 | 0 | 0.999650939 | 0.99962278 | 0.002815931 |

**Table 15:** $\lambda_1 = 500$, $\lambda_2 = 200$, $\lambda_3 = 100$, $\mu = 8$, Inventory=1 year

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 548.2725261 | 220.4955661 | 111.2319078 | 0.981183161 | 0.981183161 | 0 |
| 10 | 55.26303885 | 21.92530159 | 10.81165957 | 493.4453781 | 198.4459797 | 100.1086423 | 0.981216876 | 0.980020347 | 0.263236489 |
| 20 | 110.0902419 | 43.97487232 | 21.93488575 | 438.6182438 | 176.3963894 | 88.9853669 | 0.981250531 | 0.979638318 | 0.354686892 |
| 30 | 164.917445 | 66.02444305 | 33.05811193 | 383.7911291 | 154.3467934 | 77.86207749 | 0.981284125 | 0.979658404 | 0.357658711 |
| 40 | 219.7446481 | 88.07401379 | 44.18133812 | 328.9640439 | 132.297189 | 66.73876701 | 0.981317659 | 0.979861249 | 0.320410283 |
| 50 | 274.5718512 | 110.1235845 | 55.3045643 | 274.137006 | 110.2475712 | 55.61542281 | 0.981351133 | 0.980128209 | 0.26904326 |
| 60 | 329.3990543 | 132.1731553 | 66.42779048 | 219.3100505 | 88.1979299 | 44.4920196 | 0.981384547 | 0.980398858 | 0.216851527 |
| 70 | 384.2262573 | 154.222726 | 77.55101666 | 164.4832601 | 66.14824148 | 33.36849837 | 0.981417901 | 0.980645542 | 0.169918968 |
| 80 | 439.0534604 | 176.2722967 | 88.67424285 | 109.6568825 | 44.09843539 | 22.2446821 | 0.981451196 | 0.980858371 | 0.1304217 |
| 90 | 493.8806635 | 198.3218675 | 99.79746903 | 54.83215576 | 22.04815858 | 11.11968565 | 0.981484433 | 0.98103653 | 0.098538841 |
| 100 | 548.7078666 | 220.3714382 | 110.9206952 | 0 | 0 | 0 | 0.981517604 | 0.981183378 | 0.073529625 |

**Table 16:** $\lambda_1 = 550$, $\lambda_2 = 220$, $\lambda_3 = 110$, $\mu = 4$, Inventory=1 year

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 448.2774371 | 180.4945881 | 91.22797485 | 0.98107133 | 0.98107133 | 0 |
| 10 | 45.26315238 | 17.92536723 | 8.811480389 | 403.4498211 | 162.4450929 | 82.10508607 | 0.981112648 | 0.97965613 | 0.262173348 |
| 20 | 90.09083561 | 35.97484328 | 17.93432112 | 358.6222218 | 144.3955929 | 72.98218528 | 0.981153876 | 0.979189181 | 0.353645232 |
| 30 | 134.9185188 | 54.02431932 | 27.05716185 | 313.7946466 | 126.3460861 | 63.85926732 | 0.981195015 | 0.979212605 | 0.356833815 |
| 40 | 179.7462021 | 72.07379537 | 36.18000258 | 268.9671074 | 108.296569 | 54.73632363 | 0.981236064 | 0.979459198 | 0.319835793 |
| 50 | 224.5738853 | 90.12327141 | 45.3028433 | 224.1396258 | 90.24703548 | 45.61333876 | 0.981277023 | 0.97978432 | 0.268686468 |
| 60 | 269.4015685 | 108.1727475 | 54.42568403 | 179.312245 | 72.19747325 | 36.4902818 | 0.981317893 | 0.980114225 | 0.21666017 |
| 70 | 314.2292517 | 126.2222235 | 63.54852476 | 134.4850657 | 54.14785356 | 27.3670807 | 0.981358674 | 0.980415106 | 0.1698422 |
| 80 | 359.056935 | 144.2716995 | 72.67136549 | 89.65839054 | 36.09809022 | 18.24351923 | 0.981399366 | 0.980674823 | 0.130417891 |
| 90 | 403.8846182 | 162.3211756 | 81.79420622 | 44.83373141 | 18.04775231 | 9.11851628 | 0.981439973 | 0.980892323 | 0.098577083 |
| 100 | 448.7123014 | 180.3706516 | 90.91704695 | 0 | 0 | 0 | 0.981480478 | 0.981071653 | 0.073588582 |

**Table 17:** $\lambda_1 = 450$, $\lambda_2 = 180$, $\lambda_3 = 90$, $\mu = 4$, Inventory=1 year

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 996.7252016 | 101.4497687 | 11.82502971 | 0.981294644 | 0.981294644 | 0 |
| 10 | 100.533395 | 9.781914835 | 0.684690126 | 897.0528452 | 91.3047228 | 10.64243205 | 0.981320562 | 0.980494628 | 0.229196594 |
| 20 | 200.2058376 | 19.92692442 | 1.867237949 | 797.3805103 | 81.15966778 | 9.459821931 | 0.981346443 | 0.980167315 | 0.327208157 |
| 30 | 299.8782802 | 30.071934 | 3.049785772 | 697.7082062 | 71.01459978 | 8.277194016 | 0.981372289 | 0.980161434 | 0.336012465 |
| 40 | 399.5507228 | 40.21694359 | 4.232333594 | 598.0359483 | 60.86951231 | 7.094539402 | 0.9813981 | 0.980303799 | 0.303668558 |
| 50 | 499.2231654 | 50.36195317 | 5.414881417 | 498.3637642 | 50.72439369 | 5.911842073 | 0.981423875 | 0.980500349 | 0.25627851 |
| 60 | 598.895608 | 60.50696275 | 6.59742924 | 398.6917095 | 40.57922055 | 4.72906999 | 0.981449616 | 0.980702841 | 0.207229868 |
| 70 | 698.5680506 | 70.65197234 | 7.779977063 | 299.0199132 | 30.43393838 | 3.5461484 | 0.981475322 | 0.980888892 | 0.162734298 |
| 80 | 798.2404932 | 80.79698192 | 8.962524885 | 199.3487634 | 20.28838361 | 2.362853043 | 0.981500997 | 0.981050191 | 0.125098747 |
| 90 | 897.9129358 | 90.9419915 | 10.14507271 | 99.68019892 | 10.14173846 | 1.178062618 | 0.981526653 | 0.981185675 | 0.094621398 |
| 100 | 997.5853784 | 101.0870011 | 11.32762053 | 0 | 0 | 0 | 0.981552198 | 0.981297557 | 0.070663064 |

**Table 18:** $\lambda_1 = 1000$, $\lambda_2 = 100$, $\lambda_3 = 10$, $\mu = 4$, Inventory=1 year

| $\Phi/(\Phi+\Omega)$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\gamma_1\Omega$ | $\gamma_2\Omega$ | $\gamma_3\Omega$ | $\alpha_S$ | $\beta_S$ | Exp. Tr. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 993.6527468 | 102.877537 | 13.46971618 | 0.99963572 | 0.99963572 | 0 |
| 10 | 100.2326663 | 9.926195235 | 0.841138442 | 894.2876371 | 92.58971457 | 12.12264835 | 0.999637672 | 0.997456694 | 0.302610651 |
| 20 | 199.5978628 | 20.21398152 | 2.188155635 | 794.9225491 | 82.30188305 | 10.77556786 | 0.999639613 | 0.997557732 | 0.288861088 |
| 30 | 298.9630594 | 30.5017678 | 3.535172827 | 695.5574921 | 72.0140386 | 9.428469281 | 0.999641544 | 0.998205467 | 0.199255695 |
| 40 | 398.3282559 | 40.78955408 | 4.88219002 | 596.1924817 | 61.72617475 | 8.081343577 | 0.999643465 | 0.99877075 | 0.121089236 |
| 50 | 497.6934524 | 51.07734036 | 6.229207212 | 496.8275456 | 51.43827989 | 6.734174469 | 0.999645375 | 0.999149909 | 0.068745899 |
| 60 | 597.058649 | 61.36512664 | 7.576224405 | 397.4627399 | 41.15033073 | 5.386929403 | 0.999647275 | 0.999377706 | 0.03740279 |
| 70 | 696.4238455 | 71.65291292 | 8.923241597 | 298.0981946 | 30.86227298 | 4.039532422 | 0.999649165 | 0.999506715 | 0.019765054 |
| 80 | 795.789042 | 81.9406992 | 10.27025879 | 198.7343006 | 20.57394377 | 2.691755652 | 0.999651046 | 0.999577351 | 0.010225172 |
| 90 | 895.1542385 | 92.22848548 | 11.61727598 | 99.37301157 | 10.2845287 | 1.342459726 | 0.999652917 | 0.999615403 | 0.005205075 |
| 100 | 994.5194351 | 102.5162718 | 12.96429317 | 0 | 0 | 0 | 0.999654773 | 0.999635918 | 0.002616165 |

**Table 19:** $\lambda_1 = 1000$, $\lambda_2 = 100$, $\lambda_3 = 10$, $\mu = 8$, Inventory=1 year

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